THREE ESSAYS ON ECONOMIC CONDITIONS FOR DEMOCRATIZATION

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Three Essays on Economic Conditions for Democratization

Thesis directed by Professor Martin Boileau and Professor Anna Rubinchik

I investigate under what economic conditions democratization would be adopted in oligarchic societies in the absence of social conflict and political competition. In each essay, I construct a general equilibrium model of a delegation game and characterize the conditions in terms of economic primitives. I show that democratization occurs when there is a net positive spillover to the elite's payoff.

The benchmark delegation game considers the oligarchic regime and the democratic regime. At the status quo, the elite are entrusted with the political power over a tax policy. They decide either to sustain the power under the oligarchic regime or to delegate the power to people so that democratization occurs and the society goes under the democratic regime.

The first essay considers an environment with education. Under the oligarchic regime, the elite appropriate people's labor income. The tax decreases people's incentive to receive education and to work. The economy grows poor, and the tax revenue may be limited. Under the democratic regime, people remove the tax distortion. They pursue education and work hard, which increase the elite's capital return. Democratization would be adopted if the elite receive sufficient capital return under the democratic regime, or collect small labor income tax revenue under the oligarchic regime.

The second essay considers the presence of home production and non-exclusive public goods. Under the oligarchic regime, home production weakens the elite's appropriation on people's labor income received from market production. Under the democratic regime, people impose a capital gain tax on the elite to finance for non-exclusive public goods. Democratization would be adopted if the elite impose a small labor income tax under the oligarchic regime, or people provide abundant labor supply and impose a reasonable capital gain tax on the elite under the democratic regime.

The third essay considers a two-period economy in the absence of commitment. Under the oligarchic regime, the elite impose a savings tax on people after observing their savings behavior. When people save, the elite expropriate their saving. Under the democratic regime, the tax distortion is removed and the efficiency of the economy is improved. Democratization would be adopted if the prosperous economy increases the elite's purchasing power.

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CHAPTER 1

Introduction

I investigate under what economic conditions democratization would be adopted in oligarchic societies in the absence of conflict and political competition. In each essay, I construct a general equilibrium model and characterize the conditions for democratization in terms of economic primitives. I show that democratization occurs when there is a net positive spillover to the social elite's payoff.

Conflict is a major explanation for democratization in literature. It can be external, such as threat of revolution, or be internal, such as conflict of interest among the political parties or within the social elite. The external conflict leads to involuntary delegation of power, which is driven by the elite's strategic decisions to avoid social unrest (Acemoglu and Robinson 2000, 2001, Conley and Temimi 2001). The democratization of Britain, France, Germany and Sweden in the nineteenth century is this type. The external conflict may also lead to consolidation problem of regimes (Acemoglu and Robinson 2001). If people can initiate a revolution against non-democracy, the elite can likewise mount a coup against democracy. Political economy may be caught in a vicious circle. The unstable democracy in Latin America is the example. The internal conflict yet leads to voluntary delegation of power, which is based on majority voting rule. Consider a competition among groups with conflicting interests. A group can implement what they want only if they pool the majority of the votes. It provides incentives for the group to enfranchise the society so that they may obtain greater support from the newly-enfranchised people

(Lizzeri and Persico 2004, Llavador and Oxoby 2005, Jack and Lagunoff 2006). This argument explains the switch in political interest during the British age of reform and the enfranchisement associated with economic growth in Germany, Chile, Switzerland and Britain in the late nineteenth century.

Initiation and repression of social unrest in an authoritarian society are much more costly and devastating nowadays than foretime. Military is well-organized, weapons are ruinous, so destruction may be comprehensive. If the armed-conflict is initiated, the domestic in-humanistic behaviors agitate international condemnation and economic sanctions from other countries. The military deterrent may hold back people's intention to use force against the authority (e.g. USSR); the international pressure may discourage the rulers to use armed-force to resolve the conflict (e.g. Arab Spring); otherwise, the violence would damage the economy and jeopardize the society (e.g. African countries). In addition, most current non-democratic societies only have one dominant political party or single unshakable ruling authority (e.g. China and North Korea). Costly social unrest and impregnable authority make the probability of initiating a revolution negligible. The enfranchisement based on internal conflict of interest does not seem to be applicable to this situation either. Based on the above observations, I believe that nonbelligerent democratization deserves greater attention for the remaining non-democratic countries.

People's economic needs and political corruption are main causes of regime changes and alterations of political power. In the absence of conflict and political competition, I incorporate political corruption into economic environment. First, I construct a general equilibrium model with three types of economic agents: the consumers (the rich and the poor), the production firms, and the investment firms (for the dynamic model only). The rich are endowed with capital; the poor are the only labor suppliers. The production firms adopt a Cobb-Douglas production technology and use capital and effective labor to produce a final good. Second, I consider a delegation game with two political regimes: the oligarchic regime (the rich have the political power over the tax policy) and the democratic regime (the poor are granted the power). At the status quo, the rich are entrusted with political power over the tax policy. They can choose to sustain the oligarchic regime or to democratize society. If they sustain the oligarchic regime, they appropriate the poor's labor income (the poor's savings in the dynamic model) for personal uses. If they delegate the power to the poor, democratization occurs and the economy enters the democratic regime. Under the democratic regime, the poor decide the tax policy and how to utilize the tax revenue. When certain condition holds, unique general equilibrium exists under each regime and is characterized by economic primitives. Democratization is adopted if the rich are better off with delegation than retaining the oligarchic power.

The causality between economic growth and democracy has also been debated in literature. Lipset (1959), an adherent of Aristotle (350 B.C.), proposes that economic development is the necessary condition sustaining democracy. Barro (1999) and Papaioannou and Siourounis (2008) empirically support Lipset's hypothesis by OLS and (ordered) probit estimations. In the meantime, Robinson (2006) argues that these estimations ignore the endogeneity problem of income per capita (measure of economic growth) and simultaneous equation estimation ignores the presence of omitted variables correlated with income per capita. Hence Acemoglu et al. (2008) include fixed effects, apply IV for income per capita and use GMM estimations, but they find no evidence of a causal effect of income on democracy. On the other hand, a positive effect of democratization on economic growth is approved. Barro (1998) finds a non-linear relation between democracy and growth by 3SLS estimation with different IV: increase in political rights initially increases growth but tends to retard growth once a moderate level of democracy has been attained. Rodrik and Wacziarg (2003) show that major democratic transitions have positive effects on economic growth in the short-run, especially for the low-income countries and the ethnically diverse countries. Persson and Tabellini (2006) show that both democratization and economic liberalization promote growth; countries liberalizing economies before democratization have more significant growth. However, Przeworski and Limongi (1993) believe that political institutions matter to growth but regimes do not. Tavares and Wacziarg (2001) use simultaneous equation estimation and find that democracy may hinder growth by reducing the rate of physical capital accumulation. Even so, Persson and Tabellini (2009) find a bilateral reinforcement between economic development and democracy by innovating the concept of democratic capital.

The results of this dissertation make multiple contributions to literature. First, they shed light on the relationship between democracy and economic prosperity. Economic prosperity is the necessary condition for democratization (Lipset 1959); net positive spillover to the rich's payoff is the sufficient condition. Democratization is based on the pursuit of economic prosperity. Second, they characterize the economic reasons for democratization. Different from Acemoglu and Robinson (2000, 2001), the general equilibrium model endogenizes the cost and benefit of the economic agents' actions and the political decision (Persson and Tabellini 1994). Based on the economic primitives, these actions and decision are traceable. So is the likelihood of democratization. Third, the results in terms of economic primitives shed light on further empirical studies on democracy with econometric problems. Emergence of democracy in a country is an interactive result of political and socioeconomic factors, such as education, economic development, and protection of property right, while democracy may be also the causes of these factors. Econometric problems of endogeneity and omitted variables thus emerge (Barro 1998, 1999, Robinson 2006, Acemoglu et al. 2008). The conditions in terms of economic primitives may disseminate the ideas on the searches of appropriate IV and additional variables into the empirical studies.

Three essays in this dissertation approach the conditions for democratization from different angles and draw distinct results from one another. The model in Chapter 3 emphasizes that how the rich's appropriation on the poor's labor income affects the poor's choices of labor supply and education, which in turn affect the poor's labor productivity and the likelihood of democratization. The model in Chapter 4 considers home production, a tax haven to the poor, which mitigates the rich's appropriation on the poor's labor income under the oligarchic regime. If the rich delegate the power to the poor, the poor impose a capital gain tax on the rich instead and use the tax revenue to finance non-exclusive public goods. The likelihood of democratization may increase because of the decrease in the oligarchic gain and the provision of public goods. The model in Chapter 5 emphasizes on the effect of the absence of commitment on the economy, which in turn affects the likelihood of democratization.

At the beginning of each chapter, I will give a brief and specific introduction to the model. Please refer to appendix for the proofs of claims, lemmas, theorems and propositions.

CHAPTER 2

Review Of Literature

This chapter discusses a selection of literature, which is divided into five categories: commonly used data, determinants of democracy and economic growth, reasons for democratization, taxation, and others. Please refer to References for the complete list of literature.

2.1. Data

Polity IV project and *Freedom in the World* are the most commonly used data assessing levels of democracy nowadays. Polity projects evolved in 1970's. Polity IV is the latest version investigated by Marshal and Jaggers. It examines the qualities of authority in governing institutions, such as executive recruitment, constraints on executive authority and political competition. It codes these authority characteristics for 164 independent countries in the world from 1800 to 2010. The Polity scores range between -10 and 10. Higher scores indicate higher degree of democracy; otherwise, higher degree of autocracy. Based on these scores, each country can be categorized as autocracy (score of -10 to -6), anocracy (i.e. mixed or incoherent authority regime; score of -5 to 5) or democracy (score of 6 to 10).

Freedom in the World is an annual report published by Freedom House since 1972. It provides comparative and historical assessments of individuals' *political rights* and *civil liberties* for 195 countries and 14 disputed territories in the world. The scores of political rights measure how freely individuals are enabled to participate in political process. The assessments include if individuals have the right to vote for distinct alternatives in legitimate elections, compete for public office, join political parties and organizations, and elect representatives who have decisive impacts on public policies and are accountable to the electorate. The scores of civil liberties measure individuals' freedoms of expression and belief, associational and organizational rights, rule of law and personal autonomy without interference from the state. Each country and territory is rated by a scale of 1 to 7 for political rights and civil liberties respectively. Rating of 1 indicates the highest degree of freedom and 7 the lowest level of freedom. Based on these ratings, each country and territory is classified as "free", "partly free" or "not free".

Even though Polity IV and Freedom House data are popularly used in the literature, Bollen (1990) puts forward the problems of concept and measurement of these data sets. The conceptual problems include failing to develop an adequate theoretical definition, confounding with others, and treating democracy as a binary rather than a continuous concept. The problems of measurement include using invalid or subjective indicators, using ordinal or dichotomous measures, and failing to test reliability or validity. On the other hand, to improve the measurement, he suggests developing operational definitions for each dimension of political democracy and then following the six conventional standards of measurement (Bollen 1989, Chapter 6).

Education is another popular data in the literature. Since Mincer (1974), education has been the most commonly used measure of human capital. As a result, the quality of data is important. The educational attainment data of Barro and Lee (1993, 1996) have been widely adopted for empirical research in the past decades. They update the data in

Barro and Lee (2000) and improve the measurement of educational attainment for a broad group of countries. The up-to-date data have two advantages. First, the fill-in procedure for missing census/survey observations uses gross enrollment rates which are adjusted for the repeaters. Second, the changes of school duration over time within countries are taken into account in the construction of average years of schooling. However, the improved measurement still cannot directly assess the human skills obtained at schools, take account of the skills and experience gained after formal education, and distinguish the differences in the quality of schooling across countries. For readers' reference, they suggest multiple alternative international measures for human capital stocks, such as international test scores, international adult literacy test, estimates of market values of human capital, and OECD estimates of educational attainment.

2.2. Determinants of democracy

2.2.1. Education and protection of property rights to democracy

Based on the educational attainment data, Barro (1999) empirically tests the effect of education on democracy (the indicators of political rights and civil liberties by Freedom House) and finds a significantly positive effect. See section 2.2.2 for the details of this paper.

Glaeser et al. (2007) propose a theoretical mechanism to explain how education increases the stability of democracy and the probability of transition to democracy. The critical assumption is that education increases the benefit of civic political participation. In the battle between democracy and dictatorship, democracy needs a broad base of support from citizens but offers them weak incentives; on the contrary, dictatorship offers strong incentives to a narrow base of supporters. Education therefore increases the participation in support of the broad-based regime (democracy) relative to the participation in support of the narrow-based regime (dictatorship). This also increases the likelihood of successful democratic revolutions against dictatorships and decreases the likelihood of successful anti-democratic coups, so that democracy is more likely to be preserved.

Papaioannou and Siourounis (2008) construct a new data set of political regimes and transitions for the third wave of democratization and the 90's. They reference many political indicators (Freedom House, Polity, Przeworski et al. 1996 and 2000), electoral and political archives (e.g. Carr's Psephos election archive) and historical sources (e.g. Freedom House and Polity Project country reports) to classify countries into different groups. They focus on the countries which entered the third wave as non-democratic and examine which economic or social factors determine the democracies. They estimate the following cross-sectional probit model with maximum likelihood: $P\left(D_i = 1 \middle| X_{i,1975}, Z_i\right) = G\left(a + x_{i,1975}\beta_1 + z_i\beta_2\right)$, where $P\left(\cdot|\cdot\right)$ is the probability that initially non-democratic country i will experience a successful democratic transition D_i between 1975 and 2000; $G\left(\cdot\right)$ is a non-linear c.d.f. function of initial time-varying factors $X_{i,1975}$ (e.g. education) and time-invariant characteristics Z_i (e.g. religion). They also estimate two ordered probit models to test the intensity and the timing of democratic reforms. They find that: (1) democratization is more likely to occur and consolidate in developed and educated societies; (2) education is a significant predictor of the intensity and the timing of political reforms; (3) the effect of education on predicting democratic transitions retains when controlling for religion, fractionalization, trade openness, historical factors and proxy measures of colonial institutions; (4) religion and political institutions around independence are important drivers for the third wave; (5) trade openness and fragmentation do not significantly correlate with democratization.

Acemoglu et al. (2005) point out two problems of existing literature that looking for correlation between education and democracy. The first problem is the ignorance of within variation. They find that a given country is not likely to become more democratic as its population becomes more educated. That is, when the country fixed effects are included in the regression, the cross-sectional relationship between schooling and democracy (Barro 1998, 1999) disappears. The second problem is the omission of time effects in the time-series variation of Glaeser et al. (2004). When the year dummies are included in their regression, the impact of education on democracy disappears. Their regression is: $d_{it} = \alpha d_{i,t-1} + \gamma s_{i,t-1} + \mu_t + \delta_i + u_{it}$, where $s_{i,t-1}$ is the lagged value of average years of schooling (Barro and Lee 2000); μ_t's are the full set of time effects; δ_i's are the full set of country dummies. Their results suggest that the cross-sectional relationship in existing literature is driven by some omitted factors which influence both education and democracy but not a causal relationship. However, they cannot rule out the possibility of a very long-run causal relationship between education and democracy.

Property rights enforcement is considered an important element for institutions. Acemoglu (2008) develops a theoretical model in which protecting the incumbent producers' property rights comes at the cost of weakening the potential producers' economic opportunities. He considers two societies/regimes: the oligarchic society with well-protected property rights for the incumbents and monopolistic entry barriers for the new agents, and the democratic society with high taxes on the incumbents but low entry barriers for new agents. He shows that the oligarchic society generates greater efficiency at the beginning because, first, the incumbents have comparative advantage in production, and second, oligarchy avoids the distortionary taxation. But when the comparative advantage of entrepreneurship shifts away from the incumbents to the new agents over time, the allocation of resources in oligarchy worsens. On the contrary, although democracy creates distortionary taxation, the distortion does not go worse over time. Therefore, the oligarchic society would prosper first but decline later relative to the democratic society. Based on this framework, he also studies endogenous regime transitions. Voluntary transition from oligarchy to democracy occurs when there are conflicts among the elites. Under certain conditions, the low-skilled elites do not find entrepreneurship sufficiently profitable. They may therefore choose to end the oligarchic regime when they become the majority within the elites.

2.2.2. Economic growth to democracy

Lipset (1959), regarded as an adherent of Aristotle (350 B.C.), defines democracy and divides countries into "more" and "less" democratic groups. Based on the definition and the classification, he considers economic development and political legitimacy on the problem of stable democracy. The economics development complex comprises industrialization, wealth, urbanization and education. He argues that economic development is the necessary condition sustaining democracy (Lipset Hypothesis) and political legitimacy is the sufficient condition for stability of democracy. Compared to the variations in the systems of government, economic development and political legitimacy are more important requisites for democracy. Lipset (1959) lays the foundation for the wide range of latter-day research on democracy.

Barro (1999) formally assesses the hypothesis of Lipset/Aristotle: a higher standard of living promotes democracy. He studies a panel data over 100 countries from 1960 to 1995 with seemingly unrelated regression (SUR) method. The system of regression is: $Demo_{it} = a_0 + a_1 Demo_{i,t-T} + a_2 Demo_{i,t-2T} + a_3 \mathbf{Z}_{i,t-T} + u_{it}$, where $Demo$ is the Freedom House indicator for democracy (political rights or civil liberties); $\mathbf{Z}$ is a vector of variables which influence the extent of democracy (log of GDP per capita, primary school attainment, the gap between male and female primary schooling, urbanization rate, log of population and oil country dummy). He finds that democracy is positively related to GDP per capita and primary school attainment; negatively related to urbanization and reliance on natural resources; little related to country size. In addition, he introduces additional variables (health indicators, upper-level schooling, inequality of income and schooling, ethnolinguistic fractionalization, the rule of law, colonial history and religion) which enter one set at a time into the system of regression. Holding the standard of living constant, democracy is positively related to the middle-class share of income; little related to colonial heritage and religious affiliation. However, negative effects of Muslin and non-religious affiliations on democracy remain the same.

As discussed in section 2.2.1., Papaioannou and Siourounis (2008) and Persson and Tabellini (2009) (which will be discussed in section 2.3.3.) are also advocates of growth on democracy. However, questioning voices arise at the same time. Robinson (2006) reviews the literature on the causality from economic development to democracy. He introduces a simple model from Acemoglu and Robinson (2000, 2001, 2006) to explain the adoption of democracy: the elites more likely democratize if there is sufficient social unrest in the non-democratic regime and the elites' anticipation costs of democracy are limited. He argues that the current empirical works on the causal effect of development (measured by income per capita) on democracy have econometric flaws. For instances, OLS or dynamic probit ignores the endogeniety problem of income per capita, while simultaneous equations of development and democracy ignore the presence of omitted variables correlated with income per capita. He continues to argue that Acemoglu et al. (2005) proposes a convincing instrumental variable strategy for the issue of endogeniety. They use two potential instruments for income per capita: (1) past savings rates: they affect income per capita but should not have direct effect on democracy; (2) changes in incomes of trading partners: they create a matrix of trade shares and use the trade-share-weighted average income of other countries to construct predicted income for each country. Both instrumental-variables strategies show no evidence of a causal effect of income on democracy.

After controlling the country fixed effects or applying instrumental-variable estimations, Acemoglu et al. (2008) show that the positive effect of income per capital on democracy is removed. Their benchmark econometric model is: $d_{i,t} = \alpha d_{i,t-1} + \gamma y_{i,t-1} + \mathbf{x}'_{i,t-1}\boldsymbol{\beta} + \mu_t + \delta_i + u_{i,t}$, where $d_{i,t}$ is the democracy score of country i in period t (Freedom House and Polity IV); $y_{i,t-1}$ is the lagged value of log GDP per capita; μ_t and δ_i control the fixed time and country effects. They estimate the interested parameter γ by OLS with fixed effects, Anderson-Hsiao IV (1982, $x_{i,t-2}$ instruments for $\triangle x_{i,t-1}$ where $x \in \{d, y\}$) and Arellano-Bond GMM (1991) for various frequencies of data. For 2SLS with fixed effect estimations, they use savings rate and trade-weighted world income as instruments. All estimation results show no significantly positive effect of income on democracy. To

education	pro	Barro 1999 Glaeser, Ponzetto and Shleifer 2007 Papaioannou and Siourounis 2008
	con	Acemoglu, Johnson, Robinson and Yared 2005
property rights	pro	Acemoglu 2008
economic growth	pro	Aristotle 350 B.C. Lipset 1959 Barro 1999 Papaioannou and Siourounis 2008 Persson and Tabellini 2009
	con	Robinson 2006 Acemoglu, Johnson, Robinson and Yared 2008

Table 2.1. Factors promote democracy

explain the positive cross-country correlation between income and democracy, they propose the ideas of *divergent political-economic development paths* (i.e. some countries have embarked on a development path associated with democracy and economic growth, while others have pursued a path based on dictatorship, repression and limited growth.) and *critical junctures* (i.e. historical variations). They statistically show that the contexture of these ideas is responsible for the association between long-run economic and political changes in the literature. Nevertheless, they make the following comments necessary in interpreting their results: (1) the causal effect of income on democracy might be conditional on some other characteristics; (2) their results do not imply that democracy has no effect on growth; (3) their results do not arbitrarily suggest that there is a historical determinism in political institutions.

Table 2.1 summarizes the discussions of this section.

2.3. Determinants of economic growth

2.3.1. Education to growth

Besides the effect of democracy on growth (which will be discussed in section ?), Barro (1998, Ch.2) studies the effects of various determinants on growth. For a given starting level of real GDP per capita, he finds that growth rate increases with higher initial schooling, higher life expectancy, lower fertility, lower government consumption, better maintenance of the rule of law, lower inflation and improvements in the terms of trade. However, holding other variables constant, growth is negatively related to the initial level of real GDP per capita. Together with Barro (1999), Barro (1998, 1999) sheds light on the further empirical research on democracy and economic growth.

Glaeser et al. (2004) study the effect of institutions and human capital on growth. The benchmark regression is: $GDPG_{it} = \alpha_0 + \alpha_1 EDU_{it} + \alpha_2 INST_{it} + \alpha_3 \mathbf{Z}_{it} + u_{it}$, where $GDPG$ is growth of GDP per capita; EDU is log years of schooling (Barro and Lee 2000); $INST$ are alternative measurements of institutions, such as executive constraints, expropriation risk, autocracy, government effectiveness, judicial independence, constitutional review, plurality and proportional representation. By cross-sectional OLS estimation, they find that human capital is a more basic source for growth than institutions. Another interesting finding is that poor countries under dictatorship can accumulate both physical and human capital more quickly so as to emerge from poverty. Once they become richer, they are more likely to improve their political institutions.

2.3.2. Income equality to growth

Alesina and Rodrik (1994) study that in democracies how politics affect economic growth in an endogenous growth model with distributive conflict among the agents endowed with different capital-labor shares: $\sigma^i = \frac{l^i/l}{k^i/k}$, where (l, k) are the aggregate labor and capital respectively and the superscript indicates individual i. Suppose that the government imposes a linear tax on capital to finance for a public service which increases the productivity of private production and the returns of factors. At steady state, the growth rate on one hand is correlated with the tax on capital: given that the marginal return of capital diminishes in the tax, the growth rate first increases and then decreases as the tax increases. On the other hand, they show that at the steady state the less capital-abundant individuals prefer higher taxes on capital and the growth-maximizing tax is the ideal tax of the pure capitalists ($\sigma^i = 0$). Consider a perfectly egalitarian society where each individual has $\sigma^i = 1$ and denote $\sigma^m - 1$ to be the relevant indicator of inequality, where m stands for the median voter. By *Median Voter Theorem*, they show that the greater inequality, the higher rate of taxation, and the lower growth. In addition to the theoretical model, they present the empirical results that inequalities in income (Gini coefficients from Jain 1975 and Fields 1989) and land (Gini coefficients from Taylor and Hudson 1972) significantly retard growth (average per capita growth rate over the sample periods). This result is consistent with the findings of Persson and Tabellini (1994) and Clarke (1995).

Persson and Tabellini (1994) formulate a general-equilibrium overlapping-generation model which combines the insights of endogenous growth theory and endogenous policy theory. They suggest that inequality is harmful for growth in democracies but this relationship is not significantly present in non-democracies. The intuition is as follows. Tax policies and regulatory policies determine how much an individual can privately appropriate the fruits of his effort (so that the rest of the fruits will be redistributed to others). The ability of individuals to privately appropriate determines their incentives to accumulate capital: the higher ability to appropriate (the fewer fruits to be redistributed), the higher incentives to accumulate. The accumulation of capital then determines economic growth. Suppose that such distributional conflict is significant in a society. The political decisions are more likely to result in the policies which allow less private appropriation; in turn less capital accumulation and less growth. Note that distributional conflicts are more significant in democracies than in non-democracies. Therefore, the framework and the result above should be more prominent in democracies than in non-democracies. Take the model to the data. The authors empirically test if income inequality has significantly negative effect on growth and check the sign and the significance of this effect in democratic and non-democratic countries respectively.

Perotti (1996) surveys four main approaches in the literature on income distribution and economic growth. They all derive a negative effect of income inequality on growth. The main measure of equality is the combined share of the third and fourth income quintiles. The income inequality is measured by the sizes of the middle class, which is the combined share of the third and fourth income quintiles, from household surveys. Higher the size of the middle class is, lower income inequality is. The growth data is the average rate of growth of income per capital between 1960 and 1985. The first approach is endogenous fiscal policy (Alesina and Rodrik 1994, Persson and Tabellini

1994): it occurs because inequality drives up demand for redistribution which in turn increases distortionary taxation. The second approach is sociopolitical instability (Alesina and Perotti 1996): it is because inequality increases sociopolitical instability, which decreases incentives to invest. The third approach is borrowing constraints and investment in education (Galor and Zeira 1993): given a degree of imperfection in the capital market, increase in inequality decreases investment in human capital which in turn decreases growth. The fourth approach is the joint education/fertility decision (Galor and Zang 1993): increase in inequality increases fertility but decreases investment in human capital, so growth decreases. Perotti empirically tests these four approaches with structural models. He concludes that only the mechanisms of sociopolitical instability and the joint education/fertility decision have strong empirical supports but other two approaches do not.

2.3.3. Democracy to growth

Barro (1998, Ch.2) studies the effect of democracy (indicator of political rights by Freedom House) on economic growth (growth rate of real GDP per capita). He studies a cross-country data from 1960 to 1990 by three-stage least square estimation (with different instrumental variables). He finds a non-linear relation between democracy and growth: increase in political rights initially increases growth but tends to retard growth once a moderate level of democracy has been attained. But he warns that this result cannot conclude that more or less democracy is a critical requisite for growth. Besides democracy, he also studies the effects of other determinants on growth. For a given starting level of real GDP per capita, he finds that growth rate increases with higher initial schooling, higher life expectancy, lower fertility, lower government consumption, better maintenance of the rule of law, lower inflation and improvements in the terms of trade. However, holding other variables constant, growth is negatively related to the initial level of real GDP per capita. Together with Barro (1999), Barro (1998, 1999) sheds light on the further empirical research on democracy and economic growth.

Rodrik and Wacziarg (2005) show major democratic transitions have positive effects on economic growth in the short-run and it is especially true for the low-income countries and the ethnically diverse countries. Different from the previous works addressing long-run relationships with cross-national regression techniques, Rodrik and Wacziarg use an annual data to examine the within-country effects of democratization on growth. They use panel data techniques, include time and country fixed effects and control for other types of regime transitions. The dependent variable is growth of per capita real GDP (Penn World Tables, version 6.1); the explanatory variables are dummy variables describing regime transitions (Polity IV 2002). The major explanatory variables are: (1) *new democracy*: coded as 1 in the year(s) and subsequent five years of any major democratization, unless it is interrupted by another major regime change, in which case the dummy is coded as 1 until the interruption; (2) *established democracy*: coded as 1 for the years following the first five years of major democratic transitions; (3) *democratic transition* (the sum of the previous two dummies): takes on a value of 1 in all years following a major democratization episode until it is interrupted by another major regime change; (4) define *new autocracy*, *established autocracy* and *autocratic transition* for the regime changes of autocracy. The full sample results show that the estimated coefficient of *new democracy* is significantly positive. It also holds in the sub-sample results for low-income countries, ethnically diverse countries and Sub-Saharan African countries. In addition, they compare the growth experiences, at least nine years before and after democratization, of 24 democratizing countries. They find that democratizations tend to follow the periods of low growth rather than precede them and the volatility of growth tends to be lower in democracies.

Persson and Tabellini (2006) examine three instances that the details of democratic reforms influence the economic effects. They estimate the following panel regression by difference in differences: $y_{i,t} - y_{i,t-1} = \beta y_{i,t-1} + \phi D_{i,t} + \boldsymbol{\rho}\mathbf{x}_{i,t} + \alpha_i + \theta_t + \varepsilon_{i,t}$, where $y_{i,t}$ is (log) per capita income of country i in year t (Penn World Tables for 1960 to 2000 and the Maddison data set for 1850 to 2000), $D_{i,t}$ is a dummy variable equal to one under democracy (if the polity2 variable in Polity IV is strictly positive), $\mathbf{x}_{i,t}$ is a vector of control variables (e.g. binary indicator for years after 1989 in the former Soviet bloc, current and lagged indicators for years of war, dummy variables for continental location and socialist legal origin interacted with year dummy variables) and α_i and θ_t are country and year fixed effects. They decompose the average growth effect of political reform according to observable (dummy) features one at a time: (1) democracy, economic liberalization (Sachs and Werner 1995), democratization after liberalization, liberalization after democratization; (2) democracy, parliamentary democracy, proportional rule; (3) hazard rate out of current regime (Persson and Tabellini 2005), democracy, probability of autocracy, probability of autocracy in lagged democracy. The results with the first set of features show that both democratizations and economic liberalizations promote growth and countries liberalizing economies before extending political rights have more significant growth. The second set of features show that presidential democracy leads to faster growth than parliamentary democracy. If regressing government spending and liberalization respectively on the same set of explanatory variables, parliamentary democracy and proportional rule lead to greater government spending and less protective trade policies. The third set of features show that if taking expectations of regime change into account, democracy has stronger effect on growth.

Persson and Tabellini (2009) further propose a bilateral reinforcement between economic development and democracy. They innovate the idea of *democratic capital* to the studies of dynamics of economic and political change. Democratic capital is measured by a country's historical experience with democracy and the incidence of democracy in its neighborhood. They construct an overlapping-generations model, in which democratic capital indirectly affect capital accumulation through the probability of autocracy. In equilibrium, countries with different production efficiency (i.e. total factor productivity) endogenously sort themselves into political regimes. Based on the theoretical model, they estimate the following econometric specification by maximum likelihood estimation: $h_{i,t}^{a} = H^{a}\left(z\left(\delta\right)_{i,t-1}, f\left(\rho\right)_{i,t-1}, y_{i,t-1}, \mathbf{x}_{i,t}\right) + \psi_{i,t}$, where $h_{i,t}^{a}$ is country j's risk of exit from the regime at time t (i.e. hazard rate); a is the regime type in year $t-1$: $a = 0$ if democracy; $a = 1$ if autocracy; $z\left(\delta\right)_{i,t-1}$ is domestic democratic capital which depreciates at the rate $1-\delta$; $f\left(\rho\right)_{i,t-1}$ is democratic capital based on political conditions abroad; y is GDP; $\mathbf{x}_{i,t}$ is a number of fixed and time effects; $\psi_{i,t}$ is an error term. In addition, they estimate a regression model, which is similar to Persson and Tabellini (2006), to study the effect of the expected regime on growth by OLS: $y_{i,t} - y_{i,t-1} = \gamma^{a}\widehat{p}_{i,t}^{a} + \beta y_{i,t-1} + \boldsymbol{\sigma}\mathbf{x}_{i,t} + \omega z_{i,t} + \alpha_{i} + \varphi_{t} + \varepsilon_{i,t}$, where $\widehat{p}_{i,t}^{a}$ is the estimated $h_{i,t}^{a}$ in the previous specification.

Their results suggest a virtuous circle. Democratic capital favors economic development through physical capital accumulation, which helps consolidate democracy. In turn, this leads to the accumulation of more democratic capital, with additional positive effects on income and democratic stability. Three asymmetries are found across political regimes: (1) higher income makes democracies more stable, but does not make autocracies more precarious; (2) the probability of switching from democracy to autocracy hurts growth, but the probability of remaining in autocracy has no significant effect on growth; (3) the positive influence of democratic capital on growth is due to democracies, not to autocracies.

Przeworski and Limongi (1993) are opposed to the positive effect of democracy on economic growth. They believe that political institutions matter to growth, but regimes do not. They summarize the literature discussing the effect of regime on growth and point out the problems of these views as follows. (1) Democracy undermines property rights, but many democratic and capitalistic countries of today show this argument is too strong. (2) Property rights promote growth (North 1990), but what is the link between property rights and democracy? Can democratic institutions make a credible commitment of property rights to people? (3) Autocrats cannot credibly commit themselves (Olson 1991), but how could democratic institutions make a credible commitment? (4) States are treated as the only source of potential threat in literature, but one the other hand, strong states are required to protect property from private encroachments. (5) Individuals have dual identities, economic agents and citizens, so the resources are allocated via market mechanism and state mechanism. But the allocations decided by the market may not coincide with those by the state. Democracy may exacerbate this divergence, such that the poor suffering the consequence of private property may be more likely to expropriate the rich. In this case, using democracy as a proxy in econometric studies is not justifiable. As a result, democracy may promote growth but not via property rights. (6) Democracy does not promote growth because democracy generates an explosion of demands for current consumption so undermines investment for the future. Dictators on the contrary are more future-oriented, but the authors argue that the literature does not explicitly address why benevolent dictators would be future-oriented. (7) Democracy does not promote growth because dictatorship (e.g. state autonomy) insulates the state from particularistic pressure. But the literature does not explain why an autonomous state would behave in the interests of anyone else. (8) Democracy promotes growth because democratic rulers are less predatory than autonomous rulers. But these models over-embellish the democratic process, such as perfect information among voters, perfect competition among parties and perfect agency. (9) Statistic inferences based on standard regression models are invalid because they ignore the econometric problems of simultaneity, endogeniety and sample selection. Lastly, they argue again there is no credible commitment in political economy. Even if pre-commitment is possible, it is not optimal.

Tavares and Wacziarg (2001) introduce a new methodology to examine the empirical relationship between democracy and economic growth. Assume that democratic institutions affect growth through the following channels: political instability (measured by number of revolutions and coups per year), government-induced distortions (measured by black market premium of exchange rate), government size (measured by share of government consumption in GDP), human capital (measured by average years of secondary and higher education in population over age 25), income inequality (measured by Gini

education	pro	Barro 1998 Glaeser, La Porta, Lopez-De-Silanes and Shleifer 2004
income inequality	con	Alesina and Rodrik 1994 Persson and Tabellini 1994 Perotti 1996
democracy	pro	Barro 1998 Acemoglu, Johnson and Robinson 2001, 2002 Rodrik and Wacziarg 2005 Persson and Tabellini 2006, 2009
	con	Przeworski and Limongi 1993 Tavares and Wacziarg 2001

Table 2.2. Factors promote economic growth

coefficient), trade openness (measure by share of trade in GDP), physical capital accumulation (measured by share of investment in GDP). Growth is measured by growth rate of PPP; democracy is mainly measured by Freedom House data. Based on the assumption, they formulate a system of simultaneous equations connecting democracy and growth via these channels. They find that the overall effect of democracy on growth is negative and moderate. Decompose the total effect by channels. They find that: (1) democracy promotes growth by improving the accumulations of human capital and by lowering income inequality (less robust); (2) democracy hinders growth by reducing the rate of physical capital accumulation and by increasing government consumption (less robust). The largest off-set effect is the low rate of physical capital accumulation.

Table 2.2 summarizes the discussions of this section.

2.4. Why democratization happens

2.4.1. Involuntary democratization

Acemoglu and Robinson (2000) present a model to explain why the elites during the 19th century in the West may want to democratize societies although they have known that democracy will bring higher taxation. The authors believe that these democratizations are involuntary and driven by the elites' strategic decisions to prevent revolutions. Democratization occurs because, when the promise made by the elites in power for future transfer is not credible to the people but increases the threat of social unrest, democratization acts as a commitment for future transfer and becomes the elites' last bargaining counter to avoid revolutions. In addition, the authors apply the model to explain the potential link between democratization and Kuznets curve (figure 2.1). Increase in inequality is often associated with industrialization (part A). It also increases the threat of social unrest so that the elites are forced to democratize (summit S). Democratization ensures future redistributions, which are given back to the people in terms of welfare state and mass education. Income per capital therefore increases but inequality is reduced (part B). The authors also compare their theory to three alternative theories: the enlightenment (elites change their social values), political party competition within elites (one fraction of the enfranchised brings new groups into the political system to increase its support) and middle class drive (to shift the future balance of power). They believe that their theory provides a better interpretation for Britain, France, Germany and Sweden where the threat of revolutions was the major factor in democratization.

Based on the model in Acemoglu and Robinson (2000), Acemoglu and Robinson (2001) continue to study political transition and consolidation of democracy. In the modified model, the people have incentives to initiate a revolution in non-democracy; the elites may be also motivated to mount a coup in democracy. Four conclusions are drawn. First, inequality is the crucial determinant for political instability. It is because inequality encourages the elites to contest power in democracy (to avoid heavy taxes decided by

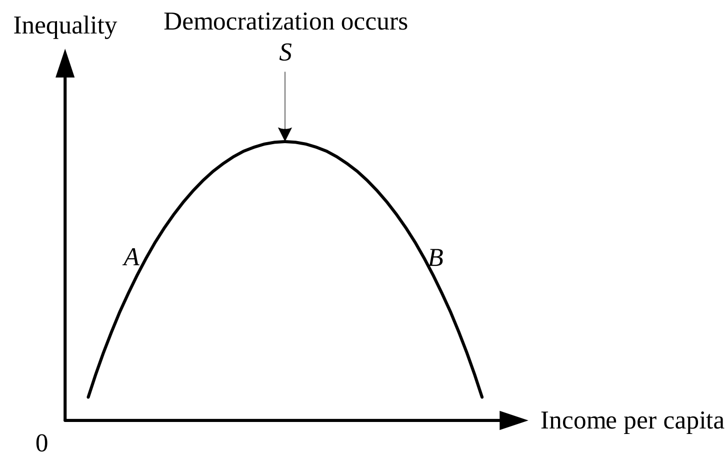


Figure 2.1. Application of Kuznets Curve in Acemoglu and Robinson (2000)

the people) and incites the people to social unrest in non-democracy (Acemoglu and Robinson 2000). Therefore, democracy is more likely to be consolidated if the level of inequality is limited. Second, there is a non-monotonic relationship between inequality and redistribution. When the inequality is sufficiently small, democracy is consolidated and redistribution is small. As inequality increases, redistribution increases at first but democracy becomes less and less consolidated so that the people are forced to reduce redistribution to avoid a coup. Third, higher inequality leads to higher fiscal volatility. It is more apparent if revolutions and coups are both less costly (e.g. Latin America). Fourth, democracy is less likely to be consolidated if redistribution is costly to the elites. Lastly, the authors suggest that asset redistributions may be able to stabilize both democratic and non-democratic regimes. However, radical asset redistributions, such as land reforms, may lead to destabilization because the elites may excuse themselves for a coup to avoid these reforms.

Acemoglu (2003) discusses three approaches to explain why inefficient policies and institutions are prevalent. The first two approaches are based on Coase Theorem: if property rights are well-defined and there are no transaction costs, economic agents will contract to achieve an efficient outcome, irrespective of who have the property rights on particular assets. Apply this theorem to political economy. The first application is Political Coase Theorem (PCT). Societies can make efficient choices for various groups and individuals regardless who or which social group has political power. If inefficient policies are chosen, political or social forces will push them back to efficient policies. The second application is "theories of belief differences," or modified PCT. Societies choose inefficient policies because the wrong beliefs of people in power. By the modified PCT, political or social forces will prevent the implementation of these inefficient policies. However, Acemoglu argues that the PCTs have overlooked the lack of political or social enforcement to push the societies back to the path of efficiency. Inefficient policies and institutions are chosen because they serve the interests of politicians or social elite groups which hold political power at the expense of others. In other words, there are interest conflicts between these two groups. Based on this observation, Acemoglu proposes an alternative approach: "theories of social conflict" (Acemoglu and Robinson 2000, 2001). These theories highlight the commitment problems in politics which in turn will undermine the potential to achieve efficient outcomes. He also argues that these theories are a more appropriate framework than the PCTs in the analysis of political and social inefficiency.

Conley and Temimi (2001) propose a model of endogenous enfranchisement with two features. First, it considers the affinities between voters. Consider two groups: the group in power and the un-enfranchised group. Individuals within each group have correlated interests. Suppose that policy proposals are randomly generated. If a proposal is passed, it produces non-exclusive benefits to each individual. Two groups' preferences over the policy proposal are negatively correlated. Second, this paper considers threat costs and initiation costs. The threat cost is imposed by the un-enfranchised group on the group in power if enfranchisement is not adopted (e.g. the associated cost of social unrest). The threat is credible because the un-enfranchised group pre-commits themselves to the initiation cost before the group in power make the enfranchisement decision. Based on these two features, Conley and Temimi show that in equilibrium (1) if the un-enfranchised group is sufficiently weak, the franchise is not granted and independent of the degree of preference conflict between two groups; (2) if the un-enfranchised group is sufficiently strong, enfranchisement depends on the degree of preference conflict but there is no monotonic relationship between these two; (3) if the un-enfranchised group is very strong, the franchise is granted and independent of the degree of preference conflict; (4) if the threat technology exhibits increasing returns to scale, the likelihood of enfranchisement increases in the degree of preference conflict. Similar to Acemoglu and Robinson (2000, 2001), these results also imply that the group in power uses enfranchisement to placate the un-enfranchised group.

2.4.2. Voluntary democratization

Lizzeri and Persico (2004) propose a voluntary franchise extension which is based on divisions within the elite. Consider a continuum of citizens of measure one. They are divided into N groups indexed by $i \in \{0, 1, ..., N\}$. Each citizen in group i is endowed with ω_i; a 100% non-distortionary tax is imposed. The first $s < N$ groups are enfranchised; the rest groups are disenfranchised. Different groups of citizens have heterogeneous preferences (or ideology) over redistribution and non-exclusive public goods. Citizens with

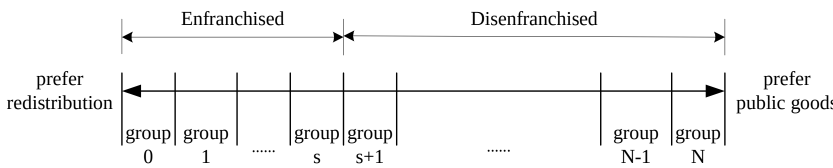


Figure 2.2. Citizens in Lizzeri and Persico (2004)

lower i prefer redistribution to public goods while citizens with higher i prefer public goods to redistribution. See figure 2.2. Suppose that two parties simultaneously compete for being elected to determine the provision of public goods and redistribution among the voters. In a repeated game, franchise expansion may occur either by referendum or by inclusion of reform in a party's platform. If the return of public goods is sufficiently large, much of resources of economy will be used to the production of public goods and little will be left for redistribution. If so, few groups within the elite will receive transfers and the remaining groups will be in favor of franchise expansion in order to increase the provision of public goods. In this case, a costless referendum will be called in the first period and will be approved. Similarly, if the return of public goods is sufficiently large and if voters are sufficiently patient, both parties will include reform in the platform in the first period (in subgame perfect equilibrium) and voters will approve it. The authors apply this analysis to explain the increased provision of public goods and the decline of special interest politics during the British age of reform. This study also account for the franchise extension in the 1867 Reform Act from the voters' motives in supporting parties which favor the reform.

Llavador and Oxoby (2005) present a model of changes in franchise with partisan competition among the elites: capitalists and landowners. The capitalists prefer to increase productivity of production, while the landowners do not. Workers are initially disenfranchised and endogenously sorted to be skilled (who work for the capitalists and prefer to promote productivity of industry) or unskilled (who work for the landowners and prefer not to promote productivity of industry). The elites then use suffrage (workers must have the minimum level of income proposed by the incumbent party to be in electorate) to implement the preferred policy. The franchise therefore expands as a trickle-down process: first to the skilled workers with higher income, then to the unskilled workers. Llavador and Oxoby identify three types of economies for the connection between enfranchisement and growth. First, when the landowners are the majority among the elites, they control the government and oppose any enfranchisement to promote productivity. The society then goes into stagnant autocracy and experiences little (or no) growth. For instances, Spain and Italy before World War I are this type. Second, when there are large numbers of capitalists and unskilled workers, the landowners win the election with universal suffrage in which the median voter is someone averse to promoting productivity. As a result, the society will have significant democratic expansion but low growth. For instances, Germany and Chile in late 19th century are this type. Third, when there are large numbers of capitalists and skilled workers, the capitalists enfranchise the workers to some point where the median voter is the median capitalist. Over time, the society will exhibit gradual franchise extension and significant growth. For instances, Switzerland and UK in the 19th century are this type.

Jack and Lagunoff (2006) develop a class of recursive games which would endogenously generate gradual franchise expansion. These games accommodate some explanations for democratization in the literature, such as external conflict (Acemoglu and

Robinson 2000 and 2001) and internal conflict (Lizzeri and Persico 2003). The key intuition behind the model is that dilution of power by an enfranchised elite is as if the pivotal voter among the elites delegates the power to another citizen who in turn becomes the pivotal voter in the new group of enfranchised elites. Franchise expansion occurs if the citizenry's private decisions have a net positive spillover to the current pivotal voters' dynamic payoff.

2.5. Optimal taxation

Ramsey (1927) develops a theory for optimal commodity taxes. He proposes that the optimal taxes should be inverse to the price elasticity of demands. For instance, if the demand for one good is less elastic than that for the other, the optimal tax rate for this good should be higher. Under certain assumptions (e.g. homothetic indifference curves, Atkinson and Stiglitz 1972), optimal commodity taxation is uniform, i.e. the optimal taxes are equated across consumption goods. This is so-called *uniform commodity taxation* in nowadays literature.

Diamond and Mirrlees (1971, part 1) show that, whatever the class of possible tax systems, optimal production is *weakly efficient* (i.e. the production plan is on the production frontier) if all possible commodity taxes are available to the government and if a poll subsidy is possible. Diamond and Mirrlees (1971, part 2) derive the tax rule: $\sum_h \beta^h x_k^h = -\lambda \sum_i p_i \frac{\partial X_i}{\partial q_k}$ for all commodity k, where β^h is the increase in social welfare from a unit increase in the income of consumer h, x_k^h is the decrease in net demand for a unit increase in consumer price q_k, λ is the non-zero Lagrange multiplier of indirect social welfare V maximization problem, p_i is the producer price of commodity i, X_i is the aggregate net demand. Note that $-V_k = \sum_h \beta^h x_k^h$ and $\lambda \frac{\partial T}{\partial t_k} = -\lambda \sum_i p_i \frac{\partial X_i}{\partial q_k}$, where $T = \sum_i t_i X_i$ is total tax revenue and t_i is the tax such that $q_i = p_i + t_i$. Therefore, at the optimum, the social marginal utility of a price change is proportional to the marginal change in tax revenue from raising that tax, calculated at constant producer prices (so consumer price elasticities enter the tax rule equation).

Saez (2001) derives a formula for optimal non-linear income tax rates using uncompensated and compensated elasticities of earnings and show the key economic effects behind the optimal tax rate formula. Three elements determine optimal tax rates. First, the *shape of the income (or skill) distribution* suggests that the government should apply high marginal rates at levels where the density of taxpayers is low compared to the number of taxpayers with higher income. The second element is *elasticity and income effects*. Let z^* be the income level at which the first-order condition of the optimal tax rate holds. When a small tax reform perturbation around the optimal tax schedule occurs, this marginal rate change in tax at z^* induces a compensated response from taxpayers earning z^* (elasticity effect). On the other hand, it also increases the tax burden of all taxpayers whose income is higher than z^* and makes them work harder (income effect). Third, *social marginal weights* represent the relative value for the government of an additional dollar of consumption at each income level, i.e. the distributive objectives of the government. The numerical simulation shows that the optimal marginal income tax rates have a U-shaped relationship with income.

Chamley (1986) studies the optimal tax on capital income in general equilibrium models of the second best. The second-best problem is the maximization of life-time utility function subject to the capital stock accumulation, non-negativity of net rate of capital return, the government's budget constraint with debts, and the first-order conditions of consumer's intratemporal and intertemporal choices. Suppose that there is no uncertainty, consumers have perfect foresight, and the government finances expenditures by imposing linear taxes on the incomes of capital and labor. He shows that if the representative individual's utility function has the form proposed by Koopmans and if the second-best dynamic path converges to a steady state, the optimal tax rate on capital income is equal to zero at the steady state. It is because, on one hand, capital income tax increases revenues on existing capital, but on the other hand, it introduces intertemporal distortions in savings. In the short run, the single-period revenue effect of the capital tax may override the savings distortions. In the long run, however, the accumulated savings distortions become predominant over the revenue effect. Therefore, there should be no tax on capital return in the long run, and the government should generate revenues from labor income tax or other available commodity taxes.

Erosa and Gervais (2002) study the optimal taxation in an overlapping generation (OLG) model in which individuals make labor-leisure choices in each period of their finite lives. Suppose that the available set of instruments to the government includes government debt and proportional taxes on consumption, labor income, and capital income. They show that the government almost always want to tax consumption goods and labor income over an individual's lifetime. One way to achieve this goal is to use age-dependent capital and labor income taxes. If these taxes cannot be conditioned on age, the optimal non-zero capital income tax derived by Ramsey approach can imperfectly imitate age-dependent consumption and labor income taxes. This non-zero capital tax is different from the result in infinitely-lived individual model (e.g. Chamley 1986) because uniform commodity taxation does not likely hold in OLG models. Since individuals' consumption and leisure increase together when they grow old in OLG models, optimal consumption taxes tend to increase over time, so that capital income taxes tend to be positive.

2.6. Miscellaneous

Commitment is a general, but restrict assumption made in optimal taxation studies. It is defined as ability of a policymaker to make binding policy choices (Golosov and Tsyvinski 2008). Persson and Tabellini (2000, Chapter. 12) explain the importance of this assumption. Consider a two-period model of a closed economy. To finance a given amount of second-period public expenditure, the government imposes capital and labor income taxes on consumers. Suppose that at the beginning of period one, before any private decision is made, the government commits to a tax structure for period two. The *ex ante* optimal policy equates the marginal distortions on the last dollar raised by each of the two tax rates are equal. However, without such commitment, it is as if the government decides the taxes at the beginning of period two, after the investment decision has made in period one. Clearly, the *ex ante* tax structure will not be *ex post* optimal, because the equilibrium capital income tax rate will be either 100% if the public expenditure is sufficiently high or as high as possible until the expenditure is fully financed. In this case, capital will be overtaxed but the labor will be lightly taxed except when capital income is fully expropriated. The social welfare will be lower in a society without commitment than in one with commitment.

Rodrik (1999) shows that there is a positive, statistically significant and robust relationship between the extent of democracy and the level of manufacturing wages. Controlling for labor productivity, income levels and other possible determinants (such as price level, schooling, urbanization, etc.), this relationship exists both across countries (cross-section regressions estimated by OLS and 2SLS) and over-time within countries (panel regressions with fixed effects). The dependent variable is the average level of dollar wages in manufacturing (Labor Market Data Base from World Bank and International Comparisons of Hourly Compensation Costs for Production Workers in Manufacturing from U.S. Bureau of Labor Statistics). The key independent variable is democracy (Freedom House and Polity III). Rodrik also provides a simple explanation based on Nash-bargaining framework for this result. He thinks that it is because the process of political participation, competition and contestation produce a wide range of legislation and institutions partial to workers' interests, and in turn increase the workers' bargaining power and/or reservation wage.

Precautionary savings are the extra savings caused by future income being random rather than determinate. Leland (1968) shows that precautionary demand for savings is positive if the utility function $u(c_1, c_2)$ is additive in c_1 and c_2, and Pratt's principle of decreasing absolute risk aversion (i.e. $u_{222} > 0$) holds. If the additivity of utility function does not hold, precautionary demand for savings is positive if the principle of decreasing risk aversion to concentration, i.e. consumers become less risk-averse in a variable as that variable becomes increasingly predominant in a constant utility bundle, holds.

CHAPTER 3

A Static Model With Endogenous Labor Productivity

In this chapter, I consider an environment with education. Educational attainment is a measure of human capital (Mincer 1974), which determines labor productivity. The model in this chapter emphasizes that how the rich's appropriation on the poor's labor income affects the poor's choices of labor supply and education, which in turn affect the likelihood of democratization.

In addition to the debate on the causality between economic growth and democracy, the effects of education on economic growth and democracy are also discussed in literature. It is approved that education promotes economic growth (Barro 1998, Glaeser et al. 2004), but the impact of education on democracy is debated. Glaeser et al. (2007) propose that education increases the benefit of civic political participation so as to increase the probability of transition to democracy. Barro (1999) and Papaioannou and Siourounis (2008) also approve there is a positive effect of education on democracy by OLS and (ordered) probit estimations. But taking fixed effects into account, Acemoglu et al. (2005) argue that the cross-section effect disappears. It suggests that the cross-sectional result is driven by omitted variables which affect both education and democracy. To better understand these factors, it is necessary to explore the theory part to see how the theory links to empirical studies. Incorporating education in the delegation game will shed light on the omitted factors for further research.

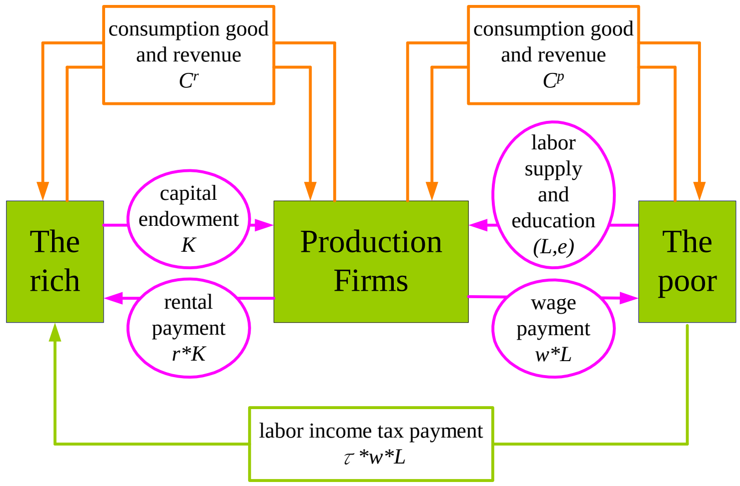


Figure 3.1. The Circular Flow of The Economy

3.1. Model

Consider a one-period closed economy with two types of consumers and production firms. Figure 3.1 shows the circular flow of the economy.

There are n^p homogeneous poor persons (superscript p, he) and n^r homogeneous rich persons (superscript r, she), who are the minority in the society ($n^r < n^p$). The rich are endowed with capital stock K; the poor are the labor suppliers L. By homogeneity, each poor person invests in educational attainment e to augment his labor productivity through a human capital generator $z(e)$, where $z > 0$ and $z' > 0$. The poor pay a distortive labor income tax payment $\tau w L$ to the rich, where (τ, w) are labor income tax rate and physical labor wage rate. The rich are expropriators. Once they collect the tax, they exclusively divide the tax revenue equally among themselves. The production firms use K and effective labor $\widetilde{L}$ to produce a final good Y, which is a consumption good C for the consumers. Normalize the market price of the final good to one.

