

TRADE OF DIFFERENTIATED GOODS: HETEROGENEITY OF
CONSUMERS, LOVE OF QUALITY AND QUANTITY, AND THE
VERTICAL TRADE

by

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Trade of Differentiated Goods:Heterogeneity of Consumers,

Love of Quality and Quantity,and the Vertical Trade

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Existing studies have found that the opening of trade results in the price-decreasing competition among horizontally differentiated goods and the quality upgrade of vertically differentiated goods, which results in the replacement of low-quality goods by high-quality goods. This paper spotlights what existing studies have overlooked and challenges these conventional results. First, this paper challenges the price decreasing competition by focusing on the heterogeneity of consumers. If each consumer has his own ideal good and the price elasticity of demand decreases with the distance from his ideal good, the opening of trade followed by the increase in the number of goods gives firms a chance to sell its product intensively to closer and less price elastic consumers. Facing lower price elastic consumers, firms will raise their prices after trade, and the price increasing competition is a result.

In these days, we sometimes observe that high-quality goods from developed countries are replaced by low-quality goods from developing countries, which contradicts with predictions of conventional studies of vertically differentiated goods. One reason why conventional studies cannot show the replacement of high-quality goods is because most studies use a Unit-Demand (UD) model. The model implicitly assumes that consumers care the quality of goods but not the quantity of it. Without the volume effect, it is less likely that inexpensive low-quality goods, which attract consumers with quantity rather than quality, beat high-quality goods. This paper uses a new Non UD model, which relaxes the assumption of the UD and shows that producers of low-quality goods steal consumers from producers of high-quality goods if the income gap between two countries is

large enough.

Besides the UD assumption, another reason why existing studies predict the quality upgrade after trade is because they assume that consumers value quality so much that the utility value of the added quality always exceeds the cost of it. This paper relaxes the assumption and considers a case where the utility increases with quality at a decreasing rate. With severe concave utility function, numerical examples show that expensive high-quality goods from a developed country are replaced by inexpensive low-quality goods from a developing country.

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Chapter 1

INTRODUCTION

I When Firms Meet Ideal Consumers

-Trade of Horizontally Differentiated Goods in the Presence of Heterogeneous Consumers-

Motivations and Research Questions Existing studies in international trade have found that the opening of trade brings countries more varieties of horizontally differentiated goods, stimulates the competition, and consequently leaves firms no room to increase their price, which is the non-surprising conclusion of most existing studies.¹ However, existing studies reach this conclusion by overlooking another change brought by trade: trade changes consumers to whom firms sell their products. This paper analyzes the forgotten change in the consumers' side by using the simplified Helpman model (1981), where each consumer has a different ideal good and the price elasticity of demand depends on the distance between the ideal good and the nearest available good.

Main Results and Contributions In my model, the price consumers face increases with the distance between their ideal goods and the good actually sold in the economy, and the price elasticity of demand increases with price. Close consumers, whose ideal goods are close to the good they actually buy have a lower price elasticity of demand than remote consumers do. With this heterogeneity, an increase in the number of firms after trade gives each firm a chance to sell its

¹Horizontally differentiated goods are differentiated based on characteristics of products such as color, while vertically differentiated goods are differentiated based on the quality of products.

product intensively to closer and less price-elastic consumers. Facing lower aggregate price elasticity of demand, firms increase their price after trade. That is, when firms face less price elastic ideal consumers, prices can increase after trade. My paper is not the first paper to show the price-increasing competition: Chen and Riordan (2008) show it in their original model. A contribution of my paper is that it shows the price-increase result can happen in a common ideal-core approach model if the forgotten heterogeneity of consumers is focused.

II Love of Quantity and Quality

-A Non-Unit Demand Model of Trade on Vertically Differentiated Goods-

Motivations and Research Questions Nowadays, we sometimes hear the news that well-known companies of producing high quality products go out of business. These companies are often said to exit the market because of low quality but inexpensive imports from developing countries.(the Wal-mart effect) This anecdote suggests that a replacement of superiors by inferiors - high quality goods in developed countries are replaced by low quality imports from developing countries - may be happening now. However, existing theoretical models do not provide satisfactory explanation for this replacement of superiors by inferiors. A typical result is that trade liberalization should result in quality upgrading in equilibrium. One reason why the replacement of superiors by inferiors has not been shown by previous studies is because most of these studies assume the unit demand: each consumer buys at most one unit of a product regardless of his income. This assumption presumes that consumers love quality but not quantity and limits the favorable effect of low price on low quality goods. Alternatively, this paper presents a new non-unit demand model, where consumers love quantity as well as quality.

Main Results and Contributions Adding consumers' love of quality into the model, this paper shows that the firm producing the high quality goods will lose its customers and profits

after trade, while the firm producing the low quality goods will steal customers and gain profits if the income gap between two countries distribution is large enough to make the volume effect of inexpensive low-quality goods work. Although the conventional unit demand model can also show that the firm producing high-quality goods lose profits after trade because of the increase in the competition, the non-unit demand model in this paper shows that the firm producing the high-quality goods lose profits not only because the increase in the competition but also because their customers are stolen by the firm producing the low-quality goods.

III: Consumer Utility and the Replacement of Superiors by Inferiors in the Quality-ladder Model -Anti-Schumpeterian Results in the Schumpeterian Model-

Motivations and Research Questions Innovations improve the quality of goods. In the process of innovations, we observe that high quality goods drive out low quality goods. However, in these days, we can also observe the opposite in the developed country: high quality goods in developed countries exit the market, while lower-quality yet inexpensive imports from developing countries enter the market and attract consumers in the developed country. The question is how and why these lower quality imports can drive out higher quality goods now. Chapter 2 answers this question by focusing on the consumers' love of quantity, while this chapter answers the same question by using a resource-consuming quality ladder model: the innovation requires more resources, while the utility of consumers increase with the product quality at a decreasing rate.

Main Results and Contributions The concavity of consumer utility plays little role under the autarky, and a typical Shumpeterian creative destruction takes place. Without trade, only expensive high-quality goods are in the developed country, while only inexpensive low-quality goods are in the developing country. However, once these two countries start free trade, the concavity of consumer utility functions affect the survival of firms. If the consumer utility increases with

the quality at a severely decreasing rate, the replacement of superiors by inferiors can be realized: firms producing high-quality expensive goods in the developed country can be extinguished by firms producing low-quality inexpensive goods in the developing country. With the strictly concave consumer utility function, all firms in the developed country except the firm producing the highest quality goods exit the market if a least developed country is a trade partner, while all firms in the developed country exit the market if a middle developed country is a trade partner.

Chapter 2

When Firms Meet Ideal Consumers

-Trade of Horizontally Differentiated Goods in the Presence of Heterogeneous Consumers-

2.1 Introduction

Existing studies in international trade have found that the opening of trade brings countries more varieties of horizontally differentiated goods, stimulates the competition, and consequently leaves firms no room to increase their price”, which is the non-surprising conclusion of most existing studies.¹ However, existing studies reach this conclusion by overlooking another change brought by trade: trade changes consumers to whom firms sell their products. This paper analyzes the forgotten change in the consumers’ side by using the simplified Helpman model (1981), where each consumer has a different ideal product and the price elasticity of demand depends on the distance between the ideal good and the nearest available good. In the model, the price consumers face increases with the distance between their ideal goods and the good actually sold in the economy. If the price elasticity of demand increases with price(distance), consumers, whose ideal goods are closer to the good actually sold in the economy, have a lower price elasticity of demand than remoter consumers.

¹Horizontally differentiated goods are differentiated based on characteristics of products such as color, while vertically differentiated goods are differentiated based on the quality of products.

With the heterogeneity of consumers, an increase in the number of firms after trade gives each firm a chance to sell its product intensively to closer and less price-elastic consumers, and the aggregate price elasticity of demand after trade decreases. Facing lower aggregate price elasticity of demand, firms increase their price after trade. That is, when firms face less price elastic ideal consumers, equilibrium prices can increase. This paper is not the first paper to predict the price-increasing competition. Chen and Riordan (2008) focus two potentially opposite effects, the price discounting effect called the market share effect and the price raising effect called the price sensitivity effect and derive conditions that the price increasing effect the competition increases price when the market structure changes from monopoly to duopoly, and the key to their result is the joint distribution of consumer valuation for two products. My paper focuses the heterogeneity of consumers with respect to price elasticity of demand as a result of the difference in ideal products, and shows that the price-increasing competition will be the result if the price elasticity of demand increases with price(distance). In spite of the price increase, it shows that consumer welfare increases with the trade liberalization.

The simplified Helpman model(1981) with specific functional forms is presented in section 4.2. My original work, the effect of heterogeneous consumers on the open economy under two approaches are discussed in section 2.3.

2.2 The Simplified Helpman Model (1981)

The Closed Economy

The Economy, Industries, and Horizontally Differentiated Goods

Consider a country where two types of goods, the composite of homogeneous goods(=numeraire goods) Y and horizontally differentiated goods X , are produced.² Both goods are produced by using labor only. The labor force of size L is supplied inelastically by consumers. Consumers are identical except that each of them has a different taste for horizontally-differentiated goods: every consumer has his own ideal horizontally-differentiated good, which is shown as a single point on the circumference of the circle in figure 2.1. The circumference of the circle in figure 2.1 shows all possible goods consumers can dream of and all possible goods the country can produce potentially. The circumference of the circle is assumed to be $2\pi r = 2\beta$.³ Moreover, consumers' ideal goods are presumed to be uniformly distributed on the circumference of the circle, and accordingly, the number of consumers at each point of the circumference of the circle becomes $n = \frac{L}{2\beta}$.

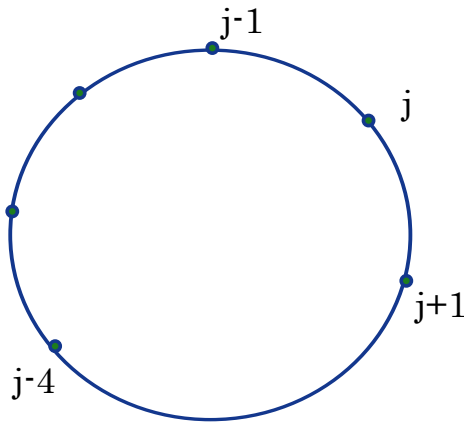


Figure 2.1: The Product Circle of Horizontally Differentiated Goods

²This paper simplified the Helpman model(1981) (based on the Lancaster model (1980)) by following ways: (1) The general homothetic utility function used in the Helpman model is changed to the stronger quasi-linear utility function, which yields the linear demand function. (2) Only one factor of production, labor, is used in this paper, while two factors of production, capital and labor, are used in the Helpman model.

³The radius of the circle is assumed to be $r\frac{\beta}{\pi}$.

Aside from the heterogeneous taste for horizontally-differentiated goods, all consumers are identical: every consumer is endowed with one unit of labor, supplies one unit of labor inelastically, owns equal shares in domestic firms, and has the same following utility function.

$$U(y, \hat{x}) = y + \alpha \hat{x} - \frac{1}{2} \hat{x}^2 \quad (2.1)$$

where y_i denotes the consumption of homogeneous goods and $\hat{x}_i$ the consumption of his ideal horizontally-differentiated goods.

Suppose that a consumer buys a non-ideal differentiated good (k). He will get lower utility than he will get from a unit of his ideal good (j). To put it another way, to achieve the same utility a unit of his ideal good (j) gives, he needs more than one unit of the non-ideal good. As the distance between the non-ideal good (k) and his ideal good increases, he needs the non-ideal good more to achieve the same utility a unit of his ideal good gives. This is the idea of the Lancaster's compensation function in figure 2.2. The Lancaster's compensation function ($h(v)$) shows how many non-ideal goods is necessary to compensate a consumer for giving up one unit of his ideal good.

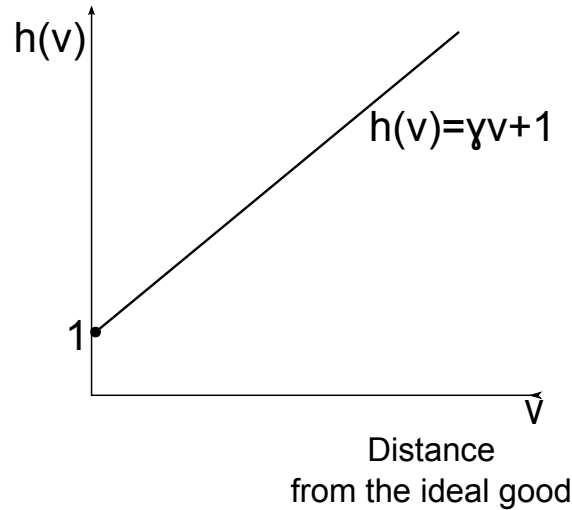


Figure 2.2: Lancaster's Compensation Function

The Lancaster's compensation function satisfies the following properties:

$$h(0) = 1^4$$

$$h'(v) \geq 0 \quad \forall v > 0$$

In words, these properties assume that the necessary amount of the non-ideal good (k) to compensate a consumer for giving up one unit of his ideal good increases with the distance (v).

With this compensation function, the consumer's preference is well defined, and every differentiated good (k), which locates at a distance of v from the ideal good can be converted into the equivalent ideal good ($\hat{x}$) units. That is,

$$\hat{x} = \frac{x_k(v)}{h(v)}$$

It should be noted that the Lancaster's compensation function also determines each consumer's effective price of his ideal product, $p_k h(v)$: if a consumer pays p_k for one unit of the good k , he will get the equivalent of $\frac{1}{h(v)}$ units his ideal products. Hence, one unit of his ideal product costs $p_k h(v)$, which implies that the further the distance from the ideal product, the higher the effective price of the ideal product becomes.

To simplify the analysis, this paper specifies the Lancaster's compensation function as follows:

$$h(v) = \gamma v + 1 \tag{2.2}$$

$$\text{where } \gamma > 0$$

Consumer's Problem- Two-Stage Maximization Problem-

Every consumer faces the two-stage maximization problem. In the first stage, each consumer allocates his income between homogeneous goods (y) and the ideal horizontally differentiated good (x).

⁴ $h(0) = 1$ means that if the good is the ideal good, 1 unit is necessary to achieve the same utility from 1 unit of the ideal good

Given the allocated income, in the second stage, each consumer chooses one variety of differentiated goods he is going to buy. As discussed later, the variety of differentiated goods actually produced in the economy is finite, while the consumer's ideal variety is infinite.⁵ Accordingly, except by coincidence, consumers cannot buy their exact ideal goods. In such a case, each consumer do its best he can do: he buys only one variety of the differentiated good with the lowest effective price ($p_j h(v)$).

First, given the utility function (4.8), the first-stage maximization problem is expressed as follows.

$$\begin{aligned} \max U(y, \hat{x}) &= y + \alpha \hat{x} - \frac{1}{2} \hat{x}^2 \\ \text{s.t. } y + p_x h(v) \hat{x} &= w \end{aligned}$$

where $\hat{x} = \frac{x_j(v)}{h(v)}$ is the consumption of differentiated goods converted into ideal good units.

Using the budget constraint, the consumption of y is expressed as $y = w - p_j \hat{x}$. With this expression, the first-stage maximization problem is rewritten as

$$\max U(y, \hat{x}) = w - p_x h(v) \hat{x} + \alpha \hat{x} - \frac{1}{2} \hat{x}^2$$

The First Order Condition (FOC) with respect to $\hat{x}$ gives the individual demand for the differentiated good j .

$$\hat{x} = \frac{x_j(v)}{h(v)} = (\alpha - p_j h(v))$$

Let the vertically differentiated good the consumer i chooses in the second-stage is x_j , the

⁵There is an infinite number of products on the circle in figure 2.1.

individual demand of the good x_j^i is given by

$$x_j^i(v) = h(v)(\alpha - p_j h(v)) \quad (2.3)$$

The above demand function is a downward-sloping linear demand function.

$$\frac{\partial x_j^i(v)}{\partial p_j} = -h(v)^2 \quad \frac{\partial^2 x_j^i(v)}{\partial p_j^2} = 0$$

With this downward-sloping demand function, the price elasticity of demand ϵ is given by:

$$\epsilon(v) = \frac{p_j h(v)}{\alpha - p_j h(v)} \quad (2.4)$$

The price elasticity of demand in eq (2.4) implies that

$$\epsilon'(v) = \frac{\alpha h'(v)}{(\alpha - p_j h(v))^2} > 0 \quad (2.4')$$

Eq (2.4') shows that the price elasticity of demand increases with the distance between the consumer i 's ideal good and the good of the lowest effective price x_j .⁶ It suggests that among all consumers of the good j , remote consumers are more price elastic than close consumers under this linear demand function, which seems reasonable. For, among all consumers of the good j , the remotest consumers of the good j are the closest consumers of rival goods, while the closest consumers of the good j are the remote consumers of rival goods.

Given the above demand and the income allocated to each good ($p_j x(v) = p_j h(v)(\alpha - p_j h(v))$, $y = w - p_j x(v)$), consumers choose the differentiated good of the lowest effective price ($p_j h(v)$) in the second stage. In this second stage, who buys each differentiated good is determined. Specifically, consumers within a distance of $v \in [\underline{\delta}, \bar{\delta}]$ buy the good j . (figure 2.3) Marginal consumers with a distance of $\underline{\delta}$ or $\bar{\delta}$ are indifferent between the good j and the rival good $j-1$ or $j+1$.

⁶If the demand function is concave rather than linear, the price elasticity of demand increases with distance in some range, while it decreases with distance in other ranges.

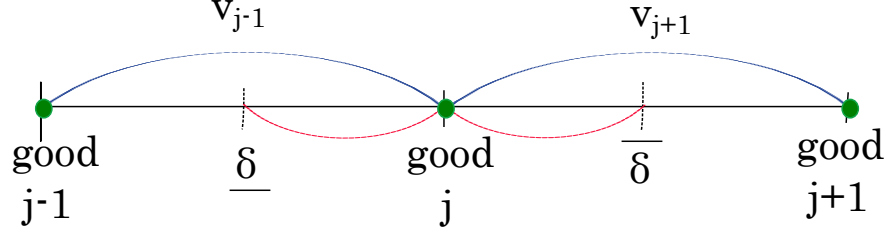


Figure 2.3: Market Share of the Good j

Precisely, marginal consumers with a distance of $\underline{\delta}$ are indifferent between the good j and the good j-1 because both goods offer them the same effective price ($ph(v)$):

$$p_j h(\underline{\delta}) = p_{j-1} h(v_{j-1} - \underline{\delta}) \quad (2.5)$$

where v_{j-1} is the distance between the good j and the good j-1.

Solving the eq (2.5) by using the special functional form $h(v) = \gamma v^2 + 1$, we get the lower market limit of the good j.

$$\underline{\delta} = \frac{p_{j-1}}{p_j + p_{j-1}} v_{j-1} - \frac{p_j - p_{j-1}}{\gamma (p_j + p_{j-1})} \quad (2.6)$$

$$\text{If } p_j = p_{j-1}, \quad \underline{\delta} = \frac{v_{j-1}}{2}$$

Similarly, the upper market limit of the good j ($\bar{\delta}$) is derived by identifying marginal consumers who face the same effective price of the good j and the good j+1:

$$p_j h(\bar{\delta}) = p_{j+1} h(v_{j+1} - \bar{\delta}) \quad (2.7)$$

where v_{j+1} is the distance between the good j and the good j+1.

The solution of the eq (2.7) gives the upper market limit of the good j.

$$\bar{\delta} = \frac{p_{j+1}}{p_j + p_{j+1}} v_{j+1} - \frac{p_{j+1} - p_j}{\gamma(p_j + p_{j+1})} \quad (2.8)$$

$$\text{If } p_j = p_{j+1}, \quad \bar{\delta} = \frac{v_{j+1}}{2}$$

Given the lower and the upper market limit of the good j , the aggregate demand is given by

$$Q_j = n \left\{ \int_0^{\underline{\delta}} h(v)(\alpha - p_j h(v)) dv + \int_0^{\bar{\delta}} h(v)(\alpha - p_j h(v)) dv \right\} \quad (2.9)$$

$n = \frac{L}{2\beta}$ is the density of consumers at each point of the circumference of the circle.

The aggregate price elasticity of demand the firm j faces is given by

$$\epsilon_j = \left\{ \int_0^{\underline{\delta}} s(v) \epsilon(v) dv + \int_0^{\bar{\delta}} s(v) \epsilon(v) dv \right\} - s(\underline{\delta}) p_j \left(\frac{\partial \underline{\delta}}{\partial p_j} \right) - s(\bar{\delta}) p_j \left(\frac{\partial \bar{\delta}}{\partial p_j} \right) \quad (2.10)$$

where $s(v) \equiv \frac{nx(v)}{Q_j}$ denotes the share of demand from consumers with a distance of v .

As shown in eq (2.10), the aggregate price elasticity consists of two parts: the weighted average of the individual price elasticity of demand (the first term) and the impact of price on the upper and the lower market limit (the second and the third term).

Since remote consumers are more price elastic than close consumers, the first term, the weighted average of the price elasticity of individual demand increases(decreases) as the firm covers remoter (closer) consumers. The second and the third term, the impact of price on the upper and the lower market limit also increases with the distance from the consumer's ideal goods unless the slope of the Lancaster's compensation function $h'(v)$ is close to zero and the distance plays no important role in product differentiation. If the slope of the Lancaster's compensation function is close to zero and the distance between two goods plays no role in product differentiation, the second and the third term in eq (2.10) approaches infinity, every firm will face the infinite elasticity of

demand, and accordingly the perfect competition will be the result. Otherwise, the aggregate price elasticity of demand increases (decreases) as the firm covers remoter (closer) consumers.⁷

Using eq(2.5), eq (2.7), and eq (2.3), the aggregate price elasticity of demand is rewritten as follows:

$$\epsilon_j = \left(\frac{np_j}{Q_j} \right) \left\{ \left(\int_0^{\underline{\delta}} h(v)^2 dv + \int_0^{\bar{\delta}} h(v)^2 dv \right) + \left(\frac{s(\underline{\delta})h(\underline{\delta})}{\gamma(p_j + p_{j-1})} + \frac{s(\bar{\delta})h(\bar{\delta})}{\gamma(p_j + p_{j+1})} \right) \right\} \quad (2.10')$$

Firms

In the economy, two industries, the homogeneous good (y) industry and horizontally-differentiated goods(x) industry produce goods by using labor. A homogeneous good (=the numeraire good) is produced with the constant returns to scale technology: one unit of labor produces one unit of the homogeneous good. Therefore, the wage rate is fixed at 1 ($w=1$). On the other hand, the other type of goods, horizontally-differentiated goods are produced with the increasing returns to scale technology. The cost function is $C(Q_j) = cQ_j + f$ where c is the constant marginal cost, and f is the fixed cost. All firms producing differentiated goods share the same production technology and the production function. Each firm in the differentiated good industry faces a two-stage game. In the first stage, the firm decides whether or not enter the market, and then, if it decides to enter the market, it chooses the differentiated good it is going to produce (=the location of the product in the product circle). In deciding the location of the product, the firm does not know that it will face free trade in the future. After deciding the location of the product, in the second stage, all firms join the Bertrand competition: they choose price simultaneously given other firms' prices.

