TEACHERS' USE OF INSTRUCTIONAL MOVES DURING TECHNOLOGY-BASED MATHEMATICAL ACTIVITIES

by

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Teachers' Use of Instructional Moves During Technology-Based Mathematical Activities

Dissertation directed by Associate Professor David C. Webb

ABSTRACT

This study investigates instructional moves by teachers in mathematics classrooms in which technology-based activities (i.e., student-oriented simulations) and features of those simulations influence classroom practices. Four teachers were studied over the course of a year as an exploratory study to build interpretive cases that described instructional practices in technology-based lessons. Teachers developed lessons using PhET simulations designed to support algebraic reasoning. Data sources included teachers' process of selecting and designing lessons, observations of teachers' non-technology and technology-based mathematical activities, and teacher interviews and reflections.

This work was based on a conceptual framework blending the ideas of Mathematical Tasks (Stein, Smith, Henningsen, & Silver, 1998), Mathematical Pedagogical Content Knowledge (Ball, Thames, & Phelps, 2008), and Technological Pedagogical Content Knowledge (Mishra & Koehler, 2006), in which teachers' instructional practices are determined by teachers' mathematical pedagogical content knowledge, task selection and design, and use of technology.

Results indicated that teachers see simulations as having significant benefits in the classroom. Teachers leveraged these opportunities by increasing class discussions, engaging in higher levels of thinking and reasoning, and focusing on mathematical representations. When teachers used simulations, the teachers spent less time in direct instruction, focused more on the mathematics, and focused more on investigations rather than drill-oriented tasks.

Technology in the classroom, however, was problematic for some teachers. The very nature of students working independently with their own devices meant that student-student interactions decreased in some lessons. Furthermore, teachers' discomfort in managing technology seems to limit ongoing use.

Specific features of the simulations that prompted instructional moves included the ability to support conceptual understanding and build student engagement. Simulations also provided a 'low floor, high ceiling,' supporting differentiation, and a dynamic responsiveness, facilitating connections between representations. On the other hand, teachers raised concerns that some features of the simulation could do the math for the students. Furthermore, the perception of simulations as being a game may impact how and when simulations are used.

The emergent use of technology in math classrooms is under-supported. For simulations to be used in a more extensive fashion in mathematics classes, professional development and curricular materials are needed to support implementation.

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Polman, who introduced me to the work of Larry Cuban, and whose research inspired me to think critically about the use of technology in new ways; Edd Taylor, who helped me consider the impact of instructional practices on student learning, and pushed me to think about rigor in my research; and Ben Shapiro, who shared key research that significantly influenced my final research direction. I owe you all a huge debt of gratitude.

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Technology is increasingly being incorporated into mathematics classrooms throughout the country largely due to three key factors: increased access to computers, tablets and other technology in the classroom; pressure from parents who want their children to develop expertise with technology; and a continued push for technology infused activities in math standards. As a result technology in classrooms is growing.

> *Teachers reported having the following technology devices either available as needed or in the classroom every day: LCD (liquid crystal display) or DLP (digital light processing) projectors (36 and 48 percent, respectively), interactive whiteboards (28 and 23 percent, respectively), and digital cameras (64 and 14 percent, respectively). Of the teachers with the device available, the percentage that used it sometimes or often for instruction was 72 percent for LCD or DLP projectors, 57 percent for interactive whiteboards, and 49 percent for digital cameras.*(Gray, Thomas, & Lewis, 2010, p. 3)

As these data suggest, while access to technology has increased, much of this technology (including projectors, whiteboards and documents cameras) remains in the hands of the teachers.

Yet, mathematics classrooms are moving towards student-centered technology, such as student use of computers, graphing calculators, and clickers. In the 2015 TIMSS study (Mullis, Martin, Foy, & Hooper, 2016), 46% of fourth grade teachers and 39% of eighth grade teachers reported that technology devices (including computers, laptops, tablets or smartphones) were available in their mathematics classroom. Students are also using technology through a myriad of available software tools. Software tools may be browser-based, such as online web resources ranging from drill type programs, to video game based learning materials, to simulations. These tools may also include stand-alone software installed directly onto the hardware. Examples of this include: typical word processing and spreadsheet software such as Microsoft Office®; game oriented software, such as programs to facilitate practice in a variety of topics; or specialized software designed as a tool for a specific purpose such as GeoGebra, a common geometry program.

1.1 TECHNOLOGY IN THE CLASSROOM

While studies have found increased access to technology, there does not seem to be consistency in how the technology is being incorporated into classroom activities. The 2015 TIMSS study (Mullis et al., 2016) found that technology was used at least monthly in eighth grade classrooms for the following activities: explore math principles and concepts (31%); practice skills and procedures (29%); and look up ideas and information (26%).

Two terms that appear repeatedly throughout literature are *technology use* and *technology integration* (M. E. Pierson, 2001; Rao, 2013; Wachire & Keengwe, 2011). While it may seem at first a simple question of semantics as the terms are not defined when used, I believe there are significant differences that play into the role of technology in classrooms. Technology is *used* when technology delivers information, often in a means that replicates work that was previously accomplished in other ways (Zbiek & Hollebrands, 2008). The technology is peripheral to the mathematical task, assessing students' knowledge, rather than creating opportunities for new educational experiences. A classroom where graphing calculators are available for routine calculations, but are not used for more in-depth study by students might be an example of technology *use*, as would some of the above uses suggested from the TIMSS data, such as practicing skills and procedures.

The second term, *technology integration*, refers to classrooms where technology plays a more regular role, with activities that are central to the mathematical task (Zbiek & Hollebrands, 2008). In this role, technology is designed to support analysis and/or mathematical understanding that could not be completed through traditional pencil and paper work. It is technology designed to support students' conceptions of mathematical ideas, and often involves the students exploring the technology without significant teacher direction. This might include classrooms where students use an applet to visualize the area of a circle, or use GeoGebra to compose and decompose a trapezoid.

I would argue, however, that there is perhaps a third term of distinction the use of technology in classrooms; I would call *technology infusion*. This classroom, where technology is infused into the mathematical tasks, would bring in a variety of technology tools that would support deep conceptual understanding. While students would be the users of the technology, teachers in these classrooms would play a strong role, both in facilitating discussions during the use of the tool, and also referring back to that experience in later class periods. The distinctions between the levels of technology involvement are shown in Table 1-1.

Table 1-1 Degrees of Technology in the classroom

Technology Use	Technology Integration	Technology Infusion
• Technology is *occasionally* present	• Technology is *routinely* present	• Technology is routinely present in *multiple forms*
• The technology *delivers information*	• The technology *supports skill development*	• The technology *supports conceptual understanding of mathematical ideas*
• The technology is *peripheral* to the mathematical task	• The technology is *central* to the mathematical task	• The technology is central *not only to the initial task, but continues to be incorporated into future lessons*
• The technology *replicates work* that was previously accomplished through other means	• The technology provides a means of analysis or understanding that is *not possible without its use*	• The technology provides a means of analysis or understanding that is *not possible without its use and is discussed by both teacher and student*
• Technology is a means of *assessment* rather than learning	• Technology is designed to support *learning, but learning is through rote, repetitive activities*	• Technology supports both *teaching and learning through construction of knowledge* activities
• Teachers or students may use the technology, but the actions are typically directed by the *teacher*	• Students *use the technology* on their own, without step-by-step guidance or support from the teacher	• *Students use the technology* on their own, without step-by-step guidance or support from the teacher; however, *teachers play an active role* in facilitating discussions about what students learned

In my experience as both a math teacher, as well as a researcher, I have found that the ways that hardware and software are used by students in classrooms vary quite a bit based on how teachers include technology in the curriculum. The amount of student interaction can vary dramatically based on whether technology is used, integrated, or infused into mathematical tasks.

You can imagine three different scenarios in a classroom where teachers are using graphing calculators to support student understanding of linear equations and slope of a line. In the first classroom, where technology is *used*, graphing calculators might be used by students to graph an equation, and calculate the slope of the line. In this case, the use of graphing calculators simply replaces the work that students could have done with pencil and paper. The calculators are used as a source of information, rather than a means to explore slope of a line. In another classroom, where technology is *integrated*, students are asked to graph a series of lines and describe the differences in the lines. Some students may come away with initial ideas regarding how the equation is related to the slope and the intercept; however, other students may simply have seen this as a game where they can use the calculator to draw random lines. There may be no actual understanding of how the changes made directly affected the lines that were drawn.

On the other hand, imagine a third classroom, where technology is *infused* into mathematical tasks. In this classroom, students are given the opportunity to explore different equations on their own. Students enter in a variety of values to see how the lines will change. As students begin to work on differing equations, the teacher becomes involved, pausing the class at strategic points within a lesson so that students come together to share their findings. A teacher might also challenge students to find an equation with a specific intercept or a specific slope in an effort to ensure that students are connecting the different parameters of a linear equation to the line that is drawn. The true infused class is actually the marriage of use and integration to elucidate true conceptual understanding which makes the teacher central to the process.

Prior research has considered the degree to which technology is being incorporated into classrooms (Bennison & Goos, 2010; Hogarty, Lang, & Kromrey, 2003), as well as how students use specific tools such as GeoGebra (Bayazit, Aksoy, & Ilhan, 2010) and SimCalc's MathWorld (Roschelle, Kaput, & Stroup, 2000); however, this research is predominantly student-oriented. While the students' interactions with technology are certainly an important part of what happens in the classroom, the teacher plays a key role as well. Specifically, research is needed to consider the ways in which the teacher engages with the students such that the teacher can interpret what the students understand, have a better sense of the range of student conceptions and make informed decisions regarding additional instructional support. We can all probably agree that in a classroom where students use calculators to simply replace pencil and paper, technology is not playing a key role in the students' conceptions of linear functions.

But what about the classroom with a more inquiry-based approach to technology? In what ways are the teachers supporting students to make sense of mathematics when students are using technology? This is a challenging problem for teachers due to the way students often use technology in mathematics classrooms: students may be working independently, and unlike more traditional activities in the classroom where teachers can walk around the room and easily scan the work students are completing, most technology tools are not set up to provide feedback to the teacher. Therefore, it is difficult for teachers to know how each student is interacting with the mathematics. As a result, teachers may choose to change or amend their instructional practices when technology is present in the classroom in response to those challenges.

1.2 MATH SIMULATIONS

This study specifically considered teachers' instructional moves during the use of pre-made student-oriented simulations designed to support mathematical understanding in the classrooms. Simulations, or "system[s] of objects, relationships and rules whose behavior resembles that of some other system" have been used (and studied) in a variety of classrooms although for the most part, simulations have predominantly been used in STEM (Science, Technology, Engineering and Math) classrooms (Doerr & Pratt, 2008, p. 261).

As one might expect, simulations range in complexity. Some are little more than a set of virtual manipulatives, such as online base 10 blocks, or an online graphing calculator (Utah State University, 2016). Others are designed to replace downloadable software, such as spreadsheets, or simulations that enable students to create data sets and see immediate changes in a bar graph or Venn diagram ("Simulation-Math," 2016).

Others, often described as applets, are tiny programs designed to illuminate a specific mathematical concept, such as exploring growth in a parabola, or making connections between concepts such as area and perimeter (University of Colorado, 2016). Still others pose a problem or challenge to be solved, through which specific mathematical concepts are brought to the forefront (Desmos, 2016). Simulations have been found to support inquiry-based approaches to science and mathematics instruction, particularly in ways that facilitate visualization and exploration of real-world phenomena (Doerr & Pratt, 2008; National Research Council, 2011; Rutten, Joolingen, & Veen, 2012).

This study investigated teachers' instructional moves during mathematical tasks created around the use of PhET[1] math simulations. PhET math simulations (or sims, as they are commonly called) are specifically designed to support algebraic understanding through student-centered use of technology. Teachers had access to five possible PhET simulations for use in this project: Function Builder (allows students to see the effects of different function changes), Expression Exchange (enables students to explore ways to combine terms), Proportion Playground (enables students to explore proportions through a variety of contexts), Unit Rates (enables students to find unit rates as well as build up to successive quantities), and Graphing Lines (supports students in understanding linear equations). See Appendix A for some screen shots of the simulations. Teachers included in this study designed lesson plans using these simulations, and then enacted these plans in their own classrooms. While the simulations all have been designed to support student conceptual understanding, as noted above, teacher enactment of mathematical tasks can significantly change what happens in the classroom. Therefore, this study focused on teachers' instructional moves when using these simulations and how these practices varied from the instructional practices used during non-technology oriented lessons ("business as usual").

