Coherent Tunable Coupling of Quantum Circuits

by

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Coherent Tunable Coupling of Quantum Circuits

Thesis directed by Dr. Raymond W. Simmonds

This thesis presents a detailed investigation of coherent tunable coupling between two coupled quantum circuits. Quantum circuits have the potential to be used as the fundamental building blocks in quantum processors. Any large scale quantum processor will be composed of a large number of these coupled circuits. The efficient implementation of quantum algorithms will be difficult without a reliable mechanism for controlling the interaction strength between coupled systems, while preserving the delicate quantum information stored in the coherent superpositions of quantum states.

We show that a flux-biased rf-SQUID can be used to coherently mediate the interaction between two coupled quantum circuits, a phase qubit and LC resonator. This interaction results from an effective mutual inductance between the qubit and resonator as a result of their direct coupling to an rf-SQUID. The sign and magnitude of this effective mutual inductance can be tuned with applied flux to the rf-SQUID, thus controlling the coupled interactions over a large range. We observe the modulation in coupling strength using measurements in both the frequency and time domains. The measurements are shown to agree well with theoretical predictions.

This thesis discusses all aspects of the experiments from a theoretical description of each component to the design, fabrication, experiment setup and measurements.

Dedication

For my beautiful wife, Olivia, and my parents. Thanks for all your support.

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Chapter 1

Introduction

1.1 What Are Quantum Circuits?

Quantum circuits are examples of macroscopic quantum systems. Macroscopic in the sense that they contain a large number of particles, yet the collective degrees of freedom describing their dynamics, e.g. the voltage across a particular branch, or the current through that branch, obey the rules of quantum mechanics. However, in order to resolve quantum behavior at the macroscopic level, the intrinsic loss in the circuit needs to be low enough that the width of the quantized energy levels is smaller than the spacing between them [4].

The way this is done in electrical circuits is by use of superconducting metals. The physics of superconductors is described by a theory proposed by John Bardeen, Leon Cooper and John Schrieffer in 1957 that came to be known as "BCS theory" [5]. When a superconducting metal is cooled below a critical temperature, T_c , part of the electrons in the metal begin to pair up into "Cooper pairs" due to a small attractive interaction between the electrons mediated by the positive lattice forming the metal. These Cooper pairs are described by a composite quantum state composed of two spin-1/2 electrons whose total spin can be an integer 0 or 1, making them bosons. Since bosons are allowed to reside in the same overall quantum state, Cooper pairs "condense" into a single macroscopic quantum state. For temperatures in the vicinity of the critical temperature, T_c , the energy of this macroscopic state is separated from the lowest-energy unpaired elections by an amount [6]

$$2\Delta(T) \simeq 6.2 \, k_B T_c \sqrt{1 - \frac{T}{T_c}}.\tag{1.1}$$

Cooper-paired electrons are able to traverse the metal in the circuit with zero electrical resistance. The remaining unpaired electrons, called quasiparticles, are still resistive, however. But in the limit that $T \ll T_c$, the available thermal energy is not enough to overcome the energy gap and the remaining quasiparticle density becomes exponentially small. In this limit, the gap energy asymptotically levels off to [6]

$$2\Delta(T) \simeq 3.52 \, k_B T_c. \tag{1.2}$$

The superconducting circuits used in this thesis are made of aluminum with a $T_c \simeq 1$ K, requiring them to be cooled in a cryostat capable of reaching temperatures in the millikelyin regime.

In order to manipulate quantum circuits they must be coupled to an external electromagnetic environment which can introduce significant dissipation, destroying our efforts to resolve quantized energy levels. Thus careful engineering is required to keep the circuit isolated enough from the environment to minimize dissipation but not so isolated that energy cannot be coupled into and out of the circuit over reasonable time-scales. In addition, circuits must be designed such that the excitation energy of the circuit is well below the superconducting gap energy, preventing the generation of quasiparticles. For superconducting aluminum this upper threshold in energy is $f_{Al} \sim 100$ GHz. Another beneficial, but not required, condition is that the available thermal energy be well below the excitation energy of the circuit so that the quantum ground state can be isolated without additional cooling techniques. For typical dilution refrigerators with base temperatures of ~ 30 mK, this means that the excitation frequency of the circuit should be well above 600 MHz. These two conditions put superconducting aluminum circuits in the microwave regime.

The simplest quantum electrical circuit is an inductor in parallel with a capacitor forming the familiar LC resonator. The generalized coordinate typically chosen is the flux, Φ , through the inductor coil, making the charge, Q, on the capacitor plate the conjugate variable. The resonant frequency is $\omega = 1/\sqrt{LC}$. However, as we show in chapter 2, the equal level spacing in an LC resonator implies that the quantum variables, $\hat{\Phi}$ and \hat{Q} , behave *almost* classically, making it very hard to distinguish quantum from classical effects in the lab. In order to more easily observe quantum effects, a non-linear element needs to be introduced in the circuit.

In superconducting circuits, this non-linearity is provided by a Josephson junction. A Josephson

junction is made by sandwiching a thin insulating barrier between two superconducting electrodes. The wave functions describing the Cooper pairs in each electrode slightly "leak" out into the barrier. When the barrier is thin enough, the leaky parts of the wave functions will overlap, allowing Cooper pairs to tunnel from one electrode to the other. The dynamics are governed by two equations, known as the "Josephson relations",

$$I = I_0 \sin \delta \tag{1.3}$$

$$\frac{d\delta}{dt} = \frac{2\pi}{\Phi_0} V, \tag{1.4}$$

where δ is the guage-invariant phase difference between the two superconducting wave functions, I_0 is the maximum possible supercurrent supported by the barrier, Φ_0 is the magnetic flux quantum, and V is the voltage across the junction [5]. We can see from these equations that if the current through the junction is changing in time, there will be a voltage across the junction. This behavior allows us to define a "Josephson inductance", L_J , by relating this voltage to the time-derivative of the current in analogy with Faraday's law of induction,

$$V = L_J(\delta)I,\tag{1.5}$$

where we have acknowledged the possibility that this inductance may be a function of the junction phase or equivalently the junction current. Comparing this expression to equation 1.4 by using the chain rule we have

$$V = L_J \dot{I}$$
$$= L_J \frac{dI}{d\delta} \dot{\delta}$$
$$= \frac{\Phi_0}{2\pi} \dot{\delta}.$$
(1.6)

Using equation 1.3 to write $dI/d\delta = I_0 \cos \delta$ we are left with

$$L_J(\delta) = \frac{\Phi_0}{2\pi} \frac{1}{I_0 \cos \delta}.$$
 (1.7)

The energy stored in this non-linear inductance will be shown in chapter 3 to be

$$E_J(\delta) = -E_J(0)\cos\delta,\tag{1.8}$$



Figure 1.1: a) Josephson junction. b) Circuit model of a Josephson junction.

where

$$E_J(0) \equiv \frac{\Phi_0}{2\pi} I_0.$$
 (1.9)

When a Josephson junction is embedded in a superconducting loop, the junction phase becomes constrained by the total flux threading the loop through the "fluxoid quantization" relation that says [5]

$$\delta = 2\pi \frac{\Phi}{\Phi_0},\tag{1.10}$$

where Φ is the total flux in the loop, having contributions from the circulating loop current as well as any externally applied flux. This constraint makes the choice of generalized coordinate for a circuit with a junction embedded in a loop of inductance, L, interchangeable between the phase of the junction and the flux in the loop. The finite surface area of the overlapping superconducting electrodes also gives a Josephson junction a self-capacitance of ~ 50 fF/ μ m² [7]. Figure 1.1 illustrates a Josephson junction and its circuit equivalent.

It was the non-linearity introduced by the Josephson junction that allowed researches to experimentally verify quantum behavior at the macroscopic level. The first evidence of this behavior came in 1981 when researches observed macroscopic tunneling from a metastable potential energy well created by a currentbiased Josephson junction [8]. When their junctions were biased at currents close to but not above the critical current, they observed statistical switching of the junction to the voltage state. They showed that this switching was not thermally induced and could only be explained by quantum tunneling through the barrier. Later in a series of experiments by Martinis *et al.* direct quantization of the energy levels in the well was observed [9, 10]. Here escape rates were measured while the junction was irradiated with microwave energy. They found that at certain frequencies, tunnel rates were enhanced due to the resonant excitation to higher energy levels in a metastable well.

In 1997, the first experiment demonstrating the superposition of two isolated states in a quantum circuit was performed by Nakamura *et al.* [11]. Here the states were a superposition of two charge states in what is called a "charge" qubit (explained in Chapter 2). Then in 2000, Friedman *et al.* and van der Wal *et al.* demonstrated the superposition of flux states in a "flux" qubit (also explained in Chapter 2) [12, 13]. Researchers quickly realized that these macroscopic quantum circuits would make prime candidates for the building blocks of quantum computers. These devices came to be known as superconducting qubits.

Superconducting qubit research has made tremendous strides in recent years. Superconducting qubits are routinely made with coherence lifetimes approaching 1 μ s and beyond [14]. Also, a number of coupled qubit experiments with fixed coupling between qubits have been performed in recent years [15, 16, 17, 18, 19, 20, 21, 22, 23]. Any real superconducting quantum computer, however, will be composed of an intricate network of many qubits coupled to each other in various ways, as well as coherent "quantum buses" that will manage the shuttling of quantum information between distant qubits. This means that it will become increasingly difficult to implement quantum information processing between many coupled quantum circuit elements with fixed coupling between elements. The need to control the coupling between various elements, such as qubit-qubit interactions or qubit-quantum bus interactions is essential. In addition, this controlled coupling must be "coherent" by preserving the delicate quantum information stored in the interacting elements.

1.2 Previous Tunable Coupling Experiments

Some of the earlier methods for implementing tunable coupling between quantum circuit elements were proposed nearly ten years ago [36, 24]. More recently, other tunable coupling schemes were proposed [25, 26, 27, 28]. At the time the experiments presented in this thesis were performed, only a few experimental demonstrations of tunable coupling existed in the literature [29, 30, 3, 2, 1]. Furthermore, only two of these experiments showed modulation of the coupling strength in the frequency domain by changing the size of the "avoided crossing" (explained later) seen in spectroscopy measurements, an effect indistinguishable from a normal-mode splitting in coupled classical circuits. They lacked the corresponding time domain data showing coherent modulation in the oscillation frequency of probability amplitudes with changing coupling strength, a strictly quantum effect [3, 1]. This is an important distinction to make. While modulation of the avoided crossing in the spectroscopy does indicate that the tunable coupling scheme is working, it does not prove that the coherent interactions between the coupled elements are not degraded by the tunable coupler. For this, time-domain measurements are required. Only one experiment showed time-domain data [2]. Experiments [30, 29] showed neither frequency or time-domain data. They measured the modulation in the coupling strength of coupled flux qubits by tracking the changes in the dc fluxes to each qubit required to keep them in their ground states as a function of the dc flux applied to the coupler.

The first tunable coupling scheme was demonstrated by Hime *et al.* in 2006 [1]. Here, the coupling strength between two coupled flux qubits was tuned by applying a bias current to the already-present readout SQUID used to readout the qubit states. The dynamic inductance of the readout SQUID modulates with this bias current, resulting in a modulation of the interaction strength between the qubits. They showed modulation in the size of the avoided crossing from a maximum of ~ 135 MHz down to ~ 20 MHz. They were not, however, able to show the avoided crossing reduce to zero because the required SQUID bias current caused the readout SQUID to switch to the voltage state prematurely. This demonstrated a lack of total control over the coupled interactions using this scheme. Figure 1.2, adapted from [1], shows the circuit used in this experiment.

In 2007, Niskanen *et al.* demonstrated time-domain modulation of the coupling strength between two coupled flux qubits using a third flux qubit, of much higher excitation energy than either of the other qubits, as the coupling element [2]. Here, the qubits had to be operated at their optimal bias points where sensitivity to flux noise is minimized, resulting in different resonant frequencies for each qubit. As such, coupled interactions could be induced only by parametrically modulating the dynamic inductance of the coupler. The largest coupling strength they measured was ~ 23 MHz. The coupling was turned off by simply removing the parametric drive. Figure 1.3, adapted from [2], shows the circuit used in this experiment.

In 2008, Fay et al. demonstrated tunable coupling between a charge and "phase" qubit (described in



Figure 1.2: Adapted from [1] a) Circuit schematic of qubits A and B with the surrounding readout SQUID. b) Optical photograph of the circuit. c) Readout SQUID bias current used to control the coupling and readout the qubit states.



Figure 1.3: Adapted from [2] a) Schematic of the coupled qubit system along with the transition frequencies of the qubits at the bias points. The center qubit is the coupler with a transition frequency, Δ_3 , much larger than Δ_1 and Δ_2 . b) Energy level diagram of the transitions achievable by applying a parametric drive at frequencies $\Delta_2 \pm \Delta_1$. c) Scanning electron microscope (SEM) image of the circuit.



Figure 1.4: Adapted from [3]. The charge qubit is formed by the Cooper pair transistor formed by the two left-most junctions E_{J1}^T and E_{J2}^T . The phase qubit is the parallel combination of the right two junctions E_{J1}^S and E_{J2}^S . The coupling is controlled by the phase difference, δ , across the charge qubit junctions, where δ can be tuned by tuning Φ_T , the total flux in that loop. Leftmost in the figure is an SEM image of the charge qubit.

chapter 3). Their circuit, adapted from [3], is shown in figure 1.4. The coupling is mediated by a Josephson interaction created by the sharing of the bottom charge qubit junction with the loop connecting to the phase qubit. By changing the flux, Φ_T to this loop, the phase across the charge qubit junctions, δ , changes, modulating the coupled interactions. Using spectroscopy measurements only, they observed the size of the avoided crossing change from a maximum of 1.1 GHz, to a minimum of 60 MHz.

1.3 Specific Contributions of This Work

We have implemented coherent tunable coupling between a superconducting phase qubit and a lumped-element LC resonator, using a third "mediating" element that fully controls the coupling strength between the qubit and resonator when on resonance [28]. We show that the coupling strength is tunable over a large range. Further, the coupling can be completely turned off with the qubit and resonator still on resonance. We present a simple model describing this tunable coupling and verify agreement with theoretical predictions using both frequency and time-domain measurements, showing explicitly that the coherent interactions are not degraded by the coupling element. In addition, we perform fast manipulations of this coupling strength on time-scales shorter than the qubit lifetime, mimicking use in a large-scale quantum processor. Finally, we present preliminary data showing that this tunable coupler can also be operated in a parametric mode, inducing off-resonant coupling between the qubit and resonator. We note that since the completion of this work, Bialczak *et al.* implemented tunable coupling between two-coupled phase qubits using a highly modular "drop-in" tunable coupler [31]. Also, Srinivasan *et al.* implemented tunable coupling between a charge qubit and resonant cavity [32].

1.4 Thesis Overview

In chapter 2, we discuss in detail the quantum description of the LC resonator. Using this simple system, we present an effective numerical technique for analyzing more complicated, non-linear, quantum circuits. We also use the LC resonator to demonstrate the fundamental differences between the two primary types of qubits; charge qubits and phase (flux) qubits. Then we delve into the theoretical description of a driven quantum LC resonator, showing how to modify the Hamiltonian to take into account the presence of an ideal current source. Finally, through explicit calculation, we show how an LC resonator is always in the "classical limit", making non-linearity necessary for qubit operation. In chapter 3, we build on the ideas developed in chapter 2 and introduce the phase qubit. We first discuss overall phase qubit operation, from initialization to state-preparation, measurement and readout. Then we discuss experimental procedures that must be done to characterize phase qubits. In chapter 4, we present a theoretical description of fixed-strength coupling between a phase qubit and LC resonator. We then introduce the vacuum Rabi oscillation, one of the workhorse measurements presented in this thesis. Finally we point out a problem with the fixed-coupling paradigm, motivating the quest to implement tunable coupling between the elements. In chapter 5, we discuss tunable coupling using a flux-biased rf-SQUID. We present a simple model that explains the origin of the effective tunable interaction mediated by the rf-SQUID. We then discuss what range of tunability we can expect and how that range depends on circuit parameters. In chapter 6, we discuss the fabrication, design, experimental setup and measurements on two circuit generations. The measurements are shown to be in good theoretical agreement with the model introduced in chapter 5. In chapter 7, we present preliminary data showing off-resonant parametric coupling. Then we discuss a modification to the rf-SQUID that will improve performance when operated in the parametric mode.

1.5 Author's Publications

- M.S. Allman, F. Altomare, J.D. Whittaker, K. Cicak, D. Li, A. Sirois, J. Strong, J.D. Teufel, and R.W Simmonds, rf-SQUID Mediated Coherent Tunable Coupling between a Superconducting Phase Qubit and a Lumped-Element Resonator, Physical Review Letters 104, 177004 (2010).
- (2) M.S. Allman, F. Altomare, J.D. Whittaker, K. Cicak, D. Li, A. Sirois, J. Strong, J.D. Teufel, and R.W Simmonds, *Time-Domain Detection of Weakly Coupled TLS Fluctuators in Phase Qubits*, arXiv:1004.2738v1 to be submitted to Applied Physics Letters (2011).
- (3) L. Tian, M.S. Allman, and R.W Simmonds, Parametric Coupling Between Macroscopic Quantum Resonators, New Journal of Physics 10, 115001 (2008).
- (4) K. Cicak, M.S. Allman, J.A. Strong, K.D. Osborn, and R.W Simmonds, Vacuum-Gap Capacitors for Low-Loss Superconducting Resonant Circuits, <u>IEEE Transactions on Applied Superconductivity</u> 19, 948 (2009).
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Chapter 2

The LC Resonator

The simplest quantum circuit is the LC resonator consisting of a single inductor in parallel with a single capacitor as shown in figure 2.1. The Hamiltonian describing this system is

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L},$$
(2.1)

where the operators, \hat{Q} and $\hat{\Phi}$, obey the commutator relation,

$$[\hat{Q}, \hat{\Phi}] = i\hbar. \tag{2.2}$$

Proceeding as we do with any quantum problem, we find energy eigenvalues and eigenvectors. The reader may recognize equation 2.1 as the Hamiltonian for a simple harmonic oscillator, the eigenvalues and eigenvectors of which can be found by defining the non-Hermitian creation and annihilation operators,

$$a = \sqrt{\frac{C\omega}{2\hbar}} \left(\hat{\Phi} + \frac{i}{C\omega} \hat{Q} \right)$$
$$a^{\dagger} = \sqrt{\frac{C\omega}{2\hbar}} \left(\hat{\Phi} - \frac{i}{C\omega} \hat{Q} \right)$$
(2.3)

where $\omega = 1/\sqrt{LC}$. Using this method, the energy eigenvalues are found to be [33],

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega,\tag{2.4}$$

where n is a non-negative integer. In anticipation of encountering more complicated Hamiltonians, however, we will discuss a numerical procedure for diagonalizing equation 2.1.



Figure 2.1: LC resonator.

2.1 Numerical Solutions of the Schrodinger Equation

We begin by projecting equation 2.1 into the continuous basis of eigenstates of the operator, $\hat{\Phi}$, yielding the familiar time-independent Schrodinger equation (TISE),

$$\left(-\frac{\hbar^2}{2C}\frac{d^2}{d\Phi^2} + \frac{\Phi^2}{2L}\right)\psi_n\left(\Phi\right) = E_n\psi_n\left(\Phi\right).$$
(2.5)

Next we remove the dimensionality by making the variable substitutions,

$$\Phi = \frac{\Phi_0}{2\pi}\phi\tag{2.6}$$

$$E_C = e^2/2C \tag{2.7}$$

$$E_L = \left(\Phi_0 / 2\pi\right)^2 \left(1 / 2L\right) \phi^2 \tag{2.8}$$

where $\Phi_0 = h/2e$ is the magnetic flux quantum. Note that with these definitions, the resonator frequency can be written as

$$\omega = \frac{4}{\hbar} \sqrt{E_C E_L}.$$
(2.9)

Equation 2.5, normalized to E_L , then becomes

$$\left(-4\frac{E_C}{E_L}\frac{d^2}{d\phi^2} + \phi^2\right)\psi_n\left(\phi\right) = \frac{E_n}{E_L}\psi_n\left(\phi\right).$$
(2.10)

We are now in a position to begin discussing an effective numerical method for dealing with more complicated, particularly non-linear, potential energy functions [34, 35]. The idea is to restrict the variable ϕ to m discreet values, ϕ_i , with a step size given by $\delta \phi = (\phi_{max} - \phi_{min})/(m-1)$. We point out that this method only works for bound states and that the endpoints, ϕ_{max} and ϕ_{min} must be chosen such that the wave functions are very close to zero there. In this case, the potential and wave function become mdimensional column vectors. An approximate expression for the second derivative, called a "3-point" stencil approximation, can be derived using the Taylor series (see appendix),

$$\frac{d^2\psi\left(\phi_i\right)}{d\phi^2} = \frac{\psi\left(\phi_i - \delta\phi\right) - 2\psi\left(\phi_i\right) + \psi\left(\phi_i + \delta\phi\right)}{\left(\delta\phi\right)^2}.$$
(2.11)

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This is a nearest neighbor operation on the column vector and can be written in matrix form as

$$\left[\frac{d^2}{d\phi^2}\right] = \frac{1}{\left(\delta\phi\right)^2} \begin{bmatrix} -2 & 1 & 0 & 0 & \cdots & 0\\ 1 & -2 & 1 & 0 & \cdots & 0\\ 0 & 1 & -2 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & 0 & 1 & -2 \end{bmatrix}.$$
 (2.12)

The TISE can now be written as a matrix eigenvalue problem,

$$\left(-4\frac{E_C}{E_L}\left[\frac{d^2}{d\phi^2}\right] + [V]\right)[\psi] = \frac{E_n}{E_L}[\psi], \qquad (2.13)$$

where [V] is the diagonal potential energy matrix who's elements are $V_{jj} = \phi_j^2$ for the case of the LC resonator. All that is left to do now is explicitly diagonalize equation 2.13, a task computational programs such as Matlab are happy to do for us. Figure 2.2 plots the results for the first seven eigenstates of the LC resonator for $E_L/E_C = 200$, showing at least qualitative agreement with theory. We can compare the simulated eigenvalues with the exact eigenvalues from theory to measure the level of quantitative agreement. The exact eigenvalues are

$$\frac{E_n}{E_L} = 4\sqrt{\frac{E_C}{E_L}} \left(n - 1 + \frac{1}{2}\right),\tag{2.14}$$

where n is a positive integer. We can write our simulated eigenvalues as

$$\frac{E_{n'}}{E_L} = 4\sqrt{\frac{E_C}{E_L}} \left(n' - 1 + \frac{1}{2}\right)$$
(2.15)



Figure 2.2: Numerical evaluation of the first seven states of the simple LC resonator showing qualitative agreement with theory.

and plot the relative error between n' and n as a function of n and m. The reason for shifting the n's by 1 is to eliminate problems associated with dividing by zero when calculating the relative error of the ground state.

Figure 2.3 plots the relative error between simulated and theoretical energy eigenvalues along with a run time analysis. We can see that by doubling m we get basically an order of magnitude improvement in error. However, each time m is doubled, the run time increases by an order of magnitude. Instead of increasing m, we can achieve significant improvement in error by using higher order point-approximations of the derivative. This will come at a significantly reduced run time cost since the size of the matrices remain fixed. For instance the "5-point" and "7-point" approximations (derived in the appendix) are

$$\psi_0'' = \frac{-\psi_{-2} + 16\psi_{-1} - 30\psi_0 + 16\psi_1 - \psi_2}{12\left(\delta\phi\right)^2}$$
(2.16)

$$\psi_0'' = \frac{2\psi_{-3} - 27\psi_{-2} + 270\psi_{-1} - 490\psi_0 + 270\psi_1 - 27\psi_2 + 2\psi_3}{180(\delta\phi)^2}$$
(2.17)

where we have used the notation $\psi_n = \psi (\phi_i + n\delta\phi)$. Figure 2.4 shows the comparison between all three approximations for m = 1600. We can see that the error is improved by almost eight orders of magnitude in



Figure 2.3: a) Percentage error between simulation and theory as a function of n for various step sizes. b) The simulation run time as a function of m.

going from the 3-point to the 7-point approximation with an almost negligible run time increase. Needless to say, the 7-point approximation will be used throughout the rest of this thesis.

Next we show how the ratio E_L/E_C affects the quantum states. Figure 2.5 plots the ground states for increasing ratios of E_L/E_C . When E_L/E_C is smaller the eigenenergies have more of a "charge-like" component and the eigenstates are more spread out in the ϕ representation. When E_L/E_C is large the energy is more "flux-like" with states more compact in ϕ . Recall though that our choice of representation of ψ was arbitrary. We could have just as well have chosen to expand the Hamiltonian into \hat{Q} eigenstates, in



Figure 2.4: a) Error comparison between the 3-point, 5-point, and 7-point approximations of the second derivative for the same step size, m = 1600. b) The corresponding run time analysis.

which case the low E_L/E_C ground states would have been more compact in the charge representation. This is because of the uncertainty relation between $\hat{\Phi}$ and \hat{Q} implied by equation 2.2. Although we used the LC resonator to illustrate the better representation in different limits, the same trend applies to qubits as well, with the Josephson energy, $E_J(0)$, replacing E_L . Qubits with low $E_J(0)/E_C$ ratios are known as "charge" qubits and those with high $E_J(0)/E_C$ ratios are known as "flux" or "phase" qubits.

The ratio $E_J(0)/E_C$ also determines what sources of noise will make the qubit vulnerable to decoherence. Decoherence in qubits is generally broken down into two broad categories: relaxation and dephasing.



Figure 2.5: Ground states of the LC resonator for $E_L/E_C = 10^0, 10^1, 10^2, 10^3, 10^4$.

Relaxation times describe the characteristic time scales over which the qubit, if placed in an excited state will spontaneously jump back down to the ground state due to resonant interaction with the environment. Dephasing times describe the characteristic time scales over which the relative phase information between the ground and excited states is lost due to lower frequency noise from the environment. Charge qubits are extremely sensitive to charge/voltage noise. Conversely, flux and phase qubits are sensitive to flux/current noise [36].

2.2 The Driven LC Resonator

2.2.1 Constructing the Hamiltonian

How do we excite the LC resonator? One way to do it would be to connect the leads to an ideal current source as in Figure 2.6. The current source's effect on the Hamiltonian can be determined by considering the total work done on the capacitor from two components, the current source and the inductor.



Figure 2.6: An LC resonator driven by an ideal current source.

This work can then be attributed to a change in a modified potential energy, one that involves the presence of the current bias. The infinitesimal work done on the capacitor as ϕ evolves from ϕ_0 to $\phi_0 + \delta \phi$ is

$$dW = \frac{dW}{d\phi} d\phi. \tag{2.18}$$

Now use the chain rule to write

$$\frac{dW}{d\phi} = \frac{\frac{dW}{dt}}{\frac{d\phi}{dt}}.$$
(2.19)

But dW/dt is just the power delivered to the capacitor,

$$P_C = V_C I_C \tag{2.20}$$

where I_C and V_C are the current through and voltage across the capacitor respectively. From Kirchov's current law, the current through the capacitor is

$$I_C = I - I_L, \tag{2.21}$$

and the voltage across the capacitor is

$$V_C = LI_L, \tag{2.22}$$

where I_L is the current through the inductor. Plugging these results back into equation 2.19 we get

$$\frac{dW}{d\phi} = \frac{1}{\phi} L \dot{I}_L (I - I_L)$$

$$= \frac{\dot{\Phi}}{\dot{\phi}} (I - I_L)$$

$$= \frac{\Phi_0}{2\pi} (I - I_L)$$

$$= \frac{\Phi_0}{2\pi} I - 2E_L \phi$$

$$= -\frac{\partial U}{\partial \phi}.$$
(2.23)

We can now see that the new potential energy should be written as

$$\hat{U}(\hat{\phi}, t) = E_L \hat{\phi}^2 - \frac{\Phi_0}{2\pi} I(t) \hat{\phi}.$$
(2.24)

The total Hamiltonian now has a time-dependent part resulting from the interaction with the current source,

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} - I(t)\hat{\Phi}.$$
(2.25)

2.2.2 The Classical Driven LC Resonator

Before delving into the solution for a driven quantum simple harmonic oscillator, let's first recall the solution to the driven classical LC resonator. From the Hamiltonian, the equations of motion are

$$\ddot{\Phi} + \omega_0^2 \Phi = \frac{1}{C} I(t) , \qquad (2.26)$$

where $\omega_0 = 1/\sqrt{LC}$. We want the solution for an arbitrary drive. The only requirement is that I(t) be well behaved enough to have a Fourier transform (FT). Applying the FT to the equation 2.26 we get

$$-\omega^2 \Phi[\omega] + \omega_0^2 \Phi[\omega] = \frac{1}{C} I[\omega], \qquad (2.27)$$

where

$$\Phi[\omega] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Phi(t) e^{-i\omega t} dt, \qquad (2.28)$$

and

$$I[\omega] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} I(t)e^{-i\omega t}dt, \qquad (2.29)$$

are the Fourier transforms of $\Phi(t)$ and I(t) respectively. Now equation 2.27 can easily be solved for $\Phi[\omega]$ to get

$$\Phi[\omega] = \frac{I[\omega]}{C \left(\omega_0^2 - \omega^2\right)}.$$
(2.30)

All that remains is to take the inverse FT to find $\Phi(t)$

$$\Phi(t) = \frac{1}{C} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{I[\omega]}{(\omega_0^2 - \omega^2)} e^{i\omega t} d\omega.$$

Inserting equation 2.29 we get

$$\Phi(t) = \frac{1}{C} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} I(t') \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{i\omega(t-t')}}{(\omega_0^2 - \omega^2)} d\omega \right] dt'.$$

The integral in brackets is the Green's function for the driven harmonic oscillator and can be calculated using contour integration (see appendix). The result is

$$G(t,t') = \sqrt{2\pi} \, \frac{\sin\left[\omega_0 \, (t-t')\right]}{\omega_0}.$$
(2.31)

Since G(t, t') = 0 for t < t' we can stop the integral at t,

$$\Phi(t) = \frac{1}{C\omega_0} \int_{-\infty}^t I(t') \sin\left[\omega_0 \left(t - t'\right)\right] dt'.$$
(2.32)

Since $Q(t) = C\dot{\Phi}(t)$ we can immediately write,

$$Q(t) = \int_{-\infty}^{t} I(t') \cos \left[\omega_0 \left(t - t'\right)\right] dt'.$$
 (2.33)

We will compare these results to the results for the driven quantum LC resonator calculated in the next section.

