Heuristics for Incorporating Simulations into Assignments

by

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The use of simulations in educational environments is a topic of growing interest, particularly in science education. While much research has been done to understand simulation use in interview settings, less has been done in the environments in which the majority of simulation use arises. The purpose of this thesis is to provide a framework for how simulations can be used in these natural environments, and analyze what can be done to promote effective use of simulations in these settings. We propose a list of heuristics or strategies that can be used when writing assignments to incorporate simulations, and additionally, provide a tentative theoretical view of how to implement these heuristics and why they work. This is done through a series of case studies that make use of the heuristics, as we first give an analysis of the heuristics that were used, and then provide a tentative theoretical view of how the heuristics were implemented, and why they work.

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Chapter 1

Introduction

We live in an age of technology. It is integrated into, and inseparable from, modern life. More than ever before, society depends upon technology to sustain its current rate of consumption, and finding a way to use technology to sustain future generations is among the most important issues of our time.

If we are to continue to use technology in modern life, we must teach others the fundamental principles that explain how technology "works." This necessarily requires a need for science, engineering, mathematics, and technology education throughout the world. Never before has the issue of educating people in these disciplines been as important as it is today; to not do so would most certainly result in tremendous problems.

Because it is patently clear that we must educate youth in these areas, the question that naturally arises is: how do we do so? Fortunately, science education researchers have been investigating these issues for many years, and there now exists a growing body of knowledge concerning the ways we can effectively educate people in these fields.

The field of science education is broad – far too broad to give a just review of here. However, a few very general findings stand out as necessary to mention. First of all, the fields of science, engineering, mathematics, and technology apparently are not learned "naturally" by the majority of human beings. We do not simply learn these disciplines by going about typical daily tasks; instead, these fields require learning to think about the world in ways that often seem counter-intuitive and "difficult." It takes years of practice and participation to learn the variety of methods involved in these disciplines, and much time must be devoted to learning their basic principles.

That being said, learning science and mathematics need not be a painstaking task. Humans carry with them a great deal of intuition and curiosity about the world around them, and this can be used as a great source of motivation to learn about science, as well as a starting place for scientific discovery.

Yet another principle in science education research is the idea that science is *culturally* learned. The act of "doing science" is largely acquired through participation in the field itself, since in doing so, people come to learn the methods and processes that are coupled with the scientific "process." We return to this topic in the following chapter because it is so important.

From these and other findings, we are able to draw some conclusions about how we ought to educate people in these fields. First, we ought to establish a sort of scientific "culture" in which students can participate in the act of science and develop some of the habits of mind associated with doing so. Second, because students carry with them intuition and curiosity about the physical world, we can attempt to appeal to those curiosities and intuitions in the curricula and methods we use to teach the subjects. Lastly, science is clearly not something simple that can be learned with relatively little effort. To keep students interested in learning these fields, and even pursue careers therein, we ought to try to make apparent the benefits and possibilities associated with pursuing those fields, and share our passion and interests in science with them.

Simulations and heuristics

The purpose of educating people in scientific fields mentioned above was largely centered around the fundamental importance of technology in modern society. Perhaps ironically, we now see the fundamental importance of technology in educating people in these scientific disciplines.

The use of computers, in particular, is a topic of growing interest in science education research circles, and in physics education research (PER), in particular.[1] Not only do computers play a fundamental role in the exchange of information, but they also provide a means for communicating ideas in ways not possible with conventional text and language.

A particular subset of the research in computer use focuses on the use of simulations in physics settings. Simulations are of great interest largely because they provide yet another means for achieving the some of the broad goals in science education listed above. More

[1] From here on, I will be talking mostly about physics education research, but note that this is not unrelated to the discussion above, and that many of the discussions in physics education research are likely to apply to science education research more broadly.

specifically, simulations have proven to be beneficial because they: are engaging and intuitive in functionality, provide a means of interacting with physics content in dynamic ways, allow students to relate formal physics concepts to the real world, appeal to intuitive and common student ideas, allow for play and experimentation, provide a platform on which discussions can be based, and much more.

The advantages of using simulations for learning physics have been demonstrated in much prior research in the field of PER, some of which will be discussed shortly. The majority of this research has been conducted by interviewing students and analyzing the ways that they interact with the simulations themselves. Through those observations, researchers have been able to extract general principles of *how* students tend to use simulations, as well as as what factors seem to promote the types of engagement that are effective for helping learn physics.

Yet while this type of research has been extremely beneficial in extracting general principles of design for simulations, and in providing knowledge about the ways students tend to use these simulations, this research provides little evidence for how these simulations are used in practice. Because the research is done in an interview setting, and not in more "natural" environments in which the majority of simulation use arises, it is difficult to draw conclusions from the research base as to how the simulations are used in the various classes in which they are implemented.

While we do not, in this thesis, attempt to provide evidence for *how* simulations are currently being used in the majority of classrooms across the country, we will explore effective ways for using simulations in these "natural" environments. In particular, we desire to develop a set of *heuristics* for how we might incorporate these simulations into assignments. In other words, in this thesis, we are interested in how we might design assignments to promote productive use of simulations in these various educational environments.

It is important to begin by defining precisely what we mean when using the term "heuristics." Believe it or not, Wikipedia provides a definition which, if modified slightly, suits our purposes quite well. According to Wikipedia, "heuristic(s) refer to experience-based techniques for problem solving, learning, and discovery. Heuristic methods are used to speed up the process of finding a satisfactory solution, where an exhaustive search is impractical" [23].

In changing this definition for our purposes, we implicitly *assume* that an exhaustive description of exactly what to do in writing an assignment is impractical, and use the following definition:

> Heuristics are research-based strategies for developing materials and assignments which help to integrate a simulation into a more general activity system. Being general strategies, they are meant to provide insights into how assignments ought to be written, and in doing so, help frame the ways we can think about writing such assignments.

We must emphasize that the heuristics are not the *sole* outcome of this thesis. Rather, the goal is to provide an explanation, based on both theory and empirical data, for *how* these heuristics should be implemented and *why* they work. In answering these questions, we draw from existing literature, as well as the case studies we conducted, to provide a general view of how we can start to effectively incorporate simulations into the various educational environments in which they are used.

In attempts to accomplish this rather lofty goal, we proceed in the following way: First, a general theoretical background, as well as an overview of relevant literature and case studies, is presented in the "Background" chapter. Next, we turn to a brief historical summary of this project, in which we situate the original intents of the project, as well as the reasons for the project evolving as it did. The following chapter presents yet more theoretical framing that will be applied to the case studies conducted. Those case studies are each given their own chapter, starting with the use of the "Build a molecule" simulation in a middle school classroom, followed by the use of the "Projectile motion" simulation in an upper-division classical mechanics course, and ending with the use of a "Quantum tunneling and wave packets" simulation in an introductory quantum mechanics course.

Chapter 2

Background

Incorporating simulations into physics assignments is a complex task that depends on a vast number of variables. The simulation, the particular students, what the students have been taught, the features of the classroom, and so on, all factor in to how assignments are written and how effective those assignments are in practice. Because this is such a complicated and multifaceted task, it is useful to have some sort of underlying theory from which we can start to organize and frame the way we think about these issues. The purpose of this chapter is to present a theoretical basis for doing just that. This framework, or more precisely, set of frameworks, is drawn from work that has been done by researchers in a variety of fields, including physics, cognitive science, psychology, cultural studies, and schools of education. Due to the interdisciplinary nature of this topic, what follows is much like a review of the findings and insights of these various fields. In this chapter, we start with a very broad theoretical framework drawn from cultural psychology, and then begin to narrow the focus to prior research in human-computer interaction and, in particular, the use of simulations.

2.1 Motivation

When I first started working on this project, I had an overwhelming suspicion that physics, a discipline which is, by nature, highly visual and conceptual, could be communicated "better" by using computers. At the time I felt that many topics in physics, from Coulomb's law to circuit theory, were inherently difficult to learn because I lacked a visual foundation for what was happening. Computers, I thought, could provide a means of communicating those visual aspects of physics in ways that are more intuitive, efficient, and effective. This intrigue inspired in me the desire to research how students might respond to visual features of computer programs, and how we might design these programs to promote student sense-making of these inherently visual and conceptual topics.

To my surprise, I quickly realized the importance of broadening my view of using computers to incorporate factors other than the mere design of the programs. At the time, I thought of the design of a simulation or computer program as being the most essential unit of analysis. My idea was that if I could study the students' reactions to a simulation, it would be possible to extract "rules" and design principles that would allow for forming some sort of theory of design. If we could just find those rules, I thought, we could design simulations for all areas of study that would allow for quick, easy, and enjoyable learning for all. However, it soon became clear that there are an incredibly large number of variables outside of the simulation itself that influence its effectiveness and use. My reductionist idealism being dampened, the questions that naturally arose were, "How broad do we actually need to be in analyzing the effectiveness of these simulations?" and "Is it even possible to know how to make a simulation effective for everyone?" As it turns out, the answer to the first question is, "pretty darn broad." In fact, the answer is so broad that it is necessary to turn to a discussion of the seemingly unrelated notion of "culture."

2.2 Culture and Psychology

The idea of culture is thought to have originated with the Greek historian Herodotus [9]. Having traveled to over fifty Greek and Persian societies in his time, he took note of the differences in each of these groups of peoples, providing evidence of their varying types of art, religion, and social practices.

The same observations are clearly present today. Yet while we are quick to acknowledge the existence of different cultures, in much the same way as did Herodotus, the influence of culture on human thought is usually less acknowledged, and its importance is often undervalued or glossed over. Further still, the colloquial use of the word "culture" likely has a variety of interpretations and perhaps remains vaguely defined as "different peoples and customs."

I want to provide a refined accounting of culture here, one that digs a little deeper into *why* these differences among people exist. This discussion will lead naturally to the importance of accounting for "context" when talking about the tasks that humans carry out, from something as typical as going to the grocery store to something as complicated as using a physics simulation. Note that the vast majority of what will follow comes directly from Michael Cole's work in *Cultural Psychology: a once and future discipline* [3].[1]

One of the reasons culture's importance is often glossed over is due to the fact that it is inherently difficult to "see." Prime examples of this are found in psychological studies that were ultimately proven to be flawed. These studies often focus on human actions in only one particular setting, and fail to understand the influence of the setting on human behavior. For instance, a psychological study of children in one cultural setting can often take certain practices of the children to be fundamental units natural to all human behavior, when in fact, a cross-cultural study will demonstrate those practices as variables among different cultures [7]. In a more physics-centric perspective, it was shown that a group of Buddhist monks had strikingly different "common misconceptions" about the nature of light and color than do American students learning the subject [18]. Had we only studied American students' misconceptions about light, we might think that these misconceptions apply to all humans, and are somehow inherent in human perception of the world. A central idea, then, is that the difficulty in seeing cultural dependencies is often due to the fact that we do not recognize these differences until we compare our habitual norms with the habitual norms of others.

The idea that culture influences thought forces us to consider what, if anything, remains the same across different cultures. To answer this question, we can turn to research done by cultural psychologists, who have developed a theory of culture based on a fairly simple foundation. Their basic premise is that all humans interact in various ways with the world around them, and that these interactions arise in two primary "types:" unmediated or "direct" interaction with the physical world, and mediated or "indirect" interaction with the world. Perhaps the best way to explain the unmediated path is to consider an infant. The infant, not capable of formal thought and reasoning, interacts with the world primarily through sensations; i.e. touch, sound, sight, smell, and taste. In this state, no higher level cognitive functions are present, since as of yet, none have been developed. It is difficult for us as adult human beings to even contemplate this state of existence because we have adapted to thinking in ways far more advanced than those of an infant. An interesting question to ask, then, is, "Where does this capability for formal thought

[1] Additionally, much of what follows is discussed in [5] and [4].

come from?" This question, which has been at the heart of research in evolutionary and cultural psychology for years, is now thought to be answerable only by considering tool use by humans.

In formal terms, cultural psychologists replace the words "tool use" with the notion of "artifact mediation." Unlike the unmediated or "direct" interaction an infant experiences in conjunction with the world around her, adult humans have the capacity to experience the world in a mediated manner through the use of "artifacts." While the word "artifact" is colloquially used to represent something found in an ancient ruin, this definition is more broad. Artifacts are, most generally, aspects of the material world that have been modified over the history of their use in human life to serve a specific purpose. This spans a wide space, and includes anything from sticks to words to gestures to ipods, and so on. The essential point is that through the use of these artifacts, humans can come to understand or experience the world in ways that are otherwise not achievable. Generally this "dual" structure of interaction with the world is represented by a meditational triangle, shown in the figure below. In it, the subject (human) interacts with an object in a direct way (like the infant) and an indirect way (through the use of an artifact). This is not to suggest that the interaction is "one or the other," but rather that both of these processes usually occur simultaneously, and work together in a productive triadic relationship. A central point is that artifacts restructure our interactions with the world, and in doing so, restructure our cognitive processes in such a way that, over time, leads to higher-level thought processes.

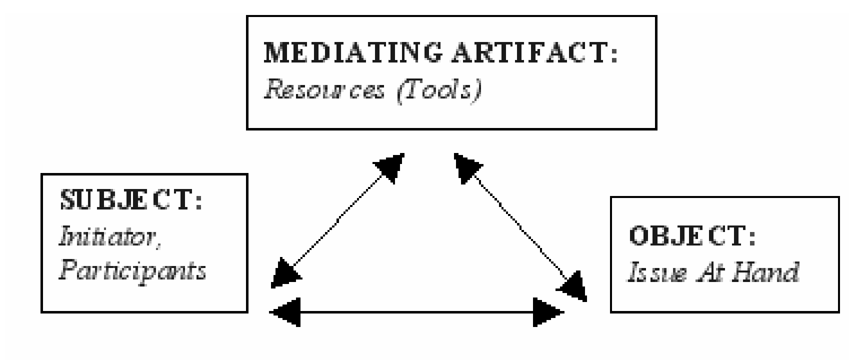


Figure 2.1: Standard mediational triangle. Taken from [19]

Crucial to this theory is the idea that artifacts are both material and ideal. They are material in that they always come in physical form, and they are ideal in the sense that they always have a particular significance. For example, an axe is of course a material object, being made out of a handle and a blade, and it is ideal in that its significance is to cut down trees. A more abstract example is words. They are material in that they are formed by movements of the mouth and the propagation of sound waves through air, and they are ideal in that each word has some meaning or significance.

Through mediated interactions with the world, humans can come to create new "worlds" that exist independently of the natural environment. These interactions provide us with the capability to imagine new situations and comprehend aspects of our own experience, and fundamentally restructure our cognitive processes.

From an anthropological perspective, the idea that tool use allows for higher-level thought processes makes a good deal of sense. Through studying fossils of human ancestors, anthropologists are able to see strong positive correlations between skull size and complex tool use. For instance, homo habilis, homo erectus, and homo sapiens are largely categorized by differences in brain size, and fossils of the ancestors with larger brain size are almost always found with a great number of tools in the vicinity. This provides us reason to believe that tool use is intimately connected with evolution.

Recently, correlations between a community's complex social structure (instead of tool use) and larger brain size have also been presented. Tool use and social structure are, of course, two interrelated areas of study, and one question is whether social structure led to the use of tools, or if tools led to the development of a complex social structure. Some take the stance that the precise reasons for humans evolving as they have is such a complex topic that it will never be described in full detail. But while the exact reasons for seeing dramatic increases in brain size in our primitive ancestors may be impossible to pinpoint, two points are clear from this research: humans evolved using artifacts that have become increasingly abstract and complex, and social interaction played a crucial role in our ancestors' abilities to share and distribute knowledge of how to use these artifacts.

So far the definition of artifacts is somewhat vague, and it is useful to further divide this notion of artifacts into categories, according to their relative levels of abstraction. This idea, drawn from Marx Wartofsky, is composed of primary, secondary, and tertiary artifacts [46]. Primary artifacts are those directly used by humans, and can include anything from cups, silverware, arrows and shoes to words, writing utensils, and cell phones. Each of these is clearly material, and they each are ideal in that they serve some particular function. They have also been modified over their incorporation into human life by becoming more suited to serve their particular "ideal" function. For instance, phones have changed from being situated in a stationary public location to phones with cords in nearly every house to cell phones with no cords in nearly everyone's pocket.

Secondary artifacts can be thought of as providing "rules" for using primary artifacts, and often primary artifacts are only effectively used in conjunction with these rules. Examples include recipes, syntax or sentence structure, dances, chants, songs, and paintings. A word, for example, only means so much alone, but with syntax we are able to create entirely new sets of meaning and ideas by following a set of 'language rules' that allow for the interpretation of these strings of words.

Tertiary artifacts can be described as "rules about rules" or "rules" about secondary artifacts. Examples include what should or should not be said in a particular situation, what actions a person should carry out at a particular event, how one should behave around a certain group of people, etc. These artifacts are not necessarily material instantiations of the world around us, but they still serve to shape our use of primary and secondary artifacts, and provide a means of "coloring" what it is we see and how we interact with the world.

The idea of artifacts lends itself to an interesting connection with work in psychology regarding the idea of schemas and scripts. Each of these are meant to help characterize cognitive processes, and if we are to acknowledge that artifacts provide a means of restructuring cognitive processes, then it is clear that artifacts allow us to form schemas and scripts.

Schemas refer to knowledge "structures" in which several different parts or elements of knowledge relate in specific ways. For example, in physics, the topic of kinematics likely constitutes a particular schema, composed of knowledge of vectors, position, velocity, etc. The different ideas presented in kinematics, from vector addition to comprehension of acceleration, all have relationships that are understood in conjunction with each other. The time derivative of position gives velocity, and the time derivative of velocity gives acceleration, and the formal treatment of all of these presents itself in the form of vector notation. Of course, all of this requires the ability to use artifacts (paper, pencil, mathematics, etc.) in order to build these cognitive structures.

Scripts are a type of schema, particularly an "event schema," and refer to different elements of knowledge that have temporal relationships among themselves. Using the topic of kinematics, scripts are a particular type of schema that allows one to predict future motion of particles under different conditions. For instance, if I throw a ball in the air, it will eventually slow and reach the top of its trajectory and then come back down. Through the use of a ball, which can be thought of as a type of artifact, we can come to form a script that allows us to predict the motion of the ball. Additionally, if we wish to describe the motion of the ball in yet more precise detail, we might like to understand the mathematical relationships of kinematics to correctly predict its motion. This necessarily requires the use of yet more artifacts, namely mathematics, pencil, paper, and so on.

Of course, schemas and scripts do not only allow us to deal with physics-specific phenomena. A more abstract example could be going to dinner with one's boss. In this case, the script and schemas used are quite different from the case of using kinematics, and some of the tertiary artifacts, such as knowledge about what to say and what not to say, will likely be incorporated into those scripts and schemas.

Upon the advent of schemas and scripts in psychological research, there was a tendency for psychologists to treat these as existing only "in the head." This notion, however, fails to give an explanation for why particular schemas and scripts are used in different situations. For an analysis of this, it is necessary to introduce the idea of context. Context, most literally, means, "the circumstances that form the setting for an event, statement, or idea, and in terms of which it can be fully understood and assessed" [20]. One way to think of it generally is by thinking of everything that influences human thought. If I am to truly account for everything going on in my head at a particular instant in time, I have to account for all of the factors - external and internal - that are affecting my thought processes at that instant. This notion of "everything going on" is the notion of context. From the micro level of something as simple as an individual's attitude or mood to something macro like the cultural norms of a particular society, all of the factors present in a given moment in time, and factors evolving throughout time, are the context of that particular situation. It is only when we look at the particular context of a situation that we can start to understand and account for individuals' activation of particular scripts and schemas.[2]

Not only does the notion of context shed light on why humans activate particular schemas and scripts depending on their situation, but it also illuminates the ways humans use artifacts in those situations. We never use an artifact in isolation from the rest of the world; our use of artifacts always takes place in a particular context. Incorporating an artifact in context usually requires thinking of context in two different, though not

[2]It is worth mentioning that this idea is intimately related to David Hammer's idea of "activation of resources" that will be presented in Chapter 4.

exclusive, ways: as that which surrounds, and as that which weaves together. Context as "that which surrounds" is represented in the figure below, and shows different "layers" of context that surround a particular activity. In this example, the student is at the center, and their actions are depicted as working on a task to acquire a concept. However, the student's task is shaped by all of the surrounding levels of context, from the particular lesson they are working on to the organization of the classroom, and all the way through community and societal factors more generally. In order to fully understand the task the student is working on, it is necessary to understand how these other layers of context influence the student, the activity, and so on.

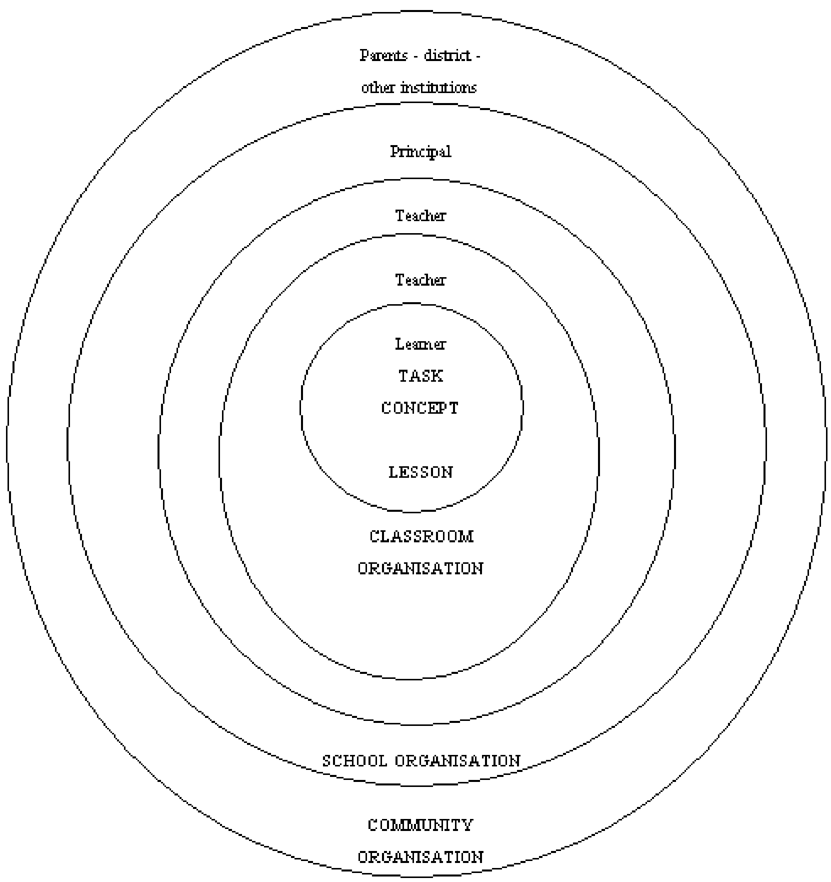


Figure 2.2: Depiction of context as "that which surrounds." Taken from [22]

Another way to think about context is as "that which weaves together." From this view, an activity, such as a task that a student is working on, is only done in cooperation with other layers of context. These layers do not necessarily just "surround" the activity, but rather they are integrated into the activity, and cannot be viewed as separate from the activity itself. Often the analogy of a rope is used to depict this. The rope, being made of smaller individual strands, is only considered a rope when the strands are woven together in a particular fashion. Similar is the notion of activity; an activity is composed of several different components, whether it be a student writing, talking with other students, etc., but the activity itself is only fully described when the interrelations of these different components or tasks are "woven together" in a contextual whole.

Because these ideas of context "surrounding" and "weaving together" are not explicitly accounted for in the mediational triangle presented earlier, theorists, particularly Yrjo Engestrom, have modified that triangle to be more inclusive of aspects other than subject, object, artifact that play a critical role in our actions in everyday life [4]. This modified triangle is referred to as an activity system, and incorporates community, rules, and division of labor in to the mix (see figure below). Community is meant to refer to the people who share the same object, rules refer to the particular social conventions and standards present in an activity system, and division of labor refers to the differing roles present in an activity system, from student to teacher, and so on. In addition, next to the triangle is the "outcome" of the relationships, meant to emphasize that all of this works to achieve some particular goal. Again, the benefit of this model is that it explicitly calls attention to features of an activity that are glanced over by the first mediational triangle presented.

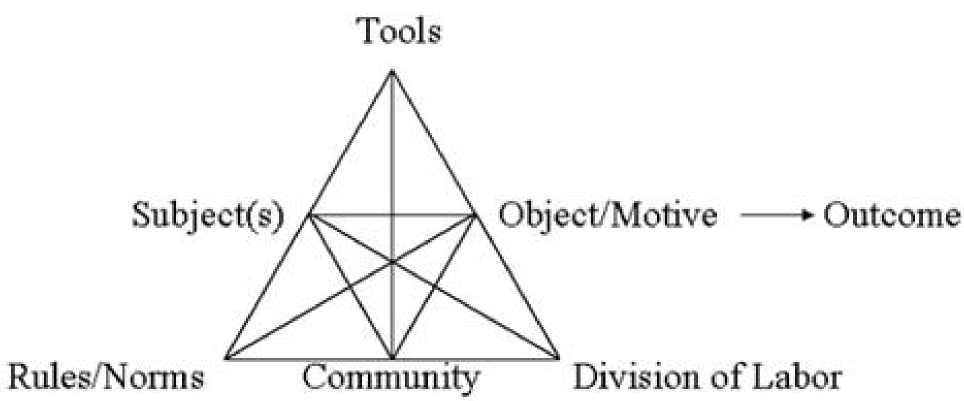


Figure 2.3: Modified mediational triangle. Taken from [4]

The reason for establishing this foundation is to acknowledge that learning occurs within context, and that an analysis of learning out of context is bound for failure. With this perspective in mind, it is possible to start to look at using computer programs, specifically simulations, as an artifact used in context. Not only do the features of a simulation matter, but the student, their prior knowledge, the teacher present, the type of activity or assignment they are working on, the arrangement of the desks in the classroom, and the students' interactions with each other, will all influence the activity and how the students use a simulation. Keeping this in mind, we turn to a discussion of literacy.

2.3 Literacy

Using a computer requires a great deal of interpretation and cognitive processing on the side of the student. In fact, there is so much cognitive processing going on that it is helpful to associate a new term with this act of interpreting and interacting with this medium. This term is literacy. While literacy is usually understood as the ability to read and write, this definition is more broad. According to diSessa, we can equate literacy with material intelligence. If we take the definition of intelligence as "the ability to acquire and apply knowledge and skills" [20], then a material intelligence is our acquisition and application of knowledge through the use of a material medium or tool. Of course, this definition clearly makes sense in terms of the colloquial use of the word "literacy," since we are acquiring or distributing information in a particular way when we read and write. However, there is no reason to limit the definition of literacy to reading and writing, since there are plenty of other forms of material intelligence that we use on an everyday basis. The ability to read a map, interpret a diagram, or pictorially represent something all are done in conjunction with some sort of material agent, and thus require a material intelligence for their effective use.

In this light, the computer is yet another material medium through which we can acquire and apply intelligence in various ways. The difference between this medium and other media, however, is that this particular medium allows for dynamic, rather than static, modes of interaction. These types of interaction, which will be described in great detail throughout this thesis, are simply not achievable with conventional texts and representations.

Before exploring the potential that lies within the dynamic capabilities of computers, it is necessary to elaborate a bit on literacy, and especially its role in cognitive processes. As diSessa describes, there are three basic "pillars" of literacy: the material, the mental or cognitive, and the social. The material pillar, of course, deals with the various symbols, representations, and interpretations of various material objects. Generally, these are both technologically dependent and designed for a particular purpose. For instance, the example of the phone given earlier falls in to this category, being both technologically dependent and "designed." Note that diSessa's classification of the material pillar of literacy bears striking resemblance to the idea of artifacts described earlier; in both cases, a material object is used for human action and is changed over the history of its use to more effectively suit its "ideal" purpose.

The cognitive pillar refers to the fact that there is a mutual dependence between the object and the subject, since for the use of a material medium to be effective, the user must be able to interpret and process the information present in that medium. Another point to make here is that establishing a literacy for interpreting and interacting with a particular medium reshapes or restructures cognitive processes in general, as those media become integrated into the cognitive structures formed. Lastly, the social pillar refers to the idea that establishing a literacy within a society requires an infrastructural change, usually distributed through the educational system, which depends on social actions and decision-making.

To elaborate briefly on this cognitive - material interaction that is so important in influencing human thought, we can look to diSessa's example of Galileo's theorems of uniform motion. In order to describe uniform motion, Galileo wrote down six theorems that describe, in full detail, the uniform motion of an object. They are:

- Theorem 1: If a moving particle, carried uniformly at constant speed, traverses two distances, then the time intervals required are to each other in the ration of these distances.

- Theorem 2: If a moving particle traverses two distances in equal intervals of time, these distances will bear to each other the same ratio as their speeds. And conversely, if the distances are as the speeds, then the times are equal.

- Theorem 3: In the case of unequal speeds, the time intervals required to traverse a given space are to each other inversely as the speeds.

- Theorem 4: If two particles are carried with uniform motion, but each with a different speed, then the distances covered by them during unequal intervals of time bear to each other the compound ratio of the speeds and time intervals.

- Theorem 5: If two particles are moved at a uniform rate, but with unequal speeds, through unequal distances, then the ratio of the time intervals occupied will be the products of the distances by the inverse ratio of the speeds.

- Theorem 6: If two particles are carried at a uniform rate, the ratio of their speeds will be the product of the ratio of the distances traversed by the inverse ratio of the time intervals occupied.

To the modern reader, this seems like a lot of unnecessary verbiage to describe something as simple as uniform motion. In fact, each of these theorems can be much more simply written with two algebraic expressions, and manipulating them in various ways:

$$d_1 = r_1 t_1 \quad \text{and} \quad d_2 = r_2 t_2$$

So why didn't Galileo use this representation? The reason is that at the time, algebra simply did not exist. If we take algebra to be a particular form of material intelligence, then it is easy to notice algebra's capacity to more aptly explain certain physical phenomena than does conventional text and language. The creation of this material intelligence now allows for nearly every student in school to be able to comprehend the idea of uniform motion, and the effects of the formation of this literacy on modern society are so fundamental and important that it is almost difficult to imagine a time when it did not exist.

The example of Galileo's theorems gives us insight into the power that lies within new literacies. The development of a new material intelligence can vastly increase of the efficiency and effectiveness of distributing and comprehending information, and as a modern society, we now find ourselves in a position of creating a new material intelligence with this potential. It is only natural to wonder what literacy or capabilities will develop in ten, twenty, or more years from now, and perhaps reasonable to suspect that computers will provide the technological basis for creating a new material intelligence that will become commonplace in society.

2.4 Simulations

While the broader discussion of literacy and the formation of a new material intelligence is incredibly interesting, it is necessary for the purposes of this research to focus more narrowly on one particular use of computers: the use of simulations. The use of simulations in physics classrooms has become more commonplace in recent years, and a great deal of research has been done to understand how to design these simulations.

So what exactly *are* simulations? Clark, et. al. use the definition of: "Computational models of real or hypothesized situations or phenomena that allow users to explore the implications of manipulating or modifying parameters within the models" [26], and this will serve as an apt starting place for this discussion. It is important to note that simulations come in a variety of types and styles, and there are different advantages to using each, depending on the situation. An analogy that is somewhat helpful is the idea of using tools in building a tree house. Different tools are helpful for different purposes. The hammer is useful for pounding nails, while a saw is necessary to cut wood, and so on. Simulations, too, can be thought of as a set of "tools" that can be more or less useful, depending on the situation present. As will become apparent, however, the use of a simulation is much more complicated than using a hammer or saw, due to the high degree of context-dependence that influences their use.

There are approximately four categories that classify simulations, according to their varying degrees of user control. These include (in order of most specific to most general) targeted simulations, "sandbox" simulations, "glass box" simulations, and "networked participatory" simulations [26]. Targeted simulations, such as PhET, TEAL and Physlets, are basically stand-alone simulations that are meant to cover a particular sub-topic in physics or another scientific discipline. These simulations are useful in that they can be used in a variety of ways, from lecture demonstrations to homework assignments, and their use is relatively quickly learned and intuitive. The amount of time needed to learn the parameters and features of these sims is minimal, and they have been shown to promote engagement and inquiry in relatively short amounts of time [26].

"Sandbox" simulations are more modifiable than targeted sims, and allow the user to control just about everything, aside from the programming of the sim itself. Examples include SimCity, SimEarth, and other Sim-brand simulations. These generally require more time for students to learn the basics of the simulation, but can also promote different types of open-ended inquiry [26].

"Glass box" models are more based around coding and require even longer periods of time for users to start to understand how to manipulate and use the simulation. The advantage to this is that once users understand how to use the program, there is a vast amount of potential for building and self-guided inquiry. Examples of this type include Logo and its successor, Boxer. With these programs, users can start to build their own models and code, or modify existing code for their own particular interests.

"Networked participatory" sims are meant for multiple users at any given time, and allow the sharing and transfer of data and other information among different users in the network. An example of this type is WISE, the Web-based Inquiry Science Environment, which allows students to collaboratively explore different simulations, collect data, and participate in forums to explain what they notice.

Again, each of these different types of simulations can be more or less helpful than others depending on the environment and the goals of the class. Since the research done in this thesis focuses entirely around the use of targeted simulations, and in particular PhET simulations, the rest of this summary will focus heavily on prior work done by researchers using PhET. To provide a better picture of how these sims have been used and studied in the past, I first provide an overview of a few particular studies conducted in the past ten or so years. Next, I will focus more specifically on the design of the simulations, and how researchers generally perceive designing these simulations. Following this is an evaluation of the research that has not been done yet, and how the goals of the research for this thesis emerge out of these gaps in our knowledge of using sims, particularly in conjunction with an assignment.

2.5 Prior research on targeted sims

Simulations as a mediational tool for discussion

The first study certainly worth presenting here was done by Valerie Otero, discussed in *Cognitive Processes and the Learning of Physics Part II: Mediated Action* [25]. The study focused on the use of a simulation in a high school physics classroom intended for juniors and seniors, and covered the topic of electric charge. The overarching goal for the activity was for students to construct a model of electric charge by analyzing different experiments and scenarios that involve charges moving from one object to another.

Students working in groups of three were led through three main "cycles," each having slightly different goals. The first cycle consisted of students working with real lab equipment to get a general feel for how charges appear to attract and repel, and how the distance between objects affects the strength of that attraction or repulsion. The second cycle asked students to construct a model that explains how rubbing insulators with various materials charges them. This was also where the simulation was first introduced. In the third and final cycle, students again experimented with the simulation and real equipment to explain charging by induction and charge polarization. Each cycle was designed to first elicit students' ideas about the phenomena, second, develop those ideas through experimenting with the sim or real equipment, and finally have them come to a consensus as a class in choosing a model that worked in describing the phenomena.

