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DESIGN OF BACKING LAYERS FOR PYRAMID ABSORBERS TO MINIMIZE LOW FREQUENCY REFLECTION

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Design of Backing Layers for Pyramid Absorbers to Minimize Low Frequency Reflection

Thesis directed by Professor Edward F. Kuester

The purpose of this project was to develop a procedure for producing optimally-absorbing backing layers to be placed behind standard pyramidal absorbers for electromagnetic anechoic chambers. The backing layers are intended to extend the usefulness of anechoic chambers using pyramid absorbers into a lower range of frequencies than currently practical without sacrificing the high-frequency performance of the chambers. In this thesis, a parametric model of the backing layers was developed which allows considerable flexibility in adjusting the reflectance while modeling physically realizable materials. The parameters of this model are subjected to a constrained optimization which finds the best parameters for a practical design.
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CHAPTER 1

INTRODUCTION

Present-generation anechoic chambers exhibit excellent broad-band suppression of reflected waves in the microwave region using pyramidal absorbers. The very low reflections from pyramidal absorbers result from the fact that incident microwaves reflect several times from the cones before finally being reflected back into free space; since a fraction of the incident wave is absorbed at each bounce, the microwaves are very much diminished by the time they reflect back from an array of absorbers.

This interpretation of the way pyramid absorbers work for microwaves results from the observation that the wavelengths involved are relatively small in comparison to the size of the absorbing structures, so that quasi-optical reflections occur.

The same type of absorber is sometimes used for lower frequency waves. At low enough frequencies, however, the waves become much longer than the spacing between adjacent absorbers. Their skin-depths in the absorbing materials likewise become long compared to the size of the pyramids. For such a structure, it no longer makes sense to model reflection in terms of a combination of reflections from individual cones. Instead, it is necessary to consider the pyramids as a layer of two-dimensionally periodic, inhomogeneous material.
Electromagnetic waves propagate into and are absorbed by this material just as any other material, with the exception that some of the incident radiation may be scattered at angles other than that prescribed by Snell's law. Dominating such terms is a pair of waves (scattered and reflected) that propagate at the Snell angles. These waves are determined by the "average" material characteristics of the layer. For typical pyramid absorbers, the reflection coefficients for an array of absorbers over metallic backing is quite large, tending toward unity in the low-frequency limit [12]. This makes pyramid absorbers of limited usefulness at for anechoic chambers to be used at lower frequencies.

The need for such a chamber, however, is growing. Modern computing devices, for example, may emit anywhere in this spectrum, and compliance with FCC regulations requires that products be tested to prevent interference with communications. In order to certify a RF enclosure for such testing, the FCC requires that the fields within the chamber be within 4 dB of open-field test conditions from 30 MHz to 300 MHz (VHF band). While this requirement may not seem overly strict, it is somewhat difficult to satisfy, especially if the same chamber is to be useful at higher frequencies as well.

At lower frequencies, it is also possible to achieve low reflection coefficients using a single-layer dispersive absorber [24]. Such absorbers can work well in a narrow range of frequencies, but because the dispersion of the absorbing material is not completely controllable, the bandwidth of these absorbers is too narrow for our purposes.

Another approach to designing anechoic chambers has been the use of multilayer absorbers. These absorbers are built to minimize reflection in a
specified range of frequencies. Design of multilayer absorbers has been successfully performed using cut-and-try methods, Smith-chart methods [23], and by numerical optimization techniques [18],[15]. In these designs, the layers have usually been composed of homogeneous materials, but this limitation narrows the achievable bandwidth for absorbers of a given size: a top layer continuously matched to the external medium has been shown to produce the best high-frequency performance [15].

The approach of this thesis is to combine the advantages of multilayer absorbers with those of pyramid absorbers. This is accomplished by replacing the top layer of a multilayer structure with a layer of pyramid absorbers. In such a structure, the effective material properties match continuously to the external medium, so good performance is expected in the range of frequencies between the design frequency and the microwave region, where quasi-optical techniques are applicable. Such a structure is shown in Figure 1.1. The advantage of this approach is that the higher frequency waves do not penetrate into the backing behind the pyramids due to their short wavelengths and skin depths, so microwave performance should be equal to that of absorbers originally designed for microwaves. The remaining layers can be adjusted so as to minimize reflection for lower frequencies.

The design proceeds in two phases:

(a) computation of the reflection and transmission properties (S parameters) of the pyramid absorbers and

(b) a search for the design which minimizes the overall reflection from the structure.

Calculating the average reflection and transmission properties of a
three-dimensionally inhomogeneous medium such as that of the cones is not trivial. However, since ordinary absorbing pyramids are two-dimensionally periodic, it is feasible to compute their averaged or "effective" permittivity and permeability accurately with respect to fields which vary slowly with distance compared to the pyramids themselves. The technique for doing this is known as homogenization. Once the averaged material properties are computed, they may be looked up as needed, and used to solve for the S-parameters the array of pyramids.

Reflection from the overall structure can then be easily calculated from the S-parameters and the known properties of the backing layers. The method of computation is given in Chapter 2. The backing layers are considered free to vary within certain practical bounds. The size and composition of these layers are controlled by a set of variables and constraints which constitute
a problem space which is a subspace of $R^N$, where $N$ is the total number of variables used to specify the backing layers. The details of this parameterization are explained in section 3.2.

Using an "objective" function which appropriately weights the function values at a variety of points, a constrained nonlinear optimization is performed on the variables. The resulting variables represent the best possible design for the backing layers, within the given constraints. The various objective functions used in these optimizations are specified and explained in Chapter 3.

Results are given in Chapter 4 for various designs. The best of these designs outperform current pyramid absorbers for anechoic chambers, both using existing materials and new materials specified by the optimization.
CHAPTER 2

COMPUTATION OF REFLECTIONS

2.1 Homogenization

An array of pyramidal absorbers such as those used in anechoic chambers constitutes an absorbing structure which is periodic in two dimensions. At frequencies for which the period is small compared to a wavelength and skin depth, the fields can be considered quasi-static. The material therefore has average properties governing the large-scale variation of the fields. Effectively, the inhomogeneity in two of the three axial directions can be averaged out, converting the actual medium to a one-dimensionally inhomogeneous, anisotropic artificial dielectric. The permittivity and permeability of the equivalent medium are intermediate between that of the absorber material and those of air.

Clearly, the effective properties vary with distance from the tips of the pyramids ($z$); to find the properties at a fixed $z$, we consider a thin transverse section so that the effective properties will be approximately homogeneous within the section. Such sections are shown in Figures 2.1 and 2.2. The media exhibit 4-fold symmetry about the $z$-axis, indicating that the average permittivity and permeability are scalar in the $x$-$y$ plane (within a section). The tensor average permittivity and permeability are diagonal, as shown:

$$\|\epsilon\| = \begin{bmatrix} \dot{\epsilon}_t & 0 & 0 \\ 0 & \dot{\epsilon}_t & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \text{ and } \|\mu\| = \begin{bmatrix} \dot{\mu}_t & 0 & 0 \\ 0 & \dot{\mu}_t & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$
Figure 2.1: Transverse Section of Rectangular Absorbers

Figure 2.2: Transverse Section of Twisted Absorbers
2.1.1 "Effective" Transverse Properties

Plane-waves incident on the array at an angle $\theta$ from the $z$-axis may be decomposed into a combination of electric, or perpendicular, and magnetic, or parallel, polarizations. For simplicity, we consider waves propagating only in the $x$-$z$ plane. The Maxwell equations for either polarization reduce to

$$\frac{dE(z)}{dz} = -j\omega\hat{\mu}_{\text{eff}}(z)H(z)$$

$$\frac{dH(z)}{dz} = -j\omega\hat{\epsilon}_{\text{eff}}(z)E(z)$$

where $E(z)$ and $H(z)$ are transverse field variables defined as follows. For the electric polarization:

$$E(z) = E_{x\text{av}}(z)$$

$$H(z) = -H_{z\text{av}}(z)$$

$$\hat{\epsilon}_{\text{eff}} = \hat{\epsilon}_{x}(z) - \frac{\mu_{0}\epsilon_{0}\sin^{2}\theta}{\hat{\mu}_{z}(z)}$$

$$\hat{\mu}_{\text{eff}} = \hat{\mu}_{x}(z)$$

and for the magnetic polarization:

$$E(z) = E_{z\text{av}}(z)$$

$$H(z) = H_{x\text{av}}(z)$$

$$\hat{\epsilon}_{\text{eff}} = \hat{\epsilon}_{x}(z)$$

$$\hat{\mu}_{\text{eff}} = \hat{\mu}_{x}(z) - \frac{\mu_{0}\epsilon_{0}\sin^{2}\theta}{\hat{\epsilon}_{z}(z)}$$

Hereafter, average fields will be assumed and the subscript $\text{av}$, denoting average fields will be dropped.
2.1.2 Absorber Geometries  Two types of absorber geometry will be considered for calculation of the effective properties. In the first, the absorbers are simple pyramids with adjacent bases. I will refer to this arrangement as "rectangular" or "square" pyramids (see Figure 2.3), since a section of the array is an array of squares. The second type of absorber consists of pyramids which are rotated 45 degrees with respect to the array. These are commonly known as "twisted" pyramids (Figure 2.4).