2.2.3 The Quantum Driven LC Resonator

Any discussion of the quantum driven LC resonator must begin with a brief discussion of coherent states. Coherent states are superpositions of energy eigenstates, constructed in such a way that their behavior most closely resembles the behavior of the classical resonator. For example the expectation value of flux and charge for an ensemble of quantum LC resonators in any energy eigenstate is zero. An ensemble of classical LC resonators however can easily be constructed in such a way as to have non-zero flux and charge ensemble averages. A question that arises is, "are there quantum states who's expectation values mimic the trajectories of the classical LC resonators through phase space?". The answer is yes and it turns out that they are eigenstates of the annihilation operator, \hat{a} ,

$$a|\alpha\rangle = \alpha|\alpha\rangle,\tag{2.34}$$

where α is the eigenvalue of the coherent state, $|\alpha\rangle$. All of the following claims not explicitly derived in the text are derived in the appendix. The projection into the number states, $|n\rangle$, is

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$
(2.35)

The expectation values of flux and charge for a coherent state are

$$\langle \alpha | \hat{\Phi} | \alpha \rangle = \tilde{\Phi} \left(\alpha + \alpha^* \right) \tag{2.36}$$

$$\langle \alpha | \hat{Q} | \alpha \rangle = -i \tilde{Q} (\alpha - \alpha^*)$$
 (2.37)

where $\tilde{\Phi} = \sqrt{\hbar/2C\omega}$ and $\tilde{Q} = \sqrt{\hbar\omega C/2}$. The product of the uncertainties in $\hat{\Phi}$ and \hat{Q} is the smallest allowable by the uncertainty principle

$$\langle \alpha | \left(\hat{\Phi} - \langle \Phi \rangle \right)^2 | \alpha \rangle = \tilde{\Phi}^2$$
 (2.38)

$$\langle \alpha | \left(\hat{Q} - \langle Q \rangle \right)^2 | \alpha \rangle = \tilde{Q}^2$$
 (2.39)

$$\tilde{\Phi}^2 \tilde{Q}^2 = \frac{\hbar^2}{4}. \tag{2.40}$$

Since coherent states are not eigenstates of the Hamiltonian, they are not stationary. Remarkably however, they evolve into other coherent states. Specifically the time evolution is

$$\begin{aligned} |\alpha, t\rangle &= e^{-i\frac{H}{\hbar}t} |\alpha\rangle \\ &= e^{-i\frac{H}{\hbar}t} \left(\exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \right) \\ &= \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} e^{-i\frac{H}{\hbar}t} |n\rangle \\ &= \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} e^{-i\omega(n+\frac{1}{2})t} |n\rangle \\ &= e^{-i\frac{\omega t}{2}} \left(\exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{(\alpha e^{-i\omega t})^n}{\sqrt{n!}} |n\rangle \right) \\ &= e^{-i\frac{\omega t}{2}} |\alpha e^{-i\omega t}\rangle \\ &= e^{-i\frac{\omega t}{2}} |\alpha'\rangle. \end{aligned}$$
(2.41)

We see that coherent states evolve into other coherent states who's eigenvalues satisfy

$$\alpha(t) = \alpha(0) e^{-i\omega t}.$$
(2.42)

The time evolution of the flux expectation value is

$$\begin{aligned} \langle \alpha, t | \hat{\Phi} | \alpha, t \rangle &= \tilde{\Phi} \left(\alpha \left(0 \right) e^{-i\omega t} + \alpha^* \left(0 \right) e^{i\omega t} \right) \\ &= \tilde{\Phi} \left(\alpha \left(0 \right) + \alpha^* \left(0 \right) \right) \cos \omega t - i \tilde{\Phi} \left(\alpha \left(0 \right) - \alpha^* \left(0 \right) \right) \sin \omega t \\ &= \langle \Phi \left(0 \right) \rangle \cos \omega t + \frac{1}{\omega C} \langle Q \left(0 \right) \rangle \sin \omega t. \end{aligned}$$

$$(2.43)$$

Similarly the charge expectation is

$$\langle \alpha, t | \hat{Q} | \alpha, t \rangle = \langle Q(0) \rangle \cos \omega t - \omega C \langle \Phi_0(0) \rangle \sin \omega t.$$
(2.44)

Note that these evolutions are precisely the evolutions of $\Phi(t)$ and Q(t) for the classical non-driven resonator.

The operator that generates coherent states from the vacuum state is called the displacement operator,

$$\hat{D}(\alpha) = \exp\left(\alpha a^{\dagger} - \alpha^* a\right).$$
(2.45)

Its effect on the $|0\rangle$ state can be seen by first using the Baker-Hausdorff formula to write [33]

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\exp\left(\alpha a^{\dagger}\right)\exp\left(-\alpha^* a\right)|0\rangle.$$
(2.46)

However, $\exp(-\alpha^* a)|0\rangle = |0\rangle$ since $a|0\rangle = 0$. So we have

$$\begin{aligned} |\alpha\rangle &= \exp\left(-\frac{1}{2}|\alpha|^2\right) \exp\left(\alpha a^{\dagger}\right)|0\rangle \\ &= \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n \left(\alpha^{\dagger}\right)^n}{n!}|0\rangle \\ &= \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n \sqrt{n!}}{n!}|n\rangle \\ &= \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle. \end{aligned}$$
(2.47)

We are now poised to consider the case of the drive quantum LC resonator which, for simplicity we will assume begins in the ground state, $|0\rangle$. The following calculations can be found in reference [37]. Here they are presented with missing steps filled in for completeness. Equation 2.25 in terms of the creation and annihilation operators is

$$\hat{H} = \hbar\omega \left(a^{\dagger}a + \frac{1}{2} \right) - I(t) \tilde{\Phi} \left(a + a^{\dagger} \right).$$
(2.48)

Now make the following definitions and use the interaction picture,

$$\hat{H}_0 = \hbar\omega \left(a^{\dagger}a + \frac{1}{2} \right) \tag{2.49}$$

$$\hat{H}_{I} = -I(t)\tilde{\Phi}\left(a+a^{\dagger}\right) \tag{2.50}$$

$$|\phi(t)\rangle = \exp\left(\frac{i\hat{H}_0 t}{\hbar}\right)|\psi(t)\rangle$$
 (2.51)

where $|\psi(t)\rangle$ is a state that obeys the Schrödinger equation. Taking the time derivative we get

$$\frac{d|\phi\rangle}{dt} = -\exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)\frac{i}{\hbar}\left(\hat{H}_{0}+\hat{H}_{I}\right)|\psi\rangle + \frac{i\hat{H}_{0}}{\hbar}\exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)|\psi\rangle$$

$$= -\exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)\frac{i}{\hbar}\left(\hat{H}_{0}+\hat{H}_{I}\right)\exp\left(-\frac{i\hat{H}_{0}t}{\hbar}\right)|\phi\rangle + \frac{i\hat{H}_{0}}{\hbar}\exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)\exp\left(-\frac{i\hat{H}_{0}t}{\hbar}\right)|\phi\rangle$$

$$= -\frac{i}{\hbar}\exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)\hat{H}_{I}\exp\left(\frac{-i\hat{H}_{0}t}{\hbar}\right)|\phi\rangle$$

$$= \frac{i}{\hbar}I(t)\tilde{\Phi}\exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)(a+a^{\dagger})\exp\left(\frac{-i\hat{H}_{0}t}{\hbar}\right)|\phi\rangle.$$
(2.52)

 But

$$\exp\left(\frac{i\hat{H}_0t}{\hbar}\right)a\exp\left(\frac{-i\hat{H}_0t}{\hbar}\right) = e^{-i\omega t}a$$
(2.53)
and

$$\exp\left(\frac{i\hat{H}_0t}{\hbar}\right)a^{\dagger}\exp\left(\frac{-i\hat{H}_0t}{\hbar}\right) = e^{i\omega t}a^{\dagger}$$
(2.54)

as is shown in the appendix. So we have

$$\frac{d|\phi\rangle}{dt} = \frac{i}{\hbar} I(t) \tilde{\Phi} \left(e^{-i\omega t} a + e^{i\omega t} a^{\dagger} \right) |\phi\rangle.$$
(2.55)

The general solution of this equation is

$$|\phi(t)\rangle = \exp\left[a^{\dagger}e^{-i\omega t}\alpha(t) - ae^{i\omega t}\alpha^{*}(t)\right]|\phi(0)\rangle$$
(2.56)

where

$$\alpha(t) \equiv \frac{i}{\hbar} \tilde{\Phi} \int_0^t I(t') e^{-i\omega(t-t')} dt'.$$
(2.57)

Transforming back to the Schrodinger picture we get

$$|\psi(t)\rangle = \exp\left[a^{\dagger}\alpha(t) - a\alpha^{*}(t)\right] \exp\left[-\frac{i\hat{H}_{0}t}{\hbar}\right] |\psi(0)\rangle.$$
(2.58)

We can see that the additional time-dependence on $|\psi\rangle$ as a result of the drive is a time-dependent displacement operation. Now let the initial state be $|0\rangle$. Then we get

$$\begin{aligned} |\psi(t)\rangle &= \exp\left[a^{\dagger}\alpha(t) - a\alpha^{*}(t)\right] \exp\left[-\frac{i\hat{H}_{0}t}{\hbar}\right]|0\rangle \\ &= e^{-i\frac{\omega}{2}t}|\alpha(t)\rangle \end{aligned}$$
(2.59)

where $\alpha(t)$ is given by equation 2.57. Now let's look at the time dependence of $\langle \alpha | \Phi | \alpha \rangle$ and $\langle \alpha | Q | \alpha \rangle$ to compare to the case of the classical resonator,

$$\begin{split} \langle \Phi(t) \rangle &= \tilde{\Phi} \left(\alpha(t) + \alpha^*(t) \right) \\ &= 2 \frac{\tilde{\Phi}^2}{\hbar} \int_0^t I(t') \sin \left[\omega \left(t - t' \right) \right] dt' \\ &= \frac{1}{C\omega_0} \int_0^t I(t') \sin \left[\omega \left(t - t' \right) \right] dt', \end{split}$$
(2.60)

and similarly

$$\langle Q(t)\rangle = \int_0^t I(t') \cos\left[\omega \left(t - t'\right)\right] dt'.$$
(2.61)

We can see that these expressions are precisely the same expressions for the case of the classical resonator, showing that a driven quantum LC resonator behaves *almost* the same way as a driven classical resonator. In



Figure 2.7: The phase space trajectories of the driven classical and quantum LC resonators.

essence the only difference is the presence of fluctuations in the quantum case. In phase space, the quantum LC resonator traces out "noisy" classical trajectories (Figure 2.7).

While the quantum LC resonator is a great system for studying the physics of coherent states, it cannot be used as a qubit by itself because of the fact that two quantum levels cannot be readily isolated using a direct, "standard" drive of the form $F(t)x_0(a + a^{\dagger})$. Physically, the reason is the equal energy spacing between each level. In order to reliably isolate two levels using a standard drive, a non-linearity needs to be introduced into the Hamiltonian resulting in sufficiently unevenly spaced levels. That is not to say that linear resonators are not useful components in quantum computing. They can be used to directly interact with a qubit in "on-chip" cavity QED experiments similar to those done in the quantum optics community. Their typically longer coherence times also make them good candidates for quantum memories [38]. They have also been used in dispersive readout of qubit states [16, 39]. In the experiments presented here the LC resonator presents a simple, easy to fabricate, quantized level structure for the qubit to exchange energy with for purposes of studying tunable coupling.

Chapter 3

The Phase Qubit

We saw in the last chapter that driven LC resonator states are coherent states. In order to isolate two computational basis states a non-linearity must be introduced to the potential energy. In phase qubits, this non-linearity is provided by a small-area Josephson tunnel junction as shown in figure 3.1. We saw in chapter 1 that a Josephson junction can be treated as a non-linear inductance,

$$L_J(\delta) = \frac{\Phi_0}{2\pi} \frac{1}{I_0 \cos \delta}.$$
(3.1)

The next task is to determine how the presence of this non-linear inductance affects the circuit Hamiltonian. The energy stored in $L_J(\delta)$ is not simply $1/2L_JI_J^2$ because the inductance is a function of the current through it. We can correctly determine the stored energy by going back to the differential limit and integrating. The first thing the reader may notice however is that the generalized coordinate for the LC resonator was the magnetic flux threading the loop of the inductor. The charge on the capacitor plate was the conjugate



Figure 3.1: A phase qubit driven by an ideal current source. The Josephson junction replaces the linear inductor of the LC resonator adding the needed non-linearity.

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variable. A Josephson junction, however, is not a loop. Nevertheless, there is a voltage generated across the junction given by equation 1.6. This can be thought of as generated by a changing magnetic flux, Φ , defined by

$$\dot{\Phi} \equiv \frac{\Phi_0}{2\pi} \dot{\delta}. \tag{3.2}$$

It shouldn't be too hard to see that with this definition δ plays the exact same role as ϕ in the LC resonator. So as to not overcomplicate things by using different symbols for the different generalized coordinates for the phase qubit and LC resonator we shall just use ϕ to represent the guage-invariant phase difference. We just have to remember that the magnetic flux associated with ϕ for the phase qubit is a convenient mathematical construct. Later, when the junction is embedded in a loop, the fluxoid quantization condition introduced in chapter 1 makes this distinction unnecessary. Following the same procedure in section 2.2.1 the infinitesimal work done on the capacitor is

$$\frac{dW}{d\phi} = \frac{V_C I_C}{\dot{\phi}}.$$
(3.3)

This time however, the current through the capacitor is

$$I_C = I - I_J \tag{3.4}$$

where I_J is the junction current given by equation 1.3. The voltage across the capacitor is

$$V_C = \frac{\Phi_0}{2\pi} \dot{\phi}.$$
(3.5)

Inserting these expression back into equation 3.3 we get

$$\frac{dW}{d\phi} = \frac{\Phi_0}{2\pi} (I - I_J)$$

$$= \frac{\Phi_0}{2\pi} (I - I_0 \sin \phi)$$

$$= -\frac{\partial U}{\partial \phi}.$$
(3.6)

The potential energy is then

$$U(\phi) = -E_J \cos \phi - \frac{\Phi_0}{2\pi} I\phi.$$
(3.7)

Note at this point we have written the constant $E_J(0)$ as E_J . This will be the notation used throughout the remainder of this thesis. When we are talking about the full phase-dependent Josephson energy, we will use $E_J(\phi)$.



Figure 3.2: A comparison between energy levels of the phase qubit and LC resonator. a) The potential energies of both the LC resonator and the Phase qubit with $I_{dc} = 0$ showing the non-linearity introduced by the junction. b) The phase qubit potential energy for $I_{dc} = 0$ and $I_{dc} = 0.5I_0$ showing how the levels are tuned with I_{dc} . The energy levels in a) and b) are calculated numerically.

The phase qubit is operated with both dc and ac current biases. The dc current tunes the level spacing between states by "tilting" the well, creating what is known as a "washboard" potential. The ac bias drives transitions between the levels. Figure 3.2 shows plots of the potential energies of both the phase qubit and LC resonator showing the effect of the junction.



Figure 3.3: A voltage source V_s in series with a bias resistor R_b and it's Norton-equivalent circuit.

3.1 The Flux-Biased Phase Qubit

Real current sources, particularly at microwave frequencies, in the lab are far from ideal. For example, the ac current source is actually an ac voltage source with a 50 Ohm source impedance. This impedance is a direct source of energy relaxation in the qubit. Figure 3.3 illustrates a typical current source used in the lab. The Norton equivalent current source model introduces a real parallel admittance that increases the energy dissipation of the qubit. Fortunately, this effect can be mitigated by replacing the direct current bias with a flux bias [40]. A geometric inductance, L, is added in parallel to the Josephson junction and then coupled through a mutual inductance, $M \ll L$, to another inductor, L_b , that terminates the current bias line. The result is that the qubit is effectively "shielded" from R_b by M and L. Figure 3.4 shows a flux-biased phase qubit and its Norton-equivalent circuit (derived in the appendix). The Norton-equivalent circuit has a modified dissipative component given by

$$R' = \left(\frac{L}{M}\right)^2 R_b \tag{3.8}$$



Figure 3.4: The flux-biased phase qubit and it's Norton-equivalent circuit.

which is several orders of magnitude larger than R_b since $M \ll L$. The consequence is that the Norton equivalent current bias gets modified, both in amplitude and phase,

$$I_N = \frac{M}{L} \frac{V_s}{\sqrt{R_b^2 + \omega^2 L_b^2}} e^{i\theta},$$
(3.9)

where θ is a frequency dependent phase shift given by

$$\tan \theta = -\frac{\omega L_b}{R_b}.\tag{3.10}$$

Additionally, the qubit inductance, L, gets modified somewhat by M and L_b ,

$$L' = L\left(1 - \frac{\omega\left(M^2/L\right)\omega L_b}{R_b^2 + (\omega L_b)^2}\right)$$
(3.11)

which is very close to L at all frequencies. We must now account for the geometric inductance L in the qubit Hamiltonian. Following the same procedures in section 1.1.2.1 we have

$$\frac{dW}{d\phi} = \frac{\Phi_0}{2\pi} (I_N - I_J - I_L)$$

$$= \frac{\Phi_0}{2\pi} \left(I_N - I_0 \sin \phi - \frac{\Phi_0}{2\pi L} \phi \right)$$

$$= -\frac{\partial U}{\partial \phi}.$$
(3.12)

where we have used the fluxoid quantization condition introduced in chapter 1,

$$\frac{\phi}{2\pi} = \frac{\Phi}{\Phi_0}$$
$$= \frac{LI_L}{\Phi_0}.$$
(3.13)

The potential energy of the flux-biased phase qubit is then

$$U(\phi) = -E_J \cos \phi + E_L \phi^2 - \frac{\Phi_0}{2\pi} I_N \phi.$$
 (3.14)

Completing the square we cast the potential energy into a more suggestive form,

$$U(\phi) = -E_J \cos \phi + E_L \left(\phi - 2\pi \frac{LI_N}{\Phi_0}\right)^2 + \frac{1}{2}LI_N^2.$$
 (3.15)

From equation 3.9 we can deduce that LI_N is simply the external flux, Φ_x , applied by the current source to L, enabling us to write the potential energy as

$$U(\phi) = -E_J \cos \phi + E_L \left(\phi - 2\pi \frac{\Phi_x}{\Phi_0}\right)^2 + \frac{\Phi_x^2}{2L}.$$
 (3.16)



Figure 3.5: A flux-biased phase qubit with $\Phi_x = 0$ and $\Phi_x = 0.75\Phi_0$ showing the lateral shift in the parabola's origin. The energy levels are not shown for clarity.

We can now see the effect of flux biasing the phase qubit. The potential energy is no longer a cosine on a background slope given by the bias current. As shown in figure 3.5, we now have the combination of a cosine with a parabola who's origin is shifted by the externally applied flux. This type of potential is referred to as a "folded washboard". While the transformation of the current bias into a flux bias primarily helps to decouple the qubit from external dissipation sources, we will see later that the parabolic potential created by the inductor also enables a convenient way to initialize and readout the qubit state. We also point out that the ratio, E_J/E_L is an important parameter in the operation of the flux-biased phase qubit. This ratio determines the number of metastable wells that exist for a given external flux to the qubit. For there to be at least one metastable well, E_J/E_L must be greater than 2. For $E_J/E_L > 9$, multiple metastable wells are present which can complicate the state reset procedure (discussed further in section 3.1.2.1).

3.1.1 Flux-Biased Phase Qubit Operation

The operation of the flux-biased phase qubit is divided into 4 distinct stages: initialization, state preparation, measurement, and readout. The four stages of operation constitute a single cycle. The measurement result of a single cycle is a single 0 or 1. The information stored in the qubit state after a single cycle is completely destroyed. These types of qubit operations are known as Non-QND operations where QND stands for "quantum non-demolition". We can gain little information about the qubit state from a single cycle, so we have to preform many cycles and histogram the results. Of course since any state other than the ground state is completely destroyed after each cycle, we have to be sure to preform the same qubit manipulations each cycle. Thus our statistical "ensemble" is a single qubit prepared the "exact" same way and repeatedly measured over many cycles (typically $\sim 10^3$).

3.1.1.1 Initialization

Initialization is the simplest of all the control operations (due to finite dissipation and thermalization with the environment). The externally applied bias flux is set to 0, centering the parabolic part of the potential energy at the origin. Here there is a single overall stable well at the origin. The goal is to get the qubit into the overall ground state. This happens spontaneously since we are in the $kT \ll \hbar\omega$ limit. The duration of the initialization stage of operation is $\sim 50 \,\mu s$.

3.1.1.2 State Preparation

The adiabatic application of an external flux ranging anywhere from $1/2\Phi_0$ to $\sim \Phi_0$, depending on the E_J/E_L , ratio takes the qubit from initialization to the state-preparation stage. The timescales for this adiabatic flux shift are $\sim 10\mu s \ll 1/2\pi\omega_p$ where,

$$\omega_p = \frac{4}{\hbar} \sqrt{E_c \left(E_L + \frac{1}{2} E_J\right)},\tag{3.17}$$

is the plasma frequency of the global minimum when the qubit is in the initialization configuration. The once global minimum now becomes metastable due to the shifting of the parabola's origin. Since the shift was adiabatic, the state now resides in the lowest level of this well. A stable well is also generated to the right of the metastable well as a result of the shift. However, the energy barrier between the two is high enough that the ground state of the metastable well will remain there for a considerable amount of time. Figure 3.6 illustrates the initialization and state-preparation stages of operation.

As the name suggests, state manipulation occurs in this stage of operation. Flux pulses of a given amplitude and phase, resonant with the energy difference between the two lowest lying levels of the metastable



Figure 3.6: The initialization and state-preparation stages of flux-biased phase qubit operation along with the applied fluxes.

well, drive transitions between these states. It is the non-linearity introduced by the junction that allows us to reliably isolate these two levels from the rest of the Hilbert space. The calculations that follow show explicitly how we are able to isolate these two levels. We write the applied Norton current as

$$I_N = I_{Ndc} + \delta I_N(t) \tag{3.18}$$

where I_{Ndc} is the dc component responsible for maintaining the metastable well, and $\delta I_N(t)$ is the rf part which manipulates the qubit state. The Hamiltonian now has a time-independent and time-dependent part

$$\hat{H(t)} = \hat{H}_0 + \hat{V}(t)$$
 (3.19)

where

$$\hat{V}(t) = -\frac{\Phi_0}{2\pi} \delta I_N(t) \hat{\phi}.$$
(3.20)

The time evolution is determined using the interaction picture,

$$|\psi(t)\rangle_I = \exp\left[\frac{i\hat{H}_0 t}{\hbar}\right]|\psi(t)\rangle_S$$
(3.21)

where $|\psi(t)\rangle_I$ is a state in the interaction picture and $|\psi(t)\rangle_S$ is a state in the Schrodinger picture. Taking the time derivative of equation 3.21 and using the Schrodinger equation we find that $|\psi(t)\rangle_I$ obeys [33],

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle_I = V_I|\psi(t)\rangle_I \tag{3.22}$$



Figure 3.7: The phase qubit during the state-preparation stage of operation, showing the first 40 eigenstates. States with significant amplitude in the right stable well are in green, those in the left metastable well are in black and those in both wells are in red. The lowest two energy states of the metastable well, states $|13\rangle$ and $|15\rangle$, are bold.

where

$$V_{I} = \exp\left[\frac{i\hat{H}_{0}t}{\hbar}\right]\hat{V(t)}\exp\left[-\frac{i\hat{H}_{0}t}{\hbar}\right].$$
(3.23)

Expanding $|\psi(t)\rangle_I$ in the eigenstates of the unperturbed Hamiltonian, \hat{H}_0 , we get a system of coupled differential equations for the level populations, $c_j(t)$ of the unperturbed eigenstate $|j\rangle$,

$$i\hbar \dot{c}_j = \sum_{m=0}^{\infty} \langle j | V_I | m \rangle c_m(t).$$
(3.24)

Equation 3.24 tells us that transitions from the state $|m\rangle$ to $|j\rangle$ can only occur if $\langle j|V_I|m\rangle \neq 0$. Figure 3.7 shows the phase qubit in the state preparation stage of operation, along with the lowest 40 eigenstates. The lowest two states of the metastable well, states $|13\rangle$ and $|15\rangle$, are bold. Since we always start in the lowest metastable state, we only need concern ourselves with other states connected to this state through V_I . We proceed by calculating all of the relevant matrix elements,

$$\langle j|V_{I}|m\rangle = e^{i\omega_{jm}t} \langle j|\hat{V}|m\rangle$$

$$= -\frac{\Phi_{0}}{2\pi} \delta I_{N}(t) e^{i\omega_{jm}t} \langle j|\hat{\phi}|m\rangle$$

$$= -\frac{\Phi_{0}}{2\pi} \delta I_{N}(t) e^{i\omega_{jm}t} \int_{-\infty}^{\infty} \psi_{j}^{*}(\phi) \phi \psi_{m}(\phi) d\phi$$

$$= -\frac{\Phi_{0}}{2\pi} \delta I_{N}(t) e^{i\omega_{jm}t} \phi_{jm}$$

$$(3.25)$$

where we have defined

$$\phi_{jm} \equiv \int_{-\infty}^{\infty} \psi_j^*(\phi) \phi \psi_m(\phi) \, d\phi \tag{3.26}$$

and

$$\omega_{jm} \equiv \frac{E_j - E_m}{\hbar}.\tag{3.27}$$

The integral in equation 3.25 can be calculated using the numerical technique described in Chapter 2. Figure 3.8 is a bar graph, with the baseline set to 10^{-5} , of the matrix elements for the lowest three metastable states with all other states up to $|50\rangle$. We can see that the coupling is only significant between these states and other metastable states. In light of these results we may now truncate the coupled differential equations in equation 3.24 to the lowest three levels of the metastable well which we will now call $|0\rangle$, $|1\rangle$, and $|2\rangle$ getting

$$i\hbar\frac{d}{dt}\begin{bmatrix} c_{0}\\ c_{1}\\ c_{2}\end{bmatrix} = -\frac{\Phi_{0}}{2\pi}\delta I_{N}(t)\begin{bmatrix} \phi_{00} & \phi_{01}e^{i\omega_{01}t} & \phi_{02}e^{i\omega_{02}t}\\ \phi_{01}e^{-i\omega_{01}t} & \phi_{11} & \phi_{12}e^{i\omega_{12}t}\\ \phi_{02}e^{-i\omega_{02}t} & \phi_{12}e^{-i\omega_{12}t} & \phi_{22}\end{bmatrix}\begin{bmatrix} c_{0}\\ c_{1}\\ c_{2}\end{bmatrix}$$
(3.28)

where we have used the fact that $\phi_{nm} = \phi_{mn}$. Remember the goal is to reliably isolate the lowest two transitions, $|0\rangle$ and $|1\rangle$. However from Figure 3.8, the $|1\rangle$ state (state $|15\rangle$, red bars, in the figure) couples just as strongly to the $|2\rangle$ state (state $|17\rangle$, blue bars in the figure) as it does the $|0\rangle$ state (state $|13\rangle$, black bars in the figure). Thanks to the nonlinearity though, $\omega_{12} \neq \omega_{01}$, and any drive tuned to the $|0\rangle \rightarrow |1\rangle$ transition will not appreciably excite the $|2\rangle$ state. As far as the $|0\rangle \rightarrow |2\rangle$ transition is concerned, it is "doubly" suppressed since, from Figure 3.8, $|\phi_{02}| << |\phi_{01}|$ and $\omega_{02} \neq \omega_{01}$.

