

## Research Article

# A Time-Dependent $\Lambda$ and $G$ Cosmological Model Consistent with Cosmological Constraints

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The prevailing constant  $\Lambda$ - $G$  cosmological model agrees with observational evidence including the observed red shift, Big Bang Nucleosynthesis (BBN), and the current rate of acceleration. It assumes that matter contributes 27% to the current density of the universe, with the rest (73%) coming from dark energy represented by the Einstein cosmological parameter  $\Lambda$  in the governing Friedmann-Robertson-Walker equations, derived from Einstein's equations of general relativity. However, the principal problem is the extremely small value of the cosmological parameter ( $\sim 10^{-52} \text{ m}^2$ ). Moreover, the dark energy density represented by  $\Lambda$  is presumed to have remained unchanged as the universe expanded by 26 orders of magnitude. Attempts to overcome this deficiency often invoke a variable  $\Lambda$ - $G$  model. Cosmic constraints from action principles require that either both  $G$  and  $\Lambda$  remain time-invariant or both vary in time. Here, we propose a variable  $\Lambda$ - $G$  cosmological model consistent with the latest red shift data, the current acceleration rate, and BBN, provided the split between matter and dark energy is 18% and 82%.  $\Lambda$  decreases ( $\Lambda \sim \tau^{-2}$ , where  $\tau$  is the normalized cosmic time) and  $G$  increases ( $G \sim \tau^n$ ) with cosmic time. The model results depend only on the chosen value of  $\Lambda$  at present and in the far future and not directly on  $G$ .

## 1. Introduction

The Newtonian gravitational parameter  $G$  is critical in both cosmology and quantum mechanics. It occurs in the former as a source term in Einstein's general relativity equations, the basis of all cosmological models. In the latter, it is fundamental to the definition of the Planck scales. The standard cosmological model assumes that  $G$  is invariant with cosmic time. Likewise, the cosmological parameter  $\Lambda$ , a well-established surrogate for dark energy density, is generally assumed to be another universal constant. However, its value of  $10^{-52} \text{ m}^2$  is 50 orders of magnitude less than what was predicted by the Glashow-Weinberg-Salam weak interaction theory [1] and 107 orders of magnitude less than what was required for grand unification [2]. If indeed  $\Lambda$  represents dark energy density, then is it reasonable for it to be cosmic time-invariant as the universe expanded 26 orders of magnitude from the Big Bang to the present? Instead, could it have decayed from much larger values in the cosmic past? The central issues explored here are as follows. Are time-dependent

$\Lambda$  and  $G$  cosmologies consistent with modern astronomical observations, especially the red shift data as reported in [3]? Are they consistent with Big Bang Nucleosynthesis (BBN)? Are they consistent with the current observed rate of acceleration?

Motivated by the huge disparity between the distance scales in the fundamental force fields and the size of the universe, Dirac [4, 5] proposed a time-dependent  $G$  cosmology. There has been considerable interest in cosmologies with variable parameters ever since. See [6–24] for recent studies and the references cited in [13] for citations to earlier research on the topic. These models considered either  $G$  or both  $G$  and  $\Lambda$  to be time-dependent. Most investigators take  $\Lambda \sim t^{-2}$  but allowed  $G$  to be either proportional or inversely proportional to time  $t$ . However, dimensional analysis [25] suggests that  $\Lambda$  and  $G$  cannot vary independently and so does the action principle [26]. Using action principle, Krori et al. [26] developed a Friedmann-Robertson-Walker (FRW) cosmology with a variable  $\Lambda$  as a function of  $G$ . Jamil and Debnath [17] extended that work with a model in which

$\Lambda \sim H^2$ , where  $H$  is the Hubble parameter. K. P. Singh and N. I. Singh [13] explored a cosmological model with  $\Lambda \sim H^2$  also, but with matter in the form of a viscous fluid. Building on Bergmann's action principle [27, 28], Esposito-Farèse and Polarski [29] developed a general scalar-tensor model that allowed for variable cosmological parameters. This model was extended by Riazuelo and Uzan [30] and used by Ellis and Uzan [31] in their critique of variable parameter cosmologies. Caldwell et al. [32] examined the possibility that there is a significant contribution to energy density of the universe from a component, like a cosmic scalar field, which has an equation of state different from that of matter, radiation, and cosmological constant.

The rest of this paper is organized as follows. Section 2 reviews the theory for cosmic time-dependent  $\Lambda$  and  $G$ . Section 3 explores the compatibility of the model with BBN. Section 4 compares the model results with the latest red shift data as summarized in [3]. Section 5 deals with the implications of variable  $\Lambda$ - $G$  cosmology. We conclude with a brief commentary on what remains unknown.

## 2. Theory

Cosmological models are based on the FRW equations for the scale factor  $a$  (e.g., [26]):

$$\begin{aligned} 3\left(\frac{\dot{a}}{a}\right)^2 &= \Lambda c^2 + \rho G, \\ 3\left(\frac{\ddot{a}}{a}\right) &= \Lambda c^2 - \frac{G}{2}\left(\rho + \frac{3p}{c^2}\right). \end{aligned} \quad (1)$$

Here,  $\rho$  is density,  $p$  is pressure, and dots denote cosmic time derivative. Note that we have omitted the curvature terms, since there is solid evidence that the universe is flat [33], as well as the shear terms that are often included [26]. As a consequence of an action principle, Krori et al. [26] showed that the use of (1) requires that  $G$  and  $\Lambda$  be both time-invariant or that  $G$  and  $\Lambda$  be both time-dependent.

In the latter case, an action principle constraint determines the covariation of  $G$  and  $\Lambda$ . This can be seen by differentiating the first equation of (1) and substituting for  $\ddot{a}$  from the second equation to obtain

$$G\left[\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right)\right] + \rho\dot{G} + c^2\dot{\Lambda} = 0. \quad (2)$$

This equation is the result of vanishing divergence of the Einstein tensor [17, 26]. The usual energy-momentum conservation equation leads to [17, 26]

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0. \quad (3)$$

The use of (3) in (2) shows that

$$\rho\dot{G} + c^2\dot{\Lambda} = 0 \quad (4)$$

so that  $G$  and  $\Lambda$  must both be time-invariant or  $G$  and  $\Lambda$  must both be time-dependent, consistent with the findings of [17, 26]. Also, recall the equation of state:

$$\frac{p}{c^2} = (\gamma - 1)\rho, \quad (5)$$

with  $\gamma = 1$  for matter and  $\gamma = 4/3$  for radiation.

