Aspects of Tachyon Condensation in String Theory: The Open and Closed Cases

by

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The final copy of this thesis has been examined by the signatories, and we find that both the
ccontent and the form meet acceptable presentation standards of scholarly work in the above
mentioned discipline.
A nonperturbative formulation of M/String theory is lacking. One phenomenon necessarily sensitive to the underlying nonperturbative dynamics is vacuum decay. Tachyonic fluctuations in perturbative string theories indicate an instability of the vacuum. We investigate aspects of tachyon condensation in both the open and closed sectors of string theory. In the case of open string tachyons we review and extend the construction of unstable D-branes as noncommutative solitons. This incorporates techniques from noncommutative field theory, string field theory, and an important conjecture due to Sen. We also extend this discussion to actions consistent with T-duality and comment on a technique developed to generate soliton solutions. For the case of closed strings we conjecture that the end point of tachyon condensation, at any non-zero coupling, involves the annihilation of space time by a bubble of nothing, resulting in a topological phase of the theory. In support of this we present a variety of situations in which there is a correspondence between the existence of perturbative tachyons in one regime and the semi-classical annihilation of space-time. Our discussion will include many supersymmetry breaking scenarios in string theory including Scherk-Schwarz compactifications, Melvin magnetic backgrounds, and noncompact orbifolds. We use this conjecture to investigate a possible web of dualities relating the eleven-dimensional Fabinger-Horava background to nonsupersymmetric string theories. Along the way we point out where our conjecture resolves some of the puzzles associated with bulk closed string tachyon condensation. We finally discuss the implications of these analyses to a nonperturbative formulation of M-theory. Included as an appendix is a discussion of the use of T-duality to map the conditions for supersymmetry preservation in D-brane systems at angles to analogous conditions on background magnetic fields. We also discuss T-dual versions of noncommutative tachyon decays resolving some ambiguities therein.
Dedication

for my mom and dad
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Chapter 1

Introduction

The primary investigation in this thesis is the question, "Where does the condensation of a tachyonic perturbative degree of freedom in string theory take us?" We will pose this question more sharply and offer some evidence for answers in a few important realizations. But let us first offer a definition and establish the context for which this question holds relevance.1

1.1 What Is A Tachyon?

In a relativistic theory of point particles, a tachyon is simply a particle which travels faster than the speed of light, i.e. \( v > c \). Another useful way of putting this is as follows. If we consider the relativistic expression for kinetic energy

\[
T = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}
\]  

then it becomes clear that for \( v > c \) the denominator becomes imaginary. If we wish to utilize an action principle which involves the integration of real valued functions, then we must accomodate this by allowing the mass parameter \( m \) to be imaginary. Thus we may equally identify a tachyonic particle as one for which \( m^2 < 0 \). Tachyons are certainly an interesting possibility in this context. Unfortunately there is considerable evidence that point particle theory is insufficient to describe our universe. So far the most successful descriptions, general relativity and the standard model, are formulated as field theories.

1 A note on notation: Due to the thesis package used to generate this document it was impossible in a number of instances to form the mathematical character "times" which signifies multiplication. In these cases I have opted to utilize "\( \otimes \)" in its stead, though technically this should be reserved for a direct product operation. Hopefully the context will clarify what is meant by "\( \otimes \)" when it appears.
In relativistic field theory, wherein “particles” usually amount to small fluctuations (suitably localized) of a continuum field, the nature of a tachyon is considerably clearer. We will consider as an example a theory with a single scalar field $\phi$. The theory is defined by specifying a Lagrangian density

$$L(\phi, \partial \phi)$$  \hspace{1cm} (1.2)

We first find a solution to the equations of motion which follow from demanding a vanishing variation of the action

$$\delta L(\phi, \partial \phi) = 0 \rightarrow \phi_0.$$  \hspace{1cm} (1.3)

Allowing the fields to fluctuate about this background,

$$\phi = \phi_0 + \delta \phi,$$  \hspace{1cm} (1.4)

one rewrites the Lagrangian density in terms of these degrees of freedom,

$$L(\delta \phi, \partial \delta \phi).$$  \hspace{1cm} (1.5)

To treat the resulting dynamics perturbatively we restrict to small fluctuations. This allows an useful power series expansion of the Lagrangian density in terms of the fluctuations. We then identify the term quadratic in the field fluctuation as the “mass” term for the particle to which it corresponds. For a Lagrangian density of the form $L = (\partial \phi)^2 - V(\phi)$, and assuming a Lorentz invariant solution $\phi_0$, the mass is simply given by

$$m^2_\phi = \frac{1}{2} \frac{\partial^2 V}{\partial \phi^2}.$$  \hspace{1cm} (1.6)

Now if in this framework we are given that some particle exhibits the property that $m^2 < 0$, then the interpretation of this is simple. The mass squared corresponds to the second derivative of the potential (again assuming a Lorentz invariant classical solution). Thus the presence of a tachyon signifies that the extremum of the potential that we have used as our vacuum is a local maximum as shown in Fig.1.1. A tachyonic fluctuation in relativistic field theory indicates that the vacuum is unstable.
This is not a situation unfamiliar to particle physicists. To explain particle masses in the standard model without spoiling gauge invariance we add to it a tachyonic field called the Higgs. The rolling of this field towards a stable vacuum state spontaneously breaks electroweak symmetry and gives masses to many of the standard model particles.

1.2 What Is String Theory?

In this section we will answer the question “What is string theory?” We will also argue that our answer is incomplete, and that the heart of this question remains unanswered.

The chief merit of string theory lies in the fact that it is at this point the only quantum mechanical theory we have come across which incorporates gravitation and yields finite amplitudes for graviton scattering. We might stop here and posit that this theory thus deserves a full understanding. But of course to demonstrate its full relevance, we must at some point embed the standard model plus Einstein-Hilbert gravity in a cosmologically viable setting as some low-energy solution to string theory. To realize its full power we should furthermore explain why the theory prefers this solution over other possibilities. While both these enterprises, accommodation and predictivity, are under intense investigation, we feel that it is also useful (and perhaps necessary) to explore the actual nature of string theory itself.

To clarify what we mean by the “nature of string theory” we will consider an analogous scenario in point particle theory.

Quantum mechanics affords us a way to answer questions like “What is the probabil-
ity for a given initial particle configuration (in-state) to evolve into a given final configuration (out-state)? This is part of the central dogma of any quantum theory. We prepare a definite experiment then list all possible outcomes. For each outcome we can compute an associated probability for its realization in any given copy of the experiment. This is all that we can do.

Relativistic quantum theory (RQT) allows us to do this in a relativistically covariant manner incorporating the creation and annihilation of particles. When we compute a probability, the answer usually includes a purely kinematic factor from considerations of phase space and more importantly a quantum mechanical amplitude or matrix element. For an interacting system this amplitude can be computed by summing a series of diagrams representing various space-time paths connecting the initial and final states as shown in Fig. 1.2. Each contribution is distinguished by the creation and annihilation of various intermediate particle states.

\[ \text{Figure 1.2: The Feynman diagram approach to scattering amplitudes. Here we have the first few terms in a } 2 \rightarrow 2 \text{ scattering process. In the first quantized approach the vertices and weights of different terms must be put in by hand. In the second quantized formalism this represents a well-defined perturbative expansion for an action including a } \phi^4 \text{ interaction.} \]

We arrange the sum in order of increasing number of vertices. For sufficiently small coupling (the perturbative regime) the amplitude can be well approximated by the first few terms. This is of course the usual Feynman diagram approach. In fact the theory in this case is defined by its Feynman expansion. This program, quantizing relativistic point particles, is often called the "first quantized" approach. Another way to think about the first quantized approach is that it is an otherwise classical relativistic point particle theory upon which we impose the noncommutativity between the underlying coordinates and their conjugate momenta. In the path integral language this means we sum over all possible particle paths connecting the initial and final configurations.

An important element of RQT is that of background dependence. Since we are limited to
the dynamics of point particles, we must assume a priori the physical background on which these small fluctuations propagate. That is we must assume a spacetime geometry and configurations for any other fields that are part of the theory. RQT then allows us to consider the behavior of small fluctuations on this otherwise fixed background.

Quantum field theory (QFT) unifies the notion of a background and the particles that propagate upon it. One proceeds by first promoting all of the degrees of freedom in the theory, excepting the space-time itself, to fields governed by some relativistically covariant Lagrangian density. The resulting field theory is then quantized directly, including now the background degrees of freedom. The resulting quantum theory can reproduce the perturbative behavior of small fluctuations on a fixed background, i.e. the Feynman expansion as shown in Fig.1.2. So QFT contains in a perturbative limit at least one RQT. However, it also affords us a method for determining the allowed backgrounds as solutions (vacua) of the equations of motion following from applying the action principle to the Lagrangian density. Furthermore we may entertain the question of in and out states corresponding to different vacua of the theory (tunneling) and other nontrivial dynamical phenomena that can not be seen at any order in a perturbative expansion (order $e^{-\frac{1}{\hbar}}$ amplitudes). On a very good day it might even allow us to treat perturbatively a theory which in one set of variables appears strongly coupled. This program is often termed the second quantized approach. Another way to think about the second quantized approach (specifically in contrast to the first quantized one) is that we begin with a classical relativistic theory of fields and impose noncommutativity between the fields and their conjugate momenta. In the path integral language this means we sum over all possible field configurations connecting the initial and final configurations.

Given that QFT generally contains more information than RQT one may ask if in fact one of these is in any sense "more correct" than the other. If our universe and everything in it were correctly described by an RQT, then perhaps we might deem a "parent" QFT superfluous. However our experience is that our world must incorporate nonperturbative phenomena beyond the content of any RQT, e.g. quark confinement. Thus between these two we know our search
for the true theory should involve at least a QFT (as the low-energy effective description of some underlying fundamental theory).

Returning now to the question “What is string theory?”, we start by reminding the reader that the well-known perturbative string theories are formulated in the first-quantized approach. That is, in some sense we begin with a theory of classical relativistic strings (a generalization of the point particle idea) propagating on some fixed background. We then quantize the theory by allowing the splitting and joining of strings to form loops of virtual states (the stringy analogue of particle creation and annihilation). Actually this is not quite the story, but the spirit is the same.²

This construction, armed with a few consistency conditions which we will discuss later, has provided us with only a handful of perturbative theories that can propagate on a Lorentz invariant vacuum. This surprising degree of uniqueness starkly contrasts the case with RQT wherein there is an unlimited number of distinct perturbative theories that can propagate on any given background (we can just keep adding new particle species). Of course this is only the beginning of string theory’s seductive story, for not only are we granted this unexpected degree of uniqueness, but in each of these theories we are also forced to reckon with a massless particle of spin two, i.e. the graviton. That quantum string theory contains gravity (necessarily and consistently) is perhaps its single greatest triumph, but I will be setting this fact aside to focus on the question at hand.

Given that we have only a first quantized prescription for quantum string theory, one may ask whether these can be embedded into a larger, nonperturbative structure. Again if we could realize the observed universe in terms of one of these few perturbative string theories then the question of a nonperturbative formulation would be only of academic interest. However there are indications that our world can not be realized in one of these formulations[13]. It seems

² To be honest we begin with a (1 + l)-dimensional field theory. One can already see why this might confuse the point. After quantizing this world-sheet theory, we turn this inside out to form a theory in spacetime by interpreting these (1 + l)-dimensional “surfaces” as string trajectories. The quantum aspect of this spacetime theory is contained in the allowance of worldsheet topologies whose spacetime interpretation is that of a loop trajectory. Altogether, we build, in this convoluted fashion, something which can in the end be interpreted as a Feynman expansion. This defines the spacetime theory.
that if string theory is going to be our T.O.E. then we need a nonperturbative formulation. Furthermore once we start looking at nonperturbative aspects of string theory (using various tricks to be discussed later) we find that the degree of uniqueness sharpens. There is now convincing evidence that all of the consistent perturbative string theories arise as limits of a single underlying nonperturbative structure which we call M-theory.

So a short answer to the question “What is string theory?” seems to be, “String theory is M-theory.”

But this is less an answer than a label for what we still don’t understand. We do not have a microscopic formulation of M-theory. While we understand both sides of $\text{QFT} \rightarrow \text{RQT}$, we only understand the right hand side of $\text{M-theory} \rightarrow \text{String Theory}$. Indeed it is not yet clear what degrees of freedom will play a central role in M-theory. Our evidence for its existence largely involves the presence of dualities. These are essentially continuous deformations that permute the various perturbative string theories among themselves. These dualities can in some cases be demonstrated within perturbation theory (T-dualities), but in many cases involve explicitly nonperturbative deformations (S-dualities) which require information beyond perturbation theory. The usual program for establishing dualities involves using the constraints from spacetime supersymmetry and the properties of BPS states.

What do we know of M-theory beyond these limits? We know that M-theory has an additional limit that involves an 11-dimensional spacetime. This comes from deducing the strong coupling properties of the type IIA string theory. We also have a low-energy effective theory that is valid away from the perturbative string limits including the 11-dimensional case. From the form of this theory in 11 dimensions it seems that membrane degrees of freedom ($2 + 1$ dimensional objects) might play an important role, though their consistent quantization has not been achieved in a general manner. There have been attempts at formulating M-theory both as string field theories (in a limited sense) and even as a quantum mechanical theory of large matrices. We will have more to say about these ideas later.

Without going into the details too deeply we should at least outline what the general
picture looks like. Figures 1.3 and 1.4 summarize the results for the supersymmetric 10 and 11-dimensional limits of M-theory.

In each case we can start with M-theory in 11 noncompact dimensions and compactify on either a circle $S^1$ of radius $R$, i.e. $S_R^1$, or a line segment $I$ of length $L$, i.e. $I_L$.\(^3\) Obviously for either $R \to 0$ or $L \to 0$ we should obtain a theory in 10 dimensions. What is not obvious is that the single coupling constant of the resulting theories is proportional to $R$ or $L$. This explains the M-theory origin of two of the perturbative string formulations, i.e. Type IIA and Heterotic $E_8 \times E_8$, and in fact identifies their coupling in terms of a geometric part of M-theory. To extend this to include the other three consistent superstring theories in $M^{10}$ we only need to add an additional circle in each case. So we now consider M-theory on either $S_R^1 \otimes S_R^1$ or $I_L \otimes S_R^1$. Now if you are really following this you might ask how it is that we are going to

\(^3\)The latter is technically called a $Z_2$ orbifold of a circle or $I \equiv S^1/Z_2$. This means we start with a circle and then project the theory onto a $Z_2$ subgroup of the $U(1)$ part of the 11-dimensional Lorentz group corresponding to the isometries of the circle. The highly nontrivial aspect of this procedure is that we are forced to introduce gauge degrees of freedom localized to the 10-dimensional boundaries of the spacetime (an $E_8$ on each "wall" to be precise). Otherwise you can just think of it as a line segment.
Figure 1.4: The standard web of dualities obtained by supersymmetric compactifications of the Horava-Witten theory [42, 56].

get anything new in $M^{10}$ by further compactification. The answer is that we use T-duality to rewrite what looks like a string theory in 9 dimensions as a string theory in 10 dimensions. Strictly speaking strings on $S^1_{R \to 0}$ or $I_{L \to 0}$ are equivalent to strings on $S^1_{R \to \infty}$. \(^4\) Lastly, owing to the presence of gauge degrees of freedom in the orbifold background of M-theory, we can also introduce Wilson lines $Y$ in the compact directions to break the gauge symmetries.\(^5\)

Altogether we obtain the structure presented in Fig.1.3 and Fig.1.4 which encompass all five perturbative superstrings in $M^{10}$ (as well as the TypeI' theory with manifestly broken translational invariance) as limits of simple compactifications of 11-dimensional M-theory. The “southeast” corner of these diagrams includes two 10-dimensional theories which are related by an S-duality transformation, i.e. $g \to \frac{1}{g}$.

The quantities we vary to move about in this picture are called moduli. Starting from

---

\(^4\) This only holds for the stringy limits, i.e. we cannot apply T-duality to the 1-dimensional compactifications of M-theory and return to 11 dimensions.

\(^5\) These are simply constant gauge field configurations (which are trivial in noncompact space) that give rise to phases for charged states as they traverse the compact direction. We can introduce a Wilson line for each independent $U(1)$ subgroup of the gauge symmetry. The broken generators are then the off-diagonal ones charged under the $U(1)$ subgroup for which we turn on the Wilson line. Note that this symmetry breaking mechanism necessarily preserves the rank of the gauge group.
M-theory these are simply geometric radii and Wilson lines, whereas in moving between stringy corners these include the coupling of the theories as well. In the low-energy effective description of M-theory (which again can be used throughout the space of vacua) these moduli manifest themselves as scalar fields. This distinguishes moduli from say the parameters that we use when defining field theories, e.g. the couplings. In principle the moduli may vary throughout the spacetime or, as in conventional field theories, take constant vev’s. The only restriction is satisfying the underlying dynamics of M-theory.

Finally we should make a few comments on the “tool” that has allowed many of these connections to be established. The $D(p)$-branes of string theory were first realized as charged $p$-brane solutions in the supergravities that describe the low-energy dynamics of the various perturbative formulations.\textsuperscript{6} Here $p$ refers to the number of spatial directions in which these objects are extended (their worldvolume includes the time direction as well). Their description was promoted to a fully microscopic one by Polchinski\textsuperscript{[55]}. In short he realized that these $p$-branes can be described by $(p+1)$-dimensional submanifolds of the spacetime on which open strings can end as shown in Fig.1.5. Thus they are called Dirichlet-branes, or $D$-branes for short, since we define them by imposing Dirichlet boundary conditions on certain worldsheet fields.

The lightest open string states on a $Dp$-brane include a massless vector state $A^\mu$ and in some cases a scalar tachyon. The massless vector should mediate a U(1) gauge theory on the $(p+1)$-dimensional brane, but it’s index $\mu$ runs over all of the dimensions of the bulk spacetime. This ambiguity is resolved by dividing the components of $A^\mu$ into those along the brane $A^i$ which mediate a U(1) gauge theory in the worldvolume of the brane, and those transverse to the brane $A^m$ which become scalars in the worldvolume theory transforming in the adjoint of the U(1). The latter serve to define the embedding of the brane into spacetime. When we bring two or more parallel $Dp$-branes together, the open strings stretched between them can become light enough to enter the massless spectrum and we find an enhancement of the gauge symmetry as shown in Fig.1.6\textsuperscript{[83]}. The low energy dynamics of these open string modes and their interactions

\textsuperscript{6} One can think of these as spatially extended supersymmetric charged black hole solutions.
Figure 1.5: A $Dp$-brane in a D-dimensional spacetime. The worldvolume extends in $x^0, x^1, \ldots, x^p$. This leaves $x^{p+1}, \ldots, x^{D-1}$ as transverse coordinates.

with the background closed string fields is neatly encoded in a Born-Infeld type action.

Figure 1.6: Three parallel $Dp$-branes in spacetime. When the separation $d$ becomes sufficiently small, the gauge symmetry will be enhanced $U(1) \otimes U(2) \rightarrow U(3)$. Note that strings stretched between different branes are oriented while those beginning and ending on the same brane are not.

The “standard” $D$-branes preserve half of the spacetime supersymmetries of the underlying closed string (or bulk) theory on their worldvolume.\footnote{“Standard” is roughly defined as isolated $Dp$-branes with $p$ even in a type IIA bulk, $p$ odd in type IIB, and $p = 1, 5, 9$ in type I.} This BPS property allows us to
calculate their tensions in perturbation theory and continue the exact result to strong coupling. This enables us to correlate the full perturbative and D-brane spectra of two S-dual theories.

In addition to the "standard" Dp-branes, there is a myriad of other possible constructions involving multiple branes oriented at various angles with respect to each other, systems of Dp-branes of different dimensionality, branes with nontrivial worldvolume field configurations, and even isolated "wrong-p" branes. In general these systems will break some or all of the underlying supersymmetry. Further discussion on some of these issues the reader may be found in the appendix.

D-branes are nonperturbative with tensions inversely proportional to the string coupling when viewed from any of the perturbative string descriptions. As such, in the first quantized formulation of string theory they comprise part of the fixed background against which we study perturbative string fluctuations. In a nonperturbative formulation these should play dynamical roles. In fact in many of the strong coupling continuations of the perturbative theories we find D1-branes replacing perturbative strings as the fundamental fluctuations.

In closing we should point out to the reader that insofar as our interests are in the nature of string theory itself, we will leave phenomenology aside. As a result we will rarely be working in less than the critical number of dimensions for perturbative strings, i.e. $D = 26$ for bosonic strings and $D = 10$ for superstrings. The compactifications we do consider will be as simple as possible (e.g. $S^1, T^2, S^1/Z_2$, etc.), and are only introduced to move about in the space of 10-dimensional string theories.

1.3 Broken SUSY

While we are confident that M-theory is a supersymmetric theory, we know that it may contain vacuum solutions with some or all of its supersymmetry spontaneously broken. Except for our brief review of Dp-branes, our discussion has involved only the supersymmetric perturbative string formulations. We have not yet tried to fit nonsupersymmetric constructions into this picture, but we can already get an idea of how this might work in the supersymmetric cases.
The five perturbative superstring theories formulated on 10-dimensional Minkowski space, $M^{10}$, exhibit different amounts of unbroken spacetime supersymmetry. The type II strings have 32 supercharges (or $N = 2$ in 10 dimensions) while the type I and heterotic theories have 16 supercharges (or $N = 1$ in 10 dimensions). If these theories are simply different vacua of M-theory, then it seems evident that it should have at least $N = 2$ supersymmetry when dimensionally reduced to $M^{10}$. In 11 dimensions this corresponds to $N = 1$ supersymmetry. This is obviously the minimal number of supersymmetries for noncompact M-theory. It also happens to be the maximal number of supersymmetries for a theory in 11 dimensions if we want to avoid having particles of spin greater than 2.

In fact in the unifying spirit of M-theory the different amounts of supersymmetry in the 10-dimensional perturbative string limits can be traced back to a choice between one of two simple M-theory vacua, c.f. Fig.1.3 and Fig.1.4. If we start with M-theory compactified on a circle $S^1$, then the resulting theories will preserve the maximal supersymmetry of M-theory, i.e. $N = 2$ in 10 dimensions. These are the type II theories. On the other hand if we start with M-theory on a line segment $S^1/Z_2$ then the resulting theories preserve only half of the maximal supersymmetries, i.e. $N = 1$ in 10 dimensions. These include the type I and heterotic strings.

The breaking of $N = 2 \rightarrow N = 1$ supersymmetry for the Type I background can also be interpreted in terms of the introduction of space-filling $D9$-branes in an otherwise closed string vacuum. As mentioned above these preserve only half of the bulk supersymmetries, i.e. $N = 1$ remains. From this background we can reach (by duality transformations) other vacua with Lorentz invariance broken by the presence of $Dp$-branes with $p < 9$. In these backgrounds the bulk theory far away from the branes preserves the full $N = 2$ supersymmetry while the worldvolume theory on the branes preserves only $N = 1$.

In light of these simple pictures of breaking $N = 2 \rightarrow N = 1$, one may consider whether there are comparatively simple realizations of perturbative strings propagating on $M^{10}$ with completely broken spacetime supersymmetry. To this end there are three possibilities we may consider:
• We can try to formulate, in the first quantized approach, consistent perturbative string theories directly in $M^{10}$ which lack spacetime supersymmetry. This leads to a relatively small number of additional theories based on closed and oriented worldsheets. These are the Type 0A and Type 0B theories (closely related to the Type IIA/IIB theories) as well as seven nonsupersymmetric heterotic theories. When considering unoriented and open worldsheets, the lack of spacetime supersymmetry removes any potential uniqueness by admitting essentially any configuration of $D$-branes.

• We can look for vacuum solutions of 11-dimensional supergravity (the low-energy description of noncompact M-theory) which completely break the spacetime supersymmetry. One can then try to argue that these reduce to the low-energy dynamics of 10-dimensional nonsupersymmetric string theories.

• As discussed earlier, we can use the properties of $D$-branes to construct consistent backgrounds which break all of the spacetime supersymmetry.

Each of these enterprises has enjoyed quite a bit of attention by string theorists. In the course of this thesis we will make use of all three of these scenarios. Before moving on let us motivate why these types of background are of interest.

In some cases the aim is phenomenological with an eye on breaking supersymmetry at the string scale. Of course this immediately presents a problem insofar as the supersymmetry breaking scale is naively $M_{SUSY} \sim M_{string} \sim M_P$. Thus supersymmetry in this framework can not protect the scalar Higgs mass (as it was designed to do) below $10^{19} GeV$, rendering it useless for phenomenology. This problem has been addressed by considering low-scale strings wherein $M_{string} \sim 1 TeV$, and also by trying to use the delicate cancellations of consistent string theories to get boson/fermion radiative correction cancellation despite a lack of spacetime supersymmetry. These constructions have their drawbacks, but at least yield some sensible reason to study phenomenology with string scale supersymmetry breaking.\footnote{We should mention that this is not the only avenue to SUSY breaking in string theory. Since string theory}
In other cases the aim is towards gaining more insight into the underlying dynamics of nonperturbative string theory. This is the root of our interest. While it may not be obvious that there is much to gain from studying M-theory's less symmetric vacua, it actually provides a setting for one of the most relevant phenomena in its nonperturbative dynamics. This is the subject to which we now turn.

1.4 Stability

The five supersymmetric perturbative vacua of string theory in $M^{10}$ are stable with vanishing cosmological constant. This is of just a reflection of the fact that any vacuum of a supersymmetric theory that preserves some of the supersymmetry will be at least a local minimum. The dualities that map out connections between the supersymmetric perturbative vacua also preserve spacetime supersymmetry along the deformation path. This tells us that in terms of the "potential" of M-theory, we are moving along a flat direction, at least perturbatively. Therefore the locus of supersymmetry preserving points in the moduli space is represented in the low-energy description by a collection of backgrounds admitting massless scalar fields with vanishing potentials. These statements may in fact be modified by nonperturbative effects which can generate potentials for the moduli at points away from the five noncompact vacua at vanishing coupling.

If we dispense with spacetime supersymmetry in string theory then the background generically becomes unstable. There are a number of ways that a theory can exhibit instability:

- Tachyonic instabilities arise for static backgrounds when we have accidently formulated the perturbative theory around a local maximum of some underlying potential.

- Semiclassical tunneling instabilities arise for static backgrounds when we have formulated the theory in a local minimum of the potential, say at $V$, that is "near" another point of lower energy $V' < V$, and the two are separated by a barrier of finite height.

\[15\]
• We could alternatively find a nontrivial potential for some modes indicating that our solution is not a consistent static background.

In all three cases the theory as formulated does not remain in the perturbative vacuum in which it began. At varying rates in time, the backgrounds will evolve way from these unstable points. The question we would like to answer is, “Where do these theories go?” The answers must take us beyond what we know in the various perturbative regions and hence, if we can find them, might serve as segways into learning more about the nonperturbative structure of M-theory.

Though we focus on the question of the fate of certain tachyonic backgrounds, we will in due course find ourselves discussing all three of the scenarios above.
Chapter 2

A Brief Account of What Follows

Before doing my best to lose the reader in the details, I will provide a brief account in words of what will follow.