We now truncate the coupled differential equations even further, leaving just the coupling between



Figure 3.8: The matrix element amplitudes $|\langle j|\phi|m\rangle|$ for the lowest three states of the metastable well, j = 13, 15, and 19, with all other states, m, for m = 1...50 showing that the metastable states only significantly couple to other metastable states.

the $|0\rangle$ and $|1\rangle$ state

$$i\hbar \frac{d}{dt} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = -\frac{\Phi_0}{2\pi} \delta I_N(t) \begin{bmatrix} \phi_{00} & \phi_{01} e^{i\omega_{01}t} \\ \phi_{01} e^{-i\omega_{01}t} & \phi_{11} \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}.$$
 (3.29)

Now let us apply an arbitrary harmonic drive of the form,

$$\delta I_N(t) = A\sin\left(\omega t + \theta\right) + A_Z,\tag{3.30}$$

and write it in terms of its in-phase and out-of-phase components,

$$\delta I_N(t) = A_X \cos \omega t + A_Y \sin \omega t + A_Z, \qquad (3.31)$$

where

$$A_X = A\sin\theta \tag{3.32}$$

$$A_Y = A\cos\theta. \tag{3.33}$$

We also project the right hand side (RHS) of equation 3.29 into the Pauli and identity matrices. The result

is

$$\begin{bmatrix} \phi_{00} & \phi_{01}e^{i\omega_{01}t} \\ \phi_{01}e^{-i\omega_{01}t} & \phi_{11} \end{bmatrix} = \phi_{01}\cos\omega_{10}t\sigma_x + \phi_{01}\sin\omega_{10}t\sigma_y + \frac{(\phi_{00} - \phi_{11})}{2}\sigma_z + \frac{(\phi_{00} + \phi_{11})}{2}I. \quad (3.34)$$

When we combine equations 3.31 and 3.34 with equation 3.29 we get terms that oscillate at frequencies $\omega + \omega_{10}$, $\omega - \omega_{10}$ and stationary terms. When the drive is close to resonance, the stationary and $\omega - \omega_{10}$ terms dominate the dynamics and we can make the rotating wave approximation by neglecting the $\omega + \omega_{10}$ terms. On resonance, we are left with

$$i\hbar\frac{d}{dt}\begin{bmatrix}c_{0}\\c_{1}\end{bmatrix} = -\frac{\Phi_{0}}{2\pi}\frac{1}{2}\left[A_{X}\phi_{01}\sigma_{x} + A_{Y}\phi_{01}\sigma_{y} + A_{z}\left(\phi_{00} - \phi_{11}\right)\sigma_{z} + A_{Z}\left(\phi_{00} + \phi_{11}\right)I\right]\begin{bmatrix}c_{0}\\c_{1}\end{bmatrix}.$$
 (3.35)

Now notice that we can define a vector with units of energy that characterizes the strength of the timedependent perturbation,

$$\vec{E}_P \equiv \frac{\Phi_0}{2\pi} \left(A_X \phi_{01}, A_Y \phi_{01}, A_z \left(\phi_{00} - \phi_{11} \right) \right)$$
(3.36)

and write equation 3.35 as

$$i\hbar \frac{d}{dt} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \frac{|E_P|}{2} \hat{n} \cdot \vec{\sigma} + A_Z \left(\phi_{00} + \phi_{11}\right) I \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$
(3.37)

where

$$\hat{n} = \frac{\vec{E}_P}{|E_P|}.\tag{3.38}$$

Equation 3.37 can be solved exactly. The solution is

$$\begin{bmatrix} c_0(t) \\ c_1(t) \end{bmatrix} = \exp\left[\frac{i}{\hbar} \frac{|E_P|t}{2} \hat{n} \cdot \vec{\sigma}\right] \begin{bmatrix} c_0(0) \\ c_1(0) \end{bmatrix}$$
(3.39)

where we have ignored the global phase factor created by the identity term. We recognize equation 3.39 as Bloch sphere rotations of the state by an angle

$$\theta = -\frac{|E_P|}{\hbar}t\tag{3.40}$$

around the axis defined by \hat{n} [41]. The angular frequency of this rotation,

$$\Omega_P = \frac{|E_P|}{\hbar},\tag{3.41}$$

is controlled by the drive amplitudes $A_{X,Y,Z}$.

More generally, we can determine the off-resonant evolution by working in the frame of the drive field itself. This amounts to applying the following unitary transformation to the Schrodinger-picture state (in the non-driven basis),

$$U_{\omega} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\omega t} \end{bmatrix}$$
(3.42)

where ω is the frequency of the drive. The state in this frame obeys,

$$\frac{d}{dt}|\psi\rangle_{\omega} = \left[-\frac{i}{\hbar}U_{\omega}\left(H_{0}+V\right)U_{\omega}^{\dagger} + \frac{dU_{\omega}}{dt}U_{\omega}^{\dagger}\right]|\psi\rangle_{\omega}$$
(3.43)

This time the "fast" terms to be neglected oscillate at frequencies ω and 2ω . Here the solution is

$$\begin{bmatrix} c_0(t) \\ c_1(t) \end{bmatrix}_{\omega} = \exp\left[\frac{i}{\hbar} \frac{|E_{\Delta}|t}{2} \hat{n} \cdot \vec{\sigma}\right] \begin{bmatrix} c_0(0) \\ c_1(0) \end{bmatrix}_{\omega}, \qquad (3.44)$$

where the new characteristic energy vector, \vec{E}_{Δ} , is

$$\vec{E}_{\Delta} = \vec{E}_P - \hbar \Delta \hat{z}, \qquad (3.45)$$

where

$$\Delta = \omega - \omega_{10} \tag{3.46}$$

is the detuning of the drive field from the transition. Imagine we start in the ground state and apply a drive where $A_Z = 0$. The solution is

$$\begin{bmatrix} c_0(t) \\ c_1(t) \end{bmatrix}_{\omega} = \begin{bmatrix} \cos\left(\frac{|E_{\Delta}|t}{2\hbar}\right) + i\sin\left(\frac{|E_{\Delta}|t}{2\hbar}\right)n_z \\ -\sin\left(\frac{|E_{\Delta}|t}{2\hbar}\right)(n_y - in_x) \end{bmatrix}.$$
(3.47)

The probability, P_1 , of exciting the $|1\rangle$ state as a function of time and detuning, is given by $|c_1(t, \Delta)|^2$ in the Schrodinger picture,

$$P_{1}(t,\Delta) = \frac{1}{2} \left(n_{x}^{2} + n_{y}^{2} \right) \left(1 - \cos \left(\frac{|E_{\Delta}|t}{\hbar} \right) \right)$$
$$= \frac{1}{2} \frac{1}{1 + \left(\frac{\Delta}{\Omega_{P}} \right)^{2}} \left(1 - \cos \Omega_{P} \sqrt{1 + \left(\frac{\Delta}{\Omega_{P}} \right)^{2}} t \right).$$
(3.48)

Figure 3.9 is a plot of equation 3.48. In the limit that $\Delta = 0$ we get the same result in equation 3.39. As



Figure 3.9: Driven Rabi oscillations as a function of time and detuning.

the detuning increases, the oscillation frequency also increases. However, the oscillation amplitude quickly decreases with increased detuning. These are the familiar driven Rabi oscillations.

In practice the qubit drive is a pulse of width Δt implying that the amplitudes A_X , A_Y , and A_Z will depend on time. This means that equation 3.44 is only valid in the small time interval δt over which the amplitudes are approximately constant. The final state after Δt is then given by successive infinitesimal rotations whose direction and rate of rotation change in time. The upshot of all of this is that care must be taken so that the width of the pulse in the frequency domain satisfies the condition

$$\Delta \omega < |\omega_{10} - \omega_{12}| \tag{3.49}$$

so that the Fourier components don't cause unwanted transitions to the $|2\rangle$ state. Pulse shaping is discussed in reference [42] where the authors find that Gaussian-shaped pulses minimize these errors. Intuitively, this result makes sense based on Fourier theory. In the time domain, we would want to use the shortest pulse possible so that operations can be performed before decoherence sets in. However, if the pulse becomes too short in time, it will have Fourier components that overlap nearby transitions causing the state to leak out of the two-state manifold. The pulse envelope that minimizes width in both the frequency and time domain is the Gaussian. In order to minimize leakage out of the two-state manifold, the width of the Gaussian pulse in time should be

$$\Delta t > \frac{1}{2|\omega_{10} - \omega_{12}|}.$$
(3.50)

3.1.1.3 Measurement

Measurement is done by applying an adiabatic (to prevent additional excitations) dc-like pulse to the qubit after the state-manipulation pulses are complete and before the qubit completely relaxes back to the ground state [43, 44]. The amplitude of this "measurement pulse" is carefully chosen such that the tunneling probability from the metastable well to the stable well is drastically different between the $|0\rangle$ and $|1\rangle$ states. Tunneling rates from metastable states has been studied extensively in superconducting circuits and are found to depend strongly on the barrier height separating the wells [8, 45, 10]. The $|1\rangle$ state being higher in energy than the $|0\rangle$ by an amount $\hbar\omega_{01}$ thus sees a smaller barrier. Another way to describe the measurement is that the well is further titled by the measurement pulse to a point where the $|1\rangle$ state wave function has significant amplitude in the stable well while the $|0\rangle$ state amplitude remains comparatively small. It is at this point that the quantum state is demolished, forcing the qubit to "decide" what tunneling rate it will assume. If the qubit chooses the $|1\rangle$ state it will escape from the metastable well with high probability. It is also this tunneling that sets the fundamental limit on our ability to distinguish the $|0\rangle$ state from the $|1\rangle$ state. Unfortunately, there is no measurement pulse amplitude such that the $|1\rangle$ state tunnels with 100% probability while the $|0\rangle$ state tunnels with 0% probability. There is always a compromise that results in errors where a small percentage of the time a $|0\rangle$ is mistaken for a $|1\rangle$ and vice versa, setting our measurement fidelity. The measurement procedure is shown in figure 3.10.

The "best" measurement pulse amplitude can be found by measuring the total tunneling probability of both the $|0\rangle$ and $|1\rangle$ states as a function of pulse amplitude. The chosen amplitude is the one that simultaneously minimizes the tunneling probability of the $|0\rangle$ state, P_{T0} and maximizes P_{T1} . The tunneling rate, Γ_i from the *i*th metastable state has been shown to be of the form

$$\Gamma_i(\Phi_{xMP}) = \frac{\omega_p}{2\pi} \left(\frac{b_i(\Phi_{xMP})}{2\pi}\right)^{1/2} \exp\left[-b_i(\Phi_x)\right]$$
(3.51)



Figure 3.10: The transition between the state-preparation and the measurement stages of operation. The measurement pulse adiabatically tilts the well to a point where the probability of a tunneling event from the $|1\rangle$ state is much greater than from the $|0\rangle$ state. The blue curve is the qubit well while still in the operation configuration. The black curve is the well during application of the measurement pulse.

where Φ_{xMP} is the measurement pulse amplitude, ω_p is the plasma frequency of the well and

$$b_i = \frac{\alpha \Delta U_i(\Phi_x)}{\hbar \omega_p} + \frac{A \left(\Delta \phi\right)^2}{\hbar R}$$
(3.52)

where $\alpha \sim 7.2$ is a geometric factor assuming a cubic approximation of the metastable well [8], and $\Delta U_i(\Phi_x)$ is the energy barrier seen by the *ith* metastable state, which obviously depends on Φ_{xMP} . The second term in b_i is the lowest order correction due to dissipation described by a parallel resistance R where A is as numerical factor of order unity and $\Delta \phi$ is the distance under the barrier, essentially the "length" of the classically forbidden region. For this qualitative discussion, we will neglect dissipation and calculate P_{T0} and P_{T1} as a function of Φ_{xMP} . Assuming the accumulated tunneling probability from the ramping up of Φ_{xMP} is negligible and that the pulse amplitude is constant in time, we divide the total time interval into N equal slices of time $\Delta t = T/N$ where the tunneling rate is constant in each interval. The total probability of the *ith* state *not* tunneling, P_{NTi} , after a time T, is

$$P_{NTi}(\Phi_{xMP}) = \lim_{N \to \infty} \left(1 - \Gamma_i(\Phi_{xMP}) \frac{T}{N} \right)^N$$
$$= \exp\left[-\Gamma_i(\Phi_{xMP}) T \right].$$
(3.53)



Figure 3.11: The simulated total tunneling probability of the $|0\rangle$ and $|1\rangle$ states as a function of the measurement pulse amplitude, Φ_{xMP} . The fidelity is maximized when the difference $P_{T1} - P_{T0}$ is maximized.

Thus we have

$$P_{Ti}\left(\Phi_{xMP}\right) = 1 - \exp\left[-\Gamma_{i}\left(\Phi_{xMP}\right)T\right]$$

$$(3.54)$$

where T is the total width of the measurement pulse in time.

Figure 3.11 is a plot of P_{T0} and P_{T1} as a function of Φ_{xMP} showing that there is no measurement pulse amplitude that perfectly distinguishes the two states. To see how this affects the overall measurement fidelity, imagine that we have a normalized superposition state,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{3.55}$$

and we want to measure the probability of finding the system in the $|1\rangle$ state. In other words, we want to measure $|\alpha|^2$. What we actually measure is the probability of a tunneling event from the metastable well which is given by

$$P_T = |\alpha|^2 \times P_{T1} + |\beta|^2 \times P_{T0}.$$
(3.56)

But $|\psi\rangle$ is a normalized state so

$$|\beta|^2 = 1 - |\alpha|^2. \tag{3.57}$$



Figure 3.12: The measured total tunneling probability of the $|0\rangle$ and $|1\rangle$ states as a function of the measurement pulse amplitude, Φ_{xMP} . Here the fidelity is limited by the decoherence mechanisms described in [46, 47].

Thus we have,

$$P_T = (P_{T1} - P_{T0})|\alpha|^2 + P_{T0}, \qquad (3.58)$$

which shows why $P_{T1} - P_{T0}$ should be maximized.

The simulation showed that the maximum theoretical fidelity is ~ 98%. In practice this fidelity is very hard to achieve due to pulse imperfections and sources of decoherence [46, 47, 43]. Figure 3.12 is a plot showing typical fidelities achievable are on the order of 70%.

3.1.1.4 Readout

Readout is the act of determining whether or not a tunneling event occurred. The readout device is a dc-SQUID inductively coupled to the qubit's geometric inductance through a mutual inductance $M_{SQ/Qb}$. Tunneling events can be determined with nearly 100% certainty so long as the dc-SQUID parameters are properly chosen. After the measurement pulse is turned off, the external qubit flux is adiabatically ramped back down from the state preparation level to the readout level at $\Phi_x = 1/2\Phi_0$ creating a symmetric double well potential shown in figure 3.13. If there was a tunneling event, the qubit will have decayed to the ground



Figure 3.13: The readout stage of operation. The metastable well is lowered by ramping the applied flux down to $\Phi_x = 1/2\Phi_0$ creating a symmetric double well. The barrier is high enough that no tunneling occurs on the timescales over which the dc-SQUID critical current is measured.

state of the right well, called $|R\rangle$. If not, it will be found in the ground state of the left well, $|L\rangle$. The astute reader may recall that the states $|L\rangle$ and $|R\rangle$ are not eigenstates of the symmetric double well potential and should thus mix with one another via tunneling through the barrier separating them [33]. For our circuit parameters however, the barrier is high enough that the tunneling rate between the two wells is entirely negligible, making $|L\rangle$ and $|R\rangle$ degenerate ground states. This means that the time to make the readout can be much longer than the characteristic lifetime of the qubit states.

For typical circuit parameters, the average phase difference between the $|L\rangle$ and $|R\rangle$ states is

$$\Delta \langle \phi \rangle \sim 2\pi. \tag{3.59}$$

Through the fluxoid quantization expression, this corresponds to a difference in loop current,

$$\Delta \langle I \rangle \sim \frac{\Phi_0}{L},\tag{3.60}$$

resulting in a flux change seen by the dc-SQUID,

$$\Delta \langle \Phi_{SQUID} \rangle \sim \frac{M_{SQ/Qb}}{L} \Phi_0. \tag{3.61}$$



Figure 3.14: a). Simulated mod-curve for an asymmetric dc-SQUID. b) Histogram of measured dc-SQUID critical currents showing the resolution of $|L\rangle$ and $|R\rangle$.

For decent resolution we keep $\Delta \langle \Phi_{SQUID} \rangle \sim 0.1 \Phi_0$ (Figure 3.14).

Once we are at the readout applied flux, the critical current of the dc-SQUID is measured by applying a bias current, I_{SQ} , and monitoring the voltage, V_{SQ} , across the leads (Figure 3.15). The parameters are chosen such that the ratio of the total loop inductance L_{SQ} to the total Josephson inductance of the two smaller SQUID junctions is less than 1. This keeps the total SQUID critical current single-valued. The third junction of critical current αI_0 is present to create an asymmetry between the two branches so that there



Figure 3.15: Schematic showing the readout dc-SQUID inductively coupled to the qubit.

will be flux sensitivity at zero flux bias applied to the SQUID [48, 40]. Figure 3.16 summarizes the entire operation cycle of the qubit.

3.1.2 Putting It All Together: Phase Qubit Characterization

3.1.2.1 Steps Measurements

Steps measurements are the first measurements made to characterize the phase qubit. We get several pieces of information from steps measurements. First and foremost we learn if our qubit and readout SQUID junctions survived the cool down process. Secondly we learn the shape of the qubit's potential energy landscape. More specifically, we learn the ratio E_J/E_L . Finally, and most importantly, we learn what voltage settings, or equivalently applied fluxes, to use for the initialization, state preparation and readout. Implied is also the knowledge of what change in voltage, ΔV , corresponds to a flux quantum change in applied flux to the qubit. From ΔV and the known bias line resistances, we can determine the mutual inductances between the qubit and bias line, as well as the qubit and readout SQUID. A complete steps data set is composed of two different measurements, a "forward" and a "reverse" measurement. In the forward measurement, two dc-bias voltages are applied in sequence. The first voltage, $V_1 = 0$, is applied for $\sim 50 \,\mu s$. Ideally, this



Figure 3.16: Summary of a single phase qubit operation cycle.

would mean no applied flux to the qubit and the potential would be in the initialization configuration. Then the voltages switches to V_2 which is swept from $0 \to +5V$. This voltage "tilts" the qubit potential energy, creating the metastable well. The duration of V_2 is the same as V_1 . However, during the application of V_2 , the critical current of the readout SQUID is measured. When the forward V_2 reaches a critical voltage, V_{Fc} , corresponding to a critical qubit applied flux, Φ_{Fc} , the $|0\rangle$ state tunnels to the right stable well resulting in an abrupt shift, or a "step", in the external flux to the SQUID. This of course manifests in an abrupt step in SQUID critical current. Statistics are generated by repeating this measurement over ~ 1000 cycles. The reverse measurement is similar except V_2 is swept from $+5 \to 0 V$. Again, V_2 will reach a critical value, V_{Rc} where tunneling occurs, stepping the SQUID critical current. However, $V_{Rc} \neq V_{Fc}$. It is the extent of this hysteretic behavior that reveals E_J/E_L .

Specifically we want to measure Φ_{Fc} and Φ_{Rc} for the *same* well. Then the amount of flux we have to apply in either direction to induce tunneling of the $|0\rangle$ state, when starting with a perfectly zeroed potential energy is given by

$$\Phi_c = \frac{1}{2} \left(\Phi_{Fc} - \Phi_{Rc} \right).$$
(3.62)



Figure 3.17: Combined steps.

In order to infer Φ_c from V_{Fc} and V_{Rc} we must know what ΔV corresponds to a flux quantum. This is simply the voltage difference between consecutive step edges in either the forward or reverse directions. This can be inferred from the periodic nature of the qubit's potential energy with applied flux. Specifically,

$$U(\phi, \Phi_x + \Phi_0) = U(\phi + 2\pi, \Phi_x).$$
(3.63)

Thus, once we know ΔV , we have

$$\Phi_c = \frac{1}{2} \frac{\Phi_0}{\Delta V} \left(V_{Fc} - V_{Rc} \right).$$
(3.64)

Now that we have Φ_c , we can determine E_J/E_L . Classically, the state escapes when there is a saddle point in the metastable well. Of course because of the zero point energy and quantum tunneling, the state escapes before the saddle point is reached but the classical calculation gives a decent enough approximation of E_J/E_L . A saddle point occurs at ϕ^* 's where both the first and second derivatives of the potential energy vanish. So we have

$$\frac{\partial U}{\partial \phi} = 2E_L \left(\phi^* + 2\pi \frac{\Phi_c}{\Phi_0} \right) + E_J \sin \phi^*$$
(3.65)

$$\frac{\partial^2 U}{\partial \phi^2} = 2E_L + E_J \cos \phi^*$$

$$= 0.$$
(3.66)

From the second derivative, we can solve for ϕ^* ,

$$\phi^* = \arccos\left(-\frac{2E_L}{E_J}\right). \tag{3.67}$$

We now plug this into the first derivative, along with our measurement of Φ_c and solve the resulting transcendental relation for E_J/E_L . For there to be at least one metastable well to operate the qubit with, E_J/E_L must be greater than 2. When $E_J/E_L \simeq 9$ multiple metastable wells begin to appear. If E_J/E_L gets much larger than 9, the qubit state can become trapped in undesired wells requiring a more complex initialization scheme to properly reset the qubit state. The device shown in figure 3.17 had $E_J/E_L \simeq 10$, but was not hindered by undesirable trapping.

= 0

Next we use the steps data to determine the initialization, state preparation and readout voltages. The state preparation voltage will be near a step edge. The readout voltage is where the potential energy is a symmetric double well. From the steps data this is half way between the step center-point and step edge. As for the initialization voltage, we are free to chose any voltage that takes us off the step we are operating on from the opposite side of our chosen step edge.

3.1.2.2 S-Curve Measurement

The next measurement is known as an "S-Curve". Essentially this measurement is used to refine the location of our step edge by measuring the tunneling probability of the $|0\rangle$ state as a function of the state preparation flux (called V_2 in the steps measurement). The main difference between this measurement and the steps measurement is the that the dc-SQUID I/V measurement is made at the fixed readout voltage level (as opposed to the V_2 level) which we obtain from the steps data. Knowledge of the step edge is



Figure 3.18: S-Curve data-a) Tunneling probability of the $|0\rangle$ state as a function of the state preparation voltage. The refined step edge is located where the tunneling probability is 50%. b) The applied bias train. The initialization and readout voltages are fixed. The state preparation voltage is swept until the well is too shallow to hold the $|0\rangle$ state.

crucial because it marks an upper limit in the state preparation level. In other words beyond this limit, the metastable well becomes too shallow to hold even the $|0\rangle$ state. Figure 3.18 is a plot of S-Curve data along with the applied bias pulse-train used.

3.1.2.3 Measurement Pulse Calibration

We don't want the state preparation flux to induce tunneling. Its job is to simply maintain the well shape or depth. Tunneling should be induced solely by a separate "measurement pulse". Thus, once we have our refined step edge via the S-Curve measurement, we again measure P_0 , but this time as a function of measurement pulse amplitude and state preparation voltage. This data is called a measurement pulse calibration. Recall from section 3.1.1.3 that to choose the ideal measurement pulse amplitude for a given state preparation flux we would also need to measure $P_{|1\rangle}$ as well but at this point in the experiment we have no way of exciting this transition since we have not measured ω_{10} . These quantities are all engineered and will vary from sample to sample. We simply choose a measurement pulse amplitude that induces tunneling from the $|0\rangle$ state approximately 10% of the time. While it is highly unlikely that this choice of amplitude will maximize fidelity, we know from theoretical considerations that it will give us enough sensitivity to reasonably distinguish the two states. The width of the measurement pulse in time needs to be long enough that when the qubit tunnels, it has time to decay far enough into the stable well that it doesn't get retrapped in the metastable well when the measurement pulse is turned off [46]. Typically a pulse width of ~ 100 ns is sufficient. The measurement pulse calibration, shown in figure 3.19, reveals an essentially linear relationship between state preparation voltage, or well depth, and the required measurement pulse amplitude to maintain a consistent tunneling rate. This calibration allows us to probe the qubit over a range of transition frequencies since energy level spacing depends on well depth.

3.1.2.4 Spectroscopy

At this point we are poised to begin probing the resonant nature of the qubit. As mentioned before, all of the qubit parameters are engineered quantities and can vary significantly from sample to sample. Thus $\omega_{10}(\Phi_{xqb})$ must be measured over the range of state preparation voltages (which we now refer to simply as "applied qubit flux", Φ_{xqb} ,) we used to calibrate the measurement pulse. We expect ω_{10} to vary quite a bit throughout this range. When the applied qubit flux is far from the step edge, the metastable well is deeper and more linear resulting in a larger ω_{10} . As we approach the step edge, the well is shallower and more nonlinear, leading to a smaller ω_{10} . This behavior is captured in a spectroscopy measurement. We apply a microwave drive to the qubit and measure the tunneling probability as a function of drive frequency and Φ_{xqb} . Since we calibrated the measurement pulse amplitude to cause the $|0\rangle$ state to tunnel 10% of the time, when the drive tone is resonant with the $|0\rangle \rightarrow |1\rangle$ transition the tunneling probability is enhanced due to population of the $|1\rangle$ state. The amplitude and length of the microwave pulse need to be chosen such that the resulting peak is not Fourier broadened. This means drive times much longer than the typical lifetime



Figure 3.19: Measurement pulse calibration-a) P_0 is measured as a function of both state preparation voltage and measurement pulse amplitude. b) The applied flux train showing the relative position of the measurement pulse. Its width however is exaggerated for clarity.

of the $|1\rangle$ state. The best drive amplitude, or equivalently drive power, is determined experimentally. We choose a drive power such that the peak height is ~ 10% above the background level. Note also that any occupied state higher than the $|1\rangle$ state will also tunnel. Depending on the non-linearity, there can be other transitions close enough in frequency to ω_{10} that they may also be excited. For example, the ω_{12} transition is close by, particularly for deeper wells. If the drive power is too high or its width is too narrow in time, the tone can be broadened to the point where its Fourier components overlap both ω_{10} and ω_{12} resulting in population of the $|2\rangle$ state. Another transition that is close by is the $|0\rangle \rightarrow |2\rangle$ two-photon transition which

is allowed due to the qubit non-linearity. Recall from figure 3.8 that $\langle 2|V|0\rangle$ is small but not insignificant. These transitions are easy to detect in spectroscopy because they occur at the geometric mean of ω_{10} and ω_{12} since

$$\frac{\omega_{02}}{2} = \frac{\omega_{10} + \omega_{12}}{2}.$$
(3.68)

Figure 3.20 shows spectroscopy data at high enough power to excite not only the $|0\rangle \rightarrow |1\rangle$ but the $|1\rangle \rightarrow |2\rangle$ and $|0\rangle \rightarrow |2\rangle$ as well. The transitions are most separated towards the right side of the plot where the well is the shallowest and most non-linear. As $\Phi_{x\,qb}$ decreases, the well becomes more harmonic, merging the spectral lines. This dominant peak is normalized to unity in each trace for clarity.

Spectroscopy measurements are not just useful for determining ω_{10} . They also reveal what other systems are strongly coupled to the qubit. When the qubit comes into resonance with another system, an avoided crossing or "splitting" occurs in the spectroscopy who's size is proportional to the coupling strength between the two. One major source of decoherence in phase qubits is spurious two-level system (TLS) fluctuators that reside in the junction tunnel barrier [49, 40, 50, 51, 52]. The locations of the more strongly coupled TLSs can be determined with a spectroscopy measurement. Typically these regions will be avoided as much as possible.

3.1.2.5 Driven Rabi Oscillations

Driven Rabi oscillations are sort of the dual to the spectroscopy measurement. Spectroscopies are taken with pulses that are long compared to the qubit lifetime and at relatively low power. In contrast, driven Rabi data is taken using shorter drive pulses and at higher powers. Using the spectroscopy data we choose a $\Phi_{x\,qb}$ in a relatively clean region free of any splittings. On one axis, the width of the drive pulse is swept from $0 \rightarrow \sim T_1$ where T_1 is the lifetime of the $|1\rangle$ state. On the other axis, the drive frequency is swept. This measurement gives us a first glance at coherent dynamics in the time domain. When the drive frequency is resonant with ω_{10} the state "Rabi flops" between the $|0\rangle$ and $|1\rangle$ state coherently. Of course this doesn't happen forever. Eventually decoherence sets in causing the state to decay to an equal classical distribution of $|0\rangle$ and $|1\rangle$. Figure 3.21 shows a typical driven Rabi oscillation. We see the characteristic



Figure 3.20: Spectroscopy-a) High power spectroscopy showing multiple transitions described in the text. b) A single line cut showing the transitions more clearly.



