

# Rapidly Rotating Bose-Einstein Condensates in and near the Lowest Landau Level

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We create rapidly rotating Bose-Einstein condensates in the lowest Landau level, by spinning up the condensates to rotation rates  $\Omega > 99\%$  of the centrifugal limit for a harmonically trapped gas, while reducing the number of atoms. As a consequence, the chemical potential drops below the cyclotron energy  $2\hbar\Omega$ . While in this mean-field quantum Hall regime we still observe an ordered vortex lattice, its elastic shear strength is strongly reduced, as evidenced by the observed very low frequency of Tkachenko modes. Furthermore, the gas approaches the quasi-two-dimensional limit. The associated cross-over from interacting- to ideal-gas behavior along the rotation axis results in a shift of the axial breathing mode frequency.

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Rotating Bose-Einstein condensates (BECs) provide a conceptual link between the physics of trapped gases and the physics of condensed matter systems such as superfluids, type-II superconductors and quantum Hall effect (QHE) materials. In all these systems, striking counterintuitive effects emerge when an external flux penetrates the sample. For charged particles this flux can be provided by a magnetic field, leading to the formation of Abrikosov flux line lattices in type-II superconductors [1], or in QHE systems to the formation of correlated electron-liquids and composite quasiparticles made of electrons with attached flux quanta [2]. For neutral superfluids, the analog to a magnetic field is a rotation of the system, which similarly spawns vortices [3]. In rotating atomic BECs, the creation of large ordered Abrikosov lattices of vortices [4] has recently become possible.

Here we examine the vortex lattice of harmonically trapped BECs approaching the high rotation limit, when the centrifugal force quite nearly cancels the radial confining force. The formal analogy of neutral atoms in this limit with electrons in a strong magnetic field has led to the prediction that quantum-Hall like properties should emerge in rapidly rotating atomic BECs [5]. In particular, the single-particle energy states organize into Landau levels, and if interactions are weaker than the cyclotron energy, primarily the near-degenerate states of the lowest Landau level (LLL) are occupied. For rotating bosons in the LLL, three regimes have been identified, distinguished by the filling factor  $\nu \equiv N_p/N_v$ , i.e., the ratio of the number of particles ( $N_p$ ) to vortices ( $N_v$ ). For high filling factors (always  $\gtrsim 500$  in our system), the condensate is in the mean-field quantum Hall regime [6–8] and forms an ordered vortex lattice ground state. With decreasing filling factor, the elastic shear strength of the vortex lattice decreases, which is reflected in very low frequencies of long-wavelength transverse lattice excitations (Tkachenko oscillations [9–13]). For filling factors

below  $\nu \approx 10$  the shear strength is predicted to drop sufficiently for quantum fluctuations to melt the vortex lattice [5,14], and a variety of strongly correlated vortex liquid states similar to those in the Fermionic fractional QHE are predicted to appear [5]. For still lower  $\nu$  exotic quasiparticle excitations obeying fractional statistics [15] are predicted.

In this Letter we report the observation of rapidly rotating BECs in the lowest Landau level, and provide evidence that the elastic shear strength of the vortex lattice drops substantially as the BEC enters the mean-field quantum Hall regime [11]. This effect is a precursor to the predicted quantum melting of the lattice at lower filling factors. Our rapidly rotating condensates spin out into a pancake shape and approach the quasi-two-dimensional regime. We observe a corresponding cross-over in the spectrum of breathing excitations along the axial direction. The high rotation limit has been studied experimentally in Ref. [16], focusing on effects of a rotating trap anisotropy, which is not present in our setup, and in Ref. [17] where the addition of a quartic term to the trapping potential led to a loss of vortex visibility.