To examine the adoption of democratization, consider two political regimes: the oligarchic regime and the democratic regime. The difference between these two regimes is who has the political power over the labor income tax rate. Under the oligarchic regime the rich are the ruling class; under the democratic regime the poor make decisions. Democratization occurs if the rich delegate the tax power to the poor[1].

Suppose that there is a commitment device for the tax decision. A delegation game is described below. At the beginning, the rich are endowed with the tax power. They can either sustain the power so that the society remains under the oligarchic regime, or adopt democratization so that the society goes under the democratic regime. Once the rich make their choice, the consumers take sequential moves. Suppose that oligarchy retains, the sequence of moves is:

stage 1: The poor decide educational attainment.

stage 2: The rich decide the tax rate and consumption.

stage 3: The poor decide labor supply and consumption.

If delegation is chosen, the sequence of moves remains the same except the tax rate is decided by the poor at stage 1. After consumers' moves, production happens, all markets clear, and the payoffs are determined. Figure 3.2 summarizes this game.

Under the oligarchic regime, the tax rate is decided at stage 2 for three reasons. First, it highlights the commitment device for the tax decision. Second, it captures how the tax affects the poor's labor supply decision. Third, it captures how the tax sways the poor pursuing education at stage 1.

[1] It is equivalent to if the rich extend the tax power to the poor, and by Medium Voter Theorem, the medium decision maker is a poor individual.

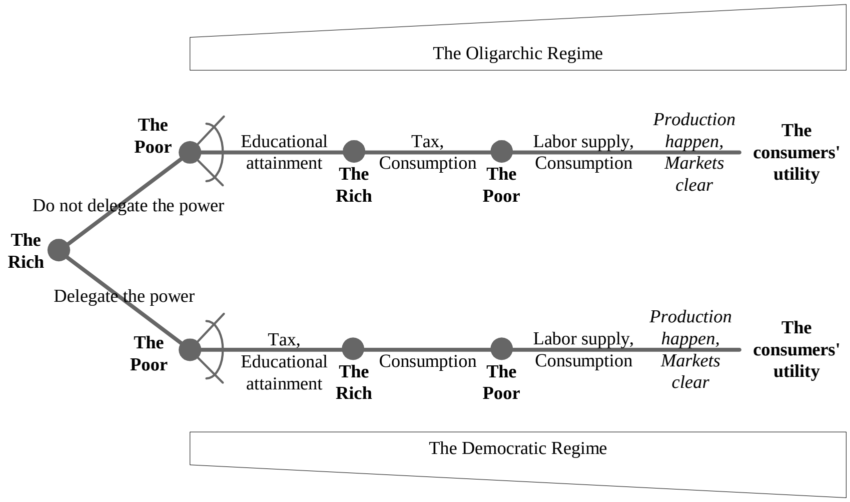


Figure 3.2. The Delegation Game

For simplicity, I make the following assumptions:

(A1): All markets are perfectly competitive.

(A2): Capital depreciates at 100% rate.

(A3): The rich are price-manipulators; the poor are price-takers.

(A4): Tax only. No subsidy or over-taxation.

(A5): CES utility function of consumption, labor supply, and education: $u(c, l, e) = [c^\rho + (1 - l - e)^\rho]^{\frac{1}{\rho}}$, where $\rho \in (0, 1)$.

(A1) and (A2) are common assumptions for simplification. (A3) seems odd at first glance but it is true in the real world. Regardless of regime types, the social elite are more knowledgeable about economy so are more able to manipulate prices than ordinary people. (A4) bounds the tax at a range of 0 and 100%. If democratization occurs, it prevents the poor from requesting a revengeful subsidy from the rich.

Denote $\widetilde{w}$ to be the effective wage rate. By (A1) and the labor market clearing condition, the following relationships hold:

Claim 1. *(1)* $\widetilde{L} = z(e)\,L$; *(2)* $w = z(e)\,\widetilde{w}$.

By homogeneity, each poor person has effective labor $z(e)\,l$. Aggregate effective labor is defined as the sum of every poor person's effective labor. Notice that the productivity in aggregate effective labor is a function of individual labor productivity (or educational attainment), which is independent of the size of cohort[2].

Effective wage is marginal product of effective labor by (A1). With e, each unit of physical labor becomes $z(e)$-fold more productive. Hence, marginal physical labor hour should be paid $z(e)$-fold higher than marginal effective labor hour.

3.2. Consumers

Consider the behaviors of the representative rich/poor person only by homogeneity within their kind. The representatives' decision-makings depend on the regime type and the stage of the sequential move, and will be discussed by the order of backward induction.

At stage 3 under either regime, the representative poor person takes as given his educational attainment, the tax rate (regardless of who decided it), and the physical wage rate. He works l hours and is paid w for each physical labor hour. After paying the tax, he has disposable labor income $(1-\tau)\,wl$. He uses disposable income to purchase consumption good c^p. He is never in debt, so his budget set is $(1-\tau)\,wl - c^p \geq 0$. He wants to find an affordable bundle of consumption and labor supply which gives him the greatest satisfaction described by $u^p(c^p, l, e)$. Hence, his utility maximization problem

[2]Consider two middle-school graduates (MG) and one college graduate (CG). Each MG attains 8-year education; each CG attains 16-year education. If the MGs collaborate, their joint productivity will still remain at the middle-school level and they cannot become as productive as the CG due to the collaboration.

(UMP) is:

$$\underset{c^p,l}{Maximize} \quad u^p\left(c^p,l,e\right)=\left[\left(c^p\right)^\rho+\left(1-l-e\right)^\rho\right]^{\frac{1}{\rho}}$$

$$subject\ to \qquad \begin{cases} (e,\tau,w) \text{ are given} \\ (1-\tau)\,wl-c^p\geq 0. \end{cases}$$

In optimum the poor person exhausts his disposable labor income on consumption good. Using utility as a measure, his optimal labor supply satisfies:

$$(1-\tau)^\rho\,w^\rho l^{\rho-1}=(1-l-e)^{\rho-1}\,. \tag{3.1}$$

The left side of (3.1) measures the marginal utility of consumption if he devotes one unit of time to working; the right side measures the corresponding marginal utility loss of leisure. The equality holds for $\rho\in(0,1)$. His consumption demand and labor supply are dependent on (e,τ,w):

$$c^p\left(e,\tau,w\right) = \frac{(1-e)\,(1-\tau)^{\frac{1}{1-\rho}}\,w^{\frac{1}{1-\rho}}}{1+(1-\tau)^{\frac{\rho}{1-\rho}}\,w^{\frac{\rho}{1-\rho}}}; \tag{3.2}$$

$$l\left(e,\tau,w\right) = \frac{(1-e)\,(1-\tau)^{\frac{\rho}{1-\rho}}\,w^{\frac{\rho}{1-\rho}}}{1+(1-\tau)^{\frac{\rho}{1-\rho}}\,w^{\frac{\rho}{1-\rho}}}. \tag{3.3}$$

In aggregation, the poor's consumption and physical labor supply are also dependent on (e,τ,w):

$$C^p\left(e,\tau,w\right) = n^p c^p\left(e,\tau,w\right); \tag{3.4}$$

$$L\left(e,\tau,w\right) = n^p l\left(e,\tau,w\right). \tag{3.5}$$

See appendix for algorithm.

I would like to explain further why I impose $\rho \in (0,1)$ on the poor person's utility function. Given (e,w), how $c^p(e,\tau,w)$ and $l(e,\tau,w)$ response to τ depends on ρ. When τ increases, consumption becomes relatively more expensive than leisure. The poor person then substitutes the cheaper leisure for consumption by working less (the substitution effect). On the other hand, the higher τ decreases their purchasing power, so he consumes less and work more (the income effect). Evidently, his consumption decreases with a higher τ, but the overall effect on leisure depends on ρ. If $\rho \to 0$, CES utility functions converge to Cobb-Douglas utility functions and the labor supply is inelastic to the tax rate. If $\rho < 0$, consumption and leisure are complements so the income effect dominates the substitution effect. In this case, the poor person works harder and the rich indulge themselves to over-tax him. If $\rho \in (0,1)$, consumption and leisure are substitutes so the substitution effect dominates the income effect. The poor person works less and the rich refrain themselves from overtaxation. Claim 2 summarizes the above discussion:

Claim 2. *(1)* $\frac{\partial c^p(e,\tau,w)}{\partial \tau} < 0$*; (2)* $\frac{\partial l(e,\tau,w)}{\partial \tau} \lesseqgtr 0$ *if* $\rho \gtreqless 0$.

Given (e,τ), the similar analysis can be applied to the effects of w on $c^p(e,\tau,w)$ and $l(e,\tau,w)$. When w increases, consumption becomes cheaper than leisure. The substitution effect makes the poor person substitute the cheaper consumption for leisure by working harder; the income effect makes him consume more and work less. The overall effect on consumption is positive, but uncertain on leisure. If $\rho \to 0$, the labor supply is inelastic. If $\rho < 0$, the income effect dominates so he works less. If $\rho \in (0,1)$, the substitution effect dominates so he works harder. Claim 3 summarizes the discussion:

Claim 3. *(1)* $\frac{\partial c^p(e,\tau,w)}{\partial w} > 0$*; (2)* $\frac{\partial l(e,\tau,w)}{\partial w} \gtreqless 0$ *if* $\rho \gtreqless 0$.

By claim 2 and claim 3, it is clear that $\rho \in (0,1)$ gives more intuitive interpretations for the poor's behaviors than $\rho \leq 0$: the poor work harder when wage is higher but work less when tax is higher. Therefore, I consider $\rho \in (0,1)$ only in this chapter.

3.2.1. The oligarchic regime

3.2.1.1. Stage 2: The rich's consumption and the tax rate. At stage 2, the representative rich person treats the poor's educational attainment e decided at stage 1 as given. She knows the poor's labor supply $L(e, \tau, w)$ decided at stage 3 and has the knowledge of how the prices are determined. She is endowed with k^r units of capital. She rents her capital endowment to the production firms and is paid r per unit. She is allotted an equal share of labor income tax revenue $\frac{1}{n^r}\tau w L(e, \tau, w)$ that the rich collect from the poor. She uses her total income to purchase c^r units of consumption good and never be in debt. Her budget set is therefore $rk^r + \frac{1}{n^r}\tau w L(e, \tau, w) - c^r \geq 0$. Given this budget set, she wants to find a pair of consumption and labor income tax rate, which is affordable and gives her the greatest satisfaction described by utility function $u^r(c^r) = c^r$. Her UMP is:

$$\underset{\tau, c^r}{Maximize} \quad u^r(c^r) = c^r$$

$$subject\ to \quad \begin{cases} e \text{ is given} \\ \text{price-manipulator} \\ rk^r + \dfrac{1}{n^r}\tau w L(e, \tau, w) - c^r \geq 0. \end{cases}$$

In optimum, the rich person exhausts her total income on consumption good. Maximizing utility in terms of consumption is as if choosing a tax which maximizes total income. Using utility as a measure, she chooses the tax which satisfies:

$$\tau(e,w,r) \in (0,1) \text{ such that } \frac{dr}{d\tau}K + \tau w\frac{\partial L}{\partial \tau} + wL + \tau\left(\frac{dw}{d\tau}L + w\frac{\partial L}{\partial w}\frac{dw}{d\tau}\right) = 0, \quad (3.6)$$

where $K = n^r k^r$ and $L = L(e,\tau,w)$. Notice that total differentials, instead of partial derivatives, are applied to capture the rich's manipulating characteristic on prices (r,w).

Condition (3.6) contains two parts. The first part measures the effect of tax on capital return; the second part on the tax revenue. Consider the second part first. The rich person knows that the raise of tax has a direct effect on the increase of tax revenue, which is wL. She also aware that there are indirect effects on the wage and the poor's willingness to work. By claim 2, the tax discourages the poor from working. It decreases the tax revenue by $\tau w\frac{\partial L}{\partial \tau}$. But if the poor work less in the economy with total capital fixed, she anticipates that the physical wage will increase. It will increase the tax revenue by $\tau\frac{dw}{d\tau}L$. In addition, higher wages encourage people to work harder by claim 3. It will increase the tax revenue further by $\tau w\frac{\partial L}{\partial w}\frac{dw}{d\tau}$. Consider the first part of tax effect. The rental rate falls while the physical wage increases. The rich will suffer a tax loss by $\frac{dr}{d\tau}K$. The rich person will choose the tax at which the total marginal gains equal the total marginal loss.

Once $\tau(e,w,r)$ is determined, the rich person's consumption (c^r) and the rich's aggregate consumption (C^r) are also dependent on (e,w,r):

$$c^r(e,w,r) = rk^r + \frac{1}{n^r}\tau(e,w,r)\,wL(e,\tau(e,w,r),w); \quad (3.7)$$

$$C^r(e,w,r) = n^r c^r(e,w,r). \quad (3.8)$$

See appendix for the detailed algorithm.

Now examine why $\tau \in (0,1)$ holds. If the rich person imposes a zero tax, she receives no tax revenue. Holding other factors constant, she can be better-off if deviating to a small positive tax. So the zero tax is not her best strategy. If she imposes a 100% tax, it will drive down labor supply to zero, which gives her zero tax revenue. But if she deviates to a tax slightly smaller than 100%, she will be better-off with positive tax revenue. So the 100% tax is not her best strategy either. Therefore, the optimal tax rate must be in the range of 0 and 100% under the oligarchic regime.

3.2.1.2. Stage 1: The poor's educational attainment. At stage 1, the representative poor person has known how he will decide his consumption $c^p(e,\tau,w)$ and labor supply $l(e,\tau,w)$ at stage 3. He does not believe that he would make any direct influences on the tax τ, even though he might had foreseen $\tau(e,w,r)$, but he does believes that he will be paid better if he is educated. His goal at this stage is to find the optimal educational attainment which will give him the highest utility. His UMP over educational attainment is therefore:

$$\underset{e}{Maximize} \quad u^p(c^p,l,e) = \left[(c^p)^\rho + (1-l-e)^\rho\right]^{\frac{1}{\rho}}$$

$$subject\ to \qquad \begin{cases} (\tau, w, \widetilde{w}) \text{ are given} \\ (1-\tau)\, wl - c^p \geq 0 \\ w = z(e)\, \widetilde{w} \end{cases}$$

where $c^p = c^p(e,\tau,w)$ and $l = l(e,\tau,w)$. In optimum the budget constraint binds. Denote $v^p(e,\tau,w)$ to be the value function of $u^p(c^p,l,e)$, i.e. $v^p(e,\tau,w) = u^p(c^p(e,\tau,w), l(e,\tau,w), e)$.

By *Envelope Theorem*, consider the direct effect of e on $v^p(e, \tau, w)$ only[3]. The poor person chooses an education level e such that:

$$(1-\tau)^{\rho}[z(e)]^{\rho-1} z'(e)\, \widetilde{w}^{\rho} l^{\rho} = (1-l-e)^{\rho-1}. \tag{3.9}$$

Educational attainment works through two channels. First, it defines his labor productivity, so it decides his disposable income and purchasing power (wealth effect). Second, it determines his leisure. Higher education he pursues, less leisure he enjoys. The left side of (3.9) is the marginal utility of consumption if the poor person receives education e; the right side is the associated marginal disutility of his leisure sacrifice. The equality holds for $\rho \in (0, 1)$.

3.2.2. The democratic regime

Under the democratic regime, the rich do not have power to decide the tax but the poor have. So the representative rich person's UMP at stage 2 is the same as under the oligarchic regime, except τ is not a choice variable any more but is taken as given. Although the rich person still has the knowledge of manipulation, she loses the opportunity when she loses the tax power. Her consumption is therefore dependent on the initial capital return and the tax payment the poor are willing to give her. The rich person's consumption (c^r) and the rich's aggregate consumption (C^r) are:

$$c^r(e, \tau, w, r) = rk^r + \frac{1}{n^r}\tau w L(e, \tau, w) \tag{3.10}$$

$$C^r(e, \tau, w, r) = n^r c^r(e, \tau, w, r). \tag{3.11}$$

[3]The indirect effects through (c^p, l) are taken into consideration at stage 3 such that $\frac{\partial u^p}{\partial c^p}\frac{dc^p}{de} = \frac{\partial u^p}{\partial l}\frac{dl}{de} = 0$.

With the tax power, the representative poor person's UMP at stage 1 is the same as under the oligarchic regime except τ is not taken as given but is a choice variable now. To maximize the utility, the poor person likes expanding his budget set as much as possible. It is straightforward that he will impose a zero tax on himself. $\tau = 0$ is therefore strictly dominant to other taxes in optimum under the democratic regime.

Notice that $\tau = 0$ always holds under the democratic regime regardless of the decision timing of the tax. Suppose that the poor switch to choose the tax at stage 3 instead. He still wants to expand the budget set, so zero tax is still strictly dominant too. In fact, regardless of the timing of decision-making, $\tau = 0$ is always the poor's optimal choice under the democratic regime. After all, no one wants to pay taxes without benefit.

The poor person chooses an education level e such that:

$$[z(e)]^{\rho-1} z'(e) \widetilde{w}^{\rho} l^{\rho} = (1 - l - e)^{\rho-1}. \tag{3.12}$$

Similar to (3.9), the left side of (3.12) is the marginal utility of consumption if the poor person imposes zero tax and receives education e; the right side is the associated marginal disutility of his leisure sacrifice. The equality holds in optimum for $\rho \in (0, 1)$. See appendix for algorithm.

3.3. Production Firms and Market Clearing Conditions

Production firms adopt constant-returns-to-scale Cobb-Douglas technology $Y = AK^{\alpha}\widetilde{L}^{1-\alpha}$, where $A \geq 1$ and $\alpha \in (0, 1)$. A is total factor productivity (TFP). It pictures overall productivity of production. α is capital share. It characterizes factor intensity of production. The firms hire effective labor $\widetilde{L}$, whose marginal product equals effective wage rate $\widetilde{w}$ by (A1), and the physical wage w is $z(e)$-fold higher than $\widetilde{w}$ by claim 1:

$$\widetilde{w} = (1-\alpha) AK^{\alpha}\widetilde{L}^{-\alpha}; \tag{3.13}$$

$$w = z(e)(1-\alpha) AK^{\alpha}\widetilde{L}^{-\alpha}. \tag{3.14}$$

All firms make zero profit in perfectly competitive markets, so the marginal product of capital equals the rental rate:

$$r = \alpha AK^{\alpha-1}\widetilde{L}^{1-\alpha}. \tag{3.15}$$

See appendix for algorithm.

There are three markets in this economy: final good market, labor market, and capital market. Consumers have no demand for investment in capital in static models, so the final good market clears when the total final good provision equals the consumers' aggregate demand for consumption:

$$AK^{\alpha}\widetilde{L}^{1-\alpha} = C^p + C^r. \tag{3.16}$$

The labor market clears when claim 1, (1) holds. Capital endowment goes rotten at the end of the day. There is no capital left for tomorrow ($K_2 = 0$).

3.4. Characterizations of General Equilibrium

A general equilibrium (GE) can be characterized under each regime. Each GE contains a set of prices $(\tau, w, \widetilde{w}, r)$ and a set of allocations $\left(c^p, c^r, l, e, C^p, C^r, L, \widetilde{L}, Y\right)$, in which all economic agents act optimally and all markets clear. For the rest of chapter, I make the following assumption:

(A6): $z(e) = \gamma e^{\theta}$, where $\gamma > 0$ and $\theta > 0$.

γ is total education productivity, which measures the overall effect of education on labor productivity. Denote $\eta_{z(e),e}$ to be elasticity of labor productivity with respect to education. By (A6), $\eta_{z(e),e} = \theta$[4]. It indicates that labor productivity increases by a constant θ% for every one percent increase in education. If $\theta > 1$, $z(e)$ exhibits increasing return in education; if $\theta = 1$, constant; if $\theta < 1$, decreasing.

Let $\psi = \rho\phi\gamma^{1-\alpha}\theta^{\alpha}$ and $\phi = (1-\alpha) A \left(\frac{K}{n^p}\right)^{\alpha}$. I will use these notations for the following discussions.

Denote x_o to be the GE value of variable x under the oligarchic regime. Equation (3.1)−(3.9), (3.13)−(3.16) and claim 1, (1) characterize the GE under the oligarchic regime. All prices and allocations can be expressed in terms of educational attainment e_o, which is governed by:

$$[\theta - (\theta + \rho) e_o] e_o^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}} = \psi [\theta - (1+\theta) e_o]^{\frac{1}{\rho}}. \tag{3.17}$$

Notice that $e_o \in \left(0, \frac{\theta}{1+\theta}\right)$ for $\rho \in (0,1)$. Equation (3.17) is derived from (3.6). The left-hand side of (3.17) describes the direct effect of the tax on the tax revenue; the right-hand side summarizes the indirect effects on capital return, physical wage, and labor supply. See appendix for algorithm.

Denote x_d to be the GE value of variable x under the democratic regime. Equation (3.1)−(3.5), (3.10)−(3.16) and claim 1, (1) characterize the GE under the democratic

[4] $\eta_{z(e),e} \equiv \frac{dz(e)}{z(e)} \div \frac{de}{e} = \frac{dz(e)}{de}\frac{e}{z(e)} = \theta$

regime. All prices and allocations can be expressed in terms of educational attainment e_d, which is governed by:

$$[\theta - (1+\theta)\, e_d]\, e_d^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}} = \frac{\psi}{\rho}\, [\theta - (1+\theta)\, e_d]^{\frac{1}{\rho}} \,. \tag{3.18}$$

Notice that $e_d \in \left(0, \frac{\theta}{1+\theta}\right)$ for $\rho \in (0,1)$. Equation (3.18) is derived from (3.12). It shows the relationship between labor supply and education in an environment without tax. See appendix for algorithm.

Given below are the conditions for the unique solution of (e_o, e_d):

Claim 4. *(Uniqueness conditions)* (e_o, e_d) *is unique if one of the following conditions holds:*

(1) $\rho < \min\left\{1, \frac{1}{(1-\alpha)(1+\theta)}\right\}$, *or*

(2) $\rho = \frac{1}{(1-\alpha)(1+\theta)} < 1 < \psi\theta^{\frac{1-\rho}{\rho}}$.

Claim 4 helps narrow down the sets of parameters for the unique solutions under two regimes. It is based on the characteristics of equation (3.17) and (3.18). Consider the parameters with which claim 4 holds. GE is unique under each regime. Table 3.1 lists the GE in terms of (e_o, e_d) only. See appendix for details.

	The Oligarchic Regime	**The Democratic Regime**
τ	$1-\rho\frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}$	0
e	$[\theta-(\theta+\rho)e_o]\,e_o^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}}=\psi\,[\theta-(1+\theta)\,e_o]^{\frac{1}{\rho}}$	$[\theta-(1+\theta)\,e_d]\,e_d^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}}=\frac{\psi}{\rho}\,[\theta-(1+\theta)\,e_d]^{\frac{1}{\rho}}$
l	$\frac{1}{\theta}e_o$	$\frac{1}{\theta}e_d$
c^p	$\frac{\psi}{\theta}\frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}e_o^{(1-\alpha)(1+\theta)}$	$\frac{\psi}{\theta}\frac{1}{\rho}e_d^{(1-\alpha)(1+\theta)}$
c^r	$\frac{\psi}{\theta\rho}\frac{n^p}{n^r}\left[\frac{1}{1-\alpha}-\rho\frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}\right]e_o^{(1-\alpha)(1+\theta)}$	$\frac{\psi}{\theta\rho}\frac{n^p}{n^r}\frac{\alpha}{1-\alpha}e_d^{(1-\alpha)(1+\theta)}$
$\widetilde{w}$	$\frac{\psi}{\gamma\rho}e_o^{-\alpha(1+\theta)}$	$\frac{\psi}{\gamma\rho}e_d^{-\alpha(1+\theta)}$
w	$\frac{\psi}{\rho}e_o^{(1-\alpha)\theta-\alpha}$	$\frac{\psi}{\rho}e_d^{(1-\alpha)\theta-\alpha}$
r	$\frac{\alpha}{1-\alpha}\frac{\psi}{\theta\rho}\frac{n^p}{K}e_o^{(1-\alpha)(1+\theta)}$	$\frac{\alpha}{1-\alpha}\frac{\psi}{\theta\rho}\frac{n^p}{K}e_d^{(1-\alpha)(1+\theta)}$
L	$\frac{n^p}{\theta}e_o$	$\frac{n^p}{\theta}e_d$
$\widetilde{L}$	$\frac{\gamma n^p}{\theta}e_o^{1+\theta}$	$\frac{\gamma n^p}{\theta}e_d^{1+\theta}$
C^P	$\frac{\psi n^p}{\theta}\frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}e_o^{(1-\alpha)(1+\theta)}$	$\frac{\psi n^p}{\theta}\frac{1}{\rho}e_d^{(1-\alpha)(1+\theta)}$
C^r	$\frac{\psi n^p}{\theta\rho}\left[\frac{1}{1-\alpha}-\rho\frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}\right]e_o^{(1-\alpha)(1+\theta)}$	$\frac{\psi n^p}{\theta\rho}\frac{\alpha}{1-\alpha}e_o^{(1-\alpha)(1+\theta)}$
Y	$\frac{\psi n^p}{(1-\alpha)\theta\rho}e_o^{(1-\alpha)(1+\theta)}$	$\frac{\psi n^p}{(1-\alpha)\theta\rho}e_d^{(1-\alpha)(1+\theta)}$

Table 3.1. Characterization of General Equilibrium in terms of Educational Attainment

3.5. Comparison of Regimes

The major difference between the oligarchic regime and the democratic regimes is the imposition of tax. Under the oligarchic regime, the brutal tax has double influences on the poor's willingness to work and in turn their desire to enhance their productivity. On the contrary, this tax distortion is completely removed with democratization. Efficiency of economy is also restored under the democratic regime. Therefore, it is intuitive that:

Claim 5. *Under the democratic regime,*

(1) The poor receive higher education, work harder, make higher income, consume more, but enjoy less leisure;

(2) Labor productivity is higher;

(3) Aggregate effective labor is greater but paid at a lower rate;

(4) Capital return is higher;

(5) Firms produce more final goods (than under the oligarchic regime).

Under the democratic regime, the remove of distortionary labor income tax has two effects on the poor's decision-making. First, it increases the poor's willingness to work, so they work harder: $l_o < l_d$. Second, it increases the poor's disposable income, so the poor are more willing to receive higher education to improve their labor productivity: $e_o < e_d \Rightarrow z(e_o) < z(e_d)$. The better-educated and hard-working workers should be paid with higher income: $wl = z(e)\widetilde{w}l$. They should also spend the income for higher consumption: $c_o^p < c_d^p$. Their only loss is to sacrifice leisure for work and education: $1 - l_o - e_o > 1 - l_d - e_d$.

Higher labor productivity gives highly-effective labor $\widetilde{L}_o < \widetilde{L}_d$, which increases total output $Y_o < Y_d$. However, highly-effective labor becomes less appreciated than the constant capital stock under the democratic regime. It is reflected on their relative return rate: $\widetilde{w}_o > \widetilde{w}_d$; $r_o < r_d$.

Although the poor make higher income under the democratic regime, their physical wage rate may not always be higher:

Claim 6. *Under the democratic regime, for each unit of physical labor supply:*

(1) The poor are paid more if and only if $\frac{1}{(1-\alpha)(1+\theta)} < 1$*;*

(2) The poor are paid the same if and only if $\frac{1}{(1-\alpha)(1+\theta)} = 1$*;*

(3) The poor are paid less if and only if $\frac{1}{(1-\alpha)(1+\theta)} > 1$ *(than under the oligarchic regime).*

The paychecks that the poor receive are physical wage. It is the augmented effective wage through labor productivity which is dependent on the poor's education. If the productivity generator $z(e)$ is responsive to education (θ is large), the better-educated poor under the democratic regime should be paid better than under the oligarchic regime. In addition, if the economy depends on labor-intensive industries (α is small), labor is a more appreciated input in production. The firms should be more willing to pay for the poor's physical labor at a higher rate. Therefore, parameters (α, θ) determine the relative size of the physical labor wage rates in two regimes.

Proposition 7. *Given a set of parameters that satisfy one of the conditions in claim 4. The poor are better off under the democratic regime if:*

$$\frac{1-e_o}{\left[\theta-(1+\theta)\,e_o\right]^{1-\rho}}<\frac{1-e_d}{\left[\theta-(1+\theta)\,e_d\right]^{1-\rho}}, \tag{3.19}$$

where $e_o \in \left(0, \frac{\theta}{1+\theta}\right)$ *is governed by (3.17) and* $e_d \in \left(0, \frac{\theta}{1+\theta}\right)$ *is governed by (3.18).*

The poor are not always better off under the democratic regime. There is a trade-off in the poor's welfare across regimes: they consume more under the democratic regime but enjoy less leisure than under the oligarchic regime. The overall effect depends on which force dominates; it depends on the values of parameters. For instance, if $\rho \leq \frac{1}{1+\theta}$, the poor are better off; otherwise, it may not be true (see appendix, proof of proposition 7). We may overlook the poor's welfare because adoption of democratization depends on the rich's choice and welfare, not the poor's, in the delegation game.

3.6. Conditions for Democratization

Based on the previous discussions, it is clear that the rich will choose to delegate the power if they are better off with the GE under the democratic regime.

Proposition 8. *Suppose that one of the uniqueness conditions holds. Democratization is adopted if:*

$$\frac{1-\alpha}{\alpha}\left[\frac{1}{1-\alpha}-\rho\frac{\theta-(1+\theta)\,e_o}{\theta-(\theta+\rho)\,e_o}\right]e_o^{(1-\alpha)(1+\theta)}<e_d^{(1-\alpha)(1+\theta)}, \tag{3.20}$$

where $e_o \in \left(0, \frac{\theta}{1+\theta}\right)$ *is governed by (3.17) and* $e_d \in \left(0, \frac{\theta}{1+\theta}\right)$ *is governed by (3.18).*

Economic primitives $\left(\alpha,\gamma,\theta,\rho,A,\frac{K}{n^p}\right)$ matter to adoption of democratization. Notice that the aggregate capital per poor person $\frac{K}{n^p}$ is significant to adoption of democratization, but the cohort size of the rich class n^r is not. The absence of n^r also implies that even an autocrat may adopt democratization. However, if each poor person is endowed with capital k^p, cohort sizes (n^p, n^r) would become significant. The relaxation of $k^p > 0$ could be for further research.

Inequality (3.20) implies that if (1) the capital return is sufficiently larger under the democratic regime, i.e. $e_d^{(1-\alpha)(1+\theta)}$ is sufficiently larger than $e_o^{(1-\alpha)(1+\theta)}$, or (2) the labor income tax revenue is small under the oligarchic regime: $\frac{1-\alpha}{\alpha}\left[1-\rho\frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}\right]e_o^{(1-\alpha)(1+\theta)}$, or (3) both hold, there is a net positive spillover to the rich's payoff with democratization such that inequality (3.20) holds. Parameters $\left(\alpha,\gamma,\theta,\rho,A,\frac{K}{n^p}\right)$ will be categorized according to these outcomes. The following exercises summarize the categorized simulation results.

To facilitate further explanations, denote $\frac{e_d}{e_o}$ to be the relative capital return across regimes, $Tax\ rev.$ to be the labor income tax revenue the rich collect under the oligarchic regime, and $\triangle\ cap.\ return$ to be the difference of capital return between two regimes: $e_d^{(1-\alpha)(1+\theta)} - e_o^{(1-\alpha)(1+\theta)}$. Democratization is therefore adopted if $\frac{\triangle\ cap.\ return}{Tax\ rev.} > 1$ by inequality (3.20).

Comparative Statics 9. *Suppose that one of the uniqueness conditions holds. Ceteris paribus, the rich receive sufficiently larger capital return under the democratic regime than under the oligarchic regime if:*

(i): *Production is labor-intensive* (α).

(ii): *Elasticity of labor productivity to education is large* (θ).

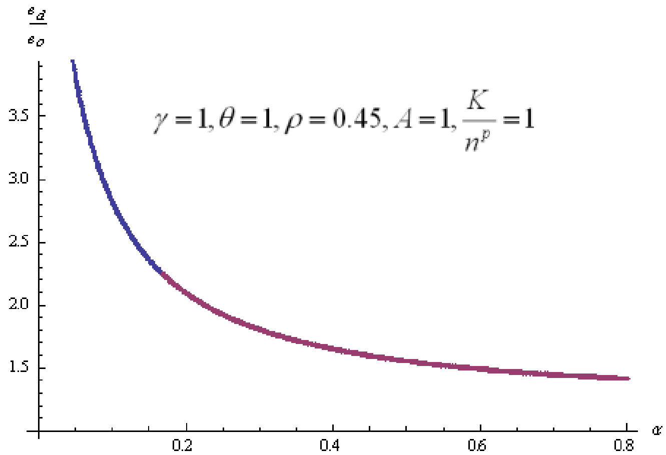


Figure 3.3. α v.s. Relative Capital Return across Regimes

Suppose that production becomes more labor-intensive, i.e. α decreases, so that $\frac{1}{(1-\alpha)(1+\theta)}$ decreases too. Physical wage rate becomes higher under the democratic regime than under the oligarchic regime as $\frac{1}{(1-\alpha)(1+\theta)}$ decreases by claim 6. That is, $\frac{w_d}{w_o}$ increases and becomes larger than 1 as α decreases. Apply the zero-profit conditions of the firms to $\frac{w_d}{w_o}$. An analogy can be drawn for the relative labor hours ratio and the relative educational attainment ratio: $\frac{l_d}{l_o} = \frac{e_d}{e_o} > 1$ increases as α decreases. Notice that $\frac{l_d}{l_o} = \frac{e_d}{e_o}$ because $\frac{e}{l} = \theta$ by table 3.1. See figure 3.3. Therefore, as production becomes more dependent on labor, the rich can receive sufficiently larger capital return under the democratic regime than under the oligarchic regime.

Increasing elasticity of labor productivity to education θ has a steady positive effect on the poor's educational attainment under the democratic regime. In the absence of labor income tax, the poor's human capital is not going to be appropriated and thereby they like to be more educated as labor productivity becomes more responsive to education. However, in the presence of labor income tax under the oligarchic regime, θ has a hump-shaped effect on the poor's education. When θ is sufficiently small, an increase in θ

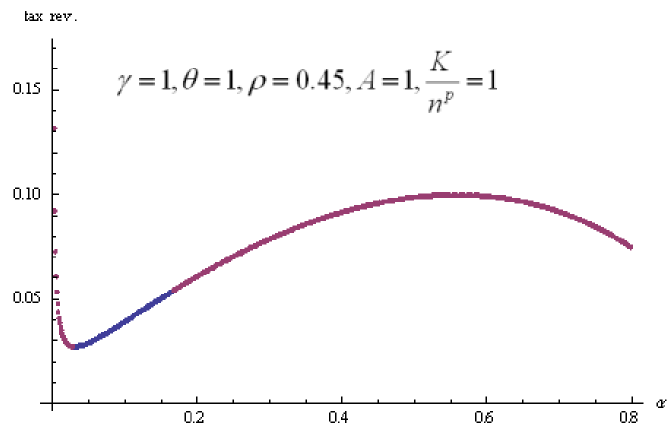


Figure 3.4. α v.s. Labor Income Tax Revenue

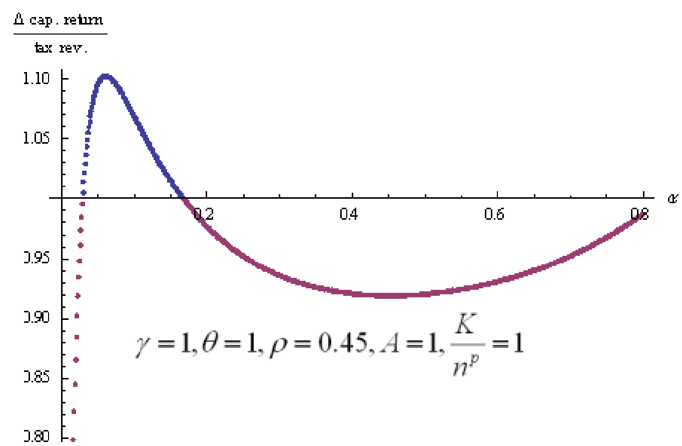


Figure 3.5. α v.s. $\frac{\triangle\ capital\ return}{tax\ revenue}$

encourages the poor to pursue education so as to work less. When θ is sufficiently large, an increase in θ holds the poor back to pursue education because they can significantly enhance human capital with little education. Figure 3.6 shows the overall effect that $\frac{e_d}{e_o}$ increases with θ. Therefore, as human capital generator becomes more responsive to education, the rich receive sufficiently larger capital return under the democratic regime than under the oligarchic regime.

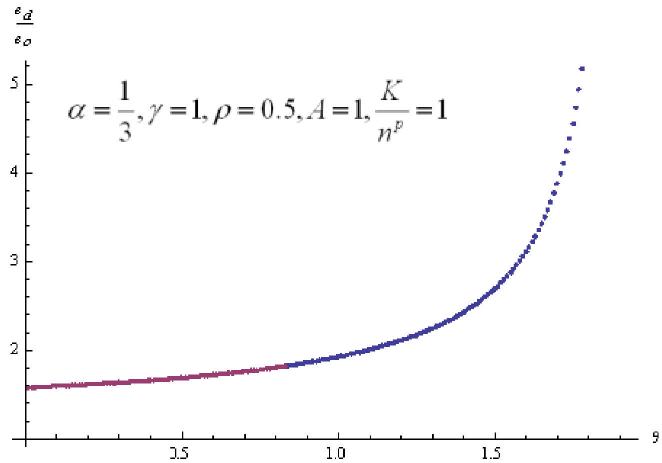


Figure 3.6. θ v.s. Relative Capital Return across Regimes

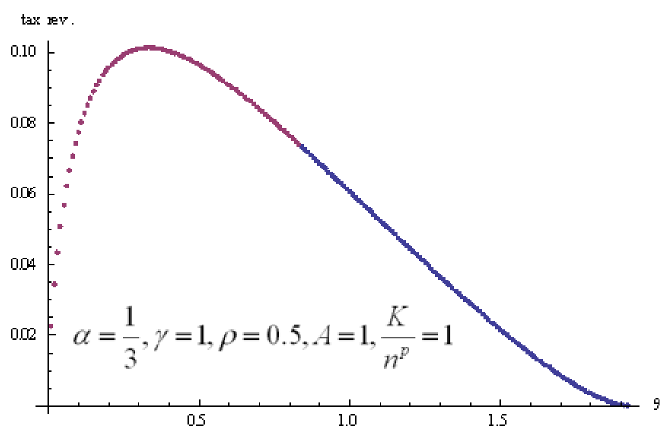


Figure 3.7. θ v.s. Labor Income Tax Revenue

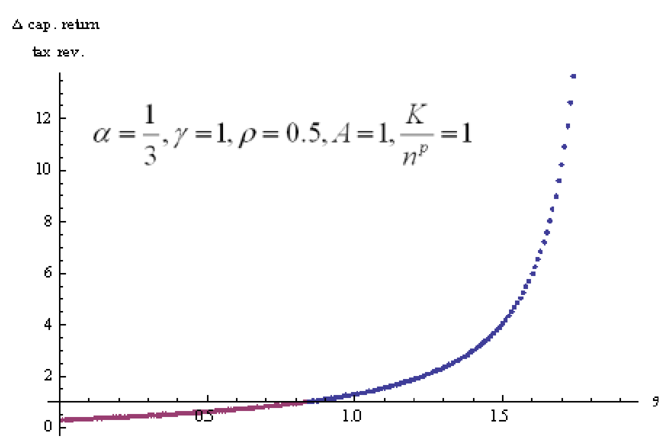


Figure 3.8. θ v.s. $\frac{\triangle\ capital\ return}{tax\ revenue}$

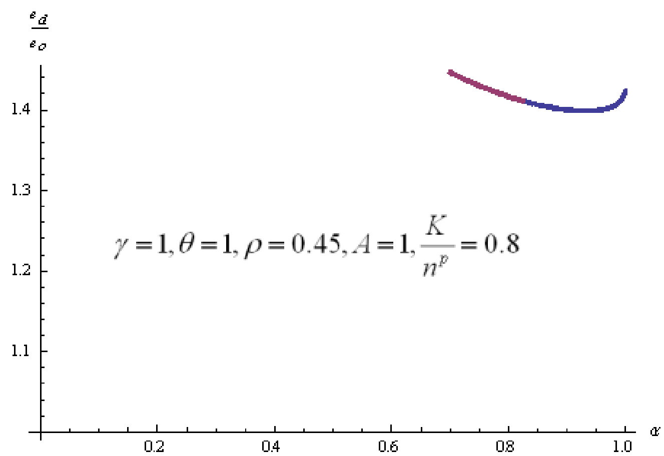


Figure 3.9. α v.s. Relative Capital Return across Regimes

Comparative Statics 10. *Suppose that one of the uniqueness conditions holds. Ceteris paribus, the rich collect small labor income tax under the oligarchic regime if:*

(i'): *production is highly capital-intensive* (α).

Suppose that production is highly capital-intensive. The production firms have little demand for labor and labor is little appreciated, which makes the physical wage rate very low. The poor then hardly work and have no incentives to be educated, but still have $\frac{e}{l} = \theta$ held. Hence the rich can only collect limited labor income tax revenue under the oligarchic regime. Figure 3.10 shows this outcome.

Comparative Statics 11. *Suppose that one of the uniqueness conditions holds. Ceteris paribus, the rich receive sufficiently larger capital return under the democratic regime and collect limited labor income tax under the oligarchic regime, if:*

(iii): *Total education productivity is low* (γ).

(iv): *Elasticity of substitution between consumption and leisure is large* (ρ).

(v): *Total factor productivity is low* (A).

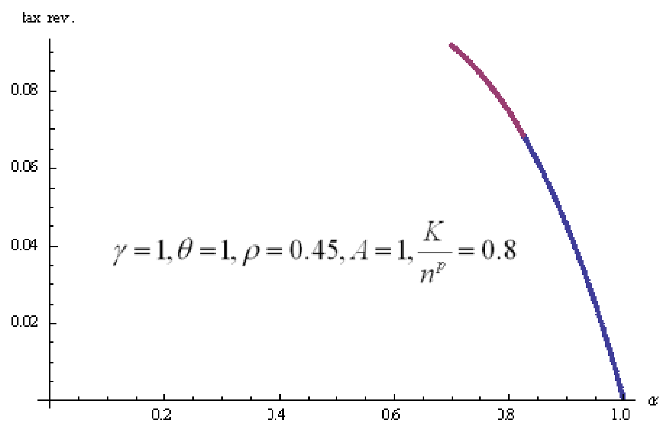


Figure 3.10. High α v.s. Relative Capital Return across Regimes

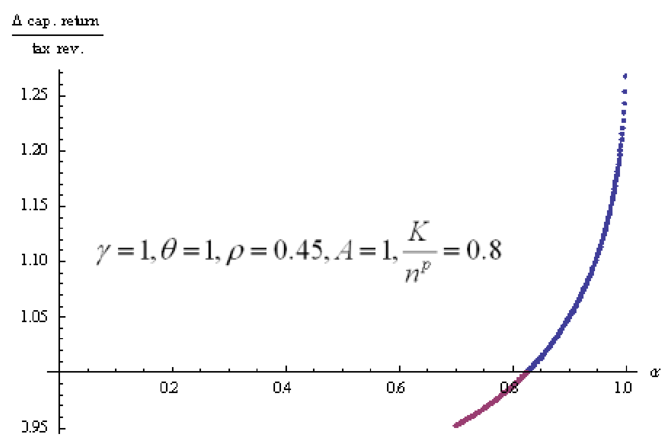


Figure 3.11. High α v.s. $\frac{\triangle \text{ capital return}}{\text{tax revenue}}$

(vi): *Capital per poor person is small* $\left(\frac{K}{n^p}\right)$.

Suppose that total education productivity γ is low. It implies that human capital generation is not overall productive. The production firms are less willing to demand for less effective labor, which makes the physical wage rate low. The poor in turn work idly and receive little education such that $\frac{e}{l} = \theta$, hence the rich can only collect limited labor income tax revenue under the oligarchic regime. See figure 3.13. In the presence of labor income tax under the oligarchic regime, the decreases in labor supply and education

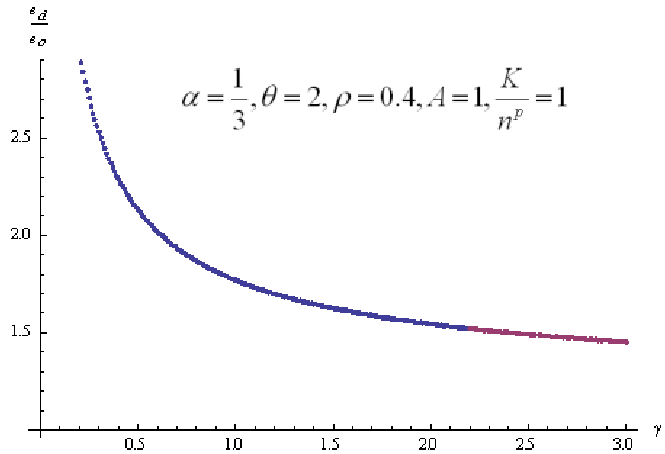


Figure 3.12. γ v.s. Relative Capital Return across Regimes

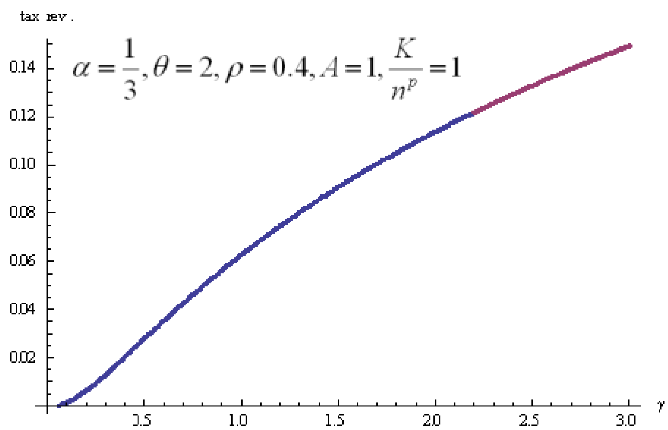


Figure 3.13. γ v.s. Labor Income Tax Revenue

are harsher than under the democratic regime. It implies that $\frac{l_d}{l_o} = \frac{e_d}{e_o}$ increases as γ decreases. See figure 3.12.

Suppose that elasticity of substitution between consumption and leisure ρ is large. It implies that consumption and leisure are substitutes to the poor. When the rich impose a labor income tax under the oligarchic regime, the poor substitute leisure for consumption. A larger ρ reinforces this substitute effect. They work idly and receive little education such that $\frac{e}{l} = \theta$. Hence, the rich can only collect limited labor income

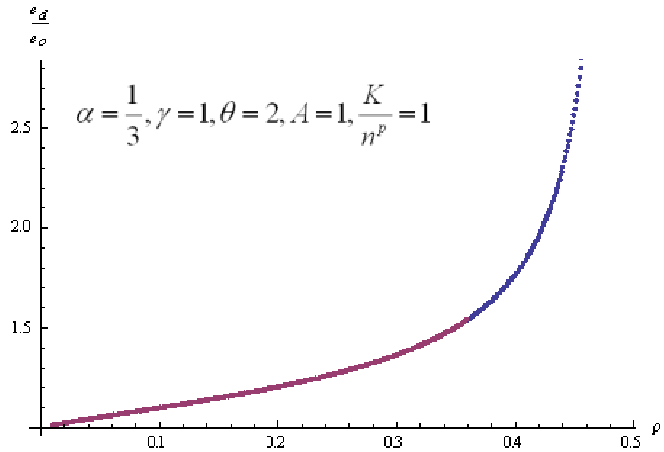


Figure 3.14. ρ v.s. Relative Capital Return across Regimes

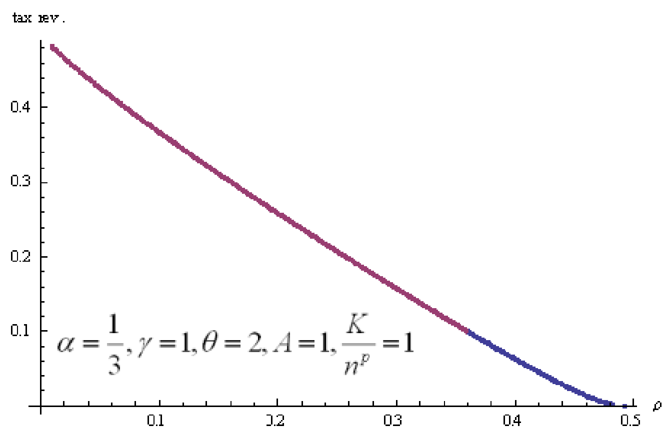


Figure 3.15. ρ v.s. Labor Income Tax Revenue

tax revenue. See figure 3.15. Under the democratic regime, this substitution effect still exists but weakens in the absence of tax distortion. It implies that $\frac{l_d}{l_o} = \frac{e_d}{e_o}$ increases as ρ increases. See figure 3.14.

Suppose that total factor productivity A is low. It implies that the final good production is not overall productive. The production firms then have lower demand for labor, which makes the physical wage rate low. The poor in turn work idly and receive little education such that $\frac{e}{l} = \theta$. Therefore, the rich can only collect limited labor income

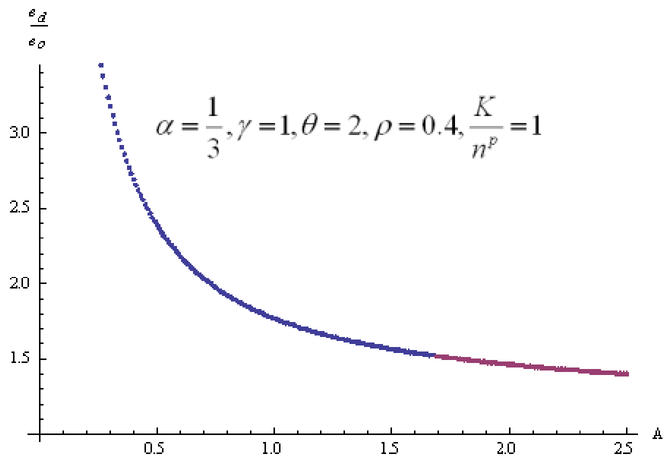


Figure 3.16. A v.s. Relative Capital Return across Regimes

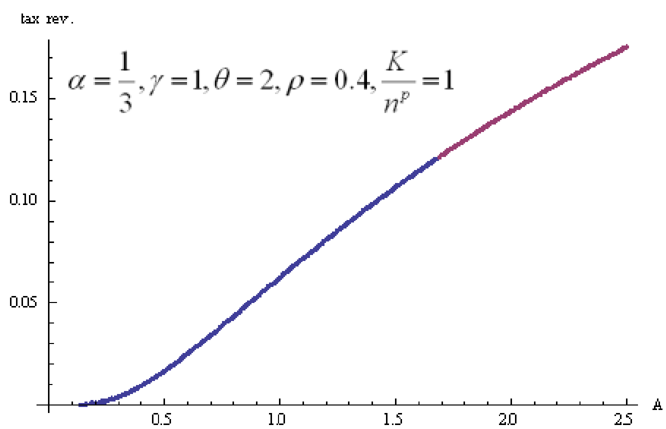


Figure 3.17. A v.s. Labor Income Tax Revenue

tax revenue under the oligarchic regime. See figure 3.17. Under the democratic regime, however, the decreases in labor supply and education are milder in the absence of tax distortion. It implies that $\frac{l_d}{l_o} = \frac{e_d}{e_o}$ increases as A decreases. See figure 3.16.

Suppose that capital per poor person is small. It implies that labor is a relatively more abundant input than capital. The abundance makes labor less valued, so the physical wage rate is low. The poor in turn work idly and receive little education such that $\frac{e}{l} = \theta$. Therefore, the rich can only collect limited labor income tax revenue under the oligarchic

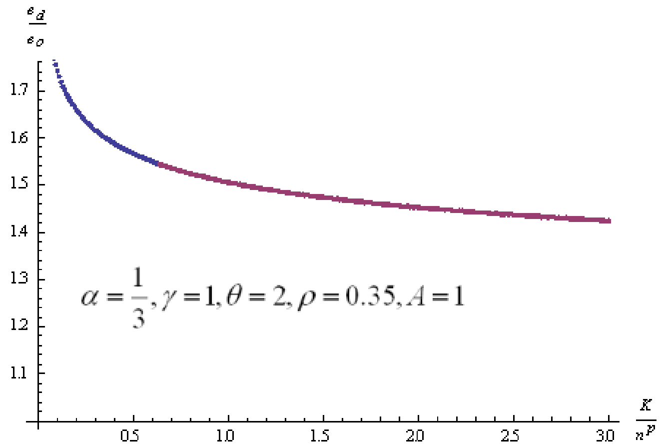


Figure 3.18. $\frac{K}{n^p}$ v.s. Relative Capital Return across Regimes

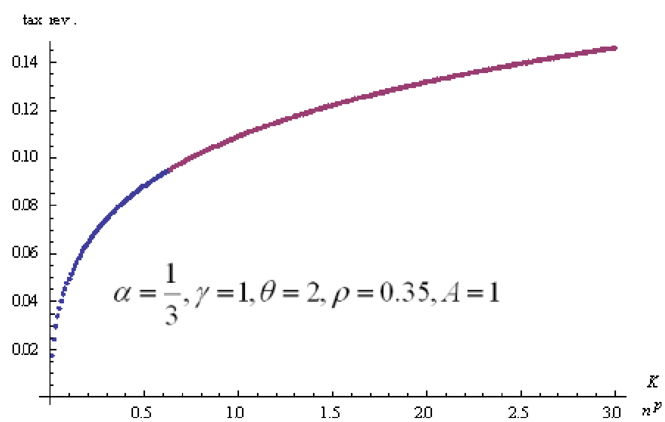


Figure 3.19. $\frac{K}{n^p}$ v.s. Labor Income Tax Revenue

regime. See figure 3.19. The decreases in labor supply and education palliate in the absence of tax distortion under the democratic regime. It implies that $\frac{l_d}{l_o} = \frac{e_d}{e_o}$ increases as $\frac{K}{n^p}$ decreases. See figure 3.18.

Comparative Statics 12. *Given a set of parameters that satisfy one of the conditions in claim 4. Ceteris paribus, the likelihood of democratization increases if:*

(i): *Production is labor-intensive or highly capital-intensive* (α)*.*

(ii): *Elasticity of labor productivity to education is large* (θ)*.*

(iii): *Total education productivity is low* (γ).

(iv): *Elasticity of substitution between consumption and leisure is large* (ρ).

(v): *Total factor productivity is small* (A).

(vi): *Capital per poor person is small* $\left(\frac{K}{n^p}\right)$.

Ceteris paribus, to see if democratization is adopted or not when a chosen parameter varies, blue color is used for the cases that democratization is adopted; red color is used for the cases that oligarchy remains.

Suppose that production becomes more labor-intensive. Figure 3.4 shows that $Tax\ rev.$ is sensitive to capital share but overall speaking it decreases. Based on figure 3.3 and figure 3.4, figure 3.5 shows that $\frac{\triangle\ cap.\ return}{Tax\ rev.} > 1$ more likely occurs as capital share decreases.

Suppose that production becomes more highly capital-intensive, figure 3.9 and figure 3.10 show that both $\frac{e_d}{e_o}$ and $Tax\ rev.$ decrease. The decaying rates of capital return and tax revenue will determine if democratization is adopted or not. Figure 3.11 shows that $Tax\ rev.$ fades out faster than $\triangle\ cap.\ return$, so that $\frac{\triangle\ cap.\ return}{Tax\ rev.}$ increases as production becomes more highly capital-intensive.