1.3 PURPOSE OF THE STUDY

In most cases, simulations do not provide tools for teachers to be able to observe student interactions[2]; as a result, teachers need new or adapted instructional practices that support moving from *using* technology to *integrating* or even *infusing* technology into mathematical tasks in order

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[1] PhET originally referred to Physics Education Technology, but now refers to a suite of interactive simulations for educational use in a variety of disciplines including mathematics.

[2] Desmos is a notable exception to this as it enables teachers to create a virtual class and to see the progress of each student assigned to that class.

to support students' deeper understanding of mathematics. More specifically, the challenge is in understanding what types of instructional moves are currently present in classrooms, and the ways in which those moves changed based on the presence of technology-based mathematical activities.

This research study was designed to better understand and articulate the ways in which teachers were engaging with technology (whether it is *used*, *integrated*, or *infused*) and the extent to which teachers make connections between technology implementation and their own instructional practices within their classrooms. By understanding these instructional moves used during technology-based lessons, researchers and educators alike will be better able to support professional development opportunities in incorporating technology into classrooms.

1.4 RESEARCH QUESTIONS

This project details the findings when considering the following research questions:

1. What observed instructional moves by teachers in math classrooms are unique to technology-based activities created with student-oriented simulations?
2. In what ways do specific design features of student-oriented simulations impact instructional moves in math classrooms?

1.5 SIGNIFICANCE OF THE STUDY

This study is contributing to the current research base in two key areas. First, the study adds to the corpus of research by specifically addressing an open question as to how teachers are supporting rich mathematical environments when bringing technology-based simulations into mathematical classrooms. Second, it details ways that student-centered simulations impact instructional moves through their design features.

2 CHAPTER 2: CONCEPTUAL FRAMEWORK

This research is predicated on a social constructivist view of teacher and student learning, that recognizes the importance of building on students' prior knowledge and beliefs (Palincsar, 1998). Additionally, this view recognizes that learning is a social activity and one that cannot be separated from socially and culturally constructed contexts. As educators become more aware of the social aspect of learning, more research has focused on group work, indicating that the effectiveness of group work varies greatly in its implementation (Palincsar, 1998). When groups work cooperatively, with rich discussions that push students' thinking, there is a strong opportunity for student learning (Cobb & Jackson, 2011b; Goldman, 2009; Jackson, Shahan, Gibbons, & Cobb, 2012; Palincsar, 2005).

In studying the instructional moves of teachers who use simulations in their math classes, it is important to take into consideration three key factors: 1) the teachers' choice of instructional practices will likely be influenced by their own mathematical knowledge (both content and pedagogical knowledge), 2) their development and enactment of mathematical tasks, and 3) their technological pedagogical knowledge (i.e. how teachers use, integrate or infuse technology into the mathematical tasks, based on that technology). These factors are discussed in the sections below.

2.1 MATHEMATICAL PEDAGOGICAL AND CONTENT KNOWLEDGE

Shulman (1998) argued that the knowledge base for teaching was far more than simply knowing the content of the material. Instead, one needed to know and understand *how* to teach the material, what he referred to as pedagogical content knowledge. Ball, Thames and Phelps (2008) took this concept a step further, and argued that there is more specialized knowledge for mathematics teaching than just content knowledge and pedagogical knowledge. The authors noted that while teachers must understand the mathematics and basic pedagogy, teachers also need knowledge of pedagogy that was specific to teaching mathematical concepts. They theorized that Shulman's theories about pedagogical content knowledge should be extended to include references to knowledge that teachers must have specifically between students and the mathematical content. The remaining two domains focused on connections teachers must make between their students and the content (what students will do, what they will find difficult, etc.) as well as between the content and teaching (creating instructional moves that will create interplay between the students and the content in appropriate ways).

This mathematical pedagogical content knowledge therefore becomes central to this study, as teachers' understandings of mathematics and student conceptions will lead to choices about instructional practices.

2.2 MATHEMATICAL TASKS

This research also draws significantly from the work of Henningsen & Stein (1997). Henningsen and Stein studied classrooms while considering the ways in which mathematical tasks changed from the original design of the task, as it was created, to the setup of the task, as the teacher introduces the task to the students, through the final enactment of those tasks, as they play out in a classroom. Their research sought to determine the ways in which teachers directly supported or inhibited the implementation of high cognitive demand tasks. Cognitive demand, defined as "the level and type of thinking that a task has the potential to elicit" (M. D. Boston & Smith, 2009, p. 122), ranges from lower levels including memorization, or procedures without connections, to higher levels such as procedures with connections and doing mathematical tasks.

> *Tasks that require students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks the demand engagement with the concepts and that stimulate students to make purposeful*

connections to meaning or relevant mathematical ideas leads to a different set of opportunities for student thinking.

Since the tasks with which students become engaged in the classroom form the basis of their opportunities for learning mathematics, it is important to be clear about one's goals for student learning. Once learning goals for students have been clearly articulated, tasks can be selected for created to match these goals. Being aware of the cognitive demand of tasks is an essential consideration in this matching. For example, if a teacher wants students to learn how to justify or explain their solution processes, she should select a task that is deep and rich enough to afford such opportunities. If on the other hand, speed and fluency are the primary learning objectives, other types of tasks will be needed. (Stein, Smith, Henningsen, & Silver, 2009, pp. 1–2)

Their framework recognizes that mathematical tasks, as enacted in the classroom, are necessarily entwined with the "goals, intentions, actions and interactions of the teachers and students" (Stein et al., 2009, p. 14); therefore, tasks cannot be considered solely as designed, but rather the focus must continue to the set up and enactment of the task in the classroom.

Henningsen and Stein's conceptual framework is shown in Figure 2-1. As displayed in the first step of the figure, this task could be one that teachers find online or in a book, or a task they designed. This task, as originally designed, is then evaluated by the teacher for its value and presumed effectiveness in meeting a learning goal, and subsequently adjusted to meet the needs of a particular classroom. Teachers can plan additional supports as needed for a particular classroom, increasing or decreasing the cognitive demands of the original tasks. These changes may be the result of the teacher's goals, the teacher's subject knowledge, the teacher's knowledge of his/her students, or the interplay between content and students (identified in the figure as *Factors Influencing Setup*).

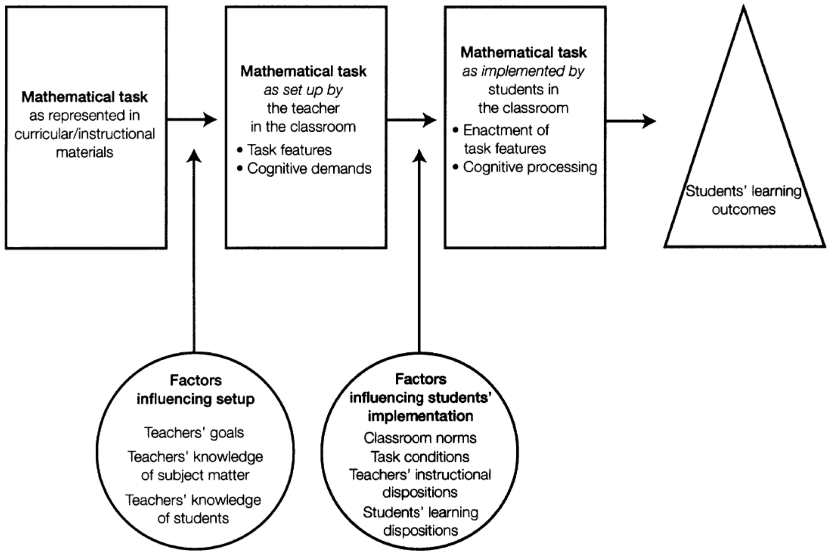


Figure 2-1 Factors influencing task setup and enactment within a classroom (Henningsen & Stein, 1997, p. 528)

In non-technology tasks, Stein et al (2009) found that teachers could increase or decrease the amount of cognitive demand in a task through their own actions (i.e. instructional practices), such as providing or withholding information, breaking the task into smaller pieces (chunking) and asking questions which prompt versus asking questions which probe (Stein et al., 1998). As shown in *Factors influencing students' implementation*, students could also significantly influence the enactment of tasks, through classroom norms, and students' learning dispositions. For example, students' levels of motivation, as well as students' prior knowledge could significantly impact the cognitive demand of tasks.

While this study does not specifically consider the cognitive demands of the tasks, it is looking at the mathematical tasks (technology based mathematical activities) from the initial planned task, through setup to the final enacted task. As noted earlier, this research considers the task as more than just the activity itself. This activity is situated in the goals, actions and interactions of the teacher and her students.

2.3 TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE

Building on these two frameworks above reminds us that teachers' are supported in the development and enactment of tasks through their knowledge of both mathematical concepts as well as their understanding of the pedagogy specific to mathematics. Other researchers (Mishra & Koehler, 2006; M. E. Pierson, 2001) contend that bringing in technology alters this relationship between pedagogy and content knowledge as teachers will need technological knowledge as well. They frame this as three interlocking circles (as shown in Figure 2.2), resulting in two additional types of knowledge for teachers: technological content knowledge (TCK) and technological pedagogical knowledge (TPK).

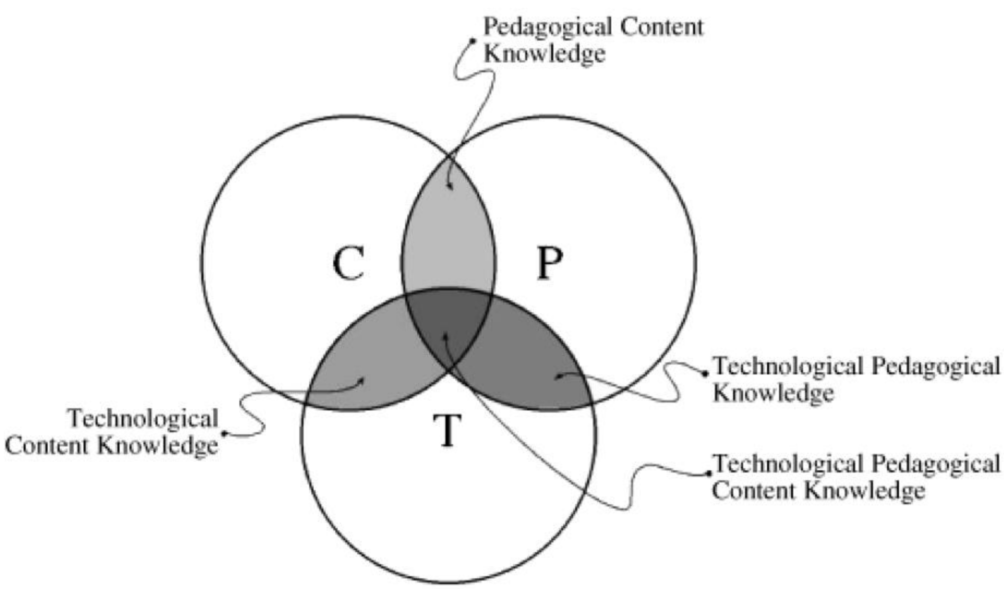


Figure 2-2 Technology and Pedagogical Content Knowledge (TPCK) Framework (Mishra & Koehler, 2006, p. 1025)

TCK is an understanding of how the "*choice of technologies affords and constrains the types of content ideas that can be taught" (Koehler & Mishra, 2009, p. 65).* TPK, on the other hand, is "an understanding of how teaching and learning can change when particular technologies are used in particular ways" (Koehler & Mishra, 2009, p. 65). It is the set of skills held by the teacher allowing her to assess the ways in which certain technologies could be best infused into a subject such as mathematics. For example, TPCK is not only being aware of the technologies, but also thinking creatively about how teachers can use these tools for individualized pedagogical purposes.

> “Thus, TPK requires a forward-looking, creative, and open-minded seeking of technology use, not for its own sake but for the sake of advancing student learning and understanding.” *(Koehler & Mishra, 2009, p. 66)*

Technological Pedagogical Content Knowledge (TPCK), therefore, is the knowledge that teachers use when incorporating technology into classrooms activities.