The longitudinal properties, $\hat{\mu}_z = \mu_{zz}$ and $\hat{\varepsilon}_z = \varepsilon_{zz}$, are known exactly. For rectangular pyramids, the "average" longitudinal permittivity and permeability are

$$\hat{\varepsilon}_z = \varepsilon_0 + \delta^2(\varepsilon_a - \varepsilon_0)$$
$$\hat{\mu}_z = \mu_0 + \delta^2(\mu_a - \mu_0)$$  \hspace{1cm} (2.10)

where $\delta = z/L$; thus $\delta^2$ represents the fraction of space filled by absorber. For
the twisted pyramids, the longitudinal properties are

\[ \dot{\epsilon}_z = \epsilon_0 + R(\dot{\epsilon}_a - \epsilon_0) \]
\[ \dot{\mu}_z = \mu_0 + R(\dot{\mu}_a - \mu_0) \]  

where

\[ R = \begin{cases} 
2(\frac{z}{L})^2 & 0 < z/L < \frac{1}{2} \\
1 - 2(\frac{L-z}{L})^2 & \frac{1}{2} < z/L < 1 
\end{cases} \]

is the filling fraction.

The transverse properties, while not known exactly in closed form, are approximated by

\[ \dot{\epsilon}_t = \epsilon_0 \left[ 1 + \delta^2 \frac{2(\dot{\epsilon}_a - \epsilon_0)}{(1 + \delta^2)\epsilon_0 + (1 - \delta^2)\dot{\epsilon}_a} \right] \]  
\[ \dot{\mu}_t = \mu_0 \left[ 1 + \delta^2 \frac{2(\dot{\mu}_a - \mu_0)}{(1 + \delta^2)\mu_0 + (1 - \delta^2)\dot{\mu}_a} \right] \]
for the rectangular pyramids and

\[
\epsilon_t = \epsilon_m \left\{ \frac{(\Delta_\epsilon - 1)(1 - g) - \Delta_\epsilon \ln[1 + g(1/\Delta_\epsilon - 1)]}{(\Delta_\epsilon - 1)(1 - g) + \ln[1 + g(\Delta_\epsilon - 1)]} \right\}^{1/2} 
\]

(2.14)

\[
\mu_t = \mu_m \left\{ \frac{(\Delta_\mu - 1)(1 - g) - \Delta_\mu \ln[1 + g(1/\Delta_\mu - 1)]}{(\Delta_\mu - 1)(1 - g) + \ln[1 + g(\Delta_\mu - 1)]} \right\}^{1/2} 
\]

(2.15)

for twisted pyramids [16], where

\[
\begin{align*}
\epsilon_m &= \epsilon_0 \\
\mu_m &= \mu_0 \\
g &= 2z/L
\end{align*}
\]

(2.16)

\[
\begin{align*}
\Delta_\epsilon &= \epsilon_a/\epsilon_0 \\
\Delta_\mu &= \mu_a/\mu_0
\end{align*}
\]

if \( z < \frac{L}{2} \) and

\[
\begin{align*}
\epsilon_m &= \epsilon_a \\
\mu_m &= \mu_a \\
g &= 2(1 - z/L)
\end{align*}
\]

(2.17)

\[
\begin{align*}
\Delta_\epsilon &= \epsilon_0/\epsilon_a \\
\Delta_\mu &= \mu_0/\mu_a
\end{align*}
\]

if \( z > \frac{L}{2} \). In the cases considered in this thesis, however, \( \mu_a = \mu_0 \), so the equations for \( \mu_t \) and \( \epsilon_t \) are unnecessary.

2.2 Characterization of Pyramid-Absorbers

Once the equivalent material properties of the medium are known, it is possible to calculate average plane-wave reflection and transmission properties of the array of absorbers. We will characterize these properties via S-parameters.
2.2.1 Computing Reflections from Pyramids  
Equations 2.1 are equivalent to the classical transmission line equations for a nonuniform line. The effective wave impedance of the medium is therefore defined as

\[ Z(z) = E(z)/H(z) \]  \hspace{1cm} (2.18)

The effective "characteristic impedance" of the medium is

\[ Z_c(z) = \sqrt{\frac{\mu_{\text{eff}}(z)}{\varepsilon_{\text{eff}}(z)}} \]  \hspace{1cm} (2.19)

and is a function of the angle of propagation as well as the material properties. (See equations 2.4, 2.5, 2.8 and 2.9). As in the case of a transmission line, the reflection coefficient and effective impedance are related by the equations:

\[ Z(z) = Z_c(z) \frac{1 + \Gamma(z)}{1 - \Gamma(z)} \quad \text{and} \quad \Gamma(z) = \frac{Z(z) - Z_c(z)}{Z(z) + Z_c(z)} \]  \hspace{1cm} (2.20)

By differentiating, it can be shown that the reflection coefficient obeys the differential equation

\[ \Gamma'(z) = 2\gamma\Gamma(z) - \frac{2Z_c'(z)}{2Z_c(z)}[1 - \Gamma^2(z)] \]  \hspace{1cm} (2.21)

which is known as the Riccati equation for \( \Gamma(z) \). The complex propagation constant, \( \gamma \), is

\[ \gamma = j\omega\sqrt{\mu_{\text{eff}}\varepsilon_{\text{eff}}} \]

Equation 2.21 is amenable to solution on a computer using a standard simultaneous differential equation solver (for the real and imaginary parts).

The effective properties used in calculating \( Z_c \) are defined by equations 2.4 and 2.5 in the case of TE-polarized waves, and by equations 2.8 and 2.9 for TM-polarized waves.
2.2.2 S-parameters of the Absorber-Array The strategy of this design method is to vary the absorbing layers behind the absorber-array while holding the properties of the pyramidal absorbers constant. It is clearly desirable, then, to know the transmission and reflection properties of the array in advance rather than to carry out a numerical solution of the Riccati equation every time a new reflection is computed during the optimization phase of a design sequence.

Neglecting waves scattered by the cones at other angles (first and higher-order effects), the layer of cones at a given angle of incidence is equivalent to a two-port network in circuit theory (Fig. 2.5).

\[
S(f, \theta) = \begin{bmatrix}
S_{11}(f, \theta) & S_{12}(f, \theta) \\
S_{21}(f, \theta) & S_{22}(f, \theta)
\end{bmatrix}
\]
The layers behind the cones, taken together, constitute another circuit-element which is equivalent to a one-port network with reflection coefficient $\Gamma_B$. In the presence of the backing layer, the total reflection from port 1 (the tips of the cones) is

$$\Gamma = S_{11} + S_{12}S_{21}\Gamma_B[1 + (S_{22}\Gamma_B) + (S_{22}\Gamma_B)^2 + \cdots]$$  \hspace{1cm} (2.22)

or, summing the geometric series,

$$\Gamma = S_{11} + \frac{S_{12}S_{21}\Gamma_B}{1 - S_{22}\Gamma_B}$$  \hspace{1cm} (2.23)

If the S-parameters are known, the reflection may be calculated for any $\Gamma_B$. Looking from port 1, $S_{12}$ and $S_{21}$ always occur as a product, so it is unnecessary to know them separately; knowing only their product is sufficient. As a result, at a given angle and frequency only three cases need to be considered to arrive at enough equations to de-embed the S-parameters.

These equations proceed from setting the backing reflectance, $\Gamma_B$, to some value and numerically integrating the Riccati equation from the bases of the cones to their tips to find $\Gamma$. Repeating the procedure for three different backing reflectances yields three equations which may then be solved for the S-parameters. It is not necessary that the backing reflectances be realizable; they need only be different from one another so that they lead to independent equations.