Our experiments take place in an axially symmetric harmonic trap with oscillation frequencies  $\{\omega_\rho, \omega_z\} = 2\pi\{8.3, 5.3\}$  Hz. Using an evaporative spin-up technique described in Ref. [18] we create condensates containing up to  $5.5 \times 10^6$  <sup>87</sup>Rb atoms in the  $|F=1, m_F=-1\rangle$  state, rotating about the vertical, z axis, at a rate  $\tilde{\Omega} = 0.95$  ( $\tilde{\Omega} \equiv \Omega/\omega_\rho$  is the rotation rate  $\Omega$  scaled by the centrifugal rotation limit  $\omega_\rho$  for a harmonically trapped gas). To further approach the limit  $\tilde{\Omega} \rightarrow 1$ , we employ an optical spin-up technique, where the BEC is illuminated uniformly with laser light, and the recoil from spontaneously scattered photons removes atoms from the condensate. Since the condensate is optically thin to the laser light, atoms are removed without position or angular-momentum selectivity, such that angular mo-

mentum per particle is unchanged. Atom loss leads to a small decrease in cloud radius which, through conservation of angular momentum, increases  $\tilde{\Omega}$ . Over a period of up to 2 seconds we decrease the number of BEC atoms by up to a factor of 100, to  $5 \times 10^4$ , while increasing  $\tilde{\Omega}$  from 0.95 to more than 0.99 [19]. At this point, further reduction in number degrades the quality of images unacceptably. Ongoing evaporation is imposed to retain a quasi-pure BEC with no discernible thermal cloud.

With increasing rotation, centrifugal force distorts the cloud into an extremely oblate shape [see Fig.1] and reduces the density significantly - thus the BEC approaches the quasi-two-dimensional regime. For the highest rotation rates we achieve, the chemical potential  $\mu$  is reduced close to the axial oscillator energy,  $\Gamma_{2D} \equiv \frac{\mu}{2\hbar\omega_z} \approx 1.5$ , and the gas undergoes a cross-over from interacting- to ideal-gas behavior along the axial direction. To probe this cross-over, we excite the lowest order axial breathing mode over a range of rotation rates. For a BEC in the axial Thomas-Fermi regime, an axial breathing frequency  $\omega_B = \sqrt{3}\omega_z$  has been predicted in the limit  $\tilde{\Omega} \rightarrow 1$  [20], whereas  $\omega_B = 2\omega_z$  is expected for a non-interacting gas.

To excite the breathing mode, we jump the axial trap frequency by 6%, while leaving the radial frequency unchanged (within  $< 0.5\%$ ). To extract the axial breathing frequency  $\omega_B$ , we take 13 nondestructive in-trap images of the cloud, perpendicular to the axis of rotation. From the oscillation of the axial Thomas-Fermi radius [21] in time we obtain  $\omega_B$ . Rotation rates are obtained [19] from the aspect ratio by averaging over all 13 images to eliminate the effect of axial breathing. As shown in Fig. 2(a), we do indeed observe a frequency cross-over from  $\omega_B = \sqrt{3}\omega_z$  to  $\omega_B = 2\omega_z$  as  $\tilde{\Omega} \rightarrow 1$ . To quantify under which conditions the cross-over occurs, we plot the same data vs.  $\Gamma_{2D}$  [Fig. 2(b)], where the chemical potential is determined from the measured atom number, the rotation rate and the trap frequencies. For  $\Gamma_{2D} < 3$ , the ratio  $\omega_B/\omega_z$  starts to deviate from the predicted hydrodynamic value, and approaches 2 for our lowest  $\Gamma_{2D} \approx 1.5$ .