The simulation, shown in the figure below, consisted of both a life-like representation of what "should" be seen to happen when rubbing various objects together in real life, and a model-like representation of the same phenomena, showing a red and blue distribution of charge, representing positive and negative charges, respectively. In this model-like representation, students were able to watch charge distribute among objects in real time, and were allowed to control the movement of the objects that they touched together. While doing this, they could notice larger amounts of blue or red "layering," which corresponds to more charge build up, and similarly, a smaller "layer" of red or blue corresponds to less charge build up.

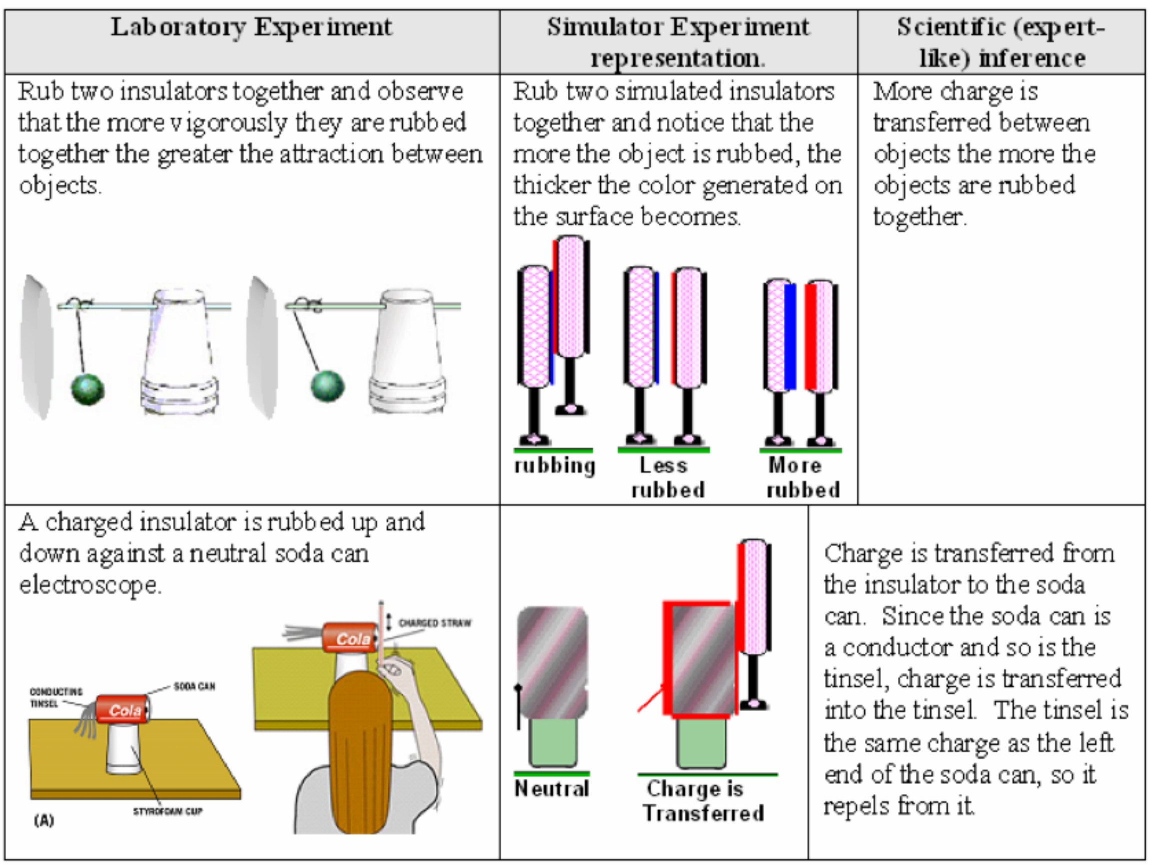


Figure 2.4: Various images of the simulation used in [25].

Otero noted two results from this study that stand out as particularly interesting. The first is that when the sim was first introduced in cycle 2, the amount of time students spent in sense-making discussions while using the simulation was greater than the amount of time they spent in sense-making discussions around the real equipment. Subsequently, however, in cycle 3, students spent a greater amount of time in sense-making discussion around the real equipment than they did around the simulation. Through looking at the nature of the conversations of the students, Otero was able to find a mechanism for the shift in the students' time spent "sense-making" around the real equipment over their time spent sense-making around the simulation.

The primary finding was that prior to using the simulation, students generally had different "models" for what charge *is*, and how it behaves in various experiments. For instance, several students thought of charge as something that can be "created." One of the reasons for this mental model is that as one rubs a cloth over a plastic rod, it charges and therefore rubbing the rod may be seen as a phenomenological mechanism of creation. On the other hand, some students held the implicit assumption that charge was simply being moved from one object to another in some fashion, and that originally there were both positive and negative charges that were free to move to different locations.

When the simulator was introduced, the students still largely held on to their original models, and it seemed that the blue and red layering was sufficiently ambiguous that the representation could be interpreted in a variety of ways. However, once the students started to discuss their models and try to come to a consensus on which model seemed to work, they were confronted with conflicting ideas that needed to be resolved.

Otero points out that the discussions held were mediated by the coloring scheme on the simulations. In the absence of that coloring scheme, the students could talk about "charge," but there was no medium through which they could articulate exactly what they meant when using it. Because the simulation allowed the students to articulate what charge meant to each of them personally, their ideas were brought to their own attention and to the attention of others, which allowed for collective evaluation of the phenomenon in question.

The ability to share different perspectives on the underlying meaning of the same representational scheme eventually led the students to sharing a model intersubjectively. Because of the discussions that took place, students were forced to consider some of the more abstract implications of their own ideas, and they were then able to collectively decide upon a model that fit the observed phenomena.

As was stated earlier, Otero shows that after the students' long sense-making discussions around the simulation in cycle 2 of the activity, the sense-making discussions in cycle 3 of the activity revolved much more around the laboratory equipment, rather than around the simulation. The mechanism for this change is the generation of this intersubjectively shared model of electric charge. After establishing this model, the students began using the simulation as more of a confirmation tool for what they noticed in the real lab equipment. Thus, more time was spent in cycle 3 applying their model to the lab equipment, since the use of the simulation was just a quick "check" to see if what they had decided upon was consistent with the simulation.

Otero's analysis brings up several key points that provide insights relevant to the present thesis. The first is that, in this case, the simulation acted as a tool that shaped the ways students interacted throughout the study. The coloring scheme, in particular, allowed students to articulate their ideas about the nature of electric charge in a way that would have been difficult, perhaps impossible, otherwise. Given the ability to articulate their ideas effectively, the students were able to publicly evaluate their ideas and come to a consensus on the nature of electric charge. In this sense, the simulation served as a mediating artifact in a particular environment. Secondly, the use of the simulation changed over time as students came to intersubjectively share a model of electric charge. Therefore, the tool being used, while never changing in its own form, became a fundamentally different "thing" as students' thinking evolved. (This suggests that studying a simulation in complete isolation would not account for its use in practice.) Lastly, in analyzing this situation, it was important to take a holistic view of the classroom interactions. The activity was a "system" with a great number of interacting parts that all factored into what the students took away from this activity. In other words, the importance of accounting for context was clearly present in this study.

Using sims to compare contrasting cases

The second study worthy of mention here involved the use of the "Faraday's Electromagnetic Lab" PhET simulation, discussed in by Dan Schwartz in *Explaining across contrasting cases for deep understanding in science: An example using interactive simulations*[28]. The underlying purpose of the study was to evaluate how students gain a "deep understanding" of the phenomenon at hand, which they characterize as "perceiving and explaining natural phenomena in terms of general principles." In their study, they compared the effectiveness of using the simulation with two different assignments present.

The first assignment used an approach commonly referred to as Predict, Observe, Explain (POE), while the second assignment used an approach that asked students to generate a general explanation (GE) of the phenomenon they witnessed. In addition, the POE and GE groups were divided in to two sub-groups, one using a measurement tool (which shows the strength of B_x, B_y and $\vec{B}$ at a particular location) and the other not using the measurement tool.

In total, the data from 80 students in an introductory physics course at a highly selective private university was collected for analysis. Students worked on the assignments in a 50-minute recitation section, and at the end of the section were asked to take a post-test to evaluate their understanding of the material.

The primary phenomenon under investigation for the assignment was electromagnetic induction, and more specifically, how a time-varying magnetic flux gives rise to an EMF, which in this case generates a current in a coil that lights a lightbulb. The simulation, shown in the figure below, allows students to move a bar magnet around the screen, and shows the direction of the magnetic field everywhere with small compass needles throughout the screen. A stronger field corresponds to a bright compass needle, and a weak field is represented by a dim compass needle.

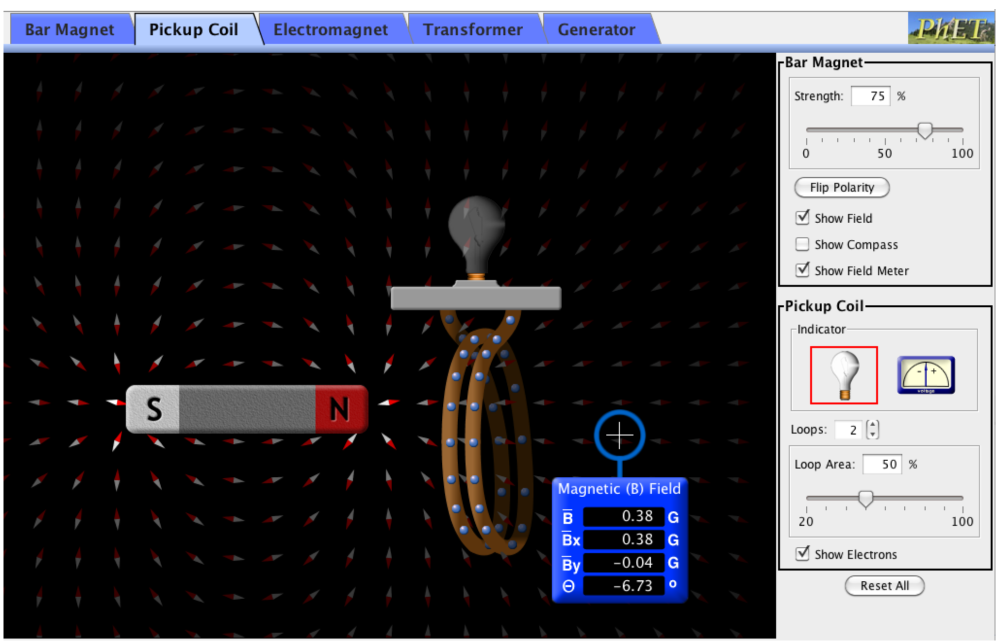


Figure 2.5: Faraday's Law sim. Taken from [15]

In each case, students were asked to analyze three different cases, shown in the figure below. The first case involves moving the north end of the magnet parallel to the area vector of the coils, the second case involves moving the magnet perpendicular to the area vector of the coils and the third case asks students to switch the polarity of the magnet (done by clicking a button on the sim). Of course, the students should notice that cases A and C cause the light bulb to turn on quite brightly while the magnet is moving, whereas case B should only cause a minimal amount of lighting of the bulb.

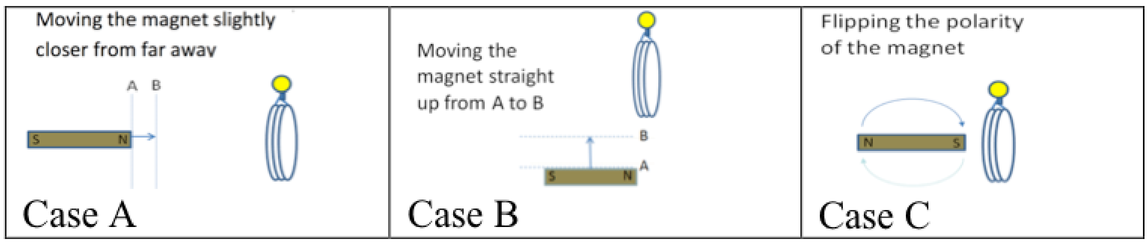


Figure 2.6: Three different scenarios of moving a bar magnet. Taken from [28]

The assignments differed in a few different ways. In the POE assignment, students were first asked to predict what would happen for each case, A, B, and C, separately. Subsequently, they were asked to use the sim to see what actually happened, specifically responding to, "explain what the light did," "explain the change in the magnetic field that caused the bulb to light." In addition, they were asked to draw the changes in the magnetic field. Each case was done at the same time, so the students were, in a sense, still comparing across contrasting cases by using this POE method.

In the GE group, students were first told what would happen in each of the three cases, and therefore the assignment did not contain a prediction phase. Instead, the students were supposed to examine these differing cases and come to a consensus in finding the general principle that explains the lighting of the bulb for all cases. Students were also provided an example of an adequate response to a different type of physics problem so that they had a better idea for what their final answers should consist of. After reading the example, they went on to playing with the sim, and were asked to draw and record their observations, and finally, give a general explanation of what must happen for the bulb to light.

According to the study, the GE assignment was superior to the POE assignment for three reasons. The first was that the GE students had higher post-test scores. In the POE group, approximately 24% of students were able to provide a "deep" explanation of induction, whereas approximately 48% of students in the GE group were able to provide a "deep" explanation.[3] These differences were statistically significant. In the study, they suggest that these post-test scores are indicative of the GE group having a "deeper" understanding of the phenomenon of induction.

An side note here is that in addition to a larger percentage of students in the GE group giving deep explanations, the students using the measurement tool in both the POE and GE groups had a higher percentage of students giving deep explanations on the post-test than did the groups not using the measurement tool. However, this difference was not statistically significant, and therefore difficult to draw conclusions on. Still, this gives some indication that the use of the measurement tool could have aided students in developing a deeper understanding.

Secondly, the GE students had a far greater tendency to give deep responses *on the assignment* when asked to provide a phenomenological description that explains all three cases. According to the study, only 1 out of the 35 students using the POE assignment wrote a deep explanation, while 14 out of 45 students using the GE assignment wrote a deep explanation. This suggests that the students using the GE were more likely to have achieved deep understanding during the assignment. The study also claims that students who wrote the deep explanation on their worksheet were much more likely to apply that same reasoning on the post test, and provide a deep explanation there.

The final reason given in the study for the superiority of GE to POE was that there was no correlation between prior student performance and post-test scores in the GE group, whereas correlations of this type did exist in the POE group. In other words, students in the POE group who had been performing well in the course prior to this assignment tended to do better than the lower-performing students on the post-test. On the other hand, there was no correlation of this type in the GE groups; both higher and lower-performing students showed similar learning gains. As they stress, this indicates that the potential for "deeper understanding was open to students of all levels" while using the GE assignment[28].

This study provides several insights that are of significant importance for this thesis. First of all, it demonstrates the potential for simulations to give students a means of

[3]Two points to make here: First, these percentages are my rough interpolation of what the average percentage of students would be from averaging the data provided in their bar graphs. Second, a "deep" explanation of induction, according to the group, had to include some sort of reference to the change in the magnitude of the component of the B-field perpendicular to the loop as being the source of EMF generation.

achieving deep understanding of physical phenomena. Secondly, it suggests that students' exploration of the simulation and what they take away from their interactions with the simulation is deeply influenced by the accompanying assignment. Third, it suggests an effective way of writing assignments; namely by asking them to look for underlying principles that explain why the simulation shows what it shows. The reasons for the superiority of this method, while not fully explainable in psychological terms, could be explained by the fact that the students using the GE assignment are looking for what stays the same across the three cases. On the other hand, the students using the POE assignment are not explicitly drawing their attention to the underlying phenomenon, but instead are noticing "surface features" of the sim that do not immediately provide insights in to the underlying physics. Thus, the motivation or goals that the students have while using the sim likely influences their interactions, and through that, their ability to pinpoint the underlying physical principle.

Replacing real lab equipment with simulations

The final study worthy of mention here tested the efficacy of using computer simulations in place of real lab equipment when using electric circuits, discussed by Noah Finkelstein et. al. in *When learning about the real world is better done virtually: A study of substituting computer simulations for laboratory equipment* [11]. In a large-enrollment introductory algebra-based physics course at a large research university, a lab session that historically had used only real equipment for an electronics lab was divided in to two different groups – one using real equipment and one using the PhET "Circuit Construction Kit" (CCK) simulation. Both groups consisted of approximately 100 students.

The study found the lab using the CCK simulation to be more effective than using real equipment, based on two main pieces of data. First, students in both groups, once finished with the laboratory, were asked to build a simple circuit *with real equipment* and then check with their TA to ensure that they had built it right. It was found that the CCK group was able to correctly build the simple circuit in a shorter amount of time than was the group that used real equipment in the lab. Note that the students using the CCK sim had no prior experience with building circuits with real equipment.

Second, students using the CCK simulation in lab showed higher performance on a three-part question on the final examination for the class concerning concepts about how the brightness of light bulbs will change when one bulb is taken out of a circuit. The final exam was taken nearly 2 months after students worked on the laboratory, and the results were statistically significant.

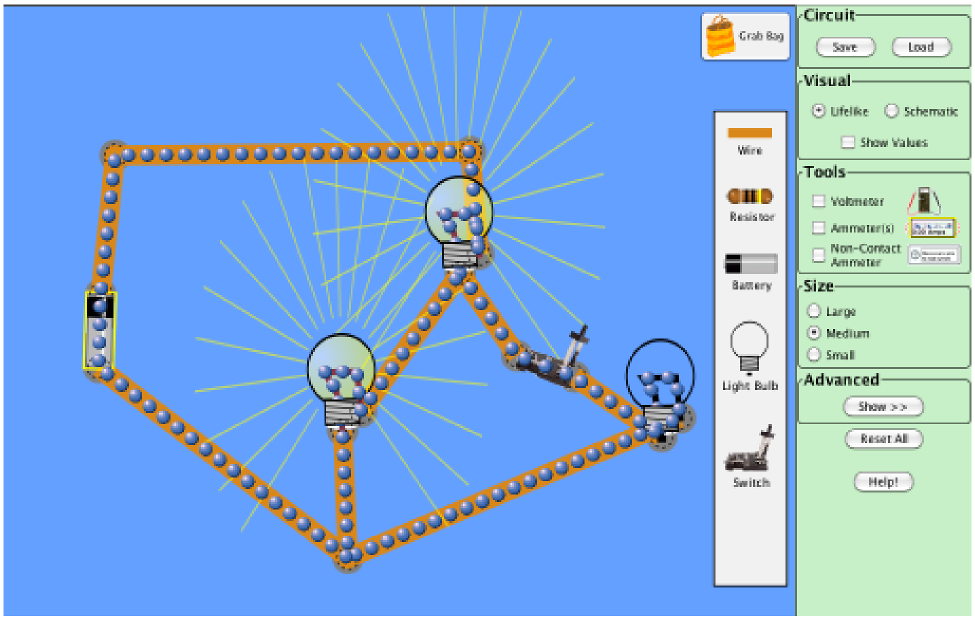


Figure 2.7: The CCK simulation. Wires, batteries, resistors, etc. can be dragged on to the blue workspace and connected in various ways to form simple circuits. The blue dots in the wires move in time, indicating the direction of the flow of electrons. Differences between a large current and a low current can be intuited by the rate at which the blue dots move[30].

While the reasons for the higher performance of students using the CCK sim were not discussed in an exhaustive fashion, field notes and observations of the students during this study provided insights into why the CCK lab was more effective. First, students using the simulation and students using real equipment were seen “messing about,” a term used to describe students’ exploration of the constraints and opportunities a system provides. For various psychological reasons, messing about is thought to be a crucial “ingredient” in helping students to develop intuition of how systems work. The difference in this study was that, though both groups were seen messing about, students using the simulation were generally messing about by building circuits on the simulator, whereas many times students using the real equipment were seen creating bracelets with the various wires and such. Thus, one observation is that the simulation constrained the students in a more productive way, allowing them to spend more time on task.

Additionally, students using real lab equipment often ran in to technical problems getting the equipment to work right. For instance, since some batteries were not working so well, students had trouble seeing parallel light bulbs light up. Often, this would force the TA to spend extra time switching out equipment, which took away from the time he or she could spend working with the students.

There are a few main points to take away from this study. First, simulations can in fact be more effective than using real laboratory equipment for developing conceptual understanding when used in the right setting. Second, simulations can guide student reasoning through the use of effective constraints (both on how students spend their time and what the students focus on in their interactions with the simulation). Third, because simulations are generally less problematic than real equipment in terms of making sure the equipment works right, the time that students and instructors spend productively working to uncover the underlying physical concepts of the topic at hand can be greater when using the simulation.

2.6 Foundations of Design of PhET Simulations

Characterizing Complexity of Simulations

So far the discussion of simulations has primarily concerned their use in different settings, as well as some of the benefits of using simulations in those settings. To provide a complete picture, however, it is also necessary to discuss how sims are designed. Because the rest of this thesis is dedicated strictly to the use of PhET sims, this section focuses on research carried out by researchers on the PhET team.

Most recently, Podolefsky et. al. suggested the possibility of characterizing simulations based on their "complexity"[17]. One question they present is "When a student sits down in front of an interactive computer simulation, what do they see?" Clearly, a student will see a number of sliders and adjustable parameters, and the visual features on the sim will influence what they feel they can initially start to play with. But as can be imagined, each student may focus on different features of the sim, and each student brings with them prior knowledge that will factor in to how they use the sim and what they take away from the sim in the end.

Podolefsky's idea is to characterize complexity on two dimensions to start with:[4] the features of the sim and prior knowledge of the student. To characterize features of the sim, he starts by creating a "complexity matrix." This matrix lists the various features of

[4] Of course, there are clearly more dimensions that are open for characterization, but these serve as a good starting place.

the sim that can be controlled by the user in rows. In columns, it lists "readouts," which are features of the simulation that change as one adjusts a certain parameter.

The example Podolefsky provides concerns the "Gravity Force Lab" simulation, shown in the figure below. Two large spheres are shown on the screen, and the user can adjust the mass of the spheres, as well as their relative distances. As they adjust these parameters, the force arrows, the pullers, and the text all change.

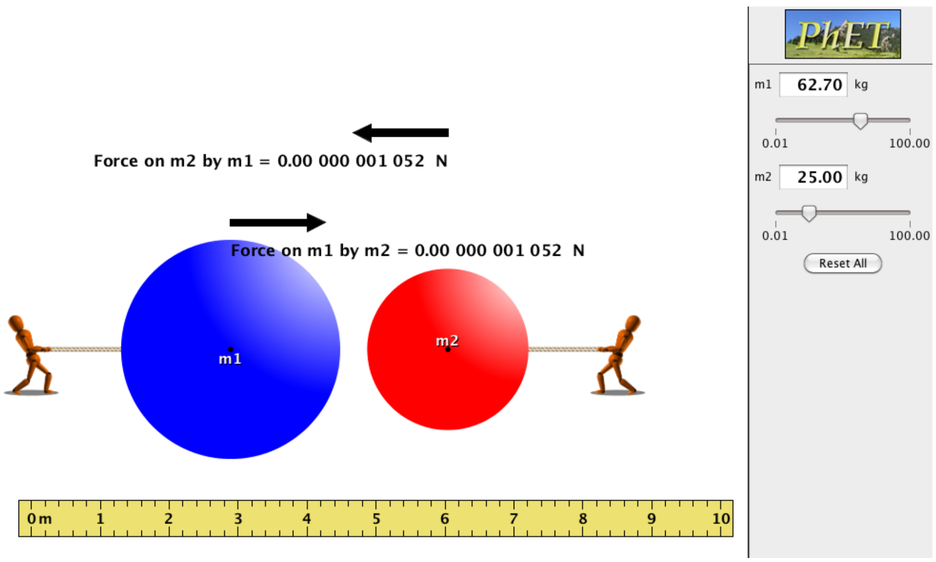


Figure 2.8: Gravity Force Lab simulation [29].

In characterizing the complexity of this simulation, Podolefsky marks a "1" in each matrix element for a feature that changes based on the user's adjustment of a parameter. For instance, changing the position of the first sphere affects F_{12}, and therefore matrix element [Position $1, F_{12}$] receives a "1." On the other hand, changing the position of the first sphere does not affect the mass of the second sphere, so that matrix element receives a "0." The complexity matrix for this simulation is shown below.

TABLE 1. Force Law Lab Sim-Based Complexity; 0 / 1 indicates whether a control (rows) affects a readout (columns).

	Position 1	Position 2	Mass 1	Mass 2	F_{12} Arrow	F_{12} Text	F_{21} Arrow	F_{21} Text	Puller 1	Puller 2	Sphere Size 1	Sphere Size 2
Position 1	-	0	0	0	1	1	1	1	1	1	1	0
Position 2	0	-	0	0	1	1	1	1	1	1	0	1
Mass 1	0	0	-	0	1	1	1	1	1	1	1	0
Mass 2	0	0	0	-	1	1	1	1	1	1	0	1

Figure 2.9: Complexity Matrix for Gravity Force Lab simulation, taken from [17].

Total complexity of the simulation, based on features of the sim alone, can be found by summing all of the matrix elements. In this case, the force law simulation receives a complexity of rating of 28. (Sims range from as low as 6 to over 100, indicating that this is a fairly simple simulation).

In order to incorporate an adjustment to this complexity rating based on the user's unique use of the simulation, Podolefsky suggests that the matrix can be "collapsed" if the student has already made sufficient connections between different representations on the sim, thus lowering the total perceived complexity. For instance, a user who already knows about forces as having some magnitude, and understands the relationship between a larger arrow and text corresponding to a larger number, will not perceive these two representations as different, and therefore the simulation will look less complex to this user than another user who has not made those connections.

Their research also provides data from interviews that illuminates the differences in student reactions to sims of varying complexity. Specifically, when students sit down in front of a sim with a high complexity rating, they initially act confused or overwhelmed at how much there is to notice and play with. They also find that when students use simulations with lower complexity rating, their actions are more predictable than their actions using a highly complex sim, an observation consistent with prior literature on human-computer interaction. Another insight is that the amount of student "learning," or perhaps better phrased, what the students "take away" from the simulation, is highly dependent on their initial interactions with the sim and the complexity they perceive in those initial moments.

The main point to take away from this research is that simulations are indeed complex, and their perceived complexity is largely influenced by students' prior knowledge. In addition, progress is being made in starting to characterize the complexity of these simulations at the micro level, and this research can (and is) being used to influence the design of simulations for future use. Still, as the authors will admit, this only accounts for two dimensions of complexity, and the use of these sims in practice forces us to add more dimensions if we are to successfully provide a full accounting for complexity. Such dimensions include the social setting in which the students are embedded, the teacher's interactions and instructions for using the sim, any assignments the students use in conjunction with the sim, and so on. The hope is that this thesis, which focuses more on the macro level of analyzing simulation use, will provide data that complements and makes use of this research in complexity, and perhaps gives insight in to how to begin characterizing complexity on these higher-level dimensions.

PhET Design Cycle

While characterizing complexity of simulations has the potential to provide deep insights for how simulations should be designed, the design of the sims themselves is described by a design cycle provided by the PhET team [34, 2]. The process always begins with content experts developing learning goals for the particular simulation, and then constructing a basic interface that appears to be clear and engaging. When the initial design is complete, PhET researchers conduct student interviews to evaluate the effectiveness of the sim. The interviews nearly always reveal interface weaknesses, pedagogical problems, and minor sim bugs. With that information, the sim is revised or perhaps redesigned in parts, and then re-tested through a second round of interviews. This cyclic process is repeated until researchers feel that the sim is sufficiently effective, and it is then tested out in a classroom setting. Further information concerning the sim collected in the classroom setting is then used to make final adjustments to the sim before it is posted online for free use world wide.

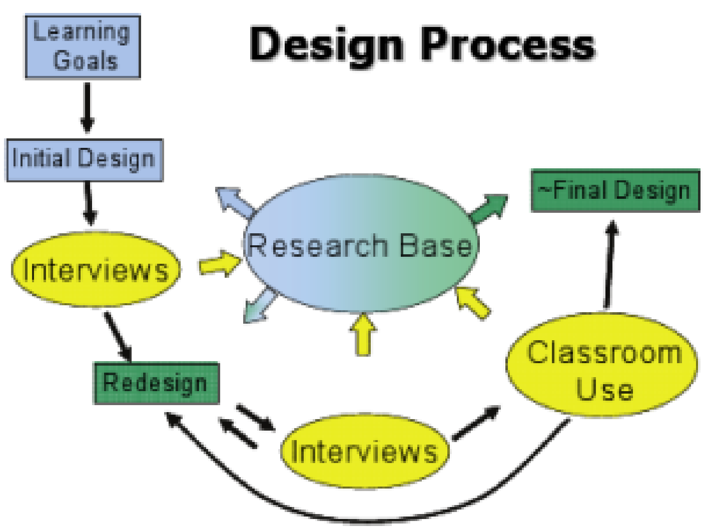


Figure 2.10: PhET Design Cycle, taken from [34].

As can be seen in the design cycle, there is a "research base" that complements the foundation for design. Podolefsky's work on complexity might be seen as fitting in to this piece of the cycle, and data from the interviews and classroom use goes in to this collective research base, as well.

The main point to take away from this discussion is that the simulations are designed through a research-based process, and this ensures that the sims themselves are more effective than we might expect from those that do not use a research-based approach.

Establishing engagement with sims

Exactly how do PhET researchers decide if a sim is effective or not? One way of measuring this is to see whether students engage and explore with the sim. In fact, Podolefsky et. al. refer to this as "engaged exploration." The term is not necessarily meant to imply that genuine scientific discovery is occurring, but rather that students are able to spend a good deal of time exploring the various features of the sim. Thus, they avoid the question of what the student is "learning," and instead ask what the students are taking away from their interactions with the sim. It may sound as though there is no clear distinction between these two views. However, the concern with asking what students are "learning" is that the question presupposes something that we will see students doing with the sim or taking away from the sim. As researchers, this view can limit our evaluation or perception of the effectiveness of the simulation. If we instead ask what the students are getting out of their time interacting with the sim, we are focusing our attention more broadly on the students and how they decide to use the simulation. This provides greater insight into the nature of students' reasoning than would checking off items on a list that we presuppose the students need to complete to learn the content.

That said, this does not imply that we don't look for evidence of student learning. Instead, research has shown that engaged exploration leads to greater learning gains in most situations[16]. Thus, promoting engaged exploration is a key concern when designing the simulations. In order to go about accomplishing this task, it is helpful to introduce a couple of new terms that organize the way we can think about simulation design.

The first term is "affordances." In short, affordances are what users *perceive* particular tools to be used for. This term (as well as the next) is drawn from Donald Norman's book, *The Psychology of Everyday Things*[1]. His definition first discusses features of things found in everyday life, including door handles, telephones, and numerous other examples. Usually when discussing this, we speak of different objects as "affording" different actions. For instance, a door handle "affords" pulling, or a coffee mug "affords" holding. When objects appear to afford some action, and they don't, this creates cognitive conflict on part of the user. For instance, how might you open the door in the picture below?

Figure 2.11: Taken from [36].

Affordances are also present in simulations. Sliders, moveable objects, etc. all afford some action on the side of the student. If a feature appears to do something, and it doesn't, this causes some discomfort on the side of the student, and can hinder their likelihood of engaged exploration.

In addition to affordances, Norman introduces the notion of constraints. These restrict the possible actions a user can take with a particular tool, and can likewise be a beneficial way of keeping students on task. An analogy that is helpful in thinking about constraints is to think of a person driving a go-cart. Usually the driver can choose where they want to drive, but they have only limited control over how fast they go. The go-cart constrains the speed of the car, and when time is up, the cart is further slowed electronically, so that everyone is moving slowly while parking next to each other. Without this constraint, there would surely be more accidents in the go-cart arena.

In terms of simulations, constraints present themselves as preventing users from accomplishing certain tasks. For instance, in the Gravity Lab sim, users can only move the spheres left or right, and can only control the mass of the objects, not the density or other features. Thus, the possible number of actions on the side of the user is limited according to the design of the simulation. This gives designers some control in determining what students do or do not work on. The goal, of course, is to establish productive constraints so that students don't worry about unnecessary or unimportant features.[5] This balance between making the sim complex enough to be engaging, while at the same time, not overwhelming, is difficult to achieve. Still, research in this area has allowed us to learn a great deal about students' interactions with simulations.

There are two additional points to make about constraints and affordances. First, these do not necessarily have to be physical in nature; they can also be conceptual or intellectual. For instance, Podolefsky refers to analogies as "cognitive affordances," which are often built into sims. Analogies afford the mapping of certain "well-known" phenomena to less understood phenomena. For instance, the "Wave Interference" PhET simulation has different tabs corresponding to different wave-like phenomena, from the most intuitive water wave model to an arguably non-intuitive photon model. This is meant to help students tie connections between the behavior of water waves and other forms of waves, while explicitly showing the differences between these models at the same time. Similarly, an example of an intellectual constraint is one's *affect* or self confidence when trying to solve physics problems; little confidence in being able to understand the material is a type of constraint that is of a different form than the physical constraints discussed earlier.

The second main point to make about this work is that the ideas of affordances and constraints bear striking similarity to the notion of artifact mediation discussed earlier. If we recall that artifacts are material objects that have been modified over the history of their use in human life to better suit a purpose, we might interpret this as saying that modifying artifacts occurs when we try to improve the object's capacity to afford a particular action. The fact that these two methods of thought are consistent provides at least some confidence that research on constraints and affordances is an integral part of this type of educational research.

[5] An example of this presented itself in the study of substituting lab equipment for the CCK sim. Several students using real equipment thought that the colors of the wire was important for the functionality of the circuit. The constraint of no wire color on the sim prevented students from raising this same thought.

Creating activities using guided inquiry

From the previous section, it should be fairly clear that promoting engaged exploration is one of the main goals of simulation design. But in addition to designing the simulations to promote engaged exploration, there are external factors that can help or hinder students' likelihood of engaging in productive ways.

The first and simplest way of externally providing impetus for engaging students with the simulation is by asking driving questions. Interviews conducted by the PhET team historically have used this particular method, and research shows that asking questions of this type can fundamentally change the way students interact with the simulations, often in a productive way [35, 16]. An example of a driving question might arise in the CCK simulation discussed earlier. For instance, one could ask, "What is the fewest number of components you can use to make a lightbulb light?" This question will direct the student's attention to completing a particular task, and questions of this nature that establish initial engagement have also proven to lead students to engaging for long periods of time.[6]

While driving questions are frequently used to promote engaged exploration with simulations, this is typically done in an interview setting. Yet we confront an unfortunate fact that the vast majority of simulation use is not done in this type of setting. Thus, it becomes crucial to understand the use of simulations in more natural environments; that is, in the contexts in which they are typically used.

