SOFTWARE FOR DETERMINING THE CAPACITY OF BOLT GROUPS UNDER ECCENTRIC LOADS

by

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Find that both the content and the form meet acceptable presentation standards

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Software for Determining the Capacity of Bolt Groups Under Eccentric Loads

Thesis directed by Professor George Hearn

In steel construction, it is not uncommon to encounter a bolted connection that supports an eccentric load. Determining the capacity of the bolted connection is dependent on the location and attitude of the applied eccentric load. There have been numerous proposed design methods for determining this capacity. These include the elastic method, the modified elastic method, the plastic method and the instantaneous center method. The instantaneous center method is the preferred method in the current AISC Steel Construction Manual (13th Edition) to design a bolt group under eccentric loads. This method, however, is complicated for design engineers since it requires an iterative analysis. Design engineers are then left with the option of laying out the bolt group to match one of the pre-populated design tables or performing a simplified and conservative elastic analysis. Neither option is particular appealing given that it is not always possible to match the design tables and design engineers are expected to provide competitive and efficient designs. The software developed in this thesis allows design engineers to quickly obtain the capacity of any bolt group using the instantaneous center method. This removes the limitations on design engineers and allows them to provide the best possible connections. The software also provides the capacity of a given bolt group using both the elastic and plastic method, in order to not limit design engineers to one particular method. While the instantaneous center method is preferred, by providing the results for all methods, the design engineer is given complete freedom to use the software in a way that works best for them.

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Chapter 1 - Introduction

General

Bolts are a common method of connecting steel framing to its supporting members. Often the bolts can be oriented such that the applied loads are in line with the center of the bolt group. However, there are some cases where the bolts cannot be oriented to allow for the loads to be concentric. If the load does not align with the center of the bolt group, it is an eccentric load, which induces a moment in the connection. Under this condition, the individual bolts resist the load in both direct shear as well as some contribution from moment, which is a function of the position of the instantaneous center of rotation. Two occurrences of this type of connection are shown in Figure 1 and Figure 2.

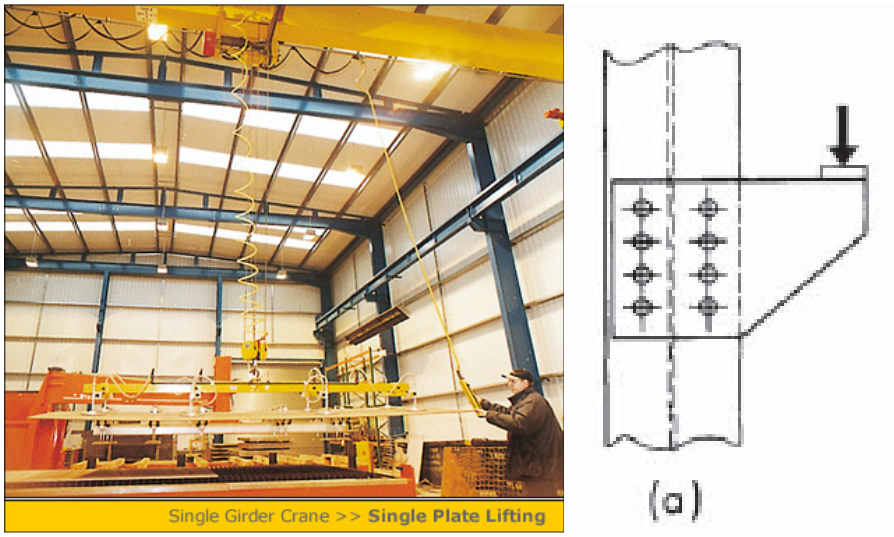


Figure 1: Beams or Girders Attached to Columns (CraneWerx n.d.) & (Kulak 1975)

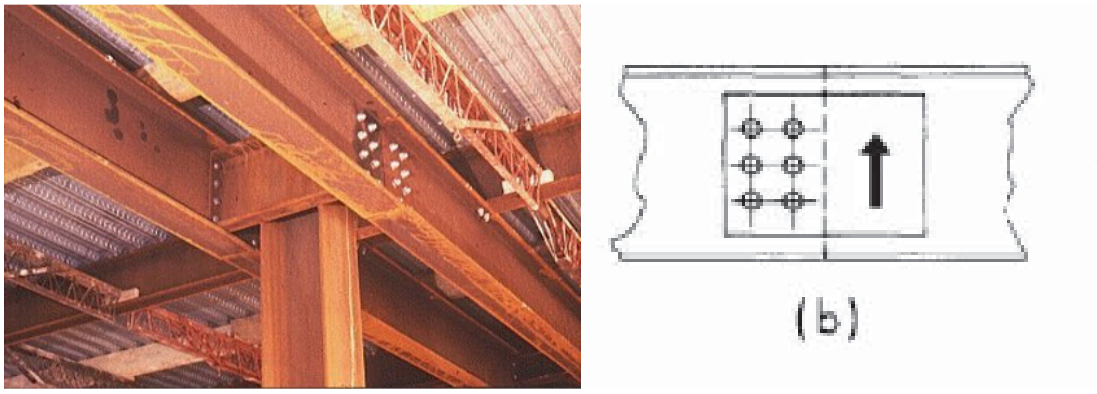


Figure 2: Web Splice (Corus Construction n.d.) & (Kulak 1975)

Various methods have been proposed for computing the strength of a bolt group subjected to an eccentric load. The American Institute of Steel Construction Steel Construction Manual (AISC Manual) has presented three different methods since the 5th edition in 1950 (*Steel* 1950). These three methods are the elastic method, the modified elastic method and the instantaneous center of rotation method (I.C.) (also referred to as the ultimate strength method). A fourth method, the plastic method, has been proposed, but has never been implemented in the AISC Manuals. The different versions of the AISC Manuals also give tables for each of the methods to assist the designer in determining the capacity of common bolt groups under specific load conditions. Using these tables, the designer can look up a bolt group coefficient, C, for a given bolt group under a specific application of load. This bolt group coefficient represents the number of bolts that are effective in resisting the eccentric shear force. Therefore, the lowest value of the bolt group coefficient is equal to the required strength divided by the available strength of a single bolt.

$$C_{min} = \frac{P_u}{\phi r_n} \qquad \text{(Eq. 1-1)}$$

Where: ϕ = strength reduction factor
C_{min} = minimum value of the bolt group coefficient
r_n = nominal shear strength per bolt

The strength reduction factor, ϕ, is given in the AISC Manual Tables 7-7 through 7-14 and has a value of 0.75 (*Manual* 2005). The nominal shear strength of a single bolt, r_n, is found in Table 7-1 of the AISC Manual (*Manual* 2005). Once the value of the bolt group coefficient is determined it is then multiplied by the design strength of a single fastener to get the capacity of the entire bolt group:

$$\phi R_n = C * \phi r_n \qquad \text{(Eq. 1-2)}$$

Where: ϕ = strength reduction factor
R_n = nominal strength of the bolt group
C = bolt group coefficient
r_n = nominal shear strength per bolt

This equation remains the same for all the different methods and versions of the AISC Manuals. The different methods produce different values for C.

Objective

This thesis will review how each of the four methods was developed and how they differ in determining the capacity of a bolt group under eccentric loads. Since the tables given in the AISC Manuals are limited to specific bolt configurations under specific applications of load, a system of equations is derived for the elastic method, plastic method and I.C. method that can be applied to any bolt pattern under any in-plane application of an eccentric point load. Software is developed using these equations to allow design engineers greater flexibility and efficiency since they are no longer required to either conform their connection to the design tables given in the AISC Manuals or to perform their own analysis. The interface and output of this software are presented to show how users can perform an analysis of a given bolt group under an eccentric load using any of the design methods. Numerous examples are executed to verify that results from the software agree with values of bolt group coefficients given in the AISC Manuals. Further analysis is then performed using examples that cannot be found in the AISC Manuals in order to show the benefit of having the software available. Lastly, the results of the examples are used to understand how the different methods, different applications of load and different bolt patterns affect the overall capacity of the bolt group.

Chapter 2 - Background

This chapter reviews the history of methods for analysis of eccentric loads on bolt groups. In 1950, the elastic method was the preferred method of analysis for eccentric loads on bolt groups. During experimental testing, however, it was determined that the elastic method provides conservative results. The modified elastic method, plastic method and I.C. method were subsequently developed in order to achieve design capacities that are more representative of the experimental test results. The equations for each of these methods will be presented in Chapter 3.

Elastic Method

In the 5th edition of the AISC Manual, the elastic method was the only method recommended for determining the capacity of a bolt group under eccentric loads (*Steel* 1950). The elastic method has been used in the design of connections since at least 1936 (Rathburn 1936). The elastic method is based on the theory that when an eccentric load is applied to a group of bolts, as shown in Figure 3, the bolt stress-strain response is linear-elastic. In other words, the bolts support an equal share of the vertical load P, plus a force due to the moment, P*e, which is proportional to its distance from the elastic centroid of the bolt group (CG) (*Manual* 1963).

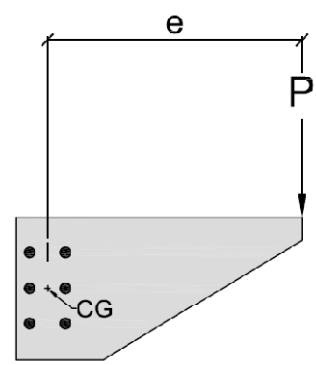
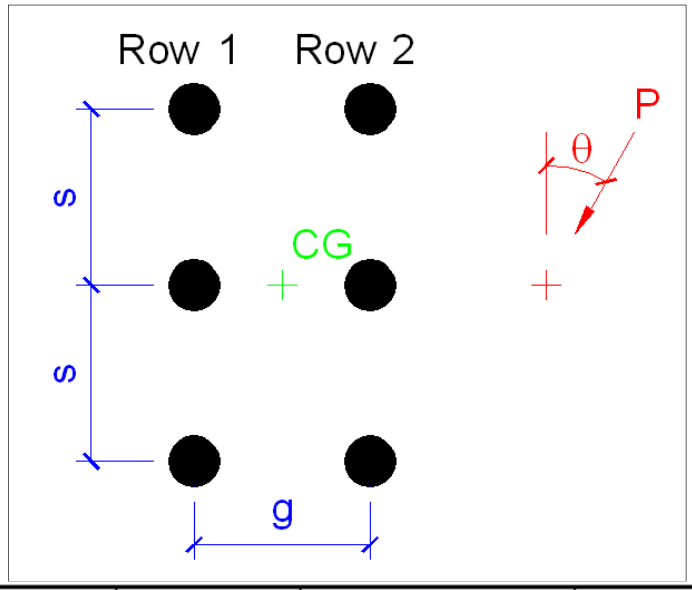


Figure 3: Eccentric Load on a Bolt Group

The 5th edition of the AISC Manual provides tables for four different cases of rivet groups under an eccentric application of load (*Steel* 1950). Table 1 is a summary of the 4 cases represented in the design tables in the 5th of the AISC Manual.

Table 1: Summary of 5th Edition AISC Design Tables for Elastic Method (*Steel* 1950)



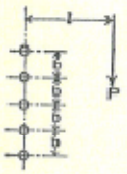
Vertical spacing, s inches	Number of Vertical Rows	Horizontal spacing, g inches	Load angle, θ degrees
3	1	N/A	0
3	2	5.5, 9.5	0
3	4	2.5-4.5-2.5, 3-5.5-3	0

Notice that the vertical spacing of rivets does not change for any configuration even with multiple vertical rows. For rivet groups with multiple vertical rows, the tables allow for two different horizontal spacing options. And all applications of load are vertical in the downward direction. These design tables were meant to be a quick reference for the most common patterns and applied loads. If the actual conditions did not meet the parameters given in this table, the designer was forced to calculate the capacity of the rivet group by hand. The equations for the elastic method were provided in the AISC Manual so that the designer could easily calculate this capacity. Since the equations were relatively simple, there was no need to provide exhaustive design tables to cover every possible scenario. The design tables provided in the 5th edition of the AISC Manual are shown in Figure 4.

RIVET GROUPS UNDER ECCENTRIC APPLICATION OF LOAD

Nomenclature:
- n = total number of rivets in any one vertical row.
- P = permissible load, acting with lever arm l.
- S = permissible load on one rivet by Specification.
- C = coefficient as tabulated below.
- P = C × S; or, knowing P, required minimum C = $\frac{P}{S}$

Case I

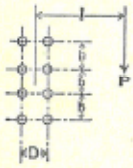


n \ l	1″	2″	3″	6″	9″	12″	15″	18″	21″	24″
2	1.7	1.2	.89	.49	.33	.25	.20	.17	.14	.12
3	2.7	2.1	1.7	.95	.65	.49	.40	.33	.28	.25
4	3.7	3.1	2.6	1.5	1.1	.82	.66	.55	.47	.41
5	4.7	4.2	3.5	2.2	1.6	1.2	.98	.82	.71	.62
6	5.8	5.2	4.6	3.0	2.2	1.7	1.4	1.1	.99	.87
7	6.8	6.3	5.6	3.9	2.8	2.2	1.8	1.5	1.3	1.2
8	7.8	7.3	6.7	4.8	3.6	2.8	2.3	1.9	1.7	1.5
9	8.8	8.4	7.7	5.8	4.4	3.5	2.8	2.4	2.1	1.8
10	9.8	9.4	8.8	6.8	5.2	4.2	3.4	2.9	2.5	2.2
11	10.9	10.4	9.8	7.8	6.1	4.9	4.1	3.5	3.0	2.7
12	11.9	11.5	10.9	8.8	7.0	5.7	4.8	4.1	3.5	3.1

In Table, b = 3″

In general, $C = \dfrac{n}{\sqrt{\left[\dfrac{6l}{(n+1)\,b}\right]^2 + 1}}$

Case II

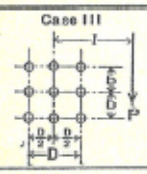


n \ l	1″		2″		3″		6″		9″		12″		15″		18″		21″		24″	
D	5½	9½	5½	9½	5½	9½	5½	9½	5½	9½	5½	9½	5½	9½	5½	9½	5½	9½	5½	9½
2	3.1	3.4	2.5	2.9	2.1	2.5	1.4	1.8	1.1	1.4	0.8	1.2	0.7	1.0	0.6	0.9	0.5	0.8	0.5	0.7
3	4.9	5.1	4.1	4.4	3.4	3.9	2.3	2.9	1.7	2.2	1.4	1.8	1.1	1.6	1.0	1.4	0.9	1.2	0.8	1.1
4	6.8	7.0	5.8	6.1	5.0	5.4	3.4	4.0	2.5	3.1	2.0	2.6	1.7	2.2	1.4	1.9	1.2	1.7	1.1	1.5
5	8.8	8.9	7.7	7.9	6.7	7.0	4.6	5.2	3.5	4.1	2.8	3.3	2.3	2.8	1.9	2.4	1.7	2.1	1.5	1.9
6	10.9	10.8	9.6	9.7	8.5	8.7	6.0	6.5	4.5	5.1	3.6	4.2	3.0	3.6	2.6	3.1	2.2	2.7	2.0	2.4
7	12.9	12.8	11.7	11.6	10.4	10.6	7.5	8.0	5.8	6.3	4.6	5.2	3.8	4.4	3.3	3.8	2.8	3.3	2.5	3.0
8	15.0	14.8	13.7	13.6	12.4	12.4	9.2	9.6	7.1	7.6	5.7	6.2	4.8	5.3	4.1	4.6	3.5	4.0	3.1	3.6
9	17.0	16.9	15.8	15.8	14.5	14.4	11.0	11.2	8.6	9.0	6.9	7.4	5.8	6.3	4.9	5.4	4.3	4.8	3.8	4.3
10	19.1	18.9	17.9	17.7	16.6	16.4	12.8	13.0	10.1	10.5	8.2	8.7	6.9	7.4	5.9	6.4	5.2	5.6	4.6	5.0
11	21.2	20.9	20.0	19.7	18.7	18.4	14.8	14.8	11.8	12.1	9.7	10.0	8.1	8.5	7.0	7.4	6.1	6.5	5.4	5.8
12	23.2	23.0	22.1	21.8	20.8	20.5	16.8	16.7	13.5	13.7	11.2	11.4	9.4	9.8	8.1	8.5	7.1	7.5	6.3	6.7

In Table, b = 3″ and D = 5½″ or 9½″

In general, $C = \dfrac{n}{\sqrt{\left[\dfrac{l\,(n-1)\,b}{D^2 + \frac{2}{3}(n^2-1)\,b^2}\right]^2 + \left[\dfrac{l\,D}{D^2 + \frac{1}{3}(n^2-1)\,b^2} + \frac{1}{2}\right]^2}}$

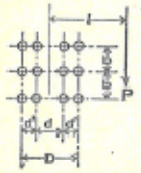
Case III



Case III, not tabulated.

In general, $C = \dfrac{n}{\sqrt{\left[\dfrac{l\,(n-1)\,b}{D^2 + \frac{2}{3}(n^2-1)\,b^2}\right]^2 + \left[\dfrac{l\,D}{D^2 + \frac{1}{3}(n^2-1)\,b^2} + \frac{1}{2}\right]^2}}$

Case IV



n \ l	1″		2″		3″		6″		9″		12″		15″		18″		21″		24″	
D	9½	11½	9½	11½	9½	11½	9½	11½	9½	11½	9½	11½	9½	11½	9½	11½	9½	11½	9½	11½
2	6.2	6.4	5.0	5.3	4.2	4.5	2.8	3.1	2.1	2.4	1.7	1.9	1.4	1.6	1.2	1.4	1.1	1.2	1.0	1.1
3	9.6	9.8	8.0	8.2	6.7	7.1	4.8	5.0	3.5	3.8	2.8	3.1	2.3	2.6	2.0	2.2	1.8	2.0	1.6	1.8
4	13.3	13.4	11.2	11.5	9.7	10.0	6.7	7.1	5.1	5.4	4.1	4.4	3.4	3.7	2.9	3.2	2.6	2.8	2.3	2.6
5	17.2	17.2	14.8	15.0	12.9	13.1	9.1	9.4	6.9	7.3	5.6	5.9	4.7	5.0	4.0	4.3	3.5	3.8	3.1	3.4
6	21.2	21.1	18.6	18.8	16.4	16.5	11.7	12.1	9.0	9.4	7.3	7.6	6.1	6.4	5.2	5.5	4.6	4.9	4.1	4.3
7	25.2	25.1	22.5	22.5	20.1	20.1	14.7	14.9	11.3	11.7	9.2	9.5	7.7	8.0	6.6	6.9	5.8	6.1	5.1	5.4
8	29.3	29.2	26.5	26.4	23.9	23.9	17.8	18.0	13.9	14.2	11.3	11.6	9.5	9.8	8.2	8.5	7.2	7.5	6.4	6.6
9	33.4	33.3	30.6	30.5	27.9	27.8	21.2	21.3	16.7	16.9	13.7	13.9	11.5	11.8	9.9	10.2	8.7	9.0	7.7	8.0
10	37.6	37.4	34.8	34.8	32.0	31.8	24.8	24.8	19.7	19.9	16.2	16.4	13.7	13.9	11.8	12.1	10.4	10.6	9.2	9.5
11	41.7	41.5	39.0	38.7	36.2	35.9	28.5	28.5	22.9	23.0	18.9	19.1	16.0	16.2	13.9	14.1	12.2	12.4	10.9	11.1
12	45.8	45.6	43.2	42.8	40.3	40.0	32.4	32.2	26.3	26.3	21.8	21.9	18.5	18.7	16.0	16.3	14.1	14.4	12.6	12.8

In Table, b = 3″
d′ = 2½″ d′ = 3″
d = 4½″ d = 5½″
d′ = 2½″ d′ = 3″
D = 9½″ D = 11½″

In general, $C = \dfrac{n}{\sqrt{\left[\dfrac{l\,(n-1)\,b}{d^2 + D^2 + \frac{2}{3}(n^2-1)\,b^2}\right]^2 + \left[\dfrac{l\,D}{d^2 + D^2 + \frac{2}{3}(n^2-1)\,b^2} + \frac{1}{4}\right]^2}}$

Figure 4: Design Tables given in the 5th Edition AISC (*Steel* 1950)

In 1963, the AISC sponsored a series of 10 tests at Lehigh University's Fritz Engineering Laboratory to compare to the results given by the elastic method (Higgins 1964). The tests were performed on groups of 3/4-inch diameter rivets in either 1 or 2 vertical rows under eccentric shear loads (Figure 5). The results of this testing are listed in Table 2.

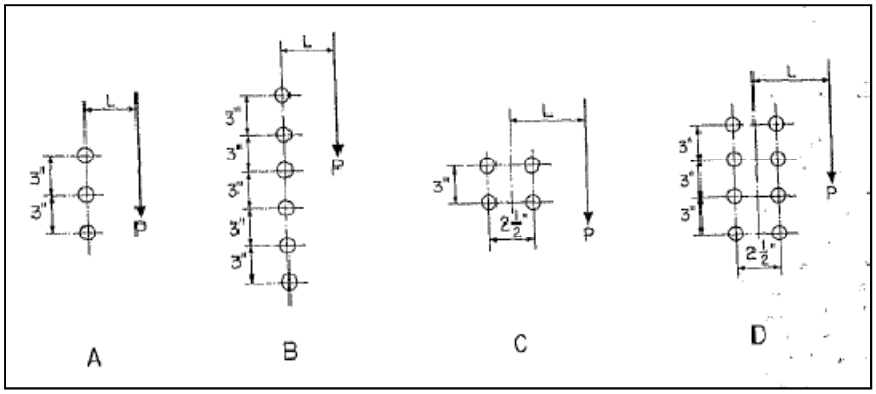


Figure 5: Rivet Patterns (Higgins 1971)

Table 2: Lehigh University Test Results (Higgins 1964)

Test Specimen	Rivet Pattern	# of rivets in one row (in)	eccentricity, e (in)	Failure Load (kips)
TP 1	A	3	2.5	216
TP 2	A	3	3.5	161
TP 3	A	3	6.5	100
TP 4	B	6	2.5	550
TP 5	B	6	4.5	440
TP 6	B	6	6.5	362
TP 7	C	2	3.5	222
TP 8	C	2	6.5	120
TP 9	D	4	3.5	568
TP 10	D	4	6.5	354

Using the elastic method, Higgins computed the nominal capacity of these connections and compared them to the test results:

Table 3: Comparison of Test Results and Elastic Method Capacities (Higgins 1964 and Higgins 1971)

Elastic Method						
Test Specimen	Rivet Pattern	# of Rivets in One Row (in)	Eccentricity, e (in)	Failure Load, P_f (kips)	Calculated Capacity, P_n (kips)	Factor of Safety = P_f/P_n
TP 1	A	3	2.5	216	49.6	4.36
TP 2	A	3	3.5	161	39.4	4.08
TP 3	A	3	6.5	100	23.4	4.27
TP 4	B	6	2.5	550	129.4	4.25
TP 5	B	6	4.5	440	97.6	4.51
TP 6	B	6	6.5	362	75.4	4.80
TP 7	C	2	3.5	222	41.8	5.30
TP 8	C	2	6.5	120	26.3	4.57
TP 9	D	4	3.5	568	116.8	4.87
TP 10	D	4	6.5	354	76.6	4.62
					Average =	**4.56**
					Std deviation =	**0.34**

The factor of safety is computed by dividing the failure load by the calculated capacity:

$$Factor\ of\ Safety = \frac{P_f}{P_n} \qquad \text{(Eq. 2-1)}$$

The elastic method gives an average factor of safety of 4.56 for these 10 tests, which is conservative since the suggested factor of safety ranges from 2.0 to 2.2 (Fisher and Beedle 1965). The current AISC Manual also uses a factor of safety of 2.0 (*Manual* 2005).

The elastic method limits the strength of the connection to the yield strength of the critical fastener. In reality, when the critical fastener reaches yield, the loads can be redistributed to the additional connectors to provide further strength in the connection. Therefore, the elastic method provides conservative results. The elastic method is still allowed by the 13th edition AISC Manual, but the Manual states, "the elastic method is simplified, but may be excessively conservative because it neglects the ductility of the bolt group and the potential for load redistribution" (*Manual* 2005). Other methods have been proposed to account for the ductility of the bolt group and the potential for load

redistribution including the modified elastic method, the plastic method and the instantaneous center method.

Modified Elastic Method

The 6th edition of the AISC Manual includes a modified elastic method for analysis of groups of bolts under eccentric shear (*Manual* 1963). The method is attributed to Higgins (Higgins 1964), and uses an effective eccentricity that is less than the distance from the center of the bolt group to the line of action of the external load as shown in Figure 6. By reducing the eccentricity, the bolt group coefficient is increased and the nominal design capacity is higher. This reduces the factor of safety since the nominal design capacity is now higher. The bolts are still assumed to behave linear-elastically and the elastic method equations are still used as is, but with the modified eccentricity.

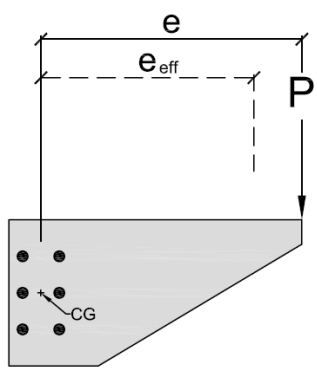


Figure 6: Illustration of Modified Elastic Method

Higgins used the results of the tests performed at Lehigh University's Fritz Engineering Laboratory in 1963 to determine an effective eccentricity, in inches, as a function of the actual eccentricity, in inches, and the number of fasteners in a single row (Higgins 1964). For fasteners equally spaced in a single column the effective eccentricity is:

$$e_{eff} = e - \frac{1 + 2n}{4} \qquad \text{(Eq. 2-2)}$$

Where: e_{eff} = effective eccentricity (inches)
e = eccentricity (inches)
n = number of bolts in one vertical row

For fasteners equally spaced in two or more columns the effective eccentricity is:

$$e_{eff} = e - \frac{1 + n}{2} \qquad \text{(Eq. 2-3)}$$

Using these equations and the results from the testing at Lehigh University (shown in Figure 5), the following safety factors are achieved:

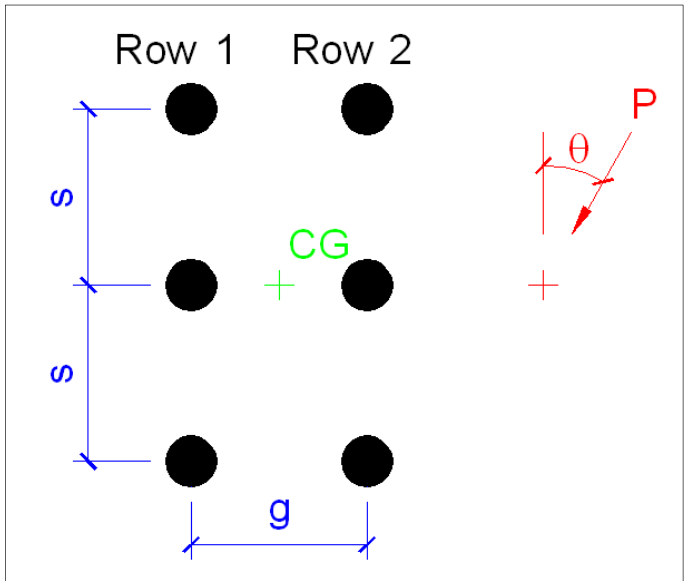
Table 4: Comparison of Test Results and Modified Elastic Method Capacities (Higgins 1964 and Higgins 1971)

Modified Elastic Method						
Test Specimen	Rivet Pattern	# of Rivets in One Row (in)	Effective Eccentricity, e_{eff} (in)	Failure Load (kips)	Calculated Capacity, P_n (kips)	Factor of Safety = P_f/P_n
TP 1	A	3	0.75	216	74.6	2.90
TP 2	A	3	1.75	161	59.9	2.69
TP 3	A	3	4.75	100	30.8	3.24
TP 4	B	6	-0.75	550	159.1	3.46
TP 5	B	6	1.25	440	150.0	2.93
TP 6	B	6	3.25	362	116.8	3.10
TP 7	C	2	2.0	222	58.0	3.83
TP 8	C	2	5.0	120	32.3	3.71
TP 9	D	4	1.0	568	183.7	3.09
TP 10	D	4	4.0	354	107.2	3.31
					Average =	**3.23**
					Std deviation =	**0.34**

By reducing the eccentricity, Higgins was able to reduce the average factor of safety, as computed in (Eq. 2-1), for these tests from 4.56 to 3.23, which is a 29.2% decrease. Thus, this method was adopted in the 6th edition of the AISC Manual (*Manual* 1963). It is unclear what factor of safety Higgins was trying to achieve. Since his equations can be scaled up or down to achieve a different result, it is likely he was comfortable with a factor of safety of around 3.0. At the time, the AISC Manuals did not provide a factor of safety for connections and there appears to have been some confusion over what should be the appropriate safety factor to use. Fisher and Beedle discussed this topic in detail and concluded that a factor of safety between 2.0 – 2.2 is appropriate (Fisher and Beedle 1965).

The 6th and 7th editions of the AISC Manual provide four design tables for bolt groups under eccentric load. Table 5 provides a summary of the 4 cases covered by the design tables in the 6th and 7th editions of the AISC Manual.

Table 5: Summary of 6th and 7th Editions AISC Design Tables for Modified Elastic Method (*Manual* 1963)



Vertical spacing, s inches	Number of Vertical Rows	Horizontal spacing, g inches	Load angle, θ degrees
3	1	N/A	0
3	2	3, 5.5	0
3	4	3-5.5-3	0

The design tables provide limited options like the tables given in the 5th edition (summarized in Table 1). The vertical spacing is still a constant value of 3-inches for every configuration. For configurations with multiple vertical rows, the design tables still provide one to two options for horizontal bolt spacing. And the loads are still applied only in the vertical downward direction. The design tables also provide the same equations given in the 5th edition, except that the effective eccentricity, e_{eff}, is used in place of actual eccentricity, e. Since the equations were relatively simple, expanded lookup tables are not required and so only the most common layouts were provided. One example of a typical design table given in AISC is shown in Figure 7.