For simplicity I chose the values of $\Gamma_B = 0$, corresponding to an exact match between port 2 and the backing layer; $\Gamma_B = -1$, for a perfectly-conducting wall and $\Gamma_B = 1$, for a magnetic wall. These choices result in the following:

$$\text{for matched reflection, } \Gamma_m = S_{11}$$  \hspace{1cm} (2.24)
for short-circuit termination, \( \Gamma_{sc} = S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}} \) \hspace{1cm} (2.25)

and for open-circuit termination, \( \Gamma_{oc} = S_{11} + \frac{S_{12}S_{21}}{1 - S_{22}} \) \hspace{1cm} (2.26)

Solving equations 2.24-2.26 for the S-parameters results in

\[
S_{11} = \Gamma_m \hspace{1cm} (2.27)
\]

\[
S_{22} = \frac{\Gamma_{sc} + \Gamma_{oc} - 2\Gamma_m}{\Gamma_{oc} - \Gamma_{sc}} \hspace{1cm} (2.28)
\]

\[
S_{12}S_{21} = 2 \frac{(\Gamma_{oc} - \Gamma_m)(\Gamma_m - \Gamma_{sc})}{\Gamma_{oc} - \Gamma_{sc}} \hspace{1cm} (2.29)
\]

The values of the S-parameters were computed and stored in data files for each angle and frequency of interest for both polarizations. The frequencies used for this procedure ranged from 30 MHz to 200 MHz in 5 MHz intervals, the angles from 0 degrees to 60 degrees in 5 degree intervals at each frequency. The program that performs these calculations is called 'cones'; there are two versions, 'Rcones' for rectangular pyramids and 'TWcones' for twisted pyramids.

2.3 Reflection From Multilayer Media

Consider a structure composed of several layers of homogeneous, isotropic dielectric materials as shown in Figure 2.6. When the structure is excited by plane waves, the average fields within the layers are also plane waves, since there is no variation of the media transverse to the z-axis. The angle of propagation in each layer is determined by Snell's law.

Assuming a single monochromatic plane wave incident from the left, the complex propagation constant, \( \gamma_i \), is defined by

\[
\gamma_i = \alpha_i + j\beta_i = j\omega\sqrt{\mu_i\varepsilon_i}
\]
Figure 2.6: Multilayer Stack of Dielectric Materials

where $\varepsilon_i$ and $\mu_i$ are the complex permittivity and permeability of the $i$th layer.

The incident wave may be decomposed into a combination of TE and TM polarized waves. The TE wave has its electric field vector perpendicular to the plane of incidence, while the TM wave has its electric fields parallel to the plane of incidence. The TE fields are

$$\vec{E}(z) = a e^{\gamma_i z} \cos \theta_i (z - z_i) + B e^{-\gamma_i z} \cos \theta_i (z - z_i)$$

(2.30)

and

$$\vec{H}(z) = \vec{H} = a \cos \theta_i \hat{n}_z (A e^{\gamma_i z} \cos \theta_i (z - z_i) - B e^{-\gamma_i z} \cos \theta_i (z - z_i))$$

$$+ a \sin \theta_i \hat{n}_z (A e^{\gamma_i z} \cos \theta_i (z - z_i) + B e^{-\gamma_i z} \cos \theta_i (z - z_i))$$

(2.31)

In the TM case, the fields are

$$\vec{E}(z) = a \cos \theta_i (A e^{\gamma_i z} \cos \theta_i (z - z_i) + B e^{-\gamma_i z} \cos \theta_i (z - z_i))$$

$$- a \sin \theta_i (A e^{\gamma_i z} \cos \theta_i (z - z_i) - B e^{-\gamma_i z} \cos \theta_i (z - z_i))$$

(2.32)
and

\[ \dot{H}(z) = \sigma T \frac{1}{\eta_i} (A_i e^{\gamma_i \cos \theta_i (z - z_i)} - B_i e^{-\gamma_i \cos \theta_i (z - z_i)}) \]  

(2.33)

\( A_i \) and \( B_i \) denote the amplitudes of the forward traveling and backward traveling waves in layer \( i \).

The tangential components of the fields must be continuous at the interfaces between the layers. The continuity conditions on the tangential components of the TE fields (equations 2.30 and 2.31) are

\[ A_{i+1} + B_{i+1} = A_i e^{\gamma_i \cos \theta_i d_i} + B_i e^{-\gamma_i \cos \theta_i d_i} \]  

(2.34)

and

\[ \frac{\cos \theta_{i+1}}{\eta_{i+1}} (A_i - B_i) = \frac{\cos \theta_i}{\eta_i} (A_i e^{\gamma_i \cos \theta_i d_i} - B_i e^{-\gamma_i \cos \theta_i d_i}) \]  

(2.35)

where \( d_i \) is the thickness of layer \( i \). For the TM fields (equations 2.32 and 2.33) the continuity conditions are

\[ (A_{i+1} + B_{i+1}) \cos \theta_{i+1} = (A_i e^{\gamma_i \cos \theta_i d_i} + B_i e^{-\gamma_i \cos \theta_i d_i}) \cos \theta_i \]  

(2.36)

and

\[ \frac{1}{\eta_{i+1}} (A_i - B_i) = \frac{1}{\eta_i} (A_i e^{\gamma_i \cos \theta_i d_i} - B_i e^{-\gamma_i \cos \theta_i d_i}) \]  

(2.37)

Equations 2.34 and 2.35 may be written in matrix form:

\[
\begin{bmatrix}
1 & 1 \\
\frac{\cos \theta_{i+1}}{\eta_{i+1}} & -\frac{\cos \theta_{i+1}}{\eta_{i+1}}
\end{bmatrix}
\begin{bmatrix}
A_{i+1} \\
B_{i+1}
\end{bmatrix} =
\begin{bmatrix}
e^u & e^{-u} \\
\frac{\cos \theta_i}{\eta_i} e^u & \frac{\cos \theta_i}{\eta_i} e^{-u}
\end{bmatrix}
\begin{bmatrix}
A_i \\
B_i
\end{bmatrix}
\]  

(2.38)

Equations 2.36 and 2.37 may similarly be written:

\[
\begin{bmatrix}
\cos \theta_{i+1} & \cos \theta_{i+1} \\
\frac{1}{\eta_{i+1}} & -\frac{1}{\eta_{i+1}}
\end{bmatrix}
\begin{bmatrix}
A_{i+1} \\
B_{i+1}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_i e^u & \cos \theta_i e^{-u} \\
\frac{1}{\eta_i} e^u & -\frac{1}{\eta_i} e^{-u}
\end{bmatrix}
\begin{bmatrix}
A_i \\
B_i
\end{bmatrix}
\]  

(2.39)

In these equations, \( u = \gamma_i \cos \theta_i d_i \).
Equations 2.38 and 2.39 can be solved for the wave amplitudes on the left hand side:

\[
\begin{bmatrix}
A_{i+1} \\
B_{i+1}
\end{bmatrix} = \begin{bmatrix}
C_{1}e^{u} & C_{2}e^{-u} \\
C_{2}e^{u} & C_{1}e^{-u}
\end{bmatrix} \begin{bmatrix}
A_{i} \\
B_{i}
\end{bmatrix}
\] (2.40)

where for the TE polarization:

\[
C_{1} = \frac{1}{2}(1 + \frac{\eta_{i+1}\cos\theta_{i}}{\eta_{i}\cos\theta_{i+1}})
\] (2.41)

\[
C_{2} = \frac{1}{2}(1 - \frac{\eta_{i+1}\cos\theta_{i}}{\eta_{i}\cos\theta_{i+1}})
\]

and for the TM polarization:

\[
C_{1} = \frac{1}{2}(\frac{\cos\theta_{i}}{\cos\theta_{i+1}} + \frac{\eta_{i+1}}{\eta_{i}})
\] (2.42)

\[
C_{2} = \frac{1}{2}(\frac{\cos\theta_{i}}{\cos\theta_{i+1}} - \frac{\eta_{i+1}}{\eta_{i}})
\]

If the forward-traveling and backward-traveling waves are known at a layer \(i\), they can be calculated at layer \(i+1\), using equation 2.40.

When the reflection coefficient \(\Gamma_{0}\) is known on one side of the multilayer structure, the total reflection coefficient \(\Gamma_{N}\) may then be calculated by multiplying several matrices of the form of 2.40. In this way, reflection or transmission of the whole structure may be characterized by a single complex \(2 \times 2\) matrix.

Ordinarily, in an anechoic chamber, a single layer of homogeneous material underlies a tapered section of absorber. This material is typically identical to the taper material. This layer is mounted on a metallic conducting wall, which shields the chamber from external radiation. In this thesis several layers of different absorbers replace the single layer of typical cones. Since the metallic wall has a reflection coefficient of approximately -1, it is simple
and straightforward to compute the plane-wave reflection coefficient at the top of each layer by means of equation 2.40. This reflection coefficient is then used as $\Gamma_B$ in equation 2.23, which gives the approximate plane-wave reflection coefficient for the array of absorbers at their tips.
CHAPTER 3

OPTIMIZATION

In this chapter, the optimization procedure is outlined and the specifications of the optimization variables are explained.