The PhET team has provided some research on this so far, but mcu of their knowledge is experientially held, and has yet to be externalized. Still, the PhET team has a list of strategies for designing activities around their sims[7][33] The list is as follows:

1. Define specific learning goals
2. Encourage students to use sense-making and reasoning
3. Connect and build on students' prior knowledge and understanding
4. Connect to and make sense of real-world experiences

[6]The other primary method used in student interviews is to ask the students to simply play with the sim, without providing a driving question. The efficacy of this method is often dependent on the particular simulations (it can be quite effective for some and not so effective for others). I will elaborate on these points throughout the thesis.

[7]Part of the motivation for creating this list of strategies was to recommend how teachers who plan to write activities and post them on the PhET website might structure the activities they write.

5. Design collaborative activities
6. Give only minimal directions on sim use
7. Require reasoning/sense-making in words and diagrams
8. Help students monitor their understanding

More details on each of these topics is provided in [33], but this at least provides a starting place.

It should be fairly clear that, while much research has been done concerning simulation design and their effectiveness in interview settings, less has been done around the use of sims in more natural environments. The goal of this thesis is to make contributions in that area by providing heuristics for how we might incorporate simulations into assignments. What follows will build upon the material provided in this background section, and provide new insights for how sims are used in a variety of environments.

2.7 Summary

This background started with a very general framing drawn from cultural psychology. It suggested that in order to address educational issues, we must think of learning not in terms of a strictly "in the head" phenomenon, but rather as a social act that is mediated through the use of artifacts. The essential point was that learning occurs within context, and that context plays the critical role of "weaving together" various components of what ought to be considered a type of ecosocial system.

Within that general framing, the idea of literacy presents itself as a natural extension of artifact mediation, being in itself a secondary artifact which allows for the interpretation of primary artifacts, and simultaneously, the formation of tertiary artifacts. The idea of computational literacy drawn from diSessa suggests that computers can provide a medium for developing a new, material intelligence that will undoubtedly play a crucial role in the cognitive processes of current and future generations.

To provide an example of how computational media may be used in a productive way, the discussion turned to the use of simulations, specifically in physics settings. Otero's research suggested that simulations can serve as a mediating artifact that allows for effective collaboration around underlying physical concepts and model-building, providing students with a way to voice their ideas about the nature of physical phenomena. Schwartz et.

al. provided an example of how simulations might be effectively used, namely by asking students to deduce the underlying physical principles that explain why the sim behaves in the ways shown. Finally, it was shown that simulations can facilitate learning more effectively than using real lab equipment in some instances; the CCK simulation effectively engaged the students and provided a model of charge movement not possible when using traditional lab equipment.

Finally, the discussion turned to the design of simulations themselves. Podolefsky's idea of characterizing complexity of sims was discussed, along with some of the implications his research has for sim design. Next, the PhET design cycle was presented with a discussion of how data is collected and used to modify simulation design. This was followed by an overview of affordances and constraints, and how they can help or hinder student engagement. In addition, the idea of engaging students through the use of driving questions and open-ended assignments was discussed briefly.

With this foundation laid, it is necessary to discuss where the discussion will lead. The rest of this thesis is dedicated to presenting case studies of various research projects conducted in a variety of environments, ranging from middle school to upper-division physics courses. Because less work has been done on understanding the effective use of simulations in natural environments, the work here presented is an attempt to provide "heuristics" or general guidelines that provide insight in to what does and does not work so well when incorporating sims into physics assignments. In addition, a framework will be presented that helps uncover the methodology behind incorporating these heuristics across different contexts.

Chapter 3

Project History

Because one of the goals for this project is to provide insights into how our thinking about writing assignments has evolved, it is necessary to give a semi-historical account of how the project started, as well as how and why it changed over time. In order to do this, this section gives a chronological summary of various findings and insights that forced us to reevaluate our goals and motives for the project along the way. While a detailed account of the findings are not included in this section, the hope is that this general framing will situate the remaining chapters of the thesis, wherein the details will present themselves and reinforce the historical framing presented here.

3.1 Initial Ideas

At the start of this project in the fall of 2010, a unique opportunity for a first case study presented itself. At the time, I was working as a learning assistant for a modern physics course, and we had planned on using a tutorial in place of one of the lectures in approximately the third quarter of the semester. The tutorial, which had already been written by a PER group member at CU, dealt with the topic of quantum mechanical tunneling. The tutorial was meant to be worked on in groups of four, and nothing other than a pen or pencil was needed to successfully complete the tutorial.

To my delight, there already existed a tunneling simulation for this particular topic, and I was quick to ask if we could try a comparative study between groups using the simulation and groups not using the simulation during the tutorial. When the time came, we asked four students (out of nearly 100) in the course to work on the tutorial in conjunction with the simulation, and we audio-recorded their conversations as a source of data for the tutorial.

There were three primary "research" questions that we hoped to answer, or at least gain insight into when we first started. These were: "Can we distinguish between student productivity when using/ not using the sim?", "What are the differences in student discourse while working on the two tutorials?", and "How do students' questions compare in the two groups? Are they more about clarification or do they probe at the underlying physical concepts?"

Unfortunately, we were unable to answer any of these questions from this initial study because students simply didn't get the chance to play with the sim. The tutorial used for both assignments was the same, and it was clear that in both groups, students spent nearly the entire time working on solving mathematical equations; namely by plugging in a wave function to the Schrödinger equation, and eventually finding a dispersion relationship that has to be satisfied for the solution to hold. Thus, in a fifty minute period, students in the simulation group were only able to spend five or so minutes actually using the sim, which was hardly enough time to gather data from.

Though we were unable to answer the questions we initially had about this study, our observations did lead us to designing a new tutorial intended to shift the amount of time students spent solving mathematical formulae to working towards a more conceptual grounding in the subject.[1] Additional revisions to this tutorial were made for the Spring 2011 semester, and the study conducted at that time is discussed in more detail later in the thesis.

The study may seem pointless to even mention, but in fact, it led us to think, "Hey, maybe we should come up with a list of strategies for including sims into assignments." It was clear that students were not reaping the benefits of using the tunneling simulation since they were stuck solving equations the entire time. One natural "strategy," then, might be to avoid asking students to spend time solving equations on the assignment. At the time, we had ideas about what might or might not be effective for writing assignments to include sims, but we had no evidence to support those ideas.

Inspired by this initial failure to collect useful data, we decided to turn to the literature to see how the research-base might apply to the present project. After doing fairly extensive research, we came up with an initial set of "heuristics" that we thought would

[1] This first revision to the tutorial was developed by myself and Sam Milton (another learning assistant), and also included several suggestions for improvement provided by Charlie Baily (one of the instructors for the course).

be important for incorporating sims into assignments. Here the initial list is presented, with details about where they came from and/ or how we thought they might apply.

1. *Ask for comparative analysis when possible*

   This heuristic was based primarily on the paper by Schwartz et. al. discussed earlier, since in it he found that asking students to compare across contrasting cases can lead to "deep understanding" of physical phenomena. Note that this heuristic does not specify *how* it should be implemented, as here we are trying to account for all possible situations in which this might arise. Unlike Schwartz's study, there may be cases in which we cannot ask students to provide a "general explanation," and might instead be forced to use something like the "predict, observe, explain" method. Thus, the idea of comparative analysis was used initially to give a sufficiently broad span, considering that these were, after all, tentative strategies.

2. *Use measurement tools when possible*

   Again, this was taken from the same study by Schwartz et. al., based on their tentative findings that using measurement tools can help students come to a deeper understanding of physical phenomena. Additionally, one can imagine cases in which a measurement tool *has* to be used, particularly if the assignment asks students to take data and conduct some sort of an experiment. Thus, one can imagine a couple of possible subcategories for this heuristic: asking students to use measurement tools to take data, or providing students with measurement tools that aid in their developing intuition about physical systems, without explicitly asking them to take data.

3. *Use the sim as a tool for mediating discussion*

   This heuristic was drawn primarily from the study done by Otero, discussed earlier. Recall that the blue and red model representation of charge allowed for the students to voice their opinions about what charge is, and how it distributes among conductors. By voicing their opinions, the students' ideas were brought both to their *own* attention, and to the attention of others, and this allowed for the public evaluation of different scientific models, thus engaging them in the practice of scientific discourse.

   One can imagine that this heuristic should be useful at *any* level, and not just for high school students. In fact, much emphasis recently in engaging college-level students in scientific discourse, as course transformations at both the lower division and upper division are specifically designed to have students collaborate in groups around tutorials, or in lectures with clicker questions[41, 42]. It should be expected that some of the insights from these other domains might be applicable here, though again the question of *how* we write assignments to engage students in discourse has yet to be discussed.

4. *Call attention to visual features on the sim*

   This heuristic was primarily an ansatz to start with. The basic idea is that in certain circumstances, it may be a good idea to point out features of the sim that students might otherwise gloss over. One way to do this is to explicitly tell them, "notice this feature, it will help you complete such and such a task." Yet another way is to ask a question centered around a particular feature. For instance, if a particular feature of the sim becomes apparent only after completing a task, one might ask "how do you know you have completed this certain task?" The primary idea was that in some cases, it might be necessary to draw attention to features that are particularly important, and should not go unnoticed.

5. *Use dynamic feedback as a tool for experimentation*

   Dynamic feedback is mentioned, in some form or another, by nearly everyone who discusses sim use. For instance, Noah Finkelstein in [6] states that simulations "emphasize causal relations by linking ideas temporally and graphically." The application of this idea, we thought, might come in the form of using that as a tool for experimentation. At this point, the definition of "experimentation" was ill-defined, but again, this was a broad idea for what *might* work, in hopes of revising or refining later.

6. *Use the sim to design a virtual experiment*

   Again, the idea of experimentation seemed particularly important at the time, though ill-defined it may have been. There were, in fact, a few ways we thought of incorporating experimentation. One way would be similar to the way the CCK simulation was incorporated into a guided lab activity, as described in the background of this thesis. Another potential way would be to ask some sort of driving question, and allowing students to design their own experiment around that, using the PhET sim. The general idea was that this heuristic might be implemented in a situation where ample time was available for students to extensively explore and use the sim in various ways.

7. *Use the sim as a device for relating formal concepts to the real world*

   This last heuristic arose out of the hope that physics concepts, which have historically been viewed by students as being unrelated to the real world [10], might be connected to the real world through the use of this virtual medium. Because sims show the repercussions of physics concepts in more real-life situations, we felt that this may be one way to tackle this long-standing problem. (An example of a sim that stands out as particularly relevant here is the "John Travoltage" PhET sim [12]. In it, one can move Mr. Travolta's foot across the carpet, causing electric charge to accumulate across his body. When he touches the door knob, he instantly gets "zapped!") Again, we aren't specifying *how* to go about doing this, but rather this was an additional idea of what we might incorporate into our assignments.

One fact that I must point out from this list is that these heuristics are entirely "topical." That is, they provide a tentative list of *what* to do, but they provide absolutely no information about *how* to implement them or *why* they ought to work. In answering these questions, I cannot help but again draw on theory to explain the how and the why. The theoretical underpinnings, which are elaborated on in the next chapter, are also tentative at this point. They are drawn largely from an "activation of resources" perspective, coupled with various theories of concept formation.

However, first it is necessary to elaborate on how these heuristics evolved to the still tentative, though more refined, list current today. The following section will read much like an overview of the case studies presented in the remainder of the thesis for precisely this reason.

3.2 Revision of Heuristics

While creating the list of heuristics was the first step of this project, we felt that we needed data to test the robustness of the heuristics, and see how well they "worked." This led to developing several different assignments and testing them out, primarily done in the spring of 2011.

In all, four case studies provided great insight into the utility of our heuristics. The first study was conducted in an upper-division classical mechanics course (PHYS 2210), and used the "Projectile Motion" PhET simulation on a homework problem concerning quadratic air drag. The second study was conducted in a middle school classroom in Texas,[2] and used the "Build a molecule" PhET simulation in conjunction with an in-class worksheet. The third study utilized the "Quantum Tunneling and Wave Packets" tutorial (discussed earlier) in an introductory modern physics course (PHYS 2130). Lastly, another homework problem was assigned in the same classical mechanics class, this time using the "Resonance" PhET sim.

The sources or types of data collected varied across these different assignments, and the interpretation of the data was done primarily in the summer of 2011. While I will not go into great detail here about the type of data collected and the specific findings, it is beneficial to illustrate the changes in the heuristics, with an overall description of why they changed. The remainder of this thesis is dedicated to the interpretation of data, so forgive the seemingly unsubstantiated nature of the claims made in the present section; the heuristics are presented this way because it will allow for the *how* and *why* associated with using these heuristics to be understood in the following chapter.

One overarching theme that became apparent in the studies was that our assignments tended to be too prescriptive. Generally, our questions were phrased as "Set up the sim this way. What do you notice?" Students generally react to these types of questions by doing precisely what the questions tell them, filling out an answer, and then moving on to the next question, without ever spending time sense-making or trying to explain the underlying physical phenomenon under question. Of course, on the side of the researcher or designer, we tend to think, "how could they not realize why I am asking the question this way?" or "why don't they see the physics here?" Notably, similar questions were raised by Mazur when he first began exploring the reasons for students learning only 1 out of 4 basic concepts in his Harvard lectures [47].

It is important to point out the similarity of these findings to those of Schwartz et. al. presented earlier. In their case, they noticed that students in the "predict, observe, explain" group were more focused on "topical" features of the simulations, and never tried to find an explanation that worked for all three cases because of it. In our case, we notice that when we explicitly ask students to perform some action with the sim, we likewise force them to look at topical features, while the underlying physics is glossed over. Additionally, it seems that these questions send them into "task completion" mode,

[2] This study was actually conducted by Emily Moore and the PhET team. I was lucky enough to have the chance to analyze some of their video files from the course.

wherein working on the assignment is much like mindlessly following a baking recipe. Therefore, a crucial finding is that *how* a question is asked can affect students' likelihood of developing a deep understanding of the phenomenon under question.

The first major changes to the initial list of heuristics were made whilst developing the projectile motion homework problem in the spring of 2011. While trying to write the problem, it was clear that, first of all, there is a great deal of mathematical complexity in the simulation itself, and secondly, only *one* of the cases in the sim can be solved analytically. Thus, since we were dealing with quadratic air drag in class, we felt that we should come up with a problem that could be solved in closed form, and at the same time shown on the simulation. It turns out that the case of shooting an object vertically upward into the air *can* be solved in closed form, so we decided to play with the sim and see what kind of questions we could write that take advantage of this particular case[3]

To our surprise, we found that when shooting a ball vertically upward, there is, in fact, a limit on the time the ball can take to get to the top of its trajectory (we found this simply by playing with the sim). So if one takes a baseball and shoots it in the air, it will always start descending after, say, five seconds, given that it is shot at high enough of an initial velocity. We felt that this was an incredibly interesting or "illuminating" case, so we decided to include it in our homework problem. This gave rise to the idea that perhaps another heuristic is "Set up the sim to look at illuminating cases." As it turns out, this heuristic has appeared in nearly all of the assignments we have written.

We also felt the need to add another heuristic, based on the use of mathematics in this homework problem. This heuristic is, "Use the sim as a sense-making device for other forms of representation." We felt that, not only could the representational form of mathematics be coordinated with use of the sim, but perhaps graphs, diagrams, and other representations could, as well.

These two heuristics were also implemented in the quantum tunneling tutorial, discussed in greater detail in Chapter 7. From both of these studies, it appeared that part of the reason for needing these additional heuristics was due to the fact that the sims were difficult to understand without the incorporation of mathematics into the assignments, or without some sort of illuminating case that makes a particular physical concept "stand out." In both cases, differential equations must be solved to understand what is shown on the sim, and as it turns out, there is often a way to emphasize a particular set-up of

[3]By saying "we" in this paragraph, I am referring to Prof. Steve Pollock (the instructor for the course) and myself. He was of great assistance in helping write and evaluate the homework problems for this class, as well as the quantum tunneling tutorial for upper-division quantum mechanics.

a sim that helps tie together these mathematical and visual representations.

Yet another heuristic that was added, based on the tunneling tutorial data, was "Set up situations that utilize 'Predict, observe, explain' or 'Elicit, confront, resolve' models. Of course, the former of these was discussed by Schwartz et. al., and they showed this method to be *less* effective than other methods in getting students to develop a deep understanding of the phenomenon at hand. However, this was applied in *one context*, and our findings in the tunneling tutorial indicate that there are instances in which this can be a useful strategy for writing assignments. The latter of these is discussed by McDermott in [44]. In fact, the University of Washington tutorials [43] are notorious for using the 'elicit, confront, resolve' model. We group these two models in the same category because they seem to be quite similar overall.

Eventually, we added another heuristic, based on research around the build-a-molecule study done in a middle school classroom. Students were asked to take a pre- and post-test before and after using the sim that assessed their understanding of chemical formulas, chemical names, and representations of chemicals. For example, students were asked to draw a picture of H_2O or $2H_2$, or write a chemical formula based on a picture of some molecule or group of molecules. As can be imagined, the students performed very poorly on the pre-test, since they had never been exposed to these ideas before. However, on the post-test, most students were drawing three-dimensional representations of chemicals, and were very successfully determining the proper chemical formulae and chemical names for various molecules.

From analyzing the data, we felt that one of the reasons for the drastic difference in student performance on the pre- and post-tests was indirectly due of the framing of the assignment itself. In the assignment, students were asked to draw pictures of the chemicals they made on the sim, and to write out the chemical names and formulae. The sim showed three-dimensional views of "completed" molecules, and it seemed that asking the students to draw the molecules helped them internalize what those molecules looked like.[4]. Thus, we felt the need to add an additional heuristic: "Ask students to recreate or re-present visual features on the sim." Part of the motivation for this was that there may be cases where simply looking at the sim is not enough to fully recognize a particular feature. For instance, if students had only been asked to use the sim without drawing molecules on their worksheets, they might have glanced over some of the more subtle aspects of what they saw. Asking them to draw something shown on the sim forced them

[4] All of this will be elaborated upon later in the thesis.

to look more carefully at what was shown, and also extended the amount of time they spent analyzing individual features. Again, this heuristic is not an entirely substantiated claim about the efficacy of asking students to write certain things on their papers; instead, it is yet another ansatz for a strategy that may prove useful when incorporating sims into assignments.

The final heuristic that was added to our initial list came from discussions with the PhET team, as well as observations of students using the projectile motion homework, as well as build-a-molecule. This heuristic is “Set up game-like situations.” The premise is that one of the effective ways to promote engaged exploration is to set up a situation that asks students to complete a certain type of task, which might be thought of as a “game-like” situation. For instance, in the build-a-molecule simulation, students are confronted with a game-like situation in the sim itself, since they have to build a certain number of molecules before moving on to the next task.

Currently, we are a bit “on the fence” about the phrasing of this particular heuristic. After discussing this with other group members, we wonder if a better phrasing of this heuristic is: “Set up explicit challenges,” and additionally, “Take advantage of implicit challenges in the sim itself.” For now, it is only important to think of these as possibilities, as the discussion will return to these options in the case studies discussed later in this thesis. They are similar enough that we can keep them in the same category for the time being.

In addition to adding heuristics, we felt the need to take some away. In particular, the original list seemed to be slightly redundant in heuristics 5 and 6. Additionally, heuristic 2 is, in several ways, already accounted for in other heuristics. Heuristics 5 and 6 are, “Use dynamic feedback as a tool for experimentation,” and “Use the sim to design a virtual experiment.” The problem with the former is that it is difficult to know how to take advantage of dynamic feedback through writing the assignment itself. Instead, dynamic feedback is a particular feature of simulations that is simply present in every sim, regardless of whether we “take advantage” of it or not. Dynamic feedback plays some sort of role in how students interpret what they see on the simulation, and perhaps also allows them to experiment in certain cases, but as of now, we do not feel that it needs to be included as an additional heuristic. Instead, we separate dynamic feedback into a new category called “features of the sim,” and leave “Use the sim to design a virtual experiment” as a heuristic. Again, the main reason for separating these is to distinguish between a strategy one can use in writing an assignment, and features of the sim that allow for that strategy to be effective (or not).

Heuristic 2 is, "Use measurement tools when possible." We eliminate this from the list of heuristics not because it is a "bad" idea, but rather because we see using measurement tools as already incorporated in to several other heuristics. For instance, in designing a virtual experiment, students may *forced* to use measurement tools and collect data. Additionally, game-like situations can give rise to students' use of measurement tools, if that game-like situation requires measurements for successful completion. Finally, the way that using measurement tools might be incorporated into the assignments is already accounted for in "Call attention to visual features of the sim." Because using measurement tools will often be a natural extension of these various categories, we do not feel the need to keep it as a separate heuristic.

With these additions (and subtractions) to the list, as well as a slight rephrasing of several of these heuristics, we are left with the following set currently in use:

1. **Set up game-like situations and/or take advantage of explicit and implicit challenges**
2. **Ask about visual features on the sim**
3. **Utilize illuminating cases**
4. **Ask students to recreate or re-present visual features on the simulation**
5. **Use the sim to coordinate other forms of representation**
6. **Ask for comparative analysis when possible**
7. **Use the sim to mediate discussion**
8. **Use the sim to relate formal concepts to the real world**
9. **Use the sim to design a virtual experiment**
10. **Set up situations that utilize 'predict, observe, explain' or 'elicit, confront, resolve' models**

One concern with this list is, "How do we know it spans the space for all possible heuristics?" In other words, how do we know that more heuristics aren't needed to account for all of the ways we might try to write assignments? The answer right now is, "we don't."

From our research thus far, these appear to be useful and/or necessary, and account for at least *most* of what we have seen. However, with more research, it may become clear that certain heuristics need to be added or removed even to this list to fully span the space.

There are three additional concerns with this list. Two of these were discussed earlier, namely *how* do we go about implementing these and *why* should these work? Both of these will be addressed in the next chapter. The other concern is that, if we choose to view learning from as a social act that occurs in context, then these heuristics provide no information about their relationship to the rest of the contextual whole. To address this issue, we came up with a new "framework" that provides perspective on how to view the creation and use of an assignment.

The framework, shown in the figure below, is read as the following: Each simulation has a particular set of features, including sliders, visual representations, dynamic animations, adjustable parameters, and so on. Those features of the simulation will affect both the environment or situation that students are situated in, as well as how the assignment is written. By "Environment/Situation," we mean everything going on in the particular environment students are situated in, ranging from their interactions with each other, the teacher's role in the classroom, the orientation of the desks in the room, and anything else that makes a situation unique. The main point is that the simulation is being incorporated into a particular environment, and the features of the sim itself have an impact on that environment.

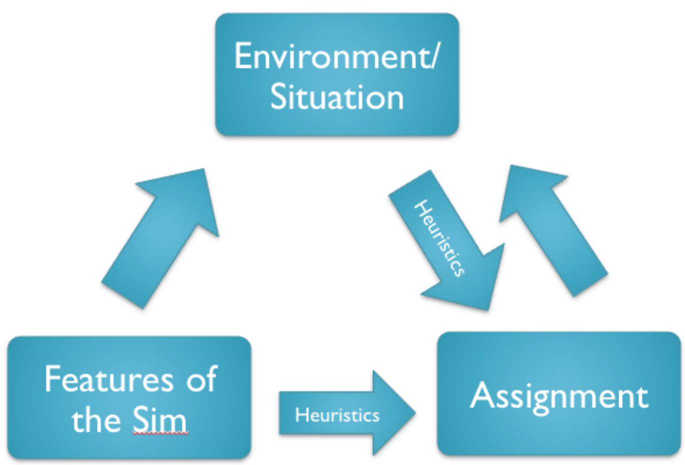


Figure 3.1: Framework for incorporating sims into assignments

In addition, the features of the sim influence how the assignment is written. Perhaps the features of one simulation are well-suited for setting up a game-like situation. In that case, the assignment will be tailored to take advantage of those features of the sim.

At the same time, the environment/ situation will impact the way the assignment is written. For instance, if one is planning on asking students to work together in class, the assignment might be written like a tutorial, rather than as a homework problem. In turn, the assignment will also impact the environment; that is, what the students are working on, their interactions with the sim, the questions they raise, the responses a teacher provides, etc.

The main point is that within this general framework, the heuristics present themselves in the two directional arrows shown. That is, based on the features of a particular simulation, some heuristics will be particularly beneficial to use, and will therefore be incorporated into the assignment. Additionally, features of the environment will determine which heuristics are used in writing the assignments. For instance, a tutorial-like environment will likely take advantage of the "use the sim to mediate discussion" heuristic, whereas it may not be a good idea to bank on using this heuristic for an individually-assigned homework problem.

3.3 Summary

As has been explained, this project started with a tentative list of heuristics that we thought *might* be useful when incorporating simulations into assignments. Through a series of case studies, we were able to "test" these heuristics, and in doing so, found ways that the list could be revised and expanded upon. However, because these heuristics did not provide any information about the contexts in which they were used, it was clear that some sort of more inclusive analysis of using the heuristics was needed. This led us to creating a framework that provides insight into where these heuristics fit into a more general activity system.

So far we have provided information on *what* the heuristics are, and *where* (in our framework) they might be implemented, but we have yet to explain *how* they should be implemented and *why* they ought to work. For the answers to these questions, it is necessary to turn to more theoretical framing, which will be elaborated upon in the next chapter.

After establishing this theoretical foundation, the case studies conducted for this project will be presented (in chapters 5, 6, and 7). In those chapters, this theoretical foundation will be further applied in attempts to explain the data collected along the way. Again, this theoretical foundation is tentative as of now, but it will at the very least serve as a good starting point for thinking about these issues of incorporating sims into assignments.

Chapter 4

More on theoretical foundations

As I have said, two primary questions still remain: *How* do we implement the heuristics? and *Why* do they work? Unfortunately, the answers to these questions are not simple. First of all, I must point out that answering *why* these heuristics work is dependent upon *how* they are implemented, and similarly, *how* we implement them depends on our theory of *why* they should work. It seems, therefore, that we are stuck in some sort of loop of contingency in answering these questions.

The way out is to first assess exactly what we hope will happen when students work on these assignments. One can imagine a wide range of topical learning goals for students, such as developing intuition, changing one's conceptual framework, developing expert-like epistemological criterion, and perhaps simply gaining an appreciation for science more broadly. Additionally, we could speak in terms of what the students should be able to *do* after working on our assignments. This could range anywhere from answering questions on an exam, to being able to act or behave like a scientist and think about issues in a scientific manner.

If we were to take the time to write out all of the desired learning goals for an assignment, we would find one commonality among them: in each we expect to see fundamental changes in the ways students think about a particular issue.[1] Yet if we want students to achieve these goals, we must address *how* this can happen. Answering this necessarily involves two components: first, we must attempt to understand the nature of student thinking, and second, we must provide a means of effectively changing students' thinking. This is no simple task. For one, student thinking is incredibly complex. Students' prior

[1] If students didn't have to change the ways they think about these issues, one might say that the students already "learned" the subject.

knowledge, beliefs, methods of utilizing resources available, and so on, all factor into how students learn physics. Additionally, providing a means for effectively changing student thinking requires yet more research, and is largely contingent upon our model of how students think.

I simply cannot provide a full accounting for how assignments and simulations change student thinking. However, I can provide insights into how certain *types* of thinking change, and how we can write assignments to promote those types of change.

To do this, I turn to two different, though not unrelated, theoretical models as a starting place for analyzing the ensuing case studies. The first of these models concerns a very particular type of conceptual change, and is referred to as "coordination class theory." The second model is an extension of this specific model, and invokes some of the fundamental ideas of coordination class theory in attempts to explain student thinking on a more general basis, for cases in which coordination class theory cannot be applied.

Before diving head-first into the topic of coordination class theory, it is necessary to provide a picture for where this will lead. Because both of these theories attempt to explain how students thinking about particular issues both occurs and changes, we are able to apply these theories to the case studies that will be presented in the following chapters.

These chapters will demonstrate that, though productive conceptual change and desirable changes in student thinking *does* occur in these studies, the change is not guaranteed. Additionally, I emphasize that, due to the fact that desirable changes in student thinking occur at different times and in different ways for every student, there is no obvious "solution" for causing this to happen for every student. My "resolution" – or perhaps better, capitulation – to this fact leads me to argue that, instead of attempting to "cause" desirable changes in every student's thinking, we must research situations in which productive changes have been shown to occur, and attempt to provide an explanation for how and why these changes can occur in those environments.

Note that this thesis is strictly concerned with changes in student thinking of two fairly specific kinds, both of which are elaborated upon in the remainder of this chapter. While other changes in student thinking are important, I *will not* address them in this thesis. Thus, when I begin to analyze *how* the heuristics ought to be implemented and *why* they work, I am grounding my claims in the theories presented in this chapter, and not in models student thinking of different sorts.

4.1 Coordination class theory

If we are to think of incorporating sims into assignments as a means of fundamentally changing the ways that students think about a given phenomenon, we need some sort of theory for how students' thinking changes. Granted, there are many different ways students' thinking might change, and it is likely not possible to account for all of the possibilities. However, because two of the primary concerns for physics are the topics of conceptual change and concept formation, we can start by looking at how these processes occur, in hopes that they will give insight into the present discussion.

There are many theories of concept formation and conceptual change, and unfortunately it is truly beyond the scope of this thesis to go into detail about the subject. However, traditionally, "concepts" have been viewed as fairly robust, unitary, entities that can be consistently applied across multiple circumstances with little variation in their application [37, 39]. An example is the concept of a chalkboard. Clearly, most people can identify a chalkboard, whether it be in a classroom, a gas station, or wherever else it might be, and this provides some confidence that the concept is robust and unitary in nature.

Similar notions of concepts have been applied in the domain of physics. Yet in this domain, it is clear that students learning the subject have much difficulty applying the concepts. If, in attempting to explain this, one were to invoke the notion of concept as a unitary, robust entity, they would be forced to state that students do not "possess" certain physics concepts if they cannot use them correctly. An example is force. If a student is asked a question about the net force acting on an object moving at constant velocity, and incorrectly states that there is some positive net force on that object, one might say that they do not "possess," or have not "acquired" the concept of force.

This claim makes a good point, which is that what the student considers as force is much different than what an expert considers as force. However, from this perspective, we still have not said *what* a concept is, and certainly know nothing about how the student will *acquire* the expert's concept of force. Furthermore, this conceptual view provides absolutely no information for how we might teach the subject.

In order to answer the question of what a concept *is*, I again turn to the work of diSessa, and in particular, his work entitled *What changes in conceptual change?* [37].[2] In this paper, diSessa first rejects the notion of concepts as unitary entities as they are

[2] In addition to this work, we also draw from diSessa's other work in coordination classes: [21], [27].

proposed in traditional literature, and in doing so, brings up a passage from McDermott and Trowbridge [38]:

> Physics instructors generally share a common interpretation of the kinematical concepts based on operational definitions and precise verbal and mathematical articulation. On the other hand, students in an introductory physics course are likely to have a wide variety of somewhat vague and undifferentiated ideas about motion based on intuition, experience, and their perception of previous instruction.
>
> Although students who were unsuccessful could generally give an acceptable definition for velocity, they did not understand the concept well enough to be able to determine a procedure they could use in a real physical situation for deciding if and when two objects have the same speed.

The above passage makes two important claims: (1) experts seem to share a common view of kinematical "concepts," and therefore, there must be a great deal of similarity in how experts determine various pieces of information in kinematics, and (2) students appear to have a variety of vague or undifferentiated ideas about those concepts that are drawn from their experiences in the world. Thus, the ways that students determine information in kinematics is very different than the ways experts determine information. Note that this passage already gives more insight into the nature of student thinking than does the previously discussed unitary notion of concept; specifically, it states that students' ideas are vague and non-differentiated, and that kinematics involves "determining" quantities in some way.

From here, diSessa proposes a model for a particular class of concepts that gives insight into the reasons for *why* students have non-differentiated and vague ideas about force and other physical quantities. These particular types of concepts are referred to as *coordination classes*. In short, coordination classes are "systematically connected ways of getting information from the world." diSessa often phrases this idea in the following way:

> the primary *task* that a concept of (blank) must do is to (fill in here).

This definition takes some getting used to, so I will try to provide examples that will help illustrate the point. Consider the following situation: John is at home, ready to drive to work, when he realizes that he cannot find his keys. How does John proceed in finding his keys? He likely checks various locations that he typically leaves his keys, and begins to think, "Where did I last put those?" Additionally, prior memories about where he has lost... and found his keys at home probably come to mind, and these might influence where he decides to look for his keys. Or perhaps he looks for the keys in a methodical fashion, starting in one area of his house, and moving to different rooms as he "checks off" the locations that the keys are not located in.

While searching for his keys, John is using a variety of strategies for determining a certain piece of information in the world. In this case, that particular information is 'key location,' or perhaps better phrased, 'location' of keys. During his time searching, John might use, at any time, his prior knowledge of where he has left his keys, the method of elimination, his recollection of sequences of events prior to losing the keys, and so on, all in attempts to determine what we will refer to as a concept. That concept is location, and this is, indeed, a coordination class.

To develop this idea, we can start by applying diSessa's phrasing to 'location.' We say that "the task that a concept of (location) must do is to (determine where in space something is situated)." The concept itself is not categorical in nature, but rather, is something that must be determined in different contexts.[3] 'Location' isn't a property of an individual object, but is instead something that we *read out* of a particular situation. Thus, the keys are not the concept at issue in this situation. The goal is not to determine "keys," but rather to determine *location* of the keys. John could equally be looking for "shoes," in which case he would still be trying to determine location.

The reason we introduce this theory is that there are many concepts in physics that, like location, are exemplary of this specific type of concept. A few to mention are: position, velocity, acceleration, force, proper time, and 'Lenz's Law.' We will elaborate on force in this section, and in the next chapter present 'molecular coefficients' as yet another example.[4]

Because there is more terminology associated with this theory, we need to take the time to define some of its terms. The first, which has been hinted at already, is *readout strategies*. These are meant to determine characteristic attributes of a particular concept in different situations. In the example above, John's readout strategies are to look at

[3]To elaborate on the point that coordination classes are not categorical in nature, consider the following: chalkboard, paper, pizza, cup. If we are to apply diSessa's phrase, we say that the task that a concept of (pizza, chalkboard, etc.) must do is to determine whether an entity in the world is a (pizza, chalkboard, etc.). Clearly, the use of coordination classes here is not all that helpful, and likely unnecessary. The reason for this is that these concepts serve the purpose of categorizing the world around us, whereas coordination classes are meant to *get* information from the world.

