PARTNERSHIP MATHEMATICS CONTENT COURSES FOR PROSPECTIVE AND PRACTICING ELEMENTARY AND MIDDLE SCHOOL TEACHERS

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Since fall 1995 the Department of Mathematical Sciences at New Mexico State University has been involved in a partnership with the Las Cruces, NM, USA, Public Schools. We offer a series of four one-semester courses attended jointly by prospective and practicing teachers of grades kindergarten through 8. The sequence covers the arithmetic of integers, rational, and real numbers, metric geometry, and algebra, with integrated use of technology. Practicing teachers act as mentors for prospective teachers, who become their apprentices. Material is organized into units that provide lesson plans for elementary and middle grades. Practicing teachers use them in their classrooms, and their apprentices observe or even teach under their mentors' supervision. The important part is that preservice teachers learn advanced mathematics in a setting in which they are going to use it.

Why the project was undertaken

There is no central agency that controls schools in the United States. Individual states, school districts, and even schools have considerable freedom in setting their own programs and curricula. Teacher licensing is done by individual states, and requirements vary. Teacher preparation is provided by Schools of Education, which are part of the college system. These Schools concentrate on pedagogical and educational issues. Courses in subject matter, such as mathematics or the sciences, are offered by departments located in Colleges of Arts and Sciences.

But elementary teachers are generalists. They have to teach all subjects, even if during their college years they do not have sufficient time to acquire enough subject matter knowledge (Stigler & Hiebert, 1999). Also the format of university courses is very different from the format of classroom work in the early and middle grades. This provides a formidable obstacle for young teachers.

Our experience with workshops and summer courses for teachers convinces us that they do not provide enough learning time to be the basis for sustained continuation of professional development. The partnership program described below seems to be a solution to this problem. It provides continuing education for practicing teachers, and at the same time offers future teachers courses in mathematics that cover topics they will need to know in their classrooms in a setting similar to the settings found in schools.

The most encouraging aspect is the response of practicing teachers who are willing to spend two evenings per week in order to continue their professional education and improve their qualifications.

Organization of the university courses

Format for the courses

The courses are taught in a laboratory format. This means that there are no lectures longer than five to ten minutes. Students sit in groups of four to six at tables, and during the class session they do the tasks assigned to them. The only whole-class activities are discussions and classroom reports (brief presentations by students and teachers). In the smaller partnership classes (12-30 students), in addition to the instructor, a teaching assistant is always present; and the larger classes (30-45) need two teaching assistants. The teaching assistants are either graduate students specializing in mathematics or education, or teachers who have already taken a number of the partnership courses.

The mentor-apprentice partnership

University undergraduate students and practicing teachers team up together in an apprentice-mentor relationship. During a semester an undergraduate student usually has two or more teacher mentors, who teach two different grade levels. Each mentor has up to four apprentices at a given time. When there are not enough mentors in the class, teachers who have previously taken the class and who are not currently enrolled at the university serve as mentors. Thus the relationship between the teachers and the university is not limited just to the semesters when teachers are actually enrolled in the university courses.

Obligations of teachers and students

Attendance at the university class sessions is obligatory for both students and teachers. They do the assigned tasks together. Seating is arranged so that in each group of four to six there is at least one teacher. If there are not enough teachers, the teaching assistants sit with some students. Both students and teachers are required to keep journals; they write about the mathematical topics in the activities covered in each session. Journals are collected about every three weeks; the instructor reads them and provides individual feed back.

Examples of journal entries and instructor comments

(1) Exploration of stars
(Journal entry written by a preservice teacher)
My understanding of the unit.
We had to divide 360 degrees by the number of points the star would have, so we would have the measure of the angle. The first star we made had five points. We divided 360 degrees by 5 to find the distance apart of the tick marks, namely, 72 degrees. For the star with six points 360 degrees was divided by 6, for an angle of 60 degrees. 360 degrees divided by 7 equals about 51.4, which we rounded to 51.5 degrees for the seven pointed star. After we measured the angles using a protractor, we had to join them up. We first numbered the points, then we joined them together. For the 5-pointed star, the first point connected to points 3 and 5. The third point also connected to the 5th point which connected to the 2nd point which connected to the 4th point, creating the star.
Mathematics. Division, angles, degrees, protractors, rounding, geometry, radius, even and odd.
My reaction. I really enjoyed this unit. I had a little trouble at first with the 5 star. I could not get the angles right, but I eventually measured them correctly. I also continued the pattern inside each of the stars, making smaller and smaller stars.