The remainder of this thesis is divided into two sections. The first section focuses on tachyon condensation in the open string sector while the second section focuses on the same issue in the closed string sector. The similarity of the two sections largely stops there. One reason for this is that while the main gist of open string tachyon condensation is well understood (and consequently our work is on honing the finer points of the process), the case for closed strings is considerably less clear. The dissimilarity goes even further. The discussion of the open string sector will make use of two techniques that are left out of the closed string discussion altogether. The first technique, that of string field theory, is in principle applicable to both the open and closed string sectors, but has proven tractable only for open strings. The second technique, that of noncommutative geometry, is available only in the open string sector. The flavor of the discussion in the closed string case will be more circumstantial. Specifically, we will be presenting evidence for and exploiting a conjecture correlating closed string tachyons and their condensation to better understood semi-classical instabilities. Coming full circle, our motivation for this conjecture will be an analogous correlation in the better understood case of open string tachyon condensation.

Despite a dissimilarity in technique, the two analyses will promote a common theme. The idea that emerges is that when we break SUSY at the string scale, the resulting instabilities
roughly lead to an annihilation of the sector of the theory in which we break supersymmetry. For open string tachyons supported on the worldvolume of an unstable $D$-brane configuration, the condensation corresponds to an annihilation of the $D$-brane(s) and consequently the open string degrees of freedom supported thereon. At the end of the day we are left with whatever supersymmetric closed (and possibly additional open) string backgrounds that came along for the ride. This simple picture, that the tachyonic open string configuration is an excitation above a supersymmetric background, has motivated the claim that string scale SUSY breaking instabilities generically tend to drive the theory back to a supersymmetric ground state. When we try to extend this discussion to a tachyonic closed string sector the picture becomes more ambiguous, potentially catastrophic, and thus considerably more interesting. Our conclusion, which contrasts the popular view, is that the closed string case is exactly parallel to the open string case. However in eliminating the closed string sector of the theory we are forced to accept an elimination of the spacetime itself. In a rough sense, $D$-branes supporting open string tachyons tend to annihilate, while spacetimes supporting closed string tachyons tend to annihilate as well.

What does this tell us about the nature of string theory? If we can argue (and we will) that these vacua with broken supersymmetry are part of the moduli space of M-theory, then M-theory should also include the vacua to which they decay. This implies that the concept of geometry in M-theory is an emergent one. A similar idea is already present in the Matrix model of M-theory. In the this formulation there is a prescription for building $Dp$-branes, or “open-string geometries”, by combining networks of fundamental “open-string geometry bits”. The common terminology is that in the Matrix model general $Dp$-branes are built out of fundamental $D0$-branes.
Chapter 3

The Open String Case [21]

The process of open string tachyon condensation is now well understood. Most of the progress in this subject has hinged on an important conjecture due to Sen [71, 68, 70, 68, 69]. We will review the content of this conjecture later and see how it has been applied to some aspects of open string tachyon condensation. Support for this conjecture has come from a variety of analyses including calculations in level-truncated open string field theory, arguments in conformal field theory, and applications of noncommutative geometry. Our discussion will focus on the last of these. The first two involve technologies sufficiently disparate from the case we will consider to preclude any discussion. For the interested reader Rastelli, et al [57] and Sen [74] provide reviews.

The presence of noncommutative geometry is limited to the open sector of string theory. It essentially arises due to the nontriviality of ordering operators inserted on the boundaries of open string worldsheets. Sen’s conjecture is applicable even in the backgrounds admitting noncommutative descriptions. This has been combined with an elegant technique for finding soliton solutions in noncommutative field theory proposed in [32], resulting in a compelling picture of unstable D-brane decay [19, 39, 84]. Several issues associated with this construction were criticized and improved on in [33, 76] and significant insight was made in [66]. Some issues however remain outstanding and this is the subject of the analysis to follow. These include various vacuum degeneracies, inexplicable solitonic configurations, the missing magnetic 21-brane soliton, and unwanted fluctuations on “good” solitonic solutions. Speculations on the
resolution of some of these issues were put forward [38, 73, 74]. In addition, a technique for
generating solutions to the equations of motion from known solutions was proposed in [37].

Our analysis will expand on the idea [66] that these outstanding issues may be resolved
by identifying the underlying degrees of freedom as $D0$-branes or $D$-instantons. Though this
is obviously closely related to a M(atrix) theory [4] interpretation, we will be working with the
unstable branes of open bosonic string theory. Unlike the BPS $D0$-brane partons of M(atrix)
theory, our fundamental constituents of all $Dp$-branes will themselves be the unstable $D0$-branes.
The instabilities of all of these $Dp$-branes are reflected in the presence of a tachyonic open string
state.

3.1 Techniques for Open Strings

In this section we will review the two techniques and an important conjecture that will
be put to use in analyzing the condensation of tachyons in the open string sector. We present
them independently, though our analysis will ultimately involve a combination of all three.\footnote{The section on string field theory is considerably more tedious than the others. The reader is encouraged to get the flavor of the discussion and not get lost in the details.}

3.1.1 String Field Theory

In the same sense that we can start with a first quantized particle theory and work our way
backwards to a quantum field formulation, we can also rewrite the content of any perturbative
string formulation in terms of a string field theory. An important limitation in both cases is
that this naive field formulation cannot confidently say anything about the theory beyond what
we already know in the perturbative region around the background with which we started. We
will discuss how to move beyond this limitation in the next section, but for now we will sketch
the ideas that go into formulating a simple string field theory. These are largely due to Witten
[82].

While string theory is most concretely envisioned of as a theory of little extended objects
flurrying about in spacetime (and indeed it is formulated as such), this is not the only way to

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[82].

While string theory is most concretely envisioned of as a theory of little extended objects
flurrying about in spacetime (and indeed it is formulated as such), this is not the only way to
think about it. We can equally consider string theory as a quantum field theory constructed in terms of a path integral over configurations of fields defined over spacetime. That this is a possibility is encouraged by the fact that if we consider the limit of strings going to zero length $l_s \to 0$, we recover a theory of fields. One simply needs to work out how to encode the "stringy" properties of string theory into the field formulation. There are two trademark stringy properties that must be encoded. One is that the multiplicity of perturbative states in spacetime (an infinite multiplicity in fact) follows from a single worldsheet theory. The other aspect is that the extended nature of strings allows them to wrap around various objects like singularities and compact dimensions of spacetime. If we stick to noncompact and singularity free backgrounds then we need only worry about the first issue.

First quantized string theory allows us to compute $S$-matrix elements in a fixed background. These data can be organized into a propagator and vertex form that looks like the perturbation theory for a field theory with an infinite number of ordinary fields.\(^2\) If, at the end of the day, our string field theory must be written explicitly in terms of an infinite number of fields, then this will not be of much use. However we suspect that since all of these fields arise from a single worldsheet theory, then they should be succinctly described in terms of a single infinite "gauge like" multiplet. We will refer to this as the string field $\Psi$.

The string field $\Psi$ is a functional of the fields defining the worldsheet theory, i.e. $X^\mu$, $\psi^\mu$, and the ghost fields $b, c, \bar{b},$ and $\bar{c}$. These are in turn functions of the worldsheet coordinates $\sigma_1, \sigma_2$. For simplicity we will focus on the open bosonic theory in the BRST-invariant form.\(^3\)

Thus the worldsheet theory is comprised of 26 $X^\mu(\sigma)$-fields and two independent ghost fields $c(\sigma)$ and $\bar{c}(\sigma)$. To see how we encode the spectrum into this string field we can expand it in terms

\(^2\) Obviously if the string spectrum contains an infinite tower of states in spacetime, then we need an infinite number of fields to accommodate these.

\(^3\) BRST quantization is merely a consistent method of quantizing a gauge theory in a gauge invariant form, i.e. without fixing the gauge. Generically this would introduce unphysical negative norm states, but these are cancelled by the introduction of Faddeev-Popov ghosts. In the case of quantizing string world sheet theories, the gauge invariance of interest is actually spacetime Lorentz invariance.
of ordinary fields in spacetime.

$$\Psi[X^\mu, c, \bar{c}] = \sum_i \Phi_i(x^\mu) \Psi_i[X^\mu, c, \bar{c}]$$  \hspace{1cm} (3.1)

The functions $\Phi_i$ describe the center-of-mass degrees of freedom for each state in the spectrum. The internal states are in turn specified by the functionals $\Psi_i$. The “stringy” aspect of the theory will be encoded in these functionals. A path integral over string field configurations $\Psi$ can now be written as an infinite product of path integrals over the center-of-mass modes for the component fields,

$$\int [d\Psi] \rightarrow \prod_i \int [d\Phi_i].$$  \hspace{1cm} (3.2)

We build the spectrum of functionals $\Psi_i$ just as we construct the spectrum on the world sheet. We start with some ground state functional $\Psi_0$ and then build the remaining states by applying raising operators constructed from the various worldsheet fields, e.g. $X^\mu \rightarrow \alpha^\mu_{-1}$. The set of functionals can be written as

$$\Psi_i = (\Psi_0, \alpha^\mu_{-1} \Psi_0, \ldots)$$  \hspace{1cm} (3.3)

where $(\ldots)$ involves ghost field raising operators and higher order combinations. To guarantee the string field be a spacetime scalar we must pair each functional with a center-of-mass mode of appropriate spacetime quantum numbers. Finally we can write the explicit string field expansion as

$$\Psi = (\phi(x) + A_\mu(x)\alpha^\mu_{-1} + \ldots)\Psi_0.$$  \hspace{1cm} (3.4)

To construct the our string field theory as a path integral over string fields

$$\int [d\Psi] \exp(iS[\Psi])$$  \hspace{1cm} (3.5)

we need to specify the spacetime string field action $S[\Psi]$. What principles can guide us in constructing such an action? Notice that the two terms shown in Eq.(3.4) include a scalar field $\phi$ and a vector field $A_\mu$. We know that a consistent spacetime theory of these fields should include a Klein-Gordon action for $\phi$ and a Maxwell action for $A_\mu$ as well as a gauge transformation for
the $A_{\mu}$. We can satisfy all of these criteria at once by constructing an action invariant under the symmetry

$$\delta \Psi = Q_B \Lambda$$  \hspace{1cm} (3.6)

where $Q_B$ is the BRST operator and $\Lambda$ is an arbitrary functional. Such an action is given by

$$S_0 = \frac{1}{2} \langle \Psi | Q_B | \Psi \rangle.$$  \hspace{1cm} (3.7)

Thus far our action will reproduce the gauge covariant kinetic terms for the component fields. To account for the explicit interaction terms (nongauge interactions) for the component fields we obviously need to add terms to the string field action. If we anticipate that these terms will be polynomial in the string field $\Psi$ then we must define a product for string fields.\(^4\) To this end consider two string fields $\Psi_1$ and $\Psi_2$. Define $\Psi_1 * \Psi_2$ as the convolution of the right half of $\Psi_1$ with the left half of $\Psi_2$. We can similarly define integration over a string field $\int \Psi$ as the convolution of the left and right sides of a single string field. To account for interactions we simply extend the symmetry transformation $\delta \Psi$ we used to construct the kinetic terms to

$$\delta \Psi = Q_B \Lambda + g \Psi * \Lambda - g \Lambda * \Psi$$  \hspace{1cm} (3.8)

We can now write the final form of the action which includes both the gauge covariant kinetic terms and the explicit interactions as

$$S = \frac{1}{2} \int \Psi * Q_B \Psi + \frac{2g}{3} \int \Psi * \Psi * \Psi.$$  \hspace{1cm} (3.9)

Suprisingly we can encode all of the explicit interaction terms in a single cubic interaction term for the string field. We have also used the definition of the $*$-product and $\int$ to rewrite the kinetic term.

Now that we have a concise form for the string field action the question turns to "What can we do with it?" The phenomenological relevance of string theory should manifest itself in

\(^4\) This is nontrivial since the string field is not an "ordinary" field defined over spacetime, i.e. we must account for its stringy structure. This is analogous to defining the multiplication of gauge multiplets as the tensor product of vector multiplication with ordinary multiplication.
the interactions of the lowest modes of the string. However the string field action is written in terms of a multiplet containing all of the string modes. To select out the behavior of a few of these will require gauge fixing the action and then integrating out an infinite number of high energy modes. A full calculation along these lines seems intractable.

Lastly we should mention that while calculating in the open string field formalism is difficult (so in fact that we will not do any calculating with it at all), the case for closed strings is considerably more dismal. Even formulating a closed string field theory has proved prohibitively difficult. Were we capable of formulating one, we would no doubt face at least similar calculational difficulties as are present in the open string case. In the open string case we will cheat, making use of noncommutative geometry to render computations tractable. Unfortunately, this tool is unavailable in the closed string sector.

3.1.2 Noncommutative Geometry in String Theory

Noncommutative geometry is simply a failure of coordinate commutativity

\[ [x, y] \neq 0. \]  

(3.10)

A familiar example for physicists is the geometry of phase space in quantum mechanics. In this case the phase space coordinates \( x \) and \( p_x \) fail to commute

\[ [x, p_x] = i\hbar. \]  

(3.11)

Of course in quantum mechanics we still have commuting coordinates for spacetime. However, we are here interested in the case for which the spacetime coordinates themselves fail to commute.

A simple physical realization of this (and one which will motivate and help abbreviate the case in string theory) is the behavior of a charged particle moving in a plane \((x, y)\) in a constant magnetic field perpendicular to the plane \( B = B\zeta \). The Lagrangian for this system is

\[ L = \frac{m}{2} \left( \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right) + qBx \frac{dy}{dt} \]  

(3.12)

where \( m \) and \( q \) specify the mass and charge respectively. If we quantize this theory and compute the spectrum we find a tower of infinitely degenerate Landau levels with energy spacing \( \Delta = \frac{qB}{m} \).
Now take the \( m \to 0 \) limit. The system is restricted to the lowest Landau level and the Lagrangian can be reduced to
\[
L_{m \to 0} = qBx \frac{dy}{dt}.
\]
(3.13)

If we compute the momentum canonically conjugate to \( y \) we find
\[
p_y = \frac{\partial L}{\partial \frac{dy}{dt}} = qBx.
\]
(3.14)

Thus the canonical commutation relations imply
\[
[x, y] = i \frac{q}{B}.
\]
(3.15)

There are a few things we should point out in this example. The coordinate noncommutativity that we described is an alternative way of describing the effect of the \( B \)-field. One can pose the scenario in terms of commuting coordinates and a \( B \)-field, or in terms of just the coordinate noncommutativity in Eq.(3.15). To make this point absolutely clear, we can add to theory some uncharged particles also moving about in the \((x, y)\) plane. For these particles \( q = 0 \) and the magnetic field obviously has no effect on their behavior. Thus the theory can be described by charged and uncharged particles in a magnetic field or in terms of a set of particles on a commuting geometry plus another set of particles on a noncommuting geometry.

Realizing coordinate noncommutativity in string theory is very similar to the example above. To do so we make use of a massless state in the string spectrum that carries two spacetime indices and is antisymmetric under their exchange. This is the 2-form \( B_{\mu\nu} \). In contrast to the familiar Maxwell field strength \( F_{\mu\nu} \) in \( U(1) \) gauge theory (which also happens to be a 2-form), the \( B_{\mu\nu} \) is only a potential, i.e. its field strength is a 3-form.\(^5\) In the same way that we promote the graviton (another element of the string spectrum) to an element of the string background (as the background metric \( g_{\mu\nu} \)), we can include in our specification of the background a nonvanishing \( B \)-field (we will henceforth suppress the indices). For simplicity we will take \( B \) to be constant throughout the spacetime. In treating the propagation of perturbative

\(^5\) Technically speaking a field strength is an exact \((p + 1)\)-form built from a \( p \)-form potential.
strings on this background we must include in the worldsheet action a term

$$\int_{\Sigma} B_{\mu\nu} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu$$

(3.16)

where \(\Sigma\) denotes the worldsheet.\(^6\) Integrating by parts this becomes

$$\int_{\partial\Sigma} B_{\mu\nu} X^\mu \partial_t X^\nu$$

(3.17)

where now \(\partial\Sigma\) denotes the worldsheet boundary and \(\partial_t\) is the derivative tangential to this boundary.

Our rigor will stop here. One should notice the similarity between Eq.(3.17) and Eq.(3.13). Since the interaction term Eq.(3.17) is a surface term it will affect only those strings whose worldsheets have boundaries. These are precisely the open strings. Thus one suspects that we may interpret the theory of open strings in the presence of \(B\)-field, as a theory of open strings propagating on a noncommutative geometry. The theory may also (and in fact must) include closed strings whose worldsheet has no boundary. Like the uncharged particles in the magnetic field example, the closed strings will be unaffected by the constant \(B\)-field. To develop the noncommutative form of the theory would require too much technology. We will simply summarize the results. The exceptionally interested reader may consult early papers on this subject [16, 25, 65] or the leviathan reference by Seiberg and Witten [67].

The closed string background is defined by the closed string metric \(g_{\mu\nu}\), the closed string coupling \(g_s\), and the constant background \(B\)-field. At low energies the dynamics can be described by a spacetime action governing fields. To write a Lagrangian for the fields in the presence of a background \(B\)-field, we need only begin with the Lagrangian without a \(B\)-field, replace the closed string parameters \(g_{\mu\nu}\) and \(g_s\) by the corresponding open string parameters \(G_{\mu\nu}\) and \(G_s^2\), and further replace all products of fields by the associative \(*\)-product of fields defined in terms of the noncommutativity parameter \(\Theta^{\mu\nu}\) [67]. What we are left with has been termed a "noncommutative field theory".\(^7\) An additional freedom in writing the Lagrangian arises from

\(^6\) Recall that perturbative strings are formulated in terms of a \((1 + 1)\)-dimensional worldsheet quantum field theory. Among others, this theory includes a set of embedding fields \(X^\mu(\sigma)\) which describe the coordinate trajectory of the string in spacetime.

\(^7\) This is not to be confused with a non-abelian gauge field theory where the noncommutativity arises solely
the choice of world-sheet regularization which may be represented by a two form \( \Phi_{\mu\nu} \). Upon choosing a particular \( \Phi_{\mu\nu} \), we may then express \( G_{\mu\nu}, G_s, \) and \( \Theta^{\mu\nu} \) in terms of \( g_{\mu\nu}, g_s, B_{\mu\nu}, \) and \( \Phi_{\mu\nu} \) using the relations

\[
\left( \frac{1}{G + \Phi} \right)^{\mu\nu} + \Theta^{\mu\nu} = \left( \frac{1}{g + B} \right)^{\mu\nu}
\]

\[
G_s^2 = g_s \sqrt{\frac{\det (G + B)_{\mu\nu}}{\det (g + B)_{\mu\nu}}}.
\]

We will see an example of this program in the analysis to come.

3.1.3 Sen's Conjecture

As we discussed earlier an important limitation in a string field formulation is that we cannot confidently say anything about the theory beyond what we already know in the perturbative region around the background with which we started. In a sense these field theories are little more than bookkeeping devices. However if we have any information beyond the perturbative picture that we can add to this naive field formulation, then combining these may indicate the route towards a more general second quantized theory.

This program has been undertaken in the case of tachyonic open string field theory. In this case we are dealing with the perturbative picture around a local maximum of some larger theory. The key piece of nonperturbative information is contained in a conjecture due to Sen.

Sen's conjecture states that the tachyonic vacuum in an open string theory on a \( D \)-brane describes the closed string vacuum without \( D \)-branes, and that various soliton solutions in this theory describe \( D \)-branes of lower dimension\[71, 68, 70, 68, 69\]. In addition Sen has argued that this result is independent of the details of the background, e.g. it holds for flat branes, wrapped branes, and \( D \)-branes in the presence of nontrivial closed string backgrounds.

There is a very simple way to picture the content of this conjecture. When we have a perturbative string theory that includes an open string sector, and if in this open string sector there is a tachyonic state, then what we are dealing with is an unstable \( D \)-brane. Furthermore due to the generators of the gauge group. Indeed we can have a noncommutative gauge field theory with a \( U(1) \) gauge symmetry. We call such a theory a noncommutative \( U(1) \).
the condensation of the tachyonic instability corresponds to an annihilation of the unstable brane leaving behind a closed string vacuum. This is the simplest case. Sen's conjecture goes on to state that in the course of annihilating to the closed string vacuum, the unstable $D$-brane may first decay to lower dimensional unstable branes which can be realized as solitonic configurations of the original tachyon.

There are at least two explicit implications of this conjecture. For the tachyon condensing to amount to the $D$-brane decaying, it must be the case that the energy density of the unstable brane, i.e. its tension, is equivalent to the difference in energy between the unstable maximum and the stable minimum of the tachyon potential as shown in Fig.3.1. In addition we must find that when the tachyon has rolled to the minimum of its potential, that the open string degrees of freedom supported by the unstable $D$-brane vanish.

![Figure 3.1: Sen's conjecture relates the height of the tachyon potential $V(T)$ with the tension $\tau_p$ of the unstable $Dp$-brane. At the minimum of the potential the open string degrees of freedom vanish leaving a closed string vacuum.](image)

There are a number of caveats as well. As we discussed in section Sec.[1.4], there are a variety of perturbative string backgrounds with unstable open string sectors. Sen's conjecture is broad enough to apply to any of these backgrounds, though the details may vary drastically.

The simplest case is the bosonic string theory in $M^{26}$. This theory can be formulated with closed strings only or as a theory with closed and open strings. In both the closed and open string sectors we have tachyonic states. What Sen's conjecture tells us is that the theory with open strings has actually a background which includes an unstable space-filling $D25$-brane.
This brane can decay into any number of unstable $Dp$-branes with $p < 25$. So while it is easiest to think of the tachyon as rolling from the top of a hill to the valley below as it condenses, we have to remember that the tachyon may vary over spacetime. In some regions it may roll to its minimum while in others it may remain at the unstable maximum. This is the scenario which will give rise to lower dimensional branes and will be made more explicit in the discussion to come. In the bosonic case $Dp$-branes for any $p$ are possible and all are unstable. The underlying closed string vacuum is tachyonic as well. Where the condensation of the closed string tachyon takes us will be taken up in the next chapter.

Another possibility is to consider unstable $D$-brane configurations on an underlying closed string vacuum which is itself supersymmetric. In this case the details are more involved and one must consider case by case scenarios.

In the subsequent analysis we will restrict ourselves to the simplest case of the bosonic string. The discussion can be carried over to the case of supersymmetric closed string backgrounds with appropriate modifications.

3.2 Putting It All Together: $D$-branes as Noncommutative Solitons

We begin by reviewing and extending the application of noncommutative soliton constructions in string field theory. In Sec.[3.2.1] we write down the effective Lagrangian governing the lowest modes in open string field theory. Discussion then turns in Sec.[3.2.2] to the issue of soliton tensions and how they essentially arise with the appearance of constituent descriptions of $Dp$-branes. Section [3.2.3] presents the equations of motion arising from the effective Lagrangian and proposes a simple set of conditions for identifying solutions. The main discussion in Sec.[3.2.4] identifies both standard solutions to the equations of motion and "spurious" ones, and motivates identifications of each with underlying string/brane configurations. Section [3.3] considers solutions arising from an alternative form of the effective Lagrangian that has been proposed by several authors [30, 8, 49]. And the last section discusses the recently proposed solution generating technique [37].
3.2.1 The Effective Lagrangian

We begin with an expression for the effective Lagrangian on a single \( Dp \)-brane in bosonic string field theory after having integrated out all modes except the tachyon \( T \) and massless \( U(1) \) gauge field \( A_\mu \) (which gives rise to the usual 2-form field strength \( F_{\mu\nu} \)).

\[
L(t) = \frac{1}{(2\pi)^{d-1}} \frac{1}{g_s} \int d^px \sqrt{|g_{\mu\nu} + F_{\mu\nu}|} - f(T') \partial_\mu T \partial^\mu T \sqrt{|g_{\mu\nu}| + ...} \tag{3.19}
\]

We have set \( 2\pi\alpha' = 1 \) and normalized \( V(T) \) such that \( V(T_{\text{max}}) = 1 \). With this normalization the coefficient in front of the action is exactly the \( D25 \)-brane tension. Our inability to actually derive this action from the full string field formulation is encoded in the unknown functions \( V(T) \) and \( f(T) \). The closed string background is defined by the closed string metric \( g_{\mu\nu} \), the closed string coupling \( g_s \), and the constant background \( B \)-field. To write the effective Lagrangian in the presence of a background \( B \)-field, we need only begin with the effective Lagrangian without a \( B \)-field Eq.(3.19), replace the closed string parameters \( g_{\mu\nu} \) and \( g_s \) by the corresponding open string parameters \( G_{\mu\nu} \) and \( G_s^2 \), and further replace all products of fields by the associative \( \star \)-product of fields defined in terms of the noncommutativity parameter \( \Theta^{\mu\nu} \) [67]

\[
A(x)B(x) \to A \star B = e^{\frac{i}{2} \Theta^{\mu\nu} \partial_\mu \partial_\nu} A(x)B(x') |_{x=x'}. \tag{3.20}
\]

Whereas in Eq.(3.19) the tachyon is neutral under the \( U(1) \) (as evidenced by the presence of ordinary derivatives in the tachyon kinetic terms), the noncommutativity induces a nonzero coupling of the tachyon to the noncommutative \( U(1) \) gauge field \( A_{\mu\nu}^{NC} \). Hence the ordinary derivatives of Eq.(3.19) should be replaced by gauge covariant ones

\[
D_\mu T = \partial_\mu T - i [A_{\mu\nu}^{NC}, T]. \tag{3.21}
\]

\(^8\) At this point one may object to the introduction of string field theory if our work makes use only of the low-energy effective action. After all, standard arguments (conformal invariance of the worldsheet in nontrivial backgrounds) were long ago used to obtain similar actions. The distinction from the present case is that for tachyonic degrees of freedom we cannot obtain useful information from string perturbation theory. This is due to the fact that string perturbation theory in the worldsheet formalism is defined only on-shell (as an S-matrix theory) and the tachyon is necessarily off-shell. We thus require an off-shell formulation of the dynamics, one such as a string field theory.
An additional freedom in writing the effective Lagrangian arises from the choice of world-sheet regularization which may be represented by a two form $\Phi_{\mu \nu}$. Putting these together we have

$$L(t) = \frac{1}{(2\pi)^{1/2}} \frac{1}{G_s^2} \int d^p x [V(T) \sqrt{\det(G_{\mu \nu} + F_{\mu \nu}^{NC} + \Phi_{\mu \nu})} - f(T) D_\mu T D^\mu T \sqrt{\det G_{\mu \nu}} + ...].$$  

(3.22)

Here we have designated the noncommutative form of the 2-form field strength by $F_{\mu \nu}^{NC}$ which is defined in terms of the now noncommutative $U(1)$ gauge field

$$F_{\mu \nu}^{NC} \equiv \partial_\mu A_\nu^{NC} - \partial_\nu A_\mu^{NC} - i[A_\mu^{NC}, A_\nu^{NC}].$$  

(3.23)

Upon choosing a particular $\Phi_{\mu \nu}$, we may then express $G_{\mu \nu}$, $g_s$, and $\Theta^{\mu \nu}$ in terms of $g_{\mu \nu}$, $g_s$, $B_{\mu \nu}$, and $\Phi_{\mu \nu}$ using the relations

$$\left( \frac{1}{G + \Phi} \right)^{\mu \nu} + \Theta^{\mu \nu} = \left( \frac{1}{g + B} \right)^{\mu \nu}$$  

$$G_s^2 = g_s \sqrt{\frac{\det(G + B)}{\det(g + B)}}$$  

(3.24)

Earlier work on the subject[39, 33] used the gauge $\Phi_{\mu \nu} = 0$. We instead follow [66] and take $\Phi_{\mu \nu} = -B_{\mu \nu}$ which leads to the exact expressions

$$\Phi_{\mu \nu} = -B_{\mu \nu}$$

$$\Theta^{\mu \nu} = \left( \frac{1}{B} \right)^{\mu \nu}$$

$$G_{\mu \nu} = -(B^{-1} B)_{\mu \nu}$$

$$G_s^2 = g_s \sqrt{\frac{\det B_{\mu \nu}}{\det g_{\mu \nu}}}$$  

(3.25)

These relations are precisely those found in the $\alpha' B_{\mu \nu} \to \infty$ limit of the description in terms of $\Phi_{\mu \nu} = 0$. In order to obtain manifest background independence, we use as our gauge degrees of freedom the $X$ fields

$$X^\mu = x^\mu + \Theta^{\mu \nu} A_\nu^{NC}$$  

(3.26)

where the coordinates $x^\mu$ satisfy $[x^\mu, x^\nu] = i \Theta^{\mu \nu}$. The effective Lagrangian for a $Dp$-brane is
then given by

\[ L(t) = \frac{1}{(2\pi)^{p+1} g_s} \int \frac{d^p x}{\sqrt{\det \Theta^\mu\nu}} [V(T) \sqrt{\det(\delta^\nu_{\mu} + g_{\mu\lambda}[X^\lambda, X^\nu])} - f(T)g_{\mu\nu}[X^\mu, T][X^\nu, T] + ...]. \]  

(3.27)

### 3.2.2 Tensions of Solitons

Utilizing ideas from [32] the authors of [19, 39] constructed noncommutative solitons on the worldvolume of a bosonic $D25$-brane and the work of Harvey et al [39] in fact identified them with $Dp$-branes of lower dimension. Part of the evidence for this identification consisted of showing that the lower dimension solitons exhibited tensions in agreement with the results for $Dp$-branes obtained by T-duality. Obtaining the correct tension though strongly suggestive, does not by itself constitute a complete demonstration that the solitonic configurations are in fact $D$-branes. The tension comes out right as a result of the formula that gives the correspondence between functions of noncommuting coordinates and operators on a Hilbert space. In particular, for $\Theta^\mu\nu$ of rank $2n$, we may group the coordinates into $n$ noncommuting pairs with $26 - 2n$ leftover commuting coordinates. Functions of the 26 coordinates can then be mapped to matrix-valued functions of the $26 - 2n$ commuting coordinates. The $*$-product gets mapped to the tensor product of operator multiplication with ordinary multiplication, and most importantly the measure of integration over the noncommutative coordinates gets mapped to a trace over the Hilbert space

\[ \int \frac{d^p x}{\sqrt{\det \Theta^\mu\nu}} \to (2\pi)^{\frac{p}{2}} T \ldots \]  

(3.28)

Considering the Lagrangian Eq.(3.27) for $p$-even. We can consider turning on a $B$-field in $p$ directions giving rise to a $\Theta^\mu\nu$ of rank $\frac{p}{2}$. Using the correspondence above (and restoring factors of $2\pi\alpha'$) we would then have

\[ L_p(t) = \frac{1}{(2\pi)^{p}(\alpha')^{\frac{p+1}{2}}} \frac{1}{g_s} \int \frac{d^p x}{\sqrt{\det \Theta^\mu\nu}} (...) \]

\[ \downarrow \]

\[ \frac{1}{(2\pi)^{p}(\alpha')^{\frac{p+1}{2}}} \frac{1}{g_s} (2\pi)^{\frac{p}{2}} (2\pi\alpha')^{\frac{p}{2}} T \ldots = \frac{1}{g_s\sqrt{\alpha'}} T \ldots \]  

(3.29)
which may be identified with the Lagrangian for \( N \to \infty \) D0-branes.