[4]Additional examples that diSessa provides are: personality, object permanence, and conservation of volume.

various places around the house and determine one thing: keys or no keys. Once this information is read out, John can go about looking in a different location.

Now, we should note that *readout strategies* are only the first primary component to coordination classes. To this, we add a second component that helps to explain what we *do* with the information that we read out in the world. This component, referred to as the *causal net*, concerns the inferences one can use to turn information that is read out *into* the information at issue. In the case of location, the causal net would specify what we do with an observation that, for instance, the keys are not located in (this place). Having read out that particular piece of information, reasoning strategies "in" the causal net will determine what to do with that information. For instance, if the keys are not in John's jacket, he might decide to look in the jeans he was wearing. Thus, the *causal net* consists of the reasoning strategies that determine when and how observations are related to the particular information at issue.

As an important side note, we should point out that there is clearly a tight, interwoven relationship between *readout strategies* and the *causal net*. In fact, we feel it is sometimes difficult to delineate which aspects of a concept fall under these two categories. Much of this difficulty lies in the fact that the *readout strategies* influence the *causal net*, just as the *causal net* influences the *readout strategies*.

Two more terms remain to be defined, both of which are performance specifications of coordination classes. The first is *integration*. This includes collecting, selecting, and combining diverse observations to determine what we wish to 'see.' The second is *invariance*. This emphasizes being able to determine the "relevant information" of a concept across a wide variety of contexts. Thus, to correctly learn a concept, one must be able to *integrate* information in a variety of situations, such that the determined information is *invariant* among the situations in which the concept is determined.

To elaborate on *invariance*, we can again consider John. If John were to have lost his dog, rather than his keys, he would again have to determine location, but in a different context. In both the case where John loses his keys *and* the case in which John loses his dog, he must determine the same *concept*, and yet, he will be forced to use different strategies for determining location in each. For instance, in determining the location of his dog, John might look near the local hot dog stand, whereas in determining the location of his keys, John might look in the coat he recently wore. Thus, to determine location *in general*, we must be able to sort through the diversity of information present in the world to determine location in different settings. If we are able to do this in a wide variety of contexts, we may say that our concept is *invariant*.

Let us now turn to the example provided by diSessa: force. This will serve to make coordination class theory more concrete, and also provide empirical evidence for why it is useful. The study to present concerns a freshman physics student, called J, who discusses various issues related to force in an interview setting. The interview is comprised of three different "episodes," each of which deals with a different question related to force. Here, we summarize two of the episodes.

Episode 1: Issues concerning *readout*

In the first episode, J is asked the following question: If gravity pulls harder on different objects, how is it that they still fall at the same rate?

The *correct* answer to this question is that the gravitational acceleration is the same for all objects, regardless of their mass. However, the *force* acting on objects varies among objects of differing mass, as can be inferred from $F = mg$. Thus, since g is the same for all objects, m is really what determines how "hard" gravity pulls on objects. This could be intuitively understood by considering holding two different objects in one's hands; perhaps a pencil in one and a bowling ball in the other. Clearly, gravity pulls "harder" on the bowling ball, as one can feel directly, even though both objects will fall at the same rate if dropped.

When responding to this question, J simply denies the fact that gravity pulls "harder" on different objects. The issue of holding a heavy item in one's hand vs. holding a light item in one's hand came up, and then the conversation proceeded in the following way:

> *J: Gravity's uniform. So gravity won't pull any harder on something that's in the same place as it will on something else.*
>
> *I: So you're not feeling the force of gravity when you hold something?*
>
> *J: You're feeling the weight of the object.*
>
> *I: The weight of the object. So that's different from the force.*
>
> *J: Right.*

Applying coordination class theory to this case, we can say that J's problem is in *readout*. When J is thinking about holding different objects in the air, she states that the *force of gravity* is the same for all objects "in the same place." Thus, she has read out "force of gravity" in place of what is actually g, the gravitational acceleration. The gravitational acceleration is the same for all objects, but the force is not.

To incorporate our previous example, this would be like John seeing a pair of scissors and thinking that they were his keys. His readout of this information is simply wrong.[5] Thus, in both cases, the problem lies in what J and John 'see.'

In this example, we also gain insight into J's *causal net*. Here, the two elements in her causal net are 'gravity (meaning gravitational force) is constant' and '$F = mg$.' J uses both of these, and in doing so, incorrectly coordinates g as a force.

Episode 2: Issues concerning the *causal net*

In the second episode,[6] J has been presented the following situation: Someone is pushing a book across a table with their hand, and the book moves at constant velocity.

In this case, J has already stated that the force of the hand on the book must be greater than the force of friction on the book, since the book is moving. The dialogue starts in the following way:

> *I: And what about the situation where I'm just pushing along like that, after I get it going?*
>
> *J: After you get it going, it's going at a constant velocity.*
>
> *I: Right. In that situation is the force on my finger greater than the force of friction?*
>
> *J:* (shakes head yes)

This passage demonstrates two aspects of coordination classes. First, it demonstrates one of J's *readout strategies*; specifically, that she (correctly) "reads out" that the book is moving at constant velocity. Second, it demonstrates part of J's *causal net*, which, in this case, contains the naïve conception that *overcoming implies motion* (or reversed, *motion implies overcoming*).[7]

[5] This may seem a ridiculous example, but consider the likelihood of something similar occurring with infants or young children.

[6] This is actually Episode 3 in [37].

[7] This statement is taken from diSessa yet again; specifically from *Towards an Epistemology of Physics* [40]. In it, he introduces the notion of phenomenological primitives (or p-prims), which are basically naïve conceptions about the world that are based in human experience.

At this point, the interviewer knows that J knows the equation $F = ma$, and he therefore tries to point out the contradiction in J's thinking. The discussion proceeds as follows:

> *J: Because of $F = ma$, if you have a constant acceleration, then you should be able to have a constant force.*
>
> *I Sounds right. If $F = ma$, constant mass.*
>
> *J: Well, then, I don't know.*
>
> *I: This becomes puzzling.*
>
> *J: Yes. See, to me, if you're applying this constant force like this, that doesn't look to me like the book is accelerating. At all.*
>
> *I: No, it doesn't.*
>
> *J: Maybe it's just accelerating at the same rate.. It seems to me that, see, if it has constant acceleration, the velocity is still increasing. It's just increasing at a constant rate, right?*
>
> *I: Say that again.*
>
> *J: If it has constant acceleration, the velocity is increasing, but it's at the same rate.*
>
> *I: Yeah, that's right.*
>
> *J: So if you're pushing this, it has constant acceleration. It still has to be getting faster and faster and faster. I don't see it getting faster and faster, unless you're applying greater and greater force.*

Here we see the dilemma that J is facing. She understands that $F = ma$ implies that if there is a force, then there must be an acceleration. At the same time, J also believes there to be a force acting on the book. Thus, from these inferences, she states that the book *should* be accelerating. Yet, she cannot see the book accelerating, and this is where the dilemma arises.

The problem is in J's *causal net*. She has the naïve conception that *motion implies overcoming*, and thus, since she sees the book moving at constant velocity, she determines that there must be a force acting on the object. Of course, this is in direct conflict with the other piece of her causal net; namely $F = ma$.

The way that J resolves this problem is by denying that the equation $F = ma$ works in all circumstances. Her formal response is as follows:

> *Like, to me, you look at $F = ma$, and there's a force, and that has to mean acceleration. But, then, it's easy to say, 'that's true'. But, I mean, there's no way it is. I guess you can just say that, you know, those darn equations aren't applicable to every single thing. They're not always true. You can't live by them.*

This fascinating response shows that, in this case, the naïve conception in J's *causal net* was so strong that she made the conscious decision that another part of her *causal net*, namely $F = ma$, must not apply in all circumstances. Thus, this example serves to show that problems can arise not only in students' *readout strategies*, but also in their *causal nets*.

These examples were meant to make coordination class theory more concrete, and to show its utility for explaining certain concepts that can arise in physics. Obviously, I have not gone into great detail in explaining the theory, nor have I applied all of the terms presented, and this perhaps leads to some dissatisfaction with the presentation of this information. The reason for presenting the theory this way is to give a general "feel" for the theory, as we will return to the topic in the next chapter, and apply it to the "Build a molecule" simulation.

The main point to take away is that coordination class theory provides insights into *how* a particular type of conceptual change occurs. It is based on the idea that concepts are not unitary, robust entities that exist in the head and can be applied across any context if one possesses it, but rather that these particular concepts are *ways* of reading out information from the world. It stipulates that there are many reasons for students carrying "vague" or "non-differentiated" ideas, and that these ideas largely arise from our naïve conceptions about the world around us, which, in turn, are based in experience and the phenomenological occurrences that we experience in our everyday lives.

4.2 Generalizing to a manifold ontology of mind

Coordination class theory required us to set aside the traditional, unitary theories of *concept* in favor of a context-sensitive view of reading out and interpreting information in particular ways. In formal terms, we might say that coordination classes introduced a manifold ontology of concepts.

Though coordination classes appear to explain conceptual change of a very specific kind, we are warned by diSessa that this theory does not account for all types of thinking and conceptual change. Nonetheless, others (including us) have found the general principles or tenets of coordination classes appealing enough to try to incorporate those principles into a more general theory of mind. We refer to this theory, for now, as being a type of *manifold ontology of mind.*

Much of this current section is drawn from work by David Hammer in *Resources, Framing, and Transfer* [39]. Though we do not follow his theoretical views to every detail, we do find his overall outlook on the problem appealing, especially when attempting to analyze changes in student thinking of a more general nature.

Resources-based view of cognition

In generalizing to a manifold ontology of mind, Hammer introduces what he calls a "resources-based view of cognitive structure." This basically postulates that humans have a variety of "resources" that they can use at any time in attempts to compile information in real time to form some sort of understanding of whatever information they are dealing with. Note that this is clearly consistent with coordination class theory, but more general; for instance, in the case of dealing with force, J drew upon the different resources she had available (including $F = ma$ and other items in her *causal net*) in order to make sense of the phenomenon at issue. The primary difference between Hammer's view and coordination class theory is that the resources need not belong to the more specific category of the *causal net*, nor do we have to specify exactly what *readout strategies* are being used.

Note also that these resources need not be "correct" views of the physical world. These can be primitive conceptions drawn from experience in the world, formal equations, graphs, and whatever else might be available to the student at a particular moment in time. Thus, the student's task while engaging with the world, or more specifically, with physics content, is to compile and assimilate information in particular ways to form a general understanding of the phenomenon at issue. When the student is able to assimilate information in an expert-like way, we might say that the student has "learned" the material. Again, this gives much more insight into *how* humans "acquire" physical concepts than do unitary perspectives of concepts.

Though I don't want to go into great detail about this view, since it will be elaborated upon in chapters 6 and 7, it is necessary to introduce one more definition. The term is "frame," and refers to a "locally coherent set of activations." The general idea is that, although humans have a vast variety of resources that *can* be used or drawn upon, we nearly always "choose" to use *particular* resources, depending on the context of the situation we find ourselves in.

For instance, if I am working on a quantum mechanics problem, I will likely activate a locally coherent set of resources that are drawn from my prior experience learning quantum mechanics. The Schrödinger equation, the Heisenberg uncertainty principle, and much of my conceptual framework for quantum mechanics will be activated because, over time, I learned to activate those particular resources in order to deal with quantum mechanics problems. On the other hand, if I am working on a classical mechanics problem, I will most likely *not* activate resources that help me deal with quantum mechanics. Thus, depending *on the context* of the situation, I will activate different resources to deal with the information at issue.

Of course, we love the idea of frames because it fits in with our notion of context which was described in chapter 2. We would argue that through the use of artifacts, we are given opportunities to perform certain actions *with* those artifacts that allow us to achieve some goal. Thus, the notion of frames is similar in that at any moment of time, the context of the situation influences the ways that we utilize the resources available to us in order to achieve some goal. Whether those resources be cognitive resources, material resources (such as a pencil, paper, or even computer simulation!), interactions with other students, or something else entirely, we can state that, because of context, we use certain *frames* and activate different resources accordingly.

Relating it back to complexity

With this general theoretical framework in place, I want to take the time to relate these ideas back to an idea introduced in chapter 2. What follows is somewhat anecdotal, as it will not be used again in this thesis, but it is worth mentioning because it provides a model for how we can relate this "activation of resources" perspective to simulation use.

Recall that Podolefsky et. al. attempt to characterize computers based on their complexity (chapter 2.6). In doing so, Podolefsky used the idea that in order to examine the complexity of a simulation, we can set up a matrix in which the rows correspond to different adjustable parameters that the user can control, and the columns correspond to features *on the sim* that can change as a result of adjusting those various parameters (these were called readouts). Of course, the central idea was that the more parameters and readouts a simulation contains, the more complex the simulation is.

Podolefsky also introduced the idea that simulation complexity can be different for every student who uses the simulation, based upon those students' prior knowledge. For instance, a student who sees several different readouts as being the "same" thing, or determining the same information, will not see the sim as being as *complex* as a student who has not yet made those connections. Thus, Podolefsky's idea was that perhaps these matrices can be "collapsed" when connections among the elements are made, such that we can account for the differences in how students *perceive* the simulation.

To us, this seems wholly consistent with an "activation of resources" perspective of mind. First of all, the fact that simulations can provide a *means* of allowing for these matrix elements to collapse implies that students are beginning to form particular *frames*. For instance, a student using the "Gravity force lab" sim who drags one of the masses apart from the other will, over time, likely make the connection between (drag objects apart $\Longleftrightarrow$ smaller force arrow $\Longleftrightarrow$ smaller quantity for force), or perhaps some other sequence of terms therein. Thus, the student will begin to activate a locally coherent source of resources in dealing with that phenomenon.

This general view of "let's see how we can collapse a student's complexity matrix" is one that I like to keep in mind while thinking about the various case studies that were conducted. Not only does it provide a relatively easy visualization for how I ought to write assignments, but it also makes me think about what must happen *for students* when they begin to interact with the simulation.

Yet, interacting with the simulation is only one way of collapsing a complexity matrix. In fact, from the studies that will be presented (i.e. projectile motion and quantum tunneling studies), it is clear that with some topics in physics, playing with a sim is not *in itself* enough to allow students to make the necessary connections. In particular, if what is shown on a simulation is determined largely by complex mathematical relationships, and students have not seen those mathematical relationships before, there is little chance that they will have the necessary *resources* available to allow them to make the connections necessary for understanding the phenomenon at hand. Thus, not only does the simulation play a role in helping students make connections, but so too do the other resources available to the student at any given time.

I hope that this general framing has provided some insight for what is to come. The intent of presenting it was not to provide a full accounting for the different types of theories that can be used to explain changes in student thinking, but rather, to provide a general framing that will be elaborated upon in the subsequent chapters.

The overall view to take away is this: Student thinking is highly dependent upon context. What the students know, the resources they have available to them, and so on will all affect how students engage in the classroom and what they take away from their interaction in that particular setting. Within that general framing, I focused on more specific *types* of changes in student thinking, one through the lens of coordination class theory, and another through the lens of a more general "manifold ontology of mind." In both, the central idea was that student thinking cannot be explained in terms of unitary, robust cognitive structures that activate across different contexts in the same way every time; such a theory is not useful for our purposes because it does not explain how students think about issues, nor why their thinking might change. In the more general view, we stated that humans have a variety of resources that they can draw upon at any given time, but that the resources that *are* activated are largely dependent upon the context of the situation.

With this general framing, let us look ahead for where we are going. The chapters that follow each deal with a particular case study. In these, we will present the relevant data collected from the studies, and then attempt to draw *heuristics*, or strategies, that we can use for writing these assignments. Following this, we elaborate on *how* the heuristics were implemented in the study, and *why* they worked. Of course, answering *why* they worked requires us to refer back to the theoretical framing presented in this chapter. With that framing, we can then turn to a discussion of where the heuristics fit into an overall activity system.

Chapter 5

Build a molecule

The PhET project at the University of Colorado recently received funding to design simulations targeted for middle school students. One of the sims created under this funding was the "Build a molecule" sim [**?**]. In addition to designing the sim, the PhET team conducted a study using the sim in a middle school classroom at a charter school in Dallas. I was fortunate enough to get to help in analyzing some of the data the team collected from this study.

The primary reason for presenting this case study first is that it provides a beautiful example of how simulations can lead to conceptual change of a very specific kind. In particular, this example looks at conceptual change through the use of diSessa's idea of coordination classes. In this case, the coordination class of interest is "molecular coefficients."

This chapter begins with an overview of the study and the environment students were situated in while using the simulation. Following is a brief introduction to molecular coefficients as coordination classes, which will provide a means for interpreting the data and results that follow in the next section. Lastly, the heuristics drawn from this study are discussed, with a follow up concerning *how* they were implemented, and *why* they apparently worked.

5.1 Overview, assignment, and simulation

Let us begin by describing the environment that students were situated in while working on the assignment. Each student had a computer and a two-page assignment in front of them, and were generally situated at desks in groups of four. The teacher was particularly careful about making sure that the entire class of around 30 students stayed on task and moved to the next "section" of the assignment together at the same time. Specifically, she often stopped the class when students were sufficiently close to moving to the next section and asked everyone to move on to the next section at that time, regardless of whether they were done with the previous section or not. From the data collected, it is clear that both the actions of the teacher *and* the environment the students were situated in influenced how the students interacted with the assignment and the simulation.

Two primary sources of data were collected from this assignment. Students took both pre- and post-assignment "tests" that assessed their comprehension of the material before and after using the simulation + assignment combination. Students' scores on these two tests are the first piece of data.

The second source of data collected was Camtasia screen capture (plus audio) videos of three different students in the classroom. Two of these students were sitting next to each other in a group of four, and one was sitting alone on one side of the table in a group of three. Through watching the Camtasia files, it was easy to take note of what the students were building, as the students' conversations with both the teacher and each other.

There were three primary learning goals for this assignment:

1. Describe the difference between a chemical name and a chemical formula

2. Distinguish between subscripts and coefficients in a chemical formula, and understand what each means

3. Use pictorial representations of molecules to generate chemical formulas

The simulation itself, part of which is shown in Figure 2, has a "work table" on which students can pull atoms from a bucket and move them around to combine them in various ways. On the right side of the simulation are boxes labeled with different molecule formulae. Once the students create a particular molecule, the box blinks and lights up, and directs the student to drag the molecule in to that particular box.

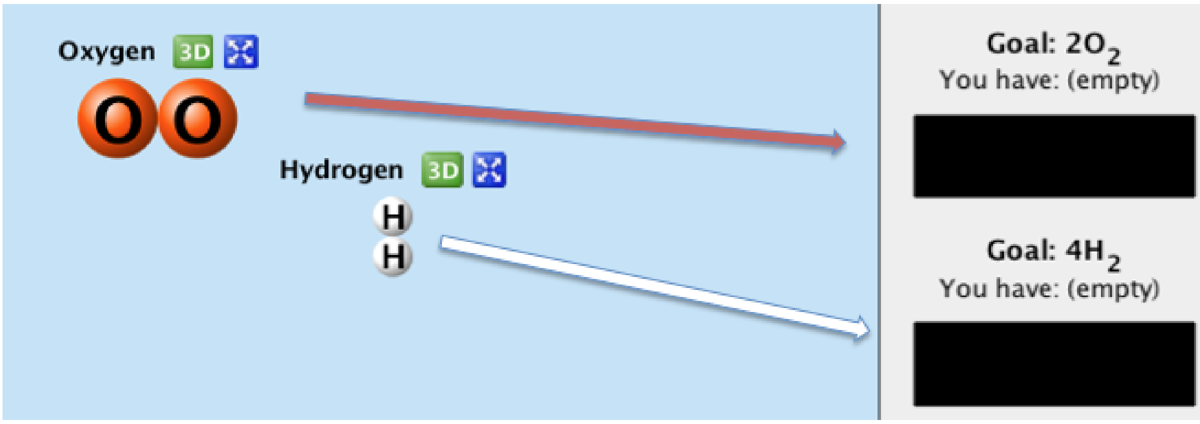


Figure 5.1: Screen capture of the build-a-molecule sim. Students create molecules on the work table and can drag them to the boxes on the right.

In addition, there are three different tabs in the sim, each of which presents a new challenge. The first tab deals strictly with single molecules, such as O_2, N_2, or NH_3. The second tab introduces coefficients, and to fill all of the boxes, students must make the correct *number* of molecules. For instance, $4H_2$ requires students to create, and move, 4 separate H_2 molecules into the same box. Only after filling all of the boxes can students move on to more complicated challenges. The third tab is completely free-form, and in the particular assignment referred to here, students were instructed to play around and try to make the largest molecule possible while using this tab.

The assignment itself contained four primary sections. The first simply asked the students to play with the sim and explore anything that looked appealing, while the remaining three sections each corresponded to using one of the three tabs.

Questions asked were presented in a tabular and open-ended structure. For instance, while using the first tab, students were given a table to fill out, in which different molecule names were to be written, along with the corresponding chemical formulas of those molecules, and a picture of what each molecule looks like.[1] The remaining questions in the assignment were presented in slightly more guided ways, but still presented a “challenge” for students to complete. For instance, students were asked to try to make $4H_2$, and then draw a picture representing $4H_2$. Additionally, students were asked to explain what the subscripts and coefficients meant. The effects of the sim and the assignment together are detailed in the *Data and Analysis* section.

[1] See the Appendix for the precise format of the questions from this assignment.

5.2 Molecular coefficients as coordination classes

Recall that coordination classes are "systematically connected ways of getting information from the world." In the case of molecular coefficients, the information that one is getting from the world is "how many" of a particular type of molecule. To most of us familiar with the idea of molecular coefficients, this seems an incredibly trivial task. When seeing a formula such as $3CO_2$, we know with no difficulty that this means three CO_2 molecules. Additionally, we know that when we see the formula $6Mg_5(SiO_4)(FOH)_2$, this means we have six Chondrodite molecules. We don't care what Chondrodite molecules are, but nonetheless, we know that that there are six of them. Our knowledge of molecular coefficients is apparently "abstract;" that is, we can apply our knowledge of molecular coefficients across different contexts without much difficulty, and this is largely why we think of this as a trivial task.

But how did we learn the meaning of molecular coefficients in the first place? How do we know that $3CO_2$ means three carbon dioxide *molecules*, and not three carbon atoms connected to two oxygen atoms that form one large conglomeration of atoms? We somehow learned to *read out* this information in a particular way. Because students in this study had not yet learned the meaning of molecular coefficients, we can look to this study as a sort of "window to the past," through which we can see just how learning this particular subject occurs.

Let us apply diSessa's terminology associated with coordination classes to molecular coefficients. First, we can say that "the primary task that a concept of (molecular coefficients) must do is to determine (the quantity of a particular type of molecule)." This will involve both *integration* and *invariance* in students' *readout strategies*; that is, we should expect to see changes in the ways students interpret the "number" of one particular type of molecule, *and* we should expect to see changes in students' strategies for reading out the number of molecules across different types of molecules. In other words, we should expect that students' initial strategies for reading out the number of molecules *will* be variant, depending on the particular molecule, but that over time, these variant strategies may be replaced with *invariant* readout strategies that determine the same information across all of the different molecules they deal with.

We must also specify the *causal net* for this case. In other words, we have to determine the reasoning strategies that determine how an observation is related to the information at issue. In this case, the causal net will determine what the coefficient in a chemical formula physically represents. Similarly, the causal net should allow students to determine an equation based on a physical picture. The reasoning strategies in both should be the same.

Through both the data collected on the pre- and post- tests in this study, as well as a detailed analysis of three students' interactions with the simulation, it will be shown that many students formed new readout strategies when dealing with molecular coefficients, and that there were significant changes in students' causal nets. After demonstrating this, we turn to the heuristics drawn from this study, and discuss how those heuristics allowed for this particular type of conceptual change to occur.

5.3 Data and Analysis

Several observations stood out as particularly important when watching the Camtasia videos of three different students. Each student generally spent several minutes at a time playing with the sim and building molecules, and would then stop playing with the sim to spend several minutes writing down various findings on the worksheets. During this time of writing, students frequently clicked on the 3-D view of the molecules in order to draw the molecules as accurately as possible. This process occurred in all sections of the assignment, and overall, students chose to build several molecules in a row before stopping to fill out the assignment.

These findings suggest that the assignment largely allowed for play and "messing about" with the sim itself. Had the assignment distracted from student engagement, we would have expected students to play with the sim for shorter periods of time, and look to the assignment for guidance as to what to do next. In addition, we would have expected students' actions to appear more hesitant and controlled when using the sim. Of course, this was not the case, even though several minutes at a time were spent writing.

During the time spent playing with the sim, affordances and constraints allowed students to quickly learn the functionality of the interface. For instance, students had little trouble figuring out that the molecules should be dragged to the boxes and that once all the boxes were filled, another "level" becomes accessible. In addition, students quickly realized that molecules can be split at different bonds by hovering the mouse over the junction between two atoms and waiting for the scissors tool to appear. These features of the sim, taken together, helped to create a game-like environment. The goal for the students using the sim was implicit in the sim itself, and this allowed for engaged exploration.

As was stated, a few students' actions stand out as superb examples of how the sim and assignment can provide a means for changing students' readout strategies in dealing with multiple molecules of the same type. In each, students were working on the second tab of the sim, in which several molecules of the same type had to be built to successfully complete the challenge.

George's attempts to build $2CO_2$

The first student, who will be referred to as George, showed great competence in applying the techniques he learned in the first tab (building individual molecules) to the second tab when he first started. The boxes to be completed were: $2CO_2$, $2O_2$, $4H_2$, and $2NH_3$. George began by first creating an O_2 molecule, then an H_2 molecule, and finally, an NH_3 molecule. He did all of this in a time span of 2 minutes and 10 seconds. However, one critical observation is that George did not spend *all* of these two minutes simply trying to build O_2, H_2, and NH_3. Instead, he first built O_2, and then attempted to try $2CO_2$ before attempting to create H_2 and NH_3.

George spent just over 50 seconds attempting to build $2CO_2$, meaning that he really only spent 1 minute and 20 seconds creating the other three molecules listed. The figure below shows George's first two attempts at creating $2CO_2$.

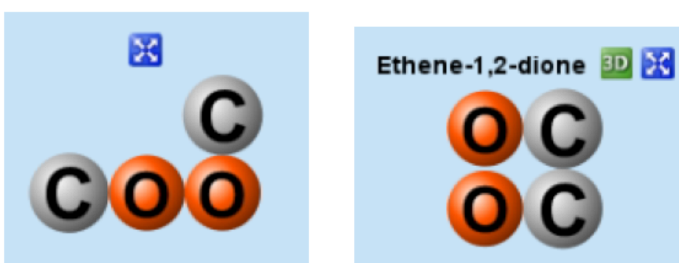


Figure 5.2: Screen capture of George's first two attempts to build $2CO_2$. His first attempt is shown on the left, and his second attempt on the right. He spent 50 seconds creating these two molecules.

From this figure, we suspect that George was associating 2C to mean C_2; that is, 2 carbon atoms bound in the same molecule. Of course, the constraints present in the sim would not allow for George to drag either of these molecules into the $2CO_2$ box, and he therefore had to reconsider his strategies. An additional observation is that after creating ethene-1,2-dione, George clicked on the 3-D representation of this molecule and stared at it for a while. It is unclear what he was thinking during this time.

Tired of attempting to work with carbon dioxide, George decided to move on to building other molecules. Within 30 seconds, he created H_2 and NH_3, and dragged both to the boxes at right. Nearly a minute later, George returned to attempting to complete $2CO_2$. His first attempt this time was to again create ethene-1,2-dione, and after doing so, he quickly separated the atoms and began to play. Shortly thereafter, George created a CO_2 molecule, saw the $2CO_2$ box light up, and dragged it into the box.

Now, it is not clear *why* George created CO_2 from watching the video. One interpretation (in fact, the one I prefer) is that George created CO_2 through random play, without consciously thinking that this was the proper way to complete the box. The are two reasons for this interpretation: first, George had initially built CO, and then appeared to semi-randomly move another oxygen atom on top of this configuration, with no indication of a deliberate action. Second, and more prominently, George's future actions indicate that he was unaware of the meaning of the coefficient in $2CO_2$ and other molecules at the time he made this first CO_2 molecule.

After dragging CO_2 to the box, George had one molecule in each of the four boxes shown on the interface. After commenting on this challenge as being "hard," he again set to work in combining oxygen and carbon in various ways. Notably, George created CO_2 a second time, after which the box on the right blinked and lit up. But instead of dragging the CO_2 molecule to the box, he created another molecule next to CO_2, and eventually combined these two into another large molecule. Both of these are shown below. After creating this large molecule, George attempted to drag this to the $2CO_2$ box, and when the sim rejected this attempt, he said "What?!!" with a clear hint of desperation in his voice.

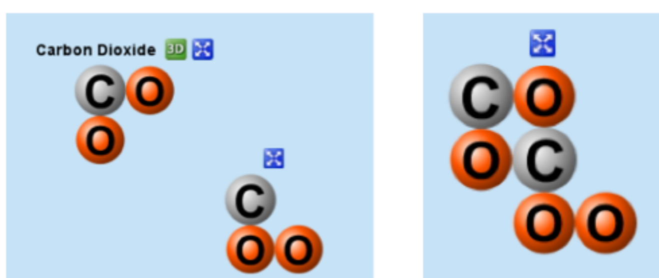


Figure 5.3: After his initial attempts, George again set to work using carbon and oxygen atoms. He built CO_2 first, the box on the right blinked and lit up, and then he built the nameless molecule next to it. Rather than dragging the CO_2 molecule to the box, he created the molecule shown in the picture at right. He then attempted to drag this molecule to the $2CO_2$ box.

The sequence of events that occurred after this are crucial, and mark a clear point of change in George's readout strategies. After creating the molecule shown in the right of Figure 4.3, George rearranged the molecules into the shape shown below. After doing this, he cut the two molecules into two separate C-O-O bonds, also shown in the figure below.[2]

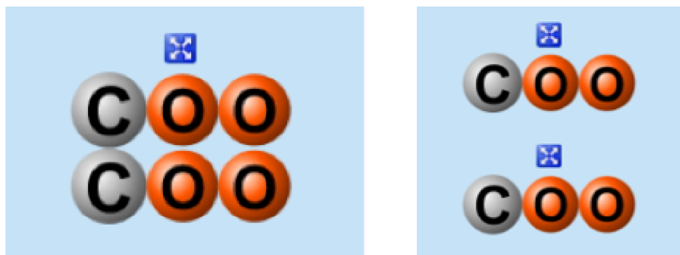


Figure 5.4: George then created the molecule on the left, and eventually separated this into the two molecules shown. After this, Jeff began talking to George, and had a fundamental influence on George's next attempts with the sim.

George then tried to drag one of the C-O-O molecules to the $2CO_2$ box, but was again unsuccessful. At this point, the student sitting next to George – Jeff, we will call him – intervened and said that George should move one of the O atoms to the left of the C atom, thus making O-C-O, which is, in fact, the correct way to make CO_2.[3] The precise dialogue was as follows:

> (George drags C-O-O to the $2CO_2$ box, and is rejected)
> *Jeff: No, cut this one off* (referring to the right O atom)
> (George cuts the carbon bond, thus leaving C and O-O. He then recombines them into C-O-O.)
> *Jeff: No, cut this one* (again pointing to the O-O bond)

[2]Note that C-O-O is *not* the same as CO_2. In addition to having the correct number of atoms in each molecule, students must also arrange them in the proper way. Understanding why these molecules must bond in particular ways is much more complicated, and the reasons are not explicitly addressed in the simulation.

[3]Another crucial observation is that Jeff – who was also recorded on Camtasia – had just completed the $2CO_2$ box 2 minutes before George got stuck. Because he had already completed this box, Jeff knew exactly how to help George out. However, this is not to say that George fully understood what the coefficient meant at the time. In fact, it is clear from watching his videos (discussed next) that he *did not* fully understand the meaning at this point, which was at precisely 9:12am.

(George cuts the O-O bond, leaving C-O, and attempts to drag C-O to the $2CO_2$ box)

Jeff: No, put this one (referring to O) *there* (to the left of C)

George: Oooohhhhh (then creates O-C-O)

(George then sees the box light up and blink, and drags CO_2 into the box. After this, he says:

George: Ooohh. Ok, I get it.

This interaction marked a clear turning point in George's interactions with the sim. After this, he appeared to fully understand the meaning of the coefficients of molecular formulae. Within 52 seconds, George built all of the 5 remaining molecules in this challenge, and completely filled all of the boxes. He did this with absolutely no hesitation. First he built O_2, then immediately after built $3H_2$, and then built the remaining NH_3 molecule. He did not work on building any other molecules during this time, and in watching the videos, it is clear that he knew exactly what to do to complete this challenge.

How are we to explain this remarkable change in George's understanding? First, we must analyze where he started. At first, his readout strategy to determine the physical structure of a molecule consisted of looking at, for example, $2CO_2$, and associating 2C with two carbon atoms bound in the same molecule. He made three different attempts at creating molecules with 2 carbon atoms in them, and then tried an incredibly interesting strategy. In Figure 4.3, George created a molecule with 2 carbon atoms and 4 oxygen atoms. This was the first conscious change in George's readout strategy. Instead of reading $2CO_2$ as what we would call C_2O_2, he read this as $2{\times}CO_2$, or what would be correctly called C_2H_4. Of course, after the sim would not allow George to place this in the box, he acted surprised and agitated that this new strategy did not work.

After this, George began interacting with Jeff. Through this interaction, George reached what might be referred to as an "aha" moment. Jeff had already completed $2CO_2$, and though Jeff himself did not fully understand the meaning of the coefficient at this time, his suggestion for George allowed George to properly adjust his readout strategies. Once he correctly understood the meaning of $2CO_2$, George was able to quickly apply this new readout strategy to the remaining molecules.

To apply diSessa's terminology, we can say that at first, George had a particular type of *integration.* He combined diverse observations of numbers in order to determine what he wished to "see." Of course, in this initial stage, George also lacked *invariance.* Having already built $1O_2$, and having placed that in the $2O_2$ box, George had applied one rule in one context (the context of $2O_2$), and was attempting to apply a different rule in another context (the context of $2CO_2$).

The changes in George's readout strategies necessarily changed George's *causal net*. He went from determining information in one way (by reading the coefficient as a subscript) to determining information in another way (by reading the coefficient as the quantity of a particular molecule). George consciously changed his causal net because he realized that this new readout strategy would work across all contexts. In doing so, George achieved *invariance*, and was then able to *integrate* information in the correct way.

