Scientific Revolution in the Development of the Rutherford-Bohr Model of the Atom

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Abstract:

1897-1914 was a period of rapid scientific progress with respect to atomic modeling. In just seventeen years Thomson, Rutherford, and Bohr would transform the world's conception of the atom from a simple, indivisible building block of matter to a semi-classical, complex system of interacting positive and negatively charged particles that could explain experimentally observed phenomena ranging from the periodicity of the elements to the large angle scattering of α-particles and the atomic spectrum of hydrogen. After offering a detailed historical investigation of this period, this thesis considers whether any of the events of period constituted a Kuhnian scientific revolution. In particular, I argue that certain difficulties in analyzing this period using Kuhn's theory suggest that, instead of analyzing this period in Kuhnian terms, we should revise Kuhn's theory of scientific revolution in light of the events of this period. In doing so, I argue that scientific revolutions actually occur in two main varieties, which can be articulated in terms of two of the main analogies Kuhn offered to explain his theory of scientific revolution. Further, I show that this revision allows us to take a thoroughly realist (though fallibilist) approach to scientific progress, scientific revolution, and science as a whole.

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<u>Introduction</u>

   Aside from the work of English chemist John Dalton, prior to 1897 the nature and

structure of the atom was primarily the subject of philosophical speculation. J.J. Thomson's

discovery of the electron, the first known subatomic constituent, abruptly changed this,

prompting Thomson to propose a model for the structure of the atom. This launched a period of

extremely productive scientific inquiry focused on teasing out nature's secrets at the smallest

known dimensions. While several other individuals contributed to the groundbreaking

experimental work of this period, three men dominated the business of atomic modeling: J.J.

Thomson, Ernest Rutherford, and Niels Bohr. In just seventeen years these men would transform

the world's conception of the atom from a simple, indivisible building block of matter to a semi-

classical, complex system of interacting positive and negatively charged particles that could

explain experimentally observed phenomena ranging from the periodicity of the elements to the

large angle scattering of α-particles and the atomic spectrum of hydrogen.

   Because of this immense progress, in ordinary terms it seems appropriate to describe this

period as revolutionary. However, this leads to a more interesting question: were the

developments of this period revolutionary in a technical, philosophical sense? In order to address

this question, in the first part of this paper I undertake a detailed historical examination of the

developments in atomic modeling from 1897 to 1914. Then, in the second part, I attempt to offer

a Kuhnian analysis of scientific progress and revolutions in this period. While I argue that at

least one of the major developments in atomic modeling during this time did, in fact, constitute a

Kuhnian scientific revolution, I argue that certain difficulties in analyzing this period in terms of

Kuhn's theory suggest that—instead of analyzing this period in Kuhnian terms—we should

revise Kuhn's theory of scientific revolution in light of the events of this period. In doing so, I argue that Kuhnian scientific revolutions actually occur in two main varieties, which can be articulated in terms of two of the main analogies Kuhn offered to explain his theory of scientific revolution: the analogy from religious conversion and the analogy from evolution. Moreover, I show that my proposed revisions are relatively consistent with the shifts in Kuhn's own thinking about his theory during his later years. However, my revised theory makes one major break with Kuhn: it admits a realist conception of revolution within science. This allows us to take a thoroughly realist (though fallibilist) approach to scientific progress, scientific revolution, and science as a whole. I then use this realist characterization of scientific progress to consider an interesting feature of scientific justification, which I discuss in terms of Rutherford's argument for the acceptance of the nuclear model of the atom over Thomson's atomic theory.

Part 1: The Atomic Models

<u>Atomic Prehistory: Democritus through Dalton</u>

It seems that almost every account of the development of atomic theory begins by discussing the origins of the English word 'atom.' Given that this bit of history is relevant to my later philosophical discussion, I see no reason to flout this tendency. The word 'atom' is derived "from the Greek adjective *atomos* or *atomon*," which means indivisible (Berryman, 2016). It is closely associated with the philosophy of the Greek atomists—Democritus chief among them. The atomists posited a theory about the fundamental nature of material reality, which held that the universe is ultimately comprised of an infinite number of indivisible particles, atoms, existing within an infinite and otherwise empty void. Many scholars believe a desire to refute Parmenides's (and his student Zeno's) assertion that change/motion is impossible motivated the atomist's theory (Berryman, 2016). As Zeno's famous paradoxes illustrate, the idea that matter

and/or space is infinitely divisible seemed to create problems for the idea that motion and change are real phenomena (Palmer 2017). Believing that change and motion were not mere illusions, the atomists argued that matter could only be divided into its constituent atoms: no further division is possible. Atoms are fundamentally indivisible.

Those familiar with infinite divisibility's role in integral and derivative calculus will not be surprised that, after the independent discovery of calculus by both Isaac Newton and Gottfried Leibniz, the idea that atoms, which occupy three-dimensional space, are indivisible began to lose popularity. This transition can be seen in Newton's writings. In his early work (see, for example, *Questiones* published in 1664), Newton expressed the belief that atoms—though infinitely divisible mathematically—are physically indivisible (Janiak, 2000). Further, in the Leibniz-Clarke correspondence, Samuel Clarke (considered to be a proxy for Newton) defends atomism within his broader argument in favor of a Newtonian worldview over a Leibnizian one (Yenter, 2017). However, in *Opticks* (published in 1704), Newton considers the idea that God could physically divide an atom along the lines of its mathematical divisibility, which Andrew Janiak takes to imply that Newton is entertaining the possibility of the physical divisibility of atoms (Janiak, 2000). Moreover, Janiak emphasizes that Newton desired empirical evidence to mediate between the philosophical positions regarding the divisibility/indivisibility of atoms—something he would never possess.

It is important to note that, while the mathematics of calculus certainly played a role, attributing this philosophical shift away from the indivisibility of the atom entirely, or even primarily, to the development of calculus is a historically naïve notion; Descartes advocated for the infinite divisibility of matter in *Météores*, which he published in 1637, decades before the development of calculus (Hatfield, 2016). Further, Leibniz's arguments in favor of the infinite

divisibility of matter are best known not as extensions of his work on calculus but as support for his philosophical theory of the monads (McDonough, 2014). The shift in thinking due to calculus was probably but one of many transformations in philosophical and mathematical thinking that led away from atomism. Regardless of the exact historical reasons for this shift towards a preference for the infinite divisibility of atoms, one thing is clear: primarily philosophical considerations, not attempts to offer empirically grounded mechanistic explanations, drove these thinkers' acceptance (or rejection) of the infinite divisibility of matter and atoms. Newton's aforementioned unfulfilled desire for empirical evidence/argument about the indivisibility of atoms only serves to emphasize this fact.

Then, in 1803, Dalton proposed his atomic theory. Based on observations of consistent mass ratios in compounds, Dalton offered what was probably the first empirically motivated model of the atom. Ironically, chemists widely opposed his theory in part because they thought his "speculation" about "unobservable" atoms was too far removed from the empirical inquiry characteristic of good chemistry (Goodman, 1969). Yet, it was the empirical observation of specific mass ratios in compounds that led Dalton to believe matter was made up of indivisible atoms of various elements and that, while the atoms of different elements have different weights and chemical properties, the atoms of a particular element are all identical. Moreover, Dalton accurately suggested that compounds are composed of atoms from two or more elements combined in particular positive integer ratios (Goodman 1969). Ultimately, as I will soon discuss in more detail, Dalton's reassertion of the indivisibility of atoms would be proven wrong by J.J. Thomson's discovery of the electron in 1897. However, his assertion that the observation of consistent mass ratios in compounds was evidence for the existence of multiple types of atoms corresponding to the various elements—though not yet understood as the constituents of the

periodic table—was a crucial step towards the proposals of modern chemistry and early atomic theory that appeared around the beginning of the 20th century. Importantly, the proponents of these theories, some of whom I shall discuss in the following sections, would attempt to explain the unifying mechanistic processes and structure underlying these heterogeneous atoms. Additionally, Dalton's atomic theory also marked a linguistic break with the use of the term 'atom' to refer to the homogenous and metaphysically and/or physically most fundamental constituent of matter. While he probably thought that his heterogeneous atoms were still the most fundamental particles, this break laid the foundation for the use of the term 'atom' as a technical piece of scientific, rather than philosophical, language.

Discovery of the Electron and Modeling the Atom: J.J. Thomson

Then in 1897, English physicist J.J. Thomson's discovery of the electron prompted him to offer a model of the subatomic structure of the atom. Even more than Dalton's contributions, this work marked the beginning of a productive period of scientific, rather than philosophical, theorizing about the nature of the atom wherein mechanistic explanations were proposed to explain empirical results. Interestingly there is a prevailing false narrative surrounding Thomson's discovery of the electron and his proposal of the "plum pudding" model of the atom.[1] This narrative purports that Thomson discovered the electron in 1897 and then did not propose an atomic model until 1904. However, Thomson actually proposed most of the salient features of his atomic model in his October 1897 paper in *Philosophical Magazine*, entitled "Cathode Rays," where he announced the discovery of the electron. He then expanded upon those initial ideas and provided a mathematically more robust description of this model in his 1904 paper, "On the Structure of the Atom," also published in *Philosophical Magazine*. Thomson made
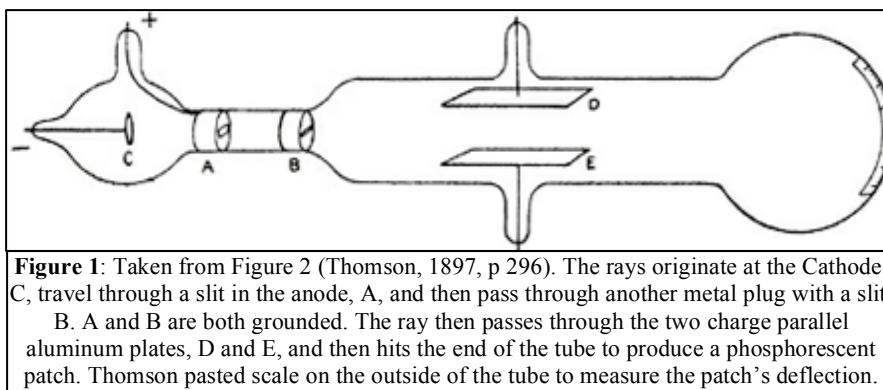
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[1] It is also a myth that Thomson used the term "plum pudding" or "raisin cake" to refer to his model. In fact, neither Thomson nor any of the scientists responding to his word used this label. Instead, this term was coined during an effort to popularize science for the public. See (Hon, 2013).

minor revisions and clarified this model in the subsequent years. He published "On the Number of Corpuscles in an Atom" in *Philosophical Magazine* in 1906 and later published modified versions of these 1904 and 1906 papers as the final chapters of his book, *The Corpuscular Theory of Matter*, in 1907. In this section, I explain that Thomson's atomic model developed in two main stages: his initial theorizing presented in the 1897 paper and the subsequent revision of his model in the 1904-1907 works. In particular, I emphasize the experimental origins of Thomson's views about atomic structure, which are based primarily in his 1897 paper.

In his 1897 paper, "Cathode Rays," Thomson set out to help resolve the debate regarding the nature of cathode rays: were they waves in the ether or particles? (Niaz, 1998). Thomson's work was, in part, a response to the results of an earlier experiment by Hertz, which found that cathode rays did not exhibit a measurable deflection in the presence of an electric field. Hertz's experiment was thought to be compelling evidence that cathode rays were not composed of charged particles but were rather waves in the ether. If the cathode rays were charged particles, they would have been deflected by the electric field when passing between the plates. While Thomson reported that he initially "got the same result, subsequent experiments showed that the absence of deflexion is due to the conductivity conferred on the rarefied gas by the cathode rays. On measuring this conductivity it was found that it diminished very rapidly as the exhaustion increased" (Thomson, 1897, 296). Thomson's explanation of Hertz results fits nicely with electrostatics. The neutral, conductive gas surrounding the cathode rays polarized as free charges in the gas moved to the edges so that the magnitude of E-field inside the volume occupied by the gas (including the region through which the cathode rays passed) was zero—or close to zero. Such an E-field would not have noticeably deflected the cathode ray even if it were composed of charged particles. Then, when he performed the experiments again "[a]t high exhaustions,"

Thomson found that the cathode rays were deflected when passing through "two aluminum plates [that] were connected

Figure 1: Taken from Figure 2 (Thomson, 1897, p 296). The rays originate at the Cathode, C, travel through a slit in the anode, A, and then pass through another metal plug with a slit, B. A and B are both grounded. The ray then passes through the two charge parallel aluminum plates, D and E, and then hits the end of the tube to produce a phosphorescent patch. Thomson pasted scale on the outside of the tube to measure the patch's deflection.

with the terminals of a battery," as would be expected if the cathode rays were composed of charged particles (see Figure 1 above) (Thomson, 1897, 297). Further, he found that, even at high exhaustion, after long periods of time the cathode ray "creeps back to its undeflected position," which is "what would happen if the space between the plates were a conductor, though a very bad one;" the free charges from cathode ray and the few gas molecules that had not been pumped out of the chamber would eventually rearrange themselves to offset the electric field from the charged plates rendering "the cathode rays free from the electrostatic force" (Thomson, 1897, 297). Thus, Thomson concluded that the cathode rays were made of charged particles.

Having argued that, contra Hertz, the constituents of cathode rays were actually particles, Thomson wanted to determine these particles' charge. Looking at the direction of the deflection, he noted that the electrostatic force deflected the cathode rays "as if they were negatively electrified" particles (Thomson, 1897, 303). Further, he observed that when the cathode rays were passed through a roughly uniform magnetic field generated by two Helmholtz coils in parallel (see figure 2)
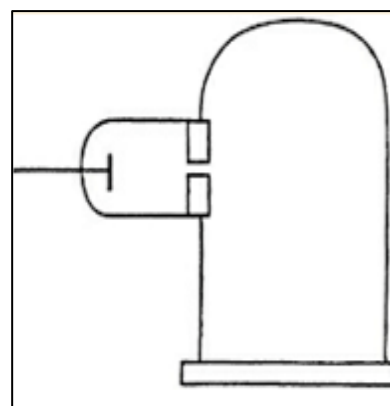
Figure 2: taken from Figure 4 (Thomson, 1897, p 301). The cathode was placed in tube connected to a bell shaped jar. A grounded anode with a slit to allow the ray to pass through was placed at the opening between the tube and the jar. The entire jar was placed between two Helmholtz coils generating a uniform magnetic field. Photographs were taken to determine the deflection of the rays.

they spread out to form a fan or magnetic spectrum. These spectra revealed that the cathode ray

particles were "acted on by a magnetic force in just the way in which this force would act on a

negatively electrified body" (Thomson, 1897, 303). Thomson believed these two facts

established that the cathode ray particles were negatively charged.

While Thomson only emphasized the conclusion that the cathode rays were negatively

charged particles of matter at this point in his paper, these magnetic spectra results had vital,

broader implications for the discovery the electron and Thomson's early atomic theory. Thomson

observed that, when he maintained the same potential difference across the circuit connected to

the Helmholtz coils generating the magnetic field, the spectra of the cathode rays passing

through the field were nearly identical irrespective of the gas in the chamber: hydrogen, air,

carbonic acid, and methyl iodide. In fact, "the photographs [of the spectra] could hardly be

distinguished from each other" (Thomson, 1897, 302). This is significant because the magnitude

of the magnetic field, B, generated by the Helmholtz coils is proportional to the current, I,

through the coils (using the Biot-Savart law and symmetry this is $B = \frac{8}{5\sqrt{5}} \frac{\mu_o n I}{R}$). Because the

circuit did not change between tests, the constant voltage ensured that the current through the

Helmholtz coils—and thus the magnetic field generated by the coils—was essentially equal for

all of the experiments that generated identical spectra (Thomson, 1897, 301).[2] Regardless of the

gas in the chamber, each cathode ray produced the same magnetic spectrum when passed

through the same magnetic field. This result led Thomson to consider the possibility that the

constituent particles of the cathode rays might not merely all be negatively charged but that they

might all actually be the same type of particle. This drove Thomson to ask, "What are these

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[2] Thomson does note that the pressure was different for each gas at this average potential difference. Though he does not discuss it, this is likely due to the varying conductivity of the gases tested. At different pressures there would be differing amounts of the gases, which could compensate for the differences in conductivity to achieve the desired potential difference across the anode and cathode.

particles? are they atoms, or molecules, or matter in a still finer state of subdivision?" (Thomson, 1897, 302). Thus, these magnetic spectrum results spurred Thomson towards the discovery of the electron and prompted him to consider atomic and, more importantly, subatomic structure

Interested in testing this suspicion that the charge carrying particles in cathode rays might all be identical, Thomson measured the mass to charge ratio of the cathode ray particles (m/e) when the cathode ray tube was filled with three different gases (air, hydrogen, and carbonic acid). He did this using two different methods. The first method compared the kinetic energy of the cathode ray particles to their deflection by a magnetic field (see appendix A). The second method relied on a comparison between the deflection of the particles due to a uniform electric field and the deflection of the particles due to a uniform magnetic field (See appendix B). Using method 1, Thomson reported mean values of m/e for two tube different configurations. In the first configuration the mean value of m/e was determined to be 4.0×10^{-8} for air, 4.2×10^{-8} for hydrogen, and $4. \times 10^{-8}$ for carbonic acid gas. In the second configuration, the mean value of m/e was determined to be 5.2×10^{-8} for air, 5.0×10^{-8} for hydrogen, and 5.4×10^{-8} for carbonic acid gas (Thomson, 1897, 307). While Thomson's second method yielded slightly larger values of m/e, around 1×10^{-7}, it too showed that the value of m/e was approximately the same for all the gases tested (Thomson, 1897, 310). Thus, both methods led Thomson to the same conclusion: "the value of m/e is independent of the nature of the gas" (Thomson, 1897, 307 & 310). This confirmed Thomson's suspicions from the magnetic spectra experiment: the carriers of charge in the cathode rays were the same regardless of the "the gas through which the discharge passes" (Thomson, 1897, 310).

Interestingly, even Thomson's largest measurement of the mass to charge ratio of the cathode ray particle, m/e $\approx 10^{-7}$, was several orders of magnitude smaller than the previously

known smallest measurement of m/e, which was m/e $\approx 10^{-4}$ for the hydrogen ion. While Thomson acknowledged that "the smallness of m/e may be due to the smallness of m or the largeness of e, or to a combination of these two," he provided reasons to think that the cathode ray particle was small compared to "ordinary molecules" (Thomson, 1897, 310). He argued that, because the mean free path (the average distance between collisions with other gas molecules) of the cathode ray particle through air is orders of magnitude larger than that of an air molecule under identical conditions,[3] the cathode ray particle must be orders of magnitude smaller than the air molecule. Putting all of this together, Thomson concluded that all cathode rays are composed of the same negatively charged particles, which are much smaller than atoms. He called these particles corpuscles; although we know them today as electrons.

Given that Thomson's "Cathode Rays" paper marks the discovery of the electron, some may find it surprising that the paper did not culminate in these conclusions about the electron. Instead, these findings were an intermediate step, propelling Thomson to propose the first version of his atomic theory. Modifying Prout's hypothesis that all larger atoms are comprised of varying numbers of the smaller hydrogen atom, Thomson argued that the observed facts about cathode rays and electrons are best explained if all atoms are made up of "some […] primordial substance X" (Thomson, 1897, 311). Thomson proposed that the corpuscle/electron was this primordial substance because, if atoms were made of corpuscles, the intense electric field around the cathode in the cathode ray tube would cause nearby gas molecules to dissociate into these charged "primordial atoms" or "corpuscles" (Thomson, 1897, 311). Under these conditions, the corpuscles would then be accelerated by the electric field and "would behave exactly like cathode rays" (Thomson, 1897, 311). Thus, the theory that atoms were made of corpuscles was consistent with Thomson's observations of cathode rays. This led to the foundational assertion of

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[3] This was shown by Philipp Lenard.

Thomson's initial atomic theory: all matter can be subdivided beyond the atomic level and all

subatomic matter—regardless of the source it is derived from, be it "hydrogen, oxygen, &c.—is

of one and the same kind; this matter being the substance from which all the chemical elements

are built up," i.e., corpuscles (Thomson, 1897, 312).

Thomson then tried to determine the configuration of the corpuscles needed to achieve

stable equilibrium within the atom. He considered two laws of force that could explain the

stability of a collection of negatively charged corpuscles in an atom despite their mutually

repellent electrostatic force. Because of the greater simplicity of the second possibility, Thomson

quickly rejected first possibility, a law proposed by Boscovich, which suggested that the particles

repel each other at close distances but attract each other when they are separated by at least "a

certain critical distance" (Thomson, 1897, 313). He embraced the second possibility: that these

"mutually repellent particles [were] held together by a central force" (Thomson, 1897, 313).[4]

However, Thomson emphasized that, because of the number of particles, the equations to

determine a stable configuration under such force were far too complex to solve mathematically.

Instead, he suggested that "we can […] obtain a good deal of insight into the general laws which

govern such configurations by the use of models" (Thomson, 1897, 313).