> *“TPCK is the basis of good teaching with technology and requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students’ prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones.” (Koehler & Mishra, 2009, p. 1029)*

Several researchers have considered the applicability of this framework in mathematics classrooms (Lee & Hollebrands, 2008; M.L. Niess, 2005; Margaret L Niess et al., 2009; Pierce & Ball, 2014; M. E. Pierson, 2001; Richardson, 2009); yet, their work is predominantly focused on measuring the current level of TPCK, or the acquisition of TPCK through a variety of professional development opportunities. One paper that stands apart from this is Guerrero’s (2010) work that identifies specific connections within teachers TPCK. First, she identifies that teachers’ TCK is impacted by their conceptions about technology and its affordances or constraints in a math classroom. Second, teachers’ TPK is influenced by their ability to adjust their instruction based on the technology used and third, their TPK is influenced by their ability to adjust their classroom management style to adapt to challenges resulting from the use of technology. The fourth component identified was the "increased responsibility teachers have to understand their mathematics in breadth and depth" (2010 p. 136). She notes as an example of this:

> *"As with instructional flexibility, depth in content knowledge provides teachers with the ability and flexibility to explore, emphasize, or deemphasize various mathematical topics that may arise in the course of instruction and investigation. When a student, using a graphing calculator, discovers an interesting fact about the slope of a tangent line while graphing quadratics, the teacher must decide if the findings are relevant and worth pursuing or tangential and best left alone. This requires content knowledge of not only functions and derivatives, but also a broader understanding of mathematics, the mathematics curricula, and where/ how derivatives fit into the scope and sequence of both" (Guerrero, 2010, p. 136)*

While the first three constructs Guerrero identifies (teachers' conceptions of technology, their willingness to adjust their instruction, and their willingness to adjust their management style) are rather generic (in that these constructs are not specific to math classrooms, nor do they represent new knowledge about technology use), it is this last construct, the increased need to deeply understand the mathematics, which begins address specific interplay between mathematics and technology; yet, even this construct is still far too general. As part of this study, it was important to tease apart ways in which TPCK lacks specificity as a framework in the mathematics classroom, and particularly in understanding the role of instructional practices. In the same way that Ball et al (2008) identified missing components in Shulman's conceptions of the interplay between content and pedagogy, I am arguing that there are similar pieces missing in describing the interplay of technology and mathematics.

Guerrero (2010), for example, considers the need for teachers to choose specific tools (e.g., graphing calculators versus Geometer's Sketchpad) based on the goals of the class to be a specific technological pedagogical content knowledge. Yet, even non-mathematically oriented TPCK research includes this as a general technology content issue - understanding and selecting the most appropriate technology tool. On the other hand, using simulations as the technology tool enabled me to study the interplay of technology and mathematics that is *not* specifically identified in TPCK. For example, my theory is that while technology provides the opportunities for relational reasoning, there is a mathematical technological pedagogical knowledge needed to support instructional moves that connect between formal/symbolic and relational representations. Similarly, the design of specific representations within the simulation may lead to instructional practices that go beyond basic use of the technology, and take into account additional knowledge bases that enable teachers to make connections between the mathematics associated with the technology and pencil/paper methods.

2.4 CONCEPTUAL MODEL

This study brings all three of these frameworks into play, drawing upon the assumptions that teachers' instructional practices are determined by teachers' mathematical pedagogical content knowledge as well as the developed and enacted task, and are mediated by the use of technology.

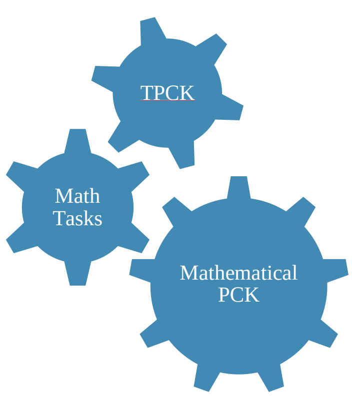


Figure 2-3 Conceptual Model

3 CHAPTER 3: LITERATURE REVIEW

Past research has indicated that teachers' instructional moves in math classrooms – the ways teachers connect past topics to new topics, take up student ideas, encourage multiple solutions, etc. – are positively correlated to improved performance by students (M. D. Boston & Wolf, 2004; Jackson & Cobb, 2010; Luna, Russ, & Colestock, 2009; Moschkovich, 1999; Stein, Engle, Smith, & Hughes, 2008). These instructional moves are consistent with a social constructivist theory of learning (as described by Palincsar, 1998), which characterizes learning as a social enterprise, where students construct mathematical expertise through discussions and interactions with the mathematical tasks rather than learning as a more individualized process where the interaction is only with a teacher or book. When considering how to assess instructional moves specific to technology-based mathematical tasks, it is important to consider the tasks, the technology, and the interplay between tasks and technology as created through simulations. In this section, I will first consider general research on the creation and enactment of mathematical tasks in the classroom, and then delve into the use of technology in classrooms (both generally as well as specifically to math classes). Finally, research specific to the use of simulations will be discussed.

3.1 MATHEMATICAL TASKS IN MATHEMATICAL CLASSROOMS

A mathematical task is a problem (or set of problems) that focus student attention on a particular mathematical concept (Stein et al., 1998). Teachers select tasks based on the tasks' ability to meet the needs of the teaching goals, with different types of tasks leading to different opportunities for student reasoning. For instance, a teacher looking to teach students how to justify their solutions to a mathematical problem will need a task that is complex enough to support this type of discussion. At a different point, where a teacher is focusing on fluency in a particular skill, a worksheet may sufficiently support student work.

Mathematical tasks have been studied in detail by Mary Kay Stein and others (Henningsen & Stein, 1997; Silver & Stein, 1996; Stein, Remillard, & Smith, 2007; Stein et al., 1998, 2009), who recognized that tasks were enacted upon by teachers, and that tasks flowed through a progression from the initial task as written or designed, to the final task as it was enacted within the classroom. Tasks significantly impact student learning since much of what students do in class is, in fact, working on tasks (Doyle, 1988; Stein et al., 1998).

It is the richness of those tasks – and the cognitive demand and mathematical knowledge required to complete those tasks – however, which ultimately determines the type of learning which occurs. Cognitive demand can be seen as a spectrum from basic memorizing and algorithms to complex thinking strategies to solve mathematical tasks (Henningsen & Stein, 1997). Cognitive demand is determined first from the actions listed in the initially designed task, but can be modified through the actions of both the teacher and the students (Henningsen & Stein, 1997; Sowder, 2007). Some researchers have argued that moving to tasks with higher cognitive thinking will require a significant shift as schools will need to "view the work of teaching as supporting students' development of core mathematical ideas and associated procedural competencies over the long term" (Cobb & Jackson, 2011a, p. 185). Furthermore, the ability of a task to engage the student in such a way that learning can be accomplished is dependent on both the actions of the teacher as well as the actions of the students (Goldman, 2009; Sowder, 2007).

Milan Sherman, in his dissertation (2011), studied the use of technology in high school classrooms using Stein's framework. His work focused specifically on understanding the changes being made to cognitive demand (from low levels such as memorizing or procedures without connections, to higher levels such as procedures with connections and doing mathematics) during task enactment in geometry-based classrooms using dynamic geometry software packages such as Geometer's Sketchpad and GeoGebra.

Sherman based his work on Pea's notion that technology could be used as an amplifier (in that it enables the user to perform tedious tasks more efficiently) or as a reorganizer (in that it facilitates changes in student thinking through its enhanced representations) (Pea, 1985). In other words, technology (as an amplifier) could be *used* in the classroom in ways that were peripheral to the task and were replicable with pencil and paper. Alternatively, technology (as a reorganizer) was *integrated* or *infused* into the mathematical task to support deeper conceptual understanding. While studying four teachers from different classrooms, he found that cognitive demand was not impacted simply through the presence of technology, but seemed instead to be associated with the intended use of the technology. Tasks that included technology only as an amplifier were less likely to maintain high cognitive demand, while tasks that included technology both as an amplifier ***and*** as a reorganizer did seem to increase or maintain cognitive demand.

While Sherman didn't study teachers' instructional moves specifically, he did note that teachers who misread students' readiness to use new technologies had tasks that declined to lower cognitive levels, while teachers who chose technologies that supported student self-monitoring had tasks that maintained their cognitive level.

Doyle (1988), who defines mathematical tasks as being any activity asked of a child in a classroom, argues that teachers play two key roles in the use of tasks – first that teachers are the selectors of the tasks, and therefore determine which tasks will enter the classroom, and second, that the teachers affect the level of cognitive demand actually enacted within a task. He raises a tension between what he refers to as "familiar work," or that work which is routine such as warm-up practices, or that which is governed by standard algorithms, and “novel work,” which requires students to find answers without standard algorithms and without explicit instructions or assistance from the teacher. He notes that this novel work (which would be comparable to the tasks in this research project) typically results in activity flow which is “slow and bumpy” with many mistakes and a high rate of non-completion. Consequently, he argues that, “novel work stretches the limits of classroom management and intensifies the complexity of the teacher’s tasks of orchestrating classroom events,” (Doyle, 1988, p. 174). As a result, he found that teachers were incentivized to use more “familiar work” in order to smooth out the curriculum flow and keep production rates high in the classroom, and acknowledged that this may result in students memorizing algorithms and interpreting problems as requiring purely computational results, rather than using logic to solve them. He ended noting that, “any attempt to reform teaching in mathematics must come to grips with the situational focuses that shape the curriculum and hold it in place as a classroom event,” (Doyle, 1988, p. 179). These recommendations were largely supported through later reform policies in both the revised NCTM standards (Heid & Blume, 2010), as well as the Common Core State Standards for Mathematics (*Common Core State Standards Initiative*, 2009).

As a result, a rich mathematical task is not only a matter of selecting those tasks requiring high cognitive demand; instructional practices play a key role in the selection and enactment of mathematical tasks. By instructional practices, I’m referring to the actions taken by the teacher during the enactment of a class. These can be less visible, such as managing basic classroom features such as organizing classroom space, and managing classroom time, or more visible, such as addressing student errors, orienting a student to a particular strategy, or eliciting ideas. Below, I discuss a number of instructional practices that have been found to support student learning in the enactment of mathematical tasks (Henningsen & Stein, 1997; Lampert, 1990).

3.1.1 Group-Worthy Tasks

It is not only the rich mathematical tasks that are an important component to student education; it is also the rich robust mathematical discourse that enables students to move from procedural understanding to conceptual mastery. To enable this rich discussion, students should have opportunities to work cooperatively in groups. As such, teachers should also select 'group-worthy' tasks to support this student-to-student discourse. Horn (2005) defines these tasks as those that illustrate important mathematical concepts, allow for multiple representations, draw effectively on the collective resources of a group, and have several possible solution paths. Boaler (2008) argued for this idea of group-worthy tasks as a means to a multidimensional classroom where success could be found in multiple ways. She noted that "when there are many ways to be successful, many more students are successful" (2008, p. 630). This argument is also made by Lotan (2003, p. 72,75) who states that "carefully constructed group learning activities can foster students' academic and social growth and help close the achievement gap," while noting that teachers must still address problems associated with "unequal access to the curriculum and unequal participation."

The use of group-worthy tasks leads to additional work for the teacher. Additional research found that these tasks required the teacher to provide roles or training to the students, enabling students to be active participants (Esmonde, 2009a). As a result, the use of group-worthy tasks does not end the role of the teacher in the classroom. In fact, Staples argues that "nearly all of the work that the teacher does with respect to *supporting students in making contributions* and *establishing and monitoring a common ground* also shapes the trajectory of the mathematical discussions while centralizing students' thinking and can be considered part of the teacher's role in guiding the mathematics" (Staples, 2007, p. 51 emphasis in original).