Jeff's attempts at $2NH_3$

Let us now turn to Jeff's interactions with the simulation. Like George, when he moved on to the second tab, he had already demonstrated a great deal of competence in building single molecules. He started by working on the $2CO_2$ box. Like George, he first made ethene-1,2-dione, indicating that he had initially associated the coefficient to mean the same thing as the subscript. However, unlike George, he quickly realized how to correct for this. With absolute precision, Jeff separated his molecule and quickly built O-C-O, and moved it to the $2CO_2$ box.

When Jeff did this, the teacher watched him, and realized that she had mistakenly asked the class to move on to the second tab ahead of time. To correct for this, she asked the entire class to go back to the first tab and continue building single molecules. Thus, after making the first CO_2 molecule, Jeff went back to the first tab, and he did not return to the second tab for nearly 9 minutes.

At 9:08am, nearly 4 minutes before interacting with George, Jeff began working on the second tab. At this time, he did not return to building the second CO_2 molecule, but instead, started working on the $4H_2$ box (notably, Jeff's boxes were the same as George's boxes, which were listed earlier).

Jeff created one H_2 molecule, looked at the 3-D picture for a few seconds, and then immediately built the remaining three H_2 molecules, one right after another. This took him approximately 35 seconds.

At 9:10am, Jeff returned to working on $2CO_2$. He seemed to know exactly what to do, as he pulled only three atoms from the buckets and connected them as O-C-O, exactly as he needed to. From this, it appeared that Jeff did, in fact, understand the meaning of the coefficients. Not only had he created 4 H_2 molecules with great efficiency, but he had also created the remaining CO_2 molecule with absolute precision.

From 9:10 to 9:14am, Jeff did not play with the sim at all. Recall that he helped George at 9:12, but it seemed that he might have also been spending time filling out the assignment at this point (he clicked on the 3-D view of the molecules a few times, indicating that he may have been trying to draw the molecules). At 9:14, he immediately built two O_2 molecules, and dragged them to the $2O_2$ box. Again, it appeared that he knew exactly what he was doing, and, more specifically, that he knew what the coefficients meant.

Later, Jeff began working on the only box that was left: $2NH_3$. The sequence of constructions he made are shown in the figure below. Although Jeff *appeared* to have understood the meaning of coefficients in his construction of other molecules, once he got to $2NH_3$, he consistently associated the 2N as meaning N_2. Recall that he showed similar difficulty in creating $2CO_2$ before, but it appeared that he had resolved that issue when he very precisely created the second CO_2 molecule and placed it in the box.

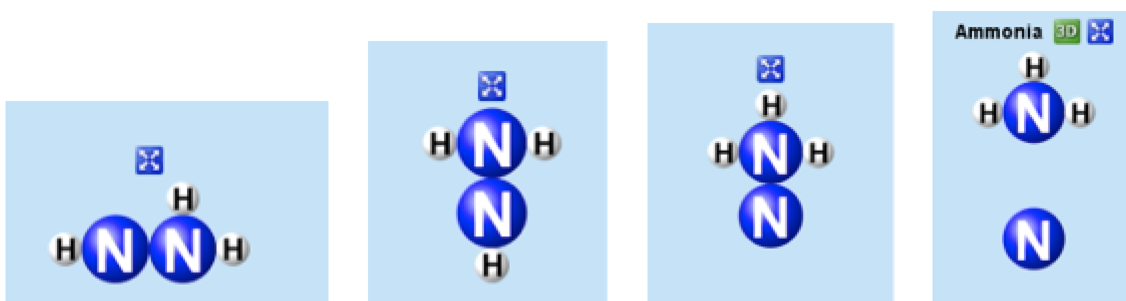


Figure 5.5: Jeff's attempts at creating $2NH_3$. Left to right shows sequential order. All of this occurred within a 2 minute window.

Within two minutes, Jeff made three different attempts at creating $2NH_3$. Eventually he separated off the bottom N atom, and placed the remaining NH_3 in the box. However, after creating 1 NH_3, Jeff never returned to making the second NH_3 molecule. The reasons for this are difficult to pinpoint. From watching the video, there appear to be two different interpretations of what happened that both seem plausible.

First, it could be that Jeff was distracted from finishing this box. One reason for this is that, earlier, George had seen Jeff make a very large molecule, and was so impressed that, after Jeff made NH_3, he asked Jeff how he made that large molecule. After George asked this question, Jeff eventually started trying to make very large molecules. Thus, the first interpretation is that George's questioning changed Jeff's goals, and therefore, Jeff did not feel the need to finish building the second NH_3 molecule.

The second interpretation is that Jeff simply thought that he had finished the box and was free to do work on something else. Note that the $2NH_3$ box *did not* turn yellow, as it does when the challenge is complete, but perhaps Jeff simply did not notice this.

In this case, the changes in Jeff's interactions with the sim are harder to explain. On one hand, his early actions indicated that he *was* correctly interpreting the coefficient. Before moving to $2NH_3$, he had correctly made 8 molecules, and had only made one mistake (when he first tried $2CO_2$). Yet once Jeff moved to $2NH_3$, he made three unsuccessful attempts in a row.

It seems that while Jeff *seemed* to show *invariance* at first, the introduction of this new context brought out a hidden difficulty that likely indicates a problem in his *causal net*. The reasoning strategies that George used in this case were different than those used in his previous attempts, and therefore, the way he determined the relevant information here was different.

Additionally, we cannot conclude that Jeff experienced the "aha" moment that George did, nor can we conclude that Jeff's concept of "molecular coefficients" changed to the expert's concept of "molecular coefficients." Because Jeff did not complete the $2NH_3$ box, and because there were no other indications in the Camtasia video that show his ability to determine the meaning of the coefficients in other contexts, there is no way to be certain that Jeff fully understood the meaning of the coefficient.

Brad's attempts at $2CO_2$ and $2NH_3$

After the class moved on to the third tab, another student – Brad, we will call him – successfully completed the $2O_2$ box in very little time.[4] Then Brad moved on to working on the $2CO_2$ box.

Like the other students, Brad initially associated 2C to mean 2 carbon atoms bound in the same molecule. His attempts are shown in the figure below.

[4]In fact, what actually happened was that when the class moved to the second tab the *first* time, he built one O_2 molecule, and then he finished building the second after the class returned to the second tab the second time around.

Figure 5.6: Brad's initial attempts at creating $2CO_2$.

After being rejected for the first two tries, Brad decided to try the $4H_2$ box instead. Within 35 seconds, he built and completed all four H_2 molecules, and then began writing on his sheet.[5] It is unclear exactly what he was writing at this moment in time.

After writing on his paper, Brad attempted to move on to the $2NH_3$ box. His first attempt was to move N_2 into the box. After, he tried three different combinations that all involved two N atoms bound together. These are shown below.

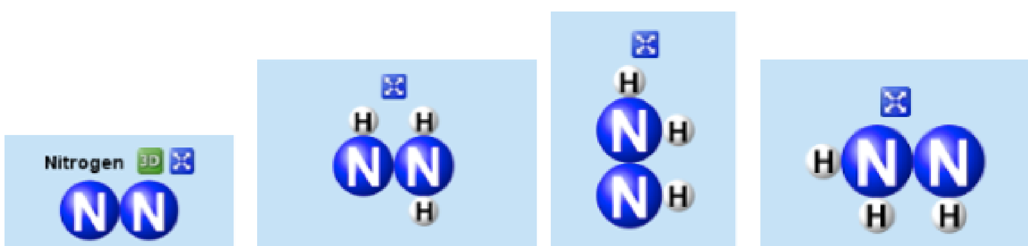


Figure 5.7: Brad's attempts at creating $2NH_3$.

Unable to complete this task, Brad again decided to move on to $2CO_2$. He formed the molecules shown in the figure below, again demonstrating the tendency to associate 2C with two carbon atoms bound in the same molecule.

[5]In this case, the scratch of Brad's pencil across the paper could be heard in the Camtasia files, and this is how we know he was writing.

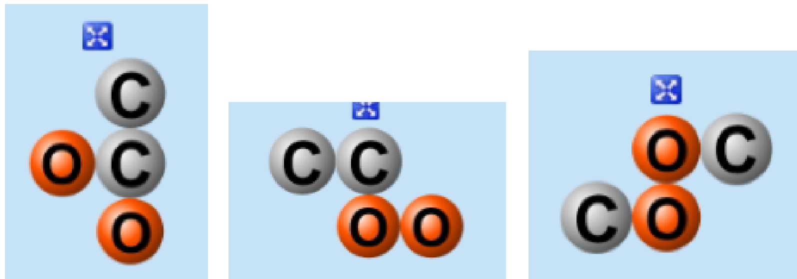


Figure 5.8: Brad's second attempts at creating $2CO_2$.

Sadly, Brad never made the necessary coordinations to successfully complete the $2CO_2$ or $2NH_3$ boxes. In fact, these boxes were left entirely blank, and eventually he moved on to the third and final challenge without ever completing these two boxes. In total, Brad spent 6 minutes 30 seconds trying to make $2NH_3$ and $2CO_2$ *after* creating $4H_2$. Thus, he spent about 7 minutes total on these two different tasks, and never successfully completed either of them.

How are we to explain the reasons for Brad's difficulty in creating $2CO_2$ and $2NH_3$? First off, note that Brad did not exhibit *invariance* among contexts. He demonstrated absolute dedication to interpreting the coefficient in $2CO_2$ and $2NH_3$ to mean 2 carbon or 2 nitrogen atoms bound in the same molecule, yet he interpreted the coefficients in $2O_2$ and $4H_4$ to mean two and four molecules of oxygen and hydrogen, respectively.

In addition, Brad showed variability in *integration*. This was present in his various attempts to create $2CO_2$ *and* $2NH_3$. Specifically, he attempted to *read out* information differently by arranging the atoms in various combinations. He also neglected information, first by eliminating one of the oxygen atoms in $2CO_2$, and then by eliminating 3 H atoms in NH_3. In both cases, he attempted to drag the molecule to the bin, indicating that he did, in fact, think of these as possibilities. Thus, the way that Brad *integrated* information varied within each context of $2CO_2$ and $2NH_3$, separately.

Unfortunately, Brad was so dedicated to his particular readout strategies that he never made all of the desired changes to his causal net. He *was* able to correctly apply the coefficient rule for diatomic molecules, such as H_2, O_2 and N_2, but the way he determined the meaning of the coefficient in that context was different than the way he determined the coefficient for other polyatomic molecules, such as $2CO_2$ and $2NH_3$. Thus, it appears that Brad carried with him different methods of determining information.

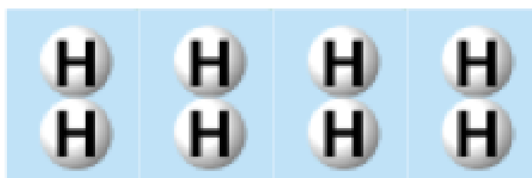
Pre- and post-test results

The Camtasia files for three different students have been analyzed in great detail, and the changes (or lack thereof) in their concepts of molecular coefficients have been discussed. But how do we know that these students are representative of the entire class?

In answering this, we can turn to the test results. The full results are not presented here, but they are included in the appendix. The results overall are incredibly positive, and the specific results shown here are a good indicator that many students showed conceptual change. 58 students total took the pre- and post- tests (there were two separate classes that this study was conducted in).

The first question of interest is the following: "Write the chemical formula below each molecule or groups of molecules."

One of the pictures was of $4H_2$, shown here:



The overall class score on the pre-test was 0%. In other words, no one got this question right on the pre-test. On the post-test, however, 63% of students answered this question correctly.[6] This is a remarkable change in scores, and indicates that this activity helped students a great deal, at least in the context of $4H_2$.

Yet another question on the pre-test asked students to *draw* $3N_2$. In this case, 17% of students answered the question correctly. Additionally, 31% drew 3 nitrogen atoms bound together, and the rest of the class (over 50%) drew something else, or nothing at all.

The corresponding question on the post-test asked students to *draw* $4N_2$ (only slightly different than the question on the pre-test). In this case, 78% of students answered correctly. Of those who did not answer correctly, 9% drew separate N_2 molecules, but not the correct amount, and 7% drew 4 nitrogen atoms bound together.

Again, the rest of the results from these tests are included in the appendix, but overall, the results indicate a very positive shift. It is certainly reasonable to suspect that, on average, students' concepts of in this study.

[6]The other common response (19%) was for H_8.

5.4 Heuristics drawn

So far, this chapter has focused on the students, and specifically, their interactions with the simulation and the conceptual change that took place as a result. Let us now step back and assess how the structure of this activity allowed for the micro-levels of conceptual change to occur.

There were four general findings in this study. First, through watching students' Camtasia videos, it was clear that the game-like situation (or implicit challenge) in the sim itself allowed for student engagement. In turn, this engagement provided a means for students to make productive use of the constraints and affordances built into the simulation itself, and all of this together allowed for conceptual change to occur. Additionally, students had little to no difficulty understanding the *goal* that they had to complete in order to move on to the next levels in the sim, and that goal was challenging enough to keep students engaged for nearly 1 hour and 20 minutes (the total time of the activity). Therefore, the game-like situation in the sim helped students to stay engaged and on task, thus increasing the total amount of time that conceptual change could occur.

Second, students showed much improved performance on the post-test, in comparison to the pre-test. Though not discussed earlier, another finding was that on the post-test, most students drew 3-D representations of molecules when a question asked them to *draw* molecules. While the reasons for students drawing 3-D representations of molecules cannot be pinpointed exactly, we do suspect that the assignment itself helped students to gain this ability.

Specifically, the questions on the assignment also asked students to *draw* pictures of molecules as they completed them on the sim.[7] We suspect that asking students to do this aided in their "internalization" of molecular structure for two reasons. First, asking students to draw pictures of molecules forced them to look more carefully at the 3-D representations, as they shifted their attention between the drawing and the representation on the screen. Second, the act of drawing itself is a type of externalization of ideas through the use of artifacts (namely, pencil and paper), and therefore, likely requires activating different cognitive structures that must coordinate with what is shown on the simulation. The main point is that the act of *drawing* the molecules on the assignment likely aided in students' abilities to draw 3-D representations on the post-test. While more research should be done to see exactly what the effects of drawing molecules really is, this provided

[7] Again, the assignment itself can be found in the appendix.

us with reasons for adding a heuristic to our list: "ask students to recreate or re-present visual features on the sim."

Third (and not discussed previously in this chapter), the explicit call of attention to visual features on the sim helped students to notice a feature of the sim that might have otherwise gone unnoticed. In this assignment, one of the first questions was "How do you know you have made a molecule?" While this may seem obvious, the purpose of the questions was for the students to realize that a molecule is only made when the chemical name appears above the molecule created, and that other conglomerations of atoms can be made which have no actual chemical name. This turned out to be important later on in the assignment when students are asked to build the largest *molecule* (not amalgam of atoms) possible. In effect, this explicit call to visual features helped to clarify ideas what constitutes creating a molecule.

Fourth, the actions of the teacher in this classroom also played a critical role in keeping students on task. Recall that the teacher collectively moved the class on to the second and third tabs, and that the class was, in a sense, obliged to follow these directions. Had the teacher not been there, students likely would have explored the sim in an unstructured manner, and perhaps would have moved on to the second or third tabs without fully understanding the first tab. We won't go into great deal about the teacher's actions here, but her actions certainly had an impact on the "when" and "what" students were working on.

These findings are suggestive of heuristics that should be used when attempting to write these assignments. These are:

1. **Set up game-like situations and/or take advantage of implicit challenges**

   This helps to engage students in the sim and allows for the constraints and affordances of the sim to guide student understanding. This can come in two flavors: allowing the game-like scenario present in the sim itself be naturally used, or setting up a game-like situation through the use of guided questions. The former of these is apparent in the build a molecule assignment; we give an example of the latter later in this paper.

2. **Ask students to recreate or re-present visual features on the sim**

   Asking students to write formulas, draw pictures, or explain something shown on the sim in words are all possible ways of fulfilling this heuristic. The key point about this is that it allows for time to reflect on the material presented, and forces the students analyze particular features of the sim more carefully. Both of these serve as mechanisms for the internalization of important ideas or concepts.

3. **Ask about visual features on the sim**

   Writing a question that addresses a feature of the sim that might otherwise be glanced over or go unnoticed can often be helpful in allowing the students to recall the usefulness of that particular feature while they engage with the sim. For example, asking the students how they knew they built a molecule was something that might have otherwise gone unnoticed. This heuristic should most likely be used sparingly, and only when there is no other way to draw attention to the importance of a particular visual feature.

Let us now assess *how* these heuristics were implemented, and *why* they apparently worked in this study. We can begin by analyzing exactly what needed to happen for students to change their concepts of molecular coefficients. As was clear from the study, students started with a set of *readout strategies* that they used to determine various types of information about molecules. Through play with the sim, students participated in the challenges built into the simulation itself, and in doing so, were able to test the 'bounds' of the simulation, and observe the rules that allow (or disallow) for successful completion of different tasks.

Because students' initial readout strategies were often inconsistent with the rules of the simulation (i.e. the correct readout strategies), they made "mistakes" while playing with the sim, and had opportunities to make revisions to their readout strategies to correct for those "mistakes." To fully complete the challenges presented in the sim, students had to change their concepts of molecular coefficients by changing the way they read out information. This is not to imply that every student developed the a fully consistent set of readout strategies, but nonetheless, it was clear that students *did* show changes in at least some of their concepts regarding molecular coefficients.

Clearly, the act of participating in engaged exploration aided in this change. So too did the acts of writing chemical names, writing chemical formulae, and drawing pictures of molecules. Thus, the factors that promoted *these* acts are equally critical in understanding how to promote the conceptual change that occurred in this study. This is where the notion of context and the implementation of heuristics come into play.

Let us list the ways the heuristics were implemented in this study. First of all, the game-like situation or challenge present in the simulation itself absorbed the heuristic: "set up game-like scenarios and/or challenges." But in conjunction with this, the assignment itself promoted participation in this game-like scenario through the formatting of the questions. The questions were not guided, but rather, open-ended, thus allowing for play and participation in the game-like scenario. Had the assignment been more guided, or even worse, prescriptive, this could have taken away from students' likelihood of engaged exploration.

Additionally, the assignment provided another means for students to interact with the content provided in the simulation; namely, by asking students to recreate or re-present visual features on the simulation itself. This forced students to externalize information in a way that simply was not achievable through experimentation with the sim alone. The benefits of asking students to do this are not well-defined as of yet, but the differences in scores on the pre- and post-tests, and more specifically, the finding that students were able to draw 3-D representations of molecules on the post-test, provides us with reason to believe that questions of this type can aid in the externalization *and* internalization of these concepts.[8]

In this study, the heuristics were strategies that helped to create a *situation* in which students had opportunities to change their readout strategies and/or causal nets. The heuristics were one component of an overall activity system in which various components interacted in specific ways to form a contextual whole. Although the end goal was for students to change their concepts of molecular coefficients and coordinate information in the correct way, all of the external factors present – the classroom environment, the teacher, the assignment, the students' interactions, and so on – factored into the ways that students engaged in the activity. Thus, the heuristics are just one component of creating a situation in which these micro-levels of conceptual change can occur. We cannot neglect the effects of context, nor can we fully account for them. However, we can draw suggestions from these studies that can be applied to similar situations, in order to promote the *types* of interactions that are shown to allow for micro-level conceptual change to occur.

[8]Though not discussed in this thesis, part of the theoretical grounding from this is backed up by Vygotsky's claims in *Thought and Language*. In it, he claims that, while external speech allows us to turn thought into words, internal speech allows us to turn speech into thought, and that both of these processes serve as key mediators in our thought processes. A similar notion is present in this study, in which external *writing* turns thought into pictures, and internal *visualization* allows us to turn thoughts into drawings or build molecules on the sim, or whatever else it may be.

Chapter 6

Projectile Motion

The second study to present took place in an upper-division classical mechanics and math methods course at the University of Colorado. Two of the focuses for the course are solving differential equations and using Newton's laws in a variety of ways, and one topic that applies both of these topics is the motion of particles subject to air drag.

This particular study emphasized the motion of a baseball being shot vertically in the air. Because this class did not have a separate time period for using tutorials, we decided to write a homework problem that used the "Projectile motion" PhET simulation [31]. A more complete description of the problem and the simulation are provided later in this chapter, but the sim itself consists of a cannon that can shoot various objects in the air, and various parameters and features can be adjusted to compare different trajectories.

This chapter will proceed in the following way: We begin with a brief overview of the topic of interest, and give a mathematical review of shooting a ball subject to quadratic air drag. In the next section, the overall format of the problem and a description of the sim is introduced. Next, an overview of the results from the studies is given, and finally, a description of the heuristics drawn from this assignment (as well as a brief theoretical interpretation of these) is discussed at the very end.

6.1 Physics overview

In most introductory physics courses, the topic of kinematics is discussed with one slight caveat: air drag is neglected. However, once physics majors reach their first upper-division classical mechanics course, the topic of air drag is added into the mix.

In general, there are two *types* of air drag that show up in the real world: linear air drag and quadratic air drag. Linear drag dominates for small particles, typically around the micron size [45]. For instance, an analysis of bacteria swimming in water will often include a linear drag term. On the scale of everyday phenomena, however, linear terms are hardly important, and the quadratic air drag term dominates. This is, indeed, a type of force, and is usually given by the following:

$$F_D = -cv^2\hat{v} \tag{6.1}$$

Here, F_D is the drag force, c is some drag "constant," v is the velocity at which the object travels, and the $-\hat{v}$ is used to indicate that the drag force always opposes the direction of velocity of the object. If we know that we are using some sort of macroscopic particle with a particular area A, and we shoot that particle through a medium with density ρ, then we can replace the drag coefficient c, as shown in the following equation:

$$F_D = -\frac{1}{2}c_0\rho_{air}Av^2\hat{v} \tag{6.2}$$

Note that a constant c_0, which is a property that is different for each object shot, is included, and that we have assumed that $\rho = \rho_{air}$, since we are, after all, interested in the case of a baseball being shot in the air.

In the homework problem, we specifically ask students to solve for the case of a ball being shot *up* in the air. Note that the case of a ball traveling from the top of its trajectory *down* to the floor requires an entirely different solution, due to the fact that the direction of the drag force changes.

After drawing a free-body diagram, we ask students to write out a differential equation for this particular case and solve it. If they choose their y-axis to point "up," they should come up with the following equation:[1]

$$m\frac{dv}{dt} = -mg - cv^2 \tag{6.3}$$

Note that I am using c, instead of that long expression in (2). Most students proceeded this way, and then substituted the necessary values later in the problem.

This is a rather nasty-looking differential equation, since it is non-linear *and* non-homogeneous. However, with a couple of clever substitutions, one can separate this and

[1] Note that students do not have to define y as pointing "up" to solve this. In fact, a few students in this study made a conscious decision to make y point "down."

solve analytically.[2]

Solving, one finds the following form for $v(t)$:

$$v(t) = -v_{term} \tan\left(\frac{gt}{v_{term}} - \lambda\right) \tag{6.4}$$

Note that $v_{term} = \sqrt{\frac{mg}{c}}$ and that $\lambda = \tan^{-1}\left(\frac{v_0}{v_{term}}\right)$, where v_{term} stands for the terminal velocity.[3] Thus, the final solution for $v(t)$ is just a shifted and inverted tangent function. When plotted, this looks something like the following:

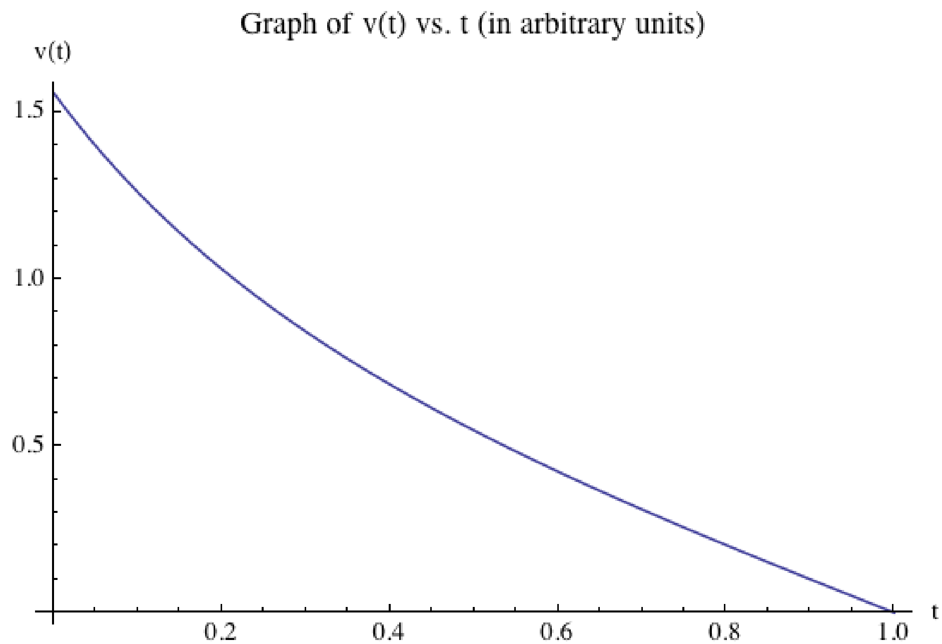


Figure 6.1: As is expected, the ball is shot at its initial positive velocity, and then rapidly slows to 0. The plot is cut off after this because a different differential equation governs its motion on the way down.

Students are not initially asked to plot this, but instead are asked to also solve for the time it takes the ball to reach the top of its trajectory. One can do this by setting $v\left(t_{top}\right) = 0$ and solving for t_{top}. Doing so, one finds:

[2]Note that in the case of a ball being shot at an *angle*, there is no analytic solution to this problem. This fact was largely what led us to writing a problem that used vertical motion instead of some angle-dependent motion.

[3]Of course, the object never reaches a constant terminal velocity in the case of being shot up, but nonetheless, the same constant appears in this equation

$$t_{top} = \frac{v_{term}}{g} \tan^{-1}\left(\frac{v_0}{v_{term}}\right) \tag{6.5}$$

If one were inclined to plot this as a function of initial velocity, they would find the following:

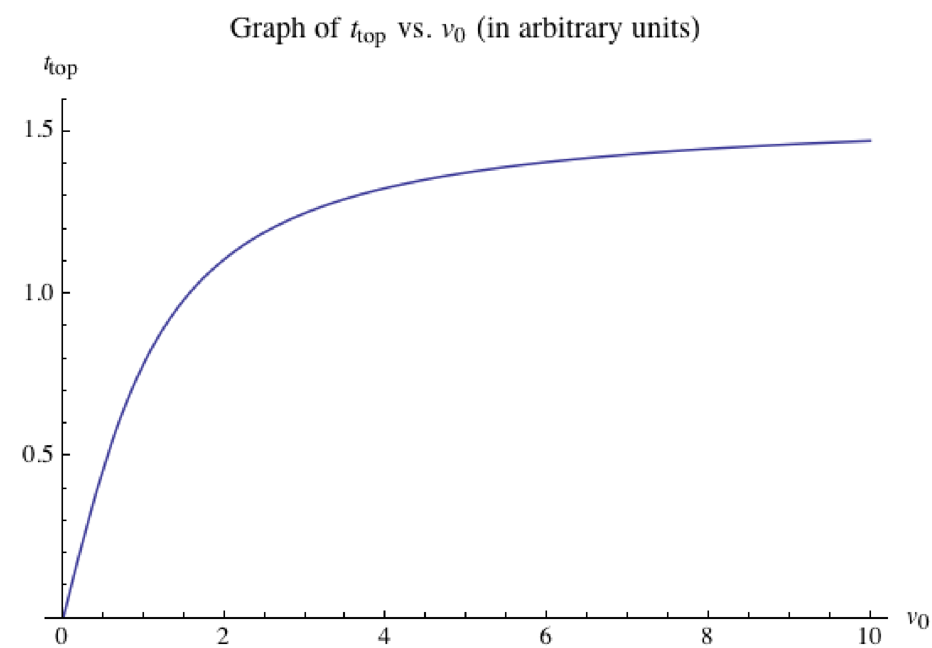


Figure 6.2: Notice the asymptotic behavior going on here.

What is this graph showing? Well, at a certain point, no matter how fast I shoot the ball in the air to begin with, it *will never* take longer than t_{max} to reach the top of its trajectory. So no matter how fast one launches a baseball in the air, it will always start descending after a set amount of time![4]

The motion of this ball is likely not intuitive for most normal beings. Therefore, it seemed like a great feature to include in the homework problem we wrote for projectile motion. The ways we "got at" this behavior are discussed next.

[4]Obviously, we are assuming that ρ_{air} is constant, and that the ball is not escaping the atmosphere.

6.2 Sim and problem overview

This study was conducted in two separate semesters. After general observations of students using the assignment in the first semester, we felt that we could make some improvements to the original assignment. Therefore, we changed specific questions and observed students using the new homework problem in the following semester. This section will review the overall structure of both problems, and the next section will discuss the changes that were made,[5] as well as the data and results that were collected.

The general structure of the problem is shown below:

Mathematical introduction to problem

1) Draw a free-body diagram, write a differential equation for this situation, and solve the differential equation for $v(t)$.

2) Solve for the time it takes the ball to reach the top of its trajectory

Parts of the problem involving the sim

3) Find the initial velocity that makes the ball reach the top of its trajectory in 3 seconds

4) Does it take longer for the ball to go from the ground to the top of its trajectory, or to go from the top of its trajectory to the ground? Explain why.

5) Using the sim, explore the time it takes the ball to reach the top of its trajectory for different values of v_0. Then use mathematics to explain the asymptotic behavior, and make a plot like that shown in Figure 5.3.

In both problems, the mathematical introductions were the exact same. The changes made revolved around the use of the use of the simulation. Let us first review the design of the simulation.

The "Projectile motion" simulation, shown below, allows users to shoot various objects out of a cannon and displays the trajectory of the object in real time. Two options can be selected on the sim: no air drag (in which case, the object's trajectory is displayed in blue), and with quadratic drag (in which case, the object's trajectory is shown in red). At 1-second intervals, a black tick-mark appears on the object's trajectory, and this

[5] See the appendix for the full version of the problems.

allows users to know when the ball reaches the top of its trajectory (at least to a fair approximation). The angle of the cannon can be adjusted to any range, and users can zoom in or out to watch the motion of the objects.

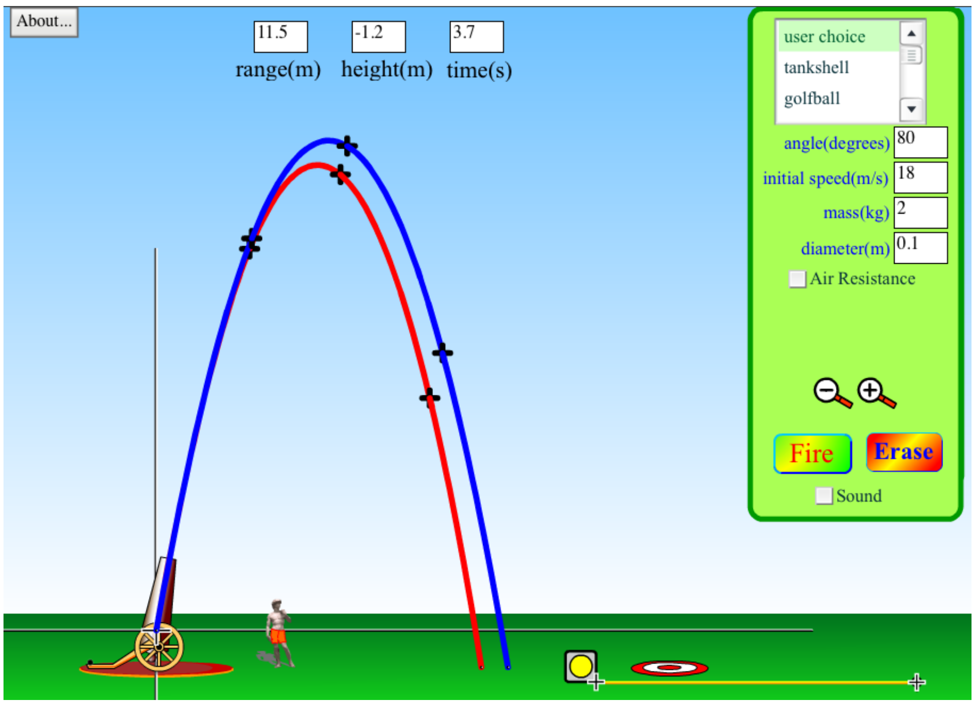


Figure 6.3: Interface of the Projectile motion sim [31]. Blue shows trajectories without air drag, red shows trajectories with quadratic drag.

Of course, this assignment only dealt with the motion of a ball *with* quadratic air drag, and only dealt with the analytically solvable case of a ball being shot vertically upwards, with no horizontal motion.

6.3 Specific differences and student responses

Differences in part (3) of the assignments

Let us now turn to the differences between part (3) in the two assignments. Part c of the problem was the first to introduce the simulation. Below, the black text was the same for both problems. The green text corresponds to the old problem, and the red part corresponds to the modified problem.

(c) On the top right, switch the object to baseball. This sim uses quadratic drag: $F_D = -\frac{1}{2}c_0 A\rho_{air}v^2\hat{v}$, where c_0 is the drag coefficient, A is the cross-sectional area of the object being shot, and ρ_{air} is the denstiy of air $= 1.3 \quad kg/m^3$.

(The sim shows you the value of c_0 and diameter it has picked for a baseball, on the right side of the simualtion). Use your formula in part b for "time to top" with these numbers to deduce what numerical initial velocity v_0 you need to get the ball to reach the top of its trajectory at precisely t=3 sec. Now test it, you can input v_0 into the sim and fire the cannon. Aim the cannon at 90° (or 88-89° if it is easier to see the trajectory) and switch on air resistance. The little + and - glasses let you zoom in or out. Does the ball reach the top at t=3 sec? (It should!)

By experimenting with the sim, what initial velocity makes the ball reach the top at approximately 3 sec?

(d) Now use your formula in part b for "time to top" to deduce what numerical initial velocity, v_0, you need to get the ball to reach the top of its trajectory at precisely $t = 3$ sec. How does your calculated value compare to your "experimental" value?

There are two main differences between these two problems. In the first problem, we asked students to first calculate t_{top}, and then see if it worked on the sim. On the second problem, we instead asked students to first *play* with the sim and experimentally find the time it takes for the ball to reach the top of its trajectory in 3 seconds. The other main difference is simply in the wording of the problem. While the first problem is verbose and prescriptive, the second is brief and more open.