3.1 Optimization Algorithm

Optimization is the name given to a set of numerical techniques which search out extrema (ordinarily minima) of a nonlinear function of many variables $F(x)$. Generally, an optimization algorithm proceeds as follows:

(a) Find a search direction $p$ along which the function is decreasing.
(b) Move a distance $\alpha$ along the search vector.
(c) Go to step (a).

The algorithm terminates if it either finds a minimum or is unable to make further progress. Different optimization procedures use different algorithms to solve each of the subproblems. The best choice of an optimization procedure depends on the character of the problem to be solved.

One important characteristic of the design problem of this thesis is that it includes constraints. For instance, the overall length of the backing is subject to some practical limits; it cannot be too large, nor can it be negative. Further, while positive values of conductivity are permitted; negative values are not. Also, for materials we wish to consider, there are practical upper and lower bounds on the permittivity. The optimization procedure used here must therefore be such as to permit consideration of upper and lower bounds on the
variables and also consideration of (at least) linear constraints.

Designing a multilayer backing for pyramid absorbers may be computationally intensive. Therefore, the algorithm should be reasonably efficient; that is, it should not require a great deal of work in each iteration.

The most powerful optimization methods require that the function be a smooth, single-valued function; these methods take advantage of the smoothness to speed convergence and estimate closeness to the solution. Further criteria must also be satisfied to guarantee a solution. These criteria are discussed in [8] and [6]. I will discuss what happens when the objective function is not smooth in subsection 4.1.2.

For the reasons described above, I chose a variable-scale optimizer, which finds a Kuhn-Tucker point subject to upper and lower bounds on the variables and to general linear and nonlinear constraints; this type of optimization algorithm is considered the most powerful. Such an optimizer (E04UCF) is provided in the Numerical Analysis Group (NAG) library of Fortran subroutines [19]. The optimization procedure is a quasi-Newton algorithm, which is suitable for finding unconstrained, linearly constrained or nonlinearily constrained minima of nonlinear functions [7].

The subroutine formulates step (a) above as a quadratic-programming problem, wherein \( F(x) \) is assumed to be of the form

\[
F(x) = \frac{1}{2} x^T A x + b^T x + c
\]  

(3.1)

at least within the constraints \( c_i \). A search is conducted within the constraints, or satisfying them as nearly as possible, for a vector \( p \) along which the objective function \( F(x) \) approaches its minimum. That is, a vector, \( p \) is found such that
the Lagrangian merit function

\[ L(x, \lambda, s) = F(x) - \sum \lambda_i (c_i(x) - s_i) + \frac{1}{2} \sum \rho_i (c_i(x) - s_i)^2 \]  \tag{3.2}

decreases most rapidly where \( \lambda_i \) are the Lagrange multipliers, \( \rho_i \) are penalty parameters (imposed for violating the constraints), and \( s_i \) are slack parameters [19]. In practice, the user specifies a certain amount of leeway in computing this optimal search direction; this allows the search routine to terminate after finding an approximate solution to the quadratic subproblem, rather than computing a more precise solution at the expense of performing many iterations.

Once the optimal search direction is found, the program computes a step-size, \( \alpha \), which yields a sufficient decrease in the objective function. This computation, like the computation of the search direction, uses curvature information stored in a matrix, \( H \), the augmented Hessian matrix (the matrix of second derivatives of the objective function, augmented by the active constraint derivatives).

This entire optimization scheme described so far rests on the assumption that the problem can be framed as an N-dimensional quadratic equation. Since, in fact, the optimization problem we wish to solve is not quadratic, the curvature in the neighborhood of the starting point is likely to be different from the curvature near the solution. Unless some compensation is made for the change in curvature, the iterates will fail to converge to the solution.

In quasi-Newton optimization techniques, the curvature information is therefore updated at the end of each iteration, so that as the iterates approach the solution, the current approximation to the Hessian approaches the Hessian at the solution, thus ensuring that the iterates approach the solution point
rapidly. The update used in E04UCF is the BFGS update:

\[
\bar{H} = H - \frac{1}{s^T H s} H s s^T H + \frac{1}{y^T s} y y^T
\]  

(3.3)

where \( s = \bar{x} - x \) is the change in \( x \); \( H \), the augmented Hessian; \( \bar{H} \), the new Hessian and \( y \), a measure of the change in the gradient.

Since the solution procedure assumes locally-quadratic objective function, the objective function must be smooth within the search region. Otherwise, it may become impossible to solve part (a) of the problem using a quadratic programming approach, and the optimization may fail.

3.2 Parameterization of the Backing Layers

In this thesis, a nonlinear optimization subroutine was chosen from the Numerical Analysis Group (NAG) library of Fortran subroutines. This subroutine minimizes a function of several variables, subject to upper and lower bounds on the variables and, if desired, to user-defined linear and nonlinear constraints.

In order to code the optimization problem, it was necessary to specify the properties of the backing layers in terms of a number of adjustable parameters, which became variables of the optimization. There are many possible choices of optimization variables which could be used to solve the design problem. One possible choice would be to specify each layer directly in terms of its S-parameters. Although this approach might sound appealing, it is in fact unsatisfactory for two reasons: first, it would require eight variables per layer (since the numbers are complex); second, and more important, it would be difficult to model the physics of the problem in a realistic manner. Modeling on this basis would be complicated by the need to compute restrictions
on the S-parameters that would be imposed by fixing the layer thickness as in a real design, and modeling of the dispersion of the absorbing media would be impossible. Further, the "optimal" design might be physically unrealizable because the material properties which would be required to manufacture the design might be unattainable. A superior approach is to make the layer thicknesses and some parameters which determine the material properties variables of the optimization. These variables are suitable for optimization because they are simply constrained to a region corresponding to realizable designs, and the dispersion of the media can be modeled parametrically.

The criteria for selection of suitable optimization variables are as follows:

- Only a small number of variables should be required per layer.
- They must be able to represent realistic values of the permittivity and permeability.
- If possible, the variables should be chosen to automatically exclude physically unrealistic behavior by imposing simple constraints.

3.2.1 Selection of Parameters In order to obtain suitable parameters, certain assumptions were made about the electromagnetic properties of the backing layers. Specifically, it was assumed that the materials were non-magnetic and consisted of absorbing foam similar to that of standard pyramid absorbers (urethane foam impregnated with graphite and fire retardants). These assumptions limit the class of functions which may reasonably be used to represent the frequency dependence of the permittivity. Even limiting the class of allowable epsilon versus frequency characteristics to those of standard absorbing-foam materials, a wide range of characteristics was producible, but
the frequency dependence of epsilon is by no means arbitrary. From the standpoint of performance, this means that the optimal design produced under these strictures may not be the best design possible if many different types of materials were considered. From the standpoint of computation, however, the simplification of the model which resulted from these assumptions about the absorber materials is justified because it resulted in an optimization problem which was smaller, and thus easier to solve, while providing useful results.

Typically, the frequency dependence of the real and imaginary parts of the permittivity were similar in a given material. Generally, they fell off at a rate somewhat less than $1/f$. It was decided that only frequency dependencies of this type would be considered, in order to limit the number of variables per layer. The bulk material properties of some standard absorbers are included in tables 4.1 and 4.2.

The permittivity was expressed in terms of the parameters $A_2, A_3, A_4, A_5$ as follows:

$$\varepsilon'_r = A_2 + 100 A_3 (f/30)^{A_5}$$  \hspace{1cm} (3.4)

$$\varepsilon''_r = -100 A_3 A_4 (f/30)^{A_5}$$  \hspace{1cm} (3.5)

where the frequency is in Megahertz. In addition, one parameter is needed to specify the thickness of each layer.

$$A_1 = d$$  \hspace{1cm} (3.6)

Thus, for each layer, there are five adjustable parameters, $A_1 - A_5$. $A_2$ represents the limiting value of permittivity as the frequency grows large. $A_3$ represents the magnitude of the frequency-dependent component of the permittivity. It is roughly proportional to the density of dopants in the material. $A_4$ is a
Table 3.1: Constraints on Optimization Parameters

\[
\begin{align*}
0.00 & \leq A_{1,i} & \leq \text{length-constraint} \\
1.10 & \leq A_{2,i} & \leq 2.00 \\
0.00 & \leq A_{3,i} & \leq 1.20 \\
0.90 & \leq A_{4,i} & \leq 2.50 \\
0.40 & \leq A_{5,i} & \leq 0.80 \\
0.00 & \leq \sum_{i=1}^{n} d_i & \leq \text{length-constraint}
\end{align*}
\]

simplified proportionality relation between the real and imaginary parts of the permittivity. It is determined by the specific combination of chemicals used to manufacture the absorber. \( A_5 \) determines the rate of rolloff of the conductivity of the material with increasing frequency. This is also dependent on the chemical composition of the absorbing material. The various parameters are scaled to comparable size for convenience.