Another key concern with the use of group-worthy tasks is the equitable access to participation and content when using cooperative learning methods. While group work can enhance the conceptual understandings of the material as discussed above, when not implemented with care, "group work may exacerbate equity issues in the classroom by supporting students who are already successful while leaving less successful students behind" (Esmonde, 2009b, p. 248). Esmonde raises important concerns about how students learn to work together, and how their actions have impact on the students with whom they are working. She found that when students worked cooperatively and equitably, there would indeed be productive learning. This learning, however, began to decrease when the students believed there was an 'expert' within their group. The issue with expertise was two-fold. First, students would look to and rely upon those experts for the 'right' answer. Second, students believed that the experts in their group were the ones who could achieve the right answer – not necessarily the ones who could explain their thought process and make connections to others' ideas. As a result, Esmonde (Esmonde, 2009b) found that "some novice students actually set aside their prior understandings and displayed less mathematical competence at this task than they had prior to their interaction with the expert" (p. 274). Consequently, students who came into the group as an expert gained the most learning from the work. She argues that this is perhaps because "the expert students were likely to take on the bulk of the work and focus much of the interaction on explaining their own ideas," (p. 274) and recognized that this focus on one student's ideas resulted in a neglect for understanding the perspectives of others.

To create more equitable group learning environments, Esmonde seemed to indicate that there are two key strategies that should be employed. First, teachers must change and disrupt the traditional group projects such that students begin to value and look for solid and rigorous explanations rather than correct answers. This can be accomplished by changing how the group tasks are graded, or by placing an emphasis when grading on multiple representations, which support the same correct answer. Second, teachers must reposition students who are not yet experts with supports for their explanations. Esmonde argues that this will move students from a position of “not understanding” to “understanding but not explaining well” (2009b, p. 276).

While Esmonde argues for changing the perspective of what makes an expert, other researchers (Boaler & Staples, 2008; Jackson & Cobb, 2010; Ladson-Billings, 2010) have argued that teachers can make group work around math tasks more equitable by adding more supports when introducing the task, thus removing the role of experts entirely. This introduction of the task must bridge the gaps in initial understanding of both the mathematical features of the task, as well as the cultural suppositions on which the task is predicated. The other point raised by some of these researchers (Jackson & Cobb, 2010) is that many students may still be unfamiliar with the goals and practices of group work. It is important to create and model norms for the classroom that facilitate cooperative learning in ways that promote equity and learning. Jackson and Cobb use research from Wood & Yackel (1990) to describe this point.

> *“Their analysis highlighted the teacher’s role in listening to students’ exchanges and interjecting to maintain the dialogue between students. Crucially, they clarified that the intent of the teacher’s interjection was not to explain one student’s solution to another student. Instead, their analysis implies that teachers should make comments or ask questions in order to support the students in verbalizing their solutions, listening to others’ solutions, and reaching consensus about solutions.” (Jackson & Cobb, 2010, pp. 19–20)*

Finally, a related norm specific to multi-lingual classrooms is determining the language that groups will use for their mathematical discussions. Researchers found that it was important for ELL students to begin their group work in their first language, but that the teacher should scaffold support toward development of English mathematical language (Jackson & Cobb, 2010) thereby connecting students' ideas from their native language to ways of expressing their ideas in English.

All of these examples illustrate ways in which instructional practices play a key role in the productive learning within math classrooms. These studies indicate that there are specific instructional moves that are often associated with productive group work in classrooms that may also be present in technology-based activities.

3.1.2 Rich Discussion of Mathematical Tasks

As noted in the previous section, inherent in creating and enacting cognitively demanding mathematical tasks in a group environment is the need for rich, robust discussions among the students. Of course, this discourse cannot occur without group-worthy tasks that invite students to collaborate and justify with and among one another to further their learning.

Consistent with the findings of researchers studying group work in mathematics, researchers studying the discussions in these groups have found that the establishment of group norms is essential to success (Hufferd-Ackles, Fuson, & Sherin, 2004). These norms must enable students to "reason, defend and prove their conceptions to one another" and teachers to "to listen to their students, to draw out students' ideas, and to encourage students to listen to each other" (Hufferd-Ackles et al., 2004, p. 113). McCrone argues that developing mathematical discussions is a complex process that involves not only a solid choice of tasks but also a "growing communicative competence" among the participants (2005, pp. 131–132). Within the process of mathematizing, students learn to use the accepted academic language inherent to mathematicians, enabling the use of formulas as well as the ability to justify various graphical representations as relevant models. Teachers play a key role in how these discussions progress, creating a microculture within the classroom as described below:

> *These sociomathematical norms are intrinsic aspects of the classroom's mathematical microculture. Nevertheless, although they are specific to mathematics, they cut across areas of mathematical content by dealing with mathematical qualities of solutions, such as their similarities and differences, sophistication, and efficiency. Additionally, they encompass ways of judging what counts as an acceptable mathematical explanation. (Yackel & Cobb, 1996, p. 474)*

Hufferd-Ackles et al (2004) also argue that the classroom will progress along a continuum as they all (teachers and students) transition to a space that encourages and facilitates rich mathematical discussions. These researchers also suggest that as these classroom norms become more commonly accepted, the responsibility for learning shifts from the teacher to the student.

> *"When student thinking began to be elicited, students became more engaged and involved in classroom discourse as speakers and listeners. Their responsibility for their own learning was indicated by their desire to ask questions in class, their eagerness to go to the board to demonstrate their understanding of problems, and their volunteering to engage in the work of and to assist struggling students at the board. Students grew to expect that their mathematics contributions would be positively received by the teacher and by other students. Having students' ideas in the classroom discourse space enabled students to help each other." (p. 108)*

While there is inherently a key piece of this responsibility that lies with the teacher, McCrone (2005) also argues that this is a shared responsibility with the student which is often overlooked, "yet has been shown to be an important factor in establishing a classroom environment that promotes mathematical discussions" (p. 132). Once students are comfortable sharing their own mathematical ideas, other researchers acknowledged an additional tension between supporting the process of mathematical discourse while facilitating discussions of mathematical content (Sherin, 2002). They found that students became so willing to explore different ways to mathematizing, that some teachers were not able to cover the mathematical curricular goals. Recognizing that the role of the teacher in facilitating group work with rich mathematical discussions is not an easy one, Sherin suggests a filtering methodology, where teachers elicit responses from students and then filter those responses in terms of the mathematical content to be covered, describing it this way:

> *In this approach, multiple ideas are solicited from the students in the initial phase. Students are encouraged to elaborate their thinking, and then to compare and evaluate their ideas with those that have already been suggested. The filtering part of the discussion comes next, as the teacher focuses the students' attention on a subset of the ideas that have been raised. In addition, the teacher may introduce a new mathematical idea or approach that the class can use to consider the focused content. This focusing on the part of the teacher is then followed by additional idea generation on the part of the students. A single class discussion may involve several cycles of this pattern. (2002, p. 227)*

In the long run, the goal becomes promoting student learning through the sharing of ideas, while teachers focus this mathematizing on the content being taught. This, therefore, becomes a learned skill for the teacher. When training pre-service teachers, researchers have found that practicing and rehearsing these strategies leads to a much higher rate of success in the classroom (Hunter & Anthony, 2012; Lampert et al., 2013). One of the other benefits of training and practicing for pre-service teachers was an increase in teacher noticing (Hunter & Anthony, 2012), where teachers choose what they attend to in a classroom discussion, and what is ignored.

Ultimately, researchers seem to agree that rich mathematical discussion comes from establishing expectations, encouraging student responses, and monitoring discussions to ensure connections between mathematical concepts (McCrone, 2005; Moschkovich, 1999; Stein et al., 2008; Yackel & Cobb, 1996). Again, these instructional practices may be similarly productive in technology-based mathematical tasks.

3.1.3 Other instructional practices

It is difficult to fully understand all of the instructional practices happening in the classroom as many of these practices occur behind the scenes. Consider this vignette (taken from Stein et al., 2008)

> *Leaves and Caterpillars Vignette*
>
> *Students in David Crane's fourth*-grade class were solving the following problem: "A fourth-grade class needs five leaves each day to feed its 2 caterpillars. How many leaves would they need each day for 12 caterpillars?" Mr. Crane told his students that they could solve the problem any way they wanted, but emphasized that they needed to be able to explain how they got their answer and why it worked.*
>
> *As students worked in pairs to solve the problem, Mr. Crane walked around the room making sure that students were on task and making progress on the problem. He was pleased to see that students were using lots of different approaches to the problem—making tables, drawing pictures, and, in some cases, writing explanations.*

This vignette is written about the teacher's instructional moves and portrays the presumed reasons for these moves. For example, it states that Mr. Crane *"walked around the rooms making sure that the students were on task and making progress on the problem"* but in an observation, no determination of Mr. Crane's intent is possible. All that is notable is that he walked around the room. It may be that as Mr. Crane walked around, he prompted students to get back on task, or commented on student progress. Yet without these verbal cues, the rest of the description is all conjectures about his intent and goals.

Therefore, the task of identifying instructional practices is a challenging one. As discussed above, researchers seem to generally agree that there are three key instructional moves for teachers: supporting mathematical classroom discussions, developing classroom norms, and making mathematical connections (both through linking representations and sense making) (Franke, Kazemi, & Battey, 2007; Jackson et al., 2012; Lampert et al., 2013; Staples, 2007; Yackel & Cobb, 1996).

Supporting rich mathematical discussions is an important instructional move in mathematics classrooms. Lampert argues that it is not enough to identify rich mathematical discussions as the goal – instead, novice teachers need skills that build up to this practice. For example, Lampert et al. (2013) used codes to note particular instructional practices that supported productive mathematizing in the math classroom, such as attending to mathematical thinking, eliciting and responding to student work, surfacing and responding to student errors, and assessing understanding.

These codes are well-aligned with Stein's framework of task enactment and the associated instructional moves, yet go beyond those specific to the mathematical portions of the task, to focusing also on task enactment and management including managing time, space, as well as body and voice use.

Stein et al (2008) have recommended instructional practices as well, suggesting that novices need to build the following five skills: anticipating student responses, monitoring student actions, discuss and summarize student work, purposefully sequencing the student work to be discussed, and helping students make connections between these different artifacts and the mathematical content. Other researchers (Baker, Gersten, & Lee, 2002; H. C. Hill, Rowan, & Ball, 2005) have suggested that the ways in which teachers provide remediation is an important aspect of instructional moves as well.

3.1.4 Summary

There is significant research that describes how teachers select and modify mathematical tasks in the classroom, and the ways in which instructional moves support student learning when using these tasks. This study takes up these ideas of the instructional moves used by teachers to support student learning, focusing on these practices in the context of technology-based math activities.

3.2 TECHNOLOGY

> In Sciences of the Artificial (1969), Herbert Simon describes a scene in which an ant is walking on a beach. Simon notes that the ant's path might be quite complex. But the complexity of the path, says Simon, is not necessary a reflection of the complexity of the ant. Rather, it might reflect the complexity of the beach. (Resnick, 1999, p. 135)

This quote is an important reminder that there are many complexities to the use of technology in classrooms. Some of the complexities are inherent in the tools being used, while others are a function of those who use the tools. This section identifies literature that elucidates the complexity of using technology in classrooms.

The degree to which teachers use technology in the classroom has been studied extensively (Al-zaidiyeen & Mei, 2010; Gray et al., 2010; Wachire & Keengwe, 2011; Zbiek, Heid, Blume, & Dick, 2007). Technology-based mathematical activities are more than just math tasks – they are mathematical tasks that are enriched by the use of specific technological tools designed to be used by students, such as graphing calculators and pre-made simulations. Understanding the use of these tools, and how that use affects mathematical tasks in the classroom, is critical.

3.2.1 Benefits of use

The most noted benefit to using technology in math classes is representational fluency (Zbiek et al., 2007). The ability to facilitate students making connections between different representations can be (depending on the design of the tool) a strong support for conceptual understanding (Heid & Blume, 2010; Hollenbeck, Wray, & Fey, 2010; Zbiek & Hollebrands, 2008). Many simulations also offer students a degree of visual clarity not possible through other mediums (Passey, 2011). Building on this idea of visual clarity, other researchers noted that the dynamic responsiveness of the tools enables students to not only manipulate objects, but also to obtain immediate feedback on their conceptions of the mathematics involved (Hollenbeck et al., 2010; Zbiek et al., 2007; Zbiek & Hollebrands, 2008). The use of technology has also been argued to support student engagement and build motivation and interest in mathematics (Laborde, 2011; Zbiek et al., 2007; Zbiek & Hollebrands, 2008).