We changed the problem in this way by implementing the heuristics we developed. Recall that one of our heuristics is: set up game-like situations and/or challenges. In the first problem, no game-like situation is present. Students simply have to calculate values and then check that they made the correct calculation by testing with the sim. On the other hand, the second question *does* present a game-like challenge by asking students to first *experimentally* determine the time it takes for the ball to reach the top of its trajectory. The main reason in writing the question this way was to get students to play with the sim more, and hopefully partake in engaged exploration.

We can now turn to the students' reactions to the questions to see how well these heuristics "worked." Unfortunately, a comparative analysis between the two problem styles is simply not possible. We did not collect sufficient data on the original problem to be able to analyze differences in students' actions.[6] However, sufficient data *was* collected for the modified problem, and this data was enough to provide insights into the effectiveness of the heuristics.

The data collected from the second problem came in two forms: field notes of students using the problems, and Camtasia video files of their interactions with the sim. We set up two homework "help" sessions that students were free to come to, and did this on two different days: on a Tuesday afternoon two days before the homework was due, and a Wednesday evening approximately 15 hours before the homework was due. Students were informed in class that if they planned on coming to one of the help sessions, they should not start working on the problem until that time. Two students showed up on Tuesday, and three students showed up on Wednesday. None of them had started working on the problem beforehand.

Students' reactions to this problem were different in the Tuesday and Wednesday groups. On Tuesday, one student – Ben, we will call him – read part c and immediately started playing with the sim. There was absolutely no hesitation in Ben's actions; he read the problem, and immediately set to work in determining the time it takes the ball to reach the top of its trajectory.[7]

On the other hand, a student in the Wednesday group – Bruce, we will call him – did not immediately start playing with the sim after reading the problem. Instead, Bruce told the other student he was working with – Alex, we will call him – that he was going to skip to part d, and then return to part c after he had calculated t_{top}. At this point, I intervened and asked Alex and Bruce to work on part c first. Additionally, I asked Bruce *why* he wanted to go on to part c, after which, he said that he didn't want to "putz around with the sim for so long."

[6]There were indications by simply observing students in the first semester that the problem was not all that engaging. Watching students gave us a "feeling" that the assignment wasn't promoting the kinds of interactions we had hoped for, but we do not have sufficient data to back up our feelings.

[7]An interesting point here is that Ben *did not* start shooting the ball vertically to begin with. Instead, he was trying to make the ball reach the top of an *angled* trajectory in three seconds. So rather than shooting the ball vertically upwards and counting three tick marks in that way, he shot the ball at around $60°$ and adjusted v_0 until he got the ball to reach the top of this angled trajectory in three seconds. Other students showed similar tendencies (in fact, 3 out of the 5 students at the help sessions did this). After I informed Ben that he was supposed to be shooting the ball vertically, he stated that that was probably "implied in the problem."

To provide some context for the situation, it should be mentioned that Bruce and Alex had other work due Thursday that they had to finish on this particular Wednesday evening. Thus, they were feeling pressed for time, and expressed this concern several times throughout the assignment. Still, they were amenable to my request that they work on part c first, and were willing to spend whatever time was necessary to finish the problem in order.

This data provides at least *some* evidence that the heuristic, "set up game-like situations or challenges," worked in the way we intended. Ben clearly 'took the bait' and immediately started playing with the sim. Bruce and Alex did not do this, but external factors were present that influenced their motives for finding the most efficient method. Had they *not* been pressed for time, they may have interacted with the sim differently.

Another question that might arise here is, "Why do we care that students play with the sim first?" There are two (not entirely unrelated) reasons for this. First, as was described in Chapter 2, driving questions, such as the one presented in the second problem above, can often lead to productive engaged exploration. This type of exploration has proven to be important in the past [16, 35], and therefore, this is something that we should try to promote in these assignments. Second, prescriptive problems, such as the one in the first assignment above, not only hinder engaged exploration, but also send students into "task completion mode." (Another example of this is presented in the next chapter, using the tunneling sim.) Our findings indicate that asking prescriptive questions is *always* less effective than asking driving questions, regardless of the topic, level of student, etc. Thus, a general principle is to *not* prescribe when asking students to use a simulation.

Differences in part (5) of the assignments

Part (5) deals with looking at the asymptotic behavior in the time it takes the ball to reach the top of its trajectory. The questions on the original and modified assignments are again shown in green and red, respectively.[8]

> **(e) Now let's look at another interesting feature of shooting an object up in the air. Start increasing the value of v_0 in the sim. Double it from what you had before, then increase it by 10, and then by 100. What is happening to the time to reach the top?**

[8]The differences in letter number were do to the fact that the modified problem replaced part c with parts c *and* d.

Use your formal mathematical results from above to explain what is happening!

(f) Again, playing with the sim, write down the initial velocity that makes the ball reach the top of its trajectory at 4 sec, then 5 sec, then 6 sec, and so on. What do you notice happening? Make a plot of t_{top} vs. v_0 and explain in words how this relates to what you see on the sim.

Once again, a notable difference between the two questions lies in style and phrasing. While the original problem is prescriptive, the modified question is more open. Also note that in the modified problem, we ask students to do the impossible. For a baseball, $t_{max} = 5$ sec, and therefore, students cannot find an initial velocity for which the ball reaches the top in 6 sec. The effects of this are discussed shortly.

In this case, both questions use the heuristic, "Utilize illuminating cases," since both ask about the asymptotic behavior. However, the modified problem implements this heuristic *in conjunction* with the heuristic, "Set up game-like situations and challenges." The reason for combining these two was to promote more play with the sim, in hopes that students would partake in engaged exploration. In fact, one of our current thoughts is that these two heuristics are "inseparable" to some extent; that is, utilizing illuminating cases *must* be done through the use of a game-like situation or challenge.

We now turn to the students' reactions to the modified problem. In the Tuesday group, Ben, who was notably further ahead of the other student – Mary, we will call her – simply did not notice the vertical asymptote. Ben thought that he had found an initial velocity for which the sim showed the ball reaching the top of its trajectory in 6 sec. Of course, Ben was mistaken, and the most likely reason for this occurrence is that Ben simply made a mistake in counting the tick marks on the sim. At any rate, he did not notice the asymptote, and therefore, Ben, Mary, and I all had a collective discussion about the asymptote. Ben and Mary were surprised that such an asymptote could exist.

In the Wednesday group, Bruce and Alex spent nearly 9 minutes attempting to get the ball to reach the top of its trajectory in 6 seconds. Each had a separate computer, and for this entire time, were individually plugging different values into the sim, discussing the results that were seen.

Bruce and Alex found that they could get the baseball to hit t_{top}= 5 sec at $v_0 = 225$ m/s, and then tried to get the ball to the top in 6 seconds. Nearly 4 minutes in to trying this, Alex stated, "I don't think it's possible." Nonetheless, both students kept trying to get the ball to reach the top in 6 seconds for another 5 minutes.

Alex eventually tried the following sequence of values: $v_0 = 20,000$ m/s, $v_0 = 60,000$ m/s, $v_0 = 100,000$ m/s, and $v_0 = 500,000$ m/s. After trying this last one, he stated, "I think your sim's broken, Danny." This then led to a discussion between Bruce and Alex as to whether 6 seconds was possible. Eventually, they asked me if it was possible, and I said, "Here's a hint – read the problem again," referring specifically to the part that says "What do you notice happening?"

After providing this hint, Alex and Bruce realized that there was an asymptote. After this, I asked them if it was possible to explain this asymptote using mathematics, and we then set to work in trying to do so. After nearly 15 minutes, we had made a plot of t_{top} vs. v_0, and could explain the asymptote of the ball via yet *another* form of representation. Thus, at the end of the assignment, Alex and Bruce had looked at the asymptotic behavior of the baseball via three different means: the visualization of the ball's motion on the sim, the mathematical equation for t_{top}, and the graph of t_{top} vs. v_0.

In terms of the effectiveness of the two heuristics mentioned earlier, it seems that implementing the "Utilize illuminating cases" heuristic through the use of a game-like situation or challenge was, in fact, quite beneficial. Rather than prescribing, and telling students how to use the sim (as was done in the original problem), we set up a situation in which the students had to *decide* how to use the sim to determine the relevant information. In asking the question this way, students had to use their own intuition and discover the asymptotic phenomenon for themselves. This difference is absolutely crucial, for reasons that will be explained shortly.

Further results: student surveys

In addition to observing students in the two help sessions, a survey was handed out to the class. In fact, this same survey was used in both semesters, and therefore, we *do* have a method for comparison through these results. The questions, as well as the average scores from both semesters, are shown below. Note students could choose integer values on a 1–5 scale, where 1 = strongly disagree, 3 = neutral, and 5 = strongly agree. Note also that N=27 students for the original assignment, whereas N=32 students for the modified assignment.

1. **Overall, the PhET sim problem from last week's homework helped me understand quadratic drag better.**

Original: 3.3 **Modified: 3.4**

2. **The PhET sim provided me a helpful visualization for this problem.**

   Original: 4.3 **Modified : 4.0**

3. **The PhET sim helped me make sense of the mathematical solution for** $v(t)$ **that I calculated earlier.**

   Original: 2.9 **Modified: 3.2**

These results indicate that there wasn't much change in the survey responses across semesters. Additionally, the rest of the survey, which asked students to give short responses to three more questions, were very similar overall.[9] Thus, it seems that the modifications to the problem did not cause any significant change *in the survey responses*.

However, it is difficult to determine exactly what these survey responses mean. First of all, we do not have a sense for *how* the simulation was used for the majority of the students. Not all students finished the problem,[10] and there is no way to know exactly what students were doing while using the sim and/or working on the problem, if they were not at one of the help sessions.

While the meaning of these results remains open for interpretation, we at least notice that there was no drastic change in students' responses across the two semesters. In any case, the more indicative and reliable data collected in this study are the Camtasia files and field notes. These provided us with the majority of information in this assignment.

6.4 Heuristics and theory

Let us now turn to the topics of which heuristics were used in this study, *how* the heuristics were implemented, and *why* they worked. The heuristics that were implemented, as well as a brief explanation of *where* these appeared in the assignment/activity, are shown below.

1. **Use the sim to mediate discussion**

   Though not mentioned previously, this heuristic certainly crawls its way out of the results presented. One especially important case where this happened was when

[9] These are *not* included in the appendix.

[10] We know this from photocopying most of the students' homework (at least for those who consented to this).

Alex and Bruce were working on part (5), which asked about the asymptotic behavior. During this period of time, Alex and Bruce were discussing what was shown on the sim, and in particular, the values for v_0 that were plugged in, along with the corresponding value for t_{top}. This likely allowed the students to stay on task, and provided a sort of "platform" for discussion, both while using the sim, and after, when trying to explain what was shown by graphing t_{top} vs. v_0.

Additionally, conversation was important in parts that did *not* use the sim. Especially noticeable was the advantage students found in discussing how to solve the differential equation in part a. (Note that students actually spent the vast majority of the total time on this part – somewhere around 50% or more of the total time.) This is not to say that students were using the sim to mediate discussion during this time, but rather, just to point out that discussion was clearly beneficial in other parts of the assignment, as well.

2. **Use the sim to coordinate other forms of representation**

   In this case, the sim served as a tool that helped "coordinate" other forms of representation. In particular, it provided a means for students to make sense of the mathematical solutions for $v(t)$ and t_{top}. Without using the sim, a likely possibility is that students would have written down the mathematical solutions for these quantities, and never given a second thought as to what they meant. With the sim, students *had* to play around and discover the asymptotic behavior. This visualization serves as a way to understand what the solution is saying, and this can then be related to the mathematical solution, thus providing a deeper sense of understanding for the phenomenon at hand. Additionally, students were asked to make plots in this assignment, and the incorporation of the sim also helped to coordinate that particular form of representation with the visualization on the sim itself.

3. **Set up game-like situations and challenges**

   One game-like situation or challenge was present in this assignment. Specifically, students were asked to determine, via experimentation, what initial velocity makes the ball reach the top of its trajectory in 3 seconds (in part c), then 4, 5, and 6 seconds (in part f). Like the "Build a molecule" study, this heuristic is useful in promoting engaged exploration of the sim. Of course, in this case, the game-like situation was not *implicit* in the simulation, but rather was *explicit* in the assignment itself. Both methods appear to be viable options, as well as tightly related (hence, they are included in the same category). However, it should be pointed out that in this case, it was difficult to get Bruce and Alex to engage in the challenge of part c, since these students wanted to move on to part d and come back to the sim later, thus decreasing the amount of time spent working on the assignment. On the other hand, no analogous occurrence presented itself in the build a molecule sim (at least from the videos watched here).

4. **Utilize illuminating cases**

   As was stated earlier, this heuristic is perhaps "inseparable" from setting up game-like situations and challenges. The reason for this in this assignment was that the illuminating case became most illuminating when students had to "discover" the phenomenon for themselves. Setting up a game-like situation therefore provides a means for allowing students to discover the illuminating case at issue. It was clear that the asymptotic behavior was non-intuitive for all of the students who showed up in the help sessions, and from Bruce and Alex's interactions, in particular, it appeared that "getting at" this illuminating case through the use of the explicit challenge, as phrased in the question, was a beneficial way of helping these students make these connections.

Let us now discuss *how* these heuristics were implemented. In the original problem, we were certainly prescriptive in implementing the "use sim to coordinate other forms of representation" *and* "utilize illuminating cases" heuristics. In both, we told the students exactly what to do, and from observations of students in the first semester, this appeared to limit their likelihood of partaking in engaged exploration.

In the modified problem, we adjusted the ways the heuristics were implemented via two means. First, we abandoned the prescriptive format of the questions. We did this by asking about the illuminating case *in conjunction* with a game-like situation or challenge. From observations of the students using the modified problem, it appeared that, for the most part, this *did* allow for productive use of the simulation, so long as students didn't skip ahead in attempts to finish the problem with the greatest efficiency.

Additionally, we changed the phrasing of the questions. Specifically, we tried to substitute brevity for verbosity. The reasons for this are not entirely unrelated to the reasons for abandoning the prescriptive format of the questions; in both, we were hoping to move students away from religiously following the instructions on the assignment as if it were a baking recipe, and instead shift their attention to engaging with the sim in an exploratory manner.

In answering *why* these heuristics worked (when implemented the second time around), we can again turn to the theoretical framing presented in chapter 4. Note that in this case, diSessa's theory of coordination classes is not entirely applicable. While coordination classes are perhaps present in the assignment itself (such as determining the forces acting on the ball), the problem overall involves concepts and ideas that cannot be explained by coordination class theory. Still, the more general notions of manifold ontologies and the importance of coordinating different "pieces of knowledge" or ideas appear to be critical in analyzing *why* these heuristics worked.

To gain a feel for this, consider the heuristic: "Use the sim to coordinate other forms of representation." Part of the reason this heuristic was implemented in this study was because this is a task that experts in the field of physics do on an everyday basis. When the expert physicist is confronted with a phenomenon, they have a variety of "tools" at their disposal for gaining an understanding of the physics that underlies that phenomenon. To the expert, the use of mathematics, graphs, visual aids, and so on, do not appear to be "separate" or unrelated ways of thinking about a particular issue. Instead, the phenomenon simply appears *as the phenomenon*, and is understood as a seemingly "abstracted" entity in itself.

Also recall that, historically, concepts have been thought of as unitary in nature. From the expert's point of view, this would only seem natural, given that their view of a particular phenomenon does not seem to be comprised of separate or unrelated components. Yet if we are to try to explain *how* experts achieved this understanding in the first place, the notion of concepts as being unitary in nature provides us with absolutely no insights; we can only say "you either have it or you don't."

This study seems to indicate that understanding a given phenomenon, such as the motion of a ball subject to quadratic air drag, is, in fact, "built" from a variety of diverse observations that are coordinated in different ways. The "deep understanding" is achieved when these various components all relate to each other in specific ways. With those connections made, we can expect that the various components of what we think of as a particular "phenomenon" will activate in sets, and that the particular activations that occur are dependent upon the context provided.

To elaborate on this, recall that, in this case, students first solved a differential equation, and then wrote down an equation for $v(t)$ and t_{top}. But at that point in time, it did not appear that students had any inclination to contemplate the physical *meaning* of those equations, nor did it appear that students were planning on plotting either of these quantities in order to gain a better "feel" for what the equations meant. With the addition of the simulation, however, students were given an opportunity to view the phenomenon via an entirely different medium. The visualization, of course, was not *separate* from the mathematics, but instead was meant to coordinate students' understanding of the mathematics with this physical picture. Additionally, the physical picture was coordinated with a plot of t_{top} vs. v_0 that students were also asked to create.

The ability to coordinate these various forms of representation is a critical skill for physicists working in the professional world. Yet getting students to *develop* those skills is a task that is very difficult to achieve. Coordinating various representations with the simulation is likely a step forward in helping students in this regard.

The ideal end result in creating an assignment like this would be for students to: (1) develop skills solving differential equations, (2) interpret the meaning of the solution to the differential equation, (3) gain a physical understanding or grasp of the phenomenon at hand, (4) be able to use various forms of representation to explain the phenomenon at hand, and (in this *particular case*) (5) explain in words what occurs when shooting a ball into the air subject to drag, and additionally, explain how that compares to the case of a ball falling. Obviously, there could be other hopeful outcomes, ranging from developing an appreciation of the phenomenon at hand to being able to transfer the information learned to other contexts. Regardless, these five seem to be among the most important.

In achieving these goals, we must acknowledge that student learning occurs within context, and that many factors *other than* the simulation or the assignments themselves influence what students take away from these assignments. In this general picture, the heuristics are meant to be one *component* of the overall activity system that students engage in. Their role is to help structure the activity and open opportunities for students to make the connections that are necessary for understanding a given phenomenon. The heuristics are certainly not an end in themselves.

In practice, a large variety of external factors (e.g. time constraints, such as the ones Alex and Bruce had) can influence how the assignment is used and what students learn. Thus, the purpose of the heuristics is not to guarantee that students use the assignment in a particular way, but rather, to suggest ways that have been shown, through empirical evidence, to promote the types of interactions that are known to be beneficial for learning physics. To make these as effective as possible, we feel the need to ground the heuristics in both theory and practice, and find a common ground between these.

Chapter 7

Modern Physics Tunneling Tutorial

The third and final study to present draws on data taken from a modern physics course at the University of Colorado. The class was comprised of approximately 150 students, and the students were primarily electrical engineering seniors or mechanical engineering sophomores. Because this course served as an introductory course, it was assumed that students had no previous experience with quantum mechanics before.

This chapter proceeds in the following way: first, we give an overview of the study and the environment students were situated in while working on these assignments. Next, the assignments themselves are discussed, along with an overview of the simulation used. Results and an analysis of those results follow, and finally, the heuristics drawn from the study, as well as a theoretical grounding of those heuristics, are presented at the end.

7.1 Environment and overview

This study took a random subset of the entire 150 students in the class (subset was 11 total), and looked at how those students used tutorials one day outside of the typical class time.[1] All 11 students met at one time in a small classroom in the physics building and were divided into four groups.

Two different tutorials were written, one using the quantum tunneling PhET sim[2] [32], and the other without a sim. Both assignments were created with the same basic structure, but different types of questions, with the exception of the introductions to the

[1] Students weren't chosen randomly, but anyone who desired to show up was free to do so. Notably, pizza was provided for those who came.

[2] For more information on the development of the tunneling sim, see [2]

tutorials, which were the same for both. Two groups of students used the PhET sim, one with 2 students and the other with 3 students. Each of these groups were sitting on one side of a table in front of a laptop computer, and the tables were separated by 6 or 7 feet. The remaining six students in the non-PhET group were situated at one large table, and though the students initially started as two separate groups, the students ended up collaborating together, and the distinction of "groups" became less obvious as time passed.

Three primary types of data were collected: field notes and observations of students' interactions, the completed assignments handed in by the students, and audio recordings of student conversations. Two audio recorders were used, one being placed on the table of the group of two students using the PhET sim, and the other placed on the table near one of the groups using the non-PhET tutorial. Dialogue was often heard across different groups in both audio recordings, and in total, three voices were present in the PhET group tutorial, while four voices were present in the non-PhET group tutorial.[3]

7.2 Assignments and sim

There were significant differences in the ways the two assignments were written, despite the fact that each assignment covered the same basic material. The underlying goal in both was to address the idea of a quantum particle (an electron) that encounters a step potential barrier, and analyze the cases of the particle having a greater energy than the potential barrier, or less energy than the potential barrier.[4] Mathematically, we could write this as:

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \text{ and } x > L \\ V_0 & \text{for } 0 \leq x \leq L \end{cases}$$

where L is the width of the potential barrier, and E (the total energy) satisfies either $E > V_0$ or $E < V_0$. Often the region of $x < 0$ will be referred to as region 1, the region under the potential barrier as region 2 and the region $x > L$ is referred to as region 3.

[3]One of the students in the PhET group of three students talked particularly loud and sometimes addressed the group of two using the PhET sim. In the non-PhET groups, two students were particularly quiet, thus only dialogue from the remaining four students was collected.

[4]Note that by "particle," I actually mean non-normalizable plane wave. There is, in fact, a "wave packet" mode on the sim, but we wanted to use the simpler case of plane wave. Pardon the sloppy language; "particle" will often refer to a non-normalizable plane wave in this chapter.

To introduce the students to the case of a quantum particle, each tutorial started with an identical section involving the case of a *classical* particle traveling over a potential barrier and back down, shown below.

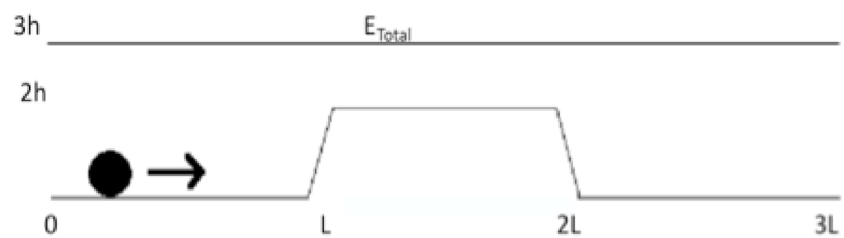


Figure 7.1: Both tutorials started by looking at the case of a classical particle, total energy $E = 3mgh$, rolling over a potential barrier of height $2h$.

This short part of the assignment was created to address the idea that the classical particle travels slower on the top of the ramp, and therefore, the likelihood of finding the particle in that region is greater than in the other two regions separately. After this, the tutorials diverge in the types of questions asked.

To get an idea for the differences in the two assignments, it is first necessary to describe the quantum tunneling PhET sim. The simulation shows 3 main windows on the interface, stacked vertically on top of each other. In addition, controls are present on the right side of the interface, allowing users to adjust various features, such as the direction of the incident wave, the parts of the solutions (real and imaginary) that are plotted, and so on. The window on the top of the simulation shows the potential barrier just described, and allows for students to adjust the total energy (E), the energy of the potential barrier (V_0), and the width of the barrier (L). The middle window shows a plot of the wave function changing in time, and the window below that shows a plot of the probability density for the particular setup that is chosen. The bottom two windows have no directly controllable features (other than a zoom in or out option), but both change simultaneously as the user adjusts the parameters shown in the top plot. A picture of the interface is shown below.

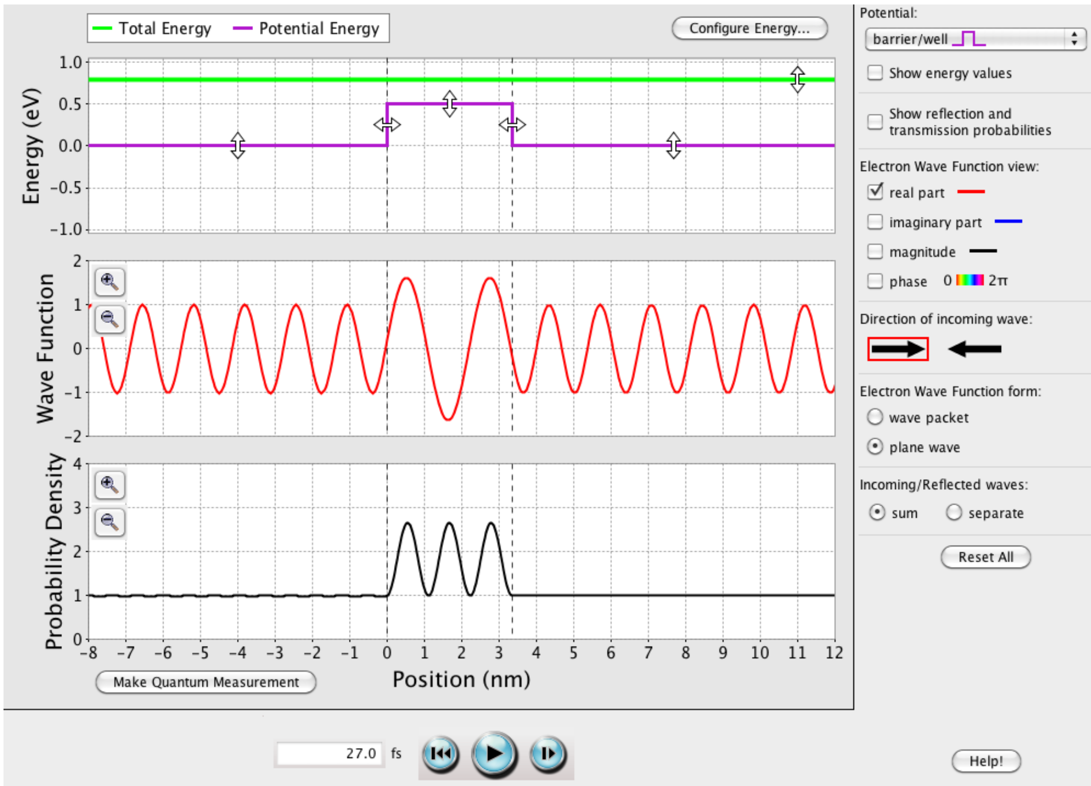


Figure 7.2: Interface of the tunneling PhET sim

After the introduction, the PhET tutorial proceeds by asking students to set up the sim with $E > V_0$, and to describe and analyze the wave function shown on the sim in the three regions. The non-PhET tutorial, on the other hand, proceeds by asking the students to solve the time-independent Schrödinger equation for the cases of $E > V_0$ and $E < V_0$, and to then use that information to eventually draw a picture of the wave function in each of the three regions. The PhET tutorial ends with the case of $E < V_0$, again asking questions primarily addressed at explaining certain features that are shown on the sim. More details about the types of questions asked in these two tutorials will be presented as we describe some of the data and results collected.

7.3 Results and analysis

After analyzing and transcribing the audio taken from the students' conversations, there was a clear indication that students using the non-PhET tutorial were *more* engaged in conversation than the group using the PhET tutorial. The four students in the non-PhET group who were recorded asked each other a total of 90 questions during the ensuing discussions, while the three students recorded on the PhET tutorial asked each other a total of 27 questions while engaged in discussions.

In addition, the *types* of questions asked by the students in the non-PhET group were radically different. In general, the non-PhET group asked questions concerning what the wave function should look like, and *why* it should look that way. In contrast, the PhET group primarily asked questions about what was shown on the interface window, without attempting to explain the underlying physics or mathematics that *would* explain why the images appear in as they do. A particularly good example of this is apparent towards the end of the tutorial when both groups are working on the case of $E < V_0$. This is the focus of the next section.

Issues concerning discourse

Recall that both tutorials began with a section on classical probabilities, and then moved on to the quantum case of $E > V_0$. The purpose of writing the assignment this way was to have the students notice the similarities between the classical and quantum cases. They are similar in that, in both cases, the kinetic energy in regions 1 and 3 are the same, and the probability density is, in general, less in regions 1 and 3 than in region 2 for both the quantum and classical cases.[5] Both assignments led students to notice these similarities through a series of questions directed towards that purpose.

However, this classical analogy completely breaks down in the case of $E < V_0$, and students can no longer use the reasoning that the amount of kinetic energy directly determines the probability of finding a particle in a particular region in space. The non-PhET group confronted this difficulty head-on when attempting to finish the sketches of the wave function for the case of $E < V_0$ in region 3. Their dilemma is demonstrated in the following dialogue:

S4: "The reasoning we used before, at least I did, was that because the velocities

[5]This is not, strictly speaking, always a true statement, since to fully understand the probability density of the quantum particle, it is necessary to calculate transmission and reflection coefficients. However, this is not a learning goal for this sophomore-level class, and we felt that the conceptual idea of a quantum particle moving "slower" in the potential barrier, thus leading to a greater probability density, was worthwhile since it bears similarity to the case of a classical particle. In the latest version of this tutorial (intended for an upper division course), we abandon asking about the probability in the potential barrier since this bears little physical significance in actual tunneling applications.

were slower in the (potential barrier), then it had a higher probability of being found there. So if they're equal, they should have an equal probability and their amplitudes should be equal, right?"

S3: "Right, well that makes sense in terms of equations, but like he said, I'm not sure you can think of it in a classical way, like $\frac{1}{2}mv^2$*."*

S4: "I know, I know that (lower amplitude in region 3) is what it should be, but I want to be able to prove it to myself."

This type of reasoning is precisely what we wanted the students to go through when trying to generate a picture of the wave function for this particular scenario. From prior knowledge, the students knew that the wave function should have a lower amplitude after tunneling occurs, but the tutorial made them confront the question of *why*.

In the analogous part of the PhET tutorial, no similar reasoning was shown. This was largely due to the fact that the students already had the wave function plotted in front of them, and were never asked to attempt to generate a plot on their own. The questions these students generated when confronted with the questions that asked about what the wave function looks like in the case of $E < V_0$ are typical of the types of questions the students tended to ask throughout the entire tutorial. Referring to the wave function in region 3, the students said:

S1: "That one is still sine right? Like if you decrease it is it still sine or is it always zero?"

S2: "Well this is technically a sine wave."

Again, these questions are primarily about what is shown on the interface itself, without ever trying to understand the underlying reasons for seeing what is shown.[6]

At this point, it becomes interesting to think about the *reasons* for the vast differences students' discourse. When designing these questions, we specifically tried to use one of the heuristics we thought would be important: "Call attention to visual features shown on the sim." Asking the students what the wave function looks like falls under this category, and yet, in this case, it was clearly not effective in getting the students to think about the underlying physics of the situation.

[6] It is possible in this example that S2 understands why the wave function is a sine wave in region 3, but they never discuss this (largely because the tutorial doesn't ask them to). Either way, it is clear that the tutorial is not helping to promote discussion around why the function looks the way it does.

We argued in chapter 3 that this heuristic should be used sparingly and only when a particular visual feature of the sim might be difficult to notice or otherwise glanced over, and the results of this assignment suggest the same. The fundamental reason for this is that asking students to notice a visual feature is different than asking about what causes that feature, or better yet, asking them to generate that feature, e.g. by drawing a picture, which would force students to coordinate other forms of knowledge and think deeply about what information is relevant for doing so.

In general, our findings indicate that questions about what the sim shows promote little discussion, whereas questions about what something will look like (be it a plot or sequence of events or something else shown on the sim), or why the sim exhibits a certain feature, are more likely to promote thought, discussion, and sense-making.

If questions that ask students to predict or explain what something will look like are important, we must understand what the important pieces of knowledge needed to make those predictions and explanations are. In this case, it was clear that the non-PhET tutorial succeeded in this when the students successfully made graphs of the wave function. Therefore, analyzing the questions leading up to the students' completion of this task should give insight into *which* pieces of knowledge were necessary for accomplishing this task.

The non-PhET tutorial first asked the students to write down general solutions to the Schrödinger equation in each of the three regions for both the cases of $E > V_0$ and $E < V_0$. Students then had to use that information, combined with subsequent questions about what the wavelength and amplitude of the wave function should look like in each region, to generate (draw) a graph of the wave function.

In contrast, the PhET tutorial never addressed solving the Schrödinger equation at all. The initial idea was that the same type of conceptual grasp (larger wavelength, less KE, higher amplitude) would be accessible in using the PhET sim, without the need for solving equations. While it is possible that students picked up on this conceptual idea, a fundamental gap in students' knowledge and reasoning about the wave function in the different regions was clearly present (not once in the tutorial did the PhET students refer to the Schrödinger equation).

A reasonable conclusion to draw about these differences in discourse is that, because the PhET group was never asked to think about the mathematics underlying the features on the sim, those students' discussions never had the potential to explore the "reasons" for what was shown on the sim. In addition, since these students were never asked to plot the wave function, they never had to coordinate the various pieces of knowledge that are necessary for completing such a task. This suggests two possible ways we ought to write assignments to use simulations.

First, the sim might be used as a way to compare and contrast features that the students were asked to generate without using the sim. In this example, we could have asked the students in the PhET group to generate a plot of the wave function and then compare it with what is shown on the simulation. In this way, the information needed to reason about the underlying physics would already have been accessed, and the sim would then serve an entirely different purpose than it did in the case presented.[7]

Secondly, these findings suggest that the sim be used as a way to coordinate different forms of representation. Here, the sim could serve as a way to coordinate the knowledge of the mathematical solutions with the representation of that solution shown on the graph in the sim. Both of these findings are, of course, heuristics, and will be summarized later in this chapter. Additionally, this tutorial has already been revised to incorporate these heuristics. This will also be mentioned briefly at the end of the chapter.

Issues concerning guidance

An additional issue with the PhET tutorial not yet described concerns the amount of guidance given to the students. Each of the cases of $E > V_0$ and $E < V_0$ started with a question asking the students to set up the sim in a particular manner. For example, the first question in using the sim for $E < V_0$ was:

> **Now, using the PhET sim, decrease the size of the wire gap to 1 dashed-line wide and increase the height of the potential energy line all the way to the top. What type of function do you see in region 1 and 3 (e.g. sinusoidal, exponential, linear, quadratic, etc.)?**

There were several negative effects of asking questions like this, as can be noticed in both the audio recordings and the field notes taken while observing the students. First, this prevented students from playing with the sim. At one point a student asked me to adjust the sim for her because she didn't know how it was supposed to be set up. This led to little "interactive engagement" while using the sim, and didn't allow for a student sense

[7] This provides evidence for the vast amount of context-dependence underlying the ways sims are used.

of "ownership" to form.[8] Both of these are described in existing literature as essential elements for effective use of simulations.

Second, this caused the students to wait for the tutorial to provide instructions on what to do next. Often times the term "cookbook lab" is attributed to labs that exhibit some of the same properties. These are notorious for being solely task-oriented activities that provide little to no guidance in concept formation.

Third, the level of guidance present set limitations on student conversations. Because the tutorial told them how to set up the sim in certain ways, there was no debate about how the sim could or should be used. No discussion was centered around ways of manipulating the sim in a way that would provide insights into the underlying physical concepts, and this prevented conversations about those physical concepts from forming.