Figure 3.21: Control qubit driven Rabi.

chevron structure who's peak is centered at ω_{10} (recall figure 3.9 in section 3.1.1.2). In addition, as in the high-power spectroscopy, we can see the $|1\rangle \rightarrow |2\rangle$ transition and $|0\rangle \rightarrow |2\rangle$ two-photon transition. Driven Rabis are useful for refining our measurement of ω_{10} that started with the spectroscopy. Beyond that, we learn what pulse lengths and amplitudes are best for generating arbitrary superpositions of $|0\rangle$ and $|1\rangle$. One pulse that is particularly useful is the " π -pulse" which fully populates the $|1\rangle$ state. From figure 3.21 we can see that at this particular drive power, a pi-pulse is ~ 10 ns.

3.1.2.6 T1 Measurements

The next measurement is the lifetime of the $|1\rangle$ state, known as a "T1" measurement. Knowledge of this parameter is important as it sets the limit on how strongly the qubit must be coupled to other systems (discussed in the following chapters) to observe coherent dynamics. Coherence times in superconducting qubits have traditionally lagged behind other better isolated systems such as trapped ions for example. But what they lack in coherence times, they make up for in the ability to couple strongly to other systems since coupling strengths are engineered. To measure T1 we simply apply a π -pulse, then sweep the delay time



Figure 3.22: T1 measurement-a) The $|1\rangle$ population decreases exponentially in time as the measurement pulse delay is swept. b) The pulse scheme. The qubit is pi-pulsed then the delay between the pi-pulse and measurement pulse is swept.

between the end of the π -pulse and the onset of the measurement pulse. When the delay time is short, a large fraction of the ensemble remains in the excited state. As the delay increases a larger and larger fraction has relaxed back to the ground state from interactions with the environment. The result is a characteristic exponential decay in P_1 . The decay rate is T1. Figure 3.22 is a T1 measurement of a typical phase qubit in our lab. The lifetime is ~ 152 ns.

Chapter 4

Fixed Coupling Between a Phase Qubit and LC Resonator

A fully-functional quantum computer will be composed of many qubits and resonators (acting as transmission and memory components) that exchange information with one another through coupled interactions. Inter-element coupling is implemented by electrostatic interactions via coupling capacitors or by magnetic interactions via coupled inductors. In traditional experiments studying these coupled interactions, whether electrostatic or magnetic in nature, the coupling strengths have been fixed by the coupling elements [15, 16, 17, 18, 19, 20, 21, 22, 23]. In this chapter we discuss fixed-strength inductive coupling between a phase qubit and lumped element LC resonator. Then we point out a problem with the fixed coupling paradigm that could potentially make operation of a large-scale quantum processor difficult.

We begin by constructing the Hamiltonian for this circuit, keeping in mind that the phase qubit is still coupled to its control and readout circuitry. We proceed as before but now must find the work done on both capacitors, the qubit's and the resonator's. The coordinates describing each system are ϕ_q , the phase across the qubit junction, and ϕ_r , the dimensionless flux unit defined in chapter 2 for an LC resonator. The work done on the qubit capacitor is

$$dW_q = P_q \, dt \tag{4.1}$$

where

$$P_q = I_{Cq} V_{Cq}$$

= $(-I_J - I_{Lq}) \left(\frac{\Phi_0}{2\pi}\right) \dot{\phi}_q.$ (4.2)



Figure 4.1: Phase Qubit coupled to a lumped element resonator through a mutual inductance, M. For simplicity, the qubit control and readout circuitry is not shown.

For the resonator capacitor we get a similar expression,

$$dW_r = P_r \, dt' \tag{4.3}$$

where

$$P_r = I_{Cr} V_{Cr}$$
$$= -I_{Lr} \left(\frac{\Phi_0}{2\pi}\right) \dot{\phi}_r.$$
(4.4)

•

The total work is then

$$dW_T = dW_q + dW_r$$

= $(P_q + P_r) dt$
= $\left(\frac{\Phi_0}{2\pi}\right) (-I_J d\phi_q - (I_{Lq} d\phi_q + I_{Lr} d\phi_r))$
= $-E_J \sin \phi_q d\phi_q - \left(\frac{\Phi_0}{2\pi}\right) (I_{Lq} d\phi_q + I_{Lr} d\phi_r)$

The junction phase is again related to the flux in the loop inductance via the fluxoid quantization relation. This time however, there is a contribution to the external loop flux from the current in the resonator's inductance. So we have

$$\left(\frac{\Phi_0}{2\pi}\right)(\phi_q + \phi_{x\,q}) = \left(L_q I_{Lq} + M I_{Lr}\right) \tag{4.5}$$

$$\left(\frac{\Phi_0}{2\pi}\right)\phi_r = \left(L_r I_{Lr} + M I_{Lq}\right). \tag{4.6}$$

We can use matrix algebra to combine these two expressions by writing

$$\begin{pmatrix} \Phi_0 \\ 2\pi \end{pmatrix} \begin{bmatrix} \phi_q + \phi_{xq} \\ \phi_r \end{bmatrix} = \begin{bmatrix} L_q & M \\ M & L_r \end{bmatrix} \begin{bmatrix} I_{Lq} \\ I_{Lr} \end{bmatrix}.$$

$$(4.7)$$

Thus the work can be written as

$$dW_T = -E_J \sin \phi_q d\phi_q - \left(\frac{\Phi_0}{2\pi}\right) \left[I\right]^T \left[d\phi\right]$$

= $-E_J \sin \phi_q d\phi_q - \left(\frac{\Phi_0}{2\pi}\right)^2 \left[\phi\right]^T \left[L\right]^{-1} \left[d\phi\right],$ (4.8)

where

$$[L] \equiv \begin{bmatrix} L_q & M \\ M & L_r \end{bmatrix}.$$
(4.9)

But since $L^{-1T} = L^{-1}$ we can write

$$[\phi]^{T}[L]^{-1}[d\phi] = \frac{1}{2}d\left([\phi]^{T}[L]^{-1}[\phi]\right).$$
(4.10)

Thus the second term is the total differential of a purely inductive potential energy. The total potential energy is then

$$U(\phi_q, \phi_r) = -E_J \cos \phi_q + \left[\phi\right]^T \left[E_L\right] \left[\phi\right]$$
(4.11)

where $[E_L]$ is the characteristic inductive energy matrix defined by

$$\begin{bmatrix} E_L \end{bmatrix} = \frac{1}{2} \left(\frac{\Phi_0}{2\pi} \right)^2 [L]^{-1} \\ = \frac{1}{2} \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{1}{(1 - k_{qr}^2)} \begin{bmatrix} \frac{1}{L_q} & -\frac{M}{L_q L_r} \\ -\frac{M}{L_q L_r} & \frac{1}{L_r} \end{bmatrix} \\ = \begin{bmatrix} E_{Lq} & -E_M \\ -E_M & E_{Lr} \end{bmatrix},$$
(4.12)
where

$$k_{qr} \equiv \frac{M}{\sqrt{L_q L_r}}.$$
(4.13)

We point out that the separate characteristic inductive energies of the qubit and resonator have been slightly increased by the factor $1/(1 - k_{qr}^2)$ which will result in a slight increase in their natural resonant frequencies due to inductive "loading" by M. This will be important later when we implement tunable coupling between the qubit and resonator and observe modulation of the resonator's resonant frequency with coupling strength. Writing the potential energy out explicitly and neglecting meaningless constants we have

$$U(\phi_q, \phi_r) = -E_J \cos \phi_q + E_{Lq} (\phi_q + \phi_{xq})^2 + E_{Lr} \left(\phi_r - \frac{E_M}{E_{Lr}} \phi_{xq} \right)^2 - 2E_M \phi_q \phi_r.$$
(4.14)

We can see that the first line is just the uncoupled qubit potential energy, the second line is the uncoupled resonator potential energy, and the third line is the interaction term. The offset in the resonator's energy can be ignored since the resonator isn't sensitive to dc flux offsets. We point out that ϕ_{xq} is time-dependent when the qubit is being driven. However, for coupled experiments the drive is only turned on when the qubit and resonator are sufficiently detuned.

The Hamiltonian for the coupled system is then

$$\hat{H} = \hat{H}_q \otimes I_r + I_q \otimes \hat{H}_r + \hat{H}_I \tag{4.15}$$

where \hat{H}_q and \hat{H}_r are the uncoupled qubit and resonator Hamiltonians respectively. The interaction part of the Hamiltonian is

$$\hat{H}_I = -2E_M \hat{\phi}_q \otimes \hat{\phi}_r. \tag{4.16}$$

We now expand the qubit part of the Hamiltonian in terms of the lowest two qubit eigenstates of the

metastable well and write the resonator Hamiltonian in terms of a and a^{\dagger}

$$\hat{H} = \begin{bmatrix} \hbar\omega_0 & 0\\ 0 & \hbar\omega_1 \end{bmatrix} \otimes I_r + I_q \otimes \hbar\omega_r \left(a^{\dagger}a + \frac{1}{2}\right)$$
$$- 2E_M \begin{bmatrix} \phi_{00} & \phi_{01}\\ \phi_{10} & \phi_{11} \end{bmatrix} \otimes \left(\frac{E_{Cr}}{E_{Lr}}\right)^{1/4} (a + a^{\dagger})$$
(4.17)

where

$$\phi_{nm} = \langle n | \hat{\phi} | m \rangle \tag{4.18}$$

are the matrix elements calculated in Chapter 3. The diagonal part of the qubit Hamiltonian can be written as a linear combination of σ_z and the identity

$$\begin{bmatrix} \hbar\omega_0 & 0\\ 0 & \hbar\omega_1 \end{bmatrix} = -\frac{\hbar}{2}\omega_{10}\sigma_z + \frac{\hbar}{2}(\omega_0 + \omega_1)I.$$
(4.19)

The geometric mean of the two qubit levels can be subtracted from the Hamiltonian, leaving only the σ_z part. Similarly, the interaction part of the qubit Hamiltonian can be written as

$$\begin{bmatrix} \phi_{00} & \phi_{01} \\ \phi_{10} & \phi_{11} \end{bmatrix} = \phi_{01}\sigma_x + \frac{1}{2}(\phi_{00} - \phi_{11})\sigma_z + \frac{1}{2}(\phi_{00} + \phi_{11})I$$

$$(4.20)$$

where we have used the fact that $\phi_{01} = \phi_{10}$.

We now use the interaction picture by applying the following unitary transformation,

$$\hat{U} = e^{\frac{i}{\hbar}\hat{H}_q t} \otimes e^{\frac{i}{\hbar}\hat{H}_r t}.$$
(4.21)

The interaction Hamiltonian in this picture is proportional to (temporarily omitting the overall constant, $2E_M$),

$$\hat{H}_{I} \sim \hat{U}\left(\phi_{01}\sigma_{x} + \frac{1}{2}\left(\phi_{00} - \phi_{11}\right)\sigma_{z} + \frac{1}{2}\left(\phi_{00} + \phi_{11}\right)I\right) \otimes \left(a + a^{\dagger}\right)U^{\dagger}.$$
(4.22)

Addressing the qubit part first we have

$$\hat{H}_{Iq} \sim e^{-i\frac{\omega_{10}}{2}\sigma_z t} \left(\phi_{01} \left(\sigma^+ + \sigma^- \right) + \frac{1}{2} \left(\phi_{00} - \phi_{11} \right) \sigma_z + \frac{1}{2} \left(\phi_{00} + \phi_{11} \right) I \right) e^{i\frac{\omega_{10}}{2}\sigma_z t}, \tag{4.23}$$

where we have expressed σ_x in terms of $\sigma^{\pm} \equiv 1/2 (\sigma_x \pm i\sigma_y)$. The first term is

$$e^{-i\frac{\omega_{10}}{2}\sigma_z t}\sigma_+ e^{i\frac{\omega_{10}}{2}\sigma_z t} = \left[\cos\frac{\omega_{10}}{2}tI - i\sin\frac{\omega_{10}}{2}t\sigma_z\right]\sigma_+ \left[\cos\frac{\omega_{10}}{2}tI + i\sin\frac{\omega_{10}}{2}t\sigma_z\right]$$
$$= e^{-i\omega_{10}t}\sigma^+$$
(4.24)

since $\sigma_z \sigma^+ = \sigma^+$ and $\sigma^+ \sigma_z = -\sigma^+$. Following the same procedure, the second term is

$$e^{-i\frac{\omega_{10}}{2}\sigma_z t}\sigma^- e^{i\frac{\omega_{10}}{2}\sigma_z t} = e^{i\omega_{10}t}\sigma^-.$$
(4.25)

The third term is trivial since σ_z commutes with the exponential

$$e^{-i\frac{\omega_{10}}{2}\sigma_z t}\sigma_z e^{i\frac{\omega_{10}}{2}\sigma_z t} = \sigma_z.$$

$$\tag{4.26}$$

Finally, the last term is

$$e^{-i\frac{\omega_{10}}{2}\sigma_z t} I e^{i\frac{\omega_{10}}{2}\sigma_z t} = I.$$
(4.27)

Combing these results, the qubit part becomes

$$\hat{H}_{Iq} \sim \phi_{01} \left(e^{-i\omega_{10}t} \sigma^+ + e^{i\omega_{10}t} \sigma^- \right) + \frac{1}{2} \left(\phi_{00} - \phi_{11} \right) \sigma_z + \frac{1}{2} \left(\phi_{00} + \phi_{11} \right) I.$$
(4.28)

The resonator part of the interaction is calculated in the appendix (see section 2.2.3)

$$\hat{H}_{Ir} \sim e^{\frac{i}{\hbar}\hat{H}_r t} \left(a + a^{\dagger}\right) e^{\frac{-i}{\hbar}\hat{H}_r t} = e^{-i\omega_r t} a + e^{i\omega_r t} a^{\dagger}.$$
(4.29)

We now combine \hat{H}_{Iq} and \hat{H}_{Ir} to get terms proportional to the following:

$$\sigma^{+} \otimes a, \ \sigma^{-} \otimes a^{\dagger}, \ \sigma_{z} \otimes a^{\dagger}, \ \sigma_{z} \otimes a, \ I \otimes a^{\dagger}, \ I \otimes a, \ \sigma^{+} \otimes a^{\dagger}, \ \sigma^{-} \otimes a.$$
(4.30)

When the qubit and resonator are close to resonance, the first 6 terms are all accompanied by fast exponentials and can be ignored under the rotating wave approximation leaving us with

$$\hat{H} = -\frac{\hbar}{2}\omega_{10}\hat{\sigma}_z \otimes I_r + I_q \otimes \hbar\omega_r \left(a^{\dagger}a + \frac{1}{2}\right) - 2\phi_{01}E_M \left(\frac{E_{Cr}}{E_{Lr}}\right)^{1/4} \left(\sigma^- \otimes a + \sigma^+ \otimes a^{\dagger}\right)$$
(4.31)

which is known as the Jaynes Cummings hamiltonian (JCH). At first glance our interaction term appears to be incorrect. This is because we order the qubit basis such that the ground state state,

$$|0\rangle = \begin{bmatrix} 1\\ 0 \end{bmatrix}. \tag{4.32}$$

The consequence is that the roles of σ^+ and σ^- are reversed.

The JCH describes the exchange of a single excitation between the qubit and resonator. The frequency of this exchange is

$$f_n = \sqrt{n} \frac{g}{\pi},\tag{4.33}$$

where

$$g = 2\frac{\phi_{01}}{\hbar} E_M \left(\frac{E_{Cr}}{E_{Lr}}\right)^{1/4},$$
(4.34)

and n is the total number of energy quanta in the coupled system. We can cast g into a more illuminating form by shifting the qubit coordinates to the center of the metastable well by writing,

$$\hat{\phi} = \phi^* + \delta\hat{\phi},\tag{4.35}$$

where ϕ^* is the center of the metastable well found by solving $\nabla U(\phi_q, \phi_r) = 0$ and $\delta \hat{\phi}$ is the displacement from ϕ^* . In analogy with equation 2.3, $\delta \hat{\phi}$, can be written as

$$\delta\hat{\phi} = \left(\frac{E_{Cq}}{E_{Lq} + \frac{1}{2}E_J\cos(\phi_q^*)}\right)^{1/4} \left(a_q + a_q^{\dagger}\right). \tag{4.36}$$

If we recall the definition of the Josephson inductance in equation 1.7, we can see that the term in the denominator is just the total inductive energy of the parallel combination of the qubit's geometric inductance with the Josephson inductance. The matrix element, ϕ_{01} , is then

$$\phi_{01} = \left(\frac{E_{Cq}}{E_{Lq} + \frac{1}{2}E_J\cos(\phi_q^*)}\right)^{1/4} \langle 0| \left(a_q + a_q^{\dagger}\right)|1\rangle.$$
(4.37)

The term, $\langle 0 | (a_q + a_q^{\dagger}) | 1 \rangle$, is of order unity since the qubit states are closely approximated by the harmonic states of the metastable well. Combining these results into equation 4.34 we are left with,

$$g = \frac{\sqrt{\omega_{pq}(\phi_q^*)\omega_r}}{2} \frac{M}{\sqrt{L_q L_r}} \frac{1}{\sqrt{1 + \frac{E_{Jq}}{2E_{Lq}}\cos\phi_q^*}},$$
(4.38)

where $\omega_{pq}(\phi_q^*)$ is the plasma frequency of the metastable well. The important thing to note about equation 4.38 is that the coupling strength is directly proportional to M.

The eigenstates and eigenenergies of the JCH can be readily found since the Hamiltonian is a 2×2 block-diagonal matrix [53]. In the basis $|m, n\rangle$ where m = 0, 1 is the qubit state and n is the number of quanta in the resonator, the eigenstates are

$$|E_0\rangle = |00\rangle$$

$$|E_n+\rangle = \sin \theta_n |0,n\rangle + \cos \theta_n |1,n-1\rangle, \quad n \neq 0$$

$$|E_n-\rangle = \cos \theta_n |0,n\rangle - \sin \theta_n |1,n-1\rangle, \quad n \neq 0$$
(4.39)

where

$$\tan 2\theta_n = \frac{2g\sqrt{n}}{\Delta} \tag{4.40}$$

and

$$\Delta = \omega_{10} - \omega_r. \tag{4.41}$$

The corresponding eigenenergies are

$$E_0 = -\frac{\hbar}{2}\Delta$$

$$E_n \pm = n\hbar\omega_r \pm \frac{\hbar}{2}\sqrt{4g^2n + \Delta^2}.$$
(4.42)

We can see that the degeneracy is lifted by the coupling, resulting in avoided level crossings in the energy spectrum of the coupled system. Figure 4.2 a) shows the avoided level crossing for the single excitation manifold as a function of the detuning. Figure 4.2 b) shows qubit spectroscopy data, as well as a theory fit, showing the avoided crossing observed as the qubit is brought into resonance with the LC resonator.

The nice thing about a 2×2 block-diagonal Hamiltonian is that the time evolution operator is also 2×2 block-diagonal. This means that if we want to determine the evolution of a state with n total excitations, we only need to concern ourselves with the corresponding 2×2 matrix describing evolution in that manifold. The time evolution operator, in the uncoupled basis, for the *n*-excitation in the manifold is

$$U_{n} = \begin{bmatrix} \cos \theta_{n} & -\sin \theta_{n} \\ \sin \theta_{n} & \cos \theta_{n} \end{bmatrix} \begin{bmatrix} e^{-i\frac{E_{n}+}{\hbar}t} & 0 \\ 0 & e^{-i\frac{E_{n}-}{\hbar}t} \end{bmatrix} \begin{bmatrix} \cos \theta_{n} & \sin \theta_{n} \\ -\sin \theta_{n} & \cos \theta_{n} \end{bmatrix}.$$
 (4.43)

Suppose we have a single system excitation that starts out in the qubit. The initial state is $|1,0\rangle$ and the



Figure 4.2: a) Avoided level crossing for a single system excitation (n = 1) as a function of the detuning, Δ . b) Normalized (for clarity) spectroscopy data along with a theory fit, showing the avoided crossing. The avoided crossing appears skewed because the qubit's frequency is changing with applied qubit flux. The theory fit gave a splitting size of $g/\pi = 48$ MHz.

state at later times is

$$|\psi(t)\rangle = \begin{bmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{bmatrix} \begin{bmatrix} e^{-i\frac{E_n+}{\hbar}t} & 0 \\ 0 & e^{-i\frac{E_n-}{\hbar}t} \end{bmatrix} \begin{bmatrix} \cos\theta_n & \sin\theta_n \\ -\sin\theta_n & \cos\theta_n \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \exp\left[-in\hbar\omega_r\right] \begin{bmatrix} \cos\frac{\Omega}{2}t - i\sin\frac{\Omega}{2}t\cos2\theta_1 \\ -2i\sin\frac{\Omega}{2}t\sin\theta_1\cos\theta_1 \end{bmatrix}.$$
(4.44)



Figure 4.3: Vacuum Rabi oscillation between the phase qubit and resonator.

The probability, P_1 , of finding the qubit with the excitation is then

$$P_{1}(t,\Delta) = |\langle 10|\psi(t)\rangle|^{2}$$
$$= \cos^{2}\frac{\Omega}{2}t + \cos^{2}2\theta_{1}\sin^{2}\frac{\Omega}{2}t \qquad (4.45)$$

where

$$\Omega = \sqrt{4g^2 + \Delta^2}.\tag{4.46}$$

This type of evolution is called a vacuum Rabi oscillation where the vacuum Rabi frequency, Ω , is a function of the detuning between the qubit and resonator. On resonance, $\Delta = 0$ and the oscillation frequency is g/π . Figure 4.3 is a plot of a vacuum Rabi oscillation with a coupling strength $g/\pi = 100$ MHz showing a characteristic chevron pattern.

In experiments where the number of coupled elements is small the interactions between coupled elements can be easily controlled by detuning. However, when the number of coupled systems grows larger, it will become increasingly difficult to control interactions with detuning alone. Frequency "crowding" will

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Figure 4.4: a) System of four coupled qubits. b) Tuning qubit 1 into resonance with qubit 3 results in undesirable temporary interaction with qubit 2.

make it difficult to bring different elements into and out of resonance without, at least temporarily, crossing a resonance with an ancillary bit at the cost of some fidelity. As a simple example, imagine a system of four coupled qubits all initially detuned from each other as shown in figure 4.4. Now imagine we want to bring qubit 1 into resonance with qubit 3 to interact for some amount of time. The problem is that qubit 2's resonant frequency lies between that of 1 and 3 resulting in undesirable coupling with qubit 2. It would be desirable to have control over both g and Δ independently. That way, g_{12} could be tuned to zero, effectively removing qubit 2 from the picture altogether. To that end, we consider using a third, "mediating" element that can directly tune the interaction strength, g, between the qubit and resonator as illustrated in figure 4.5.



Figure 4.5: Two coupled qubits who's coupling strength, g, is tuned by an external parameter, λ , via a third mediating element.

Chapter 5

Tunable Coupling Between a Phase Qubit and LC Resonator

One way to implement tunable coupling is to use a flux-biased rf-SQUID to mediate the coupling strength between the qubit and resonator [28]. A tunable effective mutual inductance between the qubit and the resonator results from their interactions with the rf-SQUID, referred to as "the coupler" (Figure 5.1). A flux change in one element will be transmitted via the circulating current in the coupler to the other element resulting in coupled interactions. When the circulating coupler current is near the critical current, the loop is no longer able to respond to flux changes in either element, effectively decoupling them from one another.

We now derive an expression for the effective mutual inductance between the qubit and resonator using a simple electrical engineering argument. Consider the transformer in figure 5.2. The voltage in the primary and secondary coils is

$$\begin{bmatrix} V_p \\ V_s \end{bmatrix} = \begin{bmatrix} L_p \dot{I}_p + M \dot{I}_c \\ -M \dot{I}_c + L_s \dot{I}_s \end{bmatrix}$$
(5.1)

But the coupler current, I_c , is governed by the relation [6]

$$I_{c} = I_{c0} \sin \phi_{c} = -I_{c0} \sin \left[\frac{2\pi}{\Phi_{0}} \left(L_{c} I_{c} + \Phi_{Tx} \right) \right],$$
(5.2)

where

$$\Phi_{Tx} = MI_p - MI_s + \Phi_x, \tag{5.3}$$



Figure 5.1: A qubit and LC resonator coupled through an rf-SQUID. An effective mutual inductance between the qubit and resonator, resulting from their interactions with the coupler, can be tuned with the applied bias flux, Φ_x .



Figure 5.2: A transformer interrupted by an rf-SQUID. For simplicity, the direct mutual inductance between the primary and secondary coils is assumed to be zero.

is the total external flux applied to the rf-SQUID loop, having contributions from the currents in the primary and secondary coils as well as the applied external flux. Differentiating equation 5.2 with respect to time and using equation 5.3, we get

$$\dot{I}_{c} = \frac{\partial I_{c}}{\partial \Phi_{Tx}} \dot{\Phi}_{Tx}
= \frac{\partial I_{c}}{\partial \Phi_{Tx}} M\left(\dot{I}_{p} - \dot{I}_{s}\right).$$
(5.4)

Now plug this result back into equation 5.1 to get

$$\begin{bmatrix} V_p \\ V_s \end{bmatrix} = \begin{bmatrix} L_p + M^2 \frac{\partial I_c}{\partial \Phi_{Tx}} & -M^2 \frac{\partial I_c}{\partial \Phi_{Tx}} \\ -M^2 \frac{\partial I_c}{\partial \Phi_{Tx}} & L_s + M^2 \frac{\partial I_c}{\partial \Phi_{Tx}} \end{bmatrix} \begin{bmatrix} \dot{I}_p \\ \dot{I}_s \end{bmatrix}.$$
 (5.5)

The effective mutual inductance between the primary and secondary coils is

$$M_{eff} = -M^{2} \frac{\partial I_{c}}{\partial \Phi_{Tx}} = -\frac{M^{2}}{L_{c}} \frac{E_{Jc}}{2E_{Lc}} \frac{\cos\left[\frac{2\pi}{\Phi_{0}}\left(L_{c}I_{c} + \Phi_{Tx}\right)\right]}{\left(1 + \frac{E_{Jc}}{2E_{Lc}}\cos\left[\frac{2\pi}{\Phi_{0}}\left(L_{c}I_{c} + \Phi_{Tx}\right)\right]\right)}.$$
(5.6)

Figure 5.3 is a plot of the coupler's circulating current along with the effective mutual inductance. What range of mutual inductances can we achieve? At first glance it may appear difficult to infer anything from equation 5.6 because of its form. Ultimately it is a function of Φ_{Tx} only since I_c is itself a function of Φ_{Tx} through the transcendental relation in equation 5.2. If the size of the current modulations in the primary and secondary coils are small enough, i.e.,

$$\delta I_p \approx \frac{dI_p}{dt} dt \tag{5.7}$$

$$\delta I_s \approx \frac{dI_s}{dt} dt, \tag{5.8}$$

then

$$\delta \Phi_{Tx} \approx \frac{d\Phi_{Tx}}{dt} dt, \tag{5.9}$$

and Φ_{Tx} and I_c in equation 5.6 can be approximated by their dc values. As such, M_{eff} is a minimum when the cosine terms are unity,

$$M_{eff\,MIN} = -\frac{M^2}{L_c} \frac{E_{Jc}}{2E_{Lc}} \frac{1}{\left(1 + \frac{E_{Jc}}{2E_{Lc}}\right)}.$$
(5.10)

This occurs when the phase of the cosine term is an integral multiple of 2π , or

$$\frac{2\pi}{\Phi_0} \left(L_c I_c + \Phi_{Tx} \right) = 2\pi n.$$
(5.11)

According to equation 5.2 however, this phase implies that $I_c = 0$. Thus we have

$$\Phi_{Tx} = n\Phi_0. \tag{5.12}$$

So $M_{eff MIN}$ occurs when the total applied flux is integral multiples of Φ_0 . We get a maximum in M_{eff} when the cosine terms are -1,

$$M_{eff\,MAX} = \frac{M^2}{L_c} \frac{E_{Jc}}{2E_{Lc}} \frac{1}{\left(1 - \frac{E_{Jc}}{2E_{Lc}}\right)}.$$
(5.13)

Following the same procedures, this occurs again when $I_c = 0$, but this time the applied flux is,

$$\Phi_{Tx} = m \frac{\Phi_0}{2},\tag{5.14}$$

where *m* is an odd integer. Finally, $M_{eff} = 0$ when the circulating current is at the critical current. Note that $M_{eff\,Max}$ appears to increase without bound as $E_{Jc}/E_{Lc} \rightarrow 2$. This is the point where the slope of $I_c(\Phi_{Tx})$ approaches infinity at $\Phi_{Tx} = n\Phi_0$. If $E_{Jc}/E_{Lc} > 2$ the circulating current becomes double valued around this region and the coupler becomes significantly more difficult to operate due to hysteresis. As such we keep the coupler in the non-hysteretic regime by requiring that $E_{Jc}/E_{Lc} < 2$. Although we get the strongest coupling when $E_{Jc}/E_{Lc} \sim 2$, we typically design $E_{Jc}/E_{Lc} \sim 1-1.5$ to allow for normal deviations from nominal values due to the fabrication process.