Instructor's comments.
Nice writing! Do you know how many different 7-point stars there are? You can make two different ones, a fat one and a skinny one; it depends on which points you join. Also did you notice that the five-point star does not "split" into two pieces, but the six-point star does "split"?

(2) Understanding long division
(Again, written by a preservice teacher)
To logically understand the simplicity of long division, we picked apart the written algorithm. Let's divide 17682 by 246. First we used a calculator to find the multiples of 246. Making a list of multiples makes it more visual to see how it works which is a very effective step. As listed below:

1	246
2	492
3	738
4	984
5	1230
6	1476
7	1722
8	1968
9	2214

Second we set up the problem as follows below, which is usually the step that is left out of solving long division:

```
 17682
-17220      (70)
   462
  -246       (1)
   216.0
  -196.8      (.8)
    19.20
   -17.22      (.07)    Instructor comment:  Now add up the numbers in
```

………. parentheses to get a quotient.

This is how a traditional long division problem would be set up:

```
       71.87
246)17682.
    -17220
       462
      -246
       216.0      Instructor comment: Traditionally the 0 on
      -196.8      17220 and the decimals (except those in the
        19.20     dividend and quotient) would not be included.
       -17.22
                     Very nice work!
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(For understanding arithmetic algorithms see also Ma, 1999.)

Students are given both obligatory and optional (extra credit) homework assignments, and they are required to make at least ten visits to the classrooms of their mentors. During their first visits they usually observe; later they co-teach; and finally they teach under the supervision of their mentors. But these decisions are made solely by the teachers and not by the course instructor. All students are required to describe each classroom visit in their journals.

Teachers do not evaluate students' performance. Evaluations and assignment of grades are done only by the instructor, who also gets input from the teaching assistants.

Example of a homework assignment

Each student selected an irregularly shaped block of wood, and the task was to find the surface area and volume of the block, to make at least three different drawings of the block, showing measurements, and to explain each step used in finding the answer. We decided that some blocks were more difficult to measure than others, so the instructor assigned a difficulty level (one = easiest; three = hardest) to each block.

This homework, by a preservice teacher, received a score of 100%, with a difficulty level of two. It included three more pages of drawings and calculations.

In order to find out the area and the volume of my chunk of wood, I did the following. I measured the length and widths for sides A, B, C, and D. For side E, I cut it to make a rectangle and a triangle. (See attached papers.) I found the length and widths for these. I then found out all the areas for these sides (see paper). I then added up all the areas & got the total area for the block (40.88 sq. in.). In order to find the volume I formed the volume of the rectangle [instructor: rectangular solid], using the formula V = L*W*H. I then found the volume of the triangle [instructor: triangular prism] by making it an imaginary rectangle [instructor: rectangular solid]. I found the volume for the [rectangular solid] and then divided by two to find the volume of the [triangular prism]. I then added the volume of the [rectangular solid] and the [triangular prism] and got the total volume of my chunk of wood (15.75 cubic inches). (See attached paper for all calculations and measurements.)

Teachers mentor their university student apprentices in classrooms with their pupils. During the students' visits they typically use materials that they have gone over together in the university courses. So the students see how the material that they have just learned is used by experienced teachers in schools. When the lessons are new and have never been classroom tested before, the teachers also administer diagnostic tests to pupils in order to assess their learning (see Testing the materials below).

Rationale for this format for the courses

We consider the education of teachers to be professional education. They will use the subject knowledge learned in college over and over again. This means that the material they are going to teach must be learned in minute detail. "General ideas" are not good enough. This suggests that mathematics courses for future teachers should be held in a laboratory format in which students work under the supervision of an instructor or an experienced teacher. In such courses the total amount of material covered is smaller than in a typical course for undergraduate students in a College of Arts and Sciences, but it is covered in more detail and with a larger stress on skills. In the future when students become teachers, they will use their subject knowledge in their classrooms, so seeing how their mentors do it, and teaching under their supervision, provides them with the practice needed in any professional education. Classroom-ready lesson plans are important for both teachers and students. For teachers they provide something that is immediately usable, and for students they connect what they learn in the university setting to what they see in the classrooms of their mentors.

Logistics

The courses are offered during the school year, when public schools are in session, so units can be tried immediately in schools. The courses are scheduled late in the afternoon or evening so that teachers can attend. The Las Cruces Teachers' Center, headed by Karin Matray, Director of Professional Development for the Las Cruces Public Schools, distributes announcements about the courses to all schools in spring and fall, thus helping to recruit teachers for the classes. Teachers' university tuition has been paid by grants. Grants have also covered the cost of tools, calculators, other supplies, and extensive Xeroxing of handouts for the classes.