Suppose that θ is initially small. When θ increases, the tax revenue increases too because of the poor's pursuit of education. When θ is sufficiently large, the poor stop pursuing education and work further less; hence the tax revenue starts decreasing. Figure 3.7 shows this hump-shaped relationship between θ and $Tax\ rev.$ Given that $\frac{e_d}{e_o}$ significantly increases with θ, figure 3.8 shows that $\frac{\triangle\ cap.\ return}{Tax\ rev.} > 1$ more likely holds when θ is large.

According to comparative statics 11, $\frac{\triangle\ cap.\ return}{Tax\ rev.} > 1$ if any of condition (iii)-(vi) holds.

3.7. Concluding Remarks

Education increases the likelihood of democratization. It is true through the following channel: under the democratic regime the poor are more motivated to pursue higher education; education increases their labor productivity; labor productivity promotes economic prosperity; economic prosperity increases the likelihood of democratization (Aristotle, Lipset 1959). This result echoes the facts observed in the real world. Figure 3.20 shows the relationship between political regimes (Polity IV[5]) and average year of total schooling[6] of countries in 2010. 134 countries are included. The trend line shows that people are more educated in democratic countries than in autocratic countries. Given the same data on schooling in 2010, figure 3.21 shows the relationship between average year of total schooling and GDP based on PPP per capita[7] of countries. 131 countries are included. The trend line shows that countries with more educated people have more prosperous economies. In turn, if the prosperity is so significant that the rich have a net positive spillover to their payoffs, they will like to democratize the societies. Figure 3.22 shows the relationship between GDP based on PPP per capita and political regimes.

[5]Data source: Polity IV Project: Political Regime Characteristics and Transitions, 1800-2010, Monty G. Marshall and Keith Jaggers, Political Instability Task Force (PITF), Societal Systems Research, and Center for Systemic Peace, December 2011.
[6]Data source: Barro, Robert and Jong-Wha Lee, April 2010, "A New Data Set of Educational Attainment in the World, 1950-2010." NBER Working Paper No. 15902.
[7]Data source: International Monetary Fund, World Economic Outlook Database, October 2012.

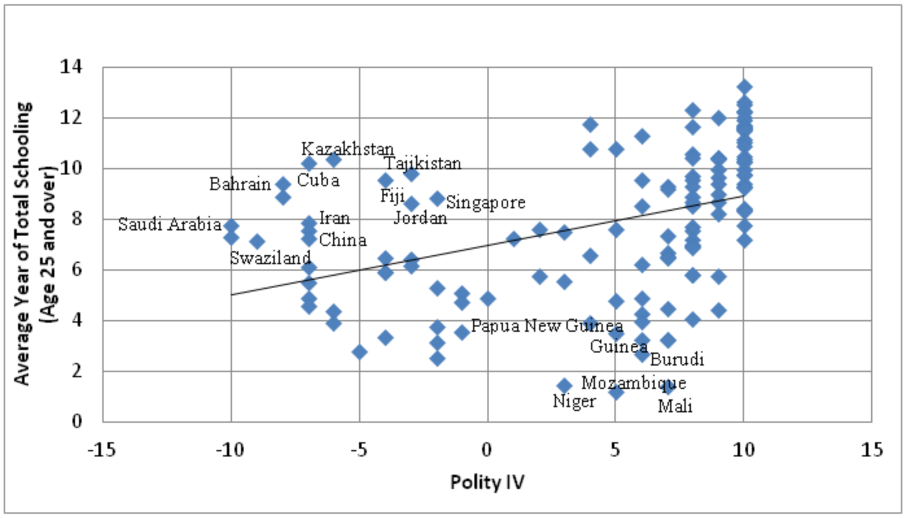


Figure 3.20. Polity IV v.s. Average Year of Total Schooling (2010)

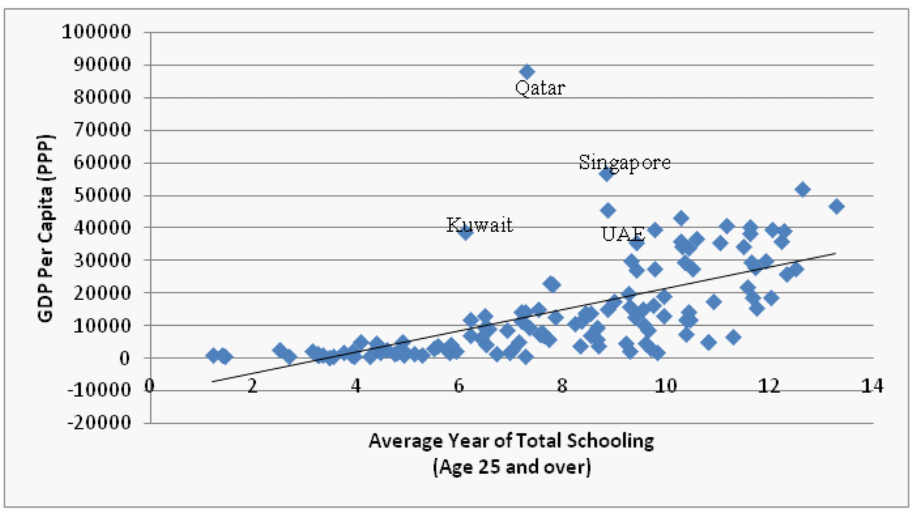


Figure 3.21. Average Year of Total Schooling v.s. GDP Per Capita (PPP) (2010)

155 countries are included. The linear trend line shows that countries with economic prosperity are in general more democratic[8].

[8]It would be more appropriate to see this interpretation if placing GDP based on PPP per capita on the horizontal axis and Polity IV on the vertical axis. But to see the positive relationship clearly, the arrangement in figure 3.22 is a better way.

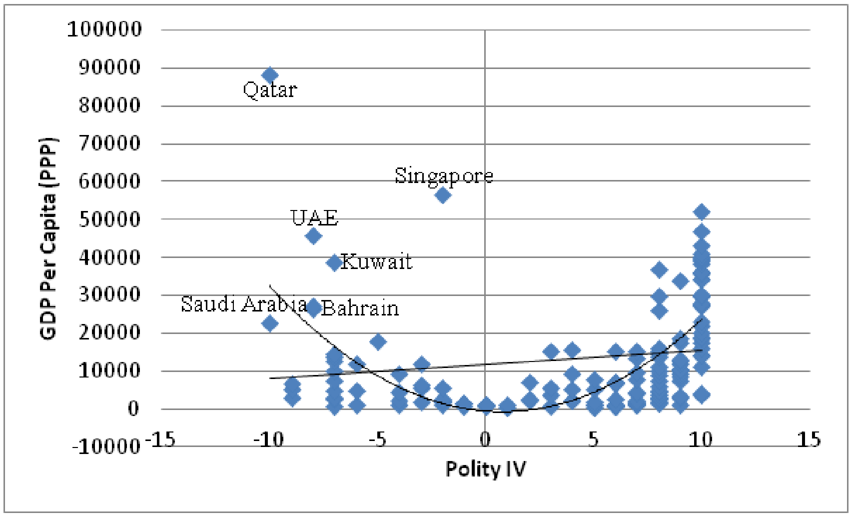


Figure 3.22. Polity IV v.s. GDP Per Capita (PPP) (2010)

In this model, economic prosperity and education are necessary but not sufficient conditions for democratization. If democratization is adopted, the poor are more educated and GDP per capita is higher. These outcomes make the democratic regime more attractive to the rich. However, they may not be sufficient to provide a net positive spillover to the rich's payoff. Notice that in this chapter education is the input of labor productivity; while in Glaeser et al. (2007) it is a factor which increases the benefit of civic participation. Even so, we both agree on the positive effect of education on democracy.

Countries dependent on capital-intensive industries are less likely democratized. The comparative statics result explains why some oil-exporting countries are not democratized even if they have educated people and outstanding economic prosperity (see the convex trend line in figure 3.22). It is because highly capital-intensive industries pay a relatively lower wage rate under the democratic regime, which discourages the poor to work hard and to be well-educated. If the society is democratized, the rich less likely receive sufficient capital return than under the oligarchic regime. It dampens the rich's-willingness-to-democratize at the first place.

Under the democratic regime, the poor may be not always paid better for each unit of physical labor hour than under the oligarchic regime. The poor's physical wage rate depends on capital share α and elasticity of labor productivity to education θ. If production is more labor-intensive and/or labor productivity is more elastic to education, the poor are paid by a better rate under the democratic regime. It echoes Rodrik (1999), although the driven reasons differ.

The timing of tax decision is the key assumption. Suppose that the price-manipulator rich decide the tax before the poor decide the educational attainment under the oligarchic regime. The optimal tax is chosen when the rich fully observe the poor's reactions to the tax (by backward induction) and take price manipulation into account. If it is positive, the zero tax under the democratic regime will not improve the rich's utility, so the rich will not adopt democratization. If it is zero, two regimes are indifferent to the rich, so the rich do not need to democratize the society either.

If production is highly capital-intensive, the likelihood of democratization increases. It is an interesting result. However, in the real world production with $\alpha > 0.9$ is hardly seen. This result may be excluded from polity implication.

Further research. If the rich are price-manipulators and like to appropriate the poor, can the poor work in an unobservable market, such as black market or work at home, to exempt themselves from being taxed by the rich? Or, after the delegation, can the poor take revenge on the rich by imposing an exclusive tax on the rich? I am going to incorporate these features into the model in the next chapter.

CHAPTER 4

Static Models With Home Production

In addition to rent appropriation, I introduce informal economy into the model.

Informal economy is also often titled as shadow, hidden, black, or underground economy. It is a label for economic activities taking place outside the framework of bureaucratic public and private sector establishments[1]. Economic activities in informal economy are important for two reasons. First, they are inevitable. In well-established capitalist societies, people divide themselves on the daily basis between work and home, and between production and consumption. In developing countries, political corruption or red tape (if there is any) fosters the activities in informal economy but hinders those in formal economy (e.g. Latin America). Beyond the boundaries of nations, bartering, usury, private loans, illegal markets, etc. have ever come to being since ancient time.

Second, informal economy plays a pivotal role connecting political regimes and economic development. Figure 4.1 and figure 4.2 provide the evidence for the connection in the real world. Figure 4.1 shows the relationship between political regimes (Polity IV[2]) and estimated sizes of informal economy as percentage of GDP[3] in 2007. 150 countries are included. The trend line shows that informal economies are more (less) prosperous in

[1]Hart, Keith. "informal economy." The New Palgrave Dictionary of Economics. Second Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan, 2008. The New Palgrave Dictionary of Economics Online. Palgrave Macmillan. 29 November 2012

[2]Data source: Polity IV Project: Political Regime Characteristics and Transitions, 1800-2010, Monty G. Marshall and Keith Jaggers, Political Instability Task Force (PITF), Societal Systems Research, and Center for Systemic Peace, December 2011.

[3]Data source: Elgin, Ceyhun and Oztunali, Oguz, "Shadow Economies around the World: Model Based Estimates," Working paper, 2012.

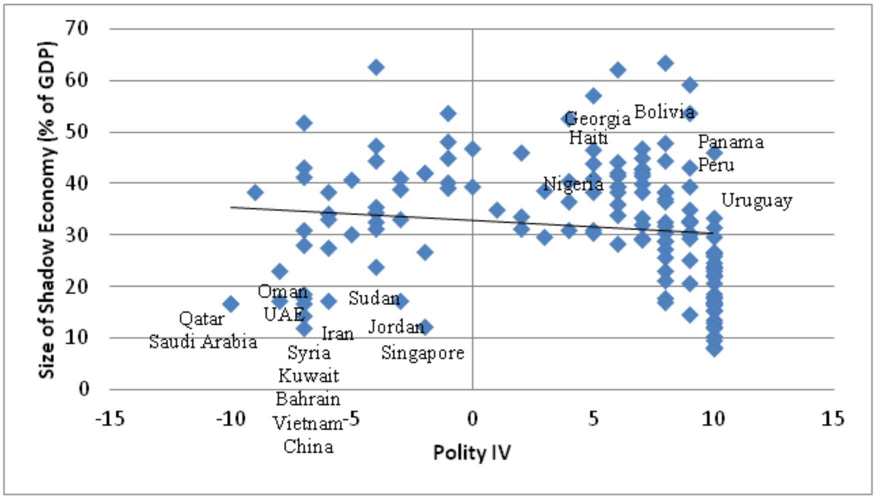


Figure 4.1. Polity IV v.s. Size of Shadow Economy as Percentage of GDP (2007)

autocratic (democratic) countries. Figure 4.2 shows the relationship between estimated sizes of informal economy as percentage of GDP and GDP based on PPP per capita[4]. 157 countries are included. The trend line shows that prosperous (recessive) informal economies deteriorate (promote) economic development. To facilitate the narration of the models based on these observations, I refer to the term of "home production" instead as the term of informal economy.

Home production is unobservable to the rich, but provides the poor an opportunity for self-sufficiency. The rich take it into account when deciding the income tax of labor, which is observable at the market. If working at the market and working at home are substitutes to the poor, a high tax will drive them away from the market but work at home. Home production is therefore like a tax haven to the poor and may turn down the rich's incentives to appropriate. The rich may be less interested in sustaining the oligarchic regime. On the other hand, if the poor are granted the power, they will impose

[4]Data source: International Monetary Fund, World Economic Outlook Database, October 2012.

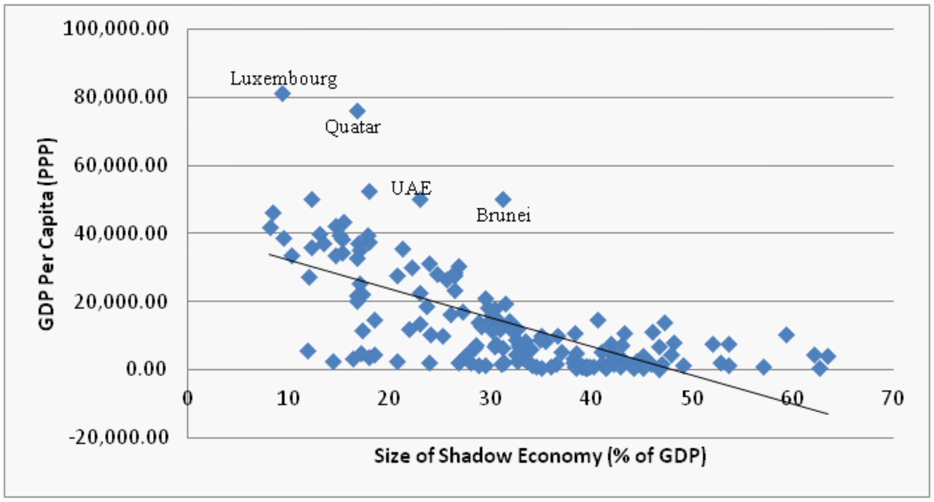


Figure 4.2. Size of Shadow Economy (% of GDP) v.s. GDP Per Capita (PPP)

a zero tax to remove the tax distortion. The zero tax encourages them to work at the market, rather than to work at home. Home production therefore declines, but market production prospers. If the rich would be better off in the more prosperous economy, they would like to democratize the society.

Two models are presented. The first model is a preliminary model. It considers labor income tax and inelastic labor supply. It also assumes that consumptions produced by two types of production are perfect substitutes. The second model is an extension of the preliminary model. In additional to the labor income tax under the oligarchic regime, it considers non-exclusive public goods financed by a capital return tax under the democratic regime. Based on these settings, I am going to characterize the conditions for each model under which the rich would like to abandon the oligarchic power, that results in large home production and sloppy economy, but embrace democracy which brings prosperity.

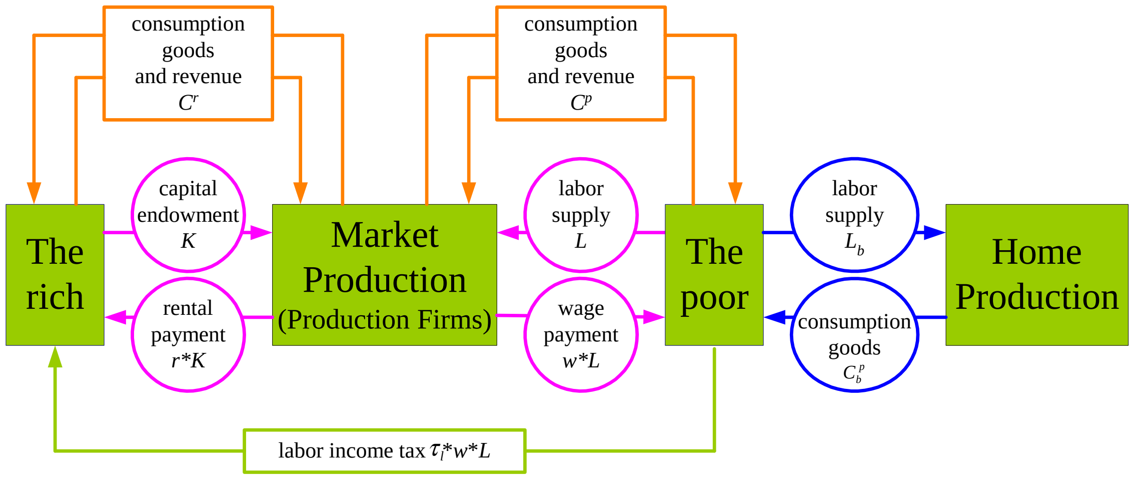


Figure 4.3. The Circular Flow of The Economy

4.1. A Preliminary Model

For simplicity, suppose that consumption goods produced by market production and home production are perfect substitutes. In equilibrium, the poor only work for one production and consume from that production. In the presence of unobservable home production, the rich dare not brutally appropriate the poor. Otherwise, the poor will turn to work at home and leave nothing to the rich at the market.

Suppose further that the poor's labor supply is inelastic. The condition for democratization can be therefore explicitly expressed in terms of economic primitives only.

4.1.1. Model

Consider a one-period closed economy with two types of consumers and two types of production. Figure 4.3 shows the circular flow of the economy.

There are n^p homogeneous poor persons (superscript p, he) and n^r homogeneous rich persons (superscript r, she). The rich class is in the minority ($n^r < n^p$). The rich are endowed with capital K; the poor are the labor suppliers (L, L_b).

There are two types of final goods, which are consumption goods to the consumers. One type is produced by market production (no subscript); the other type is produced by home production (subscript b). Market production is observable to all consumers. The market production firms require capital K and labor L as inputs and pay the input owners the associated payments (rK, wL). On the other hand, home production is only observable to the poor and requires labor L_b only. Normalize the market price of the final good produced by market production to one.

To examine the adoption of democratization, consider two political regimes: the oligarchic regime and the democratic regime. The difference between these two regimes is who has the political power over taxes. Under the oligarchic regime, the rich are the ruling class. They impose a labor income tax (τ_l) on the poor and exclusively redistribute the tax revenue $(\tau_l wL)$ equally among themselves. Under the democratic regime, the poor are granted the tax power. Democratization occurs when the rich delegate the tax power to the poor[5].

Suppose that there is a commitment device for the tax decisions. A delegation game is described below. At the beginning, the rich are endowed with the tax power. They can either sustain the power so that the society remains under the oligarchic regime, or adopt democratization so that the society goes under the democratic regime. Once the rich make their choice, the consumers take sequential moves. Suppose that the oligarchic regime sustains. The sequential moves are:

stage 1: The rich decide labor income tax (τ_l) and consumption (C^r).

[5]It is equivalent to if the rich extend the tax power to the poor, and by Medium Voter Theorem, the medium decision maker is a poor individual.

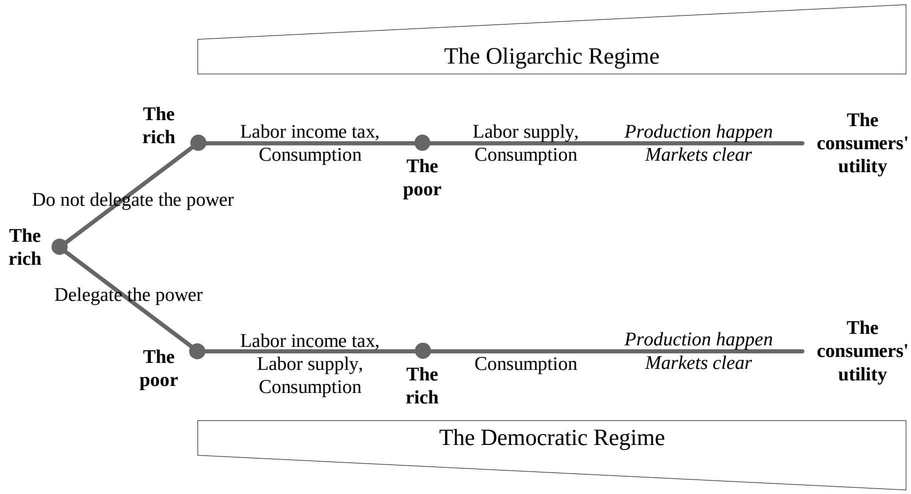


Figure 4.4. The Delegation Game

stage 2: The poor decide labor supply (L, L_b) and consumption (C^p, C_b^p).

If the rich adopt democratization, the sequential moves are:

stage 1': The poor decide labor income tax (τ_l), labor supply (L, L_b) and consumption (C^p, C_b^p).

stage 2': The rich decide consumption C^r.

After the consumers' moves, both types of production happen, all markets clear, and the payoffs are determined. Figure 4.4 summarizes this game.

For simplicity, I make the following assumptions:

(A1): All markets are perfectly competitive.

(A2): Capital depreciates at 100% rate.

(A3): All economic agents are price-takers.

(A4): Tax only. No subsidy or over-taxation.

(A5): Cobb-Douglas market production technology: $Y = AK^{\alpha}L^{1-\alpha}$, where $A \in \mathbb{R}_{++}$ and $\alpha \in (0,1)$; linear home production technology: $Y_b = BL_b$, where $B \in \mathbb{R}_{++}$.

(A6): Two types of consumption goods are perfect substitutes: $u(c, c_b) = \ln(1 + c + c_b)$.

(A7): Each poor person pays a fixed working cost $\kappa(l^*)$ if working for the market production firms.

The fixed working cost $\kappa(l^*)$ is dependent on the unavoidable labor hours l^* that each poor person has to take in order to work at the market, such as commute. It increases in l^*.

4.1.2. Consumers

4.1.2.1. The oligarchic regime. At stage 2, the representative poor person takes given the physical wage rate w and the labor income tax rate τ_l decided by the rich at stage 1. If he works l hours for the market production firms, he is paid w for each physical labor hour. After paying the tax and the fixed working cost, he has disposable labor income $(1-\tau_l)\,wl - \kappa(l^*)$. He uses disposable income to purchase consumption good c^p. He is never in debt. In the meantime, if he works l_b hours at home, he produces c_b^p units of consumption goods for self-sufficiency. The labor hours working at home are tax free. His objective is to find an affordable bundle of consumption and labor supply, which gives him the greatest satisfaction described by $u(c^p, c_b^p)$. His utility maximization problem (UMP) is therefore:

$$\underset{c^p, c^p_b}{Maximize} \ \ln\left(1 + c^p + c^p_b\right)$$

$$subject\ to \begin{cases} (\tau_l, w) \text{ are given} \\ (1 - \tau_l)\, wl - \kappa\,(l^*) - c^p \geq 0 \\ c^p_b = Bl_b \\ l + l_b = \begin{cases} 1 - l^* \text{ if } l > 0 \\ 1 \text{ if } l = 0 \end{cases} \end{cases}.$$

Suppose that the poor person works for the market production if two types of production are indifferent to him. In the optimum, his labor and consumption decisions depend on the after-tax wage rate:

$$\text{If } (1 - \tau_l)\, w \ \geq \ \frac{B + \kappa\,(l^*)}{1 - l^*}, \begin{cases} l = 1 - l^* \\ c^p = (1 - \tau_l)\, w\,(1 - l^*) - \kappa\,(l^*) \\ c^p_b = l_b = 0 \end{cases}. \tag{4.1}$$

$$\text{If } (1 - \tau_l)\, w \ < \ \frac{B + \kappa\,(l^*)}{1 - l^*}, \begin{cases} l = c^p = 0 \\ l_b = 1 \\ c^p_b = B \end{cases}. \tag{4.2}$$

If the after-tax wage rate is sufficiently large, the poor person devotes himself to market production (4.1); otherwise, he prefers being self-contained at home (4.2)[6]. In aggregation,

$$L = n^p l \tag{4.3}$$

$$C^p = n^p c^p. \tag{4.4}$$

See appendix for algorithm.

At stage 1, the representative rich person knows the poor's labor supply decided at stage 2. She does not work, but is endowed with k^r units of capital. She rents her capital endowment to the production firms and is paid r per unit. She is allotted an equal share of labor income tax revenue $\frac{1}{n^r}\tau_l wL$ collected from the poor. Her total income is $rk^r + \frac{1}{n^r}\tau_l wL$. She uses her total income to purchase consumption good c^r. Given the budget set $rk^r + \frac{1}{n^r}\tau_l wL - c^r \geq 0$, she wants to find an affordable pair of consumption and labor income tax rate which gives her the greatest satisfaction $u^r(c^r)$. Her UMP is:

$$\underset{c^r,\tau_l}{Maximize} \ \ln(1+c^r)$$

$$subject\ to \begin{cases} (r,w) \text{ are given} \\ rk^r + \frac{1}{n^r}\tau_l wL - c^r \geq 0 \\ l = \begin{cases} 0 \text{ if } (1-\tau_l)\, w < \frac{B+\kappa(l^*)}{1-l^*} \\ 1-l^* \text{ if } (1-\tau_l)\, w \geq \frac{B+\kappa(l^*)}{1-l^*} \end{cases} \end{cases}.$$

[6] $l \in (0,1)$ and $l_b \in (0,1)$ such that $l + l_b = 1 - l^*$ cannot be an equilibrium. See appendix for the proof.

Suppose that the rich person democratizes the society if two regimes are indifferent to him. In the optimum, she chooses the tax depending on the physical wage rate:

$$\tau_l = \begin{cases} 1 - \frac{B+\kappa(l^*)}{1-l^*}\frac{1}{w} \text{ if } \frac{B+\kappa(l^*)}{1-l^*} \leq w \\ 0 \text{ if } w < \frac{B+\kappa(l^*)}{1-l^*} \end{cases}. \tag{4.5}$$

The rich person prefers the poor working at the market. If the wage rate is sufficiently large, the poor work $1 - l^*$ hours at the market. To maximize the tax appropriation, the rich person imposes an as-high-as-possible tax on the poor so that they are still willing to work at the market. That is, $\tau_l \in [0, 1)$ such that $(1 - \tau_l)\, w = \frac{B+\kappa(l^*)}{1-l^*}$. However, if the wage rate is small, the poor prefer working at home. The rich person imposes a zero tax and consume nothing from the market. Once τ_l is determined, the rich person's consumption c^r and the rich's aggregate consumption C^r are also determined:

$$c^r = rk^r + \frac{1}{n^r}\tau_l wL \tag{4.6}$$

$$C^r = n^r c^r. \tag{4.7}$$

4.1.2.2. The democratic regime. At stage 2, the rich person does not have the tax power. Her UMP is:

$$\underset{c^r}{Maximize} \ \ln(1 + c^r)$$

$$subject\ to \begin{cases} (\tau_l, r, w, L) \text{ are given} \\ rk^r + \frac{1}{n^r}\tau_l wL - c^r \geq 0. \end{cases}$$

Her consumption is determined by the poor's labor supply, the tax decided by the poor at stage 1, and the final good market: (4.8). So is the aggregate consumption: (4.9).

$$c^r = rk^r + \frac{1}{n^r}\tau_l wL \tag{4.8}$$

$$C^r = n^r c^r. \tag{4.9}$$

At stage 1, the UMP of the poor person with the tax power is the same as under the oligarchic regime, except τ_l is a choice variable now. To maximize the utility, the poor person expands his budget set by imposing a zero tax on himself. Notice that $\tau_l = 0$ is strictly dominant to other taxes under the democratic regime, regardless of the timing of decision-making. In the optimum, his labor and consumption decisions depend on the wage rate:

$$\text{If } w \geq \frac{B+\kappa(l^*)}{1-l^*}, \begin{cases} l = 1 - l^* \\ c^p = w(1-l^*) - \kappa(l^*) \\ \tau_l = c_b^p = l_b = 0 \end{cases} . \tag{4.10}$$

$$\text{If } w < \frac{B+\kappa(l^*)}{1-l^*}, \begin{cases} \tau_l = l = c^p = 0 \\ l_b = 1 \\ c_b^p = B \end{cases} . \tag{4.11}$$

If the wage rate is sufficiently large, the poor person devotes himself to market production (4.10); otherwise, he prefers being self-contained at home (4.11)[7]. In aggregation,

$$L = n^p l \tag{4.12}$$

$$C^p = n^r c^p. \tag{4.13}$$

See appendix for algorithm.

4.1.3. Market production, home production and market clearing

The market production firms hire physical labor, whose marginal product equals physical wage rate, and make zero-profit:

$$w = (1-\alpha) AK^{\alpha} L^{-\alpha} \tag{4.14}$$

$$r = \alpha AK^{\alpha-1} L^{1-\alpha}, \tag{4.15}$$

where A is total factor productivity of market production; $K = n^r k^r$ is aggregate capital; α is capital share which characterizes capital intensity of market production.

The home production adopts a linear technology using labor only. For each poor person working l_b hours, he produces

$$y_b = c_b^p = Bl_b \tag{4.16}$$

[7] $l \in (0,1)$ and $l_b \in (0,1)$ such that $l + l_b = 1 - l^*$ cannot be an equilibrium. See appendix for the proof.

units of final goods for self-sufficiency, where B is total factor productivity of home production.

Consider the markets for market production only. There are three markets: final good market, labor market, and capital market. The final good market clears when the aggregate supply of final goods equals the consumers' aggregate demand for consumption goods:

$$AK^{\alpha}L^{1-\alpha} = C^{p} + C^{r}. \tag{4.17}$$

The labor market clears when the aggregate labor demand equals the aggregate labor supply of the poor class:

$$L = n^{p}l. \tag{4.18}$$

Capital endowment goes rotten at the end of the day, so there is no capital left for tomorrow ($K_2 = 0$).

4.1.4. Characterization of general equilibrium

Given a set of parameters, a general equilibrium (GE) can be characterized under each regime. Each GE contains a set of prices (τ_l, w, r) and a set of allocations $(c^p, c^p_b, c^r, l, l_b, C^p, C^r, L, Y, y_b)$, in which all economic agents act optimally and all markets clear.

Denote x_o to be the GE value of variable x under the oligarchic regime. Equation (4.1)−(4.7) and (4.14)−(4.18) characterize the GE. If $(1-\alpha)\,A\left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha} \geq \frac{B+\kappa(l^*)}{1-l^*}$, the GE is characterized in table 4.1, column 2; otherwise, the GE is characterized in table 4.1, column 4.

Denote x_d to be the GE value of variable x under the democratic regime. Equation (4.8)−(4.18) characterize the GE. If $(1-\alpha)\, A \left(\frac{K}{n^p} \frac{1}{1-l^*}\right)^{\alpha} \geq \frac{B+\kappa(l^*)}{1-l^*}$, the GE is characterized in table 4.1, column 3; otherwise, the GE is characterized in table 4.1, column 5.

	$(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha} \geq \frac{B+\kappa(l^*)}{1-l^*}$		$(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha} < \frac{B+\kappa(l^*)}{1-l^*}$	
	The Oligarchic Regime	The Democratic Regime	The Oligarchic Regime	The Democratic Regime
τ_l	$1-\frac{\frac{B+\kappa(l^*)}{1-l^*}}{(1-\alpha)A\left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha}}$	0	0	0
w	$(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha}$	$(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha}$	∞	∞
r	$\alpha A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha-1}$	$\alpha A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha-1}$	0	0
c^p	B	$(1-\alpha) A \left(\frac{K}{n^p}\right)^{\alpha} (1-l^*)^{1-\alpha} - \kappa(l^*)$	0	0
$y_b = c_b^p$	0	0	B	B
$\frac{n^r}{n^p}c^r$	$A\left(\frac{K}{n^p}\right)^{\alpha}(1-l^*)^{1-\alpha}$ $-\left[B+\kappa(l^*)\right]$	$\alpha A\left(\frac{K}{n^p}\right)^{\alpha}(1-l^*)^{1-\alpha}$	0	0
l	$1-l^*$	$1-l^*$	0	0
l_b	0	0	1	1
C^p	$n^p B$	$(1-\alpha) A n^p \left(\frac{K}{n^p}\right)^{\alpha}(1-l^*)^{1-\alpha}$ $-n^p\kappa(l^*)$	0	0
C^r	$n^p A\left(\frac{K}{n^p}\right)^{\alpha}(1-l^*)^{1-\alpha}$ $-n^p\left[B+\kappa(l^*)\right]$	$\alpha A n^p\left(\frac{K}{n^p}\right)^{\alpha}(1-l^*)^{1-\alpha}$	0	0
L	$n^p(1-l^*)$	$n^p(1-l^*)$	0	0
$\frac{Y}{n^p}$	$A\left(\frac{K}{n^p}\right)^{\alpha}(1-l^*)^{1-\alpha}$	$A\left(\frac{K}{n^p}\right)^{\alpha}(1-l^*)^{1-\alpha}$	0	0

Table 4.1. Characterization of General Equilibrium

4.1.5. Condition for democratization

Suppose that the rich choose to delegate the power if they are indifferent across regimes or better off with the GE under the democratic regime.

Theorem 13. *Democratization is adopted if*

$$(1-\alpha)\,A\left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha} \leq \frac{B+\kappa\left(l^*\right)}{1-l^*}. \tag{4.19}$$

When "=" holds, the poor work at the market and the rich receive no tax revenue. When "<" holds, the poor work at home and the rich consume nothing from the market. In either case, democratization occurs because the economy is indifferent across regimes. In addition, no tax is imposed in either regime when inequality (4.19) holds.

Comparative Statics 14. *Ceteris paribus, the likelihood of democratization increases if:*

(i): *Total factor productivity of market production is small (A is small).*

(ii): *Total factor productivity of home production is large (B is large).*

(iii): *Unavoidable labor hours for market production is large (l^* is large).*

(iv): *Capital endowment per poor person is small ($\frac{K}{n^p}$ is small).*

If market production is not overall productive (A is small), the production firms cannot pay the poor well. The low physical wage rate drives the poor to work at home.

If home production is overall productive (B is large) or the sunk cost to work at the market is large (l^* is large, so is $\kappa\left(l^*\right)$), the poor prefer working at home.

If the rich are not endowed with sufficient capital or the cohort size of the poor is large ($\frac{K}{n^p}$ is small), the poor's labor is less valued. The lower physical wage rate then drives the poor to work at home.

Let $w_{\max} = (1-\alpha) A \left[\exp\left(\frac{1}{1-\alpha}\right)\right]^{\alpha}$.

Comparative Statics 15. *Ceteris paribus, the likelihood of democratization increases if:*

(v-1): $\frac{B+\kappa(l^*)}{1-l^*} < A$*, and market production is more capital-intensive (*α *is larger).*

(v-2): $\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) > 1,\ A < \frac{B+\kappa(l^*)}{1-l^*} < w_{\max}$*, and market production is highly capital-intensive (*α *is very large).*

(v-3): $\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) > 1,\ A < \frac{B+\kappa(l^*)}{1-l^*} < w_{\max}$*, and market production is highly labor-intensive (*α *is very small).*

$\frac{B+\kappa(l^*)}{1-l^*} < A$ implies that market production is much more productive than home production ($B \ll A$), or the sunk cost to work at the market is low (l^* is small). In either case, the poor tend to work at the market. If market production becomes more capital-intensive, capital is more valued than labor. The production firms pay the poor a lower wage rate, which drives them to work at home (figure 4.5 and figure 4.6).

$\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) > 1$ implies that the rich are endowed with abundant capital stock ($\frac{K}{n^p}$ is large), or the fixed cost to work at the market is large (l^* is large). The large l^* makes $\frac{B+\kappa(l^*)}{1-l^*} > A$ easier to hold, but it does not imply that the poor prefer working at home, because $\frac{B+\kappa(l^*)}{1-l^*} < w_{\max}$ indicates that the market production is pretty productive. These two inequalities suggest that the poor are hesitated between working at the market and working at home.

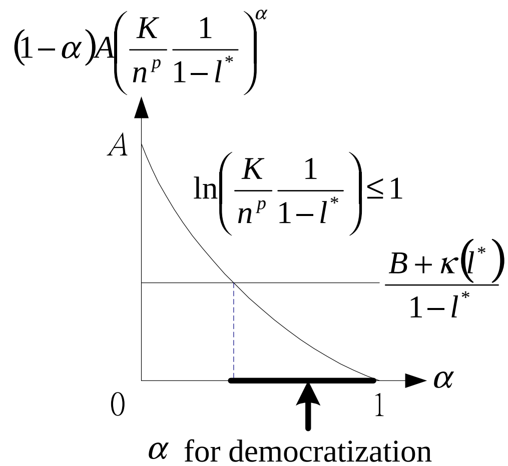


Figure 4.5. Capital-intensive Production Increases the Likelihood of Democratization (Case 1)

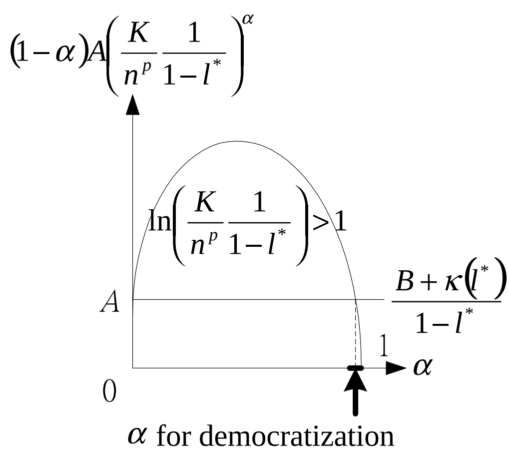


Figure 4.6. Capital-intensive Production Increases the Likelihood of Democratization (Case 2)

Suppose that market production is capital-intensive and the poor work at the market. If market production becomes capital-intensive further, capital is more valued than labor. The production firms pay the poor a lower wage rate, which drives them to work at home. See figure 4.7.

Suppose that market production is labor-intensive and the poor work at the market for the wage rate is sufficiently high. If market production becomes labor-intensive further,

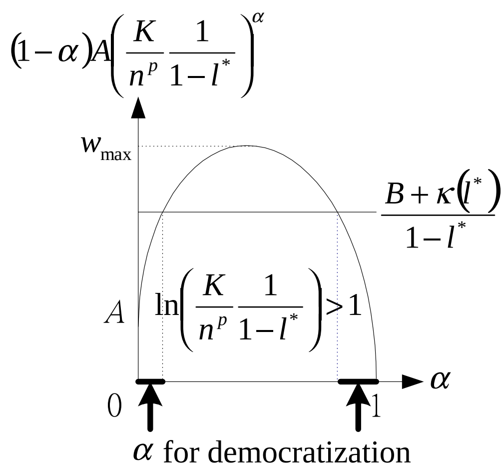


Figure 4.7. Highly Capital- or Labor-intensive Production Increases the Likelihood of Democratization (Case 3)

the wage rate drops because $\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) < \frac{1}{1-\alpha}$ more likely holds. The poor then turn to work at home. See figure 4.7 and appendix for details.

4.1.6. Concluding remarks

The presence of home production refrains the rich to appropriate the poor. Under the oligarchic regime, the presence of home production reduces the as-high-as-possible tax on the poor $\left(\frac{\partial \tau_l}{\partial B} < 0\right)$.

The oligarchic regime sustains if the rich can appropriate the poor. Because consumption goods are perfect substitutes and labor supply is inelastic, if the poor choose to work and consume at the market, productivity of total output, rental rate and wage rate are the same across regimes. If the rich impose a positive tax on the poor, the rich can purchase additional amounts of consumption goods under the oligarchic regime. They will prefer sustaining the power.

Democratization depends on the strict assumption that the rich delegate the power if they are indifferent between regimes. When inequality (4.19) holds, the rich impose a zero tax under the oligarchic regime, so that two economies are indifferent across regimes. With this strict assumption, democratization could occur; otherwise, the oligarchic regime always sustains in this model.

4.2. A Static Model with Non-exclusive Public Goods

Figure 4.8 shows the relationship between political regimes (Polity IV[8]) and corruption levels of countries (Corruption Perceptions Index, CPI[9]) in 2010. 159 countries are included. CPI measures the perceived levels of corruption in the public sectors of countries in the world. It ranges between 0 and 10. Higher CPI implies less perceived corruption. The trend line shows that more democratic countries have less observed corruption than more autocratic countries. Based on this observation, I assume that:

- Under the oligarchic regime all tax revenue are appropriated for personal use ($\Pi > 0$) and no public goods are provided ($G = 0$).
- Under the democratic regime all tax revenue are used for provision of public goods ($G > 0$) and are not appropriated at all ($\Pi = 0$).

Consider labor income tax and capital return tax. For simplicity, I boldly assume that:

- Under the oligarchic regime the rich impose labor income tax on the poor but impose a zero capital return tax on themselves, i.e. $G = 0$; $\Pi = \tau_l w L > 0$.

[8]Data source: Polity IV Project: Political Regime Characteristics and Transitions, 1800-2010, Monty G. Marshall and Keith Jaggers, Political Instability Task Force (PITF), Societal Systems Research, and Center for Systemic Peace, December 2011.
[9]Data source: 2011 Corruption Perceptions Index, Transparency International, Berlin, Germany.

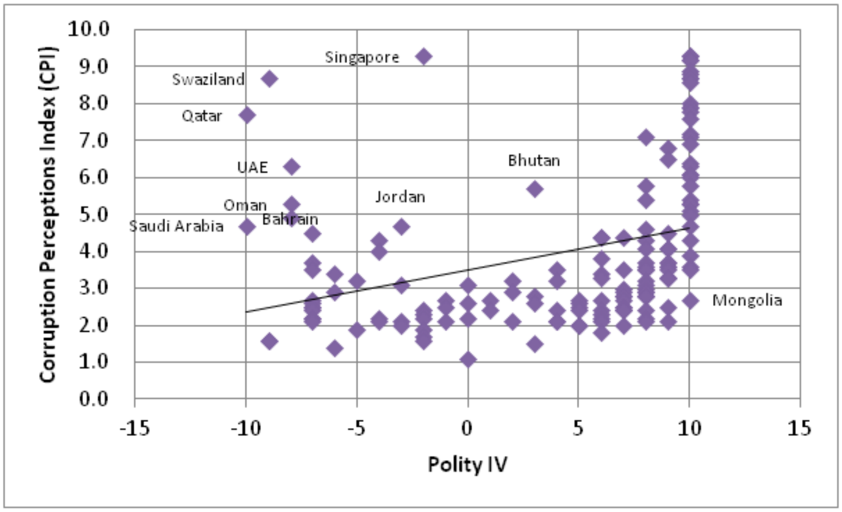


Figure 4.8. Polity IV v.s. CPI (2010)

- Under the democratic regime the poor impose capital return tax on the rich but impose a zero labor income tax on themselves, i.e. $G = \tau_k r K > 0$; $\Pi = 0$.

Although each rich person needs to pay a $\tau_k r k^r$ tax under the democratic regime, each rich individual is rewarded by the aggregate tax payment in terms of non-exclusive public goods $G = \tau_k r K$, where $K = n^r k^r$. If the tax is in an acceptable range, the rich may be more interested in delegating the power.

4.2.1. Model

Consider a one-period closed economy with two types of consumers and two types of production. Figure 4.9 shows the circular flow of the economy.

There are n^p homogeneous poor persons (superscript p, he) and n^r homogeneous rich persons (superscript r, she). The rich class is in the minority ($n^r < n^p$). The rich are endowed with capital K; the poor are the labor suppliers (L, L_b).

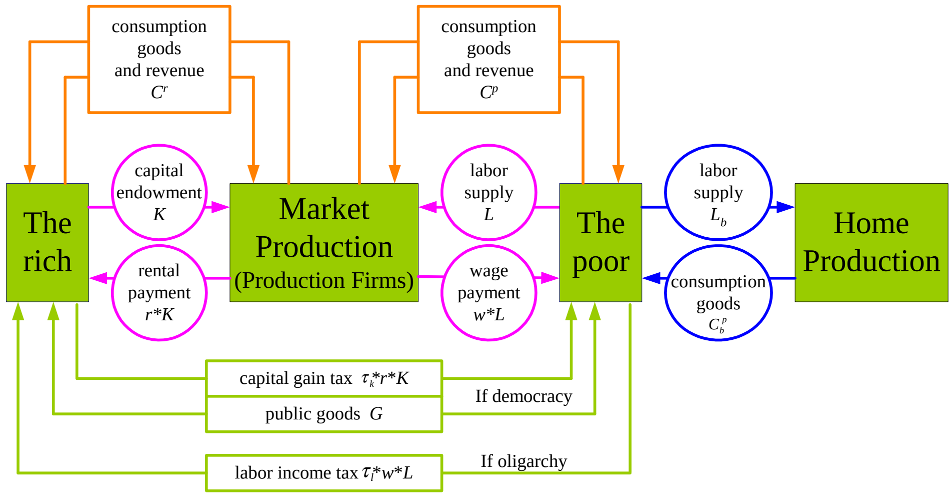


Figure 4.9. The Circular Flow of The Economy

There are two types of final goods, which are consumption goods to the consumers. One type is produced by market production (no subscript); the other type is produced by home production (subscript b). Market production is observable to all consumers. The market production firms require capital K and labor L as inputs and pay the input owners the associated payments (rK, wL). On the other hand, home production is only observable to the poor and requires labor L_b only. Normalize the market price of the final good produced by market production to one.

To examine the adoption of democratization, consider two political regimes: the oligarchic regime and the democratic regime. The difference between these two regimes is who has the political power over taxes. Under the oligarchic regime, the rich are the ruling class. They impose a labor income tax (τ_l) on the poor and exclusively redistribute the tax revenue $(\tau_l wL)$ equally among themselves. Under the democratic regime, the poor acquire the tax power. Instead, they impose a capital gain tax (τ_k) on the rich to provide a non-exclusive public good (G). Democratization occurs when the rich delegate the tax power to the poor[10].

Suppose that there is a commitment device for the tax decisions. A delegation game is described below. At the beginning, the rich are endowed with the tax power. They can either sustain the power so that the society remains under the oligarchic regime, or adopt democratization so that the society goes under the democratic regime. Once the rich make their choice, the consumers take sequential moves. Suppose that the oligarchic regime sustains. The sequential moves are:

stage 1: The rich decide labor income tax (τ_l) and consumption (C^r).

stage 2: The poor decide labor supply (L, L_b) and consumption (C^p, C_b^p).

If the rich adopt democratization, the sequential moves are:

stage 1': The poor decide capital gain tax τ_k, i.e. public goods G, labor supply (L, L_b) and consumption (C^p, C_b^p).

stage 2': The rich decide consumption C^r.

After the consumers' moves, both types of production happen, all markets clear, and the payoffs are determined. Figure 4.10 summarizes this game.

For simplicity, I make the following assumptions:

(A1): All markets are perfectly competitive.

(A2): Capital depreciates at 100% rate.

(A3): All economic agents are price-takers.

(A4): Tax only. No subsidy or over-taxation.

[10]It is equivalent to if the rich extend the tax power to the poor, and by Medium Voter Theorem, the medium decision maker is a poor individual.

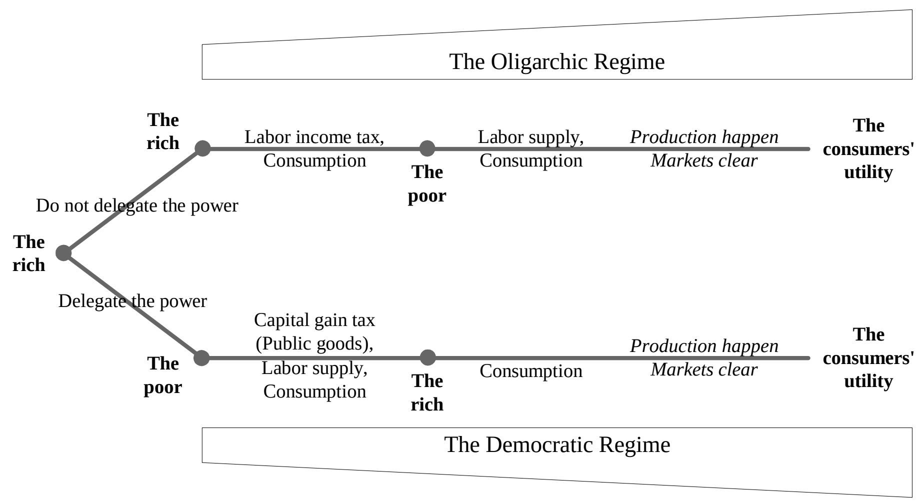


Figure 4.10. The Delegation Game

(A5): Cobb-Douglas market production technology: $Y = AK^{\alpha}L^{1-\alpha}$, where $A \in \mathbb{R}_{++}$ and $\alpha \in (0,1)$; linear home production technology: $Y_b = BL_b$, where $B \in \mathbb{R}_{++}$.

(A6): CES utility functions of consumption and labor supply:

$u(c, c_b, l, l_b) = \frac{1}{\rho} \ln \left[\gamma (1-\theta) c^{\rho} + (1-\gamma)(1-\theta) c_b^{\rho} + \theta (1 - l - l_b)^{\rho}\right]$, where $\gamma \in (0,1)$, $\theta \in (0,1)$ and $\rho \in (0,1)$[11].

(A7): Natural log utility function of public goods: $H(G) = \frac{\sigma}{\rho} \ln (1+G)$, where $\sigma \in \mathbb{R}_{++}$.

σ measures how much appreciated the public goods are. γ measures how much consumers prefer consumption goods from market production to goods from home production. θ measures how much consumers prefer leisure to consumption. ρ is the elasticity of substitution among consumption and leisure.

[11] l and l_b are perfect substitutes.

4.2.2. Consumers

Consider the behaviors of the representative rich/poor person only by homogeneity within their kind. The representatives' decision-makings depend on the regime type and the stage of the sequential move, and will be discussed by the order of backward induction.

4.2.2.1. The oligarchic regime.

Stage 2: The poor's consumption and labor supply. At stage 2, the representative poor person takes given the physical wage rate w and the labor income tax rate τ_l decided by the rich at stage 1. He works l hours and is paid w for each physical labor hour. After paying the tax, he has disposable labor income $(1-\tau_l)\,wl$. He uses disposable income to purchase consumption good c^p. He is never in debt. In the meantime, he works l_b hours at home to produce c_b^p units of consumption goods for self-sufficiency. The labor hours working at home are tax free. His objective is to find an affordable bundle of consumption and labor supply, which gives him the greatest satisfaction described by $u\left(c^p, c_b^p, l, l_b\right)$. His utility maximization problem (UMP) is therefore:

$$\underset{c^p, c_b^p, l, l_b}{Maximize}\ \frac{1}{\rho}\ln\left[\gamma\left(1-\theta\right)\left(c^p\right)^\rho + \left(1-\gamma\right)\left(1-\theta\right)\left(c_b^p\right)^\rho + \theta\left(1-l-l_b\right)^\rho\right]$$

$$subject\ to\ \begin{cases} \left(\tau_l, w\right)\ \text{are given} \\ \left(1-\tau_l\right)wl - c^p \geq 0 \\ c_b^p = Bl_b. \end{cases}$$

In the optimum, the poor person exhausts his disposable income on consumption good c^p. Using utility as a measure, his optimal labor supply satisfy:

$$\frac{\gamma(1-\theta)(1-\tau_l)^\rho w^\rho}{l^{1-\rho}} = \frac{\theta}{(1-l-l_b)^{1-\rho}} \tag{4.20}$$

$$\frac{(1-\gamma)(1-\theta)B^\rho}{l_b^{1-\rho}} = \frac{\theta}{(1-l-l_b)^{1-\rho}} \tag{4.21}$$

The left sides of these equations measure the marginal utility of consumption if he devotes one unit of time to working; the right sides measure the corresponding marginal utility loss of leisure. The equalities hold for $\rho \in (0,1)$. Denote q to be the vector of parameters $(\gamma, \theta, \rho, B)$. Let $\phi_0 = \left[\frac{(1-\gamma)(1-\theta)}{\theta}\right]^{\frac{1}{1-\rho}} B^{\frac{\rho}{1-\rho}}$ and $\phi_1 = \left(\gamma\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}} \frac{1}{1+\phi_0}$. His consumption demands and labor supply are dependent on (τ_l, w, q):

$$l(\tau_l, w, q) = \frac{\phi_1 (1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \tag{4.22}$$

$$l_b(\tau_l, w, q) = \frac{\frac{\phi_0}{1+\phi_0}}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \tag{4.23}$$

$$c^p(\tau_l, w, q) = \frac{\phi_1 (1-\tau_l)^{\frac{1}{1-\rho}} w^{\frac{1}{1-\rho}}}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \tag{4.24}$$

$$c_b^p(\tau_l, w, q) = \frac{\frac{\phi_0}{1+\phi_0}B}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}. \tag{4.25}$$

In aggregation, the poor's consumption and physical labor supply are also dependent on (τ_l, w, q):

$$L(\tau_l, w, q) = n^p l(\tau_l, w, q) \tag{4.26}$$

$$C^p(\tau_l, w, q) = n^p c^p(\tau_l, w, q). \tag{4.27}$$

See appendix for algorithm.

The imposition of labor income tax τ_l introduces two effects on consumption and leisure. The substitution effect makes the poor person substitute the cheaper leisure for consumption; the income effect makes them decrease both consumption and leisure. For $\rho \in (0,1)$, the substitution effect dominates the income effect. Therefore, the poor person works less and consume less; the rich refrain themselves from over-taxation. This discussion is summarized in the following claim:

Claim 16. *(1)* $\frac{\partial c^p(\tau_l,w,q)}{\partial \tau_l} < 0$*; (2)* $\frac{\partial l(\tau_l,w,q)}{\partial \tau_l} < 0.$

Stage 1: The rich's consumption and the labor income tax. The representative rich person knows the poor's labor supply $L(\tau_l, w, q)$ decided at stage 2. She does not work, but is endowed with k^r units of capital. She rents her capital endowment to the production firms and is paid r per unit. She is allotted an equal share of labor income tax revenue $\frac{1}{n^r}\tau_l w L(\tau_l, w, q)$ collected from the poor. Hence, her total income is $rk^r + \frac{1}{n^r}\tau_l w L(\tau_l, w, q)$. She uses her total income to purchase consumption good c^r. Given the budget set $rk^r + \frac{1}{n^r}\tau_l w L(\tau_l, w, q) - c^r \geq 0$, she wants to find an affordable pair of consumption and labor income tax rate which gives her the greatest satisfaction $u^r(c^r)$. Her UMP is:

$$\underset{c^r,\tau_l}{Maximize}\ \frac{1}{\rho}\ln\left[\gamma(1-\theta)(c^r)^\rho + \theta\right]$$

$$subject\ to\ \begin{cases} (r,w)\ \text{ are given} \\ rk^r + \frac{1}{n^r}\tau_l w L(\tau_l, w, q) - c^r \geq 0. \end{cases}$$

In the optimum, the rich person exhausts her total income on consumption good and the tax rate maximizes the tax revenue. Therefore, she chooses the tax which satisfies:

$$\tau_l (w, q) \in (0, 1) \;\; such \; that \; l (\tau_l, w, q) + \tau_l \frac{\partial l (\tau_l, w, q)}{\partial \tau_l} = 0. \tag{4.28}$$

Take w as given, the rich person knows that the raise of tax has a direct effect on the increase of tax revenue: wL. She is also aware of that the raise of tax has an indirect effect on the poor's willingness to work, which in turn reduces the tax revenue by $\tau_l w \left| \frac{\partial L(\tau_l, w, q)}{\partial \tau_l} \right|$. Write the optimal labor income tax in terms of (w, q): $\tau_l (w, q)$. She will choose $\tau_l (w, q)$ with which both effects equal.