As  $\tilde{\Omega} \rightarrow 1$ , also the dynamics in the radial plane are affected. For the highest rotation rates, interactions become sufficiently weak that the chemical potential  $\mu$  drops below the cyclotron energy  $2\hbar\Omega$ , which is only a few percent smaller than the Landau level spacing  $2\hbar\omega_\rho$ . Then,  $\Gamma_{LLL} \equiv \frac{\mu}{2\hbar\Omega} < 1$ , and the condensate primarily occupies single-particle states in the LLL. These form a ladder of near-degenerate states, with a frequency splitting of  $\epsilon = \omega_\rho - \Omega$ . The number of occupied states is  $N_{LLL} \approx \frac{\mu}{\hbar\epsilon}$ . We are able to create condensates with  $\Gamma_{LLL}$  as low as 0.6, which occupy  $N_{LLL} \approx 120$  states with a splitting  $\epsilon < 2\pi \cdot 0.06$  Hz. In this regime of near-degenerate single-particle states a drastic decrease of the lattice's elastic shear strength takes place. The elastic shear modulus  $C_2$  is predicted by Baym [11] to decrease with increasing rotation rate from its value in the "stiff" Thomas-Fermi (TF) limit,  $C_2^{TF} = n_{(\Omega)}\hbar\Omega/8$  (where  $n_{(\Omega)}$

is the BEC number density) to its value in the mean-field quantum Hall regime, of  $C_2^{LLL} \approx 0.16 \times \Gamma_{LLL} \times C_2^{TF}$ . We directly probe this shear strength by exciting the lowest order azimuthally symmetric lattice mode ( $n = 1, m = 0$ ) Tkachenko mode [9–11]). Its frequency  $\omega_{(1,0)} \sim \sqrt{C_2}$  is expected to drop by a factor  $\approx 2.5$  below the TF prediction when  $\Gamma_{LLL} = 1$ .

Our excitation technique for Tkachenko modes has been described in Ref. [10]. In brief, we shine a focused red detuned laser ( $\lambda = 850$  nm) onto the BEC center, along the axis of rotation. This laser draws atoms into the center, and Coriolis force diverts the atoms' inward motion into the lattice rotation direction. The vortex lattice adjusts to this distortion, and after we turn off the beam, the lattice elasticity drives oscillations at the frequency  $\omega_{(1,0)}$ . We observe the oscillation by varying the wait time after the excitation, and then expanding the condensate before imaging the vortex lattice along the z-axis [see Fig. 3(a),(b)]. In Fig. 3(c), we compare the measured frequencies  $\omega_{(1,0)}$  to the predictions of Ref. [11] for the TF limit and for the mean-field quantum Hall regime. For  $\tilde{\Omega} < 0.98$  ( $\Gamma_{LLL} > 3$ ), the frequency  $\omega_{(1,0)}$  follows the prediction for the TF regime, whereas by  $\tilde{\Omega} = 0.990$  ( $\Gamma_{LLL} = 1.5$ ),  $\omega_{(1,0)}$  has dropped to close to the prediction for the LLL, thus providing evidence for the cross-over to the lower shear modulus  $C_2$  predicted for the LLL.

While we are able to produce clouds with  $\Gamma_{LLL}$  measured to be substantially lower than  $\Gamma_{LLL} = 1.5$ , we are unable to accurately measure Tkachenko frequencies under these extreme conditions, due at least in part to the very weakness of  $C_2$ . The Tkachenko mode frequencies become so low that it takes multiple seconds to track even a quarter oscillation [Fig. 3(a),(b)], while at the same time the very weak shear strength means that even minor perturbations to the cloud can cause the lattice to melt and the individual cores to lose contrast [22] in a matter of seconds. These perturbations can result from residual asymmetry of the magnetic trapping potential, or from spatial structure in the optical beam used to reduce the atom number, or perhaps from thermal fluctuations. In contrast, for a "stiff" cloud of  $3 \times 10^6$  atoms at  $\tilde{\Omega} = 0.95$  ( $\Gamma_{LLL} \approx 7$ ) we observe that the lattice remains ordered, and  $\tilde{\Omega}$  can be kept constant, over the entire  $1/e$  lifetime of the BEC ( $\approx 3$  minutes).