We mentioned that the dc approximations of Φ_{Tx} and I_c are valid only if the current fluctuations in the primary and secondary coils are small enough. We can rough estimate of their size by applying the virial theorem to the LC resonator [33]. For the first excited state of the LC resonator, we have

$$\langle I^2 \rangle \sim \frac{\hbar \omega_r}{L_r}.$$
 (5.15)

The resulting fluctuations in Φ_{Tx} are then,

$$\langle \Phi_{Tx}^2 \rangle \sim \frac{M^2}{L_r} \hbar \omega_r.$$
 (5.16)

For our experiments $M \sim 1 \, pH$, $L_r \sim 1 \, nH$, and $\omega_r/2\pi \sim 10 \, GHz$ resulting in

$$\sqrt{\langle \Phi_{Tx}^2 \rangle} \sim 10 \,\mu \Phi_0, \tag{5.17}$$



Figure 5.3: The circulating current in the coupler along with the effective mutual inductance, as a function of external flux.

which is a very small fluctuation in flux to the coupler, implying that the dc approximations of Φ_{Tx} and I_c in equation 5.6 are sufficient.

Note that equation 5.5 implies that the inductances of the primary and secondary coils are modified as well. This will result in small frequency shifts of both the qubit and the resonator as we tune the coupling strength between them. We should also point out that we have neglected the self-capacitance of the coupler junction. This is equivalent to operating the coupler in the adiabatic regime where the self-resonant frequency of the coupler is much larger than either the phase qubit or resonator. This means that energy stored in the coupler is passively transferred back and forth between the resonator and qubit only, not into charge energy in the coupler [54].

Now the effective Hamiltonian describing the interactions between the phase qubit and resonator can be approximated using the Jaynes Cummings Hamiltonian in chapter 4 with a tunable g, on resonance, now given by

$$g\left(\Phi_x\right) \approx \frac{\omega_r}{2} \frac{M_{eff}\left(\Phi_x\right)}{\sqrt{L_q L_r}}.$$
(5.18)

Furthermore, as we mentioned previously the inductive energy, E_L , of both the qubit and resonator will be functions of Φ_x as well, resulting in small shifts in resonant frequency. For instance, the resonator frequency will modulate as

$$\omega_r (\Phi_x) = \frac{4}{\hbar} \sqrt{E_{Cr} E_{Lr} (\Phi_x)}$$
$$= \omega_{r0} \sqrt{1 - \frac{M_{eff} (\Phi_x)}{L_r}}$$
(5.19)

where $\omega_{r0} = 1/\sqrt{L_r C_r}$.

Chapter 6

The Experiment

This experiment was done on two different circuit generations, both fabricated on 3 inch sapphire wafers. Initial measurements were done on the first-generation circuit designed and fabricated in 2009. It employed via-style tunnel junctions and parallel plate "vacuum" shunt capacitors for the qubit and resonator designed by our group [55]. The second-generation circuit was designed and fabricated in 2010 incorporating angle-evaporated junction technology and interdigitated capacitors for both the qubit and resonator allowing a simpler fabrication process with a significantly reduced number of steps.

There are two basic measurements we use to observe coupling strength modulation: Spectroscopy and vacuum Rabi oscillations. Spectroscopy measurements are referred to as frequency domain measurements whereas vacuum Rabi oscillations are time domain measurements. Both devices were encapsulated in a two-layer Cryoperm magnetic shield to isolate external magnetic fields and measured in the same dilution refrigerator at a temperature of ~ 30 mK.

6.1 First-Generation Circuit

6.1.1 Fabrication and Design

The first-generation circuit is shown in figure 6.1. The fabrication process will be summarized in this section. The detailed steps are in the appendix. The fist step in the process is to make the vacuum capacitors. A ~ 100 nm base aluminum layer is deposited that will form the base plate for the capacitor as well as wiring cross-unders for the inductor coils. After this layer is patterned using standard photolithography, a ~ 200 nm sacrificial silicon nitride (SiNx) layer is deposited and patterned. Then another ~ 100 nm aluminum



Figure 6.1: First-generation circuit. a) Wiring schematic. b) Optical micrograph of the circuit.

layer is deposited and patterned, forming the top plate for the capacitor as well as the base layer for the junction and the remainder of the circuit, including the wiring for the inductor coils, bias line wiring and readout SQUID wiring. We note that the sacrificial layer will not be etched away until the very last step of the entire fabrication process is complete and the wafer has been diced. Vacuum-capacitor fabrication is summarized in figure 6.2.

Next is the fabrication of the via-style tunnel junctions. A silicon dioxide (SiO2) insulating layer is deposited on top of the base electrode. From here, vias are patterned into the SiO2 defining the junction areas. Next the wafer is introduced to a vacuum chamber for oxidation of the tunnel junction barrier. First an rf-plasma clean is used to remove the native oxide from the base electrode. Then oxygen is introduced into the chamber, thermally oxidizing the bare aluminum surface. Afterwards, the junction "top cap" aluminum



Figure 6.2: Vacuum capacitor fabrication. a) Capacitor base-plate. b) SiNx sacrificial layer. c) Top plate with relief holes. d) SiNx etched away.

electrode is deposited and patterned. Junction fabrication is summarized in figure 6.3.

Thus far we have created the vacuum capacitor, most of the base layer circuit wiring and the junctions. In order to complete the circuit connections, the junction top electrode as well as parts of the bias lines need to be connected to the circuit base layer. This is done by etching additional vias in the remaining bulk SiO2 insulator. Then the wafer is re-introduced to the rf-plasma clean to remove the native oxide from via holes, ensuring good connections. Finally, a ~ 100 nm aluminum wiring connection layer is deposited and patterned, completing all the circuit connections.

At this point, the circuit is complete and would "work" if you don't mind your vacuum capacitors



Figure 6.3: Via style junction fabrication process. a) Aluminum base electrode deposition. b) SiO2 deposition. c) Via is patterned. d) Native oxide removed with rf-plasma clean. e) Thermal oxidation. f) Aluminum top electrode deposition. g) Scanning electron microscope (SEM) image of the junction.

full of SiNx or the added dissipation from excess SiO2 blanketing the entire device [49]. To help minimize dissipation we etch away most of the excess SiO2, leaving only small amounts at critical locations around the junction and bias lines to help prevent shorts. Then small, $\sim 1 \times 1 \text{ nm}^2$, relief holes are etched into the top capacitor plate to allow the dry etch to remove the sacrificial layer after dicing. A protective layer of photoresist is then applied and the wafer is diced into $6.5 \times 6.5 \text{ mm}^2$ test chips. Once diced, the chips are brought back into the cleanroom where the sacrificial SiNx layer is etched away using a sulfur hexaflouride (SF₆) reactive ion etch, putting the "vacuum" in the vacuum capacitors. From here the chip is wire bonded to the sample box shown in figure 6.4 and then mounted to the DR.

With this process, the smallest area junctions that worked reliably were ~ 6 μm^2 [56]. This coupled

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c)



Figure 6.4: a) Qubit sample box with the lid off showing the test die in the center. b) Zoom-in of the test die. c) Sample box with the lid on ready to be mounted to the DR.

b)

with a typical critical current density yielded by our standard oxidation recipe of $J_0 \sim 0.2 \ \mu \text{A}/\mu\text{m}^2$ gives junctions with critical currents, $I_0 \sim 1 \ \mu\text{A}$. The corresponding Josephson energies are then $E_J \sim 2 \text{ meV}$.

The remaining circuit parameters essentially follow from the junction critical currents we can reliably achieve. Starting with the qubit geometric inductance, an $E_J/E_L \sim 9$ ratio implies $L_q \sim 1000$ pH. Gradiometric inductor coil designs were chosen to help protect the circuits from any noisy external magnetic fields. The qubit shunt capacitance is then chosen to keep the tunable frequency range of the qubit around 6 – 10 GHz. This ensures that the transition frequency will be low enough not to break Cooper pairs, given the gap frequency of ~ 100 GHz for aluminum [6]. But also, its high enough that the 30 mK thermal fluctuations of $kT \sim 600$ MHz ensure ground state isolation. Since the self-capacitance of the junction is known to be ~ 50 fF/ μ m², a ~ 6 μ m² junction will have a total self-capacitance of $C_J \sim 0.3$ pF [7]. Thus we require an additional shunt capacitance of $C_S \sim 0.3$ pF. The bias coil mutual inductances need to be as small as possible, but not so small that the bias sources cannot couple enough flux into the qubit. A happy medium is ~ 2 pH of mutual inductance for all bias coil circuitry. Given the standard bias resistances of ~ 1 kOhm, 1 volt from the bias source will apply a single Φ_0 worth of flux into the qubit or coupler. The mutual inductance between the qubit and readout SQUID was limited by the layout of the qubit inductor coil to ~ 30 pH. While not ideal, it gave decent enough separation in the SQUID histograms to readout the qubit state (see section 3.1.1.4).

The resonator inductor was chosen to match that of the qubit. The orientation of the resonator's inductor relative to the qubit's inductor was to minimize the direct mutual inductance between them. Since the resonator doesn't have a Josephson junction that contributes some inductance and capacitance, the resonator's shunt capacitor needed to be $C_r \sim 0.4$ pF, slightly larger than the qubit shunt capacitor to keep its resonant frequency in the range of the qubit.

The coupler inductance was chosen such that $E_{Jc}/E_{Lc} \sim 1.5$ implying an inductance of $L_c \sim 200$ pH. The mutual inductances between the coupler coil and the qubit and resonator were chosen to be $M_{cq} = M_{cr} \sim 60$ pH, giving us a maximum coupling strength of $g_{max}/\pi \sim 80$ MHz (equation 5.18), putting us well into the strong coupling regime for typical qubit lifetimes of ~ 150 ns.

6.1.2 Experimental Setup

Figure 6.5 shows the dilution refrigerator (DR) wiring schematic for the first generation experiment. An FPGA designed by John Martinis (now a professor at USCB) is used for all data acquisition from the readout SQUID as well as dc-bias and microwave pulse control to the qubit and coupler.

The dc-bias lines to the qubit and coupler are driven by programmable voltage sources that receive commands from the FPGA. They have a voltage range from 0 to ± 5 volts with an output bandwidth of ~ 100 kHz. The signal then enters a 2-pole RC filter at 4 K. This filter serves as both a low-pass filter, with a cut-off frequency of ~ 1 MHz, as well as provides a total bias resistance of 1 kOhm, transforming the voltage source into a current source. There are copper powder filters on the mixing chamber at 30 mK



Figure 6.5: First-generation circuit experiment wiring schematic of the dilution refrigerator.

designed to absorb line noise above 1 GHz [10, 57]. The dc-bias line is responsible for setting the overall applied dc-flux of the qubit to either the initialization, operation, or readout phase of the operation cycle (see chapter 3). For the coupler, a separate dc-bias generates the applied flux that tunes the effective mutual inductance between the qubit and resonator.

The rf-bias line is a 50 Ohm characteristic impedance line driven by an Anritsu 68369 microwave generator. This line is responsible for state manipulations of the qubit as discussed in chapter 3. The pulses are gated using either an HP 11720A pulse modulator containing a PIN diode, or I/Q mixers. Figure 6.6 shows a schematic of the two ways we generate rf-pulses. The simplest way is to use the PIN diode box as an rf-switch. A continuous-wave input signal from the microwave source is gated by a voltage signal applied to the PIN diode resulting in a square pulse shape. The drawback to this method is that there is no independent amplitude and phase control. The amplitude must be controlled by the generator itself. Another drawback is the square pulse shape itself which causes unwanted $|1\rangle \rightarrow |2\rangle$ transitions as discussed in chapter 3. A more involved way to generate pulses is to use two I/Q mixers in series to control both amplitude and phase as well as the timing and shape of the pulse. Here a continuous wave signal is input into the local oscillator of the first mixer. DC voltages from the FPGA applied at I and Q of this mixer determine both the amplitude and phase of the output. The signal is then fed into the second mixer that is responsible for the timing and shape. A timed, square envelope from the FPGA is fed into a Guassian filter. This Gaussian envelope is then fed into either I or Q of the second mixer resulting in a shaped pulse at the output. This method is used to generate Gaussian-shaped pulses. If no phase control is needed, as is the case with this experiment, the voltages applied to I and Q of the first mixer can be tuned such that the first mixer is fully "open" and the amplitude can be tuned by feeding the shaped pulse from the second mixer into a programable Hewlett Packard 11713A programmable attenuator before entering the dilution refrigerator. This significantly reduces the amount of up-front calibrations of the mixers required to perform the experiment. The pulse is then applied to the qubit using a small coupling capacitor on-chip. This small coupling capacitor serves to shield the qubit from added dissipation from the 50 Ohm microwave line in the same way that the small mutual inductances shield the qubit from the dc-bias lines. A total attenuation of 40 dB is distributed between the 4K bath and mixing chamber to absorb the black body radiation from the



Figure 6.6: RF bias line pulse gating schematic. a) A continuous wave signal from an RF source is fed into the local oscillator port of the first I/Q mixer. DC voltages applied at I and Q control the amplitude and phase. The output is fed into the second mixer local oscillator. The pulse envelope is fed into I or Q resulting in a shaped pulse at the output. b) A PIN diode box is used as a simple RF switch. The continuous wave is simply "chopped" by the gate signal. There is no phase control using this setup. The amplitude must be controlled with the signal generator.

higher temperature stages.

The measurement pulse is generated using a Stanford DG535 pulse generator with an output of 0-4 volts. Room-temperature attenuators are used at the output to maximize signal to noise. The measurement pulse is then coupled to the dc bias line with a bias tee at the 4K stage.

The qubit readout SQUID bias current is driven by another dc-voltage source using a divide-by-ten voltage divider and low pass filter at room temperature. At the 4K stage is a series 10 kOhm resistor network converting the voltage source to a current source. The current then enters copper powder filters at the 30 mK

stage before entering the SQUID. At the onset of the bias current ramp during the qubit readout sequence, a timer is started in the FPGA. When the SQUID switches, the output voltage is fed into a Stanford SR560 low-noise pre-amplifier with the gain set to 1000. From here the voltage is input into the external trigger input on a Stanford DG535 pulse generator. When the SQUID voltage reaches a certain threshold, the DG535 generates a pulse that is fed into a photo diode. The generated light pulse is then sent to the FPGA, telling it to stop the timer. The recorded time interval is then roughly proportional to the SQUID critical current.

6.1.3 Phase Qubit Characterization

As with any experiment, the qubit must first be fully characterized using the measurements discussed in section 3.1.2.

6.1.4 Coupler Characterization

The next step is to begin a coarse characterization of the coupler. The goal is to measure the coupler circulating current as a function of applied coupler flux, $\Phi_{x \ Coupler}$. Once we know the circulating current we know roughly what coupling strengths to expect for different applied coupler fluxes. To do this, we exploit the sensitivity of the $|0\rangle$ state tunneling probability, P_0 , to total applied flux to measure changes in the coupler circulating current. P_0 is a function of the equilibrium qubit junction phase ϕ_q , which is a function of total applied qubit flux. The total applied qubit flux now has contributions from the qubit bias coil and the circulating current in the coupler. The fluxoid quantization relations govern the phases of the qubit and coupler:

$$\frac{\phi_q}{2\pi} = -\frac{1}{\Phi_0} \left(L_q I_q + \Phi_{xq} + M_{qc} I_c \right)$$

$$\frac{\phi_c}{2\pi} = -\frac{1}{\Phi_0} \left(L_c I_c + \Phi_{xc} + M_{qc} I_q \right)$$

$$I_q = I_{q0} \sin \phi_q$$

$$I_c = I_{c0} \sin \phi_c.$$
(6.1)



Figure 6.7: Coarse calibration of the coupler.

Suppose we are in a situation where $P_0 = P_0^*$ with the applied fluxes balanced such that $I_c = 0$. This situation is described by

$$\frac{\phi_q^*}{2\pi} = -\frac{1}{\Phi_0} \left(L_q I_q^* + \Phi_{x\,q}^* \right) \tag{6.2}$$

$$\Phi_{xc}^* = -M_{qc}I_q^*. {(6.3)}$$

Now imagine we change the applied coupler flux by an amount $\Delta \Phi_{xc}$ but we want to keep P_0 constant. This means we have to change the applied qubit flux by an amount $\Delta \Phi_{xq}$ to compensate for the appearance of I_c . This new situation is described by

$$\Delta \Phi_{xq} = -M_{qc}I_c \tag{6.4}$$

$$\frac{\phi_c}{2\pi} = -\frac{1}{\Phi_0} \left(L_c I_c + \Delta \Phi_{xc} \right).$$
(6.5)

We can see that by keeping P_0 constant, we get a direct mapping between I_c and $\Delta \Phi_{xq}$. Inserting equation 6.4 into equation 6.5 and using equation 6.1 we get

$$\frac{\Delta\Phi_q}{MI_{c0}} = -\sin\left(\frac{1}{2}\frac{E_{Jc}}{E_{Lc}}\frac{\Delta\Phi_q}{MI_{c0}} - 2\pi\frac{\Delta\Phi_{xc}}{\Phi_0}\right)$$
(6.6)

which is the same transcendental relation governing I_c . Figure 6.7 is a plot of P_0 as a function of Φ_{xq} and Φ_{xc} . The constant color contours correspond to fixed P_0 . The background slope is due to cross-talk between



Figure 6.8: Analyzed measurement of coupler circulating current.

the coupler and qubit bias coils and can be easily accounted for. Figure 6.8 is a plot of the analyzed data taken from the $P_0^* = 0.5$ contour from figure 6.7 along with a theory fit. The theory fit gives $E_{Jc}/E_{Lc} \sim 1.02$ a 30% deviation from the design value, demonstrating the importance of allowing for, sometimes large, design parameter fluctuations due to the fabrication process.

6.1.5 Spectroscopy

Now that we know the coupler circulating current as a function of applied flux, we can use figure 6.7 as a guide and perform spectroscopy measurements at various applied coupler fluxes. We expect to an avoided crossing or splitting associated with the interaction between the qubit and LC resonator modulate with coupler flux. As shown in figure 6.9, we observe a large, ~ 97 MHz, avoided crossing when the coupler is biased near the regions of maximal slope. As expected, the splitting size is reduced to zero as the coupler current approaches the critical current. We also observed the center frequency of the avoided crossing change with applied coupler flux. This is due to the small modulation of the resonator's resonator frequency as predicted by equation 5.19.



Figure 6.9: a)-d). Spectroscopies at different coupler applied fluxes showing a maximum splitting of 97MHz in a) to no observable splitting in d) The data is normalized for clarity. e) The approximate bias points showing qualitative agreement with theory.

6.1.6 Vacuum Rabi Oscillations

While modulation in the splitting size is a good indication that the coupler is working, we do not consider spectroscopic measurements as proof of coherent quantum interactions. For this reason, we measure vacuum Rabi oscillations over the same ranges of coupler applied fluxes used in the spectroscopies. Figure 6.10 shows the measured vacuum Rabi oscillations on resonance. As with the spectroscopy measurements, we see good qualitative agreement with theory. The largest vacuum Rabi frequency measured was ~ 97 MHz when the coupler was biased at the same location that the ~ 97 MHz spectroscopic splitting was measured. As the coupler bias approached the critical current, we observed good agreement between the vacuum Rabi frequency and the spectroscopic splitting for splitting sizes above ~ 10 MHz. However, when the splitting dropped to below ~ 10 MHz, we found that the vacuum Rabi frequency began to deviate from the spectroscopic splitting, leveling off at ~ 7 MHz. Even with the coupler biased at the "zero" coupling point according to the spectroscopy, the corresponding vacuum Rabi measurement showed a residual beating (Figure 6.10 d)). If the interaction strength is truly zero here, a "vacuum Rabi" measurement should be equivalent to a T1 measurement resulting in a simple exponential decay of P_1 at a rate given by $1/T_1$. This result prompted us to begin taking vacuum Rabi measurements at other regions in the spectroscopy, tuned away from the resonator. In regions where there was a splitting due to a TLS, we expected to observe a vacuum Rabi oscillation frequency equal to the splitting size. In regions where no splitting was observable. we expected to simply measure the qubit lifetime. Figure 6.11 shows the spectroscopy over a broader range. We can clearly see the resonator splitting at ~ 7.65 GHz. We also see a large TLS splitting at ~ 8 GHz as well as a very small one at ~ 7.3 GHz. Any other TLSs, if present, are not evident. If they are present, their coupling strengths should be much less than the qubit linewidth of ~ 10 MHz. As such, any vacuum Rabi oscillation would be over-damped by the qubit decay. What we observed is illustrated in figure 6.12. When the qubit was biased at places where the splittings were evident in the spectroscopy, we observed vacuum Rabi oscillations consistent with the splitting size. Remarkably however, we observed oscillations in several locations that appeared free of splittings in the spectroscopy.

These oscillations also vary in frequency indicating a random distribution of weak coupling strengths



Figure 6.10: a)-d) Vacuum Rabi oscillations on resonance with the resonator demonstrating coherent modulation in coupling strength with applied coupler flux. e) The approximate bias points.



Figure 6.11: Spectroscopy of generation one circuit over a broader range showing two TLS splittings along with the resonator splitting. Shown on a grey scale to accentuate splittings.



Figure 6.12: a) The same spectroscopy as in figure 6.11 but with horizontal lines marking where vacuum Rabi data was taken. The black lines denote exponential decay, consistent with the spectroscopy. The red lines are where coherent oscillations were present in the vacuum Rabi data. b) The vacuum Rabi data. Counting upward from the bottom, the nth trace was taken at the qubit flux bias corresponding to the nth horizontal line in a). Exponential data are in black, oscillatory data are in red.

between the TLSs and the qubit as found for larger coupling strengths [49]. The observation of these weakly coupled TLS fluctuators is consistent with predictions based on the standard TLS model for defects in amorphous dielectric solids [49]. The expected distribution of splitting sizes given by Eq. 4 in Ref. [49] shows that the defect density scales approximately as 1/S where S is the splitting size in GHz, and the coupling strength is given by hS/2. Our measurements qualitatively agree with this prediction: as the coupling strength decreases, the defect density increases. The measurements recorded in Ref. [49] relied on traditional spectroscopic measurements with a minimum splitting resolution of 10 MHz. As for why they don't show up in the spectroscopy, we hypothesize that perhaps the long drive tone used in spectroscopy causes a saturation effect in a large fraction of the TLS ensemble, effectively decoupling them from the qubit.

We have devised a relatively rapid experimental technique for locating the position of these weakly coupled (S < 10 MHz) TLS's throughout the qubit's entire spectral range [58]. Once standard spectroscopy has been performed, we have a calibration of the resonant frequency of the qubit as a function of qubit bias flux. We can now search for coherent oscillations at each qubit frequency. Performing high resolution T_1 -scans' of time domain energy relaxation measurements will certainly reveal the TLS features as coherent oscillations but with data acquisition times that will be as long as standard spectroscopy. In order to reduce the number of data points for a given frequency range of the qubit, we choose a different approach. We hold the measure delay time τ_d fixed at a particular value, just after the maximum excitation of the qubit from the π -pulse. This value is a small fraction of the energy relaxation time of the qubit, sampling a single point early in the decay with nearly maximum probability. For a given flux, if the qubit is free from interactions with any other systems, the probability amplitude remains high. However, if the qubit is on resonance with a TLS (or any other coherent system), the probability amplitude will undergo oscillations producing a 'dip' in probability amplitude at the specific sampling point chosen. By taking a single data point for each qubit frequency, we have reduced the required number of points, spanning only the flux dimension, allowing finer resolution 'dip-scans' with fewer points and hence shorter acquisition times. Figure 6.13 illustrates this technique. We can clearly see a much higher TLS density than indicated by the broad spectroscopy data in figure **6**.11.



Figure 6.13: A time-domain dip-scan showing higher spectral TLS density than the standard spectroscopic scan in figure 6.11. The peaks correspond to regions in the qubit spectroscopy where the T_1 decay curve is exponential. The dips correspond to places where a coherent oscillation is present, identifying a TLS fluctuator in the qubit. Note that these dips occur where the standard spectroscopy curve appears to be free from any TLS fluctuators.

6.1.7 First-Generation Circuit Summary

In summary, we demonstrated coherent tunable coupling between a phase qubit and lumped-element LC resonator, using a separate, flux-biased rf-SQUID as a mediating element. Spectroscopically, the coupling strength was observed to modulate from a maximum ~ 100 MHz to zero. The vacuum Rabi oscillation frequency was observed to agree well with the spectroscopic measurements for $|g_c(\Phi_x)/\pi| \ge 7 MHz$. The residual oscillations for weaker coupling strengths were believed to be due to a high spurious TLS density in the ~ 6 μ m² junction and not the result of a residual coupling effect from the coupler. This hypothesis was supported by the observation of many spurious oscillations in time domain measurements over the entire spectral range of the qubit.

Figure 6.14 summarizes the measurements made on the generation one circuit. Overall we observed good agreement with theory. The fit to the coupler's circulating current in a) yielded the parameter $E_{Jc}/E_{Lc} \sim 1.02$. This value was then used to fit the resonator frequency vs. applied coupler flux in b) using equation 5.19. This fit yielded the uncoupled resonator frequency $f_{r0} \sim 7.709$ GHz as well as $k_{cr} \sim 0.142$. These results were then passed to the theory fit of the vacuum Rabi frequency (or spectroscopic splitting) on resonance, using equation 5.18. Note that the maximum coupling strength appears to be located at the wrong coupler bias point. According to the theory, the strongest coupling should be where the I vs. Φ_x curve has the steepest slope and the zero-coupling location should be where the slope is zero. The disparity is due to a direct coupling, g_0 , resulting from a capacitive interaction between the qubit and resonator because of their close proximity. The ability to tune the overall interaction strength to zero requires the coupler bias flux to be tuned to a sufficiently negative bare coupling value in order to cancel the positive direct coupling g_0 . The result of the fit is a direct coupling strength of $g_0/\pi \sim 53$ MHz as well as $k_{qc} \sim 0.223$.

6.2 Second-Generation Circuit

The primary motivation for designing a second-generation circuit was to employ an improved tunnel junction fabrication technology. This new technology uses double-angle evaporation to make the tunnel junctions. Double-angle evaporation can be used to reliably create junction areas many times smaller than can be achieved with the via-style process. Smaller junctions are known to reduce the junction TLS density improving overall performance [49]. In particular, it decreases the likely hood that we will have residual beating effects in the time-domain data when the coupler is biased at the "off" spot in the spectroscopy. We also wanted to reduce the direct coupling, g_0 , observed in the first generation experiment by increasing the separation distance between the qubit and resonator on chip.

Another motivation was to simplify the overall fabrication process by removing the vacuum style capacitors, replacing them with interdigitated capacitors (IDCs). While the vacuum capacitors will most likely lead to better performing devices in the future, they introduce an additional etch-step (after the chips have been diced) in the fabrication process as discussed. Additionally, because of their simple geometry, IDCs more readily reproduce their design values.



Figure 6.14: First-generation circuit data summary.

We also reduced the number of on-chip bias lines. Instead of having two separate bias lines to the qubit, one for rf and one for dc, as used in the experiments in [17, 59], we combined the two using an off-chip "home-made" dc-coupled bias tee, leaving only a single inductively coupled bias line to the qubit. This improvement not only simplified the chip layout but also allowed us to easily orchestrate the timing of different pulses because the measurement pulse, adiabatic shift pulses, and microwave pulses were all



Figure 6.15: Second-generation circuit.

combined at room temperature using a combiner. In addition, another home-made bias tee was used to add an rf line to the coupler bias to allow fast-timescale modulation of the coupling strength, mimicking actual use in a quantum computer. Another benefit of the increased bandwidth to the coupler was that it opened the door for off-resonant, parametric coupling between the qubit and resonator which is discussed in chapter 7.