The model Thomson used was based on experiments with floating magnets conducted by

Professor Mayer. The magnets were arranged so as to mutually repel one another, while being

attracted to the pole of a large magnet at the center. Mayer found that the magnets arranged

themselves in a series of concentric rings depending on the total number of magnets in the

configuration. The patterns in the number of magnets in each of these rings led Thomson to

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[4] This central force would be due to a positive charge. While Thomson makes no explicit discussion of this charge, his suggestion that electrons are the material substance from which all of the elements are built up implies that this positive charge is due to a massless positively charged space in which the electrons move. Indeed this is how other scientists interpreted Thomson's view. See (Hon, 2013).

believe that similar patterns amongst the corpuscles in atoms might explain the "the periodic law" (Thomson, 1897, 313). He thought that atoms following this model would exhibit periodic properties based on a common number of electrons in the inner ring(s), and that the total number of electrons, *n*, might be proportional to their atomic weight. Table 1, below, lists the stable configurations of the magnets. The configurations, C, are formatted such that '1.5.9.12' denotes an equilibrium configuration with one magnet in the inner ring, five magnets in the second ring, nine magnets in the third ring, and twelve magnets in the fourth ring. Expressed in this notion, Thomson's periodicity hypothesis suggested that the hypothetical series of atoms with 6 electrons (1.5), 15 electrons (1.5.9), and 27 electrons (1.5.9.12) would exhibit similar chemical and physical properties.

Table 1 (Adapted from Thomson, 1897)

1 Middle Magnet		2 Middle Magnets		3 Middle Magnets		4 Middle Magnets		5 Middle Magnets	
C	N	C	N	C	N	C	N	C	N
1 Ring									
1	1	2	2	3	3	4	4	5	5
2 Rings									
1.5	6	2.6	8	3.7	10	4.8	12	5.9	14
1.6	7	2.7	9	3.8	11	4.9	13		
1.7	8								
3 Rings									
1.5.9	15	2.7.10	19	3.7.10	20	4.8.12	24	5.9.12	26
1.6.9	16	2.7.11	20	3.7.11	21	4.8.13	25	5.9.13	27
1.6.10	17	2.8.10	20	3.8.10	21	4.9.12	25		
1.6.11	18			3.8.11	22	4.9.13	26		
				3.8.12	23				
				3.8.13	24				
4 Rings									
1.5.9.12	27	2.7.10.15	34	3.7.12.13	35	4.9.13.14	40		
1.5.9.13	28	2.7.12.14	35	3.7.12.14	36	4.9.13.15	41		
1.6.9.12	28			3.7.13.14	37	4.9.14.15	42		
1.6.10.12	29			3.7.13.15	38				
1.6.10.13	30								
1.6.11.12	30								
1.6.11.13	31								
1.6.11.14	32								
1.6.11.15	33								
1.7.12.14	34								

This is as far as Thomson took his atomic model in 1897. This initial model is significant because it was the first thoroughgoing application of mechanistic explanation to the question of atomic divisibility and the nature of subatomic matter based empirical results. While Thomson merely speculated that the positive charge responsible for the central force was uniformly distributed and massless, he appealed to known forces (namely electrostatic forces) and analogous physical phenomena (namely stable magnetic configurations) to explain the stability of his proposed atomic model, which had rings of electrons interspersed throughout a region of uninform positive charge. Furthermore, this was the first atomic model with subatomic components whose existence was supported by experimental evidence. Additionally, Thomson's model possessed an unprecedented degree of explanatory power with regards to the periodic law. Interestingly, Thomson wrongly concluded in 1897 that atomic matter was composed only of electrons. Moreover, because of his mean free path argument, Thomson clearly thought that electrons were several orders of magnitude smaller than hydrogen ions. Together these facts imply that, in 1897, Thomson incorrectly believed that atoms contained thousands of electrons.[5]

In spite of this, the next iteration of Thomson's theory—the mathematically robust atomic model to which other scientists, including Rutherford and Bohr, would respond—did not explicitly consider any configurations with n>67 electrons. The chief accomplishment of this 1904 paper was to mathematize Thomson's earlier model and demonstrate that the electrostatic forces from the negatively charged corpuscles and the uniform sphere of positive charge in the atom would lead to equilibrium states similar to the configurations suggested by the magnetic model Thomson used in 1897. These results enabled Thomson to expand his discussion of

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[5] However, as I discuss at the end of this section Thomson's belief about the number of electrons in an atom had changed by 1906. Thus, the most advanced version of his model, which Thomson presented in 1907, did not have thousands of electrons despite suggestions to the contrary. See for example: ("The Discovery," 1997).

atomic properties, especially periodicity, based on the stable configurations of the corpuscles under actual electrostatic forces.

To simplify the calculations, Thomson considered only configurations with the electrons spaced evenly around a circular ring confined to a single plane. For stationary electrons, equilibrium would be achieved only when the repellent force on an electron due to the other (n-1) electrons equaled the attractive force acting on that electron due to the sphere of positive charge:

$$(1)\ F_{attractive} = \frac{ve^2a}{b^3} = \frac{e^2}{4a^2}S_n = F_{repellent}$$

Here, $v{*}e$ is the total charge of the positively charged sphere, e is the charge of the electron, b is the radius of the positively charged sphere, a is the radius of the circle of electrons, and S_n is a sum of n-1 terms based on the geometry of the electrons spaced around the circle (see appendix C). Thomson solved this equation to find the ratio of the two radii (a/b) for $n \leq 6$ (see appendix C). He then quickly turned his attention to the equilibrium conditions for a rotating ring of electrons, which must follow:

$$(2)\ \frac{ve^2a}{b^3} = ma\omega^2 + \frac{e^2}{4a^2}S_n$$

where m is the mas of the electron and ω is the angular velocity of the ring (see appendix C). Thomson did not report any solutions to this equation, but instead used it to launch his investigation of the forces acting upon an electron that was slightly displaced from its equilibrium position on the rotating ring. For such a configuration to be stable, Thomson found that the charge must oscillate at certain frequencies, q; in a configuration with n corpuscles there will be *2n* values of q. Thomson found these frequencies by calculating the roots of a modified version of equation (2):

(3) $\left(\dfrac{3}{4}\dfrac{e^2}{a^3} S_k + L_k - L_0 - mq^2 \right)(N_0 - N_k - mq^2) = (M_k - 2m\omega q)^2 \quad k = 0, 1, 2, \dots, (n-1)$

where S, L, N, and M are previously derived quantities dependent on the displacement of the

charge and the rotation of the ring (see appendix C).

Thomson solved equation (3) for n=2 through n=6. He did this for both rotating ($\omega > 0$)

and stationary ($\omega = 0$) configurations. Interestingly, he provided additional details about the

stationary configurations while restating these results in his 1907 book. Specifically, he found

that stationary systems with 2 and 3 electrons could achieve stable equilibrium while the

confined to the plane of the ring; however stationary systems with 4 and 6 electrons the electrons

would always be unstable when confined to a plane—instead the electrons would arrange

themselves in a three-dimensional configuration within the sphere (Thomson, 1907, 106).[6]

However, even by 1907, Thomson had not solved "the general problem of finding how *n*

[stationary] corpuscles will distribute themselves [three dimensionally] inside the sphere"

(Thomson, 1907, 106). Although, in his 1904 paper, he was able to solve equation (3) for the

oscillation of slightly displaced electrons confined to the plane of a rotating ring. For n = 2 and

n=3 this equation gives positive values for q^2 for a ring rotating at any angular velocity, ω.

However, for n=4 and n=5 it only gives positive values of q^2 if ω is larger than a certain critical

value. So, provided that the ring rotates fast enough, Thomson found that the roots of equation

(3) would be real for electron configurations of n < 6, and the system could achieve a stable

rotation with the oscillating electrons arranged in a single rotating ring. However, for n=6

Thomson found that one of the values of q^2 would always be negative regardless of the value of

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[6] Interestingly, Thomson does not say in the 1904 paper or 1907 book whether 5 stationary electrons will be stable in a plane or if they need to be configured in three dimensions.

ω, thus yielding an imaginary root. He concluded that "the steady motion of 6 corpuscles in a [single] ring is unstable, however rapid the rotation" (Thomson, 1904, 251).

However, Thomson found that a rotating ring of 6 electrons could be made stable by placing an additional electron at the center of the ring, which guaranteed that the roots of q would be real. Such a configuration would have 7 total electrons with 6 electrons arranged in a ring around the final electron. Thomson found that equation (3) required that all atoms with n>5 electrons have additional electrons located inside of this ring. Further, he found that, as the number of electrons in the outer ring increased, the stability conditions for his atom required that the number of interior electrons also increase. For configurations with more than one interior electron, Thomson argued that the interior electrons would arrange themselves into a smaller concentric rotating ring. Thomson calculated the minimum number of interior electrons, p, needed to ensure the mechanical stability of an atom with n outer electrons. The results are given in table 2 below:

Table 2 (Adapted from Thomson, 1904, 254)

n	5	6	7	8	9	10	15	20	30	40
P	0	1	1	1	2	3	15	39	101	232

This showed that the number of interior electrons required for the mechanical stability of an atom quickly outgrows the number of electrons allowed in the outer ring. So, Thomson hypothesized that, in atoms with large numbers of electrons, the interior electrons would form a series of concentric rotating rings. He also predicted that if the electrons in an atom were not confined to the plane of the ring they would form a series of concentric spherical electron shells within the atom's sphere of uniform positive charge. Thomson suggested "the same kind of

properties will be associated with the shells as with the rings" (Thomson, 1904, 255). Thus, the shells in Thomson's new model would rotate at certain angular velocities, ω, and that the individual electrons would oscillate at certain frequencies, q. Thomson thought that these two distinct vibrations—constrained by the allowed values of ω and q—might explain the atomic spectra. However, Thomson did not know how many electrons there were in an atom, so he could not compare this prediction to the known frequencies of particular atomic spectra.

Thomson found that his stability conditions led to patterns in the number of electrons in a series of concentric shells. This supported Thomson's argument from 1897, which asserted that his model could explain the periodicity of atomic properties relative to their arrangement by atomic weight. Table 3, below, shows one such pattern.

Table 3 (adapted from Thomson, 1904, 258)

n	3	11	24	40	60
C	3	3.8	3.8.13	3.8.13.16	3.8.13.16.20

Thomson found that an atom with 60 electrons is arranged exactly like an atom with 40 electrons but has an additional shell containing 20 electrons. This trend continues as we move down this hypothetical family with respect to atomic mass, which is the same as moving up the family's column on the periodic table (Thomson, 1904, 258). Thomson speculated that this pattern would lead to "many points of resemblance," including similarities in atomic spectra and other chemical and physical properties, thus explaining the period law for elements in the same column in the periodic table (Thomson, 1904, 259). Thomson listed all of the stable configurations for $n \leq 100$ in his 1907 book, which reveals the existence of many such patterns (Thomson, 1907, 109-110). I reproduce this entire table in appendix 3.

Further, Thomson thought that groups of atoms with the same number of electrons in their outer ring would exhibit behavioral patterns similar to those of elements in the same period (row) on the periodic table (Thomson 1904, 259). See Table 4 below:

Table 4 (adapted from Thomson's 1904 paper and 1907 book)

n	Configuration	Valency	Electro-negativity	Corresponding Atoms from Period 2	Corresponding Atom from Period 3
59	2.8.13.16.20	+0	Neither	He	Ne
60	3.8.13.16.20	+1	Electropositive	Li	Na
61	3.9.13.16.20	+2	Electropositive	Be	Mg
62	3.9.13.17.20	+3	Electropositive	B	Al
63	3.10.13.17.20	±4	Electropositive/ Electronegative	C	Si
64	4.10.13.17.20	-3	Electronegative	N	P
65	4.10.14.17.20	-2	Electronegative	O	S
66	5.10.14.17.20	-1	Electronegative	F	Cl
67	5.10.15.17.20	-0	Electronegative	Ne	Arg

This table shows the configurations for eight different hypothetical atoms with 20 electrons in their outer shell. It includes Thomson's predictions about their valency and electronegativity and compares the atoms in this series with the elements from two actual rows of the periodic table that demonstrates the same periodic behavior as these hypothetical atoms. Thomson's model showed the mechanical stability of a series of atoms with the same number of outer electrons improved as the number of interior electrons increased. Because it had the fewest interior electrons, the n=59 atom would have been the least stable configuration and the n=67 atom would have been the most stable configuration. Consequently, Thomson suggested that the electrons in the outer shell become increasingly stable as we up in mass and across a given period. This meant that the Thomson model correctly predicted that an atom is less likely to lose its electrons as we move to the right along a row of the periodic table.

Interestingly, Thomson argued that, despite its tendency to give up an electron due to its instability, the atom with 59 electrons would not "remain permanently charged with positive electricity" (Thomson 1904, 261). He thought that the net positive charge on the atom coupled with the stability of the charged configuration with 58 electrons would prevent the loss of further electrons while attracting a new electron to replace its lost electron almost immediately. So, despite the greater instability of the n=59 configuration, Thomson proposed that the atom with n=60 electrons would be the first atom in this hypothetical series that would readily, and permanently, lose a single electron (denoted by a valency of +1 in the table above). Similarly, the n=61 atom would readily gain two electrons and the n=62 atom would gain three electrons. At the other end of the period, Thomson argued the n=67 atom is not likely to gain any electrons as the addition of a single electron would decrease the atom's stability and introduce a net negative charge to the atom, so the other electrons would repel it away. Thus, the n=67 atom "would be incapable of being permanently charged with electricity" (Thomson, 1904, 261). Instead, the atom with 66 electrons would be "the most electronegative of the series" and the first atom from the right end of this periodic row to readily gain one electron. Similarly the n=65 atom would gain two electrons while n=64 atom would readily gain three electrons. Thomson's model predicts that the atom with n=63 electrons would just as easily gain or lose four electrons. These predictions from Thomson's model corresponded nicely to the actual behavior of the real atoms found in period 2 and period 3 to which Thomson compared his hypothetical period.

Finally, Thomson considered the implications of "the radiation from the moving corpuscles," predicted by Maxwell's equations, for his atomic model (Thomson, 1904, 265). Such radiation would cause the rotating electrons to slow over time. However, Thomson's calculations showed that rings with 4 or 5 corpuscles required a minimum value of ω to maintain

a stable rotation. Thus, a decrease in ω due to radiation would threaten the stability of any atom containing one or more rings that "require[d] for [their] stability a certain amount of rotation." This would include every atom in Thomson's model except n=2 and n=3 (Thomson, 1904, 265). Thomson hypothesized that when the atom reached this critical value of ω there would be "an explosion of the corpuscles" and "as in the case of radium, a part of the atom [would] sh[oot] off" (Thomson, 1904, 265). Thus, Thomson proposed that his model could explain the radioactivity of elements like radium, although in doing so it also problematically suggested that almost all atoms would be radioactively unstable, which did not match the known facts. Thomson's suggestion that the rotating electrons' velocity would decrease slowly might be viewed as an attempt to resolve this issue in his model. If this decrease occurred slowly enough, it would explain the long lifetime of atoms, particularly those which had not been observed to undergo radioactive decay. However, it is probably better to view the discussion of the radioactivity of all atoms in Thomson's paper as the acknowledgement of a flaw in his model, which *might* be explainable, rather than an ineffective, hand waving attempt to resolve the problem.

While qualitatively similar to his initial proposal, Thomson's revised model provided the first mathematically robust mechanistic model of atomic structure based on known physical laws and experimental results. This gave it expanded explanatory power over his previous model, particularly with regards to the periodicity of the chemical and physical properties of the elements both up and down the columns and along the rows of the periodic table. In particular, it explained the variation in atoms' tendencies to gain or lose electrons as we move across a period, the increasing electronegativity of atoms as we move across a period, and the resistance of the noble gases to acquiring a net charge.

However, his new model also had a number of problems. It wrongly predicted that all elements would be radioactively unstable. Also, Thomson still incorrectly attributed "the mass of an atom [to] the sum of the masses of the corpuscles it contains" (Thomson, 1904, 258). While this suggests that there should be thousands of electrons in every atom, the status of his related belief about the number of electrons in an atom at this time is less clear. As we have seen, the largest portion of his 1904 paper was devoted to a mathematical analysis of the stable configurations for n electrons spread throughout a uniform sphere of positive charge in an atom. Thomson had explicitly avoided such analysis in his 1897 paper because he thought that the immense number of electrons made the mathematics too complicated to perform. While it is possible that Thomson simply chose to perform the difficult calculation for a manageable numbers of electrons with the intention of generalizing his results to larger numbers of electrons, there are also reasons to think that his views about the number of electrons in atoms might have been starting to change by this time. The largest value of n that Thomson explicitly considered in his 1904 paper was n=67. Moreover, though he used a relatively simple iterative graphical method to solve for the configuration of systems with n>6, Thomson gave no indication that he found or even considered the stable configurations of atoms with significantly higher values of n. While he did list the stable configurations for atoms with up to 100 electrons in his 1907 book, Thomson had indubitably revised his views about the number of electrons in the atom by 1906. In the opening sentence of "On the Number of Corpuscles in the Atom," he stated "I consider in this paper three methods of determining the number of corpuscles in an atom of an elementary substance, all of which lead to the conclusion that this number is of the same order as the atomic weight of the substance" (Thomson, 1906, 769). Additionally, by 1907 Thomson had clearly given up his belief that an atom's mass equaled the sum of its electrons' masses, as he

acknowledged that "the mass of the atom must in the main be due to its other constituent—the positive electricity" (Thomson, 1907, 162).

While it was not the only atomic model available in the subsequent years,[7] Thomson's 1904 model would remain the most popular within the scientific community until Rutherford proposed his nuclear model in 1911, which I discuss in the next section. In the intermediate years, Thomson incorporated a crucial piece of evidence into his model: the small angle scattering of α-particles when passing through "a small thickness of matter" (Rutherford, 1911, 670). This phenomenon was first observed by Rutherford in 1906; his initial conclusion was that the results were consistent with Thomson's atomic theory. Thus, the scientific community believed that Thomson's atomic model correctly predicted the observed small angle scattering behavior. However, his model would be unable to explain the large angle scattering of α-particle, which Geiger and Marsden discovered in 1909. This phenomenon ultimately provided the evidence needed to convince Rutherford to reject Thomson's model and propose his own theory.

The Scattering of the α-particle and Rutherford's Atomic Model

Coincidentally, Rutherford, who studied under Thomson at Cambridge from 1895-1898, did not initially set out to offer an atomic theory, but was led to do so by his experimental results in a manner not dissimilar to Thomson. Like Thomson, the road to Rutherford's atomic model began with investigations into the nature of particular ray, in this case the α-ray, and his experiments to determine whether it was a particle or a wave. However, unlike Thomson—whose discovery of the electron immediately led him to propose an atomic model—it would take

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[7] In particular, later in 1904, Japanese professor Hantaro Nagaoka proposed a model inspired by Maxwell's discussion of Saturn's rings that was in many ways similar to Thomson's model. His Saturnian model also relied on the electrons being equally spaced around a rotating ring and exhibited a similar radioactive instability to Thomson's model. However, Nagaoka placed all of the positive charge at the center of the atom and did not allow his electrons to be arranged in shells. Nagaoka also performed his calculation with Lagrange's equations rather than by analyzing the forces. While certainly a noteworthy contribution, discussing Nagaoka's model at greater length will not enhance my discussion.

years for Rutherford's discovery of the α-particle to lead to the scattering experiments that would prompt him to rethink atomic structure. In this section, I offer a brief overview of Rutherford's work leading to these scattering experiments and then focus on the experiments themselves. Finally, I consider the atomic model that Rutherford proposed in 1911.

In 1899, Rutherford discovered that there were two types of radiation emitted from uranium, α and β radiation (Rutherford, 1899). He spent several years trying to determine whether the α rays were particles or waves. Then, in a 1902 experiment reminiscent of Thomson's discovery of the electron, Rutherford succeeded in deflecting the α-rays with both electric and magnetic fields, demonstrating that the constituents of the α-rays were particles (Rutherford, 1903). Along with his other work on radiation, this led Rutherford and Frederick Soddy to propose their famous theory of spontaneous disintegration, which held "that elements could disintegrate and be transformed into other elements" ("Ernest Rutherford," 2014). In 1906, Rutherford published two papers reporting the results of experiments he had conducted the previous year in his Montreal lab which showed that the value of e/m was the same for all α-particles regardless of their source, thus establishing that all α-particles were identical.

In the second and more detailed of these two papers, "Retardation of the Alpha Particle from Radium in passing through Matter," Rutherford briefly discussed the "scattering or deflexion of the path of the α Particles in passing through" mica (Rutherford, 1906, Loc 19935). He had observed that some of the α-particles had "been deflected from their course through an angle of about 2" degrees (Rutherford, 1906, Loc 19947). Rutherford found this to be extraordinary as the α-particles would have to travel through an electric field perpendicular to their path of travel with a magnitude "of about 100 million volts per cm" for a length equivalent to the thickness of the mica in order to be deflected the same 2 degrees (Rutherford, 1906, Loc

19947). Appropriately, he took this result, which demonstrated "that the atoms of matter must be the seat of very intense electrical forces" to be "in harmony with [Thomson's] electronic theory of matter" (Rutherford, 1906, Loc 19947). This experiment also highlighted the α-particle's potential as a tool to probe nature at the atomic level, which would lead to Rutherford's later scattering experiments. In 1908, Rutherford was awarded a Nobel Prize for his work on radiation; he also confirmed his hypothesis that α-particles were He^{2+} by demonstrating the accumulation of helium gas in sealed tubes that were impenetrable to helium but not α-particles (Rutherford, 1908). More importantly for our purposes, in August 1908, Rutherford, Geiger, and Marsden released the preliminary results of their scattering experiments. Their 'full' results were released in April 1910 with one glaring exception—this 1910 paper glossed over their observations of the large angle scattering of the α-particle, an extremely important and surprising result that Geiger and Marsden reported in 1909.