ECCENTRIC LOADS ON FASTENER GROUPS

TABLE XII Coefficients C

Required minimum $C = \frac{P}{r_v}$

$P = C \times r_v$

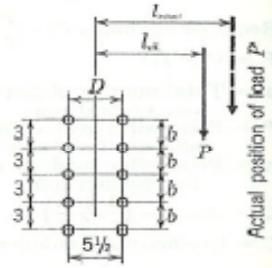
n = Total number of fasteners in any one vertical row

P = Permissible load acting with effective lever arm l_{eff}

r_v = Permissible load on one fastener by Specification

$l_{eff} = l_{actual} - \left(\frac{1+n}{2}\right)$

C = Coefficients tabulated below.



l_{eff} in.	n											
	1	2	3	4	5	6	7	8	9	10	11	12
1½	1.29	2.78	4.46	6.29	8.24	10.3	12.3	14.4	16.5	18.5	20.6	22.7
2	1.16	2.52	4.07	5.81	7.68	9.64	11.7	13.7	15.8	17.9	20.0	22.1
2½	1.05	2.29	3.74	5.37	7.15	9.06	11.0	13.1	15.2	17.2	19.3	21.4
3	.96	2.11	3.45	4.98	6.67	8.51	10.4	12.4	14.5	16.6	18.7	20.8
3½	.88	1.95	3.20	4.63	6.24	7.99	9.87	11.8	13.8	15.9	18.0	20.1
4	.81	1.81	2.98	4.32	5.84	7.52	9.33	11.2	13.2	15.2	17.3	19.4
4½	.76	1.69	2.78	4.06	5.49	7.09	8.83	10.7	12.6	14.6	16.6	18.7
5	.71	1.59	2.61	3.80	5.17	6.70	8.37	10.2	12.0	14.0	16.0	18.0
5½	.67	1.49	2.46	3.58	4.88	6.34	7.94	9.67	11.5	13.4	15.4	17.4
6	.63	1.41	2.32	3.39	4.62	6.01	7.55	9.22	11.0	12.8	14.8	16.8
6½	.59	1.34	2.20	3.21	4.38	5.71	7.19	8.80	10.5	12.3	14.2	16.2
7	.56	1.27	2.09	3.05	4.16	5.44	6.86	8.40	10.1	11.8	13.7	15.6
7½	.54	1.21	1.99	2.90	3.96	5.18	6.55	8.04	9.65	11.4	13.2	15.0
8	.51	1.15	1.90	2.77	3.78	4.95	6.26	7.70	9.26	10.9	12.7	14.5
8½	.49	1.10	1.81	2.64	3.62	4.74	6.00	7.39	8.90	10.5	12.2	14.0
9	.47	1.06	1.74	2.53	3.46	4.54	5.76	7.10	8.56	10.1	11.8	13.5
10	.43	.98	1.60	2.33	3.19	4.19	5.32	6.57	7.94	9.42	11.0	12.6
11	.40	.91	1.48	2.16	2.96	3.88	4.94	6.11	7.40	8.80	10.3	11.9
12	.37	.85	1.38	2.01	2.75	3.62	4.61	5.71	6.92	8.24	9.65	11.2
14	.33	.75	1.22	1.77	2.42	3.18	4.06	5.04	6.12	7.30	8.58	9.94
16	.29	.67	1.09	1.58	2.16	2.84	3.62	4.50	5.48	6.55	7.70	8.94
18	.26	.60	.98	1.42	1.94	2.56	3.26	4.06	4.95	5.92	6.98	8.12
20	.24	.55	.89	1.29	1.77	2.33	2.97	3.70	4.51	5.41	6.38	7.42
22	.22	.51	.82	1.19	1.62	2.13	2.73	3.40	4.14	4.97	5.87	6.84
24	.21	.47	.76	1.10	1.50	1.97	2.52	3.14	3.83	4.59	5.43	6.33

In general, $C = \dfrac{n}{\sqrt{\left[\dfrac{l_{eff}(n-1)b}{D^2 + \frac{1}{3}(n^2-1)b^2}\right]^2 + \left[\dfrac{l_{eff}D}{D^2 + \frac{1}{3}(n^2-1)b^2} + \frac{1}{2}\right]^2}}$

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Figure 7: Sample Table for Finding C given in 7th Edition of AISC Manual (*Manual* 1963)

The modified elastic method was criticized and eventually removed in the 8th Edition Manual Errata published in the Engineering Journal, Second Quarter, 1981 (Brandt 1982). Crawford and Kulak state that the modified elastic method can be criticized for the following four reasons (Crawford and Kulak 1968):

1. The number of tests (10) upon which the method is based was limited.
2. The range of eccentricities (2.5-inches to 6.5-inches) covered by the tests was limited.
3. The lack of a rational basis for the method of determining the effective eccentricity means that extrapolation beyond the range investigated was undesirable.
4. Power driven rivets were tested whereas high strength bolts are used almost exclusively in present construction methods.

Thus, the modified elastic method was removed in the 8th edition of the AISC Manual and replaced with a new method called the Instantaneous Center (I.C.) Method, also sometimes referred to as the Ultimate Strength Method.

Plastic Method

Another method that was proposed, but never implemented in the AISC Manuals, is the plastic method. It states that at failure, each fastener in the bolt group will reach its full plastic capacity regardless of its distance from the instantaneous center of rotation. This differs from the elastic and modified elastic methods, which assume that the forces in the bolts are dependent on their distance from the instantaneous center of rotation.

The plastic method was proposed by A.L. Abolitz (Abolitz 1966) and Carl L. Shermer (Shermer 1971). Since the elastic method is known to provide conservative results and the results given by both the elastic and modified elastic methods have standard deviations of 0.34 (Higgins 1971), the plastic method was proposed to obtain results that are more representative of test results. Once again the test results provided by Higgins (1964) are compared to the results given by the plastic method:

Table 6: Comparison of Test Results and Plastic Method Capacities (Higgins 1964 and Higgins 1971)

Plastic Method						
Test Specimen	Rivet Pattern	# of Rivets in One Row (in)	Eccentricity, e (in)	Failure Load, P_f (kips)	Calculated Capacity, P_n (kips)	Factor of Safety = P_f/P_n
TP 1	A	3	2.5	216	54.0	4.00
TP 2	A	3	3.5	161	43.2	3.73
TP 3	A	3	6.5	100	24.4	4.10
TP 4	B	6	2.5	550	142.1	3.87
TP 5	B	6	4.5	440	116.7	3.77
TP 6	B	6	6.5	362	94.0	3.85
TP 7	C	2	3.5	222	51.5	4.31
TP 8	C	2	6.5	120	31.8	3.77
TP 9	D	4	3.5	568	146.4	3.88
TP 10	D	4	6.5	354	97.5	3.63
					Average =	**3.89**
					Std deviation =	**0.19**

While the average factor of safety for the plastic method (3.89), as computed in (Eq. 2-1), is higher than the modified elastic method (3.23) by 20.4%, it is still 14.7% lower than the elastic method (4.56). The factor of safety resulting from the plastic method also has a standard deviation of 0.19 instead of the 0.34 standard deviation from the factor of safety resulting from the elastic methods. This means that the plastic method gives a more consistent factor of safety than both the elastic and modified elastic methods.

The plastic method was also criticized Kulak (1971; Kulak and Fisher 1967) and was never adopted by the AISC. The main criticism of the plastic method is that it does not consider the shear deformation response of the individual bolts. Typical bolt shear deformation curves are shown in Figure 8.

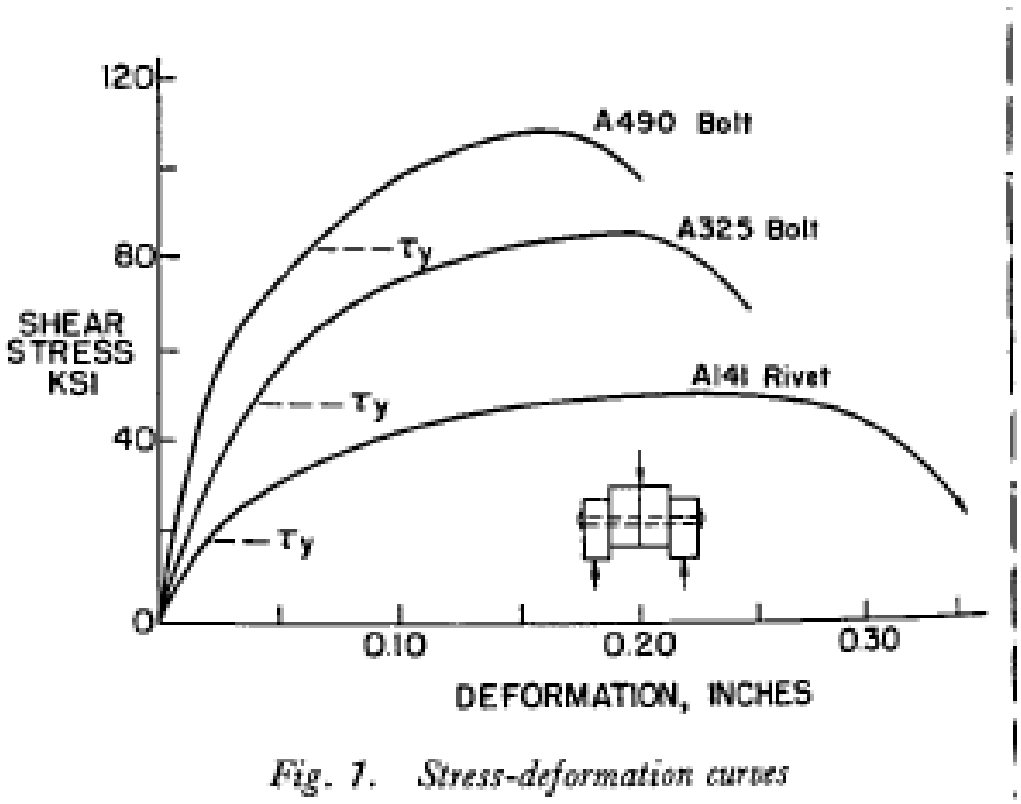


Figure 8: Shear stress-deformation curves (Kulak and Fisher 1967)

As shown in Figure 8, a shear load test is conducted by threading a bolt through multiple steel plates. A downward load is then applied to the center plate(s) to induce a direct shear into the fastener. Figure 9 shows examples of bolt failure under a shear load test. The curve in Figure 8 shows that under shear, the bolts do not have a well-defined yield point. The theoretical yield point, τ_y, is shown on the plot and it is apparent that no plateau exists at this value. Therefore, the fasteners have traditionally been assigned an allowable stress based on their ultimate shear strength (Kulak and Fisher 1967). It can also be observed that when the critical fastener reaches its maximum load, the other fasteners with

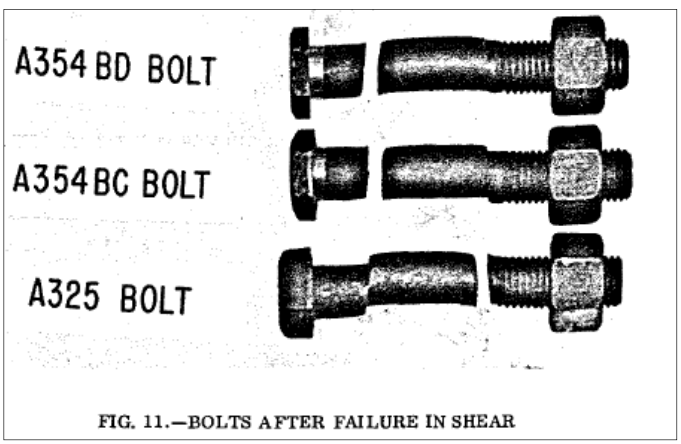


Figure 9: Bolts After Failure in Shear (Fisher & Wallaert 1965)

lesser deformation will be resisting the load with less than their ultimate capacity. The I.C. method applies these principles and was thus adopted by the AISC Manuals instead of the plastic method.

Instantaneous Center (I.C.) Method

The instantaneous center (I.C.) method is the current recommended method in the 13th Edition of the AISC Manual (*Manual* 2005). This method was developed in 1968 by S.F. Crawford and G.L. Kulak (Crawford and Kulak 1968). This method uses the inelastic load-deformation response of fasteners that was developed by Fisher (J. Fisher 1965) based on testing done by Wallaert and Fisher (Fisher and Wallaert 1965). Fisher proposed that the load-deformation response of a single fastener can be related using the following equation:

$$R = R_{ult}(1 - e^{-\mu\Delta_i})^{\lambda} \qquad \text{(Eq. 2-4)}$$

Where: R = fastener load at any given deformation
R_{ult} = ultimate load attainable by a single fastener
Δ_i = deformation of an individual bolt
μ, λ = regression coefficients
e = base of natural logarithms

Crawford and Kulak performed a series of six tests on single fasteners to verify the response given by Fisher (J. Fisher 1965) and to solve for the regression coefficients, μ and λ (Crawford and Kulak 1968). The tests were done using 3/4-inch diameter A325 bolts. The results of these tests are given in Figure 10.

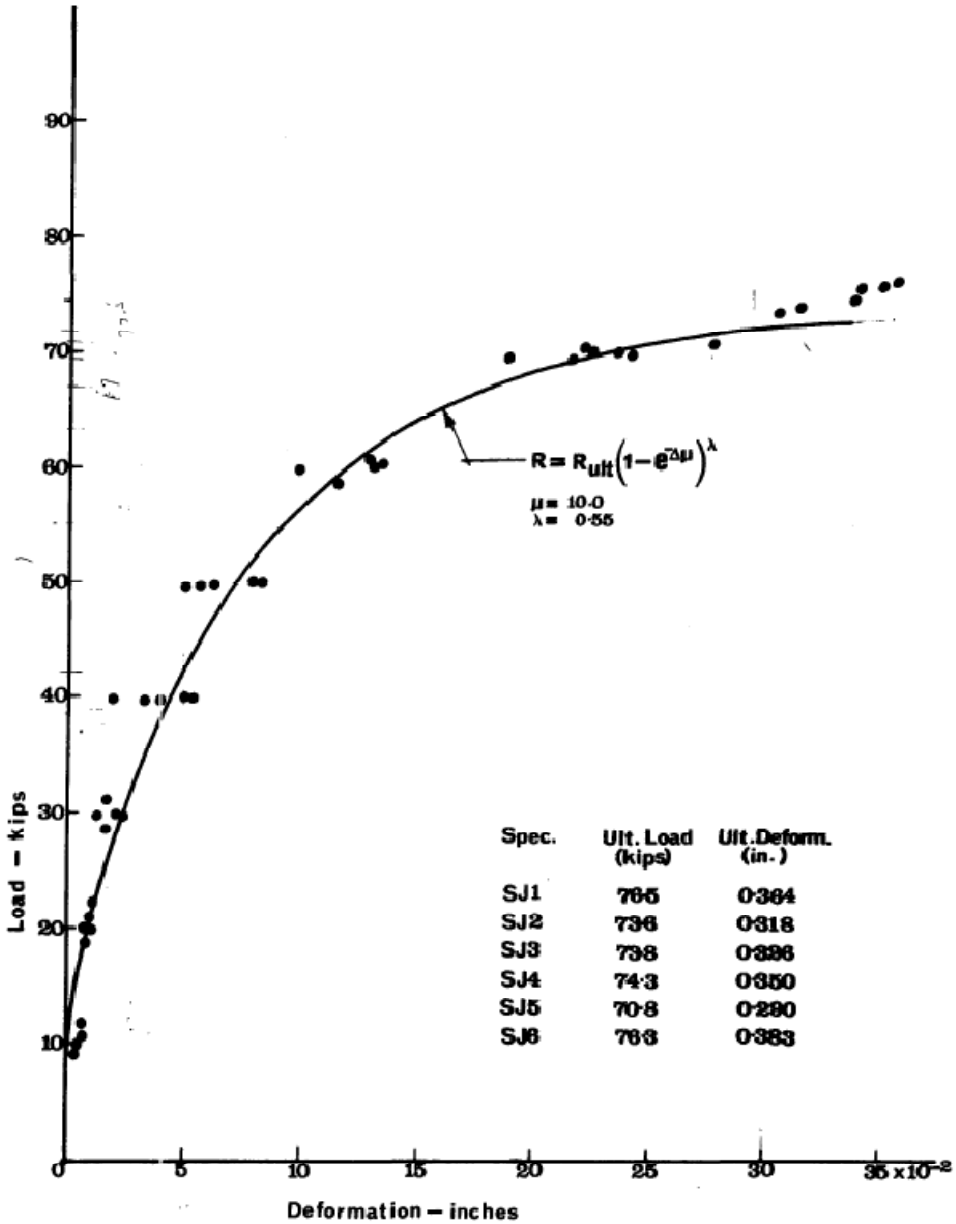


Figure 10: Load-Deformation Curves for Single Bolt Tests (Crawford and Kulak 1968)

From these tests, Crawford and Kulak proposed using regression coefficients, μ = 10.0/inch and λ = 0.55. They also determined that the ultimate capacity of a single 3/4-inch diameter A325 bolt is 74 kips at an ultimate deformation of 0.34-inches (Crawford and Kulak 1968).

The load-deformation response is used to predict the capacity of the fastener group by relating the deformation of the individual fasteners to the load that fastener exerts. The deformation of each fastener depends on its distance from the instantaneous center of rotation and each fastener carries a force that is perpendicular to the radius of rotation of the fastener as shown in Figure 11. The bolt furthest from the I.C. experiences the most deformation and therefore experiences the most force. So when the ultimate strength of the furthest fastener is reached, the capacity of the bolt group is reached.

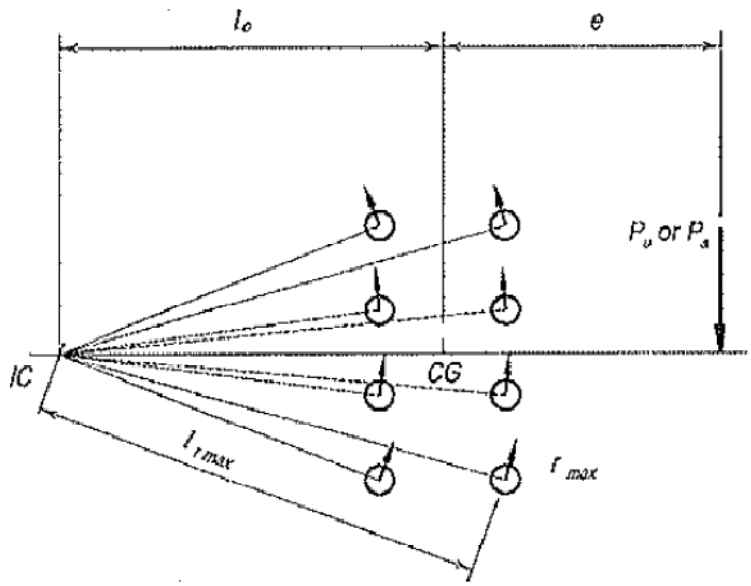


Figure 11: Deformation and Forces of Bolts for I.C. Method (*Manual* 2005)

Crawford and Kulak developed this method by not only comparing it to the testing done in 1963 at Lehigh University (Higgins 1964), but by also testing a total of sixteen specimens in eight different configurations (Crawford and Kulak 1968). A diagram of these configurations is shown in Figure 12. Tests were performed using 3/4-inch A325 bolts. The number of bolts per line varied from four to six and the load eccentricity ranged from 8-inches to 15-inches. The results of these tests are summarized in Table 7.

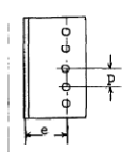
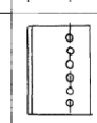
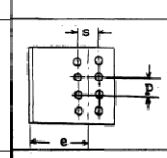
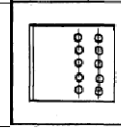
Specimen Number	Bolt Group	Eccentricity e (ins.)	p (ins.)	s (ins.)
B1		8	2-1/2	-
B2		10	3	-
B3		12	3	
B4		13	3	-
B5		15	3	-
B6		12	3	2-1/2
B7		15	3	2-1/2
B8		15	2-1/2	2-1/2

Figure 12: Diagram of Test Specimens (Crawford and Kulak 1968)

Table 7: Results from Crawford and Kulak Tests (Crawford and Kulak 1968)

Test Specimen	# of rivets in one row (in)	eccentricity, e (in)	Vertical spacing (in)	Horizontal spacing (in)	Failure Load, P_f (kips)
B1	5	8	2.5	N/A	225
B2	5	10	3	N/A	230
B3	5	12	3	N/A	190
B4	6	13	3	N/A	251
B5	6	15	3	N/A	221
B6	4	12	3	2.5	264
B7	4	15	3	2.5	212
B8	5	15	2.5	2.5	266

Crawford and Kulak then calculate a predicted load using the I.C. method and compare it to the test results, which are shown in Table 8:

Table 8: Comparison of Crawford and Kulak Test Results with I.C. Method (Crawford and Kulak 1968)

I.C. Method - Crawfurd and Kulak Test Specimens							
Test Specimen	# of rivets in one row (in)	eccentricity, e (in)	Vertical spacing (in)	Horizontal spacing (in)	Failure Load, P_f (kips)	Predicted Load, P_{ult} (kips)	P_f/P_{ult}
B1	5	8	2.5	N/A	225	252	0.894
B2	5	10	3	N/A	230	244	0.945
B3	5	12	3	N/A	190	206	0.924
B4	6	13	3	N/A	251	274	0.916
B5	6	15	3	N/A	221	239	0.925
B6	4	12	3	2.5	264	293	0.901
B7	4	15	3	2.5	212	239	0.885
B8	5	15	2.5	2.5	266	309	0.860
						Average =	**0.906**
						Std deviation =	**0.03**

Since equation (Eq. 2-4) is based on the ultimate strength of the individual fastener, the I.C. method gives a prediction of the ultimate failure load. The failure load divided by the predicted load provided by the I.C. method should then be close to unity. Table 8 shows that the I.C. method predicted the failure load to within 10-percent of unity on average with a standard deviation of 0.03. That means that the I.C.

method provides predicted loads that are consistently 10-percent conservative of the actual failure loads.

Crawford and Kulak also compare the results of the testing performed at Lehigh University against the I.C. method as follows:

Table 9: Comparison of Test Results and I.C. Method Capacities (Crawford and Kulak 1968)

I.C. Method - Lehigh Test Specimens						
Test Specimen	Rivet Pattern	# of rivets in one row (in)	eccentricity, e (in)	Failure Load, P_f (kips)	Predicted Load, P_{ult} (kips)	P_f/P_{ult}
TP 1	A	3	2.5	216	210.0	1.03
TP 2	A	3	3.5	161	166.0	0.97
TP 3	A	3	6.5	100	96.0	1.04
TP 4	B	6	2.5	550	566.0	0.97
TP 5	B	6	4.5	440	454.0	0.97
TP 6	B	6	6.5	362	358.0	1.03
TP 7	C	2	3.5	222	190.0	1.17
TP 8	C	2	6.5	120	115.0	1.04
TP 9	D	4	3.5	568	561.0	1.01
TP 10	D	4	6.5	354	367.0	0.96
					Average =	**1.02**
					Std deviation =	**0.06**

In the case of the specimens tested at Lehigh University, the I.C. method predicts the failure load to within 2-percent on average with a standard deviation of 0.06. The equations behind the methods will be reviewed in detail in the next chapter, but it is important to note the differences among the various methods and why the I.C. method is currently preferred.

Summary of Different Methods

When the elastic, modified elastic, plastic and I.C. methods are compared against each other, there is only one notable difference. All of the methods recognize that the individual bolts resist an applied eccentric load by sharing the portion of the load caused by direct shear and then have some contribution to the applied moment. The methods differ in the load-deformation models for bolts. The

elastic method and modified elastic method are the same method, except that the modified elastic method uses a reduced eccentricity. The real differences are between the elastic methods, plastic method and the I.C. method.

The elastic methods assume that the individual fasteners resist the moment by behaving linear-elastically. Thus, the capacity of the bolt group is reached when the fastener farthest from the elastic centroid of the bolt group (CG) reaches its yield stress. The plastic method assumes that all of the fasteners reach their ultimate load value regardless of their distance from the center of the group. The capacity of the bolt group is reached when the all of the fasteners are at this ultimate strength. The I.C. method assumes that bolts have a nonlinear stress-strain response. The critical fastener located furthest from the instantaneous center of rotation is given a maximum limit equal to the ultimate strength and deflection. The remaining fasteners are then evaluated for the resultant load based on their distance from the instantaneous center of rotation. Only the critical fastener will reach its failure load while remaining fasteners will have resultant forces less than failure.

Plotting the different methods on a load-deformation plot better illustrates the differences between the methods, as can be seen in Figure 13. The response of the bolts for the elastic and modified elastic methods is linear-elastic. Both the load and deformation of each individual bolt is related to its distance from the instantaneous center of rotation. The plastic method is an upper limit on the overall strength of the bolt group since it assumes that every bolt will reach an ultimate force level. The I.C. method follows a nonlinear force-displacement curve for bolts. The critical bolt reaches the upper load and deformation limits. The remaining fasteners are located somewhere on the curve below this limit.

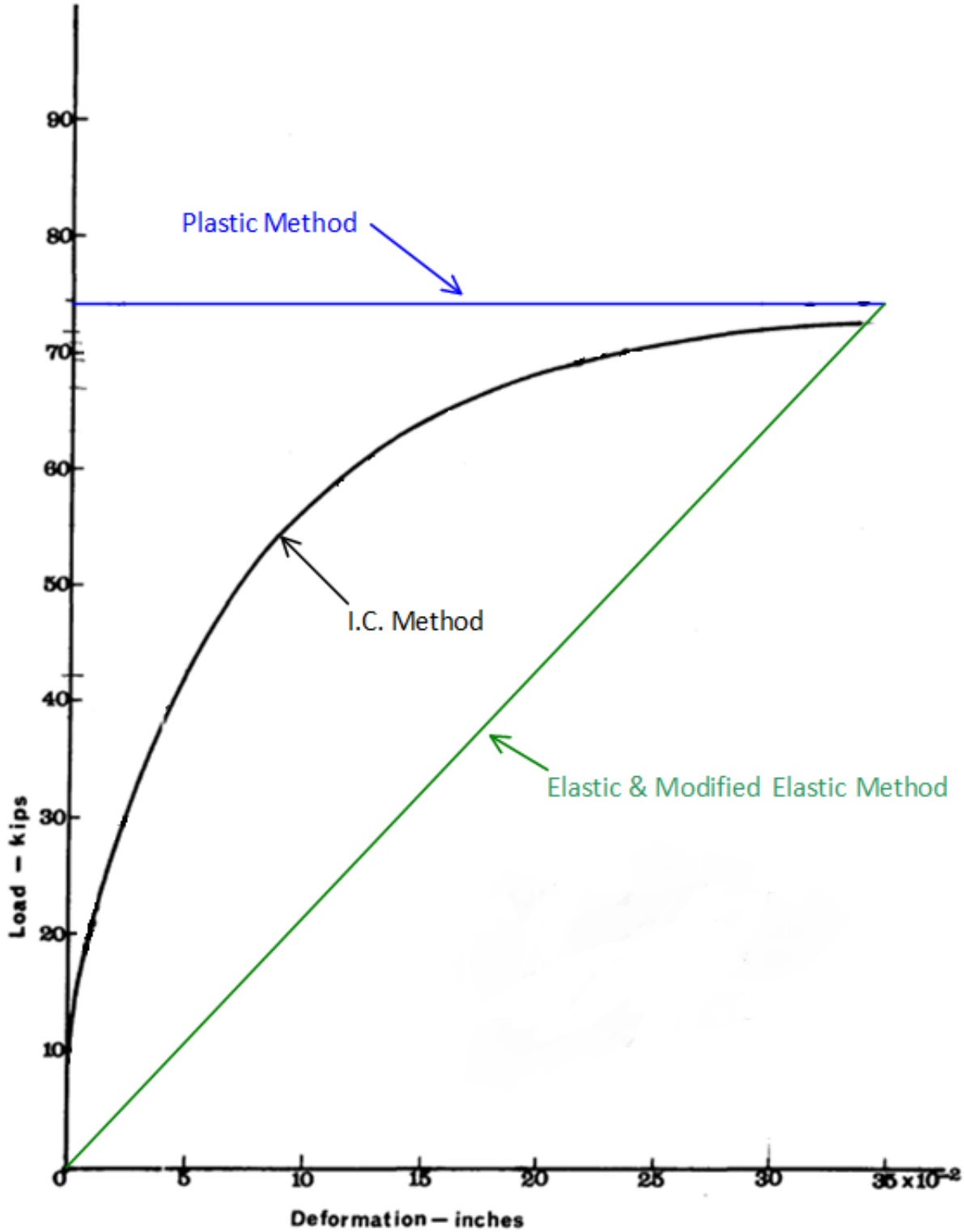
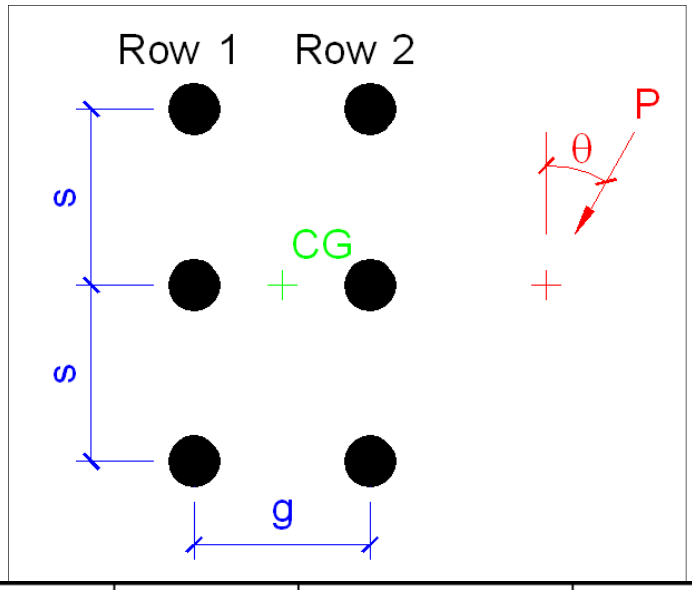


Figure 13: Load-Deformation Curve for a Single Bolt (*adapted from* Crawford and Kulak 1968)

Purpose of Software

The current AISC Manual (13th Edition) recommends the I.C. method for design purposes since it is more accurate. However, the I.C. method requires the use of pre-computed tables or an iterative solution and is therefore more complicated. The AISC Manual provides tables that represent 84 different bolt and load combinations that are commonly used in design. Table 10 is a summary of the 84 cases represented in the design tables in the 13th of the AISC Manual.