⁷Using eq(2.5), eq (2.7), the impact of price on the upper and the lower market limit are

$$\frac{\partial \underline{\delta}}{\partial p_j} = \frac{-h(\underline{\delta})}{h'(\underline{\delta})(p_j + p_{j-1})} = \frac{-h(\underline{\delta})}{\gamma(p_j + p_{j-1})}, \quad \frac{\partial \bar{\delta}}{\partial p_j} = \frac{-h(\bar{\delta})}{h'(\bar{\delta})(p_j + p_{j+1})} = \frac{-h(\bar{\delta})}{\gamma(p_j + p_{j+1})}$$

Given the location of the product and the aggregate demand eq(2.9), in the second stage, each firm maximizes the following profit function with respect to price.

$$\pi_j = n(p_j - c) \left\{ \int_0^{\underline{\delta}} h(v)(\alpha - p_j h(v))dv + \int_0^{\bar{\delta}} h(v)(\alpha - p_j h(v))dv \right\} - f \quad (2.11)$$

where c is the marginal cost of production and f is the fixed cost (sunk cost).

The F.O.C with respect to price gives us the familiar expression.

$$p_j = \left(\frac{\epsilon_j}{\epsilon_j - 1} \right) c \quad (2.12)$$

This expression (2.12) shows that a firm sets its price higher than the marginal cost, and the aggregate price elasticity of the demand is the key to determine the price.

In the first stage, each firm decides the location of the product without knowing that it will face free trade in the future.

Differentiating the profit function (2.11) with respect to v_{j-1} (the distance from two rival firms) gives the optimal location.

$$(p_j - c) \frac{\partial Q_j}{\partial v_{j-1}} = 0 \quad (2.13)$$

Since a firm sets price higher than the marginal cost [eq (2.12)], the above equation (2.13) implies that a firm chooses the midpoint from its two rival firms, at which a small change in the location does not change the aggregate demand $\left(\frac{\partial Q_j}{\partial v_{j-1}} = 0 \right)$.

The Closed Economy Equilibrium

Since all firms of horizontally differentiated goods face exactly the same maximization problem, all firms set the same price $p_j = p_{j-1} = p_{j+1} \quad \forall j$, and accordingly, the upper and the lower limit of the market are the half distance from the rival firm $\underline{\delta} = \bar{\delta} = \frac{v_{j-1}}{2} = \frac{v_{j+1}}{2}$. Moreover, firms spread equally over the circumference of the product circle, and the distance between any two

firms becomes $\frac{2\beta}{N}$ (2β = the length of the circumference of the circle and N is the number of products produced in the economy).

Using the special functional forms (the utility function (4.8) and the Lancaster's compensation function (2.2)), we get the equilibrium autarky price of the differentiated good j .

$$p^{AU} = \frac{-B + \sqrt{B^2 - 4AD}}{2A} \quad (2.14)$$

where $A \equiv \gamma^3 \underline{\delta}^3 + 3\gamma^2 \underline{\delta}^2 + 3\gamma \underline{\delta} - 3$

$$B \equiv 3\alpha + c (\gamma^3 \underline{\delta}^3 + 3\gamma^2 \underline{\delta}^2 + 3\gamma \underline{\delta} + 3)$$

$$D \equiv -3\alpha c (1 + \gamma \underline{\delta}^2)^2$$

$$\underline{\delta} = \bar{\delta} = \frac{v_{j-1}}{2} = \frac{\beta}{N}$$

Using eq(2.11) and eq(2.14), we have the profit of a firm in sector X .

$$\pi^{AU} = \frac{2n (p^{AU} - c) \{-2\gamma^2 p^{AU} \delta^3 + 3\gamma (\alpha - 2p^{AU}) \underline{\delta}^2 + 6 (\alpha - p^{AU}) \underline{\delta}\}}{6} = 0 \quad (2.15)$$

Zero profit condition Eq(2.15) and Eq (2.14) determine the autarky equilibrium price and the number of firms.

Figure 2.4 shows the equilibrium number of goods when exogenous variables are given as follows:

8

$$L = 100, c = 1 \beta = 5 \Rightarrow h(v) = 5v^2 + 1, \text{ and}$$

$$\alpha = 1000 \Rightarrow x(v) = h(v)(1000 - ph(v)) = (5v + 1) \{1000 - p^{AU}(5v + 1)\}$$

Figure 2.4 shows that the number of firms decreases with the fixed cost (f = horizontal axis).

Once the equilibrium price and the number of firms are determined, the aggregate consumer utility under autarky and the welfare can be calculated:

⁸Specifically speaking, the number of firms shown in the figure is the maximum number of firms at which every firm earns non-negative profit. That is this paper faces the integer problem. Since consumers own equal shares in firms, profits earned by horizontally differentiated goods is distributed to consumers. Consumers spend dividends on numeraire goods.(consumers' utility function is quasi-linear and there is no income effect on differentiated goods.)

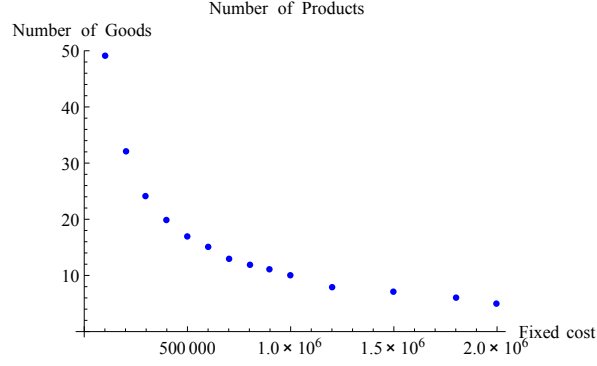


Figure 2.4: Number of Firms under Autarky

$$U^{AU} = N \left[2n \int_0^{\underline{\delta}} \left\{ w + \frac{1}{2} (\alpha - p^{AU} (\gamma v^2 + 1)^2) \right\} dv \right] \quad (2.16)$$

where w denotes the labor income ($w = 1$)

The welfare under autarky is given by

$$W^{AU} = U^{AU} + N\pi^{AU} \quad (2.17)$$

Numerical Examples: Autarky

The following table shows the autarky equilibrium when $L = 100$, $c = 1$, $\beta = 5 \Rightarrow h(v) = 5v + 1$, $\alpha = 1000$ (the reservation price), and $f = 1,200,000$. Given these exogenous variables, the zero profit condition and the price equation (2.14) determine the equilibrium number of firms and the equilibrium price are determined by the zero profit condition and the price equation: $N = 8$ and $p^{AU} = 9.12$ respectively. Substituting these variables back to eq (2.11), (2.16), and (2.17), we get the following profit, the aggregate utility of consumers and the welfare under autarky. The average effective price (=the average unit price of the ideal good) $\overline{ph(v)} = 151.55$ shows that the good consumers buy under autarky is far from their ideal one.

Autarky						
	N	p	$\overline{ph(v)}$ average effective price	π	Consumers' utility	Welfare
	8	9.12	151.55	155,732	36,331,300	37,577,200

Note: $\pi < 0$ if $N = 9$

$$\text{Average Effective Price} = \overline{ph(v)} = \left(\frac{1}{L}\right) \cdot N \underbrace{\left(\int_0^{\delta} n * ph(v) dv + \int_0^{\bar{\delta}} n * ph(v) dv \right)}_{\text{aggregate effective price}}$$

2.3 The Open Economy

Suppose that two countries suddenly start free trade. These two countries are exactly the same except horizontally-differentiated goods they produce under autarky. (figure 2.5) The opening of trade brings changes to these countries: it doubles the market size ($n \rightarrow 2n$) and the number of products ($N \rightarrow 2N$). Moreover, trade changes rival firms each domestic firm locally compete with. After the opening of trade, two rival firms, change from two domestic firms ($j - 1, j + 1$) to two foreign rival firms ($j - 1, j + 1$), which produce more similar products to the good j than two domestic rival firms. As shown in figure 2.5, except by coincidence, the distance between any two products is not equal any longer: the distance from one foreign rival firm ($j + 1^*$) is closer than the distance from the other foreign rival firm ($j - 1^*$).

The following sections discuss the impact of the above-mentioned changes on two countries under two different assumptions. First, I discuss the case with zero cost of relocation (=the cost of changing products) (the flexible location equilibrium). Second, I discuss the opposite case with an infinite cost of relocation (the sticky location equilibrium). In both cases, I discuss a short-run two-stage game without new entry or exit as well as a long-run three-stage game with new entry or exit.

In the two-stage short-run game, incumbent firms re-decide the location of their product and how far they move in the first stage. Then, in the second stage, incumbent firms set prices simultaneously. [the short-run Bertrand equilibrium]

In the two-stage long-run game, incumbent firms decide whether or not stay in the market and relocate their position if necessary, while new entrants decide whether or not enter the market and choose the location of their goods. Then, in the second stage of the long-run game, all firms set prices simultaneously. [the long-run Bertrand equilibrium]

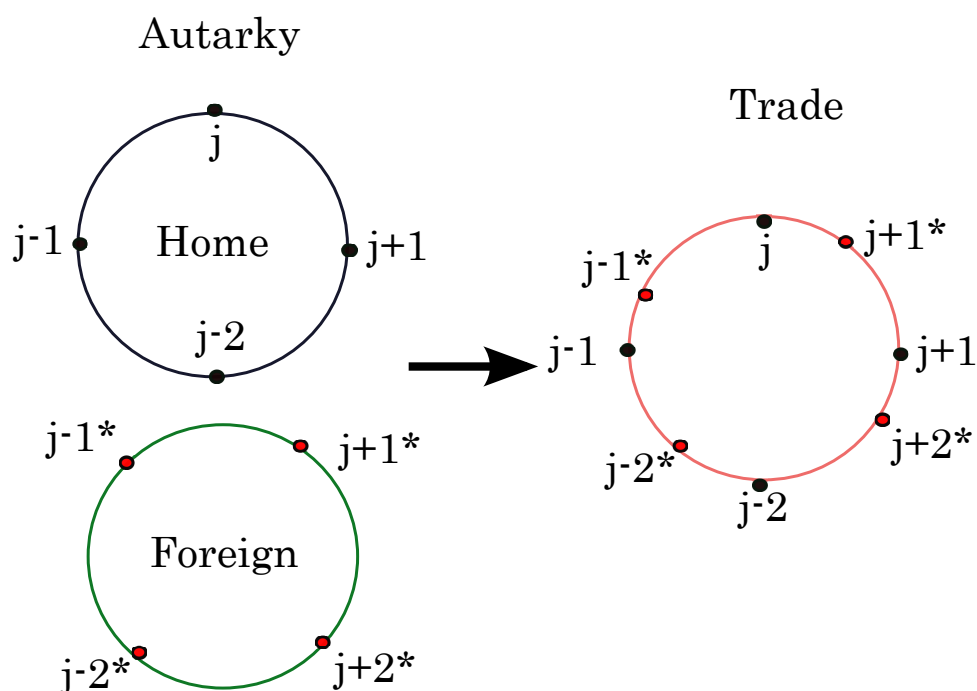


Figure 2.5: The Opening of Trade

Case I: Zero Relocation Cost

The Short-Run Equilibrium (with no new entry or exit)

If the cost of relocation is zero, what horizontally-differentiated goods were produced under autarky [the location of differentiated goods] has no importance at all. Furthermore, the asymmetry between

the distance from one foreign firm (v_{j-1}^*) and the one from another firm (v_{j+1}^*) does not cause any problem. For, thanks to zero cost of relocation, firms change their location (products) so as to regain the equidistance from two rival firms in the first stage of the short-run game.[suggested by eq (2.13)] In the short run with no new entrant, the distance between the firm (j) and its closest rival firm (v_{j-1}^* or v_{j+1}^*) is $\frac{\beta}{N}$, which is a half of the autarkic distance ($v_{j-1} = v_{j+1} = \frac{2\beta}{N}$). After regaining the equal interval between any two firms, all incumbents choose price simultaneously in the second stage of the short-run game. In this stage, firms face only two changes from their autarkic situation: the double number of consumers ($2n$) and the halved distance from the rival firms ($v_{j-1} = v_{j+1} = \frac{2\beta}{N}$). The maximization problem of incumbents in such a circumstance is given by

$$\max \pi_j = 2n(p_j - c) \left\{ \int_0^{\underline{\delta}} h(v)(\alpha - p_j h(v))dv + \int_0^{\bar{\delta}} h(v)(\alpha - p_j h(v))dv \right\} \quad (2.18)$$

The FOC with respect to p_j yields the same expression as eq(2.12).

$$p_j = \left(\frac{\epsilon_j}{\epsilon_j - 1} \right) c \quad (2.19)$$

Since all incumbents are in the same situation and face the same maximization problem, they choose the same price. As a consequence, the upper and the lower market of any differentiated good are still symmetric and become the midpoint of two firms. ($\underline{\delta} = \bar{\delta} = \frac{\beta}{2N}$) Compared with the upper and the lower market limits under autarky ($\underline{\delta}^{AU} = \bar{\delta}^{AU} = \frac{\beta}{N}$), the upper and the lower market limits after trade is shortened by half. These minor changes causes a big change in the economy. As discussed previously [eq(2.10)], the aggregate price elasticity of horizontally differentiated goods -the weighted average of individual price elasticity of demand plus the impact of price on the upper and the lower market limit- decreases(increases) as a firm sells its product to closer(remoter)

consumers.⁹ Using ($\underline{\delta} = \bar{\delta} = \frac{\beta}{2N}$) and the symmetry of the upper and the lower market limit, eq (2.10) and (2.10'), we have the new price elasticity of demand after trade.

$$\begin{aligned}
\epsilon_j &= 2 \left\{ \int_0^{\frac{\beta}{2N}} s(v) \epsilon_i(v) dv - s\left(\frac{\beta}{2N}\right) p_j \left(\frac{\partial \underline{\delta}}{\partial p_j} \right) \right\} \\
&= 2 \left\{ \int_0^{\frac{\beta}{2N}} s(v) \epsilon_i(v) dv \right\} + \frac{s\left(\frac{\beta}{2N}\right) h\left(\frac{\beta}{2N}\right)}{h'\left(\frac{\beta}{2N}\right)} \\
&= 2 \left\{ \int_0^{\frac{\beta}{2N}} s(v) \epsilon_i(v) dv \right\} + \frac{s\left(\frac{\beta}{2N}\right) h\left(\frac{\beta}{2N}\right)}{\gamma} \tag{2.20}
\end{aligned}$$

where $s(v) \equiv \frac{2nx(v)}{Q_j}$ the share of demand from consumers of a distance of v

The eq (2.20) suggests that if the distance from the closest foreign rival firms $\left(\frac{\beta}{2N}\right)$ is sufficiently large enough ($\delta \neq 0$), the slope of Lancaster's compensation is not zero, and goods are differentiated ($\gamma \neq 0$), the aggregate price elasticity of demand after trade will decrease, and accordingly firms will INCREASE their prices (!). Firms can increase their price because they sell their products to less price elastic closer consumers than they did under autarky. That is, when firms meet ideal consumers -less price elastic closer consumers-, they increase their price as long as consumers consider their products as sufficiently differentiated from their rivals' products ($h'\left(\frac{\beta}{2N}\right) = \gamma > 0$).

The short-run flexible location equilibrium without new entry or exit is given as follows:

$$p = \frac{-B + \sqrt{B^2 - 4AD}}{2A} \tag{2.21}$$

where

⁹If the distance between firms is close to zero and two goods become homogeneous, the impact of price on the upper and the lower limit becomes infinite, and consequently firms face the infinite price elasticity of demand.[The Bertrand competition of two homogeneous goods.] This paper will not discuss such a case.

$$A \equiv \gamma^3 \underline{\delta}^3 + 3\gamma^2 \underline{\delta}^2 + 3\gamma \underline{\delta} - 3$$

$$B \equiv 3\alpha + c (\gamma^3 \underline{\delta}^3 + 3\gamma^2 \underline{\delta}^2 + 3\gamma \underline{\delta} + 3)$$

$$D \equiv -3\alpha c (1 + \gamma \underline{\delta}^2)^2$$

$$\underline{\delta} = \bar{\delta} = \frac{v_j^* - 1}{2} = \frac{\beta}{2N}$$

The expression of the equilibrium price in (2.21) and that under autarky eq(2.14) are exactly the same. The only difference between the post-trade short-run equilibrium price and the autarky price is the upper and the lower market limit ($\underline{\delta} = \bar{\delta} = \frac{\beta}{2N}$).

Differentiating the equilibrium price (2.21) or eq(2.14) with respect to the market width ($\underline{\delta}$), we get the following expression.

$$\frac{\partial p_j}{\partial \underline{\delta}} = \frac{3\gamma(1+\gamma\underline{\delta})[-3(1+\gamma\underline{\delta})(E+3)(2c^2+3\alpha^2)-c\alpha(E-3)\{E+\gamma^2\underline{\delta}^2(\gamma\underline{\delta}+3)+15\}+3(1+\gamma\underline{\delta})(\alpha+2c)(\sqrt{\{3\alpha+(E+3)c\}^2+12c\alpha(1+\gamma\underline{\delta})^2(E-3)})]}{2(E-3)^2\sqrt{\{3\alpha+(E+3)c\}^2+12c\alpha(1+\gamma\underline{\delta})^2(E-3)}} \quad (2.22)$$

$$\text{where } E \equiv \gamma^3 \delta^3 + 3\gamma^2 \delta^2 + 3\gamma \delta$$

Eq (2.22) shows that $\frac{\partial p_j}{\partial \underline{\delta}} < 0$ and price must increase as the market width becomes narrower if the slope of the Lancaster's compensation function γ is sufficiently large and the market width is not too close $\underline{\delta} >> 0$.¹⁰

Using eq(2.18) , eq(2.21) and the short-run number of firms $2N$, we get the short-run profit of an incumbent firm.

$$\pi = \frac{4n(p-c)\{-2\gamma^2 p \delta^3 + 3\gamma(\alpha-2p)\underline{\delta}^2 + 6(\alpha-p)\underline{\delta}\}}{6} \quad (2.23)$$

$$\text{where } \underline{\delta} = \frac{\beta}{2N}$$

¹⁰ $\lim_{\delta \rightarrow 0} \frac{\partial p_j}{\partial \underline{\delta}} = 0, \lim_{\gamma \rightarrow 0} \frac{\partial p_j}{\partial \underline{\delta}} = 0$

The consumers' utility and welfare in this short-run flexible location equilibrium are

$$U = 2N \left[2n \int_0^{\frac{\beta}{2N}} \left\{ w + \frac{1}{2} (\alpha - p(\gamma v + 1)^2) \right\} dv \right] \quad (2.24)$$

$$W = U + N \cdot \pi \quad (2.25)$$

The numerical example of this short-run flexible location case (table 2.2, the second row) supports the above argument: Incumbents take advantage of ideal consumers- closer and less price elastic consumers- and raise its price after trade. (9.12 to 12.64) Although an increase in prices, the utility of consumers increase because the double number of differentiated goods make consumers buy goods closer to their ideal goods: the average effective price $[\overline{ph(v)}]$ decreases. (151.6 to 111.37). On the other hand, incumbent firms lose revenue after trade (1,355,730 to 1,101,740) due to the fact that the double-sized market cannot stop their sales decrease (167,045 to 98,988). Despite the revenue decrease, a large increase in the consumer utility is sufficient enough to increase the social welfare. (37,577,200 to 48,460,100)

The Long-Run Equilibrium with Zero Relocation Cost

In the long run, incumbent firms may exit the market or new firms may enter the market. The long run equilibrium is the solution of a two-stage game. In the first stage, incumbent firms decide whether or not stay in the market and decide how much they move from their current position, while new firms decide whether or not enter the market and they choose the location of their products. In the second stage, both incumbents and new entrants decide their prices simultaneously.

With zero cost of relocation, incumbents relocate their position to make distance between any two firms to be equal. $\frac{\beta}{2N}$ to $\frac{\beta}{\tilde{N}}$ [where $\tilde{N} = 2N + \text{new entrants}$].

Flexible Location (Zero Relocation Cost)						
	N	p	$\overline{ph(v)}$ average effective price	Revenue	Consumer utility	Welfare
Autarky	8	9.12	151.55	1,355,730	36,331,300	37,577,200
Short-Run Long-Run	16	12.64	111.4	1,101,740	39,646,200	48,460,100

Table 2.1: Numerical Examples with Zero Relocation Cost

Eq (2.21) and the following zero profit condition determines the number of firms and the price in the long run.

$$\pi_j = 4n(p_j - c) \left\{ \int_0^{\frac{\beta}{N}} h(v)(\alpha - p_j h(v))dv \right\} - f = 0$$

Once the number of firms and the long-run price are determined, revenue(profits),the utility of consumers, and welfare are determined in the same way as the short-run equilibrium. The numerical example (table 2.2, the third row) shows the long-run equilibrium in this flexible location case.¹¹ Since incumbent firms cannot earn enough profit to cover the entry cost f , no new firm enter the market, and the short-run equilibrium becomes the long-run equilibrium.