The main point of Geiger and Marsden's 1910 paper was to report their conclusions about the most probable scattering angle of the α-particles travelling through matter. They had

investigated the relationship between the magnitude of this angle and three parameters: the scattering material, the thickness of a scattering material, and the velocity of the α-particles (Geiger, 1910, 492). They performed their experiments in the apparatus shown in figure 3. This apparatus allowed them to produce "a very narrow beam of α-particles," which was
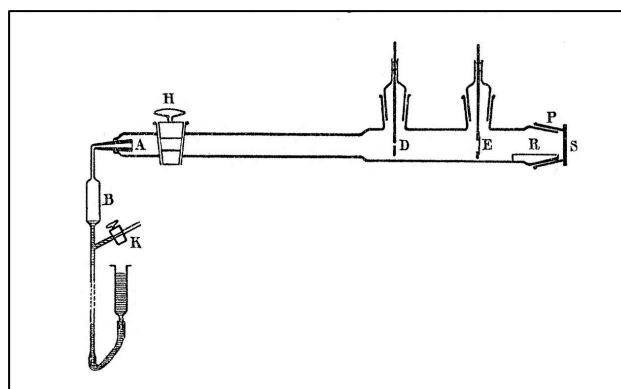
Figure 3: Taken from (Geiger, 1910, 493). The radium emanation was inserted into the exhausted bulb, b, through the stopcock K, and mercury was used to compress the emanation from bulb B into conical glass tube A where it was allowed to form a deposit on the wall. This sat for 3 hours and was then allowed to expand back into bulb B and the deposits on wall A became the α-particle source. This was allowed to sit 15 minutes to ensure that only α-particles generated from Radium C were used in the experiment. These α-particles were then passed through the narrow slit D to ensure a narrow beam of particles. They then passed through scattering foils located at D, E, or P before striking the scintillation screen S.

necessary to obtain accurate measurements of the scattering angle because the displacements

corresponding to these scattering angles were so small that a thick particle beam would obscure

them (Geiger, 1910, 492). They used a microscope to count the number of scintillations (bright

spots where the α-particles hit the screen at the end of the tube) per minute at various

displacement distances, r, from the center of the scintillation screen. They compared these

displacements to the distance between the scattering foil and the screen, s, to determine the

scattering angle, K. They then plotted the total number

of α-particles deflected through a particular scattering

angle against the scattering angle, K, or the

displacement distance, r (see figure 4). They used the

peaks of these distributions to find the most probable

scattering angle. Because of the shape of the

distributions, they noted that the average scattering

angle would "be somewhat greater than the most

Figure 4: From (Geiger, 1910, 496) Scintillation frequency versus observed distance from the center of the scintillation screen.
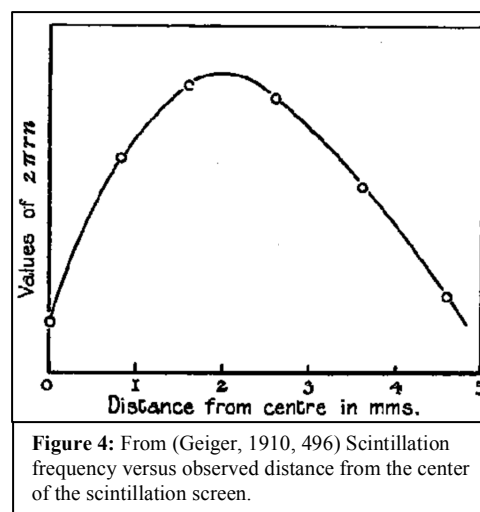
probable angle" (Geiger, 1910, 497). They performed these experiments for gold foils of various

thicknesses and then repeated the experiments for tin, silver, copper, and aluminum foils (Geiger,

1910, 501). Amongst these metals, the magnitude of the most probable scattering angle was the

greatest for gold foils at any given thickness. They also found the most probable scattering angle

for an α-particle travelling through a single gold atom to be very small, 1/200 of a degree.

Further, the largest most probable scattering angle that they reported was 7 degrees for the

scattering of α-particles passing through "35 thick gold foils" (Geiger, 1910, 493).

While Geiger did not explicitly state this, it is clear that he thought that the results of

these scattering experiments were consistent with the predictions of Thomson's atomic model—

just as Rutherford had suggested when he first observed α-particle scattering in 1906. The only results that Geiger chose to omit were those inconsistent with Thomson's model, namely the large angle scattering results from 1909, which I shall discuss momentarily. Commenting on this omission, Geiger suggested that it was "not profitable […] to discuss the assumption which might be made to account for this" (Geiger, 1910, 500). He probably said this because Geiger, Marsden, and Rutherford could offer no explanation for these results until early 1911 other than suggesting that they were statistical aberrations (Feather, 1963). The high degree of consistency between these small angle scattering results and the Thomson model greatly impacted the arguments Rutherford made for his atomic model in his 1911 paper. Thus, while Geiger himself did not do so, it is profitable to discuss these scattering results in light of the Thomson model.

First, Geiger and Marsden found that for small thicknesses the most probable scattering angle increased at a rate proportional to the square root of the thickness of the scattering foil (Geiger, 1910, 499). However, for large thicknesses, the most probable scattering angle increased linearly with the thickness of the foil. Thomson's "plum pudding" model could explain this general relationship. It suggested that an atom was comprised of a uniformly charged positive sphere with negatively charged electrons interspersed throughout. The positively charged α-particles could be deflected either by their attraction to the negatively charged elections or by their repulsion to the positive charge in the atom. Increasing the thickness of the scattering foil would increase the number of atoms through which the α-particles would have to pass. This would in turn increase the number of electrons and amount of positive charge with which the α-particles could interact, thus increasing the average number of interactions between α-particles and these subatomic constituents before the particles exited the foil. Because each interaction would divert the α-particle's trajectory slightly, increasing the number of interactions

experience by a particular α-particle would lead to larger scattering angles.[8] However, multiple

scattering was also a random walk, so the α-particles might also be randomly deflected in one

direction by one electron only to have that deflection randomly 'undone' by a different electron.

Consequently, while an increased number of scattering events could lead to a larger total

scattering angle, this random walk element would prevent these angles from getting too large,

which was consistent with Geiger and Marsden's results. Thus, Thomson's model could explain

the observed increase in scattering angle as the scattering foil's thickness increased.

Additionally, Geiger and Marsden found that the most probable scattering angle was

proportional to the atomic weight of the atoms in the scattering foil (Geiger, 1910, 502). This is

consistent with Thomson's revised atomic model, which held that the number of electrons was of

the same order as the atomic weight. Thus, Thomson's model suggested that both the number of

electrons and the total amount of positive charge would increase as the atomic weight increased.

So, in heavier atoms there would be more electrons (and more positive charge) for the α-particles

to interact with, and thus there would be more interactions between α-particles and these atomic

constituents overall, which would again lead to an increase in the scattering angle.

Finally, Geiger reported that the most probable scattering angle was inversely

proportional to the cube of the α-particles' velocity (Geiger, 1910, 504). While the Thomson

model did not necessarily explain this result on its own, this conclusion did increase the

consistency between Geiger and Marsden's results and the Thomson model. Geiger speculated

that a decrease in the α-particles' velocity might explain the comparatively "quick increase" in

the most probable scattering angle for scattering foils with greater thicknesses (Geiger, 1910,

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[8] Thomson responded to this data later in 1910, providing a mathematical model of multiple scattering that fit Geiger and Marsden's data (and Crowther's β scattering data). He developed this scattering model based on his atomic model. Thomson's proposals, as reported by Rutherford in the 1911 paper where he proposed his atomic model, match my discussion above (Rutherford, 1911, 670).

499). Given that a decrease in the α-particles' velocity corresponded to an increase in the most probable scattering angle, if the α-particles' velocity decreased with each successive interaction,[9] then shooting the α-particles through a scattering foil would cause the most probable scattering angle to increase due both to the increased frequency of the α-particles' interactions *and* to their decreased speed. Further, given that the most probable scattering angle was inversely proportional to the cube of the α-particles' velocity and that the decrease in the particles' velocity from passing through a thicker foil would be larger than the decrease in the particles' velocity from passing through a thinner foil due to the increased number of interactions, the decrease in velocity due to a thicker foil would contribute to a larger change in the α-particles' most probable scattering angle than would the decrease in velocity due to a thinner foil. Together, Thomson's model and this conclusion about the relationship between the α-particles' velocity and their most probable scattering angle implies a difference in the scattering behavior of α-particles passing through thick versus thin foils like the one that Geiger and Marsden reported. Given these conclusions, it was no surprise, then, that α-particle scattering experiments were originally perceived to be evidence for Thomson's model of the atom.

In hindsight, Geiger's omission of the large angle scattering results is particularly interesting. The experimental apparatus used to obtain the results reported in this 1910 paper (which was similar to the apparatus used for the experiments in 1908) would not permit large angle scattering measurements to be taken. Large angle scattering occurred when the particle was reflected back to the incident side of the scattering foil or plate. The apparatus shown in Figure 3 has no scintillation screen to detect particles on the incident side of the scattering foils. Further, the presumed explanatory theory, Thomson's atomic model, did not predict such large angle

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[9] Contemporary arguments from conservation of momentum suggests that this would occur, as the α-particle would transfer some of its velocity to each electron with which it interacted. See for example section 4 of (Rutherford, 1911, 676).

scattering. So, one must ask why Rutherford would have Geiger and Marsden take the time to develop an entirely separate experimental apparatus that would allow them to detect the large angle scattering effects reported in their 1909 paper when these effects were not predicted by the guiding atomic theory. Charles Baily suggests that the likely motivation for these experiments was a particular fact about β-rays, which was well known in 1908: "that directing β-rays at a metal plate would result in radiation being produced on the same side of the plate as the incident particles" (Baily, 2008, 10-11). However, historical evidence suggests that Rutherford asked Geiger and Marsden to "see if any α-particles can be scattered through a large angle" before learning about this "well-known phenomenon" (Niaz, 1998, 533). Instead numerous sources speculate that Rutherford intended this as "a training exercise" for a young Marsden (Feather, 1963, Loc 407 & Niaz, 1998, 533). This seems to be a better explanation than Baily's β-ray hypothesis, particularly given Rutherford's well-documented "surprise" and "amazement" at Geiger and Marsden's successful observation of the large angle scattering of α-particles (Pais, 1991, 123 & Feather, 1963, Loc 407). Rutherford actually described this as "the most incredible event that has ever happened to me in my life" (Feather, 1963, Loc 407).

While we may never truly know the motivation behind these famous experiments, Geiger and Marsden did conduct the experiments. They used a modified version of the setup described in their 1910 paper (see figure 3 above), turning the scattering foil into a reflector and placing the scintillation screen on the incident side of the foil (see figure 5 to the right). They found "conclusive evidence […] of the existence of a
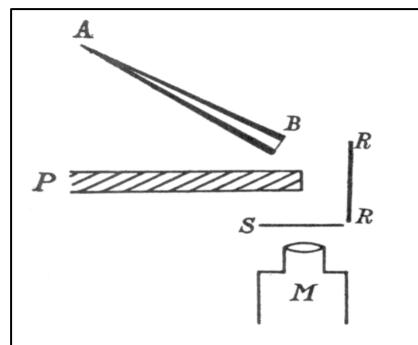
Figure 5: Taken from (Geiger, 1909, 496). The conical tube AB is similar to that in figure 3 and the radium source was prepared and inserted in the same way, except that it was not allowed to sit for 15 minutes before the first experiment as Geiger were not worried about type of α-particles that were reflected, but simply want to see if any reflection occurred. RR is the reflector and P is a thick lead plate designed to ensure that no α-particles hit the scintillation screen, S, without reflecting off of RR. M is a microscope used to observe the scintillations.

diffuse reflection [large angle scattering] of the α-particles" (Geiger, 1909, 495). In fact, Geiger and Marsden observed large angle scattering for each of the eight metals they used as a reflector; they even saw "about one scintillation per minute" in the absence of a reflector, revealing that even "the air through which the α-particles passed" could produce large angle scattering (Geiger, 1909, 497). They also found that the number of reflections (large angle scattering events) decreased as the atomic weight of the reflector also decreased. In a second experiment, Geiger and Marsden compared the number of large angle scattering events per minute to the thickness of the reflector. Based on their other scattering results that would be published in 1910, Geiger suggested that, if this really was a scattering effect, "the number of particles reflected must vary with the thickness of the reflecting screen" (Geiger, 1909, 497). The number of reflections did, in fact, increase as the thickness of the reflecting screen increased. Moreover, Geiger expressed surprise that "a layer $6x10^{-5}$ cm of gold" could turn "the high velocity and mass of the α-particle [...] through an angle of 90 [degrees]" or more; achieving a comparable effect with a magnetic field would require an "enormous field of 10^9 absolute units" (Geiger, 1909, 498). In their final experiment, Geiger and Marsden found that large angle scattering events were far less common than small angle scattering events as only "about 1 in 8000" incident α-particles underwent large angle scattering (Geiger, 1909, 499).

These results would remain ignored and unexplained in the literature until Rutherford published his famous paper, "The Scattering of α and β Particles by Matter and the Structure of the Atom," in 1911. Appropriately, he began this paper by suggesting that the theory of multiple scattering and its assumed atomic structure, i.e., Thomson's atomic model, could never explain Geiger and Marsden's large angle scattering results; multiple scattering entailed that the probability of a deflection of 90 degrees or more be "vanishingly small" (Rutherford, 1911, 670).

Instead, Rutherford proposed that a single scattering event due to a highly concentrated central charge—what would come to be called a nucleus—would best explain these results. He devoted the first part of this paper to developing a mathematical model for the large angle scattering events due to this central charge before looking at the small angle scattering. He briefly compared both results to experimental observations.

Prior to discussing his calculations, Rutherford made it clear that he would not take up the challenge the stability conditions of the atom in this paper as "this [would] depend on the minute structure of the atom, and on the motion of the constituent charged parts" (Rutherford, 1911, 671). This allowed him to sidestep the questions of radioactive instability and stable electron configurations—he actually ignored electron configurations entirely and assumed a uniformly distributed negative charge. Considering large angle/single scattering events, Rutherford first found the closest distance, *b*, to the positively charged center of the atom that an α-particle with particular mass and velocity could reach. He found this value to be on the order of 10^{-12} cm (see appendix D), which he would later propose as an outer limit for the radius of the nucleus. Because the α-particle would come so close to the central charge, Rutherford argued that any deflection in the α-particle's trajectory would be dominated by its interaction with the electric field of the central charge to such a degree that the electric field of the negative charge could be ignored for all scattering events with deflections greater than 1 degree. Thus, he could ignore the electric field due to the negative charge for his large angle scattering calculations (Rutherford, 1911, 672). Later, he would show that the effect of the central charge was so great that even the electric field near individual electrons could be completely ignored for large angle scattering events (Rutherford, 1911, 679).

Rutherford then derived a relationship between the stopping distance, *b*, and the large scattering angle, *ϕ*, for an incident particle in interacting with the central charge of an atom (see appendix D):

$$(4)\ \cot\left(\frac{\phi}{2}\right) = \frac{2p}{b},$$

p, in equation 4, is the impact parameter, which Rutherford defined as the shortest perpendicular distance between the projection of the undisturbed incident path of the particle and the center of the atom (see appendix D). Thomson then used this relationship to predict the number of α-particles, *y*, that would be observed in a unit area located a particular distance *r* from the point of incidence between the α-particles and the scattering foil:

$$(5)\ y = \frac{ntb^2 Q \csc^4\left(\frac{\phi}{2}\right)}{16r^2} = \frac{4nt(Ne)^2 E^2 Q \csc^4\left(\frac{\phi}{2}\right)}{16r^2 m^2 u^4},$$

where *n* is the number of atoms per unit volume in the scattering foil, *t* is the thickness of the scattering foil, *Ne* is the magnitude of the central charge, *E* is the charge of the α-particles, *Q* is the total number of incident particles, *m* is the mass of the α-particles, and *u* is the initial velocity of the α-particles. This equation was the crucial result of Rutherford's paper. It provided four, experimentally testable predictions for the observed frequency of α-particle scintillations at a various distances from the location where the α-particles struck the reflector. Specifically, it predicted the frequency would (i) vary according to $\csc^4(\phi/2)$; (ii) be proportional to the thickness of the scattering foil; (iii) be proportional to the square of the magnitude of the central charge, *Ne* (and thus to square of the atomic weight, which was proportional to the magnitude of the central charge); and (iv) be inversely proportional "the fourth power of velocity" (Rutherford, 1911, 675).

Rutherford then turned his attention to using his model to explain small angle or multiple scattering events. Comparing his model and Thomson's, he performed a calculation that showed it was always more probable for an α-particle to turn through particular deflection angle, even a small one, due to single scattering event than due to multiple scattering events (Rutherford, 1911, 680). He insinuated that this made single scattering (and the nuclear atom) a better explanation for small angle scattering than multiple scattering and Thomson's atomic model. Unfortunately, as physicist Norman Feather observed, Rutherford's analysis of multiple scattering was "the least convincing" part of his paper (Feather, 1963, Loc 436). Rutherford's prediction did not agree with the results of small angle scattering experiments. In fact, he only succeeded in showing that Thomson's model was as consistent (if not more consistent) with the small angle scattering results as his new model. Consequently, he suggested that "further experiments on this question [were] desirable" (Rutherford, 1911, 680). Such experimental evidence would never be obtained. In fact, by 1930, Rutherford would acknowledge that "the experiments on multiple [or small angle] scattering have indeed led to no definite conclusions about atomic structure… the best that can be done is to show that the results are in accord with the nuclear theory" ((Feather, 1963, Loc 436). Conversely, Rutherford's large angle scattering predictions would prove to be exceedingly robust. By 1913, Geiger and Marsden had experimentally verified all four of the key predictions made by Rutherford's model and equation (5). I reproduce the data from these experiments in appendix E.

Consequently, by 1913, the nucleus and Rutherford's nuclear atomic model had captured the full attention of the scientific community. By 1914, due in large part to Geiger and Marsden's experimental confirmation of Rutherford's scattering formula, the nuclear atom was almost unanimously accepted; "there was no plausible alternative to the new theory, and its quantitative

successes appeared utterly convincing" (Feather, 1963, Loc 469-470). The main holdout was

Thomson, who questioned the validity of the coulombian "law of force between electrified

bodies when the distance separating them" was on the order of the dimensions of Rutherford's

nucleus (Feather, 1963, Loc 473). However, Thomson's resistance was likely due to his personal

investment in his own atomic model rather than substantive criticism;[10] the rest of the scientific

community found the evidence of Geiger and Marsden's 1913 paper sufficient to confirm both

Rutherford's atomic model and the validity of the coulombian law at small distances. As Feather

reports:

> At the British Association meeting in Birmingham, at the Solvay Conference in Brussels
> and at an international congress in Vienna, the [model] provoked the greatest interest.
> […] [Then] in London, on March 19, 1914, the Royal Society held a special discussion
> on the subject. Except in Vienna, Rutherford was the central figure on each occasion. In
> the end, there could be no permanent opposition, for there was no plausible alternative to
> the new theory, and its quantitative successes appeared utterly convincing (Feather, 1963,
> Loc 467-470).

Given the new consensus in favor of the nuclear atom, Rutherford's model was

surprisingly light on details. It's main supposition was that atoms were composed of a system of

negatively charged electrons held together by the attractive electrostatic force from an extremely

compact and positively charged nucleus located at the center of the atom. This nucleus contained

most of the atom's mass and occupied a miniscule portion of the atom's volume. Further, in

neutral atoms, the total charge of the nucleus was equal to the total charge of the system of

electrons. Drawing from experimental data presented in 1913, Rutherford's model also posited

that the total number of electrons in an atom was roughly equal to half of its atomic weight

(Bohr, 1913a, 1). However, Rutherford's model did not even attempt to describe the

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[10] This explanation seems particularly likely given Thomson's rejection of Boscovich's law (which similarly
suggested that there might be a more complicated law with different attractive/repellent regimes based on the
relative distances between particles) in favor of the coulombian law when choosing a force law for his initial atomic
model in 1897.

configuration of the electrons or the internal structure of the nucleus. It did not even place an

outer bound on the distance of the electrons from the nucleus and thus could not even explain the

size of the atom. Consequently, while this model was the only theory that could explain the large

angle scattering of α-particles, it lacked the Thomson model's explanatory power with regards to

many chemical and physical properties like the periodic law, atomic valency, atomic

electronegativity, and the resistance of the noble gases to acquiring a net charge. Nevertheless,

his refusal to consider electron structure did allow Rutherford's model to escape the problems

related to radioactive instability due to rotating charges. In the end, it would fall to Niels Bohr, a

Dane who visited both Thomson and Rutherford in England from 1911 through 1912, to fill in

many of the 'missing' pieces in Rutherford's atomic model.