For \( p \)-odd, we can go to Euclidean space and consider turning on a \( B \)-field in \( p + 1 \) directions. Using the operator correspondence, this leads to

\[
S_p = \frac{1}{(2\pi)^p(\alpha')^{p+1}} \frac{1}{g_s} \int \frac{dtd^{p+1}x}{\sqrt{\det \Theta^{\mu\nu}}} (...) \downarrow
\]

\[
\frac{1}{(2\pi)^p(\alpha')^{p+1}} \frac{1}{g_s} (2\pi)^{p+1} (2\pi \alpha')^{p+1} T_r(\ldots) = \frac{2\pi}{g_s} T_r(\ldots)
\]

which may be identified with the Euclidean action for \( N \to \infty \) D-instantons.

It should be obvious by these identifications that obtaining the correct tension for a soliton is built into the formalism of describing functions of noncommuting coordinates by operators on a Hilbert space. The operator description of the noncommutative theory on a \( Dp \)-brane is equivalent to a constituent description.\(^9\) The process of selecting a solitonic profile is merely to select a subset of an infinitely extended collection of these constituent elements. In light of this, in order to properly identify a configuration its fluctuation spectrum should also be considered.

### 3.2.3 Equations of Motion

To connect with issues raised in [39, 33, 66] we consider the effective action Eq.(3.27) for \( p = 25 \). For the explicit construction of codimension-2n solitons it is only necessary to turn on a background \( B \)-field in 2n directions[39]. We will, however, be utilizing a maximal rank \( B \)-field. In doing so we will enable ourselves to deal with a more general set of solutions than in [39]. To do so we work in Euclidean space. Turning on the \( B \)-field in all 26 directions and using the operator correspondence outlined in the previous section we have

\[
S_{25} = \frac{2\pi}{g_s} T_r[V(\hat{T})\sqrt{\det(\delta_{\mu}^\nu + g_{\mu\lambda}[\hat{X}_\lambda, \hat{X}_\nu])} - f(\hat{T})g_{\mu\nu}[\hat{X}_\mu, \hat{T}][\hat{X}_\nu, \hat{T}] + ...].
\]

Defining

\[
M_\mu^\nu = \delta_\mu^\nu + g_{\mu\lambda}[\hat{X}_\lambda, \hat{X}_\nu]
\]
the tachyon equation of motion arising from Eq.(3.31) is given by

\[ V'(\hat{T}) \sqrt{\text{det} M_{\mu}^\nu - f'(\hat{T}) g_{\mu\nu}[\hat{X}^\mu, \hat{T}][\hat{X}^\nu, \hat{T}] + g_{\mu\nu}[\hat{X}^\mu, f(\hat{T})][\hat{X}^\nu, \hat{T}]} = 0 \]  

(3.33)

while the equation of motion from varying the X field is

\[-\frac{1}{2} [\dot{X}^\mu, (M^{-1} - (M^T)^{-1})^{\mu\nu} \sqrt{\text{det} M_{\mu}^\nu} V(\hat{T})] + [\dot{T}, [\dot{X}^\nu, \hat{T}]] = 0 \]  

(3.34)

To simplify matters we may consider a sufficient, but perhaps not necessary, set of conditions which will lead to solutions \( \hat{X}_c \) and \( \hat{T}_c \) of the equations above

\[ a. \quad V'(\hat{T}) = 0 \]

\[ b. \quad [\dot{X}^\mu, \hat{T}] = 0 \]  

(3.35)

\[ c. \quad [\dot{X}^\mu, [\dot{X}^\nu, \dot{X}^\lambda]] = 0 \]

3.2.4 Solutions

We now consider solutions to these equations. The first three solutions represent configurations that have a definite interpretation in terms of standard brane configurations [19, 39, 33, 66]. The remaining solutions have less obvious interpretations, and as such must either be accounted for in standard brane configurations or somehow excluded in this context.

An important tool in finding solutions to Eq.(3.35) is the existence of localized projector solutions to

\[ \hat{P} \ast \hat{P} = \hat{P}. \]  

(3.36)

For a function of the form \( F = \sum_{n=1}^{\infty} a_n x^n \), this implies

\[ F(\lambda \hat{P}) = F(\lambda) \hat{P}. \]  

(3.37)

The key point is that without even knowing the explicit form of \( V(\hat{T}) \) in Eq.(3.35), we can satisfy (a) by constructing a projector function \( \hat{T} \) of the form \( \hat{T} = T_{\text{ext}} \hat{P} \) where \( T_{\text{ext}} \) extremizes \( V(\hat{T}) \). This then implies

\[ V'(\hat{T}) = V'(T_{\text{ext}} \hat{P}) = V'(T_{\text{ext}}) \hat{P} = 0. \]  

(3.38)
In contrast to the obvious solution $T = T_{\text{ext}} = \text{constant}$, these projector solutions may have nontrivial spatial dependence.

One should keep in mind that the operator correspondence maps the gauge covariant derivative in a particular direction $\mu$ to a commutator term involving the $X$-fields

$$D_\mu \rightarrow -i \Theta_{\mu \nu}^{-1} [\hat{X}^\nu, \quad].$$

(3.39)

Thus if we have a background solution $\hat{X}^\mu = \lambda \hat{I}$ (where $\hat{I}$ is the identity) which necessarily commutes with all other operators, then propagating fluctuations in the $x^\mu$ direction are forbidden.

Our first solution to Eq.(3.35) takes the form

$$\hat{T}_c = T_{\text{max}} \hat{I} \quad \hat{X}_c^\mu = \hat{x}^\mu \quad \text{where} \quad [\hat{x}^\mu, \hat{x}^\nu] = \Theta^{\mu \nu} \hat{I}.$$  

(3.40)

This solution represents a uniform open string tachyon field on the worldvolume of an unstable $D25$-brane. Fluctuations about this background form a noncommutative $U(1)$ gauge theory with 26-dimensional tachyons transforming in the adjoint. As a symmetry among the operators, the noncommutative $U(1)$ is realized as a $\bigotimes_{i=1}^{13} U(N_i \rightarrow \infty)$. The tension for this configuration may be identified by inserting the background field configurations into the action;

$$S_{\text{background}} = \frac{1}{g_s} \frac{1}{(2\pi)^{26-1/2}} \int d^{26} x \sqrt{\det \Theta} V(T_{\text{max}}) \sqrt{\det(\delta^\nu_\mu + g_{\mu \lambda} \Theta^{\lambda \nu})}.$$  

(3.41)

For $V(T_{\text{max}}) = 1$, we identify the coefficient of the integral over the 26-dimensional worldvolume as the tension of the $D25$-brane.

$$\hat{T}_c = T_{\text{min}} \hat{I} \quad \hat{X}_c^\mu = \lambda \hat{I}.$$  

(3.42)

By Sen's conjecture, this uniform solution represents the stable closed string vacuum in the absence of $D$-branes. There are no propagating open string tachyon fluctuations in this background. The action with this background becomes

$$S_{\text{background}} = \frac{1}{g_s} \frac{1}{(2\pi)^{25-1/2}} \int d^{26} x \sqrt{\det \Theta} V(T_{\text{min}}) \sqrt{\det(\delta^\nu_\mu)}.$$  

(3.43)
which vanishes according to Sen's conjecture, i.e. \( V(T_{\text{min}}) = 0 \).

\[
\hat{T}_c = T_{\text{max}} \hat{P}_n + T_{\text{min}}(\hat{I} - \hat{P}_n) \quad \hat{X}_c^i = \hat{x}_i \hat{P}_n \quad i = 0, ..., p \quad \hat{X}_c^m = \lambda \hat{I} \quad m = p + 1, ..., 25
\]

(3.44)

The \( \hat{P}_n \) in this expression are projection operators onto subspaces of the Hilbert space generated by the "transverse" noncommuting coordinates \( x^m \). The tachyon profile expressed in terms of \( \hat{P}_n \) enjoys the useful property that

\[
V(\hat{T}_c) = V(T_{\text{max}}) \hat{P}_n + V(T_{\text{min}})(\hat{I} - \hat{P}_n)
\]

(3.45)

These noncommutative solitons have finite extent in the \( x^m \) directions and infinite extent in the \( x^i \) directions. Such backgrounds interpolate in the transverse directions \( x^m \) between the tachyonic vacuum in the core of the soliton and the closed string vacuum outside of the soliton. Convincing evidence has been put forward to identify these configurations with lower dimensional unstable \( Dp \)-branes[19, 39, 33]. To clarify this picture we may consider a block diagonal noncommutativity matrix

\[
\Theta = \bigoplus_{k=1}^{13} \begin{pmatrix} 0 & \theta_k \\ -\theta_k & 0 \end{pmatrix}
\]

(3.46)

where

\[
[x^{2k}, x^{2k+1}] = i\theta_k.
\]

(3.47)

Rank \( n_k \) projection operators \( P_{n_k} \) can be constructed on the Hilbert spaces \( H_k \) formed from each noncommuting pair of coordinates, so that a general projection operator takes the form

\[
P_n = P_{n_1}^{(1)} \otimes P_{n_2}^{(2)} \otimes \ldots \otimes P_{n_{\frac{25-p}{2}}}^{(25-p)}
\]

(3.48)

for \( p \) odd. A soliton of this form corresponds to the chain of decays

\[
1D25 \to n_1 D23 \to n_1 n_2 D21 \to \ldots \to \prod_{k=1}^{\frac{25-p}{2}} n_k Dp
\]

(3.49)

To concretely identify this configuration with \( n \) coincident \( Dp \)-branes, we insert the solution
back into Eq.(3.31) and use the correspondence Eq.(3.28) on $p + 1$ of the coordinates to obtain

$$S_{\text{background}} = \frac{1}{g_s} \frac{1}{(2\pi)^{\frac{p+1}{2}}} \text{Tr} \left[ \hat{P}_n V(T_{\text{max}}) + (\hat{I} - \hat{P}_n) V(T_{\text{min}}) \right] \int \frac{d^{p+1}x}{\sqrt{\det \Theta'}} \sqrt{\det (\delta_i^j + g_{ik} \Theta'^{kj})}$$

(3.50)

where

$$\Theta' = \bigoplus_{k=1}^{p+1} \begin{pmatrix} 0 & \theta_k \\ -\theta_k & 0 \end{pmatrix}$$

(3.51)

If we use $V(T_{\text{max}}) = 1$ and Sen’s conjecture, i.e. $V(T_{\text{min}}) = 0$, then (3.50) becomes

$$S_{\text{background}} = \frac{n}{g_s} \frac{1}{(2\pi)^{\frac{p+1}{2}}} \int \frac{d^{p+1}x}{\sqrt{\det \Theta'}} \sqrt{\det (\delta_i^j + g_{ik} \Theta'^{kj})}$$

(3.52)

which if compared to (3.41) is easily identified as the action for $n$ $Dp$-branes.

It should be noted that one can take the limit $\theta_k \to 0$ in both Eq.(3.41) and Eq.(3.52) after redefining the coordinates $x^{2k,2k+1} \to \frac{x^{2k,2k+1}}{\sqrt{\theta_k}}$, thus recovering the usual form of the $D$-brane action in the absence of gauge fields or $B$ fields. Thus our procedure using the background independent formalism of [66] avoids the ambiguities associated with taking a large $B$ limit.

The solutions above admit simple and elegant interpretations in terms of coincident unstable $Dp$-branes and the closed string vacuum. However, other nonsingular solutions to Eq.(3.35) exist, and thus solve the equations of motion arising from Eq.(3.31). Since the action Eq.(3.19) is expected to be an approximation to the complete string field theory in the limit where the derivatives of the gauge fields and $B$ fields are small, then these solutions must also be accounted for as configurations of perturbative and nonperturbative states in string theory. The known perturbative and nonperturbative states of bosonic string theory include the fundamental string and its magnetic dual, as well as unstable $Dp$-branes for $p = -1, \ldots, 25$. However, the string field theory action, if complete, could not only predict these states, but any possible configuration of these states consistent with the background in which the string field theory is formulated.\textsuperscript{10} Certainly a number of the smooth solutions to the full action will arise from its low energy effective form. It is via these nontrivial configurations that we will interpret the

\textsuperscript{10} In terms of unstable $D$-branes, the possible decay remnants are determined by the initial unstable background. Starting with a single unstable $Dp$-brane we can obtain remnants with arbitrary numbers of $Dp'$-branes for any $p'<p$. 
additional solutions. In our case this background is the worldvolume of an unstable \( D25 \)-brane with a constant maximal \( B \)-field.

Some additional solutions to Eq. (3.35) were first pointed out in [33]. These involve allowing different projection operators to define the transverse profiles of the tachyon and \( X \) fields. To systematically cover this set of solutions we will consider descending from the \( D25 \)-brane configuration by replacing the identity operator by projectors where appropriate. We first investigate the effect of nontrivial projection operators for the tachyon while maintaining trivial forms for \( \tilde{X}^L, \tilde{X}^T \). We then study the effects on \( \tilde{X}^L \). These results may be combined for configurations with nontrivial projection operators for both \( \tilde{T} \) and \( \tilde{X}^L \).

For now we consider operators projecting onto the subspace generated by a single pair of coordinates which we will refer to as simply \( \tilde{x}^T \), since these will in some sense be interpreted as directions transverse to the resulting system. The remaining coordinates we refer to as \( \tilde{x}^L \) since these will be roughly longitudinal to the system. We consider functions of the coordinates \( \vartheta(\tilde{x}^a) \) which may be represented by direct product operators \( \vartheta = \tilde{\vartheta}_T \otimes \tilde{\vartheta}_L \). So we have for the identity on the entire Hilbert space \( H = H_L \otimes H_T \) an expression \( \hat{I}_{L,T} = \hat{I}_L \otimes \hat{I}_T \), and for a rank \( n \) projection operator \( \hat{P}_n = \hat{I}_L \otimes \hat{P}_{nT} \). Consider a set of operators \( \hat{T}_c, \tilde{X}_c, \tilde{X}^T_c \) split into longitudinal and transverse parts. The set of general solutions of this type takes the form

\[
\begin{align*}
\hat{T}_c &= T_{\text{max}} \hat{I}_L \otimes \hat{P}_{n1T} + T_{\text{min}} \hat{I}_L \otimes (\hat{I}_T - \hat{P}_{n1T}) \\
\tilde{X}_c^L &= \tilde{x}_c^L \otimes \hat{P}_{n2T}
\end{align*}
\]

with form of the transverse \( X \) field determining two branches in the space of solutions

\[
\begin{align*}
1. & \quad \tilde{X}_c^T = \hat{I}_L \otimes \tilde{x}_c^T \quad \text{with} \quad \hat{P}_{n1T}, \hat{P}_{n2T} = \hat{0}^T \text{ or } \hat{I}_T \\
2. & \quad \tilde{X}_c^T = \lambda \hat{I}_L \otimes \hat{I}_T \quad \text{with} \quad [\hat{P}_{n1T}, \hat{P}_{n2T}] = 0.
\end{align*}
\]

We begin by descending the tachyon profile from \( \hat{I}_{L,T} \rightarrow \hat{I}_L \otimes \hat{P}_{nT} \rightarrow \hat{I}_L \otimes \hat{0}_T = \hat{0}_{L,T} \).
while keeping in mind Eq.(3.54) and holding $\tilde{X}^L$ fixed

\begin{align*}
a) \quad \hat{T}_c &= T_{\text{max}} \hat{I}_{L,T} \\
     \downarrow \\
     \hat{X}_c^L &= \tilde{x}^L \otimes \hat{I}_T \\
     \hat{X}_c^T &= \tilde{x}^T \\
\end{align*}

\begin{align*}
b) \quad \hat{T}_c &= T_{\text{max}} \hat{I}_{L,T} \\
     \downarrow \\
     \hat{X}_c^L &= \tilde{x}^L \otimes \hat{I}_T \\
     \hat{X}_c^T &= \lambda \hat{I}_L \otimes \hat{I}_T \\
\end{align*}

\begin{align*}
c) \quad \hat{T}_c &= T_{\text{max}} \hat{I}_L \otimes \hat{P}_{nT} + T_{\text{min}} \hat{I}_{L,T} (\hat{I}_T - \hat{P}_{nT}) \\
     \downarrow \\
     \hat{X}_c^L &= \tilde{x}^L \otimes \hat{I}_T \\
     \hat{X}_c^T &= \lambda \hat{I}_L \otimes \hat{I}_T \\
\end{align*}

\begin{align*}
d) \quad \hat{T}_c &= T_{\text{min}} \hat{I}_{L,T} \\
     \downarrow \\
     \hat{X}_c^L &= \tilde{x}^L \otimes \hat{I}_T \\
     \hat{X}_c^T &= \lambda \hat{I}_L \otimes \hat{I}_T \\
\end{align*}

\begin{align*}
e) \quad \hat{T}_c &= T_{\text{min}} \hat{I}_{L,T} \\
     \downarrow \\
     \hat{X}_c^L &= \tilde{x}^L \otimes \hat{I}_T \\
     \hat{X}_c^T &= \hat{I}_L \otimes \tilde{x}^T \\
\end{align*}

(3.55)

One should keep in mind that the tachyon profile will always lead to a simple tension expression which can then be used to identify the $Dp$-brane present. The $X$ field configuration on the other hand governs the propagation of various fluctuations, and so should give us information on how the constituent $Dp$-branes are assembled.

The process $a \rightarrow b$ represents a transition from a space filling $D25$-brane into a space filling stack in the $x^T$ directions of an infinite number of $D23$-branes with worldvolume extension in the $x^L$ directions, and with open strings confined to each constituent brane. That open string modes cannot propagate in $x^T$ is a consequence of the vanishing of the covariant derivative in the transverse directions

$$D_T \rightarrow -i \Theta^{-1}_{\mu \nu} [\lambda \hat{I}_L \otimes \hat{I}_L] = 0$$

In essence, propagation in the $x^T$ directions is eliminated by not allowing open strings to migrate from one $D23$-brane to the next. Propagation of fluctuations along the $D23$-branes is of course still allowed.

The transition $b \rightarrow c$ represents the decay of $\infty - n$ of the $D23$-branes into the closed string vacuum. In contrast to the standard $D23$-brane solution which implements the same projection operator for the tachyon and longitudinal $X$ fields, here we have allowed a nontrivial
projection operator for the tachyon alone. However, the resulting configuration is physically indistinguishable from the standard $D23$-brane solution Eq.(3.44). This is a simple consequence of the factors of $V(T)$ and $f(T)$ in front of the Born-Infeld and tachyon kinetic terms respectively. These give rise to an overall factor of the tachyon projection operator which acts on the $X$ fields, effectively giving $\hat{X}^L$ the same projector form as $\hat{T}$. The action evaluated for this background is precisely Eq.(3.50). In this case (with the nontrivial projector profile) we can not reinstate propagation in the $x^T$ directions in light of Eq.(3.54). This is a reflection of the simple fact that a $D25$-brane cannot be constructed out of a finite number of $D23$-branes.

The transition $c \rightarrow d$ seems to have no definite interpretation unless the assumption $f(T_{\text{min}}) = 0$ is made. With this assumption configuration $d$ is physically identical to the closed string vacuum Eq.(3.42). Without this assumption the tachyon profile will yield a vanishing tension, yet propagating fluctuations are allowed along the $x^L$ directions. One should note however that with this choice for the tachyon field, the overall coefficient $V(T_{\text{min}})$ of the Born-Infeld contribution to the action vanishes. We expect that our computations, based on a well defined action, will run into trouble in this scenario.

The transition $a \rightarrow e$ shares the complication of $a \rightarrow d$ and will need further investigation of the action to be interpreted or excluded. Sen has argued [73] that configurations $d$ and $e$ are actually equivalent descriptions of the closed string vacuum.

We may now consider the results of descending the longitudinal $X$ field $\hat{X}^L$, this time holding fixed the tachyon while maintaining Eq.(3.54).
We identify in c, d that taking the longitudinal X field projection operator to have rank 0 is effectively equivalent to setting \( X^L \propto I \).

The process \( a \rightarrow b \) is identical to the case discussed for Eq. (3.55), that is the transition from a \( D25 \)-brane into an infinite number of \( D23 \)-branes with open string ends restricted to single \( D23 \)-branes.

The transition \( b \rightarrow c \) is rather interesting. The tension is that of the space filling \( D25 \)-brane. The propagation of fluctuations in the transverse directions \( x^T \) is eliminated as in configuration \( b \). However, now the propagation of fluctuations in the longitudinal directions \( x^L \) is restricted to a region localized around the origin in \( x^T \) as described by \( \tilde{P}_{nT} \). The interpretation of this is as follows: each of \( \infty - n \) of the infinite collection of \( D23 \)-branes in configuration \( b \) undergoes a transition to an infinite collection of \( D \)-instantons with the ends of any open string confined to a single \( D \)-instanton. This confinement eliminates propagating fluctuations in the longitudinal directions \( x^L \) outside of a region localized in \( x^T \). The remaining \( nD23 \)-branes are stacked as before about the origin in \( x^T \) and admit propagating fluctuations along \( x^L \).

The transition \( c \rightarrow d \) corresponds to a transition of the remaining \( D23 \)-branes in configuration \( c \) to infinite collections of \( D \)-instantons. Since the tachyon profile has not changed, we expect an energy density identical to that of a \( D25 \)-brane. In fact the action evaluated with this
background field configuration is essentially that of Eq.(3.31), i.e.

\[ S_{\text{background}} = \frac{2\pi}{g_s} \Tr \hat{I}_L, T = \frac{2\pi}{g_s} (N \to \infty)^{13} \]  

(3.58)

Since the ends of open strings are confined to points in space-time, there are of course no propagating open string modes.

The transition \( a \to e \) is actually very closely related to the \( a \to b \) transition. In this case it may seem that the space filling \( D25 \)-brane decays to an infinite collection of \( D1 \)-branes since we have merely exchanged the roles played by the longitudinal and transverse coordinates. However, in interpreting configuration \( c \) which is extended in the \( x^L \) directions as a stack of coincident \( D23 \)-branes, we are assuming that time is an element of the \( x^L \) set of coordinates. Thus for configuration \( e \) we have a stack of two-dimensional objects extended in purely spatial directions \( x^T \) and exist only at an instant in time. These extended instantons correspond to objects resulting from \( T \)-dualizing the time coordinate on a \( D2 \)-brane. Whether this operation is well defined and the role played by the resulting objects is outside the scope of this paper.

For a discussion of such issues see [44] and references therein.

The two descent chains above may be used as the cornerstones for more complicated projector combinations. In any case one is led to a description of the \( D25 \)-brane in terms of some constituent \( Dp \)-branes which are either decayed, transversely distributed, coincident, or some combination of these which may differ for different directions.

A final set of solutions that may be considered are those which utilize operators on the transverse subspace other than \( \hat{I}, \hat{P}_n, \hat{0}, \hat{x}^T \). These must commute with any other operator acting in this subspace in order to serve as solutions to Eq.(3.35). The operators above are distinguished by possessing eigenvalues equal to either 0 or 1, for \( \hat{I}, \hat{P}_n, \hat{0} \) and values filling out the real line for \( \hat{x}^T \). Commuting operators with more diverse spectra of eigenvalues certainly exist. To this end we may consider solutions of the form

\[ \hat{T}_c = T_{\text{max}} \hat{I}_L \otimes \hat{P}_{n1T} + T_{\text{min}} \hat{I}_L \otimes (\hat{I}_T - \hat{P}_{n1T}) \]

\[ \hat{X}_c^L = \hat{x}_c^L \otimes \hat{M} \]

\[ \hat{X}_c^T = 0 \]

(3.59)
where

\[ \hat{M} \equiv \sum_{i=1}^{k} \lambda_i \hat{P}_{ni}^i \quad \text{for} \quad \hat{P}_{ni}^i \hat{P}_{nj}^j = \hat{P}_{ni}^i \delta_{ij} \quad \text{and} \quad [\hat{M}, \hat{P}_{n1T}] = 0 \]  

(3.60)

We may simplify matters by assuming

\[ Tr \hat{M} = Tr \hat{P}_{n1T} \quad \hat{M} \hat{P}_{n1T} = \hat{M}. \]  

(3.61)

to avoid the complications discussed after equation Eq.(3.53). For all \( \lambda_i \) distinct, this configuration maintains only a \( \bigotimes U(n_i) \) of the \( U(\sum_{i=1}^{k} n_i) \) present in the most degenerate case. From the gauge theory point of view this symmetry breaking corresponds to a separation of the branes.