Note:

$$\text{Average Effective Price} = \overline{ph(v)} = \left(\frac{1}{L}\right) \cdot N \underbrace{\left(\int_0^{\delta} n * ph(v)dv + \int_0^{\bar{\delta}} n * ph(v)dv \right)}_{\text{aggregate effective price}}$$

Case II: Infinite Relocation Cost

This section discusses the impact of trade under the assumption that the cost of relocation (= the cost of changing characteristics of products) is infinite. If the cost of relocation is infinite, the

¹¹Table 2.2 shows the long-run equilibrium by using the same exogenous variables as other numerical examples:

$L = 100, c = 1, \beta = 5 \Rightarrow h(v) = 5v^2 + 1, \alpha = 1000$ (the reservation price), and $f = 1,200,000$.

location incumbent firms have chosen under autarky and the difference between incumbent firms with pre-established location and new firms without any pre-established location play an important role. This section numerically shows that if two countries have developed similar products under autarky and the left-side market width and the right-side one becomes asymmetric, the asymmetry brings favorable effects to firms, while the asymmetry brings less favorable effects to consumers. If no firm enters the market, and no firm relocates its position, the opening of trade gives every incumbent firm a chance to receive benefit from less price elastic close consumers as well as the sales increase due to the double-sized market. Since firms raise their price after trade and the infinite relocation cost prevents firms to move closer to the ideal goods of every consumer, the consumer utility may deteriorate after trade in the extreme asymmetry case.

The Short-run Equilibrium with No New Entry or Exit

In the short run without new firms, all firms in the economy are incumbent firms of pre-established products. All incumbent firms ($2N$ firms) are in the same situation as shown in figure 2.6. Except by coincidence, the distance from one foreign rival firm $v_{j+1}^* = \frac{(1-\theta)\beta}{N}$ is closer than the distance from the other foreign rival firm $v_{j-1}^* = \frac{(1+\theta)\beta}{N}$ ¹²

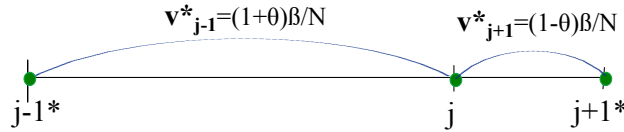


Figure 2.6: The Location of Firms after Trade

Since the cost of relocation is infinite, no existing firm changes its product and relocate its position in the first stage of the short run game. Thus, the asymmetric distance between firms hold. Given the asymmetric distance, every incumbent firm sets price simultaneously in the second

¹²This paper does not discuss the case where two countries have coincidentally developed exactly same products under autarky. If two countries have coincidentally developed exactly same products under autarky, the competition after trade will be the Bertrand competition between homogeneous goods.

stage. Since all incumbent firms face the same maximization problem, they choose the same price.

$$p = \left(\frac{\epsilon}{\epsilon - 1} \right) c$$

Accordingly, the upper and the lower limit of any incumbent firm are just the half of the distance from two close rival firms [$\underline{\delta} = \frac{(1+\theta)\beta}{2N} > \bar{\delta} = \frac{(1-\theta)\beta}{2N}$ the left market width is wider than the right one.] Given these upper and the lower limit, the aggregate price elasticity of demand is given by

$$\begin{aligned} \epsilon_j &= \int_0^{\underline{\delta}} s(v) \epsilon_i(v) dv + \int_0^{\bar{\delta}} s(v) \epsilon_i(v) dv - s(\underline{\delta}) p_j \left(\frac{\partial \underline{\delta}}{\partial p_j} \right) - s(\bar{\delta}) p_j \left(\frac{\partial \bar{\delta}}{\partial p_j} \right) \\ &= \int_0^{\underline{\delta}} s(v) \epsilon_i(v) dv + \int_0^{\bar{\delta}} s(v) \epsilon_i(v) dv + \frac{s(\underline{\delta}) h(\underline{\delta})}{2h'(\underline{\delta})} + \frac{s(\bar{\delta}) h(\bar{\delta})}{2h'(\bar{\delta})} \\ &= \int_0^{\underline{\delta}} s(v) \epsilon_i(v) dv + \int_0^{\bar{\delta}} s(v) \epsilon_i(v) dv + \frac{s(\underline{\delta}) h(\underline{\delta})}{2\gamma} + \frac{s(\bar{\delta}) h(\bar{\delta})}{2\gamma} \end{aligned} \quad (2.26)$$

$$\text{where } \underline{\delta} = \frac{(1+\theta)\beta}{2N} \quad \bar{\delta} = \frac{(1-\theta)\beta}{2N}$$

The above equation suggests that the aggregate price elasticity of demand is higher (lower) than that under zero relocation cost if the share of sales from the narrower side of the market is large (small).

The F.O.C. with respect to price $p = \left(\frac{\epsilon}{\epsilon-1} \right) c$, and the aggregate price elasticity of demand eq (2.26) when $\underline{\delta} = \frac{(1+\theta)\beta}{2N}$ and $\bar{\delta} = \frac{(1-\theta)\beta}{2N}$, give the short-run equilibrium sticky location price. (p^{asy})

$$p^{asy} = \frac{-B^{asy} + \sqrt{(B^{asy})^2 - 4A^{asy}D^{asy}}}{2A^{asy}} \quad (2.27)$$

$$\text{where } A^{asy} \equiv \gamma \{ \underline{\delta}(\underline{\delta}^2\gamma^2 + 3\underline{\delta}\gamma + 3) + \bar{\delta}(\bar{\delta}^2\gamma^2 + 3\bar{\delta}\gamma + 3) \} - 6$$

$$B^{asy} \equiv c\gamma \left\{ \underline{\delta}(\underline{\delta}^2\gamma^2 + 3\underline{\delta}\gamma + 3) + \bar{\delta}(\bar{\delta}^2\gamma^2 + 3\bar{\delta}\gamma + 3) \right\} + 6(c + \alpha)$$

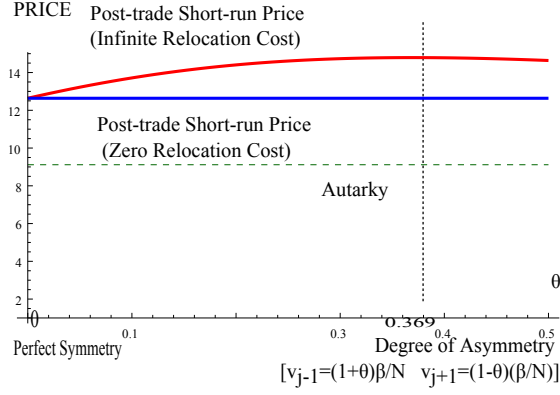
$$D^{asy} \equiv -3c\alpha \left\{ (\underline{\delta}\gamma + 1)^2 + (\bar{\delta}\gamma + 1)^2 \right\}$$

$$\underline{\delta} = \frac{(1+\theta)\beta}{2N} \quad \bar{\delta} = \frac{(1-\theta)\beta}{2N}$$

Figure 2.7 shows the post-trade short-run equilibrium price with an infinite cost of relocation, the post-trade short-run equilibrium price with zero cost of relocation when $L = 100$, $c = 1$, $\beta = 5 \Rightarrow h(v) = 5v + 1$, $\alpha = 1000$ (the reservation price), $f = 1,200,000$, and accordingly the number of products after trade becomes $2N = 2 \times 8$.

Figure 2.7 shows a numerical example with the same exogenous variables use in previous sections. The figure shows that the post-trade short-run equilibrium price in the case of an infinite relocation cost is the highest. For, the asymmetric market width and an infinite relocation cost makes the share of sales from the narrower side of the market to be so high, and accordingly firms set high price as if they cover the narrower side of the market only. With an infinite cost of relocation, the location of products each country has established under autarky plays an important role after trade. The Figure shows a non-monotonic relationship between the short-run price and the asymmetry of the market width. At first, the price increases with the asymmetry due to the fact that the share of sales from the narrower side of the market is so high that firms put more weight on the price elasticity of the narrower side of the market. However, as the market width becomes more asymmetric, the share of sales from the wider side of the market becomes larger, and thus firms care price elastic consumers in the wider side of the market and discount their price. (Figure 2.2)

Given the short-run price (p^{asy}), profits of incumbent firms, the consumers' utility, and welfare are given as follows.



Note: Lancaster's Compensation function: $\mathbf{h}(\mathbf{v})=1+5\mathbf{v}$
 Reservation price: $\alpha=100$ Population Size: $L=L^*=100$
 Fixed Cost: $\mathbf{f}=1,200,000$ Circle Circumference: $2\beta=100$
 Number of Firms under Autarky: $\mathbf{n}=\mathbf{n}^*=8$

Figure 2.7: Autarky Price, Post-Trade Price (zero relocation cost), Post-Trade Price (infinite relocation cost)

$$\pi^{asy} = 2n(p^{asy} - c) \left\{ \int_0^{\underline{\delta}} h(v)(\alpha - p^{asy}h(v))dv + \int_0^{\bar{\delta}} h(v)(\alpha - p^{asy}h(v))dv \right\} \quad (2.28)$$

$$U^{asy} = 2N \left[n \int_0^{\underline{\delta}} \left\{ w + \frac{1}{2} (\alpha - p^{asy}(\gamma v + 1)^2) \right\} dv + n \int_0^{\bar{\delta}} \left\{ w + \frac{1}{2} (\alpha - p^{asy}(\gamma v + 1)^2) \right\} dv \right] \quad (2.29)$$

$$W^{asy} = U^{asy} + N\pi^{asy} \quad (2.30)$$

The Numerical Example: the Short-Run Sticky Location Equilibrium

Table 2.2 shows the short-run equilibrium with different degree of asymmetry when the relocation cost is infinite.

Table shows that the short-run equilibrium price, increases and then decreases as products of

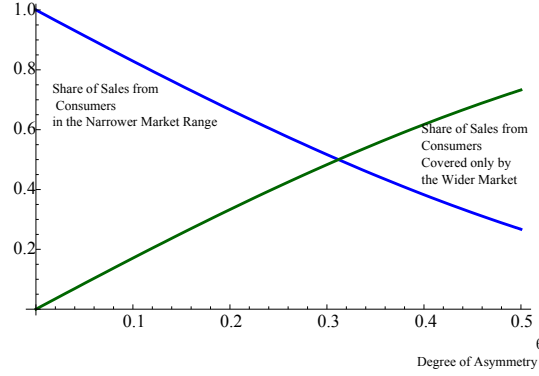


Figure 2.8: Share of Sales from Different Consumers

two countries under autarky become more similar, and the left-side market width and the right-side one becomes asymmetric. As mentioned before, the equilibrium price depends on the share of sales from remote consumers in the wider side of the market. Figure 2.2 shows that the share of sales from remote consumers in the wider side of the market is less important when the width of two side of the market is slight symmetric. As a result, firms put less weight on remote consumers, and set high price. Since no firm can move, every firm takes advantage of less price elastic consumers and continues to raise price with the degree of asymmetry. However, as the right-side market width and the left-side market width becomes too asymmetric, the sales from price elastic remote consumers in the wider side of the market becomes important, and firms have no choice but to discount their price.

With no new entry or exit, the short-run revenue is the highest when products of two countries under autarky are more similar and the market width becomes severely asymmetric. That is because firms receive benefits from covering less price elastic consumers and selling more products due to the double-size market.

On the other hand, the asymmetry brings less favorable effect to consumers. the average effective price (the third column in Table 2.2) increases as products of two countries under autarky are more

similar and the market width becomes more asymmetric. As the table shows, if two countries have established similar products under autarky and the two sides of the market is severely asymmetric, the average effective price and the consumer utility after trade is lower than those under autarky. The welfare of the economy also decreases with the degree of asymmetry. To summarize, numerical results suggest that trading with similar countries(the asymmetric market width) brings favorable effects to firms, while it brings unfavorable effects to consumers.

Case: Infinite Relocation Cost: $v_{j-1}^* = \frac{(1-\theta)\beta}{N}$ $v_{j+1}^* = \frac{(1+\theta)\beta}{N}$						
	N	p	$\overline{ph(v)}$ average effective price	π	Consumers' utility	Welfare
Autarky	8	9.12	151.55	1,355,730	36,331,300	37,577,200
Post Trade Perfect Symmetry $\theta = 0$	16	12.64	111.37	1,101,740	39,646,200	48,460,100
Post Trade Slight Asymmetry $\theta = 0.2$	16	14.41	131.45	1,267,380	37,980,100	48,119,100
Post Trade Moderate Asymmetry $\theta=0.396$	16	14.78	148.39	1,376,030	36,677,100	47,685,300
Post Trade Severe Asymmetry $\theta = 0.47$	16	14.70	154.87	1,413,160	36,191,500	47,496,800

Table 2.2: Numerical Examples:Short-Run Equilibrium with Infinite Relocation Cost

2.4 Conclusion

This paper analyzes the forgotten change in the consumers' side by using the simplified Helpman model (1981), where each consumer has a different ideal good and the price elasticity of demand depends on the distance between the ideal good and the nearest available good. In my model, the price consumers face increases with the distance between their ideal goods and the good actually sold in the economy, and the price elasticity of demand increases with price. Close consumers,

whose ideal goods are close to the good available in the economy, have a lower price elasticity of demand than remote consumers. With this heterogeneity of consumers, an increase in the number of firms after trade gives each firm a chance to sell its product intensively to closer and less price-elastic consumers, and the aggregate price elasticity of demand after trade decreases. Facing lower aggregate price elasticity of demand, firms increase their price after trade. That is, when firms face less price elastic ideal consumers, equilibrium prices can increase under both cases (the flexible location case and the sticky location case). The paper thus derives a possibly counter-intuitive result on the effect of trade liberalization on prices, though consumer welfare nevertheless increases with liberalization (consumers closer to their ideal variety). This paper also discusses the case where the cost of changing characteristic of products (the cost of relocation) is infinite. In this case, the location of the product developed under autarky becomes important. Numerical examples show that the degree of asymmetry have the opposite effects on firms and consumers. Surprisingly, trading with similar country, which creates the asymmetric market width brings favorable effects to firms, while it brings unfavorable effects to consumers. With no new entry or exit, the short-run revenue is the highest when products of two countries under autarky are more similar and the market width becomes severely asymmetric. That is because firms receive benefits from covering less price elastic consumers and selling more products due to the double-size market.

On the contrary, the asymmetry brings less favorable effect to consumers. the average effective price (the third column in Table 2.2) increases as products of two countries under autarky are more similar and the market width becomes more asymmetric. As the table shows, if two countries have established similar products under autarky and the two sides of the market is severely asymmetric, the average effective price and the consumer utility after trade is lower than those under autarky. The welfare of the economy also decreases with the degree of asymmetry. To summarize, numerical results suggest that trading with similar countries(the asymmetric market width) brings favorable

effects to firms, while it brings unfavorable effects to consumers.

Chapter 3

Love of Quantity and Quality

-A Non-Unit Demand Model of Trade on Vertically Differentiated Goods-

3.1 Introduction

Nowadays, we sometimes hear the news that well-known companies of producing high quality products go out of business. For instance, in April 2012, Aquascutum, a UK-based luxury clothing manufacturer, was sold to YGM Trading, a Chinese fashion retailer. These companies are often said to exit the market because of low quality but inexpensive imports from developing countries. (the Wal-mart effect) This anecdote suggests that a replacement of superiors by inferiors - high quality goods in developed countries are replaced by low quality imports from developing countries - may be happening now. However, existing theoretical models do not provide satisfactory explanation for this replacement of superiors by inferiors. A typical result is that trade liberalization should result in quality upgrading in equilibrium: Gabszewicz, Shaked, Sutton and Thisse (1981) predict that high quality goods will survive, while low quality goods will exit after the opening of trade, which is

the opposite to the replacement of superiors by inferiors. For another example, Flam and Helpman (1987) predict that low quality goods produced in the developed country will be replaced by the same low quality imports from developing countries, which is not the replacement of superiors by inferiors but the horizontal replacement of superiors by inferiors (the catch-up effect). One reason why the replacement of superiors by inferiors has not been shown in previous studies is because most of these studies assume the unit demand: each consumer buys at most one unit of a product regardless of his income. This assumption presumes that consumers love quality but not quantity and limits the favorable effect of low price on low quality goods. Alternatively, this paper presents a new non-unit demand model, where consumers love quantity as well as quality. Either the unit demand model or non-unit demand model can show that if the income gap between two countries is large enough, the firm producing the low-quality good in the developing country exports its goods and gain profits, while the firm producing the high-quality good in the developed country lose profit after trade. However, the non-unit demand model can show the consumer-stealing effect the conventional unit demand model cannot show. In the conventional unit demand model, the firm producing the low-quality goods exports its products to the developed country, nevertheless it does not steal any customer from the firm producing the high-quality goods. For, they sell their products to consumers in the developed country, who could not buy any good under autarky. The only reason why the firm producing the high-quality goods in the developed country loses profits after trade is because the increase in competition. On the contrary, in my model, the firm producing the high-quality goods loses profits is not only because the competition become fierce but also because some low-income consumers in the developed country, who have bought high-quality goods under autarky are attracted to the volume of products and switch to low-quality goods if the income gap between two countries is large enough to make such a volume effect to work.

The non-unit demand model with endogenous quality choice is developed in section 4.2, the

open economy non-unit model is discussed in section 3.3, and the comparison between results of the non-unit demand model with that of the traditional unit demand model is shown in section 3.4.

3.2 A Non-unit Demand Model with Endogenous Quality Choice

Autarky

Consumers

Consumers love quantity as well as quality. The utility of consumers are assumed to be measured by the consumer surplus (CS).

$$U_x \equiv CS = s_j \theta_i q_j - \frac{q_j^2}{2} - p_j q_j \quad (3.1)$$

where s_j is the quality of the good j , q_j is the quantity of good j , and θ_i is the index of the income of the consumer i .

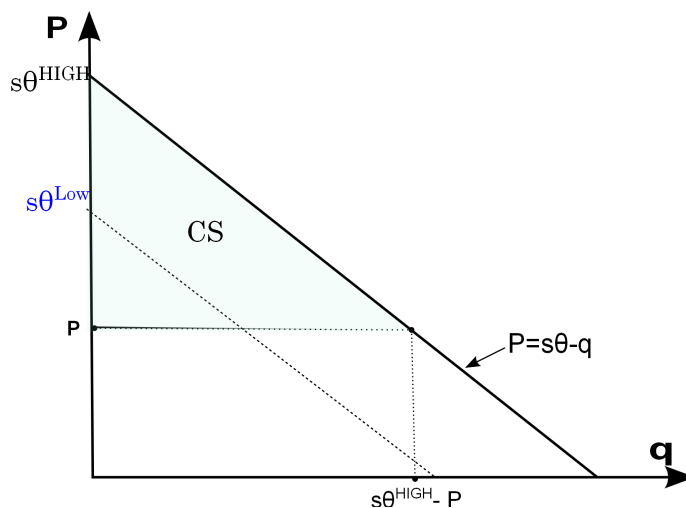


Figure 3.1: Consumer Surplus

Maximization of the above utility function (3.1) gives the following individual demand:

$$q_i = s_j \theta_i - p_j \quad \text{if } s \theta_i \geq p \quad (3.2)$$

The individual demand q_i increases with quality (s_j), income (θ_i) but decreases with price, which is a distinct feature of this model. In the conventional unit-demand model, the aggregate demand decrease with price but the individual demand does not decrease with the price, while the individual demand decreases with the price in this non-unit demand model.¹²

The consumer surplus achieved by the individual i is

$$U^i = CS^i = \frac{(s_j \theta_i - p_j)^2}{2} \quad (3.3)$$

The Firm-Monopoly-

Assume that in each country, only one firm, the monopolist, produces a vertically differentiated good under autarky. The monopolist faces a two-stage game. In the first stage, the monopolist chooses the quality of goods without knowing that it will face free trade in the future. Changing the quality of goods is assumed to be so costly that the monopolist will not change the quality of its products later. In the second stage, given the quality of products, the monopolist chooses the price of the product. Unlike the quality, price can be changed without cost.

The income of consumers θ are assumed to be uniformly distributed over the range $\theta \in [\underline{\Theta}, \bar{\Theta}]$.

³ Thus, the aggregate demand Q the monopolist faces is given as follows.

$$Q = \int_{\underline{\theta}}^{\bar{\theta}} (s \theta - p) n d\theta \quad (3.4)$$

¹ $q_i = 0$ if the price is greater than the reservation price $s \theta_i < p$

² The reservation price $s_j \theta_i$ implies that given the income θ , consumers are willing to pay more to higher quality goods.

³ Similarly, in a foreign country, the income is uniformly distributed over the range $\theta^* \in [\underline{\Theta}^*, \bar{\Theta}^*]$.

where $\underline{\theta} = \max \left\{ \underline{\Theta}, \frac{p}{\theta} \right\}$ is the endogenous lower limit, which is the higher of two values, the lowest income $\underline{\Theta}$ or $\frac{p}{\theta}$ the income of consumers whose consumer surplus is exactly zero.

$n = \frac{L}{\bar{\Theta} - \underline{\Theta}}$ is the density of consumers at each point of income distribution (L is the population size)

The Price Choice-The Second Stage- Solving backward, in the second stage, the monopolist sets the price. The marginal cost is normalized to be 0. The monopolist treats the lower limit of the market as endogenous $\underline{\theta} = \frac{p}{s}$ and chooses the price to maximize the following profits .

$$\max \pi = \int_{\underline{\theta}}^{\bar{\Theta}} p(s\theta - p)nd\theta - F(s) \quad (3.5)$$

where $F(s)$ is the cost of quality, which depends only on the quality and is independent of the quantity of production. Firms must incur the cost of quality $F(s)$ in every period.⁴

The FOC with respect to price is given by

$$\frac{(\bar{\Theta} - \underline{\theta})}{2} \{s(\bar{\Theta} + \underline{\theta}) - 4p\} = 0$$

Using the endogenous lower limit, $\underline{\theta} = \frac{p}{s}$, the above FOC gives the optimal price.

$$p = \frac{s\bar{\Theta}}{3} \quad (3.6)$$

This equation (3.6) implies that the monopolist chooses the combination of quality and price ($\frac{p}{s} = \underline{\theta} = \frac{\bar{\Theta}}{3}$) so as to make consumers with income $\frac{\bar{\Theta}}{3}$ can buy its product.

In other expressions, the market is fully covered if the income distribution is not so wide that the highest-income consumers do not earn three times more than the lowest-income consumers do. ($\bar{\theta} \leq 3\underline{\Theta}$).