3.2.2 Barriers to use

Several meta-studies illuminate common barriers to technology use. A review of articles (Hilary & Risser, 2011) from two mathematics journals conducted to discern key arguments against using technology in classrooms found three main concerns: curricular (e.g. would using technology replace basic skills that all students should have?); developmental (e.g. would the use of technology impair needed mathematical development of students and perpetuate - or create – misunderstandings?); and a perceived lack of benefits of technology based on cost of implementation. Another more general review of technology use (Jones, 2004) found that barriers included: insufficient access to technology (either because it was not available, or was poorly organized); lack of confidence, and competence; inadequate software; insufficient support for planning (both in access to materials, as well as time); and the potential for technical issues causing teacher fear.

While access to technology is often raised as a concern by teachers, studies consistently find that there is significant technology in classrooms, especially when considering computers and graphing calculators; however, studies also find that student use of technology is sporadic, and directly tied to teachers' attitudes and perceptions toward that technology. The use of technology by students (with the exception of graphing calculators in high school classes) is still quite limited (Al-zaidiyeen & Mei, 2010; Hogarty et al., 2003; Vu & McIntyre, 2013; Zbiek et al., 2007). So why isn't technology use more prevalent in the classroom? Barriers within the control of the teacher come in the form of personal concerns (teacher beliefs about roles) and management issues (teacher beliefs about practices). It comes as no surprise that the more positively one perceives technology, the more likely one is to use it in the classroom (Al-zaidiyeen & Mei, 2010; Ertmer, Addison, Lane, Ross, & Woods, 1999; Harris, Mishra, & Koehler, 2009; Li & Ma, 2010; Roschelle et al., 2010). Conversely, the more likely teachers see themselves as the authority in the classroom, the less likely that technology will be used as an investigative tool, and the more likely that that it becomes prescriptive in its use (Zbiek & Hollebrands, 2008).

Including technology in math classrooms brings its own set of challenges. In a college setting, researchers (Hora & Holden, 2013) found that science teachers (physics and biology) were more likely to use technology in classrooms; Math professors, on the other hand, saw technology as a disruption to traditional disciplinary methods. In most cases within the math classrooms, technology was primarily used to replace that which could be done with pencil and paper (for example, the use of digital tablets as a replacement for a chalkboard to facilitate storing notes online). Specifically of interest when considering integration of technology into math classrooms was the ability of the technology to facilitate writing mathematical equations and expressions with ease. Researchers speculated that as the technology grows to allow equations to be easily written through technological tools, teachers' integration of technology would grow as well. This same research found differences in the use of technology: math teachers were looking for differences between expert and novice reasoning (as described by Smith III, diSessa, & Roschelle, 1994), whereas science teachers were looking at a more general sense of conceptual understanding by the student.

As a result, understanding how teachers in this study perceived technology use within their classroom as well as understanding their curricular needs provides some insight into their curricular choices.

3.2.3 Location of the technology

While it does not directly influence student learning, the location of the technology plays a key role in how technology is integrated into classrooms. For most new curriculum implementations, the curricular resources are available in text form, or in manipulatives that remain in the classroom – this is not true for technology. While some technology, such as graphing calculators, may reside in the classroom, other technology accessible through computers may only be available when students can access the computer lab, or gain access to the computer cart. As a result, while teachers may be willing to use technology, the access to it may be severely limited – a problem made even more complex with the advent of online standardized testing – which may change how, or even if, a teacher uses technology. Furthermore, Monaghan (2004, p. 341) noted that the move to a computer lab also changed the enactment of lessons, saying, "Teachers who did not have computers in their class mainly planned 'all or nothing' computer-based lessons, i.e., all the lesson on the computer or no use of the computer at all." Therefore, capturing evidence on the location of the equipment may provide insight into both the use of technology, as well as ways in which lessons are planned and enacted.

3.2.4 Who uses the technology

As noted earlier in Chapter 1, who has hands on the technology directly affects task enactment. You can imagine two different scenarios (adapted from Hollenbeck et al., 2010). In one extreme, students each have their own device, and are encouraged to do individual explorations using the technology provided. In the other extreme, the teacher 'drives' the technology, using projection equipment to ensure all students can view the tools. In the first scenario, students directly engage with the technology – in the second scenario, only the teacher engages with the technology. In this study, I'll be focusing on computer-based simulations specifically designed to support student use.

3.2.5 Ways of Solving with Technology

One of the largest potential benefits in using technology is that students can solve mathematics problems in a variety of ways (Hollenbeck et al., 2010); yet, this is highly dependent on the tool one selects. As discussed earlier, technology can essentially be an electronic replacement of a worksheet (its only benefit is that the student will receive instant feedback if the answer is incorrect), or support a constructivist environment (as described in Simon, 1995), allowing students to develop their own strategies for solving problems (Zbiek et al., 2007). Research by Monaghan (2004) noted similar concerns when investigating teachers' use of technology in math classes. The use of graphing calculators was one way technology was incorporated; however, its use varied widely by teacher. While some teachers used it to explore a variety of graphs, others treated it more like a teacher-directed task, providing step-by-step keystrokes in a closed activity. Despite these concerns, many researchers have noted that technology can support students' ability to solve problems in a variety of ways, such as with the use of dynamic geometry software, which enables students to view and manipulate shapes in ways that are not possible without technology (Heid & Blume, 2010; Passey, 2011) leading to new ways of student reasoning.

3.2.6 Impact on students

Overall, when considering technology use in math classrooms, a recent meta-analysis of educational experiments with technology-infused mathematical tasks (Cheung & Slavin, 2013) found that the results were inconclusive. Only 36% of the research conducted used randomized experiments, and those that did had lower effect sizes. Higher effect sizes were consistently found with small sample sizes, as well as longer implementation times. Overall, educational technology had a larger effect on elementary students than high school students (related to the fact that many activities were essentially routine drills). Hoyles and Noss (1992) attribute a lack of effectiveness to what they call "Play Paradox" where students miss out on the mathematics when integrating technology. None of the studies included in this research however, was simulation based.

3.2.7 Technology and Teacher Practices

While the research specific to teacher practices inherent in the use of simulations is minimal (Zbiek & Hollebrands, 2008), there is still some research on general technology use by teachers that we can draw from. Most of the research findings agree that teaching with technology is different from teaching without technology. As mentioned earlier, teachers' beliefs about their role in the classroom will influence the tasks selected, as well as the enactment of those tasks. Those who see the teacher as the authoritarian in the classroom (what Zbiek et al (2007) referred to as "Traditional Transmission Instruction") are more likely to provide explicit instructions, and focus on the procedures involved in the use of the technology. This can lead to more focus on the technology than on the math (Monaghan, 2004). On the other hand, teachers who were more likely to use a “Constructivist Compatible Instruction” method (Zbiek et al., 2007, p. 1188) were both more likely to use technology, and use it as an investigative tool.

Many researchers also noted that teachers took on new roles when technology was used (Drijvers, Boon, & Reeuwijk, 2011; Ertmer et al., 1999; Farrell, 1996; Heid & Blume, 2010). While they used different terms in each study, the ideas that were similar were two-fold. In some classrooms, teachers’ roles were fundamentally about management, organization, and the control of the technology. On the other end of the spectrum were teachers who took on the role of facilitator, supporting student mathematizing.

Existing research also lends insight into specific teachers’ instructional moves that may support student learning. As mentioned above, taking on a more facilitator-oriented role enables teachers to focus on the mathematical objectives rather than the tech-oriented procedures (Heid & Blume, 2010). Drijvers et al (2011) found that teachers could support learning through key strategies such as questioning, summarizing, and supporting exploration and reasoning of mathematical results. Others (Ertmer et al., 1999; Zbiek & Hollebrands, 2008) noted that classroom structures needed to be adjusted to facilitate work with technology.

One of the biggest changes for teachers was the need to revamp the planning process. Since math classes are often heavily textbook dependent, the process of planning a math lesson can be straightforward for many teachers. The teachers identify the lesson number, and the problems to be covered both in class, and for homework (Monaghan, 2004). When using technology, the teaching cycle radically changes, requiring teachers to plan for significantly longer time, and find new ways to assess student progress (Drijvers et al., 2011; Ertmer et al., 1999; Monaghan, 2004). Technology changes what happens in the classroom in other ways as well. For example, teachers who successfully incorporated technology in math classes did so in ways that connected the technology to pencil/paper methods, and provided opportunities for students to solve problems in three ways – mentally, pencil and paper, and with technology – and then be able to discuss the pros and cons of each method (Ball & Stacey, 2005; Drijvers et al., 2011).

One way to conceptualize this interplay for teachers is the following equation (from Hill & Schrum, 2002):

BELIEFS + THEORY + CURRICULUM = TEACHING METHOD

Teachers' beliefs, along with their own theories about teaching and learning, coupled with the curricular goals result in a teaching method that is different with technology, as compared to teaching without technology.

Yet the research shows that there is much more to this equation. The teaching method (and associated instructional practices) are also likely influenced by the students themselves as well as the school context. Addressing that question is the central focus of my study.

3.3 GENERAL USE OF SIMULATIONS IN CLASSROOMS

As discussed earlier, simulations can support inquiry-based approaches to science and mathematics instruction, particularly in ways that support and facilitate visualization and exploration of real-world phenomena (Doerr & Pratt, 2008; National Research Council, 2011; Rutten et al., 2012). The research on the implementation of simulations, however, has been mixed. Findings suggest that STEM integrated approaches, for example, tasks that incorporate both math and technology, "benefit [the students] who already have knowledge pertinent to the integrating elements, whereas individuals with limited knowledge are less adept at building connections among conceptual structures" (National Research Council, 2014, p. 80); therefore these student-student interactions in inquiry-based simulation activities may only benefit a portion of the classroom. On the other hand, where simulations are working well in math classrooms, research has shown that there are some design features that support learning, including designing based on a proposed learning trajectory, developing a model for conceptual understanding rather than rote practice, and designing activities that promote desired strategies for the learning outcomes (Drijvers et al., 2010, 2011; Laborde, 2011; Sarama & Clements, 2008).

3.3.1 Use of simulations in mathematics classrooms

In other cases, math is often just a tool to enable the student to engage with more complex (often scientific) material. An example of this is the PhET Simulation titled Bending Light[3]. In this simulation, focused on the refraction of light through different mediums, students are provided with several mathematical tools, including a protractor to measure the angle of refraction. However, the intent of the simulation is not to highlight or study the associated mathematics, but rather to use the mathematical concepts to support scientific learning. On the other hand, simulations can be used to support students in more deeply engaging in mathematical concepts, making math the goal, rather than the tool (Drijvers et al., 2011; Ogborn, 1999). Particularly when using simulations in math classrooms, researchers have highlighted the need to understand how math is being situated within the simulation, and within the task. Ogborn (1999) argues that depending on the use of the technology, math can be a tool, such that a student needs to learn the math in order to understand the simulation. Conversely, mathematics can be the goal of the simulation, driving the need for students to learn even more mathematics.

When mathematics becomes the goal of the simulation use, such as in the case of JavaBars, Geometers Sketchpad, and MS Excel, researchers found that these technologies bridged between

[3] See Appendix B for a screen shot.

formal and informal views of mathematics, as well as differentiated and integrated teaching (Belbase, 2015).

In a paper documenting the results of the use of a virtual TI InterActive instructional environment, Bos (2009) argues that when looking at technology, researchers must evaluate whether it is "pedagogically sound, mathematically true, and cognitively defined to deepen understanding" (p. 521). Pedagogical fidelity was the ability of the technology to support learning, rather than distract from it. Mathematical fidelity is the "disconnect between what is mathematically correct and what is mathematically possible" (p. 522). Cognitive fidelity is the degree to which the technology aides in making a concept better understood. She suggests that simulations that meet these criteria will be more effective. When math is not the goal, the tool lacks math concordance for the teacher, meaning that the goal of the teacher does not match the work of the students (Zbiek & Hollebrands, 2008). Others (cf. Hoyles & Noss, 1992) took up this same idea, referring to it as play paradox, where students played with the tool, but did not take away mathematical meanings.