Now return to (1). The natural time and length scales in the problem are  $t_0 = 1/H_0 = 4.3582 \times 10^{17}$  s (13.81 Gyr) and  $a_0 = c/H_0 = 1.3066 \times 10^{26}$  m (where  $c = 2.998 \times 10^8$  m s<sup>-1</sup> and  $H_0 = 70.8$  km s<sup>-1</sup> Mpc<sup>-1</sup> =  $2.2945 \times 10^{-18}$  s<sup>-1</sup>). Using these to normalize (1) yields

$$\begin{aligned} \left(\frac{\mathfrak{R}'}{\mathfrak{R}}\right)^2 &= \Omega_\Lambda + \Omega_G(\Omega_m + \Omega_r), \\ \left(\frac{\mathfrak{R}''}{\mathfrak{R}}\right) &= \Omega_\Lambda - \frac{\Omega_G}{2}[(3\gamma - 2)(\Omega_m + \Omega_r)], \end{aligned} \quad (6)$$

where  $\mathfrak{R} = a/a_0$  and the primes denote derivatives with respect to normalized time  $\tau = t/t_0$ . Also,

$$\begin{aligned} \Omega_m &= \frac{\rho_m}{\rho_{\text{cr}}}, \\ \Omega_r &= \frac{\rho_r}{\rho_{\text{cr}}}, \\ \Omega_\Lambda &= \frac{\Lambda c^2}{3H_0^2}, \\ \Omega_G &= \frac{G}{G_0}, \end{aligned} \quad (7)$$

where subscript 0 denotes current values. The critical density of the universe is

$$\rho_{\text{cr}} = \frac{3H_0^2}{G_0} = 9.4161 \times 10^{-27} \text{ kg m}^{-3}. \quad (8)$$

Take the ‘‘vacuum’’ density as

$$\begin{aligned} \rho_\Lambda &= \Lambda c^2 G^{-1} = \rho_{\text{cr}} \Omega_\Lambda \Omega_G^{-1}, \\ \frac{\rho_{\text{tot}}}{\rho_{\text{cr}}} &= \Omega_m + \Omega_r + \left(\frac{\Omega_\Lambda}{\Omega_G}\right). \end{aligned} \quad (9)$$

Because matter density is inversely proportional to the volume and the total entropy of radiation is constant,

$$\begin{aligned} \Omega_m &= \Omega_{m0} \mathfrak{R}^{-3}, \\ \Omega_r &= \Omega_{r0} \mathfrak{R}^{-4}, \end{aligned} \quad (10)$$

so that the first equation of (6) becomes

$$\left(\frac{\mathfrak{R}'}{\mathfrak{R}}\right)^2 = \Omega_\Lambda + \Omega_G(\Omega_{m0} \mathfrak{R}^{-3} + \Omega_{r0} \mathfrak{R}^{-4}). \quad (11)$$

The consensus from modern observations [34] is as follows:

$$\begin{aligned} \Omega_{m0} &= 0.246 \pm 0.028, \\ \Omega_{\Lambda 0} &= 0.757 \pm 0.021, \\ \Omega_0 &= 1.0031 \pm 0.010, \\ Q_0 &= -0.64 \pm 0.03. \end{aligned} \quad (12)$$

Since  $\rho_r = 4\sigma T^4 c^{-3}$ , using the Cosmic Microwave Background (CMB) value of 2.725 K for  $T_0$ ,  $\Omega_{r0} = 4.9321 \times 10^{-5}$ .

In time-dependent cosmology, there is broad consensus for  $\Lambda \sim t^{-2}$  or equivalently

$$\Omega_\Lambda \sim \tau^{-2}. \quad (13)$$

However, both  $\Omega_G$  and  $\Omega_\Lambda$  must be either cosmic time-invariant or time-dependent according to (4). A general yet simple model that allows for both possibilities is the following:

$$\Omega_G = \Omega_{G1} + (1 - \Omega_{G1}) \tau^\alpha, \quad (14)$$

$$\Omega_\Lambda = \Omega_{\Lambda1} + (\Omega_{\Lambda0} - \Omega_{\Lambda1}) \tau^{-2}.$$

Here, subscript 0 indicates values at the present epoch. The parameters in (14) are not independent as they are constrained by (4). Using (7) in (4), we get  $(\Omega_m + \Omega_r)\Omega'_G + \Omega'_\Lambda = 0$  so that

$$\begin{aligned} \frac{\rho}{\rho_{cr}} &= \Omega_m + \Omega_r = -\frac{\Omega'_\Lambda}{\Omega'_G} \\ &= -\left(\frac{\beta}{\alpha}\right) \left(\frac{\Omega_{\Lambda0} - \Omega_{\Lambda1}}{\Omega_{G0} - \Omega_{G1}}\right) \tau^{\beta-\alpha}. \end{aligned} \quad (15)$$

Since density is a positive definite quantity, the right hand side of (15) must be  $<0$ . Thus, (15) is a basic constraint on time-dependent cosmologies. There is also a dynamical constraint. Substituting (15) into the first equation of (6) gives

$$\left(\frac{\mathfrak{R}'}{\mathfrak{R}}\right)^2 = \Omega_\Lambda - \Omega_G \left(\frac{\beta}{\alpha}\right) \left(\frac{\Omega_{\Lambda0} - \Omega_{\Lambda1}}{\Omega_{G0} - \Omega_{G1}}\right) \tau^{\beta-\alpha}. \quad (16)$$

Invoking  $\mathfrak{R}'/\mathfrak{R} = 1$  for  $\tau = 1$  gives

$$\left(\frac{\beta}{\alpha}\right) \left(\frac{\Omega_{\Lambda0} - \Omega_{\Lambda1}}{\Omega_{G0} - \Omega_{G1}}\right) = \frac{\Omega_{\Lambda0} - 1}{\Omega_{G0}}. \quad (17)$$

Thus, (16) simplifies to

$$\left(\frac{\mathfrak{R}'}{\mathfrak{R}}\right)^2 = \Omega_\Lambda + (1 - \Omega_{\Lambda0}) \left(\frac{\Omega_G}{\Omega_{G0}}\right) \tau^{\beta-\alpha}. \quad (18)$$

### 3. Big Bang (Primordial) Nucleosynthesis

Very important observational support for the Big Bang theory derives from Big Bang Nucleosynthesis (BBN), which, in its very early phase, led to the formation of nuclei of light elements in the universe. The standard constant  $G$ - $\Lambda$  model explains the relative abundance of light elements such as hydrogen, helium, and lithium in the universe nearly perfectly. Any alternative cosmological model proposed must do the same. A very brief summary of BBN is in order here (but see excellent reviews by Steigman [35, 36] and Olive et al. [37]).