⁴The cost of customer centers is an example of such cost.

$$\text{The market is } \begin{cases} \text{fully covered} & \text{if } \bar{\Theta} \leq 3\underline{\Theta} \\ \text{partially covered} & \text{if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

The Quality Choice-The First Stage- In the first stage, the monopolist chooses the quality of products without knowing it will face free trade in the future. As mentioned before, changing the quality of products is so costly, the monopolist will not change the quality later. The monopolist chooses the quality s so as to maximizes profits in eq (3.5).

The cost of quality $F(s)$ in eq (3.5) increases with quality, $F'(s)$, and the curvature of the quality-cost function is assumed to be greater than 1 ⁵:

$$F'(s) > 0 \text{ and } \frac{s \cdot F''(s)}{F'(s)} > 1$$

In this paper, the following special functional form is used for $F(s)$.⁶

$$F(s) = \frac{\alpha}{2}s^2 - \beta s + \frac{\beta^2}{2\alpha} \quad (3.7)$$

α in the above equation is assumed to satisfy the following condition.⁷

$$\alpha > \max \left\{ \frac{4L}{\bar{\Theta} - \underline{\Theta}} \left(\frac{\bar{\Theta}}{3} \right)^3, \frac{\bar{\Theta}(\bar{\Theta} + \underline{\Theta})L}{6} \right\}$$

With this cost function, the FOC with respect to quality (s) is given by⁸

$$\frac{p}{2}n(\bar{\Theta}^2 - \underline{\theta}^2) + p(s\underline{\theta} - p)n\left(\frac{\partial \underline{\theta}}{\partial s}\right) - \alpha s + \beta = 0 \quad (3.8)$$

⁵This is a necessary and sufficient condition to satisfy the SOC w.r.t. quality

⁶ $F'(s) = \alpha s - \beta$ and $\frac{s \cdot F''(s)}{F'(s)} = \frac{\alpha s}{\alpha s - \beta} > 1$ under this special functional form

⁷This condition is a necessary and sufficient condition for the SOC w.r.t. quality.

⁸The second term in eq (3.8) is zero either because the CS of the lower limit consumers is zero (i.e. $s\underline{\theta} - p = 0$) under the fully covered market ($\bar{\Theta} \leq 3\underline{\Theta}$) or because the lower limit becomes exogenous ($\underline{\theta} = \underline{\Theta}$ and $\frac{\partial \underline{\theta}}{\partial s} = 0$) under the partially covered market ($\bar{\Theta} > 3\underline{\Theta}$).

The Autarky Equilibrium under (Fully Covered Market) If the income distribution is not so wide that the highest-income consumers do not earn three times more than the lowest-income consumers do $\bar{\theta} \leq 3\underline{\theta}$, the market is fully covered and the lower limit of the market is equal to the lowest income in the country $\underline{\theta} = \underline{\theta}$. Substituting $\underline{\theta} = \underline{\theta}$ into eq (3.8), we have the optimal quality under fully covered market.

$$s = \frac{\beta}{\left\{ \alpha - \frac{\bar{\theta}(\bar{\theta}^2 - \underline{\theta}^2)n}{6} \right\}} \quad (3.9)$$

Substituting the above eq (3.9) back to eq (3.6), we have the optimal price under fully covered market.

$$p = \frac{\beta \bar{\theta}}{3 \left\{ \alpha - \frac{\bar{\theta}(\bar{\theta}^2 - \underline{\theta}^2)n}{6} \right\}} \quad (3.10)$$

Once the optimal price and quality are determined, the equilibrium aggregate demand and profits can be calculated as follows.

Substituting eq (3.9) and eq (3.10) into eq(3.4), we have the equilibrium aggregate demand.

$$Q = \frac{\beta (\bar{\theta} - \underline{\theta}) (\bar{\theta} + 3\underline{\theta}) n}{6 \left\{ \alpha - \frac{\bar{\theta}(\bar{\theta}^2 - \underline{\theta}^2)n}{6} \right\}} \quad (3.11)$$

Substituting eq (3.9) and eq (3.10) into eq(3.5), we have the equilibrium profits.

$$\pi = \frac{\beta^2 (9\alpha - 2\bar{\theta}^2 L)}{18 \left\{ \alpha - \frac{\bar{\theta}(\bar{\theta}^2 - \underline{\theta}^2)n}{6} \right\}^2} - \frac{\beta^2}{2\alpha} \quad (3.12)$$

The Autarky Equilibrium under the Partially Covered Market Similarly, if $\bar{\theta} > 3\underline{\theta}$, the market is not fully covered and the lower limit of the market is equal to a third of the highest income $\underline{\theta} = \frac{\bar{\theta}}{3}$. Substituting $\underline{\theta} = \frac{\bar{\theta}}{3}$ into eq (3.8), we have the optimal quality under partially covered

market.

$$s = \frac{\beta}{\left\{ \alpha - 4n \left(\frac{\bar{\Theta}}{3} \right)^3 \right\}} \quad (3.13)$$

Again substituting this optimal quality (3.13) into eq (3.6), we have the equilibrium price under the partially covered market.

$$p = \frac{\beta \bar{\Theta}}{3 \left\{ \alpha - 4n \left(\frac{\bar{\Theta}}{3} \right)^3 \right\}} \quad (3.14)$$

Substituting eq (3.13) and (3.14) back to eq (3.4) and (3.5), the equilibrium aggregate demand and the equilibrium profits under the partially covered market can be derived as follows.

$$Q = \frac{2\beta \bar{\Theta}^2 n}{9 \left\{ \alpha - 4n \left(\frac{\bar{\Theta}}{3} \right)^3 \right\}} \quad (3.15)$$

$$\pi = \frac{\beta^2}{2 \left\{ \alpha - 4n \left(\frac{\bar{\Theta}}{3} \right)^3 \right\}} - \frac{\beta^2}{2\alpha} \quad (3.16)$$

The Closed Economy Equilibrium -Summary- To summarize, the autarky equilibrium is characterized by the following equilibrium variables.

$$s = \begin{cases} \frac{\beta}{\left\{ \alpha - \frac{\bar{\Theta}(\bar{\Theta}^2 - \underline{\Theta}^2)n}{6} \right\}} & (3.9) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \frac{\beta}{\left\{ \alpha - 4n \left(\frac{\bar{\Theta}}{3} \right)^3 \right\}} & (3.13) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

$$p^{au} = \begin{cases} \frac{\beta \bar{\Theta}}{3 \left\{ \alpha - \frac{\bar{\Theta}(\bar{\Theta}^2 - \underline{\Theta}^2)n}{6} \right\}} & (3.10) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \frac{\beta \bar{\Theta}}{3 \left\{ \alpha - 4n \left(\frac{\bar{\Theta}}{3} \right)^3 \right\}} & (3.14) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

$$Q^{au} = \begin{cases} \frac{\beta(\bar{\Theta}-\underline{\Theta})(\bar{\Theta}+3\underline{\Theta})n}{6\left\{\alpha-\frac{\bar{\Theta}(\bar{\Theta}^2-\underline{\Theta}^2)n}{6}\right\}} & (3.11) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \frac{2\beta\bar{\Theta}^2n}{9\left\{\alpha-4n\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} & (3.15) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

$$\pi^{au} = \begin{cases} \frac{\beta^2(9\alpha-2\bar{\Theta}^2L)}{18\left\{\alpha-\frac{\bar{\Theta}(\bar{\Theta}^2-\underline{\Theta}^2)n}{6}\right\}^2} - \frac{\beta^2}{2\alpha} & (3.12) \text{ if } \bar{\Theta} \leq 3\underline{\Theta} \\ \frac{\beta^2}{2\left\{\alpha-4n\left(\frac{\bar{\Theta}}{3}\right)^3\right\}} - \frac{\beta^2}{2\alpha} & (3.16) \text{ if } \bar{\Theta} > 3\underline{\Theta} \end{cases}$$

All of the above variables- s , p , Q , π - increase with the highest income, the lowest income, and the population size $(\bar{\Theta}, \underline{\Theta}, L)$: it implies that the monopolist in a country with higher income and larger population size produces higher quality goods, charges higher price, sells more products, and earns more profits.⁹

3.3 The Open Economy: Duopoly

Consider the situation where two countries-a developed country (home) and a developing country (foreign)- start free trade. In each country, the monopolist produces vertically differentiated goods. Two countries have different income distributions and different population size, yet they have the same cost function and the same utility function.

This paper considers a special case: the income distribution of two countries just touches (i.e. $\bar{\Theta}^* = \underline{\Theta}$) [figure 3.2], the density of consumers is the same in both countries, (i.e. $n = \frac{L}{\bar{\Theta} - \underline{\Theta}} = \frac{L^*}{\bar{\Theta}^* - \underline{\Theta}^*} = n^*$), and both markets were fully covered under autarky. ($\bar{\Theta} \leq 3\underline{\Theta}$ in home and $\bar{\Theta}^* \leq 3\underline{\Theta}^*$ in foreign)¹⁰

The Post-Trade Bertrand Competition After free trade starts, two firms face the competition. Since the highest income (and the lowest income) in the developed country are higher than

⁹No variable faces the discontinuity problem: all variables are continuous at $\bar{\Theta} = 3\underline{\Theta}$.

¹⁰With the first two assumptions, free trade just brings the extension of income distribution from $[\underline{\Theta}, \bar{\Theta}]$ to $[\underline{\Theta}^*, \bar{\Theta}]$.

[Special Case 1] Just overlapping income distribution

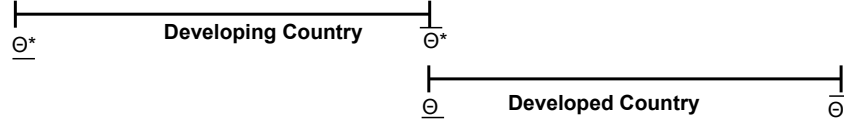


Figure 3.2: Income Distributions of Two Countries

those in the developing country by assumption, eq (3.9) suggests that the quality of goods produced in the developed country is higher than that of goods produced in the developing country. ($s > s^*$) Since the cost of changing quality is high, these two firms will not change their quality. Thus, two firms immediately start the Bertrand competition, and they set the new price simultaneously given the price of the other firm. In setting their prices, two firms set the same price in two markets because the arbitrage is possible in this model.

Consumers

After trade, consumers in both countries now have choice between the high quality goods and the low quality ones. Consumers in two countries share the same utility function, and marginal consumers achieve the same utility level by buying either of two goods. That is,

$$\frac{1}{2} \left(s\tilde{\theta} - p \right)^2 = \frac{1}{2} \left(s^*\tilde{\theta} - p^* \right)^2$$

Solving the above equation, the income of marginal consumers who are indifferent between the high quality goods and the low quality ones can be identified ¹¹:

$$\tilde{\theta} = \frac{p - p^*}{s - s^*} \quad (3.17)$$

If the income of marginal consumers is greater than the lowest income in the developed country ($\tilde{\theta} \geq \underline{\Theta}$), the firm producing the low-quality goods in the developing country will export and steal consumers

¹¹The derivation of eq (3.17) is shown in the appendix B

from the firm producing the high-quality goods, vice versa.

Firms: The Bertrand Competition

After free trade starts, two firms join the Bertrand competition and set its price simultaneously. Given the income of marginal consumers eq(3.17), the aggregate demand function and the profit function are given as follows.

$$Q^* = \int_{\underline{\theta}^*}^{\bar{\theta}} (s^* \theta - p^*) n d\theta \quad (3.18)$$

$$\pi^* = \int_{\underline{\theta}^*}^{\bar{\theta}} p^* (s^* \theta - p^*) n d\theta - F(s^*) \quad (3.19)$$

where the lower boundary of the firm producing the low-quality good becomes $\underline{\theta}^* = \max\{\underline{\theta}^*, \frac{p^*}{s^*}\}$ and

$F(s^*) = \frac{\alpha}{2} s^{*2} - \beta s^* + \frac{\beta^2}{2\alpha}$ is the cost of quality incurred in every period

The FOC with respect to price (p^*) gives the reaction function of the firm producing the low-quality goods ¹²:

$$p^* = \frac{s^* p}{3s} \quad (3.20)$$

The aggregate demand and the profit function of the firm producing the high-quality goods are given by

$$Q = \int_{\bar{\theta}}^{\bar{\Theta}} (s\theta - p) n d\theta \quad (3.21)$$

¹²There are two solutions for p^* . However, the solution in eq(3.20) is the only solution that satisfies the second order condition. Appendix shows the derivation of eq (3.20).

$$\pi = \int_{\bar{\theta}}^{\bar{\Theta}} p(s\theta - p) n d\theta - F(s) \quad (3.22)$$

where $F(s) = \frac{\alpha}{2}s^2 - \beta s + \frac{\beta^2}{2\alpha}$ is the cost of quality incurred by the firm producing the high-quality good.

The FOC with respect to price gives the reaction function of the firm producing the high-quality goods:

$$p = \frac{2 \left\{ (s - s^*)^2 \bar{\Theta} - s^* p^* \right\} \pm \sqrt{4 \left\{ \bar{\Theta} (s - s^*)^2 - s^* p^* \right\}^2 - 3s(s - 2s^*) \left\{ \bar{\Theta}^2 (s - s^*)^2 - p^{*2} \right\}}}{3(s - 2s^*)} \quad (3.23)$$

The Open Economy Equilibrium

The intersection of two reaction functions (3.20), (3.23) yields the Nash equilibrium in this special case.¹³

$$p = \frac{3s(s - s^*) \bar{\Theta} \left\{ 6s(s - s^*) - \sqrt{9(s - s^*)^2 + 16s^{*2}} \right\}}{27(s - s^*)^2 - 16s^{*2}} \quad (3.24)$$

$$p^* = \frac{s^* \bar{\Theta} \left\{ 6s(s - s^*) - \sqrt{9(s - s^*)^2 + 16s^{*2}} \right\}}{27(s - s^*)^2 - 16s^{*2}} \quad (3.25)$$

The income of marginal consumers is

$$\tilde{\theta} = \frac{(3s - s^*) \bar{\Theta} \left\{ 6s(s - s^*) - \sqrt{9(s - s^*)^2 + 16s^{*2}} \right\}}{27(s - s^*)^2 - 16s^{*2}} \quad (3.26)$$

Eq (3.22) and (3.19) give the post-trade profits:

$$\pi = np(\bar{\Theta} - \tilde{\theta}) \left\{ \frac{s}{2} (\bar{\Theta} + \tilde{\theta}) - p \right\} - \underbrace{\left\{ \frac{\alpha}{2}s^2 - \beta s + \frac{\beta^2}{2\alpha} \right\}}_{F(s)} \quad (3.22)$$

¹³The derivation of eq (3.24), (3.25), and (3.26) are shown in Appendix B.

$$\pi^* = np^*(\tilde{\theta} - \underline{\Theta}^*) \left\{ \frac{s^*}{2} (\tilde{\theta} + \underline{\Theta}^*) - p^* \right\} - \underbrace{\left\{ \frac{\alpha}{2} s^{*2} - \beta s^* + \frac{\beta^2}{2\alpha} \right\}}_{F(s^*)} \quad (3.19)$$

3.4 The Non-Unit Demand Model *vs* The Unit Demand Model: Numerical Comparison

So far, this paper has discussed the original Non-Unit Demand (Non-UD) model with endogenous quality choice. This section compares the Non-UD model with the conventional Unit Demand (UD) model with endogenous quality choice when $2\underline{\Theta} < \bar{\Theta} \leq 3\underline{\Theta}$ and $2\underline{\Theta}^* < \bar{\Theta}^* \leq 3\underline{\Theta}^*$. In this income range, autarky markets are fully covered in the Non-UD model, while they are partially covered in the UD model.^{14 15}

This section numerically shows that the income gap, the difference between the highest income in the developed country and the lowest income in the developing country is the key in both the Non-UD model and the UD model. Both models reach the same conclusion with regard to the following: the firm producing the low-quality goods in the developing country exports their products to the developed country if the income gap between two countries is large enough. However, the firm producing the high-quality goods in the developed country exports their products to the developing county and steal consumers from the firm producing the low-quality goods if the income gap is not large enough. Even if the firm producing the high-quality goods exports their products, it loses profits after trade due to the increase in competition.

Two models reach different results with regard to the consumer stealing effect of the firm producing the low-quality goods. As mentioned above, both models predict that the low-quality goods are exported to the developed country if the income gap between two countries and the income

¹⁴The income range is the only range, which allows two models to be compared meaningfully.

¹⁵The UD model with endogenous quality choice is presented in Appendix A

distribution within a country are so wide that the low-quality goods attract some consumers in the developed country. However, in the conventional UD model, consumers, who buy the low-quality goods after trade have not bought any product under autarky. On the contrary, in the Non-UD model, all consumers, who buy the low-quality goods after trade have bought the high-quality goods under autarky. The different conclusion comes from the fact that the Non-UD model considers the quantity effect as well as the price effect, while no volume effect of the low-quality good is considered in the UD model

The Non-UD Model *vs* The UD Model: Exporters and the Consumer Stealing Effect

Figure 3.5 and 3.6 show the location of marginal consumers and identifies the exporter of goods after trade in two models. In both models, the income distribution is a key determinant for the trade pattern. If the income gap between two countries and the income distribution within each country is wide enough, the firm producing the low-quality goods in the developing country attract the low-income consumers in the developed country and export its products to the developed country. However, the firm producing the high-quality goods export its product to the developing country.

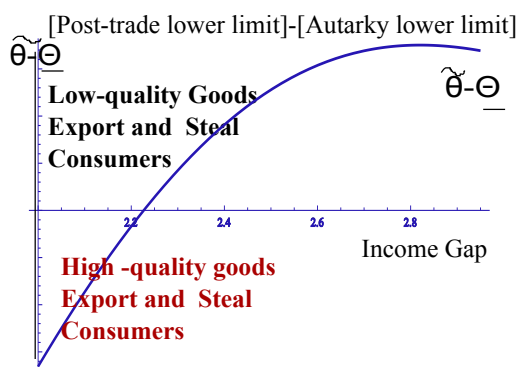


Figure 3.3: Non-UD Model: Exporters of Goods



Figure 3.4: UD Model: Exporters of Goods

In terms of the consumer stealing effect, two model reach the different conclusion. The distance between the location of the post-trade lower limit that of the autarky lower limit in Figure 3.5 shows the consumer stealing effect in the Non-UD model. If the income gap between two countries is not large enough, the firm producing the high-quality goods export its products to the developing country and steal consumers from the firm in the developing country. However, if the income gap between two countries and the income distribution within the developed country is wide enough, the firm producing the low-quality become an exporter and steal consumers, who have bought the high-quality goods before trade.

On the other hand, the conventional UD model tells a different story. The distance between the location of the autarky lower limit and that of the post-trade lower limit drawn in Figure 3.6 shows the consumer stealing effect in the UD model. As shown, the post-trade lower limit of the firm in the developed country is always lower than its autarky lower limit at any income diversity level, which means that the firm producing the high-quality goods gain new consumers rather than lose existing consumers. It also means that consumers who buy the low-quality goods after trade are consumers who have not bought any good under autarky.

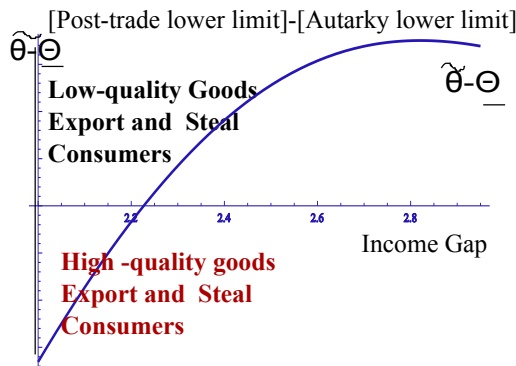


Figure 3.5: Non-UD Model: Consumer Stealing Effect?

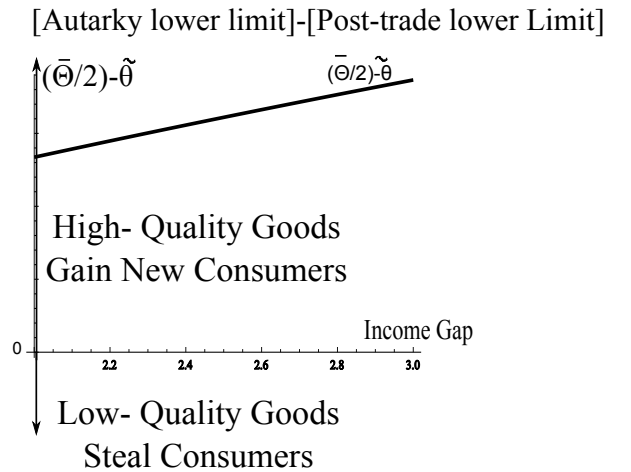


Figure 3.6: UD Model: Consumer Stealing Effect?