One example of simulation that has been studied extensively in math classrooms is the SimCalc research program that used MathWorlds to facilitate student understanding of “mathematics change and variation” (Roschelle et al., 2000, p. 5). As part of this work, a number of researchers have studied the instructional practices used by teachers (e.g., Tapper, 2013; Zahner, Velazquez, Moschkovich, Vahey, & Lara-Meloy, 2012). From this work, several common themes were apparent. The instructional practices that appeared to support student learning included alternating between whole class discussions and individual/group work time, making connections between the tool and other representations such as a story that matched the graph, and addressing student errors through additional questioning techniques (Zahner et al., 2012). Pierson’s research

(2008) found that teachers who employed class discussions with high levels of responsiveness to student ideas when using SimCalc had higher than average gains when testing students before and after the use of the SimCalc curriculum. Further research into this conclusion (Vahey, Lara-Meloy, Moschkovich, & Velazquez, 2010) found that teachers' use of mathematical language during discussions (including clarifications when needed, and interwoven with everyday language to support conceptual understanding) as well as strong connections between representations again supported student learning. These findings are consistent with other research on instructional moves that support student learning in non-technology infused math classes.

3.4 SUMMARY

This literature review, while not exhaustive, has been designed to provide an overview of prior research related to the use of instructional practices by teachers during both non-technology based tasks as well as technology-based mathematical tasks. I discussed prior research on the instructional practices found to support student learning in math classrooms, and how these practices may change from the design of the tasks through its enactment. Later, I discussed the role of technology in the classroom, and the ways in which it has been found to support or disrupt these practices. The final section gave some insight into research on using simulations in mathematics classrooms, and the types of instructional practices that continued to support student learning.

The two most similar studies to mine are those by Monaghan (2004) and Sherman (2011). Both of these studies considered teachers' choices in task selection and task enactment, with Monaghan noting that the planning and teaching cycle of technology infused tasks varied from that of non-tech based activities, and Sherman noting that teachers' choices, during the selection and enactment of technology-infused tasks, could (and often did) lessen the cognitive demand of the mathematical task. Sherman did account for some instructional moves that both maintained cognitive demand (such as requiring students to interpret and explain work, as well as pushing students to use the tool to support mathematical reasoning) as well as reduce cognitive demand (such as failing to continue to build on the learnings from the digital tool, and failing to fully utilize digital tools, keeping the technology at an amplifier stage, rather than a reorganizer of student thinking). Both of these studies, however, identify the need for a closer look at teacher practices. Furthermore, both of these studies were specifically focused on high school geometry classes. My study differs from this prior work in that it is an effort to more deeply identify and understand the instructional moves used by teachers; specifically to the use of technology-based mathematical tasks to support algebra readiness in middle and early high school.

4 CHAPTER 4: RESEARCH METHODOLOGY

In this section, I detail the methods used that enabled me to study the ways in which teachers' instructional practices varied when using technology-based mathematical activities as compared to those used during non-technology oriented lessons, and how the features of the simulations impacted those changes. As noted earlier, instructional moves are influenced by teachers' beliefs, their theories about teaching and learning, the mathematical tasks, as well as the students and school setting. Therefore, in order to separate out the influence of technology on those practices, I chose to look at teachers' practices in non-technology based activities as well as technology-based activities to answer these research questions:

1. What observed instructional moves by teachers in math classrooms are unique technology-based activities created with student-oriented simulations?
2. In what ways do specific design features of student-oriented simulations impact instructional moves in math classrooms?

In this exploratory study, I worked to build interpretive cases that described how these instructional practices in technology-using classrooms were taken into account. As Eisenhart notes,

> In order to understand whether or how x causes y, it is first necessary to know what, exactly, x and y are, and how in actual practice x can exert an influence on y. […] Without good descriptions of how teachers and students use the resources available to them or compensate for the lack of resources, and without good descriptions of the internal dynamics by which use occurs or does not occur, experimental studies to identify causal relationships will be partial if not useless, and attempts to explain why or how causal agents work as they do are likely to fail. *(2005, p. 252)*

Therefore, in order to fully answer the research questions, I completed an in-depth investigation into four teachers' practices. The first goal was to understand these teachers' typical instructional practices when teaching mathematics in two different situations: *without* simulation-based mathematical tasks, as well as *with* simulation-based mathematical tasks. The second goal was to determine how specific features of the simulations used impacted those instructional practices. In the following sections, I will discuss the study participants, the collection of data, the analysis of that data, and the validation of the data analysis.

4.1 RESEARCH DESIGN

This study draws upon the conceptual framework that teachers' instructional practices are determined by teachers' mathematical pedagogical content knowledge as well as the developed and enacted task, and are mediated by the use of technology. To determine how instructional moves changed with technology-based mathematical activities, it was essential that I interviewed the teachers to accurately describe their mathematical dispositions and educational backgrounds. Second, I considered the mathematical tasks from their creation through their enactment, and observed the teachers sufficiently to describe their typical instructional practices. Third, I considered the simulation itself and the ways that it mediated those lessons. In doing so, my analysis draws on prior research, considering how others studied mathematical tasks, the impacts of technology on those tasks, and the instructional practices used with that technology. In the sections that follow, I describe the data sources, the collection of that data, the data management, the process of coding, and the analysis of that data.

4.2 DATA SOURCES

This study used qualitative methodologies to answer the research questions. Looking back to the research questions, the first research question sought to identify the instructional practices used by teachers that were specific to technology-based mathematical tasks. In order to separate out the features of the technology that influence instructional practices, it was essential for me to understand the typical instructional practices used in classrooms without technology-based lessons, and then compare those practices with the inclusion of technology. Prior research significantly informed the range of current instructional practices, so the focus here was not on identifying the practices, but rather on recognizing the change in practices.

The second research question looked at whether there were specific features of the simulation that led teachers to use particular instructional moves. This research question was informed by many of the same data sources identified for the first research question, as looking at the enacted task for technology-based mathematical activities provides significant insight into how features of the simulation impacted instructional practices in the classroom. Additionally, the analysis focused on those features of the simulation and the intent behind the design decisions as laid out in the project's Theory of Change narrative (Singleton & Shear, 2017).

A summary of the data sources, their purpose and their timing for the investigation is provided in Table 4-1, and these sources are described in detail in the following sections.

Table 4-1 Summary of Data Sources

Research Question		Lesson Timing	Lesson Planning Artifacts	Teacher Interview	Observation Video	Enacted Task Artifacts	Observation Field Notes	Lesson Reflection	Theory of Change	Analytic Memos
1. What observed instructional moves by teachers in math classrooms are unique to differing levels (used, integrated or infused) of technology-based activities created with student-oriented simulations?	Lesson Planning	Summer	X	X						X
	Non-Technology Based Lesson	During the lesson			X	X	X			
		After the lesson					X	X		X
	Technology Based Lesson	During the lesson			X	X	X			
		After the lesson					X	X		X
2. In what ways do specific design features of student-oriented simulations impact instructional moves in math classrooms?	Simulation Feature Analysis	N/A							X	
	Technology Based Lesson	During the lesson			X	X	X			
		After the lesson					X	X		X

4.2.1 Research Question 1

4.2.1.1 Lesson Planning Artifacts

Drawing on my conceptual framework and associated literature review, I knew that teachers often planned differently when using technology, both taking longer to plan lessons, and centering the lesson on the technology (Monaghan, 2004; Zbiek & Hollebrands, 2008). Therefore, prior to the lesson being enacted, I gathered data about the planned lesson. Each of the teachers had agreed to write their own lessons using the simulations over the summer. Meetings were conducted as a group using a virtual meeting tool called Zoom. Occasionally I would meet individually with teachers who were unable to participate in the group meeting. The lesson planning itself was done individually, but the teachers were all invited to comment on each other's lessons. We discussed the lessons as a group. These discussions were perhaps more limited than I had hoped, but not entirely unexpected. It is difficult to be critical of another colleague's lessons, and particularly so when the teachers do not know each other well. As a result, most of the comments about lessons were focused on ways to improve the lesson; for example, many times it was the addition of a question to ask the students, or a question along the lines of, 'have you considered…'. I was present at all of these meetings, and used a combination of field notes and transcribed conversations to document these discussions. Additionally, all of the teachers' lessons were written using Google Docs, which were shared with me. This enabled me to see their lessons as well as other teacher comments.

4.2.1.2 Teacher Interview

From Ball et al (2008), we also know that teachers' mathematical pedagogical content knowledge plays a key role in what happens in the classroom. Specifically, at the lesson planning stage, teachers' knowledge of content and students, as well as each teacher's technological pedagogical content knowledge can influence the planning of mathematical tasks. For this reason, I interviewed the teachers prior to their enactment of technology-based lessons to better understand their own conceptions about learning and teaching, as well as their beliefs about using technology in classrooms. The first set of questions (about the teacher) were designed to ensure that I had a sense of the teachers' experience, as well as the classes they would be teaching in the fall. The second set of questions (about the content) enabled me to draw inferences about teachers' mathematical pedagogical content knowledge, while the third set of questions (about the technology) enabled me to draw inferences about the teachers' technological pedagogical content knowledge. These interviews were audio recorded and transcribed. The interview protocol is attached as Appendix C.

4.2.1.3 Observations and Enacted Lesson Artifacts

As Stein et al (1998) found, the task as enacted often differs from what was originally planned. Therefore, it was important to note any differences in what was planned versus what was enacted. I did this through observations of the class itself, collecting artifacts of student work, and teacher reflections at the end of the lesson. Each teacher was observed both during non-technology classes as well as technology based classes. For the non-technology classes, I observed the three middle school teachers for 4 class period of 45-50 minutes each. I observed the high school teacher for 3 double class periods (90 minutes each). I also observed the technology lessons: two 45-50 minute lessons for two of the middle school teachers (Christine and Sara) and three 90 minute blocks for Lily. Only one technology lesson was observed for Pat. A summary of lessons observed is shown in Table 4.3 below.

Table 4.3: Summary of Observations

<table>
<tr><th></th><th>Cary</th><th>Pam</th><th>Bethany</th><th>Rebekah</th></tr>
<tr><td>Non Tech 1</td><td>5/17/2016</td><td>10/11/2016</td><td>10/24/2016</td><td rowspan="2">1/23/2017</td></tr>
<tr><td>Non Tech 2</td><td>9/26/2016</td><td>12/7/2016</td><td>10/26/2016</td></tr>
<tr><td>Non Tech 3</td><td>9/27/2016</td><td>1/23/2017</td><td>11/16/2016</td><td rowspan="2">2/13/2017</td></tr>
<tr><td>Non Tech 4</td><td>1/19/2017</td><td>2/21/2017</td><td>1/18/2017</td></tr>
<tr><td>Tech 1</td><td>10/6/2016</td><td>3/9/2017</td><td>11/18/2016</td><td>10/21/2016</td></tr>
<tr><td>Tech 2</td><td>11/18/2016</td><td></td><td>1/18/2017</td><td>10/25/2016</td></tr>
<tr><td>Tech 3</td><td></td><td></td><td></td><td>11/29/2016</td></tr>
</table>

Observations were all video recorded, focusing on the actions of the teacher, as well as the interactions between teacher and student. For the technology-based tasks, it was more difficult to note student actions using just the video-record of the lesson. As a result, teachers agreed to allow me to ‘wander’ through the class in order to see what students are doing with the technology. My goal was to capture how most students were using the technology, as well any differences in student use. While my intent was not to study student use of the technology, it was important to note what the teacher was likely seeing as s/he also walked around the classroom. To accomplish this, two cameras were used in most lessons: a stationary camera that resided in the back of the room, as well as a small GoPro camera that was held as I walked around the classroom. The stationary camera enabled me to focus on the class as a whole, or zoom in on student-teacher interactions. The GoPro camera enabled me to capture what the teacher saw as she walked around the classroom. A microphone was hooked up to the teacher, enabling me to hear her and her conversations with students, even in noisy classrooms.

All instructional artifacts were scanned so that these artifacts could be maintained as computer files. Artifacts included student worksheets (in digital form) as well as digital photos of worksheets and information written on the board. All information was de-identified prior to collection.

4.2.1.4 Observation Field notes

Handwritten field notes were created during each observation as added insurance against technical difficulties while video recording, as well as to capture interactions that might not be visible on the recordings. Later, using the original field notes along with the recordings, detailed summaries of the lesson were developed.