Initially, up to  $\sim 1\mu\text{s}$ , the universe was a quark soup. Then, quarks combined to form ionized plasma of photons, electrons, positrons, neutrinos, protons, and neutrons.

Because of the very high prevailing temperatures, protons could combine with electrons to form neutrons. Equilibrium with the relevant nuclear reaction rates meant that the  $n/p$  ratio was  $\sim 1$ , where  $n$  is the number of neutrons and  $p$  is the number of protons. This ratio stayed roughly at unity, until the temperature dropped to about 10 MeV (per particle,  $\sim 10^{11}$  K). As the temperature fell further to about 0.8 MeV ( $8 \times 10^9$  K), neutrons could no longer form, since the weak reaction rate that made it possible became slower than the expansion rate of the universe, leading to “freeze-out” of  $n/p$  ratio at about 1/6 at  $\sim 1$  s. Neutrons, no longer being formed, began to decay to protons with a half-life of  $\sim 887$  seconds, the decay rate being independent of temperature, as long as the temperature does not fall below  $\sim 0.1$  MeV ( $\sim 10^9$  K), which occurs at  $\sim 115$  s in standard cosmology. The  $n/p$  ratio began to gradually decrease by this radioactive decay. During roughly the 114 s time delay for the temperature to fall from  $\sim 8 \times 10^9$  K to  $\sim 10^9$  K, the  $n/p$  ratio decreased from  $\sim 1/6$  to  $\sim 1/7$ . Once the temperature fell to 0.1 MeV, neutrons began to be rapidly incorporated [34] into nuclei of  ${}^4\text{He}$  with a very high efficiency of 99.99%. The resulting primordial  ${}^4\text{He}$  mass fraction is given by

$$Y_p \approx 2 \left(\frac{n}{p}\right) \left[1 + \left(\frac{n}{p}\right)\right]^{-1}. \quad (19)$$

For  $n/p$  of  $\sim 1/7$ ,  $Y_p \sim 0.25$ . Other light elements such as lithium also formed, but when the temperature fell to about 80 keV ( $\sim 8 \times 10^8$  K), nuclear reactions ceased and BBN was over. The relative abundance of light elements in the universe has since remained unaltered. Thus, in a short span of 1 s to  $\sim 1,000$  s, BBN occurred and lighter elements that we know today formed in the universe.

The observed value of the relative  ${}^4\text{He}$  abundance  $Y_p$  lies in a very narrow range of 0.228 to 0.248 thus constraining the value of  $n/p$  to between 1/7.06 and 1/7.77 at the beginning of BBN. This provides a powerful constraint on the expansion rate of the universe, and any cosmological model that does not obey this constraint cannot be valid. It is a race between the very well known rate of nuclear reactions and the expansion rate of the universe. If the change in temperature takes a much longer time than  $\sim 114$  s,  $n/p$  ratio would decrease to unacceptably small values affecting  $Y_p$  (e.g., for 1-hour time delay,  $Y_p \sim 0.01$ ). In the standard model, scale factor  $a \sim t^{1/2}$  and therefore temperature  $T \sim a^{-1} \sim t^{-1/2}$  drop from  $2.1 \times 10^{10}$  K to  $2.1 \times 10^9$  K as time increases from  $\sim 1$  s to  $\sim 115$  s. A hundredfold change in time brings about a 10-fold change in  $T$  and this leads to  $n/p$  decrease from  $\sim 1/6$  to  $\sim 1/7$  and leads to  $Y_p$  value of  $\sim 0.25$ . At  $\sim 1,000$  s, the temperature drops by a further factor of  $\sqrt{10}$  to  $\sim 6.6 \times 10^8$  and BBN ceases.

During early phases of the Big Bang, radiation dominated the universe, and most of the contribution to density was from radiation and not matter (baryons). The temperature and density of the universe are related by  $\rho = (4\sigma/c^3)T^4$

through simple thermodynamics. So, given the density, temperature is determined. During the Big Bang, density of radiation varies as  $\rho = \rho_0 \mathfrak{R}^{-4}$ . Therefore,  $T$  is given by

$$T = \left[ \frac{\rho_{\text{cr}} c^3}{4\sigma} \left( \frac{\rho_0}{\rho_{\text{cr}}} \right) \right]^{1/4} \left( \frac{\rho}{\rho_0} \right)^{1/4} = c_r \mathfrak{R}^{-1}, \quad (20)$$

where  $c_r$  is 2.725, the temperature of the Cosmic Microwave Background (CMB) radiation at present. Therefore, the normalized scale factor  $\mathfrak{R} = a/a_0$  must be such that  $\mathfrak{R}_{\text{cr1}} = c_r/T_{\text{cr1}} = 1.27 \times 10^{-10}$  for  $T_{\text{cr1}} = 2.1 \times 10^{10}$  K. Similarly,  $\mathfrak{R}_{\text{cr2}} = c_r/T_{\text{cr2}} = 1.36 \times 10^{-9}$ . These values are independent of the cosmological model used. However, the model must provide an expansion rate that yields a time difference of approximately 115–120 s between the two events, no more, no less.