Table 10: Summary of 13th Edition AISC Design Tables for I.C. Method (*Manual* 2005)



Vertical spacing, s inches	Number of Vertical Rows	Horizontal spacing, g inches	Load angle, θ degrees
3, 6	1	N/A	0, 15, 30, 45, 60 ,75
3, 6	2	3, 5.5, 8	0, 15, 30, 45, 60 ,75
3, 6	3	3, 6	0, 15, 30, 45, 60 ,75
3, 6	4	3	0, 15, 30, 45, 60 ,75

When compared to the design tables provided in previous versions of the AISC Manuals (summarized in Table 1 and Table 5) it is evident that the design tables have been significantly expanded. A total of 84 combinations are now represented instead of the four that were previously provided. This is due largely in part to the difference between the elastic method and the I.C. method. The equations given by the elastic method are simple and allow the designer to calculate the capacity for any bolt group without the use of a lookup table. However, the I.C. method is not as simple and so the design tables have been

significantly expanded to include more spacing options as well as both vertical and inclined applications of load. One example of a typical design table given in AISC is shown in Figure 14.

Table 7–7 (continued)

Coefficients C for Eccentrically Loaded Bolt Groups

Angle = 15°

Available Strength of a bolt group, ϕR_n or R_n/Ω, is determined with

$$R_n = C \times r_n$$

$\phi = 0.75$ $\Omega = 2.00$

or

LRFD	ASD
$C_{min} = \frac{P_u}{\phi r_n}$	$C_{min} = \frac{\Omega P_a}{r_n}$

where

P = required force, P_u or P_a, kips

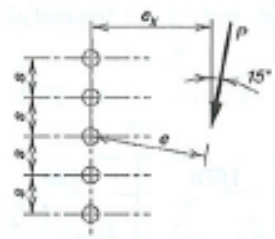
r_n = nominal strength per bolt, kips

e = eccentricity of P with respect to centroid of bolt group, in. (not tabulated, may be determined by geometry)

e_x = horizontal component of e, in.

s = bolt spacing, in.

C = coefficient tabulated below



s, in.	e_x, in.	Number of Bolts in One Vertical Row, n										
		2	3	4	5	6	7	8	9	10	11	12
3	2	1.15	2.20	3.28	4.34	5.39	6.42	7.45	8.46	9.47	10.5	11.5
	3	0.86	1.76	2.78	3.85	4.92	5.98	7.03	8.08	9.11	10.1	11.2
	4	0.67	1.42	2.35	3.36	4.41	5.48	6.55	7.61	8.67	9.72	10.8
	5	0.55	1.17	2.00	2.94	3.94	4.98	6.04	7.11	8.18	9.24	10.3
	6	0.47	0.99	1.73	2.58	3.52	4.52	5.55	6.61	7.67	8.74	9.81
	7	0.41	0.86	1.52	2.30	3.16	4.11	5.10	6.13	7.18	8.24	9.30
	8	0.36	0.75	1.35	2.06	2.86	3.74	4.69	5.68	6.70	7.74	8.80
	9	0.32	0.67	1.22	1.86	2.60	3.43	4.32	5.27	6.26	7.28	8.31
	10	0.29	0.61	1.10	1.69	2.38	3.16	4.00	4.90	5.85	6.84	7.85
	12	0.24	0.51	0.93	1.43	2.03	2.71	3.46	4.28	5.15	6.06	7.01
	14	0.21	0.43	0.81	1.24	1.76	2.37	3.04	3.78	4.57	5.41	6.30
	16	0.19	0.38	0.71	1.09	1.56	2.10	2.70	3.37	4.09	4.87	5.69
	18	0.17	0.34	0.63	0.97	1.39	1.88	2.43	3.04	3.70	4.42	5.18
	20	0.15	0.3	0.57	0.88	1.26	1.70	2.20	2.76	3.37	4.03	4.74
	24	0.12	0.25	0.48	0.73	1.06	1.43	1.86	2.33	2.86	3.43	4.04
	28	0.11	0.22	0.41	0.63	0.91	1.23	1.60	2.02	2.47	2.97	3.51
	32	0.09	0.19	0.36	0.55	0.80	1.08	1.41	1.77	2.18	2.62	3.10
	36	0.08	0.17	0.32	0.49	0.71	0.96	1.26	1.58	1.95	2.34	2.78

Figure 14: Sample Table for Finding C given in 13th Edition of AISC Manual (*Manual* 2005)

If the actual design condition does not fall into one of the combinations given in the design tables the capacity of the bolt group must be calculated by hand. This requires using either the I.C. method, which requires an iterative approach, or the elastic method, which is simpler, but provides conservative results. Either option is not very appealing for the designer. Both the additional time needed for an iterative computation and a conservative design lead to higher project costs. Therefore, a

design tool to allow for a wider range of bolt and load combinations would allow the designer to produce more cost-effective designs. The software developed in this thesis allows the designer to choose any pattern of bolts and any orientation of load. Thus, the designer is not limited by the tables and has a quick and easy application to perform calculations that lead to more cost effective designs.

Chapter 3 - Methods of Analysis of Bolt Groups

Introduction

Three methods are used to solve the problem of an eccentrically loaded bolt group. These are the elastic method, the I.C. method and the plastic method. The modified elastic method is the same as the elastic method but uses a reduced eccentricity for loads. All three methods can be solved by using a similar process. Equilibrium equations relate the applied force to the forces in the bolts. The forces in the individual bolts are related to the deformation of the individual bolts. The methods differ in the load-deformation response of bolts. This chapter reviews the equations behind each of the methods, which are used in developing the software.

Equations

Setup Info

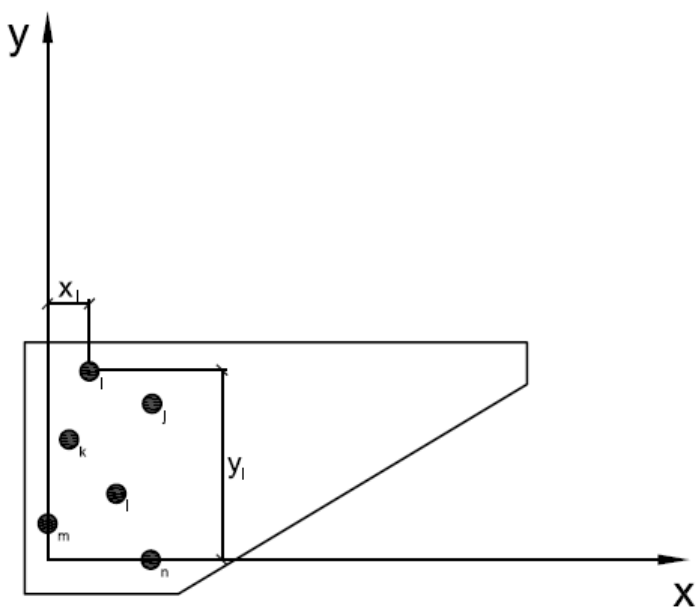


Figure 15: Example Bolt Group Layout

A set of bolts, *i, j, k...* located in any arbitrary pattern has coordinates:

$$\begin{Bmatrix} x_i & y_i \\ x_j & y_j \\ x_k & y_k \\ . & . \\ . & . \\ . & . \end{Bmatrix}$$

The origin for the bolts can be set anywhere and the analysis will not change. To keep the bolt coordinates positive, the origin is set at the intersection of the lowest bolt and the bolt that is farthest left as shown in Figure 15. The elastic center of the bolt group is found based on the locations of each bolt and total number of the bolts:

$$x_c = \frac{\sum x_i}{N} \qquad \text{(Eq. 3-1)}$$

and

$$y_c = \frac{\sum y_i}{N} \qquad \text{(Eq. 3-2)}$$

Where: x_c = x-coordinate of the elastic center of the bolt group
y_c = y-coordinate of the elastic center of the bolt group
x_i = x-coordinates of individual bolts
y_i = y-coordinates of individual bolts
N = total number of bolts

When a load is applied to the bolt group, the load magnitude and orientation are defined as follows:

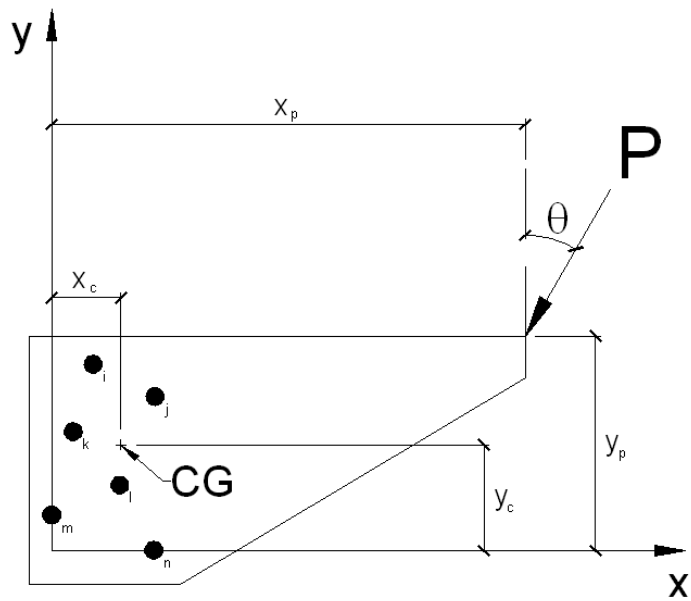


Figure 16: Load Applied to Bolt Group

Where: P = magnitude of load
θ = orientation of load (clockwise positive)
x_p = x-coordinate of load
y_p = y-coordinate of load

The applied load can be represented by a load vector {P}:

$$\{P\} = \begin{Bmatrix} P_x \\ P_y \\ M_P \end{Bmatrix} = \begin{Bmatrix} -Psin\theta \\ -Pcos\,\theta \\ Psin\theta(y_P - y_c) - Pcos\,\theta\,(x_P - x_c) \end{Bmatrix} \quad \text{(Eq. 3-3)}$$

Where: P_x = x-component of the load P (positive to the right)
P_y = y-component of the load P (positive up)
M_P = Moment caused by the load P (counterclockwise positive)

Eccentricity, e, is defined as the normal distance from the elastic centroid of the bolt group to the line of action of the applied load as shown in Figure 17. For simplicity, the bolt group coefficient is typically found using the horizontal component of eccentricity, e_x.

$$e_x = x_p - x_c \quad \text{(Eq. 3-4)}$$

Where: e_x = horizontal component of eccentricity

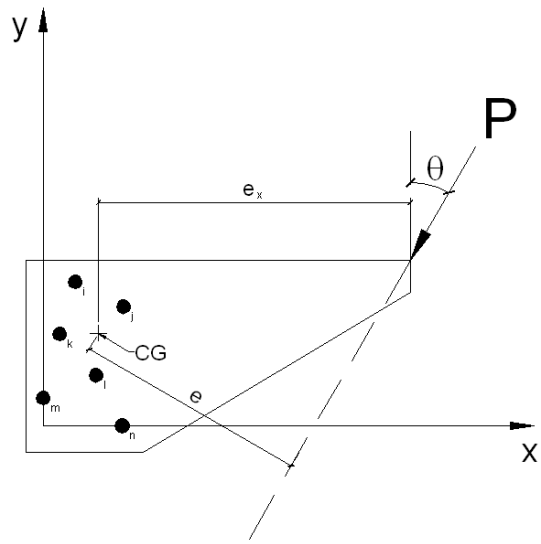


Figure 17: Eccentricity

The applied load causes a translation in the x and y directions and rotation of the connection plate as shown. These translations and rotations are constant along the length of the plate so they can be measured at any point along the plate.

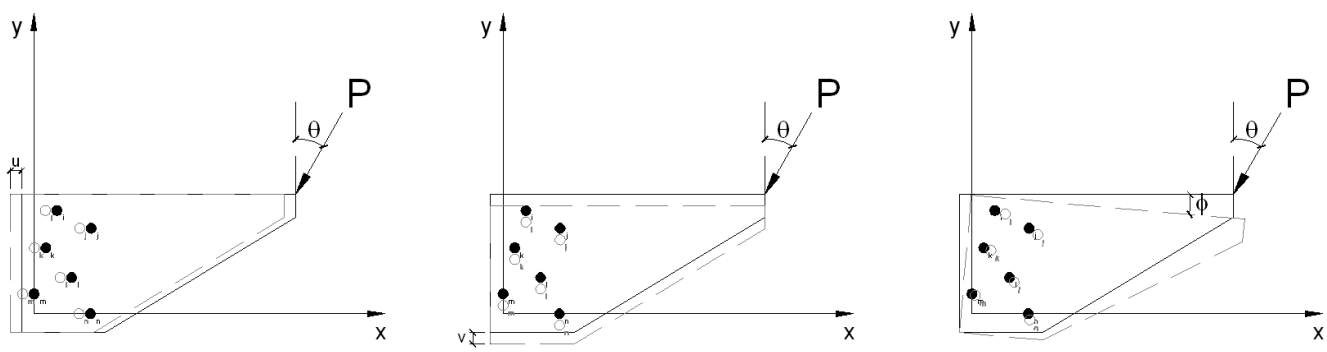


Figure 18: Translation and Rotation of Connection Plate

Where: u = translation of the connection plate in the x-direction
v = translation of the connection plate in the y-direction
ϕ = rotation of the connection plate

The point at which the connection plate experiences no vertical or horizontal translation, only rotation, is called the instantaneous center of rotation. This point can be found for the elastic, plastic and I.C. methods. The location of the instantaneous center of rotation is not required for determining the capacity of a bolt group. The coordinates of the instantaneous center of rotation can be defined as (x_r,

y_r). The translation and rotation in the connection plate, as defined in Figure 18, occurs simultaneously on the plate from the applied load, as shown in Figure 19.

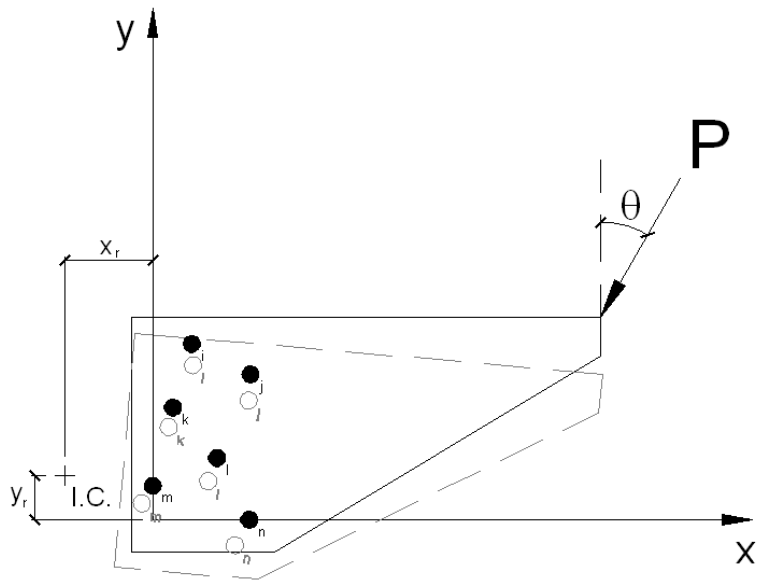


Figure 19: Coordinates of Instantaneous Center of Rotation

The translation and rotation of the plate causes shear deformations in the supporting bolts. Since the location of the instantaneous center of rotation is unknown, the analysis begins by assuming that it is located at the center of the bolt group. Horizontal and vertical components of the bolt deformations are related to the horizontal and vertical translations in the plate as well as the plate rotation.

$$\Delta_{xi} = u - (y_i - y_c)\phi \qquad \text{(Eq. 3-5)}$$

$$\Delta_{yi} = v + (x_i - x_c)\phi \qquad \text{(Eq. 3-6)}$$

Where: Δ_{xi} = x-component of an individual bolt deformation
Δ_{yi} = y-component of an individual bolt deformation

These horizontal and vertical components are combined to find the total deformation for each bolt.

$$\Delta_i = \sqrt{(\Delta_{xi})^2 + (\Delta_{yi})^2} \qquad \text{(Eq. 3-7)}$$

Where: Δ_i = deformation of an individual bolt

Each bolt has a relative distance, d_i, from the instantaneous center of rotation:

$$d_i = \sqrt{(x_i - x_r)^2 - (y_i - y_r)^2} \qquad \text{(Eq. 3-8)}$$

AISC sets a maximum deformation limit of 0.34-inches for a ¾" diameter, A325 bolt. This maximum deformation occurs in the bolt located furthest from the instantaneous center of rotation. Therefore, the individual bolt deformations are scaled based on their distance from the instantaneous center of rotation.

$$\Delta_i = \frac{d_i}{d_{max}} \Delta_{max} \qquad \text{(Eq. 3-9)}$$

Where: Δ_{max} = AISC defined maximum deformation a single bolt can achieve (0.34-inches)
d_i = distance of an individual bolt from the instantaneous center of rotation
d_{max} = maximum bolt distance from the instantaneous center of rotation

The equilibrium relation for the bolts under the applied exterior load is:

$$\{P\} + R_{ult}[D]\begin{Bmatrix} u \\ v \\ \Phi \end{Bmatrix} = 0 \qquad \text{(Eq. 3-10)}$$

Where: {P} = vector of loads P_x, P_y and M_p
R_{ult} = ultimate load attainable by a single fastener
[D] = matrix of coefficients related to geometry of the bolt group and relative deformations in bolts

The equilibrium equation can be rewritten as

$$\frac{P}{R_{ult}}\begin{Bmatrix} -\sin\theta \\ -\cos\theta \\ \sin\theta(y_P - y_c) - \cos\theta\,(x_P - x_c) \end{Bmatrix} + [D]\begin{Bmatrix} u \\ v \\ \Phi \end{Bmatrix} = 0 \qquad \text{(Eq. 3-11)}$$

Note that when P is removed from the vector {P} only geometric terms remain. Therefore, the values do not change throughout the analysis unless the locations of the bolts or the application of the load

changes. Also, note that the bolt group coefficient, C, is defined in the AISC as the load-to-ultimate bolt force ratio. It does not depend on the properties of the bolts including size, alloy or thread condition. It is also not dependent on the shear strength of the individual bolts.

$$C = \frac{P}{R_{ult}} \qquad \text{(Eq. 3-12)}$$

And

$$C\begin{Bmatrix} -\sin\theta \\ -\cos\theta \\ \sin\theta(y_P - y_c) - \cos\theta\,(x_P - x_c) \end{Bmatrix} + [D]\begin{Bmatrix} u \\ v \\ \Phi \end{Bmatrix} = 0 \qquad \text{(Eq. 3-13)}$$

Equation (Eq. 3-13) can be written

$$C\{\rho\} + [D]\{U\} = 0 \qquad \text{(Eq. 3-14)}$$

Where: {ρ} = vector of geometric terms related to load position and attitude
{U} = vector of kinematic variables u, v, Φ

Process for Calculating C

Once the locations of the bolts and the location and direction of the applied load have been identified, the bolt group coefficient, C, can be calculated. C is found by rearranging the equation (Eq. 3-14) and using the magnitude of the vectors:

$$C = \frac{|[D]\{U\}|}{|\{\rho\}|} \qquad \text{(Eq. 3-15)}$$

Note that the value of C does not depend on the strength of the individual bolts. Also note that equation (Eq. 3-15) cannot be solved directly. The kinematic variables $\{U\}$ are not known, and the matrix of bolt coefficients $[D]$ depends on kinematic variables $\{U\}$. The solution is found by iteration. Starting with assumed values for C_n and for kinematic variables $\{U\}_n$, a new vector of kinematic variables is computed $\{U^*\}_{n+1}$, and then scaled to enforce limits on bolt deformation. The process is repeated until

the kinematic variables and C no longer change. Matrix $[D]$ is updated for new kinematic variables in each iteration.

$$\{U^*\}_{n+1} = -[D]_n{}^{-1} C_n \{\rho\} \qquad \text{(Eq. 3-16)}$$

Where: $\{U^*\}_{n+1}$ = new vector of kinematic variables not constrained by a limit on bolt deformation
$[D]_n$ = matrix of bolt coefficients based on the assumed kinematic variables $\{U\}_n$
C_n = assumed initial value of bolt group coefficient

Since C is not known initially, it is assumed to be 1.00 for the first iteration. Using these kinematic variables, the bolt deformations are found using equations (Eq. 3-5) and (Eq. 3-6). The resultant bolt deformation is found using the equation (Eq. 3-7). Since a solution is sought such that the greatest bolt deformation does not exceed Δ_{max} (0.34-inches), the limit used by AISC, a scaling factor, κ is defined:

$$\kappa = \frac{\Delta_{max}}{Max[(\Delta_i)_n]} \qquad \text{(Eq. 3-17)}$$

Where: κ = scaling factor
Δ_{max} = AISC defined maximum deformation a single bolt can achieve (0.34-inches)
$(\Delta_i)_n$ = deformation of an individual bolt based on the assumed kinematic variables $\{U\}_n$
$Max[(\Delta_i)_n]$ = value of maximum deformation of an individual bolt deformation

The bolt deformations Δ_i as well as the vector $\{U\}$ are scaled using this scaling factor:

$$(\Delta_i)_{n+1} = (\Delta_i)_n * \kappa \qquad \text{(Eq. 3-18)}$$

$$\{U\}_{n+1} = \{U^*\}_{n+1} * \kappa \qquad \text{(Eq. 3-19)}$$

Where: $(\Delta_i)_{n+1}$ = iteration of the deformation of an individual bolt
$\{U\}_{n+1}$ = iteration of the vector of kinematic variables u, v, Φ with scaling factor applied

A new value for the bolt group coefficient C, can then be calculated:

$$C_{n+1} = \frac{|[D]_n\{U\}_{n+1}|}{|\{\rho\}|} \qquad \text{(Eq. 3-20)}$$

Where: C_{n+1} = iteration of the bolt group coefficient

The process is repeated until C reaches a stable value (within a specific tolerance). Once the process is complete, the location of the instantaneous center of rotation can be found:

$$x_r = x_c - \frac{v}{\Phi} \qquad \text{(Eq. 3-21)}$$

and

$$y_r = y_c + \frac{u}{\Phi} \qquad \text{(Eq. 3-22)}$$

This same process is used for all three methods. The difference in the methods is in how the different methods define the load-deformation response of the individual bolts. This yields a different matrix [D] for each method. The definition of [D] for each method is shown in the following sections.

Elastic Method

Bolt Force Components

The elastic method, unlike the plastic or I.C. method, does not require an iterative solution. This is due to the fact that the elastic method is based on a linear response based on the deformation of the bolts. Therefore, the bolt forces are based on a ratio of the bolt deformation to the maximum bolt deformation. The components of force for any bolt are

$$F_{xi} = R_e \frac{\Delta_{xi}}{\Delta_i} \qquad \text{(Eq. 3-23)}$$

and

$$F_{yi} = R_e \frac{\Delta_{yi}}{\Delta_i} \qquad \text{(Eq. 3-24)}$$

Where: F_{xi} = x-component of the force in an individual bolt
F_{yi} = y-component of the force in an individual bolt
R_e = elastic force of a single fastener, which is computed as follows:

$$R_e = R_{ult}\frac{\Delta_i}{\Delta_{max}} \qquad \text{(Eq. 3-25)}$$

The resultant elastic force of a single fastener is the linear ratio of the individual bolt deformation to the maximum bolt deformation. This corresponds with the assumed linear-elastic load deformation response of the individual bolts for this method. The coefficients of the matrix [D] are found by solving the equilibrium equations.

Moment Equilibrium

Summing moments about the center of the bolt group yields:

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum F_{xi}(y_i - y_c) - \sum F_{yi}(x_i - x_c) \qquad \text{(Eq. 3-26)}$$

The horizontal and vertical bolt forces are represented by the bolt deflections, ultimate force and maximum deformation using equations (Eq. 3-23), (Eq. 3-24) and (Eq. 3-25):

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum R_{ult}\frac{\Delta_{xi}}{\Delta_{max}}(y_i - y_c) - \sum R_{ult}\frac{\Delta_{yi}}{\Delta_{max}}(x_i - x_c) \qquad \text{(Eq. 3-27)}$$

The horizontal and vertical bolt deformations are substituted using equations (Eq. 3-5) and (Eq. 3-6):

$$\begin{aligned} 0 = -P_x(y_p - y_c) + P_y(x_p - x_c) &+ \sum R_{ult}\frac{u - (y_i - y_c)\Phi}{\Delta_{max}}(y_i - y_c) \\ &- \sum R_{ult}\frac{v + (x_i - x_c)\Phi}{\Delta_{max}}(x_i - x_c) \end{aligned} \qquad \text{(Eq. 3-28)}$$

Horizontal Equilibrium

Summing forces in the x-direction yields:

$$0 = P_x - \sum F_{xi} \qquad \text{(Eq. 3-29)}$$

Substituting for the horizontal bolt force using equations (Eq. 3-23) and (Eq. 3-25) and the horizontal bolt deformation using equation (Eq. 3-5):

$$0 = P_x - \sum R_{ult} \frac{u - (y_i - y_c)\Phi}{\Delta_{max}} \quad \text{(Eq. 3-30)}$$

Vertical Equilibrium

Summing forces in the y-direction yields:

$$0 = P_y - \sum F_{yi} \quad \text{(Eq. 3-31)}$$

Substituting for the horizontal bolt force using equations (Eq. 3-24) and (Eq. 3-25) and the horizontal bolt deformation using equation (Eq. 3-6):

$$0 = P_y - \sum R_{ult} \frac{v + (x_i - x_c)\Phi}{\Delta_{max}} \quad \text{(Eq. 3-32)}$$

Combined Equilibrium Equations

Inserting these equations into the equilibrium relation equation (Eq. 3-10):

$$\begin{Bmatrix} P_x \\ P_y \\ -P_x(y_p - y_c) + P_y(x_p - x_c) \end{Bmatrix} + R_{ult} \begin{bmatrix} -\sum \frac{1}{\Delta_{max}} & 0 & \sum \frac{1}{\Delta_{max}}(y_i - y_c) \\ 0 & -\sum \frac{1}{\Delta_{max}} & -\sum \frac{1}{\Delta_{max}}(x_i - x_c) \\ \sum \frac{1}{\Delta_{max}}(y_i - y_c) & -\sum \frac{1}{\Delta_{max}}(x_i - x_c) & -\sum\left(\frac{1}{\Delta_{max}}(y_i - y_c)^2\right) - \sum\left(\frac{1}{\Delta_{max}}(x_i - x_c)^2\right) \end{bmatrix} \{U\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(Eq. 3-33)}$$

Isolating P and dividing by R_{ult} to get C as shown in equation (Eq. 3-11) yields the following:

$$\frac{\mathrm{P}}{\mathrm{R_{ult}}}\{\rho\} + \begin{bmatrix} -\sum \frac{1}{\Delta_{max}} & 0 & \sum \frac{1}{\Delta_{max}}(y_i - y_c) \\ 0 & -\sum \frac{1}{\Delta_{max}} & -\sum \frac{1}{\Delta_{max}}(x_i - x_c) \\ \sum \frac{1}{\Delta_{max}}(y_i - y_c) & -\sum \frac{1}{\Delta_{max}}(x_i - x_c) & -\sum\left(\frac{1}{\Delta_{max}}(y_i - y_c)^2\right) - \sum\left(\frac{1}{\Delta_{max}}(x_i - x_c)^2\right) \end{bmatrix} \{U\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(Eq. 3-34)}$$

The matrix [D] for the elastic method is defined in equation (Eq. 3-34). Notice that the matrix [D] is a function of the limit Δ_{max} and the coordinates of the individual bolts, so no iterative process is required. Therefore, once the locations of the bolts and loads are known, the process for calculating C is complete.

Plastic Method

Bolt Force Components

According to the plastic method, the bolt force for any bolt that experiences non-zero deformations is the maximum force R_{ult}. Therefore, the bolt force components are based on the ratio of the deformation components:

$$F_{xi} = R_{ult}\frac{\Delta_{xi}}{\Delta_i} \qquad \text{(Eq. 3-35)}$$

and

$$F_{yi} = R_{ult}\frac{\Delta_{yi}}{\Delta_i} \qquad \text{(Eq. 3-36)}$$

The only difference between these equations and those given by the elastic method, (Eq. 3-23) and (Eq. 3-24), is that instead of using an elastic force, R_e, related to a linear deformation response of the individual bolts, the equations are based on the maximum bolt force, R_{ult}, since the plastic method assumes that every bolt reaches its ultimate capacity. The values of the matrix [D] are found by solving the equilibrium equations.

Moment Equilibrium

Summing moments about the center of the bolt group yields:

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum F_{xi}(y_i - y_c) - \sum F_{yi}(x_i - x_c) \qquad \text{(Eq. 3-37)}$$

Substituting the values from equations (Eq. 3-35) and (Eq. 3-36):

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum R_{ult}\frac{\Delta_{xi}}{\Delta_i}(y_i - y_c) - \sum R_{ult}\frac{\Delta_{yi}}{\Delta_i}(x_i - x_c) \qquad \text{(Eq. 3-38)}$$

The horizontal and vertical bolt deformations are substituted using equations (Eq. 3-5) and (Eq. 3-6):

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum R_{ult} \frac{u - (y_i - y_c)\Phi}{\Delta_i}(y_i - y_c) - \sum R_{ult} \frac{v + (x_i - x_c)\Phi}{\Delta_i}(x_i - x_c) \quad \text{(Eq. 3-39)}$$

Horizontal Equilibrium

Summing forces in the x-direction yields:

$$0 = P_x - \sum F_{xi} \quad \text{(Eq. 3-40)}$$

Substituting for the horizontal bolt force using equations (Eq. 3-35) and the horizontal bolt deformation using equation (Eq. 3-5):

$$0 = P_x - \sum R_{ult} \frac{u - (y_i - y_c)\Phi}{\Delta_i} \quad \text{(Eq. 3-41)}$$

Vertical Equilibrium

Summing forces in the y-direction yields:

$$0 = P_y - \sum F_{yi} \quad \text{(Eq. 3-42)}$$

Substituting for the horizontal bolt force using equations (Eq. 3-36) and the horizontal bolt deformation using equation (Eq. 3-6):

$$0 = P_y - \sum R_{ult} \frac{v + (x_i - x_c)\Phi}{\Delta_i} \quad \text{(Eq. 3-43)}$$

Combined Equilibrium Equations

Inserting these equations into the equilibrium relation equation (Eq. 3-10):

$$\begin{Bmatrix} P_x \\ P_y \\ -P_x(y_p - y_c) + P_y(x_p - x_c) \end{Bmatrix} + R_{ult} \begin{bmatrix} -\sum \frac{1}{\Delta_i} & 0 & \sum \frac{1}{\Delta_i}(y_i - y_c) \\ 0 & -\sum \frac{1}{\Delta_i} & -\sum \frac{1}{\Delta_i}(x_i - x_c) \\ \sum \frac{1}{\Delta_i}(y_i - y_c) & -\sum \frac{1}{\Delta_i}(x_i - x_c) & -\sum \left(\frac{1}{\Delta_i}(y_i - y_c)^2\right) - \sum \left(\frac{1}{\Delta_i}(x_i - x_c)^2\right) \end{bmatrix} \{U\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(Eq. 3-44)}$$

Isolating P and dividing by R_{ult} to get C as shown in equation (Eq. 3-11) yields the following:

$$\frac{\mathrm{P}}{\mathrm{R_{ult}}}\{\rho\} + \begin{bmatrix} -\sum \frac{1}{\Delta_i} & 0 & \sum \frac{1}{\Delta_i}(y_i - y_c) \\ 0 & -\sum \frac{1}{\Delta_i} & -\sum \frac{1}{\Delta_i}(x_i - x_c) \\ \sum \frac{1}{\Delta_i}(y_i - y_c) & -\sum \frac{1}{\Delta_i}(x_i - x_c) & -\sum \left(\frac{1}{\Delta_i}(y_i - y_c)^2\right) - \sum \left(\frac{1}{\Delta_i}(x_i - x_c)^2\right) \end{bmatrix} \{U\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{(Eq. 3-45)}$$

The matrix [D] for the plastic method is defined in equation (Eq. 3-45). Notice that the matrix [D] is a function of the individual bolt deformations, Δ_i. However, the values of Δ_i depend on the current vector of kinematic variables, {U} which depend on the matrix [D]. Therefore, an iterative process is required until the values of Δ_i, [D] and {U} stabilize to within a hundredth decimal place.