The Non-UD Model *vs* The UD Model: Profits

In terms of profits, two models reach the same conclusion: the firm producing the high-quality goods always lose profits after trade due to the increased competition, while the firm producing the low-quality goods can gain profits by exporting its products to the developed country if the income gap between two countries and the income distribution within the developed country is so wide that the low-quality goods attract some low-income consumers in the developed country. (Figure 3.7, 3.8, 3.9)

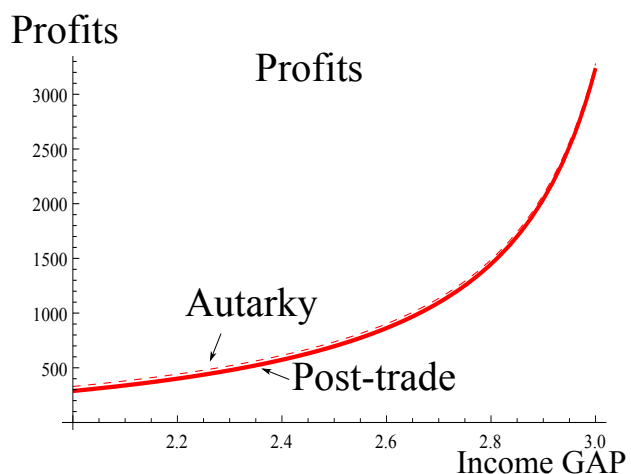


Figure 3.7: Non-UD Model: Profits of the Firm Producing the High-Quality Goods

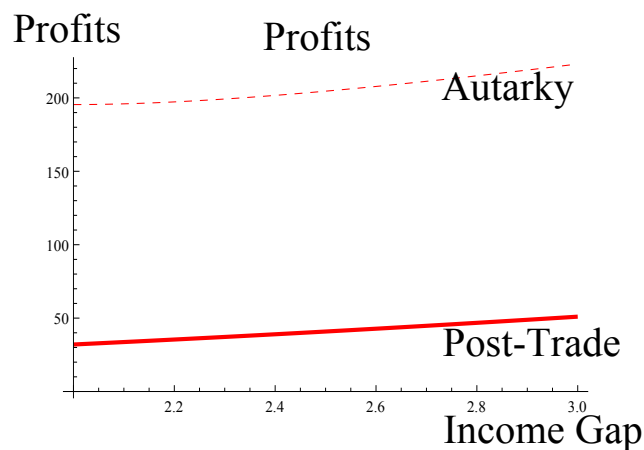


Figure 3.8: UD Model: Profits of the Firm Producing the High-Quality Goods

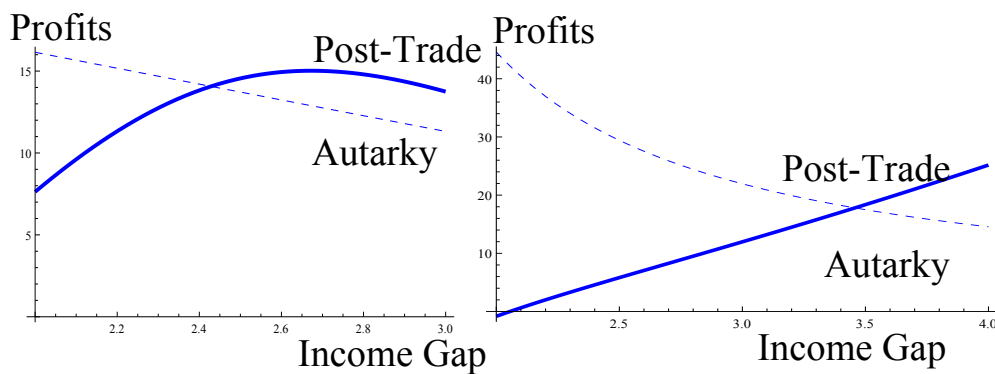


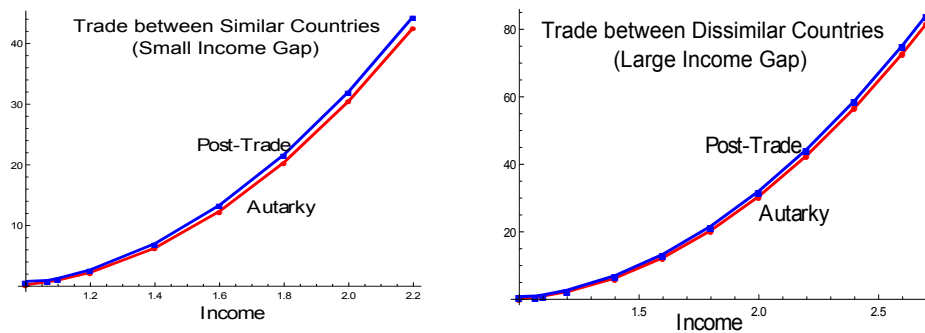
Figure 3.9: Profits of the Firm Producing the Low-Quality Goods

3.5 Numerical Examples

This section presents numerical examples corresponding to the discussion in the previous section.

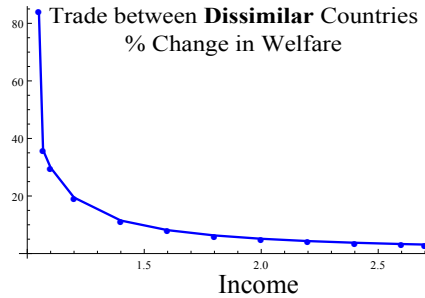
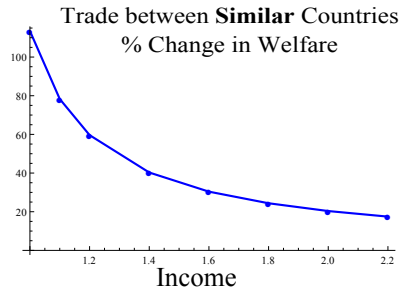
Table 3.1 shows the case where the income gap between two countries are large. As discussed in the previous section, two models reach different conclusions about the firm of low quality goods: the firm producing low-quality goods increases profits by stealing consumers from the other firm, while in the UD model, its post-trade profit gain without stealing any consumer. With or without the consumer stealing effect has different implications from trade. As shown in figure 3.5, 3.5, 3.5, every consumer will be better off after trade. In percentage, low-income consumers in the developed country will get more gain from trade than high-income consumers in the same country. In value, trade has different implications. If the income gap between two countries is small and no consumer is stolen from the firm producing the high quality goods, the gain from trade monotonically increases with income. However, if the income gap between two countries is large enough, and the firm producing low quality goods steal some consumers from the other firm, the gain from trade and income have a v-shape relationship. Marginal consumers, who are suffering the most from downgrade of their consumption will get the lowest gain from trade.

Table 3.2 shows the case where two countries are similar. As discussed in proposition ??, two models reach the same conclusion in this case.



Non-UD model: $\bar{\Theta}=135$ $\underline{\Theta}=\bar{\Theta}^*=50$ $\underline{\Theta}^*=24.5$ $L=100$ $L^*=30$ $n=n^*=1.176$												
	s	s^*	p	p^*	Q	Q^*	π	π^*	w	w^*	Exporter	consumer stealing effect
Autarky	0.142	0.036	6.4	0.6	675	22	1135	13	4019	22		
Trade	0.142	0.036	6.2	0.5	690	30	1096	15	4116	26	Low quality	YES
UD model: $\bar{\Theta}=135$ $\underline{\Theta}=\bar{\Theta}^*=50$ $\underline{\Theta}^*=24.5$ $L=100$ $L^*=30$ $n=n^*=1.176$												
	s	s^*	p	p^*	Q	Q^*	π	π^*	w	w^*	Exporter	consumer stealing effect
Autarky	0.044	0.035	3	0.9	79	29	211	26				
Trade	0.044	0.035	0.7	0.3	100	30	45	8			Low quality	NONE

Table 3.1: Non-UD model VS UD model: Large Income Gap Case



The Non-UD model: $\bar{\Theta}=110$ $\underline{\Theta}=\bar{\Theta}^*=50$ $\underline{\Theta}^*=32$ $L=100$ $L^*=30$ $n=n^*=1.67$											
	s	s^*	p	p^*	Q	Q^*	π	π^*	$\tilde{\theta}$	<i>Exporter</i>	Losers of consumers
Autarky	0.074	0.036	2.7	0.6	321	26	442	15			
Trade	0.074	0.036	2.3	0.4	367	32	400	11	49.7	high quality	low quality
The UD model: $\bar{\Theta}=110$ $\underline{\Theta}=\bar{\Theta}^*=50$ $\underline{\Theta}^*=32$ $L=100$ $L^*=30$ $n=n^*=1.667$											
	s	s^*	p	p^*	Q	Q^*	π	π^*	$\tilde{\theta}$	<i>Exporter</i>	Losers of consumers
Autarky	0.043	0.036	2.4	0.9	92	42	197	37			
Trade	0.043	0.036	0.5	0.2	116	30	35	2	40.52	high quality	low quality

Table 3.2: The Non-UD model VS The UD model: Small Income Gap Case

3.6 Conclusion

Observing that the unit demand model used by existing studies do not provide satisfactory explanation for the replacement of superiors by inferiors, this paper presents a new alternative non-unit demand model, where consumers love quantity as well as quality. Adding consumers' love of quality into the model, this paper shows that the firm producing the high quality goods will always lose profits after trade, while the firm producing the low quality goods will steal consumers from the firm producing the high quality goods and gain profits if the income gap between two countries is large enough to make room for the low-quality but inexpensive goods to attract consumers. As shown above, the contribution of this paper is to propose a new non-unit demand model that can consider consumers' love of quantity and to show that the firm producing low quality goods steal consumers from the other firm and gains profit after trade if the income gap between two countries is large enough. Numerical examples show that every consumer will gain from trade, yet trade will bring consumers heterogeneous gain from trade. If the income gap between two countries is not large enough, no consumer in the developed country changes to buy the low quality goods, the gain from trade increase with income. However, if the income gap between two countries is large enough, and low income consumers in the developed countries change to buy the low quality goods, the gain from trade and the income has a v-shape relationship: for those low-income consumers who switch to the low quality goods, the gain from trade decrease with income, while the gain from trade increases with income for those high-income consumers who do not switch to low-quality goods. The gain from trade of marginal consumers hits the lowest because marginal consumers will suffer the negative effect of giving up the high quality goods the most.

Appendix A: The Traditional Unit Demand Model with the Endogenous Quality Choice

Most models in previous studies allow neither the endogenous quality choice nor the non-unit demand, while my model allows both the non-unit demand and the endogenous once-and-all quality choice. Therefore, different results between my model and models in previous studies are explained by mixed effects of the non-unit demand and the endogenous quality choice.

To show how important the non-unit demand assumption plays in my model, this section presents the unit demand version of my model: the model presented in this section allows the once-and-for-all endogenous quality choice but imposes the unit demand assumption on consumers.

Consumers-The Unit Demand- First, the utility of consumers assumed in many previous models is given by

$$U_i = s_j \theta_i - p$$

where θ_i is the income of the consumer i .

The individual demand is simply given by

$$q_i = \begin{cases} 1 & \text{if } s\theta_i \geq p \\ 0 & \text{if } s\theta_i < p \end{cases}$$

The Firm-Monopoly under Autarky- Given the above individual demand, the aggregate demand the monopolist faces is equal to the number of consumers whose income is greater than or equal to $\underline{\theta} = \frac{p}{s}$:

$$Q^{UD} = \int_{\underline{\theta}}^{\bar{\Theta}} n \, d\theta = n \left(\bar{\Theta} - \frac{p}{s} \right) \quad (3.27)$$

The monopolist's profits is given by

$$\pi^{UD} = pQ - \underbrace{\left\{ \frac{\alpha}{2} s^s - \beta s + \frac{\beta^2}{2\alpha} \right\}}_{F(s)} \quad (3.28)$$

The monopolist faces the two-stage game: in the first stage, it chooses the quality without knowing that it will face free trade in the future, and then it sets the price in the second stage. As mentioned in previous sections, the quality choice is irreversible, while the price choice is reversible.

The Second Stage:the Price Choice In the second stage, given the quality chosen in the first stage, the monopolist sets price. The FOC with respect to price gives the optimal combination of price and quality:

$$p^{UD} = \frac{s\bar{\Theta}}{2} \quad (3.29)$$

eq (3.29) implies that under the unit demand assumption, the firm chooses price and quality to make consumers with income $\underline{\theta} = \frac{p}{s} = \frac{\bar{\Theta}}{2}$ purchase its products.¹⁶

In other expressions, eq (3.29) implies that

$$\text{The market is} \begin{cases} \text{fully covered} & \text{if } \bar{\Theta} \leq 2\underline{\theta} \\ \text{partially covered} & \text{if } \bar{\Theta} > 2\underline{\theta} \end{cases}$$

It becomes harder for the market to be fully covered under the unit demand model.

The First Stage:the Irreversible Quality Choice In the first stage, the monopolist chooses the quality of its product once and for all without knowing that free trade will start in the future.

¹⁶Compared eq(3.29) with (3.6), the monopolist sell its products less consumers under the unit demand model than it does under the non-unit demand model.

The Irreversible Quality Choice under the Fully Covered Market If the income distribution is not so diverse ($\bar{\Theta} \leq 2\underline{\Theta}$), as shown above, the monopolist covers the whole market. In such a case, the lower limit becomes **exogenous** variable, the lowest income ($\underline{\theta} = \underline{\Theta}$).

Substituting $\underline{\theta} = \underline{\Theta}$ into the profit function (3.28), the FOC with respect to s gives the optimal quality.

$$s^{UD} = \frac{\beta}{\alpha} \quad (3.30)$$

Eq(3.30) shows that the optimal quality under the exogenous lower limit is the quality, which simply minimizes the cost of quality. This optimal quality does not depend on the income distribution at all. Hence, if $\bar{\Theta} \leq 2\underline{\Theta}$ in all countries, all countries produce the same quality of goods.

The Irreversible Quality Choice under the Partially Covered Market If the income distribution is diverse enough ($\bar{\Theta} > 2\underline{\Theta}$), the monopolist chooses the combination of price and quality so as to make consumers with income $\frac{\bar{\Theta}}{2}$ or above buy its products. In this case, given the quality, the monopolist treats the lower limit as **endogenous** variable, ($\underline{\theta} = \frac{p}{s}$).

Substituting $\underline{\theta} = \frac{p}{s}$ into eq (3.28), we get the FOC with respect to s .

$$s^{UD} = \frac{\left(\frac{\bar{\Theta}^2 L}{\bar{\Theta} - \underline{\Theta}}\right) + 4\beta}{4\alpha} \quad (3.31)$$

The optimal quality under the partially covered market shows the same characteristics as the optimal quality under the non-unit demand model: the quality increases with the highest income, the lowest income, and the population size but decreases with the cost of quality.¹⁷

¹⁷The optimal quality under the partially covered market is higher than the optimal quality under the fully covered market. $s = \frac{\left(\frac{\bar{\Theta}^2 L}{\bar{\Theta} - \underline{\Theta}}\right) + 4\beta}{4\alpha} > \frac{\beta}{\alpha}$

The Closed Economy Equilibrium under the Unit Demand Model Substituting the optimal quality and price back to eq(3.27),(3.28), and (3.29), we get the closed economy equilibrium under the unit demand model.

$$s^{UD} = \begin{cases} \frac{\beta}{\alpha} & \text{(3.30) if } \bar{\theta} \leq 2\underline{\theta} \\ \frac{n\bar{\theta}^2+4\beta}{4\alpha} & \text{(3.31) if } \bar{\theta} > 2\underline{\theta} \end{cases}$$

$$p^{auUD} = \begin{cases} \frac{\beta\bar{\theta}}{2\alpha} & \text{if } \bar{\theta} \leq 2\underline{\theta} \\ \frac{\bar{\theta}(n\bar{\theta}^2+4\beta)}{8\alpha} & \text{if } \bar{\theta} > 2\underline{\theta} \end{cases}$$

$$Q^{auUD} = \begin{cases} L & \text{if } \bar{\theta} \leq 2\underline{\theta} \\ \frac{n\bar{\theta}}{2} & \text{if } \bar{\theta} > 2\underline{\theta} \end{cases}$$

$$\pi^{auUD} = \begin{cases} \frac{\beta\bar{\theta}L}{2\alpha} & \text{if } \bar{\theta} \leq 2\underline{\theta} \\ \frac{n\bar{\theta}^2(n\bar{\theta}^2+8\beta)}{32\alpha} & \text{if } \bar{\theta} > 2\underline{\theta} \end{cases}$$

note: $n \equiv \frac{L}{\bar{\theta}-\underline{\theta}}$ (the number of consumers at each point of income)

The Open Economy under the UD model

The Post-trade Bertrand Competition

Case 1: Both markets were fully covered under autarky ($\bar{\theta} \leq 2\underline{\theta}$ and $\bar{\theta} \leq 2\underline{\theta}$)

If the income distribution is not diverse in both countries ($\bar{\theta} \leq 2\underline{\theta}$ and $\bar{\theta} \leq 2\underline{\theta}$), and markets in both countries were fully covered under autarky, as shown in the previous section, the exactly same quality of goods are produced in two countries. ($s = s^* = \frac{\beta}{\alpha}$)¹⁸ If this is a case, the post trade price

¹⁸As noted before,two countries choose the same quality whose cost is the lowest.

competition between two firms is just the Bertrand competition of homogeneous goods: the price will be reduced to the marginal cost.

Case 2: Both markets were partially covered under autarky ($\bar{\theta} > 2\underline{\theta}$ and $\bar{\theta} > 2\underline{\theta}$) If the income distribution is diverse enough in both countries, as shown in the previous section, the quality chosen under autarky depends on the income distribution: the quality of goods produced in the developed country is higher than that produced in the developing country.¹⁹

$$s = \frac{n\bar{\theta}^2 + 4\beta}{4\alpha} \quad s^* = \frac{n^*\bar{\theta}^{*2} + 4\beta}{4\alpha} \quad \Rightarrow s > s^* \text{ if } \bar{\theta} > \bar{\theta}^*$$

After the opening of trade, consumers have three choices: they buy one unit of the high quality goods, buy one unit of the low quality good, or do not buy anything. The marginal consumers, who are indifferent between the low quality good and the high quality one achieve the same utility if they buy either of two goods by one unit:

$$s\tilde{\theta} - p = s^*\tilde{\theta} - p^*$$

The income of such marginal consumers is

$$\tilde{\theta} = \frac{p - p^*}{s - s^*}$$

Given the above marginal consumers, aggregate demand of the high quality goods and that of the low quality ones are derived as follows.²⁰

$$Q^{UD} = \int_{\tilde{\theta}}^{\bar{\theta}} n d\theta \quad Q^{*UD} = \int_{\underline{\theta}^*}^{\tilde{\theta}} n d\theta$$

¹⁹ $n = n^*$ by assumption

²⁰The definition of marginal consumer's income under the UD model happens to be the same as the definition under the Non-UD model.

$$\text{where } \underline{\theta}^* = \max \left\{ \underline{\Theta}^*, \frac{p^*}{s^*} \right\}$$

Facing the above aggregate demand, The low-quality goods' firm maximizes the following profit function.

$$\pi^{*UD} = \int_{\underline{\theta}^*}^{\bar{\theta}} p n d\theta - F(s^*) \quad (3.32)$$

The FOC with respect to p^* gives the reaction function of The firm producing the low-quality goods.²¹

$$p^* = \frac{s^* p}{2s} \quad (3.33)$$

Similarly, the high-quality goods' firm maximizes the following profit function.

$$\pi^{UD} = \int_{\bar{\theta}}^{\bar{\Theta}} p n d\theta - F(s) \quad (3.34)$$

The reaction function of the high-quality goods' firm is given by the FOC with respect to p :

$$p = \frac{p^*}{2} + \frac{(s - s^*)\bar{\Theta}}{2} \quad (3.35)$$

The intersection of two reaction functions (3.35), (3.33) gives the equilibrium price after trade:

$$p^{UD} = \frac{2s(s - s^*)\bar{\Theta}}{(4s - s^*)} \quad (3.36)$$

$$p^{*UD} = \frac{s^*(s - s^*)\bar{\Theta}}{(4s - s^*)} \quad (3.37)$$

Substituting the equilibrium prices (3.36), (3.37) back into $\bar{\theta}$, eq (3.34) and (3.32), we have the open economy equilibrium under the UD model:

²¹ Again, The low-quality goods' firm maximizes its profits by treating the lower limit $\underline{\theta}^* = \frac{p^*}{s^*}$ endogenous variable.

The income of marginal consumer is given by

$$\tilde{\theta}^{UD} = \frac{(2s - s^*)\bar{\Theta}}{(4s - s^*)} \quad (3.38)$$

Post-trade profits of two firms are given by

$$\pi^{UD} = \frac{4s^2(s - s^*)n\bar{\Theta}^2}{(4s - s^*)^2} - \frac{n^2\bar{\Theta}^4}{32\alpha} \quad (3.39)$$

If the quality difference between the high quality and the low quality is large enough, $[\frac{p^*}{s^*} = \frac{(s-s^*)\bar{\Theta}}{(4s-s^*)} \leq \underline{\Theta}^*$

$\Leftrightarrow (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} \geq 0]$ the market in the developing country will be fully covered after trade,

and profits of The low-quality goods' firm will be

$$\pi^{*UD} = \frac{ns^*(s - s^*)\bar{\Theta} \{s\bar{\Theta} - \{(4s - s^*)\underline{\Theta}^* - (s - s^*)\bar{\Theta}\}\}}{(4s - s^*)^2} - \frac{n^2\bar{\Theta}^{*4}}{32\alpha} \quad \text{if } (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} \geq 0 \quad (3.40)$$

If the difference between the high quality and the low quality is not large enough, $[\frac{p^*}{s^*} = \frac{(s-s^*)\bar{\Theta}}{(4s-s^*)} > \underline{\Theta}^*$

$\Leftrightarrow (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} < 0]$, the market in the developing country will continue to be partially covered

after trade, and profits of The low-quality goods' firm will be

$$\pi^{*UD} = \frac{ns^*(s - s^*)\bar{\Theta}^2}{(4s - s^*)^2} - \frac{n^2\bar{\Theta}^{*4}}{32\alpha} \quad \text{if } (4s-s^*)\underline{\Theta}^* - (s-s^*)\bar{\Theta} < 0 \quad (3.41)$$

Appendix B

The Derivation of eq (3.17)

Marginal consumers achieve the same level of CS whether they buy the high quality goods (s) or the low quality ones (s^*):

$$\frac{1}{2} (s\tilde{\theta} - p)^2 = \frac{1}{2} (s^*\tilde{\theta} - p^*)^2$$

Expanding the above equation, we get

$$(s^2 - s^{*2})\tilde{\theta}^2 - 2(sp - s^*p^*)\tilde{\theta} + (p^2 - p^{*2}) = 0$$

Solving this quadratic equation, we have two solutions:

$$\tilde{\theta} = \frac{p + p^*}{s + s^*}, \quad \frac{p - p^*}{s - s^*}$$

Under the first solution, The low-quality goods' firm faces the upward demand [$\frac{\partial \tilde{\theta}}{\partial p^*} > 0$ (larger the market share)], while both firms face the downward demand under the second solution. [$\frac{\partial \tilde{\theta}}{\partial p^*} < 0$ and $\frac{\partial \tilde{\theta}}{\partial p} > 0$ (The increase in price results in the loww of the market share)] Thus, the only economically reasonable solution is the second solution [eq (3.17)].