Once $\tau_l (w, q)$ is determined, the rich person's consumption c^r and the rich's aggregate consumption C^r are also dependent on (w, q):

$$c^r (w, q) = rk^r + \frac{1}{n^r} \tau_l (w, q) \, wL (\tau_l (w, q), w, q) \tag{4.29}$$

$$C^r (w, q) = n^r c^r (w, q). \tag{4.30}$$

See appendix for the detailed algorithm.

Examine why $\tau_l \in (0, 1)$ holds. If the rich person imposes a zero tax, she receives no tax revenue. Ceteris paribus, she can be better-off if deviating to impose a small positive tax. So the zero tax is not her best strategy. If she imposes a 100% tax, it will drive the poor to work at home and she will collect no tax. But if she deviates to some tax slightly smaller than 100%, she will be better-off with positive tax revenue. So the 100% tax is not her best strategy either. Therefore, the optimal tax rate must be in the rage of 0 and 100% under the oligarchic regime.

4.2.2.2. The democratic regime. Under the democratic regime, the rich person does not have the tax power and part of her capital return is taxed away to finance for the non-exclusive public goods. The tax payment reduces her disposable income for consumption, but the public goods increases her welfare. Her UMP is:

$$\underset{c^r}{Maximize}\ \frac{1}{\rho}\ln\left[\gamma\left(1-\theta\right)\left(c^r\right)^{\rho}+\theta\right]+\frac{\sigma}{\rho}\ln\left(1+G\right)$$

$$\text{subject to}\begin{cases}\left(\tau_k, r\right)\ \text{are given}\\ \left(1-\tau_k\right)rk^r-c^r\geq 0.\end{cases}$$

Her consumption is determined by the poor's tax choice and the final good market. So is the aggregate consumption:

$$c^r\left(\tau_k, r, q\right) = \left(1-\tau_k\right)rk^r \tag{4.31}$$

$$C^r\left(\tau_k, r, q\right) = \left(1-\tau_k\right)rK. \tag{4.32}$$

At stage 1, the poor person has the tax power. He decides the capital gain tax τ_k to finance for the non-exclusive public goods G. He considers the social benefit of public goods and the associated disutility to the rich. His social optimization problem is:

$$\underset{\tau_k}{Maximize}\ \left(n^p+n^r\right)\frac{\sigma}{\rho}\ln\left(1+G\right)+\frac{n^r}{\rho}\ln\left[\gamma\left(1-\theta\right)\left(c^r\right)^{\rho}+\theta\right]$$

$$\text{subject to}\begin{cases}G=\tau_k rK\\ \left(1-\tau_k\right)rk^r-c^r\geq 0.\end{cases}$$

The optimal level of the public goods satisfies the modified *Samuelson Condition*: the sum of the marginal benefit of public goods equals the marginal disutility of the rich who are taxed for the public goods. That is:

$$\tau_k \in (0,1) \text{ such that } \frac{\sigma}{\rho}\frac{n^p + n^r}{n^r}\frac{rK}{1+\tau_k rK} = \frac{1}{1-\tau_k}\frac{\gamma(1-\theta)(1-\tau_k)^\rho r^\rho (k^r)^\rho}{\theta + \gamma(1-\theta)(1-\tau_k)^\rho r^\rho (k^r)^\rho}. \tag{4.33}$$

If the sum of the marginal benefit of public goods is significantly large, the poor will impose $\tau_k = 1$ on the rich class. See appendix for details.

On the other hand, except the presence of labor income tax τ_l, the representative poor person's UMP is the same as the one in the previous section. In the optimum, his optimal labor supply satisfy equation (4.20) and (4.21) with $\tau_l = 0$. His optimal labor supply and consumption are functions of (w,q):

$$l(w,q) = \frac{\phi_1 w^{\frac{\rho}{1-\rho}}}{1+\phi_1 w^{\frac{\rho}{1-\rho}}} \tag{4.34}$$

$$l_b(w,q) = \frac{\frac{\phi_0}{1+\phi_0}}{1+\phi_1 w^{\frac{\rho}{1-\rho}}} \tag{4.35}$$

$$c^p(w,q) = \frac{\phi_1 w^{\frac{1}{1-\rho}}}{1+\phi_1 w^{\frac{\rho}{1-\rho}}} \tag{4.36}$$

$$c_b^p(w,q) = \frac{\frac{\phi_0}{1+\phi_0}B}{1+\phi_1 w^{\frac{\rho}{1-\rho}}}. \tag{4.37}$$

In aggregation, the poor's consumption and physical labor supply are also dependent on (w, q):

$$
\begin{aligned}
L(w,q) &= n^p l(w,q) &(4.38)\\
C^p(w,q) &= n^p c^p(w,q). &(4.39)
\end{aligned}
$$

See appendix for details.

4.2.3. Market production, home production and market clearing

The market production firms hire physical labor, whose marginal product equals physical wage rate, and make zero-profit:

$$
\begin{aligned}
w &= (1-\alpha) A K^{\alpha} L^{-\alpha} &(4.40)\\
r &= \alpha A K^{\alpha-1} L^{1-\alpha}, &(4.41)
\end{aligned}
$$

where A is total factor productivity of market production; $K = n^r k^r$ is aggregate capital; α is capital share which characterizes capital intensity of market production.

The home production adopts a linear technology using labor only. For each poor person working l_b hours, he produces:

$$y_b = c_b^p = B l_b \qquad (4.42)$$

units of final goods for self-sufficiency, where B is total factor productivity of home production.

Consider the markets for market production only. There are three markets: final good market, labor market, and capital market. Under the oligarchic regime, the final good market clears when the aggregate supply of final goods equals the consumers' aggregate demand for consumption goods:

$$AK^{\alpha}L^{1-\alpha} = C^{p} + C^{r}. \tag{4.43}$$

Under the democratic regime, the final good market clears when the net aggregate supply of final goods equals the consumers' aggregate demand for consumption goods:

$$AK^{\alpha}L^{1-\alpha} - G = C^{p} + C^{r}. \tag{4.44}$$

The labor market clears when the aggregate labor demand equals the aggregate labor supply of the poor class:

$$L = n^{p}l. \tag{4.45}$$

Capital endowment goes rotten at the end of the day, so there is no capital left for tomorrow ($K_2 = 0$).

4.2.4. Characterization of general equilibrium

A general equilibrium (GE) can be characterized under each regime. Each GE contains a tax rate (τ_l under the oligarchic regime; τ_k under the democratic regime), a pair of prices (w, r) and a set of allocations $(c^p, c_b^p, c^r, l, l_b, C^p, C^r, L, Y, y_b)$, in which all economic agents act optimally and all markets clear.

Let $\phi_2 = \left[(1-\alpha) A \left(\frac{n^r}{n^p} k^r\right)^\alpha\right]^{\frac{\rho}{1-\rho}}$ and $\phi_3 = \alpha A \left(\frac{n^p}{n^r}\right)^{1-\alpha} (k^r)^\alpha$. I will use these notations for the following discussions.

Denote x_o to be the GE value of variable x under the oligarchic regime; denote τ_l^* to be the optimal labor income tax rate. Equation (4.22)−(4.30), (4.40)−(4.43), and (4.45) characterize the GE under the oligarchic regime. All prices and allocations can be expressed in terms of τ_l^* only, which is governed by:

$$\tau_l^* \in (1-\rho, 1) \text{ such that } 1 + \phi_1 \phi_2 \left[\frac{1-\tau_l^*}{\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^\alpha}\right]^{\frac{\rho}{1-\rho}} = \frac{\rho}{1-\rho}\frac{\tau_l^*}{1-\tau_l^*}. \tag{4.46}$$

Denote x_d to be the GE value of variable x under the democratic regime; denote τ_k^* to be the optimal capital gain tax rate. Equation (4.31)−(4.39), (4.40)−(4.42), (4.44), and (4.45) characterize the GE under the democratic regime. Consider interior solutions. All prices and allocations can be expressed in terms of (τ_k^*, l_d), where $l_d \in (0,1)$ is governed by:

$$l_d^{\frac{1+\alpha\rho-\rho}{1-\rho}} + \phi_1 \phi_2 l_d - \phi_1 \phi_2 = 0; \tag{4.47}$$

$\tau_k^* \in (0,1)$ is governed by:

$$\frac{\sigma}{\rho}\left[1 + \frac{\theta}{\gamma(1-\theta)\phi_3^\rho}\frac{1}{(1-\tau_k^*)^\rho}\frac{1}{l_d^{(1-\alpha)\rho}}\right] = \frac{1}{n^p+n^r}\frac{1}{\phi_3}\frac{1}{1-\tau_k^*}\frac{1}{l_d^{1-\alpha}} + \frac{n^r}{n^p+n^r}\frac{\tau_k^*}{1-\tau_k^*}. \tag{4.48}$$

See appendix for algorithm.

Based on the characteristics of equation (4.47) and (4.48), GE is unique under each regime. Table 4.2 lists the GE in terms of $(\tau_l^*, \tau_k^*, l_d)$ only. See appendix for details.

	The Oligarchic Regime	The Democratic Regime
τ_l	$1+\phi_1\phi_2\left[\frac{1-\tau_l^*}{\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{\alpha}}\right]^{\frac{\rho}{1-\rho}}=\frac{\rho}{1-\rho}\frac{\tau_l^*}{1-\tau_l^*}$	–
τ_k	–	$\frac{\sigma}{\rho}\left[1+\frac{\theta}{\gamma(1-\theta)\phi_3^{\rho}}\frac{1}{\left(1-\tau_k^*\right)^{\rho}}\frac{1}{l_d^{(1-\alpha)\rho}}\right]=\frac{1}{n^p+n^r}\frac{1}{\phi_3}\frac{1}{1-\tau_k^*}\frac{1}{l_d^{1-\alpha}}+\frac{n^r}{n^p+n^r}\frac{\tau_k^*}{1-\tau_k^*}$
w	$\phi_2^{\frac{1-\rho}{\rho}}\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{-\alpha}$	$\phi_2^{\frac{1-\rho}{\rho}}l_d^{-\alpha}$
r	$\alpha A\left(\frac{n^r}{n^p}k^r\right)^{\alpha-1}\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$	$\frac{\phi_3}{k^r}l_d^{1-\alpha}$
c^p	$\phi_2^{\frac{1-\rho}{\rho}}\left(1-\tau_l^*\right)\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$	$\phi_2^{\frac{1-\rho}{\rho}}l_d^{1-\alpha}$
$y_b=c_b^p$	$\frac{1-\rho}{\rho}\frac{\phi_0}{1+\phi_0}\frac{1-\tau_l^*}{\tau_l^*}B$	$\frac{\phi_0}{1+\phi_0}B\left(1-l_d\right)$
c^r	$\frac{n^p}{n^r}\left[\alpha A\left(\frac{n^r}{n^p}k^r\right)^{\alpha}+\phi_2^{\frac{1-\rho}{\rho}}\tau_l^*\right]\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$	$\phi_3\left(1-\tau_k^*\right)l_d^{1-\alpha}$
l	$1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}$	$l_d^{\frac{1+\alpha\rho-\rho}{1-\rho}}+\phi_1\phi_2 l_d-\phi_1\phi_2=0$
l_b	$\frac{1-\rho}{\rho}\frac{\phi_0}{1+\phi_0}\frac{1-\tau_l^*}{\tau_l^*}$	$\frac{\phi_0}{1+\phi_0}\left(1-l_d\right)$
C^p	$n^p\phi_2^{\frac{1-\rho}{\rho}}\left(1-\tau_l^*\right)\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$	$n^p\left(1-\alpha\right)A\left(\frac{n^r}{n^p}k^r\right)^{\alpha}l_d^{1-\alpha}$
C^r	$n^p\left[\alpha A\left(\frac{n^r}{n^p}k^r\right)^{\alpha}+\phi_2^{\frac{1-\rho}{\rho}}\tau_l^*\right]\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$	$n^r\phi_3\left(1-\tau_k^*\right)l_d^{1-\alpha}$
L	$n^p\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)$	$n^p l_d$
G	–	$\alpha A n^p\left(\frac{n^r}{n^p}k^r\right)^{\alpha}\tau_k^* l_d^{1-\alpha}$
Y	$A\left(\frac{K}{n^p}\right)^{\alpha}n^p\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$	$An^p\left(\frac{K}{n^p}\right)^{\alpha}l_d^{1-\alpha}$

Table 4.2. Characterization of General Equilibrium

4.2.5. Comparison of regimes

The results in this section are primarily derived by simulation.

Simulation Result 17. *The poor are better off under the democratic regime.*

The poor are exploited without compensation under the oligarchic regime. On the contrary, the democratic regime provides a tax-free environment for them and provides non-exclusive public goods financed by other's wealth. With no doubt the poor class is better off under the democratic regime.

Simulation Result 18. *Under the democratic regime,*

(1) For market production: the poor work harder, make higher disposable income, consume more, but are paid less for each physical labor hour;

(2) For home production: the poor work less and consume less;

(3) The poor work harder;

(4) Market production produces more final goods, but home production produces less (than under the oligarchic regime).

Home production is the poor's hiding place, where they can be self-sufficient and exempt from overpaying taxes. On the contrary, market production is an exposing place, where all economic agents' behaviors and transaction prices are shown to the public. The absence of labor income tax in observable market production encourages the poor to work harder under the democratic regime ($l_d > l_o$). It also makes their disposable income higher ($w_d l_d > (1 - \tau_l^*) \, w_o l_o$) so that they can consume more ($c_d^p > c_o^p$). A lower physical wage rate is the only disadvantage to the poor because the abundance of labor supply makes labor become less valued than the constant capital stock ($w_d < w_o$). In addition, because of the poor's devotion to market production, they work less at home ($l_{b,d} < l_{b,o}$) so consume less from home production $\left(c^p_{b,d} < c^p_{b,o}\right)$. Consider both types of production together. The poor prefer to work more under the democratic regime without tax distortion ($l_d + l_{b,d} > l_o + l_{b,o}$).

Lastly, the poor's devotion to market production under the democratic regime ($l_d > l_o$) implies that the market production firms can produce more final goods; on the other hand, home production produces fewer consumption goods than under the oligarchic regime.

Simulation Result 19. *Under the democratic regime, the rich are paid more for each unit of capital, make lower disposable capital gain, and consumes less (than under the oligarchic regime).*

Compare to the poor's abundant labor supply, the rich's capital becomes more valued under the democratic regime. However, it incurs the poor's jealousy to impose a high capital gain tax, which decreases the rich's disposable capital gain and consumption. On the other hand, the poor use public goods, which are financed by the rich's capital tax payment, to compensate the rich for their loss. This compensation therefore increases the likelihood of democratization.

4.2.6. Condition for democratization

Based on the previous discussion, it is clear that the rich will choose to delegate the power if they are better off with the GE under the democratic regime. Suppose that the rich choose to delegate the power if they are indifferent across regimes.

Proposition 20. *Democratization is adopted if:*

$$\left[\frac{(n^p+n^r)\sigma\phi_3}{\rho}\left(1-\tau_k^*\right)^{1-\rho} l_d^{(1-\alpha)(1-\rho)}\right]^{\sigma}\left[\frac{\theta}{\gamma(1-\theta)\phi_3^{\rho}}+\left(1-\tau_k^*\right)^{\rho} l_d^{(1-\alpha)\rho}\right]^{1+\sigma} \geq \frac{\theta}{\gamma(1-\theta)\phi_3^{\rho}}+\left(1+\frac{1-\alpha}{\alpha}\tau_l^*\right)^{\rho}\left(1-\frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{(1-\alpha)\rho}, \tag{4.49}$$

where $\tau_l^* \in (0,1)$, $l_d \in (0,1)$, *and* $\tau_k^* \in (0,1)$ *are governed by (4.46), (4.47), and (4.48) accordingly.*

The left side of inequality (4.49) is a rich person's utility under the democratic regime over consumption and public goods. The right side is her utility under the oligarchic regime over consumption associated with rent appropriation. The rich prefer the democratic regime, if the public goods can considerably compensate for their utility losses of oligarchic appropriation and capital gain tax payment.

Economic primitives $\left(\alpha, \gamma, \theta, \rho, \sigma, \frac{n^p}{n^p+n^r}, \frac{n^r}{n^p+n^r}, n^p+n^r, k^r, A, B\right)$ matter to adoption of democratization. Inequality (4.49) implies that democratization is more likely adopted if: (1) the rich impose a small labor income tax rate τ_l^* under the oligarchic regime, or (2) the poor provide abundant labor supply for market production and impose a low capital gain tax rate τ_k^* under the democratic regime l_d. The economic primitives will be categorized according to these outcomes. The following exercises summarize the categorized simulation results.

Comparative Statics 21. *Ceteris paribus, the rich impose a small labor income tax rate under the oligarchic regime if:*

(i): *Market production is capital-intensive (α is large).*

(ii): *The poor prefer home production to market production (γ is small).*

(iii): *People prefer leisure to consumption (θ is large).*

If the labor income tax rate is low under the oligarchic regime, rent appropriation is small[12]. Sustaining the oligarchic power becomes less attractive to the rich.

Suppose that the market production becomes more capital-intensive. The production firms have lower demand for labor, but higher demand for capital. The lower demand for labor drives down the wage rate and the firms hire fewer labor hours. On the other hand, the higher demand for capital increases the rich's capital return. Therefore, the rich become less interested in appropriating the poor and impose a lower labor income tax under the oligarchic regime (figure α-1).

Suppose that the poor prefer home consumption to market consumption. It implies that the poor prefer working at home to working at the market. To prevent driving the poor further away from the market, the rich therefore impose a lower labor income tax under the oligarchic regime (figure γ-1).

Suppose that the poor prefer leisure to consumption. It implies that the poor prefer taking more time off from the market and the home work. To prevent driving the poor further away from the market, the rich therefore impose a lower labor income tax under the oligarchic regime (figure θ-1).

[12] $\tau_l^* w_o l_o = \phi_2^{\frac{1-\rho}{\rho}} \tau_l^* \left(1 - \frac{1-\rho}{\rho} \frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$ by table (4.2). $\frac{d\tau_l^* w_o l_o}{d\tau_l^*} > 0$.

α v.s. $(\tau_l^*, l_d, \tau_k^*)$ and $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\gamma = 0.7, \theta = 0.1, \rho = 0.3, \sigma = 0.1, A = 1, B = 1, \frac{n^p}{n^p+n^r} = 0.95, \frac{n^r}{n^p+n^r} = 0.05, n^p + n^r = 10000, k^r = 1.$

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0,1)$.

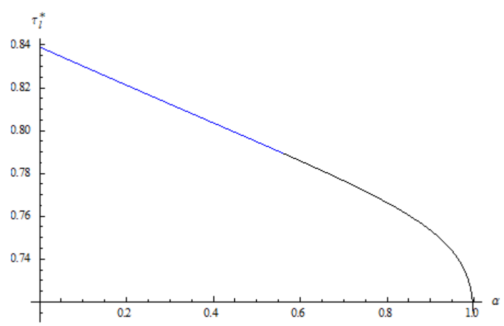


Figure α-1

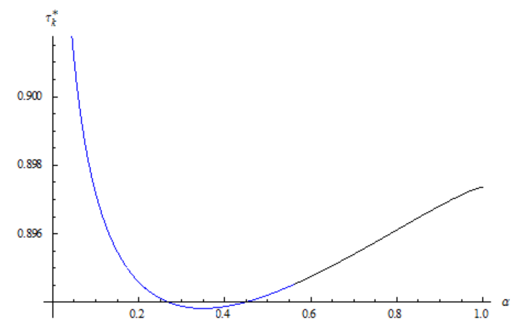


Figure α-3

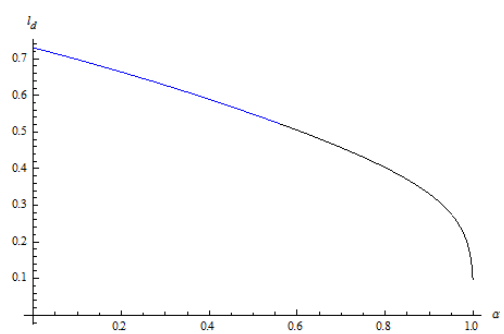


Figure α-2

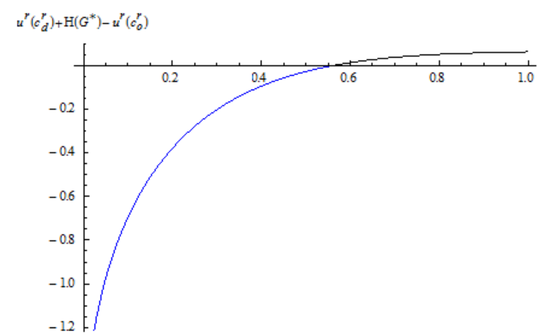


Figure α-4

γ v.s. $(\tau_l^*, l_d, \tau_k^*)$ and $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\alpha = 0.3, \theta = 0.3, \rho = 0.5, \sigma = 0.1, A = 1, B = 1, \frac{n^p}{n^p+n^r} = 0.95, \frac{n^r}{n^p+n^r} = 0.05, n^p + n^r = 10000, k^r = 100.$

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0,1)$; red: oligarchy sustains with $\tau_k^* = 1$.

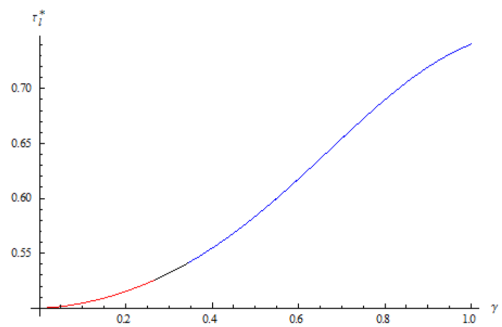


Figure γ-1

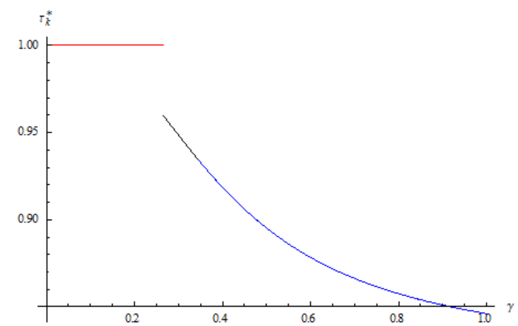


Figure γ-3

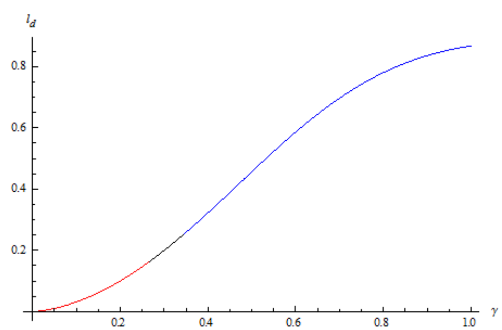


Figure γ-2

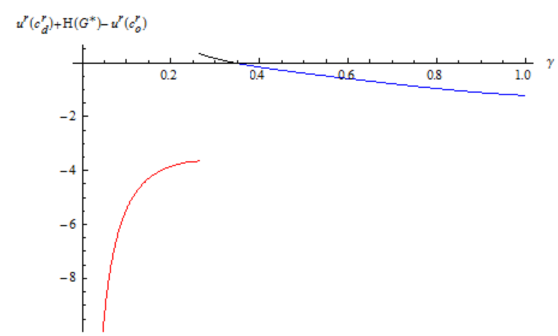


Figure γ-4

θ v.s. $(\tau_l^*, l_d, \tau_k^*)$ and $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\alpha = 0.3, \gamma = 0.7, \rho = 0.3, \sigma = 0.1, A = 1, B = 1, \frac{n^p}{n^p+n^r} = 0.95, \frac{n^r}{n^p+n^r} = 0.05, n^p + n^r = 10000, k^r = 100.$

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0,1)$; red: oligarchy sustains with $\tau_k^* = 1$.

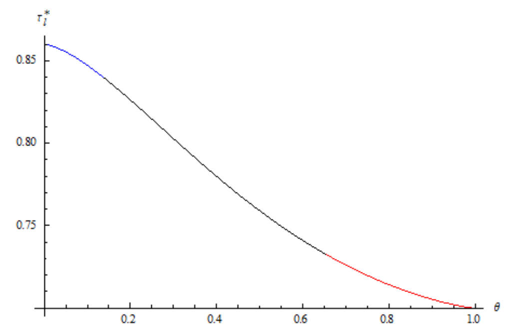


Figure θ-1

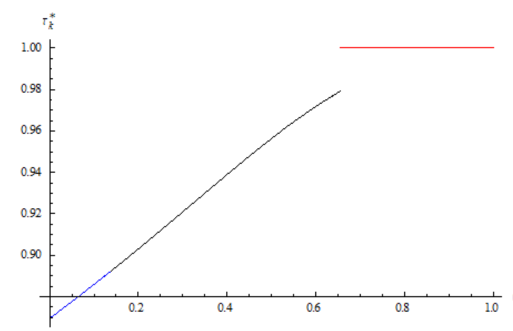


Figure θ-3

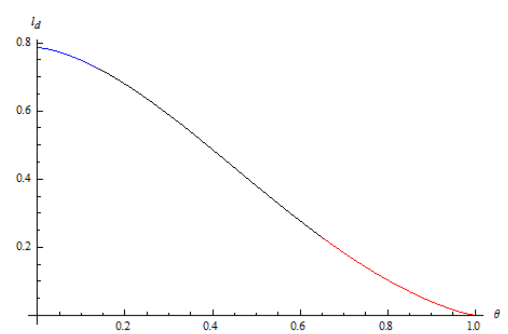


Figure θ-2

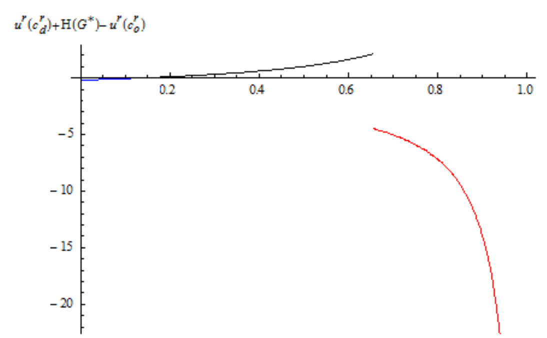


Figure θ-4

Comparative Statics 22. *Ceteris paribus, the poor provide abundant labor supply and impose a low capital gain tax rate on the rich under the democratic regime if:*

(iv): *The cohort size of the rich in minority is large* $\left(\frac{n^r}{n^p+n^r} < \frac{1}{2}\right.$ *is large*$\left.\right)$.

(v): *Each rich person is endowed with large capital stock* (k^r *is large*).

If under the democratic regime the poor work sufficiently hard and impose a low capital gain tax on the rich, the capacity of market production increases, so the rich can consume more and pay less tax. Delegating the power to enter the democratic regime becomes more attractive to the rich.

Suppose that the size of the rich class increases (i.e. the size of the poor class decreases). It increases the relative value of the poor's physical labor to the rich's capital endowment. Under the democratic regime in the absence of labor income tax, the increase in physical wage rate encourages the poor to work harder for market production (figure $\frac{n^r}{n^p+n^r}$-2). Their devotion to market production increases not only the capacity of market production but also the relative value of capital. In turn, they impose a lower capital tax rate but may enable to provide more of public goods (figure $\frac{n^r}{n^p+n^r}$-3).

Suppose that each rich person is endowed with larger amount of capital. It decreases the relative value of the rich's capital endowment to the poor's physical labor. If the rich delegate the power, the higher physical wage rate and the absence of labor income tax encourage the poor to work harder for market production (figure k^r-2). The capacity of market production increases; so does the relative value of capital. Therefore, the poor may impose a lower capital tax rate but collect sufficient tax revenue to provide significant public goods (figure k^r-3).

$\frac{n^r}{n^p+n^r}$ **v.s.** $(\tau_l^*, l_d, \tau_k^*)$ **and** $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\alpha = 0.3, \gamma = 0.7, \theta = 0.1, \rho = 0.3, \sigma = 0.1, A = 1, B = 1, \frac{n^p}{n^p+n^r} = 1 - \frac{n^r}{n^p+n^r}, n^p + n^r = 10000, k^r = 100.$

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0,1)$.

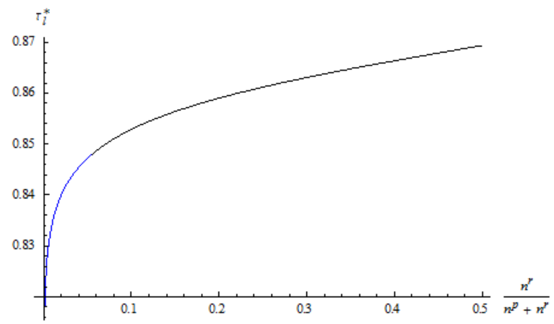


Figure $\frac{n^r}{n^p+n^r}$-1

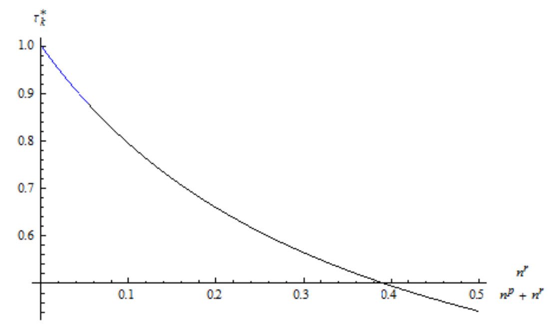


Figure $\frac{n^r}{n^p+n^r}$-3

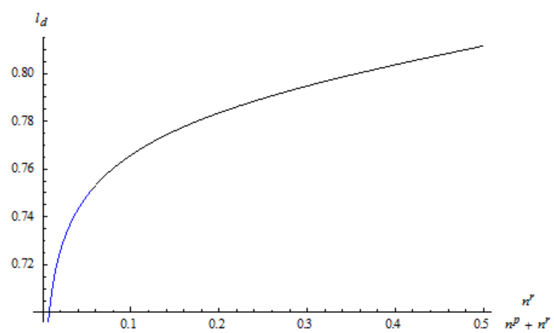


Figure $\frac{n^r}{n^p+n^r}$-2

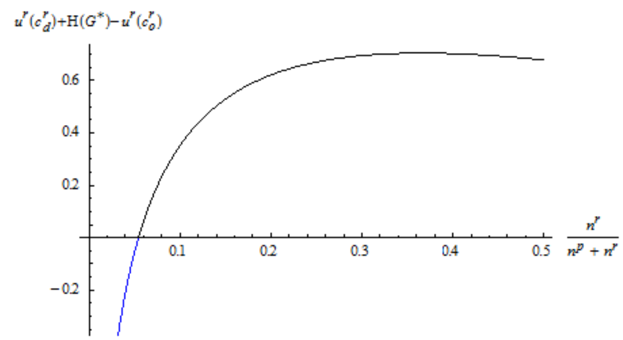


Figure $\frac{n^r}{n^p+n^r}$-4

k^r **v.s.** $\left(\tau_l^*, l_d, \tau_k^*\right)$ **and** $u^r\left(c_d^r\right) + H\left(G^*\right) - u^r\left(c_o^r\right)$

Let $\alpha = 0.3, \gamma = 0.3, \theta = 0.1, \rho = 0.3, \sigma = 0.1, A = 1, B = 1, \frac{n^p}{n^p+n^r} = 0.95, \frac{n^r}{n^p+n^r} = 0.05, n^p + n^r = 10000.$

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0, 1)$.

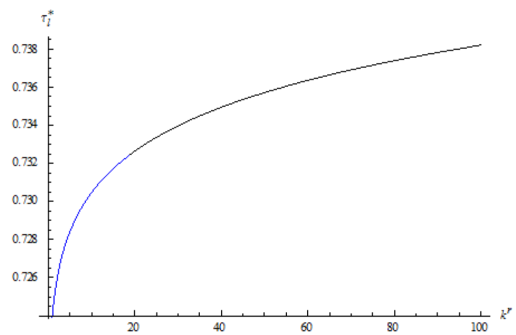


Figure k^r-1

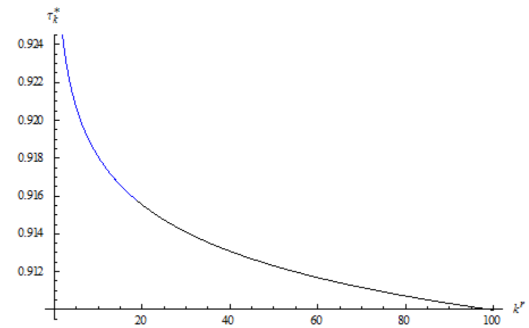


Figure k^r-3

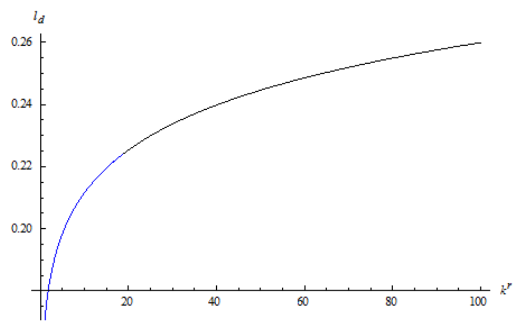


Figure k^r-2

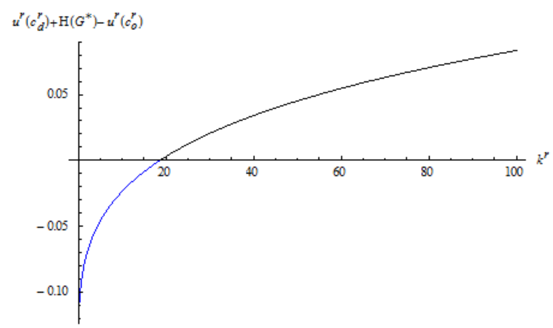


Figure k^r-4

Comparative Statics 23. *Ceteris paribus, the likelihood of democratization increases if:*

(i): *Market production is capital-intensive (α is large).*

(ii): *The poor prefer home production to market production (γ is small).*

(iii): *People prefer leisure to consumption (θ is large).*

(iv): *The cohort size of the rich class is large $\left(\frac{n^r}{n^p+n^r} < \frac{1}{2}\right.$ is large$\left.\right)$.*

(v): *Each rich person is endowed with large capital stock (k^r is large).*

(vi): *Public goods are appreciated (σ is large).*

(vii): *Population is large ($n^p + n^r$ is large).*

Comparative statics 21 explains that the poor are less devoted to market production if (i), (ii), or (iii) holds. It is also true under the democratic regime (figure α-2, figure γ-2, and figure θ-2), and what's more, the poor may abuse the tax power to impose a higher capital gain tax on the rich (figure α-3, figure γ-3, and figure θ-3). The rich certainly do not like being taken advantage of. But to each rich person, enjoying non-exclusive public goods under the democratic regime may be a better idea than sharing the diluted labor income tax revenue with others under the oligarchic regime. In this situation, the marginal utility of public goods dominates the marginal disutility of capital tax and oligarchic appropriation, and democratization is adopted (figure α-4, figure γ-4, and figure θ-4).

The rich's capital endowment becomes less valuable than the poor's labor supply if (iv) or (v) holds such that $\frac{n^r}{n^p}k^r$ increases. The rich's capital return thus decreases. Under the oligarchic regime, the rich excuse themselves for imposing a higher labor income tax on the poor to cover up their loss of capital gain (figure $\frac{n^r}{n^p+n^r}$-1 and figure k^r-1). By sharing the higher tax revenue, the rich are happy with the oligarchic payoff. However, if the rich delegate the power, the marginal utility of non-exclusive public goods financed by aggregate capital tax may dominate the marginal utility of diluted labor tax and make up the marginal disutility of lower capital tax payment. In this situation, the rich will like to democratize the society (figure $\frac{n^r}{n^p+n^r}$-4 and figure k^r-4).

It is trivial that democratization is more likely adopted if people appreciate public goods, or if public goods benefit greater civilian population. The left side of (4.49) strictly increases if σ or $n^p + n^r$ increases, while the right side remains constant because τ_l^* and l_d are independent of σ and $n^p + n^r$ by (4.46) and (4.47) (figure σ-1, figure $n^p + n^r$-1, figure σ-2, figure $n^p + n^r$-2). However, large σ or $n^p + n^r$ increases the demand for public goods; in turn, it increases the capital tax rate (figure σ-3, figure $n^p + n^r$-3). Even so, the increase in public goods may make up the utility loss because of higher capital tax. The rich will like to democratize the society (figure σ-4, figure $n^p + n^r$-4).

σ v.s. $(\tau_l^*, l_d, \tau_k^*)$ and $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\alpha = 0.3, \gamma = 0.7, \theta = 0.1, \rho = 0.3, A = 1, B = 1, \frac{n^p}{n^p+n^r} = 0.95, \frac{n^r}{n^p+n^r} = 0.05, n^p + n^r = 10000, k^r = 100.$

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0,1)$.

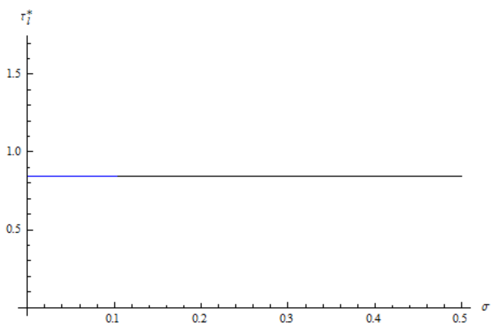


Figure σ-1

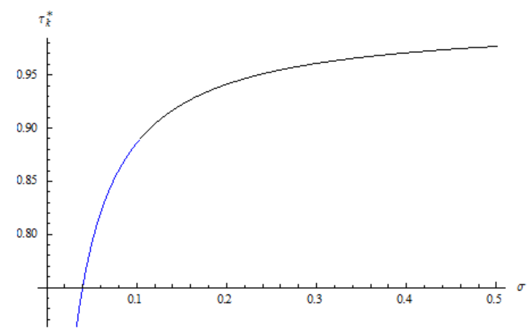


Figure σ-3

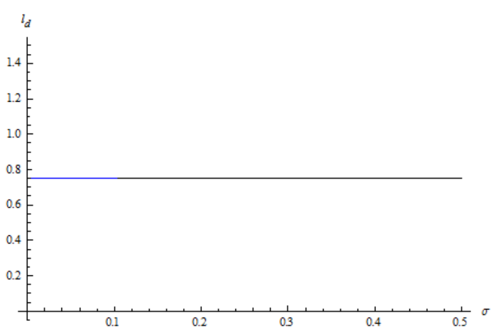


Figure σ-2

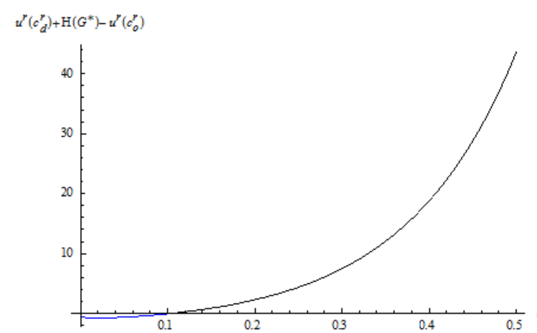


Figure σ-4

$n^p + n^r$ v.s. $(\tau_l^*, l_d, \tau_k^*)$ and $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\alpha = 0.3, \gamma = 0.3, \theta = 0.3, \rho = 0.3, \sigma = 0.1, A = 1, B = 1, \frac{n^p}{n^p+n^r} = 0.95, \frac{n^r}{n^p+n^r} = 0.05, k^r = 100$.

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0, 1)$.

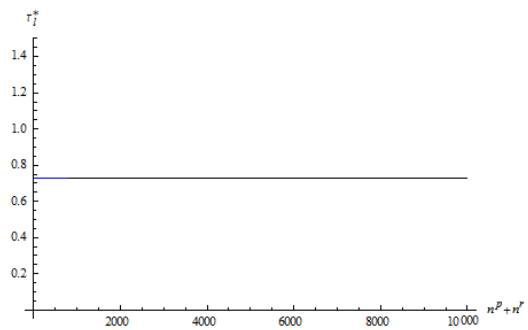


Figure $(n^p + n^r)$-1

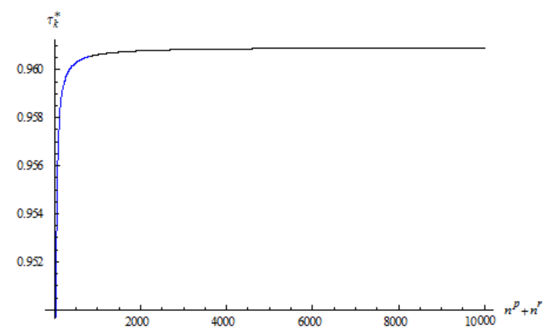


Figure $(n^p + n^r)$-3

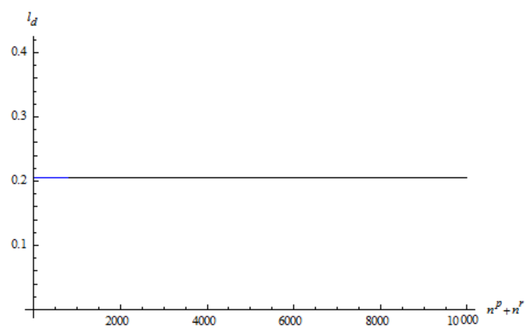


Figure $(n^p + n^r)$-2

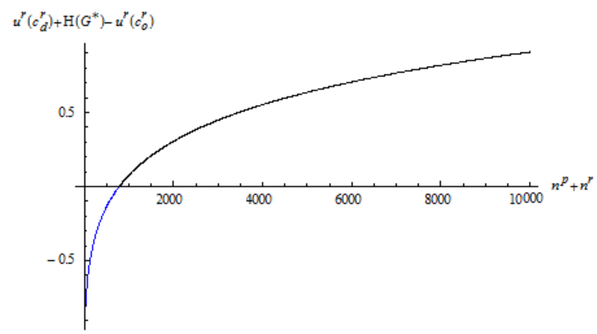


Figure $(n^p + n^r)$-4

4.2.7. Discussions

Consider $\tau_k^* \to 1$. Under the democratic regime, except $c_d^r \to 0$ and $G^* \to n^r \phi_3 l_d^{1-\alpha}$, all prices and allocations remain the same as before. See appendix for algorithm.

Comparative Statics 24. *Ceteris paribus, $\tau_k^* \to 1$ if:*

(i): *Market production is labor-intensive (α is small).*

(ii): *The poor significantly prefer home production to market production (γ is very small).*

(iii): *People significantly prefer leisure to consumption (θ is very large).*

(iv): *The cohort size of the rich is small ($\frac{n^r}{n^p+n^r} < \frac{1}{2}$ is small).*

(v): *Each rich person is endowed with small capital stock (k^r is small).*

(vi): *Public goods are significantly appreciated (σ is very large).*

(vii): *Population is significantly large ($n^p + n^r$ is very large).*

Labor-intensive production requires labor. The production firms have higher demand for labor and have lower demand for capital. The former promotes the wage rate and hire more labor hours; the latter devalues the rental rate. A lower capital return decreases the capital tax revenue, while there are more public goods available on the market. It gives the poor an excuse to increase the capital tax rate to purchase extra public goods. Therefore, when production is more labor-intensive, the poor tend to expropriate the rich (figure α-5).

The poor with small γ and large θ prefer idling around and taking advantage of the rich. As $\frac{\gamma}{\theta}$ significantly decreases, the poor tend to expropriate the rich to offset their idleness (figure (γ, θ)).

A decrease in the size of the rich class (i.e. an increase in the size of the poor class) increases the relative value of the rich's capital to the poor's labor. The poor hence work less at the market, the volume of market production decreases, and so does the public goods. To cover up their loss, the poor take advantage of the wealthy rich in minority by imposing a higher capital tax. Therefore, the poor tend to expropriate the rich as $\frac{n^r}{n^p+n^r}$ decreases (figure $\frac{n^r}{n^p+n^r}$-5).

The size of market production shrinks with the decrease in each rich person's capital endowment. Both wage payment and capital return decrease; the poor work less at the market; the volume of public goods decreases. To cover up their loss the poor take advantage of the poorer rich by imposing a higher capital tax. Therefore, the poor tend to expropriate the rich as k^r decreases (figure k^r-5).

It is trivial that $\tau_k^* \to 1$ more likely holds if people appreciate public goods, or if public goods benefit greater civilian population. Large σ or $n^p + n^r$ increases the demand for public goods, which in turn increases the capital tax rate. Therefore, the rich are more likely expropriated with the increase in either σ or $n^p + n^r$.

$\tau_k^* = 1$: α v.s. τ_k^* & $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\gamma = 0.3$, $\theta = 0.7$, $\rho = 0.1$, $\sigma = 0.5$, $A = 1$, $B = 1$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$, $k^r = 100$.

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0, 1)$; red: oligarchy sustains with $\tau_k^* = 1$.

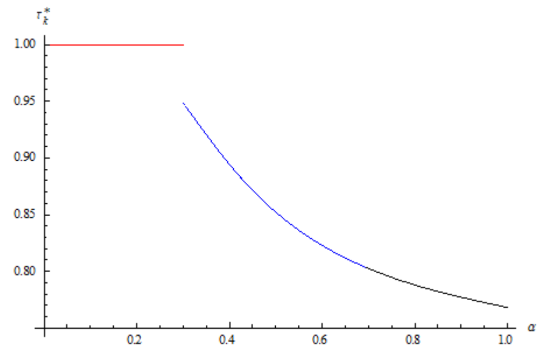


Figure α-5

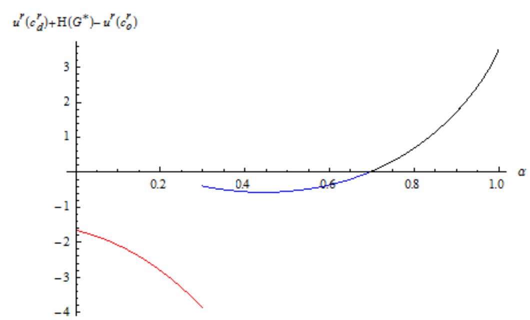


Figure α-6

$\tau_k^* = 1$: (γ, θ) v.s. τ_k^*

Let $\alpha = 0.3$, $\rho = 0.3$, $\sigma = 0.1$, $A = 1$, $B = 1$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$, $k^r = 100$.

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0, 1)$; red: oligarchy sustains with $\tau_k^* = 1$.

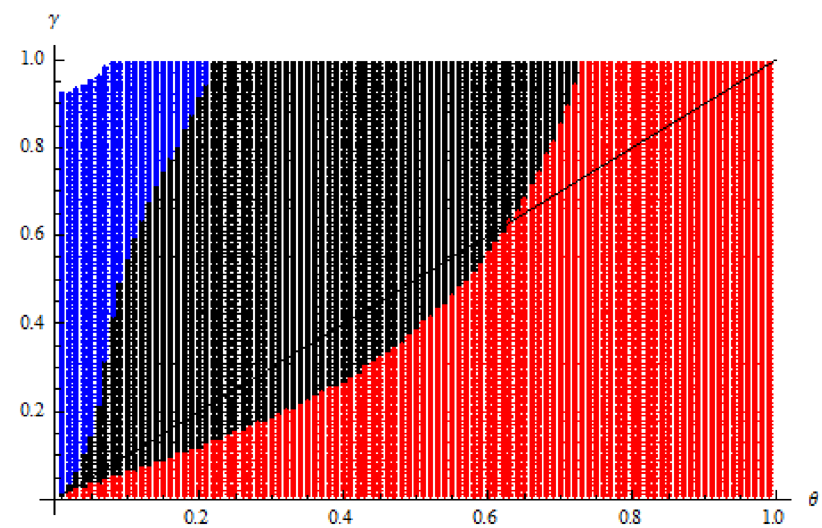


Figure (γ, θ)

$\tau_k^* = 1$: $\frac{n^r}{n^p+n^r}$ v.s. τ_k^* & $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$

Let $\alpha = 0.3$, $\gamma = 0.7$, $\theta = 0.3$, $\rho = 0.5$, $\sigma = 0.3$, $A = 1$, $B = 1$, $\frac{n^p}{n^p+n^r} = 1 - \frac{n^r}{n^p+n^r}$, $n^p + n^r = 10000$, $k^r = 100$.

Black: democratization is adopted; red: oligarchy sustains with $\tau_k^* = 1$.

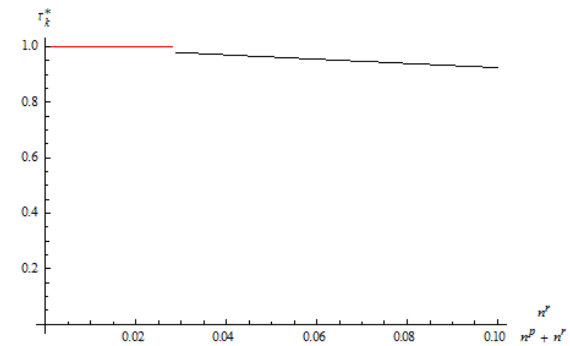


Figure $\frac{n^r}{n^p+n^r}$-5

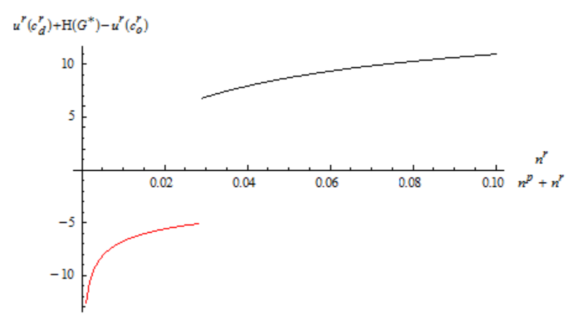


Figure $\frac{n^r}{n^p+n^r}$-6

$\tau_k^* = 1$**: k^r v.s. τ_k^* & $u^r(c_d^r) + H(G^*) - u^r(c_o^r)$**

Let $\alpha = 0.3$, $\gamma = 0.3$, $\theta = 0.1$, $\rho = 0.3$, $\sigma = 0.7$, $A = 1$, $B = 1$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$.

Black: democratization is adopted; red: oligarchy sustains with $\tau_k^* = 1$.

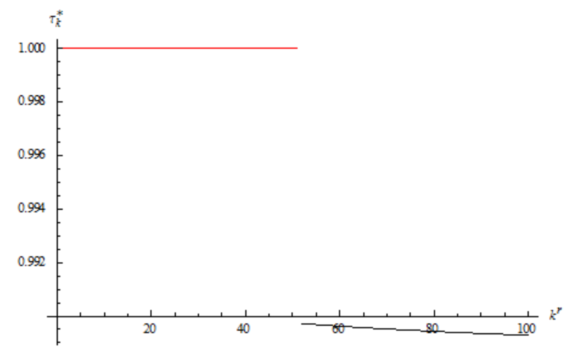


Figure k^r-5

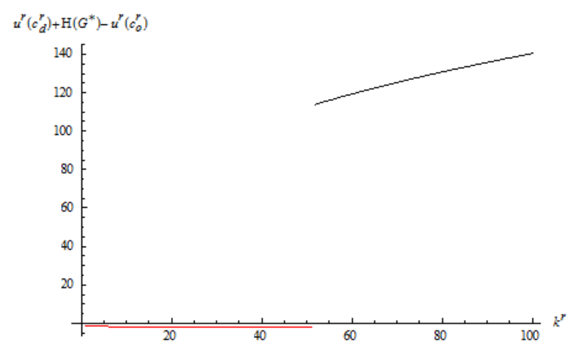


Figure k^r-6

Simulation Result 25. *Oligarchy sustains if* $\tau_k^* \to 1$.

No one likes expropriation taxes. It is especially true when the incentive behind the expropriation taxes is opportunism. The rich would prefer sustaining the oligarchic power and sharing the diluted labor income tax, rather than being expropriated by the opportunistic poor under the democratic regime. See figure x-5 and figure x-6 where $x \in \left\{\alpha, \gamma, \theta, \frac{n^r}{n^p+n^r}, k^r\right\}$.

The conditions for democratization is sensitive to the changes of economic primitives. Some special cases promote democratization but do not fit the simulation results in the previous sections. Given below are two examples:

Simulation Result 26. *Special cases for likelihood of democratization:*

(1) The likelihood of democratization increases when k^r *decreases, if* α *and* $\frac{\sigma}{\rho}$ *are small and* $\frac{\gamma}{\theta}$ *is large.*

(2) $\tau_k^* = 1$ *occurs when* α *increases, if* $\frac{n^r}{n^p+n^r}$ *and* k^r *are small and* $\frac{\gamma}{\theta}$ *is large.*

Consider the first case. The poor with large $\frac{\gamma}{\theta}$ and relatively large ρ prefer working to leisure, market production to home production, and they are sensitive to relative price changes. When the market production is labor-intensive, they work hard at the market. But when each rich person's capital endowment decreases, the size of market production shrinks and the firms hire fewer labor hours (figure k^r-special-2). The poor then turn to work at home. To avoid driving them further from the market, the rich will decrease the labor income tax under the oligarchic regime (figure k^r-special-1). The rich are therefore less interested in the oligarchic regime but more likely adopt democratization (figure k^r-special-4).

Consider the second case. The poor with large $\frac{\gamma}{\theta}$ prefer working to leisure and prefer market production to home production. They like to work hard at the market, but the value of their labor will depreciate because of their diligence and the market production's increasing dependence on capital. They then turn to work at home, and in the meantime, they tend to take advantage of the small size of the rich class with little capital endowment by imposing an extreme capital tax (figure α-special-1 and figure α-special-2).

Decrease in k^r may increase likelihood of democratization

Let $\alpha = 0.3$, $\gamma = 0.7$, $\theta = 0.1$, $\rho = 0.9$, $\sigma = 0.3$, $A = 1$, $B = 1$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$.

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0, 1)$.