6.2.1 Fabrication and Design

The second-generation circuit is shown in figure 6.15. The fist step is the deposition and patterning of a ~ 100 nm base aluminum layer that will serve as a wiring "cross-under" layer for the inductor coils. After this layer is patterned using standard photolithography, a ~ 200 nm SiO2 wiring insulation layer is deposited and patterned. The difference here is that most of the SiO2 is removed, leaving only enough to cover the wiring cross-unders. Instead of using vias to make the connections to the top layer, we leave small tabs exposed at the ends of the wiring cross-unders. These tabs are then rf-cleaned to remove the native oxide ensuring a good connection with the top layer. The ~ 100 nm aluminum top layer is then deposited and patterned, forming all of the circuit components except for the junctions. This part of the fabrication



Figure 6.16: Second-generation circuit base layer fabrication. a) Wiring cross-unders. b) SiO2 wiring insulation. c) Circuit base layer.

is summarized in figure 6.16 showing a simplified version of the qubit for clarity.

The next step is to add the junctions. As mentioned, this is done using a double-angle shadow evaporation deposition [60]. The first step in this process is to coat the wafer with a layer of lift-off resist (LOR) of thickness, $h \sim 2 \,\mu$ m (it will be measured more precisely later). Next a $\sim 1 \,\mu$ m thick layer of photo resist is applied on top of the LOR. A bridge of pre-determined width, $w \sim 1.5 \,\mu$ m, and length, $l \sim 1 \,\mu$ m, is then patterned over the location where the junction is to be located. These dimensions, along with the LOR thickness h, will ultimately be used to calculate the deposition angle required to give the desired junction area. When the top-layer resist is developed, the underlying LOR is also developed away in the region under the pattern, exposing the circuit base layer underneath. At this point, the LOR thickness is measured using a profilometer. Now that h is known, the required deposition angle θ is calculated using simple geometry assuming that the deposition is unidirectional over the entire wafer. The formula for θ is

$$\tan \theta = \frac{w+O}{2h},\tag{6.7}$$

where O is the amount of overlap needed to get the desired junction area based on l (Area = $O \times l$). This stage of the process is summarized in figure 6.17.

The wafer then enters the evaporation chamber where two depositions are performed at angles $\pm \theta$ with respect to a line normal to the wafer surface. Before the first deposition, ion-milling is done to remove native oxide from the exposed connecting tabs, ensuring a good connection with the circuit base layer. After


Figure 6.17: a) Circuit base layer. b) LOR deposition. c) Photo resist deposition. d) Shadow bridge is patterned. Dimensions h, w, and l, are shown.



Figure 6.18: a) Ion milling and first angle deposition. b) Thermal oxidation. c) Second angle deposition. d) Three junctions are formed. The center junction is the smallest and thus the largest influence on the circuit. The overlap, O, is shown. e) SEM image of the junction.

the first deposition, oxygen is introduced into the chamber, thermally oxidizing the surface. Since the first deposition was performed under vacuum, no milling is needed to remove native oxide, keeping the oxidizing surface much cleaner. Once the oxide is formed the second angle deposition is done finishing the junction. The angle evaporation is summarized in figure 6.18. Note in d) that actually three junctions are formed in the process. The two outer junctions are many times larger in area than the center junction, contributing very little to the dynamics. From here the wafer is diced and test chips are wire bonded to the same sample box used in the first-generation circuit.

The design parameters were chosen to give similar performance as the first-generation circuit. The standard angle evaporation oxidation recipe we used consistently yielded current densities of $J_0 \sim 1.1$

 μ A/ μ m². As mentioned before, the junction area should be as small as possible. We chose a consistently reproducible area of ~ 0.5 μ m² giving a nominal critical current of $I_0 \sim 0.5 \mu$ A and a Josephson energy of $E_J \sim 1$ meV.

The qubit inductance was chosen to maintain the standard $E_{Jq}/E_{Lq} \sim 9$ requiring $L_q \sim 2500$ pH. Because of the larger inductance however, a stronger mutual inductance between the qubit and readout dc-SQUID was required to maintain the histogram separation. A tri-lobe gradiometric design for the qubit inductance was thus incorporated to allow both lobes of the readout SQUID to overlap with two of the qubit coil lobes (Figure 6.15). Using this design, we were able to get the mutual inductance between the qubit and readout SQUID up to an acceptable ~ 140 pH. The third (middle) lobe of the qubit coil was to allow sufficient coupling between the qubit and coupler. To keep the qubit operating frequency in the 6 – 10 GHz range, the qubit shunt capacitance needed to be $C_s \sim 0.4$ pF. The coupler inductance was chosen to keep $E_{Jc}/E_{Lc} \sim 1$ requiring $L_c \sim 300$ pH. The mutual inductances between the coupler coil and qubit and resonator coils were chosen to be $M_{cq} = M_{cr} \sim 75$ pH, keeping the maximum coupling strengths well into the strong coupling regime. Finally, the resonator inductor employed a standard 2-lobe gradiometer with $L_r \sim 1900$ pH and $C_r \sim 0.3$ pF with a resulting design resonant frequency of ~ 6.7 GHz.

6.2.2 Experimental Setup

Figure 6.19 shows the DR wiring for the second-generation circuit experiment. The second-generation circuit was measured on the same DR as the first generation with slightly modified wiring. The primary modification was the addition of the home-made dc-coupled bias tees on qubit and coupler flux lines at 30 mK. As mentioned, the increased bandwidth to qubit and coupler bias lines allowed us to introduce fast time-scale adiabatic shift pulses using a Tektronix AWG610 arbitrary waveform generator for the qubit and a Tektronix AWG520 arbitrary waveform generator for the coupler.

6.2.3 Second-Generation Circuit Summary

As with the first-generation circuit, we observed modulation in the coupling strength using both spectroscopy and vacuum Rabi measurements. However, the residual beating effects at lower coupling strengths





Magnetic Shield





Figure 6.20: a) Spectroscopy showing a maximum splitting of ~ 50 MHz. b) The corresponding vacuum Rabi oscillations. c) Spectroscopy where the coupling strength is tuned to zero showing no splitting. d) The corresponding time-domain data showing the expected exponential decay. e) Line cut of the data in d) along with an exponential fit giving $T_1 \sim 146$ ns, consistent with typical qubit lifetimes.

were not observed. Indeed, as the spectroscopic splitting shrank to zero, the vacuum Rabi oscillation frequency smoothly transitioned to an exponential decay as expected. Figure 6.20 a) and b) shows measurements in both the frequency and time-domain at the maximum coupling strength for this device of ~ 50 MHz. Figure 6.20 c) and d) show the same measurements when the coupling was tuned to zero. We can see in d) that the time-domain measurement shows an exponential decay as expected.

We also point out, in figure 6.20 b), a new pulse sequence used to generate the vacuum Rabi oscillation. Here the slow dc applied flux to the coupler was tuned such that the coupling strength between the qubit and resonator was zero. The qubit was then pi-pulsed while on resonance with the resonator. Immediately



Figure 6.21: The pulse sequence applied to the qubit and coupler for the vacuum Rabi data in figure 6.20 b).

after the pi-pulse, a fast adiabatic shift pulse was applied to the coupler, changing the coupling strength to 50 MHz. This required a compensation shift pulse to the qubit due to the additional influence the change in the coupler circulating current had on the qubit's bias flux and it's resonance frequency. On top of this compensation pulse was another shift pulse used to control the detuning between the qubit and resonator. The pulse sequence is show in figure 6.21.

An important measurement we were able to perform with the second-generation circuit was a direct measurement of the energy lifetime, or T1, of the LC resonator. This measurement was done by first putting the qubit into the excited state with the qubit detuned from the resonator and the coupling strength at the maximum 50 MHz. Then a 10 ns adiabatic shift pulse was applied to the qubit placing the qubit on resonance with the resonator for a half vacuum Rabi cycle, transferring the excitation into the resonator, performing a "state-swap" operation. Then after "hold-time" with the excitation in the resonator, another state-swap is performed, bringing what was left of the excitation back to the qubit. The qubit was then measured. The result is shown in figure 6.22. The lifetime of the resonator was $T_{1 Resonator} \sim 265$ ns.

Figure 6.23 summarizes the measurements made on the second-generation circuit. The second generation design also agreed well with predictions. The maximum coupling strength of ~ 50 MHz was not quite as large as the generation one circuit because of the significantly larger inductances used for the qubit and resonator compared with the relatively small mutual inductances between them. Note however that the



Figure 6.22: a) Lifetime measurement of the resonator. b) Pulse sequence applied to the qubit.

location of the maximum and minimum coupling strengths are located closer to the maximum and minimum slopes of the I_c vs. Φ_{xc} curve, indicating a weaker direct coupling $g_0 \sim 6.3$ MHz.



Figure 6.23: Second-generation circuit data summary.

Chapter 7

Future Directions

The first-and second-generation circuit experiments showed that the coupling strength between two quantum circuits on resonance could be tuned by applying a dc external flux bias to a mediating rf-SQUID. There is another mode of operation made possible by the mediating rf-SQUID where off resonant coupling between the qubit and resonator is induced through the application of an rf drive to the coupler. This is known as parametric coupling. The required rf-drive frequency is the detuning, Δ , between the elements. To allow the qubit and resonator to exchange energy when they are off-resonance, a "pump tone" at the difference frequency or detuning is introduced through the mediating element in order to make up for the energy difference between the two systems. The rate of this exchange is controlled by the rate at which pump photons enter and leave the system. The way this works is as follows. Imagine the coupler is dc-biased to a region where $g(\Phi_{xC})$ has a large slope. For example, in the first-generation circuit (Figure 6.14 c)), this would be somewhere near $\Phi_{xC \ dc} \sim 0.4 \ \Phi_0$. Now let us apply a small amplitude (relative to Φ_0) rf signal on top of the dc bias. The total applied flux to the coupler is then

$$\Phi_{xC}(t) = \Phi_{xC\ dc} + \delta\Phi\cos\Delta t. \tag{7.1}$$

Since the drive amplitude is small, the first order response from g is

$$g(t) = g_0 + \delta g \cos \Delta t$$
$$= g_0 + \frac{\delta g}{2} \left(e^{i\Delta t} + e^{-i\Delta t} \right),$$

where

$$\delta g = \frac{dg}{d\Phi_{x\,C}} \delta \Phi \tag{7.2}$$

and $dg/d\Phi_{xC}$ is the slope of the curves in figures 6.14 c) and 6.23 c).

Now recall the derivation of the JCH in chapter 4. On resonance the dominant terms were found to be $\sigma^- a$ and $\sigma^+ a^{\dagger}$. The rest were neglected under the RWA. In the off-resonant case these terms oscillate at frequencies $\pm \Delta$. When Δ becomes large enough the oscillations are fast enough that they contribute little to the dynamics as can be seen by considering equation 4.44 in the limit of large detuning. In the parametric mode however, the fast oscillations at large detuning are countered by the time-dependence of g leading to stationary terms again,

$$H_{I} = -ig(t) \left[e^{i\Delta t} \sigma^{-} a + e^{-i\Delta t} \sigma^{+} a^{\dagger} \right]$$

$$= -ig_{0} \left[e^{i\Delta t} \sigma^{-} a + e^{-i\Delta t} \sigma^{+} a^{\dagger} \right]$$

$$- i \frac{\delta g}{2} \left[\left(e^{i\Delta t} + e^{-i\Delta t} \right) e^{i\Delta t} \sigma^{-} a + \left(e^{i\Delta t} + e^{-i\Delta t} \right) e^{-i\Delta t} \sigma^{+} a^{\dagger} \right]$$

$$\approx -i \frac{\delta g}{2} \left[\sigma^{-} a + \sigma^{+} a^{\dagger} \right].$$
(7.3)

In contrast with the resonant coupling case, the coupling strength in the parametric case is governed by the *modulation* in g due to the parametric drive. How much modulation we can achieve for a given parametric drive amplitude is, to lowest order, limited by the slope $dg/d\Phi_{xC}$. Ideally, one would operate at the points of inflection on the g vs. Φ_{xC} curves.

Figure 7.1 shows preliminary parametric coupling data on the second-generation circuit. Here the applied qubit flux was held fixed such that the detuning between the qubit and resonator was $\Delta \sim 2\pi \times 480$ MHz. The coupler was dc-biased near an inflection point of the g vs. Φ_{xC} curve in figure 6.23 c), where $g_0 \sim \pi \times 15$ MHz, putting the qubit and resonator in the far-detuned limit. Qubit spectroscopy was taken as a function of pump frequency for different pump powers. The splitting size, S, grew larger with increased pump power as expected, until it saturated at $S \sim 8$ MHz at a room temperature pump power of -3 dBm. Figure 7.1 b) shows the corresponding vacuum Rabi oscillation. From equation 7.3 the maximum coupling strength of 8 MHz corresponds to $\delta g = \pi \times 16$ MHz which is in decent agreement with what we would expect from figure 6.23 c) at $g_0 \sim \pi \times 15$ MHz. One way to increase the maximum coupling strength is to use a coupler with a larger E_{Jc}/E_{Lc} ratio, so that the slope of the coupler's circulating current vs.



Figure 7.1: Preliminary parametric coupling data. a) Qubit spectroscopy showing avoided crossing. b) Corresponding vacuum Rabi data.

applied flux curve (figure 6.23 a)) increases at $\Phi_{xc} = 0.5 \Phi_0$. This leads to larger inflection point slopes in the corresponding g vs. Φ_{xC} curve. Figure 7.2 shows simulated plots of coupler circulating current and the resulting coupling strength as a function of applied flux. We can see that as $E_{Jc}/E_{Lc} \rightarrow 2$, $dg/d\Phi_{xC}$ increases significantly. One thing to keep in mind however is that the location of the inflection point moves to a larger and larger g_0 with increasing E_{Jc}/E_{Lc} . For example when $E_{Jc}/E_{Lc} = 1.8$ the inflection point is at $g_0 \approx 280$ MHz. To put the qubit and resonator in the far detuned limit requires $\Delta >> g_0$ MHz. Given that the qubit has a tunable operating frequency range of only a few GHz means we may be limited in how



Figure 7.2: a) Coupler circulating current vs. applied flux for different E_{Jc}/E_{Lc} ratios. b) Coupling strength vs. applied flux for different E_{Jc}/E_{Lc} ratios.

close to the inflection point we can operate.

The most direct way to increase E_{Jc}/E_{Lc} is to increase the coupler junction's critical current, increasing E_{Jc} . However, precisely controlling critical currents directly with fabrication is difficult. What we propose to do is replace the coupler's junction with a dc-SQUID with it's own flux bias coil as shown in Figure 7.3 [29]. The embedded dc-SQUID behaves as a single Josephson junction with a critical current that depends on ϕ_x given by

$$I_{dc-SQUID}(\phi_x) = \sqrt{I_+^2 \cos^2 \frac{\phi_x}{2} + I_-^2 \sin^2 \frac{\phi_x}{2}}$$
(7.4)



Figure 7.3: The single coupler junction is replaced with an embedded dc-SQUID with a separate bias coil. The Josephson energy of the coupler can be tuned with ϕ_x .

where $I_{\pm} = I_{01} \pm I_{02}$ is the sum and difference of the individual critical currents of the junctions comprising the dc-SQUID. This gives us direct control over the effective Josephson energy of the coupler, allowing us to tune $E_{Jc\,eff}/E_{Lc}$ arbitrarily close to 2.

Chapter 8

Conclusion

In conclusion, we have shown that a flux-biased rf-SQUID can be used to coherently modulate the interaction strength between a phase qubit and lumped element resonator. Measurements verifying agreement with theory, in both the frequency domain and time domain, were done on two circuit generations. The first-generation circuit spectroscopy measurements showed the coupling strength modulate from a maximum ~ 100 MHz to zero. The vacuum Rabi oscillation frequency was observed to agree well with the spectroscopic measurements for $|g_c(\Phi_x)/\pi| \geq 7$ MHz. The residual oscillations for weaker coupling strengths were attributed to spurious TLSs in the junction barrier and not the result of a residual coupling effect from the coupler. This hypothesis was supported by the observation of many spurious oscillations in time domain measurements over the entire spectral range of the qubit. A capacitive offset coupling of $g_0 \sim 53$ MHz was observed due to the close proximity of the inductor coils. Fortunately, the changing sign of the effective mutual inductance mediated by the rf-SQUID could be used to cancel this direct coupling and reduce the overall coupling to zero. Also the bias line filtering to the coupler and qubit used for the first-generation circuit prevented fast time-scale modulation of the coupling strength.

A second-generation circuit was designed to improve the overall performance. In particular, smaller area angle-evaporated junctions were used to reduce the TLS defect density, reducing the likelihood of observing residual beating effects in the time domain when the coupling between the qubit and resonator was "off". Indeed as the coupling strength was tuned to zero, the vacuum Rabi oscillations smoothly transitioned into exponential decay as expected. To reduce the direct capacitive coupling observed in the first-generation circuit, the spatial separation between the inductor coils was increased. The offset coupling was reduced to $g_0 \sim 6.3$ MHz. Bias-tees were used to couple rf-lines to the dc-bias lines of the qubit and coupler. This allowed fast time-scale modulation of the coupling strength, mimicking use in a quantum processor. Additionally, the increased bandwidth to the coupler bias allowed parametric modulation of the coupling strength, inducing off-resonant coupling between the qubit and resonator. Preliminary frequencydomain and time-domain parametric coupling data showed that the coupler could indeed be operated in this mode. Chapter 9

Appendix

9.1 Calculations

9.1.1 Stencil Approximation of The Second Derivative

Here we derive the expression for the stencil approximation of the derivative. This is all based on the Taylor series expansion of a function:

$$f(x_0 + \delta x) = \sum_{n=0}^{\infty} f^{(n)}(x_0) \frac{\delta x^n}{n!}$$
(9.1)

where $f^{(n)}(x_0)$ is the nth derivative evaluated at x_0 . Consider the function in integer multiples of some small quantity, h, away from x_0 . In general we have

$$f(x_0 + mh) = \sum_{n=0}^{\infty} f^{(n)}(x_0) \frac{m^n h^n}{n!}$$
(9.2)

where m is any integer, positive, negative, or zero. First, let's calculate $f(x_0 \pm h)$:

$$f_{\pm 1} = f_0 \pm f_0^{(1)} + \frac{1}{2} f_0^{(2)} h^2 \pm \frac{1}{6} f_0^{(3)} h^3 + O\left(h^4\right)$$
(9.3)

where we have used the shorthand notation, $f_{\pm n}^{(m)} \equiv f^{(m)} (x_0 \pm nh)$.

Now lets, add the two expansions together to get

$$f_1 + f_{-1} = 2f_0 + f_0^{(2)}h^2 + O(h^4).$$
(9.4)

Solving for $f_0^{(2)}$ we have

$$f_0^{(2)} = \frac{f_1 - 2f_0 + f_{-1}}{h^2} + O\left(h^2\right)$$
(9.5)

This is called the three point approximation of $f^{(2)}(x_0)$ for obvious reasons. The error is of order h^2 . We can improve on this approximation by carrying equation 9.4 out to fifth order and also calculating $f_2 + f_{-2}$ out to fifth order. We have

$$f_1 + f_{-1} = 2f_0 + f_0^{(2)}h^2 + \frac{1}{12}f_0^{(4)}h^4 + O(h^6)$$
(9.6)

$$f_2 + f_{-2} = 2f_0 + 4f_0^{(2)}h^2 + \frac{16}{12}f_0^{(4)}h^4 + O(h^6)$$
(9.7)

Now we eliminate the h^4 term by multiplying equation 9.6 by 16 and subtracting the two equations. Then solving for $f_0^{(2)}$ we have

$$f_0^{(2)} = \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2} + O\left(h^4\right)$$
(9.8)

This is called the 5-point stencil approximation. It has an error of order h^4 . We can get the nth-point stencil approximation by calculating $f_n + f_{-n}$ out to order h^{n+1} . The resulting error will be of order h^{n-1} .

9.1.2 Green's Function in Equation 1.31

The goal here is to calculate the integral,

$$\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{i\omega(t-t')}}{(\omega_0^2-\omega^2)}d\omega\right]$$
(9.9)

using contour integration. Consider the following integral in the complex plane.

$$\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{iz(t-t')}}{(\omega_0^2-z^2)}dz\right]$$
(9.10)

As the integral stands now, it has poles on the real axis at $z_{\pm} = \pm \omega_0$. The problem can be simplified by adding a damping term, $b\dot{\Phi}$, to the original equation of motion, equation 2.26. The FT of this term is $ib\omega\Phi[\omega]$ and thus the new integral becomes

$$\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{iz(t-t')}}{(\omega_0^2+ibz-z^2)}dz\right]$$
(9.11)

Now the poles have been shifted upward to

$$z_{\pm} = \pm \omega_0 \left(1 - \frac{b^2}{8\omega_0^2} \right) + i\frac{b}{2}.$$
(9.12)

We now apply Cauchy's Theorem for integration around the contour [61]. We must decide which contour



Figure 9.1: Contour used to calculate integral.

to use. If we use the blue contour, the total integral is zero since the function is analytic over the entire region. If we use the green contour, the total integral is proportional to the sum of the residues at z_{\pm} . Which contour we chose is the one that makes the contribution from the semi-circular arc go to zero in the limit that $|z| \to \infty$. This is entirely determined by the behavior of the $e^{iz(t-t')}$ term. Consider the case when t < t'. In that case, the integral along either arc is of the form

$$\left[\int_{\theta_0}^{\theta_0+\pi} \frac{e^{-i|T|R\,e^{i\theta}}}{(\omega_0^2+ibRe^{i\theta}-R^2e^{i2\theta})} iRe^{i\theta}d\theta\right]$$
(9.13)

where $T \equiv t - t'$ and we have used polar coordinates defined by $z = Re^{i\theta}$ and $dz = iRe^{i\theta}d\theta$. If we choose the top arc then $\theta_0 = 0$. If we choose the bottom arc $\theta_0 = \pi$. After some algebra, the real and imaginary parts of the integral can be written as

$$\begin{aligned} \Re\left[\int\right] &= \int \frac{e^{|T|R\sin\theta} \left[R^2b\cos\left(\theta - |T|R\cos\theta\right) - R\omega_0^2\sin\left(\theta - |T|R\cos\theta\right) - R^3\sin\left(\theta + |T|R\cos\theta\right)\right]}{R^4 - 2bR^3\sin\theta + R^2\left(b^2 - 2\omega_0^2\cos2\theta\right) - 2bR\omega_0^2\sin\theta + \omega_0^4} d\theta \\ \Im\left[\int\right] &= \int \frac{e^{|T|R\sin\theta} \left[R\omega_0^2\cos\left(\theta - |T|R\cos\theta\right) - R^2b\sin\left(|T|R\cos\theta\right) - R^3\cos\left(\theta + |T|R\cos\theta\right)\right]}{R^4 - 2bR^3\sin\theta + R^2\left(b^2 - 2\omega_0^2\cos2\theta\right) - 2bR\omega_0^2\sin\theta + \omega_0^4} d\theta. \end{aligned}$$

In the limit of large R, the leading contributions are given by

$$\Re \left[\int \right] = -\int \frac{e^{|T|R\sin\theta}\sin\left(|T|R\cos\theta\right)}{R} d\theta$$
$$\Im \left[\int \right] = -\int \frac{e^{|T|R\sin\theta}\cos\left(|T|R\cos\theta\right)}{R} d\theta.$$

Now if we are integrating around the upper arc, $0 < \theta < \pi$, making $\sin \theta > 0$ which results in the numerators increasing exponentially as $R \to \infty$. In the lower arc, $\sin \theta < 0$ and the numerators decrease exponentially. Thus when t < t' we should choose the lower arc to complete our contour. What's more is that the entire function is analytic over the region enclosed by the lower contour. Thus when t < t' we have the simple result

$$\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{i\omega(t-t')}}{(\omega_0^2+ib\omega-\omega^2)}d\omega\right] = 0$$
(9.14)

This result is a manifestation of causality. The response of the oscillator at time t is affected only by the drive at time t' < t. For the case t > t' the exact opposite situation occurs. The leading contributions to the integral are of the form $e^{-|T|R\sin\theta}$ requiring integration along the upper arc. Only this time the function is not analytic in the entire region. In this case we are left with the residues at z_{\pm} . So we have

$$\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{i\omega(t-t')}}{(\omega_0^2+ib\omega-\omega^2)}d\omega\right] = 2\pi i \left(\text{Sum of the residues at } z_{\pm}\right)$$
(9.15)

Before finding the residues, first not that the denominator can be factored into

$$\left(\omega_0^2 + ibz - z^2\right) = -(z - z_+)(z - z_-) \tag{9.16}$$

It should be clear now that the poles at z_{\pm} are simple poles. As such the residues are given by

$$R[z_{\pm}] = \lim_{z \to z_{\pm}} \left[\frac{1}{\sqrt{2\pi}} \frac{(z - z_{\pm}) e^{iz|T|}}{-(z - z_{\pm}) (z - z_{-})} \right]$$
(9.17)

So we have

$$R[z_{+}] = -\frac{1}{\sqrt{2\pi}} \frac{e^{iz_{+}|T|}}{(z_{+} - z_{-})}$$
(9.18)

$$R[z_{-}] = \frac{1}{\sqrt{2\pi}} \frac{e^{iz_{-}|T|}}{(z_{+} - z_{-})}$$
(9.19)

(9.24)

Plugging the residues back into equation 9.15 we have

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{i\omega(t-t')}}{(\omega_0^2 + ib\omega - \omega^2)} d\omega = \sqrt{2\pi} i \frac{\left(e^{iz_-|T|} - e^{iz_+|T|}\right)}{(z_+ - z_-)} \\
= \sqrt{2\pi} i \frac{e^{-\frac{b|T|}{2}} \left(e^{-i|T|\omega_0 \left(1 - \frac{b^2}{8\omega_0^2}\right)} - e^{i|T|\omega_0 \left(1 - \frac{b^2}{8\omega_0^2}\right)}\right)}{2\omega_0 \left(1 - \frac{b^2}{8\omega_0^2}\right)} \\
= \sqrt{2\pi} \frac{e^{-\frac{b|T|}{2}} \sin\left[|T|\omega_0 \left(1 - \frac{b^2}{8\omega_0^2}\right)\right]}{\omega_0 \left(1 - \frac{b^2}{8\omega_0^2}\right)} \tag{9.20}$$

Now all we have left to do is allow $b \to 0$ to get

$$G(t,t') = \sqrt{2\pi} \, \frac{\sin\left[\omega_0 \, (t-t')\right]}{\omega_0} \tag{9.21}$$

9.1.3 Coherent States

9.1.3.1 The projection of a coherent state into the number states

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$
(9.22)

Assuming the eigenstate exists, it can be written as

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle.$$
(9.23)

Now apply the annihilation operator and set the result equal to a constant, α , times the original state

$$a\left(\sum_{n=0}^{\infty} c_n |n\rangle\right) = \sum_{n=0}^{\infty} c_n a |n\rangle$$

$$= \sum_{n=0}^{\infty} c_n \sqrt{n} |n-1\rangle$$

$$= \sum_{n=1}^{\infty} c_n \sqrt{n} |n-1\rangle$$

$$= \sum_{m=0}^{\infty} c_{m+1} \sqrt{m+1} |m\rangle$$

$$= \alpha\left(\sum_{m=0}^{\infty} c_m |m\rangle\right)$$

This requires that

$$c_{m+1} = \frac{\alpha}{\sqrt{m+1}} c_m \tag{9.25}$$

We start this recursion relation with c_0 , which will later be chosen such that the state is properly normalized. By trying the first few $c_{m's}$ it is easy to see that the recursion relation is satisfied if

$$c_m = \frac{\alpha^m}{\sqrt{m!}} c_0 \tag{9.26}$$

So we have

$$|\alpha\rangle = c_0 \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \alpha^n |n\rangle \tag{9.27}$$

Now all that's left to do is normalize to find c_0

$$\begin{aligned} \langle \alpha | | \alpha \rangle &= |c_0|^2 \sum_n \sum_m \frac{1}{\sqrt{n!m!}} \alpha^{*n} \alpha^m \delta_{nm} \\ &= |c_0|^2 \sum_m \frac{1}{m!} \alpha^{*m} \alpha^m \\ &= |c_0|^2 \sum_m \frac{1}{m!} |\alpha|^{2m} \\ &= |c_0|^2 e^{|\alpha|^2} \\ &= 1 \rightarrow \\ |c_0| &= \exp\left[-\frac{1}{2} |\alpha|^2\right] \end{aligned}$$
(9.28)

9.1.3.2 Expectation values of flux and charge in coherent states.

Equation 2.3 can be inverted to give expressions of $\hat{\Phi}$ and \hat{Q} in terms of the creation and annihilation operators. The result is

$$\hat{\Phi} = \tilde{\Phi} \left(a + a^{\dagger} \right)$$

$$\hat{Q} = -i\tilde{Q} \left(a - a^{\dagger} \right)$$
(9.29)

