Using surface-wave spectroscopy to characterize tilt modes of a vortex in a Bose-Einstein Condensate

P. C. Haljan, B. P. Anderson*, I. Coddington, and E. A. Cornell*
JILA, National Institute of Standards and Technology and Department of Physics,
University of Colorado, Boulder, Colorado 80309-0440
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A vortex in a condensate in a nonspherical trapping potential will in general experience a torque. The torque will induce tilting of the direction of the vortex axis. We observe this behavior experimentally and show that by applying small distortions to the trapping potential, we can control the tilting behaviour. By suppressing vortex tilt, we have been able to hold the vortex axis along the line of sight for up to 15 seconds. Alternatively, we can induce a 180° tilt, effectively reversing the charge on the vortex as observed in the lab frame. We characterize the vortex non-destructively with a surface-wave spectroscopic technique.

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The decay of "persistent" supercurrents, be they in superfluids or superconductors, is intimately connected to the dynamical behavior of vortices. A magnetically trapped, gas-phase Bose-Einstein condensate (BEC) provides a useful laboratory for characterizing the microscopic behavior of individual vortices subject to various controlled perturbations [1–4]. In one recent experiment, a vortex core in a near-spherical condensate "vanished" [3] from view apparently without moving out to the edge of the sample and annihilating there. Another group [2] found that the empirical critical rotation velocity for the formation of a vortex in an elongated condensate is much higher than can be accounted for by a simple model of the vortex as a rigid line-defect without any dynamics along its length. Feder *et al.* [5] explain the latter observation by showing that the higher rotation rate is necessary to suppress the growth of anomalous normal modes ("bending" modes) of the vortex. In this paper we study the lowest odd-order normal mode, which in our near-spherical geometry corresponds not to a bend but to a tilting of the vortex orientation. We show that the "vanishing" vortex of ref. [3] was in fact due to tilting of the vortex away from the line of sight. Such uncontrolled tilting was caused by residual asphericity in the condensate's confining potential. By tailoring the asphericity, we have learned to control the tilting dynamics.

The tilting behaviour of a vortex in a condensate, confined in a slightly aspheric, parabolic potential, is discussed in detail by Svidzinsky and Fetter [6], starting from the Gross-Pitaevskii equation. Only a brief summary of their theoretical results is given here. The vortex direction satisfies a set of equations similar in form to the Euler equations for rigid body rotation, familiar from classical mechanics. As a result, the tilting dynamics of a quantized vortex in a confined BEC are reminiscent of the behaviour of a freely spinning rigid body as seen in the body-fixed frame [7]. In the case of the spinning rigid body, there are two stable axes about which precession of the spin direction will occur, namely the axes with the largest and smallest moments of inertia. The intermediate axis is unstable and no precession occurs about that direction. If the spin direction lies initially near the intermediate axis, it will evolve away from its initial orientation. Similarly, for a quantized vortex in a BEC, tilt precession of the vortex is predicted to occur about two stable axes, given in this case by the tight and weak directions of confinement. The unstable axis corresponds to the direction of intermediate trapping strength.

The tilting dynamics of a vortex are constrained by two integrals of motion, one in particular corresponding to conservation of energy. Physically, this implies that, as the vortex tilts, its direction follows an angular trajectory which is a contour of constant vortex length, or equivalently constant energy. As is the case for all the normal modes of a vortex, the precession frequency for a vortex tilting about a stable axis is predicted to scale with the rate of vortex fluid circulation evaluated at the condensate edge. For our typical conditions, this fluid rotation is near 0.3 Hz. The theoretical frequency for tilt precession depends additionally on the magnitude and character of the trap asymmetries and the initial orientation of the vortex.

In our experiment, we first use a wavefunction engineering technique to make singly quantized vortices in a two-component BEC [1,8]. The two components, which are two different hyperfine levels of ^{87}Rb, are magnetically confined together in a nominally spherical, harmonic TOP trap [9], parameterized by a trapping frequency $\omega_{trap}/2\pi = 7.8(1)$ Hz. The vortex formation process leaves one component in the circulating state while the other component, which is non-rotating, fills out the core. The vortex is formed initially aligned along the line of sight. We take a non-destructive picture of the two-component vortex to record the initial displacement of the core with respect to the center of the condensate cloud. We then create a bare vortex by selectively removing the fluid filling the core [3], which shrinks down to a size below our imaging resolution.