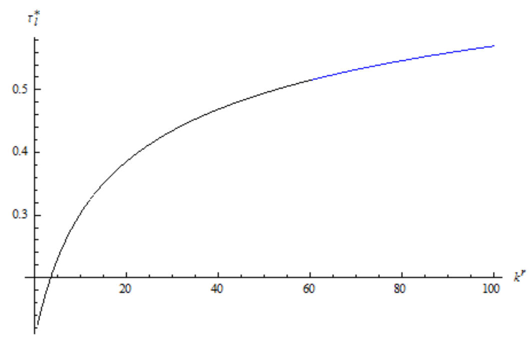


Figure k^r-special-1

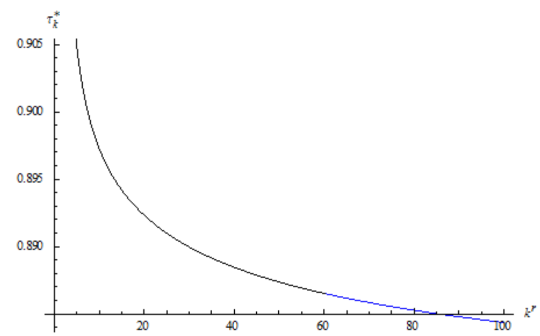


Figure k^r-special-3

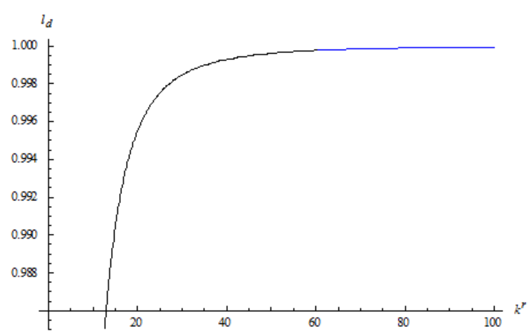


Figure k^r-special-2

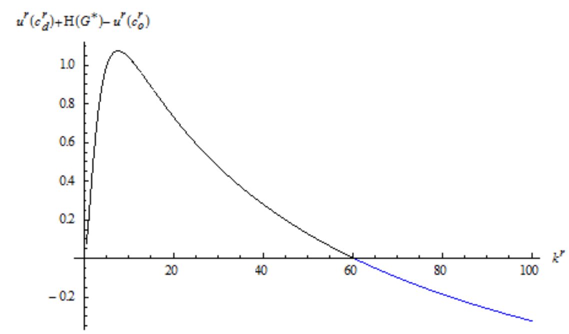


Figure k^r-special-4

$\tau_k^* = 1$ may occur when α increases

Let $\gamma = 0.9$, $\theta = 0.1$, $\rho = 0.7$, $\sigma = 0.9$, $A = 1$, $B = 1$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$, $k^r = 1$.

Black: democratization is adopted; red: oligarchy sustains with $\tau_k^* = 1$.

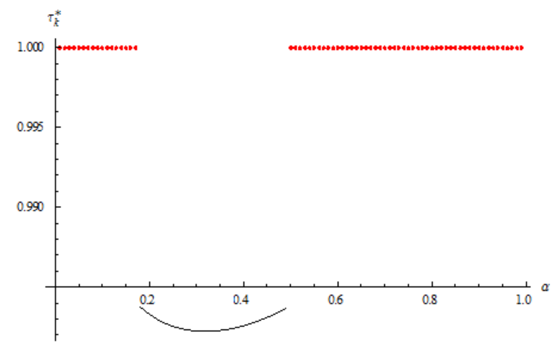


Figure α-special-1

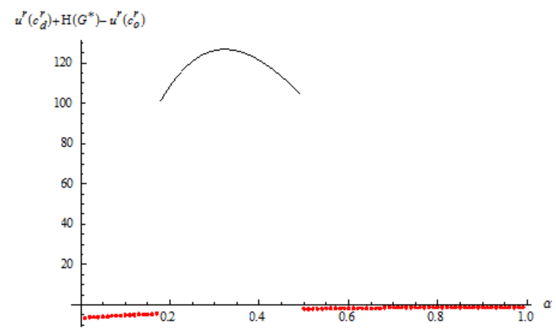


Figure α-special-2

4.2.8. Concluding Remarks

The relative size of (l, l_b) *depends on economic primitives, instead of regimes.* There is a common belief that under a particular regime people work harder for one type of production than the other. The following claim shows that it is not true. The poor's devotion to a particular type of production depends on how much the poor like to work (θ), how much the poor prefer working for one production to another (γ), how sensitive the poor are to relative price change (ρ), etc.

Claim 27. *The relative size of* (l, l_b) *depends on parameters:*

(1) $l_d \gtreqless l_{b,d}$ *if and only if* $\left(\frac{\phi_0}{1+2\phi_0}\right)^{\frac{\alpha\rho}{1-\rho}} \lesseqgtr \frac{1+\phi_0}{\phi_0}\phi_1\phi_2$.

(2) $l_o \gtreqless l_{b,o}$ *if and only if* $\left(\frac{\phi_0}{1+2\phi_0}\right)^{\frac{\alpha\rho}{1-\rho}} \lesseqgtr \frac{1+\phi_0}{\phi_0}\phi_1\phi_2\left[\frac{\rho(1+\phi_0)}{1+\phi_0+(1-\rho)\phi_0}\right]^{\frac{\rho}{1-\rho}}$.

The presence of home production refrains the rich to appropriate the poor; the absence of an analogic tax haven for the rich encourages the poor to take advantage of the rich. The former is more prominent if α and θ are large or γ is small, while the latter may result in $\tau_k^* \to 1$ on the rich. The extreme tax may be avoided if the poor tax themselves too under the democratic regime.

(A, B, ρ) *do not have monotonic effects on democratization and* $\tau_k^* \to 1$. For instance, figure (A, B)-1 shows that, regardless of B, democratization is adopted if A is in a *moderate* range, but figure (A, B)-2 shows that democratization is adopted if A is *sufficiently large.* On the other hand, figure (A, B)-3 indicates that there is some positive relationship between (A, B) for democratization. The effect of ρ is also indecisive. See appendix for a primitive explanation. Further research is required.

(A, B) v.s. democratization and oligarchy

Let $\alpha = 0.3$, $\gamma = 0.7$, $\theta = 0.5$, $\rho = 0.5$, $\sigma = 0.1$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$, $k^r = 100$.

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0, 1)$; red: oligarchy sustains with $\tau_k^* = 1$.

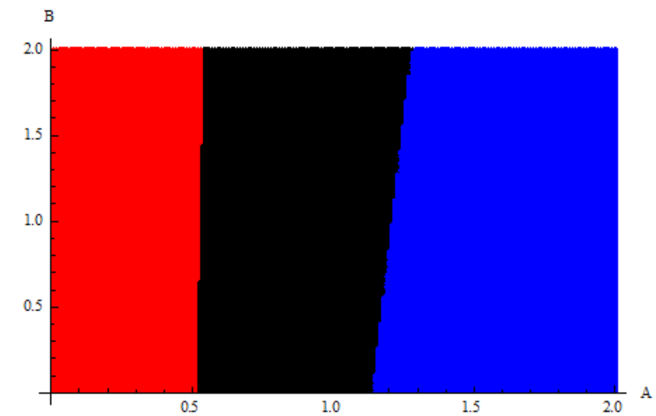


Figure (A, B)-1

(A, B) v.s. democratization and oligarchy

Let $\alpha = 0.3$, $\gamma = 0.7$, $\theta = 0.3$, $\rho = 0.7$, $\sigma = 0.5$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$, $k^r = 100$.

Black: democratization is adopted; red: oligarchy sustains with $\tau_k^* = 1$.

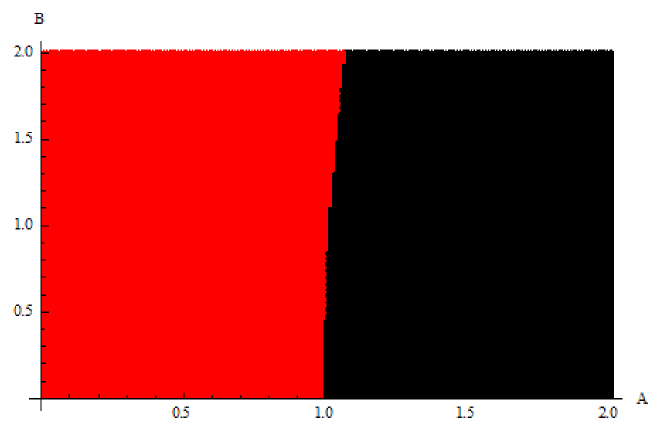


Figure (A, B)-2

(A, B) v.s. democratization and oligarchy

Let $\alpha = 0.3$, $\gamma = 0.1$, $\theta = 0.1$, $\rho = 0.5$, $\sigma = 0.1$, $\frac{n^p}{n^p+n^r} = 0.95$, $\frac{n^r}{n^p+n^r} = 0.05$, $n^p + n^r = 10000$, $k^r = 100$.

Black: democratization is adopted; blue: oligarchy sustains with $\tau_k^* \in (0, 1)$; red: oligarchy sustains with $\tau_k^* = 1$.

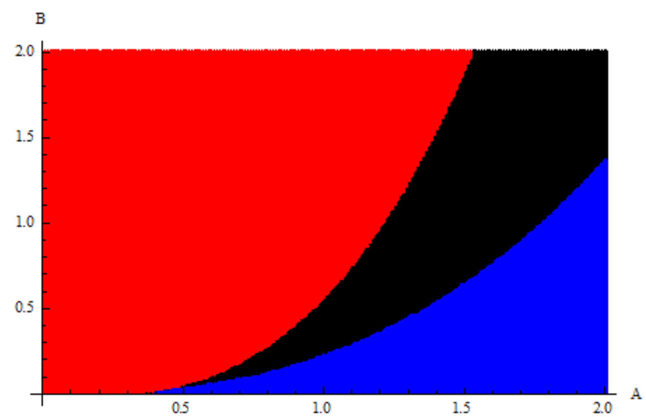


Figure (A, B)-3

Further research. So far, I assume that the rich can credibly commit to the policy they decide. But in reality, there is no such credible commitment in political economy (Przeworski and Limongi 1993). I am going to incorporate this feature into the model in the next essay.

CHAPTER 5

A Two-period Model Without Commitment

Commitment is a general assumption in optimal taxation literature (Ramsey 1927, Mirrlees 1971, Chamley 1986, Chari et al. 1994). It is defined as ability of a policymaker to make binding policy choices (Golosov and Tsyvinski 2008). But in reality the existence of such assumption is questioned because a government can change the fiscal policy any time in compliance with legislative procedure. Therefore, the timing without commitment is more plausible in political economy (Przeworski and Limongi 1993).

Suppose that the rich cannot commit under the oligarchic regime. To capture this idea, I consider a two-period economy and a savings tax imposed on the poor in period one only. After the markets clear in period one, the poor's savings are determined; before the markets clear in period two, the rich decide the tax after observing the poor's savings. If the rich choose to delegate the power, they prefer the equilibrium with the tax chosen by the poor. To the poor, democratization is endogenized as a credible commitment to their preferred tax.

5.1. Model

Consider a two-period closed Arrow-Debreu economy with two types of consumers, investment firms, and production firms. Figure 5.1 shows the circular flow of the economy.

There are n^p homogeneous poor persons (superscript p, he) and n^r homogeneous rich persons (superscript r, she), who are the minority in the society ($n^r < n^p$). The

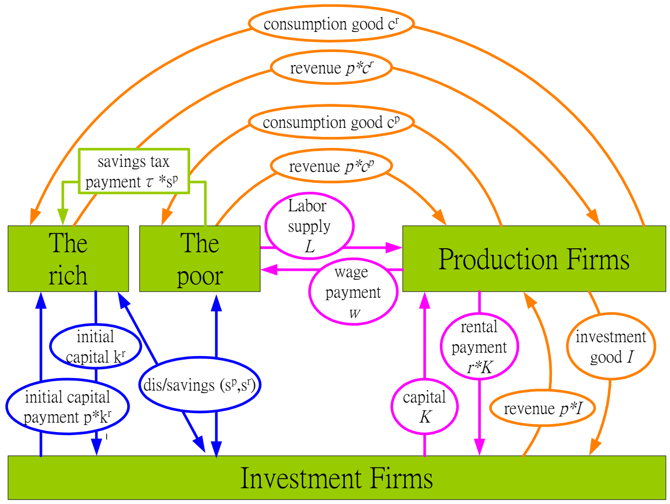


Figure 5.1. The Circular Flow of The Economy

rich are endowed with capital stock K; the poor are the labor suppliers L. The rich are also endowed with political power to impose a savings tax τ on the poor's first-period savings S_1^p. Once they collect the tax, they equally share the total tax revenue among themselves. The production firms hire the poor and borrow capital from the investment firms to produce a final good Y. The final good is convertible. It is a consumption good to the consumers or an investment good to the investment firms. The investment firms govern capital formation. They collect the rich's capital endowment and the consumers' savings, and purchase the investment good for capital accumulation.

Figure 5.2 explains the timing of the economic activities in this Arrow-Debreu economy. Consider period one. Production Y_1 happens overnight. It uses the rich's capital endowment K_1 and the poor's effective labor supply L_1^e. The consumers then make their consumption decisions (C_1^r, C_1^p) and the savings decisions (S_1^r, S_1^p). If they save, they deposit their savings in the investment firms, and the firms use their savings to purchase more investment goods. If they over spend, they borrow from the investment firms, and

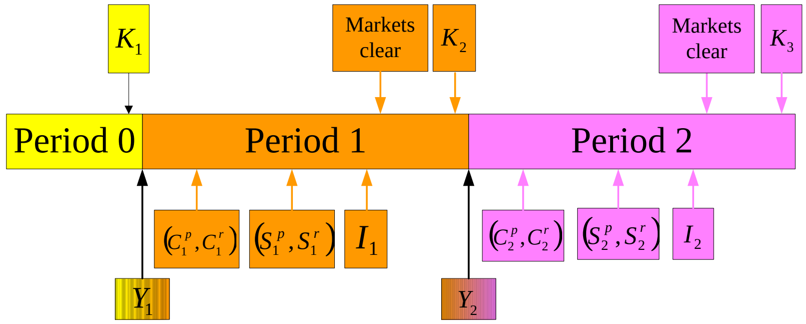


Figure 5.2. The Timing of The Economic Activities

the firms retrieve part of the capital in circulation from the market to loan the poor. In either case, after the markets clear, the investment firms have the capital stock ready for next-period production via the capital accumulation process $K_2 = I_1 + (1 - \delta) K_1$. The economy enters period two hereafter and repeat the above economic activities.

To examine the possibility of democratization, consider two political regimes: the no-commitment regime and the delegation regime. Under the no-commitment regime, the rich have the tax power but do not commit to any tax policies. Under the delegation regime, the poor are granted the tax power.

At the status quo (the no-commitment regime), the rich with the tax power can choose to either delegate the power to the poor or sustain the no-commitment regime. If the rich delegate the power, the society goes under the delegation regime and the poor choose the savings tax $\tau \in [0, 1]$. If the rich sustain the no-commitment regime instead, they decide the savings tax $\tau \in [0, 1]$ at the beginning of period two, after the poor's period-one decisions have been made, to capture the idea that they do not commit. This game is solved by backward induction. When all markets clear at the end of each regime, the consumers' payoff/utility are revealed.

For simplicity, I make the following assumptions:

(A1): All markets are perfectly competitive.

(A2): All economic agents are price-takers.

(A3): Tax only. No subsidy or over-taxation.

(A4): Natural log utility functions of consumption and leisure: $\sum_{t=1}^{2} \beta^{t-1} u(c_t, 1 - l_t) = \sum_{t=1}^{2} \beta^{t-1} [(1 - \theta) \ln c_t + \theta \ln (1 - l_t)]$, where $\beta \in (0, 1]$ and $\theta \in (0, 1)$.

(A5): Cobb-Douglas market production technology: $Y = AK^{\alpha} (L^{e})^{1-\alpha}$, where $A \in \mathbb{R}_{++}$ and $\alpha \in (0, 1)$.

(A6): In period one, aggregate effective labor is aggregate physical labor: $L_1^e = L_1$. In period two, aggregate effective labor is aggregate physical labor augmented by a labor productivity $z \in \mathbb{R}_{++}$: $L_2^e = zL_2$.

(A3) implies $\tau \in [0, 1]$. β captures how much the consumer values the happiness/utility of period two relative to period one. θ measures how much the consumer prefers leisure to consumption. A measures the total factor productivity of production. α is the capital share of production, which measures how much the production technology depends on capital.

5.2. Consumers

Consider the behaviors of the representative rich/poor person only by homogeneity within their kind. The representatives' decision-makings will be discussed by the regime type.

5.2.1. The no-commitment regime

The representative rich person (she) has two inherent gifts. She is endowed with initial capital stock k^r and has the political power to decide the savings tax τ. If she chooses to stay under the no-commitment regime, this tax power remains in the elite hierarchy.

Before production occurs, she sells her ownership of capital to the investment firms and obtain initial wealth $p_0 k^r$. To purchase consumption goods, she may expand her budget set by expropriating the poor via her tax power. Denote s_t^p to be the poor person's savings in period $t \in \{1, 2\}$, which are:

$$s_t^p = w_t l_t - p_t c_t^p,$$

such that the poor's aggregate savings are:

$$S_t^p = n^p s_t^p.$$

After observing the poor's saving behavior, she takes prices (p_0, p_1, p_2) and the poor's savings S_1^p given, so as to maximize her utility subject to her budget. The formal utility maximization problem (UMP) is:

$$\underset{\tau, c_1^r, c_2^r}{Maximize} \;\; \ln c_1^r + \beta \ln c_2^r$$

$$subject\ to \begin{cases} p_0 k^r + \frac{1}{n^r}\tau S_1^p - p_1 c_1^r - p_2 c_2^r = 0 \\ 0 \leq \frac{1}{n^r}\tau S_1^p \\ 0 \leq \tau \leq 1 \\ (p_0, p_1, p_2, S_1^p) \text{ given,} \end{cases}$$

where p_0 is the given price of initial capital and (p_1, p_2) are the prices of consumption goods in period one and in period two[1].

If the poor save $S_1^p \geq 0$, the representative rich person will impose a 100% tax on the poor to increase her budget. But if the poor borrow $S_1^p < 0$, she will choose a zero tax so as not to transfer resources to the poor. Her demands for consumption goods depend on her total wealth (the initial wealth and the tax revenues) and the prices of consumption goods. The formal expressions of her demands and tax decision are:

$$\tau = \begin{cases} 0 \text{ if } S_1^p < 0 \\ 1 \text{ if } S_1^p \geq 0 \end{cases} \tag{5.1}$$

$$c_t^r = \frac{\beta^{t-1}}{1+\beta} \frac{p_0 k_1^r + \frac{1}{n^r}\tau S_1^p}{p_t}, \tag{5.2}$$

for $t \in \{1, 2\}$. The tax decision rule (5.1) shows that the rich would expropriate the poor if they do not commit.

[1]Utility functions $(1-\theta)\ln c_1^r + \beta(1-\theta)\ln c_2^r$ and $(1-\theta)\ln c_1^r + \beta(1-\theta)\ln c_2^r$ describe the same ordinal ranking of preference.

The representative poor person (he) likes consumption goods (c_1^p, c_2^p) and leisure $(1 - l_1, 1 - l_2)$. He is not endowed with any initial capital, but he is willing to work (l_1, l_2) to afford consumption goods. For each unit of his labor supply, he is paid w_t for $t \in \{1, 2\}$.

Under the no-commitment regime, the poor person does not have bargaining power over the tax rate but chooses his savings. Recall $s_1^p = w_1 l_1 - p_1 c_1^p$. For $t \in \{1, 2\}$, take the wage rates and the prices of consumption goods (w_t, p_t) given, he maximizes his utility subject to his after-tax budget and the rich's tax decision rule. The formal UMP is:

$$\underset{c_t^p, l_t}{Maximize} \ \sum_{t=1}^{2} \beta^{t-1} \left[(1-\theta) \ln c_t^p + \theta \ln (1 - l_t)\right]$$

$$subject\ to \begin{cases} (1-\tau)\left(w_1 l_1 - p_1 c_1^p\right) + w_2 l_2 - p_2 c_2^p = 0 \\ \tau = \begin{cases} 0 \text{ if } w_1 l_1 - p_1 c_1^p < 0 \\ 1 \text{ if } w_1 l_1 - p_1 c_1^p \geq 0 \end{cases} \\ (w_1, w_2, p_1, p_2) \text{ given.} \end{cases}$$

If the rich choose $\tau = 0$ and w_2 is sufficiently larger than w_1 $\left(\beta < \frac{w_2}{w_1}\right)$, the poor person's demands and labor supply (c_t^p, l_t) for $t \in \{1, 2\}$ are:

$$c_t^p = \frac{\beta^{t-1}(1-\theta)}{1+\beta} \frac{\sum_{t=1}^{2} w_t}{p_t} \tag{5.3}$$

$$l_t = 1 - \frac{\beta^{t-1}\theta}{1+\beta} \frac{\sum_{t=1}^{2} w_t}{w_t}, \tag{5.4}$$

such that $s_1^p < 0$. However, if w_1 is sufficiently larger than w_2 $\left(\frac{w_2}{w_1} \leq \beta\right)$ when the rich choose $\tau = 0$, the poor person's choices in (5.3) and (5.4) give $s_1^p \geq 0$. The rich will deviate to impose a 100% tax instead, and the poor will change their minds as well.

If the rich choose $\tau = 1$, the poor person should spend every penny he earns at the end of period one so pay nothing to the rich. In this case, he works equal hours in both periods and squanders his paycheck of the period on consumption goods. His demands for consumption goods and labor supply are formally expressed as follows:

$$c_t^p = (1-\theta)\frac{w_t}{p_t} \tag{5.5}$$

$$l_t = 1-\theta, \tag{5.6}$$

for $t \in \{1,2\}$.

5.2.2. The delegation regime

Under the delegation regime, the poor decide the tax rate and the tax payment, and the rich lose the bargaining power over the tax matters. Recall $S_1^p = n^p s_1^p = n^p \left(w_1 l_1 - p_1 c_1^p\right)$. Take the prices of consumption goods and the poor's tax payment as given, the representative rich person's UMP is:

$$\underset{c_1^r, c_2^r}{Maximize} \ \ln c_1^r + \beta \ln c_2^r$$
$$subject\ to \begin{cases} p_0 k^r + \frac{1}{n^r}\tau S_1^p - p_1 c_1^r - p_2 c_2^r = 0 \\ \left(p_0, p_1, p_2, \tau S_1^p\right) \text{ given,} \end{cases}$$

where $p_0 k^r + \frac{1}{n^r}\tau S_1^p$ is her total wealth. Her demands for consumption goods are:

$$c_t^r = \frac{\beta^{t-1}}{1+\beta} \frac{p_0 k^r + \frac{1}{n^r}\tau S_1^p}{p_t}, \tag{5.7}$$

for $t \in \{1, 2\}$.

The UMP of the representative poor person with tax power is:

$$\underset{\tau, c_t^p, l_t}{Maximize} \sum_{t=1}^{2} \beta^{t-1} \left[(1-\theta)\ln c_t^p + \theta \ln(1-l_t)\right]$$

$$subject\ to \begin{cases} (1-\tau)\left(w_1 l_1 - p_1 c_1^p\right) + w_2 l_2 - p_2 c_2^p = 0 \\ \tau \in [0,1] \\ (w_1, w_2, p_1, p_2)\ \text{given.} \end{cases}$$

It is straightforward that the representative poor person will impose a zero tax on himself, regardless of the timing of the tax decision and his savings behaviors. Then he can freely choose how many hours to work and what amount of consumption goods to afford in each period. For $t \in \{1, 2\}$, his choices are:

$$\tau = 0 \tag{5.8}$$

$$c_t^p = \frac{\beta^{t-1}(1-\theta)}{1+\beta} \frac{\sum_{t=1}^{2} w_t}{p_t} \tag{5.9}$$

$$l_t = 1 - \frac{\beta^{t-1}\theta}{1+\beta} \frac{\sum_{t=1}^{2} w_t}{w_t}, \tag{5.10}$$

regardless of $s_1^p \in \mathbb{R}$.

5.3. Firms and Market Clearing

There are two types of firms. The production firms control the production of the convertible final goods; the investment firms govern capital formation.

The production firms are profit-maximizers. They rent capital K_t from the investment firms and hire labor L_t^e from the poor households to produce final goods. For each unit of output they earn p_t from the buyers; for each unit of capital they pay r_t to the investment firms; for each unit of effective labor they pay w_t^e to the poor. Their formal profit maximization problem (PMP) is:

$$\underset{K_t, L_t^e}{Maximize} \ \sum_{t=1}^{2} p_t A K_t^{\alpha} \left(L_t^e\right)^{1-\alpha} - r_t K_t - w_t^e L_t^e.$$

The first-order conditions are:

$$r_t = p_t \alpha A K_t^{\alpha-1} \left(L_t^e\right)^{1-\alpha} \tag{5.11}$$

$$w_t^e = p_t \left(1-\alpha\right) A K_t^{\alpha} \left(L_t^e\right)^{-\alpha} \tag{5.12}$$

$$w_t = z^{t-1} w_t^e, \tag{5.13}$$

where w_t is the physical wage rate. In period t, each unit of physical labor becomes z^{t-1}-fold more productive. Hence, marginal physical labor hour should be paid z^{t-1}-fold higher than marginal effective labor hour.

The price of an input equals the market value of its marginal product. For instance, if one unit of increase in capital increases output by $MP_{K_t} = \alpha A K_t^{\alpha-1} \left(L_t^e\right)^{1-\alpha}$ units, the market value of this marginal product is $p_t MP_{K_t}$. If r_t, the price of this additional unit of capital, is higher than its market value, the firms will rent less capital; if r_t is lower than its market value, the firms will like to rent more capital. In either case, the firms will adjust the capital rental until r_t equals the market value of the marginal product of capital. The same logic applies to the wage rates w_t^e.

The investment firms are profit maximizers. At the beginning, they pay the rich p_0K_1 to collect their initial capital for production. In each period, they collect the poor's savings or loan to the poor and purchase investment goods for capital accumulation $(I_t > 0)$ or retrieve the capital in circulation from the market $(I_t < 0)$. At the end of each period, they have newly accumulated capital stock ready for the production firms to rent in the next period. By doing so, they receive capital rent r_tK_t from the production firms every period. Their profit maximization problem is therefore:

$$\underset{I_1, I_2}{Maximize} \; -p_0K_1 + r_1K_1 - p_1I_1 + r_2K_2 - p_2I_2,$$

where $K_{t+1} = I_t + (1-\delta)\,K_t$.

The investment firms decide the size of investment according to three rules. First, the marginal cost of investment good that the investment firms pay today must equal the marginal benefit they will receive in the future. Second, because there is no period three and the investment goods are convertible, it is unnecessary to leave any capital in circulation at the end of period two. The investment firms will retrieve all after-depreciation capital from the market at the end of period two and sell it to the consumers as consumption goods. The retrieval then becomes additional benefit of the investment to the investment firms. Third, all investment firms make zero profit at the end of period two. Therefore, the investment decisions of the investment firms must satisfy the following conditions:

$$p_0 = r_1 + (1-\delta) r_2 + (1-\delta)^2 p_2 \quad (5.14)$$

$$p_1 = r_2 + (1-\delta) p_2. \quad (5.15)$$

In each period, there are three markets: final goods, labor, and capital markets. Given the initial capital stock $K_1 = n^r k^r$, the market clearing conditions are:

$$Y_t = n^p c_t^p + n^r c_t^r + I_t \quad (5.16)$$

$$L_t^e = z^{t-1} n^p l_t \quad (5.17)$$

$$K_{t+1} = I_t + (1-\delta) K_t, \quad (5.18)$$

where $Y_t = AK_t^{\alpha} (L_t^e)^{1-\alpha}$ and $K_3 = 0$.

5.4. Characterization of General Equilibrium

Multiple general equilibria (GE) may be characterized under each regime. Each GE contains a tax rate τ, a set of prices (p_t, w_t^e, w_t, r_t), and a set of allocations (l_t, c_t^p, c_t^r, I_t, K_{t+1}, L_t, L_t^e, C_t^p, C_t^r, Y_t), in which all economic agents act optimally and all markets clear. For simplicity, assume that:

(A7): Capital depreciates at 100% rate: $\delta = 1$.

Claim 28. *(The uniqueness of GE when $\delta = 1$)*

(1) Under the delegation regime, there is a unique general equilibrium in which $\tau = 0$, and the poor save in period one.

(2) Under the no-commitment regime, there is a unique general equilibrium in which $\tau = 1$, and the poor neither save nor borrow in both periods.

Denote $x^i_{t,g}$ to be the GE value of variable x (of consumer i) in period t under regime g, where $i \in \{p, r\}$, $t \in \{1, 2\}$ and $g \in \{d, nc\}$. It is straightforward that $\tau = 0$ constructs a unique GE under the delegation regime. Combine $\tau = 0$ and equation (5.7) ~(5.18). Table 5.1, column 3 and column 4 summarize the GE in reduced form under the delegation regime. By table 5.1, $s^p_{1,d} > 0$[2].

Under the no-commitment regime, two tax rates would construct general equilibrium: $\tau = 0$ and $\tau = 1$. Suppose that $\tau = 0$. If so, $\tau = 0$ must be *ex-ante* and *ex-post* optimal. But claim 28, (1) states that the poor save when $\tau = 0$. The rich will deviate to $\tau = 1$ after the poor save. So $\tau = 0$ is not *ex-post* optimal. On the contrary, $\tau = 1$ such that $s^p_1 = 0$ is *ex-ante* and *ex-post* optimal, because given their opponents' actions, neither the rich nor the poor will deviate. Therefore, $\tau = 1$ generates the unique GE. Combine $\tau = 1$ and equation (5.2), (5.5), (5.6), and (5.11) ~(5.18). Table 5.1, column 1 and column 2 summarize the GE in reduced forms under the no-commitment regime.

5.5. Condition for Democratization

Based on the previous discussions, it is clear that the rich will adopt democratization if and only if they are better off with the GE under the delegation regime.

[2]It is because the poor's utility function is Cobb-Douglas type. If their preferences are described by other utility functions, such as CES utility functions, the poor may not always save under the delegation regime when $\delta = 1$.

	The No-Commitment Regime		The Delegation Regime
τ_{nc}	1	τ_d	0
$l_{1,nc}$	$1-\theta$	$l_{1,d}$	$\frac{(1+\alpha\beta)(1+\beta)(1-\theta)}{(1+\alpha\beta)(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$
$l_{2,nc}$	$1-\theta$	$l_{2,d}$	$\frac{(1+\beta)(1-\theta)}{(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$
$K_{2,nc}=I_{1,nc}$	$\frac{\alpha\beta}{1+\beta}AK_1^{\alpha}L_{1,nc}^{1-\alpha}$	$K_{2,d}=I_{1,d}$	$\frac{\alpha\beta}{1+\alpha\beta}AK_1^{\alpha}L_{1,d}^{1-\alpha}$
$c_{1,nc}^p$	$\frac{1-\alpha}{n^p}AK_1^{\alpha}L_{1,nc}^{1-\alpha}$	$c_{1,d}^p$	$\frac{1+\alpha\beta+\beta}{(1+\alpha\beta)(1+\beta)}\frac{1-\alpha}{n^p}AK_1^{\alpha}L_{1,d}^{1-\alpha}$
$c_{2,nc}^p$	$\frac{1-\alpha}{n^p}AK_{2,nc}^{\alpha}\left(L_{2,nc}^e\right)^{1-\alpha}$	$c_{2,d}^p$	$\frac{1+\alpha\beta+\beta}{1+\beta}\frac{1-\alpha}{n^p}AK_{2,d}^{\alpha}\left(L_{2,d}^e\right)^{1-\alpha}$
$c_{1,nc}^r$	$\frac{\alpha}{1+\beta}\frac{1}{n^r}AK_1^{\alpha}L_{1,nc}^{1-\alpha}$	$c_{1,d}^r$	$\frac{\alpha}{1+\beta}\frac{1}{n^r}AK_1^{\alpha}L_{1,d}^{1-\alpha}$
$c_{2,nc}^r$	$\frac{\alpha}{n^r}AK_{2,nc}^{\alpha}\left(L_{2,nc}^e\right)^{1-\alpha}$	$c_{2,d}^r$	$\frac{1+\alpha\beta}{1+\beta}\frac{\alpha}{n^r}AK_{2,d}^{\alpha}\left(L_{2,d}^e\right)^{1-\alpha}$
$L_{1,nc}^e=L_{1,nc}$	$n^p(1-\theta)$	$L_{1,d}^e=L_{1,d}$	$\frac{(1+\alpha\beta)(1+\beta)(1-\theta)n^p}{(1+\alpha\beta)(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$
$Y_{1,nc}$	$AK_1^{\alpha}L_{1,nc}^{1-\alpha}$	$Y_{1,d}$	$AK_1^{\alpha}L_{1,d}^{1-\alpha}$
$p_{1,nc}$	$\frac{1}{\alpha AK_1^{\alpha-1}L_{1,nc}^{1-\alpha}}p_0$	$p_{1,d}$	$\frac{1}{\alpha AK_1^{\alpha-1}L_{1,d}^{1-\alpha}}p_0$
$w_{1,nc}^e=w_{1,nc}$	$\frac{1-\alpha}{\alpha}\frac{K_1}{L_{1,nc}}p_0$	$w_{1,d}^e=w_{1,d}$	$\frac{1-\alpha}{\alpha}\frac{K_1}{L_{1,d}}p_0$
$r_{1,nc}$	p_0	$r_{1,d}$	p_0
$K_{3,nc}=I_{2,nc}$	0	$K_{3,d}=I_{2,d}$	0
$L_{2,nc}^e$	$zn^p(1-\theta)$	$L_{2,d}^e$	$\frac{(1+\beta)(1-\theta)zn^p}{(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$
$L_{2,nc}$	$n^p(1-\theta)$	$L_{2,d}$	$\frac{(1+\beta)(1-\theta)n^p}{(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$
$Y_{2,nc}$	$AK_{2,nc}^{\alpha}\left(L_{2,nc}^e\right)^{1-\alpha}$	$Y_{2,d}$	$AK_{2,d}^{\alpha}\left(L_{2,d}^e\right)^{1-\alpha}$
$p_{2,nc}$	$\frac{1}{\alpha AK_{2,nc}^{\alpha-1}\left(L_{2,nc}^e\right)^{1-\alpha}}p_{1,nc}$	$p_{2,d}$	$\frac{1}{\alpha AK_{2,d}^{\alpha-1}\left(L_{2,d}^e\right)^{1-\alpha}}p_{1,d}$
$w_{2,nc}^e$	$\frac{\beta(1-\alpha)}{\alpha(1+\beta)}\frac{K_1}{L_{2,nc}^e}p_0$	$w_{2,d}^e$	$\frac{\beta(1-\alpha)}{\alpha(1+\alpha\beta)}\frac{K_1}{L_{2,d}^e}p_0$
$w_{2,nc}$	$\frac{\beta(1-\alpha)}{\alpha(1+\beta)}\frac{K_1}{L_{2,nc}}p_0$	$w_{2,d}$	$\frac{\beta(1-\alpha)}{\alpha(1+\alpha\beta)}\frac{K_1}{L_{2,d}}p_0$
$r_{2,nc}$	$p_{1,nc}$	$r_{2,d}$	$p_{1,d}$

Table 5.1. Characterization of General Equilibria in Reduced Form

Lemma 29. *Under the delegation regime, the rich are better off if and only if they have greater purchasing power.*

The relative price of final goods $\frac{p_1}{p_2}$ decreases after democratization by table 5.1: $\frac{p_{1,d}}{p_{2,d}} < \frac{p_{1,nc}}{p_{2,nc}}$. It has two effects to the representative rich person. The substitution effect (*S.E.*) makes the rich person consume more in period one but less in period two after democratization while holding her purchasing power constant. The income effect (*I.E.*) is

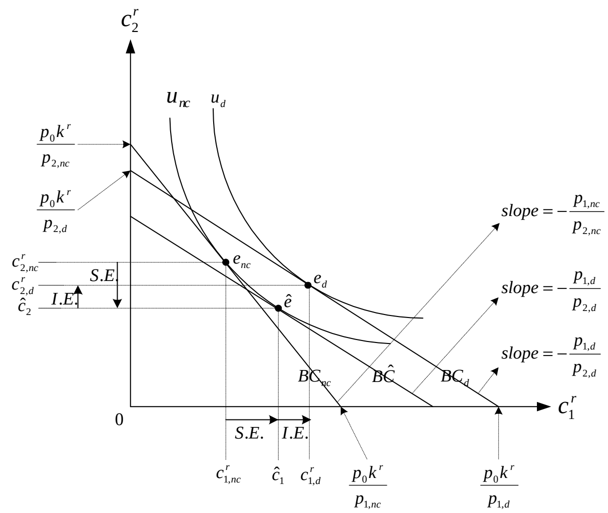


Figure 5.3. Democratization Increases Purchasing Power

uncertain because the prices of consumption goods in both periods change after democratization. However, given that consumption goods are normal goods in this chapter[3], figure 5.3 indicates that if the rich person's purchasing power increases (positive *I.E.*), she will be better off after democratization, and vice versa.

Let $u^r(c_1^r, c_2^r) = \ln c_1^r + \beta \ln c_2^r$ and $u_t^r(c_1^r, c_2^r) = \frac{\partial u^r(c_1^r, c_2^r)}{\partial c_t^r}$. Denote $\widehat{I}$ be the income level to exactly afford $\widehat{e} = (\widehat{c}_1, \widehat{c}_2)$. According to figure 5.3:

Lemma 30. *The rich's purchasing power increases after democratization if and only if:*

$$p_0 k^r > p_{1,d}\widehat{c}_1 + p_{2,d}\widehat{c}_2,$$

where $u^r(\widehat{c}_1, \widehat{c}_2) = u^r\left(c_{1,nc}^r, c_{2,nc}^r\right)$ *and* $\frac{u_1^r(\widehat{c}_1,\widehat{c}_2)}{p_{1,d}} = \frac{u_2^r(\widehat{c}_1,\widehat{c}_2)}{p_{2,d}}$.

[3] $\frac{\partial c_t^r}{\partial p_0 k^r} > 0$.

The representative rich person has $p_0 k^r$ dollars to afford e_d. Her purchasing power increases after democratization if and only if $p_0 k^r > \widehat{I}$ hold. Given $(p_{1,d}, p_{2,d})$, $\widehat{I}$ is determined by $\widehat{e}$, which gives the same utility with e_{nc} and is optimal at relative price $\frac{p_{1,d}}{p_{2,d}}$. Furthermore, let $f(\alpha, \beta, \theta) = \left[\frac{1+\alpha\beta}{1+\alpha\beta\theta+\beta}\right]^{\beta} \left[\frac{(1+\alpha\beta)(1+\beta)}{(1+\alpha\beta)(1+\beta)-\alpha\beta^2\theta}\right]^{1+\alpha\beta}$:

Theorem 31. *(The condition for democratization when $\delta = 1$)*

Democratization is adopted if and only if the rich have greater purchasing power after delegation, i.e.

$$f(\alpha, \beta, \theta) > 1. \tag{5.19}$$

If capital is not storable at all, adoption of democratization depends only on the capital share of production, the discount factor of consumers' utility, and the poor's preference of leisure to consumption.

Comparative Statics 32. *Ceteris paribus, the likelihood of democratization increases if:*

(1) Production is more capital-intensive (α is large).

(2) The poor prefer leisure to consumption (θ is larger).

(3) Consumers like the second-period utility as much as the first-period (β is larger).

When production becomes more capital-intensive, the production firms demand fewer labor hours in both periods under the delegation regime. The overall effect on total output is uncertain: it depends on the size of initial capital stock and total factor productivity. So are the effect on the prices of consumption goods $(p_{1,d}, p_{2,d})$ and the relative price $\frac{p_{1,d}}{p_{2,d}}$. Yet, it is still true that consumption goods in period one is relatively cheaper under the delegation regime than under the no-commitment regime: $\frac{p_{1,d}}{p_{2,d}} < \frac{p_{1,nc}}{p_{2,nc}}$. To hold the utility under the no-commitment regime constant, the rich person substitutes consumption goods of period one for period two if she adopts democratization: $\widehat{c}_1 > c^r_{1,nc}$, $\widehat{c}_2 < c^r_{2,nc}$. With the new relative price $\frac{p_{1,d}}{p_{2,d}}$, she has greater purchasing power if she can consume more than while holding the utility under the no-commitment regime constant: $\widehat{c}_t < c^r_{t,d}$ for $t \in \{1,2\}$. The relative size $\frac{c^r_{t,d}}{\widehat{c}_t}$ depends on the compound relative prices $\left(\frac{p_{1,nc}}{p_{1,d}}\right)^{\frac{1}{1+\beta}} \left(\frac{p_{2,nc}}{p_{2,d}}\right)^{\frac{\beta}{1+\beta}}$, which increase as α increases by table 5.1 so that $\widehat{c}_t < c^r_{t,d}$ for $t \in \{1,2\}$ more likely holds. It implies that the rich person's fixed initial wealth has greater purchasing power after democratization. Despite the uncertain effects on prices and production, the capital-owner rich are more likely better off with capital-dependent production technology.

The poor with larger θ prefer leisure to working. Given a set of $(\alpha, \beta, \theta_L)$, suppose that the rich's purchasing power does not change after democratization. Refer to figure 5.4. Suppose that θ increases such that $0 < \theta_L < \theta_H < 1$. With θ_H, the poor work less; less capital is accumulated for period two; final goods production shrinks in both periods; final goods become more expensive in both periods; the rich consume less in both periods. The relative prices change too such that $\frac{p_{1,d,\theta_H}}{p_{2,d,\theta_H}} < \frac{p_{1,d,\theta_L}}{p_{2,d,\theta_L}} < \frac{p_{1,nc,\theta_L}}{p_{2,nc,\theta_L}} < \frac{p_{1,nc,\theta_H}}{p_{2,nc,\theta_H}}$. If the rich person's new budget constraint after democratization goes through the convex set of $u_{nc,\theta_H} = \widehat{u}_{\theta_H}$, her purchasing power increases. For example, e_{d,θ_H} with which the rich are better off even though the final goods become more expensive.

The increases in prices because of the increase in θ have two effects on the rich person's utility. They make the rich person worse off under the no-commitment regime

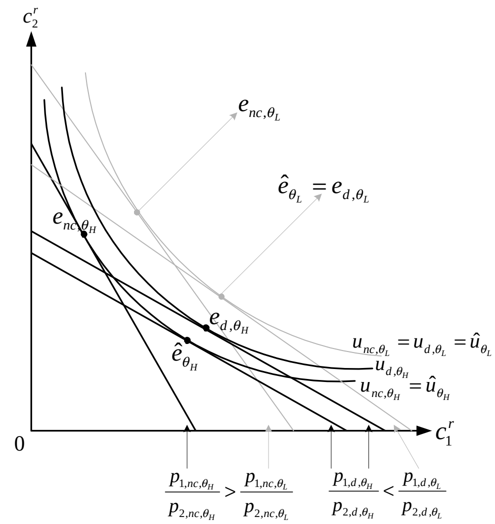


Figure 5.4. Larger θ Increases The Likelihood of Democratization

$(u_{nc,\theta_H} < u_{nc,\theta_L} = u_{d,\theta_L} = \widehat{u}_{\theta_L})$, but they make the rich person's happiness easier to be improved when they are already in a less happy situation $(u_{d,\theta_H} > u_{nc,\theta_H} = \widehat{u}_{\theta_H})$. Therefore, a larger θ creates a purchasing power improvement for the rich, so that they would be easier to be better off under the delegation regime even though consumption goods become more expensive.

Unlike (α, θ), the discount factor of consumers' utility β does not have a monotonic effect on the likelihood of democratization (figure 5.5)[4]. However, a larger β makes the economic activities in period two more compatible to in period one. It makes $\frac{c_{t,d}^r}{\widehat{c}_t}$ more likely larger than 1.

Simulation Result 33. *The input-intensity of production* (α) *is a more dominant effect than the people's preferences* (β, θ) *on the likelihood of democratization.*

[4]Let $g(\alpha,\beta,\theta) = \frac{1+\alpha\beta}{1+\alpha\beta\theta+\beta} < 1$; $h(\alpha,\beta,\theta) = \frac{(1+\alpha\beta)(1+\beta)}{(1+\alpha\beta)(1+\beta)-\alpha\beta^2\theta} > 1$; $f(\alpha,\beta,\theta) = [g(\alpha,\beta,\theta)]^{\beta}[h(\alpha,\beta,\theta)]^{1+\alpha\beta} > 0$. The likelihood of democratization increases iff $\frac{\partial f(\alpha,\beta,\theta)}{\partial\beta} \gtrless 0$ iff $\ln[h(\alpha,\beta,\theta)]^{\alpha} + \frac{\alpha\beta}{1+\alpha\beta}\left[\frac{1-\theta}{1+\alpha\beta} + \frac{\theta}{1+\beta}\left(1+\frac{1+\alpha\beta}{1+\beta}\right)\frac{h(\alpha,\beta,\theta)}{g(\alpha,\beta,\theta)}\right] \gtrless \frac{\beta}{(1+\alpha\beta)^2}$.

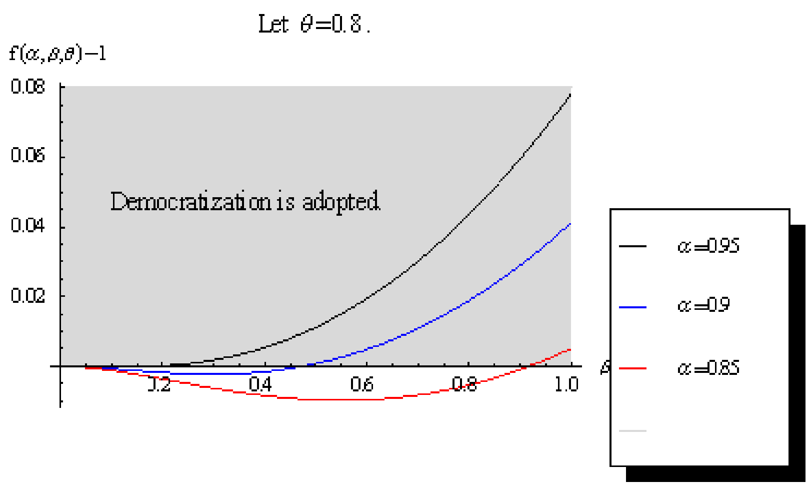


Figure 5.5. Larger β Increases Likelihood of Democratization

Let $\alpha \in \{0.8, 0.85, 0.9\}$. Table 5.2 shows adoption of democratization requires lower (β, θ) when α increases. It implies that α plays a more dominant role than (β, θ) for democratization. Notice that having (5.19) held requires an extraordinary large α: $\alpha > 0.8$. It is not realistic. In the next section, I'll relax the critical assumption on capital depreciation rate and show that democratization could occur when production is labor-intensive.

5.6. Generalization

Suppose that capital do not fully depreciate. Denote $x^{i*}_{t,g}$ to be the GE value of variable x of consumer i in period t under regime g for $i \in \{p, r\}$, $t \in \{1, 2\}$, and $g \in \{d, nc\}$.

Given $\delta \in [0, 1)$, the delegation regime has a unique general equilibrium in which $\tau = 0$. Reduced-form solution is not available, but all allocations and prices can be

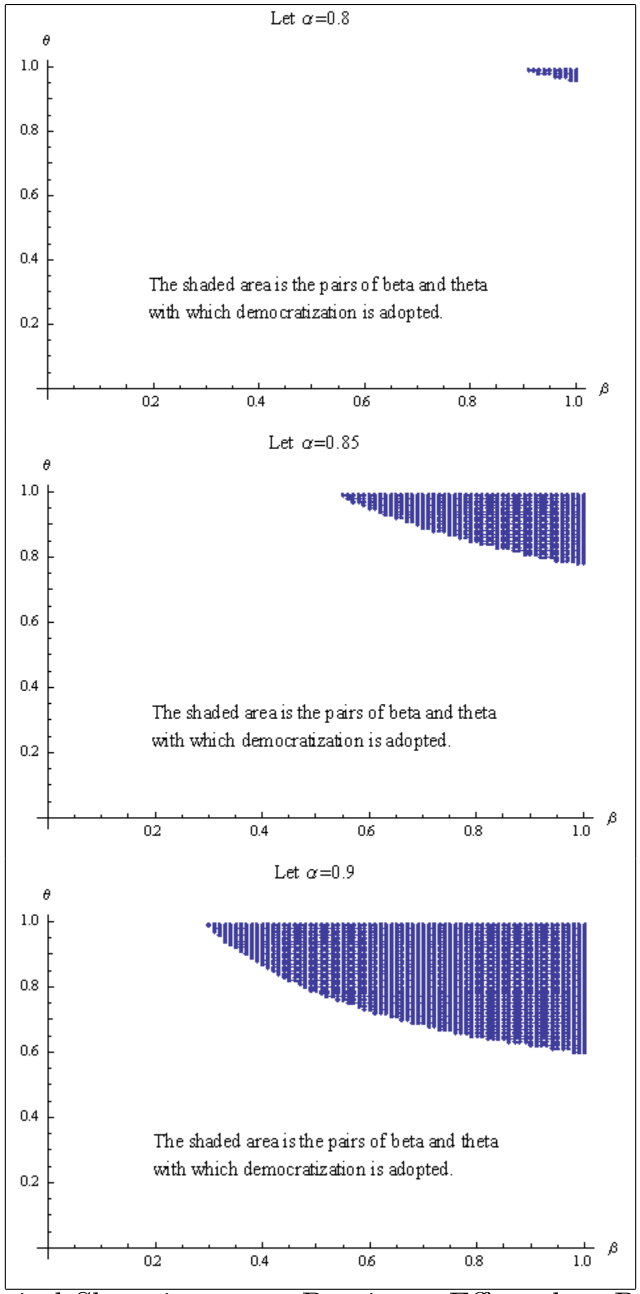


Table 5.2. Capital Share is a more Dominant Effect than People's Preferences on Democratization

expressed in terms of $l^*_{1,d} \in \left(0, 1 - \frac{\theta}{1+\beta-\beta\theta}\right)$, which is implicitly determined by:

$$1 + \frac{z^{1-\alpha}\left[\Omega\left(l^*_{1,d}\right) l^*_{1,d}\right]^{\alpha}}{1-\delta+\alpha A z^{1-\alpha}\left[\Omega\left(l^*_{1,d}\right)\right]^{\alpha-1}} \left(\frac{K_1}{n^p}\right)^{-\alpha} = \frac{1+\beta}{\theta}\left(1 - l^*_{1,d}\right), \tag{5.20}$$

where $\Omega\left(l_1\right)=\frac{\beta}{1+\beta}\frac{K_1}{n^p}\left[1-\delta+A\left(\frac{K_1}{n^p}\frac{1}{l_1}\right)^{\alpha-1}\left(\frac{1-\alpha+\beta}{\beta}-\frac{(1-\alpha)(1+\beta)(1-\theta)}{\beta\theta}\frac{1-l_1}{l_1}\right)\right]\frac{(1+\beta)(1-l_1)-\theta}{(1+\beta-\beta\theta)(1-l_1)-\theta}$. The allocations and the prices under the delegation regime are summarized in table 5.3. See appendix for characterization.

The necessary condition for democratization is that the poor save under the delegation regime so that the rich impose $\tau=1$ under the no-commitment regime. Otherwise, two regimes are indifferent to the rich, and they have no motivation to democratize the society. But if $\delta\in[0,1)$, the poor may not always save under the delegation regime. Their savings behavior depends on how hard they work in period one:

Lemma 34. *(The poor's savings and labor supply in period one)*

(1) $s_{1,d}^{p*}>0$ *if and only if* $l_{1,d}^{*}>1-\theta$*;*

(2) $s_{1,d}^{p*}=0$ *if and only if* $l_{1,d}^{*}=1-\theta$*;*

(3) $s_{1,d}^{p*}<0$ *if and only if* $l_{1,d}^{*}<1-\theta$*.*

In period one, the poor save if and only if each of them works more than $1-\theta$ hours. Associated with (5.20), the necessary condition is:

Claim 35. *(The necessary condition for democratization) Democratization is adopted only if:*

$$\frac{z^{1-\alpha}\left[\Omega\left(1-\theta\right)\left(1-\theta\right)\right]^{\alpha}}{1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(1-\theta\right)\right]^{\alpha-1}}\left(\frac{K_1}{n^p}\right)^{-\alpha}<\beta, \tag{5.21}$$

where $\Omega\left(1-\theta\right)=\frac{\beta}{1+\beta}\frac{1}{1-\theta}\frac{K_1}{n^p}\left[1-\delta+\alpha A\left(\frac{1}{1-\theta}\frac{K_1}{n^p}\right)^{\alpha-1}\right]$*, so that the poor save in period one under the delegation regime.*

Let $\Omega\left(l_1\right)=\frac{\beta}{1+\beta}\frac{K_1}{n^p}\left[1-\delta+\frac{1-\alpha+\beta}{\beta}A\left(\frac{K_1}{n^p}\frac{1}{l_1}\right)^{\alpha-1}-\frac{(1-\alpha)(1+\beta)(1-\theta)}{\beta\theta}A\left(\frac{K_1}{n^p}\frac{1}{l_1}\right)^{\alpha-1}\frac{1-l_1}{l_1}\right]\frac{(1+\beta)(1-l_1)-\theta}{(1+\beta-\beta\theta)(1-l_1)-\theta}$.	
$l_{1,d}^*=\frac{L_{1,d}^*}{n^p}=\frac{L_{1,d}^{e*}}{n^p}$	$1+\frac{z^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)l_{1,d}^*\right]^{\alpha}}{1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}}\left(\frac{K_1}{n^p}\right)^{-\alpha}=\frac{1+\beta}{\theta}\left(1-l_{1,d}^*\right)$
$l_{2,d}^*=\frac{L_{2,d}^*}{n^p}=\frac{1}{z}\frac{L_{2,d}^{e*}}{n^p}$	$1-\frac{\beta\theta\left(1-l_{1,d}^*\right)}{(1+\beta)\left(1-l_{1,d}^*\right)-\theta}$
$\frac{I_{1,d}^*}{n^p}$	$\Omega\left(l_{1,d}^*\right)l_{2,d}^*-(1-\delta)\frac{K_1}{n^p}$
$\frac{K_{2,d}^*}{n^p}$	$\Omega\left(l_{1,d}^*\right)l_{2,d}^*$
$c_{1,d}^{p*}$	$\frac{(1-\alpha)(1-\theta)}{\theta}A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha}\left(1-l_{1,d}^*\right)$
$c_{2,d}^{p*}$	$\frac{\beta(1-\alpha)(1-\theta)}{\theta}A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha}\left(1-l_{1,d}^*\right)\left[1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}\right]$
$c_{1,d}^{r*}$	$\frac{k^r}{1+\beta}\left[1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}\right]$
$c_{2,d}^{r*}$	$\frac{\beta k^r}{1+\beta}\left[1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}\right]\left[1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}\right]$
$\frac{Y_{1,d}^*}{n^p}$	$A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha}l_{1,d}^*$
$p_{1,d}^*$	$\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}p_0$
$w_{1,d}^{e*}=w_{1,d}^*$	$\frac{(1-\alpha)A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha}}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}p_0$
$r_{1,d}^*$	$\frac{\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}p_0$
$\frac{I_{2,d}^*}{n^p}$	$-(1-\delta)\,\Omega\left(l_{1,d}^*\right)l_{2,d}^*$
$K_{3,d}^*$	0
$\frac{Y_{2,d}^*}{n^p}$	$Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha}l_{2,d}^*$
$p_{2,d}^*$	$\frac{1}{1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}p_0$
$w_{2,d}^{e*}$	$\frac{(1-\alpha)Az^{-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha}}{1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}p_0$
$w_{2,d}^*$	$\frac{(1-\alpha)Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha}}{1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}p_0$
$r_{2,d}^*$	$\frac{\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}}{1-\delta+\alpha Az^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}p_0$

Table 5.3. Characterization of General Equilibrium under the Delegation Regime (Generalization)

Let $\Omega(1-\theta) = \frac{\beta}{1+\beta}\frac{1}{1-\theta}\frac{K_1}{n^p}\left[1-\delta+\alpha A\left(\frac{1}{1-\theta}\frac{K_1}{n^p}\right)^{\alpha-1}\right]$.	
$l^*_{1,nc} = \frac{L^*_{1,nc}}{n^p} = \frac{L^{e*}_{1,nc}}{n^p}$	$1-\theta$
$l^*_{2,nc} = \frac{L^*_{2,nc}}{n^p} = \frac{1}{z}\frac{L^{e*}_{2,nc}}{n^p}$	$1-\theta$
$\frac{I^*_{1,nc}}{n^p}$	$\Omega(1-\theta)(1-\theta) - (1-\delta)\frac{K_1}{n^p}$
$\frac{K^*_{2,nc}}{n^p}$	$\Omega(1-\theta)(1-\theta)$
$c^{p*}_{1,nc}$	$(1-\alpha)A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha}(1-\theta)$
$c^{p*}_{2,nc}$	$(1-\alpha)Az^{1-\alpha}\left[\Omega(1-\theta)\right]^{\alpha}(1-\theta)$
$c^{r*}_{1,nc}$	$\frac{k^r}{1+\beta}\left[1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}\right]$
$c^{r*}_{2,nc}$	$\frac{\beta k^r}{1+\beta}\left[1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}\right]\left[1-\delta+\alpha Az^{1-\alpha}\left[\Omega(1-\theta)\right]^{\alpha-1}\right]$
$\frac{Y^*_{1,nc}}{n^p}$	$A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha}(1-\theta)$
$p^*_{1,nc}$	$\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$
$w^{e*}_{1,nc} = w^*_{1,nc}$	$\frac{(1-\alpha)A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha}}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$
$r^*_{1,nc}$	$\frac{\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$
$\frac{I^*_{2,nc}}{n^p}$	$-(1-\delta)\Omega(1-\theta)(1-\theta)$
$K^*_{3,nc}$	0
$\frac{Y^*_{2,nc}}{n^p}$	$Az^{1-\alpha}\left[\Omega(1-\theta)\right]^{\alpha}(1-\theta)$
$p^*_{2,nc}$	$\frac{1}{1-\delta+\alpha Az^{1-\alpha}[\Omega(1-\theta)]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$
$w^{e*}_{2,nc}$	$\frac{(1-\alpha)Az^{-\alpha}[\Omega(1-\theta)]^{\alpha}}{1-\delta+\alpha Az^{1-\alpha}[\Omega(1-\theta)]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$
$w^*_{2,nc}$	$\frac{(1-\alpha)Az^{1-\alpha}[\Omega(1-\theta)]^{\alpha}}{1-\delta+\alpha Az^{1-\alpha}[\Omega(1-\theta)]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$
$r^*_{2,nc}$	$\frac{\alpha Az^{1-\alpha}[\Omega(1-\theta)]^{\alpha-1}}{1-\delta+\alpha Az^{1-\alpha}[\Omega(1-\theta)]^{\alpha-1}}\frac{1}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$

Table 5.4. Characterization of General Equilibrium under the No-commitment Regime (generalization)

Consider the parameters which satisfy the necessary condition. There is a unique GE under the no-commitment regime in which $\tau = 1$. Reduced-from solution is available. The allocations and the prices are summarized in table 5.4.