In retrospect, it is easy to wonder why we wrote the tutorial in this particular way. As it turns out, the main reason for telling the students how to set up the sim was based on our "interpretation" of a heuristic that we felt would be important: "Set up the sim to look at illuminating cases." As has already been shown in this thesis, illuminating cases can be used as a means for providing unique insights into underlying physics concepts, and in this study, we implemented this by *telling* the students about these particular cases. In the question taken from the assignment written above, we ask the students to set up the sim with only one dashed-line of width because this is a case in which tunneling occurs, and some of the wave function leaks in to region 3 (if the barrier is too wide, none of the wave function leaks over).

The problem with writing the question this way is that it took away from other heuristics that were important. For instance, the students felt unable to play or engage in game-like situations with the sim because of the way the question was presented. We still think that the heuristic of looking at illuminating cases can be useful, but crucial to its success is the way it is implemented. This study provides yet more evidence that this heuristic will be most effective if it is implemented *in conjunction* with a game-like situation or challenge. For instance, in the latest version of the tutorial, we have rephrased the question regarding the illuminating case of tunneling to say: "How can you maximize the amount of transmission to region 3?" In this way, students may come to understand an illuminating case through a game-like situation.

[8] "Ownership" is a term frequently used by the PhET team to describe how comfortable students feel while using a simulation – when comfortable, they feel like they "own" the sim, in a certain sense.

7.4 Heuristics and theory

Although this chapter has pointed out several deficiencies with the PhET tunneling tutorial, we in no way intend to communicate the message that sims are "bad." Rather, we view this tutorial as evidence of the complexity of the challenges present when writing assignments to use simulations. This complexity stems from the features of the sim that influence the ways students interact with the content, the environmental or situational aspects that influence the students' use of the sim, and the nature of the assignment given to the students that influences both the environment and the students' interactions with the simulation. Before discussing the theoretical underpinnings that elaborate on this view, we summarize the heuristics drawn from this particular case study.[9]

1. **Set up situations that utilize 'Predict, Observe, Explain' or 'Elicit, confront, resolve' models**

   These models, described in existing literature [44, 28], use the idea of asking the students to think about a particular phenomenon through some sort of prediction, and then compare their predictions with the actual answers in order to gain perspective on the underlying concepts. In the non-PhET case, these models were implemented by asking students to draw graphs of the wave function. However, the 'observe and explain' stages or the 'confront and resolve' stages were not present. These could be integrated in to the assignment by asking them to compare what they drew with what is shown on the sim.

2. **Use the sim to coordinate other forms of representation**

   We again find that this heuristic appears, as it seems that the sim should be used as a means to coordinate different representations of the same physical phenomena. An example is the coordination of mathematics and the plot of the wave function in the tunneling sim. Both the plot and the mathematics describe the same physical situation, but each allows for different ways of looking at that phenomena. This also helps to satisfy a skill necessary for all professional physicists: to be able to effectively utilize many forms of representation.

3. **Utilize illuminating cases**

[9] Additionally, to see the implementation of these heuristics in the creation of an *upper division* quantum tunneling tutorial, see the very end of the appendix.

Since often times particular scenarios in physics give unique insight in to the underlying physical concepts involved, this heuristic can often be implemented for that purpose. The way in which this heuristic is implemented is crucial to its success, and it is likely that combining this heuristic with a game-like situation is an effective mode of implementation. An example was briefly described above.

4. **Set up game-like situations or challenges**

   This heuristic seems to present itself in nearly all situations, and using it promotes effective use of the simulation through play, interactive engagement, creating a sense of ownership, and possibly in mediating discussions. This was also described in the build-a-molecule example earlier in the paper.

5. **Use sim to mediate discussion**

   Once again, this heuristic seems to be important, if implemented effectively. Though the types of discussion around the simulation, in this case, were not what we wished would have happened, the potential for improving this through the incorporation of other heuristics (e.g. setting up game-like situations or challenges and incorporating predict, observe, and explain stages) is obviously there. In order to implement this more effectively, we might look to other studies for guidance in how to do so.

We again turn to an evaluation of *how* these heuristics should be implemented, and *why* they should work. We begin with the use of 'predict, observe, explain' and 'elicit, confront, resolve' models. As was shown through a comparison of the tutorials presented, students using the non-PhET tutorial, who were asked to generate a wave function on their own, were forced to think carefully about what the wave function should look like in the three different regions. By "thinking carefully," we mean that students had to combine diverse pieces of information or *coordinate* knowledge in particular ways to achieve a common goal. This proved to be a difficult task, and one that inspired much conversation and discussion. Thus, in this *prediction* or *elicit* stage, students were, in fact, coordinating different forms of representation (specifically mathematics and graphs) *and* were using the these forms of representation as a mediator in their discussions.

For the non-PhET group, this proved to be incredibly useful. Yet it appeared that this heuristic could potentially be fully implemented by introducing the sim after the *predict* phase. That is to say that the 'confront, resolve' or 'observe, explain' stages could be incorporated through the use of the simulation, in which, students could observe the dynamic plot of the wave function on the simulation and explain the differences between their predictions and that plot. In fact, the most recent version of this tutorial (intended for upper division quantum mechanics) utilizes this precise strategy. It can be found at the end of the appendix.

By using POE or ECR models, the sim can simultaneously incorporate some of the other heuristics listed above. For instance, in confronting and resolving the differences between the sim and their plots, students would likely be coordinating other forms of representation, engaging in explicit challenges, and discussing features of the simulation.

Of course, all of these heuristics *should be* implemented in ways such that they enable students to coordinate multiple pieces of knowledge. Like the projectile motion study, solving mathematical equations is likely not, in itself, a means *and an end* in getting students to develop a deep understanding of the phenomenon at hand. But with the incorporation of plots, visual features on the sim, etc. (i.e. other forms of representation), the likelihood that students will make the coordinations necessary for developing a deep understanding of this phenomenon vastly increases.

Along those same lines, we now know that the likelihood of students interacting with the sim and the other resources available to them in a productive way drastically increases when the phrasing of the questions is left open, and not prescriptive or closed. Thus, some of the findings from the previous two studies mentioned should be considered as potentially applicable in this study, as well.

At this point, we hope that *how* the heuristics ought to be implemented and *why* they ought to work is clear to at least some extent. The fact is that there is no "clear-cut" strategy that one can use for every assignment to be written. For this reason, we emphasize that these are *heuristics*, and that the context of the situation will *always* shape the ways students use the assignments developed with these heuristics. Nonetheless, the heuristics do shape the ways the assignments are written, and thus provide a very "controllable" means for shaping educational environments that are notoriously difficult to "control."

Chapter 8

Conclusion

Integrating simulations into physics assignments is clearly a complex task. Variables such as the type of simulation, the particular students, the environment students are situated in, and so on, all factor in to how students use the simulation, and what they take away from their interactions with it. The goal of this project was to provide insights into how to go about creating assignments to promote effective use of simulations in physics learning.

We began work on this project by creating a tentative initial list of heuristics based on theoretical framing and literature review that we thought would be important when integrating simulations into assignments. Through a series of case studies where we created, implemented, and evaluated the use of simulations in learning environments, we were able to refine that list into its current form:

1. **Set up game-like situations and/or take advantage of explicit and implicit challenges**

   This heuristic is primarily meant to help students engage with the simulation. The idea is that with a game-like situation or challenge present, students must attempt to achieve some goal, and in doing so, must explore various features of the simulation that allow them to "see" the physics (or other rules) present in the simulation. Examples from this thesis are: filling the boxes in the "Build a molecule" simulation and making the baseball reach the top of its trajectory in a set amount of time in the "Projectile motion" simulation. In both of these cases, students knew the goal that they had to complete, and through using the simulation, were able to come to realize either the rules of how to create molecules or the asymptotic behavior in t_{top}.

2. **Ask about visual features on the sim**

   We include this heuristic as a type of "safe guard" for features of the sim that might otherwise go unnoticed. Though it is not meant to be used exhaustively, we feel that there are cases in which explicitly addressing a feature on the simulation can be helpful. In the case of the "Build a molecule" study, a feature that would have otherwise gone unnoticed was knowing when a molecule was actually created. Because not all conglomerations of atoms are molecules with formal names, students needed a push in the right direction to realize this.

3. **Utilize illuminating cases**

   Often times, while using a sim, there is some action that can be performed that is particularly interesting or "illuminating" in displaying the underlying physics in the simulation. In the case of the "Projectile motion" study, the illuminating case was the asymptotic behavior in the time the ball takes to reach the top of its trajectory. In the case of tunneling, a particularly illuminating set up is when the reflected wave is eliminated, and 100% transmission occurs. In writing assignments, we can try to take advantage of these illuminating cases, and ask about the underlying physics of what is shown on the interface. Currently, we suspect that illuminating features should be presented through the use of a game-like situation or challenge, because in doing so, students can come to "discover" the illuminating case for themselves.

4. **Ask students to recreate or re-present visual features on the sim**

   We include this heuristic because of the results obtained from the "Build a molecule" study. In it, students had to draw molecules, and in doing so, were forced to look more carefully at the representations, which we suspect helped them to internalize what those molecules looked like. Obviously, more research is needed to understand exactly what the effects of asking students to re-present different features on the simulation are, but for now, this remains yet another option for how assignments can be written.

5. **Use the sim to coordinate other forms of representation**

   This heuristic is meant to stress the importance of relating what is shown on the simulation with the mathematics, graphs, etc. that are used in more traditional physics problems. With the visualizations on the simulation, students are given yet another way of "looking at" the physics of a particular problem. Because being able to use and understand multiple forms of representations is a skill important for professional physicists, we feel that explicitly addressing this in assignments is necessary.

6. **Ask for comparative analysis when possible**

   Though only described briefly in chapters 2 and 3 of this thesis, we keep this heuristic as yet another option. The idea came from Schwartz et. al. [28] from the study concerning the use of the electromagnetic induction simulation. The general idea was that asking students to compare different scenarios on the simulation will help to direct their attention to the differences and similarities of those cases, and in doing so, emphasize the physical principles that explain why those differences and similarities exist.

7. **Use the sim to mediate discussion**

   Originally based on the paper by Otero [25], this heuristic is meant to emphasize that simulations can be used as a platform for discussion. The visual and dynamic features can allow students to discuss why they are seeing what is shown on the sim, opening up possibilities for discussions that would otherwise not take place. The simulation is a mediator in the sense that it provides a common reference for discussions that revolve around what is shown.

8. **Set up situations that utilize 'predict, observe, explain' or 'elicit, confront, resolve' models**

   The ECR model in writing tutorials is one that was originally developed by the University of Washington [44]. The POE model is similar enough that we include it in the same category. The general idea is that prior to using the simulation, we can ask students to make a prediction for what will be shown, and then ask them to use the sim to compare and explain the differences between their predictions and what is shown on the simulation. The tunneling tutorial gave us reasons for thinking that this heuristic might be particularly useful, due to the fact that the nonPhET group reaped benefits from predicting what the wave function should look like, while the PhET group did not reap those same benefits. Thus, by including a predict phase, and then using the simulation to compare the differences, we feel that the advantages of both the sim and the nonPhET tutorial can be incorporated.

9. **Use the sim to relate formal concepts to the real world**

   Though only discussed briefly in chapter 2, we keep this heuristic as yet another option. It is meant to address the common finding that students see physics concepts as not applying to the real world. Because some simulations show real-life representations, we feel that it may be possible to help relate formal concepts to the real world through the use of these representations. Obviously, more research needs to be done to understand exactly how to implement this heuristic, and exactly what its effects are.

10. **Use the sim to design a virtual experiment**

    Again, this heuristic remains another tentative strategy for how we can write assignments. The idea is that, given enough time, students may be able to use the sim to design and carry out an experiment in which they make inferences and attempt to draw conclusions from the data they collect. Again, more work is needed to understand exactly how to implement this heuristic, and obviously, none of the studies presented in this thesis used this in the creation of the assignments. Nonetheless, we leave this as a possible option open for future study.

In addition to this list, we created a framework that emphasizes the importance of accounting for both the features of the simulations and the environment when going about creating an assignment that uses a simulation. This framework, represented in the figure below, stresses that the simulation has some features that will affect both the environment and the way the simulation is written. Additionally, the environment will influence how the assignment is written, and in turn, the assignment will affect the environment. The key point is that the heuristics can be seen as being implemented in the arrows shown in the figure below. That is, based on certain features of the sim, we can pick heuristics that will allow us to write the assignment to take advantage of those features, and based on the environment students are situated in, we can take advantage of other heuristics in creating these assignments.

Obviously, the use of this framework can be elaborated upon as more more case studies are conducted, and ideally, we will eventually be able to start to point out exactly which heuristics should be implemented in the directional arrows for particular features of the simulation and different environments.

With both the list of heuristics and this framework, we felt that there was an additional need for explaining *how* to implement the heuristics, and *why* they work. To answer these

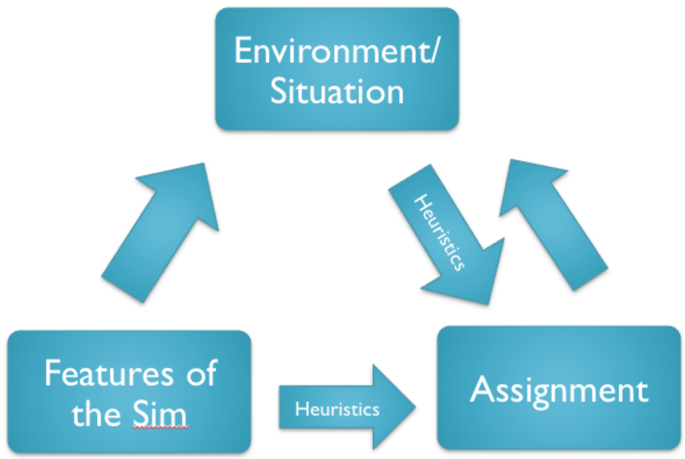


Figure 8.1: Framework for implementing heuristics

questions, we turned to both our case studies and more theoretical framing. In particular, we looked at coordination class theory and a generalization of that theory, which was referred to as a manifold ontology of mind. The central premise in both was that students' thinking cannot be effectively characterized by robust, unitary concepts that exist inside the head, but rather, that students have a variety of different resources available to them that, taken together, shape the ways they think about different problems.

It should be emphasized that the theoretical views presented are still tentative, and that more data should be collected to begin to refine and analyze the use of these heuristics in more detail. Nonetheless, we feel that this provides a decent starting place for what a solution to writing assignments effectively might look like. Future work will consist of more case studies and attempts to further characterize how to implement the heuristics. With more data, we should be able to draw more conclusions for how assignments that incorporate sims ought to be written.

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Appendix

Note that materials in this appendix are presented in the same order as in the thesis.

Build a molecule

Projectile motion

Quantum Tunneling Tutorials

Name: ________________________ 5___

Build a Molecule Computer Sim

Learning Goals:

1. Describe the difference between a molecule name and chemical formula.
2. Distinguish between the coefficient and subscript in a chemical formula.
3. Use pictorial representations of molecules to generate chemical formulas.

First Tab

1. Make a molecule:
   a. How do you know you made a molecule? ____
   b. Write the **name** of some molecules you made (ex. Water).

   ______________________________ ______________________________
   ______________________________ ______________________________
   ______________________________ ______________________________

2. Molecule Names and Chemical Formulas:
   a. Compare the name and chemical formula for some molecules:

Molecule Name	Drawing	Chemical Formula

Second Tab

3. Make Many
   a. Fill all the collection boxes and then complete the questions for each Goal.

Goal: $4H_2$	
Draw it!	
What does the big '4' in $4H_2$ mean?	
What does the little '$_2$' in $4H_2$ mean?	

Goal: $2CO_2$	
Draw it!	
What does the big '2' in $2CO_2$ mean?	
What does the little '$_2$' in $2CO_2$ mean?	

Goal: $2O_2$	
Draw it!	
What does the big '2' in $2O_2$ mean?	
What does the little '$_2$' in $2O_2$ mean?	

Goal: $2NH_3$	
Draw it!	
What does the big '2' in $2NH_3$ mean?	
What does the little '$_3$' in $2NH_3$ mean?	

Third Tab Challenge

4. What's the biggest molecule you can make?
   a. Molecule Name: ______________________________
   b. Chemical formula: ____________________________

5. Can you make a molecule that can be broken into smaller molecules?
   a. Big molecule **name**: ____________________________
   b. Big molecule **chemical formula**: ___________________
   c. Smaller molecule **names**: _________________________
   d. Smaller molecule **chemical formulas**: _________________

Build a Molecule Pre-Lab (Statistics)

1. We use symbols to represent atoms.

a. What is the chemical symbol for the atom Hydrogen?________
b. What is the chemical symbol for atom Oxygen?__________
c. What is the chemical symbol for the atom Carbon?__________

		N	N %
1a	Correct	43	74.1%
	Incorrect	12	20.7%
	N/A	3	5.2%
1b	Correct	44	75.9%
	Incorrect	12	20.7%
	N/A	2	3.4%
1c	Correct	39	67.2%
	Incorrect	17	29.3%
	N/A	2	3.4%

About 3/4 of the students could write the correct chemical symbols from the element names.

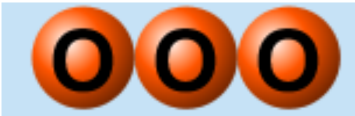
2. We use chemical formulas to represent individual molecules and groups of molecules. Write the chemical formula below each molecule or groups of molecules

a. 

		N	N %
2a	Correct	33	57%
	Name, not Symbols	14	24%
	other	10	17%
	N/A	1	2%

b. 

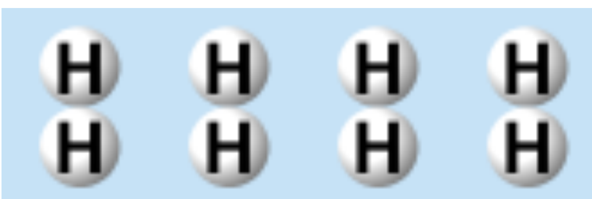
		N	N %
2b	Correct	37	64%
	Name, not Symbols	14	24%
	other	7	12%
	N/A	0	0%

c. 

		N	N %
2c	Correct	29	50%
	Name, not Symbols	0	0%
	other	24	41%
	N/A	5	9%

d. 

		N	N %
2d	Correct	31	53%
	Name, not Symbols	24	41%
	other	2	3%
	N/A	1	2%

e. 

		N	N %
2e	Correct	0	0%
	Name, not Symbols	15	26%
	H8, all or part	28	48%
	Other	10	17%
	N/A	5	9%

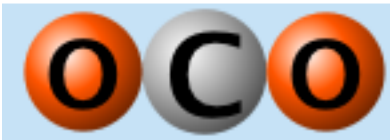
3. Try it!

a. Draw CO_2	
b. Draw $2H_2O$	
c. Draw $3N_2$	

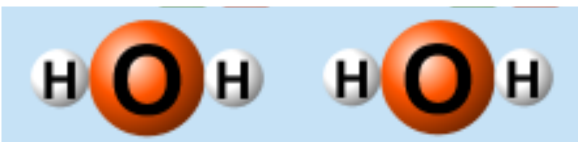
		Count	Column N %
Pre3a	2 O, 1 C	43	75%
	Other	7	12%
	N/A	7	12%
Pre3b	2 separate H2O	14	24%
	2 H, 2 O	17	29%
	Other	21	36%
	N/A	6	10%
Pre3c	3 separate N2	10	17%
	3 N Together	18	31%
	other	18	31%
	N/A	12	21%

Build a Molecule Post-Lab (Statistiscs)

1. We use chemical formulas to represent individual molecules and groups of molecules. Write the chemical formula below each molecule or groups of molecules.

a. 

		N	N %
1a	Correct	50	88%
	Name, not Symbol	5	9%
	Other	1	2%
	N/A	1	2%

b. 

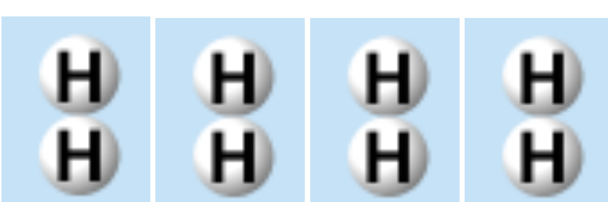
		N	N %
1b	Correct	38	67%
	Name, not Symbol	4	7%
	Molecular Formula, no coefficient	3	5%
	other	9	16%
	N/A	3	5%

c. 

		N	N %
1c	Correct	36	63%
	Name, not Symbol	4	7%
	Molecular Formula, no coefficient	5	9%
	O6 as all or part of answer	7	12%
	Other	5	9%
	N/A	0	0%

d. 

		N	N %
1d	Correct	40	70%
	Name, not Symbol	2	4%
	Molecular Formula, no coefficient	1	2%
	N6 as all or part	9	16%
	Other	4	7%
	N/A	1	2%

e. 

		N	N %
1e	Correct	36	63%
	Name, not Symbol	3	5%
	Molecular Formula, no coefficient	4	7%
	H8 as all or part	11	19%
	Other	3	5%
	N/A	0	0%

2. Try it!

a. Draw $2CO_2$	
b. Draw $3H_2O$	
c. Draw $4N_2$	
d. Draw $2NH_3$	

		N	N %
2a	2 separate CO2	43	74%
	1 CO2	3	5%
	2C, 2O together	3	5%
	Other	9	16%
2b	3 separate H2O	39	67%
	Separate H2O	8	14%
	3H, 2O together	7	12%
	Other	3	5%
	N/A	1	2%
2c	4 separate N2	45	78%
	Separate N2	5	9%
	4 N together	4	7%
	Other	3	5%
	N/A	1	2%
2d	2 separate NH3	39	67%
	Separate NH3	2	3%
	2 N, 3 H together	5	9%
	Other	10	17%
	N/A	2	3%

3. Molecule Names vs. Chemical Formulas

a. Give an example of a molecule name: ___________________________

b. Give an example of a chemical formula: _________________________

c. What is the difference between a molecule name and a chemical formula? _________________________

____

		N	N %
3a	Correct	46	81%
	Incorrect	11	19%
	N/A	0	0%
3b	Correct	49	86%
	Incorrect	8	14%
	N/A	0	0%
3c	Correct	19	33%
	Formula Correct	9	16%
	Name Correct	1	2%
	Other	28	49%
	N/A	0	0%

PhET sim problem for ball being shot up in the air.

1. In your textbook (section 2.4), Taylor solves for the case of a baseball being dropped from a high tower subject to quadratic air resistance, $F_D = -cv^2\hat{v}$. Let's look at the case of a ball being shot *up* at an initial speed v_0.

   (a) Draw a free body diagram for a ball moving vertically upwards, subject to quadratic air drag. Write down a differential equation for this situation and solve this differential equation for $v(t)$. Make a rough sketch of $v(t)$ vs. t, and briefly discuss any key features.

   (b) Using your result from part a, find an expression for the time it takes to reach the top of the trajectory. (It will look simpler if you write it in terms of terminal velocity, which satisfies $v_t^2 = mg/c$.)

   (c) Now download the PhET simulation at: `http://phet.colorado.edu/en/simulation/projectile-motion`. On the top right, switch the object to baseball. This sim uses quadratic drag: $F_D = -\frac{1}{2}c_0 A\rho_{air}v^2\hat{v}$, where c_0 is the drag coefficient, A is the cross-sectional area of the object being shot, and ρ_{air} is the denstiy of air $= 1.3 \quad kg/m^3$. **(The sim shows you the value of c_0 and diameter it has picked for a baseball, on the right side of the simualtion). Use your formula fin part b for "time to top" with these numbers to deduce what numerical initial velocity v_0 you need to get the ball to reach the top of its trajectory at precisely t=3 sec. Now test it, you can input v_0 into the sim and fire the cannon. Aim the cannon at 90° (or 88-89° if it is easier to see the trajectory) and switch on air resistance. The little + and - glasses let you zoom in or out. Does the ball reach the top at t=3 sec? (It should!)**

   (d) When you fired the ball on the PhET sim, did it take longer for the ball to go from the ground to the top of the trajectory or from the top of the trajectory to the ground? Explain why this is the case.

   (e) Now let's look at another interesting feature of shooting an object up in the air. Start increasing the value of v_0 in the sim. Double it from what you had before, then increase it by 10, and then by 100. What is happening to the time to reach the top? Use your formal mathematical results from above to explain what is happening!

   (f) Play with the PhET sim a little more and explore anything you are interested in. Write down one question that you have about something you notice when playing with the sim.

PhET sim problem for ball being shot up in the air.

1. In your textbook (section 2.4), Taylor solves for the case of a baseball being dropped from a high tower subject to quadratic air resistance, $F_D = -cv^2\hat{v}$. Let's look at the case of a ball being shot *up* at an initial speed v_0.

   (a) Draw a free body diagram for a ball moving vertically upwards, subject to quadratic air drag. Write down a differential equation for this situation and solve this differential equation for $v(t)$. Make a rough sketch of $v(t)$ vs. t, and briefly discuss any key features.

   (b) Using your result from part a, find an expression for the time it takes to reach the top of the trajectory. (It will look simpler if you write it in terms of terminal velocity, which satisfies $v_t^2 = mg/c$.)

   (c) Now download the PhET simulation at: `http://phet.colorado.edu/en/simulation/projectile-motion`. On the top right, switch the object to baseball. This sim uses quadratic drag: $F_D = -\frac{1}{2}c_0 A\rho_{air}v^2\hat{v}$, where c_0 is the drag coefficient, A is the cross-sectional area of the object being shot, and ρ_{air} is the denstiy of air $= 1.3 \quad kg/m^3$. (The sim shows you the value of c_0.) **By experimenting with the sim, what initial velocity makes the ball reach the top at approximately 3 sec?**

   (d) Now use your formula in part b for "time to top" to deduce what numerical initial velocity, v_0, you need to get the ball to reach the top of its trajectory at precisely $t = 3$ sec. How does your calculated value compare to your "experimental" value?

   (e) When playing with the PhET sim, does it seem to take longer for a ball to go from the ground to the top of a trajectory or from the top of the trajectory to the ground? Explain why this is the case.

   (f) Again, playing with the sim, write down the initial velocity that makes the ball reach the top of its trajectory at 4 sec, then 5 sec, then 6 sec, and so on. What do you notice happening? Make a plot of t_{top} vs. v_0 and explain in words how this relates to what you see on the sim.

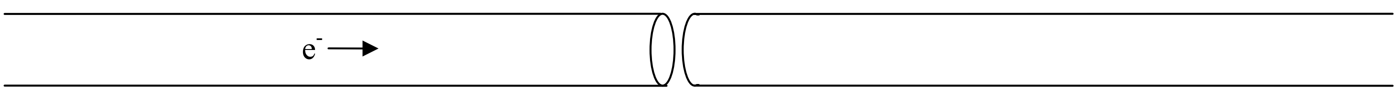
   Extra Credit: Play with the PhET sim a little more and explore anything you are interested in. Write down one question that you have about something you notice when playing with the sim.

Name____________________Student ID____________________

Tutorial: Quantum Tunneling

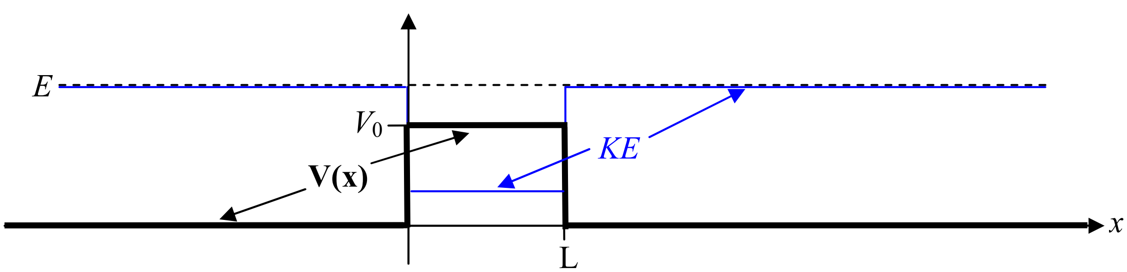
In this tutorial you will explore the physics of an electron traveling through an air gap in a wire – first in the case in which the electron has enough energy to get through the gap classically, and then in the case in which it does not. If you're paying attention, you should be surprised by some of the results.

Consider an electron initially moving to the right through a very long smooth copper wire with a small air gap in the middle. (See figure below.) The work function of copper is V_0.



PART I: $E > V_0$: Suppose the electron shown above has an initial energy $E > V_0$.

1. In the space below, sketch a graph of the potential energy V of the electron as a function of horizontal position x. Define $V = 0$ inside the wire. Once you have $V(x)$ sketched, use a dashed line to show the energy of an electron that satisfies the $E > V_0$ condition.



2. For the region in the copper wire to the left of the air gap, write down the general solution for $\Psi(x,t)$. Plug it into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

The general solution is: $\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$

Plugging into the Schrodinger Equation gives: $(-\hbar^2/2m)(-k^2)\Psi(x,t) + 0 = i\hbar(-iE/\hbar)\Psi(x,t)$

This simplifies to: $\hbar^2 k^2/2m = E$

For the region in the air gap, write down the general solution for $\Psi(x,t)$. Is the value of k here the same as the value of k in the previous region? Why or why not? If not, call it k' to distinguish it from the k above. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

The general solution is: $\Psi(x,t) = (Ae^{ik'x} + Be^{-ik'x})e^{-iEt/\hbar}$

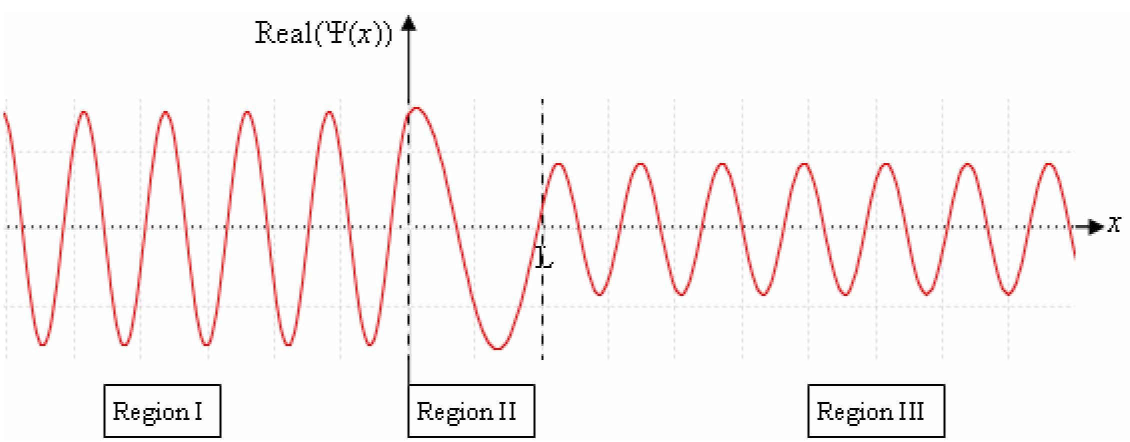
Plugging into the Schrodinger Equation gives: $(-\hbar^2/2m)(-k'^2)\Psi + V_0\Psi = i\hbar(-iE/\hbar)\Psi$

This simplifies to: $\hbar^2 k'^2/2m + V_0 = E$. The k here is not the same as the k above because it is related to the total energy E in a different way than the k above. Another way to say this is that the kinetic energy in the gap is different than the kinetic energy in the wire.

For the region in the copper wire to the right of the air gap, write down the general solution for $\Psi(x,t)$. Is the value of k here the same as either of the values of k above? Why or why not? If not, call it k'' to distinguish it from the k's above. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

Because the potential here is the same as in the left wire, the solution is the same, the k is the same, and the energy is the same: $\hbar^2 k^2/2m = E$

3. In the plot below, sketch the shape of the real part of the wave function at $t = 0$ in each region for the case where $E > V_0$. The air gap starts at $x = 0$ and ends at $x = $ L. Don't worry about the relative magnitudes of the waves in the different regions, but think carefully about the general shape of the graph in each region.



4. What is the basic shape of the real part of the wave function in each of the three regions? For example, is it linear, constant, quadratic, exponential, sinusoidal, or something else?

It's sinusoidal in all three regions.

5. Is the total energy of the electron to the right of the air gap *greater than*, *less than*, or *equal to* the energy of the electron to the left of the air gap? Explain how you arrived at your answer.

Equal to. Energy must be conserved, so the total energy can't change.

6. Fill in the values for the potential, kinetic, and total energy of the particle in each of the three regions in the table below. Your answers should be in terms of E and V_0.

	Left wire	Air gap	Right wire
Potential Energy	0	V_0	0
Kinetic Energy	E	$E-V_0$	E
Total Energy	E	E	E

7. On top of the graph you drew in question 1, now sketch the kinetic energy KE of the electron as a function of position. Be sure to label each of the energies clearly.

8. Write an equation that relates the kinetic energy of a particle to its deBroglie wavelength.

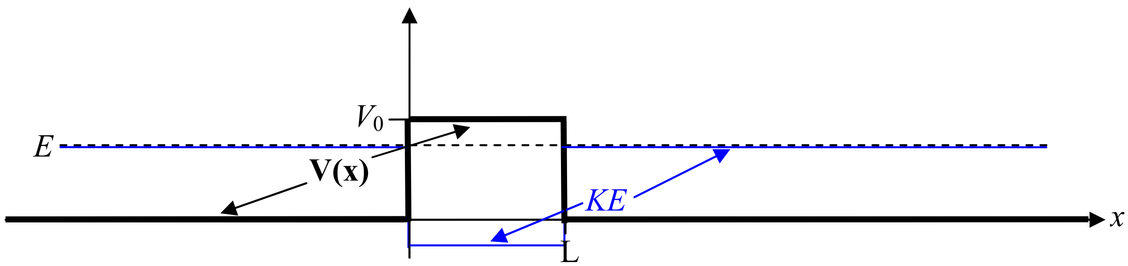
$KE = p^2/2m = h^2/2m\lambda^2$

9. Are the wave functions you sketched in question 3 consistent with your equation in question 8 and your kinetic energies in question 6? Resolve any discrepancies.

My equation in question 8 tells me that the as the kinetic energy increases, the wavelength decreases, and vice versa. In question 6 I said that the kinetic energy in the right wire is equal to the kinetic energy in the left wire, and this is consistent with my drawing, which shows the same wavelength in these two regions. I also said that kinetic energy is smaller in the gap, and this is also consistent with my drawing, which shows a larger wavelength in this region.

PART II: *E* < *V*$_0$: Now suppose the electron has an initial kinetic energy $E < V_0$.