Finally we discuss observations related to two other predicted consequences of rapid rotation. First, the radial condensate density profile has been predicted to change from the parabolic Thomas-Fermi (TF) profile to a Gaussian profile in the LLL [6]. We create clouds with varying values of  $\Gamma_{LLL}$  down to as small as 0.6, and after 3 sec equilibration time acquire images of the radial density profile. These images fit better to a TF profile than to a Gaussian, showing no signs of a cross-over in the radial density profile as the LLL is entered.

On the theoretical side, it has been pointed out [12] that, whereas the predictions of Ref. [6] are based on an assumption that the areal density of vortices is uniform, a very small deviation from uniformity could account for an overall TF density profile. Furthermore, according to Ref. [8], a Gaussian radial profile is to be expected only if  $Na/R_z \ll 1$  ( $N$ : number of atoms in the BEC,  $a$ : scattering length,  $R_z$ : axial BEC radius), whereas our measurements were performed on condensates with  $Na/R_z \gtrsim 26$ . It certainly seems reasonable to expect a TF profile for a given direction when the chemical potential is much greater than the confining frequency in that direction, and for our condensates with  $\Gamma_{LLL} = 0.6$ , the chemical potential  $\mu/\hbar \approx 2\pi 10$  Hz while the (centrifugally weakened) effective radial trap frequency  $\omega_{eff} = \sqrt{\omega_p^2 - \Omega^2} \approx 2\pi 1$  Hz.

Second, there has been a debate about the possibility of vortex core overlap in the high rotation limit [7,8,23]. To achieve adequate imaging resolution, we expand the BEC before imaging the vortex lattice. Our expansion procedure [18] leads to nearly pure two-dimensional (2-d) expansion (a thirteen-fold expansion in radius occurs while the axial size changes by  $\lesssim 25\%$ ). We expect that under these conditions, the ratio of the vortex core area to the lattice's unit cell area should be preserved throughout the expansion process. We fit a 2-d Gaussian to the missing optical density associated with each vortex core. The vortex radius  $r_v$  is defined to be the RMS radius of the 2-d Gaussian, which is 0.60 times its FWHM. We then define the fractional area  $\mathcal{A}$  occupied by the vortices to be  $\mathcal{A} = n_v \pi r_v^2$ , where  $n_v$  is the areal density of vortices. In the limit of many vortices, the theoretical prediction for  $n_v$  is  $m\Omega/(\pi\hbar)$ .

To determine a theoretical value for the vortex core size, we perform a numerical simulation of the Gross-Pitaevskii equation of a BEC containing an isolated vortex. We obtain  $r_v = 1.94 \times \xi$  with  $\xi = (8\pi na)^{-\frac{1}{2}}$ , where  $a$  is the scattering length and  $n$  is the density [24]. The resulting prediction for  $\mathcal{A}$  can be expressed as  $\mathcal{A} = 1.34 \times (\Gamma_{LLL})^{-1}$ . This value exceeds unity for  $\Gamma_{LLL} < 1.34$ , which has led to the prediction that vortices should merge in the LLL limit. An alternate treatment due to Baym and coworkers [8] on the other hand predicts that  $\mathcal{A}$  saturates at 0.225 in the LLL. Our data for  $\mathcal{A}$  are plotted in Fig. 4. For  $\Gamma_{LLL}^{-1} < 0.1$  the data agree reasonably well with our numerical result. For larger  $\Gamma_{LLL}^{-1}$  the data clearly show saturation of  $\mathcal{A}$  at a value close to the LLL limit [8], rather than a divergence of  $\mathcal{A}$  as  $\tilde{\Omega} \rightarrow 1$ . Further details on vortex core structure will be provided in a future publication [25].

In conclusion, we have created rapidly rotating BECs in the lowest Landau level. The vortex lattice remains ordered, but its elastic shear strength is drastically reduced. In expansion-images we find no divergence of vortex core area as  $\tilde{\Omega} \rightarrow 1$ , as well as no deviation from

a radial Thomas-Fermi profile. Additionally, our rapidly rotating BECs approach the quasi-two-dimensional limit. We remain far from the regime in which quantum fluctuations [14] should destroy the lattice, but observing the effects of thermal fluctuations [26,27] in this reduced-dimensionality system may be possible.