If the rich's purchasing power increases after democratization, they will like to democratize. Refer to figure 5.6. Denote e^*_{nc} to be the representative rich person's optimal

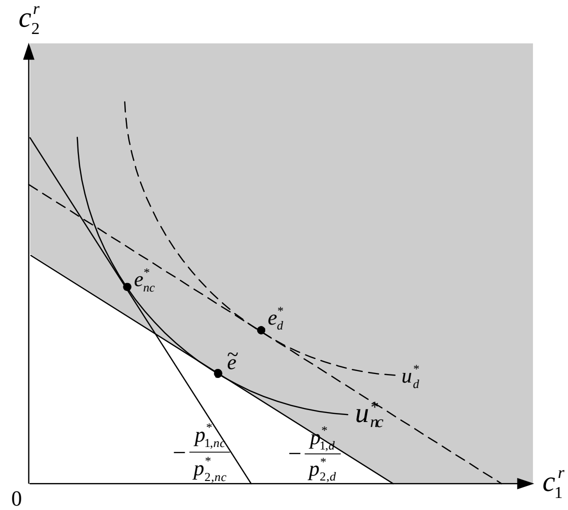


Figure 5.6. The Rich Person's Purchasing Power Increases After Democratization.

bundle with the relative price $\frac{p_{1,nc}^*}{p_{2,nc}^*}$. e_{nc}^* gives her utility u_{nc}^*. Denote $\widetilde{e} = (\widetilde{c}_1, \widetilde{c}_2)$ to be the optimal bundle when she faces the new relative price $\frac{p_{1,d}^*}{p_{2,d}^*} < \frac{p_{1,nc}^*}{p_{2,nc}^*}$ while holding her utility constant at u_{nc}^*. $\widetilde{e}$ satisfy $\frac{u_1^r(\widetilde{c}_1,\widetilde{c}_2)}{p_{1,d}^*} = \frac{u_2^r(\widetilde{c}_1,\widetilde{c}_2)}{p_{2,d}^*}$ and $u^r(\widetilde{c}_1, \widetilde{c}_2) = u^r\left(c_{1,nc}^{r*}, c_{2,nc}^{r*}\right)$. The rich person's purchasing power increases after democratization if she has sufficient money to afford $\widetilde{e}$ with the new prices, i.e. $p_0 k^r > p_{1,d}^* \widetilde{c}_1^r + p_{2,d}^* \widetilde{c}_2^r$. For instance, the dashed part in figure 5.6.

Proposition 36. *(The sufficient condition for democratization) Democratization is adopted if:*

$$\frac{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l_{1,d}^*}\right)^{\alpha-1}}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}} > \left[\frac{1-\delta+\alpha A z^{1-\alpha}\left[\Omega\left(1-\theta\right)\right]^{\alpha-1}}{1-\delta+\alpha A z^{1-\alpha}\left[\Omega\left(l_{1,d}^*\right)\right]^{\alpha-1}}\right]^{\frac{\beta}{1+\beta}}, \tag{5.22}$$

so that the rich's purchasing power increases after delegation.

The rich's purchasing power depends on the relative prices of final goods $\left(\frac{p_{1,nc}^*}{p_{1,d}^*}\right)^{\frac{1}{1+\beta}} \times \left(\frac{p_{2,nc}^*}{p_{2,d}^*}\right)^{\frac{\beta}{1+\beta}}$. If it is larger than 1, the rich's purchasing power increases. (5.22) implies that if capital is storable, besides (α, β, θ), the adoption of democratization depends on capital depreciation rate (δ), total factor productivity (A), labor productivity (z), and capital endowment per worker $\left(\frac{K_1}{n^p}\right)$. Democratization is adopted if and only if claim 35 and proposition 36 hold.

Simulation Result 37. *If democratization is adopted when $\delta = 1$, it is also adopted when $\delta \in [0, 1)$, but not vice versa.*

Simulation result 37 implies that the parameters which satisfy (5.19) is a proper subset of the parameters which satisfy (5.21) and (5.22). From now on, consider the cases that adoption of democratization failed when $\delta = 1$ but the parameters satisfy the necessary condition (5.21).

Comparative Statics 38. *Suppose that $\delta \in [0, 1)$; (5.19) fails; (5.21) holds. Ceteris paribus, the likelihood of democratization increases if:*

(1) Production is more capital-intensive (α is larger).

(2) Consumers like the second-period utility as much as the first-period (β is larger).

(3) The poor prefer leisure to consumption (θ is larger).

(4) Capital depreciates at lower rate (δ is smaller).

(5) Capital-endowment-per-worker is abundant ($\frac{K_1}{n^p}$ is larger).

(6) Physical labor is less productive in period two (z is smaller).

(7) Total factor productivity is smaller (A is smaller).

Similar to the case with $\delta = 1$, it is not surprising that the likelihood of democratization increases if (α, β, θ) are larger. But when $\delta \in [0, 1)$, the necessary condition (5.21) more likely holds than (5.19). It imposes fewer restrictions on (α, β, θ). (α, β, θ) can be smaller to satisfy (5.21) than (5.19). For instances, $\alpha \approx 0.3$, $\beta \approx 0.3$, $\theta \approx 0.4$.

When δ or z decreases, the necessary condition (5.21) more likely holds (figure x v.s. necessary condition for $x \in \{\delta, z\}$). With smaller δ, the poor know there are more capital left as consumption goods at the end of period two; with smaller z, the poor know that they will be paid at a lower rate for each physical labor hour in period two. In either case, the poor work less in period two and work harder in period one (figure x v.s. $l^*_{1,d}$ for $x \in \{\delta, z\}$). A larger $l^*_{1,d}$ and a constant $l^*_{1,nc}$ increase $\frac{p^*_{1,nc}}{p^*_{1,d}} > 1$. A smaller $l^*_{2,d}$ and a constant $l^*_{2,nc}$ decrease $\frac{p^*_{2,nc}}{p^*_{2,d}}$, but the impact is smaller than in period one because $K^*_{2,d} > K^*_{2,nc}$. Therefore, the rich's purchasing power more likely increases (figure x v.s. sufficient condition for $x \in \{\delta, z\}$).

In period one, the increase in A or $\frac{K_1}{n^p}$ decreases the rental rate, and increases the wage rate and the price of final good. The production firms have higher demand for capital and lower demand for labor. Under the delegation regime, the poor work less in period one, which further decreases the rental rate, and increases the wage rate and the price of final good, and work harder in period two. Under the no-commitment regime, the poor's working hours are not affected by the increase in A or $\frac{K_1}{n^p}$. The percentage increase in the price of final goods is not as large as under the delegation regime, so $\frac{p^*_{1,nc}}{p^*_{1,d}}$ decreases.

In period two, the increases in $p^*_{1,g}$ for $g \in \{nc, d\}$ increase the rental rate, the wage rate, and the price of final good. In addition, a larger A or $\frac{K_1}{n^p}$ increases $K^*_{2,g}$ for $g \in \{nc, d\}$, but under the delegation regime this increase is less significant because the poor work less and investment goods are more expensive in period one. Recall that under the delegation regime the poor work harder in period two, but under the no-commitment regime their working hours are not affected by the increase in A or $\frac{K_1}{n^p}$ such that $l^*_{2,nc} > l^*_{2,d}$. Under the no-commitment regime, the percentage increase in input payments is higher than under the delegation regime. The price of final good increases further, and the percentage increase is higher under the no-commitment regime. So $\frac{p^*_{2,nc}}{p^*_{2,d}}$ increases. Therefore, when A or $\frac{K_1}{n^p}$ increases, $\left(\frac{p^*_{1,nc}}{p^*_{1,d}}\right)^{\frac{1}{1+\beta}} \left(\frac{p^*_{2,nc}}{p^*_{2,d}}\right)^{\frac{\beta}{1+\beta}}$ is more likely larger than 1; the rich's purchasing power more likely increases.

5.7. Comparison of Regimes

Claim 39. *(Comparison of regimes when $\delta = 1$.)*

(1) $l_{1,d} > l_{1,nc} = l_{2,nc} > l_{2,d}$ *such that* $l_{1,d} + l_{2,d} < l_{1,nc} + l_{2,nc}$.

(2) $w_{1,nc} > w_{1,d} > w_{2,d} > w_{2,nc}$.

(3) $K_{2,d} > K_{2,nc}$.

(4) $Y_{t,d} > Y_{t,nc}$ *for* $t \in \{1, 2\}$.

(5) $\sum_{t=1}^{2} w_{t,d} l_{t,d} > \sum_{t=1}^{2} w_{t,nc} l_{t,nc}$.

(6) $\sum_{t=1}^{2} \beta^{t-1} u^p \left(c^p_{t,d}, 1 - l_{t,d}\right) > \sum_{t=1}^{2} \beta^{t-1} u^p \left(c^p_{t,nc}, 1 - l_{t,nc}\right)$.

To have $S^p_{1,d} > 0 > S^p_{2,d}$ under the delegation regime, the poor work harder in period one than in period two: $l_{1,d} > l_{2,d}$. Compare to $l_{1,nc} = l_{2,nc}$ such that $S^p_{1,nc} = S^p_{2,nc} = 0$ under the no-commitment regime, $l_{1,d} > l_{1,nc} = l_{2,nc} > l_{2,d}$.

$w_{1,nc} > w_{1,d}$ for $l_{1,d} > l_{1,nc}$; $w_{2,d} > w_{2,nc}$ for $l_{2,nc} > l_{2,d}$. $w_{1,d} > w_{2,d}$ by table 5.1.

$Y_{1,d} > Y_{1,nc}$ such that $p_{1,d} < p_{1,nc}$ for $l_{1,d} > l_{1,nc}$.

α v.s. $l^*_{1,d}$, Necessary Condition, and Sufficient Condition
Let $\beta = 0.98$, $\delta = 0.05$, $\theta = 0.9$, $A = 1$, $z = 1$, $\frac{K_1}{n^p} = 10$.
Blue: democratization is adopted; red: oligarchy is sustained; green: necessary condition fails.

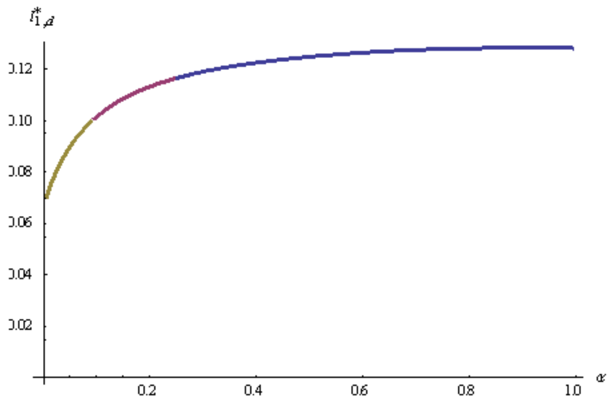


Figure α v.s. $l^*_{1,d}$

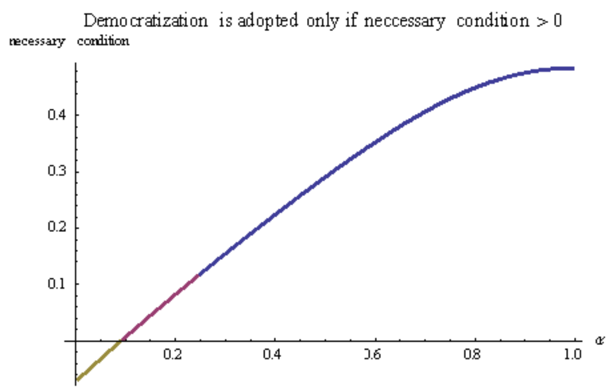


Figure α v.s. Necessary Condition

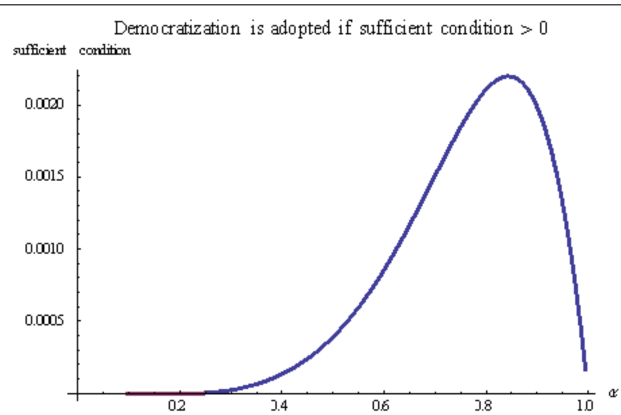


Figure α v.s. Sufficient Condition

$\boldsymbol{\beta}$ v.s. $l^*_{1,d}$, Necessary Condition, and Sufficient Condition
Let $\alpha = \frac{1}{3}$, $\delta = 0.1$, $\theta = 0.9$, $A = 1$, $z = 1$, $\frac{K_1}{n^p} = 9$.
Blue: democratization is adopted; red: oligarchy is sustained; green: necessary condition fails.

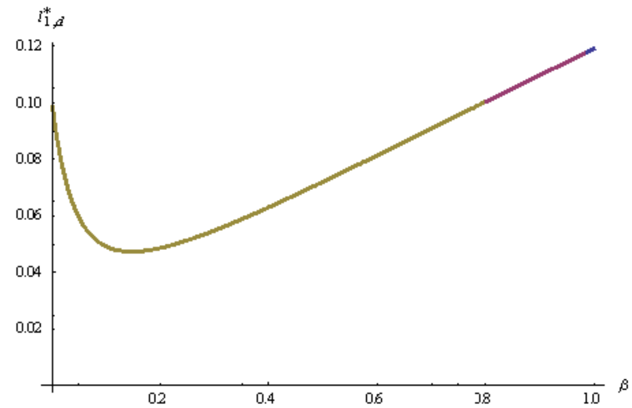


Figure β v.s. $l^*_{1,d}$

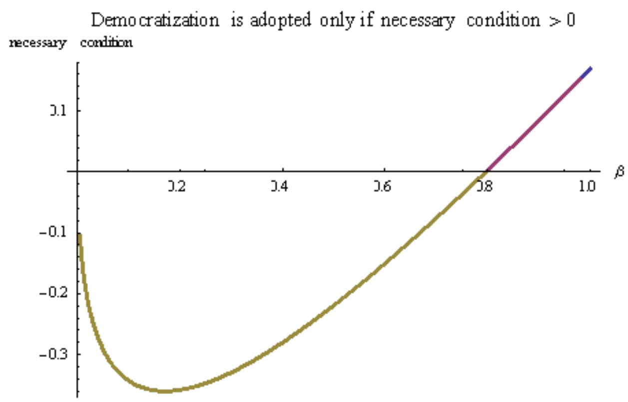


Figure β v.s. Necessary Condition

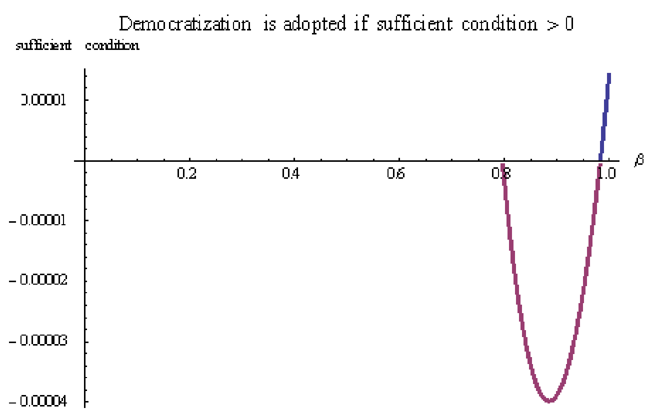


Figure β v.s. Sufficient Condition

θ v.s. $l^*_{1,d}$, Necessary Condition, and Sufficient Condition
Let $\alpha = \frac{1}{3}$, $\beta = 0.98$, $\delta = 0.1$, $A = 1$, $z = 1$, $\frac{K_1}{n^p} = 9$.
Blue: democratization is adopted; red: oligarchy is sustained; green: necessary condition fails.

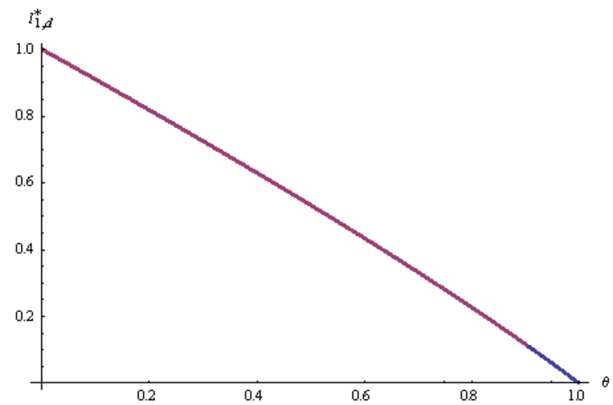


Figure θ v.s. $l^*_{1,d}$

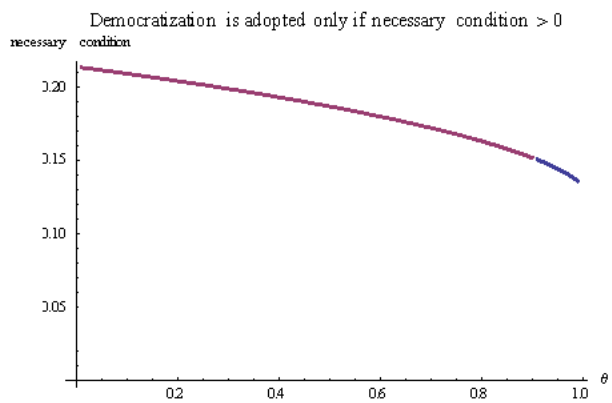


Figure θ v.s. Necessary Condition

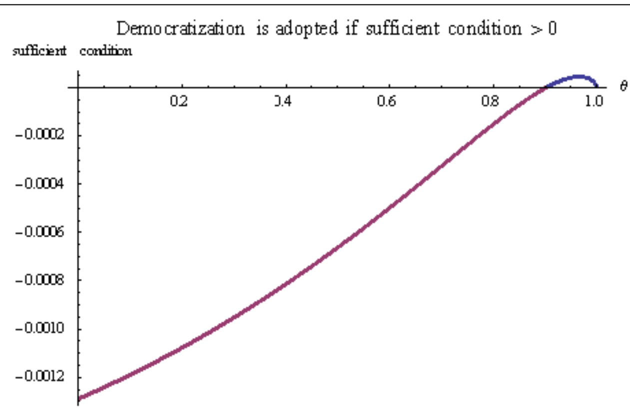


Figure θ v.s. Sufficient Condition

δ v.s. $l^*_{1,d}$, Necessary Condition, and Sufficient Condition
Let $\alpha = \frac{1}{3}$, $\beta = 0.98$, $\theta = 0.9$, $A = 1$, $z = 1$, $\frac{K_1}{n^p} = 9$.
Blue: democratization is adopted; red: oligarchy is sustained; green: necessary condition fails.

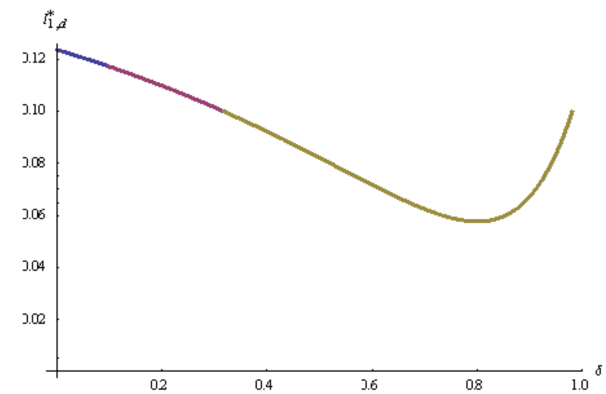


Figure δ v.s. $l^*_{1,d}$

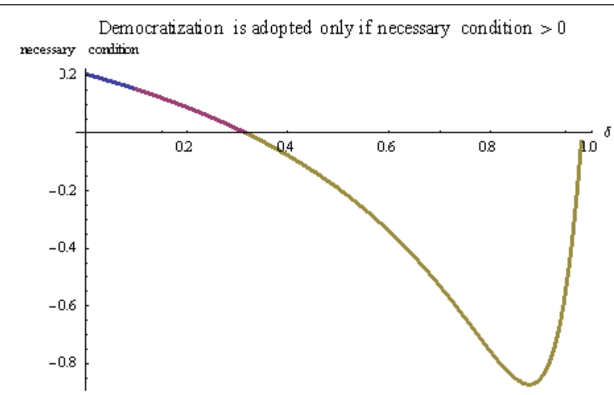


Figure δ v.s. Necessary Condition

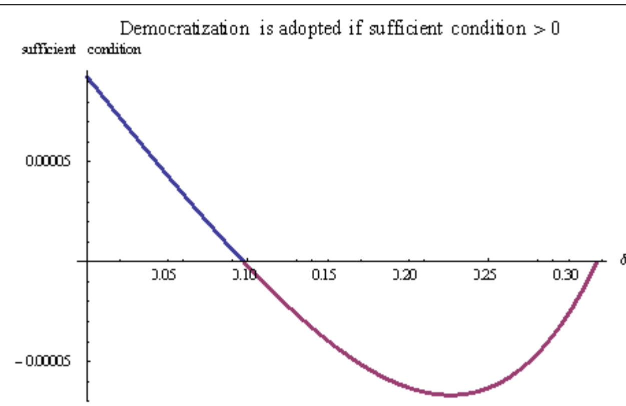
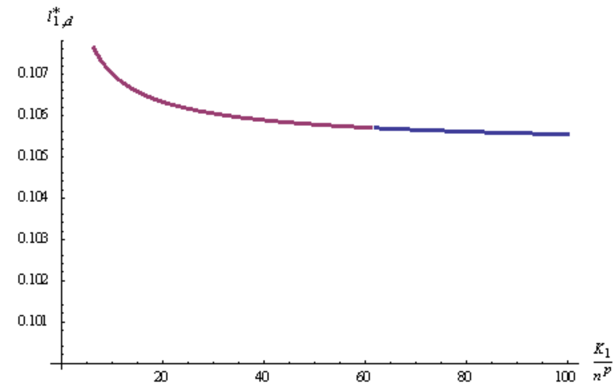
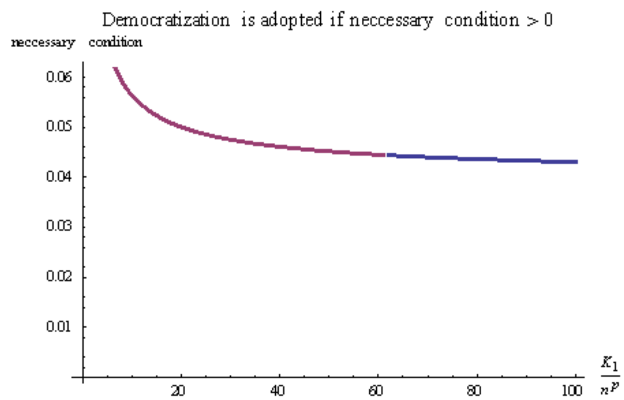
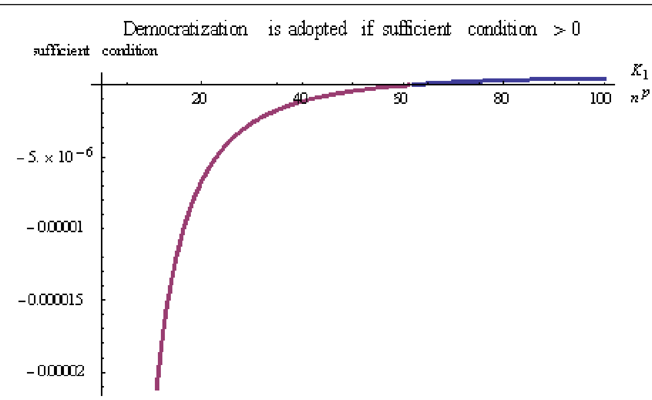


Figure δ v.s. Sufficient Condition

$\frac{K_1}{n^p}$ v.s. $l^*_{1,d}$, Necessary Condition, and Sufficient Condition
Let $\alpha = 0.3$, $\beta = 0.9$, $\delta = 0.1$, $\theta = 0.9$, $A = 1$, $z = 1$.
Blue: democratization is adopted; red: oligarchy is sustained; green: necessary condition fails.
 Figure $\frac{K_1}{n^p}$ v.s. $l^*_{1,d}$
 Figure $\frac{K_1}{n^p}$ v.s. Necessary Condition
 Figure $\frac{K_1}{n^p}$ v.s. Sufficient Condition

z v.s. $l^*_{1,d}$, Necessary Condition, and Sufficient Condition
Let $\alpha = \frac{1}{3}$, $\beta = 0.98$, $\delta = 0.05$, $\theta = 0.9$, $A = 1$, $\frac{K_1}{n^p} = 10$.
Blue: democratization is adopted; red: oligarchy is sustained; green: necessary condition fails.

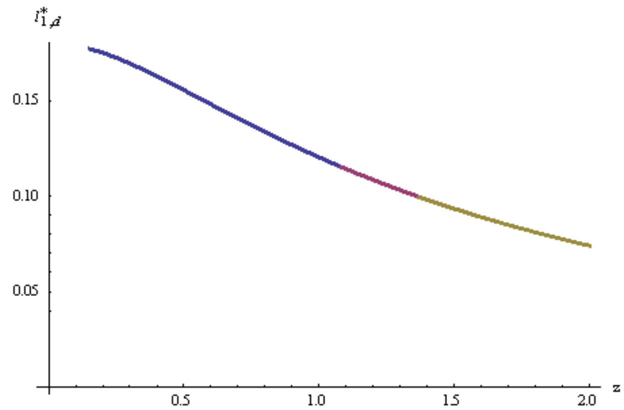


Figure z v.s. $l^*_{1,d}$

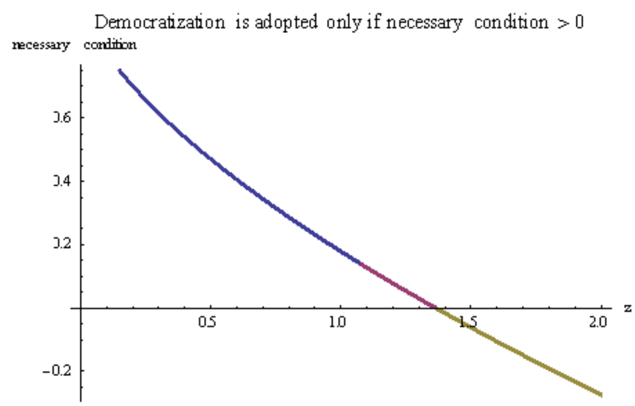


Figure z v.s. Neccessary Condition

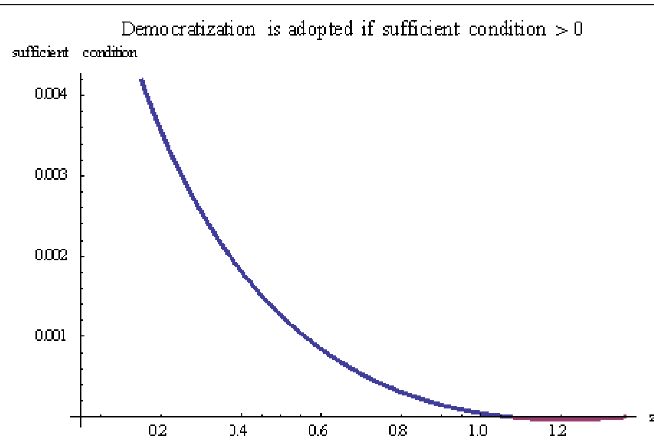


Figure z v.s. Sufficient Condition

A v.s. $l^*_{1,d}$, Necessary Condition, and Sufficient Condition
Let $\alpha = 0.4$, $\beta = 0.98$, $\delta = 0$, $\theta = 0.8$, $z = 1$, $\frac{K_1}{n^p} = 10$.
Blue: democratization is adopted; red: oligarchy is sustained; green: necessary condition fails.

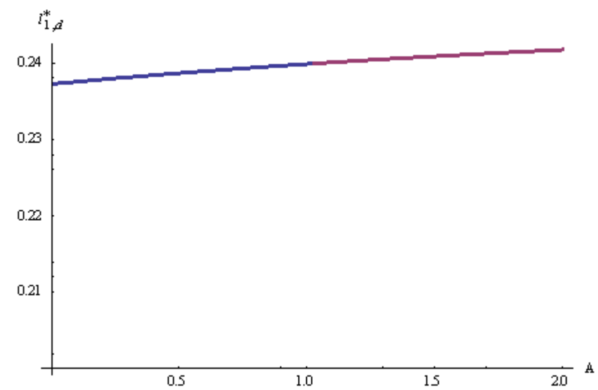


Figure A v.s. $l^*_{1,d}$

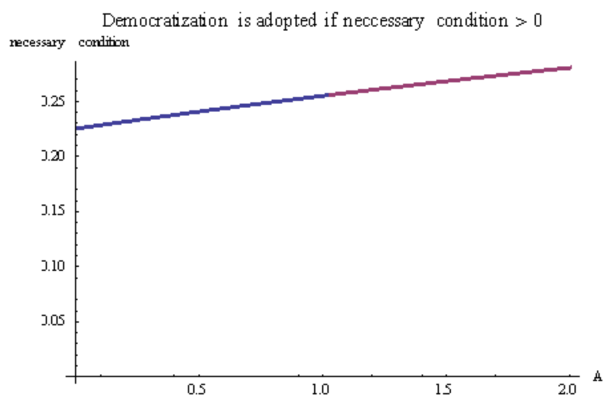


Figure A v.s. Neccessary Condition

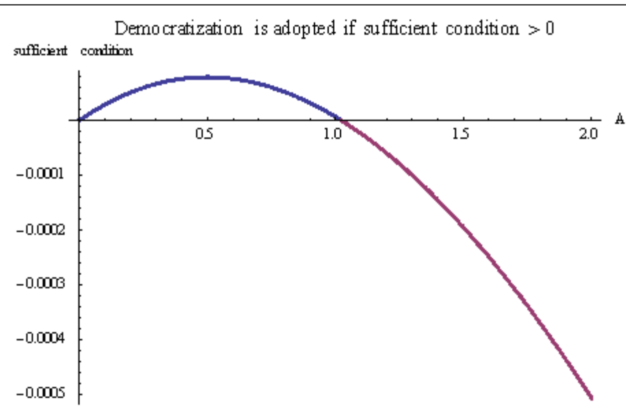


Figure A v.s. Sufficient Condition

The democratic economy creates greater savings $\left(S^p_{1,d} > 0 = S^p_{1,nc}\right)$ and produces cheaper final goods $(p_{1,d} < p_{1,nc})$, so the capital stock for period two is greater under the delegation regime $(K_{2,d} > K_{2,nc})$.

$Y_{2,d} > Y_{2,nc}$ for $K_{2,d} > K_{2,nc}$, which increases the capacity of production in period two, although $l_{2,nc} > l_{2,d}$.

Wage rate is a function of capital per worker: $w_t = c\frac{K_t}{L_t}p_{t-1}$ for $t \in \{1,2\}$, where $c = \frac{1-\alpha}{\alpha}$. The paycheck that the poor receive is $w_t L_t = cp_{t-1}K_t$. Given the same capital endowment in period one, the poor earn equal money in both regimes $(w_{1,d}L_{1,d} = w_{1,nc}L_{1,nc} = cp_0K_1)$. Suppose that under regime $g \in \{d, nc\}$, the percentage of capital in final good output $\left(\frac{I_{1,g}}{Y_{1,g}} = \frac{K_{2,g}}{Y_{1,g}}\right)$ is λ_g. In period two, the paycheck that the poor receive is $w_{2,g}L_{2,g} = \frac{c}{\alpha}\lambda_g p_0 K_1$[5]. By Table 5.1, $\lambda_d = \frac{\alpha\beta}{1+\alpha\beta} > \lambda_{nc} = \frac{\alpha\beta}{1+\beta}$. Therefore, under the delegation regime the poor receive a higher paycheck in period two and make more money throughout their life.

Tax expropriation distorts the economy under the no-commitment regime; democratization restores the efficiency of economy. Therefore, the poor are better off under the delegation regime.

If $\delta \in [0, 1)$ and the necessary condition (5.21) holds, the results of claim 39 hold by simulation.

5.8. Concluding Remarks

When capital does not fully depreciate, democratization could be adopted if production is labor-intensive or consumers prefer consumption to leisure. Although the increases

[5] $w_{2,g}L_{2,g} = cp_{1,g}K_{2,g} = c\frac{\lambda_g Y_{1,g}}{\alpha\frac{Y_{1,g}}{K_1}}p_0 = \frac{c}{\alpha}\lambda_g p_0 K_1$

in (α, β, θ) increase the likelihood of democratization, democratization could be adopted if, for instance, $\alpha \approx 0.3$ (figure α v.s. sufficient condition) or $\theta \approx 0.4$[6].

Under the delegation regime, the poor may be not always paid better for each unit of physical labor hour than under the no-commitment regime. It is true by claim 36, (2).

[6]Democratization is adopted if $\alpha = 0.9$, $\beta = 0.9$, $\delta = 0$, $\theta = 0.4$, $A = 1$, $z = 1$, $\frac{K_1}{n^p} = 10$.

CHAPTER 6

Conclusion

In the absence of conflict and political competition, the three essays show that democratization could be adopted in an oligarchic society with brutal appropriation on the workers' labor income (chapter 3), retaliatory capital gain tax on the elite (chapter 4), or in the absence of credible commitment (chapter 5). This positive result can hold even if the social elite are self-interested and the people are not well-educated.

Within the framework of the models, the necessary condition for democratization requires economic prosperity, i.e. the poor save (claim 35). The sufficient condition requires a net positive spillover to the elite's payoff (proposition 8, theorem 13, proposition 20, theorem 31, proposition 36). If both conditions hold, the elite voluntarily relinquish the oligarchic power to pursue a better economic outcome.

Consider the static model with endogenous labor productivity. Under the oligarchic regime, the rich brutally appropriate the poor's labor income. The tax decreases the poor's incentive to receive education and to work. The labor productivity is low; the economy shrinks; the tax revenue may be limited. Under the democratic regime, the poor with the tax power remove this tax distortion, so they are motivated to pursue education and to work, which increase the rich's capital return. The economy thus prospers. Democratization would be adopted if the rich receive sufficient capital return under the democratic regime (comparative statics 9), collect small labor income tax under the oligarchic regime (comparative statics 10), or both (comparative statics 11).

Consider the static model with non-exclusive public goods. Under the oligarchic regime, the rich appropriate the poor's labor income. The existence of home production weakens the appropriation, so sustaining the oligarchic power may be less attractive to the rich. Under the democratic regime, the poor work hard and impose a capital gain tax on the rich to finance for non-exclusive public goods. Each rich person pays a finite tax but is rewarded the benefit of aggregate tax revenue. Delegation of the power may become attractive to the rich. Democratization would be adopted if the rich impose a small labor income tax under the oligarchic regime (comparative statics 21), or the poor provide abundant labor supply and impose a reasonable capital gain tax on the rich under the democratic regime (comparative statics 22).

Consider the two-period model without commitment. Under the no-commitment regime, the rich impose a savings tax on the poor after observing their savings behavior. If the poor save, the rich expropriate their savings and it turns down the poor's savings to zero by backward induction. Little capital dooms the economy. Under the delegation regime, the tax distortion is removed and the efficiency of the economy is improved. Democratization would be adopted if the prosperous economy increases the rich's purchasing power (theorem 31, comparative statics 38). Democratization is endogenized as a credible commitment to the poor.

The conditions for democratization are characterized in terms of economic primitives. The comparative statics of the likelihood of democratization are summarized in table 6.1. Overall, three models conclude consistently, except the capital share α and capital endowment per poor person $\frac{K_1}{n^p}$.

In chapter 3, labor-intensive production makes the physical wage rate higher under the democratic regime than under the oligarchic regime (claim 6). It encourages the poor to work harder under the democratic regime (claim 3, (2)), and the rich receive higher capital return (figure 3.3). In chapter 4, home production refrains the rich from appropriation (figure α-1). On the other hand, capital-intensive production increases the rich's capital return (figure α-2), so the provision of public goods is abundant under the democratic regime (figure α-3). In chapter 5, the input-dependence of production affects the relative prices of final goods $\left(\frac{p^*_{1,nc}}{p^*_{1,d}}, \frac{p^*_{2,nc}}{p^*_{2,d}}\right)$, which determines the rich's purchasing power $\left(\frac{p^*_{1,nc}}{p^*_{1,d}}\right)^{\frac{1}{1+\beta}} \left(\frac{p^*_{2,nc}}{p^*_{2,d}}\right)^{\frac{\beta}{1+\beta}}$. Notice that although more capital-intensive production increases the rich's purchasing power in chapter 5, democratization would occur if production is labor-intensive (figure α v.s. $l^*_{1,d}$, necessary condition, and sufficient condition).

In chapter 3, a small $\frac{K_1}{n^p}$ decreases the poor's physical wage rate, so they work idly and receive little education, which decreases the rich's appropriation on labor income. In chapter 4, a large $\frac{K_1}{n^p}$ (either $\frac{n^r}{n^p+n^r}$ or k^r is large) increases the relative value of the poor's physical labor to the rich's capital endowment. The poor are encouraged to work harder under the democratic regime (figure $\frac{n^r}{n^p+n^r}$-2, figure k^r-2). The capacity of market production and the relative value of capital increase. The poor may enable to provide sufficient public goods by imposing a lower capital tax (figure $\frac{n^r}{n^p+n^r}$-3, figure k^r-3). In chapter 5, a large $\frac{K_1}{n^p}$ decreases $\frac{p^*_{1,nc}}{p^*_{1,d}}$; increases $\frac{p^*_{2,nc}}{p^*_{2,d}}$ so that $\left(\frac{p^*_{1,nc}}{p^*_{1,d}}\right)^{\frac{1}{1+\beta}} \left(\frac{p^*_{2,nc}}{p^*_{2,d}}\right)^{\frac{\beta}{1+\beta}}$ increases.

Suppose that the necessary condition holds. The results of the comparison of regimes are summarized in table 6.2. Three models conclude that under the democratic/delegation regime, the production firms produce more final goods and the poor make better life-time labor income than under the oligarchic/no-commitment regime.

Economic primitives	Description	Chapter 3	Chapter 4	Chapter 5
$\alpha \in (0,1)$	capital share	small	large	large
$\beta \in (0,1]$	utility discount factor	-	-	large
$\gamma \in \mathbb{R}_{++}$	total edu prod	small	-	-
$\gamma \in (0,1)$	pref of home prod over mkt prod	-	small	-
$\delta \in [0,1]$	capital dep. rate	-	-	small
$\theta \in \mathbb{R}_{++}$	elasticity of labor prod to edu	large	-	-
$\theta \in (0,1)$	pref of leisure to consumption	-	large	large
$\rho \in (0,1)$	constant elasticity of substitution	large	?	-
$\sigma \in \mathbb{R}_{++}$	pref over public goods	-	large	-
$A \in \mathbb{R}_{++}$	TFP	small	?	small
$B \in \mathbb{R}_{++}$	TFP of home prod	-	?	-
$z \in \mathbb{R}_{++}$	labor prod	-	-	small
$\frac{K}{n^p} \in \mathbb{R}_{++}$	capital endnt per poor	small	large	large
$n^p + n^r \in \mathbb{R}_{++}$	population	-	large	-

Table 6.1. Summary of Comparative Statics

	Chapter 3	Chapter 4	Chapter 5
The poor's utility	$u_d^p \gtreqless u_o^p$ if $\frac{1-e_o}{[\theta-(1+\theta)e_o]^{1-\rho}} \lesseqgtr \frac{1-e_d}{[\theta-(1+\theta)e_d]^{1-\rho}}$, where (e_o, e_d) are governed by (3.17) and (3.18).	$u_d^p > u_o^p$	$u_d^p > u_{nc}^p$
Labor supply	$l_d > l_o$	$l_d > l_o; l_{b,d} < l_{b,o}$	$l_{1,d} > l_{1,nc} = l_{2,nc} > l_{2,d}$
Labor income	$w_d l_d > w_o l_o$	$w_d l_d > w_o l_o$	$\sum_{t=1}^{2} w_{t,d} l_{t,d} > \sum_{t=1}^{2} w_{t,nc} l_{t,nc}$
Non-leisure time	$l_d + e_d > l_o + e_o$	$l_d + l_{b,d} > l_o + l_{b,o}$	$l_{1,d} + l_{2,d} < l_{1,nc} + l_{2,nc}$
Total output	$Y_d > Y_o$	$Y_d > Y_o; Y_{b,d} < Y_{b,o}$	$Y_d > Y_o$
Wage rate	$w_d \gtreqless w_o$ if $\frac{1}{(1-\alpha)(1+\theta)} \lesseqgtr 1$	$w_d < w_o$	$w_{1,nc} > w_{1,d} > w_{2,d} > w_{2,nc}$

Table 6.2. Summary of Comparison of Regimes

Taiwan is an example of the main ideas expressed in this dissertation. First, the elite voluntarily democratize the society. Chiang Ching-Kuo, the last appointed president (1978-1988) under the martial law, launched a series of economic projects[1] in 1970's, which contributed to the economic miracle in 1980's. During his presidency, he promoted Taiwan-born citizens, who later became the executors of democratization, into

[1]For instances, Fourteen Major Construction Projects, Ten Major Construction Projects, and Twelve New Development Projects.

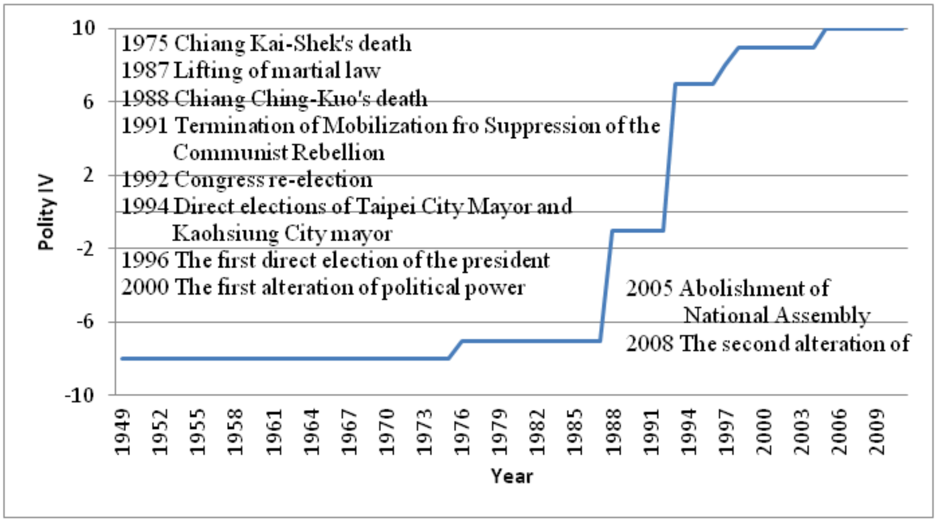


Figure 6.1. Taiwan: Polity IV Scores since 1949

government services and lift the martial law in 1987. Second, democratization promotes economic development and is driven by the pursuit of further development. Democratization differentiates the power of the government over the economy to the microeconomic decision-makers, so the economy can operate through the market mechanism with less intervention. The free market prospers the economy. The rapid growth increases people's demand for differentiation of the power. In addition to the economic liberalization, political liberalization reinforces the interaction between political and economic activities. It contributes to further economic growth. See figure 6.1[2] and figure 6.2[3].

(South) Korea is another example. After Korean War, Korea experienced a long period of political instability. The government was under the different controls of dictatorship until late 1980's. In the meantime, the government under the leadership of military dictator Park developed significant public infrastructure, which was the foundation for

[2]Data source: Polity IV Project: Political Regime Characteristics and Transitions, 1800-2010, Monty G. Marshall and Keith Jaggers, Political Instability Task Force (PITF), Societal Systems Research, and Center for Systemic Peace, December 2011.

[3]Data source: International Monetary Fund, World Economic Outlook Database, October 2012.

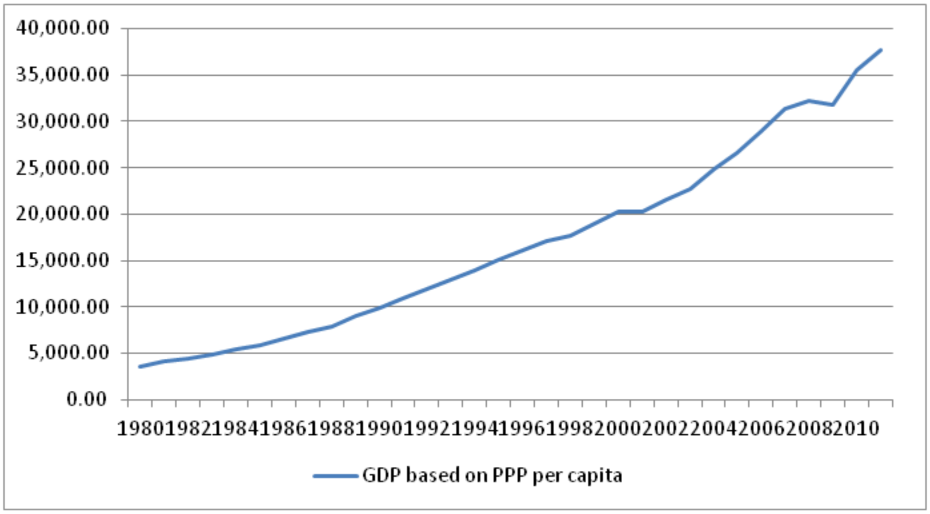


Figure 6.2. Taiwan: GDP per capita from 1980 to 2011

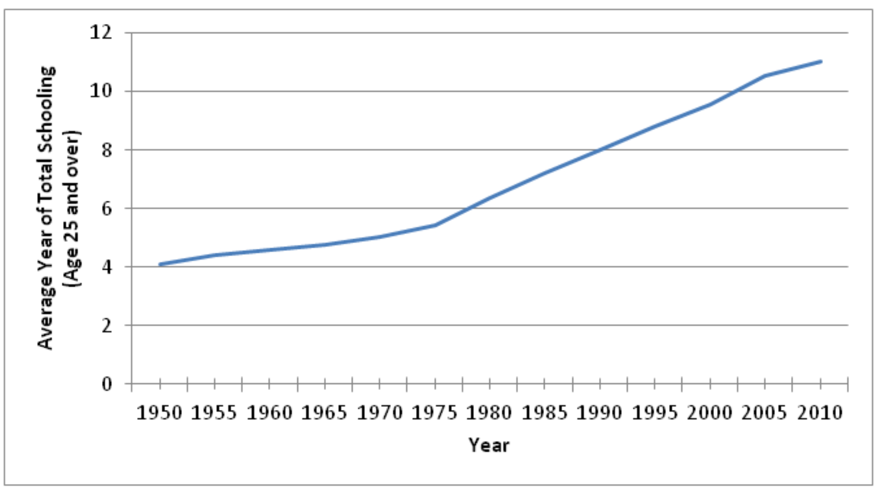


Figure 6.3. Taiwan: Average Year of Total Schooling (Age 25 and over)

economic development (Miracle on the Han River). Since 1990's, military influence has faded out from Korean politics. Over the years, the presidents, who are directly elected by the people, carried out a variety of economic reforms and made successful decisions during global financial crises. Korea therefore becomes one of the countries that are able to avoid recession but have significant economic growth. See figure 6.6 and figure 6.7.

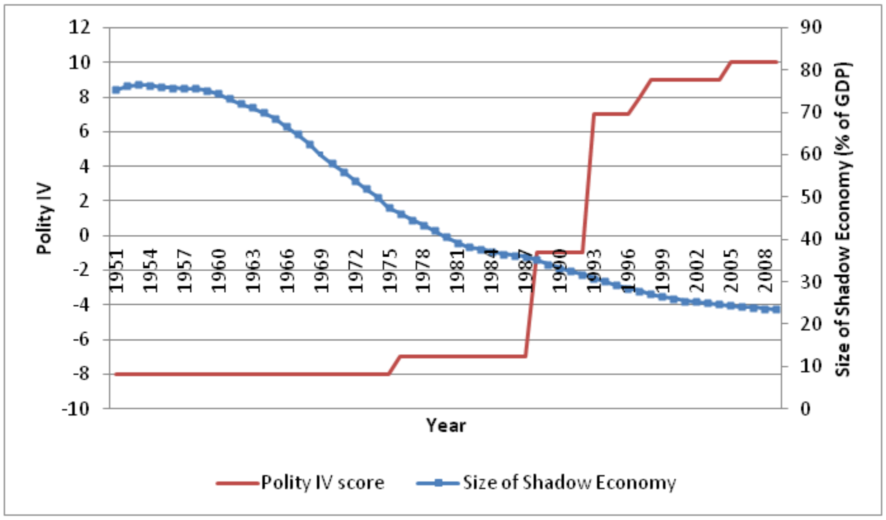


Figure 6.4. Taiwan: Polity IV v.s. Size of Informal Economy (% of GDP)

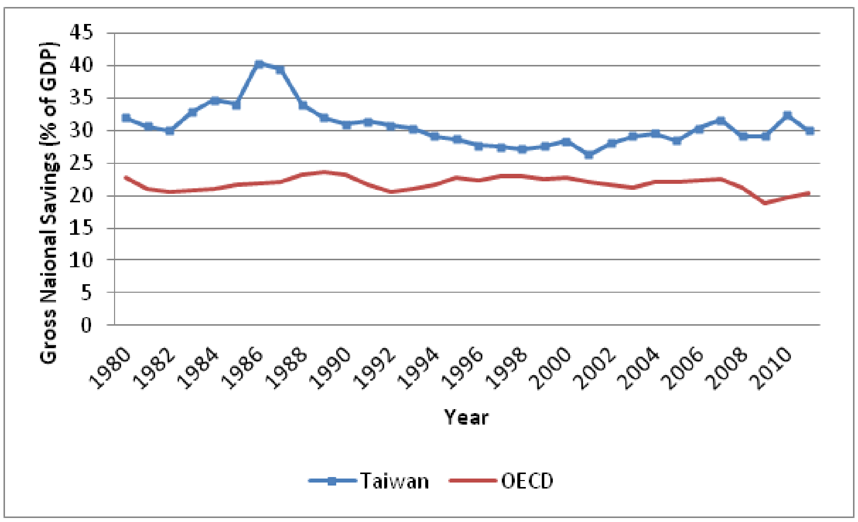


Figure 6.5. Taiwan v.s. OECD: Gross National Savings (% of GDP)

In additional to the similarity in political transition, Taiwan and Korea both have well-educated people (figure 6.3 and figure 6.8)[4], shrinking informal economy (figure 6.4 and figure 6.9)[5], and outstanding savings rate for future production (figure 6.5 and figure

[4]Data source: Barro, Robert and Jong-Wha Lee, April 2010, "A New Data Set of Educational Attainment in the World, 1950-2010." NBER Working Paper No. 15902.

[5]Data source: Elgin, Ceyhun and Oztunali, Oguz, "Shadow Economies around the World: Model Based Estimates," Working paper, 2012.

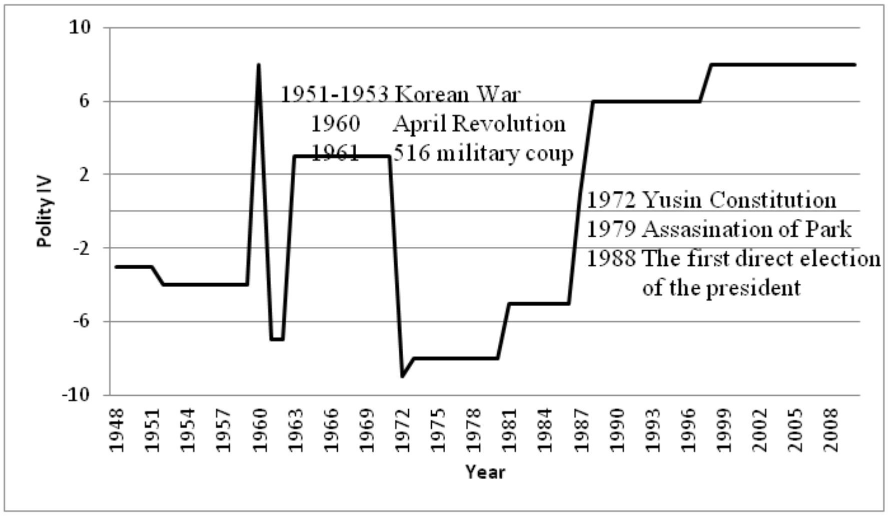


Figure 6.6. Korea: Polity IV Scores since 1948

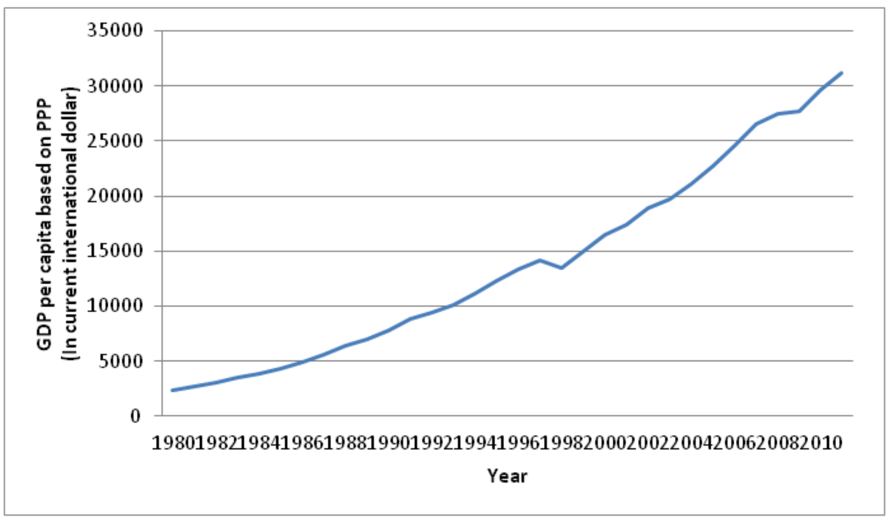


Figure 6.7. Korea: GDP Per Capita since 1980

6.10)[6] during democratization. These facts echo the different angles that this dissertation tries to picture for democratization.