The adjustable parameters for each layer, \( A_{1,i} - A_{5,i} \), are stored in a single vector \( \mathbf{x} \), the vector of free variables that is used to compute \( F(\mathbf{x}) \).

3.2.2 Constraints on the Variables The values of the variables were subjected to constraints in order to prevent them from taking on unrealistic or unreasonable values. These constraints, like the variables themselves, were selected to model behavior similar to that of ordinary absorber materials. In addition, an overall constraint was enforced on the length of the backing section. The length of each layer was required to be less than or equal to the overall length. Conveniently, the choice of parameters eliminates the need for nonlinear constraint functions, although such functions are easy to add using E04UCF. The constraints used are shown in Table 3.1.

Alternatively, it is quite feasible to optimize the backing layers using
fixed material properties. This may be especially desirable where computation time is expensive, or where there is some uncertainty as to whether materials can be inexpensively made to order. This may, in practice, often be the case because process controls on impregnation of urethane foam with carbon and fire retardants are crude. In this case, we simply bypass the parameterization stage in the optimization program and use the measured properties of available materials. The same program is easily used for both cases, provided the upper and lower bounds on the appropriate variables are set equal to one another, allowing no variation. Loss of computing efficiency due to carrying excess variables in these highly-restricted cases is small.

3.3 Objective Functions

The optimization problem is to minimize a function of many variables that are subject to various bounds. The function to be minimized is called the objective function. There are a wide variety of available functionals which we might want to consider as objective functions and that might suit our purpose of reducing overall reflection to within the required limits. A suitable general form for such functionals is

$$F(x) = \int_{f_1}^{f_2} \int_{\theta_1}^{\theta_2} W_1(f, \theta)W_2(\Gamma(x, f, \theta))d\theta df \quad (3.7)$$

where $W_1$ and $W_2$ are non-negative weighting functions. $W_1(f, \theta)$ governs the weighting which is given to different frequencies and angles of incidence. Since computing definite integrals every iteration of the optimization would be time-consuming, it was decided that a sampling function could be used for $W_1$, which reduces equation 3.7 to

$$F(x) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_f} W_2(\Gamma(x, \theta_i, f_j)) \quad (3.8)$$
$W_2(\Gamma)$ is a second weighting function which is used to give varying emphasis to points based on the magnitude of reflection. When adjusting the variables, $x$, reductions of the reflection in one region of the frequency range often caused increases in other areas. If we take $W_2(\Gamma) = |\Gamma|$, it is quite possible to get a low value of the objective function even when the reflection at a small number of points is large. Since this may create a bad chamber resonance at a single frequency, it is inadvisable to use this choice of weighting. If, however, the weighting is chosen as a power of the reflection, the weighting favors the higher points, and an optimum will be found which has a low overall reflection, but may not be as low as possible at every point.

With this in mind, we reconstructed the computation of the objective function to compute norms of the reflection coefficient.

$$F(x) = \sqrt[4]{\frac{1}{N_\theta N_f} \sum_{i=1}^{N_\theta} \sum_{j=1}^{N_f} |\Gamma(x, \theta_i, f_j)|^k}$$  \hspace{1cm} (3.9)

These functionals are smoother than the simple sums of powers which would be given by adhering to the formula 3.8, so the optimization usually converges in fewer steps. In order to accommodate objective functions of this type, it is necessary to alter the general form:

$$F(x) = U \left( \int_{f_1}^{f_2} \int_{\theta_1}^{\theta_2} W_1(f, \theta) W_2(\Gamma) d\theta df \right)$$  \hspace{1cm} (3.10)

$U$ must be smooth, increasing function of its argument.

### 3.4 LFmin

The program LFmin implements all of the described functions in Fortran. The functional units of LFmin are listed below, along with their functions.
EPS  Provides bulk material properties for the backing layers and defines the parameterization of these properties. If the backing is fixed, EPS looks them up from an array.

FUN  Computes plane-wave amplitude reflection coefficients from the complete absorbing structure as it is currently configured.

OBJN0 and OBJNA  Compute the objective functions and their gradients. The gradients are estimated by finite-differencing the function FUN. OBJN0 is used for cases in which the angle is fixed at normal incidence, while OBJNA computes norms over a range of incidence angles.

E04UCF  (Provided by Numerical Analysis Group) conducts the search for the optimum value of the objective function.

LFmin  Defines the size of the problem (number of layers), specifies constraints on the variables and the linear constraint on total length of the backing layers, sets the sampling points for frequency and angle, sets the order of the norms for the objective function and selects the appropriate files for material data and S-parameters of the pyramids in the top layer. All of these features are set at runtime. When the search for the minimum terminates, LFmin reports the final values of the objective function, the final vector of variables, and the gradient of the objective function at this point.
CHAPTER 4

OPTIMIZATION RESULTS

Once a program had been developed which could minimize the reflections from a hybrid pyramid-multilayer design, it was desirable to see what degree of improvement upon existing designs could be made.

The design problems presented in this chapter concentrate on the frequency range from 30 MHz to 100 MHz. It was generally observed that designs performing well in this range also performed adequately at higher frequencies, as is easily seen by inspection of the various figures.

In an anechoic chamber, reflections occur at all possible angles, and regardless of the placement of transmit and receive antennas within the chamber, some of the oblique reflections are important. For this reason, it is important to consider off-normal reflections when designing absorbers. There seems little point in trying to include angles close to 90° in our sampling functions, since as the angle of incidence approaches the normal, the magnitude of the reflections always tends to unity. From a practical standpoint, it seems that angles greater than 45° are relatively unimportant anyway; any ray-path from the transmit to receive antenna must include at least one reflection at an incidence angle of less than 45°, unless the chamber is long and thin, in which case more than one type of absorber should be used.

Because the length constraints used in the design problems were small relative to the longer wavelengths, it was expected that a small number of
backing layers would be needed to approach the best possible design. For this reason, the optimization program, LFmin, was designed to accept no more than five backing layers. In early experiments, when a large number of backing layers was used, the optimization program usually either reduced some of the layer thicknesses to zero or set the materials of adjacent layers equal to one another. Effectively, LFmin reduced the actual number of backing layers to one, two or three for optimal cases. Therefore, it was decided at this point to limit our consideration to solutions to the two-layer and three-layer problems and thereby save computing time.

Comparison of some two-layer and three-layer sample problems showed that the solutions to three-layer problems were, if not identical to the two-layer solutions, only marginally better. We therefore decided to concentrate on two-layer optimization problems.

4.1 Designs Using Rectangular Pyramids

Originally, we wished only to investigate the efficacy of the method, so it was considered sufficient to demonstrate improvement using the rectangular pyramid geometry. It was assumed that the qualitative results proceeding from this investigation would also be applicable to twisted-pyramids or any other similar absorbing-cone geometry.

Absorbers of this type are made in a variety of sizes. For each size, a different material is used to make the pyramids absorb sufficiently well in the microwave spectrum. Table 4.1 shows the relative permittivities of several samples of absorber manufactured by Rantec, Inc. Since they are absorptive, the permittivities are complex and strongly frequency dependent. All of these materials are non-magnetic.
Table 4.1: Bulk Properties of some Rantec Absorbers

<table>
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<th>Frequency (MHz)</th>
<th>R1</th>
<th></th>
<th>R2</th>
<th></th>
<th>R3</th>
<th></th>
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The material labeled R1 in Table 4.1 was used for pyramids having a total length of 48 inches, consisting of 40 inches of taper and 8 inches of bulk material. At the frequencies we wish to consider, these absorbers are quite short compared to a wavelength. As one might expect, their performance at frequencies below 100 MHz is poor (see Figure 4.1). Since the absorbers are designed to be used at much higher frequencies, low frequency reflections were not considered important in their design.

The material labeled R2 in Table 4.1 is used for 72 inch absorbers. These absorbers have 62 inches of taper and 10 inches of bulk absorber. With their greater length, they require less doping to achieve the same performance at high frequencies as the 48 inch absorbers. They do not perform as well as the smaller absorbers below 200 MHz (see Figure 4.2).