This paper deals exclusively with the dynamics of bare vortices in a single-component BEC. The bare vortex state is formed with about $2 \cdot 10^5$ atoms at a temperature of $T/T_c = 0.8(1)$ where the critical temperature T_c is 20(6) nK. The number of atoms in the condensate is determined (in the Thomas-Fermi limit) from the condensate radius R, equal to 20.8(4) μm on average. Following the creation process and core removal, the condensate is held for a variable holding time in its confining potential, and then probed for the presence of the vortex.

We have previously reported that the visibility of vortex cores was lost after a holding time of about 1 s in a nominally spherical trap [3]. In those experiments, the confining potential was suddenly removed, and the condensate allowed to expand ballistically before imaging. The presence of a vortex was detected in the expansion image as a dimple in the condensate's density distribution. The topological nature of a quantized vortex means that the only way for a BEC to rid itself of a vortex is for the core to make its way to the edge of the cloud and annihilate there; the vortex cannot gradually spin down. Our vortices, however, seemed to disappear without any visible radial motion of the cores outward. We did notice that the contrast of the cores as observed in expansion decreased at longer holding times before being lost altogether. These results could be explained by a tilting of the vortex away from the line of sight. Because the vortex core is such a narrow feature, only small deviations from the line of sight ($\sim 20°$) are necessary for contrast to be lost below the noise threshold in the expansion images.

In order to circumvent this imaging limitation, we have implemented an alternative method of vortex detection that is more robust against tilting of the vortex from the line of sight, and that has the additional advantage of being sensitive to the handedness of the vortex circulation. This detection technique uses the idea that the collective excitation frequencies of a trapped condensate are sensitive to the presence of a vortex [10]. We make use of quadrupolar surface-wave excitations, that is, surface-wave excitations which carry angular momentum $l = 2$. (The angular momentum of these perturbative excitations should not be confused with the angular momentum per particle of the bulk of the condensate, which in the presence of a well-centered vortex approaches $l = 1$). The projection of angular momentum of an $l = 2$ surface-wave excitation onto the axis of the vortex direction can have components $m = 0, \pm1, \pm2$. In the absence of a vortex, the counter-propagating $\pm m$ modes are degenerate, but in the presence of a vortex the handedness of the fluid flow breaks time-reversal symmetry and lifts the degeneracy. The use of surface-wave spectroscopy for vortex detection was first demonstrated experimentally by Chevy *et al.* [4].

To detect a vortex, we excite an $l = 2$, $m_z = 0$ excitation about the vertical $\hat{z}$ axis by modulating the trapping potential for a single cycle. Along the line of sight, the $\hat{x}$ axis, the $m_z = 0$ mode projects onto a superposition of $m_x = +2$, $m_x = -2$, and $m_x = 0$ modes. The $m_x = +2$ and $m_x = -2$ superposition may be thought of as a standing-wave of clockwise and counterclockwise surface waves. Along the line of sight we observe the cloud alternately stretch along two principal axes, first vertical, then horizontal. The excitation period is measured to be 11.0(2) Hz in agreement with the predicted value $\sqrt{2}\omega_{trap}$ [11]. The presence of a vortex along the line of sight induces a splitting of the clockwise and counterclockwise wave velocities, so that the nodes of the standing-wave are not completely fixed. As a result we observe a precession of the principal axes of the quadrupolar excitation. We record the precession in a sequence of seven non-destructive images, strobed at half the quadrupolar excitation period. A typical data set is shown in Fig. 1. Opposite vortex circulation clearly leads to opposite precession of the principal axes, as is evident in Figs. 1(b) and (c).