Interpreting the separation in eigenvalue space as a spacetime distance la M(atrix) theory [4], this configuration can be identified with a collection of \( k \) non-coincident stacks of \( D23 \)-branes.

### 3.3 Alternative Lagrangians

By demanding consistency with T-duality several authors [30, 8, 49] have obtained alternate forms of our starting point Eq.(3.19) which differ by including the tachyon kinetic term under the square root

\[ L(t) = \frac{1}{(2\pi)^{p-1}} \frac{1}{g_s} \int d^p x V(T) \sqrt{\det(g_{\mu\nu} + F_{\mu\nu} + \partial_{\mu} T \partial_{\nu} T)}. \]  

(3.62)

An obvious advantage in considering actions of this form is the automatic vanishing of the tachyon kinetic term for \( T = T_{\text{min}} \). We may repeat the analysis above for this modified effective Lagrangian. Turning on a maximal rank \( B \)-field and using the operator correspondence we obtain

\[ S_{25} = \frac{2\pi}{g_s} Tr V(T) \sqrt{\det(\delta \nu^\mu + g_{\mu\lambda}[\hat{X}^\lambda, \hat{X}^\nu] + g_{\mu\nu}[\hat{X}^\mu, \hat{T}][\hat{X}^\nu, \hat{T}])}. \]  

(3.63)

Define

\[ W = \delta \nu^\mu + g_{\mu\lambda}[\hat{X}^\lambda, \hat{X}^\nu] + g_{\mu\nu}[\hat{X}^\mu, \hat{T}][\hat{X}^\nu, \hat{T}]. \]  

(3.64)

The tachyon equation of motion is now given by

\[ V'(\hat{T}) \sqrt{\det W_{\mu}^\nu + [\hat{X}_\mu, [\hat{X}_\nu, \hat{T}]] W^{-1\mu\nu} \sqrt{\det W_{\mu}^\nu V(T)} - [\hat{X}_\nu, W^{-1\mu\nu} \sqrt{\det W_{\mu}^\nu V(T)} [\hat{X}_\nu, \hat{T}]] = 0 \]  

(3.65)
while the equation of motion from varying the $X$ field is

$$[\hat{X}_\mu, (W^{-1} - (W^T)^{-1})^{\nu\mu} \sqrt{\text{det} W_\mu^\nu V(\hat{T})} + [\hat{T}, [\hat{X}_\mu, \hat{T}]W^{-1\mu\nu} \sqrt{\text{det} W_\mu^\nu V(\hat{T})}]
\hspace{1cm} + [\hat{T}, W^{-1\mu\nu} \sqrt{\text{det} W_\mu^\nu V(\hat{T})}][\hat{X}_\mu, \hat{T}] = 0. \quad (3.66)$$

Again we may consider a set of sufficient conditions for a solution of these equations

$$V'(\hat{T}_c) = 0
\hspace{1cm} [\hat{X}_\mu', [\hat{X}_\nu', \hat{T}_c]] = 0 \quad (3.67)
\hspace{1cm} [\hat{X}_\mu', [\hat{X}_\nu', \hat{X}_\lambda']] = 0.
$$

These conditions admit all of the configurations discussed in the previous section as solutions since $\hat{X}_c'$ and $\hat{T}_c$ satisfying

$$[\hat{X}_c', \hat{T}_c] = 0 \quad (3.68)$$

certainly describe a subset of the solutions of Eq.(3.67). However, the conditions Eq.(3.67) admit now a larger set of solutions including, for example, configurations satisfying

$$[\hat{X}_c', \hat{T}_c] \propto \hat{I}. \quad (3.69)$$

Such solutions are considerably more difficult to explicitly construct than those in the preceding discussion. However, owing to the advantage of the automatically vanishing tachyon kinetic term for $T = T_{min}$ it would be worthwhile to investigate these solutions further.

### 3.4 Generating Solutions: The Shift Operation

Recently a technique was introduced to facilitate finding solutions to the equations of motion for noncommutative gauge theories from known solutions by acting with an "almost" gauge transformation [37]. This method was applied to vacuum solutions in open string field theory to obtain solitonic field configurations which might then be interpreted as $Dp$-branes.

There are a few issues regarding this construction which we feel should be discussed.
• The shift operation formulation of the solution generating technique began by observing that a transformation obeying

\[ \hat{U}^\dagger \hat{U} = \hat{I} \]
\[ \hat{U} \hat{U}^\dagger = \hat{P} \]  \hspace{1cm} (3.70)

where \( \hat{P} \) is a projection operator, when applied to the fields \( \hat{\theta}^i \) in an equation of motion would result in new field configurations obeying the same equation of motion.

• Tensions and the tachyon

Solutions to Eq.(3.70) only exist for infinite dimensional \( \hat{U} \). The authors of [37] construct an infinite dimensional representation with the shift operators

\[ \hat{S} = \sum_{k=0}^{\infty} |k + 1 > < k| \]  \hspace{1cm} (3.71)

which satisfy

\[ \hat{S}^\dagger \hat{S} = I, \quad \hat{S} \hat{S}^\dagger = \hat{I} - \hat{P}_n \]  \hspace{1cm} (3.72)

where \( \hat{P}_n \) are projection operators onto the first \( n \) states. The effect of \( \hat{U} = \hat{S}^n \) on the matrix representation of a field \( \hat{\theta} \) is a “southeast shift”. The idea proposed in [37] is that by acting on the closed string vacuum field configurations with \( \hat{U} \) defined above one may generate configurations corresponding to \( Dp \)-branes. To see this in action, we will look at the effect of \( \hat{U} = \hat{S}^n \) on the vacuum tachyon field configuration discussed in Sec.[3.2.4]. We will merely consider trying to build a pair of \( D23 \)-branes from the closed string vacuum, so that the corresponding projection operator is nontrivial in the subspace \( H_k \) generated by \( [\hat{x}^{24}, \hat{x}^{25}] = i\theta \hat{I} \). The tachyon vacuum configuration transforms as follows
The resulting configuration may be identified with the tachyon configuration corresponding to a pair of $D_{23}$-branes only if we have arranged that $T_{\text{max}} = 0$. For this mechanism to work for a choice of $T_{\text{max}} \neq 0$, it would be necessary for the shift to produce an upper left diagonal block $\text{diag}(T_{\text{max}}, \ldots, T_{\text{max}})$. However, a "shift" operation accommodating nonzero "northwest" elements can not be constructed, and so this procedure exhibits a peculiar dependence on the value of what one might have expected to be an arbitrary choice. If the choice $T_{\text{max}} = 0$ is made, then one obtains the correct tension for the $D_{23}$ pair in light...
of Eq.(3.45).

The point is that an equation of motion of the form \( F(\vartheta) = 0 \) will give rise to an equation \( \tilde{U}F(\vartheta)\tilde{U}^\dagger = 0 \) under the action of the shift transformation. But this is not the same as \( F(\tilde{U}\vartheta\tilde{U}^\dagger) = 0 \) unless \( F(0) = 0 \) is true as well.

- \( X \) fields Let us now investigate the result of the shift transformation on the gauge field configurations corresponding to the closed string vacuum in our formalism. Applying \( \tilde{U} = S^2 \) to \( \tilde{X}_{\text{vac}}^\mu \) we have

\[
\tilde{X}_{\text{vac}}^\mu = 0 \rightarrow \tilde{S}^2 \tilde{X}_{\text{vac}}^\mu \tilde{S}^2 = 0
\]

(3.75)

where \( \hat{0} \) represents the null matrix. The result above will pose a problem when we compare the shifted \( \tilde{X}_{\text{vac}}^\mu \) to the expected \( X \) field configuration for a pair of \( D23 \)-branes (see Eq.(3.44))

\[
\tilde{X}_{D23}^i = \begin{pmatrix}
\dot{x}^i \\
0 \\
\dot{x}^i \\
0 \\
0 \\
\vdots
\end{pmatrix}
\]

\[i = 0, \ldots, 23\]

(3.76)

\[
\tilde{X}_{D23}^m = \begin{pmatrix}
0 & 0 & 0 & \ldots \\
0 & 0 & 0 & \ldots \\
0 & 0 & 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

\[m = 24, 25\]

It appears that in order for this technique to work the shift operation would have to distinguish between components of the \( X \) field, and produce a nonzero "northwest" block for the components along the brane. In addition, the transformation would have to distinguish between the tachyon and \( X \) fields and produce appropriate "northwest" blocks for each.
This issue does not arise in [37]. In that work a choice is made for the closed string vacuum configuration which effectively reverses the situation above, that is

\[ \hat{X}_{vac}^\mu = \hat{x}^\mu. \]  

(3.77)

$X$ field configurations for the $D$-branes are identified as

\[ \hat{X}^i_{D23'} = \begin{pmatrix} 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & \ldots \\ 0 & \hat{x}^i & 0 & \ldots \\ 0 & 0 & \hat{x}^i & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad i = 0, 1 \]  

(3.78)

\[ \hat{X}^m_{D23'} = \begin{pmatrix} \hat{x}^m & 0 & 0 & 0 & \ldots \\ 0 & \hat{x}^m & 0 & 0 & \ldots \\ 0 & 0 & \hat{x}^m & 0 & \ldots \\ 0 & 0 & 0 & \hat{x}^m & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad m = 2, \ldots, 25 \]

Again, the solution generating transformation seems to depend on the choice of $0$ for the diagonal $X$ field terms representing $D$-branes. This identification arises directly from the initial choice Eq.(3.77) for the vacuum $X$ field configurations. In [37] it is proposed that these configurations afford an extension of the $\hat{X}^\mu = 0$ vacuum to appropriate configurations for arbitrary noncommutativity $\theta$. These configurations reproduce the correct expressions for $D$-brane tensions, however as discussed in Sec.[3.2.2], the evidence for identifying these as the correct configurations should take into account the spectrum of fluctuations as well. The configuration Eq.(3.77) will admit a spectrum of fluctuations identifiable with that of the closed string vacuum if one of two conditions hold. Either one works in the $\alpha' B \to \infty$ limit or one conjectures that the coefficient function for the tachyon kinetic term in the action vanishes for $\hat{T} = T_{min} \hat{t}$, i.e. $f(T_{min}) = 0$. The actions discussed in Sec.[3.3] automatically enforce the latter of these.
3.5 Summary and Conclusions

If open string field theory has any hope of providing a nonperturbative definition of open string theory, then it should predict all possible vacua of the theory. There is sufficient evidence from dualities that various D-brane configurations exhaust the set of open string vacua. To this end we expect that all solutions of the string field equations of motion should find some interpretation in terms of D-brane configurations. We have demonstrated that a large class of the explicit solutions constructed via techniques from noncommutative geometry have such an interpretation. A main feature of these constructions involved viewing higher dimensional $Dp$-branes to be composed of lower dimensional $Dp$-branes. In particular, infinite configurations of D-instantons allow one to account for the tension of higher dimensional branes, while forbidding propagating fluctuations. Though the general solutions become complicated very quickly, we expect that the simple ideas presented here can be used to construct any configuration required.

Extending the bosonic string field effective action to include modifications consistent with $T$-duality seems to enlarge the space of solutions. Explicit construction of these new solutions is difficult, and their interpretations will certainly not be straightforward. On the other hand an advantage to this form of the action is that the tachyon kinetic term automatically vanishes for $T = T_{\text{min}}$.

The shift symmetry solution generating technique may provide some important insight into noncommutative tachyon condensation. There is certainly an appeal to the generation of solutions from solutions via a single well defined transformation. The construction is reminiscent of $T$-duality and it would be nice to have a better understanding of the issues discussed in Sec.[3.4].
Chapter 4

The Closed String Case [22]

In contrast to the open string scenario, the case for closed string tachyons is much less understood. Previous work on systems with closed string tachyons [36, 1, 20] have pointed towards the conclusion that the endpoint of their decay is a supersymmetric closed string vacuum much like the case for open string tachyons.\(^1\) However not all tachyons are equal. We believe that if the coupling is non-zero, closed string tachyons will have a more drastic effect on the theory than the open string tachyons for the following reason. For open string string tachyons to arise one must have \(D\)-branes in some closed string background (space time). According to Sen the height of the tachyon potential is given by the tension of the relevant \(D\)-brane(s). At the location of the decaying \(D\)-brane(s) one has (before the decay happens and for non-zero coupling) positive curvature. (In the case of say \(D9-D9\) in a IIB background one would have dS space to begin with). After the decay one would have a flat closed string background. On the other hand the bulk closed string tachyon of nonsupersymmetric Type 0B exists already in a flat background. This means that the flat space background corresponds to the unstable point (a maximum or a saddle point) of the tachyon potential. Even if there is a minimum to the tachyon potential the end point of the decay will not be one of the known stable flat space

\(^1\) Exceptional cases include the analysis [43] of the closed string tachyon in the purely bosonic theory and the analysis [5] of the thermal tachyon associated with the Hagedorn transition. Though the authors of [43] make no explicit conjecture for the fate of the theory after tachyon condensation, the implications of their discussion are along the lines presented in this paper. In [5] the authors present an argument for the decay of Wick-rotated Type 0A into supersymmetric IIB. They first localize the thermal tachyon by imposing an AdS background geometry, then demonstrate that the endpoint of tachyon condensation should involve a large AdS black hole (as large as the tachyonic region) with the spacetime behind the horizon excluded. The theory at the transverse boundary of the original spacetime is then identified as Euclidean Type IIB by a simple spin structure argument. Though this conclusion is reminiscent of what we will discuss in this paper, the interpretation of the final state of the theory as IIB on a boundary is not entirely clear to us.
backgrounds of string theory. This is illustrated in Figure 4.1.

![Diagram of tachyon potentials](image)

Figure 4.1: The tachyon potentials for an unstable D-brane in dS space on the left versus the flat space tachyon of Type 0A on the right.

In [36] it has been conjectured that the end point of the decay of Type 0A/B due to the bulk tachyon in these theories is the supersymmetric Type IIA/B theory. These arguments however are dependent on the equivalence of certain M-theory backgrounds with Type 0 and Type II in certain Melvin magnetic backgrounds (we will review these arguments later). Precisely at the point where one has the flat OA background however the region in which this equivalence holds shrinks to zero. This renders the picture of magnetic flux shielding considerably less trivial.

On the other hand in a paper by David et al [20], sigma model RG arguments have been used to show that the end point of the decay of 0A in flat space is IIA in flat space. How then can we reconcile the argument of the previous paragraph with this claim?

The point is that sigma model arguments are made in a particular background and give a set up in which perturbation theory around that background can be done. One can calculate S-matrix elements for arbitrary numbers of particles on the assumption that the coupling is so weak that the back reaction on the background can be ignored. Of course if the coupling were exactly zero there would be no back reaction and as the tachyon slides down the potential there will be no change in the background and it is consistent to argue that the end point is indeed IIA. However in this paper we are interested in the question of what happens to non-supersymmetric
theories at finite (non-zero) coupling. In this case one really needs to take into account the discussion of the previous paragraph. In fact even if the tachyon potential bottomed out, at any non-zero coupling the best that one could hope for is to end up with a SUSY string theory (say IIA for 0A) in AdS space.

What then might be the endpoint of this decay (for any theory with bulk tachyons - not just 0A/B) at non-zero coupling? For now we note that another endpoint seems plausible, that of space-time annihilation. The motivation for this conjecture comes from a semi-classical argument first presented by Witten [81] (to be reviewed) and directly parallels the case for $D$-$\overline{D}$ annihilation (also to be reviewed). This instability disappears in the zero coupling limit and so may only be associated with a tachyonic instability at non-zero coupling.

The outline of this chapter is as follows. We begin by reviewing the tools relevant for the closed string analysis. These include a review of the consistency conditions for closed string theories, which allow us to treat even tachyonic closed string vacua as potentially interesting string backgrounds. We then present the program for analyzing the semiclassical instability of gravitational vacua and review the simplest case of a 5-dimensional Kaluza-Klein vacuum first discussed by Witten [81]. We go on to mention a strongly suggestive scenario present in the well understood open string case, and use this to motivate a conjecture relating tachyonic and semiclassical instabilities in general. The rest of the chapter is devoted to gathering evidence for this conjecture by systematically investigating backgrounds with closed string tachyons and possibly related semiclassical instabilities. The conjectured relationship between these two is also used to draw out a web of dualities between nonsupersymmetric string vacua. We will first apply the conjecture in systems with an eleven-dimensional starting point. These are important because their perturbative limits involve the well known ten-dimensional string theories. Starting in Sec.[4.2] with semi-classically unstable circle and interval compactifications of M-theory we identify the tachyonic perturbative limits involving Type 0A/B and nonsupersymmetric heterotic strings on flat backgrounds, Melvin magnetic backgrounds, and noncompact orbifolds. We then move on in Sec.[4.3] to similar considerations for ten-dimensional starting points which admit a
greater degree of control and in many instances may be related to the eleven-dimensional cases
by a “9-11” flip duality. Along the way we will encounter several situations for which recent
analyses have led to conflicting conclusions and we will discuss these issues.

4.1 Techniques for Closed Strings

4.1.1 Consistency Conditions

One of the appealing features of perturbative string theory is the level of uniqueness that
emerges from its most basic requirement, consistency. We will very briefly review three of the
primary sources of constraint on possible perturbative formulations. This subject comprises a
large part of the history of string theory. The interested reader is invited to peruse the standard
reference texts [34, 35, 53, 54].

4.1.1.1 Conformal Invariance

Recall that a perturbative string theory is formulated in terms of a quantum field theory
on a (1 + 1)-dimensional worldsheet.

\[ S_{WS} \sim \int d^2 \sigma \sqrt{\text{det} g^{ab}} (\partial_a X^\mu \partial_b X_\mu + \psi^\mu \Gamma_a \partial_b \psi_\mu + \cdots). \] (4.1)

This theory includes (among others) a set of scalar fields \( X^\mu \) which carry gauge indices \( \mu \) for
an internal \( SO(1, D - 1) \) symmetry on the worldsheet. The theory is turned inside out by
interpreting these \( X^\mu \) fields as embedding coordinates describing the propagation of the string
in a \( D \)-dimensional Lorentz invariant target spacetime (hence the \( SO(1, D - 1) \) symmetry). One
may ask what other fields can be included in the worldsheet theory and furthermore if there is
anything constraining the number of \( X^\mu \) fields, i.e. the dimensionality of spacetime. In short, one
finds that to consistently quantize the worldsheet theory on a Lorentz invariant target spacetime
requires the worldsheet theory to be a conformal field theory (CFT) of “critical” central charge.\(^2\)

Conformal invariance determines the form of the action, or how we combine the fields. What

\(^2\) By allowing non-Lorentz invariant spacetimes the central charge condition can be relaxed [23], but we will
not be discussing such “noncritical” formulations here.
about the content? What fields can we include? This is where the central charge comes in. After some specifications of the worldsheet topology (with or without boundary, oriented or unoriented) we can add worldsheet fermions, worldsheet bosons, and ghosts as needed to obtain the required central charge. One can compute the contribution for each type of field. A consistent closed string theory combines a set of these with total central charge \((26, 26)\). Since the central charge for \(n\) \(X^\mu\)-fields is \(n\) times that of a single \(X^\mu\)-field, we see how the dimension of spacetime arises as a consistency condition for the worldsheet theory. Once we have a consistent CFT, we can quantize the worldsheet theory and determine the spectrum in spacetime (this is a step removed from the worldsheet spectrum, i.e. a field on the worldsheet is not a field in spacetime). Lo and behold we find that for any theory based on closed world sheets, there is a graviton.

### 4.1.1.2 Modular Invariance

We now move on to the remaining consistency condition relevant for strings based on closed and oriented worldsheets. In the next section we will modify this argument for strings with unoriented worldsheets and those with boundaries.

So far we have laid out the conditions for the consistent quantization of the worldsheet theory. To interpret this as a quantum theory in spacetime (via the embedding prescription) we directly construct the worldsheets that can be interpreted as terms in a Feynman expansion of processes in spacetime. Hence we are necessarily working in a first quantized program for the theory in spacetime. This is compactly expressed in terms of the Polyakov path integral which includes a sum over worldsheets of successively higher genus.

\[
\int [dgdx] \exp(-SW_S - \lambda \chi).
\]

We have added to the worldsheet action \(S_W S\) a 2D gravitational term \(\lambda \chi\). This turns out to be nondynamical (it is a total derivative on the worldsheet) and hence depends only on the worldsheet topology. This is characterized by an invariant \(\chi\) which measures the genus of the

---

\(3\) The central charge \((26, 26)\) is appropriate for closed strings which have no worldsheet boundary. In this case we are free to add independent left and right moving degrees of freedom usually distinguished by a () over the right movers.
worldsheet. The genus in turn simply counts the number of handles on the worldsheet which in spacetime is interpreted as the number of quantum loops in the diagram.

Since we are now dealing with a quantum theory that includes a graviton we should ask if this theory suffers the uncontrollable divergences encountered when trying to formulate such a theory in terms of spacetime fields. It can be demonstrated that this question is answered once and for all at the one-loop level (basically the argument that works at one loop in string theory can be extended with some plumbing to multi-loop levels). For closed strings the one loop diagram is a torus (assuming vacuum to vacuum). We will skip the details and provide only the results sufficient to understand why the string amplitude is rendered finite. In quantum field theory the corresponding one loop vacuum to vacuum amplitude, the so called partition function, is given by

$$ Z_{S^1}(m_i^2) = iV_d \int_0^\infty \frac{dl}{2l} (2\pi l)^{-d/2} \exp(-m_i^2 l/2). \quad (4.3) $$

where $m_i$ specifies the particle mass and $l$ parameterizes the size of the circle. UV divergences arise when we consider the behavior as $l \to 0$. If we naively apply this result to strings then we must sum over the full string spectrum $\Sigma_i Z_{S^1}(m_i^2)$.\footnote{We are not doing string theory at this point. We are merely including the spectrum of the string in spacetime in a field theory calculation.} Doing this can be encoded by introducing yet another "circle" (the string itself) which we will parameterize by $\theta$. Combining the two variables into a single complex coordinate $\theta + \frac{i l}{\alpha'} \equiv 2\pi \tau = 2\pi (\tau_1 + i \tau_2)$, the one loop amplitude now involves integrating over both cycles making the measure now

$$ \Sigma_i Z_{S^1}(m_i^2) = \int_R \frac{d\tau d\bar{\tau}}{4\tau_2} \cdots \quad (4.4) $$

The integration region $R$ includes all $\tau_2 > 0$ and $|\tau_1| < \frac{1}{2}$. The divergence as $l \to 0$ persists, now guised as $\tau_2 \to 0$, and is in fact worsened by the multiplicity of diverging contributions from the string states. If we calculate the same amplitude in string theory we find an expression that is strikingly similar

$$ Z_{T^2} = \int_F \frac{d\tau d\bar{\tau}}{4\tau_2} \cdots \quad (4.5) $$
However the string result can in certain situations have a symmetry called modular invariance which is generated by the two transformations

\[ \tau \rightarrow -\frac{1}{\tau} \]  
(4.6)

\[ \tau \rightarrow \tau + 1. \]  
(4.7)

If this is indeed the case, then integrating over a region including both \( \tau \to 0 \) and \( \tau \to \infty \) constitutes an overcounting since these are related by the transformation Eq.(4.6). In fact it is easy to see that if our result exhibits modular invariance, then we can consistently restrict the integration region \( F \) to \( \tau_2 > 1 \) and \( |\tau_1| < \frac{1}{2} \). Notice now that the dangerous region \( \tau_2 \to 0 \), or \( l \to 0 \), is removed. This is the way in which string theory can render the one loop amplitude finite.

Of course we could do this in field theory by simply imposing a cutoff, but doing so while maintaining general covariance is exceedingly difficult. String theory manages this in a subtle way. The theory is cutoff by the string length, but general covariance is preserved.

Modular invariance of the partition function arising in a conformally invariant theory of critical central charge defined on closed and oriented worldsheets limits us to exactly thirteen theories. Nine of these lack spacetime supersymmetry. The remaining four are known as types IIA/B, heterotic \( SO(32) \) and heterotic \( E8 \otimes E8 \). If you are wondering why we have not discussed open strings as a possible feature, it turns out that these cannot interact consistently with closed and oriented strings.

### 4.1.1.3 Tadpole Cancellation

Reversing the orientation of the string worldsheet simply amounts to a parity operation \( \Omega \) which interchanges left and right movers. To build a perturbative closed string theory based on unoriented worldsheets, we may begin with an oriented theory invariant under \( \Omega \), and project the spectrum on to those states invariant under orientation reversal. That is, while the worldsheet action is invariant under reversal of orientation of the worldsheet, some of the solutions do not
share this symmetry. In an oriented theory these states are allowed, but in an unoriented theory they must be projected out. The only supersymmetric string based on a worldsheet theory invariant under \(\Omega\) is the type IIB. The projection operator that does the trick is
\[
P \equiv \frac{1}{2}(1 + \Omega). \tag{4.8}
\]
States odd under \(\Omega\) have zero eigenvalue under this projection. Since we started with type IIB we already know that we are working with a conformal theory of critical central charge. The only issue left to consider is the one loop amplitude which we must check for finiteness. What is the effect of projecting the partition function by \(P\)? Consider
\[
Z_{T^2}^{\text{II}B} \rightarrow \frac{1}{2}(1 + \Omega)Z_{T^2}^{\text{II}B} = Z_{T^2} + Z_K. \tag{4.9}
\]
The first term in the projector acts trivially as multiplication by a constant and so modular invariance of \(Z_{T^2}\) is ensured by the invariance of \(Z_{T^2}^{\text{II}B}\). What about \(Z_K \equiv \frac{1}{2}\Omega Z_{T^2}^{\text{II}B}\)? This partition function arises from a worldsheet theory with the topology of a Klein-bottle (hence the subscript). A Klein-bottle has no analogue of the modular group of transformations so this partition function certainly can not exhibit modular invariance. Thus there is no reason to restrict the region of integration and we find ourselves once again plagued by the divergences from the \(l \to 0\) region.

However all is not lost. For we can, in a consistent way, couple a theory based on unoriented closed worldsheets with additional sectors based on worldsheets with a boundary. That is, we can add open strings. These will give additional contributions to the partition function now in the form of a cylinder amplitude \(Z_C\) and a Mobius-strip \(Z_M\) (one loop open string analogues of the torus and Klein-bottle respectively). The net partition function
\[
Z^I = Z_{T^2} + Z_K + Z_C + Z_M. \tag{4.10}
\]
is finite if the divergences of the last three terms cancel among themselves. It may not be clear that there is any freedom to adjust things if we find that the divergences do not cancel. However, when we introduce open strings we also introduce gauge degrees of freedom on their
endpoints and we have some flexibility in choosing the gauge symmetry. In spacetime we can think of this as a freedom in choosing the $D$-branes in our background that will support the open strings. In the unoriented case we are relegated to an $SO(n)$ gauge group so our only freedom is in specifying the rank of the gauge symmetry, i.e. in choosing the number of $D$-branes, c.f. Fig.1.6. For $n = 32$ one can show that the divergences from the last three terms cancel, and so this leads to the unique supersymmetric unoriented closed+open string theory called type I $SO(32)$. This construction of building type I from type IIB is called an orientifolding.