Quantum Conditions and Atomic Modeling: Niels Bohr and the Rutherford-Bohr Model

After visiting Thomson and Rutherford's labs in 1912, Bohr returned to Denmark

inspired to write a series of three papers that would "explain some of the properties of matter on

the basis of [Rutherford's] atom-model" (Bohr, 1913a, 1). Given this motivation and Bohr's

frequent collaboration with Rutherford throughout the writing process, it is unsurprising that the

final model became known as the Rutherford-Bohr model.[11] Because Bohr had "little enthusiasm

for experimental work," he did not make any notable experimental contributions to the atomic

project (Baily, 2008, 17). However, his theoretical contributions were groundbreaking.

Considering results from other areas of physics, Bohr recognized "the inadequacy of the classical

electrodynamics in describing systems of atomic size" (Bohr, 1913a, 2). So, he introduced

"Planck's constant, or as it often is called the elementary quantum of action" to his atomic

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[11] Rutherford was actually the one who "communicated" or submitted all three of Bohr's papers to Philosophical Magazine.

theory. This allowed him to create a dramatically improved version of Rutherford's atomic

model that could explain an unprecedented number of experimental results and empirical laws.

Bohr introduced these quantum considerations to his model via two principal

assumptions. First, he assumed that ordinary mechanics could be used to describe "the dynamical

equilibrium of [atomic] systems in their stationary states" but could not be used to describe the

transition of atomic "systems between different stationary states" (Bohr, 1913a, 7). Second, he

assumed that the transition between stationary states could be described by the emission of one

photon with energy $E=hv$. This was the same equation that Planck used to explain black-body

radiation and that Einstein used to explain the photoelectric effect.

In order to develop the mathematics needed to describe the stationary states of his atom

and also to compare his proposal to the predictions of 'ordinary mechanics,' Bohr first

considered a classical model for hydrogen. This system had one electron in an elliptical orbit

around a nucleus with a positive charge equal to the magnitude of the charge on the electron. In

this case, and in the case of atoms with many more electrons, Bohr 'built' his atom one electron

at a time. He did this by performing his analysis in terms of the work, W, required to remove

each successive electron from the atom and take it an infinite distance away from the atom. Bohr

suggested, the magnitude of this work, W, was equivalent to the energy emitted during the

process of binding a free electron to the nucleus. In the case of hydrogen, this was the energy

emitted in binding one free electron to an isolated hydrogen nucleus to form a neutral hydrogen

atom. Using the known behaviors of orbital systems defined by forces inversely proportional to

the square of the distance, Bohr reported the frequency, ω, and the major axis, $2a$, of the

electron's orbit in terms of W:

$$(6) \quad \omega = \frac{\sqrt{2}}{\pi} \frac{W^{\frac{3}{2}}}{eE\sqrt{m}}, \quad 2a = \frac{eE}{W}$$

where e is the charge of the electron, E is the charge of the nucleus, and m is the mass of the electron. However, Bohr explained that, once radiation was taken into account, these values would change as the electron would spiral inward at an increasing velocity and the total energy of the system would decrease until the electron collided with the nucleus. Bohr noted that the predicted magnitude of this radiation would far exceed the observed values; so, classical electrodynamics could not describe the behavior of an electron orbiting a nucleus.

Instead, Bohr built upon J.W. Nicholson's attempt to introduce quantum constraints into the picture. While Nicholson's proposed constraint—that the total energy of an atomic system with only one ring of electrons must equal $h\omega$—could explain certain ratios in the wavelengths of "different sets of lines of the coronal spectrum," it could not account for the fact that atomic emission occurred in finite amounts of homogenous radiation (Bohr, 1913a, 6). As soon as Nicholson's atom started radiating, ω would instantaneously decrease thus causing the energy (and frequency) of the radiation to decrease. Instead, Bohr used his two principle assumptions along with two "more special assumptions" to derive a different constraint (Bohr 1913a, 8). Once again he built his atom by binding successive free electrons to the nucleus. Bohr assumed that the frequency, v, of the radiation emitted during the process of binding an electron would equal half the frequency, $\omega/2$, of the electron's resultant bound orbit around the nucleus. Combining this assumption with Planck's formula, Bohr further assumed that the energy, W, emitted in binding the electron to the nucleus could be given by:

$$(7) \; W = \tau h \frac{\omega}{2},$$

where τ is an integer representing the various stationary states of the atom. Bohr suggested that these stationary states would correspond to variously sized concentric rings in which the electron could orbit the nucleus without constantly radiating energy. For the hydrogen atom, $\tau=1$ would

correspond to the electron orbiting in the ring closest to the nucleus and $\tau=2$ would correspond to the electron orbiting in the next ring out. Once the electron was bound in one of these stationary states, Bohr's first principle assumption suggested that ordinary mechanics could be used describe the system. Thus, equation (7) could be substituted into the ordinary orbital equations from (6). Solving for W, ω, and $2a$, Bohr obtained new equations to describe an electron's orbit:

$$(8) \quad W = \frac{2\pi^2 m e^2 E^2}{\tau^2 h^2}, \quad \omega = \frac{4\pi^2 m e^2 E^2}{\tau^3 h^3}, \quad 2a = \frac{\tau^2 h^2}{2\pi^2 m e E}$$

These equations suggested that the energy, W, required to free an electron from a neutral hydrogen atom was greatest when $\tau=1$ and the electron is in the ring closest to the nucleus. Because this configuration would require the most work to disassemble, Bohr suggested it was the most stable configuration for a neutral hydrogen atom. He then used the known values of m, e, and h to calculate W, ω, and $2a$ for hydrogen. In this he found his first agreement with experimental results; his values were "of the same order of magnitude as the linear dimensions of the atoms, the optical frequencies, and the ionization potentials" observed for hydrogen (Bohr, 1913a, 5).

Bohr then used the results in (8) to derive the Rydberg equation for the emission spectrum of hydrogen (see appendix F):

$$(9) \quad \nu = \frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{\tau_2{}^2} - \frac{1}{\tau_1{}^2}\right)$$

Here ν is the frequency of the homogenous radiation emitted when an electron moves from an orbital ring further from the nucleus, defined by τ_1, to an orbital ring closer to the nucleus, defined by τ_2. This implied that τ_2 must be less than τ_1 for the atom to emit radiation. By successfully deriving this equation, Bohr demonstrated that his model could account for many previously unexplained phenomena. By setting $\tau_2=2$ and allowing τ_1 to vary equation (9)

predicted the Balmer series. Setting τ_2=3 and again allowing τ_1 to vary produced the Paschen

series. Moreover, Bohr showed that the value of the constant, $\frac{2\pi^2 me^4}{h^3}$, in front of the parentheses

was 3.1×10^{15}. Remarkably, this value agreed with the observed value of 3.290×10^{15} within the

"uncertainties due to experimental errors in the constants" (Bohr, 1913a, 9). Further, this

derivation of the Rydberg equation also allowed Bohr to explain why hydrogen's spectrum only

had 12 spectral lines when observed in laboratory tubes but had 33 lines when observed in

coronal settings. Bohr's theory suggested that the observation of a greater number of spectral

lines would require that the hydrogen atom take configurations corresponding to greater values

of τ. Given the dependence 2a (which was a good estimate of the diameter of an atom) on τ^2,

large values of τ would require atomic diameters on the same scale as the mean distance between

gas molecules at extremely low tube pressures. Thus, achieving the atomic configurations needed

to observe more spectral lines would require that the hydrogen gas have a lower density than

could ever be obtained in the volume of a typical laboratory tube.

Bohr then used his model to derive the Rydberg formula for the emission spectrum of

helium:

$$(10) \; \nu = \frac{2\pi^2 me^4}{h^3} \left(\frac{1}{\left(\frac{\tau_2}{2}\right)^2} - \frac{1}{\left(\frac{\tau_1}{2}\right)^2} \right)$$

This allowed his model to explain even more experimental results. Setting τ_2=3 and allowing τ_1

to vary led to the spectral series observed by Fowler in his "experiments with vacuum tubes,"

and fixing τ_2=4 gave the Pickering series in which every second line is "identical with a line in

the Balmer series of the hydrogen spectrum" (Bohr, 1913a, 10). This result allowed Bohr to

explain Pickering's observation that the spectral lines corresponding to the Balmer series were

more intense than the rest of the spectral lines in the Pickering series emitted from the star

Puppis. Bohr's model suggested a simple explanation: both hydrogen and helium were present in the star. Moreover, while the spectral equations for the rest of the known elements were too complicated and numerous for Bohr to feasibly derive them all, he ingeniously demonstrated that his model could account for them as well. His theory correctly predicted that the term in front of the parentheses in the spectral equation would be the same constant, $\frac{2\pi^2 m e^4}{h^3}$, for every element. This value corresponded to the universal constant in Rydberg's empirically derived spectral law.

Importantly, all of these spectral results rendered Bohr's special assumptions unnecessary; he could obtain his model using only the data for the atomic spectra (which led to the Rydberg equation) and his two principal assumptions (see Appendix G). While this result significantly reduced the number of assumptions required by his model, Bohr was also very interested in the implications of result for the physical meaning of his theory:

> We are thus led to assume that the interpretation of the equation [7] is not that the different stationary states correspond to an emission of different numbers of energy-quanta. But that the frequency of the energy emitted during the passing of the system from a state in which no energy is yet radiated out to one of the different stationary states, is equal to different multiples of *ω/2*, where *ω* is the frequency of revolution of the electron in the state considered (Bohr, 1913a, 14).

Bohr then turned his attention to the absorption of radiation by atomic systems. Here, he was able to show that his atomic theory predicted another important empirical result also not described by classical electromagnetic theory: the photoelectric effect (see appendix H). Similarly, Bohr's model showed that an atom would only absorb radiation with certain distinct frequencies until the energy of the incident radiation exceeded the threshold energy given by the photoelectric effect (see appendix H). At that point, the atom would emit an electron with a kinetic energy, T, determined by Einstein' equation for the photoelectric effect:

$$(11)\ T = h\nu - W$$

This two-fold absorption behavior perfectly described the results of "some experiments of R.W. Wood on absorption of light by sodium vapor," (Bohr, 1913a, 219). Moreover, these absorption predictions derived from the Bohr model could also explain Rutherford's observation that, in small angle scattering experiments with β-particles, high speed electrons travelling through an atom only "lose energy in distinct finite quanta" (see appendix H) (Bohr, 1913a, 221).

Bohr then worked to find the stable equilibrium states of his atom. Importantly, Bohr realized that the equations in (8) guaranteed that all of the electrons in stationary states (and thus all of the electrons in equilibrium states) would have a particular angular momentum, L:

$$(12) \; L = \frac{h}{2\pi}$$

Moreover, in his calculations he was able to use "an adaptation of Thomson's analysis [which I considered earlier] given by Nicholson" (Bohr, 1913a, 23).[12] This was possible because Bohr's first principal assumption required that classical electrodynamics govern the atom's stationary states, so, like Thomson, Bohr could perform his analysis in terms of the electronic forces acting upon individual electrons arranged in a series of concentric rings. Determining the equilibrium states required him to find the stability conditions for displacements of the electrons in the plane of the ring and for displacements perpendicular to the ring. While Nicholson had been unable to find appropriate stability conditions for displacements in the plane of the ring, Bohr's model resolved this issue. It implied that the stability conditions for all electron displacements were intimately tied to the "mechanism of the binding the electrons" to the nucleus (Bohr, 1913a, 23). Thus, Bohr proposed his first stability condition: all electrons in stable atomic state must obey the "universal constancy of angular momentum" given in equation (12), which was a product of the particular binding mechanism in his model. Further, he argued that, within a group of

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[12] See my discussion of Thomson's analysis, found in "On the Structure of the Atom," above (Thomson, 1904).

neighboring atomic configurations, the configuration that emitted the most energy during its formation—i.e., the configuration in which sum of all the values of W for the successive bindings of each of its electrons was greatest—would require the most energy to break apart. Thus, his second stability condition required that atomic equilibrium states correspond to configurations that emitted more energy during their formation than their neighboring stationary states. Furthermore, Bohr found that, when applied to electron displacements perpendicular to the plane of the ring, these stability conditions were identical to those found in Thomson's "ordinary mechanical calculations" (Bohr, 1913a, 23).

Bohr continued this discussion in the second paper of his trilogy, also published in 1913. Arguing that the formation of helium atoms from α-particles (discovered by Rutherford in 1908), supported the essential correctness of his assumption "that the cluster of electrons is formed by the successive binding [of electrons] initially nearly at rest" to the nucleus, Bohr applied the stability results from his first paper to atoms with more electrons and greater nuclear charge (Bohr, 1931b, 476). He did this both for neutral atoms and for charged atoms. Then, Bohr used his stability conditions along with the equations from (8)—which were modified so that the charge of the nucleus was given by E=ne, where n was the number of electrons in the neutral atom—to look at the possible configurations of hydrogen, helium, lithium, and beryllium. Where experimental data was available, he found good agreement between his predicted values of a, ω, and W and the experimental results. For the expected configuration of the neutral atoms, he calculated that the radius of helium was smaller than the radius of hydrogen and the radius of beryllium was smaller than lithium. Moreover, he also found that neutral lithium and beryllium atoms would be expected to have to two electron rings and thus be larger than hydrogen and

helium. These fit with the known periodic trends for atomic radii to decrease across a period and

increase down a group.

He then considered stable atomic configurations for atoms containing greater numbers of

electrons. Bohr's model predicted that any atom with a nucleus of charge ne and only one ring of

electrons would only be stable if there were n<8 electrons in that singular ring. However, he also

found that atoms with more than one ring if electrons could be stable with rings containing n≤8

electrons. Using these and other stability concerns, Bohr proposed configurations for the first 24

neutral elements, which are shown in the table below:

Table 5: Bohr's Atomic Configurations—Taken from (Bohr, 1913b, 497)

n	C	n	C	n	C
1	1	9	4.4.1	17	8.4.4.1
2	2	10	8.2	18	8.8.2
3	2.1	11	8.2.1	19	8.8.2.1
4	2.2	12	8.2.2	20	8.8.2.2
5	2.3	13	8.2.3	21	8.8.2.3
6	2.4	14	8.2.4	22	8.8.2.4
7	4.3	15	8.4.3	23	8.8.4.3
8	4.2.2	16	8.4.2.2	24	8.8.4.2.2

As this table shows, the Rutherford-Bohr model predicted the general "periodicity with period 8"

observed for the lighter atoms. Further, Bohr found that the outer electrons were bound more

weakly when there were more rings between them and the nucleus. Thus, his model also

correctly predicted the increasing electropositive character observed when moving down the

columns of the periodic table. Further, this consideration of a greater number of atoms confirmed

something Bohr had noticed in his consideration of the configurations of the lightest four

elements: the Bohr model accurately predicted that atomic radii would decrease across a period

and increase down a group. Thus, like Thomson's model, the Bohr model explained a number of

the periodic trends in chemical and physical behavior of the elements. However, Bohr's model

struggled to explain the behavior of "atoms of higher atomic weight[s]" especially once "we meet with the iron group" (Bohr, 1913b, 497).

Near the end of this second paper, Bohr turned his discussion to focus on three previously unexplained phenomena that his atomic model correctly predicted. He first considered the characteristic Röntgen radiation produced by various atoms. Bohr built upon Thomson's hypothesis that this radiation was generated when "electrons in inner rings [of the atom] were removed by some agency, e.g. by impact of cathode particles" (Bohr, 1913b, 498). He focused on the K-type characteristic Röntgen radiation, which was the hardest to generate. Bohr thought that this would correspond to the freeing of an electron bound in the atom's innermost ring. Based on the work required to free such an electron, he calculated the minimum velocity needed for a cathode ray particle to produce this K-type radiation in various atoms. He found this velocity to be proportional to $2.1 \times 10^8 N$, where N is the charge of the atom's nucleus. Whiddington had experimentally determined that this velocity was proportional to $10^8 A$, where A is the atomic weight, "for elements form Al to Se" (Bohr, 1913b, 499). Given that $A/2=N$ for light elements, these results demonstrated remarkable agreement. Further, Bohr argued that radioactive phenomena, specifically α-particles and β-particles, originated in the nucleus. Because they were positively charged, there was nowhere else that α-particles could originate. The case of the high-speed electrons emitted as β-particles, however, was less clear. Certain atoms produced these radioactive particles spontaneously. So, Bohr argued that the β-particles could not be electrons ejected from the orbiting rings due to the absorption of radiation. He also ruled out α-particles ejected from the nucleus hitting one of these external electrons, knocking it free, and causing it to accelerate to a high speed because scattering experiments did not demonstrate α-particles causing the emission of electrons. Thus, he postulated that the only way

for an electron to be spontaneously emitted from an atom at high speed was for it to originate in

the nucleus. Bohr correctly believed that his suggestion that α-particles and β-particles originated

in the nucleus would explain two empirical laws:

1. Whenever an α-particle is expelled the group in the periodic system to which the resultant product belongs is two units less than that to which the parent body belongs.
2. Whenever a β-particle is expelled the group of the resultant body is 1 unit greater than that of the parent. (Bohr, 1913b, 502).

Bohr's third and final paper on the structure of the atom, published in 1914, focused on

the implications of his model for the systems of two or more atoms. He suggested when two

atoms formed a molecule the electrons in their outermost rings would form a singular ring

rotating about both nuclei. However, Bohr realized that he could not build molecules up one

electron at a time, as he had atoms. The two positively charged nuclei would repel one another if

electrons were not present. Thus, he suggested that molecules were combinations of fully formed

atomic systems with electrons bound to each atom's respective nucleus prior to the two atoms

bonding. He suggested two mechanisms for this bond to occur. First, two atoms with electron

rings rotating in parallel planes could approach one another and, as they neared, the electrons in

both atoms would be displaced away from the space in between the nuclei, opening a gap in both

rings that would allow the resulting two half ring configurations to join into a single outer

electron ring orbiting both nuclei. Alternatively, the two atoms could become charged, one with

a positive charge, the other with a negative charge, and be attracted to one another. Once joined,

Bohr thought that the electronic forces would cause the atom's outer electrons to form one

orbiting ring.

Interestingly, while Rutherford's nuclear proposal was widely accepted, the Rutherford-

Bohr model "had a fairly adverse reception in the scientific community;" although the model has

been treated more favorably in historical retrospect (Niaz, 1998, 540). It was abandoned after the

introduction of quantum mechanics as Bohr's electron orbits, which had definite radii and definite angular momentum, were shown to violate the Heisenberg uncertainty principle. However, the pre-quantum lack of acceptance was somewhat surprising; Bohr's model was a theoretical masterpiece predicting and explaining an unprecedented litany of observed phenomena and empirical laws. This was largely due to Bohr's willingness to introduce quantum considerations to describe atomic behavior during the transitions between stationary states. This enabled his model to explain the "paradoxical stability of the Rutherford atom" and allowed him to fill in many of its missing details, particularly the configurations and behaviors of electrons orbiting the nucleus (Niaz, 1998, 539). Further, this allowed Bohr to make predictions of the linear dimensions, rotational frequencies, and ionization energies of atoms that were consistent with the best observations of the time.

Moreover, his model explained the large angle scattering of α-particles, the radioactive stability of most atoms, the behavior of unstable radioactive atoms after emitting α-particles and β-particles, the atomic spectra (including Rydberg's universal constant and the Balmer, Paschen, and Pickering series), the failure to observe certain spectral lines in laboratory settings that were observed in stellar contexts, the photoelectric effect, the dual regimes in which atoms absorbed radiation, the quantized nature of the β-particle's energy loss when travelling through an atom, the periodicity of the elements (specifically the 8 period periodicity of the light elements and the observed trends in atomic radii and the electropositive character of the elements), and the stable configurations of light atoms. However, the model failed to make good predictions about the stable configurations of the heavy atoms, particularly those occurring after the iron group. Importantly, the Rutherford-Bohr model explained these phenomena in a mechanistic manner grounded upon electronic forces, which was consistent with Thomson's electronic theory of

matter. However, it also introduced a novel element into these mechanistic explanations by treating newly discovered behaviors, specifically the quantized behavior of radiation, and the associated laws (namely $E=hv$) as physically fundamental—on par with electronic forces. Ultimately, the expansive explanatory power of Bohr's model shows just how far atomic modeling was able to advance in the 17 years following Thomson's discovery of the electron.