4.2.1.5 Teacher reflections

Originally, I planned to ask teachers to write a reflection about the lesson, with prompts specific to each lesson. The teachers all asked, instead, that we just talk after the lesson as any changes were fresh in their minds. We did this for all of the observations. I would ask teachers about their lesson, what went well, what did not, and what they changed and why they made those changes. These portions of the conversations were all recorded and transcribed enabling me to reflect on their answers.

Additionally, after the lesson, specific moments from the observation video were reviewed and discussed by both the teacher and researcher. The goal here was twofold. First, I wanted a member-check to ensure that my interpretation of their actions was appropriately described.

Second, I wanted to look more deeply into specific moments when teachers have chosen particular instructional moves.

4.2.1.6 Summary

To address RQ#1 (understanding the variance in instructional practices), the final data sources included an interview for each teacher, as well as field notes from a series of lesson planning meetings. For each teacher, both non-technology based and technology based lessons were observed. This aligned with the validation studies of the MQI that found that a minimum of two observations are necessary for consistency in scores, with slight improvements in the consistency with four observations. Student artifacts along with teacher reflections complemented the lesson data sources, providing me with multiple sources to support confirmation of my claims.

4.2.2 Research Question 2

The second research question looked at whether there were specific features of the simulation that led teachers to use particular instructional moves. To help determine this, I used three different sources of information: observations, teacher reflections, and an analysis of the projects' Theory of Change (Singleton & Shear, 2017).

Through the observations, I was able to see how the simulation functioned in the classroom and reflect on how this operationalization was reflected in the theory of change. Furthermore, teachers spent a lot of time thinking about and talking about how the simulation worked in their classroom. These conversations were very informative in understanding the features of the simulation that impacted instructional moves.

I also used the results of an organizational framework for a related PhET study creating a Theory of Change for PhET (See Appendix D and E). A theory of change can be thought of in this way:

> *Mindfully or not, people are theorists of change. That is, they are theorists insofar that they engage in a mental process by which they develop ideas that allow them to explain why events ought to occur (Turner, 1982). As a way of managing the uncertainty of everyday living, people rely on personal theories, or predictive assumptions, about the best ways to achieve desired effects. Personal theories about what kinds of action will bring about desired changes and why some actions work best are but some of many forms of a person's tacit knowledge and thus typically remain unstated. For the purposes of this paper, we work from this definition: A theory of change is a predictive assumption about the relationship between desired changes and the actions that may produce those changes. Putting it another way, "If I do x, then I expect y to occur, and for these reasons."* (Connolly & Seymour, 2015, p. 1)

As a group, the PhET leadership team met with a consultant from SRI who aided the team in delineating their own theory of change. As I have worked with PhET as a graduate researcher this past year, I was included in this process, enabling me to fully understand the theory of change as well as the discussions behind it. This theory of change document (although still in working form) provided significant information from the team (designers, PIs and researchers) about not only how the design of the simulations should impact student learning objectives, but also how the design features of the simulation should support specific outcomes in classrooms.

Therefore, to answer RQ#2 (concerning the features of the technology related to instructional practices), detailed information taken from the Theory of Change document was used along with teacher reflections and observations. This provided with me insight on not only the intended features of the simulations, but also how those features were enacted in classrooms.

4.3 SAMPLE

The unit of analysis for this research was the teacher, when planning and enacting a mathematical task within the classroom. Because I am studying changes in teachers' instructional practices from non-technology based mathematical activities to technology based mathematical tasks, it was essential that teachers were chosen who were willing to plan with and later use the PhET simulations within their classrooms. Furthermore, since the simulations are designed to support algebra readiness, I needed teachers who taught in the years prior to an algebra 1 course. As a result, I limited my selection criteria to those teachers who taught any math class from grade 6 through Algebra 1.

This research was conducted with four teachers from four different schools, from two different school districts in the foothills of the Rocky Mountains. As researchers generally have found that the actual implementation of technology-based tasks in classrooms remains low despite increased access to technology (Doerr & Pratt, 2008), it was important to find teachers who were willing to implement these lessons in their classrooms. All were selected based on their interest in participating in research with PhET mathematics simulations, and were recommended by other math education researchers or PhET team members. None of these teachers was known to me at the start of the research project. While I originally proposed to study three teachers, a high school teacher indicated interest in working with PhET shortly after the prospectus was approved. Since the other teachers were all middle school teachers, I included this new teacher, as she offered a slightly different venue and perspective on the use of technology in math classrooms.

Each teacher taught a section of the observed class prior to my observations. This enabled the teachers to try out lessons before I came to observe them. As a result, teachers were not using the technology for the first time in any observation.

4.4 DATA COLLECTION

Data collection began in May of 2016 and continued through early March of 2017, with a total of 22 classes observed at the four different school sites. Summer planning sessions occurred first, with the teachers meeting four times over the course of the summer. Interviews were also conducted for all four teachers, with some teachers participating in a second follow-up interview for clarification. Coding was completed later, as all interviews were first transcribed.

Field notes were written after each observation, and initial coding of the observation was done immediately after the observation. The coding of those observations was completed a second time after all observations were conducted as this enabled me to look for and correct any inconsistencies in the coding process.

Analytic memos were generated throughout the project, as a way to document and reflect on the significance of observations and interviews. A time line of these activities appears in the Figure 4.1 below.

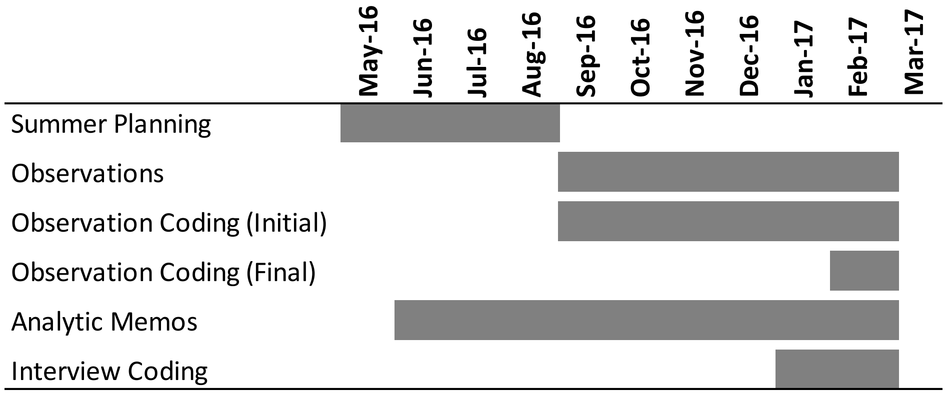


Figure 4-1 Timeline of Data Collection

4.5 DATA CODING

There were two key types of data that were analyzed through coding: observations and interviews/reflections. The observations were systematically coded to be able to determine changes in instructional practices from non-technology based activities to technology based activities. The interviews and reflections were coded, enabling me to view patterns among and between these interactions. In the sections that follow, I will describe the process I used to code both the observations and the interviews/reflections.

4.5.1 Data coding of the Observation

Much of the data analysis would come from the observation itself, since the goal was to identify changes in the instructional practices in non-technology and technology based activities. As mentioned earlier, field notes were written to support what was seen during the observation. However, it is difficult to write field notes with enough consistency to enable comparison between different classes and different teachers. As a result, I wanted to consider existing protocols as a means for this data analysis.

Thinking back to the theoretical framework discussion in Chapter 2, I noted that I was bringing together three different frameworks in an effort to fully capture the actions in a technology infused mathematical task. These three frameworks – Ball et al's (2008) articulation of Mathematical Knowledge for Teaching, Henningsen & Stein's (1997) conceptions of the role and use of Mathematical Tasks, and Koehler and Mishra's (2006) conceptions of Technological Pedagogical Knowledge -- were then broadened to consider technology-based mathematical tasks in classrooms.

Continued work by each of these research teams has led to some framework-specific observation tools designed to support their use. The MQI (Mathematical Quality of Instruction) tool was designed to support conceptions of Mathematical Knowledge of Teaching, and the IQA was designed to support knowledge about cognitive demand in the creation and enactment of mathematical tasks. I discuss the pros and cons of using both of these tools for this specific study along with the ICOT (the ISTE Classroom Observation Tool) which is designed to capture technology use.

4.5.1.1 MQI – Mathematical Quality of Instruction

One common means of observing mathematical tasks in the classroom has been through the Mathematical Quality of Instruction (MQI) tool (Hill et al., 2005). This tool specifically notes interactions between the teacher, the student, and the content and seeks to address connections specific to the Mathematical Knowledge for Teaching from Ball's framework of math teaching. Hill and her team defined a series of instructional moves that could be documented throughout a series of classes, and then could result in an assessment as to the quality of mathematical instruction occurring in the classroom. In large part, the MQI considers the verbal interactions between students and teachers, either as part of whole class discussions, or if possible, one-on-one discussions.

The MQI as a tool is designed to be used in conjunction with a video-recording of the class, rather than during the observation itself.

> *Each recorded lesson is divided into roughly equal-length (e.g., 5 or 7.5 minute) segments for scoring. Two raters independently give each segment a score for each of these five MQI elements. This allows the raters to accurately capture important events in the lesson as they occur. The raters also each give the whole lesson an overall MQI score.*

The MQI includes five dimensions of instruction: Classroom Work is Connected to Mathematics, Richness of the Mathematics, Working with Students and Mathematics, Errors and Imprecision, and Common Core Aligned Student Practices. The *Classroom Work is Connected to Mathematics* dimension is a standalone construct with a yes/no answer. The remainder of the dimensions have subscales. The six subscales for *Richness of Mathematics* capture the extent to which teachers and/or students (1) explicitly link and connect between different representations of mathematical ideas or procedures; (2) provide mathematical explanations focusing on why rather than how; (3) attend to the meaning of numbers and operations; (4) discuss multiple procedures or solutions (5) notice, extend or generalize a mathematical pattern based on examples; and (6) fluency of use of mathematical language. The *Working with Students and Mathematics* dimension captures whether teachers can understand and respond to students' mathematical contributions and errors. The *Errors and Imprecision* dimension is focused on capturing the teacher's errors, as well as problematic use of mathematical language and notation. The last dimension, *Common Core Aligned Student Practices*, incorporates the eight student practices identified in the CCSSM. Each category is then scored on a four-point scale as not present (0), low (1), medium (2) or high (3).

The MQI framework has much to offer in terms of noting ways in which teachers can support a productive math class. Most notably, it goes beyond just identifying moments of the classroom discussions and considers the content of those discussions and the degree to which these discussions focus on key constructs such as mathematical sense making and generalizations. On the other hand, the MQI is problematic when considering the use of technology (and specifically simulations used by students) in a classroom. Even though the MQI relies on what is said between the teacher and students, in a tech-based classroom where much of the time may be spent with students working independently on the simulation, this mathematical work would not captured within the MQI framework.

While each section of the lesson is given a score of not-present, low, medium and high, it is still the raters' judgment as to the overall score for the lesson itself (using the same scale of not present, low, medium or high). In the training for the MQI certification, the trainers stress that the scores are not an average of the sub category specific scores, but are rather the raters' judgment of the overall class. I believe this is a weakness of the tool, as the tool appears to partition the lesson into manageable segments in order to bring it together as a whole; yet, the scores of the partitioned pieces are not necessarily related to the overall class score. One way to address this concern is with a new option within the tool (referred to as the MQI-Lite) whereby one can simply score the overall categories and not score the sub-categories. While this does eliminate the possible confusion, it also loses specificity in documenting teachers' instructional moves.

The other challenge I found with the MQI as I began to code the observations is the tension between the goals of the tool (to assess teaching) and the goals of the study (to assess *changes* in teaching). As a result, I used features of the MQI to help me articulate instructional moves likely to be seen in a math classroom, but did not use the tool to evaluate teacher effectiveness.

4.5.1.2 IQA - Instructional Quality Assessment

A second option for an observational protocol is the Instructional Quality Assessment, or IQA. This tool came from studies of cognitively demanding tasks in group-work, which led many researchers to take up how to evaluate classrooms in a way that recognized and valued high cognitive demand, rich discussions, and quality group work. Many of these research efforts were based on case studies, surveys or self-reported results (M. D. Boston & Wolf, 2006). The TIMSS Video Study analyzed 231 eighth grade classrooms (100 in the US) and found that although 95% of teachers felt they were aware of "current ideas about teaching and learning mathematics" and "believed that these ideas were reflected in their teaching," nearly 80% of the teachers just "stated" the mathematical ideas rather than developed them (Hiebert, 2013, p. 5). In fact, to exemplify the differences between teachers' intentions and enactment of mathematical activity, in the TIMSS Video Study of mathematics classrooms there were '*no* instances or working through proofs or reasoning deductively in U.S. lessons" (Hiebert, 2013, p. 5), a goal that is at the heart of what it means to do mathematics.