Starting a partnership program requires the approval of three groups.

• The School District must allow undergraduate students to visit classrooms, and it must allow teachers to use the lesson plans we provide. It also must give us access to the work of pupils in schools so we can assess what they learn.

• The University's College of Education must approve the courses as appropriate for both present and future teachers.

• The University's Department of Mathematical Sciences must approve the content of the courses, and it must agree that the same course can be attended simultaneously by graduate students (teachers) and undergraduates (future teachers).

The content of the courses was chosen in close collaboration with the School District. The District identified middle school algebra and the use of technology as two critical issues that needed to be included in any attempt to improve the mathematical education of its pupils. And the integration of mathematics and science education, which will be the topic of the next course in the series, is high on the District's priority list.

Course material

Units

The material is organized into units. Most of the units are based on a specific task that cannot be completed without applying mathematical knowledge and skill.

> Example.
> (A class project.) From poster board construct 20 cubes having volumes, 1, 2, 3, …, 20 cubic inches. In order to do the task the students must learn the formula for the volume of a cube. They have to compute cube roots. And they to have enough geometric skills and knowledge to draw the plans and assemble the cubes.
> Each unit is accompanied by a lesson plan for its use in school classrooms.

Students and teachers are provided with approximately 50 units each semester. (See Baggett & Ehrenfeucht, 1995; 1998; 2000). Both students and teachers go through them either in the university class or as assignments at home. The teachers use lessons of their choice from the university class in their own classrooms. A typical unit requires one or two periods of school time.

There is no textbook that covers the mathematical topics in the abstract; all topics are learned in the context of some application.

Mathematical content

The sequence of courses covers the following mathematical topics:

- Arithmetic of real numbers and its subsets, rational numbers, integers and whole numbers.
- Three dimensional metric geometry, namely geometry based on the concept of distance. This is a modern version of the "practical geometry" of the past.
- Algebra presented in the Newtonian (and not Eulerian) tradition. It doesn't stress the "formal aspects" of algebra, but relates it to numerical techniques and physical quantities.

All of these topics are presented in a mutually consistent way.

> Example.
> Unacceptable: 3 is the next number after 2 (it is unacceptable because $2 < 2.5 < 3$).
> Acceptable: 3 is the next integer after 2.

All topics are present in all four courses. But the first course focuses on arithmetic, the second on geometry, the third on algebra, and the fourth on the use of technology.

Technology

The use of four-operation calculators is integrated even with mathematical topics that are usually taught in kindergarten and first grade (Baggett & Ehrenfeucht, 1992). Scientific calculators are used with materials containing algebra, and graphing calculators are used for computationally complex tasks and "computer simulations". Computer workstations are multi-purpose devices used for mathematical tasks, internet searches, editing, and so on. The use of technology in the courses is divided as follows. The first two courses use four-operation calculators, the third one adds scientific calculators, and the fourth one adds graphing calculators and a computer lab.

Designing algorithms and learning the rudiments of programming starts early. Already in the first university course students learn how to compute a cube root on a four-operation calculator. They use the iterative procedure based on the fact that (the cube root of n) = lim Z(k), where Z(k+1) = (the fourth root of n*Z(k)). In the third course, which focuses on algebra, students are shown a procedure to solve equations using Newton's method.

Without modern technology, the introduction of mathematical topics and the order of their presentation are severely limited by the very low pace at which children acquire minimal skills in written computation. Technology allows one to select topics on the basis of children's intellectual development and their interests, rather than the level of their computational skills.

The role of measurements and use of tools

One way to build number sense in the early grades is to show pupils that numbers come from measurements, which has always been done in learning mathematics as a vocation (Hawney, 1807; Daboll, 1812; Pike, 1827). Thus measurements with common standard measuring tools, rulers, measuring tapes, protractors, scales, measuring cups and thermometers are important parts of lessons for all grades. And their proper and skillful use is considered important. Also tools used in constructions in metric geometry are not limited to compass and straight edge, but include rulers, protractors, and even more complex draftsman's tools such as French curves.

Rationale

The purpose of mathematics in grades K to 8 is twofold:
(1) to provide a solid knowledge of "everyday" mathematics, and
(2) to provide a background for future study for pupils who will later choose "mathematically intensive" careers.