I.C. Method

Bolt Force Components

The load-deformation response of a single fastener in double shear can be represented using the following equation, which was developed by Fisher (J. Fisher 1965):

$$R = R_{ult}(1 - e^{-\mu\Delta_i})^{\lambda} \quad \text{(Eq. 3-46)}$$

Where: R = fastener load at any given deformation
R_{ult} = ultimate load attainable by a single fastener
Δ_i = deformation of an individual bolt
μ, λ = regression coefficients
e = base of natural logarithms

As a simplification a new term, $g(\Delta_i)$ is introduced:

$$R = R_{ult} * g(\Delta_i) \quad \text{(Eq. 3-47)}$$

Where: $g(\Delta_i) = (1 - e^{-\mu\Delta_i})^{\lambda}$

The bolt force components, like the plastic method, are based on the ratio of the deformation components:

$$F_{xi} = R_{ult} * g(\Delta_i) * \frac{\Delta_{xi}}{\Delta_i} \quad \text{(Eq. 3-48)}$$

and

$$F_{yi} = R_{ult} * g(\Delta_i) * \frac{\Delta_{yi}}{\Delta_i} \quad \text{(Eq. 3-49)}$$

The only difference between these equations and those given by the elastic method, (Eq. 3-23) and (Eq. 3-24), and plastic method, (Eq. 3-35) and (Eq. 3-36), is that the resultant bolt force as defined by Fisher in (Eq. 3-46) is used to represent the non-linear response of the individual bolts rather than the maximum bolt force, R_{ult}, representing a plastic response of the individual bolts or the elastic force, R_e, representing a linear response of the individual bolts. The values of the matrix [D] is found by solving the equilibrium equations.

Moment Equilibrium

Summing moments about the center of the bolt group yields:

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum F_{xi}(y_i - y_c) - \sum F_{yi}(x_i - x_c) \quad \text{(Eq. 3-50)}$$

Substituting the values from equations (Eq. 3-48) and (Eq. 3-49):

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum R_{ult} * \mathrm{g}(\Delta_\mathrm{i}) * \frac{\Delta_{xi}}{\Delta_i}(y_i - y_c) - \sum R_{ult} * \mathrm{g}(\Delta_\mathrm{i}) * \frac{\Delta_{yi}}{\Delta_i}(x_i - x_c) \quad \text{(Eq. 3-51)}$$

The horizontal and vertical bolt deformations are substituted using equations (Eq. 3-5) and (Eq. 3-6):

$$0 = -P_x(y_p - y_c) + P_y(x_p - x_c) + \sum R_{ult} * \mathrm{g}(\Delta_\mathrm{i}) * \frac{u - (y_i - y_c)\Phi}{\Delta_i}(y_i - y_c) - \sum R_{ult} * \mathrm{g}(\Delta_\mathrm{i}) * \frac{v + (x_i - x_c)\Phi}{\Delta_i}(x_i - x_c) \quad \text{(Eq. 3-52)}$$

Horizontal Equilibrium

Summing forces in the x-direction yields:

$$0 = P_x - \sum F_{xi} \quad \text{(Eq. 3-53)}$$

Substituting for the horizontal bolt force using equations (Eq. 3-48) and the horizontal bolt deformation using equation (Eq. 3-5):

$$0 = P_x - \sum R_{ult} * \mathrm{g}(\Delta_\mathrm{i}) * \frac{u - (y_i - y_c)\Phi}{\Delta_i} \quad \text{(Eq. 3-54)}$$

Vertical Equilibrium

Summing forces in the y-direction yields:

$$0 = P_y - \sum F_{yi} \quad \text{(Eq. 3-55)}$$

Substituting for the horizontal bolt force using equations (Eq. 3-49) and the horizontal bolt deformation using equation (Eq. 3-6):

$$0 = P_y - \sum R_{ult} * \mathrm{g}(\Delta_\mathrm{i}) * \frac{v + (x_i - x_c)\Phi}{\Delta_i} \quad \text{(Eq. 3-56)}$$

Combined Equilibrium Equations

Inserting these equations into the equilibrium relation equation (Eq. 3-10):

$$\begin{Bmatrix} P_x \\ P_y \\ -P_x(y_p - y_c) + P_y(x_p - x_c) \end{Bmatrix} + R_{ult} \begin{bmatrix} -\sum \frac{g(\Delta_i)}{\Delta_i} & 0 & \sum \frac{g(\Delta_i)}{\Delta_i}(y_i - y_c) \\ 0 & -\sum \frac{g(\Delta_i)}{\Delta_i} & -\sum \frac{g(\Delta_i)}{\Delta_i}(x_i - x_c) \\ \sum \frac{g(\Delta_i)}{\Delta_i}(y_i - y_c) & -\sum \frac{g(\Delta_i)}{\Delta_i}(x_i - x_c) & -\sum\left(\frac{g(\Delta_i)}{\Delta_i}(y_i - y_c)^2\right) - \sum\left(\frac{g(\Delta_i)}{\Delta_i}(x_i - x_c)^2\right) \end{bmatrix} \{U\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{(Eq. 3-57)}$$

Isolating P and dividing by R_{ult} to get C as shown in equation (Eq. 3-11) yields the following:

$$\frac{\mathrm{P}}{\mathrm{R_{ult}}}\{\rho\} + \begin{bmatrix} -\sum \frac{g(\Delta_i)}{\Delta_i} & 0 & \sum \frac{g(\Delta_i)}{\Delta_i}(y_i - y_c) \\ 0 & -\sum \frac{g(\Delta_i)}{\Delta_i} & -\sum \frac{g(\Delta_i)}{\Delta_i}(x_i - x_c) \\ \sum \frac{g(\Delta_i)}{\Delta_i}(y_i - y_c) & -\sum \frac{g(\Delta_i)}{\Delta_i}(x_i - x_c) & -\sum\left(\frac{g(\Delta_i)}{\Delta_i}(y_i - y_c)^2\right) - \sum\left(\frac{g(\Delta_i)}{\Delta_i}(x_i - x_c)^2\right) \end{bmatrix} \{U\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{(Eq. 3-58)}$$

The matrix [D] for the I.C. method is defined in equation (Eq. 3-58). Notice that, similar to the plastic method, the matrix [D] is a function of the individual bolt deformations, $g(\Delta_i)$ and Δ_i. Therefore, an iterative process is required. As the vector of kinematic variables change with each iteration, the values of $g(\Delta_i)$ and Δ_i also change. The solution for C is found when these variables stabilize to within a hundredth decimal place.

Summary

These equations are used in development of the software that will be shown in the next chapter. The process for calculating C is the same for all three methods. The only difference among the methods is in how the matrix [D] is defined, which is how the individual bolts deform under the applied load. Therefore, the software performs the same operations for all of the methods using the different [D] matrices.

Chapter 4 - Software Development

Description

The software for analysis of bolt groups has two purposes: to look up the shear strength of a bolt using predefined tables, and to calculate the bolt group coefficient, C, and the location of the instantaneous center of rotation using the elastic, plastic and I.C. methods for a group of bolts under a specified eccentric load. These two aspects of the software are independent. While both the individual bolt shear strength and the bolt group coefficient are required to ultimately determine the strength of the bolt group, the software does not tie the two together and each piece can be determined individually. This allows the designer greater freedom to use the software as needed.

The software was developed using Microsoft Visual C++ 2010 Express. It works on Windows-based operating systems. The file used to open the software is a stand-alone executable file so no other software is required to open it or use it.

Other software packages currently available to do these calculations range from an individual's spreadsheet to complete steel connection design and detailing software. BoltGroup located at http://yakpol.net/BoltGroup.html and the Microsoft Excel VBA code provided at http://engineersviewpoint.blogspot.com/2010/01/test.html are examples of simpler spreadsheet style calculators that are available. The BoltGroup spreadsheet costs $50 and closely resembles the software being created in this thesis. The VBA code is given for free; however, when input, the code does not appear to work. The more sophisticated software packages include RISAConnection and RAMConnection. These are stand-alone design and detailing software packages that are capable of designing all types of steel connections. They are expensive, but are significantly more involved than the software developed in this thesis.

Using the Software

The program is opened by double-clicking the executable file, JerodIC.exe. When the program opens, the following user interface is shown:

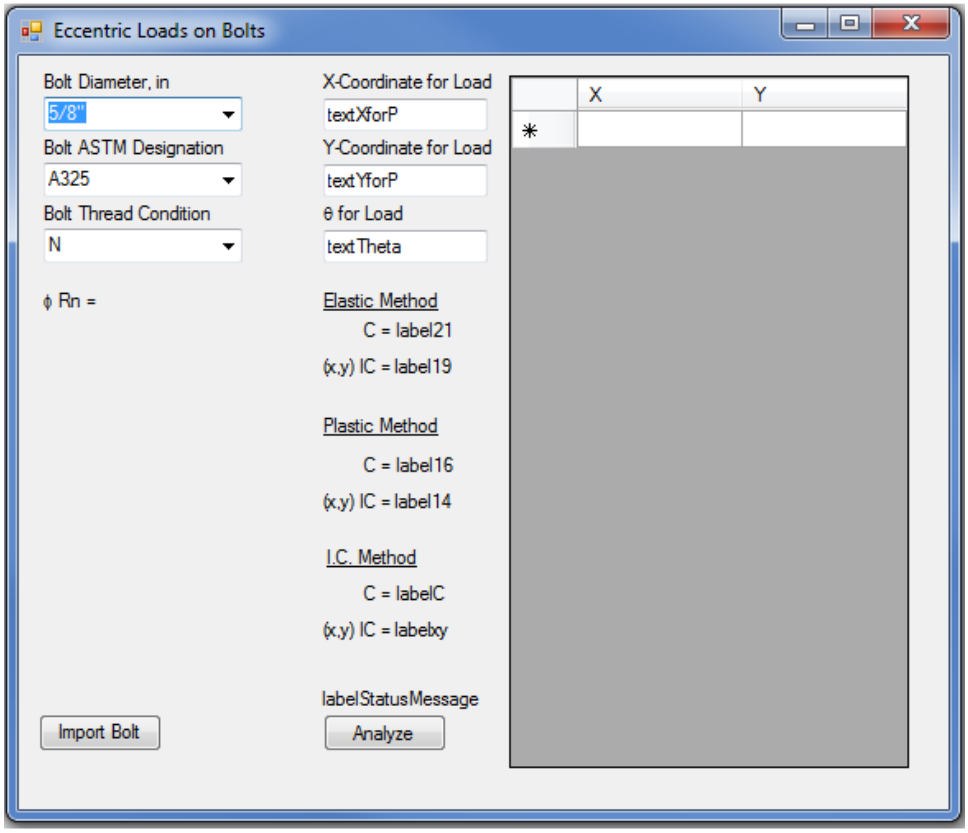
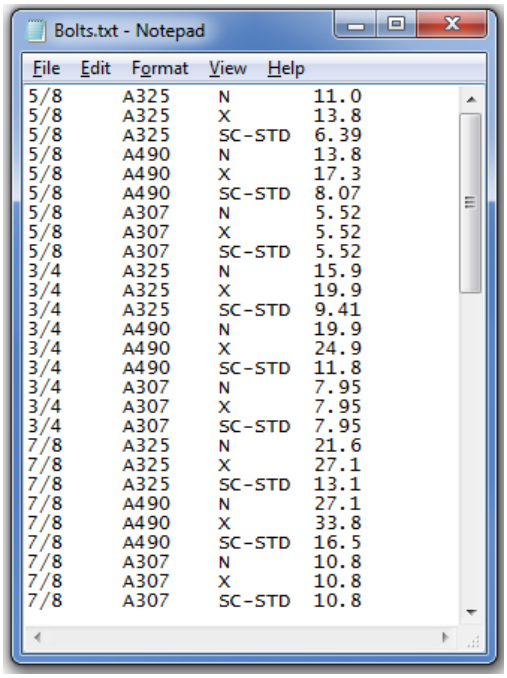


Figure 20: Jerod IC User Interface

The user interface is separated into two sections. The left column determines the shear strength of an individual bolt, and the columns on the right side calculate the bolt group coefficient and locate the I.C. for the three different methods.

Individual Bolt Shear Strengths

The shear strength of a bolt is based on the bolt diameter, the ASTM designation and the thread condition. Once these input parameters are defined, the individual bolt shear strength can be read directly from a predefined table. One such table, shown in Figure 21, has already been generated using the information given in the tables of the 13th edition of AISC Manual (*Manual* 2005). Each of the columns is tab-delimited in order to be read properly by the software. The first column is the bolt



Bolts.txt - Notepad

File Edit Format View Help

5/8	A325	N	11.0
5/8	A325	X	13.8
5/8	A325	SC-STD	6.39
5/8	A490	N	13.8
5/8	A490	X	17.3
5/8	A490	SC-STD	8.07
5/8	A307	N	5.52
5/8	A307	X	5.52
5/8	A307	SC-STD	5.52
3/4	A325	N	15.9
3/4	A325	X	19.9
3/4	A325	SC-STD	9.41
3/4	A490	N	19.9
3/4	A490	X	24.9
3/4	A490	SC-STD	11.8
3/4	A307	N	7.95
3/4	A307	X	7.95
3/4	A307	SC-STD	7.95
7/8	A325	N	21.6
7/8	A325	X	27.1
7/8	A325	SC-STD	13.1
7/8	A490	N	27.1
7/8	A490	X	33.8
7/8	A490	SC-STD	16.5
7/8	A307	N	10.8
7/8	A307	X	10.8
7/8	A307	SC-STD	10.8

Figure 21: Sample Bolt File based on values from 13th Ed of AISC Manual (*Manual* 2005)

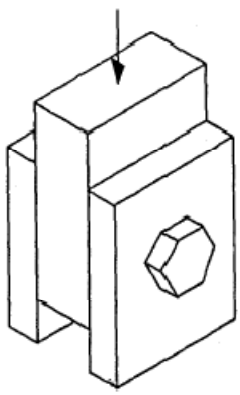


FIG. 2.—SHEAR TEST OF INDIVIDUAL BOLT

Figure 22: Shear Test of Individual Bolt (Crawford & Kulak 1971)

diameter in inches written in fraction form, the second column is the ASTM designation, the third column is the thread condition and the fourth column is the shear strength of the bolt in single shear in kips. The shear values are found by performing a shear test on the individual bolts as shown in Figure 22. The strength of the bolts given in column four are already multiplied by the reduction factor required by the Load Reduction Factor Design (LRFD) method. The software is currently limited to the bolts given in the AISC Manual; however, it would not be difficult to modify the software to allow the user to create different tables to apply to rivets, other bolt types, dowels or any other type of fastener.

In order for the software to look up the individual bolt strength, the user must first import the predefined table. This is accomplished by clicking the "Import Bolt" button in the lower left corner of the user interface. This button opens a separate window for the user to choose the appropriate predefined text file containing the input parameters and the resulting design shear strength. Once the user selects a diameter, ASTM designation and thread condition, the individual bolt strength is given. When the file is located and opened, the complete path is displayed below the "Import Bolt" button to signify that the file has been properly imported, as shown in Figure 23:

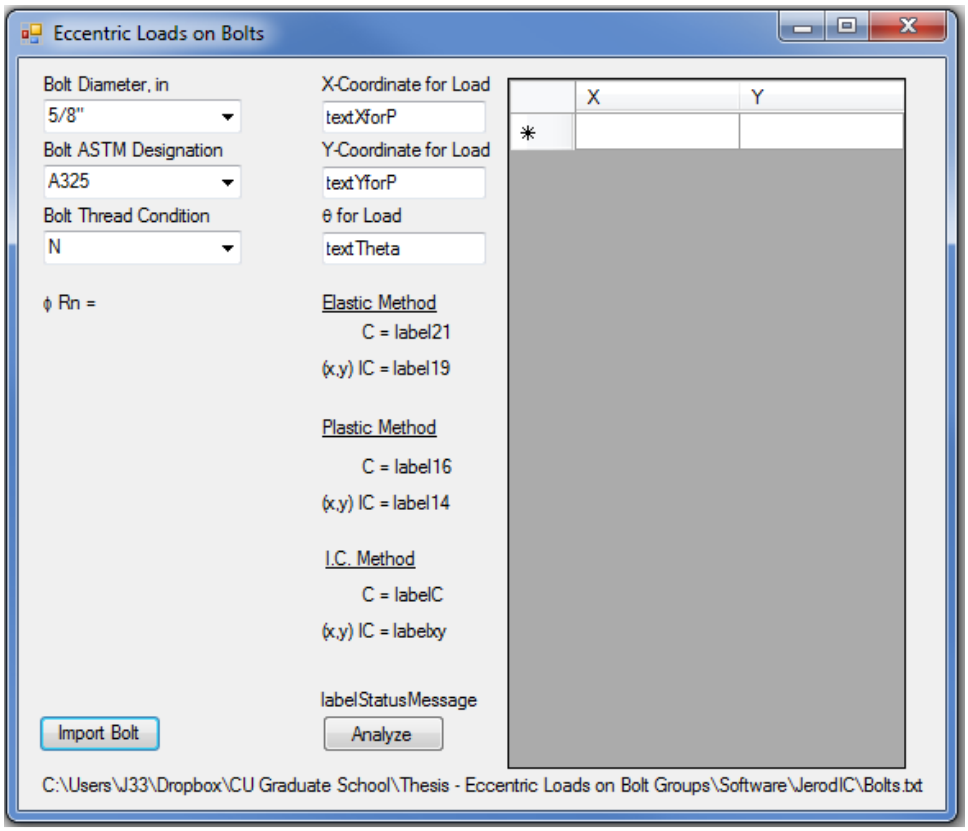


Figure 23: Bolt File Imported

Once the file is imported, the user can select the appropriate input parameters by using the three dropdown menus in the top left of the user interface. The drop down menus are titled "Bolt Diameter, in", "Bolt ASTM Designation", and "Bolt Thread Condition". When the appropriate input parameters are chosen the shear strength of the bolt is given. The result is shown in the text "ϕRn = ...". Figure 24 shows one example that gives a shear strength of 33.8 kips for a 7/8-inch diameter A490 bolt with an X thread condition.

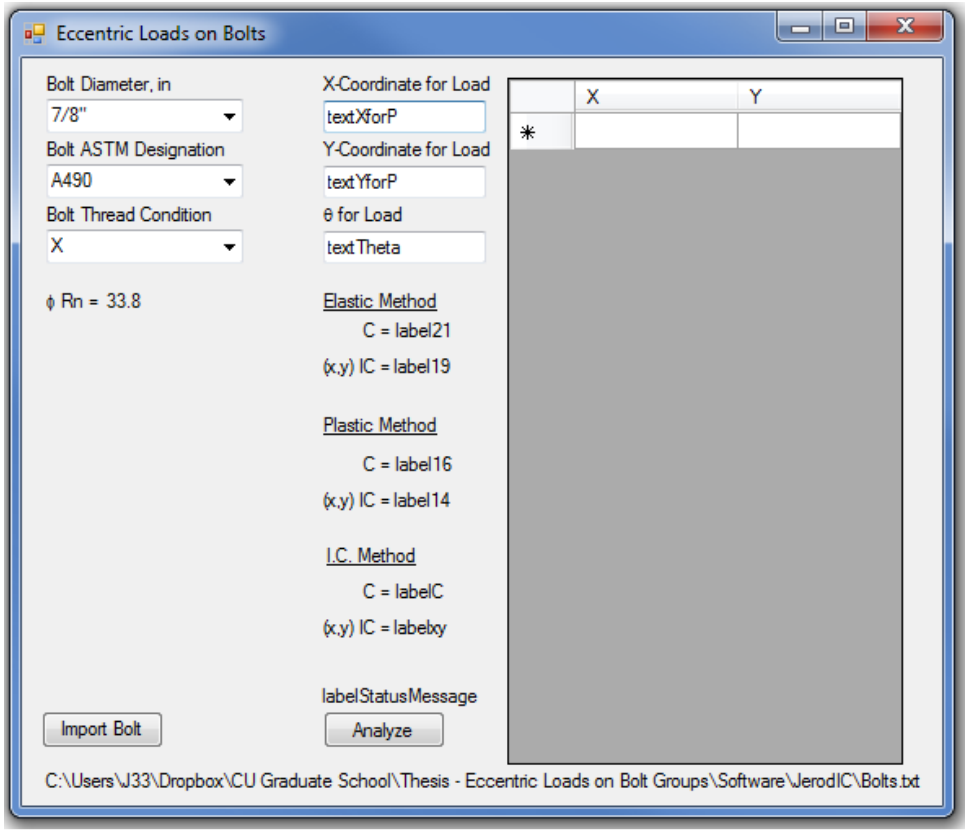


Figure 24: Bolt Strength Given Once Parameters are Chosen

The parameters in the dropdown menus are limited to those given in the current AISC Manual since that is how the predefined tables were set up. The results are also limited to a LRFD design shear strength in kips. However, minor modifications to the software can be made to allow the user to change the input parameters, change the design method (ASD, LSD, etc.) and/or change the units of the given shear strength.

Notice that a solution for the shear strength of an individual bolt is determined without having to specify a bolt pattern or calculate anything with regards to the bolt group coefficient or I.C. location. The individual bolt shear strength is given by a simple table lookup using the input parameters. Therefore, one use for this software is to quickly determine the shear strength of an individual bolt without having to manually go through the tables in the code.

C and Location of Instantaneous Center of Rotation (I.C.)

To find C for a bolt group, the user must input x and y-coordinates for each bolt in the bolt group and an x-coordinate, y-coordinate and an attitude, θ, for the applied load. The coordinates are not specific to any unit system as long as the units remain consistent. The angle, θ, must be entered in degrees. Based on the bolt locations and applied load location and orientation, the software gives C for the elastic method, plastic method and I.C. method. The software also gives the x and y-coordinate of the instantaneous center of rotation (I.C.) for all three methods.

The bolt group can be defined using the table input on the far right side of the user interface. There is a column for x-coordinates with the heading "X" and a column for y-coordinates with the heading "Y". The user should enter an x-coordinate and a y-coordinate for each bolt in the group. An example bolt pattern is shown in Figure 25, which has a total of four bolts with a vertical spacing of 3-inches and a horizontal spacing of 2-inches. In order to enter the x and y-coodinates for these bolts, the user must determine what origin they will use. How the origin is defined will not alter the results as long as the location of the origin remains consistent. For the bolt pattern given in Figure 25, the user may choose to set the origin at the center of the bolt pattern as shown in Figure 26. The x and y-coordinates for the bolts would then be entered as shown in the table of Figure 26.

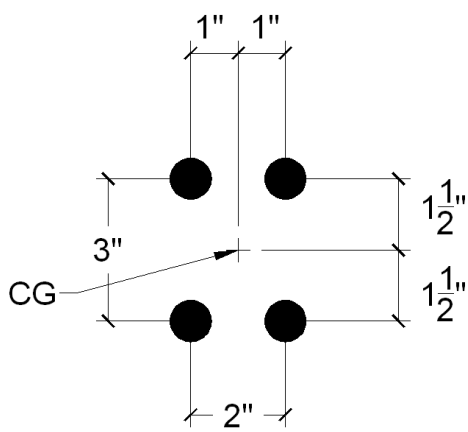
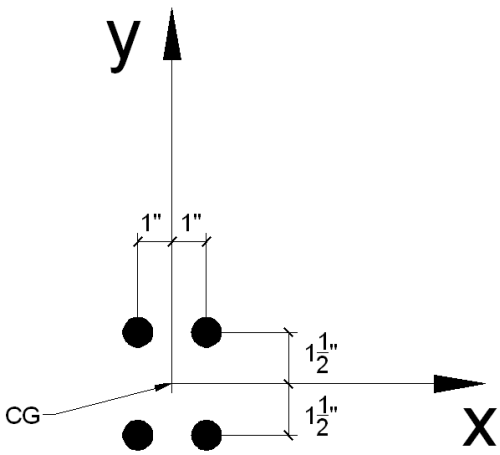


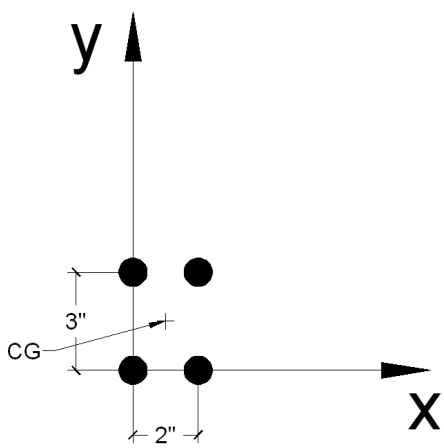
Figure 25: Example Bolt Pattern



X	Y
-1	1.5
-1	-1.5
1	1.5
1	-1.5

Figure 26: Bolt Coordinates with Origin at Centroid

Alternately, the user may choose to set the origin at the traditional zero point located at the bottom left corner of the bolt group as shown in Figure 27. This would lead to the x and y-coordinates being entered as shown in Figure 27.



X	Y
0	0
0	3
2	0
2	3

Figure 27: Bolt Coordinates with Origin at Zero

Notice that since the bolt pattern is comprised of four bolts, the table has a total of four rows, one row for each bolt. The order of the rows does not matter as long as all of the bolts are included.

The user must not only define the location of the individual bolts, but they must also specify the location and orientation of the applied load. This is entered in the text boxes located in the center

column of the user interface. In order to define the location of the applied load, the user must enter an x-coordinate, y-coordinate and angle for the applied load. The x-coordinate for the applied load is entered in the text box titled "X-Coordinate for Load". When the program is first opened, the text box is filled in with the default text "textXforP". The user should replace the default text with the x-coordinate of the applied load. The y-coordinate for the applied load is entered in the text box titled "Y-Coordinate for Load" which has the default text "textYforP". Both the x and y-coordinates of the applied load should maintain the same origin choice that was used for the bolt coordinates. If the same example bolt pattern has a load applied to it as shown in Figure 28, the user can still choose to set the origin at the middle or at the traditional zero point. The location of the origin chosen by the user will determine the x and y-coordinates of the applied load. For the load application shown in Figure 28, the user would input an x-coordinate of 4 and a y-coordinate of 2.5 if the origin were set to the middle. If the origin were set to the traditional zero point, the user would input an x-coordinate of 5 and a y-coordinate of 4.

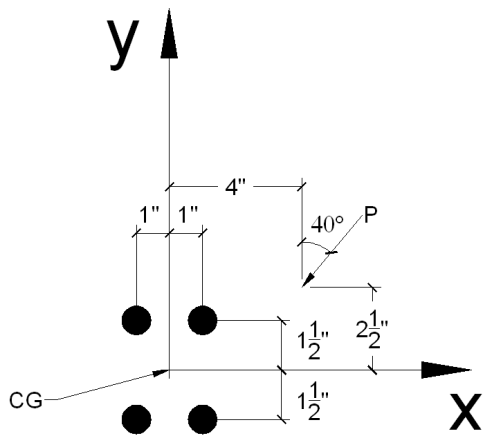


Figure 28: Example Bolt Pattern with Load Applied

Lastly, the user must input an angle for the applied load, in degrees, to define its orientation. The angle, θ, is defined from the vertical axis with clockwise positive as shown in Figure 28. Therefore, an angle equal to zero degrees means that the load is applied vertically in the negative y-direction. The user enters the angle of the applied load in the text box titled "θ for Load" which has the default text "text Theta".

When the user has completed entering the bolt location information and the applied load location and orientation information they are ready to perform the analysis. The analysis is done when the user clicks the "Analyze" button. If all of the inputs have been completed appropriately, the label titled "labelStatusMessage" disappears and the software calculates C and coordinates of the I.C. simultaneously for the elastic, plastic and I.C. methods. Values for C are given in the "C = ..." label and the coordinates of the I.C. are given in the "(x,y) IC = ..." label under each of the different methods. The coordinates of the I.C. are based on the origin defined by the user since it is calculated based on how the bolts and applied load were defined. The output for the bolt pattern and applied load shown in Figure 28 with the origin chosen at the traditional zero point is shown in Figure 29. A red box has been added to show the location of the results.