#### 4. Solutions

We put  $\Omega_{G0} = 1$  and  $\Omega_{G1} = 0$  so that

$$\Omega_G = \tau^\alpha. \quad (21)$$

Equation (18) becomes

$$\left( \frac{\mathfrak{R}'}{\mathfrak{R}} \right)^2 = \Omega_{\Lambda 1} + (1 - \Omega_{\Lambda 0}) \tau^\beta. \quad (22)$$

It is remarkable that this equation does not contain the gravitational parameter  $G$  directly. Now, consider the power law behavior of (22) as  $\tau \rightarrow 0$ . Let

$$\mathfrak{R} = b\tau^\varepsilon. \quad (23)$$

Then, the left hand side of (22) is  $\varepsilon^2 \tau^{-2}$ . Therefore,

$$\begin{aligned} \beta &= -2, \\ \varepsilon &= \sqrt{1 - \Omega_{\Lambda 1}} \end{aligned} \quad (24)$$

so that

$$\Omega_\Lambda \sim (\Omega_{\Lambda 0} - \Omega_{\Lambda 1}) \tau^{-2}. \quad (25)$$

From (17),

$$\alpha = \frac{\beta(\Omega_{\Lambda 0} - \Omega_{\Lambda 1})}{(\Omega_{\Lambda 0} - 1)}. \quad (26)$$

The value of  $\varepsilon$  is the negative of the value of the deceleration parameter  $Q$  at  $\tau = 1$ . This therefore fixes the value of  $\Omega_{\Lambda 1}$  at 0.60, the same as the standard model. The value of  $b$  must be  $\sim 10$  for model consistency with BBN (see Table 1). This leaves only the value of  $\Omega_{\Lambda 0}$  as a free parameter. The value of  $\alpha$  is of course determined by the chosen value of  $\Omega_{\Lambda 0}$ .

The accepted consensus value for  $\Omega_{\Lambda 0}$  is 0.73. However, the model results do not agree well with red shift data (see Figure 1) for this value. The best agreement is obtained for  $\Omega_{\Lambda 0} = 0.82$ . Table 1 contrasts the values of cosmological parameters for the two models.

TABLE 1: The value of various cosmological parameters in the two models.

Parameter	Standard model	Variable $G$ - $\Lambda$ model
$\Omega_{\Lambda 0}$	0.73	0.82
$\Omega_{\Lambda 1}$	—	0.60
$\Omega_{m0}$	0.27	0.18
$\Omega_{r0}$	$4.9321 \times 10^{-5}$	$4.9321 \times 10^{-5}$
$Q_{\tau=1}$	-0.6	-0.6
$\alpha$	0	2.5
$\beta$	0	-2
$t_{\text{cr1}}$ (s)	1.0	2.6
$T_{\text{cr1}}$ (K)	$2.147 \times 10^{10}$	$2.147 \times 10^{10}$
$t_{\text{cr2}}$ (s)	115	109.8
$T_{\text{cr2}}$ (K)	$2.0 \times 10^9$	$2.0 \times 10^9$
$\Delta t_{\text{cr}}$ (s)	114	107.2
$(p/n)_1$	6	6
$(p/n)_2$	7.56	7.52
$Y_p$	0.234	0.235

Another critical test of a cosmological model is red shift observational data. The distance modulus is given by

$$\mu(z) = 5 \log \left[ \frac{(1+z)a(z)}{10 \text{ pc}} \right], \quad (27)$$

where  $z$  is the red shift and

$$a(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{F(z')}}. \quad (28)$$

Here,

$$\begin{aligned} F(z) &= (1+z)^2 (1 + \Omega_m z) - z(2+z)\Omega_\Lambda \\ &\quad + \Omega_r (1+z)^4 \end{aligned} \quad (29)$$

based on Frieman et al. [34] and Spergel et al. [33].  $\Omega_r$  is too small ( $4.9321 \times 10^{-5}$ ) to matter as computations indicate but is included anyway. Rewriting (28) in terms of  $\mathfrak{R}$ ,

$$a(z) = \frac{c}{H_0} \int_{\mathfrak{R}}^1 \frac{d\mathfrak{R}}{\sqrt{\mathfrak{F}(\mathfrak{R})}} \quad (30)$$

with

$$\begin{aligned} \mathfrak{F}(\mathfrak{R}) &= \mathfrak{R}^2 \left[ 1 + \Omega_m (\mathfrak{R}^{-1} - 1) \right] + (\mathfrak{R}^4 - \mathfrak{R}^2) \Omega_\Lambda \\ &\quad + \Omega_r. \end{aligned} \quad (31)$$

Figures 1 and 2 summarize the salient results from the proposed variable  $G$ - $\Lambda$  model. Figure 1(a) shows  $\mu$  plotted against  $z$  up to  $z = 2$ . Observational data from [3] are superimposed as in Figure 4 of [34]. In Figure 1(b), the residuals in the distance modulus  $\Delta\mu$  relative to the open universe with  $\Omega_{m0} = 0.3$  and  $\Omega_{\Lambda 0} = 0$  (blue curve) are plotted as in Figure 4 of [34], but with data from [3] superimposed.