We think that these two different goals can be best achieved if mathematics is taught in a unified and consistent way as an applied science. So we do not "explain" mathematics in terms of the manipulation of physical objects; instead we present it as "problem solving tool" for a variety of practical problems (Freudenthal, 1973; Nunes, Schliemann & Carraher, 1993). This approach serves both students who like mathematics, and those who struggle with abstract concepts and become "math phobic".

Our approach to technology is also pragmatic. Technology is extensively used by adults in all tasks requiring mathematics; therefore it should be integrated into school learning. We treat it again as a problem solving tool, and not as a "teaching aid." Four-operation calculators are the easiest to use, and are the most useful in combination with mental calculations. When pupils start to rely on written formulas and learn some algebra, the most useful are scientific calculators. After this background pupils start to explore the variety and versatility of more advanced technology using graphing calculators or workstations.

Extensive handouts are used for the courses. Recently we gave an (anonymous) questionnaire to students asking them if a textbook would be useful for these courses. The large majority answered no. The reason given most often was that textbooks in college courses mainly help instructors, but not students.

Use of the materials in classrooms

Grade level

Teachers use the lesson plans provided in the university courses in their classrooms, which range from kindergarten through eighth grade. But different grades require material with different mathematical content and different difficulty. This problem is resolved in two ways. First, teachers decide which material is appropriate for their pupils. And second, most units can be adapted to many grade levels.

> Example.
> Basic task. Students are given a baseball that just fits in a cubic box, a package of rice, and scales. Question: What percentage of the volume of the box is filled by the ball? (The surprising answer is pi/6, or about 52%.)
> Method. Weight the ball in the box, fill the empty space with rice, and weigh again. Weigh the empty box, and the box filled with rice. Compute the answer using a simple calculator.
>
> This lesson is suitable for grades 5 through 8. For earlier grades, the problem may be formulated in terms of part of the volume rather than percentage, and students may be given measuring cups, to avoid the difficulty of computing the ratio of volumes from weights.
>
> In grades 7 and 8, it can be an introduction to the formula for the volume of a sphere, or even to computing volumes by Cavalieri's principle, using the definite integral on graphing calculators.

Remark.
Even very experienced teachers can rarely prepare their own lesson plans. But they are usually skilled in adapting lessons to the level of their pupils.

Testing the materials

We assess pupils' skill and the knowledge they gain from particular lessons that were presented in classrooms of participating teachers. The assessments are non-intrusive; they are either follow-up activities with diagnostic value, or writing assignments that assess the children's understanding and recall.

Examples of children's recalls

First graders did two tasks: (1) They drew triangles by making three dots and connecting them with straight segments using a ruler. Later they colored the patterns they made. (2) They were given a non-convex 9-gon and were asked to measure its sides in inches and sixteenths of inches and write the lengths next to each side. Their spoken recalls were taken two days later. Here are two:
Subject 1. I learned to make lines with a ruler, to connect dots and measuring and about fractions. I liked the coloring best.
Subject 2. I learned you can make anything out of triangles. And how to measure lines. I liked doing the triangles.
(See Baggett & Ehrenfeucht, 1999.)

Exploration of stars (see also Examples of journal entries and instructor comments (1) above) was taught in a sixth grade class, and recalls were requested two days later. Here is a recall of a sixth grader:

Stars
Tools used-
-calculator
-compass
-ruler
-protractor
Directions- First you need to get all of your tools. Then you set your compass at four inches. It needs to be four inches because the paper's width is eight inches. Now you make your circle. Now you have to pick how many points you want for your star. For example you picked a five point star. You have to measure where the points are going to be by dividing 360 by 5. You get 72. Now you use your protractor by measuring where 72 is. When you finish doing that you can start making your star. For the last step draw lines from 1 to 3, 3 to 5, 5 to 2, 2 to 4, and 4 to 1. Now you have a star. I thought this was fun. At home I even tried making a harder one. I hope we do this in class again.

Remarks.

Children remember good lessons well. And procedures are usually remembered better than facts and conclusions (Engelkamp, 1998). A lack of understanding is demonstrated by incoherence in the description of what was done and poor recall.

In this school district, different schools use different textbooks and adhere to different teaching philosophies. Good lessons can be incorporated in a variety of curricula, and fit different teaching styles.

Examples of teachers' and students' opinions from course evaluations
(These comments came from anonymous course evaluations from fall 1999.)

It challenged me to understand more about math and not just math on paper.
I felt challenged throughout the course because it made me see math in a different way.
All of the activities made me enjoy this class, and gave me insight about math in a new way.
The activities were great. I like that the instructor continually gave us feedback on our journals.
Handouts were extremely useful.
Lesson plans for units were most helpful.
Invaluable lesson plans that I will most assuredly utilize.