Part 2: Philosophical Implications

Scientific Revolution: Kuhn's Antirealist View

As I have shown, the work of Thomson, Rutherford, and Bohr from 1897-1914 resulted in profound and rapid changes to the world's conception of the atom, most notably those associated with the discovery of the electron and the discovery of the nucleus. Thus, in ordinary terms, it seems appropriate to say that this period was in some sense revolutionary. In this section, I consider the notion of scientific revolution in a more technical context—namely based on the work of Thomas Kuhn—in order to evaluate whether a scientific revolution did occur during this period in the next section. Kuhn first articulated his views on scientific revolution in his 1962 masterpiece, *The Structure of Scientific Revolution*. Before discussing the particulars of his view, it is important to emphasize that some people think that Kuhn was only discussing revolutions *within* science and not revolutions *to* science (i.e., he is not theorizing about *the* scientific revolution). In an introductory essay to the 50th Anniversary edition of *Structure*, Ian Hacking explains this quite well:

> A first point to notice […] is that Kuhn was not talking about *the* scientific revolution.[13] That was quite a different kind of event from the revolutions whose structure Kuhn

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[13] By "*the* scientific revolution" Hacking is referring to the Copernican or Newtonian scientific revolution of the 17th century, often credited with emergence of the modern scientific discipline, replete with experiments, the scientific method, mathematization of natural philosophy, and mechanistic rather than teleological explanations of the natural world (i.e., *the* revolution to science from philosophy and other disciplines).

postulated. Indeed shortly before he published Structure, he had proposed that there was a "second scientific revolution." It took place during the early years of the nineteenth century; whole new fields were mathematized. Heat, light, electricity, and magnetism acquired paradigms, and suddenly a whole mass of unsorted phenomena began to make sense. This coincided with—went hand-in-hand with—what we call the industrial revolution. It was arguably the beginning of the modern technoscientific world in which we live. But, no more than the first scientific revolution, did this second revolution exhibit the "structure" of *Structure* (Hacking, 2012, xiii).

However, not everyone interprets Kuhn in this way.

With this scope distinction in mind, let us turn to Kuhn's theory. Coincidentally, Kuhn's view of scientific progress is analogous to Bohr's view of atomic behavior. Bohr divided atomic behavior into the equilibrium of stationary states and the rapid transitions between states; Kuhn divided scientific progress into periods of normal science and revolutionary science. For Kuhn, normal science progressed in a steady, well defined, and cumulative manner according to a paradigm, much like the predictable movement of electrons in Bohr's stationary states. However, just as the transitions between states led to discrete energy jumps in Bohr's model, progress during revolutionary science for Kuhn was anything but steady.

In *Structure*, paradigms are defined as past scientific achievements that "attract an enduring group of adherents" to "provide models from which spring particular coherent traditions of scientific research"—i.e., paradigms are accepted and foundational scientific theories and methodologies that define a particular field of research (Kuhn 1962/2012, 10-11). Kuhnian paradigms are also sometimes described as worldviews. The point of normal science was to solve problems using known techniques accepted within the field defined by the paradigm. Consequently, Kuhn suggested that periods of normal science progressed in a manner similar to solving a puzzle. The scientist "knows what he wants to achieve" and "expects to have a reasonable chance of solving the puzzle" based on familiar methods and strategies (Kuhn, 1962/2012, 97; Bird, 2011).

Conversely, Kuhn argued that revolutionary science is always preceded by a "malfunction" in the current paradigm leading to a crisis in which at least some members of the scientific community lose faith in that paradigm. This leads to a period of improvisation and innovation in which novel theories and new paradigms are proposed. This period is filled with debates over what counts as "legitimate methods, problems, and standards of solution" (Kuhn, 1962/2012, 48). Ultimately, each individual scientist must to make their own decision between the competing paradigms, which define "alternate ways of practicing science" (Kuhn, 1962/2012, 162). So, the initial proponents of each paradigm will continue to develop and argue for their view, demonstrating what science would look like within their paradigm. If they are persuasive, more scientists will be converted to their paradigm. Further, if their paradigm is the one destined to win out in the end, new supporters and favorable arguments will continue to accumulate behind the paradigm, until there are "only a few elderly hold-outs" left (Kuhn, 1962/2012, 158). At this point, the field will be defined by the new paradigm and the scientific community will return to a period of normal science. For this reason, Kuhn also called scientific revolutions paradigm shifts.

Importantly, Kuhn thought that these paradigm shifts led to fundamental discontinuities that interrupted the stable progress of normal science that occurred within a paradigm. This arose from his belief that opposing paradigms were fundamentally "incommensurable," which meant that the proponents of the different paradigms would "fail to make complete contact with each other's viewpoints" (Kuhn 1962/2012, 147). He gave three reasons for the incommensurability of rival paradigms. First, he thought that the proponents of competing paradigms would disagree as to which problems fell within the productive domain of inquiry and that they would use fundamentally different criteria to evaluate proposed solutions to scientific problems and

puzzles. He thought that the latter would include using profoundly different methodologies to conduct experiments and test predictions. Further, Kuhn recognized that a shared history would often lead the proponents of rival paradigms to use the same words. He thought that each group would add new experimental and/or theoretical content to this historical usage, which would give these shared terms slightly, or even radically, different meanings between paradigms. Finally, Kuhn thought that, at a fundamental level, "the proponents of competing paradigms practice[d] their trades in different worlds" (Kuhn, 1962/2012, 149). Consequently, he suggested that

> before they can hope to communicate fully, one group or the other must experience the conversion that we have been calling a paradigm shift. Just because it is a transition between incommensurables, the transition between competing paradigms cannot be made a step at a time, forced by logic and neutral experience. Like the gestalt switch [or a religious conversion], it must occur all at once (though not necessarily in an instant) or not at all (Kuhn, 1962/2012, 149).

> This points to another important feature of Kuhn's theory. He thought that paradigm-

neutral (or theory-neutral) considerations, such as logical arguments and empirical results, were insufficient to resolve the conflicts between paradigms.[14] Instead, each group would need to use

> its own paradigm to argue in that paradigm's defense. The resulting circularity does not, of course, make the arguments wrong or even ineffectual. The man who premises a paradigm when arguing in its defense can nonetheless provide a clear exhibit of what scientific practice will be like for those who adopt the new view of nature. That exhibit can be immensely persuasive, often compellingly so. Yet, whatever its force, the status of the circular argument is only that of persuasion. It cannot be made logically or even probabilistically compelling for those who refuse to step into the circle. The premises and values shared by the two parties to a debate over paradigms are not sufficiently extensive for that (Kuhn, 1962/2012, 94).

Thus, Kuhn believed that the ultimate selection between paradigms during a scientific revolution was not concerned with which theory was 'right' or 'true' but was entirely a question of which paradigm was more persuasive to the scientific community. Given the number of such paradigm shifts throughout the history of science, this led Kuhn to adopt an antirealist position about

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[14] Allan Franklin shown that this is not always the case. See "Are Paradigms Incommensurable?" (Franklin, 1984).

scientific progress as a whole. He used an analogy to evolution to express this conclusion in the final chapter of *Structure*. The evolutionary process, which determines the fittest organisms through natural selection, "move[s] steadily *from* primitive beginnings but *toward* no goal" (Kuhn, 1962/2012, 171). Similarly, Kuhn suggested that science moves from past theories towards "the fittest way to practice future science" via selection by paradigm conflict (Kuhn, 1962/2012, 171). Moreover, he thought the belief in "a permanent fixed scientific truth" towards which science progresses was problematically on par with teleological theories of evolution.

Did a Scientific Revolution Occur Between 1897 and 1914?

We are now in a position to assess whether a scientific revolution occurred in the realm atomic theory between 1897 and 1914. Incidentally, this is not simply a question of whether or not the entire period constituted a revolution. Instead, there are at least two, and perhaps three, developments within this period that might be best described by Kuhn's revolutionary science. The two main candidates are: 1.) The development of Thomson's electronic theory of matter and 2.) Rutherford's proposal of the nuclear atom. One might also suggest that the difficulties in accounting for the radioactive stability of moving electrons, which led Bohr to introduce his quantum constraints, are evidence that this period does not fall within the domain of normal science, but rather is part of the crisis leading to the quantum revolution. I will not discuss this possibility in this paper, as it would require a more substantial consideration of the events after the Bohr model than this paper provides. I first consider the possibility that both of the main candidates were scientific revolutions. I then consider interpretations that would suggest that only one of these events is best described as revolutionary science.

At first glance it seems like the proposal and subsequent acceptance of Thomson's electronic theory of matter clearly constitutes a Kuhnian paradigm shift, and thus should be

described by revolutionary science. The existing paradigm was Dalton's indivisible atom, which

suggested that atoms could not be divided and that there were different types of atoms

corresponding to the various elements, which could recombine in integer ratios to form

compounds. Thomson's discovery that cathode rays were made of particles much smaller than

any known atom challenged both Dalton's atomic paradigm and the aether paradigm for cathode

rays, thus throwing the field into crisis.[15] Alternatively, with respect to the nature of cathode

rays, it could be said that the field was already in crisis and Thomson's experiments and

subsequent paradigm proposal helped move the field back towards normal science. Importantly,

all of Thomson's arguments that electrons were atomic constituents used his new paradigm as a

premise—just as Kuhn suggested. His first argument, that these particles were much smaller than

a charged hydrogen atom and thus *could* be subatomic constituents, relied on his calculations of

the smallness of m/e, which he showed to be much smaller than any known particle, and the

mean free path calculations, which revealed that the cathode ray particles would travel orders of

magnitude further than air molecules between collisions and thus suggested that the smallness of

m/e was due at least in part to m being smaller than that of any atom. Both of these were

inextricably linked to Thomson's new particle hypothesis.

Further, Thomson also offered a second argument. He contended that the behavior of the

cathode rays could be explained by his atomic hypothesis if the electric field near the cathode

was strong enough to rip electrons from the gas molecules and accelerate them through the air in

a beam, thus resulting in a ray of high-speed particles with the appropriate value of m/e. This

argument also used Thomson's new atomic theory as a premise. Moreover, the initial atomic

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[15] Although as Baily notes, very few scientists were engaged in the field of atomic modeling at this time. So, the field, which was thrown into crisis, constituted a very small portion of the overall scientific community. The presence of the crisis in such a narrow group accords with Kuhn's discussion in chapter 9 of *Structure* (Kuhn, 1962, 92).

model that Thomson presented in 1897 can be seen as a proposal of what science would look like in the new field defined by his paradigm. It showed that the stability of atoms could be accounted for in terms of known electromagnetic forces and that the stable configurations of electrons could account for the periodic law. Consequently, all of Thomson's initial arguments posited his paradigm as premise in order to persuade others to convert to his viewpoint, which is thoroughly in line with Kuhn's discussion of revolutionary science.

Thomson's 1904 paper then expanded upon his paradigm and improved the arguments in its favor. Notably it also mathematized his model, something Hacking suggests Kuhn viewed as characteristic of revolutions *to* science. He showed that calculations based on attractive electronic forces would give rise to stability configurations similar to his original proposal. This gave his model expanded explanatory power. Instead of just predicting periodic behavior in general, these calculations correctly predicted that periodicity in the chemical and physical properties of the elements would occur both across the rows and up/down the columns of the periodic table. In particular, it explained the variations in atomic valency and trends in the electronegativity of atoms and also suggested an explanation for the resistance of the noble gases to acquiring a net charge.

Historical evidence suggests that after this 1904 paper, Thomson's model became widely accepted within the scientific community—he had persuaded enough scientists to convert to paradigm. If such a case were revolutionary, Kuhn suggests that there should then be a return to normal science. Thomson's subsequent revisions of his model—dramatically reducing the number of electrons and attributing mass to the positive charge—can be seen as progress following such a return to normal science. Both demonstrated progress characteristic of normal science via experiments that were in line with Thomson's paradigm and related to his cathode

ray experiments. In particular, they looked at the scattering of Röntgen radiation (X-rays), the dispersion of light, and the gaseous absorption of β-rays. Further, Rutherford's suggestion that the small angle scattering of α-particles was evidence for Thomson's view shows that other members of the scientific community were engaging in atomic science defined by his paradigm. Thus, it seems that the proposal of Thomson's atomic model led to a paradigm shift after which his paradigm came to define normal science.

Moreover, Geiger's attempts to avoid discussing the large angle scattering results he and Marsden published in 1909 is also indicative of the influence of Thomson's paradigm over normal science. This paradigm created expectations about the types of phenomena that would occur in scattering experiments. Given that the statistical data agreed to an immense degree with these predictions, Geiger and Marsden tried to convince themselves that there was some sort statistical anomaly occurring in the large angle scattering experiments rather than a real conflict with Thomson's paradigm. In this, they exhibited what Kuhn called "the most striking feature of […] normal research problems[:] […] how little they aim to produce major novelties, conceptual or phenomenal" (Kuhn, 1962/2012, 35). However, behind the scenes a crisis was brewing in the Rutherford lab.

As this small segment of the atomic modeling community looked at their 1909 results, they could not in good conscience write them off as statistical anomalies. Thus, despite Geiger's line in the 1910 paper indicating their frustrations and lack of success, they kept innovating and searching for new explanations until Rutherford developed his nuclear model of the atom and presented it in 1911. The atypical organization of this paper, pointed out by Feather, illustrates how large the large angle scattering crisis loomed in Rutherford's' mind (Feather, 1963, Loc 426-434). Despite the fact that he had studied the small angle scattering first and he would have

to convince the scientific community—which was operating in a period of normal science defined by the Thomson model—that his model could do at least as well as Thomson's model (especially in explaining the small angle scattering results), Rutherford did not discuss multiple scattering at the beginning of the paper. Feather suggests that this is because Rutherford knew that his small angle scattering sections were the least convincing and did not want "to play into the hands of others" (read Thomson here) (Feather, 1963, Loc 432). Rutherford's desire to avoid this issue is evidence of the hold that Thomson's atomic model had on the scientific community. It also shows that Rutherford's innovation during this period of crisis extended even to his paper's structure; he broke with convention in an attempt to make his new paradigm more persuasive.

In this 1911 paper, Rutherford began with the large angle scattering phenomena and used it to argue for the abandonment of the Thomson model's distribution of positive charge in favor of a compact nucleus. Rutherford's scattering formula provided a compelling vision for the future of atomic science. Consequently, when Geiger and Marsden confirmed all of four of the formula's predictions the scientific community took notice. This led to the widespread acceptance of the nuclear model of the atom. As I indicated earlier, Thomson was the only major holdout. This is an excellent example of Kuhn's description of a few elderly holdouts that remain resistant to a new paradigm even after a revolution/paradigm shift has occurred in the field. Bohr's efforts to further elucidate Rutherford's model can then be seen as steady progress in this new period of normal science—though given his introduction of quantum considerations, Bohr's work may have been more revolutionary than steady. Thus, this initial characterization of the period in atomic theorizing from 1897-1914, suggests that it contained two paradigm shifts, the one to Thomson's atomic model and a second to Rutherford's nuclear model.

However, it is also possible to argue that the establishment of Thomson's model did not constitute a revolutionary paradigm shift *within* science, but rather was a revolution *to* science. As Hacking emphasizes in his introductory essay, his interpretation of Kuhn's description of revolutionary science in *Structure* does not include the latter case. Interestingly, in *Structure* Kuhn suggested that the "subatomic phenomena unknown before the twentieth century" show that a new theory does not need to conflict with an old theory if it deals with previously unknown phenomena (Kuhn, 1962/2012, 95).[16] This fits nicely with Hacking's interpretation of Kuhn and seems to be evidence of Kuhn's belief that the acceptance of Thomson's theory would not require the conflict between paradigms inherent to a revolution *within* science. Further, given that Thomson was the first scientist to offer mechanistic explanation of subatomic structure (and subsequently mathematize this explanation), it could be argued that his work better resembles the events of *the* first scientific revolution and *the* second scientific revolution, which Hacking discusses in the introduction and which Kuhn wrote about prior to *Structure.* If this is the case, Thomson did not trigger a paradigm shift but rather blazed the trail to create the initial scientific paradigm for atomic theory. This requires that we not grant Dalton's atomic theory the status of paradigm. We could, however, view Dalton as the initial proponent of the mechanization of atomic theory. On this view, Thomson then finished this task and completed atomic theory's revolution *to* science. Under this characterization, my earlier discussion of the establishment of Thomson's paradigm fits nicely with Kuhn's suggestion that pre-paradigm periods are filled with the same "deep debates over legitimate methods, problems, and standards of solution" that occur during the periods of crisis associated with scientific revolutions (Kuhn, 1962/2012, 48). Despite

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[16] However, it is interesting to note that Kuhn associates these phenomena and the accompanying development of a new theory that does not conflict with an old one with the development quantum theory. Elsewhere in *Structure* he treats the development of quantum theory as a scientific revolution in which the old mechanics were rejected (see Kuhn, 1962, 89).

this similarity, Hacking's interpretation of Kuhn suggests that the establishment of the first scientific paradigm marks the beginning of normal science and is not a case of revolution within science. So, on this view, Thomson's establishment of the initial atomic paradigm began the first period of normal science, which was followed by the first revolution in this field and the only paradigm shift during this period: the shift to the nuclear atom.

However, it is still possible to view the initial adoption of Thomson's model as a revolutionary event *within* science, as Kuhn's treatment of quantum theory elsewhere in *Structure* seems to imply (see, for example Kuhn, 1962/2012, 89).[17] This would require us to treat Dalton's atomic theory as the accepted scientific paradigm prior to Thomson. On such an interpretation, we could also argue that Rutherford and Bohr both worked in period of normal science fully defined by Thomson's electronic theory of matter. This would require us to assert that Rutherford's proposal of the nucleus did not undermine the fundamental aspect of Thomson's paradigm. This is in harmony with the developments of the period if we consider Thomson's proposal that atoms were made of oppositely charged subatomic constituents governed by the laws of electrodynamics to be the essence of his atomic paradigm. On this view, Rutherford's 1911 paper did not propose a revolutionary paradigm but rather offered a solution to the problem of large angle scattering within the realm normal science defined by Thomson's electronic theory of matter. Given that Rutherford, Geiger, and Marsden were conducting scattering experiments, this is not a far-fetched conclusion. These scattering experiments were well within the domain of the field defined by Thomson's paradigm and small angle scattering experiments were originally taken to support his view. Moreover, Rutherford's nuclear proposal merely changed the distribution and size of the positive charge in Thomson's model; it did not move away from the fundamental electro-dynamic explanation for atomic structure but rather

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[17] This is evidence against Hacking's interpretation.

resolved a problem with it. Further, Bohr's work is naturally seen as a continuation of Rutherford's work. So it too can be seen as normal scientific progress wherein Bohr worked to resolve the problems in the nuclear version of Thomson's atomic model using known methods and techniques. This seems to fit with Bohr's own attitudes towards Thomson's model at the time. Rather than rejecting Thomson's model as a conflicting paradigm, Bohr considered Thomson the "world master in the design of atomic models" and used a modified version of Thomson's analysis to find the equilibrium configurations for his own atomic model (Niaz, 1998, 540). While I not entirely convinced by this view, on this characterization there would only be one paradigm shift in atomic theory during this period: the original shift to Thomson's model.

Combining this view's interpretation of Rutherford's nuclear model as part of normal science with the previous suggestion that the development of Thomson's model instantiated, rather than revolutionized, scientific atomic theory suggests a fourth option: there was no scientific revolution during this period. Thomson's model merely started a period of normal science that extended through the work of Bohr and eventually was replaced by the quantum paradigm when the quantum mechanics was introduced. I do not find this view convincing. I think that the development and adoption of Rutherford's nuclear model is best described in the language of scientific revolution as in the first two cases I considered. Further, I think that, if Kuhn distinguishes between revolutions *to* science and revolutions *within* science in *Structure* as Hacking suggests, then this distinction is applied inconsistently in his discussion of quantum theory and the associated discoveries of subatomic particles.[18] This inconsistency makes the suggestion that Thomson's model was not revolutionary problematic. However, I will reserve a

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[18] In particular, Kuhn treats the development of quantum mechanics as a revolutionary event while simultaneously suggesting that Plank's discovery of quantum blackbody radiation was not revolutionary. It seems inconsistent to suggest that Plank's work, which introduced the quantum of action, was not part of the quantum revolution.

more detailed discussion of this problem for the next section. I conclude that there was at least one Kuhnian scientific revolution, the nuclear revolution, during this period of atomic modeling.

Revolution *to* Science and Revolution *within* Science: Revising Kuhn's Model

As I said in the last section, I think that Kuhn's discussion of scientific revolution in *Structure*, particularly the inconsistencies in his treatment of the development quantum theory as both revolutionary and non-revolutionary makes it unclear how to classify the development of Thomson's atomic model according to Kuhn's theory.[19] Consequently, in this section, I suggest that these difficulties in analyzing the proposal and adoption of Thomson's atomic model indicate that, rather than analyzing the events of this period in terms of Kuhn's work, we should revise Kuhn's theory of scientific revolution in a way that harmonizes it with the developments in atomic modeling during this period in science.[20] In particular, I suggest that if Kuhn did intend to focus solely on revolution *within* science as Hacking suggests, the notion of revolution *to* science nonetheless crept into his discussion. This is most apparent in Kuhn's gestalt switch or religious conversion analogy. Alternatively, if Kuhn did intend to include revolution *to* science in his discussion in *Structure*, he is inconsistent in doing so given his treatment of the early subatomic discoveries, which expanded the domain of science to a new subatomic field, as non-revolutionary. Further, I argue that Kuhn's move away from his gestalt switch analogy and towards his evolutionary analogy and smaller conceptions of scientific revolution is evidence of Kuhn's desire to clarify his conception of revolution *within* science in his later years. From this, I offer a revised version of Kuhn's theory of scientific revolution which suggests there are two

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[19] On Hacking's interpretation this inconsistency is found in the application of the distinction between revolution *to* science and revolution *within* science to quantum theory. However, even if Hacking's interpretation of the scope of Kuhn's discussion of scientific revolutions in *Structure* is wrong, this inconsistency is still present in Kuhn's work. See footnote 18.