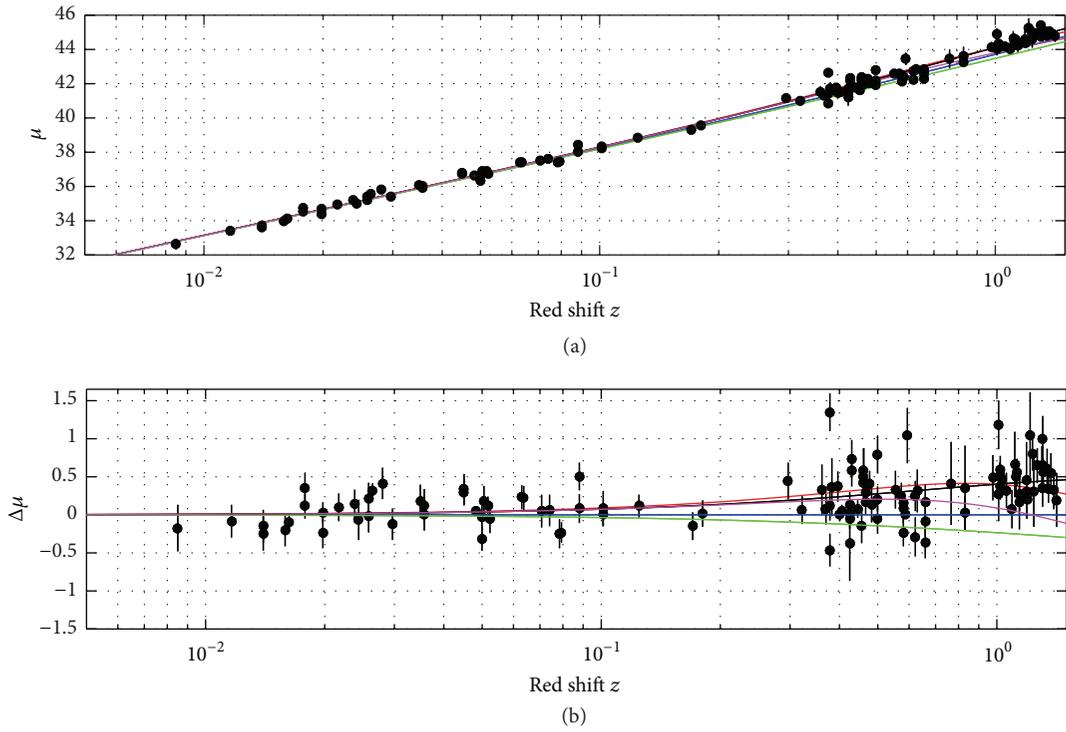


FIGURE 1: (a) Variation of distance modulus  $\mu$  with red shift  $z$ . (b) The residuals in distance modulus relative to the open universe (shown by the blue curve) that expands forever. In both panels, data from [3] are superposed as in Figure 4 of [34]. The green curve shows matter-only universe that collapses on itself. Black curve shows the current constant  $G-\Lambda$  model with  $\Omega_{\Lambda 0}$  of 0.73. The red and magenta curves correspond to the proposed variable  $G-\Lambda$  model with  $\Omega_{\Lambda 0}$  of 0.82 and 0.73, respectively (see the text for more details).

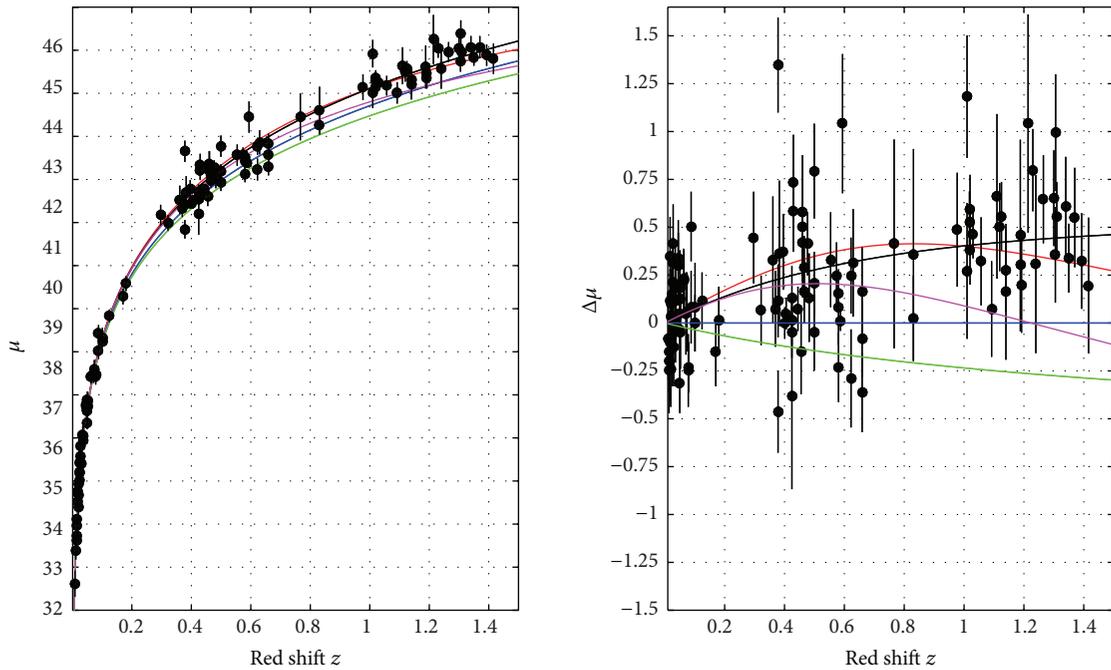


FIGURE 2: As in Figure 1 but on a linear scale.

In Figures 1(a) and 1(b), the  $x$ -axis is in log scale to accentuate the current epoch. Figure 2 is the same as Figure 1, except that the  $x$ -axis is linear to highlight the region around  $z = 1$  as in [3]. There are 5 cases displayed in Figures 1 and 2:

- (1)  $\Omega_{m0} = 1, \Omega_{\Lambda0} = 0$  (green curve): this is the matter-only universe, which will collapse on itself because of the gravitational attraction of the mass of the universe.
- (2)  $\Omega_{m0} = 0.3, \Omega_{\Lambda0} = 0$  (blue curve): this is the so-called open universe. It will expand forever.
- (3)  $\Omega_{m0} = 0.27, \Omega_{\Lambda0} = 0.73$  (black curve): this is the current standard constant  $G$ - $\Lambda$  cosmological model.
- (4)  $\Omega_{m0} = 0.27, \Omega_{\Lambda0} = 0.73$  (magenta curve): this is the proposed variable  $G$ - $\Lambda$  cosmological model with current values for the parameters.
- (5)  $\Omega_{m0} = 0.18, \Omega_{\Lambda0} = 0.82$  (red curve): this is also the proposed variable  $G$ - $\Lambda$  cosmological model except for the value of  $\Omega_{\Lambda0}$  which better fits the observed red shift data.

Note that the observations [3] extend to  $z = 1.4$  only. There is a significant overlap between the standard model and the model proposed here from low values of  $z$  to about  $z \sim 1$ , but divergence starts above this value. The residuals make it very clear that both models agree until  $z \sim 0.3$  but then diverge. The model proposed here (red curve) rises slightly above the standard model (black curve) until  $z \sim 1$  and then dips down significantly beyond  $z \sim 1$ . This behavior provides a testable hypothesis. It provides a crucial test of the proposed time-dependent cosmology when data is collected from future observational campaigns for red shift values beyond  $z \sim 1.4$ . This can be seen in Figure 3, which extends the model results to  $z \sim 100$ .