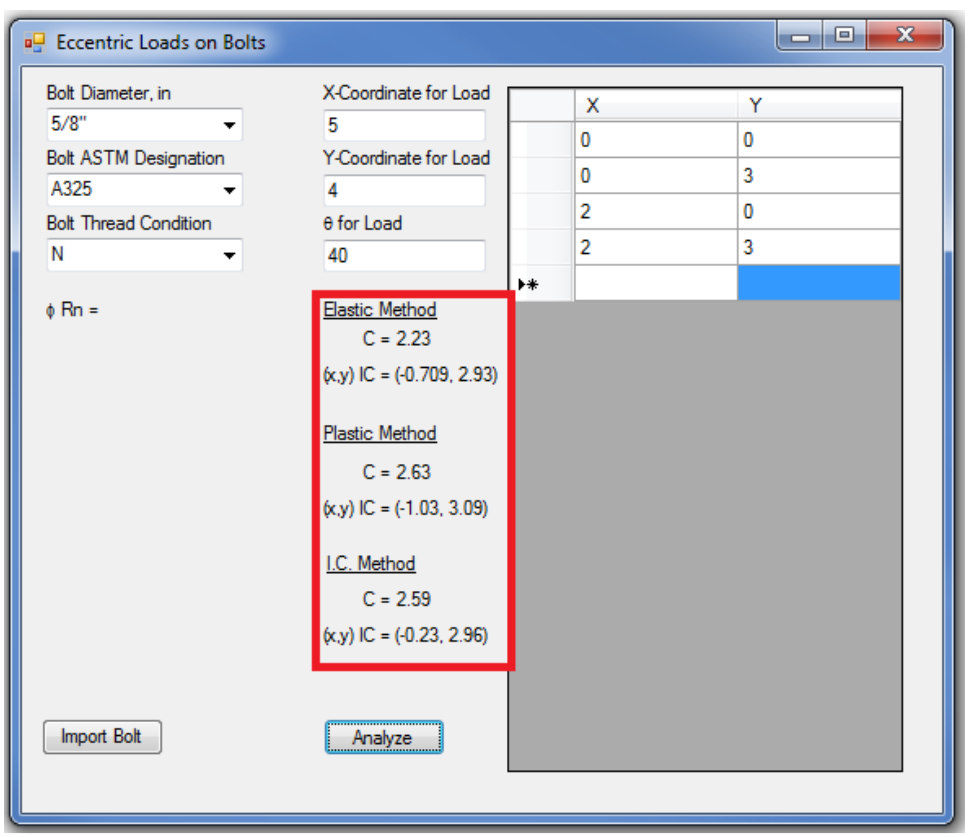


Figure 29: C and I.C. Location Output

Notice that in Figure 29 the calculations for C and I.C. locations in the software were completed in the software without a value of ϕR_n being found on the left. While both of these values are needed to ultimately determine the capacity of the bolt group, they can be calculated separately in the software. This allows users greater flexibility to use the software to meet their needs.

If the user does not complete all of the inputs required to perform the calculations, the label titled "labelStatusMessage" above the "Analyze" button will give one of two error messages. If there are no bolt coordinates entered, the error will state "Bolts not ready" as shown in Figure 30:

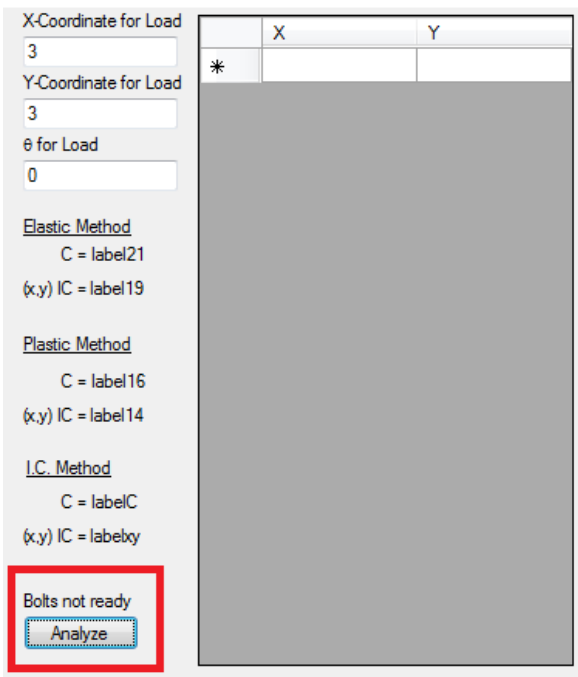


Figure 30: Bolts Not Ready Error

If the user does not complete all of the inputs for the applied load, the error will state "Load not ready" as shown in Figure 31:

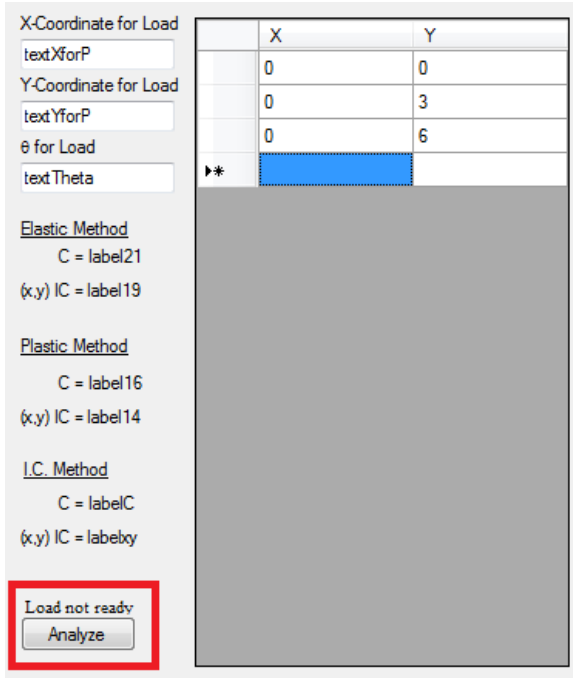


Figure 31: Load Not Ready Error

These are the only two error messages given in the software. In order to proceed, the user must complete all of the required inputs. Without this information, the software cannot perform the necessary calculations.

Limitations

For determining the individual bolt shear strengths, the software is currently limited to the values given in Table 11.

Table 11: Bolt Inputs

Bolt Diameter	Bolt ASTM Designation	Bolt Thread Condition
5/8"	A325	N
3/4"	A490	X
7/8"	A307	SC-STD
1"		
1-1/8"		
1-1/4"		
1-3/8"		
1-1/2"		

Any combination of these inputs can be chosen and the shear strength of an individual bolt in single shear will be generated based on the values given in the 13th edition of the AISC Manual (*Manual* 2005). Modifications to the software will be required if other inputs are desired. The software could easily be customized for any fastener type, diameter, ASTM designation or thread condition.

For determining C and the I.C. location, the software seemingly has no limitations. There is no limit to the number of bolts the user can enter, although the calculations could be delayed due to the iterative process that is required. The user is also not limited to a range of load locations or orientations. The applied load, however, can only be a single point load. The software does not allow for distributed loads or multiple point loads, only single point loads. The coordinates are not limited by units, but the unit system must be consistent. Likewise, the coordinates are not limited by a specific origin, but the origin must remain consistent. The bolts can appear in any order with no change on the results. Therefore, the software appears to be able to compute C and the I.C. location for an unlimited configuration of bolt groups and applied single point loads. Numerous examples have been run through the software, as shown in the next section, and results were obtained for every combination of bolt pattern and load application.

Examples

Numerous examples are run through the software in order to check the validity of the software, verify the functionality of the software, as well as compare and contrast the different methods. These examples are based on common code configurations, historical testing configurations and other random configurations. While an infinite number of examples could have been included, these examples vary all of the input parameters and allow for interesting comparisons between the different methods.

Code Verification Examples

The first set of examples that are run through the software are based on the configurations given in the AISC Manuals in order to check the functionality and validity of the software. The elastic

method and the I.C. method are the only two methods given in both the software and the AISC Manuals, so they are the only methods that can be verified using the software. The first set of configurations that are run through the software is comprised of bolts in a single vertical row, shown in Figure 32. The number of bolts, attitude of the applied load, spacing and eccentricity are varied to check multiple conditions.

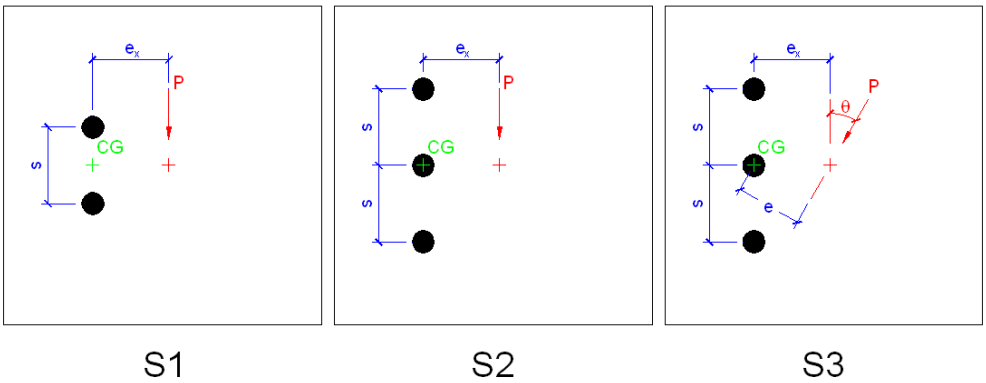


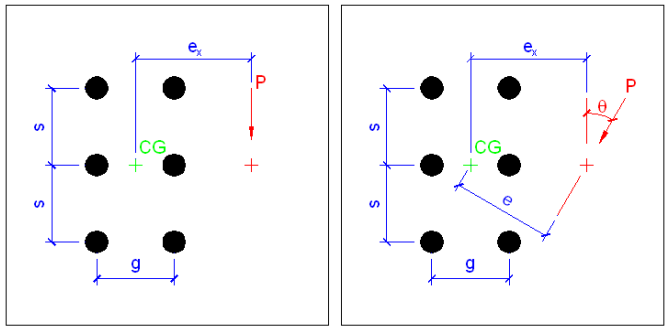
Figure 32: Single Vertical Row Configurations

The bolt group coefficients for both the elastic method, C_e, and the I.C. method, C_{ic}, are calculated and compared against the values given in the AISC Manuals to verify that results given are accurate. The results given in Table 12 for the elastic method matched exactly with what is given in the 7th edition of the AISC Manual (*Manual* 1950) and the results for the I.C. method matched exactly with what is given in the 13th edition of the AISC Manual (*Manual* 2005).

Table 12: Single Vertical Row Results

Single Row Code Verification Examples					
Bolt Configuration	e_x (in)	s (in)	θ (degrees)	Elastic C_e	IC C_{ic}
S1	2	3	0	1.20	1.18
	4			0.70	0.69
	8			0.37	0.36
	16			0.19	0.18
	32				0.09
	2	6			1.63
	4				1.18
	8				0.69
	16				0.36
	32				0.18
S2	2	3	0	2.12	2.23
	4			1.34	1.40
	8			0.73	0.73
	16			0.37	0.37
	32				0.18
	2	6			2.71
	4				2.23
	8				1.40
	16				0.73
	32				0.37
S3	2	3	30		2.20
	4				1.50
	8				0.84
	16				0.42
	32				0.21
	2	6			2.66
	4				2.20
	8				1.50
	16				0.84
	32				0.42

The next set of configurations that are run through the software are comprised of bolts in double vertical rows, shown in Figure 33. In this case the number of bolts remains constant, but the vertical spacing, attitude of the applied load and eccentricity are varied.



D1 D2

Figure 33: Double Vertical Row Configurations

The output of the bolt group coefficients for both the elastic method, C_e, and the I.C. method, C_{ic}, are calculated and shown in Table 13. These values are compared to the design tables in the AISC Manuals to verify that results given are accurate. The results given in Table 13 for the elastic method matched exactly with what is given in the 7th edition of the AISC Manual (*Manual* 1950) and the results for the I.C. method matched exactly with what is given in the 13th edition of the AISC Manual (*Manual* 2005).

Table 13: Double Vertical Row Results

Double Row Code Verification Examples					
Bolt Configuration	e_x (in)	s (in)	θ (degrees)	Elastic C_e	IC C_{ic}
D1	2	3	0	3.88	4.48
	4			2.66	3.06
	8			1.58	1.78
	16			0.86	0.95
	32				0.49
	2	6			5.39
	4				4.44
	8				2.87
	16				1.57
	32				0.81
D2	2	3	30		4.52
	4				3.29
	8				2.00
	16				1.08
	32				0.56
	2	6			5.31
	4				4.42
	8				3.08
	16				1.78
	32				0.93

These examples confirm that the software is functioning properly and is providing valid results for both the elastic and I.C. methods. Therefore, further investigation can be done to compare and contrast the different methods as well as look for any interesting patterns when the input parameters are varied.

Historical Examples

One interesting investigation is done using the examples from Higgins of the testing done at Lehigh University, shown in Figure 5, (Higgins 1964) and the examples from the testing done by Crawford and Kulak, shown in Figure 12, (Crawford and Kulak 1968). The bolt group coefficients are calculated for each of the different methods. Even though the software does not provide output for the modified elastic method, the elastic method can be used by substituting the effective eccentricity, which can be calculated by hand, for the actual eccentricity. A table summarizing the C values allows for quick comparison of how the methods differ in computing the strength of the different bolt configurations.

Table 14: Comparison of C Values and Nominal Capacities for Elastic, Modified Elastic, Plastic and I.C. Methods

Comparison of Historical Tests										
Source	Test Specimen	Failure Load (kips)	Elastic Method		Modified Elastic Method		Plastic Method		I.C. Method	
			C Coefficient	Predicted Load (kips)	C Coefficient	Predicted Load (kips)	C Coefficient	Predicted Load (kips)	C Coefficient	Predicted Load (kips)
Tests Performed at Lehigh University (Higgins 1965)	TP 1	216	1.87	206	2.81	309	2.05	226	1.98	218
	TP 2	161	1.49	164	2.26	248	1.63	179	1.56	172
	TP 3	100	0.88	97.0	1.16	128	0.92	102	0.90	98.6
	TP 4	550	4.88	537	5.87	645	5.37	591	5.23	575
	TP 5	440	3.68	405	5.65	622	4.41	485	4.22	464
	TP 6	362	2.84	313	4.40	484	3.54	389	3.35	369
	TP 7	222	1.57	172	2.18	240	1.86	205	1.77	195
	TP 8	120	0.99	109	1.22	134	1.13	124	1.08	119
	TP 9	568	4.40	484	6.94	764	5.49	603	5.21	573
	TP 10	354	2.85	313	4.05	445	3.60	396	3.40	374
Crawford and Kulak Tests (Crawford & Kulak 1968)	B1	225	1.49	221	2.15	318	1.84	272	1.73	256
	B2	230	1.44	213	1.91	283	1.77	262	1.67	246
	B3	190	1.21	179	1.54	228	1.49	209	1.40	207
	B4	251	1.56	231	2.03	300	2.00	295	1.86	276
	B5	221	1.36	202	1.71	254	1.75	259	1.63	241
	B6	264	1.69	250	2.08	300	2.12	313	2.00	295
	B7	212	1.38	204	1.63	241	1.72	254	1.62	240
	B8	266	1.72	254	2.11	312	2.20	326	2.08	307

From the results given in Table 14, it can be seen that the elastic method consistently gives the lowest values of C, the modified elastic method gives the highest values of C and the plastic and I.C. methods give C values somewhere in between. Therefore, the elastic method would be the most conservative in

its estimation of the strength and the modified elastic method would likely be too liberal in its estimation of strength. If the modified elastic method is ignored, the plastic method gives the highest values of C, which suggests that the plastic method is also more liberal than the I.C. method.

Variable Input Examples

Another interesting investigation is done by using examples where all of the input parameters are varied to look for any interesting patterns. These examples vary the following input parameters:

- Eccentricity
- Number of bolts
- Vertical spacing
- Attitude of the applied load
- Number of rows
- Vertical position of an inclined load
- Symmetry of vertical spacing

For each of the examples, the following output results are recorded for comparison. The zero point for these examples is set at the elastic centroid of the bolt group, so all coordinates are calculated from this location.

- Elastic
    - Bolt group coefficient, C_e
    - x-coordinate of the I.C.
    - y-coordinate of the I.C.
- Plastic
    - Bolt group coefficient, C_p
    - x-coordinate of the I.C.
    - y-coordinate of the I.C.
- I.C.
    - Bolt group coefficient, C_{ic}
    - x-coordinate of the I.C.
    - y-coordinate of the I.C.

This information is plotted to compare how the results change for the different examples. Plots of the C/C ratios for each method are also created to compare how the different methods relate to each other.

S1 - (2) Bolts in a Single Row with Vertical Load

The first configuration to be investigated contains two bolts in a single vertical row as shown in Figure 34. This configuration is designated S1. This configuration is analyzed using multiple values for horizontal component of eccentric length, e_x, and vertical spacing, s. These results are then plotted to allow for analysis of the results.

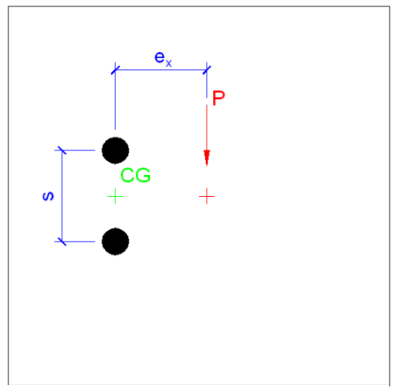


Figure 34: S1 Configuration

Figure 35 shows a plot of C vs. the eccentricity for all three methods at vertical spacing of s = 3-inches.

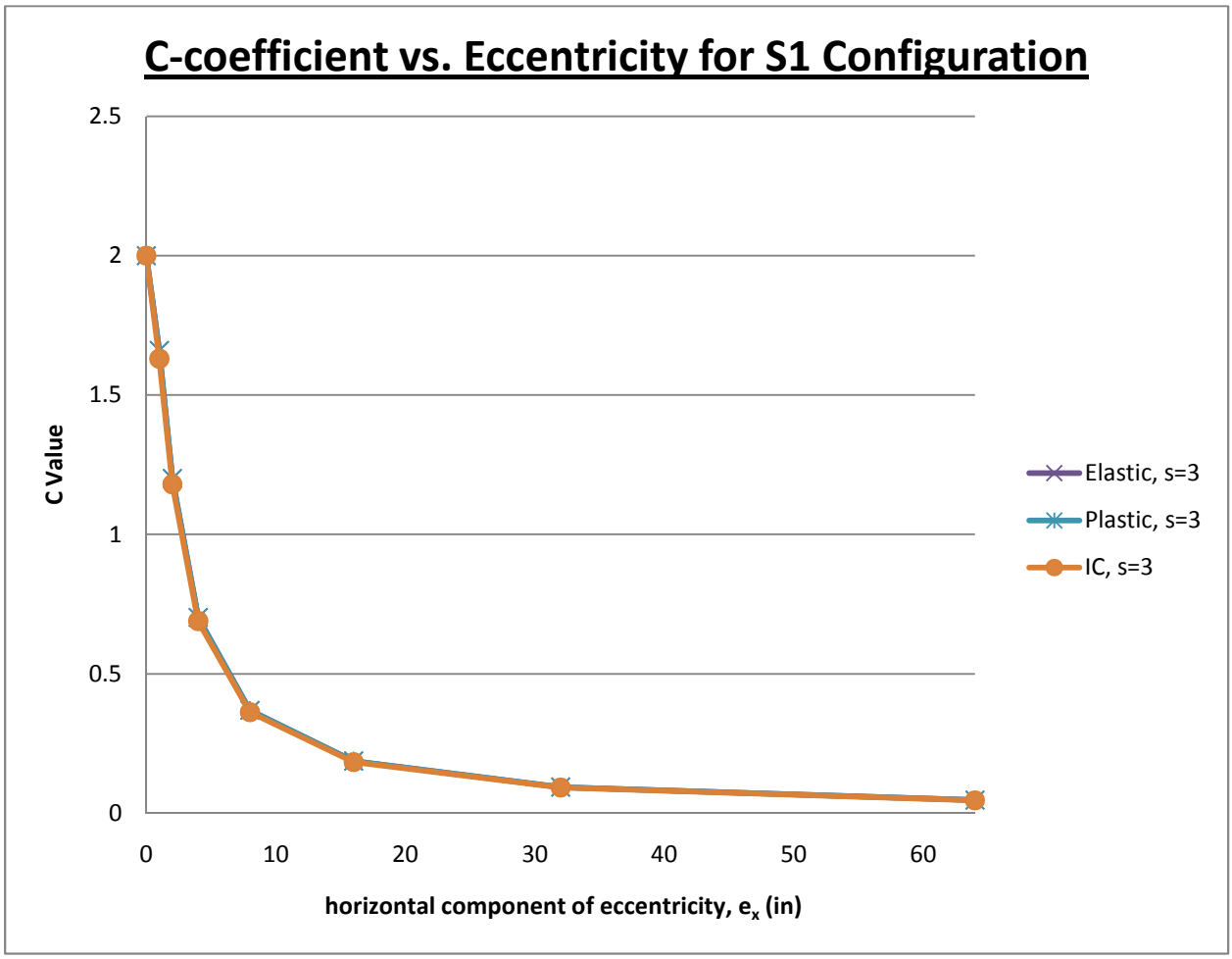


Figure 35: S1 - C vs. e_x for Different Methods

For all three methods, C begins at a value of 2.0 when the eccentricity is equal to zero and then converges to a value of zero for C when the eccentricity is very large. This makes sense since the bolts

will support pure shear when the eccentricity is equal to zero and will be limited to the number of bolts (2 in this case) times the ultimate strength of the bolt. As eccentricity increases, the bolts are subjected to forces due to the applied moment, which decreases the capacity of the bolt group. As the eccentricity becomes very large, the forces in the bolts due to moment are too high for the bolt configuration to support and thus the capacity converges to zero. Figure 36 shows a plot of C vs. the eccentricity for the IC method only at vertical spacing of s = 2, 3, 5 and 6-inches.

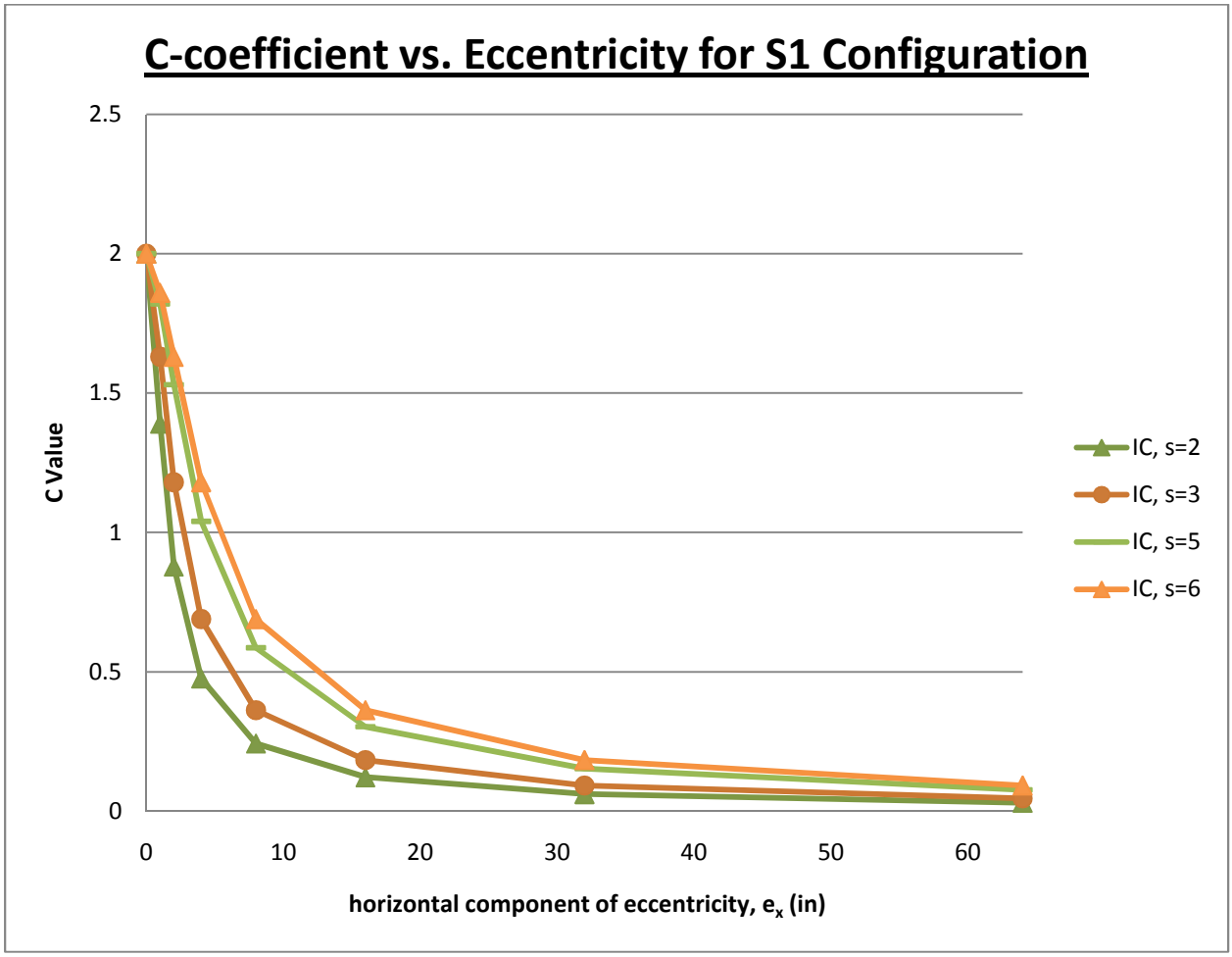


Figure 36: S1 - C vs. e_x for Different Spacings

All plots begin and end at the same locations, but as the spacing increases, the plot moves further from the origin meaning that a higher spacing approaches a C coefficient equal to zero at a slower rate than a smaller spacing.

Figure 37 plots the x-coordinate of the I.C. vs. eccentricity using for various spacing of bolts.

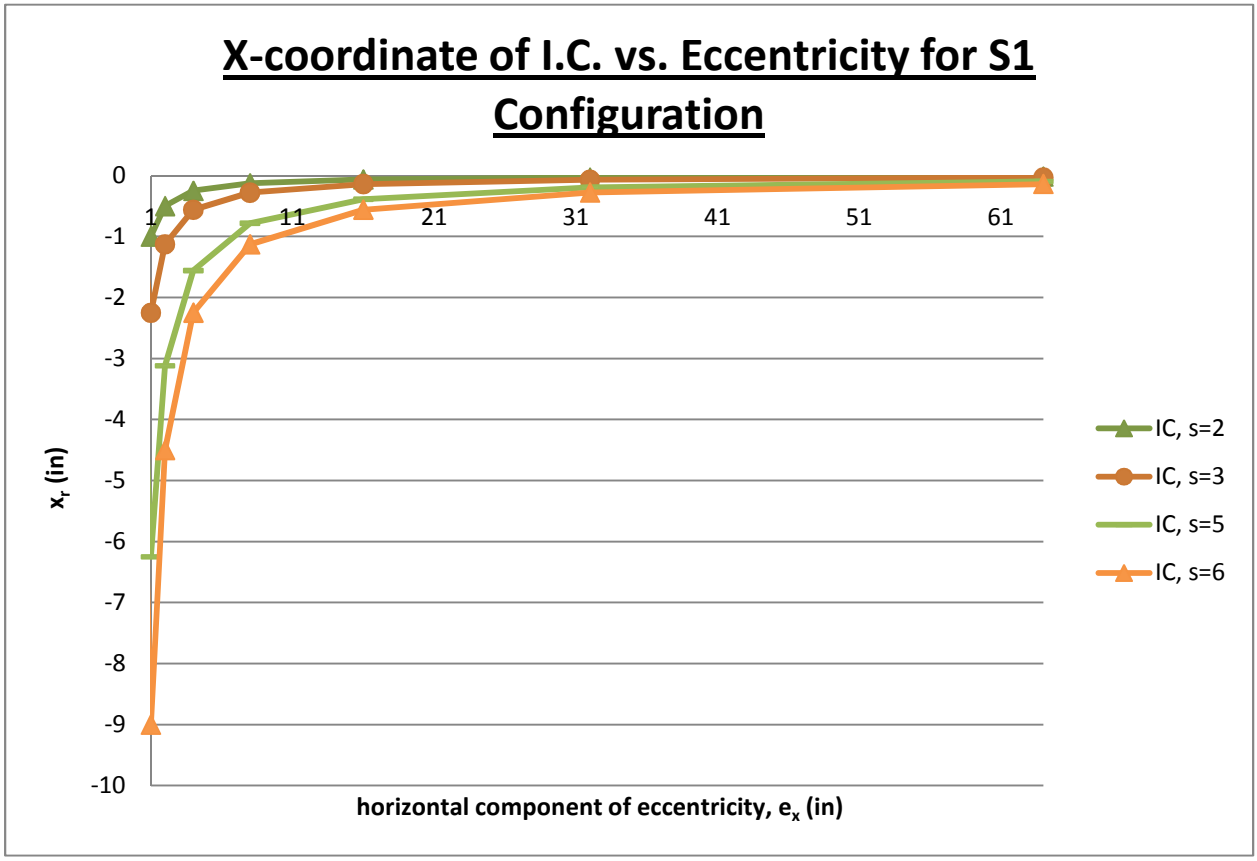


Figure 37: S1 - x_r vs. e_x

When the eccentricity is equal to 0-inches, x_r converges to negative infinity the closer the eccentricity gets to zero. As the eccentricity increases, x_r converges to a value of zero or the elastic centroid of the bolt group. The values only converge to the elastic centroid when the bolt group has a symmetrical pattern. When the pattern is not symmetric, the different methods converge to separate points. This will be discussed further for the A1 configuration. For a small eccentricity, the contribution from moment is minimal compared to the contribution from shear. Therefore, it is understandable that the location of the I.C. is far from the elastic centroid. Similarly, for very large eccentricities, the connection is governed by rotation so the I.C. would be located closer to the elastic centroid. It is difficult to see the effects of the different methods from Figure 35 since they seem to be right on top of each other. In

order to compare the differences in the methods, the ratio of bolt group coefficients are plotted (Figure 38 - Figure 40).