In his master's thesis [12], Holloway describes a new design produced by increasing the permittivity and conductivity of the material and changing
Figure 4.1: Low-frequency performance of Rantec 48" absorbers

Figure 4.2: Low-frequency performance of Rantec 72" absorbers
Figure 4.3: Low-frequency performance of Holloway absorbers

the ratio of taper to bulk absorber in the 72 inch pyramids. These new cones performed significantly better than the standard cones below 100 MHz. The material properties of Holloway's design are listed under R3 in Table 4.1; their tapers are 48 inches (1.22 meters) in length, with 24 inches (0.61 meters) of bulk absorber. The improved performance of the Holloway design is shown in Figure 4.3. Even with Holloway's improved design, however, reflections may be large enough to make a chamber unsuitable for FCC testing, especially for frequencies approaching the lower limit of 30MHz.

4.1.1 Designs Using Standard Tapers Using the same tapered sections already existing in Rantec absorbers, it was hoped that optimizing a multilayer backing would produce sufficiently small reflections to prevent any unwanted resonances. Since the constraint on total thickness of the absorbing layer was more relaxed and more than one material was allowed
in the new cone designs, it was expected that further improvement could be made over Holloway's design, as well as current commercial absorber designs.

Most of these pyramid absorbers used only one material to make both the backing layers and the pyramids. This has the advantage that the transition of average characteristic impedance from the base of the pyramids into the backing layer is smooth. However, this is not necessarily the best thing to do from the standpoint of low frequency wave reflection; sometimes partial reflection from the backing layers can reduce overall reflection if it is out of phase with the portion of the wave which is incrementally reflected by the pyramids. Different backing materials may be found to outperform the absorber material used for the pyramids, even when the pyramids are themselves optimized for a single-material design.

In the first phase of this project, only 40", 48" and 62" tapers were considered. These tapers correspond to the taper lengths of standard Rantec absorbers and to Holloway's absorbers. Underneath these layers, we modeled several layers of bulk absorber with adjustable thicknesses and material properties, as in equations 3.4 and 3.5. Various values were considered for the constraints on the variables. The final constraints (Table 3.1) were selected to provide fair modeling of actual absorber materials while providing sufficient flexibility to improve performance substantially.

Optimization runs using LFmin were made for some two-layer cases with 40" and 62" tapers, as in the standard square cones; the materials used in these tapers were R1, R2, R3 and R4. The earliest version of LFmin used only the second power (RMS or $L^2$) norm. The best results for 40" tapers are shown in figures 4.4 and 4.5. These results show considerable improvement
Figure 4.4: Optimized Case with 40° Tapers in E-Polarization

Figure 4.5: Optimized Case with 40° Tapers in M-Polarization
over standard absorbers, at least near 30 MHz. Unfortunately, the reflections are large at 45 degree incidence in the electric polarization, which may make them impractical for many chamber shapes. The materials and dimensions are as follows:

taper material: R2
layer 1 length: 0.364 meters
layer 1 material: R2
layer 2 length: 1.059 meters
layer 2 material: see Fig. 4.6
total length: 2.439 meters or 96 inches

This optimization was run with the layer 1 material fixed and the layer 2 material variable. Compare Figure 4.7, which was run with two layers of variable material and the same tapers as Figure 4.4. The result is considerably
better in the frequency range over which the optimization was run, but the reflections increase at higher frequencies. This is an unusual result, based on comparison with the other cases, but it illustrates that considerable care must be taken in selecting which of several designs coming out of LFmin should be used. It also demonstrates that, in some cases, relaxing the restrictions on a design may not dramatically improve the results.

In the following cases, only the E-polarization reflections are shown, because in the frequency-range from 30-100 MHz, the E-polarized wave is almost always more strongly reflected than the M-polarized wave in critical cases. LFmin is capable of sampling both polarizations, but only the electric polarization was used here. The restriction of the sampling to electric polarizations only is justified by comparison of the E and H polarization reflections for each optimal case. If the magnetic polarization had turned out to be worse than the
electric, we would have re-run LFmin using sampling over both polarizations. but it is more efficient to consider only one polarization whenever possible.

The best designs using 48" and 62" tapers turned out to be considerably better than those for 40" tapers. The optimum for 48" tapers is shown in Figure 4.8. gradual tapers. For 62" tapers, the best design is shown in Figure 4.9. Both of these designs include one layer of material identical to the pyramids and one with variable permittivity. The parameters for these materials are listed in Appendix A.

It is sometimes possible to improve on these results substantially by trying other methods. We found that using an RMS norm as the objective function is not always the best choice. In many cases, minimizing the $L^2$ norm resulted in reflection curves that, though generally low, exhibit relatively high peaks; these are "paid for" by very low reflections at other frequencies and
angles. These peaks are potential trouble spots for a design to be used in an anechoic chamber, as they can lead to intolerable chamber resonances. An example of this kind of minimum is shown in Figure 4.5.

4.1.2 Other Norms To alleviate this problem, a new kind of objective function had to be found. We would prefer to be able to minimize the highest peak of the reflection coefficient within the range of frequencies and angles under consideration; if the peak reflection is made sufficiently low, larger than minimal reflections at other frequencies or angles can be tolerated. To find the highest peak, $\text{LF}_{\text{min}}$ must calculate the reflections at several sample points and select the highest one.

When we attempted to find minima of objective functions of this sort, the optimization algorithm almost always came to a stop before locating the minimum; repeated application of the optimizer ($\text{LF}_{\text{min}}$) with a max-norm
objective function eventually converged to a consistent result, which we take as a good approximation to the actual minimum. To see why the optimization stops, we consider what the optimization subroutine “sees”. The optimizer “knows” the last computed value of the objective function and its first derivatives, and has an estimate of the second derivatives, all of which it needs to compute the optimal search direction. As it adjusts the variables, it manages to lower the objective function, which is the reflection at the highest point. The other points shift with each iteration, some decreasing, others increasing. After a few iterations, one of the increasing points rises to the level of the maximum point. When there are two or more maximum points at the same level, some of the first derivatives of the objective function become discontinuous. The corrections to the Hessian matrix which are made automatically each iteration become unpredictable (and large) in the region around such a point, and the search for the optimal descent direction becomes extremely inaccurate, making further progress slow and difficult. Effectively, this means that many iterations are required to solve for the optimal search direction, so the process of finding these minima is quite slow.

It was desirable, therefore, to select an objective function which places a continuously-varying weight on each point according to its reflection. As mentioned in section 3.3, this is easily accomplished by selecting an integer order norm of the vector of reflection coefficients. The norm order is selected at runtime. These other norms are a compromise between the RMS norm and the max or “infinity” norm. It was found that using a 20th order norm compared favorably with the max-norm while retaining sufficient smoothness for a reasonably quick optimization.
Figure 4.10 shows the results of optimization runs using LFmin with three different objective functions, using identical tapers, constraints and initial data. This figure shows that using a max norm or a 20th order norm can yield significant improvement over a 2nd order norm for the worst case reflection. Minimizing the 20th-order norm produces almost the same reflection characteristic as the max-norm, because a 20th-order norm heavily favors the maximum value. The high correlation of the two curves is no accident; in fact, the two points in $R^{10}$ are, as expected, quite close. They are, respectively,

$$x_{min_{20}}^T = [0.5844, 1.3539, 0.5000, 2.4950, 1.3000, 0.2670, 1.1000, 0.0028, 2.2200, 0.4000]$$

and

$$x_{min_{\infty}}^T = [0.5635, 2.0000, 0.5000, 2.5000, 1.3000, 0.3115, 1.1000, 0.0143, 2.5000, 0.4000]$$

LFmin found the first vector $x_{min_{20}}^T$ in 29 major iterations (solutions to the local quadratic search direction subproblem) involving 44 evaluations of the objective function. The search for the optimal max-norm took 40 major iterations and required 177 evaluations of the objective function. The max-norm itself is also more expensive to evaluate because comparisons are slower to compute than arithmetic functions, so more computation time is spent per iteration. Further, because the gradient is usually discontinuous in the neighborhood of a solution, the stopping points are not Kuhn-Tucker points, so it is difficult to evaluate whether they are local minima.
Figure 4.10: Comparison of Norm Orders

4.1.3 Designs Using Longer Tapers  Examination of Figures 4.4, 4.8 and 4.9 shows that lengthening the tapered section of the absorbers resulted in somewhat lower overall reflections, presumably because more of the incident waves penetrate into the backing layers, due to lower rates of incremental reflection. Due to this effect, it was decided to investigate what would happen with still more gradual tapering of the pyramids; even the best cases for above standard taper lengths above can yield reflection coefficients large enough at some frequencies to cause a resonance outside of the ±4 dB limits for FCC certification. In order to guarantee that the absorbers will perform within specifications when they are installed in a chamber, we should have reflection coefficients as low as possible, but adding the various reflection magnitudes for the many possible third-order reflections indicates that the individual magnitude reflection coefficients should all be below about 20 percent. It seems clear
that this will not be achieved with tapers of 62" or less.