4.1.1.4 Orbifolds

The last consistency device we want to discuss is the technique of orbifolding. Orbifolding has actually pervaded most of our discussion thus far, but we have not specifically defined the procedure. Simply put, an orbifold is a method of deforming a theory based on a modular invariant partition function while maintaining consistency. The program is to choose some global symmetry of the worldsheet theory $G$ and project onto the states invariant under it.

$$P \equiv \frac{1}{2} (1 + G).$$

(4.11)

This is strikingly analogous to orientifolding, though here we stick with oriented worldsheets. As in the orientifold case, we find that the naive projection ruins the finiteness of the theory. For orientifolds this happens because the unoriented worldsheets do not admit a modular group. For orbifolds we still have a modular group, but the projected theory is simply not invariant. The projected partition function contains the orginal modular invariant term plus a new piece from the nontrivial part of the projection operator as in Eq.(4.9). To cure this we must add new sectors to the theory. Though this is similar in spirit to adding open strings in the orientifold case, these new “twisted” strings in the orbifold case are again closed and oriented. The net result for orbifolds reconstitutes modular invariance.

Orbifolding actually allows us to build the four relevant string theories in terms of orbifolds of each other. Simple compactifications of string theory can be phrased in the language
of orbifolds, and the construction also gives rise to some of the more interesting singular, but nonetheless consistent, background geometries. The construction has considerably larger applications and has proven to be one of the most fruitful programs in string theory.

\subsection{The Bounce}

In this section we review the semi-classical stability analysis of gravitational vacua. The simplest example in this type was worked out by Witten [81]. Here we outline the general strategy.

"Semi-classical" essentially means that we work in a classical gravity formalism, i.e. Einstein-Hilbert, supplemented by the possibility of barrier penetration. We will not need the machinery of perturbative quantum gravity in this analysis so we leave it aside. We will come back to this issue in the context of string theory later.

The program begins with some vacuum solution of Einstein-Hilbert gravity in D-dimensions. The relevant action is

\[ S = M_p^{D-2} \int d^D x \sqrt{\det G} R. \tag{4.12} \]

A familiar solution to the equations of motion following from this action is D-dimensional Minkowski space \( M^D \). This is locally characterized by the metric solution

\[ ds^2_M = -dt^2 + \sum_{i=1}^{D-1} dx_i^2 \]  \tag{4.13}

and has a noncompact topology. Another solution to the equations of motion is the Kaluza-Klein vacuum. Locally this vacuum looks like \( M^D \) sharing with it the metric Eq. (4.13). The distinction from \( M^D \) is topological. In the simplest we can take \( M^{D-1} \otimes S^1_R \). The metric for this vacuum can be written as

\[ ds^2_{KK} = -dt^2 + \sum_{i=2}^{D-1} dx_i^2 + dw_R^2 \]  \tag{4.14}

to remind the reader that the \( w_R \) coordinate corresponds to the compact \( S^1_R \) part of this solution.

Is the Kaluza-Klein vacuum semi-classically stable? In principle one could analyze the full space of vacua, find another solution with lower energy and directly compute tunneling
properties. However comparing vacuum energies in general relativity is rather subtle, and so here we take a simpler route that is familiar from field theory. The program, which we will elaborate with an example, is as follows (working in D dimensions):

1. First Euclideanize the D-dimensional vacuum solution \( V_M \) under consideration, i.e. \( V_M \rightarrow V_E \).

2. Euclideanize the action from which this vacuum solution arose, i.e. \( S_M \rightarrow S_E \).

3. Look for another solution \( B_E \) to the Euclideanized action \( S_E \) with the same asymptotic behavior as the solution under consideration, i.e. \( B_E(r \rightarrow \infty) \sim V_E(r \rightarrow \infty) \). Such a solution is called a bounce and mediates tunneling between the Minkowski versions of the original vacuum \( V_M \) and a new vacuum \( V'_M \) still to be determined.

4. In the general theory of semiclassical vacuum decay in D dimensions the original vacuum \( V_M \) decays into a new vacuum \( V'_M \) which agrees with the \( B_E \) on a \( (D-1) \)-dimensional hypersurface which is taken as the \( t = 0 \) hypersurface for the post decay geometry. To describe the new vacuum \( V'_M \), one looks for a \( (D-1) \)-dimensional surface of zero extrinsic curvature \( S_0 \) in \( B_E \) and uses this as the initial data in an analytic continuation of \( B_E \) to Minkowski space, i.e. \( B_E|_{S_0} \rightarrow V'_M|_{t=0} \).

5. The post decay evolution is determined by considering \( V'_M(t) \) for \( t > 0 \).

6. Lastly the decay rate per unit volume per unit time can be calculated by evaluating \( \Gamma \sim e^{-S_E(B_E)} \).

We will now review the application of this program to the simplest 5-dimensional Kaluza-Klein vacuum. Again the action is simply an Einstein-Hilbert term Eq.(4.12) in 5 dimensions and the original Minkowski vacuum is described by Eq.(4.14) with a single compact coordinate \( w_R \) of radius \( R \).

1. Euclideanizing the solution \( t \rightarrow i\tau \) we have

\[
\begin{align*}
  ds^2_{KK} &= d\tau^2 + dw_R^2 + dx^2 + dy^2 + dz^2.
\end{align*}
\]  

(4.15)
We indicate a Euclidean signature metric by $d\tilde{s}^2$. Introducing polar coordinates to describe the $R^4$ spanned by $(\tau, x, y, z)$ this becomes

$$d\tilde{s}_{KK}^2 = dr^2 + r^2 d\Theta^2 + dw_R^2$$

(4.16)

where now $r$ runs from 0 to $\infty$ and $d\Theta^2$ is the line element of the three sphere.

(2) At this point we won’t need the explicit Euclideanized form of the Einstein-Hilbert action.

(3) To find a second solution from the Euclideanized Einstein-Hilbert action we can simply take a second familiar Minkowski solution and rotate it to Euclidean signature. The solution we use is the 5-dimensional Schwarzschild solution. Euclideanizing this and identifying it as our bounce solution we find

$$ds_B^2 = \frac{1}{1 - \frac{\alpha}{r^2}} dr^2 + r^2 d\Theta^2 + (1 - \frac{\alpha}{r^2})dw_R^2.$$ 

(4.17)

If the location of the nontrivial metric factors seem surprising it is because this metric would not follow from the same continuation from Minkowski space that we used to obtain $d\tilde{s}_{KK}^2$ from $ds_{KK}^2$. This difference is irrelevant. Note that this has the correct asymptotic property, i.e. $ds_B^2(r \to \infty) \sim d\tilde{s}_{KK}^2$. The additional parameter $\alpha$ can be determined by demanding the metric be nonsingular at $r = \sqrt{\alpha}$, i.e. the location of the Euclidean black hole horizon. To rewrite things in terms of purely exterior coordinates we introduce a new radial coordinate $\lambda$ satisfying

$$r = \sqrt{\alpha} + \lambda^2$$

(4.18)

where $\lambda$ runs from 0 to $\infty$. Expanding the nontrivial terms in Eq.(4.17) about the point $\lambda = 0$ we find

$$2\sqrt{\alpha}(d\lambda^2 + \frac{\lambda^2}{\alpha}dw_R^2)$$

(4.19)

which should be compared to the standard metric describing a plane

$$d\rho^2 + \rho^2 d\phi^2.$$ 

(4.20)
The metric Eq.(4.20) is nonsingular if $\phi$ has a period of $2\pi$, otherwise the metric describes a cone whose tip is obviously a metric singularity. To avoid a conic singularity in Eq.(4.19) we thus require the coordinate $w_R$, which has period $2\pi R$ in the Kaluza-Klein geometry, to also have period $2\pi \sqrt{\alpha}$. Simultaneously satisfying these periodicity requirements forces us to set $\sqrt{\alpha} = R$.

The requirement of setting $\sqrt{\alpha} = R$ in order to render the bounce nonsingular may have seemed tedious, but the end result should not be surprising. We started off with a vacuum $M^{D-1} \otimes S^1_R$ uniquely specified by the parameter $R$. The decay of this background is to be mediated by a bounce solution which takes the form of a Euclidean Schwarzchild black hole metric. The Schwarzchild metric describes an infinite set of black hole solutions distinguished by a single parameter ($\alpha$ in our case) which essentially describes the black hole mass. In avoiding the conical singularity above we are simply choosing the single Euclidean Schwarzchild metric which consistently glues into the Euclideanized Kaluza-Klein spacetime, hence the determination of $\alpha$ in terms of $R$.

The most important feature of this analysis, and one that we will come back to discuss in a moment, is that the coordinate $r$ in the metric Eq.(4.17) has a minimum value $r_{min} = \sqrt{\alpha}$. Our bounce solution only makes use of the exterior black hole geometry. This is a legitimate geometry for a bounce insofar as it is nonsingular and geodesically complete. More importantly it will eventually provide a finite action which implies that it is relevant for mediating the decay of the vacuum in a finite time.

(4) To continue the bounce solution back to Minkowski space we must find a surface of zero extrinsic curvature. There are a number of possibilities. Anticipating some simplifications later on we break up $d\Theta^2 = d\theta^2 + \sin^2 \theta d\Omega^2$ and use the surface $\theta = \frac{\pi}{2}$. The continuation to Minkowski space can be affected by $\theta \rightarrow \frac{\pi}{2} + i\psi$. This yields

$$ds_B^2 = \frac{1}{1 - \frac{R}{r^2}} dr^2 - r^2 d\psi^2 + r^2 \cosh^2 \psi d\Omega^2 + (1 - \frac{R}{r^2}) dw_R^2. \quad (4.21)$$

(5) At this point we have followed the program but we are left with the metric Eq.(4.21)
which is supposed to tell us about the post-decay spacetime. However, it is less than
obvious what this spacetime is. Furthermore it is not even clear how it is connected
to the pre-decay spacetime. This largely owes to the fact that when we continued to
Euclidean space we used one coordinate, and when we continued back to Minkowski
space we used a different one. To clarify this picture consider the asymptotic form of
Eq.(4.21),

\[ ds_B^2(r \to \infty) \sim dr^2 - r^2 d\psi^2 + r^2 \cosh^2 \psi d\Omega^2 + dw_R^2 \]  

(4.22)

and define

\[ \dot{r} = r \cosh \psi \]

\[ t = r \sinh \psi. \]  

(4.23)

Using that \( dr^2 - r^2 d\psi^2 = d\hat{r}^2 - dt^2 \) the expression Eq.(4.22) becomes

\[ ds_B^2(r \to \infty) \sim -dt^2 + d\hat{r}^2 + \hat{r}^2 d\Omega^2 + dw_R^2. \]  

(4.24)

So asymptotically far from the decay "seed" we recover the flat Kaluza-Klein vacuum
with which we started. When we are far from a nucleated bubble of the true vacuum,
things should still approximate the false vacuum. We should also expect the full post-
decky metric to get decidedly more complicated as we approach the decaying region. The
full form of the metric in this region is not very illuminating. What is interesting however
is the question of what lies inside the nucleated bubble. The coordinates defined in
Eq.(4.23), which is the coordinate system asymptotically reproducing the Kaluza-Klein
geometry, satisfy

\[ \dot{r}^2 - t^2 = r^2. \]  

(4.25)

A trivial rearrangement of this expression yields

\[ \dot{r}(t) = \sqrt{r^2 + t^2}. \]  

(4.26)

Now recall that the radial coordinate \( r \) has a minimum value \( r_{\text{min}} = \sqrt{\alpha} \). This sets
a minimum for \( \dot{r}(t) \) which is itself minimized at \( t = 0 \), i.e. \( \dot{r}_{\text{min}}(t = 0) = \sqrt{\alpha} \). For
$t > 0$ the $\tilde{r}$ coordinate minimum expands with time. This is Witten’s bubble of nothing. Inside the region $\tilde{r} < \tilde{r}_{\text{min}}(t)$ the metric degrees of freedom simply cease to exist. The bubble surface has a topology $R^2 \otimes S^2$. Naively one might think that simply removing a sphere from a spacetime will necessarily create singular boundaries and geodesic incompleteness. However, the extra compact dimension plays a highly nontrivial role in smoothing out what would otherwise be a “sharp” cut in the manifold, c.f. Fig.4.2.

![Figure 4.2](image)

**Figure 4.2:** Removing sections of manifolds. In (a) removing a segment from the line $R^1$ introduces boundaries. In (b) the extra compact dimension in $R^1 \otimes S^1$ can be used to smooth out the remaining space after removing a section, rendering it boundary free.

At this point we may justify having called the the metric Eq.(4.21) a “bounce” solution. In contrast to more familiar instantons, bounce solutions mediate tunneling between a static vacuum and a time dependent one as shown in Fig.4.3. The semiclassical decay corresponds to the formation of the bubble itself, while the subsequent evolution, i.e. growing of the bubble, is governed by purely classical dynamics.

![Figure 4.3](image)

**Figure 4.3:** Two tunneling events. In (a) a bounce mediates the tunneling between a metastable vacuum on the left and a point of nonvanishing slope on the right. After tunneling the background evolves with time. In (b) an instanton mediates the decay from the metastable vacuum on the left to the more stable vacuum on the right. The vacuum before and after tunneling in this case is otherwise static.
Lastly we compute the decay rate per unit volume per unit time and find

\[ \Gamma \sim e^{-S_F(B_F)} \sim e^{\frac{-R}{128 \times M_P^3}} \]  

reflecting, as one might expect, that this instability becomes less relevant for larger values of \( R \), vanishing as we approach noncompact 5-dimensional Minkowski space, i.e. \( M^4 \otimes S^1_R \rightarrow \infty \).

As a final note we should mention the nontrivial impact of adding fermions to this analysis. The simplest Kaluza-Klein vacuum \( M^{D-1} \otimes S^1 \) is not simply connected. If we pick two points on the spacetime and draw a path connecting them, then there are two distinct possibilities that cannot be continuously deformed into one another, i.e. they are homotopically inequivalent. This is easily seen by considering the \( S^1 \) itself. For fermions this implies that we have a freedom in defining the periodicity along the \( S^1 \) factor. The choices may be cast as a simple +1 or -1 phase acquired by a fermion as it encircles the \( S^1 \). That is, we have two distinct ways of adding fermions to a theory defined on \( M^{D-1} \otimes S^1 \). This is referred to as choosing the spin structure. On the other hand, on simply connected geometries, for which all paths connecting two points are homotopically equivalent, there is a single allowed spin structure. The surface of Witten's bubble is simply connected (recall that it involves a two-sphere and one can easily convince oneself that this is simply connected) and thus we should determine the unique spin structure admitted on the post decay geometry. More importantly since this post-decay geometry must asymptotically reproduce the original unstable Kaluza-Klein vacuum, we should determine which choice of spin structure on the \( S^1 \) factor is compatible with the spin structure on the bubble geometry. The answer is that we must choose -1 spin-structure, or fermions anti-periodic on the \( S^1 \). This in turn implies that a theory with periodic fermions on the Kaluza-Klein vacuum is stable against this semi-classical process. In supersymmetric theories, which necessarily include fermions, this corresponds to the choice of fermion periodicity that preserves the spacetime supersymmetry.

This reflects what we already know, i.e. that supersymmetric vacua are stable.

The example presented here will be relevant for some of the discussion to follow. However
we will generally be dealing with 1-dimensional compactifications specified by not one but two parameters. These vacua are subject to a similar analysis. In this case the relevant black hole solution must include two parameters to be uniquely matched to the decaying geometry. Such solutions are already known in the form of D-dimensional Kerr solutions which describe spinning black holes.

4.1.3 A Conjecture for Closed String Tachyons

Sen's conjecture gave us a foothold on understanding the role of tachyons in the open string sector. Numerous techniques have been successfully applied to the study of the tachyon condensate directly, and its properties seem fairly well understood. There is however in addition a natural picture that emerges wherein the open string tachyonic instability is the perturbative manifestation (for certain values of the background moduli) of an instability which also admits a semi-classical description (for other values of the background moduli). The simplest example of this type is the $D-\bar{D}$ system. In this case for small enough separation between the two branes there exists an tachyonic open string mode which mediates annihilation of the pair à la Sen. Now consider separating the two branes. The strings stretched between them (the spectrum of which included the tachyon) gain an energy proportional to the distance between the branes. For a large enough separation $d$ (usually $d \sim l_s$) the tachyonic mode becomes massive. However, the spacetime supersymmetry is still completely broken and so we may ask if the resulting background is still unstable. There are actually two instabilities that we find. On one hand there is an attractive force between the two branes which will act to pull them together rendering the tachyon tachyonic again. However there is also a possibility for the branes to annihilate directly at separations larger than the string scale. For large enough $d$ one can work with the low-energy effective action governing the open string modes on the $D$-branes which takes the Born-Infeld form. Callan and Maldecena [14], and independently Savvidy [63], analyzed the resulting dynamics and found a semi-classical instability towards annihilation. The analyses included

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5 A $\bar{D}$-brane is simply a $D$-brane rotated by $\pi$. This operation can be shown to reverse the sign of the charge carried by the brane and completely breaks the spacetime supersymmetry.
solving the equations of motion from the separate actions on each brane to find half-bounce solutions and then gluing these together to form a solution over the full system. An extensive foray into Born-Infeld dynamics would take us too far afield. Fortunately the results of their analyses can be demonstrated pictorially as shown in Fig. 4.4. Note that the end result of this process is similar to the final state after tachyon condensation. The existence of the tachyon at short distances and the possibility of tunneling for large distances both rely on the anti-alignment of the $D$-branes. The probability for nucleating a throat is uniform over the surface of the branes as is the presence of the tachyonic instability. Furthermore both processes exhibit localization of the decay “seed” for nontrivial relative orientations. These similarities make it seem natural to identify these two processes as different manifestations of a single nonperturbative instability.

This is the principle motivation for the following conjecture: Suppose we have a theory $A$ on a background $X$ which admits a semi-classical instability (determined by an analysis of the low-energy effective field theory). Now also assume that this semi-classical instability varies smoothly as the moduli determining $X$ are varied. If by adjusting the moduli of $X$ we reach a region in which the semi-classical analysis is invalid (but otherwise would lead to an instability), and we are instead afforded a perturbative description of the quantum theory $Y$, then the semi-

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6 Of course the construction of noncommutative solitons made use of Born-Infeld dynamics, but the flavor of that analysis is sufficiently different from that in [14],[63] to necessitate too lengthy a discussion.
solving the equations of motion from the separate actions on each brane to find half-bounce solutions and then gluing these together to form a solution over the full system. An extensive foray into Born-Infeld dynamics would take us too far afield. Fortunately the results of their analyses can be demonstrated pictorially as shown in Fig.4.4. Note that the end result of this

\[ \text{(a)} \quad \text{(b)} \quad \text{(c)} \]

Figure 4.4: For two 1-dimensional surfaces governed by an action \( S \sim \text{length} \) the solutions (a) and (b) are degenerate in energy. If we begin with the solution in (a), then it may semiclassically tunnel to solution (b) by an appropriate bounce, see Fig.4.3. The action principle (length minimization) that governs the dynamics will subsequently force the “hole” to expand as shown. Note that for oriented surfaces the vacuum in (b) can only be formed from anti-aligned surfaces.

process is similar to the final state after tachyon condensation. The existence of the tachyon at short distances and the possibility of tunneling for large distances both rely on the anti-alignment of the D-branes. The probability for nucleating a throat is uniform over the surface of the branes as is the presence of the tachyonic instability. Furthermore both processes exhibit localization of the decay “seed” for nontrivial relative orientations. These similarities make it seem natural to identify these two processes as different manifestations of a single nonperturbative instability. This is the principle motivation for the following conjecture: Suppose we have a theory \( \mathbf{A} \) on a background \( \mathbf{X} \) which admits a semi-classical instability (determined by an analysis of the low-energy effective field theory). Now also assume that this semi-classical instability varies smoothly as the moduli determining \( \mathbf{X} \) are varied. If by adjusting the moduli of \( \mathbf{X} \) we reach a region in which the semi-classical analysis is invalid (but otherwise would lead to an instability), and we are instead afforded a perturbative description of the quantum theory \( \mathbf{Y} \), then the semi-

\[ ^{6} \text{Of course the construction of noncommutative solitons made use of Born-Infeld dynamics, but the flavor of that analysis is sufficiently different from that in [14],[63] to necessitate too lengthy a discussion.} \]
classical instability should be reflected by a tachyonic instability in the perturbative description $Y$. In addition, the endpoints of both instabilities are to be identified.

We may consider a strong and weak form of the conjecture above distinguished as follows:

Strong: A semiclassical instability as described above predicts the existence of a tachyonic perturbative description of the resulting theory and we should identify the endpoint of condensation of the tachyon with that of the semiclassical instability.

Weak: A semiclassical instability as described above can be related to the tachyonic mode whenever it exists in a perturbative description of the resulting theory by identification of the endpoints of the instabilities.

The weak form allows for theories which do not reduce to tachyonic perturbative descriptions. We will see examples of each case below.

The conjecture above will in many cases involve extrapolations from strong coupling regions. As the systems are non-supersymmetric, such extrapolations are unprotected and hence we do not know how to prove them. Support for this conjecture comes from three directions. (1) The close analogy to open string tachyons in unstable $D$-brane systems for which there is much support. (2) The existence of closed string tachyons in perturbative limits of systems exhibiting semi-classical instabilities. (3) The fact that flat space decay endpoints do not seem plausible for finite coupling as argued above.

### 4.2 Applications of the conjecture for 11D $\rightarrow$ 10D

In this section we will discuss circle and interval compactifications of M-theory. The semi-classical instabilities arise in the eleven-dimensional low-energy gravity theory as a result of the Kaluza-Klein structure of the vacuum. We adapt several results from [17] to the case of ten noncompact dimensions and discuss our own ideas on the relevance of the semiclassical decay evolution. Identification of perturbative string limits requires an extrapolation from strong to weak coupling and in the absence of supersymmetry is unprotected.
4.2.1 Twisted Circle $M^{10} \otimes S^1_{R,B}$

Consider eleven-dimensional M-theory on a background which locally resembles $M^{10} \otimes S^1$

$$ds^2_{11} = -dt^2 + d\rho^2 + \rho^2 d\phi^2 + dy_1dy_i + dx^2_{11} \quad i = 3, ..., 9$$

but differs globally by the nontrivial identifications:

$$x_{11} \sim x_{11} + 2\pi n_1 R$$

$$\phi \sim \phi + 2\pi n_1 BR + 2\pi n_2$$

where $n_1, n_2 \in Z$. These identifications merely impose a rotation by $n_1 BR$ on any state as it is transported around the compact direction $n_1$ times. We designate such a “twisted” circle by $S^1_{R,B}$. Let us choose a periodic spin structure for the $S^1_R$. The twist parameter $B$ takes values $0 \leq |B| < \frac{2}{R}$. For $B = 0$ this is a supersymmetric compactification while for $B \neq 0$ the spacetime supersymmetry is completely broken. The effective theory governing the low-energy dynamics will generically incorporate Einstein-Hilbert gravity. It has been known for some time that gravity on a Kaluza-Klein background of this form exhibits a semi-classical instability towards the annihilation of spacetime (first discussed for five dimensions in [26, 27] and later extended to eleven dimensions in [17]). This instability is mediated by a bounce solution that takes the form of an eleven-dimensional Euclidean Kerr black hole solution:

$$ds^2_{11} = (1 - \frac{\mu}{r^6})dx_0^2 - \frac{2\mu \sin^2 \theta}{r^6} dx_0 d\phi + \frac{\Sigma}{r^2 - \alpha^2 - \mu^{-6}} dr^2 + \Sigma d\Omega^2$$

where $\Sigma \equiv r^2 - \alpha^2 \cos^2 \theta$, $\mu$ is the black hole mass parameter, $\alpha$ is a single complexified angular momentum parameter, and we have written $d\Omega_7$ as $d\chi^2 + \sin^2 \chi d\Omega_6$ for later convenience. The identifications Eq.(4.29) are most easily expressed in eleven-dimensional $SO(2)$-coordinates on $S_1$:

$$t, \rho, \phi^{(2\pi)}, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{11}$$

but the bounce metric is more easily expressed in $SO(9)$-coordinates on $S_8$:

$$x_0, r, \theta^{(\frac{\pi}{2})}, \phi^{(2\pi)}, \chi^{(2\pi)}, \theta_1^{(\pi)}, \theta_2^{(\pi)}, \theta_3^{(\pi)}, \theta_4^{(\pi)}, \theta_5^{(\pi)}, \theta_6^{(\pi)}.$$
To match the bounce solution Eq. (4.30) to the unstable background Eq. (4.28, 4.29) the background parameters must satisfy:

\[
R = \frac{\mu}{4r_H^7 - 3\alpha^2 r_H^5} \quad (4.31)
\]

\[
B = \frac{\alpha r_H^6}{\mu} - \frac{\alpha}{|\alpha| R} \quad (4.32)
\]

where \(r_H\) is the location of the Euclidean black hole horizon satisfying:

\[
r_H^2 = \alpha^2 + \frac{\mu}{r_H^6}. \quad (4.33)
\]

The coordinate singularity sets a lower bound on the range of the radial coordinate

\[
r \geq r_H. \quad (4.33)
\]

To estimate the decay rate we evaluate the Euclidean action \(I\) for the bounce solution Eq. (4.30) and calculate \(\Gamma \sim e^{-I}\). This evaluates to:

\[
\Gamma \sim e^{-\frac{e^5 \mu B}{G_{11}}}. \quad (4.34)
\]

where \(G_{11}\) is the eleven-dimensional Newton's constant. Evaluating the decay rate in terms of the background parameters \(R, B\) involves untangling expressions Eq. (4.31, 4.32) which can be quite difficult. The task simplifies for two important parameter regions [27]:

For \(|B| \sim 0\) the expression for \(\mu\) reduces to \(\mu = (\frac{\alpha}{2})^7 \frac{R}{|B|}\) which clearly diverges for \(|B| \to 0\). The decay rate vanishes rendering the theory stable against the semi-classical instability.

For \(|B| = \frac{1}{R}\) (corresponding to \(\alpha = 0\)) the expression simplifies to \(\mu = (4R)^8\). One can demonstrate that this is actually a minimum of \(\mu(B)\) and hence represents the most unstable background.