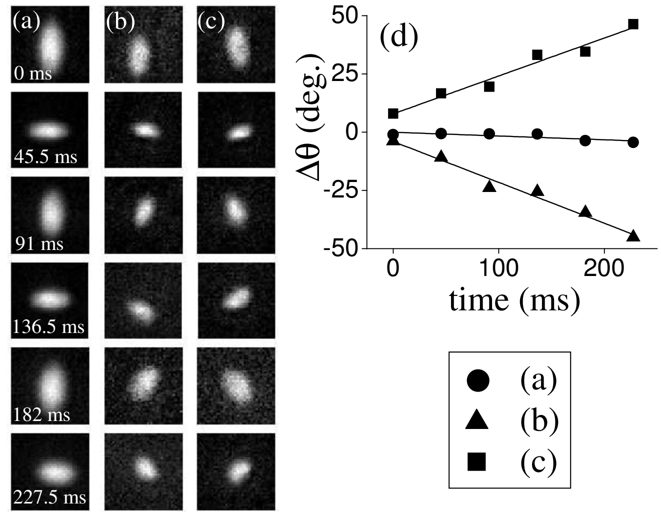

FIG. 1. Using surface excitations for *in situ* detection of a vortex in a confined BEC. (a), (b) and (c) are each a series of non-destructive images of the quadrupolar mode, after excitation. The pictures are strobed at 45.5 ms, half the excitation period. (a) is the case of a vortex free condensate; (b) and (c) show the excitations in the presence of a vortex whose core is normal to the plane of the page. The vortices in (b) and (c) have opposite handedness. The principal axes of the ellipse-shaped quadrupolar mode precess in the direction of the fluid flow. The images are each fit to an elliptical distribution with orientation $\Delta\theta$ of the principal axes. The orientation, expressed as an angular deviation from the vertical and horizontal axes, is plotted versus time in (d) for each of the cases (a), (b) and (c). A linear fit has been applied to the data to determine a precession frequency of the principal axes, -0.49(4) Hz for (b) and 0.45(5) Hz for (c).

For a vortex whose direction is tilted with respect to the line of sight, we still expect precession of the quadrupolar principal axes to be induced about the vortex direction. The main limitation to vortex detection in this case is that there must be sufficient projection of the quadrupolar precession onto the imaging camera. In the extreme case where the vortex is oriented at 90° to the line of sight, any quadrupolar precession will also be about an axis perpendicular to the line of sight, in other words, not readily observable on the imaging camera.

Armed with our new detection method, we revisited the topic of longevity of vortices in our nominally spherical trap. The precession of surface waves clearly revealed the presence of vortices at 1.5 s and at 2 s holding time, well after the vortices ceased to be visible in the expansion images [3]. Given that the surface-wave probe for vortices should be relatively robust to small tilts of the vortices with respect to the line of sight, we came to the tentative conclusion that our vortices were in fact tilting away from the line of sight during the first few seconds after their creation. This behavior could be accounted for by small ($\leq 3\%$) residual asymmetries in our trapping potential.

Further study of the tilting required better characterization of the trap potential, and we achieved this by deliberately introducing tailored deformations to the confining potential, deformations that were certain to overwhelm the residual asymmetry. The results of three such experiments are presented here: first, the suppression of vortex tilt; second, the generation and observation of vortex tilt about a stable axis; and, third, the manipulation of a vortex's orientation through controlled deformations of the BEC's confining potential.