The proposed model is also consistent as to the expansion rate of the universe. Recall the deceleration parameter  $Q$  given by

$$Q = -\frac{a\ddot{a}}{(\dot{a})^2} = -\frac{\mathfrak{R}\mathfrak{R}''}{(\mathfrak{R}')^2} \quad (32)$$

with the current value of  $Q$  the same as the standard model. From (22), it can be seen that as  $\tau \rightarrow \infty$ ,  $\mathfrak{R}' \sim \sqrt{\Omega_{\Lambda 1}}\mathfrak{R}$  and therefore  $Q \rightarrow -1$ . This is the same limit as in the standard constant  $G$ - $\Lambda$  model. From (21),

$$\frac{1}{G} \frac{dG}{dt} = \frac{\alpha H_0}{\tau} \quad (33)$$

so that the current value is simply  $\alpha H_0$ . With  $\alpha = 2.5$ ,  $dG/dt$  is  $1.8 \times 10^{-10} \text{ yr}^{-1}$ . Current estimates vary widely (Table 1 of Ray et al. [8]), with negative values of as much as  $-2.5 \times 10^{-10} \text{ yr}^{-1}$  to positive values of as much as  $+4 \times 10^{-10} \text{ yr}^{-1}$ . The value from the proposed model is positive and lies within the range of these estimates. If the present change in  $G$  with time can be measured more precisely, it would be another test of the variable  $G$ - $\Lambda$  cosmological model proposed here.

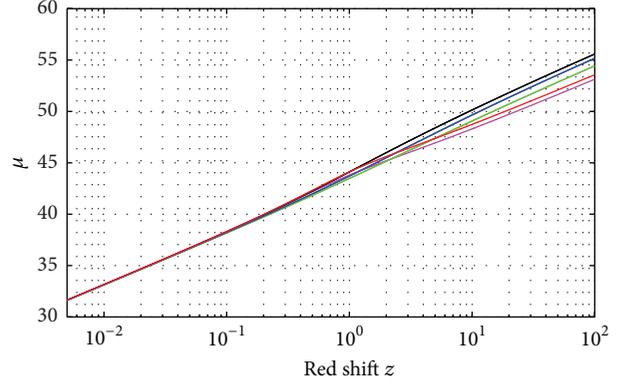


FIGURE 3: As in Figures 1 and 2, but model results extended to red shift  $z = 100$ . If at all it is possible to extend observational data to even a red shift  $z \sim 5$ , that would provide strong evidence to validate or reject the proposed model.

## 5. Implications of Variable $G$ - $\Lambda$ Cosmology

In cosmology, the three important parameters are the Hubble parameter  $H$ , the cosmological “constant”  $\Lambda$ , and the speed of light  $c$  (which figures prominently in both cosmology and quantum physics). It is possible to form a salient cosmological nondimensional number

$$K = \frac{\Lambda c^2}{H^2} \quad (34)$$

out of these three. Using  $\Lambda \sim 1.318 \times 10^{-52} \text{ m}^{-2}$  and  $H \sim 70.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  ( $2.2945 \times 10^{-18} \text{ s}^{-1}$ ), the current value is  $K_0 \sim 1.722$ . Interestingly, in the proposed cosmology, in the limit  $\tau \rightarrow 0$ ,  $\mathfrak{R} = b\tau^p$ ,  $H = H_0 p\tau^{-1}$ , and

$$\Lambda = \Lambda_0 (\Omega_{\Lambda 0} - \Omega_{\Lambda 1}) \tau^{-2} \quad (35)$$

so that

$$K = K_0 \left( \frac{\Omega_{\Lambda 0} - \Omega_{\Lambda 1}}{p^2} \right) = K_0 \quad (36)$$

and the salient cosmological number  $K$  remains invariant with cosmological time.

If  $\Lambda$  is representative of dark energy, since  $\Lambda^{-1/2}$  is a length scale and  $a = c/H$  is the scale factor, this suggests that  $\Lambda \sim a^{-2}$  obeys an “inverse square law,” somewhat akin to the radiative flux from a star and the gravitational force of a heavenly body.

Whether the physical “constants” such as Newtonian gravitational constant  $G$  and Planck constant  $\hbar$  have remained invariant throughout the history of the universe from the beginning of the Big Bang to the present epoch is a very important issue. The only true constant is most likely the speed of light  $c$ , which underpins relativity and space-time concepts. We also regard the constancy of light speed as inviolable. The same is not necessarily true for the other “constants” we deal with in cosmology and quantum physics. However, Planck scales once again underpin quantum physics and must be regarded as sacrosanct. Consequently, if the variable  $G$ - $\Lambda$  cosmology presented above is held valid or at least worthy of

consideration, then it is essential to elaborate on its implications, since it requires a cosmic time-dependent parameter  $G$ , which occurs prominently in definition of Planck scales. However, if we postulate that the Planck constant  $\hbar$  has also been cosmic time-dependent, then the product  $\hbar G$  or the ratio  $\hbar/G$  must remain invariant. Recall the various Planck scales, length, time, mass, energy, temperature, and entropy:

$$\begin{aligned}
 \ell_P &= \left( \frac{\hbar G}{c^3} \right)^{1/2}, \\
 t_P &= \left( \frac{\hbar G}{c^5} \right)^{1/2}, \\
 m_P &= \left( \frac{\hbar c}{G} \right)^{1/2}, \\
 E_P &= \left( \frac{\hbar c^5}{G} \right)^{1/2}, \\
 \Theta_P &= \left( \frac{\hbar c^5}{G k^2} \right)^{1/2}, \\
 S_P &= \frac{E_P}{\Theta_P} = k,
 \end{aligned} \tag{37}$$

where  $k$  is the Boltzmann constant (note that Stefan-Boltzmann constant  $\sigma = \pi^2 k^4 / 60 \hbar^3 c^2$  remains time-invariant). As  $\tau \rightarrow 0$ ,  $G$  also  $\rightarrow 0$ , and if  $\hbar G$  is invariant, Planck scales  $m_P$ ,  $E_P$ , and  $t_P$  become singular at  $\tau = 0$ . If  $\hbar/G$  were to remain invariant, the Planck length and time scales go to zero, but Planck scales  $m_P$ ,  $E_P$ , and  $t_P$  need not become singular. The only Planck scales independent of  $G$  are the Planck-Boltzmann entropy  $S_P$  and the speed of light  $c = \ell_P t_P^{-1}$ .