$$\tilde{\theta} = \frac{p - p^*}{s - s^*} \quad (3.17)$$

The Derivation of the reaction function of the low-quality goods' firm-eq(3.20)-

After trade, The firm producing the low quality goods maximizes the following reaction function by treating the lower limit as an endogenous variable $\underline{\theta}^* = \frac{p^*}{s^*}$:

$$\pi^* = \int_{\underline{\theta}^*}^{\bar{\theta}} p^*(s^*\theta - p^*) n d\theta - F(s^*) \quad (3.19)$$

where $\bar{\theta} = \frac{p-p^*}{s-s^*}$ the upper limit

The FOC with respect to p^* yields the following quadratic equation:

$$3s^2 p^{*2} - 4ss^* p^* + s^{*2} p^2 = 0$$

This quadratic equation gives two roots:

$$p^* = \frac{s^* p}{3s}, \quad \frac{s^* p}{s}$$

However, the Second Order Condition(SOC) requires that

$$\begin{aligned} \frac{ns}{s^*(s-s^*)^2} (3sp^* - 2s^*p) &< 0 \\ \Leftrightarrow 3sp^* - 2s^*p &< 0 \\ \Leftrightarrow p^* &< \frac{2s^*p}{3s} \quad [\text{SOC}] \end{aligned}$$

The only solution that satisfies the above SOC is the first root.

$$p^* = \frac{s^* p}{3s} \quad (3.20)$$

The Derivation of the Open Economy Equilibrium

Substitute the reaction function of The low-quality goods' firm $p^* = \frac{s^* p}{3s}$ [eq(3.20)] into the reaction function of the high-quality goods' firm eq (3.23), we get the following quadratic equation.

$$(s - 2s^*) \left\{ 27(s - s^*)^2 - 16s^{*2} \right\} p^2 - 36s\bar{\Theta}(s - 2s^*)(s - s^*)^2 p + 9s^2(s - 2s^*)\bar{\Theta}^2(s - s^*)^2 = 0$$

If $s \neq 2s^*$, the above quadratic equation becomes as follows.

$$\left\{27(s-s^*)^2 - 16s^{*2}\right\} p^2 - 36s\bar{\Theta}(s-s^*)^2 p + 9s^2\bar{\Theta}^2(s-s^*)^2 = 0$$

Solving this quadratic equation, we have two solutions.

$$p = \frac{3s(s-s^*)\bar{\Theta} \left\{6s(s-s^*)^2 \pm \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}}$$

Numerical examples show that the positive root solution $(p = \frac{3s(s-s^*)\bar{\Theta} \left\{6s(s-s^*)^2 + \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}})$

is so high that the high-quality goods' firm face no demand, and the other negative root solution always bring higher profits to the firm. Therefore, the solution, which maximizes profits of the high-quality goods' firm is the negative root solution:

$$p = \frac{3s(s-s^*)\bar{\Theta} \left\{6s(s-s^*)^2 - \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}} \quad (3.24)$$

The Nash equilibrium price of the low quality products is

$$p^* = \frac{s^* p}{3s} = \frac{s^* \bar{\Theta} \left\{6s(s-s^*)^2 - \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}} \quad (3.25)$$

The income of marginal consumers is

$$\tilde{\theta} = \frac{p - p^*}{s - s^*} = \frac{(3s - s^*)\bar{\Theta} \left\{6s(s-s^*)^2 - \sqrt{9(s-s^*)^2 + 16s^{*2}}\right\}}{27(s-s^*)^2 - 16s^{*2}} \quad (3.26)$$

Chapter 4

Consumer Utility, Vertical Trade, and the Replacement of Superiors by Inferiors in the Quality-ladder Model

-Anti-Schumpeterian Results in the Schumpeterian Model-

4.1 Introduction

Innovations improve the quality of goods constantly. In the process of the development, we observe that high quality goods drive out low quality goods. However, in these days, we also observe the opposite in the developed country: high quality goods in developed countries exit the market, while lower-quality yet inexpensive imports from developing countries enter the market and attract consumers in the developed country. It suggests that the new replacement of superiors by inferiors, where low-quality inexpensive imports drive out high-quality expensive goods produced in the developed country may be happening now. The question is how and why the new replacement of superiors by inferiors happens NOW. This paper answers the question by using a resource-consuming quality ladder model: the innovation of the next highest quality good requires more resources, while the utility of consumers increase with the product quality at a decreasing rate.

The concavity of consumer utility plays little role under the autarky, while it becomes important after trade. Under autarky, a typical Shumpeterian creative destruction takes place: every time a new highest quality good arrives, the firm producing the lowest quality good exit the market and the economy devotes more resource to innovate the next highest quality goods, which drives up the wage. Without trade, only expensive high-quality goods are distributed in the developed country in the advanced development stage, while only inexpensive lower quality goods are distributed in the developing country in the early development stage. However, once two countries in different development stages start free trade, consumers, specifically the concavity of their utility functions, and the development stage of the trade partner affect the survival of firms. If the consumer utility increases with the quality at a mildly decreasing rate (a mild concave utility function), no replacement of superiors by inferiors will be realized: high-quality products will drive low-quality products out of the market as Gabszewicz et.al (1981) predict. However, if the consumer utility increases with the quality at a severely decreasing rate (a severe concave utility function), the replacement of superiors by inferiors will be realized: low-quality inexpensive goods from the developing country attract consumers, and firms producing high-quality expensive goods in the developed country will be extinguished by firms producing low-quality inexpensive goods in the developing country. Numerical results show that firms from the developing country drive out all but the firm producing the highest quality good if the developed country start trade with a least developed country. If the trade partner is a middle-developed country, all firms in the developed country exit the market and all firms from the middle-developed country survive.(perfect replacement of superiors by inferiors). For, with a strongly concave utility function, the quality advantage of firms in the developed country is not large enough to cancel their price disadvantage.

Since consumers receive benefit of buying inexpensive imports without downgrading their consumption so much, numerical results show that trading with a middle-developed country improves

the utility of consumers at any income level more than trading with a least-developed country does while no producer prefers a middle-developed country as a trade partner.

The basic closed economy model is presented in section 4.2, and the timing of opening trade on the economy is discussed in section 4.3.

4.2 The Quality Ladder Model

The Economy, Technological Progress and the Labor Market

Consider a closed economy where two sectors produce two types of goods: one sector produces differentiated goods (X), and the other sector produces numeraire goods, the composite of homogeneous consumption goods (Y). Both sectors produce goods by using labor only. Labor is mobile between sectors, but labor is immobile between countries. Consumption goods sector (Y) is a mature industry, and no new technology will arrive. These consumption goods are produced with the decreasing returns to scale technology.

$$F(L) = 2\phi s_t L^{\frac{1}{2}}$$

where ϕs_t is the technology spillover from the other sector. ($\phi > 0$) When a new good of quality (s_t) (such as semiconductor, LCD panel, etc) is invented in the other sector (X), the productivity of the sector (Y) will increase. The production function is the neoclassical production function: $F'_Y(L_Y) > 0$, $F''_Y(L_Y) < 0$, $\lim_{L_Y \rightarrow 0} F'_Y(L_Y) = \infty$, and $\lim_{L_Y \rightarrow \infty} F'_Y(L_Y) = 0$.

The sector Y is in the perfect competition, and every firm in the sector maximizes the following profit function:

$$Max \Pi_y = 2\phi s_t L^{\frac{1}{2}} - w_t L - f$$

The F.O.C. determines the labor demand of the sector Y .

$$L_Y^D = \left(\frac{\phi s_t}{w_t} \right)^2 \quad (4.1)$$

The other sector, the vertically differentiated goods sector, X is a growth sector, and new technology improves the quality of the product constantly. The process of the technological development is exogenous, and the technological progress is the quality-ladder type: the exogenous innovation arrives constantly and improves the quality of the product at a constant rate λ . Thus, the quality of the t -th generation product becomes

$$s_t = s_0 \lambda^t$$

where s_0 is the quality of the first-generation good.

Every vertically differentiated good is produced with the increasing returns to scale technology and it maximizes the profit function below:

$$\pi_j = (p_j - \beta w_t) Q_j(L_j) - f \quad \forall j \in X \quad (4.2)$$

where Q_j is the quantity of a vertically differentiated good j , $\beta > 0$ denotes constant marginal cost, and f is a negligible amount of a small fixed cost. The labor requirement for production in the sector X is given by

$$L_X^D = \sum_{j \in X} (\beta Q_j) \quad (4.3)$$

In addition to production, every time the new technology arrives, the growth sector X devotes labor to introduce the new technology. The labor requirement for introducing the new technology increases with quality at an increasing rate. Specifically, the labor requirement for introducing the new t -th generation good is given by

$$L_S^D = \psi s_t \quad (4.4)$$

This Equation maps quality to cost in labor units, thus s becomes the index of cost of added quality. Eq (4.1), (4.3), and (4.4) give the aggregate labor demand in the economy.

$$L^D = \left(\frac{\phi s_t}{w_t} \right)^2 + \beta N + \psi s_t \quad (4.5)$$

Where N denotes the number of consumers, who are assumed buy one unit of a vertically differentiated good at most.(unit demand) Thus,the total variable cost of producing differentiated goods becomes βN if every consumer buys a single unit of a differentiated good. Since the vertically-differentiated goods sector requires more labor input to introduce the new good, and the technical spillover from the sector X to the other sector Y also increases the labor demand, the labor demand expands every period (Figure 4.1).¹

Unlike the labor demand, the labor supply is assumed to be fixed in every period. The population size does not change, and all consumers supply their labor endowment inelastically. Each worker is endowed with different skill(labor productivity), and their labor productivity (skill) is uniformly distributed over the range $\theta \in [\underline{\Theta}, \bar{\Theta}]$ Thus, the aggregate labor supply in the economy is constant at

$$\begin{aligned} L^S &= \int_{\underline{\Theta}}^{\bar{\Theta}} \theta d\theta \\ &= \frac{(\bar{\Theta}^2 - \underline{\Theta}^2)}{2} \end{aligned} \quad (4.6)$$

Eq (4.5) and eq (4.6) give us the equilibrium wage per unit of labor productivity (skill).

$$w_t = \frac{\phi s_t}{\sqrt{\left(\left(\frac{\underline{\Theta} + \bar{\Theta}}{2} \right) - \beta \right) N - \psi s_t}} \quad (4.7)$$

Eq (4.7) implies that a worker(consumer) with skill θ_i earns $w_t \theta_i = \frac{\phi s_t \theta_i}{\sqrt{\left(\left(\frac{\underline{\Theta} + \bar{\Theta}}{2} \right) - \beta \right) N - \psi s_t}}$ when the economy produces the $t - th$ generation product.

¹In every period, the total labor demand in the economy increases by $2 \frac{\phi s_t}{w_t}^2 \left(\frac{\Delta s_t}{s_t} - \frac{\Delta w_t}{w_t} \right) + \psi s_t \frac{\Delta s_t}{s_t}$

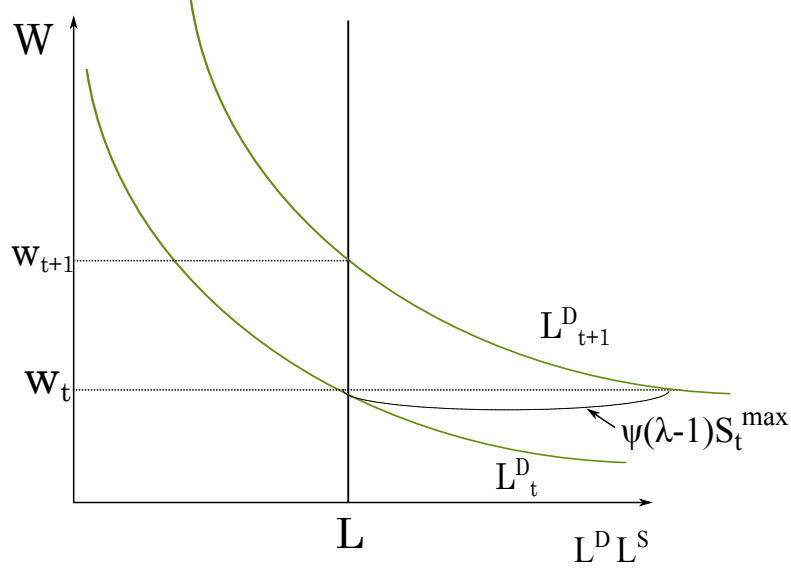


Figure 4.1: The Labor Market

Consumers

Except their productivity (and income), all consumers are identical, and they share the same utility function.

$$U(x, y) = y^{1-\gamma} u_x^\gamma \quad (4.8)$$

where $\gamma > 0$ becomes the share of income spent on the differentiated good. Eq 4.8 shows that the utility of consumers depends on the consumption of the good Y and the sub-utility u_x from the consumption of good X . The sub-utility increases with the quality of the good X but decreases with the price. Specifically,

$$u_x = \alpha w_t \theta_i s_j^\mu - p_j \quad (4.9)$$

where s^μ ($0 < \mu \leq 1$) is the utility value of s the index of the cost of added quality. μ measures the concavity of utility with respect to quality: the lower the μ , the more concave the utility becomes.

The income of the consumer i , $w_t\theta_i$ enters the sub-utility function (4.9) to reflect that high-income consumers value the quality of products more than low-income consumers do.

Every consumer faces a two-stage game. In the first stage, each consumer allocates his income between the vertically differentiated good X and other consumption goods Y . In the second stage, he buys at most one unit of a vertically differentiated good, which maximizes his sub-utility. Thus, the consumer i 's individual demand for the differentiated good x_j is given by

$$x_j = \begin{cases} 1 & \text{if } \alpha w_t \theta_i s_j^\mu - p_j \geq \alpha w_t \theta_i s_k^\mu - p_k \quad \forall k \in J \\ 0 & \text{otherwise} \end{cases}$$

A consumer ($\tilde{\theta}_j$) is indifferent between the good j and the lower-quality good $j - 1$ if and only if both goods give them the same utility level:

$$\alpha w_t \tilde{\theta}_j s_j - p_j = \alpha w_t \tilde{\theta}_j s_{j-1} - p_{j-1}$$

Rearranging the above equation, the skill (the labor productivity) of marginal consumers who regard the good j and the lower quality good $j - 1$ indifferent is given by

$$\tilde{\theta}_j = \frac{p_j - p_{j-1}}{\alpha w_t (s_j^\mu - s_{j-1}^\mu)} \quad (4.10)$$

Similarly, the skill (the labor productivity) of marginal consumers who regard the good j and the higher-quality good $j + 1$ indifferent is given by

$$\tilde{\theta}_j = \frac{p_{j+1} - p_j}{\alpha w_t (s_{j+1}^\mu - s_j^\mu)} \quad (4.11)$$

With the unit-demand assumption, the aggregate demand for the good j , Q_j is the number of consumers with labor productivity $\theta \in [\tilde{\theta}_j, \tilde{\theta}_j]$:

$$\begin{aligned}
Q_j &= [\tilde{\theta}_j - \tilde{\theta}_j] \\
&= \frac{1}{\alpha w_t} \left(\frac{p_{j+1} - p_j}{s_{j+1}^\mu - s_j^\mu} - \frac{p_j - p_{j-1}}{s_j^\mu - s_{j-1}^\mu} \right)
\end{aligned} \tag{4.12}$$

Firms

Each Firm, which produces a vertically differentiated good with an increasing returns to scale technology faces a two-stage game. In the first stage, a new highest-quality good arrives, and every existing firm decides whether to stay or exit the market. Firms exit the market if and only if their expected profit in the coming period is negative. In the second stage, all remaining firms join the Bertrand competition and set prices simultaneously given prices of other firms.

Solving backward, in the second stage of the game, each firm sets the price to maximize the profit function below.²

$$\pi_j(L_j) = (p_j - \beta w_t) \left(\frac{p_{j+1} - p_j}{s_{j+1}^\mu - s_j^\mu} - \frac{p_j - p_{j-1}}{s_j^\mu - s_{j-1}^\mu} \right) - f \quad \forall j \in X$$

Since the technological progress constantly improves the quality of the vertically good x_j by $\lambda\%$, we have the F.O.C below.³

$$p_j = \frac{1}{2} \left\{ \beta w_t + \left(\frac{\lambda^\mu}{\lambda^\mu + 1} \right) p_{j-1} + \left(\frac{1}{\lambda^\mu + 1} \right) p_{j+1} \right\} \tag{4.13}$$

The above F.O.C shows that the price depends on the marginal cost βw_t at time t and the weighted average of prices of rival firms (strategic complement).⁴

²This profit function combines the profit function in eq (4.2) with the aggregate demand in eq (4.12).

³The quality of next higher quality good $j + 1$ is $s_{j+1} = \lambda s_j$

⁴The weight depends on the quality ladder λ . The larger the quality difference λ , the more weight is put on the price of lower-quality goods.

Similarly, the firm producing the highest-quality good faces the exogenous upper bound, the exogenous highest skill, $\bar{\Theta}$ and maximizes the profit function below:

$$\begin{aligned}\pi_{max} &= (p_{max} - \beta w_t) \left\{ \bar{\Theta} - \tilde{\Theta}_{max-1} \right\} - f \\ &= (p_{max} - \beta w_t) \left\{ \bar{\Theta} - \frac{p_{max} - p_{max-1}}{s_{max}^\mu \left(1 - \frac{1}{\lambda^\mu}\right)} \right\} - f\end{aligned}\quad (4.14)$$

FOC for the firm producing the highest quality good is given by

$$p_{max} = \frac{1}{2} \left\{ \beta w_t + p_{max-1} + s_{max}^\mu \left(1 - \frac{1}{\lambda^\mu}\right) \right\} \quad (4.15)$$

As shown in eq (4.15), the firm producing the highest quality goods raises its price as its marginal cost, the price of its rival firm, or the quality difference (λ) increases.

Like the firm producing the highest quality product, the firm producing the lowest quality good s_{min} may face an exogenous market limit. Its lower bound is the higher of two values: the lowest skill in the economy $\underline{\Theta}$ (exogenous variable) or the skill of marginal consumers who are indifferent between buying the lowest-quality good and buying nothing $\frac{p_{min}}{s_{min}^\mu}$. (i.e. $\tilde{\Theta}_{min} = \max\left\{\underline{\Theta}, \frac{p_{min}}{s_{min}^\mu}\right\}$) The firm producing the lowest-quality good maximizes its profit as if its lower boundary were endogenous

$\frac{p_{min}}{s_{min}^\mu}$.

$$\begin{aligned}\pi_{min} &= (p_{min} - \beta w_t) \left\{ \tilde{\Theta}_{min} - \tilde{\Theta}_{min} \right\} - f \\ &= (p_{min} - \beta w_t) \left\{ \frac{p_{min+1} - p_{min}}{s_{min+1}^\mu \left(1 - \frac{1}{\lambda^\mu}\right)} - \frac{p_{min}}{s_{min}^\mu} \right\} - f\end{aligned}\quad (4.16)$$

FOC for the firm producing the lowest quality good is given by

$$p_{min} = \frac{1}{2} \left\{ \beta w_t + \frac{p_{min+1}}{\lambda^\mu} \right\} \quad (4.17)$$

The firm producing the lowest quality goods increases its price with its marginal cost and the price of its rival firm, but it lowers its price with the quality difference.

Above reaction functions, (4.13),(4.15) and (4.17) show that firms compete locally:price of one firm depends linearly on price(s) of its rival firm(s) producing one lower-quality goods and/or one higher-quality ones, which is summarized in eq (4.18).⁵

$$\begin{bmatrix}
 1 & \frac{-1}{2\lambda^\mu} & 0 & 0 & \dots & \dots & \dots & \dots & 0 \\
 \frac{-\lambda^\mu}{2(\lambda^\mu+1)} & 1 & \frac{-1}{2(\lambda^\mu+1)} & 0 & \dots & \dots & \dots & \dots & 0 \\
 0 & \frac{-\lambda^\mu}{2(\lambda^\mu+1)} & 1 & \frac{-1}{2(\lambda^\mu+1)} & 0 & \dots & \dots & \dots & 0 \\
 0 & 0 & \frac{-\lambda^\mu}{2(\lambda^\mu+1)} & 1 & \frac{-1}{2(\lambda^\mu+1)} & 0 & \dots & \dots & 0 \\
 \vdots & & & & & & & & \vdots \\
 \vdots & & & & & & & & \vdots \\
 0 & \dots & \dots & \dots & \dots & 0 & \frac{-\lambda^\mu}{2(\lambda^\mu+1)} & 1 & \frac{-1}{2(\lambda^\mu+1)} \\
 0 & \dots & \dots & \dots & \dots & \dots & 0 & \frac{-1}{2} & 1
 \end{bmatrix}
 \begin{bmatrix}
 p_1 \\
 p_2 \\
 p_3 \\
 p_4 \\
 \vdots \\
 \vdots \\
 p_{max-1} \\
 p_{max}
 \end{bmatrix}
 =
 \begin{bmatrix}
 \frac{\beta w_t}{2} \\
 \frac{\beta w_t}{2} \\
 \frac{\beta w_t}{2} \\
 \frac{\beta w_t}{2} \\
 \vdots \\
 \vdots \\
 \frac{\beta w_t}{2} \\
 \frac{\beta w_t}{2} + \frac{\alpha s_{max}^\mu (1 - \frac{1}{\lambda^\mu}) \bar{\Theta}}{2}
 \end{bmatrix}
 \quad (4.18)$$

The Bertrand equilibrium is the solution of the eq (4.18). For instance, if six different-quality goods, the first-generation (s_1) to the sixth-generation goods (s_6), compete in the economy, the Bertrand equilibrium is given as follows:

$$\begin{bmatrix}
 p_1 \\
 p_2 \\
 p_3 \\
 p_4 \\
 p_5 \\
 p_6
 \end{bmatrix}
 = \frac{w_t}{\lambda}
 \begin{bmatrix}
 \lambda^{-\mu} \left\{ \alpha \bar{\Theta} s_0^\mu \lambda^{5\mu} (\lambda^\mu - 1) + \beta (32\lambda^{5\mu} + 112\lambda^{4\mu} + 186\lambda^{3\mu} + 177\lambda^{2\mu} + 96\lambda^\mu + 24) \right\} \\
 2\alpha \bar{\Theta} s_0^\mu \lambda^{5\mu} (\lambda^\mu - 1) + 3\beta (16\lambda^{4\mu} + 52\lambda^{3\mu} + 73\lambda^{2\mu} + 52\lambda^\mu + 16) \\
 \alpha \bar{\Theta} s_0^\mu \lambda^{5\mu} (4\lambda^{2\mu} - \lambda^\mu - 3) + \beta (56\lambda^{4\mu} + 172\lambda^{3\mu} + 222\lambda^{2\mu} + 141\lambda^\mu + 36) \\
 2\alpha \bar{\Theta} s_0^\mu \lambda^{5\mu} (4\lambda^{3\mu} + 2\lambda^{2\mu} - 3\lambda^\mu - 3) + 3\beta (20\lambda^{4\mu} + 59\lambda^{3\mu} + 73\lambda^{2\mu} + 45\lambda^\mu + 12) \\
 \alpha \bar{\Theta} s_0^\mu \lambda^{5\mu} (16\lambda^{4\mu} + 20\lambda^{3\mu} - 3\lambda^{2\mu} - 21\lambda^\mu - 12) + \beta (62\lambda^{4\mu} + 178\lambda^{3\mu} + 216\lambda^{2\mu} + 135\lambda^\mu + 36) \\
 2\alpha \bar{\Theta} s_0^\mu \lambda^{5\mu} (16\lambda^{5\mu} + 32\lambda^{4\mu} + 15\lambda^{3\mu} - 21\lambda^{2\mu} - 30\lambda^\mu - 12) + 3\beta (21\lambda^{4\mu} + 59\lambda^{3\mu} + 72\lambda^{2\mu} + 45\lambda^\mu + 12)
 \end{bmatrix}$$

⁵These firms have strategic complementary relationships.

where $\Lambda = 64\lambda^{4\mu} + 176\lambda^{3\mu} + 216\lambda^{2\mu} + 135\lambda^\mu + 36$

Closed Economy Equilibrium

The solution to the matrix (4.18) determines the equilibrium price in the economy.⁶

Once Bertrand Equilibrium prices are determined, profits of each firm (4.2), (4.14), (4.16) is determined:

$$\pi_j(L_j) = \begin{cases} (p_{min} - \beta w_t) \left\{ \frac{p_{min+1} - p_{min}}{s_{min+1}^\mu \left(1 - \frac{1}{\lambda^\mu}\right)} - \frac{p_{min}}{s_{min}^\mu} \right\} - f & (4.16) \quad \text{if } j = min \\ (p_{max} - \beta w_t) \left\{ \bar{\Theta} - \frac{p_{max} - p_{max-1}}{s_{max}^\mu \left(1 - \frac{1}{\lambda^\mu}\right)} \right\} - f & (4.14) \quad \text{if } j = max \\ (p_j - \beta w_t) \left(\frac{p_{j+1} - p_j}{s_{j+1}^\mu - s_j^\mu} - \frac{p_j - p_{j-1}}{s_j^\mu - s_{j-1}^\mu} \right) - f & (4.2) \quad \text{otherwise} \end{cases}$$

The Equilibrium wage per unit of skill is determined to clear the labor market:

$$w_t = \frac{\phi s_t}{\sqrt{\left(\frac{\Theta + \bar{\Theta}}{2} - \beta\right) N - \psi s_t}} \quad (4.7)$$

Numerical Example in the Closed Economy

This section assigns numerical values to exogenous variables (Table 4.1), and shows how the technological progress replaces obsolete goods with new higher quality goods (Shumpeterian creative destruction), while the resource-consuming technological progress drives up the labor demand and wage.