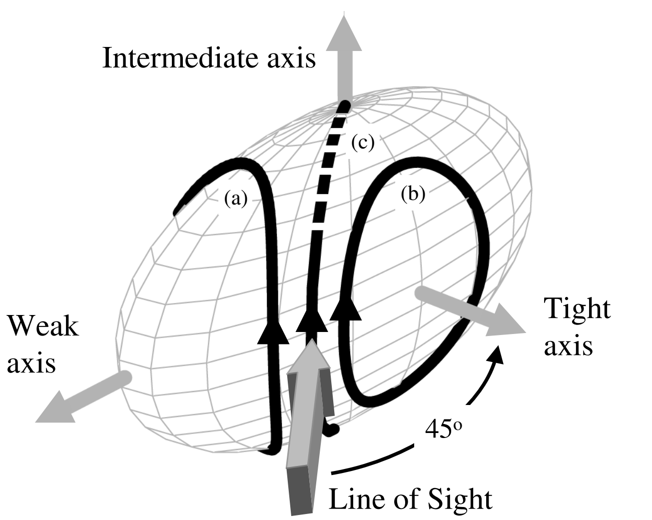

FIG. 2. Possible tilting behaviour of a vortex in a confined BEC is illustrated for the conditions of the second tilting experiment (see text). The axes of symmetry for the triaxial confining potential are indicated. Three possible trajectories for a tilting vortex, initially aligned near the line of sight, are traced out on the surface of the condensate. Depending on the initial orientation of the vortex and the relative values of the confinement strengths, the vortex direction could (a) precess about the weak axis, (b) precess about the tight axis, or (c) drift towards the intermediate axis along a 'saddle' line.

In the first experiment, the spherical confining potential is squeezed by 10% along the line of sight following the creation of a vortex in the BEC [12,13]. This places the vortex along the tight, stable axis of the trap. As a result, tilting of the vortex away from the line of sight should be suppressed. After a variable holding time in the squeezed trap, the confining potential is returned to spherical symmetry and the condensate is then quickly probed for the presence of the vortex. Using both the surface-wave and expansion techniques, we have detected vortices lasting for holding times up to 15 s. This is a significant improvement over our previously published limit of 1 s. Although visibility has been restored at long times by holding the condensates in a squeezed trap, it is important to note that only well-centered vortices have been found to survive to the longest times. This is consistent with a model of initially offset cores spiraling out of the condensate under the influence of thermal damping [14].

The second experiment proceeds in the same way as the first except the spherical trap is now squeezed along a horizontal axis at 45° to the line of sight. The squeezing process produces a triaxial confining potential, where the tight and weak directions of confinement lie in the horizontal plane while the intermediate axis is vertical. The measured trap asymmetries together with the initial vortex direction indicate that the vortex should precess about the weak axis, although slightly different conditions could lead to rather different behavior (see Fig. 2 [15]). In any case, as the vortex tilts away from the line of sight to a maximum excursion of 90° from the imaging axis, the visibility of the vortex will gradually disappear. Eventually, if the vortex precesses about the stable axis and back into the line of sight, it should become visible again in a 'revival of visibility'. This effect is seen in Fig. 3, where vortex visibility is plotted as a function of

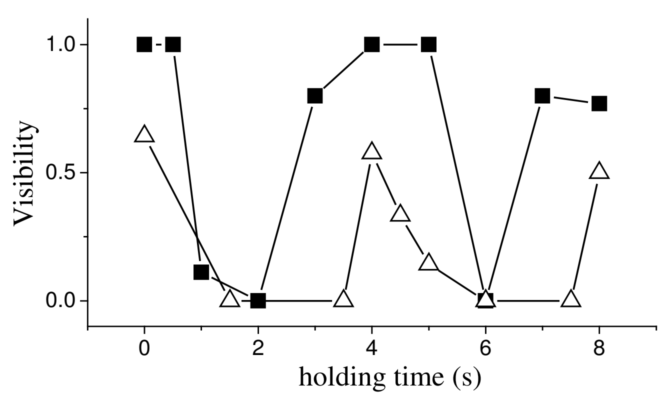

FIG. 3. Visibility of a vortex in a BEC is plotted versus the time the BEC is held in a trap squeezed along an axis at 45° to the line of sight. Visibility is the probability of detecting a vortex over several shots. Filled squares indicate vortex detection with the surface-wave technique; open triangles indicate detection with the expansion technique.

holding time in the squeezed trap. Visibility is defined as the probability of detecting a vortex at a given time and is obtained from a set of approximately ten shots for each point plotted. Vortices whose initial offset from trap center exceeds a prescribed value of $\sim$0.15 R are omitted from the analysis. This is done to reduce blurring of the revival due to possible position-dependent tilt rates, and, moreover, to prevent loss of contrast at long times due to the drift of cores out of the condensate. The vortex detection is accomplished with both surface-wave and expansion techniques. The revival measured with expansion imaging is much more narrowly resolved in time, as is to be expected from this method's greater sensitivity to alignment of a vortex with the line of sight.