## 6. Discussion

It is remarkable that the proposed variable  $G$ - $\Lambda$  cosmological model depends only on the values of cosmological parameter at present ( $\Lambda_0$ ) and far into the future ( $\Lambda_1$ ) and not at all on the gravitational parameter  $G$ , which underpins much of what happens in the universe. As such, the solutions are not restricted to any phase of the evolution of the universe, such as matter-dominated, radiation-dominated, and inflation phases, and hence are uniformly valid for all time. For the choice made above for these two parameters, the various cosmic times of importance are as follows:

- Freeze-out:  $\tau \sim 5.9058 \times 10^{-18}$ .
- Start of BBN:  $\tau \sim 2.5184 \times 10^{-16}$ .
- End of BBN:  $\tau \sim 2.8723 \times 10^{-16}$ .
- End of radiation phase:  $\tau \sim 3.0 \times 10^{-6}$ .
- End of the Dark Age:  $\tau \sim 2.1 \times 10^{-5}$ .

The acceptability of the proposed model depends very much on the acceptable split between matter and dark energy.

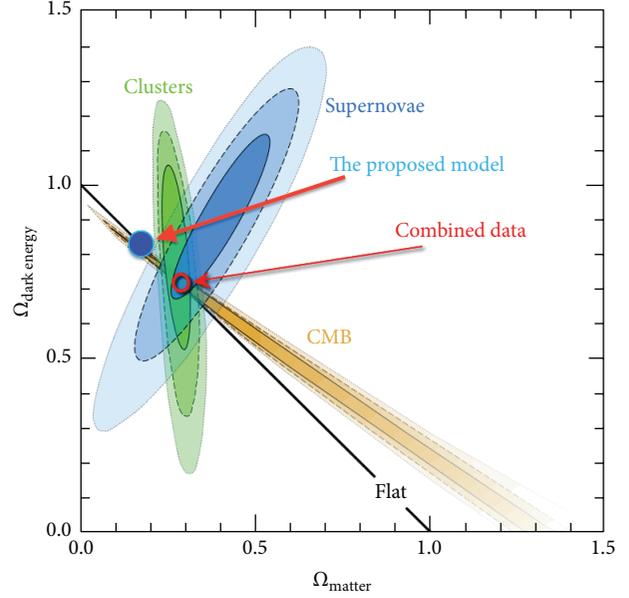


FIGURE 4: Existing observational data on dark energy/matter split from various sources displaying the current consensus that relies on constant  $G$ - $\Lambda$  cosmology as well as the proposed variable  $G$ - $\Lambda$  cosmology. Adapted from <http://www.hep.ucl.ac.uk/darkMatter/>.

The current consensus appears to be 27% matter and 73% dark energy. However, we propose 18% and 82%, respectively. As can be seen from Figure 4, this is at the very edge of what may be considered reasonable, according to data available at present.

Reexamining issues such as the formation of stars after the Big Bang may shed more light on the viability of the proposed model. The current ideas assume a time-invariant  $G$  [38]. What happens if  $G$  is several orders of magnitude smaller at the end of the Dark Age? How does this affect the initial star formation process? How does a time-dependent  $G$  affect star formation over cosmic time? These issues are worthy of further exploration. Numerical simulations of initial star formation at much lower values of  $G$  would be helpful. Can the existing observational data from various sources be shown to be consistent with matter being only 18%? Is the proposed model consistent with observations on star formation? The answers to such questions remain unexplored but are beyond the scope of this study.

## Competing Interests

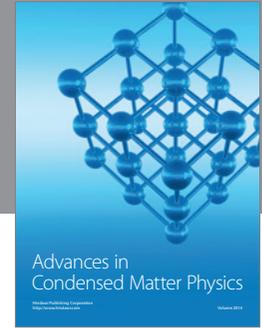
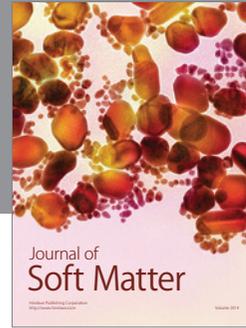
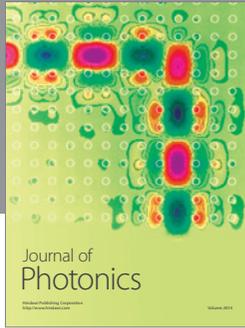
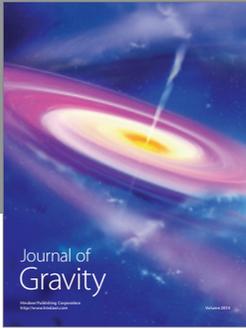
The author declares that there are no competing interests regarding the publication of this paper.