Even though the democratization in Taiwan seems peaceful, there were some suppression incidents. The 228 Incident in 1947 and The Kaohsiung Incident in 1979 are two large-scale events. The former resulted in a massacre of civilians; the latter is regarded as

[6]Data source: International Monetary Fund, World Economic Outlook Database, October 2012.

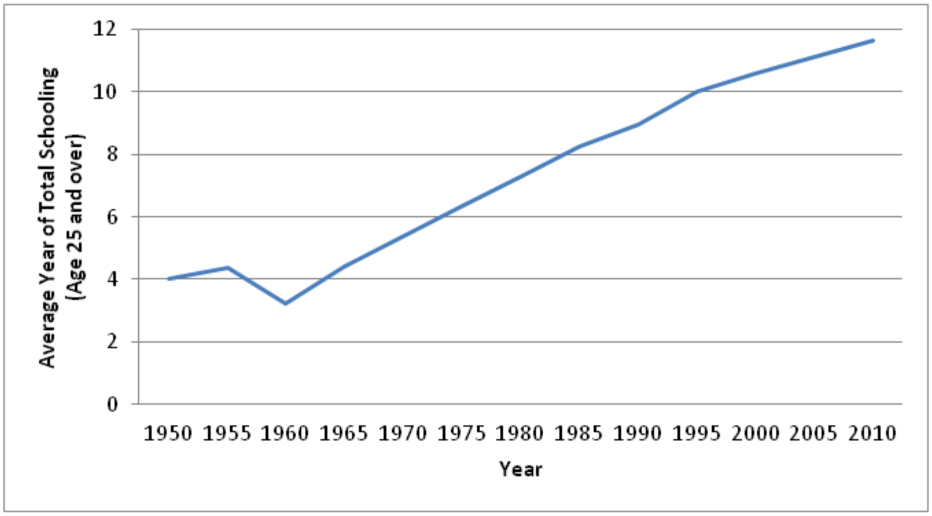


Figure 6.8. Korea: Average Year of Total Schooling (Age 25 and over)

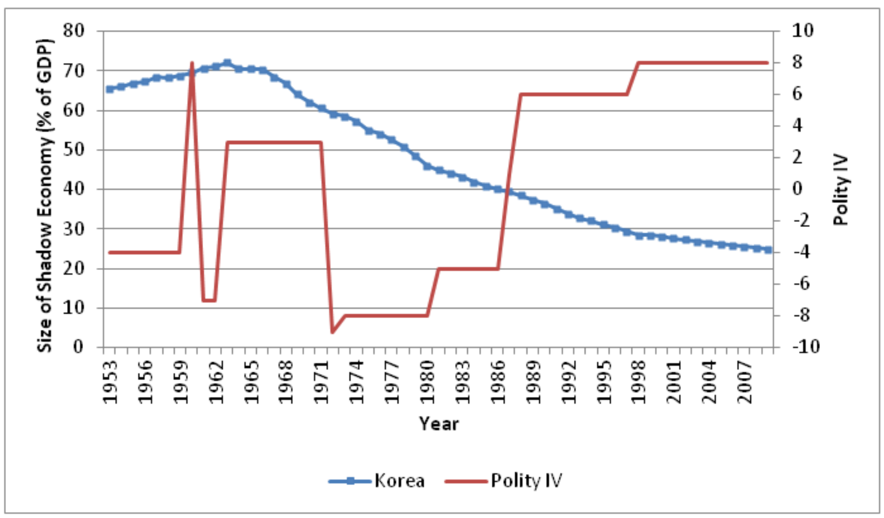


Figure 6.9. Korea: Polity IV v.s. Size of Shadow Economy (% of GDP)

the critical incident pushing Taiwan onto the democratization path. Atonement, reparations and memorials to the victims and their families have made by the government since 1992. Besides the quantifiable indemnification, the greater social loss is unquantifiable distrust or antagonism between peoples, which turns into grudges. Up to some point, the discontentment may result in another violent incident or be manipulated by politicians in

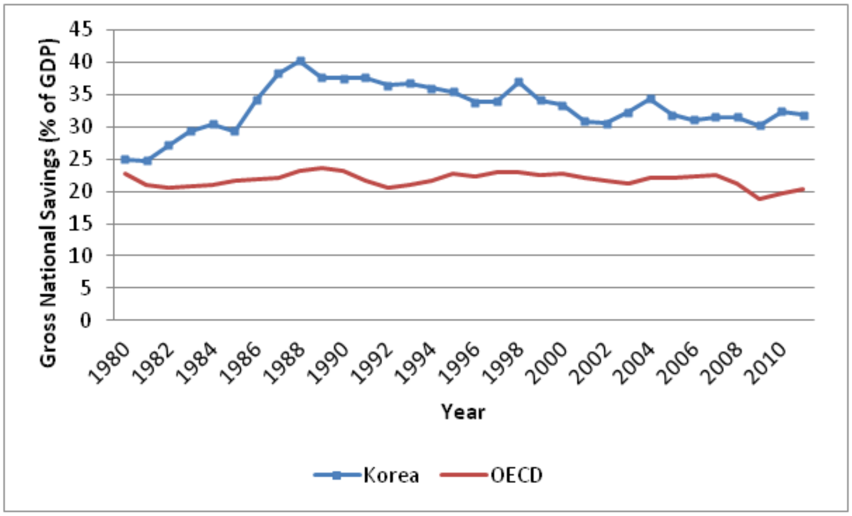


Figure 6.10. Korea v.s. OECD: Gross National Savings (% of GDP)

elections. To minimize the total social loss, I believe that peaceful democratization like Taiwan and Korea deserves greater attention.

References

[1] Acemoglu, Daren and Robinson, James A. *Economic Origins of Dictatorship and Democracy.* Cambridge, UK: Cambridge University Press, 2006.

[2] Acemoglu, Daron "Oligarchic Versus Democratic Societies." *Journal of the European Economic Association*, Mar. 2008, Vol. 6, No. 1, pp. 1-44.

[3] Acemoglu, Daron "Why not a Political Coase Theorem? Social Conflict, Commitment, and Politics." *Journal of Comparative Economics*, Dec. 2003, Vol. 31, Iss. 4, pp. 620-652.

[4] Acemoglu, Daron and Robinson, James A. "A Theory of Political Transitions." *The American Economic Review*, Sep. 2001, Vol. 91, No. 4, pp. 938-963.

[5] Acemoglu, Daron and Robinson, James A. "Why did the West Extend the Franchise? Democracy, Inequality, and Growth in Historical Perspective." *Quarterly Journal of Economics*, Nov. 2000, Vol. 115, No. 4, pp. 1167-1199.

[6] Acemoglu, Daron; Johnson, Simon; Robinson, James A. and Yared, Pierre "From Education to Democracy?." *AEA Papers and Proceedings*, May 2005, Vol. 95, No. 2, pp. 44-49.

[7] Acemoglu, Daron; Johnson, Simon; Robinson, James A. and Yared, Pierre "Income and Democracy." *The American Economic Review*, Jun. 2008, Vol. 98, Iss. 3, pp. 808-842.

[8] Alesina, Alberto and Rodrik, Dani "Distributive Politics and Economic Growth." *The Quarterly Journal of Economics*, May 1994, Vol. 109, No. 2, pp. 465-490.

[9] Aristotle. *Politics*, translated by Benjamin Jowett. Download from: classics.mit.edu/Aristotle/politics.html.

[10] Atkeson, Andrew, Chari, V.V. and Kehoe, Patrick J. "Taxing Capital Income: A Bad Idea." *Federal Reserve Bank of Minneapolis Quarterly Review*, Summer 1999, Vol. 23, No. 3, pp. 3-17.

[11] Atkinson, A. B. and Stiglitz, J. E. "The Structure of Indirect Taxation and Economic Efficiency." *Journal of Public Economics*, Apr. 1972, Vol. 1, Iss. 1, pp. 97-119.

[12] Barro, Rober J. "Determinants of Democracy." *Journal of Political Economy*, 1999, Vol. 107, No. 6, Pt. 2, pp. 158-183.

[13] Barro, Robert J. and Lee, Jong-Wha "International Data on Educational Attainment: Updates and Implications." *Center for International Development at Harvard University, Working paper No. 42*, Apr. 2000.

[14] Barro, Robert J. and Sala-I-Martin, Xavier. *Economic Growth*. New York, NY: McGraw-Hill Companies, Inc., 1995.

[15] Barro, Robert J. *Determinants of Economic Growth: A Cross-Country Empirical Study*. Cambridge, MA: The MIT Press, 1997.

[16] Benhabib, Jess, Rogerson, Richard and Wright, Randall "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *Journal of Political Economy*, Dec. 1991, Vol. 99, No. 6, pp. 1166-1187.

[17] Biglaiser, Glen and Brown, David S. "The Determinants of Economic Liberalization in Latin America." *Political Research Quarterly*, Dec. 2005, Vol. 58, No. 4, pp. 671-680.

[18] Blanchard, Olivier Jean and Fischer, Stanley. *Lectures on Macroeconomics*. Cambridge, MA: The MIT Press, 1989.

[19] Boileau, Martin. *Lecture Notes for Macroeconomic Theory I*, Department of Economics, University of Colorado at Boulder, fall 2004.

[20] Boix, Carles. *Democracy and Redistribution*. Cambridge, UK: Cambridge University Press, 2003.

[21] Bollen, K. A. *Structural Equations with Latent Variables*. New York: Wiley, Wiley Series in Probability and Mathematical Statistics, 1989.

[22] Bollen, Kenneth A. "Political Democracy: Conceptual and Measurement Traps." *Studies in Comparative International Development*, Mar. 1990, Vol. 25, No. 1, pp. 7-24.

[23] Booth, Alison and Coles, Melvyn "The Impact of Fiscal Policy on Labor Supply and Education in an Economy with Household and Market Production." *Center for Applied Macroeconomic Analysis, Working Paper 8*, Apr. 2997.

[24] Chamley, Christophe "Optimal Taxation of Capital Income in General Equilibrium with Infinite lives." *Econometrica*, May 1986, Vol. 54, No. 3, pp. 607-622.

[25] Chen, Shaohua, Datt, Gaurav and Ravallion, Martin "A Program for Calculating Poverty Measures from Grouped Data." Povcal Software, PovCalNet Online Poverty Analysis Tool, The World Bank.

[26] Chiang, Alpha C. *Fundamental Methods of Mathematical Economics*. New York, NY: McGraw-Hill Companies, Inc., 1984.

[27] Conley, John P. and Temimi, Akram "Endogenous Enfranchisement When Groups' Preferences Conflict." *The Journal of Political Economy*, Feb. 2001, Vol. 109, No. 1, pp. 79-102.

[28] De Bartolome, Charles. *Lecture Notes for Fundamental Theory of Public Economics*, Department of Economics, University of Colorado at Boulder, spring 2006.

[29] De La Croix, David and Michel, Philippe. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*. Cambridge, UK: Cambridge University Press, 2002.

[30] Deininger, Klaus and Squire, Lyn "A New Data Set Measuring Income Inequality." *The World Bank Economic Review*, Sep. 1996, Vol. 10, No. 3, pp. 565-591.

[31] Diamond, Peter A. and Mirrlees, James A. "Optimal Taxation and Public Production I: Production Efficiency." *The American Economic Review*, Mar. 1971, Vol. 61, No. 1, pp. 8-27.

[32] Diamond, Peter A. and Mirrlees, James A. "Optimal Taxation and Public Production II: Tax Rules." *The American Economic Review*, Jun. 1971, Vol. 61, No. 3, pp. 261-278.

[33] Elgin, Ceyhun and Oztunali, Oguz, "Shadow Economies around the World: Model Based Estimates," Working paper, 2012.

[34] Erosa, Andres and Gervais, Martin "Optimal Taxation in Life-Cycle Economies." *Journal of Economic Theory*, Aug. 2002, Vol. 105, Iss. 2, pp. 338-369.

[35] *Freedom in the World*, Freedom House, annual report since 1972.

[36] Friedman, Milton. *Capitalism and Freedom*. Chicago, IL: The University of Chicago Press, 1962.

[37] Glaeser, Edward L.; La Porta, Rafael, Lopez-De-Silanes, Florencio and Shleifer, Andrei "Do Institutions Cause Growth?." *Journal of Economic Growth*, Sept. 2004, Vol. 9, No. 3, pp. 271-303.

[38] Glaeser, Edward L.; Ponzetto, Giacomo A. M. and Shleifer, Andrei "Why Does Democracy Need Education?." *Journal of Economic Growth*, 2007, Vol. 12, No. 2, pp. 77-99.

[39] Golosov, Mikhail and Aleh Tsyvinski, "optimal fiscal and monetary policy (with commitment)", "The New Palgrave Dictionary of Economics", Eds. Steven N.

Durlauf and Lawrence E. Blume, Palgrave Macmillan, 2008, The New Palgrave Dictionary of Economics Online, Palgrave Macmillan. 24 September 2012, DOI:10.1057/9780230226203.1223

[40] Golosov, Mikhail and Aleh Tsyvinski, "optimal fiscal and monetary policy (without commitment)", "The New Palgrave Dictionary of Economics", Eds. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan, 2008, The New Palgrave Dictionary of Economics Online, Palgrave Macmillan. 24 September 2012, DOI:10.1057/9780230226203.1224

[41] Greenwood, Jeremy and Hercowitz, Zvi "The Allocation of Capital and Time over the Business Cycle." *Journal of Political Economy*, Dec. 1991, Vol. 99, No. 6, pp. 1188-1214.

[42] Greenwood, Jeremy, Rogerson, Richard and Wright, Randall. *Household Production in Real Business Cycle Theory*. Frontiers of Business Cycle Research.

[43] Gujarati, Damodar N. *Basic Econometrics*. New York, NY: McGraw-Hill Companies, Inc., 1995.

[44] Hu, Yunfang and Mino, Kazuo "Fiscal Policy, Home Production and Growth Dynamics." *Munich Personal RePEc Archive Paper No. 17017*, Aug. 2009.

[45] Jack, William and Lagunoff, Roger "Dynamic Enfranchisement." *Journal of Public Economics*, May 2006, Vol. 90, Iss. 4, pp. 551-572.

[46] Leland, Hayne E. "Saving and Uncertainty: The Precautionary Demand for Saving." *The Quarterly Journal of Economics*, Aug. 1968, Vol. 82, Iss. 3, pp. 465-473.

[47] Lipset, Seymour Martin "Some Social Requisites of Democracy: Economic Development and Political Legitimacy." *The American Political Science Review*, Mar. 1959, Vol. 53, No. 1, pp.69-105.

[48] Lizzeri, Alessandro and Persico, Nicola "Why did the Elites Extend the Suffrage? Democracy and the Scope of Government, with an Application to Britain's "Age of Reform." May 2004, *The Quarterly Journal of Economics*, Vol. 119, Iss. 2, pp. 707-765.

[49] Llavador, Humberto and Oxoby, Robert J. "Partisan Competition, Growth, and the Franchise." *The Quarterly Journal of Economics*, Aug. 2005, Vol. 120, Iss. 3, pp. 1155-1189.

[50] Marshall, Monty G. and Jaggers, Keith. *Polity IV Project: Political Regime Characteristics and Transitions, 1800-2010*. Societal-Systems Research Inc. and the Center for Systemic Peace, 2011.

[51] Mas-Colell, Andreu; Whinston, Michael D. and Green, Jerry R. *Microeconomic Theory*. New York, NY: Oxford University Press, Inc.: 1995.

[52] McGrattan, Ellen R., Rogerson, Richard and Wright, Randall "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy." *International Economic Review*, May 1997, Vol. 38, No. 2, pp. 267-290.

[53] Mincer, Jacob. *Schooling, Experience, and Earnings*. New York, New York, Columbia University Press, 1974.

[54] Mirrlees, J. A. "An Exploration in the Theory of Optimal Income Taxation." *Review of Economic Studies*, Apr. 1971, Vol. 38, No. 2, pp. 175-208.

[55] Myles, Gareth D. *Public Economics*. Cambridge, UK: Cambridge University Press, 1995.

[56] Papaioannou, Elias and Siourounis, Gregorios "Economic and Social Factors Driving the Third Wave of Democratization." *Journal of Comparative Economics*, Sept. 2008, Vol. 36, Iss. 3, pp. 365-387.

[57] Parente, Stephen L., Rogerson, Richard and Wright, Randall "Homework in Development Economics: Household Production and the Wealth of Nations." *Journal of Political Economy*, Aug. 2000, Vol. 108, No. 4, pp. 680-687.

[58] Perli, Roberto "Indeterminacy, Home Production, and the Business Cycle: A Calibrated Analysis." *Journal of Monetary Economics*, Feb. 1998, Vol. 41, Iss. 1, pp. 105-125.

[59] Perotti, Roberto "Growth, Income Distribution and Democracy: What the Data Say." *Journal of Economic Growth*, Jun. 1996, Vol. 1, No. 2, pp. 149-187.

[60] Persson, Torsten and Tabellini, Guido "Democracy and Development: The Devil in the Details." *The American Economic Review*, May 2006, Vol. 96, No. 2, pp. 319-324.

[61] Persson, Torsten and Tabellini, Guido "Democratic Capital: The Nexus of Political and Economic Change." *American Economic Journal: Macroeconomics*, Jul. 2009, Vol. 1, Iss. 2, pp. 88-126.

[62] Persson, Torsten and Tabellini, Guido "Is Inequality Harmful for Growth?." *The American Economic Review*, Jun. 1994, Vol. 84, No. 3, pp. 600-621.

[63] Persson, Torsten and Tabellini, Guido. *Political Economics: Explaining Economic Policy*. Cambridge, MA: The MIT Press, 2000.

[64] Przeworski, Adam and Limongi, Fernando "Political Regimes and Economic Growth." *The Journal of Economic Perspectives*, Summer 1993, Vol. 7, No. 3, pp. 51-69.

[65] Przeworski, Adam. *Democracy and The Market: Political and Economic Reforms in Eastern Europe and Latin America.* Cambridge,UK: Cambridge University Press, 1991.

[66] Przeworski, Adam; Alvarez, Michael E.; Cheibub, Jose Antonio and Limongi, Fernando. *Democracy and Development: Political Institutions and Well-Being in the World, 1950-1990.* Cambridge, UK: Cambridge University Press, 2000.

[67] Rios-Rull, Jose-Victor "Working in the Market, Working at Home, and the Acquisition of Skills: A General Equilibrium Approach." *The American Economic Review*, Sep. 1993, Vol. 83, No.4, pp. 893-907.

[68] Robinson, James A. "Economic Development and Democracy." *Annual Review of Political Science*, Jun. 2006, Vol. 9, No. 1, pp. 503-527.

[69] Rodrik, Dani "Democracies Pay Higher Wages." *The Quarterly Journal of Economics*, Aug. 1999, Vol. 114, No. 3, pp. 707-738.

[70] Rodrik, Dani and Wacziarg, Romain "Do Democratic Transitions Produce Bad Economic Outcomes?." *The American Economic Review*, May 2005, Vol. 95, No. 2, pp. 50-55.

[71] Romer, David. *Advanced Macroeconomics.* New York, NY: McGraw-Hill Companies, Inc., 1996.

[72] Rupert, Peter, Rogerson, Richard and Wright, Randall "Estimating Substitution Elasticities in Household Production Models." *Economic Theory*, Feb. 1995, Vol. 6, No. 1, pp. 179-193.

[73] Saez, Emmanuel "Using Elasticities to Derive Optimal Income Tax Rates." *Review of Economic Studies*, Jan. 2001, Vol. 68, Iss. 1, pp. 205-229.

[74] Sargent, Thomas J. *Dynamic Macroeconomic Theory.* Cambridge, MA: Harvard University Press, 1987.

[75] Simon, Carl P. and Blume Lawrence. *Mathematics for Economists.* New York, NY: W.W. Norton & Company, Inc., 1994.

[76] Starr, Ross M. *General Equilibrium Theory: An Introduction.* Cambridge, UK: Cambridge University Press, 1997.

[77] Tavares, Jose and Wacziarg, Romain "How Democracy Affects Growth?." *European Economic Review*, Aug. 2001, Vol. 45, Iss. 8, pp. 1341-1378.

[78] Tirole, Jean. *The Theory of Industrial Organization.* Cambridge, MA: The MIT Press, 1988.

[79] Varian, Hal R. *Microeconomic Analysis*. New York, NY: W.W. Norton & Company, Inc., 1992.

[80] Zax, Jeffrey. *Lecture Notes for Local Public Economics*, Department of Economics, University of Colorado at Boulder, fall 2006.

Appendix

1. Chapter 3

Proof of claim 1

(1) Denote (e_i, l_i) to be poor person i's educational attainment and physical labor hours. His effective labor hours are $z(e_i) l_i$. Aggregate effective labor hours are $\sum_{i=1}^{n^p} z(e_i) l_i$; production firms demand $\widetilde{L}$ hours. Effective labor market clears when $\widetilde{L} = \sum_{i=1}^{n^p} z(e_i) l_i$. By homogeneity, $\widetilde{L} = n^p z(e) l = z(e) L$, where L is aggregate physical labor hours.

(2) Suppose that the production firms adopts final good production technology $Y\left(K, \widetilde{L}\right)$, where $\widetilde{L} = z(e) L$. By $(A1)$, $\widetilde{w} = \frac{\partial Y\left(K,\widetilde{L}\right)}{\partial \widetilde{L}}$. Similarly, $w = \frac{\partial Y\left(K,\widetilde{L}\right)}{\partial L} = \frac{\partial Y\left(K,\widetilde{L}\right)}{\partial \widetilde{L}} \frac{\partial \widetilde{L}}{\partial L} = z(e) \widetilde{w}$.

Algorithm for the representative poor person's decision making at stage 3

Take (e, τ, w) as given, the poor person maximizes $\{[(1-\tau) wl]^\rho + (1-l-e)^\rho\}^{\frac{1}{\rho}}$ with respect to l. The first-order condition $(f.o.c.)$ of l gives (3.1). Rewrite (3.1) to derive $l(e, \tau, w) = \frac{(1-e)(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}{1+(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}$. Plug $l(e, \tau, w)$ back to the budget constraint for $c^p(e, \tau, w) = \frac{(1-e)(1-\tau)^{\frac{1}{1-\rho}} w^{\frac{1}{1-\rho}}}{1+(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}$.

Proof of claim 2

- $\frac{\partial c^p(e,\tau,w)}{\partial \tau} = -\frac{1}{1-\tau} \frac{\frac{1}{1-\rho} + (1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}{1+(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} c^p(e, \tau, w) < 0.$
- $\frac{\partial l(e,\tau,w)}{\partial \tau} = -\frac{\rho}{1-\rho} \frac{1}{1+(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \frac{l(e,\tau,w)}{1-\tau} \lesseqgtr 0 \text{ if } \rho \gtreqless 0.$

Proof of claim 3

- $\frac{\partial c^p(e,\tau,w)}{\partial w} = \frac{1}{1-\rho} \frac{1+(1-\rho)(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}{1+(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \frac{c^p(e,\tau,w)}{w} > 0.$
- $\frac{\partial l(e,\tau,w)}{\partial w} = \frac{\rho}{1-\rho} \frac{1}{1+(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \frac{l(e,\tau,w)}{w} \gtreqless 0$ if $\rho \gtreqless 0.$

The oligarchic regime: Stage 2: The rich's consumption and the tax rate

The representative price-manipulator rich person takes e given and maximizes $rK + \tau wL(e,\tau,w)$ with respect to τ. Take total differential of τ with respect to (r,w) and $L(e,\tau,w)$ to derive (3.6).

The oligarchic regime: Stage 1: The poor's educational attainment

Denote $v^p(e,\tau,w)$ to be the value function of $u^p(c^p,l,e)$, i.e.

$$v^p(e,\tau,w) = \{[(1-\tau)\, wl(e,\tau,w)]^\rho + [1-l(e,\tau,w)-e]^\rho\}^{\frac{1}{\rho}},$$

where $w = z(e)\,\widetilde{w}$. The representative poor person takes (τ,w) as given and maximizes $\{[(1-\tau)\, z(e)\,\widetilde{w}l(e,\tau,w)]^\rho + [1-l(e,\tau,w)-e]^\rho\}^{\frac{1}{\rho}}$ with respect to e. By *Envelope Theorem*, the indirect effects through (c^p,l) are taken into consideration at stage 3 such that $\frac{\partial u^p}{\partial c^p}\frac{dc^p}{de} = \frac{\partial u^p}{\partial l}\frac{dl}{de} = 0$. So consider the direct effect of e on $v^p(e,\tau,w)$ only. Take $l(e,\tau,w)$ as given and take partial derivative of $v^p(e,\tau,w)$ with respect to e. It is (3.6).

The democratic regime: Stage 2: The rich's consumption and the tax rate

The representative price-manipulator rich person takes (e,τ) given to maximize $rk^r + \frac{1}{n^r}\tau wL(e,\tau,w)$. Without the tax power, c^r is determined by the poor and the markets. So is C^r.

The democratic regime: Stage 1: The poor's educational attainment

Denote $v^p(e,\tau,w)$ to be the value function of $u^p(c^p,l,e)$, i.e.

$$v^p(e,\tau,w)=\{[wl(e,\tau,w)]^\rho+[1-l(e,\tau,w)-e]^\rho\}^{\frac{1}{\rho}},$$

where $w=z(e)\widetilde{w}$ and $\tau=0$. The representative poor person takes w as given and maximizes $\{[z(e)\widetilde{w}l(e,0,w)]^\rho+[1-l(e,0,w)-e]^\rho\}^{\frac{1}{\rho}}$ with respect to e. By *Envelope Theorem*, consider the direct effect of e only. Take $l(e,0,w)$ as given and take partial derivative of $v^p(e,0,w)$ with respect to e. It is (3.12).

Production firms' decision making

The production firms search $\widetilde{L}$ which maximizes profit $\Pi\left(\widetilde{L}\right)=AK^\alpha\widetilde{L}^{1-\alpha}-rK-\widetilde{w}\widetilde{L}$. Profit is maximized when $\frac{\partial\Pi(\widetilde{L})}{\partial\widetilde{L}}=(1-\alpha)AK^\alpha\widetilde{L}^{-\alpha}-\widetilde{w}=0$ holds. By zero-profit condition, $\alpha AK^{\alpha-1}\widetilde{L}^{1-\alpha}-r=0$ must hold too.

Equation (3.17) for characterization of GE under the oligarchic regime

Step 1: $\frac{dw}{d\tau}=-\alpha\frac{w}{l}\left(\frac{\partial l}{\partial\tau}+\frac{\partial l}{\partial w}\frac{dw}{d\tau}\right)=-\frac{\alpha\frac{w}{l}\frac{\partial l}{\partial\tau}}{1+\alpha\frac{w}{l}\frac{\partial l}{\partial w}}=\frac{\frac{\alpha\rho}{1-\rho}\frac{w}{1-\tau}}{1+\frac{\alpha\rho}{1-\rho}+(1-\tau)^{\frac{\rho}{1-\rho}}w^{\frac{\rho}{1-\rho}}}>0$: At stage 2, the rich takes e as given. Recall that $w=z(e)(1-\alpha)AK^\alpha[z(e)L(e,\tau,w)]^{-\alpha}$. Apply *Implicit Function Theorem (IFT)* to $F(\tau,w)\equiv w-(1-\alpha)A\left(\frac{K}{n^p}\right)^\alpha[z(e)]^{1-\alpha}[l(e,\tau,w)]^{-\alpha}=0$. By *IFT,* claim 2, and claim 3, $\frac{dw}{d\tau}=-\frac{F_\tau}{F_w}=-\frac{\alpha\frac{w}{l}\frac{\partial l}{\partial\tau}}{1+\alpha\frac{w}{l}\frac{\partial l}{\partial w}}=\frac{\frac{\alpha\rho}{1-\rho}\frac{w}{1-\tau}}{1+\frac{\alpha\rho}{1-\rho}+(1-\tau)^{\frac{\rho}{1-\rho}}w^{\frac{\rho}{1-\rho}}}>0$. Take total derivative of w with respect to τ: $\frac{dw}{d\tau}=-\alpha\frac{w}{l}\left(\frac{\partial l}{\partial\tau}+\frac{\partial l}{\partial w}\frac{dw}{d\tau}\right)$.

Step 2: $\frac{dr}{d\tau}=\alpha\frac{n^p}{K}w\left(\frac{\partial l}{\partial\tau}+\frac{\partial l}{\partial w}\frac{dw}{d\tau}\right)<0$: At stage 2, the rich takes e as given. Take total derivative of $r=\alpha A\left(\frac{K}{n^p}\right)^{\alpha-1}[z(e)]^{1-\alpha}[l(e,\tau,w)]^{1-\alpha}$ with respect to τ.

Step 3: $1+(1-\tau)^{\frac{\rho}{1-\rho}}w^{\frac{\rho}{1-\rho}}=\frac{\rho}{1-\rho}\frac{\tau}{1-\tau}$: Plug step 1, step 2, claim 2 and claim 3 into condition (3.6). Simplify it.

Step 4: $(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}} = \frac{\frac{z(e)}{z'(e)}}{1-e-\frac{z(e)}{z'(e)}}$: Condition (3.1) and (3.6) imply $l = \frac{z(e)}{z'(e)}$. Equate $l = \frac{z(e)}{z'(e)}$ and $l(e,\tau,w)$. Rewrite $(1-\tau)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}$ in terms of e only.

Step 5: $w = (1-\alpha) A \left(\frac{K}{n^p}\right)^{\alpha} [z(e)]^{1-\alpha} \left[\frac{z(e)}{z'(e)}\right]^{-\alpha}$: Plug $l = \frac{z(e)}{z'(e)}$ into $w = (1-\alpha) A \left(\frac{K}{n^p}\right)^{\alpha} \times [z(e)]^{1-\alpha} l^{-\alpha}$.

Step 6: $1-\tau = \frac{1}{(1-\alpha)A\left(\frac{K}{n^p}\right)^{\alpha}} \frac{\left[\frac{z(e)}{z'(e)}\right]^{-1+\alpha+\frac{1}{\rho}}}{[z(e)]^{1-\alpha}\left[1-e-\frac{z(e)}{z'(e)}\right]^{-1+\frac{1}{\rho}}}$: Combine step 4 and step 5.

Step 7: $\frac{1}{1-\rho} + \frac{\frac{z(e)}{z'(e)}}{1-e-\frac{z(e)}{z'(e)}} = \frac{\rho\phi}{1-\rho} \frac{[z(e)]^{1-\alpha}\left[1-e-\frac{z(e)}{z'(e)}\right]^{\frac{1-\rho}{\rho}}}{\left[\frac{z(e)}{z'(e)}\right]^{\frac{1+\alpha\rho-\rho}{\rho}}}$: Plug step 4 and step 6 back to step 3. Simplify it.

Step 8: $[\theta-(\theta+\rho)e]\, e^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}} = \psi [\theta-(1+\theta)e]^{\frac{1}{\rho}}$: Apply $z(e) = \gamma e^{\theta}$ into step 7. Rewrite it to the form of (3.17).

Equation (3.18) for characterization of GE under the democratic regime

Step 1: $l = \frac{z(e)}{z'(e)}$ by (3.1), (3.12) and $\tau = 0$.

Step 2: $\frac{w^{\frac{\rho}{1-\rho}}}{1+w^{\frac{\rho}{1-\rho}}} = \frac{z(e)}{z'(e)}\frac{1}{1-e}$: Equate $l = \frac{z(e)}{z'(e)}$ and $l(e,0,w) = \frac{(1-e)w^{\frac{\rho}{1-\rho}}}{1+w^{\frac{\rho}{1-\rho}}}$. Rewrite it.

Step 3: $w = (1-\alpha) A \left(\frac{K}{n^p}\right)^{\alpha} [z(e)]^{1-\alpha} \left[\frac{z(e)}{z'(e)}\right]^{-\alpha}$: Plug $l = \frac{z(e)}{z'(e)}$ into $w = (1-\alpha) A \left(\frac{K}{n^p}\right)^{\alpha} \times [z(e)]^{1-\alpha} l^{-\alpha}$.

Step 4: $\phi^{\frac{\rho}{1-\rho}} \left[1-e-\frac{z(e)}{z'(e)}\right] [z(e)]^{\frac{(1-\alpha)\rho}{1-\rho}} = \left[\frac{z(e)}{z'(e)}\right]^{\frac{1+\alpha\rho-\rho}{1-\rho}}$: Plug step 3 into step 2. Simplify and rewrite it.

Step 5: $[\theta-(1+\theta)e_d]\, e_d^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}} = \frac{\psi}{\rho} [\theta-(1+\theta)e_d]^{\frac{1}{\rho}}$: Apply $z(e) = \gamma e^{\theta}$ into step 4. Rewrite it to the form of (3.18).

Proof of claim 4

Consider $e \in \left(0, \frac{\theta}{1+\theta}\right)$ only. Let $f_{oL}(e) = [\theta-(\theta+\rho)e]\, e^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}}$ and $f_{oR}(e) = \psi[\theta-(1+\theta)e]^{\frac{1}{\rho}}$. e_o is unique if and only if $f_{oL}(e)$ intersects $f_{oR}(e)$ only once in the range of $\left(0, \frac{\theta}{1+\theta}\right)$. Notice that $\lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{oR}(e) = 0 < \lim_{e\to 0^+} f_{oR}(e) = \psi\theta^{\frac{1}{\rho}}$; $f'_{oR}(e) < 0 < f''_{oR}(e)$, but the shape of $f_{oL}(e)$ depends on parameters. If $\rho < \min\left\{1, \frac{1}{(1-\alpha)(1+\theta)}\right\}$, $\lim_{e\to 0^+} f_{oL}(e) = \theta > 0$; $\lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{oL}(e) > 0$; $f'_{oL}(e) \gtreqless 0$ if $e \lesseqgtr \frac{\theta}{\theta+\rho}\frac{1-(1-\alpha)(1+\theta)\rho}{1-(1-\alpha)(1+\theta)\rho+\rho}$. Uniqueness of solution holds. If $\rho = \frac{1}{(1-\alpha)(1+\theta)}$, $\lim_{e\to 0^+} f_{oL}(e) = \theta > \lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{oL}(e) = \frac{\theta(1-\rho)}{1+\theta} > 0$; $f'_{oL}(e) = -(\theta+\rho) < 0 = f''_{oL}(e)$. To have unique solution, additional condition is necessary. It is, $\lim_{e\to 0^+} f_{oR}(e) > \lim_{e\to 0^+} f_{oL}(e)$, i.e. $1 < \psi\theta^{\frac{1-\rho}{\rho}}$. Therefore, if $\rho = \frac{1}{(1-\alpha)(1+\theta)} < 1 < \psi\theta^{\frac{1-\rho}{\rho}}$, uniqueness of solution holds. If $\frac{1}{(1-\alpha)(1+\theta)} < \rho$, $\lim_{e\to 0^+} f_{oL}(e) = \infty$; $\lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{oL}(e) > 0 > f'_{oL}(e)$. Either no solution, or tangent solution, or two solutions. Disregard this case.

Consider $e \in \left(0, \frac{\theta}{1+\theta}\right)$ only. Let $f_{dL}(e) = [\theta - (1+\theta)e]\, e^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}}$ and $f_{dR}(e) = \frac{\psi}{\rho}[\theta - (1+\theta)e]^{\frac{1}{\rho}}$. e_d is unique if and only if $f_{dL}(e)$ intersects $f_{dR}(e)$ only once in the range of $\left(0, \frac{\theta}{1+\theta}\right)$. Notice that $\lim_{e\to 0^+} f_{dR}(e) > 0 = \lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{dR}(e)$; $f'_{dR}(e) < 0 < f''_{dR}(e)$, but the shape of $f_{dL}(e)$ depends on parameters. If $\rho < \min\left\{1, \frac{1}{(1-\alpha)(1+\theta)}\right\}$, $\lim_{e\to 0^+} f_{dL}(e) = \lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{dL}(e) = 0$; $f'_{dL}(e) \gtreqless 0$ if $e \lesseqgtr \frac{\theta}{1+\theta}\frac{1-(1-\alpha)(1+\theta)\rho}{1-(1-\alpha)(1+\theta)\rho+\rho}$. Uniqueness of solution holds. If $\rho = \frac{1}{(1-\alpha)(1+\theta)}$, $\lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{dL}(e) = 0 < \lim_{e\to 0^+} f_{dL}(e)$; $f''_{dL}(e) = 0 > f'_{dL}(e)$. To have unique solution, additional condition is necessary. It is, $\lim_{e\to 0^+} f_{dR}(e) > \lim_{e\to 0^+} f_{dL}(e)$, i.e. $\psi\theta^{\frac{1-\rho}{\rho}} > \rho$. Therefore, if $\rho = \frac{1}{(1-\alpha)(1+\theta)} < \min\left\{1, \psi\theta^{\frac{1-\rho}{\rho}}\right\}$, uniqueness of solution holds. If $\frac{1}{(1-\alpha)(1+\theta)} < \rho$, $\lim_{e\to 0^+} f_{dL}(e) = \infty$; $\lim_{e\to\left(\frac{\theta}{1+\theta}\right)^-} f_{dL}(e) = 0 > f'_{dL}(e)$. Either no solution, or tangent solution, or two solutions. Disregard this case.

Therefore, (e_o, e_d) are unique if one of the conditions holds: $\rho < \min\left\{1, \frac{1}{(1-\alpha)(1+\theta)}\right\}$ or $\rho = \frac{1}{(1-\alpha)(1+\theta)} < 1 < \psi\theta^{\frac{1-\rho}{\rho}}$.

Characterization of GE under the oligarchic regime

- $l_o = \frac{z(e_o)}{z'(e_o)} = \frac{e_o}{\theta} \Rightarrow L_o = n^p l_o = \frac{n^p}{\theta} e_o \Rightarrow \widetilde{L}_o = \gamma e_o^{\theta}\frac{n^p}{\theta} e_o = \frac{\gamma n^p}{\theta} e_o^{1+\theta}$

- $Y_o = AK^\alpha \left(\widetilde{L}_o\right)^{1-\alpha} = \frac{\psi n^p}{(1-\alpha)\theta\rho} e_o^{(1-\alpha)(1+\theta)}$
- $\widetilde{w}_o = (1-\alpha) A \left(\frac{K}{n^p}\right)^\alpha \left(\gamma e_o^\theta\right)^{-\alpha} \left(\frac{e_o}{\theta}\right)^{-\alpha} = \frac{\psi}{\gamma\rho} e_o^{-\alpha(1+\theta)} \Rightarrow w_o = \gamma e^\theta \frac{\psi}{\gamma\rho} e_o^{-\alpha(1+\theta)} = \frac{\psi}{\rho} e_o^{(1-\alpha)\theta-\alpha}$
- $r_o = \alpha A \left(\frac{K}{n^p}\right)^{\alpha-1} \left(\gamma e_o^\theta\right)^{1-\alpha} \left(\frac{e_o}{\theta}\right)^{1-\alpha} = \frac{\alpha}{1-\alpha} \frac{\psi}{\theta\rho} \frac{n^p}{K} e_o^{(1-\alpha)(1+\theta)}$
- $\tau_o = 1 - \rho \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}$
  - $\tau_o = 1 - \frac{1}{(1-\alpha)A\left(\frac{K}{n^p}\right)^\alpha} \frac{\left[\frac{z(e_o)}{z'(e_o)}\right]^{-1+\alpha+\frac{1}{\rho}}}{[z(e_o)]^{1-\alpha}\left[1-e-\frac{z(e_o)}{z'(e_o)}\right]^{-1+\frac{1}{\rho}}} = 1 - \frac{1}{\phi} \frac{\left(\frac{e_o}{\theta}\right)^{-1+\alpha+\frac{1}{\rho}}}{\left(\gamma e_o^\theta\right)^{1-\alpha}\left(1-e_o-\frac{e_o}{\theta}\right)^{-1+\frac{1}{\rho}}}$
  - $\tau_o = 1 - \frac{\psi}{\phi\gamma^{1-\alpha}\theta^\alpha} \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}$ by $\frac{1}{\psi}\left[\theta - (\theta+\rho) e_o\right] e_o^{\frac{1-(1-\alpha)(1+\theta)\rho}{\rho}} = \left[\theta - (1+\theta) e_o\right]^{\frac{1}{\rho}}$.
  - $\tau_o = 1 - \rho \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}$ by $\frac{\psi}{\phi\gamma^{1-\alpha}\theta^\alpha} = \rho$.
- $c_o^p = (1-\tau_o) w_o l_o = \rho \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o} \frac{\psi}{\rho} e_o^{(1-\alpha)\theta-\alpha} \frac{e_o}{\theta} = \frac{\psi}{\theta} \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o} e_o^{(1-\alpha)(1+\theta)}$
- $C_o^p = \frac{\psi n^p}{\theta} \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o} e_o^{(1-\alpha)(1+\theta)}$
- $C_o^r = Y_o - C_o^p = \frac{\psi n^p}{\theta\rho} \left[\frac{1}{1-\alpha} - \rho \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}\right] e_o^{(1-\alpha)(1+\theta)}$
- $c_o^r = \frac{\psi}{\theta\rho} \frac{n^p}{n^r} \left[\frac{1}{1-\alpha} - \rho \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o}\right] e_o^{(1-\alpha)(1+\theta)}$

Characterization of GE under the democratic regime

- $l_d = \frac{z(e_d)}{z'(e_d)} = \frac{e_d}{\theta} \Rightarrow L_d = \frac{n^p}{\theta} e_d \Rightarrow \widetilde{L}_d = \gamma e_d^\theta \frac{n^p}{\theta} e_d = \frac{\gamma n^p}{\theta} e_d^{1+\theta}$
- $Y_d = AK^\alpha \left(\widetilde{L}_d\right)^{1-\alpha} = \frac{\psi n^p}{(1-\alpha)\theta\rho} e_d^{(1-\alpha)(1+\theta)}$
- $\widetilde{w}_d = (1-\alpha) A \left(\frac{K}{n^p}\right)^\alpha \left(\gamma e_d^\theta\right)^{-\alpha} \left(\frac{e_d}{\theta}\right)^{-\alpha} = \frac{\psi}{\gamma\rho} e_d^{-\alpha(1+\theta)} \Rightarrow w_d = \gamma e_d^\theta \frac{\psi}{\gamma\rho} e_d^{-\alpha(1+\theta)} = \frac{\psi}{\rho} e_d^{(1-\alpha)\theta-\alpha}$
- $r_d = \alpha A \left(\frac{K}{n^p}\right)^{\alpha-1} \left(\gamma e_d^\theta\right)^{1-\alpha} \left(\frac{e_d}{\theta}\right)^{1-\alpha} = \frac{\alpha}{1-\alpha} \frac{\psi}{\theta\rho} \frac{n^p}{K} e_d^{(1-\alpha)(1+\theta)}$
- $c_d^p = w_d l_d = \frac{\psi}{\theta\rho} e_d^{(1-\alpha)(1+\theta)} \Rightarrow C_d^p = \frac{\psi n^p}{\theta\rho} e_d^{(1-\alpha)(1+\theta)}$
- $C_d^r = Y_d - C_d^p = \frac{\psi n^p}{(1-\alpha)\theta\rho} e_d^{(1-\alpha)(1+\theta)} - \frac{\psi n^p}{\theta\rho} e_d^{(1-\alpha)(1+\theta)} = \frac{\alpha}{1-\alpha} \frac{\psi n^p}{\theta\rho} e_d^{(1-\alpha)(1+\theta)} \Rightarrow c_d^r = \frac{\alpha}{1-\alpha} \frac{\psi}{\theta\rho} \frac{n^p}{n^r} e_d^{(1-\alpha)(1+\theta)}$

Proof of claim 5

- $e_o < e_d$: e_o such that $f_{oL}(e_o) = f_{oR}(e_o)$. $f_{dL}(e_o) < f_{oL}(e_o) = f_{oR}(e_o) < f_{dR}(e_o)$ for $\rho \in (0,1)$. For $f'_{dL}(e_o) > 0 > f'_{dR}(e_o)$, e increases until $f_{dL}(e_d) = f_{dR}(e_d)$. Hence, $e_o < e_d$.
- $l = \frac{e}{\theta} \Rightarrow l_o < l_d$. $1 - l - e = 1 - \frac{e}{\theta} - e = 1 - \frac{1+\theta}{\theta}e \Rightarrow 1 - l_o - e_o > 1 - l_d - e_d$
- $z(e) = \gamma e^{\theta} \Rightarrow z(e_o) < z(e_d)$. $\widetilde{L} = \frac{\gamma n^p}{\theta} e^{1+\theta} \Rightarrow \widetilde{L}_o < \widetilde{L}_d$.
- $wl = \frac{\psi}{\theta\rho} e^{(1-\alpha)(1+\theta)} \Rightarrow w_o l_o < w_d l_d$. $\widetilde{w} = \frac{\psi}{\gamma\rho} e^{-\alpha(1+\theta)} \Rightarrow \widetilde{w}_o > \widetilde{w}_d$
- $r = \frac{\alpha}{1-\alpha} \frac{\psi}{\theta\rho} \frac{n^p}{K} e^{(1-\alpha)(1+\theta)} \Rightarrow r_d < r_d$
- $c_o^p = \frac{\psi}{\theta} \frac{\theta-(1+\theta)e_o}{\theta-(\theta+\rho)e_o} e_o^{(1-\alpha)(1+\theta)} < \frac{\psi}{\theta} \frac{1}{\rho} e_d^{(1-\alpha)(1+\theta)} = c_d^p$.
- $Y = \frac{\psi n^p}{(1-\alpha)\theta\rho} e^{(1-\alpha)(1+\theta)} \Rightarrow Y_o < Y_d$

Proof of claim 6

$w = \frac{\psi}{\rho} e^{(1-\alpha)\theta-\alpha}$. $w_d \gtreqless w_o$ iff $\theta \gtreqless \frac{\alpha}{1-\alpha}$ iff $\frac{1}{(1-\alpha)(1+\theta)} \lesseqgtr 1$.

Proof of proposition 7

The representative poor person is better off under the democratic regime if he gains higher utility than under the oligarchic regime, i.e. $[c_o^\rho + (1 - l_o - e_o)^\rho]^{\frac{1}{\rho}} < [c_d^\rho + (1 - l_d - e_d)^\rho]^{\frac{1}{\rho}}$. Plug (c_o, c_d, l_o, l_d) in table 3.1 and apply condition (3.17) and (3.18) into this inequality. Simplify and rearrange it to derive $\frac{1-e_o}{[\theta-(1+\theta)e_o]^{1-\rho}} < \frac{1-e_d}{[\theta-(1+\theta)e_d]^{1-\rho}}$, where (e_o, e_d) are governed by (3.17) and (3.18) respectively.

Let $g(e) = \frac{1-e}{[\theta-(1+\theta)e]^{1-\rho}}$. $\lim_{e\to 0^+} g(e) > 0$; $\lim_{e\to\left(\frac{\theta}{1+\theta}\right)^+} g(e) = \infty$; $g'(e) > 0$ if $\rho \leq \frac{1}{1+\theta}$. Notice that $\rho \leq \frac{1}{1+\theta}$ implies $\rho < \min\left\{1, \frac{1}{(1-\alpha)(1+\theta)}\right\}$, so the unique (e_o, e_d) exists. $e_o < e_d$ by claim 5. Therefore, $g(e_o) < g(e_d)$, so the poor are better off. However, if $\frac{1}{1+\theta} < \rho$ and the condition for uniqueness of (e_o, e_d) holds, $g'(e) \lesseqgtr 0$ for $e \lesseqgtr \underline{e}$, where $\underline{e} = 1 - \frac{1}{(1+\theta)\rho} \in \left(0, \frac{\theta}{1+\theta}\right)$. $g(e_o) < g(e_d)$ cannot be concluded even though $e_o < e_d$ by claim 5.

Proof of proposition 8

Democratization is adopted iff $u^r(c_o^r) < u^r(c_d^r)$ iff $c_o^r < c_d^r$ iff $C_o^r < C_d^r$. Plug (C_o^r, C_d^r) in table 3.1 into $C_o^r < C_d^r$. Rewrite it to inequality (3.20) holds, where (e_o, e_d) are governed by (3.17) and (3.18) respectively.

2. Chapter 4

2.1. A Preliminary Model

The representative poor person's decision making under the oligarchic regime

Suppose that the representative poor person works and consumes at the market if two types of production is indifferent to him. (c^p, c_b^p) are perfect substitutes. The poor person either works and consumes at the market only or works and consumes at home only.

Suppose $(1-\tau_l)\,w \geq \frac{B+\kappa(l^*)}{1-l^*}$. If the poor person works and consumes at the market only, he is better off than working at home or working for both types of production. That is, $\ln[1+(1-\tau_l)\,w\,(1-l^*)-\kappa\,(l^*)] \geq \max\{\ln(1+B), \ln[1+(1-\tau_l)\,wl-\kappa\,(l^*)+Bl_b]\}$, where $l>0$, $l_b>0$, and $l+l_b=1-l^*$.

Suppose $(1-\tau_l)\,w < \frac{B+\kappa(l^*)}{1-l^*}$. If the poor person works and consumes at home only, he is better off than working at the market or working for both types of production. That is, $\ln(1+B) > \max\{\ln[1+(1-\tau_l)\,wl-\kappa\,(l^*)+Bl_b], \ln[1+(1-\tau_l)\,w\,(1-l^*)-\kappa\,(l^*)]\}$, where $l>0$, $l_b>0$, and $l+l_b=1-l^*$.

Claim 40. *$l \in (0,1)$ and $l_b \in (0,1)$ such that $l+l_b = 1-l^*$ cannot be an equilibrium.*

Proof. Suppose that $l \in (0,1)$ and $l_b \in (0,1)$ such that $l + l_b = 1 - l^*$ in equilibrium. Suppose $\lambda \in (0,1)$ such that $l = \lambda(1-l^*)$ and $l_b = (1-\lambda)(1-l^*)$. To have this pair of (l, l_b) to be optimal, $\ln[(1-\tau_l)\,w\lambda(1-l^*) - \kappa(l^*) + B(1-\lambda)(1-l^*)] > \max\{\ln B, \ln[(1-\tau_l)\,w(1-l^*) - \kappa(l^*)]\}$. That is, $B > (1-\tau_l)\,w > \frac{B - B(1-\lambda)(1-l^*) + \kappa(l^*)}{\lambda(1-l^*)}$. However, $\frac{B - B(1-\lambda)(1-l^*) + \kappa(l^*)}{\lambda(1-l^*)} < B$. Therefore, $l \in (0,1)$ and $l_b \in (0,1)$ such that $l + l_b = 1 - l^*$ cannot be an equilibrium. □

The representative poor person's decision making under the democratic regime

Let $\tau_l = 0$. The reasoning is the same as "The representative poor person's decision making under the oligarchic regime."

Characterization of GE

Suppose $(1-\alpha)\,A\left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha \geq \frac{B+\kappa(l^*)}{1-l^*}$.

- $l_o = l_d = 1 - l^*$ and $l_{b,o} = l_{b,d} = 0$, for
  - $w_o = w_d = (1-\alpha)\,A\left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha \geq \frac{B+\kappa(l^*)}{1-l^*}$;
  - $(1-\tau_{l,o})\,w_o = \frac{B+\kappa(l^*)}{1-l^*}$, where $\tau_{l,o} = 1 - \frac{\frac{B+\kappa(l^*)}{1-l^*}}{(1-\alpha)A\left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha} \geq \tau_{l,d} = 0$.
- $r_o = r_d = \alpha A\left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^{\alpha-1}$
- $\frac{Y_o}{n^p} = \frac{Y_d}{n^p} = A\left(\frac{K}{n^p}\right)^\alpha(1-l^*)^{1-\alpha}$; $y_{b,o} = y_{b,d} = c^p_{b,o} = c^p_{b,d} = 0$.
- $\begin{cases} c^p_o = (1-\tau_{l,o})\,w_o(1-l^*) - \kappa(l^*) = B \\ c^p_d = w_d(1-l^*) - \kappa(l^*) = (1-\alpha)\,A\left(\frac{K}{n^p}\right)^\alpha(1-l^*)^{1-\alpha} - \kappa(l^*) \end{cases}$
- $\begin{cases} \frac{n^r}{n^p}c^r_o = \frac{n^r}{n^p}\left(r_o k^r + \frac{1}{n^r}\tau_{l,o}w_o L_o\right) = A\left(\frac{K}{n^p}\right)^\alpha(1-l^*)^{1-\alpha} - [B + \kappa(l^*)] \\ \frac{n^r}{n^p}c^r_d = \frac{n^r}{n^p}r_d k^r = \alpha A\left(\frac{K}{n^p}\right)^\alpha(1-l^*)^{1-\alpha} \end{cases}$
- $C^p_g = n^p c^p_g$; $C^r_g = n^r c^r_g$; $L_g = n^p l_g$ for $g \in \{o, d\}$.

Suppose $(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha < \frac{B+\kappa(l^*)}{1-l^*}$.

- Even if $\tau_{l,o} = 0$ and the poor work at the market, the physical wage rate is too low to keep them working at the market for $(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha < \frac{B+\kappa(l^*)}{1-l^*}$.
- Regardless of regimes, $l_g = 0$, $l_{b,g} = 1$, $\tau_{l,g} = 0$ for $g \in \{o,d\}$.
- $w_g = \infty$; $r_g = 0$; $c_g^p = c_g^r = C_g^p = C_g^r = L_g = Y_g = 0$; $y_{b,g} = c_{b,g}^p = B$ for $g \in \{o,d\}$.

Proof of theorem 13

Suppose $(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha \geq \frac{B+\kappa(l^*)}{1-l^*}$. Democratization is adopted if $\alpha A \left(\frac{K}{n^p}\right)^\alpha (1-l^*)^{1-\alpha} \geq A \left(\frac{K}{n^p}\right)^\alpha (1-l^*)^{1-\alpha} - [B+\kappa(l^*)]$, i.e. $(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha \leq \frac{B+\kappa(l^*)}{1-l^*}$. Suppose $(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha < \frac{B+\kappa(l^*)}{1-l^*}$. The general equilibria under two regimes are the same. The rich's utility are indifferent between regimes. Therefore, democratization is adopted if $(1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha \leq \frac{B+\kappa(l^*)}{1-l^*}$.

Comparative statics of α

By table 4.1, $\frac{\partial w}{\partial \alpha} \gtreqless 0$ iff $\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) \gtreqless \frac{1}{1-\alpha}$. By theorem 13, democratization is adopted if $w = (1-\alpha) A \left(\frac{K}{n^p}\frac{1}{1-l^*}\right)^\alpha \leq \frac{B+\kappa(l^*)}{1-l^*}$.