(9.30)

where $\tilde{\Phi}=\sqrt{\hbar/2C\omega}$ and $\tilde{Q}=\sqrt{\hbar\omega C/2}.$ So we have

$$\langle \alpha | \hat{\Phi} | \alpha \rangle = \tilde{\Phi} \langle \alpha | (a + a^{\dagger}) | \alpha \rangle$$

$$= \tilde{\Phi} (\langle \alpha | a | \alpha \rangle + \langle \alpha | a^{\dagger} | \alpha \rangle)$$

$$= \tilde{\Phi} (\langle \alpha | a | \alpha \rangle + \langle \alpha | a | \alpha \rangle^{*})$$

$$= \tilde{\Phi} (\alpha + \alpha^{*}).$$

$$(9.31)$$

Similarly

$$\langle \alpha | \hat{Q} | \alpha \rangle = -i \tilde{Q} \langle \alpha | (a - a^{\dagger}) | \alpha \rangle$$

$$= -i \tilde{Q} \left(\langle \alpha | a | \alpha \rangle - \langle \alpha | a^{\dagger} | \alpha \rangle \right)$$

$$= -i \tilde{Q} \left(\langle \alpha | a | \alpha \rangle - \langle \alpha | a | \alpha \rangle^{*} \right)$$

$$= -i \tilde{Q} \left(\alpha - \alpha^{*} \right).$$

$$(9.32)$$

For $\langle \alpha | \hat{\Phi}^2 | \alpha \rangle$ we have

$$\langle \alpha | \hat{\Phi}^2 | \alpha \rangle = \tilde{\Phi}^2 \langle \alpha | \left(a^2 + a^{\dagger 2} + a a^{\dagger} + a^{\dagger} a \right) | \alpha \rangle$$

$$= \tilde{\Phi}^2 \left(\langle \alpha | a^2 | \alpha \rangle + \langle \alpha | a^{\dagger 2} | \alpha \rangle + \langle \alpha | a a^{\dagger} | \alpha \rangle + \langle \alpha | a^{\dagger} a | \alpha \rangle \right).$$

$$(9.33)$$

The last two operators can be related using the commutation relation $[a, a^{\dagger}] = 1$ to give

$$\langle \alpha | \hat{\Phi}^{2} | \alpha \rangle = \tilde{\Phi}^{2} \left(\langle \alpha | a^{2} | \alpha \rangle + \langle \alpha | a^{\dagger 2} | \alpha \rangle + 1 + 2 \langle \alpha | a^{\dagger} a | \alpha \rangle \right)$$

$$= \tilde{\Phi}^{2} \left(\alpha^{2} + \alpha^{*2} + 1 + 2\alpha\alpha^{*} \right)$$

$$= \tilde{\Phi}^{2} \left(1 + (\alpha + \alpha^{*})^{2} \right).$$

$$(9.34)$$

Similarly for the charge we have

$$\langle \alpha | \hat{Q}^2 | \alpha \rangle = \tilde{Q}^2 \left(1 + (\alpha - \alpha^*)^2 \right).$$
(9.35)

Thus the uncertainties are

$$\langle \alpha | \left(\hat{\Phi} - \langle \Phi \rangle \right)^2 | \alpha \rangle = \tilde{\Phi}^2$$
 (9.36)

$$\langle \alpha | \left(\hat{Q} - \langle Q \rangle \right)^2 | \alpha \rangle = \tilde{Q}^2$$

$$(9.37)$$

$$\tilde{\Phi}^2 \tilde{Q}^2 = \frac{\hbar^2}{4}. \tag{9.38}$$

9.1.3.3 Equations 2.53 and 2.54

$$\exp\left(\frac{i\hat{H}_0t}{\hbar}\right)a\exp\left(\frac{-i\hat{H}_0t}{\hbar}\right) = e^{-i\omega t}a\tag{9.39}$$

The left hand side of the above equation is just the definition of $a(t)_H$ in the Heisenberg picture obeying the following equation of motion

$$\frac{da_H}{dt} = -i\omega a. \tag{9.40}$$

The solution is $e^{-i\omega t}a$. Another more direct way to show this is to simply act on an arbitrary state $|\psi\rangle$ projected into the number basis

$$\exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)a\exp\left(\frac{-i\hat{H}_{0}t}{\hbar}\right)|\psi\rangle = \exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)a\exp\left(\frac{-i\hat{H}_{0}t}{\hbar}\right)\sum_{n=0}^{\infty}\langle n|\psi\rangle|n\rangle$$

$$= \exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)a\sum_{n=0}^{\infty}e^{-i\left(n+\frac{1}{2}\right)\omega t}\langle n|\psi\rangle|n\rangle$$

$$= \exp\left(\frac{i\hat{H}_{0}t}{\hbar}\right)\sum_{n=0}^{\infty}e^{-i\left(n+\frac{1}{2}\right)\omega t}\langle n|\psi\rangle\sqrt{n}|n-1\rangle$$

$$= \sum_{n=0}^{\infty}e^{i\left(n-1+\frac{1}{2}\right)\omega t}e^{-i\left(n+\frac{1}{2}\right)\omega t}\langle n|\psi\rangle\sqrt{n}|n-1\rangle$$

$$= \sum_{n=0}^{\infty}e^{-i\omega t}\langle n|\psi\rangle\sqrt{n}|n-1\rangle$$

$$= e^{-i\omega t}a\sum_{n=0}^{\infty}\langle n|\psi\rangle|n\rangle$$

$$= e^{-i\omega t}a|\psi\rangle. \tag{9.41}$$

Equation 2.54 is just the Hermitian conjugate of 2.53.

9.1.3.4 Derivation of Equation 2.58

This derivation follows directly from the derivation of the Baker-Hausdorff Formula in Appendix 4A from reference [37] with missing steps filled in and an actual motivation for the transformation they use. Consider a state vector governed by the following first order differential equation

$$\frac{d|\phi\rangle}{dt} = \left[A(t) + B(t)\right]|\phi\rangle \tag{9.42}$$

where A(t) and B(t) are operators that obey the following commutation relations

$$[A(t), A(t')] = 0 (9.43)$$

$$[B(t), B(t')] = 0 (9.44)$$

$$[A(t), B(t')] = f(t, t')$$
(9.45)

$$[A(t''), f(t, t')] = 0 (9.46)$$

$$[B(t''), f(t, t')] = 0 (9.47)$$

(9.48)

Now imagine for a moment that this was an ordinary scalar equation and A(t) and B(t) were simple scalar functions

$$\frac{dy}{dt} = \left[A(t) + B(t)\right]y \tag{9.49}$$

This equation is separable and can easily be solved. The solution is

$$y(t) = \exp\left[\int_0^t \left(A(t') + B(t')\right) dt'\right] y(0)$$
(9.50)

After some thought it should be clear that the only reason this works is because scalar functions always commute at different times. To see this, let's define

$$F(t) = A(t) + B(t)$$
 (9.51)

$$H(t) = \int_0^t \left(A(t') + B(t') \right) dt'$$
(9.52)

$$= \int_0^t F(t')dt' \tag{9.53}$$

(9.54)

and note that

$$\frac{dH}{dt} = F(t) \tag{9.55}$$

Plugging this back into our solution we have

$$y(t) = \exp \left[H(t)\right]y(0)$$

= $\left(\sum_{n=0}^{\infty} \frac{H^n}{n!}\right)y(0)$ (9.56)

Now to show it's a solution, we take the time derivative

$$\frac{dy}{dt} = \left(\sum_{n=0}^{\infty} \frac{1}{n!} \frac{dH^n}{dt}\right) y(0) \tag{9.57}$$

Now the key here is since H(t) commutes with itself at different times, it also commutes with it's derivative. Thus we can write

$$\frac{dH^n}{dt} = nH^{n-1}\frac{dH}{dt}.$$
(9.58)

Inserting this resut we get

$$\frac{dy}{dt} = \left(\sum_{n=0}^{\infty} \frac{1}{n!} n H^{n-1} \frac{dH}{dt}\right) y(0)
= \frac{dH}{dt} \left(\sum_{n=1}^{\infty} \frac{1}{(n-1)!} H^{n-1}\right) y(0)
= \frac{dH}{dt} \left(\sum_{m=0}^{\infty} \frac{1}{(m)!} H^m\right) y(0)
= \frac{dH}{dt} \exp[H(t)] y(0)
= [A(t) + B(t)] y.$$
(9.59)

Now if H(t) does not commute with itself at different times, it does not commute with it's derivative. To see this, let

$$[H(t), H(t')] = g(t, t')$$
(9.60)

where g(t, t') = 0 for t = t'. Now the commutator with it's derivative is given by

$$\begin{bmatrix} H, \frac{dH}{dt} \end{bmatrix} = \lim_{\Delta t \to 0} \frac{H(t)H(t + \Delta t) - H^2(t) - H(t + \Delta t)H(t) + H^2(t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{H(t)H(t + \Delta t) - H(t + \Delta t)H(t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{g(t, t + \Delta t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{g(t, t) + \frac{\partial g}{\partial t'}\Delta t + O(\Delta t^2)}{\Delta t}$$
$$= \frac{\partial g}{\partial t'}|_{t=t'}.$$
(9.61)

As such, equation 9.58 is invalid making equation 9.50 also invalid. The way around this problem is to apply the following transformation to $|\psi\rangle$

$$|U(t)\rangle = \exp\left[-\int_0^t B(t')dt'\right]|\phi(t)\rangle.$$
(9.62)

Taking the time derivative and applying equation 9.42 we get

$$\frac{d|U\rangle}{dt} = \exp\left[-\int_0^t B(t')dt'\right]A(t)\exp\left[\int_0^t B(t')dt'\right]|U\rangle.$$
(9.63)

Now we apply the Baker-Hausdorff Formula,

$$\exp\left[-G\right]A\exp\left[G\right] = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} [G, A]_n \tag{9.64}$$

where $[G, A]_n$ is the recursive commutator defined by

$$[G, A]_0 = A (9.65)$$

$$[G, A]_1 = [G, A] \tag{9.66}$$

$$[G, A]_n = [G, [G, A]_{n-1}], (9.67)$$

to the right hand side of equation 9.63. We get

$$\exp\left[-\int_{0}^{t} B(t')dt'\right]A(t)\exp\left[\int_{0}^{t} B(t')dt'\right] = A(t) - \int_{0}^{t} f(t,t')\,dt'.$$
(9.68)

Plugging this result back in we have

$$\frac{d|U\rangle}{dt} = \left[A(t) - \int_0^t f(t, t') dt'\right] |U\rangle.$$
(9.69)

The solution of this transformed expression is the same for the scalar case since the terms in the brackets commute at different times:

$$|U(t)\rangle = \exp\left[\int_{0}^{t} A(s)ds - \int_{0}^{t} ds \int_{0}^{s} f(s,s') ds'\right] |U(0)\rangle.$$
(9.70)

Transforming back to $|\phi\rangle$ we get

$$|\phi(t)\rangle = \exp\left[\int_0^t B(t')dt'\right] \exp\left[\int_0^t A(t')dt'\right] \exp\left[-\int_0^t ds \int_0^s f(s,s')\,ds'\right] |\phi(0)\rangle. \tag{9.71}$$

Specifically for equation 2.58,

$$A(t) = \frac{i}{\hbar} \tilde{\Phi} I(t) e^{i\omega t} a^{\dagger}$$
(9.72)

and

$$B(t) = \frac{i}{\hbar} \tilde{\Phi} I(t) e^{-i\omega t} a.$$
(9.73)

So we have

$$\int_0^t A(t')dt' = \frac{i}{\hbar}\tilde{\Phi}a^{\dagger} \int_0^t I(t')e^{i\omega t'}dt'$$
$$= \frac{i}{\hbar}\tilde{\Phi}a^{\dagger}e^{i\omega t} \int_0^t I(t')e^{-i\omega(t-t')}dt', \qquad (9.74)$$

$$\int_0^t B(t')dt' = \frac{i}{\hbar}\tilde{\Phi}a \int_0^t I(t')e^{-i\omega t'}dt'$$
$$= \frac{i}{\hbar}\tilde{\Phi}ae^{-i\omega t} \int_0^t I(t')e^{i\omega(t-t')}dt'$$
(9.75)

and

$$f(s,s') = -\frac{\tilde{\Phi}^2}{\hbar^2} I(s) I(s') e^{i\omega(s-s')} \left[a^{\dagger}, a\right]$$
$$= \frac{\tilde{\Phi}^2}{\hbar^2} I(s) I(s') e^{i\omega(s-s')}$$
(9.76)

since $[a^{\dagger}, a] = -1$. Now define

$$\alpha(t) \equiv \frac{i}{\hbar} \tilde{\Phi} \int_0^t I(t') e^{-i\omega(t-t')} dt'.$$
(9.77)

Plugging these results back into equation 9.71 we get

$$|\phi(t)\rangle = \exp\left[-ae^{-i\omega t}\alpha^*(t)\right] \exp\left[a^{\dagger}e^{i\omega t}\alpha(t)\right] \exp\left[-\frac{\tilde{\Phi}^2}{\hbar^2}\int_0^t ds \int_0^s I(s)I(s')e^{i\omega\left(s-s'\right)} ds'\right] |\phi(0)\rangle.$$
(9.78)

The third exponential in the above expression is just a global phase and can be ignored. We are left with

$$|\phi(t)\rangle = \exp\left[-ae^{-i\omega t}\alpha^*(t)\right] \exp\left[a^{\dagger}e^{i\omega t}\alpha(t)\right] |\phi(0)\rangle.$$
(9.79)

Now we need to transform back to the Schrodinger picture by applying

$$|\psi(t)\rangle = \exp\left[-i\frac{H_0}{\hbar}t\right]|\phi(t)\rangle.$$
 (9.80)

The result is

$$|\psi(t)\rangle = \exp\left[-i\frac{H_0}{\hbar}t\right] \exp\left[-ae^{-i\omega t}\alpha^*(t)\right] \exp\left[a^{\dagger}e^{i\omega t}\alpha(t)\right]|\psi(0)\rangle.$$
(9.81)

The left-most exponential removes the $e^{\pm i\omega t}$ factors in the other two exponentials the following way

$$\exp\left[-i\frac{H_0}{\hbar}t\right]\exp\left[-ae^{-i\omega t}\alpha^*(t)\right] = \sum_{n=0}^{\infty} \frac{(-1)^n e^{-in\omega t}\alpha^{*n}}{n!} \exp\left[-i\frac{H_0}{\hbar}t\right]a^n$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n e^{-in\omega t}\alpha^{*n}}{n!} e^{in\omega t}a^n \exp\left[-i\frac{H_0}{\hbar}t\right]$$
$$= \left(\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{*n}}{n!}a^n\right) \exp\left[-i\frac{H_0}{\hbar}t\right]$$
$$= \exp\left[-a\alpha^*(t)\right] \exp\left[-i\frac{H_0}{\hbar}t\right],$$
(9.82)

where we have repeatedly applied

$$\exp\left[-i\frac{H_0}{\hbar}t\right]a = e^{i\omega t}a\exp\left[-i\frac{H_0}{\hbar}t\right]$$
(9.83)

which can be inferred from equation 2.54. We are left with

$$|\psi(t)\rangle = \exp\left[-a\alpha^{*}(t)\right]\exp\left[a^{\dagger}\alpha(t)\right]\exp\left[-i\frac{H_{0}}{\hbar}t\right]|\psi(0)\rangle.$$
(9.84)

Finally, we can combine the first two exponents using the well known formula [33]

$$\exp\left[A+B\right] = \exp\left[A\right]\exp\left[B\right]\exp\left[-\frac{1}{2}\left[A,B\right]\right].$$
(9.85)

Ignoring the accompanying global phase we get equation 2.58

$$|\psi(t)\rangle = \exp\left[a^{\dagger}\alpha(t) - a\alpha^{*}(t)\right] \exp\left[-i\frac{H_{0}}{\hbar}t\right]|\psi(0)\rangle.$$
(9.86)

9.1.4 Derivation of The Norton-Equivalent Circuit of The Flux-Biased Phase Qubit

We first start by transforming the circuit using the T-equivalent model of coupled inductors (see Figure 9.2). Next we find the Thevnin equivalent impedance and voltage from Z looking back toward the generator. The Thevnin voltage is calculated by shorting the source voltage and calculating the resulting impedance parallel to Z. Then the Norton current is given by the Thevnin voltage divided by the Thevnin impedance.

First we have the series combination of R_b and $L_b - M$ in parallel with M.

$$Z_1 = j\omega (L_b - M) + R$$
$$Z_2 = j\omega M$$



Figure 9.2: T-equivalent model of coupled inductors.

The parallel combination is

$$\begin{split} Z_{1||2} &= \frac{(j\omega\,(L_b - M) + R)\,j\omega M}{j\omega L_b + R} \\ &= \frac{-\omega^2 M\,(L_b - M) + Rj\omega M}{j\omega L_b + R} \\ &= \frac{(-\omega^2 M\,(L_b - M) + Rj\omega M)}{\omega^2 L_b^2 + R^2} \frac{(R - j\omega L_b)}{1} \\ &= \frac{(-\omega^2 M\,(L_b - M)\,(R - j\omega L_b) + Rj\omega M\,(R - j\omega L_b))}{\omega^2 L_b^2 + R^2} \\ &= \frac{\left(\omega^2 L_b RM - R\omega^2 M L_b\left(1 - \frac{M}{L_b}\right) + \left[R^2 j\omega M + j\omega^3 M L_b^2\left(1 - \frac{M}{L_b}\right)\right]\right)}{\omega^2 L_b^2 + R^2} \\ &= \frac{R\omega^2 M^2 + jM\omega\left[R^2 + \omega^2 L_b^2\left(1 - \frac{M}{L_b}\right)\right]}{R^2 + \omega^2 L_b^2} \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j\frac{M\omega\left[R^2 + \omega^2 L_b^2 - \omega^2 L_b M\right]}{R^2 + \omega^2 L_b^2}. \end{split}$$

Now $Z_{1||2}$ is combined in series with L - M to get

$$\begin{split} Z_{Thev} &= Z_{1||2} + j\omega \left(L - M \right) \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \frac{M\omega \left[R^2 + \omega^2 L_b^2 - \omega^2 L_b M \right]}{R^2 + \omega^2 L_b^2} + j\omega \left(L - M \right) \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \frac{M\omega \left[R^2 + \omega^2 L_b^2 - \omega^2 L_b M \right]}{R^2 + \omega^2 L_b^2} + j\omega \left(L - M \right) \frac{\left(R^2 + \omega^2 L_b^2 \right)}{R^2 + \omega^2 L_b^2} \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \frac{M\omega \left(R^2 + \omega^2 L_b^2 - \omega^2 L_b M \right) + \omega \left(L - M \right) \left(R^2 + \omega^2 L_b^2 \right)}{R^2 + \omega^2 L_b^2} \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \frac{\omega \left(-M^2 \omega^2 L_b + L R^2 + L \omega^2 L_b^2 \right)}{R^2 + \omega^2 L_b^2} \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \frac{\omega \left(-M^2 \omega^2 L_b + L \left(R^2 + \omega^2 L_b^2 \right) \right)}{R^2 + \omega^2 L_b^2} \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \omega \left(\frac{-M^2 \omega^2 L_b + L \left(R^2 + \omega^2 L_b^2 \right)}{R^2 + \omega^2 L_b^2} \right) \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \omega \left(\frac{-M^2 \omega^2 L_b}{R^2 + \omega^2 L_b^2} + L \right) \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} - j \frac{M^2 \omega^3 L_b}{R^2 + \omega^2 L_b^2} + j \omega L \\ &= \frac{R\omega^2 M^2}{R^2 + \omega^2 L_b^2} + j \omega L \left(1 - \frac{M^2 \omega^2 \left(L_b / L \right)}{R^2 + \omega^2 L_b^2} \right) \\ &= R_{Thev} + j \chi_{Thev}. \end{split}$$

We see that the Thevnin impedance is a series composition of a real part R_{Thev} and a reactive part χ_{Thev} . For typical phase qubit parameters, $\chi_{Thev} >> R_{Thev}$. In such situations, the series combination of R_{Thev} and χ_{Thev} is well approximated by a parallel combination of two impedances, R' and $\Delta \chi$ given by

$$\frac{1}{Z_{Thev}} = \frac{1}{R_{Thev} + j\chi_{Thev}}$$

$$= \frac{1}{R_{Thev} + j\chi_{Thev}} \frac{R_{Thev} - j\chi_{Thev}}{R_{Thev} - j\chi_{Thev}}$$

$$= \frac{R_{Thev} - j\chi_{Thev}}{R_{Thev}^2 + \chi_{Thev}^2}$$

$$= \frac{R_{Thev} - j\chi_{Thev}}{\chi_{Thev}^2 (1 + \delta^2)}$$

$$= \frac{R_{Thev} - j\chi_{Thev}}{\chi_{Thev}^2} (1 - \delta^2)$$

$$= (1 - \delta^2) \frac{R_{Thev}}{\chi_{Thev}^2} - \frac{j(1 - \delta^2)}{\chi_{Thev}}$$

$$= \frac{1}{R'} + \frac{1}{j\Delta\chi},$$

where

$$\delta \equiv \frac{R_{Thev}}{\chi_{Thev}} << 1. \tag{9.87}$$

Keeping only to second order in δ we have

$$R' = \frac{L^2}{M^2} R \left[1 + \frac{\omega^2 L_b^2}{R^2} \left(1 - 2 \left(\frac{M^2}{L_b L} \right) + \left(\frac{M^2}{L_b L} \right)^2 \right) \right]$$
$$\approx \frac{L^2}{M^2} R \left(1 + \frac{\omega^2 L_b^2}{R^2} \right).$$

since for typical qubit parameters $M^2/L_bL \ll 1$. The reactive part, to second order in δ , is

$$\begin{aligned} \Delta \chi &= \omega L \left[1 - \frac{M^2 \omega^2 L_b}{L \left(R^2 + \omega^2 L_b^2\right)} \left(1 - \frac{R^2 M^2}{L L_b \left(R^2 + \omega^2 L_b^2\right)} + \frac{R^2 \omega^2 M^4}{L^2 \left(R^2 + \omega^2 L_b^2\right)^2} \right) \right] \\ &\approx j \omega L \left(1 - \frac{M^2 \omega^2 L_b}{L \left(R^2 + \omega^2 L_b^2\right)} \right), \end{aligned}$$

which we can see amounts to just a small perturbation on L. We define

$$L' = L \left(1 - \frac{M^2 \omega^2 L_b}{L \left(R^2 + \omega^2 L_b^2 \right)} \right).$$
(9.88)

The Thevnin voltage is given by the open-circuit voltage at Z,

$$V_{Thev} = V_s \frac{j\omega M}{j\omega L_b + R_b}$$

= $V_s \frac{j\omega M (R_b - j\omega L_b)}{R_b^2 + \omega^2 L_b^2}$
= $V_s \frac{(j\omega M R_b + \omega^2 M L_b)}{R_b^2 + \omega^2 L_b^2}$
= $V_s \left(\frac{\omega^2 M L_b}{R_b^2 + \omega^2 L_b^2} + \frac{j\omega M R_b}{R_b^2 + \omega^2 L_b^2}\right).$

The Norton current is then

$$\begin{split} I_{N} &= \frac{V_{Thev}}{Z_{Thev}} \\ &= V_{s} \left(\frac{1}{R'} - \frac{j}{\Delta \chi} \right) \left(\frac{\omega^{2} M L_{b}}{R_{b}^{2} + \omega^{2} L_{b}^{2}} + \frac{j \omega M R_{b}}{R_{b}^{2} + \omega^{2} L_{b}^{2}} \right) \\ &= V_{s} \left(\frac{M^{2} R}{L^{2} \left(R^{2} + (\omega L_{b})^{2} \right)} - \frac{j}{\omega L'} \right) \left(\frac{\omega^{2} M L_{b}}{R_{b}^{2} + \omega^{2} L_{b}^{2}} + \frac{j \omega M R_{b}}{R_{b}^{2} + \omega^{2} L_{b}^{2}} \right) \\ &= \frac{V_{s} M R_{b}}{L' \left(R_{b}^{2} + \omega^{2} L_{b}^{2} \right)} \left[1 + \frac{M^{2} L' \omega^{2} L_{b}}{L^{2} \left(R^{2} + \omega^{2} L_{b}^{2} \right)} \right] - j \omega \frac{1}{L'} \frac{V_{s} M L_{b}}{\left(R_{b}^{2} + \omega^{2} L_{b}^{2} \right)} \left[1 - \frac{L' M^{2} R_{b}^{2}}{L^{2} L_{b} \left(R^{2} + \omega^{2} L_{b}^{2} \right)} \right] \\ &\approx \frac{V_{s} M R_{b}}{L \left(R_{b}^{2} + \omega^{2} L_{b}^{2} \right)} - j \omega \frac{V_{s} M L_{b}}{L \left(R_{b}^{2} + \omega^{2} L_{b}^{2} \right)}. \end{split}$$

The Norton current has a magnitude,

$$|I_N| = \frac{M}{L} \frac{V_s}{\sqrt{R_b^2 + \omega^2 L_b^2}},$$
(9.89)



Figure 9.3: Norton-equivalent circuit model of a flux-biased phase qubit.

and a frequency-dependent phase

$$\tan \theta = -\frac{\omega L_b}{R_b}.\tag{9.90}$$

The Norton equivalent model of the flux-biased phase qubit is shown in figure 9.3.

9.1.5 The Effect of Transmission Lines and Attenuators

In the lab all generators are coupled to the experiment by matched transmission lines, with some amount of attenuation. The goal here is to show how their presence affects the response of the qubit in both the frequency and time domain. Then using these results we calculate a modified Thevnin equivalent voltage source and source impedance that incorporates their affects.

Figure 9.4 shows the qubit bias line connected to the current source via a transmission line with an attenuator. Here the length of the transmission line is Δx which is typically much longer than the wavelength of the signal it contains. For simplicity, the attenuator is assumed to have no electrical length. The current and voltage along the transmission line obey the one-dimensional wave equation

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 V}{\partial t^2} \tag{9.91}$$

$$\frac{\partial^2 I}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 I}{\partial t^2} \tag{9.92}$$

where ν is the speed of signal propagation along the line. The boundary conditions are set by the generator and qubit,

$$V(-\Delta x, t) = V_s(t) - I(-\Delta x, t)Z_0$$
(9.93)

$$V(0,t)_R = L_b \frac{\partial I_R}{\partial t}_{x=0} + M \frac{dI_L}{dt}, \qquad (9.94)$$



Figure 9.4: An ideal voltage source of impedance Z_0 coupled to the qubit via a matched, lossless transmission line and matched attenuator.

where $V(0,t)_R$ and $I(0,t)_R$ are the voltage and current to the *right* of the attenuator, as seen by the qubit, and I_L is the current in the qubit inductor, L. Also Z_0 is assumed to be purely real. Now we make the assumption that the voltage and currents along the line, as well as the source voltage, can be written as Fourier transforms

$$V(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} V[x,\omega] \exp[i\omega t] d\omega$$
(9.95)

$$I(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} I[x,\omega] \exp[i\omega t] d\omega$$
(9.96)

$$V_s(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} V_s[\omega] \exp[i\omega t] d\omega.$$
(9.97)

We proceed by taking the Fourier transform of the wave equations. The result is a pair of ordinary differential equations

$$\frac{d^2 V[x,\omega]}{dx^2} + \beta^2 V[x,\omega] = 0$$
(9.98)

$$\frac{d^2 I[x,\omega]}{dx^2} + \beta^2 I[x,\omega] = 0$$
(9.99)

where we have defined

$$\beta^2 \equiv \frac{\omega^2}{\nu^2}.\tag{9.100}$$

The general solutions are

$$V[x,\omega]_{L} = V_{0}^{+} \exp[-j\beta x] + V_{0}^{-} \exp[j\beta x]$$
(9.101)

$$I[x,\omega]_L = \frac{V_0^+}{Z_0} \exp[-j\beta x] - \frac{V_0^-}{Z_0} \exp[j\beta x], \qquad (9.102)$$

where we have explicitly noted that these waves correspond to the left of the attenuator. Applying the Fourier transform to the boundary conditions we get

$$V[-\Delta x,\omega] = V_s[\omega] - I[-\Delta x,\omega]Z_0$$
(9.103)

$$V[0,\omega]_R = I[0,\omega]_R Z_{qb}. (9.104)$$

For convenience, we will break $V[0, \omega]_R$ and $I[0, \omega]_R$ into incident and reflected amplitudes in analogy with the general solution of the wave equation,

$$V[0,\omega]_R = V_R^+ + V_R^- \tag{9.105}$$

$$I[0,\omega]_R = I_R^+ + I_R^-$$

= $\frac{V_R^+}{Z_0} - \frac{V_R^-}{Z_0}.$ (9.106)

Now, V_0^+, V_0^-, V_R^+ and V_R^- are related by the attenuator,

$$V_{R}^{+} = \alpha V_{0}^{+} \tag{9.107}$$

$$V_0^- = \alpha V_R^- \tag{9.108}$$

where

$$\alpha = 10^{-\frac{G}{20}} \tag{9.109}$$

where G is the attenuation of the attenuator in dB [62].