[20] While this revised theory will certainly apply to this period, I suspect that it will also generalize to all scientific revolutions throughout history.

types of scientific revolution that fall under Kuhn's theory of paradigm shift and correspond to

his two analogies—although I do modify his evolutionary analogy. I also argue that these two

types of revolutions do not need to be mutually exclusive and that their combination can resolve

the inconsistencies in Kuhn's treatment of quantum theory.

As my discussion of the atomic prehistory indicated, prior to Dalton and Thomson,

atomic nature was the subject of philosophical rather than scientific debate. Then, Dalton

proposed that the observed mass ratios in compounds could be explained by the combination of

various indivisible atoms of different elements in whole number ratios. Subsequently, Thomson

showed that the atomic constituents of Dalton's elements were, in fact, divisible and offered a

mechanistic and mathematical explanation of their structure in terms of electrodynamics. This

picture suggests that the development of Dalton and Thomson's models occurred during the

transition to science from philosophy in the realm of atomic theory. Thus, on Hacking's

interpretation, Kuhn's model of scientific revolution in *Structure* should not apply to it. Kuhn

does, however, suggest that pre-paradigm periods are filled with similar debates to those that

occur during the periods of crisis inherent scientific revolutions (Kuhn, 1962/2012, 48). Thus, it

seems that the initial establishment of a scientific paradigm is preceded by a similar events as a

revolution *within* science. This might explain the impulse to analyze this development as a

revolution *within* science on Hacking's interpretation of Kuhn.

Interestingly, in his later years Kuhn changed his characterization of this initial revolution

to science by suggesting that "even the schools of the pre-paradigm period" possessed paradigms

(Kuhn 1977, 295 footnote 4). This meant the initial establishment of a paradigm within science

would also involve a conflict between paradigms, namely conflict between the pre-scientific

paradigm and the scientific one. Thus, the period leading to the establishment of an original

scientific paradigm would feature the same debates as the crisis period preceding a scientific revolution. It would also feature the conflict between paradigms characteristic of a Kuhnian scientific revolution. On this view, it seems like the only difference between revolutions *to* science and revolutions *within* science is in the type of paradigm (scientific or non-scientific) rejected during the switch. This is either evidence against Hacking's interpretation of *Structure* or evidence that Kuhn's views on this distinction had shifted since he wrote *Structure*. Either way, the result is that, in his later years, Kuhn viewed revolutions *to* science in much the same way he viewed revolutions *within* science.

Moreover, around this time Kuhn also distanced himself from extreme interpretations of his views on incommensurability, which focused on his assertion that the proponents of the competing theories practice science "in different worlds" (Kuhn, 1962/2012, 149). These radical interpretations held that rival paradigms were "*incommensurable* in the sense that they are so radically distinct that they cannot be compared" (Liston, n.d.).[21] The proponents of such interpretations latched onto Kuhn's description of scientific revolution and paradigm shift as a gestalt switch or religious conversion. They also fully embraced Kuhn's initial suggestion that there were *no* theory neutral considerations to help scientists decide between paradigms, thus emphasizing the conversion between paradigms involved in scientific revolution. However, Kuhn distanced himself from this idea in his later work (see the postscript of the 1970 version of *Structure* and *The Essential Tension*). As Michael Liston explains:

> Paradigm transitions and incommensurability, Kuhn argues, are never as total as the radical interpretation assumes: enough background (history, instrumentation, and every-day and scientific language) is shared by P- and P*-adherents to underwrite good reasons they can employ to mount persuasive arguments. Moreover, he lists several properties any theory should have—accuracy (of description of experimental data), consistency (internal and with accepted background theories), scope (T should apply beyond original intended applications), fecundity (T should suggest new research strategies, questions,

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[21] Again Franklin provides reasons to doubt this claim. See (Franklin, 1984).

problems), and simplicity (T should organize complex phenomena in a simple tractable structure). Application of these criteria accounts for progress and theory choice. However, these are "soft" values that guide choices rather than "hard" rules that determine choices (Liston, n.d.).

As time went on, Kuhn realized that the hard interpretation of his view was not consistent with revolution *within* science. This pushed him away from his gestalt switch metaphor and towards his evolutionary metaphor. It also led him to offer a smaller and less dramatic conception of revolution *within* science, which resembled the jumps of macroevolution rather than religious conversion (Nickles, 2017).

I believe that these transitions in Kuhn's views point towards a solution to the problem we have faced in trying to analyze the development of Thomson's atomic model using Kuhn's theory. Instead of excluding revolutions *to* science from Kuhn's discussion in *Structure* (Hacking's interpretation) or treating them as identical to revolutions *within* science (the opposite of Hacking's interpretation), we should include them both in our overarching theory of scientific revolution as paradigm shift and clarify the ways in which each should be treated distinctly. I have already shown that Kuhn's later views entailed that revolutions *to* science involve a conflict between paradigms and feature debates like those characteristic of periods of crisis. Thus, revolutions *to* science have all of the key features that Hacking suggests Kuhn assigns only to revolutions *within* science. This high degree of similarity suggests that revolutions *to* science should occur in a similar fashion to Kuhn's account of revolutions *within* science. Thus, I believe that a failure to adequately and simultaneously understand both the similarity and the distinction between these types of scientific revolutions can explain the inconsistency in Kuhn's treatment of quantum theory. This implies that the distinction between the two in Kuhn's work is not the scope distinction that Hacking suggests that it is. While this points to a potential cause of the inconsistencies in Kuhn's treatment of quantum theory, it still seems that we need to find a way

to distinguish between revolutions *to* science and revolutions *within* science while including both in our overarching theory of scientific revolutions in order to resolve this inconsistency. Because paradigm shifts occur in each case, it seems to me that the largest difference between the two situations is that a scientific paradigm and a non-scientific paradigm are more incommensurable (and incompatible) than two scientific paradigms. Consequently, a shift from a non-scientific paradigm to a scientific one will be much more pronounced and jarring than a shift between two scientific paradigms.

Consider for example, the transition from the philosophical consideration of the nature of the atom to the scientific inquiry of Thomson. The former used primarily *a priori* methods to answer a question of fundamental metaphysics: is there a smallest constituent of matter that cannot be subdivided? The latter focused on the physical reality of specific class of particles' divisibility or indivisibility and then set out to explain that result in mechanistic terms thoroughly consistent with the findings of *a posteriori* methods. Further, these two disciplines meant entirely different things by the term atom. So, it does not seem to be a leap to suggest that the two paradigms are not only incommensurable but also in some sense incompatible. Thus, the step from these philosophical ponderings to scientific investigation *did* resemble religious conversion or a gestalt switch in a way that a transition between two scientific paradigms does not. Furthermore, it seems that this transition does not result in scientific progress. This is because such a shift is best seen as an assertion about the scope of science, specifically that it extends to the field of interest (atomic theory in our example). Thus, this truly is a 'conversion' event. Before the revolution occurred the topic was outside of the realm of science, afterwards it is within the scientific domain. Science, and scientific progress, can only begin after the field of

interest belongs to the scientific domain, i.e., after the initial paradigm is established in the revolution *to* science.

Consequently, let us call these revolutions *to* science that fall under our overarching theory of scientific revolution Type 1 revolutions.[22] Further, let us say that they are described by Kuhnian paradigm shifts that are consistent with his religious conversion metaphor. After this type of revolution occurs, there will be subsequent sub-paradigm shifts within science. These will result in the rejection of field specific sub-paradigms, but not the general scientific paradigm itself. While there will always be sub-paradigm dependent concerns in deciding between competing theories (thus leading to the need to premise the paradigm that Kuhn discusses), there will also always be the "'soft' values" inherent to the scientific paradigm "that *guide* choices" between sub-paradigms (Liston, n.d.). In essence, I am suggesting that science as a whole possesses sub-theory neutral criteria to adjudicate conflicts between two scientific paradigms.[23, 24] These overarching criteria reduce the extremity of revolutions *within* science compared to Type 1 revolutions and give science a degree of continuity on the whole. Tying this back to Kuhn's analogies, the revolutions associated with sub-paradigm shifts then will resemble macroevolution and the even smaller changes of normal science will resemble microevolution. I will call these revolutions Type 2 revolutions. I consider the revolution from Newtonian to relativistic mechanics to be a Type 2 revolution. While this was a major paradigm shift, it was in some sense just a rapid, but continuous progression within of normal science as a whole. I say this because, while relativity showed that Newtonian mechanics was incorrect for velocities near

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[22] I use this term to avoid relying on the phrase 'revolution *to* science,' which I equate with Hacking's interpretation of Kuhn and, consequently, is distinct from the modified view I am proposing.

[23] However, as Kuhn suggests these criteria will not be theory neutral. This is because they will be dependent on the paradigm of science itself, which is theory-laden.

[24] This idea is supported by Allan Franklin's discussion in "Are Paradigms Incommensurable" (Franklin, 1984).

the speed of light, it also showed that Newtonian mechanics was a limiting case of Eisteinian

mechanics. This gave this revolution a degree of continuity not present in a Type 1 revolution.

However, I do not think that Type 1 and Type 2 revolutions are mutually exclusive. In

fact, their compatibility allows us to explain the difficulties in analyzing Thomson's work in

light of Kuhn's comments on quantum theory. On the one hand, quantum theory stems from the

Type 1 revolution leading to the investigation of "unknown" subatomic phenomena (Kuhn,

1962/2012, 95). This accounts for the dramatic discontinuities in the quantum revolution.

However, Kuhn also suggests that quantum theory arose in response the need to find a

"substitute for classical mechanics," or a Type 2 revolution (Kuhn, 1962/2012, 89). Thus, the

quantum revolution was a composite Type 1-2 revolution—i.e., it was the product of both a

revolution *to* science and a revolution *within* science. So, if Kuhn intended to only discuss

revolution *within* science in *Structure*, it is unsurprising that at times he refers to the

development of quantum theory as a revolution (i.e., a revolution *within* science from classical to

quantum mechanics) and at others suggests it was merely the development of a new theory that

did not conflict with past theories (i.e., it was a revolution *to* science). Thus, based on this new

characterization, I similarly conclude that the period in atomic modeling from 18978-1914

contained one Type 1 (the development of Thomson's model) and one Type 2 revolution (the

shift from Thomson's model to Rutherford's nuclear atom).

My revision of Kuhn's account to include Type 1 and Type 2 revolutions has another

interesting philosophical implication: it suggests that realist, cumulative scientific progress might

be possible across Type 2 scientific revolutions. On my view, all Type 2 revolutions occur within

the scientific paradigm, which provides soft standards and guides all of science. Thus, even Type

2 revolutions are in some sense part of normal science as a whole, which on Kuhn's view should

progress in a relatively steady manner. While Type 1 revolutions will certainly disrupt this progress, on the whole, this modified view holds more hope for a realist conception of science than Kuhn's original theory. By realist scientific progress, I do not mean that science asymptotically approaches objective fixed physical reality.[25] Instead, I mean something along the lines of Peter Godfrey-Smith's scientific realism. This view holds that:

> we all inhabit a common reality, which has a structure that exists independently of what people think and say about it, except insofar as reality is comprised of thoughts [and that] one actual and reasonable aim of science is to give us accurate descriptions (and other representations) of what reality is like (Goddfrey-Smith, 2003, 176).

I believe that the general success of the scientific venture gives us reasons to believe science and adopt a realist perspective towards it. At the same time, I agree with Allan Franklin "that science is fallible and that it doesn't always yield truth about the world and its entities" (Franklin, 2001, 12). Thus, while we are justified in believe what science tells us about reality, we must always recognize that these beliefs can be proven wrong.

Furthermore, this move towards a realist account of scientific revolution is not motivated by a metaphysical agenda, although I will freely admit that I am inclined towards scientific realism. Instead, it is driven by a desire to make my revisions to Kuhn's theory responsive to the developments of this period of early atomic modeling. For instance, Bohr was very interested in clarifying the physical meaning of equation (7) in his theory (reproduced below).

$$(7) \; hv = W = \tau h \frac{\omega}{2},$$

After discharging his special assumptions, he emphasized that this equation did not mean that different stationary states (corresponding to different values of τ) corresponded to the emission of "different numbers of energy quanta" during the formation of the atom (Bohr, 1913a, 14).

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[25] This is but one example of the many versions of scientific realism that exists that are distinct from the realist view I develop here.

Instead he emphasized that this equation reflected that physical fact that the frequency of the energy emitted in the transition from a stationary state of the atom in which no energy has yet been emitted to another stationary state "is equal to different multiples of *ω/2*, where *ω* is the frequency of revolution of the electron in the state considered" (Bohr, 1913a, 14). This shift in his interpretation of this equation did not enhance the explanatory power of Bohr's model with respect to the atomic spectra, the formation of atoms, or any other phenomenon. Nor did it improve its ability to predict phenomena. So it served no antirealist or instrumentalist end. Rather it seems that Bohr, in making this comment, was entirely concerned with increasing the accuracy of his model's description of physical reality. Similarly, it appears that many of the critics of Bohr's atomic model rejected his theory because they thought (or hoped) it did not reflect reality. Lorentz suggested "the existence of quanta in the aether is impossible" and Otto Stern said, "If that nonsense is correct which Bohr has just published, then I will give up being a physicist" (Holton, 1986; Holton, 1993; both as cited in Niaz, 1998, 540). While it is perhaps unsurprising to find that these scientists had such realist tendencies, this does not discredit the notion that offering a realist account of scientific revolution is responsive to the progression of science (in the historical rather than revolutionary sense) during this period of atomic modeling.

Consequently, because my revised version of Kuhn's theory ties Type 2 revolutions to his evolutionary analogy, which has explicitly antirealist elements, I must also modify this analogy. The original analogy will not permit this potential notion of cumulative realist scientific progress across Type 2 revolutions. The issue is that Kuhn's evolutionary analogy identifies fixed physical reality with a teleological goal for the evolutionary process. However, this seems to conflate with general realist position with the more specific position that holds that science asymptotically approaches fixed physical reality—a view that many realists, including me, reject.

Instead, I believe that the general realist position holds that science can give us an accurate picture of reality, not because science approaches reality, but because it is *responsive* to reality. This realist position suggests that science can lead to general laws that describe fixed physical reality because reality constrains experimental results in a way that reflects the laws that describe its nature. If science is to be equated to the organisms in Kuhn's evolutionary analogy, then this position suggests that fixed physical reality will be a selection pressure, which helps determine which theories survive and which are rejected. This general characterization seems to be more accurate because it can also account for the more specific realist opinion I described above. If fixed physical reality were the only selection pressure, then it seems like it would be reasonable to think that our scientific descriptions and theories would asymptotically approach that fixed physical reality.

However, I do not think that fixed physical reality is the only selection pressure acting on our theories. As Kuhn suggests, there is a social, epistemic component. The community of scientists must accept a theory or it will be abandoned. Thus, communal epistemic justification plays an essential role in the transition between paradigms. As Kuhn indicates, scientists must find the arguments for a specific paradigm persuasive. This introduces human biases and errors to the paradigm selection process. It also makes the progression of science reliant on human ingenuity and argumentation. Because human senses are not suited to directly observe all scientific phenomena this also introduces the use of instrumentation and the need to consider instrumental errors. Scientists generally trust their instruments to certain a degree of confidence. Instrumental problems, inaccuracies, or limitations can lead to faulty scientific conclusions. Thus, scientific instrumentation introduces additional epistemic considerations as it affects the formation of scientific beliefs and the degree of justification for or confidence in those beliefs. In

the end, this epistemic selection pressure can lead to the acceptance of an incorrect paradigm in a way that the pressure of fixed physical reality cannot. This can account for the fallibility of science and the delays and breaks in cumulative scientific progress not due to Type 1 revolutions.

At the same time, the need to prove a theory to a community of scientists can help raise our confidence in our scientific beliefs. This is because the influence of one scientist's bias can be offset by other views. Additionally, one set of experimental results can be verified or contradicted by an experiment conducted by another scientist. As various scientists successive experiments agree, this provides better evidence for a theory. Ultimately, I believe that Kuhn became so focused on this epistemic selection pressure that he neglected the pressures of physical reality, which led to his antirealist conclusions. By introducing the additional selection pressure of fixed physical reality to Kuhn's theory, we can offer a realist, yet fallibilist, account of scientific revolution that includes many of the pivotal features of Kuhn's account, but also resolves the inconsistency that made it difficult to apply his theory to the development of the Thomson model of the atom.

This modification to Kuhn's evolutionary analogy reveals an interesting feature of scientific argument and justification evident in Rutherford's 1911 paper. As Norman Feather noted, despite the fact that Rutherford's analysis of small angle scattering was "the least convincing" part of his paper, Rutherford still implied that his nuclear model provided a better explanation for this phenomena than Thomson's model (Feather, 1963, Loc 436).[26] Conversely, by 1930, Rutherford acknowledged that "the experiments on multiple [or small angle] scattering have indeed led to no definite conclusions about atomic structure… the best that can be done is

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[26] Indeed, if anything, Rutherford showed that Thomson's data had better agreement with Crowther's data as he had to appeal to additional results obtained by Marsden to justify the agreement of his theory with Crowther's data (Rutherford, 1911, 680).

to show that the results are in accord with the nuclear theory" (Feather, 1963, Loc 436). This argument in Rutherford's 1911 paper highlights the epistemic pressures present in a Type 2 scientific revolution. Rutherford knew that his theory had better agreement with physical reality for large angle scattering experiments, so he presented that data first though he had conducted the experiments later. However, the same could not be said of the small angle results. Yet Rutherford knew that the scientific community accepted small angle scattering as compelling evidence in favor of Thomson's model. So, in order for his theory to be accepted, he had to convince the scientific community that his model could explain scattering in general. This led him to perform a calculation that made a persuasive claim but, at best, had a tenuous correspondence with physical reality. After his model had already been accepted and his theory was no longer subject to the same epistemic selection pressures he acknowledged that his argument did not agree with physical reality.

This is an example of the epistemic pressures tied to the need for communal acceptance of a theory leading to an argument that did not agree with physical reality. However, despite its inaccuracies, the argument Rutherford generated in response to this epistemic pressure reinforced the physically legitimate pressure of the large angle scattering results. Thus, the demands of this epistemic pressure led Rutherford to offer an inaccurate argument, which incidentally contributed to the justification of his physically accurate theory that each atom has a nucleus. While this did not impede scientific progress in this case, this points to the fallibility of a scientific process in which progress/revolution is responsive to such epistemic pressures. These epistemic pressures can lead to the acceptance of physically inaccurate theories in other situations. Consequently, errors due to such epistemic pressures prevent my realist view of science from progressing steadily or asymptotically towards reality. Yet, because science is

responsive to physical reality, we are justified in believing its claims about physical reality until they are proven wrong.

<u>Conclusion</u>

Based on my examination of the events leading to the development of the Rutherford-Bohr model of the atom, beginning with Thomson's discovery of the electron in 1897 and continuing through the publication of Bohr's trilogy, I conclude that this period demonstrated a Type 1 revolution *to* science (the development and acceptance of Thomson's model) and a Type 2 revolution *within* science (the switch to Rutherford's nuclear model). These classifications are based on my revisions to Kuhn's notion of scientific revolution intended to harmonize his theory with the events of this period. Further, I also conclude that my revised version of Kuhn's theory is compatible with a realist, though fallibilist, view of scientific progress. This view of scientific progress leads to the belief that scientific paradigms are subject to epistemic and physical selection pressures. The combination of these selection pressures entails that scientific theories/paradigms must be responsive to the justificatory demands of the scientific community and the pressures of fixed physical reality. This can lead scientists to offer different types of arguments for a paradigm that are responsive to these different selection pressures. Moreover, as Rutherford's 1911 paper on the nuclear model of the atom shows, this epistemic pressure can lead to physically inaccurate arguments and introduce errors to science.