⁶Of course, the number of firms in the equilibrium is determined by exogenous variables such as the distribution of skill, the quality ladder λ , other parameters such as consumers' quality discount parameter μ .

$\lambda = 1.2$	Every innovation improves the highest quality goods by 20 %
$s_0 = 10$	Quality of the first-generation good s_0 is 10. Thus, the quality of the t-th generation goods $s_t = s_0\lambda^t = 10(1.2)^t$
$\theta \in [50, 700]$	Lowest productivity is $\underline{\Theta} = 50$ and the highest one is $\bar{\Theta} = 700$ Labor productivity of a worker is uniformly distributed over the range.
$\beta = 1$	One unit of labor input produces one unit of any vertically differentiated good. The marginal cost of any differentiated good = w_t
$\alpha = 0.2$	Income of individuals ($w_t\theta_i$) is scaled down to $(0.2w_t\theta_i)$ and mapped to the utility value of the added quality.
$\mu = 0.002$	Consumer Utility level w.r.t quality is strictly concave $s_j^\mu = s_j^{0.002}$ the utility value of added quality decreases at an increasing rate.
$\phi = 10$	Technological progress in the X sector improves the productivity in the Y sector by $\phi s_t = 10s_t$,
$\psi = 30$	The labor requirement for developing the next highest quality good is $\psi s_t = 30s_t$

Table 4.1: Exogenous Variables in the Numerical Example

Without trade, as shown in table 4.2, the competition for higher quality escalates, and the Shumpeterian creative destruction repeats within a country. During the repetition of the Shumpeterian creative destructions, more and more labor force is devoted to both sectors, and the wage level increases more and more. As a result, only expensive and high-quality goods survive in the developed country. On the other hand, the developing country in the early development stage has not devoted much labor resource into the improvement in quality yet, and accordingly only low-quality yet inexpensive goods are distributed.

Table 4.2 shows the variety of goods available in the economy, prices of these goods, the market share of these goods at each stage of technological progress.

For instance, when the highest quality good in a country is $s_5 = s_0\lambda^5$ (the third row in Table 4.2), the wage rate $w_5 = 0.505$ (=the marginal cost), three products s_3, s_4 , and s_5 are distributed in the economy, lower quality goods s_2 has just exited the market, and even lower quality goods s_1 have already exited it. The lowest quality good in the economy s_3 is sold to consumers in $\theta \in [50, 60.4]$

at the price $p_3 = 0.508$. The utility of these consumers ranges from $u_x \in [4.576, 5.633]$, while the producer of s_3 earns $\pi_3 = 0.03$. The second lowest quality good in the economy s_4 is sold to consumers in $\theta \in [60.4, 292.1]$ at the price $p_4 = 0.510$. The utility of these consumers ranges from $u_x \in [5.633, 29.200]$, while the producer of s_4 earns $\pi_4 = 1.16$. The highest quality good in the economy s_5 is sold to high-end consumers in $\theta \in [292.1, 700]$ at the price $p_5 = 0.521$. The utility of these consumers ranges from $u_x \in [29.20, 70.70]$, while the producer of s_5 earns the highest profits among three firms $\pi_5 = 6.53$.

As shown in Table 4.2, without trade, the quality competition among domestic firms will lead to Schumpeterian creative destructions. Every time the new highest-quality good arrives in an economy, the lowest-quality good in the economy exits the market. (In the particular numerical example, in every period, only top 3 goods survive in the economy.) As innovation improves the product quality, it uses more resources, improve labor productivity, and drives up wage. As a result, in a developed country in an advanced development stage, its wage level is high and only expensive high-quality goods survive in the market. A developed country in the advanced development stage, $t = 30$ for instance, only three expensive high quality goods $s_{28}(p_{28} = 57.49)$, $s_{29}(p_{29} = 57.75)$, and $s_{30}(p_{30} = 58.98)$ survive, and the wage rate is high $w_{30} = 57.3$. On the contrary, in a developing country in the early development stage, $t = 5$ for instance, the wage level is low ($w_5 = 0.51$) and only inexpensive low-quality goods [$s_3(p_3 = 0.508)$, $s_4(p_4 = 0.510)$, and $s_5(p_5 = 0.521)$] are distributed in the country.

Stage t	w	Surviving Firms								
3	w_3	s_1 : market coverage $\theta \in [50, 60.4]$			s_2 : $\theta \in [60.4, 292.1]$			s_3 : $\theta \in [292.1, 700]$		
		p_1	π_1	Consumer Utility	p_2	π_2	Consumer Utility	p_3	π_3	Consumer Utility
	0.35	0.352	0.02	$u_x \in [3, 4]$	0.354	0.93	$u_x \in [4, 20]$	0.361	4.49	$u_x \in [20, 49]$
4	w_4	s_2 : market coverage $\theta \in [50, 60.4]$			s_3 : $\theta \in [60.4, 292.1]$			s_4 : $\theta \in [292.1, 700]$		
		p_2	π_2	Consumer Utility	p_3	π_3	Consumer Utility	p_4	π_4	Consumer Utility
	0.42	0.423	0.02	$u_x \in [4, 5]$	0.425	0.93	$u_x \in [5, 24]$	0.434	5.30	$u_x \in [24, 59]$
5	w_5	s_3 : market coverage $\theta \in [50, 60.4]$			s_4 : $\theta \in [60.4, 292.1]$			s_5 : $\theta \in [292.1, 700]$		
		p_3	π_3	Consumer Utility	p_4	π_4	Consumer Utility	p_5	π_5	Consumer Utility
	0.505	0.508	0.03	$u_x \in [5, 6]$	0.510	1.16	$u_x \in [6, 29]$	0.521	6.53	$u_x \in [29, 71]$
$\dots$										
10	w_{10}	s_8 : market coverage $\theta \in [50, 60.4]$			s_9 : $\theta \in [60.4, 292.1]$			s_{10} : $\theta \in [292.1, 700]$		
		p_8	π_8	Consumer Utility	p_9	π_9	Consumer Utility	p_{10}	π_{10}	Consumer Utility
	1.26	1.266	0.05	$u_x \in [11, 14]$	1.27	2.31	$u_x \in [14, 73]$	1.299	15.5	$u_x \in [73, 177]$
20	w_{20}	s_{18} : market coverage $\theta \in [50, 60.4]$			s_{19} : $\theta \in [60.4, 292.1]$			s_{20} : $\theta \in [292.1, 700]$		
		p_{18}	π_{18}	Consumer Utility	p_{19}	π_{19}	Consumer Utility	p_{20}	π_{20}	Consumer Utility
	7.97	7.999	0.34	$u_x \in [73, 89]$	8.035	16.00	$u_x \in [89, 463]$	8.207	98.30	$u_x \in [463, 1120]$
28	w_{28}	s_{26} : market coverage $\theta \in [50, 60.4]$			s_{27} : $\theta \in [60.4, 292.1]$			s_{28} : $\theta \in [292.1, 700]$		
		p_{26}	π_{26}	Consumer Utility	p_{27}	π_{27}	Consumer Utility	p_{28}	π_{28}	Consumer Utility
	37.5	37.61	1.6	$u_x \in [342, 421]$	37.78	74.4	$u_x \in [421, 2183]$	38.59	461.3	$u_x \in [2183, 5284]$
30	w_{30}	s_{28} : market coverage $\theta \in [50, 60.4]$			s_{29} : $\theta \in [60.4, 292.1]$			s_{30} : $\theta \in [292.1, 700]$		
		p_{28}	π_{28}	Consumer Utility	p_{29}	π_{29}	Consumer Utility	p_{30}	π_{30}	Consumer Utility
	57.3	57.49	2.4	$u_x \in [524, 644]$	57.75	113.8	$u_x \in [644, 3338]$	58.98	705.3	$u_x \in [3338, 8082]$

Table 4.2: Closed Economy Equilibrium

4.3 The Open Economy

Suppose that two countries start free trade. These two countries are exactly the same except their development stage. The stage of development defines the least-developed, the middle-developed, and the developed country. One country (home) is the developed country in the advanced development stage, and only high-quality and expensive goods survive in the market. On the contrary, the other country (foreign) is the least-developed country (the middle-developed country) in the early development (the middle development) stage, and accordingly only low-quality yet inexpensive goods are distributed in the market. The difference in technological development stages results in the difference in wage (w, w^*) , income $(w\theta, w^*\theta)$, and the marginal cost, which affects all firms in the same country $(\beta w, \beta w^*)$. Except the stage of the technological development, two countries are the same :both countries have the same production technology (one unit of any differentiated good is produced with one unit of labor input. That is, $\beta = 1$), the population size in two countries are the same $N = N^*$, the skill of workers (productivity) are uniformly distributed over the same range $\theta \in [\underline{\Theta}, \bar{\Theta}]$, $\theta^* \in [\underline{\Theta}, \bar{\Theta}]$, and consumers(workers) share the same utility function.

Although trade offers consumers a wider variety of goods, consumer's problem does not change after the opening of trade. To maximize his sub-utility (4.9), each consumer chooses the best differentiated good from whatever available in the market and buys at most one unit of the good.

$$x_j = \begin{cases} 1 & \text{if } \alpha w_t \theta_i s_j^\mu - p_j \geq \alpha w_t \theta_i s_k^\mu - p_k \ \forall k \in J \\ 0 & \text{otherwise} \end{cases}$$

As in autarky (4.10), (4.11), marginal consumers, who are indifferent between the good j and another good k ($k < j$) are defined as follows:

The lower/upper limit between any two firms ($j > k$)

$$\tilde{\theta}_{j,k} = \frac{p_j - p_k}{s_j^\mu - s_k^\mu} \quad (4.10)*, \quad (4.11)*$$

The lower limit for the firm producing the lowest quality becomes

$$\tilde{\theta}_{min} = \min \left\{ \underline{\Theta}, \frac{p_{min}}{s_{min}^\mu} \right\} \quad (4.19)$$

The upper limit for the firm producing the highest quality is the highest skill in the economy.

$$\tilde{\theta}_{max} = \bar{\Theta} \quad (4.20)$$

It should be noted that the firm j faces zero demand if any one of the following condition holds.

1. The quality of the good chosen by the lowest-income consumers is higher than that of the good j . Thus, the upper limit of the firm j is lower than the skill of the lowest-income consumers in the economy

$$\tilde{\theta}_{j,j+1} < \underline{\Theta} \quad [Zero\ demand\ condition\ 1]$$

2. The goods chosen by the highest-income consumers is lower than the good j . Thus, the lower limit of firm j is higher than the highest productivity in the economy

$$\bar{\Theta} < \tilde{\theta}_{j-1,j} \quad [Zero\ demand\ condition\ 2]$$

3. The good j will never be the most preferred good to anyone. (as shown in Figure 4.2) No consumer prefers the good j to the lower quality good $j - 1$ or the higher quality good $j + 1$. Thus, the lower limit of the higher quality good $j + 1$ overtakes the lower limit of the good j , and the location of the lower limit of the higher quality good $j + 1$ is lower than that of the good j .

$$\tilde{\theta}_{j-1,j+1} < \tilde{\theta}_{j-1,j} \quad [\text{Zero demand condition 3}]$$

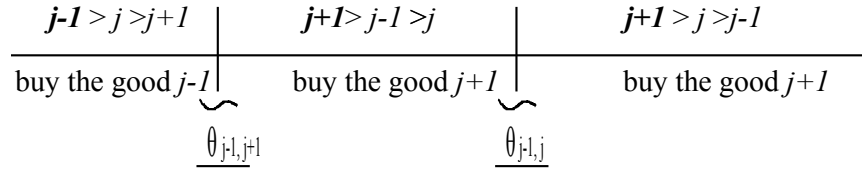


Figure 4.2: Zero Demand Condition 3

Once free trade starts, firms can sell their products in their home market and/or the foreign market. No arbitrage can work between domestic and foreign markets, thus the price discrimination is possible. Since this paper focuses on the market in the developed country, the following sections discuss the developed country only.

The game after trade is basically the same as that under autarky. In the first stage, each firm decides whether or not join the competition in the domestic market and/or the foreign market. A firm will exit the market if no consumer buys its products even if it discounts its price to the marginal cost. (w_t for the developed country and w_t^* for the developing country) In the second stage, all firms in the market join the Bertrand competition.

Suppose that before free trade starts, $k + 1$ varieties of vertically differentiated goods ($j = 0, 1, \dots, k$) are distributed in each country.⁷ If all firms join the Bertrand competition after trade, firms from the developing country maximize the following profit function in the developed country market.

⁷ $s = s_{t-k}, s_{t-(k-1)}, \dots, s_t$ $s^* = s_{t^*-k}^*, s_{t^*-(k-1)}^*, \dots, s_{t^*}^*$

Profit functions of firms from the developing country

$$\pi_j^* = \begin{cases} \left(p_j^* - \beta w_{t^*} \right) \left\{ \tilde{\theta}_{j,j+1} - \underline{\theta}_j \right\} - f & (4.16)* \quad \text{if } j = t^* - k \\ \left(p_j^* - \beta w_{t^*} \right) \left\{ \tilde{\theta}_{j,j+1} - \underline{\theta}_{j-1,j} \right\} - f & (4.2)* \quad \text{if } j = t^* - k + 1, t^* - k + 2, \dots, t^* \end{cases}$$

Profit functions of firms from the developed country

$$\pi_j = \begin{cases} \left(p_j - \beta w_t \right) \left\{ \tilde{\theta}_{j,j+1} - \underline{\theta}_{j-1,j} \right\} - f & (4.2) \quad \text{if } j = t - k, t - k + 1, \dots, t - 1 \\ \left(p_j - \beta w_t \right) \left\{ \tilde{\Theta} - \underline{\theta}_{j-1,j} \right\} - f & (4.14) \quad \text{if } j = t \end{cases}$$

where $\tilde{\theta}, \underline{\theta}$ are the skill (productivity) of marginal consumers, derived from (4.10), (4.19), (4.11) and (4.20).

Differentiating the above profit functions with respect to its price, we have the FOCs:

$$p_j^* = \begin{cases} \frac{1}{2} \left\{ \beta w_{t^*} + \frac{p_{j+1}^*}{\lambda^\mu} \right\} & (4.17)* \quad \text{if } j = t^* - k \\ \frac{1}{2} \left\{ \beta w_t^* + \left(\frac{\lambda^\mu}{\lambda^\mu + 1} \right) p_{j-1}^* + \left(\frac{1}{\lambda^\mu + 1} \right) p_{j+1}^* \right\} & (4.13)* \quad \text{if } j = t^* - k + 1, t^* - k + 2, \dots, t^* - 1 \\ \frac{1}{2} \left\{ \beta w_{t^*} + \lambda^\mu \left(\frac{\lambda^{\gamma\mu} - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t^*-1}^* + \left(\frac{\lambda^\mu - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t-k} \right\} & (4.21)* \quad \text{if } j = t^* \end{cases}$$

where $\gamma \equiv (t - k) - t^*$ denotes the difference in quality between the lowest quality good in the

developed country and the highest quality good in the developing country.⁸⁹

$$p_j = \begin{cases} \frac{1}{2} \left\{ \beta w_t + \lambda^{\gamma\mu} \left(\frac{\lambda^\mu - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t^*}^* + \left(\frac{\lambda^{\gamma\mu} - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t-k+1} \right\} & (4.22) \quad \text{if } j = t - k \\ \frac{1}{2} \left\{ \beta w_t + \left(\frac{\lambda^\mu}{\lambda^\mu + 1} \right) p_{j-1} + \left(\frac{1}{\lambda^\mu + 1} \right) p_j \right\} & (4.13) \quad \text{if } j = t-k+1, t-k+2, \dots, t-1 \\ \frac{1}{2} \left\{ \beta w_t + p_{t-1} + s_t^\mu \left(1 - \frac{1}{\lambda^\mu} \right) \right\} & (4.15) \quad \text{if } j = t \end{cases}$$

The open economy Bertrand equilibrium is the solution of linear relationship among firms in the market (4.17)*,(4.13)*,(4.13),(4.21),(4.22),and (4.15)

8

$$p_{t^*}^* = \frac{1}{2} \left\{ \beta w_{t^*} + \lambda^\mu \left(\frac{\lambda^{\gamma\mu} - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t^*-1}^* + \left(\frac{\lambda^\mu - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t-k} \right\}^* \quad (4.21)$$

9

$$p_{t-k} = \frac{1}{2} \left\{ \beta w_t + \lambda^{\gamma\mu} \left(\frac{\lambda^\mu - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t^*}^* + \left(\frac{\lambda^{\gamma\mu} - 1}{\lambda^{(\gamma+1)\mu} - 1} \right) p_{t-k+1} \right\} \quad (4.22)$$

$$\begin{bmatrix}
1 & \frac{-1}{2\lambda^\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-\lambda^\mu}{2(\lambda^\mu+1)} & 1 & \frac{-1}{2(\lambda^\mu+1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & & \ddots & & & & & & \vdots & \\
0 & 0 & \frac{-\lambda^\mu(\lambda^\gamma-1)}{\lambda(\gamma+1)\mu-1} & 1 & \frac{-(\lambda^\mu-1)}{\lambda(\gamma+1)\mu-1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-\lambda^{\gamma\mu}(\lambda^\mu-1)}{\lambda(\gamma+1)\mu-1} & 1 & \frac{-(\lambda^{\gamma\mu}-1)}{\lambda(\gamma+1)\mu-1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-\lambda^\mu}{2(\lambda^\mu+1)} & 1 & \frac{-1}{2(\lambda^\mu+1)} & 0 & 0 & 0 \\
\vdots & & & & & & \ddots & & \vdots & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} & 1 & 0
\end{bmatrix}
\begin{bmatrix}
p_{t^*-k} \\
p_{t^*-k+1} \\
\vdots \\
p_{t^*} \\
p_{t-k} \\
p_{t-k+1} \\
\vdots \\
p_t
\end{bmatrix}
=
\begin{bmatrix}
\frac{\beta w_{t^*}}{2} \\
\frac{\beta w_{t^*}}{2} \\
\vdots \\
\frac{\beta w_{t^*}}{2} \\
\frac{\beta w_t}{2} \\
\frac{\beta w_t}{2} \\
\vdots \\
\frac{\beta w_t}{2} + \Omega
\end{bmatrix}
\quad (4.23)$$

where $\Omega \equiv \frac{\alpha s_h^\mu (1 - \frac{1}{\lambda^\mu}) \bar{\Theta}}{2}$ and $\gamma \equiv t - k - f$ the quality difference between the highest quality good

in the developed country and the lowest quality good in the developing country.

Numerical Examples in an Open Economy: Replacement of Superiors by Inferiors

Consider a developed country, where the 30-th generation good has been invented and a least developed country, where the 5-th generation product has just been invented (a middle developed country, where the 13-th generation product has invented), start free trade. Using the same exogenous values (Table 4.1) as those used in the previous section, the 3rd, the 4th, and the 5th generation goods are in the least developed country (the 11-th, the 12-th, and the 13-th generation goods are in the middle developed country), and the 28th, the 29th and the 30th generation goods are in the developed country.

This section focuses the market in the developed country and numerically shows that the replacement of superiors by inferiors will actually happen if the utility function of consumers is strictly

concave with respect to quality and the utility value of the added quality decreases at an increasing rate. Whether or not all firms in the developed country are extinguished by firms from the developing country depends on the trade partner. If the utility gain of the added quality decreases at an increasing rate (strictly concave utility function), and the trade partner is the least developed country in the very early development stage, only the producer of the highest quality good in the developed country barely survives. However, all firms in the developed country exit the market if the utility function of consumers is strictly concave and the trade partner is the middle developed country in the middle development stage. Numerical results also show that all consumers prefer the middle developed country as a trade partner to the least developed country, while no firm in the developed country prefer the middle developed country as a trade partner.