I will use the activities for the duration of my career.
This class will definitely help me in the future.
Great content! I can actually use it in my own teaching.
Everything I learned I know I will use later on.
The activities could be used immediately with no modifications.
I was glad to take a class with no textbook.
The course built my confidence in my math ability.
I really learned so much about going into the classrooms and in the different ways teachers teach and children learn.
It was great getting to work outside of class with other teachers.

Conclusions

Teachers taking the courses have directly and successfully used materials presented in the university classes in their classrooms. From earlier programs we have been involved in, we know that giving teachers only written materials, or discussing them in a few workshops, is not sufficient for a program to succeed. Having a university course, and having both prospective and practicing teachers attend it and go through materials they will use with children, is the key.

Impact on the School District

Data about the District

The Las Cruces School District enrolls approximately 16000 elementary and middle school students in 29 schools. It employs 692 elementary teachers and 333 middle school teachers. The elementary teachers are generalists and teach all subjects. Middle school teachers are at least partially specialists. The District is multi-ethnic and bilingual (Spanish and English).

Teachers taking our university courses

Since 1995, the first two courses of the series have been offered 10 times, and the last two have been offered 5 times. During this period 148 different teachers (109 of them elementary) have taken the courses, with an average enrollment of 14 teachers per course. Teachers do not have to take the lower level courses in order to take the more advanced ones.   So many of them just take the advanced ones.

Here are the data for five years:

Number of courses taken:	1	2	3	4
Number of teachers:	104	32	6	6

This gives approximately 3.5 teachers per school who have taken at least one course. But the distribution is not uniform and the number of teachers in one school varies from 0 to 8.

Ending teachers' professional isolation

Finishing a university course does not end teachers' professional development. Contact with the university is sustained in three different ways.

- Some teachers volunteer to mentor university students who take the same class in following semesters.
- A yearly summer workshop and "reunion" for all participants is organized by the School District.
- Teachers take an active part in preparing and running a yearly spring Mathematics Education Institute for college mathematics instructors of courses for teachers, organized by the Department of Mathematical Sciences and the College of Education. The fourth annual Institute is planned for four days in spring 2001. In the three previous Institutes, about 75 university instructors from New Mexico, other parts of the United States, and Central America have come to Las Cruces to see the partnership program in action. Teachers, including those who are not currently taking a partnership course, allow Institute attendees to visit their classes and see units from the courses presented to their pupils.

Conclusion

The impact on the School District is already visible and significant. Approximately 16% of the elementary teachers have taken at least one university course from the series, and they keep contact with each other and with the university. Many plan to take additional university courses. So this program seems to bring sustained professional development.

Impact on other courses for teachers

In contrast, the impact on other mathematics courses for teachers that are offered at the college level is very small. Most courses are taught by instructors with master's degrees in mathematics, who have a heavy teaching load. Often they are not prepared to teach more advanced topics, and they are unwilling to teach a class in a laboratory format, because it requires much more time to prepare. For the regular faculty these courses are not attractive because they carry little prestige, and they are difficult to teach because very few mathematicians have any experience in elementary education. Finally, on the national level there is very little consensus about almost anything involving mathematics in schools. All opinions about the content and methods of school mathematics are challenged.

Plans for the future and general conclusions

We are publishing a third book of classroom materials sufficient for a one-semester university course focusing on algebra and geometry (Baggett & Ehrenfeucht, in press). In fall 2000 we are offering a fifth course in the series. It combines mathematics and science (mainly physics) and again will be taught in a lab format in a high technology setting.

We already know that average pupils in elementary schools can understand and master mathematics that is more advanced than is usually taught to them, providing that their teachers have a good knowledge of these topics. (See also Butterworth, 1999; Dehaene, 1997; Wynn, 1995; Gelman & Gallistel, 1986.) But we still do not have sufficient data from kindergarten classes, and from middle school algebra classes.

Our current estimate is that elementary teachers need at least four semesters (and preferably more) of college courses in mathematics to be adequately prepared to teach this subject. And what courses in mathematics they take is of crucial importance. We think courses that cover material related to topics taught in schools should have priority. We also think this can best be achieved if teachers' subject matter education doesn't end in college, but when they continue to take university courses throughout their careers as teachers.

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