For this reason, we also wished to study longer tapers, to see if the more gradual transition from air dielectric to 100% absorber could achieve our objective with a reasonable overall length of the absorbing structure. We eventually decided on trial taper lengths of 1.8, 2.0 and 2.2 meters, or 70.9", 78.7" and 86.6" respectively. The total length constraint for the designs produced was 2.7432 meters, or 9 feet.

Using these taper lengths, S-parameters were generated for the various materials in Table 4.1. A variety of optima were generated using $L^2$ and $L^{20}$ norm objective functions. In the following cases, the samples were taken every 10 MHz from 30MHz to 100MHz and every 15° from 0° to 45° in the E-polarization only.

First, we wished to see what could be done with existing materials only. S-parameter for the 1.8, 2.0 and 2.2 meter tapers were generated for each of the materials in Table 4.1. Using these with one backing material of the same material and one of another material, we computed optima for all of the 72 possible 2-layer cases.

The results for these cases are quite good, and the designs produced should be sufficient for use in anechoic chambers for frequencies down to 30 MHz. Their E-polarization reflections are shown in Figures 4.11, 4.12 and 4.13. The design specifications for these absorbers are listed in the Appendix.

To compare what happens when LFmin has the freedom to continuously adjust the permittivity and conductivity of the materials, I used the same tapers as above with one backing layer of material identical to the pyramids and one freely-varying layer. There are 24 such cases. The best of these cases
Figure 4.11: 9ft Absorbers with Fixed Backing and 1.8m Square Pyramids

Figure 4.12: 9ft Absorbers with Fixed Backing and 2.0m Square Pyramids
Figure 4.13: 9ft Absorbers with Fixed Backing and 2.2m Square Pyramids is shown in Figure 4.14, which is quite similar in performance to the best case with fixed backing materials. It might seem that this minimum would be more expensive to find than the minimum for the fixed-material case, but since a smaller number of cases need to be computed, it is actually faster to find the minimum with continuously varying materials, even though each minimum is more expensive to compute in this manner. It is then possible to redo the optimization using a fixed material with properties similar to the “optimal” material, if materials cannot be readily made to order. Presuming the fixed material is fairly close to the “optimal” material, the behavior of the second design will be similar to that of the first.

Larger total length constraints were also considered, but it was felt 9 foot absorbers performed sufficiently well, while being sufficiently short to provide useful designs.
Figure 4.14: 9ft Absorbers with Free Backing and 2.2m Square Pyramids

4.2 Designs Using Twisted Pyramids

LFmin uses a file of S-parameters for the topmost layer of the stack of materials to be optimized. Conceptually at least, these S-parameters can be computed or measured and stored for any fixed device which is to be used as the top layer so the same program works without modification, except for the input data, for many different problems. This feature makes LFmin useful for comparison of different tapered absorber geometries, and easily adaptable to new designs for the uppermost (high frequency) absorbers.

The adaptability of LFmin was very useful when we were approached by IBM and Rayproof Corps. to design some new absorbers for IBM's Boca Raton chambers. These chambers originally used standard Rayproof 96" absorbers, but the design resulted in fields outside the FCC certification limits at 60 MHz. These absorbers use a twisted-pyramid geometry; the tapers are
Table 4.2: Bulk properties some Rayproof absorber materials

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2.2 meters long, with the remaining .24 meters of solid bulk material.

The complex permittivities of several Rayproof absorber materials were measured and supplied to us by IBM. Some of these data are listed in Table 4.2. The material labeled C1 was used to manufacture the Boca Raton absorbers.

Figure 4.15 shows the E-polarization reflections computed for the original design. These absorbers were optimized by Rayproof using a cut and try method to minimize reflection in a waveguide at 120 MHz. Effectively, this means that the reflection should be minimal for Rayproof's original design at 120 MHz and 31° in the electric polarization. The figure shows that this is in fact true according to our calculations as well. The figure also shows the reflection peak at 60 MHz that caused the chamber to fail FCC certification tests. Optimizing the reflection for the single frequency and angle of incidence found a reflection null, but the reflection at lower frequencies increases rapidly.
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<th>Frequency (MHz)</th>
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meters long, with the remaining .24 meters of solid bulk material.

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Figure 4.15: Computed Reflections for Original Boca Raton Absorbers (E) lower frequencies increases rapidly.

We wanted to improve on the original design by removing the existing backing layer (the solid layer behind the tapers) and allowing LFmin to find the best possible 96\degree or shorter absorber design using the existing tapers.

The S-parameters of the original tapers were computed using equation 2.21 by the program TWcones, which is a version of cones for the twisted-pyramid absorbers. These S-parameters were stored in data files and used by LFmin.

Since the largest reflections were the source of the problem, the $L^{20}$ objective function was used over a range of angles from normal to 45 degree incidence. The frequencies were sampled from 30 to 100 MHz at 10 MHz intervals.

Minima were computed for 96\degree absorbers with materials fixed and
with "freely" varying materials. The $L^{20}$ objective functions for each case are shown in table 4.3 (cases 0-4). In this table, the "layers" column shows how many layers were allowed at the start of LF_{min}. If the layers number is followed by a star, the actual number of layers was reduced by LF_{min} for the optimum case. The "free" column shows materials that were allowed to vary. Materials are numbered from the bottom (nearest to the chamber wall) to the top (nearest to the cones). Case 0 in this table is the original design. This table shows that little can be done to improve on this design within the 96" length constraint, since even the most general case produces an objective value only 1.2 percent lower than the original design. Relaxation of this constraint seems to be indicated.

By increasing the length constraint to 108", or 9 feet, the norm can be decreased substantially. Even using only the C1 bulk material (case 4), some improvement can is made. The most general case (case 9) is very good, since reflection must be almost everywhere below 20 percent. Case 7, with layer 1 of C1 material, is nearly as good, and would be simpler to manufacture, since it requires only two materials, including the tapered sections. The reflections for this design are shown in Figure 4.16

Increasing the length constraint still further has little effect on the optimized objective function. Case 10 shows the same problem, subject to a 120" (10 foot) length constraint. The minimum of the objective function lies within this constraint, not on it, so it seems unlikely that still larger constraints would improve on this optimum.

To make a fair comparison of twisted pyramids to rectangular pyramids, it was necessary to consider a variety of cases. To this end, we used
Table 4.3: $L^{20}$ Optima for 2.2 meter C1 Twisted Pyramids

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<td>108&quot;</td>
<td>108&quot;</td>
<td>1</td>
<td>1</td>
<td>0.2050</td>
</tr>
<tr>
<td>7</td>
<td>108&quot;</td>
<td>108&quot;</td>
<td>2</td>
<td>1</td>
<td>0.2037</td>
</tr>
<tr>
<td>8</td>
<td>108&quot;</td>
<td>108&quot;</td>
<td>2*</td>
<td>2</td>
<td>0.2050</td>
</tr>
<tr>
<td>9</td>
<td>108&quot;</td>
<td>108&quot;</td>
<td>2</td>
<td>1,2</td>
<td>0.2016</td>
</tr>
<tr>
<td>10</td>
<td>108.3&quot;</td>
<td>120&quot;</td>
<td>2</td>
<td>1,2</td>
<td>0.2015</td>
</tr>
</tbody>
</table>

To solve for the S-parameters of 1.8, 2.0 and 2.2 meter tapers consisting of materials C1, C3, C4 and C5 from Table 4.2. This gave a considerable selection of pyramids to consider. For each set of pyramids, minima of both the $L^2$ and $L^{20}$ objective functions were generated using $\text{L}_{\text{Fmin}}$, with both of two backing layers consisting of fixed materials. The materials used for the backing layers were C1, C3, C4, C5 and C7. The total number of cases considered was 96. Only the $L^{20}$ minima are included here because it was felt this objective function produced more useful designs.