Baseline Case: Mildly Concave Utility Function $\mu = 0.4$, $s^\mu = s^{0.4}$ and Trade with a Least Developed Country $t = 5$

First, as a benchmark case, this section discusses the case, where the utility gain of the added quality decreases at a moderate increasing rate and the utility function of consumers is mildly concave. Immediately after the opening of trade, six varieties of goods ($s^{*3}, s^{*4}, s^{*5}, s^{28}, s^{29}, s^{30}$) are in the developed country. If all of these six firms compete in the developed country, the Bertrand competition will determine the market coverage of each firm (the location of marginal consumers) shown in the upper half of the Table 4.3. As shown, if the utility of consumers increases with the quality at a mildly decreasing rate (the utility gain of consumers decreases at a mildly increasing rate), the location of marginal consumers of all firms from the developing country is lower than the lowest income in the developed country. That is, all three firms of the developing country face

zero demand.¹⁰ Even if all these three firms in the developing country set their prices at their marginal cost βw_{t^*} , the lowest income of consumers who choose their products are lower than the lowest income in the developed country and firms of the developing country still face zero demand as shown in the bottom half of the Table 4.3. As a result, no firm in the developing country will enter the developed country market in the first stage of the game.

Mildly Concave Utility Function $\mu = 0.4$: $t^* = 5, t = 30$				
$\tilde{\theta}_{3,4}$	$\tilde{\theta}_{4,5}$	$\tilde{\theta}_{5,28}$	$\tilde{\theta}_{28,29}$	$\tilde{\theta}_{29,30}$
$\underline{\Theta} - 49.5$	$\underline{\Theta} - 47.8$	$\underline{\Theta} - 44.7$	$\underline{\Theta} + 14.8$	$\underline{\Theta} + 244.6$

Mildly Concave Utility $\mu = 0.4$: $t^* = 5, t = 30$ s_3^*, s_4^* , and s_5^* are sold at its marginal cost βw_{t^*}				
$\tilde{\theta}_{3,4}$	$\tilde{\theta}_{4,5}$	$\tilde{\theta}_{5,28}$	$\tilde{\theta}_{28,29}$	$\tilde{\theta}_{29,30}$
-	-	$\underline{\Theta} - 44.6$	$\underline{\Theta} + 14.8$	$\underline{\Theta} + 244.6$

Table 4.3: The Location of Marginal Consumers in a Mildly Concave Utility Function Case

Since there is no entry from the developing country, no change will be brought to the developed country after trade. All three firms in the developed country will not change their prices after trade, maintain the same market share, as shown in Table 4.2. In this case, Schumpeterian creative destructions in the developed country will be repeated forever.

Case of Strictly Concave Utility Function: $\mu = 0.002$, $s^\mu = s^{0.002}$

Suppose that the utility function of consumers is strictly concave and the quality of a product increases the utility of consumers at a rapidly decreasing rate. This section shows the strictly

¹⁰As shown in the upper half of the Table 4.3, the location of marginal consumers for firms in the developing country are below the lowest labor productivity $\underline{\Theta}$.

concave utility function of consumers will cause an anti-Schumpeterian creative destruction, the replacement of superiors by inferiors if two countries in different development stages start trade. Numerical examples show that low-quality inexpensive goods from the developing country will extinguish some or all of high-quality expensive goods in the developed country. How many high-quality goods in the developed country survive after the start of trade depends on the trading partner. This section uses numerical example and shows that if the trade partner is a least developed country in the early development stage ($s^* \leq 12$), the highest quality good (s_{30}) will barely survive but other two goods in the developed country exit the market. If the trade partner is change to in the middle developed country in the middle development stage ($s^* > 12$), the quality advantage of goods produced in the developed country is not sufficient enough to cover the price disadvantage, and all goods produced in the developed country are extinguished by lower quality goods produced in the developing country, and Schumpeterian creative destructions will finally stop in the developed country.

Case 1: Strictly Concave Utility Function $\mu = 0.002$ and Trade with a Least Developed Country: $t^* \leq 12$

Given that the utility function of consumers is strictly concave with respect to the quality of products ($s^{0.002}$), two countries, a developed country in the advanced development stage ($t = 30$) and a developing country in the early development stage ($t^* = 5$) start trade. If six firms ($s_3^*, s_4^*, s_5^*, s_{28}, s_{29}, s_{30}$) join the Bertrand competition, as the upper half of Table 4.4 shows, two producers of the lowest and the second lowest quality goods in the developed country (s_{28}, s_{29}) and a producer of the lowest quality good in the developing country (s_3^*) face zero demand: the *Zero Demand Condition 3* holds for two firms in the developed country (s_{28}, s_{29}) and the *Zero Demand*

Condition 1 holds for a firm in the developing country (s_3^*), and thus they face zero demand.¹¹ If the firm in the developing country s_3^* sets its own marginal cost βw_{t^*} , as shown in the bottom of Table 4.4, the highest income of consumers, who buys s_3 is higher than the lowest income in the developed country, which means that it will now face positive demand. However, other two firms in the developed country (s_{28}, s_{29}) will still face zero demand even if they set their prices at their own marginal cost βw_t , which is much higher than the marginal cost in the developing country.¹²

Thus, when two countries, a developed country in an advanced development stage ($t = 30$) and a least developed country in an early development stage ($t^* = 5$) start trade, two firms producing relatively high-quality goods (s_{28} and s_{29}) in the developed country will exit the market in the first stage of the game. As a result, in this case, free trade will polarize goods in the market: three firms producing low quality goods in the developing country (s_3^*, s_4^*, s_5^*) and one firm producing the highest quality good in the developed country (s_{30}) will survive and join the Bertrand competition in the second stage.

Strictly Concave Utility $\mu = 0.002$: $t^* = 5, t = 30$				
$\tilde{\theta}_{3,4}$	$\tilde{\theta}_{4,5}$	$\tilde{\theta}_{5,28}$	$\tilde{\theta}_{5,29}$	$\tilde{\theta}_{5,30}$
$\underline{\Theta} - 4.5$	$\underline{\Theta} + 177.4$	$\underline{\Theta} + 509.4$	$\underline{\Theta} + 498.1$	$\underline{\Theta} + 489.6$

Strictly Concave Utility $\mu = 0.02$: $t^* = 5, t = 30$ s_3^*, s_{28} , and s_{29} are sold at its marginal cost βw_{t^*} or βw_t				
$\tilde{\theta}_{3,4}$	$\tilde{\theta}_{4,5}$	$\tilde{\theta}_{5,28}$	$\tilde{\theta}_{5,29}$	$\tilde{\theta}_{5,30}$
$\underline{\Theta} + 26.4$	$\underline{\Theta} + 179.2$	-	$\underline{\Theta} + 497.5$	$\underline{\Theta} + 489.6$

Table 4.4: The Location of Marginal Consumers in the Strictly Concave Utility Function Case

The Bertrand equilibrium in this case is shown in Table 4.5. Since firms producing similar

¹¹ s_{28} , and s_{29} face zero demand if $\tilde{\theta}_{5,28} > \tilde{\theta}_{5,29}$ and $\tilde{\theta}_{5,29} > \tilde{\theta}_{5,30}$.

¹² Again, the *Zero Demand Condition 3* holds for s_{29} : $\tilde{\theta}_{5,29} > \tilde{\theta}_{5,30}$

quality goods in the developed country (s_{28}, s_{29}) will exit the market after trade, the firm producing the highest quality good in the developed country (s_{30}) will raise its price ($p_{30} = 58.98 \rightarrow p_{30} = 66.54$). That is because, the opening of trade drives similar quality goods out, and its product is more differentiated from other goods. ($s_3, s_4, \text{ and } s_5$) Due to the price increase, the firm s_{30} will lose a part of its market share ($\theta \in [292.1, 700] \rightarrow \theta \in [612.0, 700]$). Nevertheless, the increase in the profit margin overwhelms the decrease in the market share, and its profit π_{30} will increase after trade. ($\pi_{30} = 705.3 \rightarrow \pi_{30} = 817$)

Strictly Concave Utility Function and Least Developed Trading Partner $\mu = 0.002, t^* = 5, t = 30$							
<u>Firm Exit</u> : Two Domestic Firms: s_{28} and s_{29}							
<u>Surviving Firms</u> : Three Foreign Firms s_3^*, s_4^*, s_5^* & One Domestic Firm s_{30}							
s_3 : market coverage $\theta \in [\underline{\Theta} = 50, 85.5]$		s_4 : $\theta \in [85.5, 256.3]$		s_5 : $\theta \in [256.3, 612.0]$		s_{30} : $\theta \in [612.0, 700.0]$	
p_3^*	π_3^*	p_4^*	π_4^*	p_5^*	π_5^*	p_{30}	π_{30}
0.505 (0.508)	0 (0.03)	0.86 (0.510)	148 (1.16)	1.94 (0.521)	490 (6.53)	66.54 (58.98)	817 (705.3)

Table 4.5: Strictly Concave Utility & Trade with the Least Developed Country: Bertrand Equilibrium

() shows autarky prices or profits.

On one hand, as shown above, all firms surviving in the developed country will gain profit after trade. On the other hand, not all consumers in the developed country will improve their utility after trade. Figure 4.3 shows the change in the consumer utility after trade. The consumer utility will improve for majority of consumers $\theta \in [\underline{\Theta} = 50, 540.47]$. These consumers will improve their utility by buying lower-quality imports at lower prices. However, consumers of the high income (productivity) $\theta \in [540.47, 700]$ will be hurt after trade: some high-income consumers $\theta \in [540.47, 612.05]$ will be hurt by downgrading their consumption (s_{30} to s_5^*). Other high-income consumers $\theta \in [612.05, 700]$ will be hurt by the price increase (p_{30}).

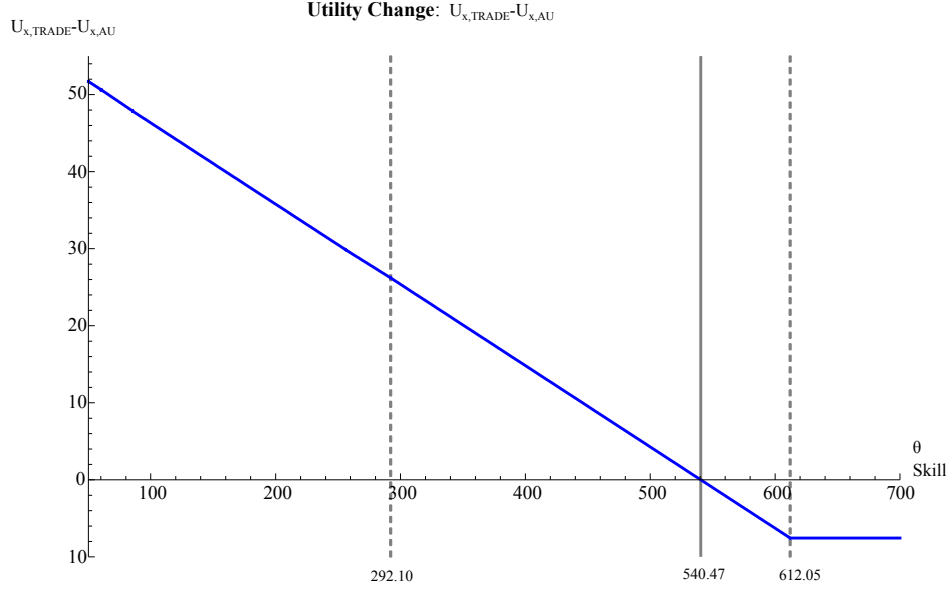


Figure 4.3: Case 1: Strictly Concave Utility $\mu = 0.02$ and Trade with the Least Developed Country $t^* = 5$

Case 2: Strictly Concave Utility Function $\mu = 0.02$ and Trade with a Middle Developed Country: $t^* > 12$ Next, let's change the trading partner from the least developed country ($t^* = 5$) to a middle developed country in the middle development stage ($t^* = 13$). If all six firms ($s_{11}^*, s_{12}^*, s_{13}^*, s_{28}, s_{29}, s_{30}$) stay in the market, as the upper table in table 4.6 shows, the lowest income of consumers, who buy high-quality goods in the developed country is higher than the highest income in the developed country, which means that all firms in the developed country will face zero demand.¹³

If all firms in the developed country s_{28}, s_{29} , and s_{30} set their marginal cost βw_t , as shown in the bottom of Table 4.6, even the highest income consumers in the developed country prefer lower quality goods from the middle-developed country, and producers in the developed country still face zero demand. (The location of marginal consumers who are indifferent between s_{13}^* and

¹³the *Zero Demand Condition 3* holds for two firms in the developed country (s_{28}, s_{29}) $\tilde{\theta}_{13,28} > \tilde{\theta}_{13,29}$ and $\tilde{\theta}_{13,29} > \tilde{\theta}_{13,30}$ and the *Zero Demand Condition 2* holds for the firm producing the highest quality good $\tilde{\theta}_{13,30} > \bar{\theta}$.

s_{30}) exceeds the highest skill $\bar{\Theta}$ in the developed country.¹⁴

Thus, if the utility of consumers increases with quality at a rapidly decreasing rate $\mu = 0.002$, and the trade partner is in the middle development stage $t^* \geq 13$, all firms in the developed country will be extinguished by firms in the developing country. In this case, the perfect replacement of superiors by inferiors will be realized, and finally, the Schumpeterian creative destructions will stop in the developed country.

Strictly Concave Utility $\mu = 0.002$ & Trade with the Middle Developed Country $t^* = 13, t = 30$				
$\tilde{\theta}_{3,4}$	$\tilde{\theta}_{4,5}$	$\tilde{\theta}_{5,28}$	$\tilde{\theta}_{5,29}$	$\tilde{\theta}_{5,30}$
$\underline{\Theta} + 15.2$	$\underline{\Theta} + 275.6$	$\bar{\Theta} + 111.7$	$\bar{\Theta} + 85$	$\bar{\Theta} + 59.7$

Strictly Concave Utility t $\mu = 0.002$ & Trade with the Middle Developed Country $t^* = 13, t = 30$ s_{28}, s_{29} and s_{30} are sold at its marginal cost βw_t				
$\tilde{\theta}_{3,4}$	$\tilde{\theta}_{4,5}$	$\tilde{\theta}_{5,28}$	$\tilde{\theta}_{5,29}$	$\tilde{\theta}_{5,30}$
$\underline{\Theta} + 9.9$	$\underline{\Theta} + 248.9$	-	-	$\bar{\Theta} + 41.7$

Table 4.6: The Location of Marginal Consumers

The actual Bertrand competition in the second stage of the game will be done by three firms from the developing country, and the equilibrium in the developed country is shown in Table 4.7. As the table shows, facing high-income consumers without any developed country firms, three developing country firms($s_{11}^*, s_{12}^*, s_{13}^*$) set higher price and earn higher profit in the developed country than they do in its own country.

Figure 4.4 (solid line) shows the change in consumer utility in the developed country. Since all high-quality goods in the developed country will be replaced by lower quality goods from the country in the middle development stage, all consumers in the developed country will downgrade

¹⁴ Zero Demand Condition 2 holds for s_{30} .

their consumption. As in the previous case (Figure 4.3), the majority of consumers ($\theta \in [50, 482.6]$) will improve their utility after trade, while some high-income consumers ($\theta \in [482.6, 700]$) will be hurt by downgrading their consumption. Comparing the middle developed country (solid line in Figure 4.4) with the least developed country (dashed line), Figure 4.4 clearly shows that at every income level, trade with the middle developed country brings more benefit to consumers than that with the least developed country does. If the developed country trades with the least developed country, some high-income consumers will be hurt by downgrading their consumption from the highest quality goods to inexpensive but much lower-quality imports, and other high-income consumers will be hurt by the increase in the price of the highest quality goods. However, if the developed country trade with the middle-developed country, high-income consumers will not have to downgrade their consumption so much, and accordingly all consumers will benefit from inexpensive imports. As shown, consumers at any income level prefer trade with the middle-developed country, while firms prefer trade with the least-developed country. The overall effect of trade is summarized in Table 4.8. Trade improves the aggregate consumer surplus at the cost of producers. If the developed country trades with the middle developed country, all firms in the developed country will exit the market, while all consumers will improve their utility. If the developed country trades with the least developing country, some consumers will be hurt by the downgrade of their consumption, but the aggregate consumer surplus will improve and the firm producing the highest quality good in the developed country will survive.

4.4 Conclusion

This paper focuses the trade-off between the utility gain from the improvement of quality and the cost of the improvement and answers how and why high-quality expensive goods in the developed country are replaced by low-quality inexpensive goods from the developing country in the quality-

Strictly Concave Utility and Trade with the Middle Developed Country $\mu = 0.002, t^* = 13, t = 30$								
<u>Firm Exit</u> : Three Domestic Firms: s_{28}, s_{29} and s_{30}								
<u>Surviving Firms</u> : Three Foreign Firms $s_{11}^*, s_{12}^*, s_{13}^*$								
s_{11}^* : market coverage $\theta \in [\underline{\Theta} = 50, 58.4]$			s_{12}^* : $\theta \in [58.4, 291.7]$			s_{13}^* : $\theta \in [291.7, 700]$		
p_{11}^*	π_{11}^*	u_x	p_{12}^*	π_{12}^*	u_x	p_{13}^*	π_{13}^*	u_x
2.43 (2.19)	20 (0.09)	[575.08, 672.57]	2.68 (2.20)	625 (4.31)	[672.57, 3367.7]	3.91 (2.25)	1596 (26.75)	[3367.7, 8087.1]

Table 4.7: Strictly Concave Utility Function & Trade with the Middle Developed Country: Bertrand Equilibrium

() shows prices or profits in the developing country market.

	Producer Surplus $\pi_{28} + \pi_{29} + \pi_{30}$	Consumer Surplus u_x
Autarky	821.5	2.784×10^6
Trade with a Least Developed Country	813	2.808×10^6
Trade with a Middle Developed Country	0	2.815×10^6

Table 4.8: Producer Surplus, Consumer Surplus in the Developed Country

ladder model. Under a closed economy, all firms use the same domestic labor force and share the same cost, the cost of the added quality is reflected equally in home goods, and thus the concavity of utility function plays no role under trade. Without trade, the Schumpeterian creative destruction repeats: every time a new highest quality good arrives, the arrival drives the lowest quality good out and the resource-consuming technological progress will drive up the wage and the marginal cost in the country. As a result, only high-quality expensive goods survive in the developed country, while low-quality inexpensive goods are distributed in the developing country.

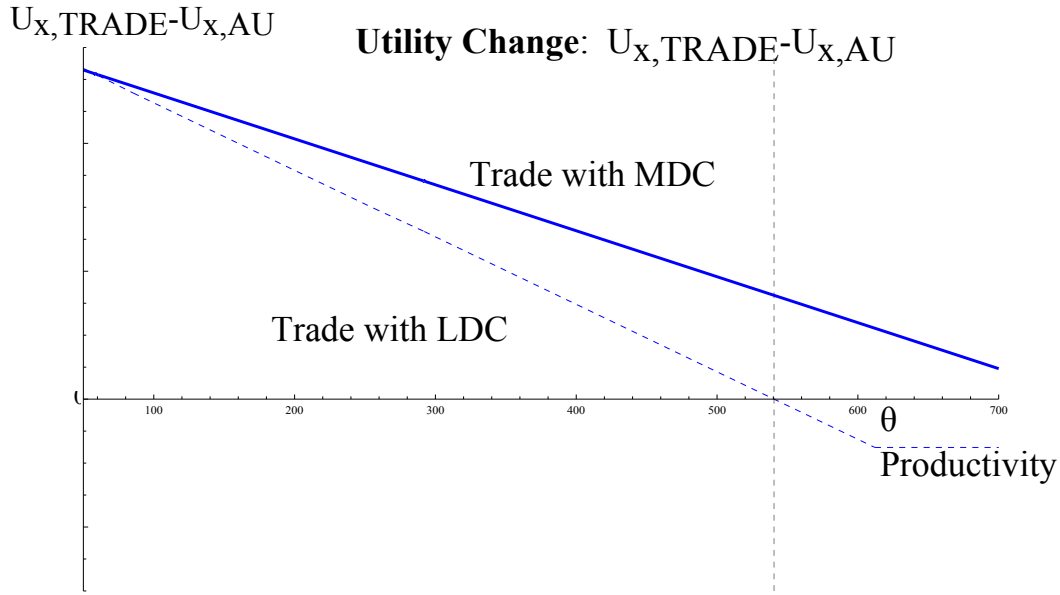


Figure 4.4: Utility Change: Strictly Concave Utility and Trade with the Middle Developed Country

However, once two countries in different development stages start free trade, low-quality inexpensive goods, which reflect low cost of the added quality in the developing country as well as high-quality expensive goods, which reflect high cost of the added quality in the developed country are available in the market. In such a case, the concavity of the consumer utility function plays an important role. If the consumer utility increases with the quality at mildly decreasing rate and the utility value of the added quality always exceed the cost of it, no replacement of superiors by inferiors will be realized, and Schumpeterian creative destruction will repeat forever. However, if the utility of consumers increases the quality of products at a rapidly decreasing rate and the cost of the added quality exceeds the utility value of the added quality, the quality advantage of goods in the developed country is not large enough to make up the difference in price. As a result, low-quality inexpensive goods from the developing country drive out high-quality expensive goods in the developed country. Recently, such a phenomenon is actually reported in the developed country. For instance, in April 2012, Aquascutum, a UK-based luxury clothing manufacturer, was sold to YGM Trading, a Chinese fashion retailer.

Numerical results show that only the firm producing the highest quality good in the developed country barely survives if the trade partner is the least-developed country, while all firms in the developed country exit the market (the perfect replacement of superiors by inferiors) if the trade partner is the middle-developed country.

For consumers, trade with the middle-developed country is more preferable. If the country trade with the middle-developed country, consumers will not downgrade their consumption so much, and accordingly the benefit of buying inexpensive imports outweighs the damage from downgrading their consumption for the majority of consumers.

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