In fact we have found with some numerical work that the decay rate is a monotonically decreasing function of \(R\) for fixed \(B \neq 0\) indicating the expected stability of the decompactified theory and in turn the maximum instability of the theory as \(R \to 0\). For fixed \(R\) one can also demonstrate that the decay rate is a monotonically increasing function for \(B\) increasing from \(0 \to \frac{1}{R}\) beyond which it monotonically decreases as \(B\) approaches \(\frac{2}{R}\).
The evolution of the background Eq.(4.28,4.29) after the decay is determined by finding a zero-momentum surface in the bounce solution and using this as initial data for an analytic continuation back to Lorentzian signature. Such a zero-momentum surface is given by \( \chi = \frac{\pi}{2} \), so we may continue Eq.(4.30) by sending \( \chi \to \frac{\pi}{2} + i\tau \) to obtain

\[
\begin{align*}
\frac{ds_{11}^2}{r^6} &= \left(1 - \frac{\mu}{r^6} \right) dx^2_0 - \frac{2\rho \sin \theta}{r^6} d\phi + \frac{\Sigma}{r^2 - \alpha^2 - \mu r^6} dr^2 + \Sigma d\theta^2 \\
&+ \frac{\sin^2 \theta}{\Sigma} [(r^2 - \alpha^2) \Sigma - \frac{\mu}{r^6} \alpha^2 \sin^2 \theta d\phi^2 + r^2 \cos^2 \theta d\Omega_6] \\
&= (4.35)
\end{align*}
\]

To get a feel for what the metric above describes let us first identify the spatial infinity limit with the pre-decay geometry. This is nontrivial owing to the double analytic continuation \((t \to ix_0, \chi \to \frac{\pi}{2}+i\tau)\) that we have used to get to this expression. Just after the decay the geometry far from the decay nucleus should be in its pre-decay form. Evaluating Eq.(4.35) for \( r \to \infty \) we find

\[
\begin{align*}
\frac{ds_{11}^2}{r \to \infty} &\sim dx^2_0 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - r^2 \cos^2 \theta d\tau^2 + r^2 \cos^2 \theta \cosh^2 \tau d\Omega_6. \\
&= (4.36)
\end{align*}
\]

In this form it is not obvious that this metric describes asymptotically flat space. To see this we first introduce radial coordinates \((\tilde{\rho}, \tilde{r})\) defined by

\[
\begin{align*}
\tilde{\rho} &= r \sin \theta \\
\tilde{r} &= r \cos \theta
\end{align*}
\]

and then introduce “flat” coordinates

\[
\begin{align*}
\tilde{\tau} &= \tilde{r} \cosh \tau \\
\tilde{\tau} &= \tilde{r} \sinh \tau.
\end{align*}
\]

In these coordinates Eq.(4.36) takes the form

\[
\begin{align*}
\frac{ds_{11}^2}{r \to \infty} &\sim dx^2_0 + d\tilde{\rho}^2 + \tilde{\rho}^2 d\phi^2 + d\tilde{r}^2 + \tilde{r}^2 d\Omega_6 - d\tilde{\tau}^2 \\
&= (4.39)
\end{align*}
\]

which clearly describes a flat eleven-dimensional spacetime.
The full post-decay metric expressed in these flat coordinates is extremely complicated, however the most striking feature of the decay scenario is easily seen as a result of Eq.(4.38).

Note that the coordinate redefinitions imply

\[ \tilde{r}^2 + \tilde{p}^2 - \tilde{\tau}^2 = r^2. \]  

(4.40)

A trivial algebraic rearrangement of Eq.(4.40) combined with the coordinate minimum for \( r \) in Eq.(4.33) implies the existence of a totally geodesic submanifold which is growing in time

\[ (\tilde{r}^2 + \tilde{p}^2)_{\text{min}} = r_H^2 + \tilde{\tau}^2. \]  

(4.41)

For the coordinate region inside of the expanding bubble \( \tilde{r}^2 + \tilde{p}^2 < r_H^2 + \tilde{\tau}^2 \) the metric degrees of freedom cease to exist. This is simply a deformed version of Witten’s “bubble of nothing” annihilation of spacetime.\(^7\) It is a difficult picture to consider but is strikingly reminiscent of the idea of a purely topological phase of gravity. One should again consider the corresponding story for unstable open string theories in which the decay (either via condensation of tachyons or semi-classical processes) often leads to an annihilation of the open string degrees of freedom. For unstable D-branes one always has the closed string vacuum to leave behind, but for an unstable closed string vacuum the natural result seems, though perfectly analogous, considerably more catastrophic.

A great deal of discussion has been aimed at elucidating the picture of this semi-classical decay in terms of a dimensionally reduced theory\(^{[26, 27, 36, 17]}\). This has led to a number of seemingly strange equivalences. A very simple example involves two disparate ten-dimensional descriptions of the same eleven-dimensional process.\(^8\) In one case the decay involves spacetime falling into a pointlike singularity at an ever increasing rate, while the other description resembles the (considerably less catastrophic) shielding of a Kaluza-Klein magnetic field via pair produc-

\(^7\) Witten analyzed the spherically symmetric case corresponding to the Euclidean-Swarzchild bounce (the \( \alpha = 0 \) form of (4.30)). In that case the expanding bubble has a spherically symmetric geometry. Though the expanding surface in (4.41) is in terms of a coordinate sphere, the geometry of the expanding surface as measured by the metric will be deformed from an \( SO(9) \) isometry to \( SO(2) \otimes SO(7) \).

\(^8\) The example here is the shifted and unshifted reductions to ten-dimensions of the critically twisted circle background first discussed in [26]. These reductions will be discussed in more detail in the next section.
tion of magnetic monopoles. While these are certainly very interesting results, we take here the view that exactly when a Kaluza-Klein reduction becomes appropriate we lose the eleven-dimensional classical gravity approximation used in these calculations. At sufficiently small length scales quantum M-theory effects become important. The appropriate quantum description in many cases will be in terms of a perturbative string theory on the reduced background. We shouldn’t concern ourselves with the dimensionally reduced picture of the semi-classical instability. Instead we should look for perturbative manifestations of this instability. Why then identify the endpoints of the semi-classical and perturbative decays? Again a chief motivation is the analogy with unstable open string theories where we see the semiclassical instability described in [14, 63] go over to the tachyonic instability elucidated by Sen in [70, 68, 69].

If the size of the radius shrinks below the eleven-dimensional Planck length $R < l_P$ then the eleven-dimensional gravity approximation used above breaks down. However we are in most cases afforded a description of the resulting dynamics in terms of weakly coupled string theory. For $B = 0$ this of course reduces to supersymmetric Type IIA strings in a flat background. The bounce action above diverges for this particular case reflecting that the eleven-dimensional theory is actually supersymmetric and hence stable. We will now move on to the unstable $B \neq 0$ cases and discuss their perturbative limits.

4.2.2 Melvin Models

For values of $0 \leq |B| < \frac{1}{R}$ and $R < l_P$ we may reduce the background Eq.(4.28,4.29) along the Killing vector $l = \partial_{x_{11}} - B \partial_\phi$ to obtain a Melvin magnetic flux tube background \cite{26, 17} which is described by:

$$ds^2_{10} = \Lambda^\frac{1}{2} (-dt^2 + d\rho^2 + dy_idy_i) + \Lambda^{-\frac{1}{2}} \rho^2 d\phi^2$$

(4.42)

$$e^{\frac{4\phi}{\Lambda}} = \Lambda = 1 + \rho^2 B^2$$

(4.43)

$$A_\phi = \frac{B \rho^2}{2\Lambda} \quad \Rightarrow \quad \frac{1}{2} F_{\mu\nu} F^{\mu\nu}|_{\rho=0} = B^2$$

(4.44)

9 We would like to thank Steve Giddings for discussion on this point.
where \( \tilde{\phi} \equiv \phi - B_{x_{11}} \). This curved ten-dimensional background incorporates an axially symmetric RR two-form field strength parameterized by its central \( \rho = 0 \) value \( B \), and a nontrivial dilaton which grows as we move away from the \( \rho = 0 \) hyperplane for \( B \neq 0 \). To determine the perturbative content of the theory we should recall that the eleven-dimensional starting point was M-theory on a flat Kaluza-Klein background. For the periodic choice of spin structure on the \( S^1_R \) factor and for \( B = 0 \) this reduces to Type IIA strings on \( M^{10} \) as discussed above. For \( B \neq 0 \) we should then obtain Type IIA strings propagating on the Melvin background.

Quantizing strings on the background Eq.(4.42,4.43,4.44) faces the twin difficulties of incorporating RR flux and a curved geometry and is beyond current understanding. Applying the strong form of the conjecture discussed in the introduction would however imply that the corresponding closed string fluctuations should admit at least one tachyonic mode whose condensation would also lead to the annihilation of the spacetime.

The theory for \( |B| \neq 0 \) is continuously connected to the supersymmetric Type IIA vacuum. It may therefore seem natural that condensation of a closed string tachyon would in this case relax the value of \( |B| \) to zero, restoring the supersymmetric vacuum. However the Melvin magnetic flux would represent an excited state in the Type IIA theory, decaying by flux dissipation. However the Melvin background does not merely constitute weakly coupled Type IIA string theory with some additional unstable flux. As one can see from the nontrivial dilaton profile Eq.(4.43) a description in terms of any weakly coupled string theory will only be possible in the spatial region \( R < \rho < \frac{1}{|B|} \), see Fig.4.5.

For \( \rho < R \) we invalidate the Kaluza-Klein ansatz, while for \( \rho > \frac{1}{|B|} \) the string coupling becomes strong. In either case we must utilize the eleven-dimensional description.

---

10 By Type IIA strings we mean a theory of closed oriented world sheets with the standard Type IIA GSO constraints quantized in this particular background.

11 This is the identification made in [36].

12 This has been the motivation behind several assertions that the tachyonic Melvin background decays into the "underlying" supersymmetric vacuum.
4.2.3 Critical Melvin and Type 0A: The Scherk-Schwarz Circle

For $|B| = \frac{1}{R}$ the effect of the twisted identifications Eq.(4.29) is to accompany a $2\pi R$ translation in $x_{11}$ (generated by $n_1 \rightarrow n_1 + 1$) with a $2\pi$ rotation in $\phi$. This forces fermions to pick up a $-1$ when transported around the compact circle (so called Scherk-Schwarz (SS) boundary conditions [64]) and leaves bosons unaffected. Our starting point was a periodic spin structure on the $S^1_R$ so the net effect of $|B| = \frac{1}{R}$ is to exactly reverse this choice of spin structure. Thus one may consider the "critical" case of $|B| = \frac{1}{R}$ in either of two ways:

a. Periodic spin structure on $S^1_R$ and $|B| = \frac{1}{R}$.

This case will again reduce to Type IIA strings propagating on a Melvin magnetic background. However in this critical case (Fig.4.6) the theory is nowhere described by a weakly coupled ten-dimensional string theory since the relevant region shrinks to zero.

b. Antiperiodic spin structure on $S^1_R$ and $|B| = 0$.

In this case the resulting ten-dimensional background is flat $M^{10}$. Bergman and Gaberdiel have considered M-theory compactified on a Scherk-Schwarz circle and conjectured that
the appropriate perturbative degrees of freedom are Type 0A strings. Combined with standard results from T-duality, they developed a network of dualities for closed oriented nonsupersymmetric strings (Fig. 4.7) akin to supersymmetric case (Fig. 1.3). While their conjecture is still unverified it agrees with ours in the sense that the spectrum of Type 0A strings on \( M^{10} \) admits a closed string tachyon. The endpoint of condensation of the Type 0A closed string tachyon would then be identified with the annihilation of spacetime.

Thus far only the special cases \( |B| = 0, \frac{1}{R} \) admit reliable perturbative information. The presence of RR flux and the curvature of spacetime for \( |B| \neq 0, \frac{1}{R} \) renders even a spectrum calculation beyond our reach, however, we will later discuss similar models in nine dimensions for which the full spectrum is trivially obtained.

\footnote{This relies on interpolating orbifolds of the form \( \Sigma_R = \frac{S^+_{10}}{(-1)^{F_s} \otimes S} \) where \( F_s \) is the spacetime fermion number and \( S \) generates a half shift around the \( S^+_{10} \). This amounts to compactifying on a circle of radius \( R \) with Scherk-Schwarz boundary conditions. The key to this construction is that in the limit \( R \to 0 \) the orbifold factorizes into a compactification on a circle of radius \( 2R \) and a twist by \( (-1)^{F_s} \). The compactification gives IIA on \( S^3_{2R} \) and then applying \( (-1)^{F_s} \) twists this into 0A on \( S^3_{2R} \). The factor of 2 arises from the half shift and becomes irrelevant in the decompactification limit. An extensive application of these interpolating models for nonsupersymmetric string theories is in [10][11].}
4.2.4 Noncompact Orbifolds

When the twist parameter takes special values of the form $|B| = \frac{1}{N_R}$ or $|B| = \frac{1}{R} + \frac{1}{N_R}$, we can perform $SL(2, Z)$ transformations on the $(x_{11}, \phi)$ compactification torus and reduce to a ten-dimensional background of the form[36]:

$$ds_{10}^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + \frac{r^2}{N^2} d\phi^2$$

(4.45)

$$e^{\frac{4\phi}{3}} = N^2$$

(4.46)

$$A_\phi = \frac{1}{2} R \frac{N - 1}{N}$$

(4.47)

The resulting spacetime is that of a noncompact orbifold with the fundamental region a cone of deficit angle $\frac{2\pi}{N}$. The dilaton in this case is constant throughout the spacetime. If we start with periodic spin structure on the $S^1_R$ then for $|B| = \frac{1}{N_R}$ the correct perturbative degrees of freedom involve Type 0A strings. For periodic spin structure on the $S^1_R$ and $|B| = \frac{1}{R} + \frac{1}{N_R}$ we transform to $B' = B - B \frac{1}{|B|} \frac{1}{R}$ and reverse the spin structure as in section 4.2.3. This reduces to Type IIA strings propagating on the orbifold background Eq.(4.45,4.46,4.47).

Even though the geometry is locally flat, string quantization on this background is difficult owing to the presence of the RR Wilson line. Our conjecture would imply the existence of a tachyonic instability in the perturbative description for either $|B| = \frac{1}{N_R}$ or $|B| = \frac{1}{R} + \frac{1}{N_R}$ with the endpoint of its condensation involving the annihilation of spacetime.

This result seems to contradict the conclusions reached in [1] where it was found that the effect of tachyon condensation in noncompact orbifolds is to “un-orbifold” the theory restoring the “underlying” supersymmetric closed string vacuum. This deserves some discussion. Among the arguments in [1] was the observation that the orbifold fixed plane represents a curvature singularity in a locally flat spacetime which may be viewed as a localized excitation above the underlying background. In fact for the special cases $Z_N$ with $N$ odd it was pointed out that the closed string tachyons are localized to the orbifold fixed plane and by an analogy with open
string tachyons localized to unstable $D$-branes represent an instability towards decay of the localized energy density restoring the supersymmetric vacuum. The elegant analysis in [1] is sound, however when one tries to apply their conclusions to the present scenario we find some obstacles.

First of all the analysis was performed in the zero coupling limit. As we argued in the introduction, for zero coupling the tachyon may condense without affecting the underlying background. We expect that for non-zero coupling the tachyon will have a more dramatic effect on the background. Secondly to connect these orbifolds to the twisted circle compactifications discussed above one must include the RR Wilson line Eq.(4.47). In [1] the spectrum of the theory was computed without incorporating any Wilson line. As we have mentioned earlier including the RR Wilson line is difficult, however one can circumvent this difficulty by looking at the corresponding situation obtained by compactifying a ten-dimensional theory on a twisted circle. In this case the Wilson line will arise in the NSNS sector and the spectrum can be evaluated exactly. We will discuss these in more detail in Sec.[4.3] but for now we point out that one important effect of including the Wilson line is the localization of closed string tachyons for any value of $0 < |B| < \frac{1}{R}$ (particularly $|B| = \frac{1}{NR}$ with $N$ even or odd). In addition the "curvature singularity as a localized excitation above a flat background" argument needs to be reconsidered. For a Wilson line of the form Eq.(4.47) the ten-dimensional geometry actually lifts to a flat eleven-dimensional geometry which is everywhere regular. Probing distances very close to the orbifold fixed plane invalidates the Kaluza-Klein ansatz and we should replace the reduced theory by its eleven-dimensional interpretation.

4.2.5 The Scherk-Schwarz Interval $M^{10} \otimes I_L^{SS}$

Consider now Horava-Witten (HW) theory [42, 41], i.e. eleven-dimensional M-theory compactified on a line segment of length $L$. In addition to the bulk degrees of freedom anomaly cancellation requires an $E8$ gauge theory to live on each ten-dimensional wall bounding the bulk spacetime. Fabinger and Horava (FH) considered the scenario that results from reversing the
chirality of fermions living on one of the walls[29]. This breaks the spacetime supersymmetry and renders the theory unstable. FH demonstrated the existence of an attractive casimir force between the walls and then went on to discuss a semi-classical instability towards formation of a wormhole-like tube connecting the two walls, the interior of which has no metric degrees of freedom. This tube grows radially outward eating up both the E8 walls and the bulk spacetime. This system is equivalent to a compactification of M-theory on a Scherk-Schwarz circle of radius \( \frac{L}{\pi} \) followed by a \( Z_2 \) orbifolding. This may be viewed as a \( Z_2 \) orbifolding of the critically twisted circle \( S^1_{\frac{L}{\pi}, \frac{L}{\pi}} \) discussed in Sec.[4.2.3]. The relevant bounce solution is simply the \( Z_2 \) invariant form of Eq.(4.30) evaluated at \( R_{FH} = \frac{L}{\pi} \) and \( B = \frac{\pi}{L} \).

\[
ds^2_{11} = (1 - \frac{\mu}{R_8^3})dx^2_{11} + (1 - \frac{\mu}{R_8^3})^{-1}dr^2 + r^2(d\chi^2 + \sin^2\chi d\Omega_8) \tag{4.48}
\]

For the critical case we may easily express the mass parameter \( \mu \) in terms of the background parameters

\[
\mu = \left( \frac{4L}{\pi} \right)^8. \tag{4.49}
\]

Borrowing expression Eq.(4.34) for the decay rate we find

\[
\Gamma \sim e^{-\frac{11^1L}{5\pi^2G_{11}}}. \tag{4.50}
\]

Analysis of the post decay evolution proceeds along the lines of Sec.[4.2.1]. The picture is that of a \( Z_2 \) projection of Witten's spherically symmetric bubble of nothing expanding in time. We now discuss two possible perturbative limits of the FH scenario.

### 4.2.6 The Case of the Shrinking Interval \( M^{10} \otimes I_L^{\text{SS}} \rightarrow_0 \)

Consider the situation where the two E8 walls come together. For \( L \sim L_P \) the eleven-dimensional gravity approximation breaks down. We might anticipate a result similar to the HW case for which the appropriate description as the two E8 walls come together is in terms of weakly coupled Heterotic \( E8 \otimes E8 \) string theory \( H^{\text{sysy}}_{E8\otimes E8} \) on \( M^{10} \). For the case of FH the resulting perturbative string description must have broken supersymmetry. Furthermore, our conjecture
in its strong form would imply that the resulting string theory should have a tachyonic mode which mediates the annihilation of spacetime. There are seven candidate nonsupersymmetric heterotic string theories [47]. Their relevant properties are summarized in Table 4.1.

Table 4.1: The following table lists the seven nonsupersymmetric heterotic string theories based on modular invariant partition functions. For each theory with a tachyon we indicate how the tachyon transforms under the gauge symmetry. We also specify whether the fermion spectrum of the theory is chiral or not.

<table>
<thead>
<tr>
<th>Gauge Symmetry</th>
<th>Tachyon Representation</th>
<th>Chiral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{SO(16) \otimes SO(16)}$</td>
<td>tachyon-free</td>
<td>yes</td>
</tr>
<tr>
<td>$H_{SO(32)}$</td>
<td>$(32_e)$</td>
<td>no</td>
</tr>
<tr>
<td>$H_{SO(8) \otimes SO(24)}$</td>
<td>$(8_e, 1)$</td>
<td>yes</td>
</tr>
<tr>
<td>$H_{SU(16) \otimes SO(2)}$</td>
<td>$(1, 2_e)$</td>
<td>yes</td>
</tr>
<tr>
<td>$H_{SO(16) \otimes E_8}$</td>
<td>$(16_e, 1)$</td>
<td>yes</td>
</tr>
<tr>
<td>$H_{E_7 \otimes SU(2) \otimes E_7 \otimes SU(2)}$</td>
<td>$(1, 2, 1, 2)$</td>
<td>yes</td>
</tr>
<tr>
<td>$H_{E_8}$</td>
<td>$(1)$</td>
<td>no</td>
</tr>
</tbody>
</table>

To identify the best candidate theory we consider the membrane world-volume anomaly analysis of HW[41]. For a topologically stabilized membrane wrapping a large $S^1_{x^9}$ and stretched between the two walls, the right-moving $8''$ fermions induce a three-dimensional gravitational anomaly since the world-volume has orbifold singularities, i.e. it is not a smooth manifold. To cancel this anomaly one must add left-moving current algebra modes with $c = 16$. Since the anomaly is localized and evenly distributed between the two boundaries of the world-volume, the current algebra modes should be evenly distributed between the two ends as well. In the HW case spacetime supersymmetry is preserved, and the only supersymmetric string theory with this world-sheet structure is the $H^{susy}_{E_8 \otimes E_8}$ theory. If the spacetime supersymmetry is broken, as in the case at hand, then we should look for nonsupersymmetric strings with this world-sheet structure. Only two of the seven cases above are of this type; the $H_{E_8 \otimes SO(16)}$ and $H_{SO(16) \otimes SO(16)}$ theories. There are two additional reasons which lead to the choice of $H_{E_8 \otimes SO(16)}$ as the $L \to 0$ limit of FH. Motivated by our conjecture, we choose the only one of these two that is tachyonic.

14 Right-moving here is simply — chirality under the $SO(1,1)$ isometry of the $(t,x^9)$ cylinder, and $8''$ is the conjugate spinor of $SO(8)$.

15 More precisely, the $H^{susy}_{E_8 \otimes E_8}$ makes use of the left-moving current algebra in two independent sets, e.g. in the free fermionic construction the current algebra is carried by two independent sets of 16 left moving fermions. The $H^{susy}_{SO(32)}$ on the other hand makes use of a single set of 32 left moving fermions.
This would follow from the strong form of the conjecture. An indication that this is plausible was worked out in [29] where the mass of a membrane state stretched between the two walls was calculated and shown to become tachyonic when the two walls are sufficiently close, i.e. $L < l_p$.

Of course the membrane energy calculation becomes invalid precisely in this regime, but it does seem indicative of a continued instability of the theory. Furthermore, the GSO constraints which lead to the $H_{E8@SO(16)}$ theory differ from those that lead to the $H_{E8@E8}^{susy}$ theory by a twist of $\exp(i\pi F_B)$ which affects only half of the left moving current algebra modes.\footnote{In the free-fermionic construction the world-sheet fields are divided into 8 left and 8 right-moving bosons $(\partial X^\mu, \bar{\partial} X^\mu)$, 8 right-moving fermions $\bar{\psi}^\mu$, and two sets of 16 left-moving fermions $(\psi_A, \psi_B)$. The $H_{E8@E8}^{susy}$ GSO constraints involve $\exp(i\pi F_A) = \exp(i\pi F_B) = \exp(i\pi \bar{F}) = +1$, whereas the $H_{E8@SO(16)}$ construction uses $\exp(i\pi F_A) = \exp(i\pi F_B + i\pi \bar{F}) = +1$, where $F_A, F_B, \bar{F}$ are the world-sheet fermion numbers for $\psi_A, \psi_B, \bar{\psi}^\mu$ respectively.} It may seem odd that the gauge group is broken asymmetrically as the two-walls come together since there seems to be no "preferred" wall. However both $E8$ and $SO(16)$ require 16 current algebra fermions on the corresponding worldsheet so in a sense the walls are on equal footing. The effect of "flipping" one of the $E8$ walls on the M-theory side must translate into a modification of the GSO projection on one of the two sets of 16 current algebra fermions on the heterotic string worldsheet. In any case, the two walls coming together ventures through intermediate coupling regions (for which there is no known description) unprotected by supersymmetry, rendering the specific mechanism behind the spacetime gauge symmetry breaking difficult to study.

### 4.2.7 The Case of the Shrinking Transverse Circle $M^9 \otimes S^1_{R \to 0} \otimes I^{SS}_{L=finite}$

Now consider the FH background keeping the Scherk-Schwarz interval length $L$ large and further compactifying the theory on a transverse circle $S^1_R$ with a periodic choice of spin structure. For $R < l_p$ we should be able to describe the system by a nonsupersymmetric variant of the familiar Type I' theory (Type 0') as shown in Fig.4.8.

In the familiar supersymmetric case Type I' is obtained from IIA compactified on a circle by dividing out by the $Z_2$ symmetry $g = I \Omega$ where $\Omega$ is the world sheet orientation reversal and $I : x^9 \to -x^9$. In the original M-theory picture of the FH construction the reversal of
orientation of one of the $E_8$ walls may be accomplished by dividing the theory by an additional $Z_2$ symmetry generated by $g' = S(-1)^F g$ giving

$$M^9 \otimes S_R^1 \otimes S_L^1 / (Z_2 \otimes Z_2')$$

(4.51)

where in the M-theory case the first $Z_2$ is generated by just the reflection. Thus the theory that we get on reduction to ten dimensions $R \to 0$ should be Type IIA on $M^9 \otimes S_R^1 / (Z_2 \otimes Z_2')$ where the two symmetries are generated by $g$ and $g'$. This is equivalent to a theory defined in [3, 46] and has an orientifold 8-plane at one fixed point and an anti-orientifold at the other fixed point.

There is no need to add D-branes to cancel RR tadpoles but getting flat space would require the cancellation of the NSNS tadpoles and thus would entail the presence of 16 $D8$-branes and 16 $\overline{D8}$-branes with the passage to the corresponding M theory case possible when the former are coincident with the orientifold plane and the latter with the anti-orientifold plane. Thus this limit of the FH theory is simply an orientation reversed version of the Type I' theory. We should then really be considering an $S^1$ compactification of FH with a Wilson line $Y$ which breaks the $E_8 \otimes E_8$ gauge symmetry to $SO(16) \otimes SO(16)$. This has a potentially tachyonic mode coming
from the twisted sector [7], with mass

$$m^2 = \frac{L^2}{4\pi^2 \alpha'} - \frac{2}{\alpha'}$$

(4.52)

which becomes tachyonic when $L < 2\pi \sqrt{2\alpha'}$. This state clearly survives the $g$ and $g'$ projections.

It should be stressed that this is a closed string tachyon coming from the lowest winding mode of the twisted sector of the theory. In addition of course the theory has open string tachyons coming from the open strings stretched between the $D\bar{D}$ when they get within a distance $\pi \sqrt{2\alpha'}$ of each other. The $D$-branes are attracted to each other and will annihilate due to this, leaving us with a background that will have a negative cosmological constant (due to the negative tension of the orientifold anti-orientifold system). What would one expect to be the end point of the decay of the closed string tachyon?

The answer to this according to our conjecture should be obtained from the picture of semi-classical vacuum bubble decay that one has when $L > 2\pi \sqrt{2\alpha'}$ in analogy with the corresponding M-theory FH case. Thus as in the FH case one expects this theory to be subject to space time annihilation.