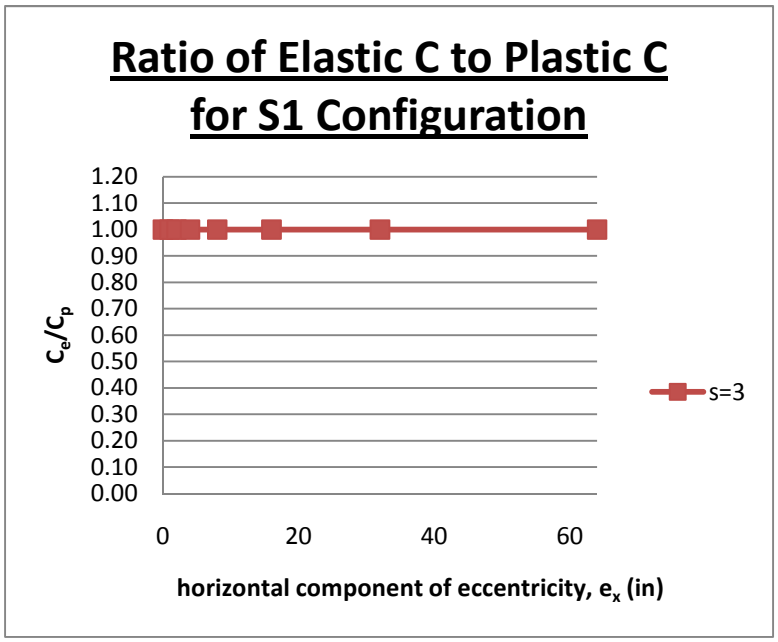


Figure 38: S1 - C_e/C_p

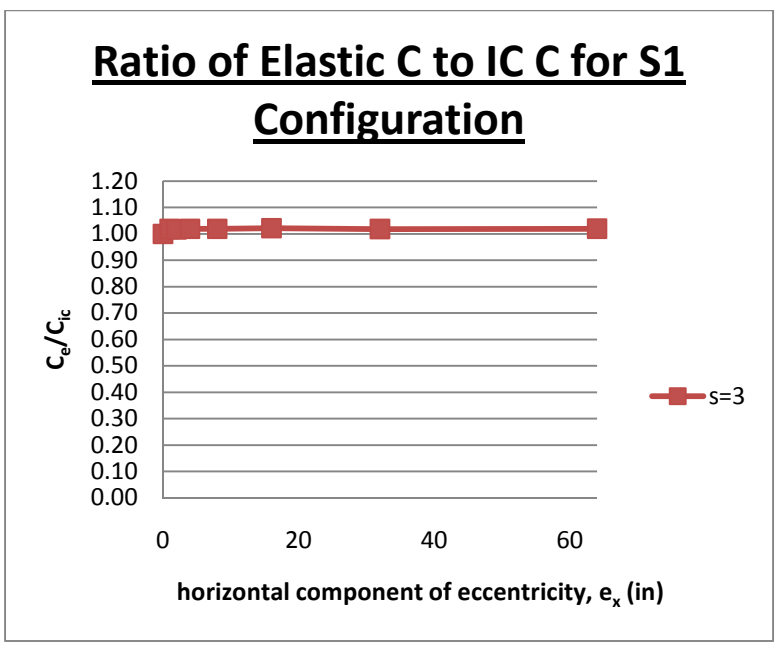


Figure 39: S1 - C_e/C_{ic}

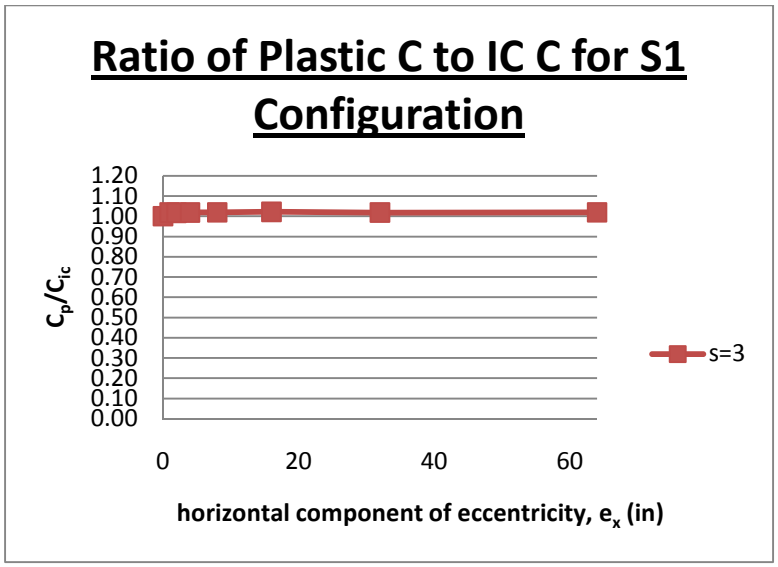


Figure 40: S1 - C_p/C_{ic}

For this bolt configuration all three methods give the same values for C. Since there are only two bolts equally spaced, the distance to the I.C. will be identical for both bolts, which means that the bolts will both reach the maximum deformation of 0.34-inches. Therefore, all three methods have identical results.

S2 - (3) Bolts in a Single Row with Vertical Load

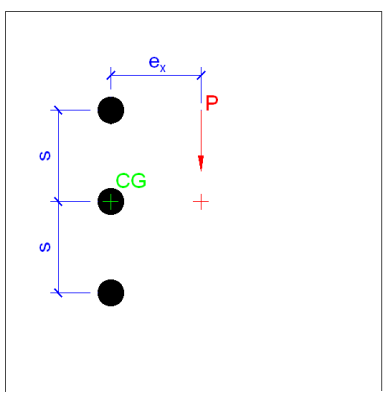


Figure 41: S2 Configuration

The next configuration, designated S2, contains three bolts in a single vertical row as shown in Figure 41. Horizontal component of eccentricity, e_x, and vertical spacing, s, are varied similar to the S1 configuration, but the addition of another bolt allows for a comparison between the S1 and S2 configurations. The ratios of bolt group coefficients are plotted to compare the differences (Figure 42 - Figure 44).

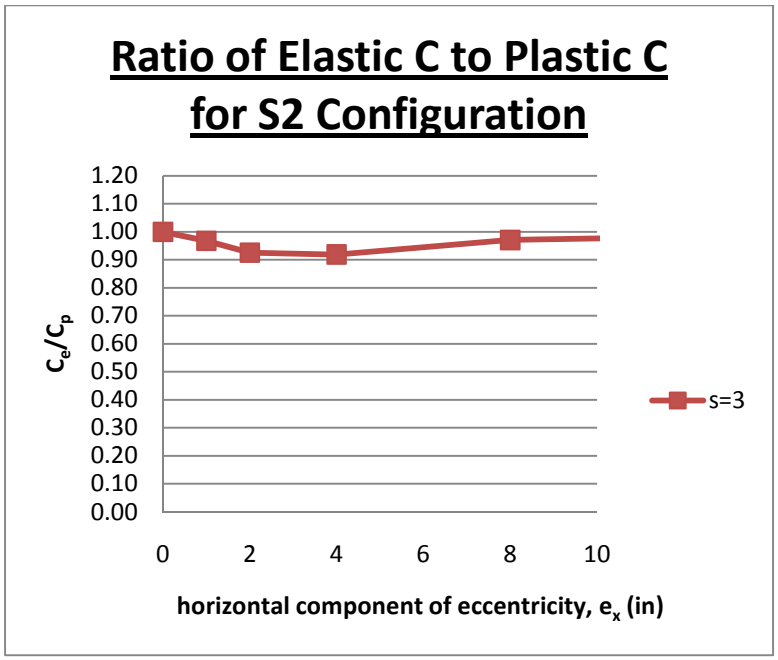


Figure 42: S2 - C_e/C_p

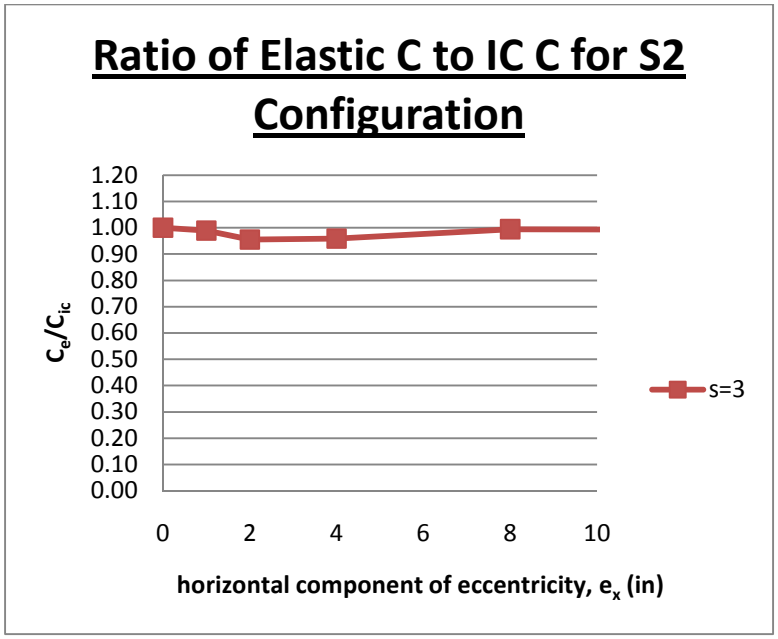


Figure 43: S2 - C_e/C_{ic}

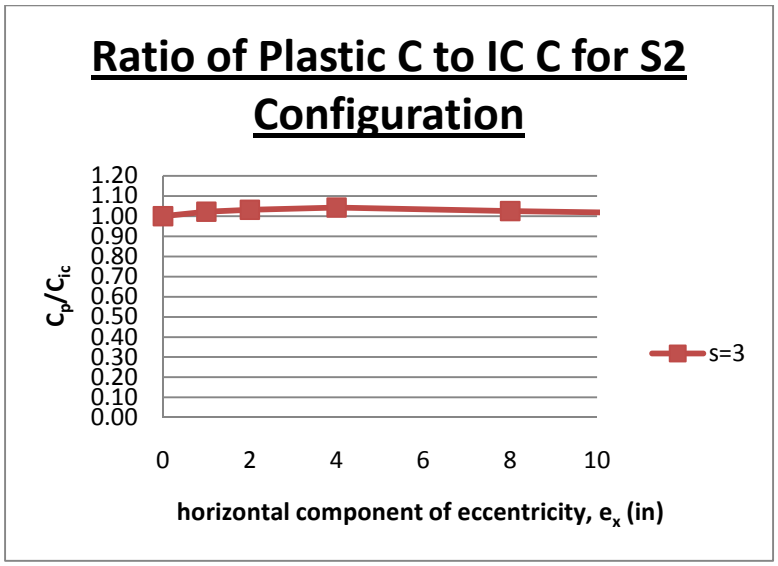


Figure 44: S2 - C_p/C_{ic}

For this bolt configuration, the methods do not give the same values for C as they did in the S1 configuration. With the addition of the third bolt, the distance from the I.C. and thus the individual bolt deformations will not be equal. Therefore, the methods give differing results. The majority of the differences in the C/C ratios occur at smaller eccentricities with little to no differences at zero or large eccentricities. The C_e/C_p ratio ranges from 0.91 - 1.00, the C_e/C_{ic} ratio ranges from 0.95 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.04.

S3 - (3) Bolts in a Single Row with Inclined Load

In order to review the effects of inclined load, the S3 configuration, as shown in Figure 45, is analyzed and compared to the S2 configuration. The only difference between the S2 configuration and the S3 configuration is that the load being applied on the S3 configuration is at an angle θ = 30-degrees. The variations in both the eccentricity and vertical spacing are kept constant between the two configurations.

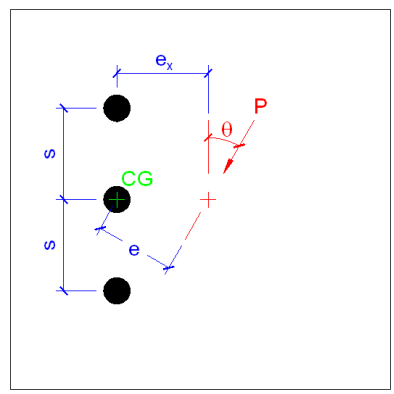


Figure 45: S3 Configuration

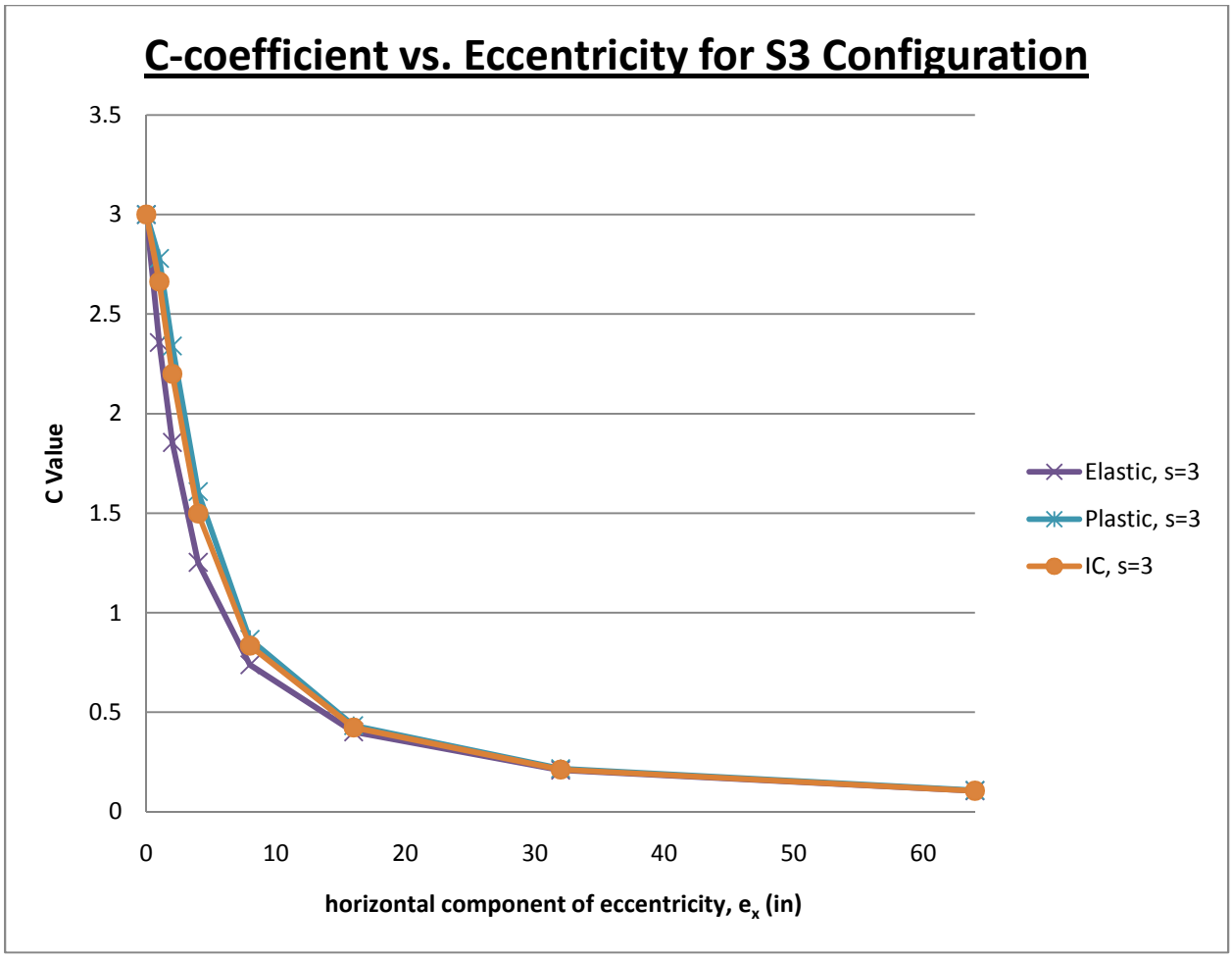


Figure 46: S3 - C vs. e_x for Different Methods

Comparing this plot against the same plot from the S1 configuration (Figure 35) reveals a couple of differences. Similar to the plot for S1, this plot begins at a value of 3, equal to the number of bolts being used, and converge to a value of zero. Unlike the S1 plot, however, the different methods do not provide identical results. This is due to the load being applied at an incline. This effect can be seen further in the C/C ratio plots.

Since the load for the S1 and S2 configurations is vertical, the values of y_r are equal to zero. Once the attitude of the load changes, the y_r coordinate shifts from the elastic centroid. Figure 47 shows a plot of the y_r coordinates for the S3 condition.

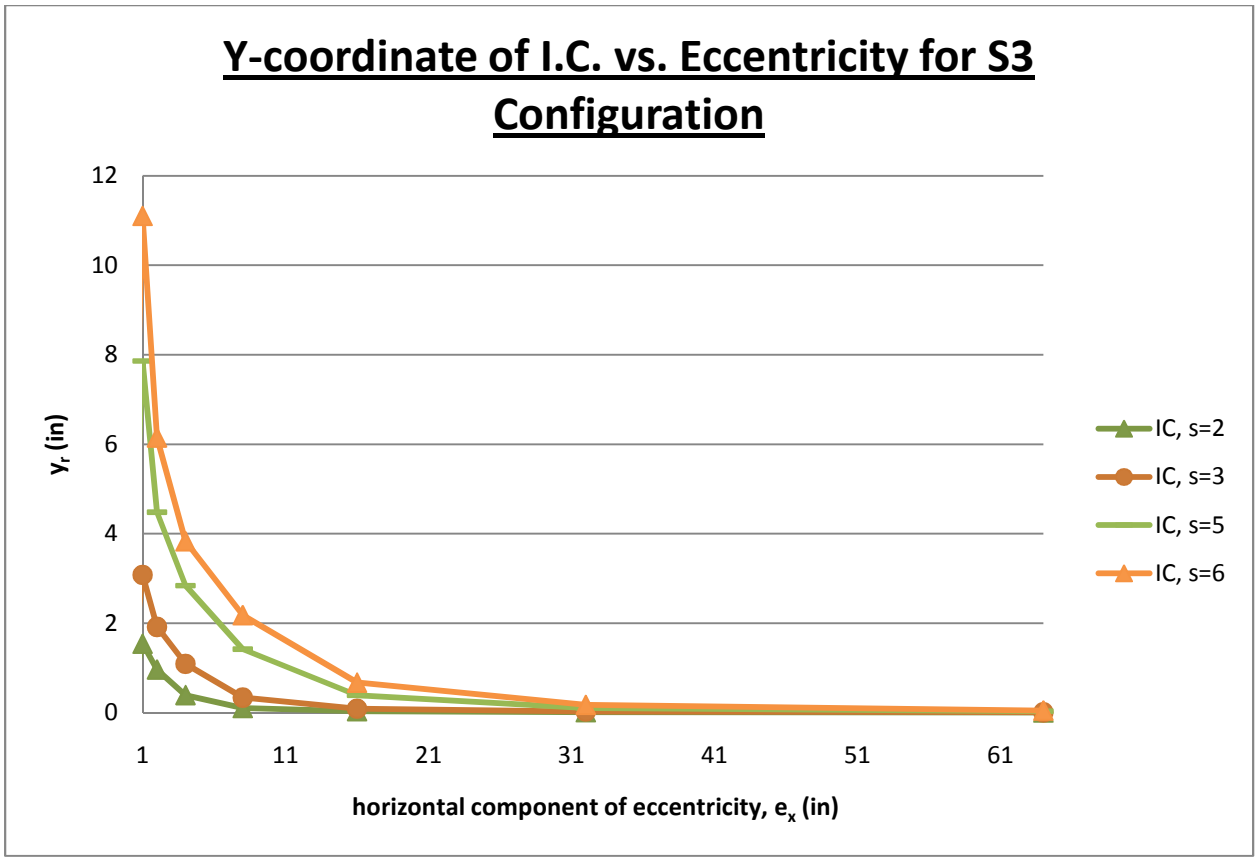


Figure 47: S3 - y_r vs. e_x

The values for y_r begin at some value, which appears to increase with the spacing, and then converges to zero or the elastic centroid of the bolt group. Like the x_r coordinate, the values only converge to the elastic centroid if the bolt group is symmetric as will be discussed further in the A1 configuration.

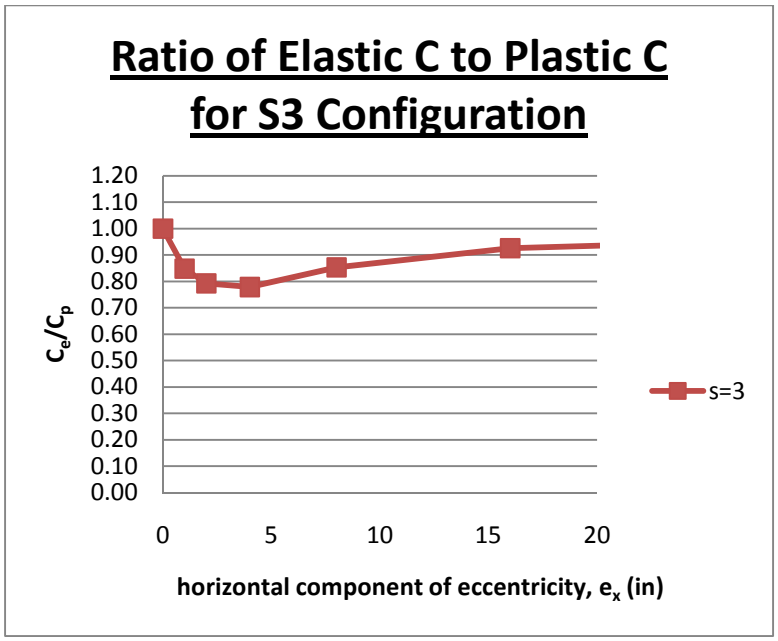


Figure 48: S3 - C_e/C_p

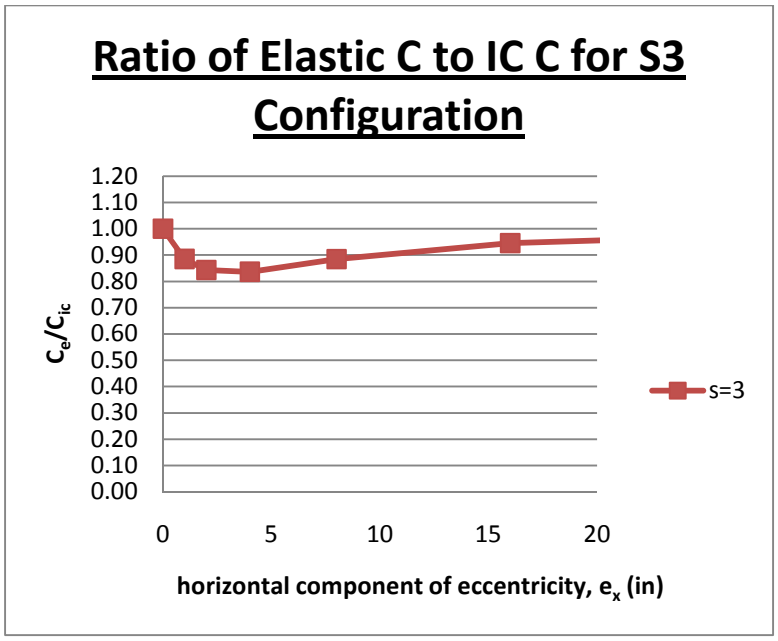


Figure 49: S3 - C_e/C_{ic}

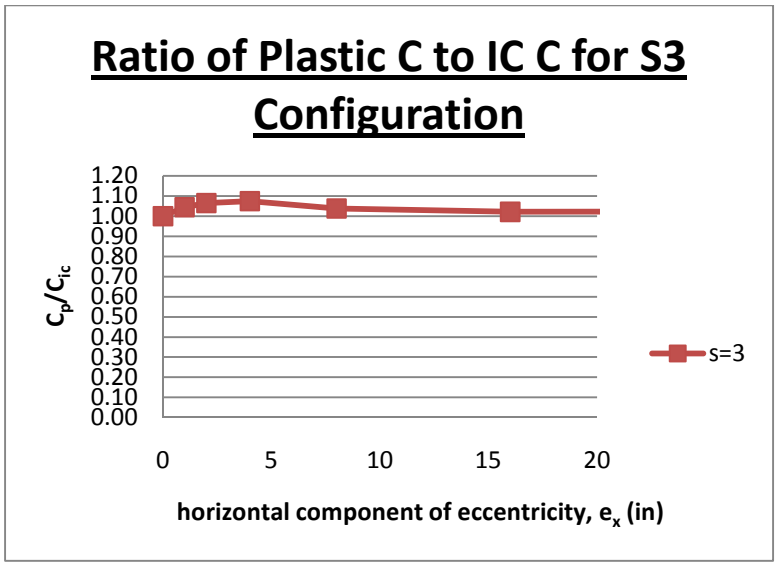


Figure 50: S3 - C_p/C_{ic}

The C/C ratio plots for the S3 configuration have the same general shape as those from the S2 configuration. The range of C/C ratios increases from the S2 configuration as a result of the inclined load. The C_e/C_p ratio ranges from 0.78 - 1.00, the C_e/C_{ic} ratio ranges from 0.83 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.07.

D1 - (3) Bolts in Double Rows with Vertical Load

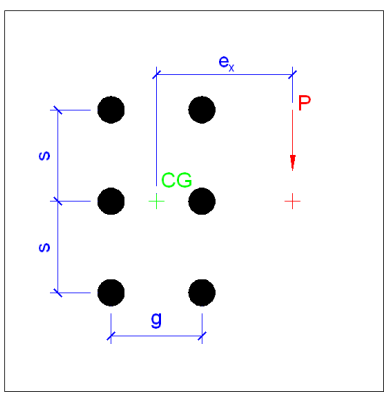


Figure 51: D1 Configuration

Configuration D1, shown in Figure 51, is analyzed and compared to the S2 configuration in order to determine the difference between bolts in a single row versus multiple rows. Like the other examples, the origin is set at the elastic centroid of the bolt group. The variations of the horizontal component of eccentricity, e_x, and vertical spacing, s, are the same as the S2 configuration. The horizontal spacing, g, is held constant at a dimension of 3-inches.

Similar to the effects of an inclined load, the addition of another row of bolts results in the different methods providing different results. As shown in Figure 52, the plots for the plastic and IC methods are nearly identical whereas the plot for the elastic method is slightly lower.

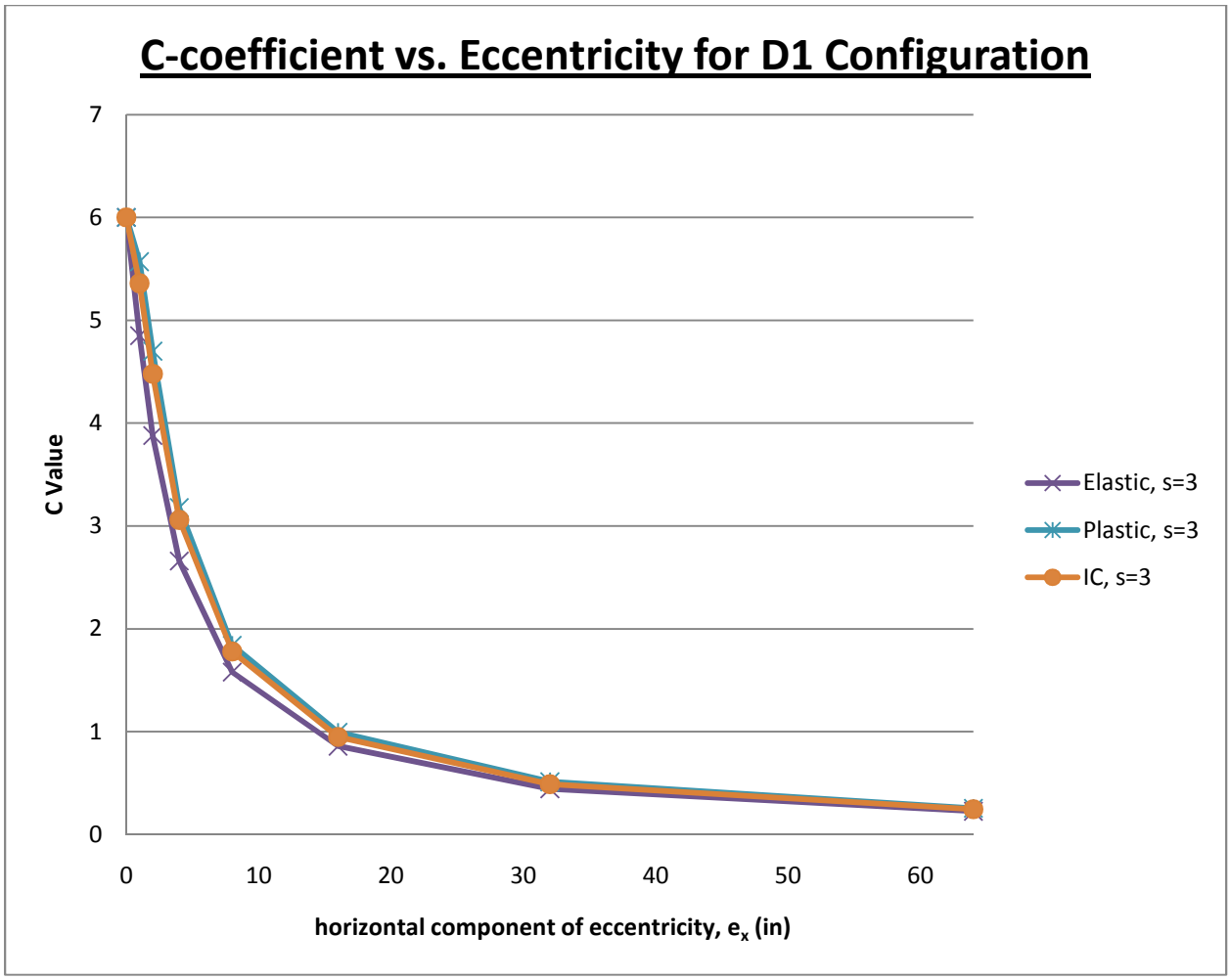


Figure 52: D1 - C vs. e_x for Different Methods

The C/C ratio plots also show how the various methods differ:

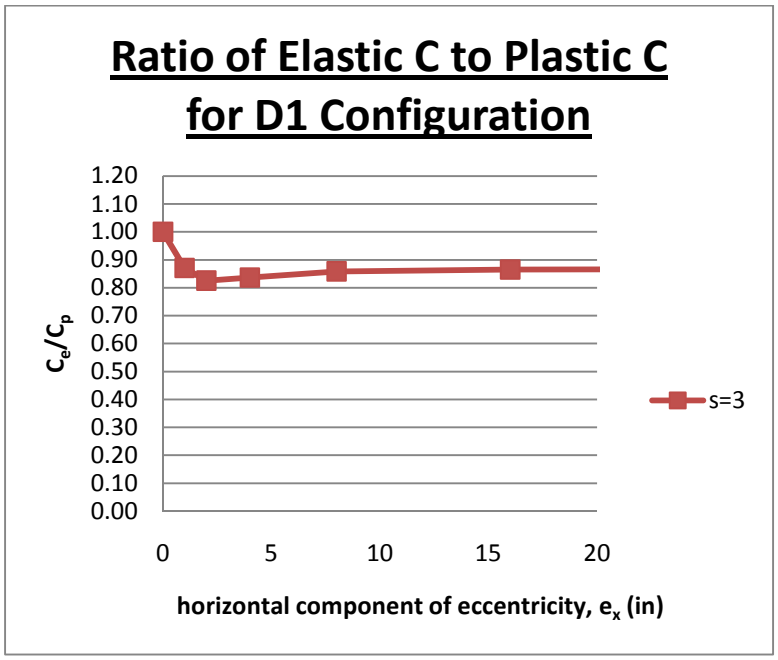


Figure 53: D1 - C_e/C_p

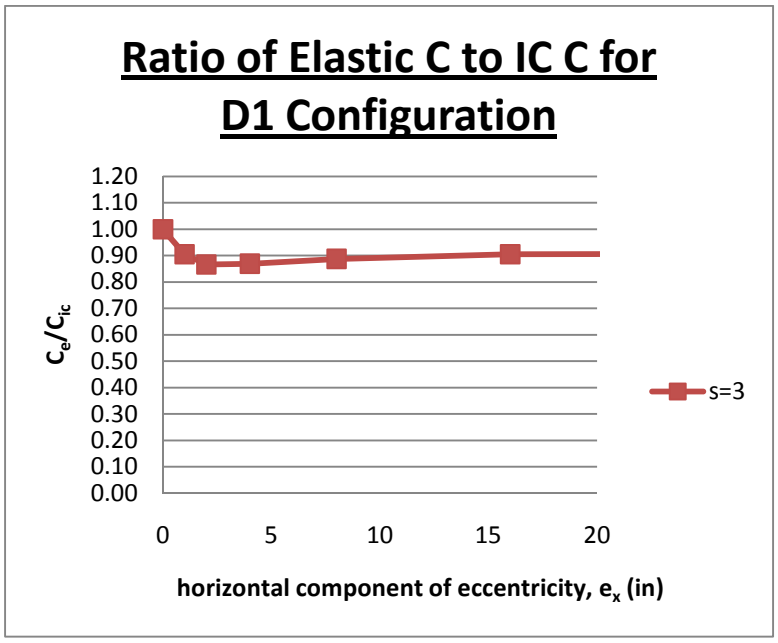


Figure 54: D1 - C_e/C_{ic}

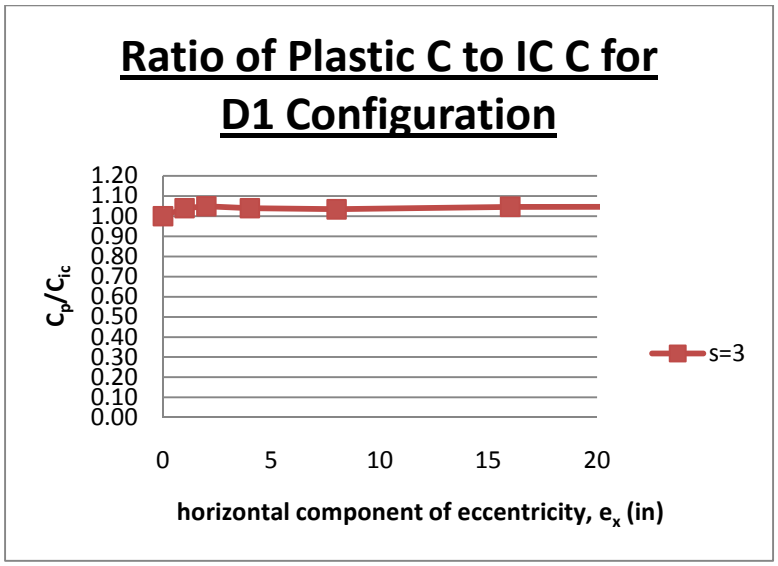


Figure 55: D1 - C_p/C_{ic}

The addition of another row of bolts increases the overall range of C/C ratios. The greatest difference occurs between the elastic method and the other two methods since both the C_e/C_p and C_e/C_{ic} ratios increase by approximately 10-percent from the S2 configuration (Figure 42 - Figure 43), whereas, the C_p/C_{ic} ratio only increases by 1.2-percent from the S2 configuration (Figure 44). For the D1 configuration, the C_e/C_p ratio ranges from 0.82 - 1.00, the C_e/C_{ic} ratio ranges from 0.87 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.06.