Figures 4.17, 4.18 and 4.19 show the best cases for each taper length; they parallel Figures 4.11, 4.12 and 4.13. Comparing the two pyramid geometries (square and twisted pyramids) shows that, given the same overall length constraints and taper lengths, comparable results can be produced by optimizing the backing layers: neither geometry is clearly superior.
Figure 4.16: Reflections from 108" Optimized Absorbers Using 2.2m Tapers

Figure 4.17. Optimal 9ft Absorbers with Fixed Backing and 1.8m Twisted Pyramids
Figure 4.18. Optimal 9ft Absorbers with Fixed Backing and 2.0m Twisted Pyramids

Figure 4.19. Optimal 9ft Absorbers with Fixed Backing and 2.2m Twisted Pyramids
CHAPTER 5

SENSITIVITY OF COMPUTED MINIMA

In Chapter 4, it was demonstrated that considerable improvement may be gained by optimizing a multilayer backing for a set of pyramid absorbers. Unfortunately, the optimal designs cannot be produced exactly, because it is impossible to control the manufacture of the absorbing materials to very high precision, such as that used in computing the minima. Also, the absorbing materials can absorb water from the air, which alters their conductivity and permittivity, sometimes substantially. It is therefore necessary to see what effects variations in the manufacturing process might have on the overall performance of a wall of absorbers. If small variations in the materials or dimensions produced large variations in the reflections, the designs would be too sensitive to be of any practical importance.

It seemed impractical to test the sensitivity of every design in this thesis, especially since the process of using Rcones or TWcones to compute the S-parameters takes a considerable amount of time and would need to be re-run for each design with altered material properties. Instead, a family of variations on a single design was generated for comparison with one original (optimal) design. (This design minimizes the $L^2$ objective function over 30-100 MHz and normal incidence). For each section of the absorber (Pyramids, Layer #1 and Layer #2), a $+5\%$ variation was made in its length, real and imaginary permittivity. The effects of these perturbations on the magnitude
of the normal reflection coefficient are shown in Table 5.1; column 0 of this
table shows the reflection coefficients for the original (optimal) design. The
reflections for the perturbed design are almost the same as for the optimal
design, which demonstrates that the design is not overly sensitive, and could
presumably be manufactured successfully.

While some of the minima may be more sensitive than this case, it
seems reasonable to expect that sensitivity of the designs to manufacturing
variations will not be too severe. However, the sensitivity of any design should
be tested before it is manufactured and installed in a chamber at considerable
expense.
### Table 5.1: Effects of variations at Normal Incidence

<table>
<thead>
<tr>
<th>Freq. MHz</th>
<th>Magnitude of reflection coefficient for design #</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>.098</td>
</tr>
<tr>
<td>40</td>
<td>.138</td>
</tr>
<tr>
<td>50</td>
<td>.145</td>
</tr>
<tr>
<td>60</td>
<td>.112</td>
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<tr>
<td>70</td>
<td>.080</td>
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<tr>
<td>80</td>
<td>.072</td>
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<tr>
<td>90</td>
<td>.069</td>
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<tr>
<td>100</td>
<td>.059</td>
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<tr>
<td>120</td>
<td>.042</td>
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<tr>
<td>140</td>
<td>.045</td>
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<tr>
<td>160</td>
<td>.035</td>
</tr>
<tr>
<td>180</td>
<td>.034</td>
</tr>
<tr>
<td>200</td>
<td>.040</td>
</tr>
</tbody>
</table>

**design #** | +5% variation in:  
0 | optimized design “A”  
1 | pyramids: length  
2 | pyramids: $e'$  
3 | pyramids: $e''$  
4 | Layer 1: length  
5 | Layer 1: $e'$  
6 | Layer 1: $e''$  
7 | Layer 2: length  
8 | Layer 2: $e'$  
9 | Layer 2: $e''$  

### Design A parameters

<table>
<thead>
<tr>
<th>Section</th>
<th>material</th>
<th>length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>C1</td>
<td>0.315 m</td>
</tr>
<tr>
<td>Layer 2</td>
<td>C5</td>
<td>0.081 m</td>
</tr>
<tr>
<td>Tapers</td>
<td>C1</td>
<td>2.200 m</td>
</tr>
</tbody>
</table>

Twisted Pyramid Geometry  
1st layer is adjacent to the metallic wall.  
2nd layer is adjacent to the tapers.
BIBLIOGRAPHY


APPENDIX A

PARAMETERS OF DESIGNS INCLUDED IN THIS THESIS

The following are the parametric specifications of the designs shown in this thesis. The layer thicknesses in the "size" column are in meters. Parameters A1–A4 are as specified by equations 3.4 and 3.5.

Table A.1: Specifications of Absorbers for Fig 4.4

<table>
<thead>
<tr>
<th>layer</th>
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<th>A3</th>
<th>A4</th>
</tr>
</thead>
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<td>1.0589</td>
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<td>.00757</td>
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<td>0.4000</td>
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Table A.2: Specifications of Absorbers for Fig. 4.7

<table>
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<th>size</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
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<td>1.1000</td>
<td>.00000</td>
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<td>0.40000</td>
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Table A.3: Specifications of Absorbers for Fig. 4.8

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<th>A2</th>
<th>A3</th>
<th>A4</th>
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</thead>
<tbody>
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<td>0.4000</td>
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</table>


tapers: 48" of R2 material

tapers: 40" of R2 material


tapers: 48" of R2 material
Table A.4: Specifications of Absorbers for Fig. 4.9

<table>
<thead>
<tr>
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<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
<td>0.3403</td>
<td>R2 material</td>
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<td></td>
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</tbody>
</table>

Table A.5: Specifications of Absorbers for Fig. 4.11

<table>
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<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>R3 material</td>
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<td>0.297</td>
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Table A.6: Specifications of Absorbers for Fig. 4.12

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</thead>
<tbody>
<tr>
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<td>R3 material</td>
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<tr>
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<td>0.3340</td>
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Table A.7: Specifications of Absorbers for Fig. 4.13

<table>
<thead>
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<th>A2</th>
<th>A3</th>
<th>A4</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.064</td>
<td>R3 material</td>
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Table A.8: Specifications of Absorbers for Fig. 4.14

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
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Table A.9: Specifications of Absorbers for Fig. 4.16

<table>
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<td>0.1592</td>
<td>C1 material</td>
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</table>
Table A.10: Specifications of Absorbers for Fig. 4.17

<table>
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<td>1</td>
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<td>2</td>
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<td>C1 material</td>
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</tbody>
</table>

tapers: 1.8m of C1 material

Table A.11: Specifications of Absorbers for Fig. 4.18

<table>
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<tbody>
<tr>
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</tr>
<tr>
<td>2</td>
<td>0.330</td>
<td>C1 material</td>
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</tbody>
</table>

tapers: 2.0m of C1 material

Table A.12: Specifications of Absorbers for Fig. 4.19

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<tr>
<td>1</td>
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<td>0.254</td>
<td>C1 material</td>
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</tbody>
</table>

tapers: 2.2m of C1 material
APPENDIX B

COMPARISON OF MEASURED AND FITTED MATERIAL PROPERTIES

When "freely-varying" materials are used in the absorber designs of this thesis, the materials are specified by a set of four continuously varying parameters. These parameters allow substantial flexibility in adjusting the dispersion curves. This flexibility is sufficient to provide us with useful designs, but does not represent the full range of characteristics found in the measured data.

The parametric model was tailored to fit the various Rayproof materials as closely as possible. Accordingly, the "best fits" match the measured properties almost exactly for these materials. The most lightly doped (least absorptive) Rayproof material is C1; figures B.1 and B.2 show the real and imaginary parts of the measured permittivity and the best fit of the parametric permittivity model to these data. The fit is very good, showing that the parametric material model can fit at least some real frequency data. In fact the fits for all of the Rayproof materials are comparable. Figures B.3 and B.4 show the best fit to the real and imaginary parts of the C7 material (the most absorptive of the samples). All of the Rayproof materials fit similar curves because they are made by the same process.

The Rantec materials are made by a somewhat different process and as a result, do not easily fit the same functional description as the Rayproof
Figure B.1: Measured vs. Parametric Permittivity of C1 Material (real)

Figure B.2: Measured vs. Parametric Permittivity of C1 Material (imaginary)
Figure B.3: Measured vs. Parametric Permittivity of C7 Material (real)

Figure B.4. Measured vs. Parametric Permittivity of C7 Material (imaginary)
materials. An example of the “best” fit to the Rayproof data is shown in figures B.5 and B.6. Since this fit is not very good, it might be helpful to make a new set of parameters for the Rantec process.
Figure B.5: Measured vs. Parametric Permittivity of R1 Material (real)

Figure B.6. Measured vs. Parametric Permittivity of R1 Material (imaginary)