In the region for which the lowest mode becomes tachyonic $L < 2\pi \sqrt{2\alpha'}$ the nine-dimensional theory should be replaced by an appropriate T-dual description. \[17\] For the T-dual of Type $0'$ one expects a perturbative description in terms of a nonsupersymmetric analog of Type I theory. Before discussing this theory in detail we use the strong form of our conjecture to anticipate some of its features. The semi-classical instability in eleven-dimensions annihilates both the gauge degrees of freedom and the spacetime. In the compactification at hand the gauge degrees of freedom and the spacetime are described by two different sectors of the theory (the former by open strings and the latter by closed strings). The single semi-classical annihilation instability of the eleven-dimensional theory should then descend to two tachyonic instabilities, one leading to the annihilation of the gauge degrees of freedom and the other leading to the

\[17\] In the supersymmetric case the appropriate T-dual description is the Type I theory. This theory can also constructed from Type IIB by gauging the world-sheet parity $\Omega$ (Type IIB/$\Omega$) and adding 32 $D9$-branes to cancel the resulting massless RR and NSNS tadpoles. At strong coupling this theory is described by the weakly coupled $H^\text{weak}_{SO(32)}$ theory. See Fig.1.4.
annihilation of spacetime.\footnote{This is in contrast to the discussion in Sec.[4.2.6]. For heterotic strings it is difficult to imagine an annihilation of the gauge degrees of freedom (now carried by closed strings) without losing all closed string degrees of freedom.}

To construct the T-dual theory we permute the two $Z_2$ symmetries. In the strict $L \to 0$ limit one is then left with Type 0B/$\Omega$, which has been called Type 0 theory (a nonsupersymmetric analog of Type I). This theory has been constructed in [9, 62, 6, 7] and indeed contains both open string tachyons charged under the gauge symmetry as well as a closed string tachyon (the Type 0B tachyon survives the $\Omega$ projection). The massless NSNS tadpole contribution for Type 0B/$\Omega$ is twice that in Type IIB/$\Omega$, so to formulate the Type 0B/$\Omega$ theory in flat space (which we expect for the T-dual description of Type 0' in flat space) requires the addition of 64 $D9$-branes. The absence of massless RR tadpoles implies that 32 of these should be $D9$-branes and the other 32 $\overline{D9}$-branes. However the Type 0 theories exhibit two types of $D_p$-brane for any given $p$ distinguished by their charge under the twisted sector fields [6, 7]. If we designate these by $Dp'$ and $Dp''$, then the massless tadpoles can be cancelled by adding $n$ $D9' - \overline{D9'}$ pairs and $32 - n$ $D9'' - \overline{D9''}$ pairs. The resulting gauge symmetry is then $SO(n) \otimes SO(n) \otimes SO(32-n) \otimes SO(32-n)$. Of course we originally had the 16 $D8$-branes and 16 $\overline{D8}$-branes of Type 0', but these are standard Type II $D8$-branes. For finite radius on the T-dual side the situation can be described by the splitting of Type II $Dp$-branes into pairs of Type 0 $Dp'/Dp''$-branes on the dual circle [45], c.f. Fig.4.9.\footnote{We thank Oren Bergman for several useful discussions on this and related issues.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{doubled_d_branes.png}
\caption{The doubling of D-branes by T-duality as discussed in [45].}
\end{figure}

This gauge symmetry enhancement is an unusual feature of Scherk-Schwarz compactification.
cations. The point is that for a Scherk-Schwarz compactification there are new twisted sector states in the theory which only become light as $R \to 0$. In the closed string sector these states form an essential part of the conjectured relationship between Type $0A$ and M-theory [7] where they become the additional NSNS and RR fields of Type $0A$ (relative to Type IIA). In the case of open strings they lead to the gauge symmetry enhancement discussed above.

4.2.8 Horava-Witten on a Scherk-Schwarz Circle $M^9 \otimes I_L \otimes S^{SS}_R$

We finally consider taking the supersymmetric HW theory and further compactifying on a Scherk-Schwarz circle, as in Fig.4.10. This picture may seem a bit suprising. The semi-

![Diagram of Horava-Witten theory compactified on a Scherk-Schwarz circle.](image)

Figure 4.10: Horava-Witten theory compactified on a Scherk-Schwarz circle.

classical instability at large $R$ towards Witten-type bubble formation (which annihilates all degrees of freedom) descends to the closed string tachyonic instability of the bulk Type $0A$ theory in addition to the open string tachyonic instability of the boundary gauge degrees of freedom. This separation of the tachyonic instability into spacetime and gauge parts is similar to what we saw in Sec.4.2.7. The increase in rank of the gauge groups on the walls arises
from the addition of 32 branes required to cancel massless tadpoles. Again this is similar to the discussion in Sec.[4.2.7]. The fact that the orientifold fixed planes for Type 0A carry no RR charge is consistent with the case for Type 0B/Ω. In addition the absence of RR flux in the interval prevents the possible shifting of the Type 0A tachyon mass via an effect similar to the one found in [48], which might have otherwise rendered the theory non-tachyonic invalidating the implications of our conjecture. We will refer to this theory as Type 0". An interesting question in this case concerns the picture that results as these two walls come together. In this case the we may write the M-theory background as

\[ M^9 \otimes S^1_r/Z_2 \otimes S^1_r/Z'_2 \]  

(4.53)

where the two symmetries are generated by reflection and and by \( g' \) defined just before Eq.(4.51). The above differs from the latter by which factors the discrete symmetries act on. However, in the limit that we are taking, i.e. \( R, L \rightarrow 0 \), the orbifold factorizes

\[ (M^9 \otimes S^1_r/Z_2 \otimes S^1_{r'})/(\mathbb{Z}_2) \]  

(4.54)

So we expect that this system will reduce to the system discussed in Sec.[4.2.7]. We can already see that the gauge symmetry will be \( SO(32) \otimes SO(32) \) with one factor carried by the 32 D8-branes and the other factor carried by the 32 D8-branes. The web of dualities conjectured for supersymmetry breaking compactifications of the Horava-Witten background is presented in Fig.4.11.

### 4.2.9 Type 0 at Strong Coupling

Our analysis thus far has been based on perturbative constructions and nonperturbative relations motivated by the tachyon/semi-classical instability conjecture. At this point we will take an aside from the main line of this paper to complete the picture that seems to emerge. If we are bold enough to push the admittedly speculative results of Sec.[4.2.7] to strong coupling we may expect that the S-dual of the Type 0 theory discussed above will involve a nonsupersymmetric heterotic string theory with a tachyonic instability which is charged under the gauge
symmetry. The task then is to identify which of the seven candidate theories is appropriate. We should point out the similarity between the limits of FH that we have been considering and the standard picture for the HW background, c.f. Fig.1.4. Where the Type 0' theory resulted from compactifying the FH theory on a circle with a Wilson line $Y$ breaking the $E_8 \otimes E_8$ to $SO(16) \otimes SO(16)$, we can consider also compactifying the $H_{E_8 \otimes SO(16)}$ theory on a circle with the same Wilson line $Y$. The resulting T-dual description is the $H_{SO(32)}$ theory [31]. This leads us to conjecture that the S-dual of Type 0 is described by the $H_{SO(32)}$ string as in Fig.4.12.

An unusual feature of this S-duality proposal is the change in rank of the gauge symmetry group. Though S-dualities of this type are know in field theory, we know of no such example in string theory. Of course the S-duality relationship that we have proposed has its geometric origin in a compactification torus which involves a Scherk-Schwarz cycle. The subtleties involved in a strict zero-radius limit for a Scherk-Schwarz circle should manifest itself when taking the strong coupling limit of Type 0. This mismatch in gauge group rank was pointed out in a closely related context in [10, 11]. It also motivated the authors of [6] to conjecture that the strong

Figure 4.11: The conjectured web of dualities obtained from Scherk-Schwarz compactifications of the Horava-Witten background.
Figure 4.12: The conjectured web of dualities obtained from simple compactifications of the Fabinger-Horava background.

coupling dual of the ten-dimensional Type 0 theory is the $D = 26$ bosonic string compactified on the $SO(32)$ lattice since this is the only possible closed string theory with a rank 32 gauge group. Our conjecture stems from a larger scheme of dualities (presented in Fig.4.12) akin to the familiar web of dualities shown in Fig.1.4.

A standard technique for supporting S-duality conjectures is to find a stable soliton that becomes light in the strong coupling limit and identify its fluctuation spectra with that of the fundamental degrees of freedom in the dual theory. In Type I/Heterotic $SO(32)$ duality for instance the massless fluctuations of the Type I $D$-string (with mass inversely proportional to the string coupling) are identified with world-sheet fields of the $F$-string in $H_{SO(32)}^{susy}$ [56]. In particular the $DD$ open string modes become the $F$-string fields with spacetime quantum numbers while the $DN$ open string modes go over to the current algebra degrees of freedom.

Trying to apply this reasoning to the present case immediately confronts an ambiguity in that there are two types of non-tachyonic $D$-string present in Type 0 [6, 7]. Furthermore the
fluctuation spectrum on either $D$-string does not match up with the worldsheet structure of the $H_{SO(32)}$ $F$-string. The resolution of this ambiguity has already been suggested in [7] based on observations noted earlier in [48]. The appropriate soliton to consider is a bound state of the two $D$-strings present in the theory. In particular the modes of open strings stretched between the two $D$-strings give rise to the worldsheet fermions carrying a spacetime vector index in the dual theory. These bound states are very interesting on their own in so far as they are very BPS-like despite being non-supersymmetric. For example when parallel two of these bound states exhibit no force on one another in a manner analogous to BPS $D$-strings. In addition the bound state is decoupled from all of the twisted sector fields in the theory including the bulk tachyon [48]. Similar proposals have been discussed for the self-duality of Type 0B [7, 18]. A thorough understanding of these soliton bound states would provide considerable support for the picture that we have outlined.

4.3 Applications of the conjecture for 10D $\to$ 9D

We now turn to applications of the conjecture for compactifications from ten to nine dimensions. Some advantages over the previous discussion are that any Kaluza-Klein gauge field will now reside in the NSNS sector of the perturbative string descriptions and there is no coupling interpretation for compact dimensions (thus avoiding problems with strong coupling extrapolations).

Perhaps the most important aspect of starting in ten-dimensions is that we have at our disposal the full quantum theory (as opposed to its low-energy effective field theory limit in the eleven-dimensional case). Since the $M^9 \otimes S^1_{R,B}$ is flat (though globally nontrivial), string quantization on this background is straightforward. We can always reduce the theory to nine-dimensions to obtain the corresponding curved NSNS Melvin backgrounds, but for our purposes the spectrum calculation in ten-dimensions will suffice.

There are numerous supersymmetric ten-dimensional starting points. We will first briefly present the ten-dimensional version of the twisted circle semi-classical instability which parallels
Sec.[4.2.1]. This analysis will apply to any perturbative string theory compactified on a twisted circle. We will then move on to a case by case analysis of the small $R$ limit. Type IIA/B exhibit similar behavior as do the two heterotic theories. Type I we discuss on its own.

### 4.3.1 Twisted Circle $M^8 \otimes S^1_{R,B}$

Our discussion of the semi-classical instability of twisted circle compactifications in 11D carries over to this case with little change. We will quickly highlight the results. The ten-dimensional geometry is flat with the nontrivial identifications

$$x_9 \sim x_9 + 2\pi n_1 R$$

$$\phi \sim \phi + 2\pi n_1 BR + 2\pi n_2.$$

The ten-dimensional Euclidean Kerr bounce solution is given by

$$ds^2_{10} = (1 - \frac{\mu}{r^5})dt^2 - \frac{2\mu \sin^2 \theta}{r^5} d\tau d\phi + \frac{\Sigma}{r^2 - \alpha^2 - \mu r^{-5}} dr^2 + \Sigma d\theta^2$$

$$+ \frac{\sin^2 \theta}{\Sigma} (r^2 - \alpha^2) \Sigma - \frac{\mu}{r^5} \alpha^2 \sin^2 \theta |d\phi^2 + r^2 \cos^2 \theta d\Omega_6.$$

To match this bounce solution to the twisted circle background the black hole mass and angular momentum parameters $(\mu, \alpha)$ must satisfy

$$R = \frac{2\mu}{7r_H^6 - 5\alpha^2 r_H^4}$$

$$B = \frac{\alpha r_H^5}{\mu} - \frac{\alpha}{|\alpha| R},$$

where the horizon radius $r_H$ now satisfies

$$r_H^2 = \alpha^2 + \frac{\mu}{r_H^5}.$$

The decay rate is given by

$$\Gamma \sim e^{-\frac{15\pi^4 \mu R}{1120 r_H}},$$

The post decay evolution of these ten-dimensional theories may be addressed by repeating the analysis of Sec.[4.2.1] everywhere replacing $d\Omega_6 \rightarrow d\Omega_5$. The picture is that of an expanding

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$^{20}$ The semiclassical instability arises in the gravity sector which is common to all five perturbative string theories.
bubble of nothing with surface isometry group $SO(2) \otimes SO(6)$. Two aspects of the semi-classical instability are important for the discussion in the next section. First of all the analytically continued bounce solution (bubble) constructed in Sec.[4.2.1] is centered about $\rho = 0$ in the plane defining the twist parameter $B$. It is difficult to imagine an off-axis bounce (centered around $\rho \neq 0$) having the correct asymptotic form to be glued into the decaying spacetime, i.e. $SO(2)$ isometry defined about $\rho = 0$. In addition the pre-decay geometry is translationally invariant along the hyperplane, so one would expect the semi-classical decay to proceed by nucleating bubbles with a uniform distribution along the $\rho = 0$ hyperplane. These bubbles will expand off the hyperplane eventually affecting the geometry at all points in the spacetime. For the critically twisted case $B = \frac{1}{R}$ the identifications Eq.(4.55) act trivially on the spacetime and the $U(1) \otimes SO(6,1) \otimes SO(2)$ isometry is restored to a full $U(1) \otimes SO(8,1)$. The $\rho = 0$ hyperplane is no longer distinguished and so the geometry will decay by production of spherically symmetric bubbles nucleated throughout the spacetime.

4.3.2 Type IIA/B

The $R < \sqrt{\alpha'}$ limit of the twisted circle compactification works out very nicely for the Type IIA/B starting points. The spectra of these theories has been analyzed in detail in [61, 60] and we will recount only a few important aspects of their results. Our purpose is to compare these results with the semiclassical instability described above offering support for our conjecture that these instabilities are related.

For $-B| \neq 0, \frac{1}{R}$ the 9+1-dimensional Lorentz invariance of uncompactified theory is broken to 6+1-dimensional Lorentz invariance by the twisted compactification. These theories contain tachyonic states in the winding sectors for $R < 2\alpha'|B|$. Combining this with the limited range of the twist parameter $0 \leq |B| \leq \frac{1}{R}$, we see that the largest value of $R$ for which the theory is tachyonic is $R = \sqrt{2\alpha'}$ which occurs for the critical twist $|B| = \frac{1}{R}$. When $B = 0$ the theory is supersymmetric and there are of course no tachyonic modes. For any $|B| \neq 0$ the lowest mass state is a $(w = 1)$ winding mode in the NS+NS+ sector with a negative mass shift
due to its angular momentum in the $\phi$-plane. This arises from a gyromagnetic interaction term in the string Hamiltonian of the form

$$-\frac{2B R \omega}{\alpha'} (\hat{J}_R - \hat{J}_L)$$

(4.60)

where $\hat{J}_R, \hat{J}_L$ are the angular momentum operators in the $\phi$-plane. In terms of world-sheet oscillator excitations it is the same tensor fluctuation that in flat space gives rise to the graviton.

For $|B| \neq 0, \frac{1}{R}$ the winding states (closing only up to $n_1 = w$ in eq.(4.55)) must not only stretch over the circle, but must also stretch to accommodate the arc-length subtended by $2\pi wBR$. This clearly depends on the distance $\rho$ from the hyperplane about which $\phi$ is defined giving a winding energy contribution to the string mass of the form

$$\delta m^2 = \frac{w^2 R^2}{\alpha'^2} (1 + \rho^2 B^2).$$

(4.61)

The tachyonic states in the theory are necessarily winding states and for $B \neq 0$ it is clear that any finite negative mass contribution will be cancelled for sufficiently large values of $\rho$. The tachyonic states are thus effectively localized about $\rho = 0$. This fits in nicely with our conjecture relating the semi-classical instability for large $R$ to the tachyonic instability for small $R$. In both cases the decay seed is localized to the distinguished hyperplane.

One can go even further and analyze the perturbative spectrum for the critical case $|B| = \frac{1}{R}$. Naively the argument based on Eq.(4.61) would seem to again imply localization of twisted states. However a careful treatment of string quantization reveals that for a critical twist the shift in normal ordering constant restores the zero mode structure in the $\phi$-plane [60, 61]. The tachyons are no longer localized to the $\rho = 0$ hyperplane in accord with the delocalization of semi-classical bubble production for the large radius critically twisted circle. This result is not surprising insofar as we can consider the critically twisted case in terms of trivial circle compactification with a reversal of spin structure on the $S^1_R$. One can argue that the $R \to 0$ limit of Type IIA/B on a critically twisted (Scherk-Schwarz) can be described by Type 0A/B on $M^9 \otimes S^1_{2R \to 0}$ which are better described by the T-dual Type0B/A theories on $M^{10}$.\(^\text{21}\) This leads\(^\text{21}\) These arguments again rely on interpolating orbifolds.
one to pose the following question. Suppose we start with Type 0A string theory on $M^{10}$ which has its usual flat-space tachyon. We wish to connect this tachyon to a semi-classical instability. There appear to be two candidates. Either M-theory on a critically twisted circle or Type IIB string theory on a critically twisted circle of vanishing radius. Though both instabilities lead to the annihilation of spacetime, the first proceeds via an eleven-dimensional bubble geometry while the latter proceeds via a ten-dimensional bubble geometry. This essentially becomes a question of limits. Though $M^{10}$ resembles in many ways $M^{9} \otimes S_{R \to \infty}^{1}$ there are global distinctions (for example quantization conditions). If we are interested in the Type 0A tachyon in strictly $M^{10}$ then we should identify it with the M-theory instability since the limiting theory is fully ten-dimensional (the compactification radius playing the role of the coupling). The Scherk-Schwarz compactification of IIB only approaches 0A on $M^{10}$ as $M^{9} \otimes S_{R \to \infty}^{1}$. However one may still suppose that a similar ambiguity would hold for Type 0A on $M^{9} \otimes S_{R}^{1}$ where $R$ is finite and nonzero. In this case either M-theory on $M^{9} \otimes S_{R}^{1} \otimes S_{R}^{1}$ or IIB on $M^{9} \otimes S_{2R}^{SS}$ would seem to work. However the latter does not T-dualize to Type 0A on $M^{9} \otimes S_{R}^{1}$, but rather to Type 0A on $M^{9} \otimes S_{2R}^{SS, 22}$. Again the appropriate instability is the M-theory one.

4.3.3 Heterotic $SO(32)/E8 \otimes E8$

The twisted circle semi-classical instability is generic. Any theory with a gravity sector formulated on this background will decay by nucleating (possibly deformed) bubbles of nothing. If the twisted circle radius is small enough the semi-classical calculation is no longer valid and we believe the same underlying instability should emerge in whatever description becomes appropriate. We can push this further and say that since the semi-classical instability is generic then, at least for twisted circle compactifications, the corresponding tachyonic instabilities should be generic. For theories with gauge degrees of freedom this can be a nontrivial issue.

Consider taking either of the supersymmetric heterotic theories and compactifying on a

22 The dual Scherk-Schwarz circle may be formally written as $S_{R}^{1} \otimes S_{2R}^{SS}$, where $f_{R}$ is the right moving worldsheet fermion number [7].
twisted circle. The nontrivial boundary conditions for arbitrary $B$ will affect both bosons and fermions winding around the compact circle, but should leave massless the nonwinding gauge bosons of the heterotic theory. The gauge symmetry should thus remain unbroken. It would be natural then to expect that the tachyonic instability will be neutral under the heterotic gauge symmetry. We expect the spectrum of heterotic strings on a twisted circle to be very similar to the spectrum of Type II strings on a twisted circle barring the usual differences between the spectra of the free theories. A cursory investigation of the spectrum of these theories in [60] supports this picture. In particular the negative mass contributions attributable to the gyromagnetic interaction term renders the lowest NS+NS+ states tachyonic. This gauge neutral tachyonic state is exactly what we expect from our conjecture relating it to the semi-classical instability. The story changes considerably if we include a Wilson line along the twisted circle which breaks the heterotic gauge symmetry to some subgroup. In this case the semi-classical instability is associated with a gauge symmetry breaking compactification and we expect the corresponding tachyons to transform nontrivially under the unbroken gauge group.

The critical case without a Wilson line poses an interesting puzzle for the heterotic strings. Consider setting $|B| = \frac{1}{R}$ and sending the compactification radius to zero. By the interpolating orbifold argument the resulting description can be described in terms of a nonsupersymmetric string theory on $M^{10}$ that is the T-dual of the result of twisting the orginal theory by $(-1)^{F_s}$. Surveying the Table 4.1 in Sec.[4.2.6] we find that there is no flat space theory with gauge group $E8 \otimes E8$. In addition the theory with gauge group $SO(32)$ exhibits a tachyon which transforms nontrivially under the gauge group in contradiction to the expectations outlined above. The resolution of this puzzle is what makes the heterotic case so interesting. The two supersymmetric heterotic string theories are actually invariant under a twist by $(-1)^{F_s}$ as pointed out in [24]. Combined with the self-duality of these theories under T-duality, the result is that the $R \to 0$ limit of a critically twisted compactification of these theories returns the original theory on $M^9 \otimes S^1_{\alpha'/R \to \infty}$. In this case the association is not between a semi-classical instability on one hand and a tachyonic instability on the other, but rather between semi-classical instabilities
both large and small compactification radii. This case represents the single exception we have found to the strong form of our conjecture. If we include a Wilson line along the critically twisted circle, then the $R \to 0$ limit can be described by the T-duals of the nonsupersymmetric heterotic theories and the delocalized semi-classical instabilities descend to the bulk tachyons.

Our conjecture then implies that the fate of these theories after condensation of the closed string tachyon involves the annihilation of spacetime.

This poses a resolution to an issue raised by Suyama [80, 77]. Working under the assumption that condensation of the tachyon in the nonsupersymmetric heterotic theories takes the theories to a stable supersymmetric background the only choices for the endpoint involve the gauge symmetries $SO(32), E8 \otimes E8$. It was pointed out that condensation of a tachyon transforming nontrivially under a gauge symmetry should reduce the rank of the gauge group. However, the ranks of the gauge symmetries for the nonsupersymmetric heterotic strings are already as small or smaller than in the two supersymmetric cases. Our conclusion, i.e. that condensation of the tachyon leads to an annihilation of the spacetime, avoids this puzzle entirely.

A notable exception is the $H_{SO(16) \otimes SO(16)}$ theory which exhibits no tachyonic instability. This theory may be obtained as the T-dual of a Scherk-Schwarz orbifold of either $H_{SO(32)}^{\text{susy}}$ or $H_{E8 \otimes E8}^{\text{susy}}$ with appropriate Wilson lines. One expects a semiclassical instability towards spacetime annihilation, however in this case it is unclear to what the semi-classical instability descends. However this case is unique in that this particular Scherk-Schwarz compactification generates a positive cosmological constant. For all other Scherk-Schwarz compactifications the cosmological constant is negative driving the theory towards compactification and thus towards the tachyonic regime. In this case the positive cosmological constant generates a potential which pushes the theory towards decompactification thereby restoring the supersymmetric background with which we began.
4.4 Summary and Conclusions

Our aim in this chapter has largely been twofold. On the one hand we have taken seriously the idea that semi-classical gravitational instabilities in supersymmetry breaking compactifications may in certain limits reduce to perturbative instabilities signalled by the appearance of a closed string tachyon. In making this identification it is then natural to identify the endpoint of condensation of the tachyon with the endpoint of the semi-classical instability. In every case that we have considered the endpoint involves an annihilation of the metric degrees of freedom. We have further made a case for the naturalness of such a catastrophic fate by comparing these theories to those theories exhibiting open string tachyons for which extensive evidence has been presented. In both cases the corresponding degrees of freedom are annihilated.

On the other hand we have used this connection between semi-classical instabilities and tachyons to explore a possible web of dualities involving nonsupersymmetric string theories. In particular the eleven-dimensional origins of many nonsupersymmetric ten-dimensional string backgrounds has been conjectured and the overall picture appears to hang together quite nicely. Our discussion of the limits of the Fabinger-Horava theory constitute to our knowledge the first attempt to extend the 0A/M-theory relation of Bergman and Gaberdiel [7] to the heterotic theories.

A by product of our arguments is that Scherk-Schwarz compactification is not a very useful tool for constructing phenomenological SUSY breaking theories. In this the usual problem has been that the radius $R$ of the compactification circle would tend to zero because of the potential that develops at one loop (and higher) [58]. Thus the system approaches the tachyonic regime. However one might imagine that this modulus is stabilized either by classical flux terms or by some non-perturbative quantum effect. One would of course want this stabilization to occur at some $R > \sqrt{\alpha'}$, in order to avoid having a tachyon (and also usually to get smaller than string scale SUSY breaking). However at such radii the semi-classical instability of Witten that we discussed extensively in this paper takes over. It may be possible of course that the
fluxes (or quantum effects) are such as to stabilize the radius at a large enough value such that the semi-classical decay lifetime is larger than the age of the universe, but this strikes us as being somewhat unnatural.

There are some outstanding issues associated with our conclusions. First of all a quantitative description of the condensation of closed string tachyons in a vein similar to the open string case would put all of these speculations on a much firmer footing. The bounce solutions for Scherk-Schwarz orbifolds with Wilson lines have not yet been obtained. These might shed light on a nonperturbative framework for the “other” nonsupersymmetric heterotic theories.\textsuperscript{23}

\textsuperscript{23} The equivalence of supersymmetric heterotic strings on SUSY-breaking Melvin backgrounds to nonsupersymmetric heterotic strings has been thoroughly investigated in [80][77][79][78].
Chapter 5

Discussion

A nonperturbative definition of string theory is lacking. There is now overwhelming evidence that its known perturbative supersymmetric formulations constitute different vacua of some underlying theory. There is also support, some of which has been presented in this thesis, that the nonsupersymmetric perturbative formulations are part of this space of vacua as well. Any evidence as to what degrees of freedom should play a fundamental role in M-theory would constitute enormous progress.

In the limit of decoupling the open string sector of the theory from the closed string background, it seems that open string field theory is a promising candidate for nonperturbatively defining the dynamics of these degrees of freedom. Open strings are supported by the existence of D-branes in a closed string background. A background independent formulation thus requires not only a string field theory reproducing the perturbative dynamics of open strings on a given D-brane configuration, but must also accommodate the various D-brane configurations themselves as solutions. The idea of building general Dp-branes as networks of D0-branes or D-instantons has been used in this thesis to clarify several issues in the nonperturbative process of open string tachyon condensation. This constituent brane formalism was originally motivated by the Matrix model of M-theory, but has more recently been incorporated into attempts at constructing background independent string field theory in the so-called “sliver-state” formalism [57]. In this formalism open string field theory is formulated starting from the closed string vacuum itself. While we cannot explore the details of these developments here, we should like to point out
the fundamental view that for open strings, the best nonperturbative formulation is in terms of a pre-open-string geometry. That is, one in which the theory is formulated before explicitly introducing $D$-branes.

In the case of closed strings there is considerably more room for debate. Even before venturing into a nonperturbative formulation for closed string theory, there is already a disparity in the views regarding the endpoint of tachyon decay. A lack of direct evidence leaves the question open. The two popularly held views each reflect a distinct suggestion of what might go into a full definition of closed string theory.

On the one hand there is the widely held view that closed string tachyon condensation leads one back to a supersymmetric closed string vacuum state. This conclusion is in literal agreement with the case of open string tachyon condensation. In some sense the two processes intersect with a common stable remnant. A great deal of effort has gone into arguing for this scenario. We have already expressed our reservations on many of these arguments. A notion that pervades this line of reasoning is the absoluteness of spacetime. The nonperturbative vacuum transitions brought about by a condensing tachyon maintains the underlying geometry, and simply exchanges an excited state of the theory defined thereupon with a less excited state. In this light, it seems that M-theory should be formulated in terms of some degrees of freedom propagating on an underlying dynamical geometry.

On the other hand there is the less commonly held view that closed string tachyon condensation leads to an annihilation of the metric degrees of freedom, thereby eliminating the background geometry. This conclusion parallels the case of open string tachyon condensation and therefore never intersects with its stable vacuum. This is the view supported by the latter half of this thesis. Our arguments have been circumstantial and have involved a fair amount of speculation, but, as we have suggested, really no more so than the arguments posed for the alternative view. If this mode of decay is to be included as a part of the vacuum structure of M-theory, then it seems that M-theory should be formulated in terms of degrees of freedom that preceed the emergence of geometry. What does this leave? In the absence of geometry, it is hard
to imagine anything left save topology. Perhaps M-theory, in its most fundamental formulation, can be posed in a purely topological language, with geometry emerging in a derived fashion, much like the open-string-geometries built from the "sliver state". If indeed this is the case, then surely we face a fundamental shift in the way in which we think about spacetime and the physics thereon.