D2 - (3) Bolts in Double Rows with Inclined Load

The D2 configuration is exactly the same as the D1 configuration except that the load is inclined to an angle of θ = 30-degrees. Horizontal component of eccentricity, e_x, vertical spacing, s, and horizontal spacing, g, are all identical to the D1 configuration. Comparisons of the D2 configuration can then be made to both the D1 configuration for the differences with the inclined load as well as to the S3 configuration for the additional row of bolts.

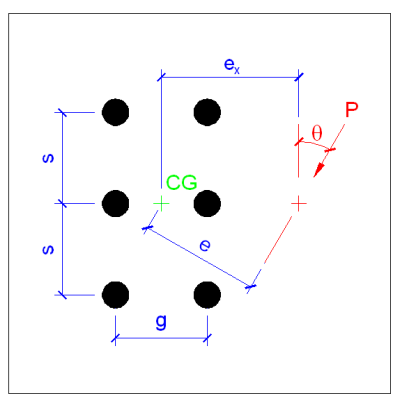


Figure 56: D2 configuration

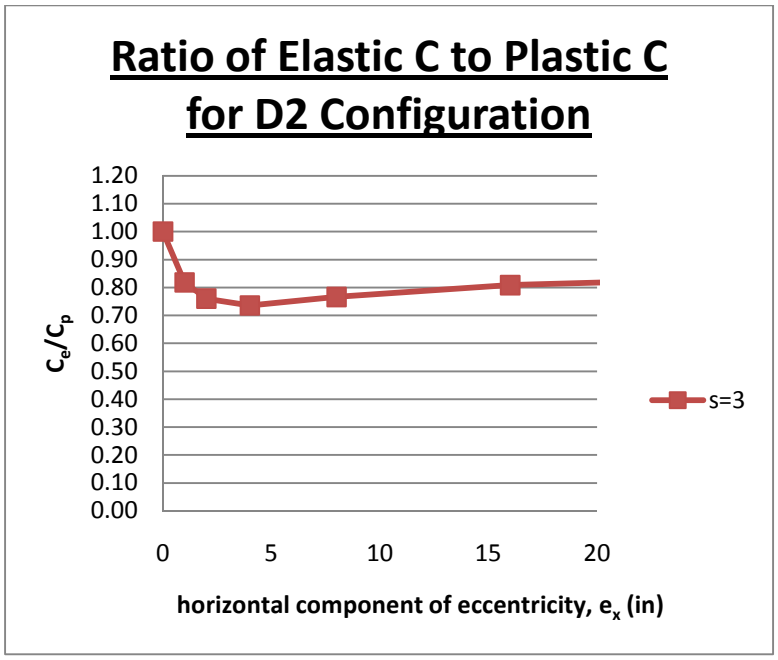


Figure 57: D2 - C_e/C_p

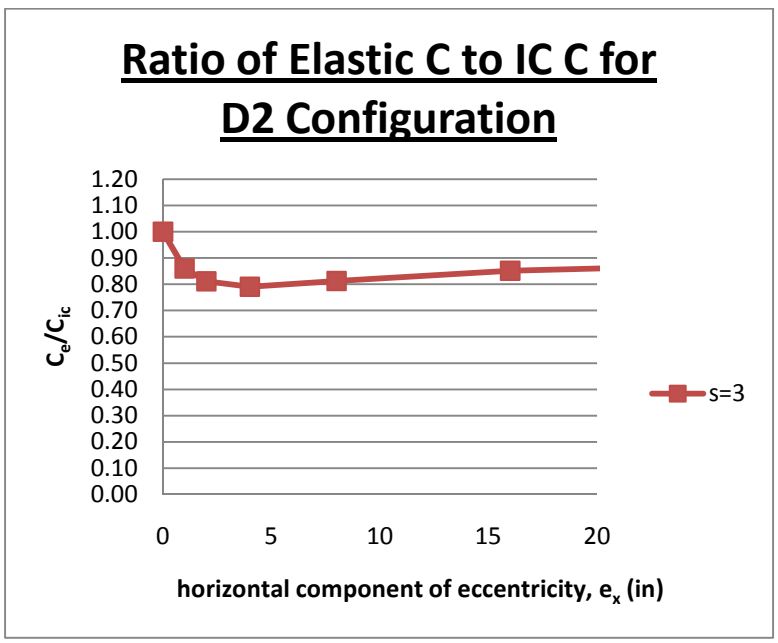


Figure 58: D2 - C_e/C_{ic}

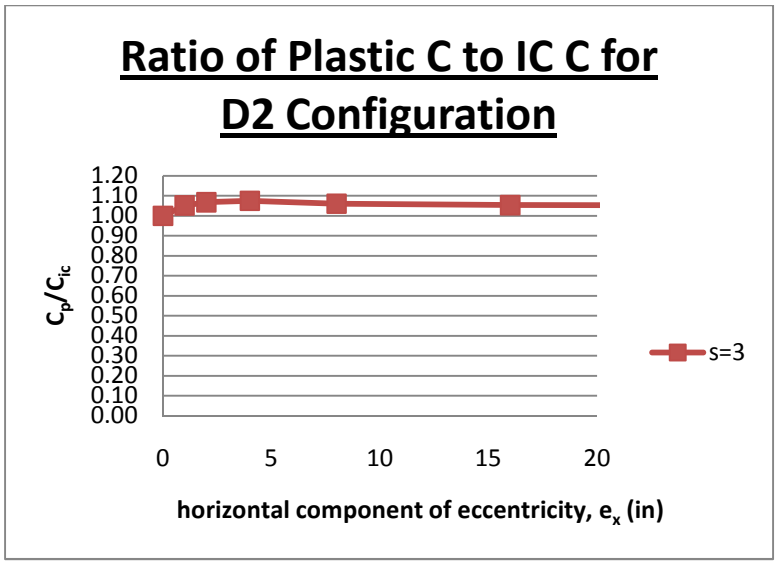


Figure 59: D2 - C_p/C_{ic}

The C/C ratio plots for the D2 configuration have the same general shape as those from the previous examples. Similar to the S3 configuration, the effect of going from a vertical load to the inclined load increases the range of C/C ratios. The ratios including the elastic method increase by 7 to 9-percent from the D1 configuration (Figure 53 - Figure 54) while the C_p/C_{ic} ratio increases only 2.1-percent (Figure 55). When compared to the S3 configuration, the additional row of bolts increases the range of C_e ratios by 4-percent (Figure 48 - Figure 49) and the C_p/C_{ic} ratio increases by only 0.2-percent (Figure 50). The D2 configuration C_e/C_p ratio ranges from 0.74 - 1.00, the C_e/C_{ic} ratio ranges from 0.79 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.08.

V1 - (3) Bolts in a Single Row with Varied Inclined Load

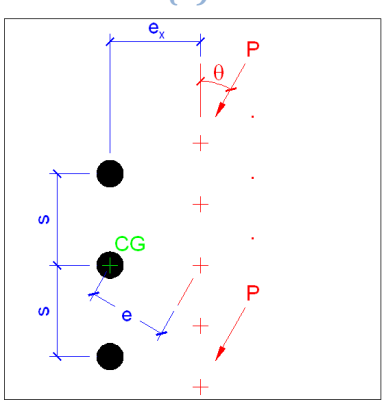


Figure 60: V1 Configuration

All of the examples thus far have had either a vertical load or an inclined load placed in line with the x-coordinate of the elastic centroid of the bolt group. The V1 and V2 configurations were created to investigate the effects of varying the location of an inclined load. The horizontal component of eccentricity, e_x, vertical spacing, s, and attitude of the load, θ, for the V1 configuration is identical to that of

the S3 configuration. The only difference is that now the y-coordinate of the applied load, y_p, will be changed to equal values of -6, -3, 0, 3 and 6. These dimensions are based on the origin being set at the elastic centroid.

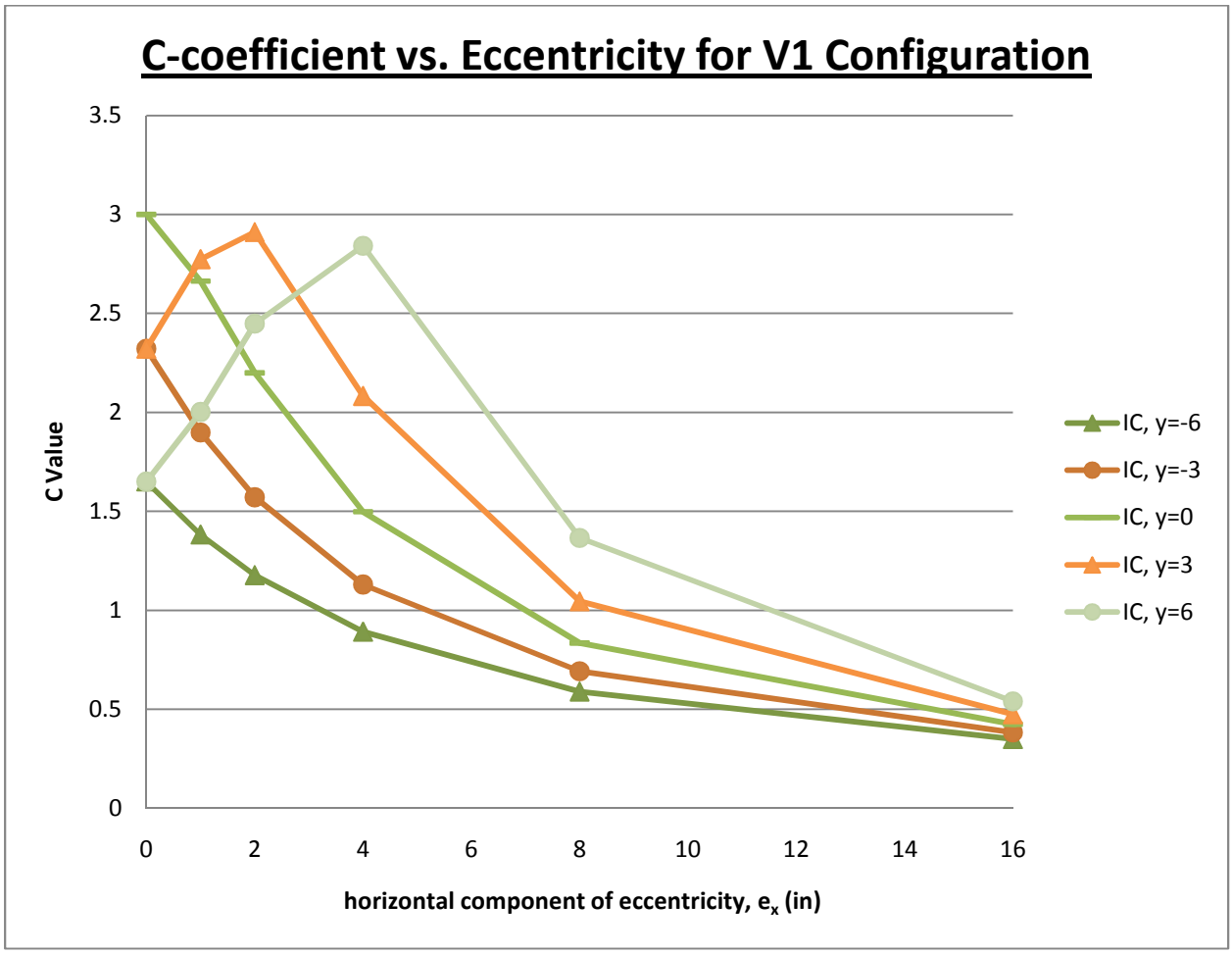


Figure 61: V1 - C vs. e_x

The plot of C vs. e_x for the V1 condition provides interesting insights into how the load is being applied to the bolt groups. As expected, when y_p is equal to zero, the plot is identical to the S3 configuration. When the y_p value is negative, the plot begins at a C value less than 3 and then follows the same general path to zero. When the y_p value is positive, the plot begins at a C value lower than 3, increases until it hits a value of 3 and then follows the same general path to zero from there. A C value of 3, in this case, means that the connection is in pure shear since the capacity of the bolt group cannot be more than the

number of bolts times the ultimate shear strength of the each bolt individually. Any value lower than 3, in this case, means that the bolts are experiencing moment from the eccentric load and reducing the capacity of the connection.

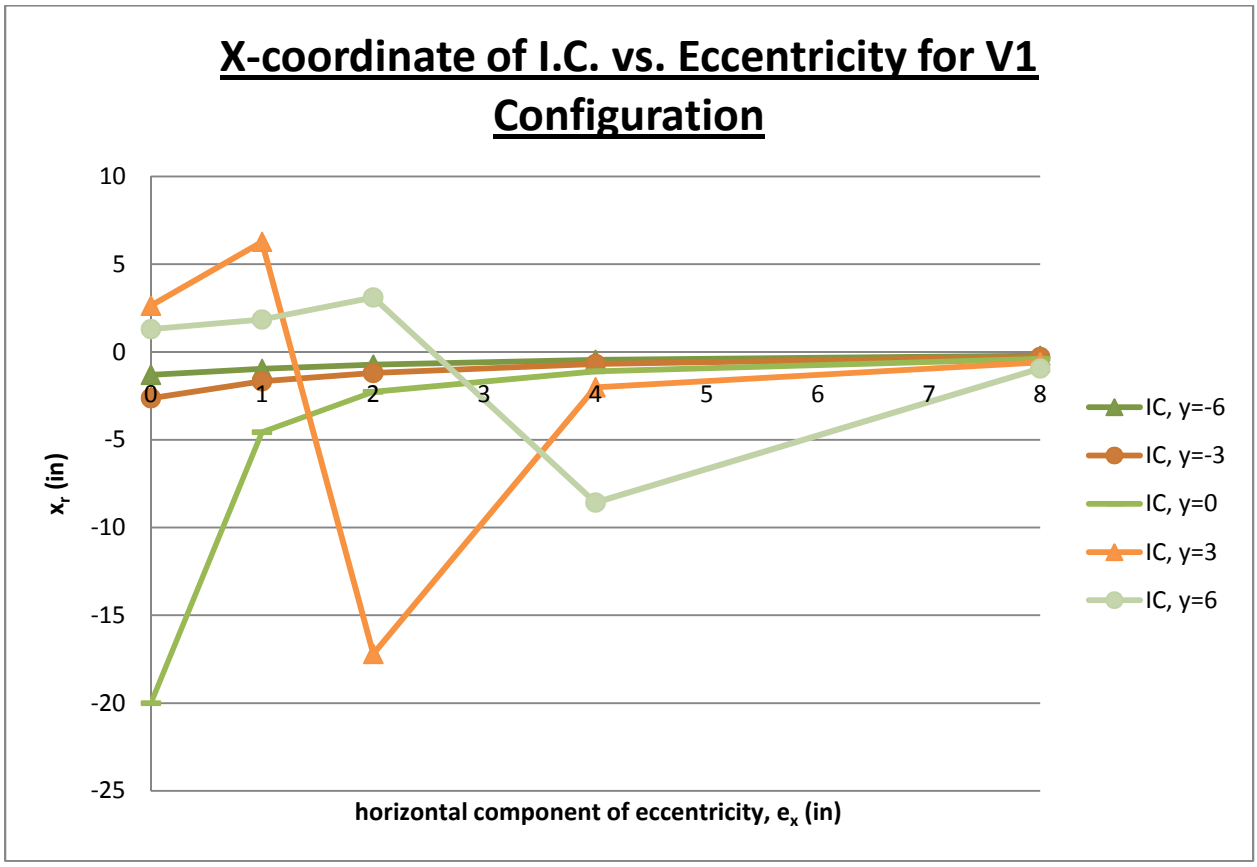


Figure 62: V1 - x_r vs. e_x

The x_r and y_r plots for the V1 configuration shown in Figure 62 and Figure 63 are deceiving for the condition where the y_p values are positive.

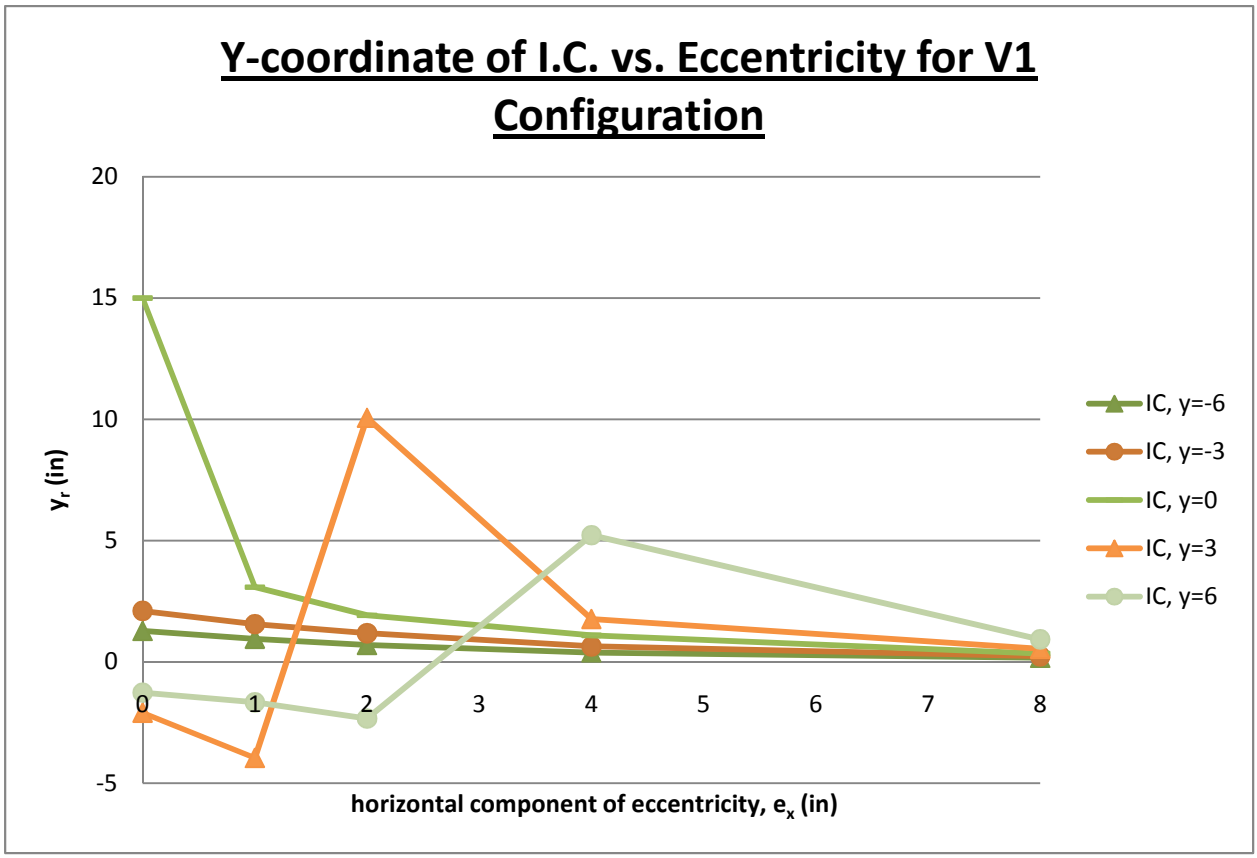


Figure 63: V1 - y_r vs. e_x

As discussed for the C vs. e_x plot for the V1 configuration, there exists a point where the connection is in pure shear and the C value equals 3. At this same point, the value of both x_r and y_r approach ±infinity. Therefore, the x_r and y_r values should be ±infinity for y_p = 3-inches when the eccentricity is a little more than 1-inch and for y_p = 6-inches when the eccentricity is around 2.5-inches. Once the plots are beyond the point at which the connection is in pure shear, the plot begins to converge to zero as the eccentricity increases.

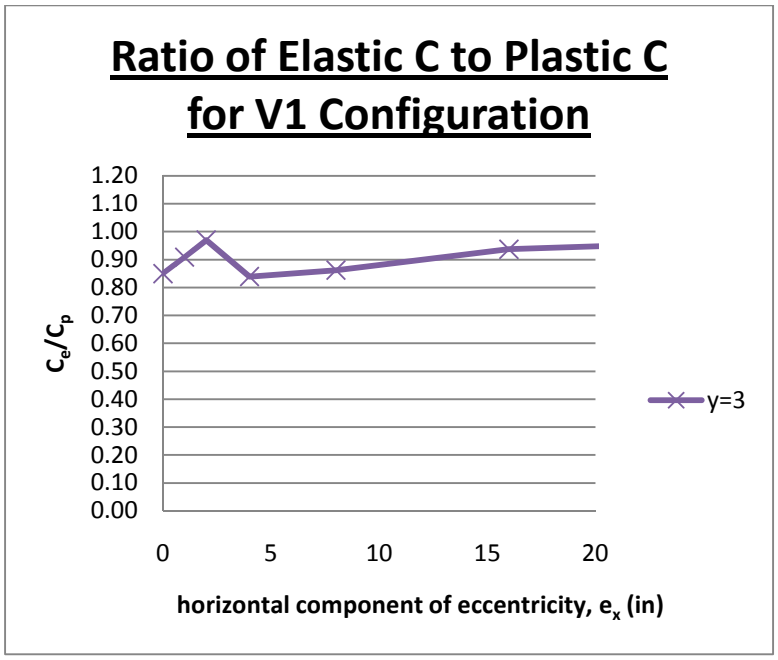


Figure 64: V1 - C_e/C_p

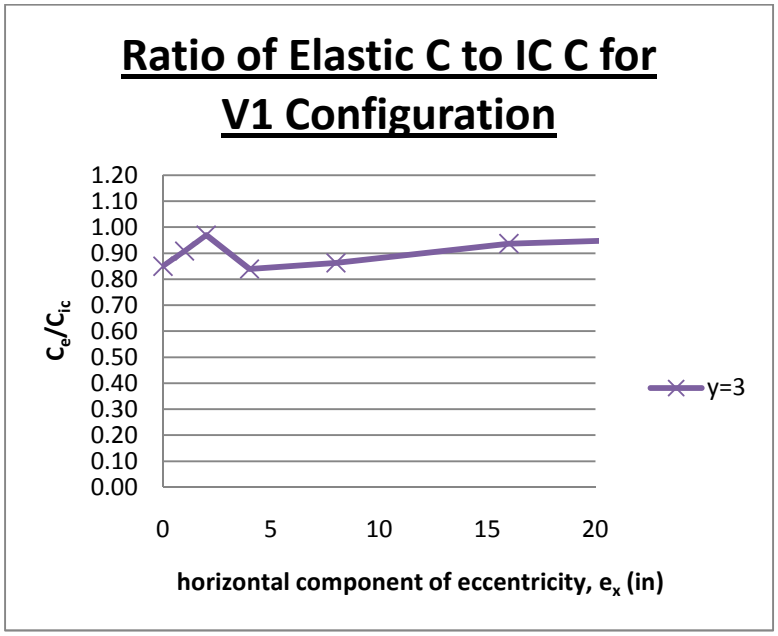


Figure 65: V1 - C_e/C_{ic}

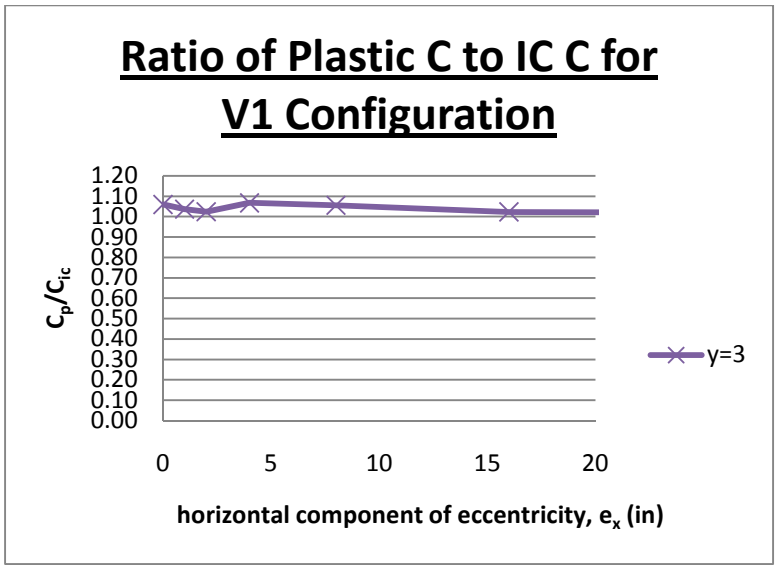


Figure 66: V1 - C_p/C_{ic}

The C/C ratio plots for the V1 configuration also reflect the transition from pure shear to a combination shear and moment condition for both the y_p = 3-inch and y_p = 6-inch conditions. The inflection points in the curves for these conditions occur at the same values of eccentricity as the maximum C value (Figure 61) and as the inflection points of the x_r and y_r coordinates (Figure 62 & Figure 63). For the V1 configuration, the C_e/C_p ratio ranges from 0.78 - 1.00, the C_e/C_{ic} ratio ranges from 0.83 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.08. When the ranges of the C/C ratios for the V1 configuration are compared to the ranges of C/C ratios for the S3 configuration (Figure 48 - Figure 50) they are identical, which means that as long as the bolt configuration and attitude of the applied load do not change the range of C/C ratios will not change.

V2 - (3) Bolts in Double Rows with Varied Inclined Load

The V2 configuration adds a second row of bolts to the V1 configuration as shown in Figure 67. The horizontal component of eccentricity, e_x, vertical spacings, s, and attitude of the load, θ, remain the same. The second row of bolts is added with a constant horizontal spacing of 3-inches. This allows the V2 configuration to be

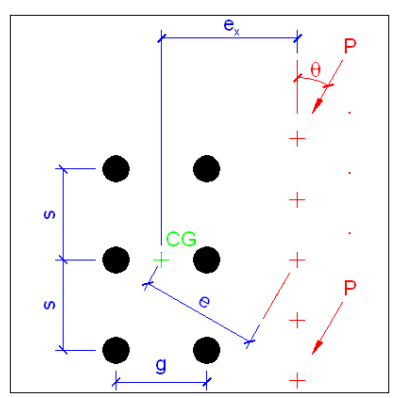


Figure 67: V2 Configuration

compared to the V1 configuration for the additional row of bolts as well as the D2 configuration for the varying location of the inclined load.