We have eight unknowns to solve for: $V_0^+, V_0^-, V[0, \omega]_R, I[0, \omega]_R, V_R^+, V_R^-, V[-\Delta x, \omega]$ and $I[-\Delta x, \omega]$. Using the Fourier transformed boundary conditions and the general solutions at x = 0 and $x = -\Delta x$, along with the attenuator relation, we have eight equations and can solve for the unknowns. A convenient way of solving this system is to use the reflection coefficient defined by

$$\Gamma_{qb} = \frac{V_R^-}{V_R^+} \tag{9.110}$$

$$= \frac{Z_{qb} - Z_0}{Z_{qb} + Z_0}.$$
(9.111)

Applying this to the attenuator relations we find

$$V_0^+ = \frac{1}{\alpha} V_R^+$$
(9.112)

$$V_0^- = \alpha \Gamma_{qb} V_R^+. \tag{9.113}$$

Now plug these expressions into the $x = -\Delta x$ boundary conditions along with the general solutions for the wave equation at $x = -\Delta x$ to get

$$V_R^+ = \frac{\alpha V_s[\omega]}{2} \exp\left[-j\beta\Delta x\right]. \tag{9.114}$$

Now all that is left is to plug this result into the expressions for $V[0, \omega]_R$ and $I[0, \omega]_R$ using the definition of Γ_{qb} . We get

$$V[0,\omega]_R = \alpha V_s[\omega] \frac{Z_{qb}}{Z_{qb} + Z_0} \exp\left[-j\beta\Delta x\right]$$
(9.115)

$$I[0,\omega]_R = \frac{\alpha V_s[\omega]}{Z_{qb} + Z_0} \exp\left[-j\beta\Delta x\right],\tag{9.116}$$

which, except for the factor, α , and the overall phase, $\exp[-j\beta\Delta x]$, is exactly the same result we would have gotten in the absence of the transmission line and attenuator. We can see how this phase affects the time-domain signals by taking the inverse transforms

$$V(0,t)_{R} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \alpha V_{s}[\omega] \frac{Z_{qb}}{Z_{qb} + Z_{0}} \exp[-j\beta\Delta x] \exp[j\omega t] d\omega$$

$$= \frac{\alpha}{\sqrt{2\pi}} \int_{-\infty}^{\infty} V_{s}[\omega] \frac{Z_{qb}}{Z_{qb} + Z_{0}} \exp[j\omega \left(t - \frac{\Delta x}{\nu}\right)] d\omega$$

$$= \alpha V_{No TL} \left(t - \frac{\Delta x}{\nu}\right), \qquad (9.117)$$

where $V_{NoTL}(t)$ is the qubit response in the absence of the transmission line and attenuator. Following the same procedure for the current we get

$$I(0,t)_R = \alpha I_{No\,TL} \left(t - \frac{\Delta x}{\nu} \right). \tag{9.118}$$

In the time domain, as we might expect, the transmission line has simply caused a time delay in the qubit response. Similarly, the attenuator has scaled the response by the factor, α . In fact, we can in effect "lump" the transmission line and attenuator in with the generator impedance by applying Thevnin's theorem at the qubit bias coil. Of course when we find the open circuit voltage here, the reflection coefficient at the qubit is now unity. Following the same procedure as above leads to

$$V_{Thev} = \alpha V_s \exp[-j\beta \Delta x]. \tag{9.119}$$

The Thevnin impedance is found by shorting the source and calculating the impedance in parallel with Z_{qb} . Ideally all components looking back toward the generator are perfectly matched so the Thevnin impedance



Figure 9.5: The Thevnin-equivalent qubit circuit incorporating effects from the transmission line and attenuator.

is simply

$$R_{Thev} = Z_0. \tag{9.120}$$

Figure 9.5 shows the original circuit and the lumped-element equivalent using Thevnin's theorem. In general, any parasitic impedances along the way to the qubit will only delay the sent signal as long as these impedances are perfectly matched to the generator and any acquired phase shifts are linear in frequency. In reality, there are imperfect impedances in the qubit drive lines. In particular, poor SMA connections throughout the line can result in signal distortion. This is why care must be taken to properly torque all connectors. Another source of impedance mis-match are at the wire bonds connecting the chip to the feed lines.

9.2 Home Made DC-Coupled Bias Tee

The dc-coupled bias tee used in the generation two experiment was to allow long-duration (~ 1 μ s) dc shift pulses with rise times of ~ 2 ns to be coupled to the qubit through the 50 Ohm rf lines. Most commercial bias tees use a dc-block on the rf side to protect the rf source from the dc current coming in from the dc side of the bias tee. The low-frequency cut-off for typical commercial bias tees is ~ 100 kHz. As such a dc shift pulse through the rf side will decay with a time constant of $\tau = 1/100$ kHz = 10 μ s. Getting the correct pulse shape to the qubit would then require some compensation at room temperature. In our circuits the dc block is simply not needed. Since the qubit bias coil is superconducting, all of the dc current gets shorted to that branch, making a dc block unnecessary (figure 9.6). Even if current were to couple to the rf side, the 40 dB worth of attenuation between the bias tee and room temperature equipment provides


Figure 9.6: a) Commercial bias tee with dc-block. b) DC-coupled bias tee.

a sufficient path to ground. We decided to make our own using 6 μ H broad-band conical inductors from Piconics. These inductors were designed so that resonances associated with parasitic capacitances in the inductor coils occur only at frequencies above ~ 13 GHz, giving good rf isolation in the relevant frequency ranges of the signals applied to the qubit.

9.3 Fabrication

9.3.1 First-Generation Circuit

Sapphire Wafer

1. Deposit base aluminum layer in SIS system

LL Vent, Mount wafer on platen, LL Pumpdown $(10:00 or < 5x10^{-}7T orr)$

Transfer wafer into process chamber

Check recipe ShaneBElayer

rfclean
60 - 15
mTorr, 60 W, 60 s

ShaneBElayer - line 240 deposition power 300 W

ShaneBElayer - line 825 deposition time 525 sec (150nm)

Record parameters on DATA SHEET

2. Pattern vacuum capacitor base holes

Clean spinner nozzle with acetone and IPA and purge it 5 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec

Spin 1 micron resist SPR 660L: 3200 rpm (setting 324) 40 sec Bake on hotplate 95 C for 60 sec with vacuum on. Expose on stepper: Job File: jb080514.qblcwithdrive.dli Reticles: ALIGN, BH Layer 1 "bh" Expose at 275 mJ/cm^2 Post-bake on hot plate 110 C for 60 sec with vacuum on. Spin-develop with MF-701 for 60 sec. Inspect under microscope Wet-etch Al using Al etchant type A at 48-49 C for 15 sec or until it clears (set hotplate 65-70 C) Inspect and re-dip in 8 s increments to get the Al holes to clear Inspect under microscope Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with ACETONE then IPA while spinning dry. Inspect under microscope 3. Pattern vacuum capacitor base outline and base wire Clean spinner nozzle with acetone and IPA and purge it 5 times. Spin HMDS at 3500 rpm (setting 381) for 35 sec Spin 1 micron resist SPR 660L: 3200 rpm (setting 324) 40 sec

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job File: jb080514.qblcwithdrive.dli

Reticles: ALIGN, BC

Layer 1 "bc"

Expose at 275 mJ/cm^2

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop with MF-701 for 60 sec.

Inspect under microscope

Wet-etch Al using Al etchant type A at 48-49 C (set hotplate 65-70 C)(15sec)

Inspect under microscope

Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with

ACETONE then IPA while spinning dry.

Inspect under microscope. Record Over etch amount.

Measure etched thickness using profilometer (100 nm)

Do O2 ash.

4. Deposit SiNx sacrificial layer

Glue sapphire wafer onto "spider" wafer (skip this step if using Si wafer)

Clean teflon chuck with acetone and IPA and blow it dry.

Mount teflon chuck and carefully place the sapphire wafer face (polished-side) down on the chuck (to spin resist on back of it).

Spin "glue" resist SPR 220-3, 2500 rpm, 35 sec (dispense manually). There should be no resist on the face of the sapphire wafer after spin.

Place the sapphire wafer, resist side down, onto a "spider" wafer with flats aligned, and push the edges together until the two are "glued" together

Bake on hotplate 95 C for 10 min with vacuum on (sapphire face up)

Inspect sapphire wafer under microscope for surface cleanliness

Spin-clean the wafer with acetone and IPA to clean resist from the edges of the wafer.

Ash 2:00 to make insulator stick better.

Deposit Insulator using PlasmaQuest ECR

Run process BIGCLEAN with the cleaning wafer in the machine.

Is the chamber manometer zeroed?

Is the chamber being heated?

Run a dummy wafer to get microwave power tuned

Load wafer and run your process SiNx RFcln (edit deposition time to deposit SiNx for 120 sec)

5. Pattern sacrificial layer

Spin HMDS at 3900 rpm (setting 390) for 35 sec

Evacuate HMDS fumes from spinner/Bake on hotplate 95 C for 60 sec with vacuum on.

Clean spinner nozzle with acetone and IPA and purge it 5 times.

Spin 1 micron resist SPR 660L: 2200 rpm (setting 215) 35 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: jb080514.qblcwithdrive.dli, clearout.

Reticles: SE, EH, ALIGN.

Layers "se", "eh".

Expose at 200 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop in MF-26A for 60 sec (make sure settings are as they are labeled on the machine).

Inspect under microscope.

Condition chamber by running JASnitride.prc for (5 min).

Etch through Insulator using AXIC. Use proper recipe (JASnitride.prc). Use acetone to make clean

spot for laser beam thickness monitor. Align beam (8-12 V).

Pump to base pressure p = 6x10-5 torr.

Start etch and watch graph for wavy curve.

Sketch etch curve below and where the laser monitor was relative to the flat on the wfr.

Inspect under microscope.

Strip resist.

Clean asher without wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Ash wafer 50 sccm O2 50 W (subtract any offset) 3:00 min.

Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with ACETONE then IPA while spinning dry.

Inspect under microscope.

Measure etched thickness using profilometer.

Record thickness in ECR log book.

6. Deposit 2nd aluminum layer for circuit base layer.

Same procedure as first aluminum layer.

7. Pattern ground plane holes.

Clean spinner nozzle with acetone and IPA, and purge it 10-20 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec.

Spin resist SPR 660L, 3200 rpm ' $\,$ 1micron, (setting 302), 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job file: jb080514.qblcwithdrive.dli.

Reticles: BG

Layer 3 "bg"

Exposure dose: 300 mJ/cm2

Post-bake on hot plate 110 C for 60 sec with vacuum on. Spin-develop in MF-701 for 60 sec (make sure settings are as they are labeled on the machine).

Inspect under microscope.

Wet Etch Aluminum: Heat Al etchant type A to 48-49 C (set hot plate to 65-70 C settings).

Use tripod and dip in etchant until 2 sec after it clears (15 sec).

Inspect under microscope.

Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with

ACETONE then IPA while spinning dry.

Inspect under microscope.

8. Pattern circuit base layer.

Clean spinner nozzle with acetone and IPA, and purge it 10-20 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec.

Spin resist SPR 660L, 3200 rpm ' 1micron, (setting 302), 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job file: jb080514.qblcwithdrive.dli.

Reticles: B.

Layer 3 "b".

Exposure dose: 300 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop in MF-701 for 60 sec (make sure settings are as they are labeled on the machine).

Inspect under microscope.

Wet Etch Aluminum:

Heat Al etchant type A to 48-49 C (set hot plate to 65-70 C settings).

Use tripod and dip in etchant until 2 sec after it clears (15 sec).

Inspect under microscope.

Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with ACETONE then IPA while spinning dry.

Inspect under microscope.

9. Deposit wiring insulator SiO2.

Glue sapphire wafer onto "spider" wafer (skip this step if using Si wafer).

Mount Teflon chuck and carefully place the sapphire wafer face (polished-side) down on the chuck (to spin resist on back of it).

Spin "glue" resist SPR 220-3, 2500 rpm, 35 sec (dispense manually). There should be no resist on the face of the sapphire wafer after spin.

Place the sapphire wafer, resist side down, onto a "spider" wafer with flats aligned, and push the edges together until the two are "glued" together.

Bake on hotplate 95 C for 10 min with vacuum on (sapphire face up).

Inspect sapphire wafer under microscope for surface cleanliness.

Spin-clean the wafer with acetone and IPA to clean resist from the edges of the wafer.

Ash 2:00 to make insulator stick better.

Deposit Insulator using PlasmaQuest ECR.

Run process BIGCLEAN with the cleaning wafer in the machine.

Load wafer and run your process SiO2 RFcln (edit deposition time to deposit SiO2 for 250 sec).

Ultrasound "dirty" acetone (1 min), separate wafers (skip this step if using Si wafer).

Ultrasound "dirty" acetone (1 min), Ultrasound "clean" acetone (1 min), spray HEAVELY with

ACETONE then IPA while spinning dry.

Inspect under microscope.

10. Pattern insulator for tunnel junction layer.

Clean spinner nozzle with acetone and IPA and purge it 10 times.

Spin HMDS and Resist SPR 660L.

Thin layer HMDS: 3500 rpm (setting 332) 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

1.23 micron Resist: 2200 rpm (setting 227) 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job file: jb080514.qblcwithdrive.dli, endpoint, (clearout).

Reticles: JI, FLOOD, (ALIGN for optional clearout).

Layer 3 "ji" (1, 2, 3, 4, 12, 34, or all)", "try" (1, 2, 3, 4, or all).

Exposure dose: 300 mJ/cm2.

Post-bake on hot plate 110 C for 70 sec with vacuum on.

Spin-develop in MF-701 for 70 sec (make sure settings are as they are labeled on the machine).

Inspect under microscope.

Etch through Insulator using AXIC. Use proper recipe (SiO2.prc).

Condition chamber by running sio2.prc for (30 min).

Pump to base pressure p = 6x10-5 torr.

Start etch and watch graph for wavy curve (see c)).

Sketch curve below and where the laser monitor was relative to the flat on the wfr.

Strip resist:

Clean Asher without wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Use Asher with wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Ultrasound "dirty" acetone (3 min), Ultrasound "clean" acetone (3 min), spray HEAVELY with ACETONE then IPA while spinning dry.

Inspect under microscope.

11. Oxidize and deposit aluminum JC layer.

Run process "Shane Junction" in SIS system.

 $\mathbf{P}=10$ torr. $\mathbf{T}=5065$ sec.

12. Pattern junction conductor layer.

Clean spinner nozzle with acetone and IPA, and purge it 10-20 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec.

Spin resist SPR 660L, 3200 rpm ' 1micron, (setting 302), 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job file: jb080514.qblcwithdrive.dli, endpoint.

Reticles: JC, FLOOD.

Layer 3 "jc" (1, 2, 3, 4, 12, 34, or all)", "all" to remove aluminum that covers future endpoints.

Exposure dose: 300 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop in MF-701 for 60 sec (make sure settings are as they are labeled on the machine).

Inspect under microscope.

Wet Etch Aluminum:

Heat Al etchant type A to 48-49 C (set hot plate to 65-70 C settings).

Use tripod and dip in etchant until 2 sec after it clears (15 sec).

Inspect under microscope.

Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with ACETONE then IPA while spinning dry.

Inspect under microscope.

Record resistance measurements.

13. Pattering top electrode wiring insulating layer.

Clean spinner nozzle with acetone and IPA, and purge it 10-20 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec.

Spin resist SPR 660L, 3200 rpm ' thick, (setting 227), 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job file: jb080514.qblcwithdrive.dli, endpoint (wi).

Reticles: WI, FLOOD.

Layer 3 "wi" (1, 2, 3, 4, 12, 34, or all)".

Exposure dose: 300 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop in MF-701 for 60 sec (make sure settings are as they are labeled on the machine).

Inspect under microscope.

Etch through Insulator using AXIC. Use proper recipe (SiO2.prc).

Condition chamber by running SiO2.prc for (15-20 min).

Pump to base pressure p = 6x10-5 torr.

Start etch and watch graph for wavy curve (see c)).

Sketch etch curve below and where the laser monitor was relative to the flat on the wfr.

Inspect under microscope.

Strip resist:

Clean Asher without wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Use Asher with wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Ultrasound "dirty" acetone (3 min), Ultrasound "clean" acetone (3 min), spray HEAVELY with

ACETONE then IPA while spinning dry.

Inspect under microscope.

14. Deposit aluminum top electric wiring layer.

Run process "BE layer" in SIS system.

15. Pattern top electrode wiring layer.

Clean spinner nozzle with acetone and IPA, and purge it 10-20 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec.

Spin resist SPR 660L, 3200 rpm ' 1micron, (setting 302), 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job file: jb080514.qblcwithdrive.dli.

Reticles: WC.

Layer 3 "wc" (1, 2, 3, 4, 12, 34, or all)".

Exposure dose: 300 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop in MF-701 for 60 sec (make sure settings are as they are labeled on the machine).

Inspect under microscope.

Wet Etch Aluminum:

Heat Al etchant type A to 48-49 C (set hot plate to 65-70 C settings).

Use tripod and dip in etchant until 2 sec after it clears (15 sec).

Inspect under microscope.

Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with

ACETONE then IPA while spinning dry.

Inspect under microscope.

16. Pattern insulator etch layer.

Clean spinner nozzle with acetone and IPA, and purge it 10-20 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec.

Spin resist SPR 660L, 3200 rpm ' thick, (setting 227), 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job file: jb080514.qblcwithdrive.dli.

Reticles: IE.

Layer 3 "IE" (1, 2, 3, 4, 12, 34, or all)".

Exposure dose: 300 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop in MF-701 for 60 sec (make sure settings are as they are labeled on the machine).

 $In spect \ under \ microscope.$

Etch through Insulator using AXIC. Use proper recipe (sio2.prc).

Use acetone to make clean spot for laser beam thickness monitor. Align beam (8-12 V).

Condition chamber by running sio2.prc for (15-20 min).

Pump to base pressure p = 6x10-5 torr.

Start etch and watch graph for wavy curve (see c)).

Sketch etch curve below and where the laser monitor was relative to the flat on the wfr.

Strip resist:

Clean Asher without wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Use Asher with wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Ultrasound "dirty" acetone (3 min), Ultrasound "clean" acetone (3 min), spray HEAVELY with

ACETONE then IPA while spinning dry.

Inspect under microscope.

Measure etched thickness using profilometer.

17. Pattern vacuum capacitor top holes layer.

Clean spinner nozzle with acetone and IPA and purge it 5 times.

Spin HMDS at 3500 rpm (setting 381) for 35 sec.

Spin 1 micron resist SPR 660L: 3200 rpm (setting 302) 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job File: jb080514.qblcwithdrive.dli.

Reticle: TH.

Layer 4 "th".

EXPOSE AT 360 ?J/cm2 FOCUS -0.4.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop with MF-701 for 60 sec.

Inspect under microscope.

Wet-etch Al using Al etchant type A at 48-49 C for 15 sec or until it clears (set hotplate 65-70 C).

Inspect and re-dip in 8 s increments to get the Al holes to clear (23) s.

Inspect under microscope.

Ultrasound "dirty" acetone (2 min), Ultrasound "clean" acetone (2 min), spray HEAVELY with ACETONE then IPA while spinning dry.

Inspect under microscope.

18. Dice wafer.

Spin protective layer of resist:

Spin resist SPR 660L, 3200 rpm ' 1micron, (setting 302), 40 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Dice wafer on a dicing saw using resinoid blade and parameters: 060/246/246/8/35/90/100/3000/3.

19. Create vacuum capacitory by dry etching sacrificial layer.

Strip resist on a spinner with acetone and IPA from individual chips to be processed further.

You can either use IPE RIE SF6 etch to remove sacrificial layer, or use XeF2 dry chemical etcher.

If using IPE RIE SF6 etch (it will heat the whole chip):

Dry etch in IPE RIE to remove SiNx between plates.

Prepare several extra dies as some will be needed for etch calibration check.

Pre-condition machine by running plasma for 5-10 min without chips, process kcsf6.prc.

Run process kcsf6.prc to etch SiNx (avoid etching for longer than 3-4 min since chips get hot. If longer

etching is needed, stop the process and re-run it again several times).

Inspect under microscope.

20. Select test dies for SEM inspection.

21. Select dies for cool down.

9.3.2 Second-Generation Circuit

Sapphire wafer.

1. Deposit Aluminum base layer.

Use recipe "QubitBaseCleanAldep" (sub-recipes: qbsubClean60W, qbsubAldep100nm) in SIS system.

2. Pattern alignment marks.

Spin HMDS at 3900 rpm (setting 390) for 35 sec.

Evacuate HMDS fumes from spinner/Bake on hotplate 95 C for 60 sec with vacuum on.

Clean spinner nozzle with acetone and IPA and purge it 3 times.

Spin 1 micron resist SPR 660L: 2800 rpm 35 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job File: jb100920.qblctunm.gen2.msa.

Reticles: ALIGN, Layer 0 "PM".

Expose at 180 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop with MF-26A for 60 sec.

Inspect under microscope.

Etch Al using Trion etcher:

Use recipe kcAlvertical for (60s) (no endpoint detection).

He press: 5.0 torr He flow: 2.0 sccm RIE power: 200 W.

Pressure: 30 torr Cl2 flow: 10 sccm BCl3 flow: 30 sccm 5 nm/s (100 nm/20 s).

Passivate in DI water for 2 min, Sonicate in DI water for 2 min at 50V, spin dry.

Ultrasound at 50V in acetone (2 min), IPA (2 min), H2O washer.

Inspect under microscope.

Measure etched thickness with profilometer.

2. Pattern wiring cross-under layer.

Spin HMDS at 3900 rpm (setting 390) for 35 sec.

Evacuate HMDS fumes from spinner/Bake on hotplate 95 C for 60 sec with vacuum on.

Clean spinner nozzle with acetone and IPA and purge it 3 times.

Spin 1 micron resist SPR 660L: 2800 rpm 35 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: Job File: jb100920.qblctunm.gen2.msa.

Reticles: BC, Layer 2 "bc".

Expose at 180 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop with MF-26A for 60 sec.

Inspect under microscope.

Etch Al using Trion etcher.

Use recipe kcAlvertical for (25s) (includes 3s overetch).

He press: 5.0 torr He flow: 2.0 sccm RIE power: 200 W.

Pressure: 30 torr Cl2 flow: 10 sccm BCl3 flow: 30 sccm 5 nm/s (100 nm/20 s).

Passivate in DI water for 2 min, Sonicate in DI water for 2 min at 50V, spin dry.

Ultrasound at 50V in acetone (2 min), IPA (2 min), H2O washer.

Inspect under microscope.

Measure etched thickness with profilometer.

3. Deposit SiO2 insulator.

Glue sapphire wafer onto "spider" wafer:

Spin "glue" resist SPR 220-3, 2500 rpm, 35 sec (dispense manually) on spider.

Glue sapphire wafer onto a "spider" wafer with flats aligned, push, bake 95 C for 10 min.

Spin-clean the wafer with acetone and IPA to clean resist from the edges of the wafer.

Ash 2:00 to make insulator stick better.

Deposit Insulator using PlasmaQuest ECR:

Run process BIGCLEAN if many SiO2 deps (¿1000nm) have been done before.

Follow ECR instructions for loading, running, etc.

Load wafer and run your process SiO2RFcln (edit deposition time to deposit SiO2 for 190 sec).

Ultrasound at 50V acetone (1 min), separate wafers (skip if Si wafer).

Ultrasound at 50V in acetone (2 min), IPA (2 min), H2O washer. Ash 2:00 min.

4. Pattern insulator layer.

Spin HMDS at 3900 rpm (setting 390) for 35 sec.

Evacuate HMDS fumes from spinner/Bake on hotplate 95 C for 60 sec with vacuum on.

Clean spinner nozzle with acetone and IPA and purge it 3 times.

Spin 1.23 (thick) micron resist SPR 660L: 2000 (or 2200?) rpm 35 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: jb100920.qblctunm.gen2.msa, clearout, endpoint

Reticles: SE, ALIGN, FLOOD, Layer 3 "se", endpoint layer, clearout.

Expose at 130 mJ/cm2 (Sparse amnt of Al on BC layer = less exposure for SiO_2).

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop in MF-26A for 60 sec (make sure settings are as they are labeled on the machine). Inspect under microscope.

Etch through Insulator using AXIC. Use proper recipe (SiO2.prc).

Condition chamber by running sio2.prc for (30 min).

Pump to base pressure p = 6x10-5 torr.

Start etch and watch graph for wavy curve (see c)).

Sketch etch curve below and where the laser monitor was relative to the flat on the wfr.

Inspect under microscope.

Strip resist: Ash wafer: 50 sccm O2 50 W (subtract any offset) 3:00 min.

Ultrasound at 50V in acetone (2 min), IPA (2 min), H2O washer.

Inspect under microscope.

Measure thickness with profilometer.

Record thickness in ECR log book. Deposition rate.

5. Deposit aluminum circuit base layer.

Use recipe "QubitBaseCleanAldep" (sub-recipes: qbsubClean60W, qbsubAldep100nm) in SIS system.

6. Pattern circuit base layer and ground plane holes.

Spin HMDS at 3900 rpm (setting 390) for 35 sec.

Evacuate HMDS fumes from spinner/Bake on hotplate 95 C for 60 sec with vacuum on.

Clean spinner nozzle with acetone and IPA and purge it 3 times.

Spin 1 micron resist SPR 660L: 2800 rpm 35 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Expose on stepper: jb100920.qblctunm.gen2.msa.

Reticles: B2R1, B2R2, B2R3, BG.

Layers 4, "b".

Expose at 200 mJ/cm2.

Post-bake on hot plate 110 C for 60 sec with vacuum on.

Spin-develop with MF-26A for 60 sec.

Inspect under microscope.

Etch Al using Trion etcher.

Use recipe kcAlvertical for (25s) (includes 3s overetch).

He press: 5.0 torr He flow: 2.0 sccm RIE power: 200 W.

Pressure: 30 torr Cl2 flow: 10 sccm BCl3 flow: 30 sccm 5 nm/s (100 nm/20 s).

Passivate in DI water for 2 min, Sonicate in DI water for 2 min at 50V, spin dry.

Ultrasound at 50V in acetone (2 min), IPA (2 min), H2O washer.

Measure etched thickness using profilometer.

Inspect under microscope.

7. Make tunnel junctions using aluminum shadow evaporation.

Resist Prep:

HMDS, 3900 rpm for 35s, bake 60 s at 95 deg C

Apply smooth puddle half-wafer diameter of LOR 20B

500 rpm @ 500 rpm/s for 5 s

3000 rpm @ 10,000 rpm/s for 45 s

Clean wafer edge while spinning fast with Nano EBR remover-PG soaked cloth

Bake @ 170 deg C for 5 min.

Photoresist

bake @ 95 deg C for 1 min.

Expose at $180 M J/cm^2$

post-bake @ 110 deg C for 1 min.

Spin develop: 5 s pre-wet, 26A for 1 min, 45 rinse, spin 20 sec.

 $\mathrm{O2}$ ash at 50 W and 50 sccm of O2 for 30 sec.

Inspect

Measure resist height of stack.

Deposition:

Ion Mill 40 seconds:

Ion Tech, Inc. MPS-300 FC

Cathode Filament Current = 3.67 A

Discharge Current = 0.40 A

Beam Current = 32 mA

Accelerator Current = 2 mA

Neutralizer Current = 51 mA

Discharge Voltage = 55.0 V

Beam Voltage = 300 V

Accelerator Voltage = 950 V

Filament Current = 3.74 A

Dep angle was +/-19.2 deg

 $\mathrm{Pm}=6.930\times10^{-8}$

 $\mathrm{PLL} = 1.071 \times 10^{-6}$

Deposit: Al @ 3.8 A/s to 75 nm

Oxidize: 750 mTorr, for 10 min.

Deposit: Al @ 3.8 A/s to 150 nm

Cleaning:

NO ULTRASOUND!

Liftoff in Acetone overnight (¿3 hours).

Clean Acetone for 5 min.

Clean Acetone for 5 min.

Nano-remover PG at 80 deg C for 30 min.

Nano-remover PG at 80 deg C for 30 min.

Clean Isopropyl for 5 min.

Clean Isopropyl for 5 min.

Inspect under microscope.

8. Record resistance measurements.

9. Dice wafer.

Spin protective layer of resist:

Spin HMDS at 3900 rpm (setting 390) for 35 sec.

Evacuate HMDS fumes from spinner/Bake on hotplate 95 C for 60 sec with vacuum on.

Spin 1 micron resist SPR 660L: 2200 rpm (setting 215) 35 sec.

Bake on hotplate 95 C for 60 sec with vacuum on.

Bake on hotplate 110 C for 60 sec with vacuum on.

Dice wafer on a dicing saw using resinoid blade: 060/3000/3000/246/246/8/35/90/100/3/15000.

10. Select dies for SEM inspection and cooldown.

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