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the measured BEC aspect ratio to the trap aspect ratio (see e.g. [20]). At our lowest values of  $\Gamma_{2D}$  we correct for quantum-pressure contributions to axial size.

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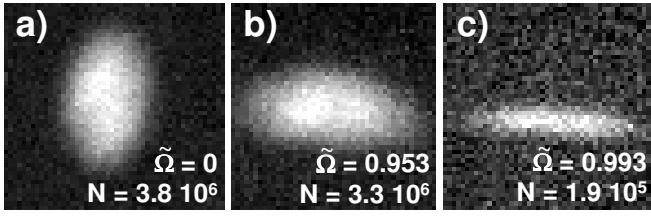


FIG. 1. Side view images of BECs in trap. (a) Static BEC. The aspect ratio  $R_z/R_\rho = 1.57$  ( $N = 3.8 \times 10^6$  atoms) resembles the prolate trap shape. (b) After evaporative spin-up,  $N = 3.3 \times 10^6$ ,  $\tilde{\Omega} = 0.953$ , (c) evaporative plus optical spin-up,  $N = 1.9 \times 10^5$ ,  $\tilde{\Omega} = 0.993$ . Due to centrifugal distortion the aspect ratio is changed by a factor 8 compared to (a).

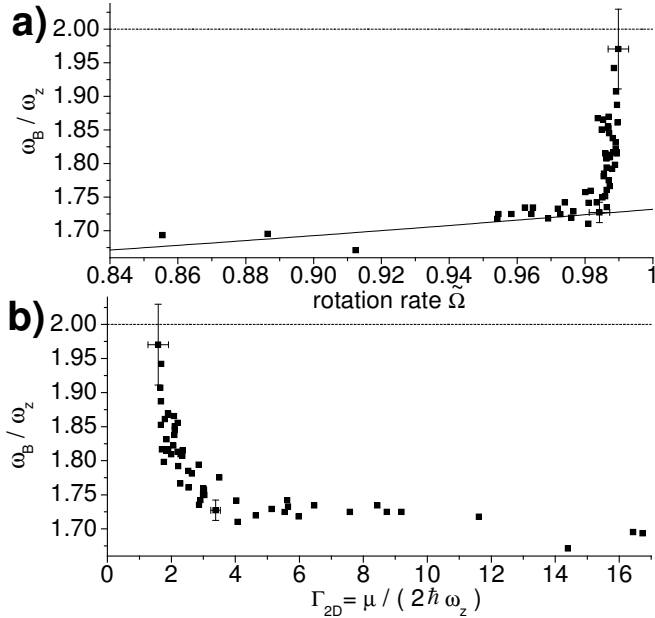


FIG. 2. Measured axial breathing frequency  $\omega_B/\omega_z$  (a) as a function of rotation rate  $\tilde{\Omega}$  and (b) vs.  $\Gamma_{2D}$ . Solid line: Prediction for the hydrodynamic regime [20]; Dashed line: ideal gas limit. For  $\tilde{\Omega} > 0.98$  ( $\Gamma_{2D} < 3$ ) a cross-over from interacting- to ideal-gas behavior is observed. Representative error bars are shown for two data points.