10. In the space below, sketch a graph of the potential energy V of the electron as a function of horizontal position x. Define where $V = 0$ inside the wire. Once you have $V(x)$ sketched, use a dashed line to show the energy of an electron that satisfies the $E < V_0$ condition.



11. For the region in the copper wire to the left of the air gap, write down the general solution for $\Psi(x,t)$. Plug it into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

The general solution is: $\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$

Plugging into the Schrodinger Equation gives: $(-\hbar^2/2m)(-k^2)\Psi(x,t) + 0 = i\hbar(-iE/\hbar)\Psi(x,t)$

This simplifies to: $\hbar^2 k^2/2m = E$

For the region in the air gap, write down the general solution for $\Psi(x,t)$. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

The general solution is: $\Psi(x,t) = (Ae^{\alpha x} + Be^{-\alpha x})e^{-iEt/\hbar}$

Plugging into the Schrodinger Equation gives:

$$(-\hbar^2/2m)(\alpha^2)\Psi(x,t) + V_0\Psi(x,t) = i\hbar(-iE/\hbar)\Psi(x,t)$$

This simplifies to: $-\hbar^2\alpha^2/2m + V_0 = E$

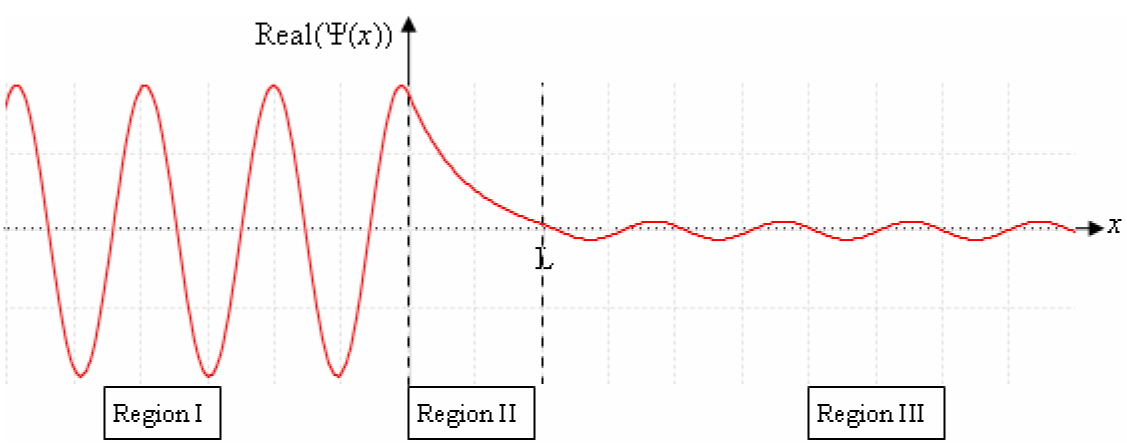
For the region in the copper wire to the right of the air gap, write down the general solution for $\Psi(x,t)$. Plug your solution into the Schrodinger Equation to make sure it works and solve for the total energy E of the electron.

The general solution is: $\Psi(x,t) = (Ae^{ikx} + Be^{-ikx})e^{-iEt/\hbar}$

Plugging into the Schrodinger Equation gives: $(-\hbar^2/2m)(-k^2)\Psi(x,t) + 0 = i\hbar(-iE/\hbar)\Psi(x,t)$

This simplifies to: $\hbar^2k^2/2m = E$

12. In the plot below, sketch the shape of the real part of the wave function at $t = 0$ in each region for the case where $E < V_0$. The air gap starts at $x = 0$ and ends at $x =$ L. Don't worry about the relative magnitudes of the waves in the different regions, but think carefully about the general shape of the graph in each region.



13. What is the basic shape of the real part of the wave function in each of the three regions? For example, is it linear, constant, quadratic, exponential, sinusoidal, or something else?

It's sinusoidal in regions I and III and it's a decaying exponential in region II.

14. It is often stated that a particle can quantum mechanically tunnel through a barrier. Explain what is meant by this.

In this example, although classically the electron does NOT have enough energy to get out of the metal and into the air gap, there is a solution to the Schrodinger equation in this region. This means there is a non-zero probability to find the electron in the air gap. The probability decays exponentially as you go farther into the gap, but if the gap is thin enough, the electron can "tunnel" through the gap into the copper wire on the right, wire it has enough energy to stay.

15. Consider an electron that has tunneled through the barrier. Is the energy of the electron to the right of the air gap *greater than*, *less than*, or *equal to* the energy of the electron to the left of the air gap? Explain how you arrived at your answer.

Equal to. Energy must be conserved, so the total energy can't change.

16. Fill in the values for the potential, kinetic, and total energy of the particle in each of the three regions in the table below. Your answers should be in terms of E and V_0.

	Left wire	Air gap	Right wire
Potential Energy	0	V_0	0
Kinetic Energy	E	$E-V_0$	E
Total Energy	E	E	E

17. On top of the graph you drew in question 10, now sketch the kinetic energy KE of the electron as a function of position. Be sure to label each of the energies clearly.

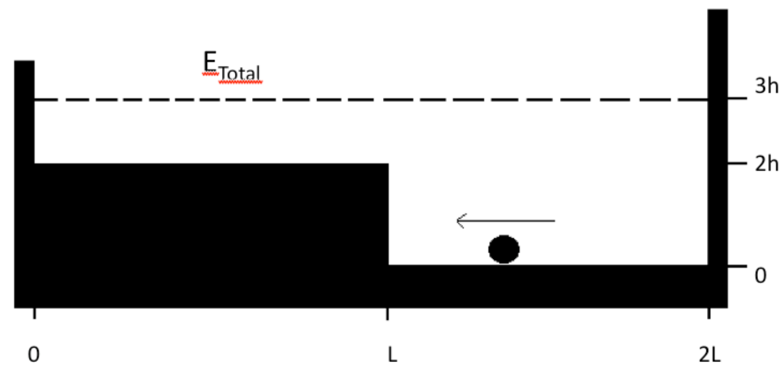
18. Do you notice anything unusual about the kinetic energy?

The kinetic energy is negative in the air gap! This is because the particle doesn't have enough total energy to be in this region, but it is "borrowing" some kinetic energy to compensate for having a potential energy greater than its total energy. There is no classical analogue to the situation, and if you try to think of the electron as a classical particle moving around in this region, it won't work. But we can describe the behavior of the wave function in this region without any problem and it accurately predicts the results of experiments.

PHYS 2130 Tunneling Tutorial

PART A: CLASSICAL PROBABILITY

A ball rolls back and forth in a frictionless track with very steep sides. Two levels are joined by a steep ramp, and the ball is able to slide up the ramp without a loss in total energy. Assume that the amount of time the ball spends going up the ramp is negligible and that the ball rolls forever with perfectly elastic collisions between the walls.



1) Sketch the gravitational potential energy between 0 and 2L below. Also include a dashed line representing the total energy of the ball.

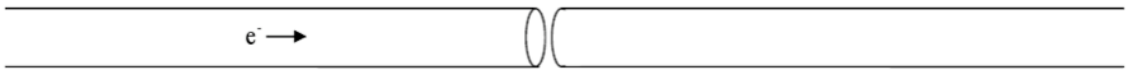
2) Is the kinetic energy of the ball on the top ramp greater than, less than or equal to the kinetic energy on the bottom half of the ramp?

3) Is the time the ball spends on the top half of the ramp greater than, less than or equal to the time it spends on the bottom half?

4) If someone were to take a photograph of the ball at a random time, how do the probabilities of the ball being on the top of the ramp and the ball being on the bottom of the ramp in the photograph compare?

PART B: Electron in a wire

Consider an electron moving to the right through a very long smooth copper wire with a small air gap in the middle:



Assume that the work function of the wire is V_0 and that $V = 0$ inside the wire.

1) If $\mathbf{E > V_0}$, draw a graph of the potential energy of the situation shown above. Also draw a dashed line indicating the total energy of the electron.

2) If you had to take a guess, what types of solutions to the Schrodinger equation would you expect to get in a) the left side of the wire, b) the right side of the wire, and c) inside the air gap? (We are looking for answers like: sines and cosines, real exponentials, quadratic, linear, and so on)

3) Prove It!

Here is the Time-Independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

We can rewrite this as:

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E - V(x))}{\hbar^2}\psi$$

Now, assuming that a solution to this equation is:

$$\psi = Ae^{kx} + Be^{-kx}$$

Plug this in and solve for k.

Check your answer with an instructor before moving on!

4) If E > V(x), is k real or imaginary? What does this mean in terms of the types of solutions you expect to have in a) the left wire, b) the right wire, and c) in the air gap?

5) Rank, in order of magnitude the values of k. (For instance, if you think that k_1 is greater than k_2 and k_3, write: $k_1 > k_2 > k_3$) (Let regions 1, 2 and 3 represent the left wire, air gap, and right wire, respectively.)

6) Recall that $k = \frac{2\pi}{\lambda}$. Use this and your answer above to compare the wavelengths of your solutions in each of the above 3 regions.

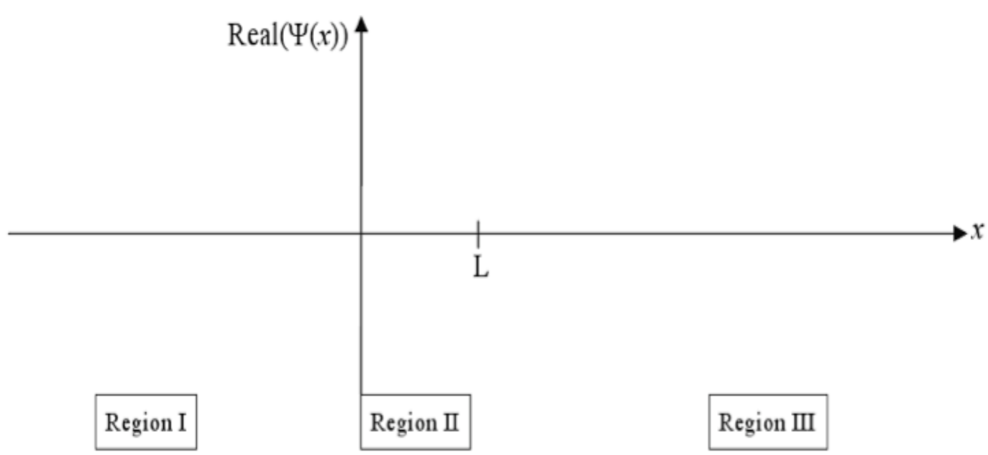
7) Now consider the kinetic energy of the electron, which is given by:

$$KE = \frac{\hbar^2 k^2}{2m} = \frac{h^2}{2m\lambda^2}$$

What does this tell you about the kinetic energy in regions 1, 2 and 3?

8) Now remember back to the first part of the tutorial dealing with the ball on the track. Was the ball more likely to be found when the kinetic energy was large or small? By the same reasoning, what does this tell you about the probability of the electron being *in the left wire* vs. being in the air gap?

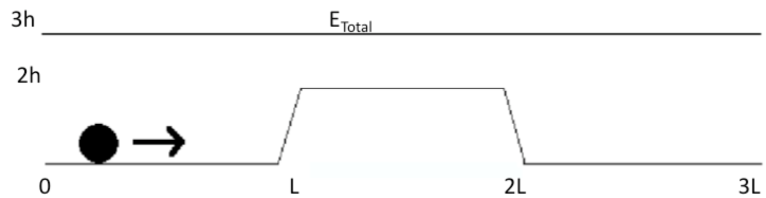
9) With that information in mind, draw a graph of the wave function below. (Hint: think about what ψ^2 tells you in terms of the probabilities, and then use that to draw ψ.)



Name:__________________________________ Student ID:___________________________

Tutorial: Quantum Tunneling

PART A: CLASSICAL PARTICLE



A ball of mass *m* rolls to the right on a flat, frictionless surface with total energy $E = 3mgh$. The ball soon encounters a sloped surface and rolls up to height $2h$. After, the ball rolls back down the ramp, always staying in contact with the surface.

1) Is the total energy of the ball as it rolls from 0 to 3L increasing, decreasing, or staying the same?

2) Sketch the kinetic energy, gravitational potential energy, and total energy of the ball between 0 and 3L. Scale your graph with multiples of mgh.

3) Is the amount of time the ball spends between L and 2L greater than, less than, or equal to the amount of time it spends between 0 and L? How does it compare to the amount of time it spends between 2L and 3L? (Ignore the time the ball spends on the ramp.)

4) Now imagine that we take a photograph of the ball at some random time. Is the probability of finding the ball between 0 and L greater than, less than or equal to the probability of finding it between L and 2L? Why?

Name:__________________________________ Student ID:__________________________

PART B: SOLUTIONS TO SCHRÖDINGER'S EQUATION

The time-independent Schrödinger equation is given by:

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\psi(x) = E_{TOT}\psi(x)$$

This can be rewritten as:

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}\left(E - V\right)\psi = \frac{2m}{\hbar^2}\left(V - E\right)\psi$$

1) If $\boldsymbol{E} < \boldsymbol{V}$, will the solutions to Schrödinger's equation be real exponentials or complex exponentials? [Hint: Is the quantity on the right-hand side positive or negative in this case?]

2) Write down the most general solution to Schrödinger's equation for the case when $\boldsymbol{E} < \boldsymbol{V}$.

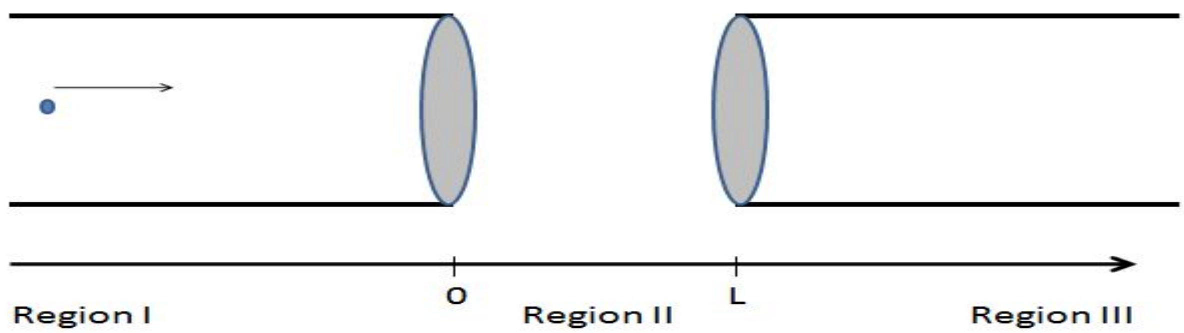
3) If $\boldsymbol{E} > \boldsymbol{V}$, will the solutions to Schrödinger's equation be real exponentials or complex exponentials? [Again, consider whether the right-hand side is positive or negative.]

4) Write down the most general solution to Schrödinger's equation for the case when $\boldsymbol{E} > \boldsymbol{V}$.

Name:___________________________________ Student ID:___________________________

PART C: ELECTRON IN A WIRE ($E > V_0$)

Consider an electron with total energy $\boldsymbol{E}$ moving to the right through a very long smooth copper wire with a ***small*** air gap in the middle:



Assume that the work function of the wire is V_0 and that $V = 0$ inside the wire.

1) If $\mathbf{E} > \mathbf{V_0}$, draw a graph of the electron's potential energy in all three regions. Also draw a dashed line indicating the total energy of the electron.

2) In each of the three regions, are the solutions to Schrödinger's equation real exponentials or complex exponentials? Write down a solution for each of the three regions corresponding to an electron traveling to the right.

Region I:

Region II:

Region III:

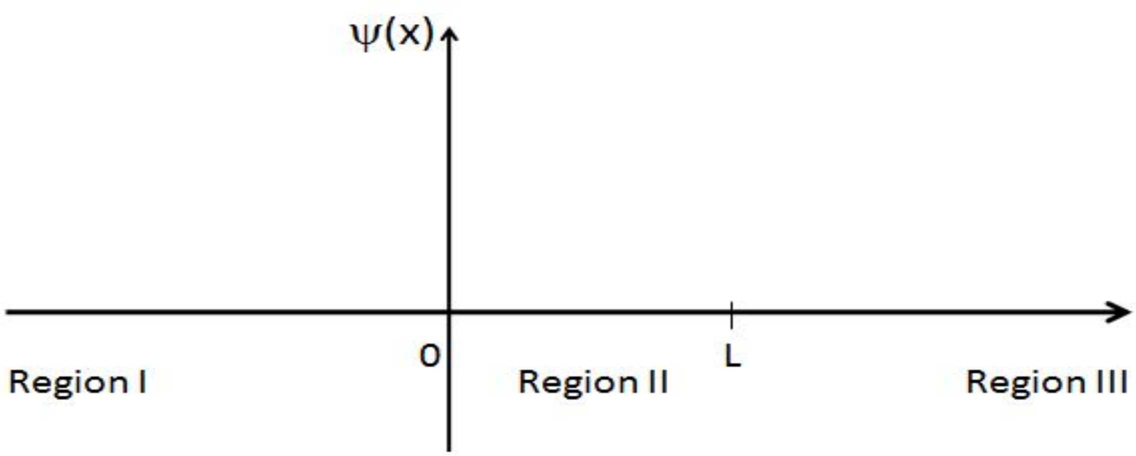
Name:__________________________________ Student ID:__________________________

3) How does the kinetic energy of the electron compare in each of the three regions? Rank the kinetic energies in the three regions ($\boldsymbol{KE_1}$, $\boldsymbol{KE_2}$ & $\boldsymbol{KE_3}$) from high to low.

4) How does the deBroglie wavelength of the electron compare in each of the three regions? Rank the wavelengths in the three regions ($\boldsymbol{\lambda_1}$, $\boldsymbol{\lambda_2}$, $\boldsymbol{\lambda_3}$) from largest to smallest. If the wavelength is not defined in a particular region, then say so.

5) How does the amplitude of the electron's wave function compare in each of the three regions? [Hint: think about $|\psi(x)|^2$ what tells you in terms of probabilities].

6) With this information in mind, sketch the ***real part*** of the electron's wave function in all three regions:



Name:__________________________________ Student ID:__________________________

PART D: ELECTRON IN A WIRE ($E < V$)

Consider the same situation as in **Part C**, but now the total energy E of the electron is ***less than*** the work function V_0.

1) If $\mathbf{E < V_0}$, draw a graph of the electron's potential energy in all three regions. Also draw a dashed line indicating the total energy of the electron.

2) In each of the three regions, are the solutions to Schrödinger's equation real exponentials or complex exponentials? Write down a solution for each of the three regions corresponding to an electron traveling to the right.

Region I:

Region II:

Region III:

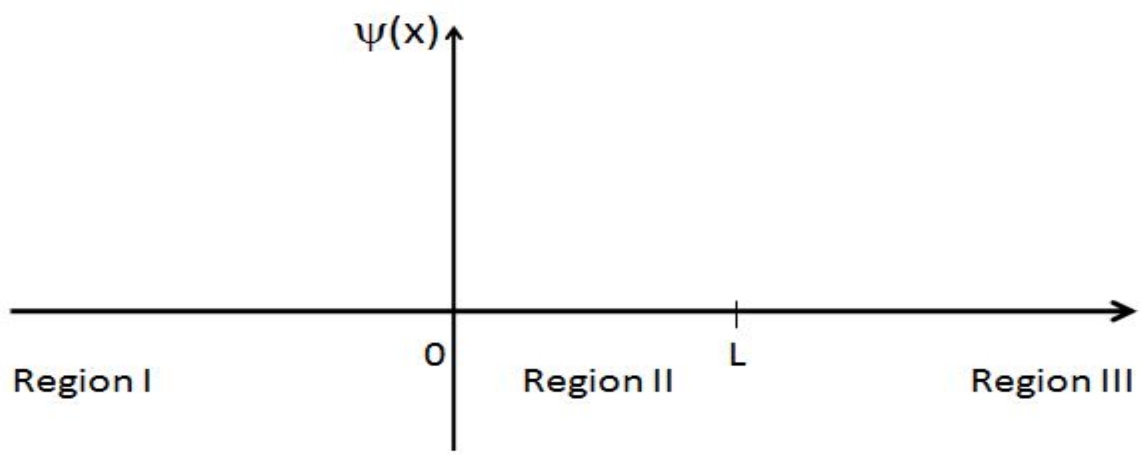
3) How does the kinetic energy of the electron compare in each of the three regions? Rank the kinetic energies in the three regions (KE_1, KE_2 & KE_3) from high to low.

Name:________________________________ Student ID:_________________________

4) How does the deBroglie wavelength of the electron compare in each of the three regions? Rank the wavelengths in the three regions (λ_1, λ_2, λ_3) from largest to smallest. If the wavelength is not defined in a particular region, then say so.

5) How does the amplitude of the electron's wave function compare in each of the three regions? [Hint: think about $|\psi(x)|^2$ what tells you in terms of probabilities]. Explain what physical meaning we can make from the shape of the wave function in Region II.

6) With this information in mind, sketch the ***real part*** of the wave function for this electron:



Name:________________________________ Student ID:_________________________

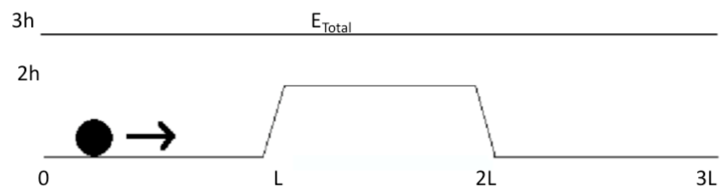
7) Using the solution to **#6**, what conclusions can you make about the possible position of the particle? How is this different than a classical particle in the same situation? Can you offer an explanation of why classical objects (people) don't exhibit the same property, called tunneling?

Name:______________________________ Student ID:____________________________

Tutorial: Quantum Tunneling with PhET Simulation

Part A: Classical Particle

A ball of mass *m* rolls to the right on a flat, frictionless surface with total energy $E = 3mgh$. The ball soon encounters a sloped surface and rolls up to height *2h*. After, the ball rolls back down the ramp, always staying in contact with the surface.



1) Is the total energy of the ball as it rolls from 0 to 3L increasing, decreasing, or staying the same?

2) Sketch the kinetic energy, gravitational potential energy, and total energy of the ball between 0 and 3L. Scale your graph with multiples of mgh.

3) Is the amount of time the ball spends between L and 2L greater than, less than, or equal to the amount of time it spends between 0 and L? How does it compare to the amount of time it spends between 2L and 3L? (Ignore the time the ball spends on the ramp.)

4) Now imagine that we take a photograph of the ball at some random time. Is the probability of finding the ball between 0 and L greater than, less than or equal to the probability of finding it between L and 2L? Why?

Name:______________________________ Student ID:____________________________

PART B: Quantum Particle with E > V (Using PhET sim)

1) Observe the plot of the wave function for plane wave solutions for the case where E > V. Where does the time-dependent oscillation come from mathematically? Describe how the frequency of oscillation changes as you change E.

2) Now widen the width of the wire gap (where V > 0) to 3.5 dashed-lines wide. How does the wavelength of the wave function in this region compare to the wavelength in the region to the left? How about to the region on the right? Lastly, how do the wavelengths in the regions on the left and right compare to each other?

3) What does your answer to (2) tell you about the kinetic energy of the particle in each of these three regions? Be sure to discuss this with your group members.

4) Now refer back to the classical particle case. How does the kinetic energy of the classical particle from L to 2L relate to the kinetic energy wave function in the air gap? What does this say about the relationship between wavelength and kinetic energy?

Name:______________________________ Student ID:____________________________

5) Now let's look at the amplitude of the wave function. What does the amplitude of ψ (or $|\psi|^2$) tell you?

6) If we were to make a measurement of position of the particle, would we be more likely to find it in the region to the left of the air gap or in the air gap (for now ignore the region to the right of the air gap)?

7) In this case of E > V, explain how measurements of position for a quantum particle compare to taking a photograph of a classical particle.

PART C: Classical Particle with E < V

Now imagine that the same ball from PART A has an initial total energy of E = 1.5mgh, while the height of the hill remains at 2h.

1) What happens to the ball as it starts to go up the hill? Is it possible for the ball to be found between L and 2L? How about between 2L and 3L?

Name:_______________________________ Student ID:______________________________

PART D: Quantum Particle with E < V

1) Now, using the PhET sim, decrease the size of the wire gap to 1 dashed-line wide and increase the height of the potential energy line all the way to the top. What type of function do you see in region 1 and 3 (e.g. sinusoidal, exponential growth, exponential decay, linear, quadratic, etc)?

2) What type of function is shown inside the wire gap (e.g. sinusoidal, exponential decay, exponential growth, linear, etc.)? Hint: It might be more obvious if you look at the wave function when the air gap is very wide… but return to 1 dashed-line wide for the next question!

3) How do the wavelengths of the wave function on the left and right of the air gap compare to each other? What does that tell you about the kinetic energy of the particle in each of those regions?

4) Now refer back to PART C with the classical particle. How does the kinetic energy of the classical particle in regions 2 and 3 compare to the kinetic energy of the quantum particle in regions 2 and 3?

Name:______________________________ Student ID:____________________________

5) What does the amplitude of ψ (or $|\psi|^2$) tell you about finding a particle in regions 2 or 3?

6) If we were to make a measurement of the particle's position with $E < V$, which region would we be most likely to find it in? Compare this to the case of the classical particle.

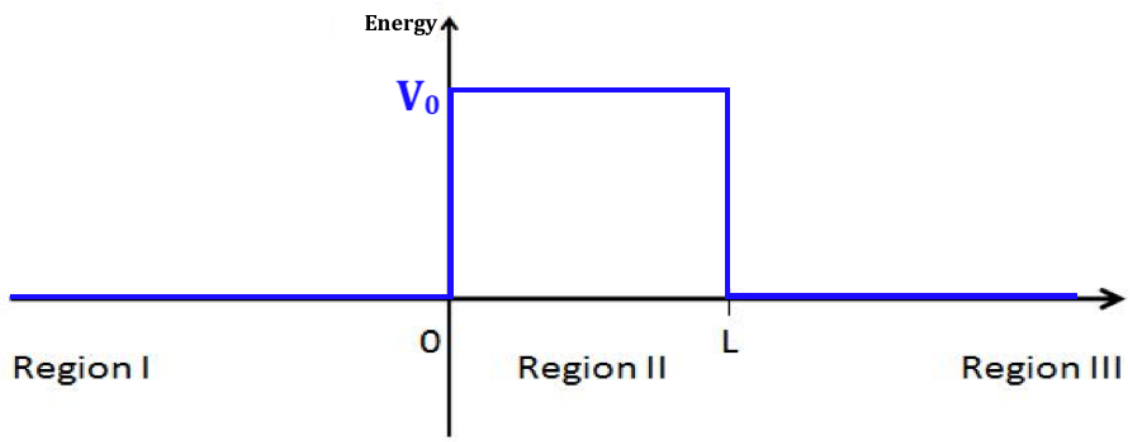
PHYS 3220 PhET Quantum Tunneling Tutorial

Part I: Mathematical Introduction

Recall that the Schrödinger Equation is $i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \hat{H}\Psi(x,t)$. Usually this is solved by first assuming that $\Psi(x,t) = \psi(x)\phi(t)$, from which we obtain the solution $\phi(t) = e^{-iEt/\hbar}$ and are left with the following equation to solve for the spatial dependence:

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E - V(x))\psi$$

1. Consider a potential region such as the one shown in the figure below. Given that $E > V_0$, write down a general solution of the Schrödinger Equation for each region. Define any constants that will simplify your solution.

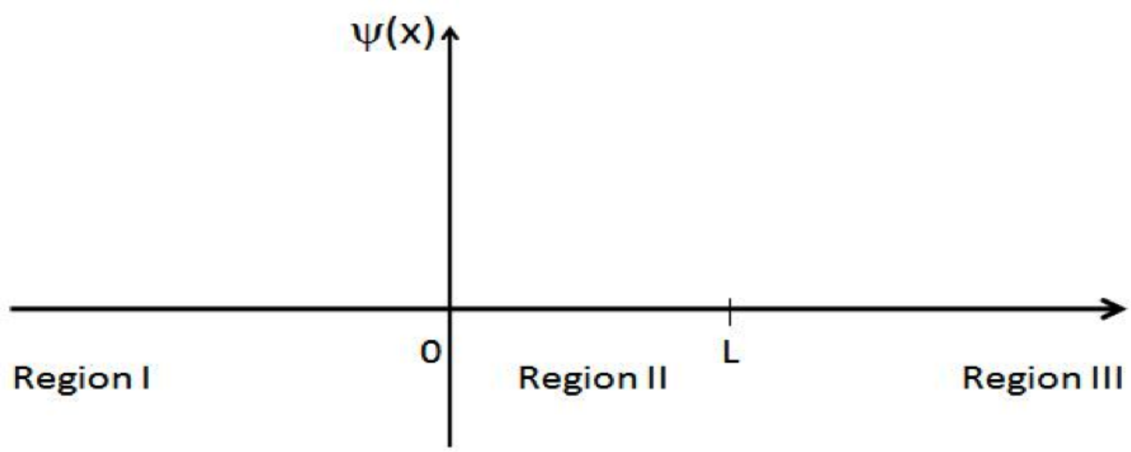


2. How many boundary conditions are needed to completely specify this situation?

3. If a right-going plane wave with amplitude A originating from $x \to -\infty$ is incident upon the barrier, what simplifications can be made in your above equations? Which of your unspecified constants (if any) are now specified completely?

4. What are the remaining boundary conditions for this system? (A simple mathematical formula or explanation in words are both acceptable.)

5. Using this information, do your best to make a plot of the wave function for the case of $E > V_0$.



6. In the graph that you just drew, did you account for the wavelength and amplitude differences in the three regions? (Don't change your graph, just think about it!)

    (a) Rank the magnitude of the wavelengths for the three regions.

    (b) How do you expect the amplitude to compare across the three regions? Give a brief qualitative explanation.

Part II: Plane Wave of $E > V_0$ using PhET sim

Download the Tunneling PhET sim, found at: `http://phet.colorado.edu/sims/quantum-tunneling/quantum-tunneling_en.jar`. Play with the sim for a bit, and then switch to "Plane Wave" mode to answer the following questions.

Notice: For this tutorial, you may find it very useful to switch between using the "Separate" and "Sum" representations on the sim!!!

1. Comparing your findings in Part I to the sim:
   (a) What are the main differences between your plot of the wave function and what is shown?

   (b) Do your predictions for wavelength and amplitude agree with what you see? If not, why were your predictions wrong?

2. You should be able to see the wave function in Region 1 bob up and down.
   (a) What causes this? (You might find the 'Notice' at the top of the page helpful!)

(b) List *all* parameters that you can adjust to eliminate this "bobbiness." Is there only one way to do this, or are there several different ways?

3. Play with the sim and maximize the amount of transmission to Region 3.

   (a) What parameters affect the amount of transmission in this region? List them all. Again, is there only one way to maximize the amount of transmission, or are there multiple ways?

   (b) How does the case of maximum transmission compare to "eliminating the bobbiness" in region 1? Give a brief qualitative explanation of why this is the case.

   (c) Often times the probability of transmission is denoted by the variable T, and takes the following form:

$$T = \frac{1}{1 + \frac{V_0 \sin^2(k_2 L)}{4E(E - V_0)}}$$

   According to this equation, what condition must be satisfied for maximum transmission to occur?

(d) How many variables does the Transmission probability depend on (don't forget to think about what k_2 depends on)? Does this account for everything you found in 3a?

4. Is there any way to set up the sim such that there is a time-dependence in the probability density? Use the fact that $\Psi_{\text{region } j} = A_j e^{i(k_j x - \omega_j t)} + B_j e^{-i(k_j x + \omega_j t)}$ to justify your answer.

5. (a) Based on your result from 4, which regions can show sinusoidal probability densities in the spatial dependence?

   (b) Is there any way to make Region 3 have a sinusoidal probability density?

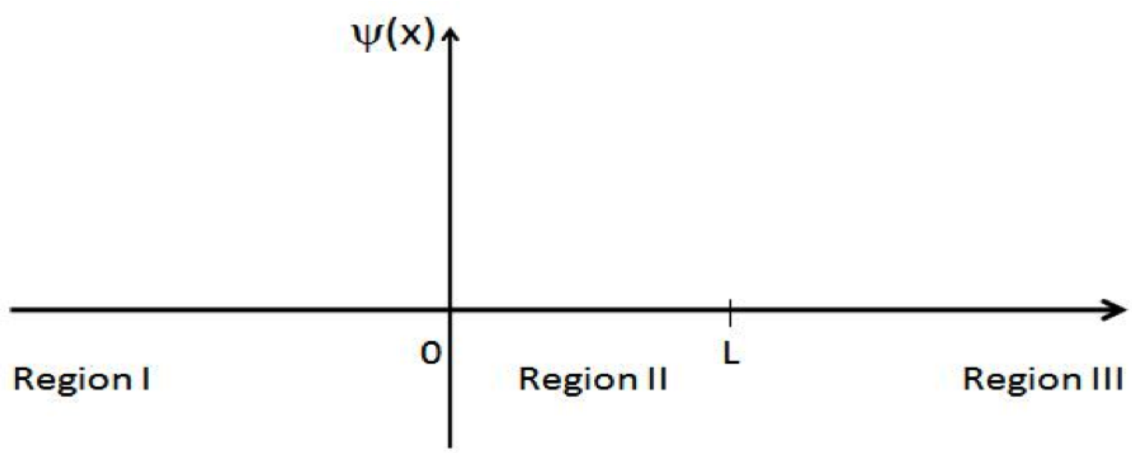
   (c) Under what conditions can you have a sinusoidal probability density?

Part III: Case of $E < V_0$

1. For the case of $E < V_0$, write down the most general solutions to the Schrödinger Equation for each of the three regions. Define any constants that help simplify your answers.

2. This time, assume that a left-going plane wave with *fixed amplitude* originating from $x \to +\infty$ is incident upon the barrier. Which variables are now "fixed" or completely specified? List the remaining boundary conditions using a simple mathematical expression.

3. Do your best to plot $\psi(x)$ vs. x across all three regions.



Using the sim for $E < V_0$

1. What parameters can you adjust to maximize the amount of transmission to Region 1 in the sim? List all of the possible ways.

2. (a) When $E < V_0$, is there any way to completely eliminate the reflected wave in Region 3?

   (b) Assuming that the potential barrier, V_0, has *some* finite width, is there any way to get 100% transmission in this case? Why is this the case?

3. Looking at the wave function in the potential barrier, is there any similarity to the case of $E > V_0$, where there was a reflected wave and a transmitted wave? Is the full wave function a sum of solutions or just one particular solution?