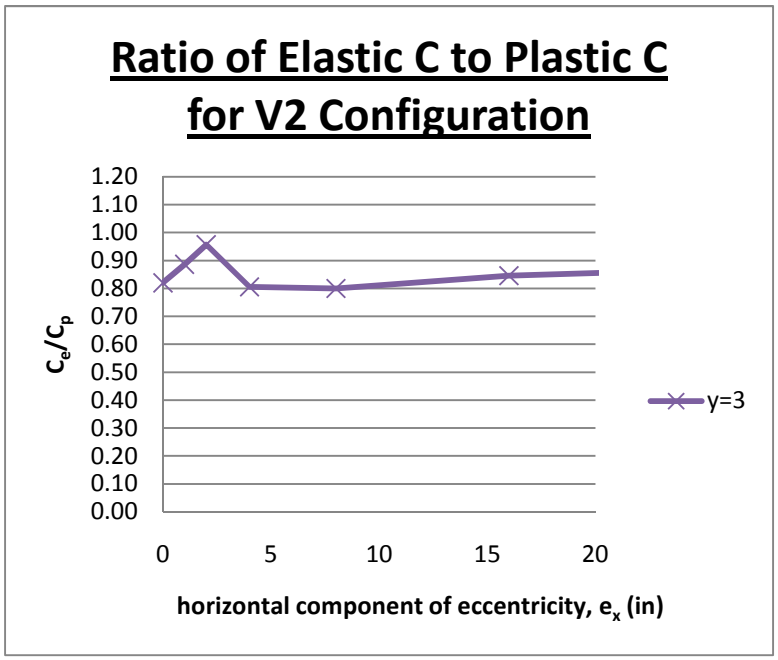


Figure 68: V2 - C_e/C_p

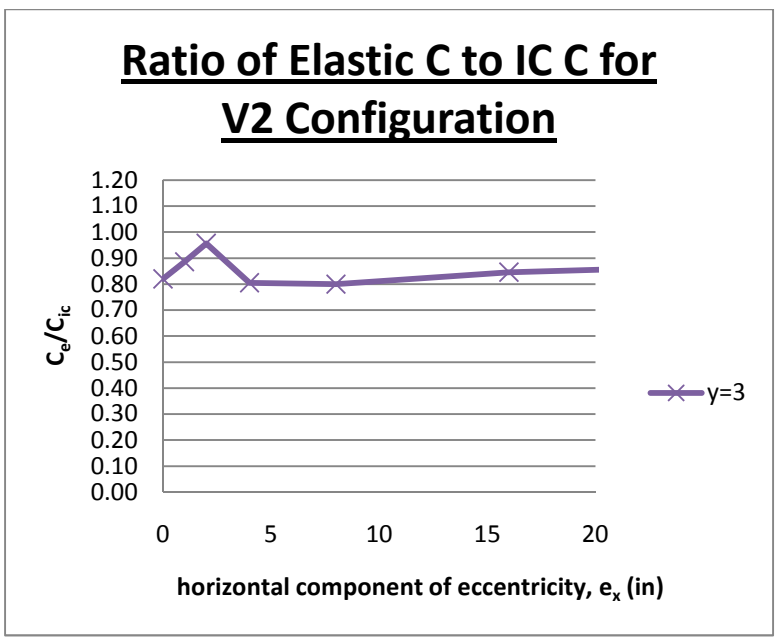


Figure 69: V2 - C_e/C_{ic}

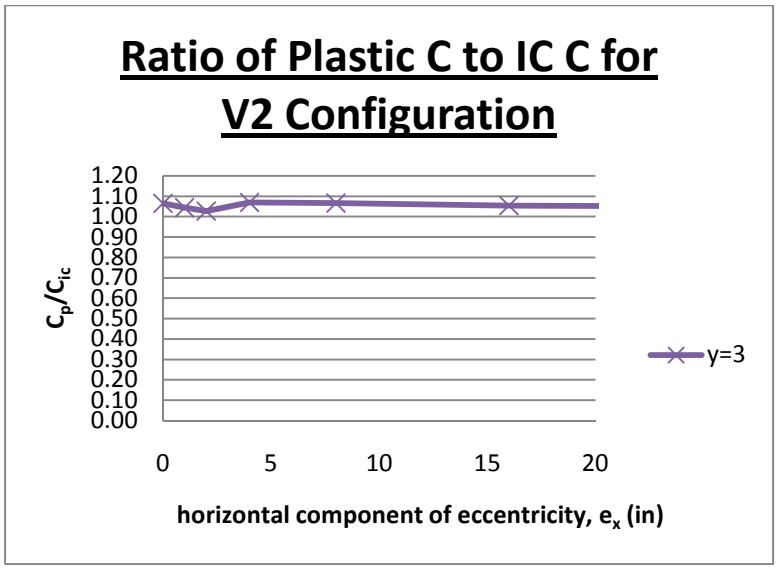


Figure 70: V2 - C_p/C_{ic}

Similar to the V1 configuration, the C/C ratio plots for the V2 configuration reflect the transition from pure shear to a combination shear and moment condition for both the y_p = 3-inch and y_p = 6-inch conditions. For the V2 configuration, the C_e/C_p ratio ranges from 0.74 - 1.00, the C_e/C_{ic} ratio ranges from 0.79 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.08. When the ranges of the C/C ratios for the V2 configuration are compared to the ranges of C/C ratios for the D2 configuration (Figure 57 - Figure 59) they are identical, which demonstrates again that as long as the bolt configuration and attitude of the applied load do not change the range of C/C ratios will not change.

A1 - (3) Bolts in Asymmetric Row with Vertical Load

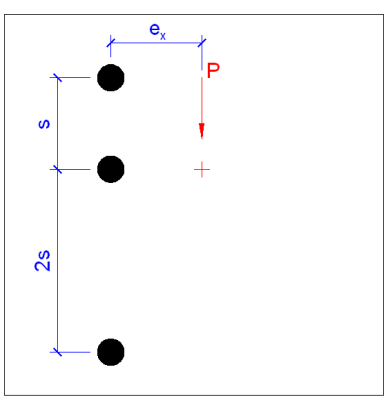


Figure 71: A1 Configuration

All of the previous examples have had bolt patterns that are symmetric about the elastic centroid. Configuration A1 is used to compare a bolt pattern that is asymmetric. For this pattern, the origin is set at the center bolt instead of the elastic centroid. The number of bolts and horizontal component of eccentricity, ex, are identical to the S2 configuration. However, the vertical spacing of the bolts, s, is varied as shown in Figure 71, where the values of s = 2, 3, 5 and 6.

When the bolts are symmetric and the load is applied vertically, the y_r value is equal to zero. For asymmetric bolt configurations, the y_r value is no longer zero because the location of the location of the I.C. has to be adjusted to account for the unbalanced bolt deformation and resultant bolt forces. As shown in Figure 72, the elastic method always gives a constant value for y_r and both the plastic and I.C. methods approach a constant value for y_r as the eccentricity increases.

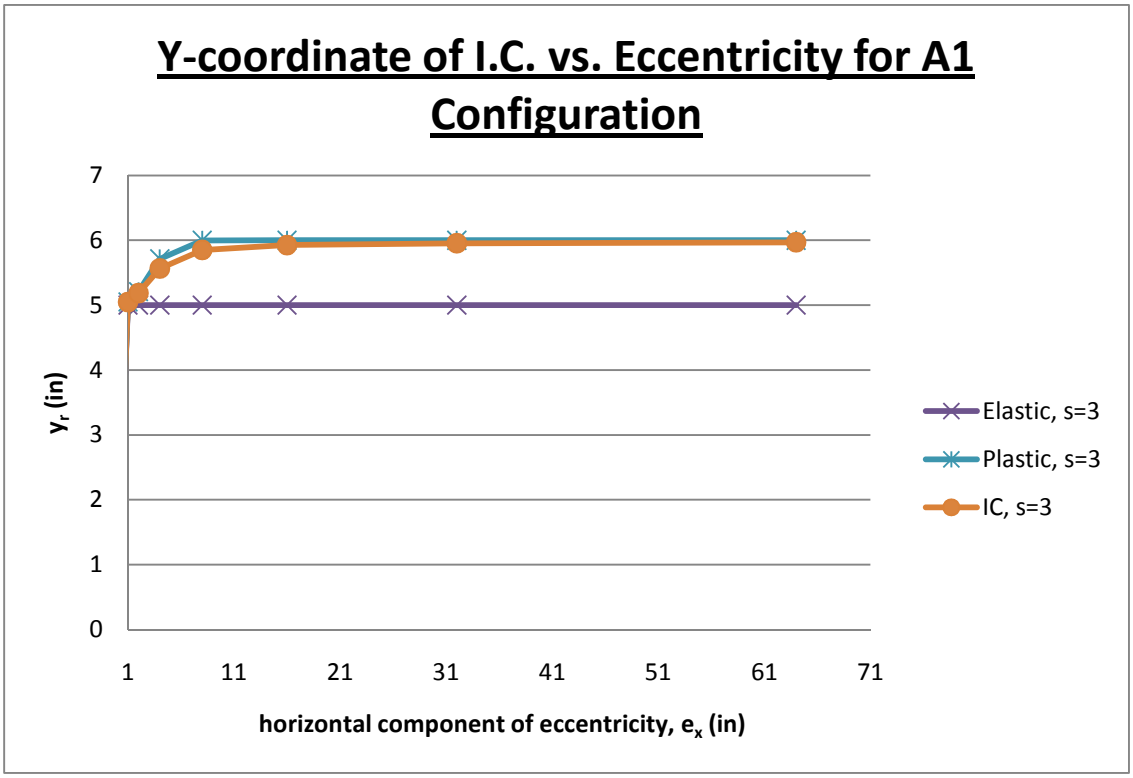


Figure 72: A1 - y_r vs. e_x for Different Methods

Figure 73 shows how as the spacing increases so do the values for y_r.

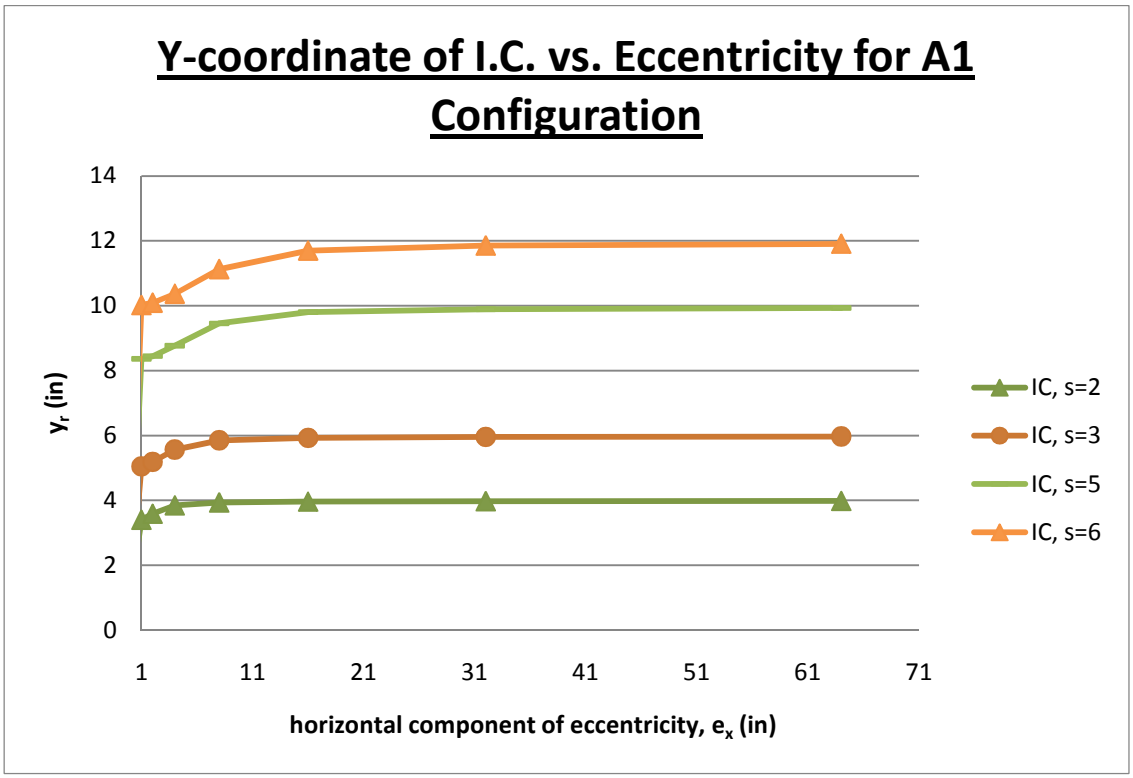


Figure 73: A1 - y_r vs. e_x for Different Spacings

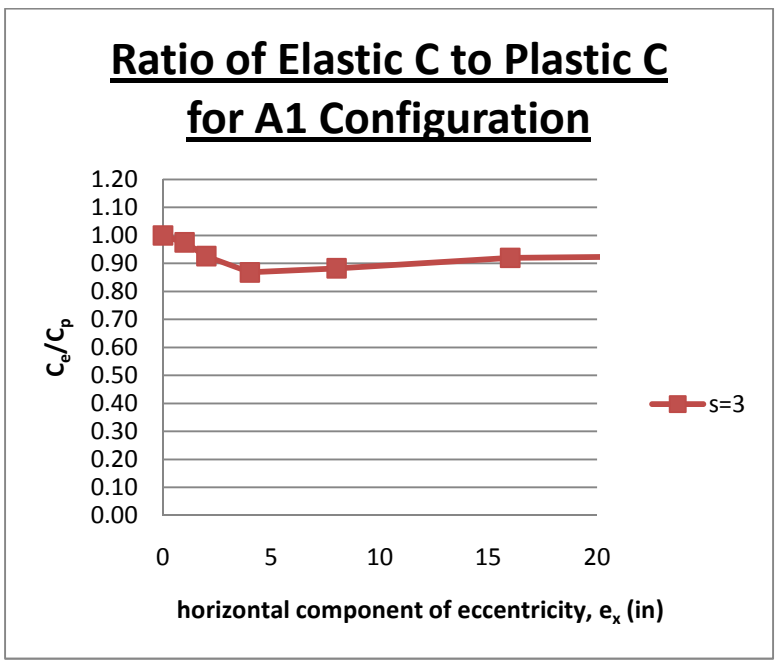


Figure 74: A1 - C_e/C_p

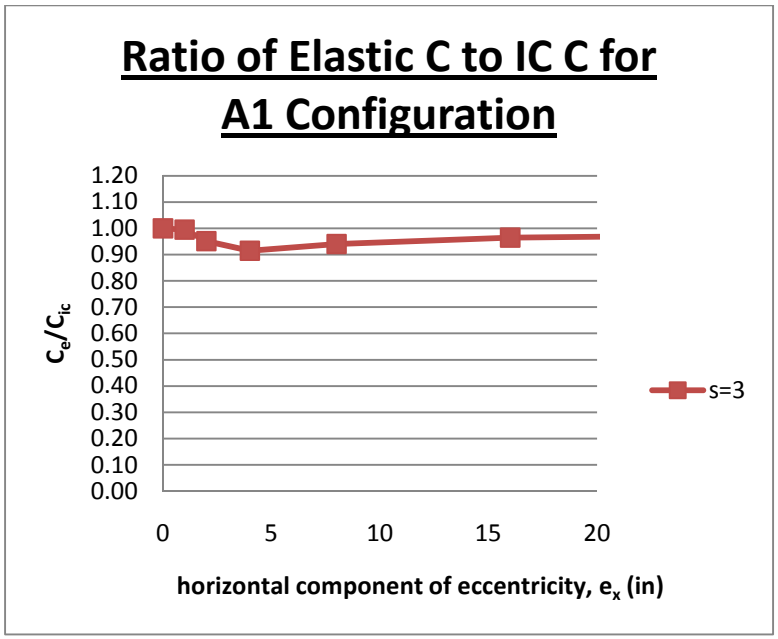


Figure 75: A1 - C_e/C_{ic}

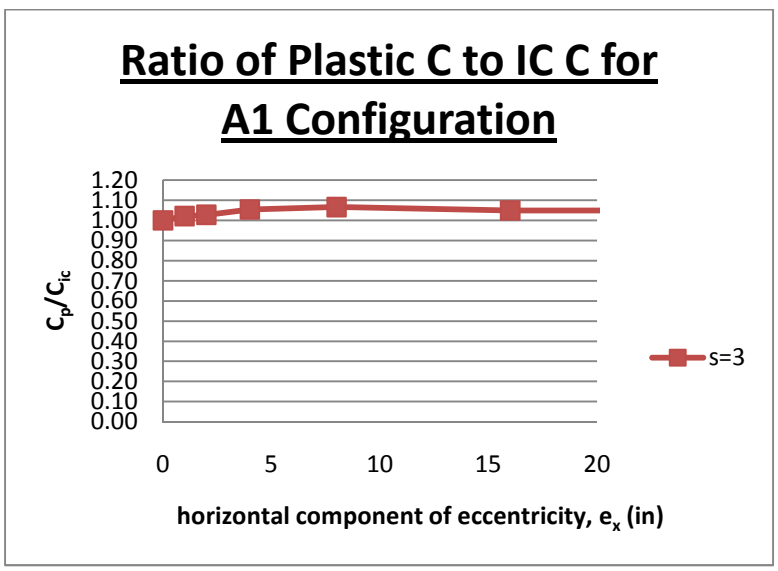


Figure 76: A1 - C_p/C_{ic}

The C/C ratio plots for the A1 configuration are similar in shape to the S3 configuration (Figure 48 - Figure 50). The range of values is slightly lower than the S3 configuration. For the A1 configuration, the C_e/C_p ratio ranges from 0.86 - 1.00, the C_e/C_{ic} ratio ranges from 0.91 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.07.

AV1 - (3) Bolts in Asymmetric Row with Varied Inclined Load

The last example that is analyzed contains both an asymmetric bolt pattern and an inclined load with various positions. The bolt pattern is identical to the A1 configuration with an s = 3-inches only and the varying inclined load is applied the same as for the V1 configuration at y_r = -6, -3, 0, 3 and 6-inches. For this condition, the origin was set to be at the center bolt instead of the elastic centroid.

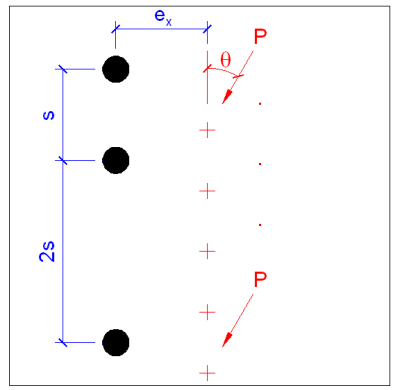


Figure 77: AV1 Configuration

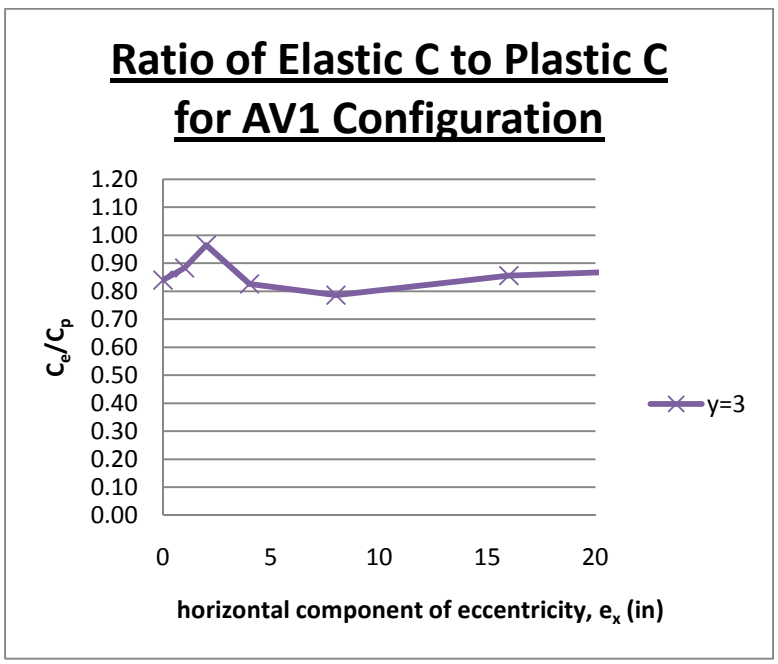


Figure 78: AV1 - C_e/C_p

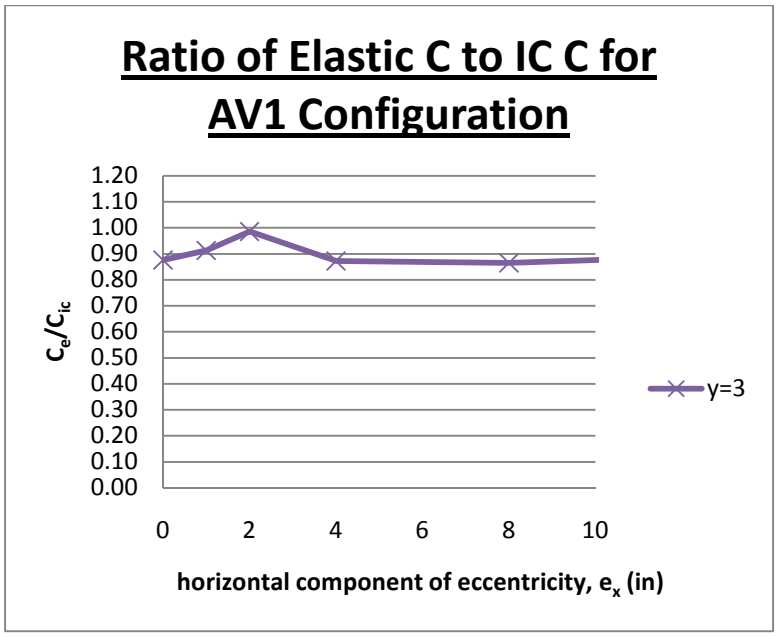


Figure 79: AV1 - C_e/C_{ic}

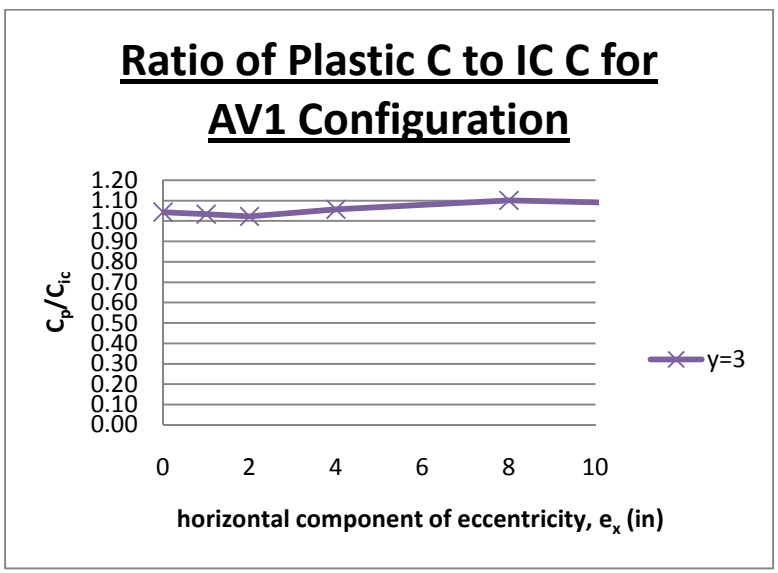


Figure 80: AV1 - C_p/C_{ic}

The C/C ratio plots for the AV1 configuration, similar to the V1 plots (Figure 64 - Figure 66), reflect the transition from pure shear to a combination shear and moment condition. The inflection points in the curves for these conditions occur at the same values of eccentricity as the maximum C value and as the inflection points of the x_r and y_r coordinates. For the AV1 configuration, the C_e/C_p ratio ranges from 0.78 - 1.00, the C_e/C_{ic} ratio ranges from 0.85 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.10. When the ranges of the C/C ratios for the AV1 configuration are compared to the ranges of C/C ratios for the V1

configuration they are within 1 to 3-percent. This suggests that asymmetric bolt patterns have minimal effects on the overall range of C/C ratios.

Examples Summary

A variety of examples have been reviewed throughout this chapter in order to check the validity of the software, verify the functionality of the software as well as compare and contrast the different methods. These examples came directly out of the code, historical testing as well as numerous examples that were chosen to allow for variation of the input parameters.

From the code verification examples, it was observed that the software gave identical results to the design tables in the AISC Manuals. The code only provides values for the elastic method and I.C. methods, so those are the only methods that can be verified. All results for these two methods, however, were identical to those given in the code. This confirmed that the software is functioning properly and is providing valid results for both the elastic and I.C. methods.

The examples based on historical testing as well as the variable input examples showed that the elastic method consistently gives the lowest values of C, the modified elastic method gives the highest values of C and the plastic and I.C. methods give C values somewhere in between. Since the modified elastic method was determined to be overly-liberal in its estimation of strength it was not included further in the comparison of the methods.

A summary of the C/C ranges for the variable input examples is given in Table 15.

Table 15: Variable Input Example C/C ranges

Configuration	C_e/C_p		C_e/C_{ic}		C_p/C_{ic}	
	Min	Max	Min	Max	Min	Max
S1	1.000	1.000	1.000	1.025	1.000	1.025
S2	0.912	1.000	0.949	1.000	1.000	1.043
S3	0.776	1.000	0.833	1.000	1.000	1.074
D1	0.824	1.000	0.866	1.000	1.000	1.055
D2	0.736	1.000	0.791	1.000	1.000	1.076
V1	0.776	1.000	0.834	1.000	1.000	1.075
V2	0.736	1.000	0.791	1.000	1.000	1.075
A1	0.864	1.000	0.914	1.000	1.000	1.069
AV1	0.782	0.995	0.853	0.998	1.003	1.100

This table also shows that the C_e/C_p ratio ranges from values of 0.74 - 1.00, the C_e/C_{ic} ratio ranges from 0.83 - 1.00 and the C_p/C_{ic} ratio ranges from 1.00 - 1.10. The elastic method is the most conservative method since the ratios containing C_e are always less than 1.00. The plastic method can also be considered the most liberal since it is always larger than both the elastic method and the I.C. method.

Table 15 also shows the effects of varying the different inputs. The S1, S2, D1 and A1 configurations have the tightest C/C ratios since these configurations have loads applied vertically. When attitude is applied to the load, such as in the S3, D2, V1, V2 and AV1 configurations, the C/C ratio ranges get larger, which suggest that the methods differ most when an inclined load is applied. Asymmetric bolt patterns also had a detrimental effect on the C/C ratios. The C/C ratio ranges for the A1 configuration are larger than those of the S2 configuration, so asymmetric bolt configurations also affect the methods differently. One input parameter that had no effect on the C/C ranges is the location of the applied load. The C/C ratios for the S3 configuration are identical to the V1 configuration and the C/C ratios for the D2 configuration are identical to the V2 configuration.

Chapter 5 - Conclusion

Up until 1981, the elastic method was the preferred method of analysis for determining the capacity of a bolt group under eccentric loads. The AISC Manuals, at that time, provided both design tables and simple equations for the design engineer to easily calculate the bolt capacity. Once the IC method was adopted, the equations no longer were simple since they required an iterative analysis. Thus, the design engineer had the choice of forcing their design to fit within one of the pre-populated design tables, performing a complex iterative analysis or using the elastic method, which was identified as being overly conservative. If their bolt pattern did not conform to the design tables, the design engineer was left with few options for designing a competitive connection without spending a significant amount of design time. Therefore, the main purpose of this thesis was to develop a tool that would allow design engineers to quickly calculate the capacity of a bolt group under eccentric loads using the currently recommended IC analysis method.

This software tool was successfully developed and presented in this thesis. Numerous examples were then executed in the software to verify that results match those given in the AISC Manuals and to understand how the different methods, different applications of load and different bolt patterns affect the overall capacity of the bolt group. Overall it was determined that the elastic method consistently gives the lowest values of C and the plastic method consistently gives the highest values of C. This means that the elastic method is the most conservative and the plastic method is the most liberal. The IC method provides values that are between the elastic and plastic method suggesting that it is the most accurate method. This makes sense since it takes into account the non-linear response of the individual bolt deformations. The software developed in this thesis will finally provide engineers with the tool necessary to complete the design of any bolt pattern under any eccentric in-plane point load using any method of analysis.

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Appendix – Definition of Variables

C = bolt group coefficient

C_{min} = minimum value of bolt group coefficient

C_n = assumed initial value of bolt group coefficient

C_{n+1} = iteration of the bolt group coefficient

CG = elastic centroid of the bolt group

[D] = matrix of coefficients related to geometry of the bolt group and relative deformations in bolts

$[D]_n$ = matrix of bolt coefficients based on assumed kinematic variables $\{U\}_n$

d_i = distance of an individual bolt from the instantaneous center of rotation

d_{max} = maximum bolt distance from the instantaneous center of rotation

e = eccentricity

e_x = horizontal component of eccentricity

e = base of natural logarithms

e_{eff} = effective eccentricity

F_{xi} = x-component of the force in an individual bolt

F_{yi} = y-component of the force in an individual bolt

I.C. = instantaneous center of rotation

$Max[(\Delta_i)]_n$ = value of maximum deformation of an individual bolt

M_P = moment due to load P

N = total number of bolts

n = number of bolts in one vertical row

P = magnitude of load

{P} = vector of loads P_x, P_y and M_p

|P| = magnitude of the vector {P}

P_f = failure load of a bolt group

P_n = nominal capacity of a bolt group

P_{ult} = predicted failure load of a bolt group

P_x = x-component of load

P_y = y-component of load

R = fastener load at any given deformation

R_e = elastic force of a single fastener

R_n = nominal strength of the bolt group

r_n = nominal strength per bolt

R_{ult} = ultimate load attainable by a single fastener

s = vertical spacing of bolts

{U} = vector of kinematic variables u, v, Φ

$\{U\}_n$ = assumed vector of kinematic variables u, v, Φ

$\{U^*\}_{n+1}$ = iteration of the vector of kinematic variables u, v, Φ without scaling factor applied

$\{U\}_{n+1}$ = iteration of the vector of kinematic variables u, v, Φ with scaling factor applied

u = translation of the connection plate in the x-direction

v = translation of the connection plate in the y-direction

x_c = x-coordinate of the elastic center of the bolt group

x_i, x_j, x_k ... = x-coordinates of individual bolts

x_p = x-coordinate of load

x_r = x-coordinate of the instantaneous center of rotation

y_c = y-coordinate of elastic center of the bolt group

y_i, y_j, y_k ... = y-coordinates of individual bolts

y_p = y-coordinate of load

y_r = y-coordinate of the instantaneous center of rotation

Δ_i = deformation of an individual bolt

$(\Delta_i)_n$ = deformation of an individual bolt based on assumed kinematic variables $\{U\}_n$

$(\Delta_i)_{n+1}$ = iteration of the deformation of an individual bolt

Δ_{max} = AISC defined maximum deformation a single bolt can achieve (0.34-inches)

Δ_{xi} = x-component of an individual bolt deformation

Δ_{yi} = y-component of an individual bolt deformation

κ = scaling factor

λ = regression coefficient

μ = regression coefficient

Φ = rotation of the connection plate

ϕ = load-reduction factor

$\{\rho\}$ = vector of geometric terms related to load position and attitude

θ = orientation of load (clockwise positive)

τ_y = theoretical yield shear stress