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## References

- [1] P. Langacker, "Grand unified theories and proton decay," *Physics Reports*, vol. 72, no. 4, pp. 185–385, 1981.
- [2] N. Suzuki, D. Rubin, C. Lidman et al., "The Hubble space telescope cluster supernova survey: V. Improving the dark-energy constraints above  $z > 1$  and Building an early-type-hosted supernova sample," *The Astrophysics Journal*, vol. 746, no. 1, article 85, 2012.
- [3] E. S. Abers and B. W. Lee, "Gauge invariance in classical field theories," *Physics Reports*, vol. 9, no. 1, pp. 6–13, 1973.
- [4] P. Dirac, "A new basis for cosmology," *Proceedings of the Royal Society of London*, vol. 165, no. 921, 1938.
- [5] P. A. Dirac, "Cosmological models and the large numbers hypothesis," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 338, no. 1615, pp. 439–446, 1974.
- [6] C. P. Singh and A. Beesham, "Particle creation in higher dimensional space-time with variable  $G$  and  $\Lambda$ ," *International Journal of Theoretical Physics*, vol. 51, no. 12, pp. 3951–3962, 2012.
- [7] C. P. Singh, "Early universe with variable gravitational and cosmological constants," *International Journal of Theoretical Physics*, vol. 45, no. 3, pp. 531–540, 2006.
- [8] S. Ray, U. Mukhopadhyay, and S. B. Choudhury, "Dark energy models with a time-dependent gravitational constant," *International Journal of Modern Physics. D. Gravitation, Astrophysics, Cosmology*, vol. 16, no. 11, pp. 1791–1802, 2007.
- [9] R. Bali and M. Kumawat, "C-field cosmological models with variable  $G$  in FRW space-time," *International Journal of Theoretical Physics*, vol. 48, no. 12, pp. 3410–3415, 2009.
- [10] J. Grande, J. Sola, I. L. Fabris, and J. C. Shapiro, "Cosmic perturbations with running  $G$  and  $\Lambda$ ," *Classical Quantum Gravity*, vol. 27, no. 10, pp. 1–29, 2010.
- [11] U. Mukhopadhyay, P. P. Ghosh, and S. Ray, "Higher dimensional dark energy investigation with variable  $\Lambda$  and  $G$ ," *International Journal of Theoretical Physics*, vol. 49, no. 7, pp. 1622–1627, 2010.
- [12] J. Grande, J. Solà, S. Basilakos, and M. Plionis, "Hubble expansion and structure formation in the 'running FLRW model' of the cosmic evolution," *Journal of Cosmology and Astroparticle Physics*, vol. 2011, no. 8, article 007, pp. 1–41, 2011.
- [13] K. P. Singh and N. I. Singh, "Viscous cosmological model with variable gravitational and cosmological constants," *International Journal of Theoretical Physics*, vol. 50, no. 8, pp. 2486–2492, 2011.
- [14] S. Ray, F. Rahaman, U. Mukhopadhyay, and R. Sarkar, "Variable equation of state for generalized dark energy model," *International Journal of Theoretical Physics*, vol. 50, no. 9, pp. 2687–2696, 2011.
- [15] C. P. Singh, "Cosmological models with time-varying gravitational and cosmological 'constants,'" *Astrophysics and Space Science*, vol. 331, no. 1, pp. 337–342, 2011.
- [16] C. P. Singh and A. Beesham, "Early universe cosmology with particle creation: kinematics tests," *Astrophysics and Space Science*, vol. 336, no. 2, pp. 469–477, 2011.
- [17] M. Jamil and U. Debnath, "FRW cosmology with variable  $G$  and  $\Lambda$ ," *International Journal of Theoretical Physics*, vol. 50, no. 5, pp. 1602–1613, 2011.
- [18] J. A. Belinchón, "Bianchi I in scalar and scalar-tensor cosmologies," *Central European Journal of Physics*, vol. 10, no. 4, pp. 789–798, 2012.
- [19] E. Piedipalumbo, P. Scudellaro, G. Esposito, and C. Rubano, "A matter-dominated cosmological model with variable  $G$  and  $\Lambda$  and its confrontation with observational data," *General Relativity and Gravitation*, vol. 44, no. 10, pp. 2477–2501, 2012.
- [20] C. P. Singh and V. Singh, "Power-law expansion and scalar field cosmology in higher derivative theory," *International Journal of Theoretical Physics*, vol. 51, no. 6, pp. 1889–1900, 2012.
- [21] S. D. Tade and M. M. Sambhe, "Higher dimensional FRW cosmology with variable  $G$  and  $\Lambda$ ," *International Journal of Theoretical Physics*, vol. 51, no. 2, pp. 447–454, 2012.
- [22] L. Kantha, "What if the gravitational constant  $G$  is not a true constant?," *Physics Essays*, vol. 25, no. 2, pp. 282–289, 2012.
- [23] E. L. D. Perico, J. A. S. Lima, S. Basilakos, and J. Solà, "Complete cosmic history with a dynamical  $\Lambda=\Lambda(H)$  term," *Physical Review D*, vol. 88, Article ID 063531, 2013.
- [24] J. Sadeghi, B. Pourhassan, M. Khurshudyan, and H. Farahani, "Time-dependent density of modified cosmic chaplygin gas with variable cosmological constant in non-flat universe," *International Journal of Theoretical Physics*, vol. 53, no. 3, pp. 911–920, 2014.
- [25] A. Narimani, A. Moss, and D. Scott, "Dimensionless cosmology," *Astrophysics and Space Science*, vol. 341, no. 2, pp. 617–629, 2012.
- [26] K. D. Krori, S. Chaudhury, and A. Mukherjee, "Cosmologies with variable  $G$  and  $\Lambda$  from action principle," *General Relativity and Gravitation*, vol. 32, no. 8, pp. 1439–1447, 2000.
- [27] P. G. Bergmann, "Comments on the scalar-tensor theory," *International Journal of Theoretical Physics*, vol. 1, no. 1, pp. 25–36, 1968.
- [28] B. Boisseau, G. Esposito-Farèse, D. Polarski, and A. A. Starobinsky, "Reconstruction of a scalar-tensor theory of gravity in an accelerating universe," *Physical Review Letters*, vol. 85, no. 11, pp. 2236–2239, 2000.
- [29] G. Esposito-Farèse and D. Polarski, "Scalar-tensor gravity in an accelerating universe," *Physical Review D*, vol. 63, no. 6, Article ID 063504, 20 pages, 2001.
- [30] A. Riazuelo and J.-P. Uzan, "Cosmological observations in scalar-tensor quintessence," *Physical Review D*, vol. 66, Article ID 023525, 2002.
- [31] G. F. R. Ellis and J.-P. Uzan, "c is the speed of light, isn't it?" *American Journal of Physics*, vol. 73, no. 3, pp. 240–247, 2005.
- [32] R. R. Caldwell, R. Dave, and P. J. Steinhardt, "Cosmological imprint of an energy component with general equation of state," *Physical Review Letters*, vol. 80, no. 8, pp. 1582–1585, 1998.
- [33] D. N. Spergel, L. Verde, H. V. Peiris et al., "First-year Wilkinson Microwave Anisotropy Probe (WMAP): observations: determination of cosmological parameters," *The Astrophysical Journal*, vol. 148, no. 1, article 175, 2003.
- [34] J. A. Frieman, M. S. Turner, and D. Huterer, "Dark energy and the accelerating universe," *Annual Review of Astronomy and Astrophysics*, vol. 46, pp. 385–432, 2008.
- [35] G. Steigman, "Light element nucleosynthesis," 2000, <https://arxiv.org/abs/astro-ph/0009506>.
- [36] G. Steigman, "Primordial nucleosynthesis in the precision cosmology era," *Annual Review of Nuclear and Particle Science*, vol. 57, pp. 463–491, 2007.
- [37] K. A. Olive, G. Steigman, and T. P. Walker, "Primordial nucleosynthesis: theory and observations," *Physics Reports*, vol. 333–334, pp. 389–407, 2000.
- [38] N. Yoshida, T. Hosokawa, and K. Omukai, "Formation of the first stars in the universe," *Progress in Theoretical and Experimental Physics*, Article ID 01A305, 2012.



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