Suppose $\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) \leq 1 < \frac{1}{1-\alpha}$ such that $\frac{\partial w}{\partial \alpha} < 0$. Given $\frac{B+\kappa(l^*)}{1-l^*} \in \mathbb{R}_{++}$. (1) If $\frac{B+\kappa(l^*)}{1-l^*} \geq A$, democratization is always adopted for $w \leq \frac{B+\kappa(l^*)}{1-l^*}$. (2) If $\frac{B+\kappa(l^*)}{1-l^*} < A$, w is more likely smaller than $\frac{B+\kappa(l^*)}{1-l^*}$ as α increases (figure 4.5).

Suppose $\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) > 1$. $\frac{\partial w}{\partial \alpha} \gtreqless 0$ iff $\ln\left(\frac{K}{n^p}\frac{1}{1-l^*}\right) \gtreqless \frac{1}{1-\alpha}$. Let $w_{\max} = (1-\alpha) A \left[\exp\left(\frac{1}{1-\alpha}\right)\right]^\alpha$. Given $\frac{B+\kappa(l^*)}{1-l^*} \in \mathbb{R}_{++}$. (1) If $\frac{B+\kappa(l^*)}{1-l^*} \geq w_{\max}$, democratization is always adopted for $w \leq \frac{B+\kappa(l^*)}{1-l^*}$. (2) If $A < \frac{B+\kappa(l^*)}{1-l^*} < w_{\max}$, democratization is adopted if α is sufficiently large and increases or α is sufficiently small and decreases (figure 4.7). (3) If $\frac{B+\kappa(l^*)}{1-l^*} \leq A$, democratization is adopted if α is sufficiently large and increases (figure 4.6).

2.2. A Static Model with Non-exclusive Public Goods

The representative poor person's decision making under the oligarchic regime

The representative poor person finds (l, l_b) to maximize $\frac{1}{\rho}\ln[\gamma(1-\theta)(1-\tau_l)^\rho w^\rho l^\rho + (1-\gamma)(1-\theta)B^\rho l_b^\rho + \theta(1-l-l_b)^\rho]$. The *f.o.c.*'s are $\frac{\gamma(1-\theta)(1-\tau_l)^\rho w^\rho}{l^{1-\rho}} = \frac{\theta}{(1-l-l_b)^{1-\rho}}$ and $\frac{(1-\gamma)(1-\theta)B^\rho}{l_b^{1-\rho}} = \frac{\theta}{(1-l-l_b)^{1-\rho}}$. Let $\phi_0 = \left[\frac{(1-\gamma)(1-\theta)}{\theta}\right]^{\frac{1}{1-\rho}} B^{\frac{\rho}{1-\rho}}$. Rewrite l_b in terms of l only: $l_b = \frac{\phi_0}{1+\phi_0}(1-l)$. Notice that $\frac{dl_b}{dl} < 0$. Let $\phi_1 = \left(\gamma\frac{1-\theta}{\theta}\right)^{\frac{1}{1-\rho}}\frac{1}{1+\phi_0}$. Plug $l_b = \frac{\phi_0}{1+\phi_0}(1-l)$ back to the *f.o.c.* of l to derive l in terms of only: $l(\tau_l, w, q) = \frac{\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \in (0,1)$. It gives $l_b(\tau_l, w, q) = \frac{\frac{\phi_0}{1+\phi_0}}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} \in (0,1)$. Plug this pair of (l, l_b) back to the budget constraints to derive: $c^p(\tau_l, w, q) = \frac{\phi_1(1-\tau_l)^{\frac{1}{1-\rho}} w^{\frac{1}{1-\rho}}}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}$ and $c_b^p(\tau_l, w, q) = \frac{\frac{\phi_0}{1+\phi_0}B}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}$.

Proof of claim 16

$\frac{\partial c^p(\tau_l,w,q)}{\partial\tau_l} = -\frac{1}{1-\rho}\frac{1+(1-\rho)\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}}\frac{c^p}{1-\tau_l} < 0$; $\frac{\partial l(\tau_l,w,q)}{\partial\tau_l} = -\frac{\rho}{1-\rho}\frac{1}{1-\tau_l}\frac{l}{1+\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}}} < 0$ for $\rho \in (0,1)$.

The representative poor person's capital tax decision under the democratic regime

The representative poor person chooses τ_k to maximize $(n^p + n^r)\frac{\sigma}{\rho}\ln(1+\tau_k rK) + \frac{n^r}{\rho}\ln\{\gamma(1-\theta)[(1-\tau_k)rk^r]^\rho + \theta\}$. The *f.o.c.* of τ_k is $\frac{\sigma}{\rho}\frac{n^p+n^r}{n^r}\frac{rK}{1+\tau_k rK} = \frac{1}{1-\tau_k}\frac{\gamma(1-\theta)(1-\tau_k)^\rho r^\rho(k^r)^\rho}{\theta+\gamma(1-\theta)(1-\tau_k)^\rho r^\rho(k^r)^\rho}$, where $\tau_k \in (0,1)$.

The representative poor person's consumption and labor supply under the democratic regime

The representative poor person chooses $\tau_l = 0$ and finds (l, l_b) to maximize $\frac{1}{\rho}\ln[\gamma(1-\theta) \times w^\rho l^\rho + (1-\gamma)(1-\theta) B^\rho l_b^\rho + \theta(1-l-l_b)^\rho]$. The $f.o.c.$'s are $\frac{\gamma(1-\theta)w^\rho}{l^{1-\rho}} = \frac{\theta}{(1-l-l_b)^{1-\rho}}$ and $\frac{(1-\gamma)(1-\theta)B^\rho}{l_b^{1-\rho}} = \frac{\theta}{(1-l-l_b)^{1-\rho}}$. Rewrite l_b in terms of l only: $l_b = \frac{\phi_0}{1+\phi_0}(1-l)$. Plug $l_b = \frac{\phi_0}{1+\phi_0}(1-l)$ back to the $f.o.c.$ of l to derive l in terms of only: $l(w,q) = \frac{\phi_1 w^{\frac{\rho}{1-\rho}}}{1+\phi_1 w^{\frac{\rho}{1-\rho}}} \in (0,1)$. It gives $l_b(w,q) = \frac{\frac{\phi_0}{1+\phi_0}}{1+\phi_1 w^{\frac{\rho}{1-\rho}}} \in (0,1)$. Plug this pair of (l, l_b) back to the budget constraints to derive: $c^p(w,q) = \frac{\phi_1 w^{\frac{1}{1-\rho}}}{1+\phi_1 w^{\frac{\rho}{1-\rho}}}$ and $c_b^p(w,q) = \frac{\frac{\phi_0}{1+\phi_0}B}{1+\phi_1 w^{\frac{\rho}{1-\rho}}}$.

Equation (4.46) for characterization of GE under the oligarchic regime

Step 1: $l_o = 1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*} \in (0,1)$ if $\tau_l^* \in (1-\rho, 1)$: By (4.28), $\phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}} = \frac{\rho}{1-\rho}\frac{\tau_l}{1-\tau_l} - 1$. Plug this equation into (4.22). Simplify it.

Step 2: $w_o = \phi_2^{\frac{1-\rho}{\rho}}\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{-\alpha}$: Plug step 1 into (4.40).

Step 3: $\tau_l^* \in (1-\rho, 1)$ is governed by $1 + \phi_1\phi_2\left[(1-\tau_l^*)\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{-\alpha}\right]^{\frac{\rho}{1-\rho}} = \frac{\rho}{1-\rho}\frac{\tau_l^*}{1-\tau_l^*}$: Plug step 2 into (4.28).

Characterization of GE under the oligarchic regime

- $l_o = 1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*} \in (0,1)$ for $\tau_l^* \in (1-\rho, 1)$
- $l_{b,o} = \frac{1-\rho}{\rho}\frac{\phi_0}{1+\phi_0}\frac{1-\tau_l^*}{\tau_l^*}$: Plug l_o into (4.23).
- $w_o = \phi_2^{\frac{1-\rho}{\rho}}\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{-\alpha}$
- $r_o = \alpha A\left(\frac{n^r}{n^p}k^r\right)^{\alpha-1}\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$: Plug l_o into (4.41).
- $c_o^p = \phi_2^{\frac{1-\rho}{\rho}}(1-\tau_l^*)\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$ plug (w_o, l_o) into (4.24).
- $y_b = c_{b,o}^p = \frac{1-\rho}{\rho}\frac{\phi_0}{1+\phi_0}\frac{1-\tau_l^*}{\tau_l^*}B$: Plug $1 + \phi_1(1-\tau_l)^{\frac{\rho}{1-\rho}} w^{\frac{\rho}{1-\rho}} = \frac{\rho}{1-\rho}\frac{\tau_l}{1-\tau_l}$ into (4.25).
- $c_o^r = \frac{n^p}{n^r}\left[\alpha A\left(\frac{n^r}{n^p}k^r\right)^\alpha + \phi_2^{\frac{1-\rho}{\rho}}\tau_l^*\right]\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$: Plug (w_o, r_o, l_o) into (4.29).
- $L_o = n^p\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)$ by (4.45).
- $C_o^p = n^p\phi_2^{\frac{1-\rho}{\rho}}(1-\tau_l^*)\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$ by (4.27).

- $C_o^r = n^p \left[\alpha A \left(\frac{n^r}{n^p} k^r\right)^\alpha + \phi_2^{\frac{1-\rho}{\rho}} \tau_l^*\right] \left(1 - \frac{1-\rho}{\rho} \frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$ by (4.30).
- $Y_o = A \left(\frac{K}{n^p}\right)^\alpha n^p \left(1 - \frac{1-\rho}{\rho} \frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$

Equation (4.47) and (4.48) for characterization of GE under the democratic regime

Plug (4.40) into (4.34). Simplify and rewrite it: $l_d \in (0,1)$ such that $l_d^{\frac{1+\alpha\rho-\rho}{1-\rho}} + \phi_1\phi_2 l_d - \phi_1\phi_2 = 0$. Rewrite (w,r) in terms of l only: $w_d = \phi_2^{\frac{1-\rho}{\rho}} l_d^{-\alpha}$; $r_d = \frac{\phi_3}{k^r} l_d^{1-\alpha}$. Plug step 2 into (4.33). Simplify it: $\frac{\sigma}{\rho}\left[1 + \frac{\theta}{\gamma(1-\theta)\phi_3^\rho} \frac{1}{\left(1-\tau_k^*\right)^\rho} \frac{1}{l_d^{(1-\alpha)\rho}}\right] = \frac{1}{n^p+n^r} \frac{1}{\phi_3} \frac{1}{1-\tau_k^*} \frac{1}{l_d^{1-\alpha}} + \frac{n^r}{n^p+n^r} \frac{\tau_k^*}{1-\tau_k^*}$.

Let $f(\tau_k) = \frac{\sigma}{\rho}\left[1 + \frac{\theta}{\gamma(1-\theta)\phi_3^\rho} \frac{1}{\left(1-\tau_k^*\right)^\rho} \frac{1}{l_d^{(1-\alpha)\rho}}\right]$. $\lim_{\tau_k \to 0^+} f(\tau_k) > 0$; $\lim_{\tau_k \to 1^-} f(\tau_k) = \infty$; $f'(\tau_k) > 0$; $f''(\tau_k) > 0$. Let $g(\tau_k) = \frac{1}{n^p+n^r} \frac{1}{\phi_3} \frac{1}{1-\tau_k^*} \frac{1}{l_d^{1-\alpha}} + \frac{n^r}{n^p+n^r} \frac{\tau_k^*}{1-\tau_k^*}$. $\lim_{\tau_k \to 0^+} g(\tau_k) > 0$; $\lim_{\tau_k \to 1^-} g(\tau_k) = \infty$; $g'(\tau_k) > 0$; $g''(\tau_k) > 0$. There is some $\tau_k^* \in (0,1)$ such that $f(\tau_k^*) = g(\tau_k^*)$. Otherwise, $\tau_k^* \to 1$.

Characterization of GE under the democratic regime

- $w_d = \phi_2^{\frac{1-\rho}{\rho}} l_d^{-\alpha}$; $r_d = \frac{\phi_3}{k^r} l_d^{1-\alpha}$.
- $c_d^p = (1-\alpha) A \left(\frac{n^r}{n^p} k^r\right)^\alpha l_d^{1-\alpha}$; $C_d^p = n^p (1-\alpha) A \left(\frac{n^r}{n^p} k^r\right)^\alpha l_d^{1-\alpha}$.
- $l_{b,d} = \frac{\phi_0}{1+\phi_0}(1-l_d)$; $y_{b,d} = c_{b,d}^p = \frac{\phi_0}{1+\phi_0} B (1-l_d)$.
- $c_d^r = \phi_3 (1-\tau_k^*) l_d^{1-\alpha}$: Plug r_d into (4.31). $C_d^r = n^r \phi_3 (1-\tau_k^*) l_d^{1-\alpha}$.
- $G^* = n^r \phi_3 \tau_k^* l_d^{1-\alpha} = \alpha A n^p \left(\frac{n^r}{n^p} k^r\right)^\alpha \tau_k l^{1-\alpha}$: Plug r_d into $G = \tau_k r K$. Simplify it.
- $Y_d = A n^p \left(\frac{K}{n^p}\right)^\alpha l_d^{1-\alpha}$

Proof of proposition 20

Democratization is adopted if $\frac{1}{\rho} \ln\left[\gamma(1-\theta)(c_d^r)^\rho + \theta\right] + H(G^*) \geq \frac{1}{\rho} \ln\left[\gamma(1-\theta)(c_o^r)^\rho + \theta\right]$, i.e. $(1+G^*)^\sigma \left[\theta + \gamma(1-\theta)(c_d^r)^\rho\right] \geq \theta + \gamma(1-\theta)(c_o^r)^\rho$. Plug $G^* = n^r \phi_3 \tau_k^* l_d^{1-\alpha}$, $c_o^r =$

$\left(\phi_3 + \frac{n^p}{n^r}\phi_2^{\frac{1-\rho}{\rho}}\tau_l^*\right)\left(1 - \frac{1-\rho}{\rho}\frac{1-\tau_l^*}{\tau_l^*}\right)^{1-\alpha}$, and $c_d^r = \phi_3\left(1-\tau_k^*\right)l_d^{1-\alpha}$ into the previous inequality. Rewrite it to derive inequality (4.49).

Characterization of GE under the democratic regime where $\tau_k^* = 1$

Consider $\tau_k^* = 1$. Denote $x_{d'}$ to be the GE value of variable x under the democratic regime. Plug (4.40) into (4.34) to write l: $l_{d'} \in (0,1)$ such that $l_{d'}^{\frac{1-\rho+\alpha\rho}{1-\rho}} + \phi_1\phi_2 l_{d'} - \phi_1\phi_2 = 0$. When l_d is determined, $w_{d'} = \phi_2^{\frac{1-\rho}{\rho}} l_{d'}^{-\alpha}$; $r_{d'} = \frac{\phi_3}{k^r} l_{d'}^{1-\alpha}$; $\frac{Y_{d'}}{n^p} = A\left(\frac{n^r}{n^p}k^r\right)^{\alpha} l_{d'}^{1-\alpha}$; $l_{b,d'} = \frac{\phi_0}{1+\phi_0}\left(1-l_{d'}\right)$; $c_{b,d'}^p = \frac{Y_{b,d'}}{n^p} = Bl_{b,d'}$; $c_{d'}^p = w_{d'}l_{d'}$; $c_{d'}^r \to 0$; $\frac{G^{*\prime}}{n^p} \to \frac{n^r}{n^p}\phi_3 l_{d'}^{1-\alpha}$. Notice that, except $\left(\tau_k^*, c_{d'}^r, \frac{G^{*\prime}}{n^p}\right)$, all prices and allocations are the same as the case of $\tau_k^* \in (0,1)$.

Proof of claim 27

(1) By table (4.2), $l_d \gtreqless l_{b,d}$ iff $l_d \gtreqless \frac{\phi_0}{1+2\phi_0}$. By (4.47), $l_d \gtreqless \frac{\phi_0}{1+2\phi_0}$ iff $\left(\frac{\phi_0}{1+2\phi_0}\right)^{\frac{\alpha\rho}{1-\rho}} \lesseqgtr \frac{1+\phi_0}{\phi_0}\phi_1\phi_2$.

(2) By table (4.2), $l_o \gtreqless l_{b,o}$ iff $\tau_l^* \gtreqless 1 - \frac{\rho(1+\phi_0)}{1+\phi_0+(1-\rho)\phi_0}$. By (4.46), $\tau_l^* \gtreqless 1 - \frac{\rho(1+\phi_0)}{1+\phi_0+(1-\rho)\phi_0}$ iff $\left(\frac{\phi_0}{1+2\phi_0}\right)^{\frac{\alpha\rho}{1-\rho}} \lesseqgtr \phi_1\phi_2\frac{1+\phi_0}{\phi_0}\left[\frac{\rho(1+\phi_0)}{1+\phi_0+(1-\rho)\phi_0}\right]^{\frac{\rho}{1-\rho}}$.

A primitive explanation for the effect of ρ on democratization

The effect of ρ on democratization is indecisive because (first of all) its effect on labor supply is not clear. Consider the democratic regime first. Recall (4.47) and let $F(l) = l^{\frac{1-\rho+\alpha\rho}{1-\rho}} + \phi_1\phi_2 l - \phi_1\phi_2$. Notice that $\frac{\partial\phi_1\phi_2}{\partial\rho} \gtreqless 0$ iff $\frac{\gamma(1-\theta)}{\theta}(1-\alpha)A\left(\frac{n^r}{n^p}k^r\right)^{\alpha} \gtreqless \left[\frac{(1-\gamma)(1-\theta)B}{\theta}\right]^{\frac{\phi_0}{1+\phi_0}}$ and $F'''(l) \gtreqless 0$ iff $\alpha \gtreqless \frac{1-\rho}{\rho}$. The former captures the "altitude" of $F(l)$; the latter captures the "curvature" of $F(l)$. Either condition or both affect the size of l_d, which in turns affects the size of τ_k^* through (4.48). Apply this logic to the oligarchic regime. the effect of ρ on τ_l^* is also uncertain according to (4.46). Therefore, it is hard to conclude that there is a monotonic effect of ρ on the likelihood of democratization.

3. Chapter 5

The representative rich person's decision making under the no-commitment regime

The representative rich person takes (p_0, p_1, p_2, S_1^p) given and maximizes $\pounds_{nc}^r = \ln c_1^r + \beta \ln c_2^r + \lambda_{nc}^r \left(p_0 k^r + \frac{1}{n^r}\tau S_1^p - p_1 c_1^r - p_2 c_2^r\right)$ with respect to (τ, c_1^r, c_2^r). The *f.o.c.*'s of (c_1^r, c_2^r) show $c_2^r = \beta \frac{p_1}{p_2} c_1^r$. Plug it back to the budget constraint to derive (5.2) for $t \in \{1, 2\}$. The optimal tax decision on the given S_1^p is (5.1) with which her wealth is maximized.

The representative poor person's decision making under the no-commitment regime

Suppose $\beta < \frac{w_2}{w_1}$ and $\tau = 0$. The representative poor person takes (w_1, w_2, p_1, p_2) given and maximizes $\pounds_{nc}^p = \sum_{t=1}^2 \beta^{t-1} \left[(1-\theta)\ln c_t^p + \theta \ln(1-l_t)\right] + \lambda_{nc}^p \left(w_1 l_1 - p_1 c_1^p + w_2 l_2 - p_2 c_2^p\right)$ with respect to (c_t^p, l_t). The *f.o.c.*'s of (c_t^p, l_t) show $c_2^p = \beta \frac{p_1}{p_2} c_1^p$, $1 - l_2 = \frac{w_1}{w_2}(1 - l_1)$, and $\frac{c_t^p}{1-l_t} = \beta^{t-1} \frac{1-\theta}{\theta} \frac{w_t}{p_t}$ for $t \in \{1, 2\}$ such that (5.3) and (5.4) hold, and $w_1 l_1 - p_1 c_1^p = \frac{w_1}{1+\beta}\left(\beta - \frac{w_2}{w_1}\right) < 0$.

Suppose $\beta \geq \frac{w_2}{w_1}$ instead. If the poor person chooses (5.3) and (5.4), $w_1 l_1 - p_1 c_1^p \geq 0$. The rich deviate to $\tau = 1$. The poor person also deviate to $w_t l_t - p_t c_t^p = 0$ for $t \in \{1, 2\}$, which gives (5.5) and (5.6).

The representative rich person's decision making under the delegation regime

The representative rich person takes $(p_0, p_1, p_2, \tau S_1^p)$ as given and maximizes $\pounds_d^r = \ln c_1^r + \beta \ln c_2^r + \lambda_d^r \left(p_0 k^r + \frac{1}{n^r}\tau S_1^p - p_1 c_1^r - p_2 c_2^r\right)$ with respect to (c_1^r, c_2^r). The *f.o.c.*'s of (c_1^r, c_2^r) gives $c_2^r = \beta \frac{p_1}{p_2} c_1^r$. Plug it back to the budget constraint to derive (5.7).

The representative poor person's decision making under the delegation regime

The representative poor person takes (w_1, w_2, p_1, p_2) as given and maximizes $\pounds_d^p = \sum_{t=1}^{2} \beta^{t-1} \left[(1-\theta)\ln c_t^p + \theta \ln(1-l_t)\right] + \lambda_d^p \left[(1-\tau)(w_1 l_1 - p_1 c_1^p) + w_2 l_2 - p_2 c_2^p\right]$ with respect to (τ, c_t^p, l_t), where $\tau \in [0,1]$. First, $\tau = 0$ is time-consistent. The *f.o.c.*'s of (c_t^p, l_t) give $c_2^p = \beta \frac{p_1}{p_2} c_1^p$, $1 - l_2 = \frac{w_1}{w_2}(1 - l_1)$, and $\frac{c_t^p}{1-l_t} = \beta^{t-1} \frac{1-\theta}{\theta} \frac{w_t}{p_t}$ for $t \in \{1, 2\}$ such that (5.9) and (5.10) hold.

Proof of claim 28

(1) $\tau = 0$ gives the unique $l_{1,d} \in (0,1)$ such that $l_{2,d} \in (0,1)$. Other allocations and prices are also unique and strictly positive. Plug $\left(c_{1,d}^p, l_{1,d}, p_{1,d}, w_{1,d}\right)$ in Table 5.1 into s_1^p. $s_{1,d}^p = \frac{1-\alpha}{1+\alpha\beta} \frac{\beta^2}{1+\beta} \frac{K_1}{n^p} p_0 > 0$ such that $S_{1,d}^p > 0$.

(2) Under the no-commitment regime, the rich impose either $\tau = 0$ or $\tau = 1$. To be an equilibrium, τ needs to be *ex-ante* and *ex-post* optimal. $\tau = 0$ is not *ex-post* optimal because the rich will deviate to $\tau = 1$ after observing $s_{1,d}^p > 0$. $\tau = 1$ such that $s_1^p = 0$ is *ex-ante* and *ex-post* optimal because, take their opponents' action as given, no consumers want to deviate. Therefore, there is a unique GE under the no-commitment in which $\tau = 1$ such that $s_1^p = 0$.

Characterization of GE under the delegation regime ($\delta = 1$)

Step 1: $p_1 = \frac{1}{\alpha A (n^p)^{1-\alpha} K_1^{\alpha-1} l_1^{1-\alpha}} p_0$; $p_2 = \frac{1}{\alpha A z^{1-\alpha} (n^p)^{1-\alpha} K_2^{\alpha-1} l_2^{1-\alpha}} p_1$: Plug (5.11) into (5.14); plug (5.11) into (5.15).

Step 2: $w_1^e = w_1 = \frac{1-\alpha}{\alpha} \frac{1}{n^p} \frac{K_1}{l_1} p_0$; $w_2^e = \frac{1-\alpha}{\alpha} \frac{1}{z} \frac{1}{n^p} \frac{K_2}{l_2} p_1$; $w_2 = \frac{1-\alpha}{\alpha} \frac{1}{n^p} \frac{K_2}{l_2} p_1$; $r_1 = p_0$; $r_2 = p_1$: Plug step 1 into (5.12) and (5.13); plug step 1 into (5.11).

Step 3: $A\left(n^{p}\right)^{1-\alpha}K_{1}^{\alpha}l_{1}^{1-\alpha}=\frac{1}{1+\beta}\frac{p_{0}K_{1}+(1-\theta)n^{p}(w_{1}+w_{2})}{p_{1}}+K_{2}$: Plug (5.9) and (5.7) into $C_{1}=n^{p}c_{1}^{p}+n^{r}c_{1}^{r}$: $C_{1}=\frac{1}{1+\beta}\frac{p_{0}K_{1}+(1-\theta)n^{p}(w_{1}+w_{2})}{p_{1}}$. Plug (5.18) and C_1 into (5.16).

Step 4: $Az^{1-\alpha}\left(n^{p}\right)^{1-\alpha}K_{2}^{\alpha}l_{2}^{1-\alpha}=\frac{\beta}{1+\beta}\frac{p_{0}K_{1}+(1-\theta)n^{p}(w_{1}+w_{2})}{p_{2}}$: Plug (5.9) and (5.7) into $C_{2}=n^{p}c_{2}^{p}+n^{r}c_{2}^{r}$: $C_{2}=\frac{\beta}{1+\beta}\frac{p_{0}K_{1}+(1-\theta)n^{p}(w_{1}+w_{2})}{p_{2}}$. Plug $K_3=0=I_2$ and C_2 into (5.16).

Step 5: $l_{1}=1-\frac{\theta}{1+\beta}\frac{w_{1}+w_{2}}{w_{1}}$ by (5.10).

Step 6: $l_{1,d}=\frac{(1+\alpha\beta)(1+\beta)(1-\theta)}{(1+\alpha\beta)(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}\in(0,1)$: Rewrite step 5: $w_{1}+w_{2}=\frac{1+\beta}{\theta}w_{1}\left(1-l_{1}\right)$. Plug it into step 3: $K_{2}=A\left(n^{p}\right)^{1-\alpha}K_{1}^{\alpha}l_{1}^{1-\alpha}-\frac{1}{1+\beta}\frac{p_{0}K_{1}+\frac{(1+\beta)(1-\theta)n^{p}}{\theta}w_{1}(1-l_{1})}{p_{1}}$. Rewrite step 4: $p_{2}l_{2}^{1-\alpha}=\frac{\beta}{1+\beta}\frac{p_{0}K_{1}+\frac{(1+\beta)(1-\theta)n^{p}}{\theta}w_{1}(1-l_{1})}{Az^{1-\alpha}(n^{p})^{1-\alpha}K_{2}^{\alpha}}$. Plug step 1 into this equation to derive $K_{2}=\frac{\alpha\beta}{1+\beta}\frac{p_{0}K_{1}+\frac{(1+\beta)(1-\theta)n^{p}}{\theta}w_{1}(1-l_{1})}{p_{1}}$. Equate these two K_2; plug step 1 and step 2 into this equation.

- $I_{1,d}=K_{2,d}=\frac{\alpha\beta}{1+\alpha\beta}A\left(n^{p}\right)^{1-\alpha}K_{1}^{\alpha}l_{1,d}^{1-\alpha}$: Plug $l_{1,d}$ back to one of the K_2 equations.
- $K_{3,d}=I_{2,d}=0$
- $l_{2,d}=\frac{(1+\beta)(1-\theta)}{(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}\in(0,1)$: Recall $w_{1}+w_{2}=\frac{(1+\beta)(\gamma+\theta)}{\theta}w_{1}\left(1-l_{1}\right)$. Plug step 1, step 2, $l_{1,d}$ and $K_{2,d}$ into this equation.
- $p_{1,d}=\frac{1}{\alpha A(n^{p})^{1-\alpha}K_{1}^{\alpha-1}l_{1,d}^{1-\alpha}}p_{0}$; $p_{2,d}=\frac{1}{\alpha Az^{1-\alpha}(n^{p})^{1-\alpha}K_{2,d}^{\alpha-1}l_{2,d}^{1-\alpha}}p_{1,d}$: Plug $l_{1,d}$ back to step 1. Plug l_d and $K_{2,d}$ back to step 1.
- $w_{1,d}=w_{1,d}^{e}=\frac{1-\alpha}{\alpha}\frac{1}{n^{p}}\frac{K_{1}}{l_{1,d}}p_{0}$;$w_{2,d}^{e}=\frac{1-\alpha}{\alpha}\frac{\beta}{1+\alpha\beta}\frac{K_{1}}{n^{p}}\frac{1}{z}\frac{1}{l_{2,d}}p_{0}$; $w_{2,d}=\frac{1-\alpha}{\alpha}\frac{\beta}{1+\alpha\beta}\frac{K_{1}}{n^{p}}\frac{1}{l_{2,d}}p_{0}$; $r_{1,d}=p_{0}$; $r_{2,d}=p_{1,d}$: Plug $l_{1,d}$, $l_{2,d}$, $K_{2,d}$ back to step 2.
- $c_{1,d}^{p}=\frac{(1-\alpha)(1+\alpha\beta+\beta)}{(1+\alpha\beta)(1+\beta)}\frac{1}{n^{p}}A\left(n^{p}\right)^{1-\alpha}K_{1}^{\alpha}l_{1,d}^{1-\alpha}$; $c_{2,d}^{p}=\frac{(1-\alpha)(1+\alpha\beta+\beta)}{1+\beta}\frac{1}{n^{p}}Az^{1-\alpha}\left(n^{p}\right)^{1-\alpha}K_{2,d}^{\alpha}l_{2,d}^{1-\alpha}$: Plug $p_{1,d}$, $p_{2,d}$, $w_{1,d}$, and $w_{2,d}$ back to (5.9).
- $c_{1,d}^{r}=\frac{\alpha}{1+\beta}\frac{1}{n^{r}}A\left(n^{p}\right)^{1-\alpha}K_{1}^{\alpha}l_{1,d}^{1-\alpha}$; $c_{2,d}^{r}=\frac{\alpha(1+\alpha\beta)}{1+\beta}\frac{1}{n^{r}}Az^{1-\alpha}\left(n^{p}\right)^{1-\alpha}K_{2,d}^{\alpha}l_{2,d}^{1-\alpha}$: Plug $p_{1,d}$ and $p_{2,d}$ back to (5.7).

- $L^e_{1,d} = L_{1,d} = \frac{(1+\alpha\beta)(1+\beta)(1-\theta)n^p}{(1+\alpha\beta)(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$; $L_{2,d} = \frac{(1+\beta)(1-\theta)n^p}{(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$; $L^e_{2,d} = \frac{(1+\beta)(1-\theta)zn^p}{(1+\beta)(1-\theta)+(1+\alpha\beta+\beta)\theta}$
- $Y_{1,d} = AK_1^\alpha L_{1,d}^{1-\alpha}$; $Y_{2,d} = AK_{2,d}^\alpha L_{2,d}^{1-\alpha}$

Characterization of GE under the no-commitment regime ($\delta = 1$)

- $l_{1,nc} = l_{2,nc} = 1-\theta$; $L_{1,nc} = L^e_{1,nc} = L_{2,nc} = n^p(1-\theta)$; $L^e_{2,nc} = zn^p(1-\theta)$.
- $p_{1,nc} = \frac{1}{\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}}p_0$; $w^e_{1,nc} = w_{1,nc} = \frac{1-\alpha}{\alpha}\frac{1}{1-\theta}\frac{K_1}{n^p}p_0$; $r_{1,nc} = p_0$: Plug (5.11) and $l_{1,nc}$ into (5.14); plug $(p_{1,nc}, l_{1,nc})$ into (5.12), (5.13), and (5.11).
- $Y_{1,nc} = A(n^p)^{1-\alpha}K_1^\alpha(1-\theta)^{1-\alpha}$
- $c^p_{1,nc} = (1-\alpha)A\left(\frac{K_1}{n^p}\right)^\alpha(1-\theta)^{1-\alpha}$; $c^r_{1,nc} = \frac{k^r}{1+\beta}\alpha A\left(\frac{K_1}{n^p}\frac{1}{1-\theta}\right)^{\alpha-1}$: Plug $(p_{1,nc}, w_{1,nc})$ into (5.3) and (5.2).
- $C_{1,nc} = \frac{1-\alpha\beta+\beta}{1+\beta}A(n^p)^{1-\alpha}K_1^\alpha(1-\theta)^{1-\alpha}$
- $I_{1,nc} = K_{2,nc} = \frac{\alpha\beta}{1+\beta}A(n^p)^{1-\alpha}K_1^\alpha(1-\theta)^{1-\alpha}$ by (5.16) and (5.18).
- $p_{2,nc} = \frac{1}{\alpha AK_{2,nc}^{\alpha-1}\left(L^e_{2,nc}\right)^{1-\alpha}}p_{1,nc}$; $w^e_{2,nc} = \frac{(1-\alpha)\beta}{\alpha(1+\beta)}\frac{K_1}{L^e_{2,nc}}p_0$; $w_{2,nc} = \frac{(1-\alpha)\beta}{\alpha(1+\beta)}\frac{K_1}{L_{2,nc}}p_0$; $r_{2,nc} = p_{1,nc}$: Plug (5.11) into (5.15); plug $\left(p_{2,nc}, L^e_{2,nc}\right)$ into (5.12), (5.13), and (5.11).
- $Y_{2,nc} = Az^{1-\alpha}(n^p)^{1-\alpha}(K_{2,nc})^\alpha(1-\theta)^{1-\alpha}$
- $c^p_{2,nc} = \frac{1-\alpha}{n^p}AK_{2,nc}^\alpha\left(L^e_{2,nc}\right)^{1-\alpha}$; $c^r_{2,nc} = \frac{\alpha}{n^r}AK_{2,nc}^\alpha\left(L^e_{2,nc}\right)^{1-\alpha}$; $C_{2,nc} = AK_{2,nc}^\alpha\left(L^e_{2,nc}\right)^{1-\alpha}$: Plug $(p_{2,nc}, w_{2,nc})$ into (5.3) and (5.2).
- $I_{2,nc} = K_{3,nc} = 0$

Proof of lemma 29

The relative price of final goods $\frac{p_1}{p_2}$ decreases after democratization by table 5.1: $\frac{p_{1,d}}{p_{2,d}} < \frac{p_{1,nc}}{p_{2,nc}}$. It has two effects to the representative rich person:

(1) Substitution effect (*S.E.*): *S.E.* measures the quantity changes of two goods when their relative price changes and the consumer's utility (or purchasing power) remains constant. With $\frac{p_{1,d}}{p_{2,d}} < \frac{p_{1,nc}}{p_{2,nc}}$ and strictly quasi-concave utility, it makes the rich person consume more in period one but less in period two after democratization while holding her purchasing power constant.

(2) Income effect (*I.E.*): *I.E.* measures the quantity changes of two goods when the consumer's purchasing power changes. If goods are normal, the consumer purchases more/less in both periods when their purchasing power increases/decreases. Consumption goods in this paper are normal because $\frac{\partial c_t^r}{\partial p_0 k^r} > 0$, but the change in the rich person's purchasing power is uncertain because the prices of both goods change after democratization.

Refer to figure 5.3. On the c_1^r-c_2^r plane, e_{nc} is the rich person's optimal consumption bundle with relative price $\frac{p_{1,nc}}{p_{2,nc}}$ under the no-commitment regime. It gives her utility u_{nc}. If she delegates the power, she is facing the new relative price $\frac{p_{1,d}}{p_{2,d}}$ where $\frac{p_{1,d}}{p_{2,d}} < \frac{p_{1,nc}}{p_{2,nc}}$. $\widehat{e}$ is her optimal consumption bundle with the new relative price $\frac{p_{1,d}}{p_{2,d}}$ while holding her utility level constant at u_{nc}. The quantity changes in (c_1^r, c_2^r) from e_{nc} to $\widehat{e}$ are the *S.E.* Suppose that the rich person's purchasing power increases after democratization. In this case, suppose that BC_d is her new budget constraint. To maximize the utility, she deviates to e_d which is the optimal consumption bundle with the new relative price $\frac{p_{1,d}}{p_{2,d}}$. It gives her utility u_d. The quantity changes in (c_1^r, c_2^r) from $\widehat{e}$ to e_d are the *I.E.* Clearly, she is better off for $u_d > u_{nc}$.

Consider the opposite direction. Suppose that the rich person is better off after democratization. It implies that she deviates to another bundle which provides higher utility after democratization. For example, e_d gives utility u_d where $u_d > u_{nc}$. If the rich person's purchasing power remains the same after democratization, i.e. $BC_d = \widehat{BC}$, e_d is not affordable. To make e_d affordable, she must have greater purchasing power to expand the budget set, i.e. BC_d is parallel on the right side of $\widehat{BC}$.

Notice that adoption of democratization does not require if the rich person has sufficient increase in purchasing power to afford e_{nc}. Consider the following two cases. First, suppose that the representative rich person's purchasing power increases but is not sufficient to afford e_{nc} (figure .1). Suppose that she chooses bundle a or bundle b, which gives her utility u_{nc}. But neither is optimal because $\frac{u_1^r\left(c_1^r, c_2^r\right)}{p_{1,d}} \neq \frac{u_2^r\left(c_1^r, c_2^r\right)}{p_{2,d}}$ where $u_t^r\left(c_1^r, c_2^r\right) = \frac{\partial u^r\left(c_1^r, c_2^r\right)}{\partial c_t^r}$. She will buy more of the good with larger $\frac{u_t^r\left(c_1^r, c_2^r\right)}{p_{t,d}}$ until "=" holds. In the meantime, she makes herself better off after democratization. Second, suppose that the representative rich person's purchasing power increases and is exactly sufficient to afford e_{nc} (figure .2). e_{nc} is not optimal any more for $\frac{u_1^r\left(c_{1,nc}^r, c_{2,nc}^r\right)}{p_{1,d}} > \frac{u_2^r\left(c_{1,nc}^r, c_{2,nc}^r\right)}{p_{2,d}}$. She will consume more in period one but consume less in period two until $\frac{u_1^r\left(c_{1,d}^r, c_{2,d}^r\right)}{p_{1,d}} = \frac{u_2^r\left(c_{1,d}^r, c_{2,d}^r\right)}{p_{2,d}}$. In the meantime, she makes herself better off after democratization.

Proof of lemma 30

By figure 5.3.

Proof of theorem 31

Step 1: $\widehat{e} = \binom{\widehat{c}_1}{\widehat{c}_2} = \left(\begin{matrix} \left(\beta \frac{p_{1,d}}{p_{2,d}}\right)^{-\frac{\beta}{1+\beta}} \left(c_{1,nc}^r\right)^{\frac{1}{1+\beta}} \left(c_{2,nc}^r\right)^{\frac{\beta}{1+\beta}} \\ \left(\beta \frac{p_{1,d}}{p_{2,d}}\right)^{\frac{1}{1+\beta}} \left(c_{1,nc}^r\right)^{\frac{1}{1+\beta}} \left(c_{2,nc}^r\right)^{\frac{\beta}{1+\beta}} \end{matrix}\right)$: By table 5.1, solve $u^r\left(\widehat{c}_1, \widehat{c}_2\right) = u^r\left(c_{1,nc}^r, c_{2,nc}^r\right)$ and $\frac{u_1^r(\widehat{c}_1, \widehat{c}_2)}{p_{1,d}} = \frac{u_2^r(\widehat{c}_1, \widehat{c}_2)}{p_{2,d}}$ for $(\widehat{c}_1, \widehat{c}_2)$.

Step 2: Plug step 1 into $p_0 k^r > p_{1,d}\widehat{c}_1 + p_{2,d}\widehat{c}_2$. By table 5.1, it implies (5.19).

Proof of proposition 32

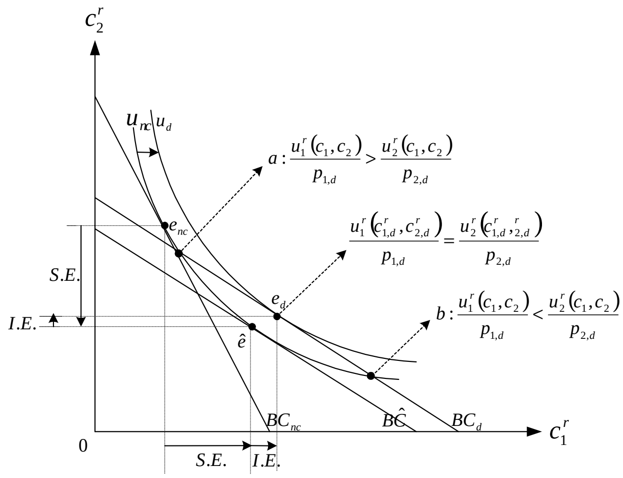


Figure .1. The representative rich person has greater but insufficient purchasing power to afford e_{nc}

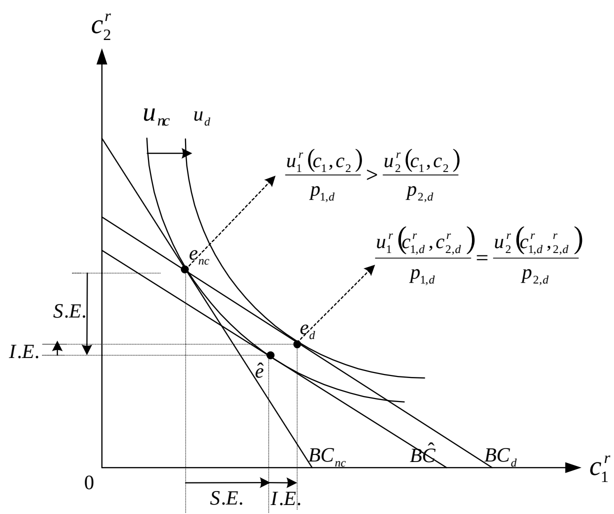


Figure .2. The representative rich person has greater and exactly sufficient purchasing power to afford e_{nc}

$\frac{\partial f(\alpha,\beta,\theta)}{\partial x} > 0$ for $x \in \{\alpha, \theta\}$.

Equation (5.20) for characterization of GE under the delegation regime

$$(\delta \in [0,1))$$

Let $\Omega(l_1) = \frac{\beta}{1+\beta}\frac{K_1}{n^p}\left[1-\delta+A\left(\frac{K_1}{n^p}\frac{1}{l_1}\right)^{\alpha-1}\left(\frac{1-\alpha+\beta}{\beta}-\frac{(1-\alpha)(1+\beta)(1-\theta)}{\beta\theta}\frac{1-l_1}{l_1}\right)\right]\frac{(1+\beta)(1-l_1)-\theta}{(1+\beta-\beta\theta)(1-l_1)-\theta}$.

Step 1: $p_1 = \frac{1}{1-\delta+\alpha A(n^p)^{1-\alpha}K_1^{\alpha-1}l_1^{1-\alpha}}p_0$; $w_1^e = w_1 = \frac{(1-\alpha)A(n^p)^{-\alpha}K_1^{\alpha}l_1^{-\alpha}}{1-\delta+\alpha A(n^p)^{1-\alpha}K_1^{\alpha-1}l_1^{1-\alpha}}p_0$; $r_1 = \frac{\alpha AK_1^{\alpha-1}(n^p)^{1-\alpha}l_1^{1-\alpha}}{1-\delta+\alpha A(n^p)^{1-\alpha}K_1^{\alpha-1}l_1^{1-\alpha}}p_0$: Plug (5.11) into (5.14) and (5.15) to derive p_1. Plug p_1 into (5.13) and (5.11).

Step 2: $c_1^r = \frac{k^r}{1+\beta}\left[1-\delta+\alpha A\left(n^p\right)^{1-\alpha}K_1^{\alpha-1}l_1^{1-\alpha}\right]$; $Y_1 = \left(n^p\right)^{1-\alpha}AK_1^{\alpha}l_1^{1-\alpha}$: Plug p_1 into (5.7).

Step 3: $I_1 = K_2 - (1-\delta)K_1$ and $I_2 = -(1-\delta)K_2$ by capital accumulation and $K_3 = 0$.

Step 4: $p_2 = \frac{1}{1-\delta+\alpha AK_2^{\alpha-1}\left(L_2^e\right)^{1-\alpha}}p_1$; $w_2^e = \frac{(1-\alpha)Az^{-\alpha}(n^p)^{-\alpha}K_2^{\alpha}l_2^{-\alpha}}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}p_1$; $w_2 = zw_2^e$; $r_2 = \frac{\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}p_1$: Plug (5.11) into (5.14) and (5.15) to derive p_2. Plug p_2 into (5.13) and (5.11).

$$\text{Step 5:}\begin{cases} c_1^p = \frac{1-\theta}{1+\beta}\left[(1-\alpha)A\left(n^p\right)^{-\alpha}K_1^{\alpha}l_1^{-\alpha} + \frac{(1-\alpha)Az^{1-\alpha}(n^p)^{-\alpha}K_2^{\alpha}l_2^{-\alpha}}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}\right] \\ c_2^p = \frac{\beta(1-\alpha)(1-\theta)}{1+\beta}A\left(n^p\right)^{-\alpha}\left[\left(1-\delta+\alpha Az^{1-\alpha}\left(n^p\right)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}\right)K_1^{\alpha}l_1^{-\alpha} + z^{1-\alpha}K_2^{\alpha}l_2^{-\alpha}\right] \\ c_2^r = \frac{\beta k^r}{1+\beta}\left[1-\delta+\alpha A\left(n^p\right)^{1-\alpha}K_1^{\alpha-1}l_1^{1-\alpha}\right]\left[1-\delta+\alpha Az^{1-\alpha}\left(n^p\right)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}\right] \end{cases}:$$

Plug step1 and step 4 into (5.9). Plug step 4 into (5.7).

Step 6: Plug (c_1^p, c_1^r, I_1) into $Y_1 = AK_1^{\alpha}L_1^{1-\alpha}$; plug (c_2^p, c_2^r, I_2) into $Y_2 = AK_2^{\alpha}\left(L_2^e\right)^{1-\alpha}$; plug (w_1, w_2) into $1 - l_2 = \beta\frac{w_1}{w_2}(1-l_1)$; plug (w_1, w_2) into (5.10):

(key1): $\frac{1-\alpha+\beta}{1+\beta}A\left(n^p\right)^{1-\alpha}K_1^{\alpha}l_1^{1-\alpha} = \frac{(1-\alpha)(1-\theta)}{1+\beta}A\left(n^p\right)^{1-\alpha}\left[K_1^{\alpha}l_1^{-\alpha} + \frac{z^{1-\alpha}K_2^{\alpha}l_2^{-\alpha}}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}\right] - \frac{\beta(1-\delta)}{1+\beta}K_1 + K_2$

$$\textbf{(key2):}\left\{\begin{array}{c} \frac{Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha}l_2^{1-\alpha}}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}} \\ = \left\{\begin{array}{c} \frac{\beta(1-\alpha)(1-\theta)}{1+\beta}A\left(n^p\right)^{1-\alpha}\left[K_1^{\alpha}l_1^{-\alpha} + \frac{z^{1-\alpha}K_2^{\alpha}l_2^{-\alpha}}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}\right] \\ +\frac{\beta K_1}{1+\beta}\left[1-\delta+\alpha A\left(n^p\right)^{1-\alpha}K_1^{\alpha-1}l_1^{1-\alpha}\right] - \frac{(1-\delta)K_2}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}} \end{array}\right\} \end{array}\right\}$$

(key3): $1 - l_2 = \beta K_1^{\alpha}l_1^{-\alpha}\frac{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}{z^{1-\alpha}K_2^{\alpha}l_2^{-\alpha}}(1-l_1)$

(key4): $l_1 = \frac{1+\beta-\theta}{1+\beta} - \frac{\theta}{1+\beta}\frac{1}{(1-\alpha)A(n^p)^{-\alpha}K_1^{\alpha}l_1^{-\alpha}}\frac{(1-\alpha)Az^{1-\alpha}(n^p)^{-\alpha}K_2^{\alpha}l_2^{-\alpha}}{1-\delta+\alpha Az^{1-\alpha}(n^p)^{1-\alpha}K_2^{\alpha-1}l_2^{1-\alpha}}$

Step 7: $\frac{z^{1-\alpha}\left(\frac{K_2}{l_2}\right)^{\alpha}}{1-\delta+\alpha A z^{1-\alpha}(n^p)^{1-\alpha}\left(\frac{K_2}{l_2}\right)^{\alpha-1}} = \left[\frac{1+\beta}{\theta}(1-l_1)-1\right]\left(\frac{K_1}{l_1}\right)^{\alpha}$: Rewrite (key4).

Step 8: $l_2 = 1-\frac{\beta\theta(1-l_1)}{(1+\beta)(1-l_1)-\theta}$: Plug step 7 into w_2 to rewrite w_2 in terms of l_1 only.

Plug this w_1 into the poor's Euler's equation $1-l_2=\beta\frac{w_1}{w_2}(1-l_1)$.

Step 9: $\frac{1}{n^p}\frac{K_2}{l_2}=\Omega(l_1)\Rightarrow\frac{\frac{K_2}{l_2}}{\frac{K_1}{l_1}}=\Omega(l_1)\left(\frac{1}{n^p}\frac{K_1}{l_1}\right)^{-1}$: Plug step 7 into (key1).

Step 10: $l_1 = 1-\frac{\theta}{1+\beta}\left[1+\frac{z^{1-\alpha}}{1-\delta+\alpha A z^{1-\alpha}\left(\frac{1}{n^p}\frac{K_2}{l_2}\right)^{\alpha-1}}\left(\frac{\frac{K_2}{l_2}}{\frac{K_1}{l_1}}\right)^{\alpha}\right]$: Rewrite (key4).

Step 11: $1+\frac{z^{1-\alpha}\left[\Omega\left(l^*_{1,d}\right)l^*_{1,d}\right]^{\alpha}}{1-\delta+\alpha A z^{1-\alpha}\left[\Omega\left(l^*_{1,d}\right)\right]^{\alpha-1}}\left(\frac{K_1}{n^p}\right)^{-\alpha} = \frac{1+\beta}{\theta}\left(1-l^*_{1,d}\right)$: Plug step 9 into step 10.

Characterization of GE under the no-commitment regime ($\delta\in[0,1)$)

Plug $l^*_{1,d}$ back to step 8 for $l^*_{2,d}$. Plug $l^*_{2,d}$ back to step 9 for $K^*_{2,d}$. Plug $\left(l^*_{1,d}, l^*_{2,d}, K^*_{2,d}\right)$ back to step 1 to step 5 for $\left(p^*_{t,d}, w^{e*}_{t,d}, w^*_{t,d}, r^*_{t,d}, c^{p*}_{t,d}, c^{r*}_{t,d}, Y^*_t, I^*_t\right)$.

Proof of lemma 34

Let $f(l_1) = 1+\frac{z^{1-\alpha}[\Omega(l_1)l_1]^{\alpha}}{1-\delta+\alpha A z^{1-\alpha}[\Omega(l_1)]^{\alpha-1}}\left(\frac{K_1}{n^p}\right)^{-\alpha}$; $g(l_1)=\frac{1+\beta}{\theta}(1-l_1)$. $f(l_1)$ is a hyperbola which is discontinuous for $l_1\in\left(1-\frac{\theta}{1+\beta-\beta\theta}, 1-\frac{\theta}{1+\beta}\right)$; $g(l_1)$ is linear downward-sloping in $l_1\in(0,1)$. There are two l^*_1 such that $f(l^*_1)=g(l^*_1)$: $l^*_1\in\left(0,1-\frac{\theta}{1+\beta-\beta\theta}\right)$ and $l^*_1=1-\frac{\theta}{1+\beta}$. If $l^*_1=1-\frac{\theta}{1+\beta}$, $l^*_2\to-\infty$ by table 5.3. Only $l^*_1\in\left(0,1-\frac{\theta}{1+\beta-\beta\theta}\right)$ gives stable and unique GE. Therefore, $l^*_{1,d}\in\left(0,1-\frac{\theta}{1+\beta-\beta\theta}\right)$ is stable and unique such that (5.20) holds.

Let $s^{p*}_{1,d}=w^*_{1,d}l^*_{1,d}-p^*_{1,d}c^{p*}_{1,d}$. By table 5.3, $s^{p*}_{1,d}=\frac{(1-\alpha)A}{\theta}\frac{\left(\frac{K_1}{n^p}\frac{1}{l^*_{1,d}}\right)^{\alpha}}{1-\delta+\alpha A\left(\frac{K_1}{n^p}\frac{1}{l^*_{1,d}}\right)^{\alpha-1}}\left[l^*_{1,d}-(1-\theta)\right]p_0 \gtreqless 0$ iff $l^*_{1,d}\gtreqless 1-\theta$.

Proof of claim 35

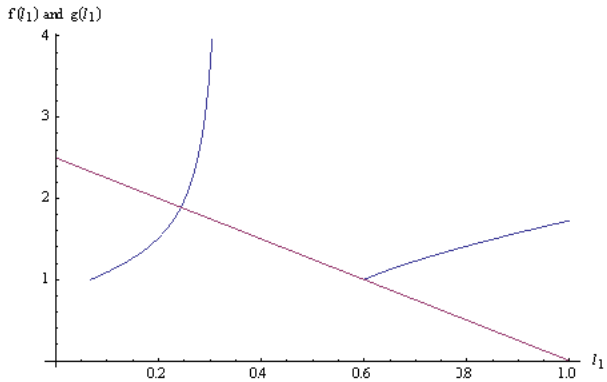


Figure .3. Stable and Unique $l_{1,d}^*$

The necessary condition holds such that $l_{1,d}^* \in \left(1-\theta, 1-\frac{\theta}{1+\beta-\beta\theta}\right)$. By figure .3, $l_{1,d}^* \in \left(1-\theta, 1-\frac{\theta}{1+\beta-\beta\theta}\right)$ if and only if it is on the right-side of $l_1 = 1-\theta$, i.e. $f(1-\theta) < g(1-\theta)$. Simplify this inequality to derive (5.21).

Proof of proposition 36

Step 1: $\widetilde{e} = \binom{\widetilde{c}_1}{\widetilde{c}_2} = \left(\begin{matrix} \left(\beta\frac{p_{1,d}}{p_{2,d}}\right)^{-\frac{\beta}{1+\beta}} \left(c_{1,nc}^r\right)^{\frac{1}{1+\beta}} \left(c_{2,nc}^r\right)^{\frac{\beta}{1+\beta}} \\ \left(\beta\frac{p_{1,d}}{p_{2,d}}\right)^{\frac{1}{1+\beta}} \left(c_{1,nc}^r\right)^{\frac{1}{1+\beta}} \left(c_{2,nc}^r\right)^{\frac{\beta}{1+\beta}} \end{matrix}\right)$: By table 5.3 and table 5.4, solve $u^r(\widetilde{c}_1, \widetilde{c}_2) = u^r\left(c_{1,nc}^r, c_{2,nc}^r\right)$ and $\frac{u_1^r(\widetilde{c}_1,\widetilde{c}_2)}{p_{1,d}} = \frac{u_2^r(\widetilde{c}_1,\widetilde{c}_2)}{p_{2,d}}$ for $\widetilde{c}_1, \widetilde{c}_2$.

Step 2: Plug step 1 into $p_0 k^r > p_{1,d}\widetilde{c}_1 + p_{2,d}\widetilde{c}_2$. By table 5.3 and table 5.4, it implies (5.6).

Alternative: The rich's purchasing power increases, if $\frac{c_{1,d}^*}{\widetilde{c}_1} = \left(\frac{p_{1,nc}^*}{p_{1,d}^*}\right)^{\frac{1}{1+\beta}} \left(\frac{p_{2,nc}^*}{p_{2,d}^*}\right)^{\frac{\beta}{1+\beta}} > 1$.

Proof of claim 39

By table 5.1.