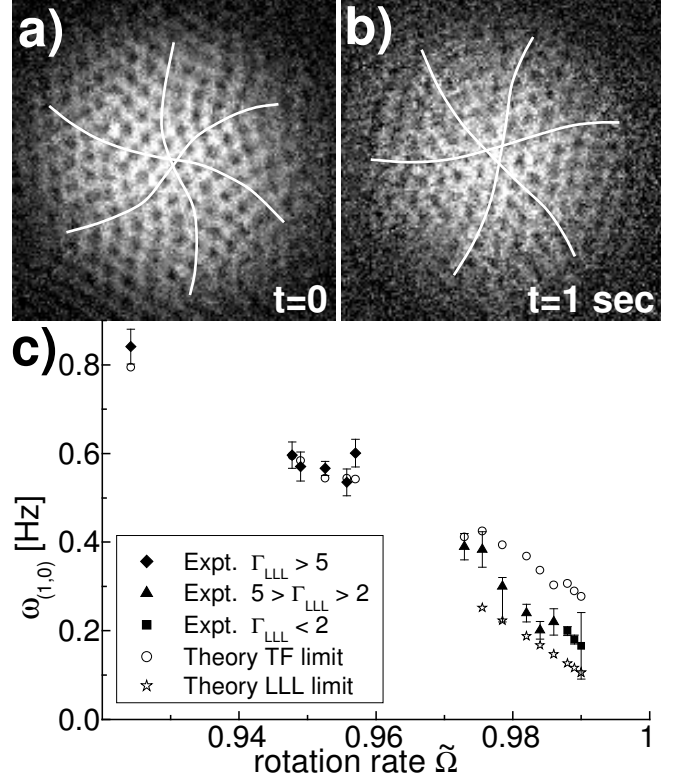


FIG. 3. (a), (b) Tkachenko mode at  $\Gamma_{LLL} = 1.2$  ( $N = 1.5 \times 10^5$ ,  $\tilde{\Omega} = 0.989$ ): (a) directly after excitation, (b) after 1 sec - the lattice oscillation has not yet completed 1/4 cycle. (c) Comparison of measured Tkachenko mode frequency  $\omega_{(1,0)}$  (solid symbols) vs.  $\tilde{\Omega}$  to theory [11], using vortex lattice shear modulus  $C_2^{TF}$  in the Thomas-Fermi (TF) limit (circles), and  $C_2^{LLL}$  in the mean-field quantum Hall regime (stars). Note that both  $N$  and  $\Gamma_{LLL}$  decrease as  $\tilde{\Omega}$  increases. For  $\Gamma_{LLL} \approx 3$  (reached at  $N = 7.8 \times 10^5$ ,  $\tilde{\Omega} \approx 0.978$ ) the data cross over from the TF to the quantum Hall prediction.

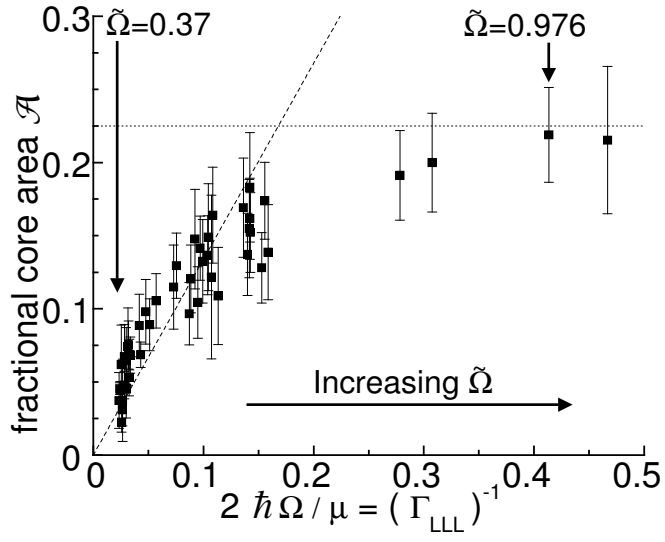


FIG. 4. Fraction of the condensate surface area occupied by vortex cores,  $\mathcal{A}$ , measured after condensate expansion (squares), plotted vs. the inverse of the lowest Landau level parameter,  $(\Gamma_{LLL})^{-1} = 2\hbar\Omega/\mu$ . The data clearly show a saturation of  $\mathcal{A}$ , as  $\tilde{\Omega} \rightarrow 1$ . Dashed line: prediction (see text) for the pre-expansion value at low rotation rate. Dotted line: result of Ref.[8] for the saturated value of  $\mathcal{A}$  in the LLL.