Of course in the end we all look for some foothold, like Sen's conjecture for open strings, that will allow us to reason with confidence once and for all what happens when the closed string tachyon condenses. Until then I feel a sort of kindred spirit to a certain world leader presently holding steadfastly to his beliefs, despite their contrast with the prevailing world opinion.¹

In hopes of defying any further comparison, I will let Nietzsche have the last word, "What convinces is not necessarily true, it is merely convincing: a note for asses."[51]

¹ On the day this thesis was completed, the President of the United States declared war on the regime in control of Iraq. This measure was advanced in the absence of consent by a majority of the global population.


Appendix A

Branes with B-fields and their T-duals

A.1 Introduction

Recently it has proven quite fruitful to consider $D$-brane systems in the presence of a background NS-NS two form potential $B_{\mu\nu}$. The presence of background $B$-fields has been shown to modify the conditions for supersymmetry in $Dp - Dp'$ systems [15, 85]. In most cases configurations supersymmetric in the absence of a $B$-field remain supersymmetric only for special nonzero $B$-fields. In other cases the presence of a nonvanishing $B$-field actually gives rise to new supersymmetric configurations not present in the $B \to 0$ limit. When a $B$-field is turned on along the spatial worldvolume coordinates of a $D$-brane, the system admits a low energy description in terms of a noncommutative gauge theory [67]. Techniques from noncommutative field theory [32] have been used to extract solitonic solutions to these low energy effective actions [19, 39], and these have been argued to represent lower dimensional $D$-branes [39]. Similar methods have been used to study the decays of unstable $D$-brane systems to BPS states [2, 50].

When considering these analyses it is useful to keep in mind that $D$-brane systems in the presence of background $B$-fields have T-dual descriptions in terms of "tilted" and "intersecting" branes. We will make these notions more precise in the next section. At times we may gain a simpler and more intuitive picture of the physics at hand in these T-dual systems. Our goal in this work is to present the T-dual versions of several recent analyses. This paper is the first step in an attempt to address still open questions in these subjects from this point of view.
This appendix is organized as follows. We begin by reviewing the relationships between tilted branes, branes in a background $B$-field (or with a world-volume magnetic field), and noncommutative geometry. In order to implement T-duality in our discussion we must consider all test systems on a torus whose radii we may take to infinity. In Sec.[A.3] we use these dual descriptions to explore the conditions for supersymmetry in $D$-brane systems with background $B$-fields. We then turn to unstable $D$-brane sytems and consider their fate in the T-dual picture. In particular we consider the systems discussed in [2, 50].

When we refer to T-duality without further explanation, we are referring to the $R \to \alpha'/R$ transformation of a coordinate axis. A D-string wrapping a two-torus $T^2$ on the $X^1$ cycle $n$ times and the $X^2$ cycle $k$ times will be referred to as an $(n,k)$ wound D1.

During the preparation of Sec.[A.3], two preprints appeared discussing similar results [52, 12]. We repeat these overlapping results for completeness.

### A.2 T-duality at an angle, Branes with $B$-fields, and NC geometry

We begin by reviewing some well known facts. For concreteness we will only consider $Dp$-branes with $p = 0,2$ in type IIA and $p = 1$ in type IIB.

1. For small enough separation a system of $k$ $D0$-branes and $n$ $D2$-branes is unstable to decay into a BPS state. The endpoint of this decay is the dissolving of the $D0$'s into a constant magnetic flux $F$ over the $D2$ worldvolume. This flux lives in the central $U(1)$ of the $U(n)$ gauge theory on the $D2$-branes [54].

2. A collection of $n$ $D2$-branes wrapping a two-torus $T^2$ with $k$ units of magnetic flux $F$ is T-dual to a single $D1$-brane wrapping the dual torus $T^2^*$ $n$ times around one cycle and $k$ times around the other [54, 40].

3. A $D2$-brane with a constant gauge invariant two-form flux $M$ along its spatial worldvolume dimensions exhibits low energy physics which may be described by a $(2+1)$-dimensional noncommutative $U(1)$ gauge theory [67].
The gauge invariant flux $M$ has two sources, either the NS-NS antisymmetric two form potential $B$ or the $U(1)$ field strength $F$ associated with the massless open string vector state living on the $D2$-brane worldvolume. On $T^2$ the magnetic flux $F$ is quantized, whereas the $B$-field may take any value. On the other hand one can argue that owing to the gauge invariance of $M$ the $B$-field may change only in integer shifts [67]. The specific relation to tilted branes is between a $D$-brane with a background magnetic flux $F$ and a diagonally wound $D(p-1)$-brane.

For some arbitrary split of the gauge invariant flux $M = B + F$ we may relate the $Dp$ on $T^2$ to a diagonally wound $D(p-1)$ on $T^{2*}$ with a background two form potential $B^*$. Characterizing the wrapping by $(n,k)$, the gauge invariance of $M$ provides an equivalence between two systems $(n,k)$ with $B^*$ and $(n',k')$ with $B'^*$ on $T^{2*}$. For simplicity, we will use the gauge invariance to shift everything into the magnetic field $F$, thus eliminating any residual background fields in the T-dual picture.

To clarify the first two statements we may consider two pictures of the decay of a single $D0$-$D2$ system on a two-torus as illustrated in Fig. A.1. On $T^2$ we see the $D0$ dissolve into a single unit of quantized magnetic flux on the $D2$-brane. On $T^{2*}$ we see two perpendicular $D1$-branes decay into a single diagonally wrapped $D1$. We will later consider modifying this simple picture in two ways. First by adding a large magnetic flux $F$ on $T^2$ and then tilting the system before T-dualizing. We will find that these modifications describe compactified versions of the systems analyzed in [2, 50].

These two descriptions of the $D0$-$D2$ decay provide us with little in the way of quantitative results. We may therefore seek to describe this system and its decay in a field theoretic setting with well-defined computations. Open string field theory potentially provides such a setting. Unfortunately it is sufficiently undeveloped that we do not even have a precise form of the low energy effective action controlling the dynamics of the lowest level excitations. However, as pointed out in [32], when a theory is defined on a noncommutative space, we may extract interesting results from it with minimal knowledge of its form. This has been put to use in constructing solitonic solutions to the open string field equations of motion. Various evidence
has been presented to identify these with $D_p$-branes [39, 19]. The connection between open string field theory on a noncommutative geometry and open string theory in the presence of a background $B$-field was developed in [67].

We thus see the intimate connection between T-duality at an angle, branes with background $B$-fields, and noncommutative geometry. The nonlocality of string field theories on noncommutative spaces is directly related to the nonlocal minimum length winding modes of tilted $D$-branes wrapping torii. For large $B$-fields these modes may become light enough to enter the low-energy dynamics. See Fig.A.2.

These multiple alternatives for describing processes such as the decay of the $D0-D2$ certainly allow us to sharpen our insight. However, one may inquire as to whether the quantitative analysis of these decays in the noncommutative framework provides a more informative handle on the subject. What we will find is that one really gains very little from the noncommutative
framework in regards to the difficult questions in D-brane decay. What emerges is instead a reiteration of the idea that $Dp$-branes may be viewed as composite states of lower dimensional $D$-branes.

### A.3 BPS $Dp - Dp'$ systems

To elucidate further the relationship between T-duality at an angle and branes with $B$-fields we may consider the mapping of a very simple property of $D$-brane systems, i.e. the conditions for supersymmetry. Toroidal compactification and subsequent T-duality transformations should preserve the number of conserved supersymmetries of a system. Specifically we want to compare the conditions for supersymmetry in a system of two branes rotated with respect to one another and in the T-dual $Dp - Dp'$ system in the presence of a background $B$-field.

The analysis of conserved supersymmetries for a $D0-Dp$ system with a $B$-field (as in [85]) seems strikingly reminiscent of the analysis for rotated brane configurations. Yet the notion of the relative orientation of a $D0$ and a $Dp$-brane bears no meaning. It is only in the appropriate T-dual picture that the connection becomes apparent.

We will consider two parallel $D4$-branes in type IIA and rotate one of them with respect to other. Taking the two $D4$-branes initially aligned with the $x^{02468}$ coordinate axes we will rotate in the $x^1 - x^2$ plane by $\phi^{12}$, the $x^3 - x^4$ plane by $\phi^{34}$, the $x^5 - x^6$ plane by $\phi^{56}$, and the
\( x^7 - x^8 \) plane by \( \phi^{78} \). The conditions for supersymmetry in this system are discussed in [54].

We list the conditions on the four angles and the number of supersymmetries preserved in each case. The indices \( i, k, m, p \in (1, 3, 5, 7) \) and \( j, l, n, q \in (2, 4, 6, 8) \).

i. \( \phi^{ij} + \phi^{kl} + \phi^{mn} + \phi^{pq} = 0 \mod 2\pi \ 2 - \text{susys} \)

ii. \( \phi^{ij} + \phi^{kl} + \phi^{mn} = \phi^{pq} = 0 \mod 2\pi \ 4 - \text{susys} \)

iii. \( \phi^{ij} + \phi^{kl} = \phi^{mn} + \phi^{pq} = 0 \mod 2\pi \ 4 - \text{susys} \)

iv. \( \phi^{ij} + \phi^{kl} = \phi^{mn} = \phi^{pq} = 0 \mod 2\pi \ 8 - \text{susys} \)

v. \( \phi^{ij} = \phi^{kl} = \phi^{mn} = \phi^{pq} = 0 \ 16 - \text{susys} \)

When \( k \) of the angles are \( \frac{\pi}{2} \) and the rest are zero, we may count the number of ND coordinates to determine the supersymmetry of the system.

For simplicity we will consider T-dual systems of \( D0-Dp \) branes for \( p = 2, 4, 6, 8 \). To obtain these systems we simply set some of the \( \phi^{ij} \) to \( \frac{\pi}{2} \) and then T-dualize in \( x^2, x^4, x^6, \) and \( x^8 \). This construction leads to \( D0-Dp \) systems without a \( B \)-field.

To obtain \( D0-Dp \) systems with a background \( B \)-field we must consider deviations from the \( \frac{\pi}{2} \) rotation which we denote \( \delta \phi^{ij} \). The original rotation angle we call \( \phi_0^{ij} \), and the total rotation angle is then \( \phi^{ij} = \phi_0^{ij} - \delta \phi^{ij} \). Before T-dualizing, the conditions for supersymmetry are expressed in terms of the relative orientation of the \( D4 \) pair. After T-dualizing, these will become conditions on the \( B \)-field components along the worldvolume of the \( Dp \)-brane. Taking the \( B \)-fields to have a skew-diagonal form

\[
B = \bigotimes_k \begin{pmatrix} 0 & b_{k,k+1} \\ -b_{k,k+1} & 0 \end{pmatrix}
\]  
(A.3.1)

we can relate the \( B \)-field induced by T-dualizing a coordinate \( x^j \) after rotation in the \((x^i, x^j)\) plane to the angle of rotation \( \phi^{ij} \) by \( b_{ij} = \cot \phi_0^{ij} - \delta \phi^{ij} \). See Fig.A.3.

- \( D4 - D4 \) : \( \phi_0^{12} = \frac{\pi}{2} \), \( \phi_0^{34} = \phi_0^{56} = \phi_0^{78} = 0 \) : \( x_0^{2468} - x_0^{61468} \)
Figure A.3: The T-dual of a pair of D4-branes rotated in a single plane defined by $x^i - x^j$.

\[ D0 - D2 : x^0 - x^{012} \]

In the absence of a $B$-field the $D0$-$D2$ system is not supersymmetric. This can be seen in a number of ways. One can simply count the number of Neumann-Dirichlet coordinates ND as described in [54]. Supersymmetry requires this be a multiple of four. The $D0$-$D2$ system has two, $x^1$ and $x^2$ in our example, and is hence not supersymmetric.

Turning on a $B$-field along $x^1$ and $x^2$ allows one to modify the conditions for supersymmetry as discussed in [15, 85]. It is recognized through this analysis that the $D0$-$D2$ becomes supersymmetric only in the presence of a diverging $B$-field, i.e. $b_{12} \to \infty$. In the T-dual picture this diverging $B$-field simply "undoes" the original rotation by $\frac{\pi}{2}$,

\[ \phi^{12} = \frac{\pi}{2} - \cot^{-1}(b_{12} \to \infty) \to 0 \quad \text{(A.3.2)} \]

bringing the two $D4$'s parallel and hence restoring supersymmetry. The final system is a pair of $D0$-branes. There are thus sixteen unbroken supersymmetries for this system.

- $D4 - D4 : \phi_0^{12} = \phi_0^{34} = \frac{\pi}{2}, \quad \phi_0^{56} = \phi_0^{78} = 0 : x^{02468} - x^{01368}$

\[ D0 - D4 : x^0 - x^{01234} \]

The $D0$-$D4$ system is supersymmetric in the absence of a $B$-field. The number of $\frac{\pi}{2}$ rotations is two, hence the number of ND coordinates is four.
Turning on a magnetic field along $x^1 - x^2$ and $x^3 - x^4$ may adjust the number of supersymmetries. In this case the total rotation angles sum to

$$\phi^{12} + \phi^{34} = \frac{\pi}{2} + \delta\phi^{12} + \frac{\pi}{2} + \delta\phi^{34} \quad (A.3.3)$$

If $\delta\phi^{12}$ and $\delta\phi^{34}$ are supplementary angles, then condition (iv) is met and the system preserves eight supersymmetries. This condition is exactly the requirement that the magnetic field on the $D4$-branes be anti-self dual, i.e. $b_{12} = -b_{34}$.

- $D4 - D4 : \phi_0^{12} = \phi_0^{34} = \phi_\phi = \phi_7^8 = 0 : x^{02468} - x^{01358}$

  $D0 - D6 : x^0 - x^{0123456}$

The $D0$-$D6$ system has three ND coordinates and is hence not supersymmetric in the absence of a $B$-field.

Introducing a $B$-field in the six spatial worldvolume coordinates of the $D6$-brane modifies this. There are numerous possibilities for how one may turn on the $B$-field, so we will catalogue them in order of decreasing number of conserved supersymmetries.

$$\phi^{12} + \phi^{34} = \phi^{56} = \phi^7_8 = 0 \quad 8 - susys \quad (A.3.4)$$

To satisfy this condition we need $b_{56} \to \infty$. The remaining $B$-field components would then have to satisfy $b_{12} = -b_{34}$. This system is actually equivalent to the $D0$-$D4$ discussed above since the diverging $b_{56}$ reverses the initial $\frac{\pi}{2}$ rotation in the $x^5 - x^6$ plane.

$$\phi^{12} + \phi^7_8 = \phi^{34} = \phi^{56} = 0 \quad 8 - susys \quad (A.3.5)$$

Since we are restricted to $\phi^7_8 = 0$ this condition would require all three of $b_{12}$, $b_{34}$, and $b_{56}$ to diverge. This system would then satisfy condition (v) and hence would preserve sixteen supersymmetries.
Again since $\phi^{78} = 0$, this condition reduces to (iv) as discussed above.

$$\phi^{12} + \phi^{34} + \phi^{56} = \phi^{78} = 0 \quad 4 - susys$$  \hspace{1cm} (A.3.7)

This case is interesting. If we take all of the initial $\phi^{12}_{0} = \phi^{34}_{0} = \phi^{56}_{0} = \frac{\pi}{2}$ then the sum of the full rotation angles becomes

$$\phi^{12} + \phi^{34} + \phi^{56} = \frac{3\pi}{2} + \delta\phi^{12} + \delta\phi^{34} + \delta\phi^{56}.$$  \hspace{1cm} (A.3.8)

For this sum to vanish we require $\delta\phi^{12} + \delta\phi^{34} + \delta\phi^{56} = \frac{\pi}{2}$ mod $2\pi$, which gives an implicit condition on the three independent $B$-field components. This condition was discussed in [85]. This is the only case of supersymmetry enhancement in the $D0$-$D6$ system via a finite $B$-field.

$$\phi^{12} + \phi^{34} + \phi^{78} = \phi^{56} = 0 \quad 4 - susys$$  \hspace{1cm} (A.3.9)

Requiring $\phi^{78} = 0$ effectively reduces this to condition (iv).

$$\phi^{12} + \phi^{34} + \phi^{56} + \phi^{78} = 0 \quad 2 - susys$$  \hspace{1cm} (A.3.10)

This reduces to condition (ii).

- $D4 - D4 : \phi^{12}_{0} = \phi^{34}_{0} = \phi^{56}_{0} = \phi^{78}_{0} = \frac{\pi}{2} : \ x^{02468} - x^{01357}$

$\downarrow$

$D0 - D8 : x^{0} - x^{012345678}$

The $D0$-$D4$ system requires a restricted $B$-field to remain supersymmetric, while the $D0$-$D6$ system requires a special $B$-field to become supersymmetric. The $D0$-$D8$ system in a sense exhibits both of these properties. In the absence of a $B$-field the $D0$-$D8$ system is supersymmetric.
Turning on a $B$-field in all eight of the $D8$'s spatial worldvolume directions we obtain several possible configurations.

$$\phi^{12} + \phi^{34} = \phi^{56} = \phi^{78} = 0 \quad 8 - susys \quad (A.3.11)$$

This condition requires turning on infinite $b_{56}$ and $b_{78}$. The remaining components must satisfy $b_{12} = -b_{34}$. This system is equivalent to the $D0$-$D4$ system discussed above.

$$\phi^{12} + \phi^{34} = \phi^{56} + \phi^{78} = 0 \quad 4 - susys \quad (A.3.12)$$

This condition is satisfied with finite $B$-fields if the components obey $b_{12} = -b_{34}$ and $b_{56} = -b_{78}$.

$$\phi^{12} + \phi^{34} + \phi^{56} = \phi^{78} = 0 \quad 4 - susys \quad (A.3.13)$$

This condition requires $b_{78} \to \infty$. The condition on the remaining $B$-field components is given implicitly by $\delta \phi^{12} + \delta \phi^{34} + \delta \phi^{56} = \frac{\pi}{2} \mod 2\pi$. This system is equivalent to the $D0$-$D6$ system discussed above.

$$\phi^{12} + \phi^{34} + \phi^{56} + \phi^{78} = 0 \quad 2 - susys \quad (A.3.14)$$

This condition can be satisfied in two distinct ways with a finite $B$-field.

$$\phi_0^{12} = \phi_0^{34} = \frac{\pi}{2} \quad \phi_0^{56} = \phi_0^{78} = -\frac{\pi}{2} \quad (A.3.15)$$

In this case the condition above becomes

$$\delta \phi^{12} + \delta \phi^{34} + \delta \phi^{56} + \delta \phi^{78} = 0 \mod 2\pi. \quad (A.3.16)$$

This condition includes in its solutions the special case $b_{12} = b_{34} = b_{56} = b_{78} = 0$, i.e. it is continuously connected to the case of vanishing $B$-field.

A second possibility is to choose

$$\phi_0^{12} = \phi_0^{34} = \phi_0^{56} = \frac{\pi}{2} \quad \phi_0^{78} = -\frac{\pi}{2}. \quad (A.3.17)$$

With this choice the condition for supersymmetry becomes

$$\delta \phi^{12} + \delta \phi^{34} + \delta \phi^{56} + \delta \phi^{78} = \pi \mod 2\pi. \quad (A.3.18)$$
This again represents a condition on $b_{12}$, $b_{34}$, $b_{56}$, and $b_{78}$ to preserve two supersymmetries, however $B$-fields satisfying this constraint cannot be continuously dialed to zero.

### A.4 Unstable $Dp - Dp'$ systems and their decays

The tool of noncommutative geometry provides assistance in constructing soliton solutions in situations where we do not even know the full form of the low energy effective action. Unfortunately it does little to elucidate the nature of the closed string vacuum to which these solitons should asymptote. This “nothing” state should support closed string excitations only and thus exhibit an insensitivity to the background $B$-field initially required for these constructions. Eliminating residual open string fluctuations in the “nothing” and formulating a background independent construction has lead to interesting new ideas and insights. To probe this tool in a more concrete setting, several authors have looked at the construction of solitonic solutions using noncommutative geometry in situations that supposedly do not share these complications. Unstable $D0-D2$ and $D2-\overline{D2}$ systems with background $B$-fields have been considered in detail [2, 50]. These scenarios are straightforwardly related to the simple process discussed in Sec.[A.2]. We discuss each below.

- **$D0-D2$**

We may consider the effect of adding a large magnetic flux $F$ to the $D0-D2$ picture in Fig.A.1 along the spatial worldvolume directions of the $D2$-brane. Since the $D2$ wraps a two-torus $T^2$, the flux which we will turn on along the $D2$’s spatial worldvolume directions must be added in quantized amounts.

$$\int d^2xF = 2\pi k \quad q \in \mathbb{Z} \tag{A.4.1}$$

This is obviously related to the fact that in the T-dual picture, the tilted D1-brane must wrap the torus an integer number of times. Turning on a large magnetic flux, i.e. $k \to \infty$, the T-dual picture of the decay is as shown in Fig.A.4.
One can see that with such a large wrapping, the relative angle between the two $D1$-branes approaches zero. The T-dual picture of this process thus reduces to the decay of a system of two very nearly parallel $D1$-branes. The endpoint of this decay will be a BPS state consisting of a $(k+1,1)$ wrapped $D1$-brane. In the $\theta \to 0$ limit this process should implement only the lowest modes of strings stretched between the two $D1$-branes. This is merely another formulation of the limit which [2] uses to ensure a valid low energy description, i.e. a vanishingly small ratio of the string scale to the noncommutativity scale. The tower of parametrically light $0$-$2$ excitations discussed in [2] is exactly the T-dual of the tower of light excitations between two $D1$-branes intersecting at an angle. In particular, the base of both towers is a single complex tachyonic state representing the instability of the system to decay.

- $D2 - \overline{D2}$

The $D2 - \overline{D2}$ system analyzed in [50] at first appears to encompass all of the complications associated with the “nothing” state. Certainly, for this system the endpoint of annihilation would be the closed string vacuum with no residual $D$-branes present. However, the particular magnetic field configuration implemented in this discussion actually reduces this system to a much more straightforward one, particularly in the T-dual picture. The authors of [50] set out to follow the decay of the $D2 - \overline{D2}$ via noncommutative
gauge theory in the limit of a large background $B$-field. They note that instead of a single large $B$-field, what is needed is a large background $B$-field supplemented by a particular magnetic field configuration on the $D2$, i.e. $\bar{F} = -2B$. This magnetic field is chosen to effectively reverse the $D0$-brane charge for the Wess-Zumino term in the $D2$-brane action.

$$S_{\text{D2}} = -i \mu_2 \int d^3 x C^{(1)} \wedge 2\pi \alpha'(\bar{F} + B) \rightarrow i \mu_2 \int d^3 x C^{(1)} \wedge 2\pi \alpha'(B)$$

This enables both the $D2$ and $\bar{D}2$ to be described in terms of $D0$-branes. Using the well known low-energy description of $D0$-branes a la Matrix Theory [4], the dynamics of both the $D2$ and $\bar{D}2$ may be described with the same underlying Lagrangian. This allows one to follow the decay of the $D2 - \bar{D}2$ system to its final state, a collection of $D0$'s.

We may consider the T-dual of this system. We begin by wrapping both the $D2$ and $\bar{D}2$ on a torus $T^2$. These would normally tangentially T-dualize to a $D1 - \bar{D}1$ system. However, we want in addition to put a magnetic flux $F$ on $D2$ and an equal and opposite flux $\bar{F} = -F$ on $\bar{D}2$. We will use magnetic fluxes exclusively in contrast to [50] to avoid the complications discussed in Sec.[A.2]. On $T^2$ these fluxes are quantized and again correspond to integer windings of the T-dual $D1$-branes. Turning on large $(k \to \infty)$ $F$ and $\bar{F}$, the T-dual picture of the decay is as shown in figure A.5.

Figure A.5: T-dual picture of the $D2-\bar{D}2$ decay. In the large $B$ limit considered by [50], $k \to \infty$ and the angle $\theta \to 0$.

Notice again that the diverging flux on $T^2$ corresponds to a vanishingly small intersection
angle $\theta$ on $T^2$. We are again dealing with a system of two nearly parallel D1-branes. This limit allows [50] to describe the system on $T^2$ in terms of the low energy effective theory for D0-branes, i.e. noncommutative Yang-Mills. The price of describing the $D2 - \overline{D2}$ in terms of D0-branes only is that the T-dual system in a sense trivializes. Analyzing the fluctuation spectrum of the the system on $T^2$, [50] found a tower of parametrically light $2 - \overline{2}$ states. These are again T-dual to the tower of light states between two D1-branes at a nonvanishing angle. Both towers exhibit a complex tachyonic base signalling the instability of the system. The endpoint of this decay will be a $(2k,0)$ wound D1-brane. T-dualizing back to the original torus, we see that this corresponds to a collection of $2k$ D0-branes. These are precisely the decay remnants found in [50]. They correspond to the flux that was added to the $D2 - \overline{D2}$ system by hand to facilitate the description in terms of noncommutative gauge theory.

These discussions may lead one to identify the two processes analyzed in [2, 50]. In fact if we begin with a D0-D2 system on $T^2$ with $k$ D0-branes, 2 D2-branes, and $k$ units of gauge invariant flux (in the central $U(1)$ of the $U(2)$), we may T-dualize this system to a $(k,0)$ wound D1 and a $(k,2)$ wound D1 on $T^2$. We can perform a basis change of the compactification lattice using an element of the $SL(2,Z)$ subgroup of general T-duality transformations $O(2,2,Z)$ to shift this system to a $(k,-1)$ wound D1 and a $(k,1)$ wound D1. This is exactly the T-dual of the system studied in [50].

The relation between these two processes seems a bit surprising. In the D0-D2 case we see a D0 dissolve into a magnetic flux on a D2. In the $D2 - \overline{D2}$ case we see the magnetic flux on a pair of D2's form vortices, or D0-branes, upon the decay of the $D2 - \overline{D2}$ pair. In fact it was pointed out in [50] that the formation of vortices usually requires positive energy and hence would not constitute a decay mode. However, by carefully normalizing the gauge theory solutions for these vortices it was shown that the system energy actually decreases. In the T-dual picture the decay in both cases is that of intersecting D1-branes and no discrepancy
A.5 Conclusions

The tools of noncommutative field theory have been used to gain numerous insights into nonperturbative phenomena in string theory. This framework allows one to work in the familiar stomping grounds of conventional field theory while maintaining remnants of the underlying stringy behavior. While having proved to be a useful program, there are numerous issues which remain unresolved. In most situations of interest the noncommutative field theory arises as the effective theory for open strings in the presence of a background two-form flux. We may then consider the T-dual representation of these scenarios and develop a map between results in the noncommutative field theory and the low-energy description of the dual system of tilted and intersecting brane configurations. In this work we have taken a few steps in this direction by considering the mapping of the conditions for supersymmetry and by constructing the T-duals of certain brane annihilation processes. These dual representations reduce either to familiar results in the case of supersymmetry, or to simpler processes in the case of brane annihilation. Interesting avenues for further work include a more detailed mapping of the construction of noncommutative solitons from the noncommutative field theory framework, and analysis of the role played by generalized T-duality transformations in the effective theory.