Appendix A: Thomson's first measurement of m/e

In his first set of measurements of m/e, Thomson used the kinetic energy of the cathode ray particles, the total charge of all of the particles responsible for this heating, and the radius of deflection of the cathode ray in a uniform magnetic field to determine m/e for the particles in three different gasses: air, hydrogen, and carbonic acid. He determined the kinetic energy of the cathode ray by measuring the temperature increase of an object with a known thermal capacity after the cathode ray particles collided with the object. Thomson assumed that all of the kinetic energy from the cathode ray was converted to heat upon colliding with the solid body. He calculated the total charge of the cathode ray particles responsible for this heating by using an electrometer to measure the potential difference between two coaxial cylinders. The negatively charged cathode ray particles passed through the inner cylinder, giving it a negative electric potential, and this was compared to the electric potential of the outer, grounded and electrically isolated coaxial cylinder. Thomson had some issues keeping the cylinders electrically isolated from one another because of the conductive nature of the gasses with the cathode ray running through them. Thomson measured the current through the two Helmholtz coils used to generate the magnetic field to determine its magnitude and photographed the deflection of the cathode ray. He used this photograph to measure the radius of deflection. Because some of the charge from the cathode ray caused the tube itself to become charged and glow—which disturbed the measurement of the total charge of the cathode ray particles—Thomson repeated this experiment with three similar, but different tube setups. The first and third setups resembled the cathode ray tube in figure 1 (see page 5). The only differences were that the charged plates (E and D) were removed and the coaxial cylinders were placed at that location instead. The Helmholtz coils were positioned on opposite sides outside of the tube. The second tube configuration resembled that of figure 2 (see page 6) with coaxial cylinders added. This setup did not exhibit the charging/glowing issue. Because the second tube had to use sealing wax for some connections, this setup did not permit comparable exhaustion and the pressures were different between the two setups. The third tube configuration was very similar to the first, but the cathode ray was passed through a smaller slit so that fewer particles entered the tube. Thomson did not test all of the gases in the third tube. While the values of m/e varied between the tube configurations, the average ratios of m/e for chambers filled with air, hydrogen, and carbonic acid were comparable for the two configurations in which all of the gases were tested. I reproduce Thomson's results for these three tube configurations at the end of this appendix. Thomson used the following equation to compute m/e from these measurements:

$$(A1) \quad \frac{m}{e} = \frac{QI^2}{2KE}$$

Q is the total charge of the cathode ray particles, KE is the total kinetic energy of these particles, and I is the product of the radius of deflection of the cathode ray (r) and the magnitude of the magnetic field (B) responsible for that deflection (Thomson, 1897). While Thomson gives the initial equations needed to derive equation 1 (equations 2,3, and 6), he does not actually show the derivation in his paper. I do so here.

Thomson started with the equation for the total charge of the particles from the cathode ray:

$$(A2) \quad Ne = Q,$$

where N is the total number of particles and e is the charge on the particles. Moreover he knew that the total kinetic energy of these N particles was:

$$(A3)\ KE = \frac{1}{2}Nmv^2,$$

where m is the mass of the particle and v is the particles velocity. Further, the force on a particle from a perpendicular uniform magnetic field:

$$(A4)\ F_B = e\vec{v}\times\vec{B} = evB\sin\theta = evB,$$

Given Newton's second law and the fact that a moving charge acted upon by a perpendicular magnetic field will undergo uniform circular motion, we know:

$$(A5)\ F_B = ma = m\left(\frac{v^2}{r}\right) = evB,$$

This expression can be rearranged to give the third equation Thomson presents in his paper:

$$(A6)\ \frac{mv}{e} = Br = I,$$

where I is a convenient abbreviation for the product Br. In order to obtain equation A1 we start by dividing equation A3 by equation A2, which gives:

$$(A7)\ \frac{KE}{Q} = \frac{\frac{1}{2}Nmv^2}{Ne} = \frac{1}{2}\frac{m}{e}v^2,$$

We then substitute equation A6 into the right hand side of this result, which yields:

$$(A8)\ \frac{KE}{Q} = \frac{1}{2}\left(\frac{mv}{e}\right)v = \frac{1}{2}(Br)v = \frac{1}{2}Iv,$$

Solving this expression for v we obtain:

$$(A9)\ v = \frac{2KE}{QI},$$

Substituting this result into equation A7 gives:

$$(A8)\ \frac{KE}{Q} = \frac{1}{2}\frac{m}{e}\left(\frac{2KE}{QI}\right)^2$$

Finally, solving this expression for m/e gives us equation A1:

$$(A1)\ \frac{m}{e} = \frac{QI^2}{2KE}$$

The results of Thomson's experiments using this method are summarized in table 1 on the next page:

Table 1 (Adapted from Thomson 1897 p. 306)

Gas	Value of KE/Q	Value of I (=Br)	Value of m/e	Value of v
Tube 1				
Air	4.6E+11	230	5.7E-08	4.E+09
Air	1.8E+12	350	3.4E-08	1.E+09
Air	6.1E+11	230	4.3E-08	5.4E+09
Air	2.5E+12	400	3.2E-08	1.2E+10
Air	5.5E+11	230	4.8E-08	4.8E+09
Air	1.E+12	285	4.E-08	7.E+08
Air	1.E+12	285	4.E-08	7.E+09
Hydrogen	6.E+12	205	3.5E-08	6.E+09
Hydrogen	2.1E+12	460	5.E-08	9.2E+09
Carbonic Acid	8.4E+11	260	4.E-08	7.5E+09
Carbonic Acid	1.47E+12	340	4.E-08	8.5E+09
Carbonic Acid	3.0E+12	480	3.9E-08	1.3E+10
Tube 2				
Air	2.8E+11	175	5.3E-08	3.3E+09
Air	4.4E+11	195	4.7E-08	4.1E+09
Air	3.5E+11	181	4.7E-08	3.8E+09
Hydrogen	2.8E+11	175	5.3E-08	3.3E+09
Air	2.5E+11	160	5.1E-08	3.1E+09
Carbonic Acid	2.E+11	148	5.4E-08	2.5E+09
Air	1.8E+11	151	6.3E-08	2.3E+09
Hydrogen	2.8E+11	175	5.3E-08	3.3E+09
Hydrogen	4.4E+11	201	4.6E-08	4.4E+09
Air	2.5E+11	176	6.1E-08	2.8E+09
Air	4.2E+11	200	4.8E-08	4.1E+09
Tube 3				
Air	2.5E+11	220	9.E-08	2.4E+09
Air	3.5E+11	225	7.E-08	3.2E+09
Hydrogen	3.E+11	250	1.0E-07	2.5E+00

Appendix B: Thomson's second measurement of m/e

Thomson used a similar setup to that shown in Figure 1 to compare the deflection of the cathode ray due to an electric field with its deflection due to a magnetic field. These fields were setup so that the deflection due to each would be perpendicular to the cathode rays undisturbed path and velocity of travel (v). The only difference is that two coils were placed outside of the tube with diameters equal to the length, l, of the plates (see D and E in figure 1) used to generate the electric field. The coils were placed so as to cover the area produced by the plates and create a uniform magnetic field in the same region as the electric field. The strength of the magnetic field was determined by measuring the current through the coils. The deflection due to the electric field was first determined in by measuring the change in position of the phosphorescent patch produced by the cathode ray at the end of the tube. After verifying that the deflection of the cathode ray was proportional to the strength of the electric field, Thomson adjusted the magnetic field so that it had a nonzero magnitude but produced no deflection (ϕ=0). Then, he measured the electrostatic deflection, θ, and used this along with the magnitude of the E-field and B-field to determine m/e according to the following equation:

$$(B1) \quad \frac{m}{e} = \frac{B^2 l}{E\theta},$$

Thomson arrived at this equation by obtaining equations for the angles of deflection due to the magnetic and the electric fields. He found equations for these angles based on a comparison between the particles' deflection velocity, v_E and v_B, and the cathode rays particles' undisturbed velocity, v. Importantly, v was perpendicular to both v_E and v_B. I start by deriving the equation for the deflection angle, θ, due to the electric field. Thomson derived the deflection velocity due to the electric field from the acceleration of the particles, a_e, due to the constant electrostatic force in the region between the charged plates and the time, t, it took the particle to traverse the length, $l,$ of the electric field:

$$(B2) \quad v_E = \Delta v_\perp = a_e t = \left(\frac{Ee}{m}\right)\left(\frac{l}{v}\right),$$

E is the magnitude of the electric field, e is the charge of the cathode ray particles, m is the particles mass, and v is the undisturbed velocity of the cathode ray particles. Thomson then used trigonometry to find the deflection angle, θ, from the perpendicular particle velocities, v and v_E.

$$(B3) \quad \tan \theta = \frac{v_E}{v} = \frac{\left(\frac{Ee}{m}\right)\left(\frac{l}{v}\right)}{v} = \frac{Eel}{mv^2},$$

Given that the fields were set so as to give small angles of deflection that would be measureable with an apparatus similar to that shown in figure 1, we can use the small angle approximation for tangent to give:

$$(B4) \quad \theta = \frac{Eel}{mv^2},$$

We can undertake a similar derivation of the angle of deflection, ϕ, due to the magnetic field, B.

$$(B5) \quad v_B = \Delta v_\perp = at\left(\frac{Bev}{m}\right)\left(\frac{l}{v}\right) = \left(\frac{Be}{m}\right)l,$$

Again, using trigonometry we find:

$$(B6) \quad \tan \phi = \frac{v_B}{v} = \frac{\left(\frac{Be}{m}\right)l}{v} = \frac{Bel}{mv},$$

And by the small angle approximation:

$$(B7) \quad \phi = \frac{Bel}{mv},$$

In order to find the undisturbed velocity of the cathode ray and calculate m/e we divide equation B7 by equation B4:

$$(B8) \quad \frac{\phi}{\theta} = \frac{\frac{Bel}{mv}}{\frac{Eel}{mv^2}} = \frac{Bv}{E},$$

Solving for v this gives:

$$(B9) \quad v = \frac{\phi}{\theta}\frac{E}{B},$$

Then we substitute this solution for *v* in equation B7 to obtain:

$$(B10)\ \phi = \frac{Bel}{mv} = \frac{Bel}{m\left(\frac{\phi}{\theta}\frac{E}{B}\right)} = \frac{e}{m}\frac{B^2 l\theta}{E\phi},$$

Solving for m/e gives:

$$(B11)\ \frac{m}{e} = \frac{B^2 l\theta}{E\phi^2},$$

However, Thomson configured the experiment so that *ϕ=0*. Thus, we can use a Maclaurin approximation for *v(ϕ/θ)* from equation B7. This yields:

$$(B12)\ v = \frac{E}{B},$$

Substituting this result into equation B4 we find:

$$(B13)\ \theta = \frac{Eel}{mv^2} = \frac{Eel}{m\left(\frac{E}{B}\right)^2} = \frac{e}{m}\frac{B^2 l}{E},$$

Solving for m/e we finally obtain equation B1:

$$(B1)\ \frac{m}{e} = \frac{B^2 l}{E\theta},$$

The results of this experiment are summarized in table 2 below:

Table 2 (Adapted from Thomson 1897 p. 309)

Gas	*θ*	*B*	*E*	*l*	*m/e*	*v*
Air	8/110	5.5	1.5E+10	5	1.3E-07	2.8E+09
Air	9.5/110	5.4	1.5E+10	5	1.1E-07	2.8E+09
Air	13/110	6.6	1.5E+10	5	1.2E-07	2.3E+09
Hydrogen	9/110	6.3	1.5E+10	5	1.5E-07	2.5E+09
Carbonic Acid	11/110	6.9	1.5E+10	5	1.5E-07	2.2E+09
Air	6/110	5	1.8E+10	5	1.3E-07	3.6E+09
Air	7/110	3.6	1.E+10	5	1.1E-07	2.8E+09

Thomson emphasized that while this method was easier and "probably more accurate" than his first method for determining m/e, it could not be used over as wide of a range of pressures of the gases (Thomson, 1897, p. 310). Though he does not say so, it is likely that these pressure issues contributed in part to the low number of reported results for carbonic acid and hydrogen.

Appendix C: The Mathematics of Thomson's Revised Model

Though Thomson just gives the equation, we can find the force on a single electron due to the sphere of positive charge using Gauss's law. The circle of electrons and the sphere of positive charge both share the same center point. The radius of the circle of electrons is a and the radius of the sphere is b. The total charge of the positive sphere is ve. Thus Gauss's law suggests that the magnitude of the electric field is given by:

$$(C1)\ E(4\pi a^2) = \frac{ve\,\frac{4}{3}\pi a^3}{\varepsilon_0\,\frac{4}{3}\pi b^3} = \frac{ve}{\varepsilon_0}\frac{a^3}{b^3},$$

Solving for E this gives:

$$(C2)\ E = \frac{ve}{\varepsilon_0}\frac{a^3}{b^3}\frac{1}{(4\pi a^2)} = \frac{vea}{4\pi\varepsilon_0 b^3},$$

Using the electric field we find the magnitude of the attractive force:

$$(C3)\ F_{attractive} = Eq = \frac{1}{4\pi\varepsilon_0}\frac{ve^2a}{b^3}$$

For the repellent force we can start by considering the simplest case where n=2 and the electrons are on opposite sides of the circle. The Coulomb force is:

$$(C4)\ F_{repellent} = \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{(2a)^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4a^2}$$

For n charges the geometry will become more complicated and we must multiple equation C4 by:

$$(C5)\ S_n = \csc\frac{\pi}{n} + \csc\frac{2\pi}{n} + \ldots + \csc\frac{(n-1)\pi}{n},$$

Such that equation C4 becomes:

$$(C6)\ F_{repellent} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4a^2}S_n$$

In order for this configuration to be stable the radially attractive force (equation C3) must be equal in magnitude to this radial repellent force (equation C6):

$$(C7)\ \frac{1}{4\pi\varepsilon_0}\frac{ve^2a}{b^3} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4a^2}S_n,$$

Simplifying we obtain the stability equation for n stationary electrons in the sphere of uniform positive charge.

$$(C8)\ \frac{ve^2a}{b^3} = \frac{e^2}{4a^2}S_n$$

Thomson solved this equation to find the ratio a/b for n=2 through n=6:

n	2	3	4	5	6
a/b	.5	.5773	.6208	.6505	.6726

He then modified equation C8 to account for a rotating ring of electrons. In order to achieve stable uniform circular motion Newton's second law requires:

$$(C9)\ F_{net} = ma_{circular} = ma\omega^2 = F_{attractive} - F_{repellent}$$

where $a_{circular}$ is the acceleration of the circular motion, m is the mass of the electron, and ω is the angular velocity of the rotating ring/charges. Rearranging this expression, Thomson gives the stability equation for a rotating ring:

$$(C10)\ \frac{ve^2 a}{b^3} = ma\omega^2 + \frac{e^2}{4a^2} S_n$$

He then devotes the next several pages to the mathematics required to modifying this expression to find the stability conditions for an electron displaced from its equilibrium position in a rotating ring. The resultant equation is:

$$(C11)\left(\frac{3}{4}\frac{e^2}{a^3}S_k + L_k - L_0 - mq^2\right)(N_0 - N_k - mq^2) = (M_k - 2m\omega q)^2$$

Stability of this configuration requires that the value of ω be chosen such that, q^2, is always positive and the values of q are all real. Equation C11 is written in terms of other equations Thomson previously derived:

$$L_k = \frac{e^2}{8a^3}\left(\cos\frac{2k\pi}{n}\left(\frac{1}{\sin\frac{\pi}{n}} + \frac{1}{\sin^3\frac{\pi}{n}}\right) + \cos\frac{4k\pi}{n}\left(\frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin^3\frac{2\pi}{n}}\right) + \cos\frac{6k\pi}{n}\left(\frac{1}{\sin\frac{3\pi}{n}} + \frac{1}{\sin^3\frac{3\pi}{n}}\right) + ...\right),$$

$$N_k = \frac{e^2}{4a^3}\left(\cos\frac{2k\pi}{n}\frac{\cos\frac{\pi}{n}}{\sin^2\frac{\pi}{n}}\left(\cot\frac{\pi}{n} + \frac{1}{2}\tan\frac{\pi}{n}\right) + \cos\frac{4k\pi}{n}\frac{\cos\frac{2\pi}{n}}{\sin^2\frac{2\pi}{n}}\left(\cot\frac{2\pi}{n} + \frac{1}{2}\tan\frac{2\pi}{n}\right) + ...\right),$$

$$M_k = \frac{ne^2}{8a^3}\left(\sin\frac{2k\pi}{n}\frac{\cos\frac{\pi}{n}}{\sin^2\frac{\pi}{n}} + \sin\frac{4k\pi}{n}\frac{\cos\frac{2\pi}{n}}{\sin^2\frac{2\pi}{n}} + \sin\frac{6k\pi}{n}\frac{\cos\frac{3\pi}{n}}{\sin^2\frac{3\pi}{n}} + ...\right),$$

where k= 0, 1, 2,…, (n-1)

Thomson's solutions to this equation indicated that rings with more than six electrons required that additional electrons be placed in the center. From these results, he used a graphical method to determine the configurations according to this model for n ≤ 100. He published all 100 configurations in his 1907 book. I reproduce this table on the next page:

Table 3 Configurations of n Electrons in an Atom (adapted from Thomson, 1907, 109-110)

n	C	n	C	n	C	n	C	n	C
1	1	21	1.8.12	41	3.9.13.16	61	3.9.13.16.20	81	2.8.13.16.20.22
2	2	22	2.8.12	42	3.9.13.17	62	3.9.13.17.20	82	3.8.13.16.20.22
3	3	23	2.8.13	43	3.10.13.17	63	3.10.13.17.20	83	3.9.13.16.20.22
4	4	24	3.9.13	44	4.10.13.17	64	4.10.13.17.20	84	3.9.13.17.20.22
5	5	25	3.9.13	45	4.10.14.17	65	4.10.14.17.20	85	3.10.13.17.20.22
6	1.5	26	3.10.13	46	5.10.14.17	66	5.10.14.17.20	86	3.10.13.17.20.23
7	1.6	27	4.10.13	47	5.10.15.17	67	5.10.15.17.20	87	4.10.13.17.20.23
8	1.7	28	4.10.14	48	5.11.15.17	68	5.10.15.17.21	88	4.10.14.17.20.23
9	1.8	29	5.10.14	49	1.5.11.15.17	69	5.11.15.17.21	89	5.10.14.17.20.23
10	2.8	30	5.10.15	50	1.5.11.15.18	70	1.5.11.15.17.21	90	5.10.15.17.20.23
11	3.8	31	5.11.15	51	1.6.11.15.18	71	1.5.11.15.18.21	91	5.10.15.17.21.23
12	3.9	32	1.5.11.15	52	1.7.11.15.18	72	1.6.11.15.18.21	92	5.11.15.17.21.23
13	3.10	33	1.6.11.15	53	1.7.11.16.18	73	1.7.11.15.18.21	93	5.11.15.17.21.24
14	4.10	34	1.7.11.15	54	1.7.12.16.18	74	1.7.11.16.18.21	94	1.5.11.15.17.21.24
15	5.10	35	1.7.11.16	55	1.7.12.16.19	75	1.7.12.16.18.21	95	1.5.11.15.18.21.24
16	5.11	36	1.7.12.16	56	1.8.12.16.19	76	1.7.12.16.19.21	96	1.6.11.15.18.21.24
17	1.5.11	37	1.8.12.16	57	2.8.12.16.19	77	1.8.12.16.19.21	97	1.7.11.15.18.21.24
18	1.6.11	38	2.8.12.16	58	2.8.13.16.19	78	1.8.12.16.19.22	98	1.7.11.16.18.21.24
19	1.7.11	39	2.8.13.16	59	2.8.13.16.20	79	2.8.12.16.29.22	99	1.7.12.16.18.21.24
20	1.7.12	40	3.8.13.16	60	3.8.13.16.20	80	2.8.13.16.19.22	100	1.7.12.16.19.21.24

Appendix D: The Mathematics of Rutherford's Nuclear Model

Rutherford performed a number of calculations pertaining to the large angle scattering of an α-particle due to a single interaction with a highly concentrated central charge. Because this central charge was so concentrated, he suggested it could be treated as a point charge. While Rutherford was not wedded to the idea that the nucleus had be positively charged and the surrounding corpuscle charge be negative, he performed his calculations treating the central a positive charge positive and the surrounding charge as negative. He initially ignored the proposed corpuscular configuration of the other charges, treating it as a uniform field outside the nucleus (later he would show that even if this charge was arranged as electrons the central force was so great that this effect could be ignored. He began by using the electric force, *X*, and the electric potential, *V*, at a distance, *r*, from the center of an atom with a highly concentrated (point) positive central charge, *Ne*, surround by a uniform distribution of negative electricity, *Ne*, arranged in a sphere of radius, *R*, to find the closest distance an α-particle could approach relative to the central point charge.

He found the electric field to be:

$$(D1)\ X = Ne\left(\frac{1}{r^2} - \frac{r}{R^3}\right),$$

where the left term inside the parentheses is due to the central charge (which can be obtained with the electrostatic equation for the E-field of a point charge) and the right term is due to the uniformly distributed negative charge (this can be obtained using Gauss's law). Similarly, he found the electric potential to be:

$$(D2)\ V = Ne\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^3}{3R^3}\right),$$

Appealing to conservation of energy, he then suggested that all of the kinetic energy of an α-particle with mass m and charge E that was shot at the center of this atom with velocity u would be converted to potential energy when the α-particle obtained its closest position (distance b) to the center of atom. Thus:

$$(D3)\ KE_i = \frac{1}{2}mu^2 = PE_{at\ b} = VE = NeE\left(\frac{1}{b} - \frac{3}{2R} + \frac{b^3}{3R^3}\right)$$

Using experimentally determined values for m, u, and E and assuming $Ne=100e$ (where e is the charge of an electron), Rutherford found $b=3.4\text{x}10^{-12}$ cm for gold or platinum.