The frequency of the tilt mode, determined from the data in Fig. 3, is 0.25(2) Hz. The theory of Svidzinsky and Fetter [6] predicts the lowest-order odd-parity vortex mode should have a frequency below 0.3 Hz for our values of the Thomas-Fermi radius, confinement asymmetry, and initial vortex angle. Uncertainties in the confinement asymmetry preclude a more precise determination of the theoretical tilting frequency. The uncertainties are such that the vortex could lie on an initial trajectory arbitrarily close to a 'saddle' line, where the revival time should diverge (see Fig. 2). The high contrast of the two revivals in Fig. 3 clearly indicates, however, that the shot-to-shot fluctuations of the confinement asymmetry and of the initial position of the vortex are small.

The third and final tilting experiment demonstrates the reversal of vortex handedness. The vortex, as usual, is formed along the line of sight. The direction of the vortex is then flipped 180° by deforming the trapping potential in two steps, as illustrated in Fig. 4. From the perspective of the line of sight, the 'lab frame,' the vortex has effectively flipped its handedness. Surface-wave detection is used to verify this, with pictures similar to Figs. 1(b) and (c) being obtained before and after the flipping process respectively. Manipulation of vortex direction may prove to be a useful technique in a TOP trap. For example, maneuvering a vortex to an arbitrary direction followed by a second stage of wavefunction engineering could produce a condensate wavefunction with more complicated, three-dimensional topological structure.

In conclusion, we have described a non-destructive method of vortex detection and used it to characterize the tilt modes of a bare vortex in a trapped BEC. Control over the tilt of a vortex has been demonstrated, including the suppression of tilting altogether. The suppression of tilting, which maintains the visibility of vortices out to long times, now permits the study of the lifetime of vortices at finite temperature.

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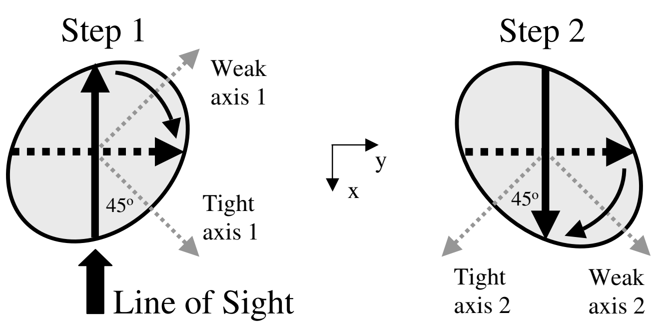

FIG. 4. A sequence of trap deformations to reverse the direction of a vortex is shown. The figure shows a horizontal crosssection of the BEC as seen from above. The vortex is initially formed along the line of sight and aligned with the -$\hat{x}$ axis. In step 1, the trap is squeezed in the horizontal plane along tight axis 1. The vortex is then allowed to precess for 1.5 s, at which point it should be aligned with the $\hat{y}$ axis. The new direction of the vortex is indicated by a dotted arrow. In step 2, the trap is squeezed along tight axis 2 and the vortex allowed to precess another 1.5 s until it is aligned with the $\hat{x}$ axis. To an observer looking down the line of sight, the vortex has effectively reversed its handedness. Changing the time allowed for vortex precession by 0.5 s inhibited the effect. For the correct timing, the flipping process was found to be successful 80% of the time.

* Quantum Physics Division, National Institute of Standards and Technology.

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[12] A 10% tightening of the spherical trap is achieved by turning off the component of the rotating TOP bias field along the direction of the imaging axis. This leaves an

oscillating bias field perpendicular to the line of sight. The BEC equilibrium position is sagged so far vertically that the oscillating quadrupole zero of the squeezed TOP trap does not pass through the cloud.

[13] The confining potential is squeezed over 200 ms, slow enough to avoid inducing large-amplitude shape oscillations of the BEC but fast enough so as not to interfere significantly with the tilting dynamics.

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[15] For ease of presentation, Fig. 2 shows the trajectories for a vortex with initial orientaion opposite to that in the experiment and Fig. 4. This only reverses the *direction* of the trajectories.