Having argued that he only needed to consider the effects of the central charge for large angle scattering, he then calculated the deflection angle of the α-particle. He considered the physical situation depicted in the diagram below. In it, an α-particle enters an atom initially travelling along the trajectory from P to O and then, after being deflected by the central charge located at position S, exits the atom along the path from O to P'. Given that this interaction was governed by a the Coulomb force due to the central charge (assumed to be a point charge), which falls off at a rate inversely proportionate to the square of the distance, Rutherford determined that the particle would undergo a hyperbolic 'orbit.' Drawing a line from the central charge, S, to the apse of this hyperbola, A, Rutherford defined the angle formed by POA to be θ.
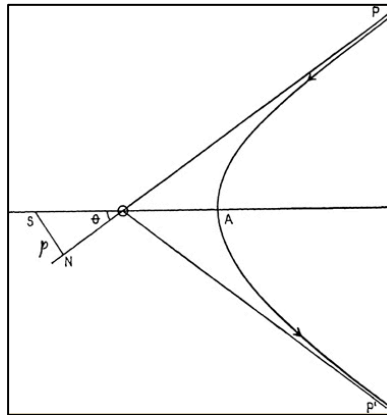
If the α-particle initially has velocity, V, Rutherford used angular momentum to determine that the α-particle's velocity at A, v, could be given by:

$$(D4)\ pV = \overline{SA}v$$

Here p is the impact parameter, which Rutherford defined as the shortest perpendicular distance between the projection of the undisturbed incident path (OP) of the particle and the center of the

atom, S. $\overline{SA} = |\overrightarrow{SA}|$, the distance from the center of the atom to apse of the α-particle's trajectory. Conservation of energy then gives:

$$(D5) \; \frac{1}{2}mV^2 = \frac{1}{2}mv^2 + \frac{NeE}{\overline{SA}},$$

which can be rewritten as:

$$(D6) \; \frac{1}{2}mv^2 = \frac{1}{2}mV^2\left(1 - \frac{\frac{NeE}{\overline{SA}}}{\frac{1}{2}mV^2}\right) = \frac{1}{2}mV^2\left(1 - \left(\frac{NeE}{\frac{1}{2}mV^2}\right)\frac{1}{\overline{SA}}\right)$$

Using equation D3, we can rewrite equation D6 as:

$$(D7) \; \frac{1}{2}mv^2 = \frac{1}{2}mV^2\left(1 - \frac{b}{\overline{SA}}\right),$$

Solve it for v:

$$(D8) \; v^2 = V^2(1 - \frac{b}{\overline{SA}}),$$

which be written as:

$$(D9) \; \frac{v^2}{V^2} = \left(1 - \frac{b}{\overline{SA}}\right),$$

rearranging equation D4, we obtain:

$$(D10) \; \frac{p}{\overline{SA}} = \frac{v}{V},$$

and combining equations D9 and D10 gives:

$$(D11) \; \frac{v^2}{V^2} = \left(1 - \frac{b}{\overline{SA}}\right) = \frac{p^2}{\overline{SA}^2}$$

after rearranging this produces:

$$(D12) \; p^2 = (\overline{SA} - b)\overline{SA}$$

Considering the geometry of this situation, Rutherford saw that:

$$(D13) \; \overline{SA} = \overline{SO} + \overline{OA} = p\csc(\theta) + p\cot(\theta) = p(\csc(\theta) + \cot(\theta)) = p\cot\left(\frac{\phi}{2}\right),$$

where $\phi = 2\theta$ and is the angle through which the α-particle is deflected. Substituting this result into equation 12 gives:

$$(D14) \; p^2 = p\cot\left(\frac{\phi}{2}\right)\left(p\cot\left(\frac{\phi}{2}\right) - b\right)$$

Solving this expression for b gives:

$$(D15) \; b = \frac{p^2\left(\cot^2\frac{\phi}{2} - 1\right)}{p\cot\left(\frac{\phi}{2}\right)} = p\left(\cot\left(\frac{\phi}{2}\right) - \tan\left(\frac{\phi}{2}\right)\right) = 2p\cot(\phi)$$

Thus we arrive at the equation (4) in the main text:

$$(D16) \; \cot\left(\frac{\phi}{2}\right) = \frac{2p}{b}$$

Rutherford then used this result to predict the number of α-particles, y, that would be observed in a unit area located a distance r from the point of incidence between the α-particles and the scattering foil.

He started by determining the probability, D, for an α-particle to enter an atom and penetrate to a distance from the center of the atom equivalent to the atom's impact parameter, p:

$$(D17)\ D = \pi p^2 nt,$$

where πp^2 is the cross sectional area of the spherical region distance p from the center of the atom, n is the number of atoms per unit volume for the material being used as a scattering foil, t is the thickness of the foil, and thus nt is the number of atoms per unit area for the foil material. Taking the derivative of this expression we find:

$$(D17)\ \frac{dD}{dp} = 2\pi p nt$$

Thus the probability dD of the α-particle striking the atom at a location between p and dp is:

$$(D19)\ dD = 2\pi p nt\, dp$$

We begin to rewrite this probability in terms of the scattering angle ϕ. This gives the probability dD of α-particle being deflected at scattering angle ϕ. We start by rearranging and then differentiating equation D16 to give:

$$(D21)\ p = \frac{b}{2}\cot\left(\frac{\phi}{2}\right),$$

$$(D22)\ \frac{dp}{d\phi} = \frac{b}{4}\csc^2\left(\frac{\phi}{2}\right),$$

$$(D23)\ dp = \frac{b}{4}\csc^2\left(\frac{\phi}{2}\right)d\phi,$$

Substituting equations D21 and D23 into equation D19 gives:

$$(D24)\ dD = 2\pi nt \left(\frac{b}{2}\cot\left(\frac{\phi}{2}\right)\right)\left(\frac{b}{4}\csc^2\left(\frac{\phi}{2}\right)d\phi\right) = \frac{\pi}{4}ntb^2 \cot\left(\frac{\phi}{2}\right)\csc^2\left(\frac{\phi}{2}\right)d\phi$$

Rutherford used this result to find the number of α-particles, y, turned through a particular angle, ϕ, such that they are observed striking the scintillation screen at a distance, r, away from the point where the beam of α-particles strikes the reflector. The unit area on the scintillation screen where these particles fall is given by:

$$(D25)\ A_s = C_s d\rho_s = 2\pi \rho_s d\rho_s = 2\pi(r\sin\phi)rd\phi = 2\pi r^2 \sin(\phi)d\phi$$

where C_s is the circumference of the circle in which the α-particles deflected through angle, ϕ, strike the scintillation screen and ρ_s is the radius of this circle. Thus the total number of particles falling in this area is given by:

$$(D26)\ y2\pi r^2 \sin(\phi)d\phi = QdD$$

where Q is the total number of α-particles that strike the reflector. Solving this expression for y gives:

$$(D27)\ y = \frac{QdD}{2\pi r^2 \sin(\phi)d\phi} = \frac{Q\left(\frac{\pi}{4}ntb^2 \cot\left(\frac{\phi}{2}\right)\csc^2\left(\frac{\phi}{2}\right)d\phi\right)}{2\pi r^2 \sin(\phi)d\phi}$$

Using the double angle formula for sine gives:

$$(D27) \ y = \frac{Q\left(\frac{\pi}{4}ntb^2 \cot\left(\frac{\phi}{2}\right)\csc^2\left(\frac{\phi}{2}\right)d\phi\right)}{4\pi r^2 \sin\left(\frac{\phi}{2}\right)\cos\frac{\phi}{2}d\phi} = \frac{ntb^2Q}{16r^2 \sin^4\left(\frac{\phi}{2}\right)}$$

Rewriting this result we obtain equation (5) in the main text, Rutherford's scattering formula:

$$(D28) \ y = \frac{ntb^2Q \csc^4\left(\frac{\phi}{2}\right)}{16r^2} \ or \ given \ b = \frac{2NeE}{mu^2}, y = \frac{4nt(Ne)^2E^2Q \csc^4\left(\frac{\phi}{2}\right)}{16r^2m^2u^4}$$

Appendix E: Experimental Verification of Rutherford's Large Angle Scattering Formula

In his 1911 paper, Rutherford derived the following equation for the number of α-particles, y, that would be observed a distance r from the point of incidence between the α-particles and the scattering foil:

$$(E1) \ y = \frac{4nt(Ne)^2E^2Q \csc^4\left(\frac{\phi}{2}\right)}{16r^2m^2u^4}$$

This equation led to the four crucial predictions of Rutherford's model.

The predicted that the observed frequency of α-particle scintillations at a given distance, r, from the center of incidence on the reflector would:
1. Vary according to $\csc^4(\phi/2)$.
2. Be proportional to the thickness of the scattering foil.
3. Be proportional to the square of the atomic weight given that the magnitude of the central charge, Ne, was proportional to the atomic weight.
4. Be inversely proportional the fourth power of velocity, u.

Geiger and Marsden tested and confirmed all four predictions. The results are produced below.

1. First, they showed that the distribution of scintillations for large angle scattering did vary according to $csc^4(\phi/2)$. The counted scintillations at various large scattering angles (and thus distances from the center of incidence) and then compared the variation in these frequencies for various values of ϕ to $csc^4(\phi/2)$ by multiplying the number of observed scintillations, N, by $csc^4(\phi/2)$. Their results are produced in the table on the next page:

Table 4: Taken from (Geiger, 1913, 610)

I.	II.	III. SILVER	IV.	V. GOLD	VI.
Angle of deflexion, ϕ	$\dfrac{1}{\sin^4\phi/2}$	Number of scintilla-tions, N	$\dfrac{N}{\sin^4\phi/2}$	Number of scintilla-tions, N	$\dfrac{N}{\sin^4\phi/2}$
150	1.15	22.2	19.3	33.1	28.8
135	1.38	27.4	19.8	43.0	31.2
120	1.79	33.0	18.4	51.9	29.0
105	2.53	47.3	18.7	69.5	27.5
75	7.25	136	18.8	211	29.1
60	16.0	320	20.0	477	29.8
45	46.6	989	21.2	1435	30.8
37.5	93.7	1760	18.8	3300	35.3
30	223	5260	23.6	7800	35.0
22.5	690	20300	29.4	27300	39.6
15	3445	105400	30.6	13200	38.4
30	223	5.3	0.024	3.1	0.014
22.5	690	16.6	0.024	8.4	0.012
15	3445	93.0	0.027	48.2	0.014
10	17330	508	0.029	200	0.0115
7.5	54650	1710	0.031	607	0.011
5	276300			3320	0.012

This shows that the value of $N\csc^4(\phi/2)$ was "approximately constant" for all ϕ, which confirmed the first prediction in Rutherford's scattering formula (Geiger, 1913, 610-611). However, Geiger noted that the value of $N\csc^4(\phi/2)$ began to increase once ϕ became sufficiently small, around 37.5 degrees in the table above. Given that this prediction was for large angle scattering, this did not seem to be too problematic.

2. Next, Geiger and Marsden compared the frequency of large angle scattering events at particular distances to the thickness of the foil. They did this by looking at the number of scintillations, N, at a particular distance from the center of incidence for various thicknesses, T, of three materials: gold, copper, and aluminum. They reported their results for gold, reproduced in the table below:

Table 5: Taken from (Geiger, 1913, 616)

I. Number of Foils.	II. Air equivalent. T in cm	III. Number N of scintillations per minute	IV. Ratio $\frac{N}{T}$
1	0.11	21.9	200
2	0.22	38.4	175
5	0.51	84.3	165
8	0.81	121.5	150
9	0.90	145	160

The provided a graph of *N* versus *T* for all three metals:

Graph 1 Taken from (Geiger, 1913, 617)
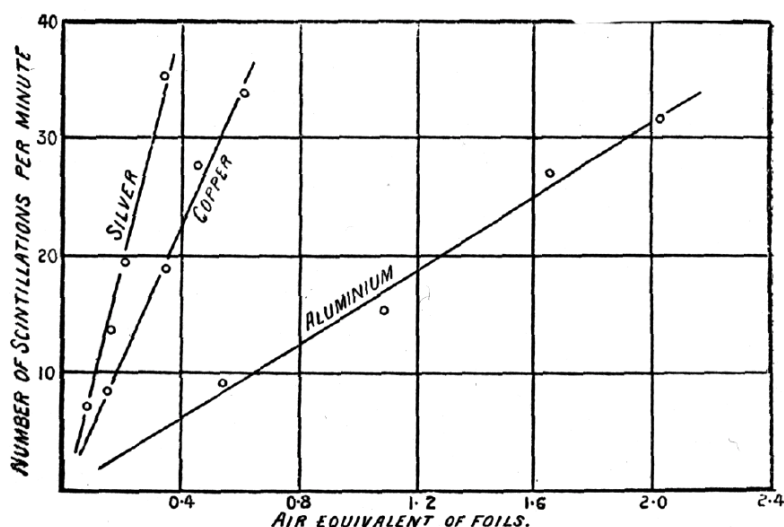
Thus, Geiger and Marsden confirmed Rutherford's second prediction by "prov[ing] that for small thickness of matter the scattering is proportional to the thickness" (Geiger, 1913, 616).

3. Then, looking at the number of scintillations, *N*, at a particular distance from the center of incidence of α-particles with the reflector for foils made of various materials, they obtained results "that indicate[d] the essential correctness of the [third] assumption that the scattering per atom is proportional to the square of the atomic weight," A (Geiger, 1913, 623). However as the table below shows, they actually found N to vary more proportionally with $A^{3/2}$.

Table 6: Taken from (Geiger, 1913, 622)

I.	II.	III.	IV.	V.	VI.
Substance	Air equivalents of foils used	Total number of scintillations counted for each substance.	Number N of scintillations at same angle and for same air equivalent	$A^{3/2}$	$N \times A^{2/3}$
Gold	0.52 0.68	1200	2400	2770	0.85
Platinum	0.54, 0.625	1000	2900	2730	1.05
Tin	0.51, 1.15	1400	1290	1300	0.99
Silver	0.38, 0.435	600	1060	1120	0.95
Copper	0.495, 0.61	1300	570	507	1.12
Aluminium	0.45, 0.52, 1.06	1600	151	144	1.05
Carbon	0.55, 0.57	400	57	41.6	1.37

However, Geiger explained in a note underneath this table that these results relied Bragg's experimental findings to determine the air equivalent to the thickness of the foil. After Geiger

and Marsden had already calculated these results based on Bragg's law, another member of Rutherford's lab, Richardson, used a new scintillation method to show that Bragg's air equivalents were off by a factor of $A^{1/2}$. Using Richardson's results, Geiger showed that the values of N/A^2 (essentially an improved column VI) were: "Au 3.1, Pt 3.4, Sn 3.4, Ag 3.4, Cu 3.95, and Al 3.4" (Geiger, 1913, 621-622). This modification improved their results and confirmed the third prediction derived from Rutherford's scattering formula: N varied proportionately with A^2.

4. Finally, Geiger and Marsden passed the α-particles through used sheets of mica before allowing them to collide with the reflector. This allowed them to slow the particles and observe the impact of the particles speed, v, on the number of scintillations observed at a particular distance from the particles' center of incidence with the reflector. By multiplying the number of observed scintillations by the relative value of v^4 associated with the number of mica sheets introduced before the α-particles reached the deflector, they successfully showed that "in every case the scattering was found to vary at a rate more nearly proportional to the inverse fourth power of the velocity than to any other integral power" (Geiger, 1913, 625). I produce their results in the table below:

Table 7: Taken from (Geiger, 1913, 624)

I.	II.	III.	IV.	V.
Number of sheets of mica	Range R of α particles after leaving mica	Relative values of $1/v^4$	Number N of scintillations per minute	Nv^4
0	5.5	1.0	24.7	25
1	4.76	1.21	29.0	24
2	4.05	1.50	33.4	22
3	3.32	1.91	44	23
4	2.51	2.84	81	28
5	1.84	4.32	101	23
6	1.04	9.22	255	28

Appendix F: Bohr's Derivation of the Rydberg Equation for Hydrogen

Bohr derived the Rydberg equation for hydrogen, beginning with the result for W found in equation (8) in the main text:

$$(F1) \ W = \frac{2\pi^2 m e^2 E^2}{\tau^2 h^2}$$

Setting the charge of the nucleus equal to the charge of one electron (E=e) this becomes:

$$(F2) \ W = \frac{2\pi^2 me^4}{\tau^2 h^2}$$

Since W corresponded to the energy radiated out during the formation of a stable state, Bohr suggested that the energy emitted in the transition from the stationary state corresponding to $\tau=\tau_1$ to the stationary state corresponding to $\tau=\tau_1$ would be:

$$(F3) \ W_{\tau_2} - W_{\tau_1} = \frac{2\pi^2 me^4}{\tau_2^2 h^2} - \frac{2\pi^2 me^4}{\tau_1^2 h^2} = \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)$$

From this, if we assume that the radiation emitted during this transition between states is homogenous (as is observed) and use Planck's theory that the energy emitted is equal to hv, we obtain:

$$(F4) \ W_{\tau_2} - W_{\tau_1} = \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right) = hv$$

Solving this for v gives Bohr's version of the Rydberg equation for hydrogen:

$$(F4) \ v = \frac{2\pi^2 me^4}{h^3}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)$$

Appendix G: Eliminating Bohr's Special Assumptions

The mathematical significance of the elimination of Bohr's special assumptions is that it eliminated the restriction of W in the equation (7) in the main text:

$$(G1) W = \tau h \frac{\omega}{2}$$

Instead, using only his second principal assumption (i.e., the transition between stationary states is described by the emission of one photon of energy $E=hv$), Bohr could propose the more general quantum restriction:

$$(G2) W = f(\tau)h\omega$$

Given that principal assumption one suggests that ordinary mechanics could describe the stationary states of an atomic system, Bohr used the equations in (6) in the main text,

$$(G3) \ \omega = \frac{\sqrt{2}}{\pi}\frac{W^{\frac{3}{2}}}{eE\sqrt{m}}, \quad 2a = \frac{eE}{W},$$

to obtain:

$$(G4) \ W = \frac{2\pi^2 me^2 E^2}{f^2(\tau)h^2}, \quad \omega = \frac{4\pi^2 me^2 E^2}{f^3(\tau)h^3}$$

Fitting this equation for W to the experimentally obtained Rydberg equations and accompanying spectral data, Bohr could then perform a calculation similar to his derivation of the spectral formula (see appendix F) to show:

$$(G5) f(\tau) = \tau/2$$

This is equivalent to equation G1 above. Substituting in this value, equation G4 becomes equivalent to equation (8) in the main text:

$$(G6) \quad W = \frac{2\pi^2 m e^2 E^2}{\tau^2 h^2}, \quad \omega = \frac{4\pi^2 m e^2 E^2}{\tau^3 h^3}, \quad 2a = \frac{\tau^2 h^2}{2\pi^2 m e E}$$

Appendix H: Obtaining the Photoelectric Effect from Bohr's Model

In the versions of the Rydberg equations derived from Bohr's model, the relative values of τ_1 and τ_2 determine whether an atomic system transitioning between two stationary states would absorb or emit radiation. For atoms transitioning from the state associated with τ_2 to the state associated with τ_1, the atom would emit radiation for $\tau_2 > \tau_1$, which corresponds to an electron moving to a ring closer to the nucleus from a ring further from the nucleus. Conversely, the model predicted that atoms would absorb radiation for $\tau_1 > \tau_2$, which corresponds to an electron moving to a ring further away from the nucleus. This implied that an atom transitioning between two stationary states could only absorb radiation of certain frequencies, namely frequencies corresponding to energies equal to the change in energy of the atomic system due to the transition between various states. However, Bohr suggested that atomic absorption could also occur during transitions "between one of the stationary states and a state in which the electron is free" (Bohr, 1913a, 17). Recalling that values of W for each state were determined by the energy required to unbind an electron from the atomic system and that the finite differences in these values led to the Rydberg equations (see appendix F), Bohr suggested that any transition from a stationary state of the atom to a state where the electron was free would require the atom to absorb radiation with energy, given by $h\upsilon$, greater than W for the electron being freed. At the same time, conservation of energy would require that $h\upsilon$ equal the total change in energy of the atomic system. So, Bohr suggested that the excess energy, $h\upsilon$-W, not required to unbind the electron from the nucleus would be imparted to the newly freed electron in the form of kinetic energy. Thus, Bohr's model accurately predicted Einstein's equation for the photoelectric effect:

$$(H1) \quad T = h\upsilon - W$$

where T is the kinetic energy the electron ejected from the atom due to the photoelectric effect. Consequently, the Rutherford-Bohr model of the atom suggested that the behavior of atomic systems absorbing radiation would fall into two distinct regimes depending on the energy of the radiation. In a first regime, where the energy of the radiation was less than W, the atom would only absorb radiation of discrete frequencies. In the second regime, where the energy of the radiation was greater than W, the atom would exhibit absorb radiation of all frequencies and also emit an electron with kinetic energy equal to $h\upsilon$-W. The existence of these two regimes fits nicely the results of R.W. Wood's experiments on sodium gas's absorption of light. Similarly, Bohr's model could also explain Rutherford's observation that, in small angle scattering experiments with β-particles, high-speed electrons travelling through an atom only "lose energy in distinct finite quanta" (Bohr, 1913a, 221). Given the small angle of deflection, the free, high-speed electrons must have been colliding with bound electrons. Just in the case of absorbed radiation, Bohr's model suggests that in such collisions the bound electron could only gain energies corresponding to a transition between stationary states. So, the bound electron could only gain distinct finite quanta of energy. By conservation of energy, the β-particle could, consequently, only lose distinct finite quanta of energy.

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