The Instructional Quality Assessment (IQA) Toolkit incorporates a series of rubrics to quantitatively observe and describe two main constructs: academic rigor and accountable talk (M. D. Boston & Wolf, 2006). Used in both mathematics classrooms as well as reading comprehension classes, the tool has been demonstrated to have a "moderate to high level of reliability" and the overall classroom scores obtained when using this protocol were "significantly associated in mathematics, but not in reading comprehension," (Junker et al., 2006, p. 1).

This tool has an advantage over the MQI for this study in that the MQI covers each aspect of mathematical tasks (as defined by Stein et al., 1998) – selection of the task, implementation of the task, student discussion following the task, as well as rigor of the teacher questioning. On the other hand, while the IQA focuses on specific aspects of mathematical discussions in more detail - for example, considering student and teacher participation in the discussions, the amount of linking to mathematical concepts, and the effectiveness of those conversations to improving knowledge and rigorous thinking (M. D. Boston & Wolf, 2006) - unlike the MQI, the IQA simply acknowledges the presence of these teacher moves, and not the degree to which these practices move the class forward in support of the mathematics (i.e., in the IQA, observers would note that the teacher pressed students for explanations; in the MQI, the observer would note the frequency with which this was done).

While this tool does support the analysis of the cognitive demand of the task and student questioning, the MQI is not the best choice for this study as it does not give as much specificity to impact and intent of the instructional moves, and is thought to have a bias toward reform-oriented teaching styles, whereas the MQI is impartial to teaching styles (M. Boston, Bostic, Lesseig, & Sherman, 2015). As a result, this tool was not used at all in this study.

4.5.1.3 ICOT – ISTE Classroom Observation Tool

Whereas both the MQI and the IQA are designed specifically for math classrooms, neither specifically accounts for the inclusion of technology in the classroom. ISTE (the International Society for Technology in Education) has addressed the need for such a protocol through the development a different framework for considering technology used in classrooms. In this framework, similar to the MQI, pedagogical styles remain neutral with no preference being given for example, between different approaches to learning, nor to differences in pedagogical styles. Based on standards specifically related to technology (the NETS-S [National Educational Technology Standards for Students]), rather than standards for mathematics classrooms, this protocol captures technologies based on their type, as well as the user (teacher versus student) of the technology. When using this tool, observers note whether each standard was not addressed, addressed, or met. Standards include:

- Creativity

- Communication
- Information Fluency
- Thinking
- Citizenship
- Technology Operations
- Computational Thinking Skills

Additionally, for each three-minute segment of the class, observers note both who was using the technology, and whether the technology was being used 'for learning.' While this tool does capture the use of technology, it is distinctly separated from the mathematizing that occurs in the classroom, and is more focused on the types of technology used, rather than the implementation of that technology.

4.5.1.4 Summary of Tool Analysis

As noted above, none of these tools is able to fully assess teachers' instructional moves throughout technology-based mathematical tasks. Many researchers, such as Monaghan (2004) and Sherman (2011), chose to forgo any observation tool and instead chose to use field notes taken during the observation. I was reluctant to simply use field notes, as it would be difficult to compare similar factors across lessons and teachers. On the other hand, none of the available tools fully supported the framework I was using in this study. Each tool supported ways to identify particular constructs related to the research questions, but did not fully capture critical details of the lesson. Since the MQI is least dependent on instructional styles, I used features from that tool as a way to articulate instructional moves, but not to compare classes or evaluate teaching effectiveness. In addition to that tool, however, I created my own constructs specifically designed for this study to address the aspects specific to the use of technology in the classroom.

4.5.2 Observation Constructs

Potential codes to use in the analysis of teachers' instructional moves were deductively generated based both on the conceptual framework and information summarized from past research findings. A more inductively generated understanding of teachers' instructional moves was later generated based on the proposed data sources and the initial attempts at coding the observations.

Therefore, I began my coding structure by considering the constructs included in the MQI (See Appendix F for full listing of the constructs included in the MQI). For example, I included teacher practices related to student engagement, mathematical contributions, and mathematical language irrespective of whether technology is used. On the other hand, as discussed above in Section 4.4.1.1, the MQI does not include some dimensions that are specific to the use of technology. Therefore, the initial coding structure not only included constructs from the MQI, but also additional constructs centered around four dimensions generated using prior research related to math classrooms and technology-based activities. These dimensions include classroom characteristics, teacher characteristics, student characteristics, and mathematical characteristics. A codebook was written to support this coding structure and is included in Appendix G, but I have summarized the dimensions, constructs and subconstructs in the sections below.

4.5.2.1 Teacher Information

The first dimension, teacher information, focused on the instructional practices of the teacher. Specifically, *Teacher Information,* captured two visible actions of teachers that are not captured through the MQI – *Teacher is* (location) and *Teacher Verbal* (types of verbal utterances). In *Teacher Is*, I coded where teacher was located in the classroom (at her desk, walking around, in the font of the room, with students, or other (such as outside of the room, talking with another teacher, etc.). With *Teacher Verbal*, I captured the types of utterances made by the teacher. This was not intended to facilitate a discourse analysis – rather, when used in conjunction with details on the focus of the class, it is intended to support later identification of the role of the teacher in the classroom (Farrell, 1996). The types of utterances included silent, questioning (when asking students mathematical questions), direct instruction, supporting (offering encouragement), remediation, providing answers (either directly or through leading questions), managing behavior, providing instructions (for the task), general administration (such as talking about grading papers) and technical assistant (where the teacher is explaining how to use the technology).

4.5.2.2 Class discussions

Since prompting class discussions is a specific, and common, instructional move, this also became a dimension to be considered. Within this dimension, I included four constructs. The first, *Students talking*, captured not just that students were involved in math talk, as noted in the construct *Doing*, but also what types of talking they are engaged in. This construct included the five sub constructs: answering questions (direct responses to teacher questions with correct/incorrect answers, discussing questions (in response to teachers asking students to share their answers with one another), justifying answers, restating ideas, or reflecting on the lesson. During these discussions, I also captured who was doing most of the talking in *Who Talks,* and whether either the teacher or student was building on previously stated ideas in *Who Builds*. Finally, I captured the *Type of Discussion*, noting whether the discussion was primarily a procedural discussion (talking about the steps one might take to solve a problem) or a rich mathematical discussion (focusing on building conceptual knowledge).

4.5.2.3 Student Information

The next dimension focuses on the students. While this research is specifically focused on teachers' instructional moves, as other researchers have noted, the enactment of a task is invariably altered by the students (McCrone, 2005; Stein et al., 1998). Therefore, having insight into the actions of the students also lent insight into choices made by the teacher.

The first construct within this dimension captures *Student Engagement Patterns* by gender and by race. Changes in engagement of students could potentially lead to changes in teachers' instructional practices. My goal here was to ensure that I considered whether there were gendered and racial differences in engagement when studying instructional practices. No differences were noted in engagement, and therefore, segmenting the engagement data by race and gender did not further enlighten the results. This category was therefore collapsed to simply identify engagement for the majority of the students.

The next two constructs within this dimension look at how the students are working in the classroom. As mentioned earlier, the solitary nature of student technology use can lead to inaccurate descriptions of classrooms using the MQI. Instead, I chose to capture how students are working (individual work vs. group work, vs. discussions (small group and whole class)) in the *Working* as well as what the majority of students are *Doing*.

Again, the purpose of looking at what the students are doing is so that I can better understand the instructional practices of the teacher. Therefore, I captured what the *majority* of the students were doing at any time. As mentioned earlier, I used both a primary camera, and a secondary GoPro as I walked around to support this data. The subcontracts within this construct included doing mathematics (with or without technology), talking (mathematics or non-mathematics related), listening (to instructions of an assignment or to the lesson itself), waiting on help, gathering supplies, or other non- mathematics (intended to cover non math activities such as the time spent standing during announcements).

4.5.2.4 Mathematical Focus

The next dimension, Mathematical Focus, includes four constructs within mathematical classrooms. The first one, *Focus of Class* was designed to capture the predominant focus of that segment of the class time. Prior research has found that the focus of the class often changes with the inclusion of technology. In many cases, the focus moves from mathematics to the technology tool being used. This construct, *Focus of Class,* categorized whether the work is mathematical in nature, technological in nature, administrative in nature or other (defined in the same light as non-mathematical work from the MQI, including activities such as attending to a fire drill, lockdown drill etc. (H. C. Hill et al., 2005)).

The second construct was *Type of Task*. In this construct I included three possible sub constructs, with the type of tasks being drill (focusing on fluency in a particular skill), investigation (focusing on building conceptual understanding), and assessment (assessing student knowledge). In line with the MQI, I included a construct on *Math Content*, with a desire to capture whether the content was both clear and correct. Since this was evaluative rather than comparative, it was not ultimately used. The final construct in this dimension was *Focus of Math*. Again, these sub constructs were informed by constructs used in the MQI. As students were engaged in mathematical tasks, I identified the goal of the task as one of the following choices: linking between representations, generalizing patterns, sense making (where the focus is on number sense), multiple solutions, precise language (where the teacher or activity is focused on building academic language), press for evidence (where the teacher or activity is focused on justification of answers), and explanations (with two sub constructs to specifically capture explaining how, using procedural explanations, and explaining why, focusing on conceptual understanding).

4.5.2.5 Technology Focus

The dimension of *Technology Focus* adds to knowledge about the classroom and the use of technology. There are three key constructs within this dimension: *technology arrangement*, *technology used* and *software used*. As mentioned earlier, in some classes, the teacher projects the tool on a whiteboard, but the students don't actually ever have their hands on the keyboard. In this project, teachers agreed to use a tool designed specifically for student use. Therefore, the *technology arrangement* construct (whether each student has his/her own device (One-to-one), or whether devices are shared between students (Pairs) or shared in some other fashion (Other)), and who was using the technology (teacher and or student) enabled me to capture times when the teachers controlled student use of the resource in a variety of ways. The technology used and software used enabled me to capture both the device and the software being employed in the classroom.

4.5.2.6 Remediation and Reflection

While *remediation* from a mathematical sense is included in the MQI, and has been shown to support strong student conceptual understanding (H. C. Hill et al., 2008, 2005), the process of remediation can be disrupted with technology use as the focus becomes using the tool, rather than the mathematical concepts (Monaghan, 2004). Thus I looked at who was being remediated (*Remediation of*) (an individual, a small group, or the whole class), what was being remediated (*Remediation on*) (math or technology) and by whom (*Remediation by*)(student or teacher).

The other construct is *Reflection*. This refers to the need for students to be thoughtful about their own learning, and is not captured through the MQI. The intentional use of reflection aids students in making connections between the technology used and their conceptual understanding (Clements, Sarama, Yelland, & Glass, 2008; Sarama & Clements, 2008). Sub constructs were used to designate the types of reflections seen in the classroom, such as individual reflections, when teachers specifically asked students to stop and think about some portion of the task versus student-student reflections where students were asked to share their reflections with a table partner. I also included sub constructs for small group reflection (since many classes were in groups larger than just two students) and whole class reflection. While I hypothesized (and the research supported) that these were key instructional practices, neither construct was used more than once. As a result, while I included these in my coding structure, these constructs were not effective in helping me answer the research questions.

4.5.2.7 Final Observation Video Coding Structure

As mentioned earlier, through the triangulation of these three constructs (teacher actions including location and verbal clues, focus on the lesson, and who uses the technology), enabled me to capture the teachers' instructional practices.

The final coding structure for the observations is shown in the Figure 4-2. This coding structure was used for all videos. For each set of constructs, the class was analyzed using the same time increments as the MQI (5 minute increments), providing me with granulated data throughout the whole class period, and is consistent with prior work in this field (Farrell, 1996; Zbiek & Hollebrands, 2008).

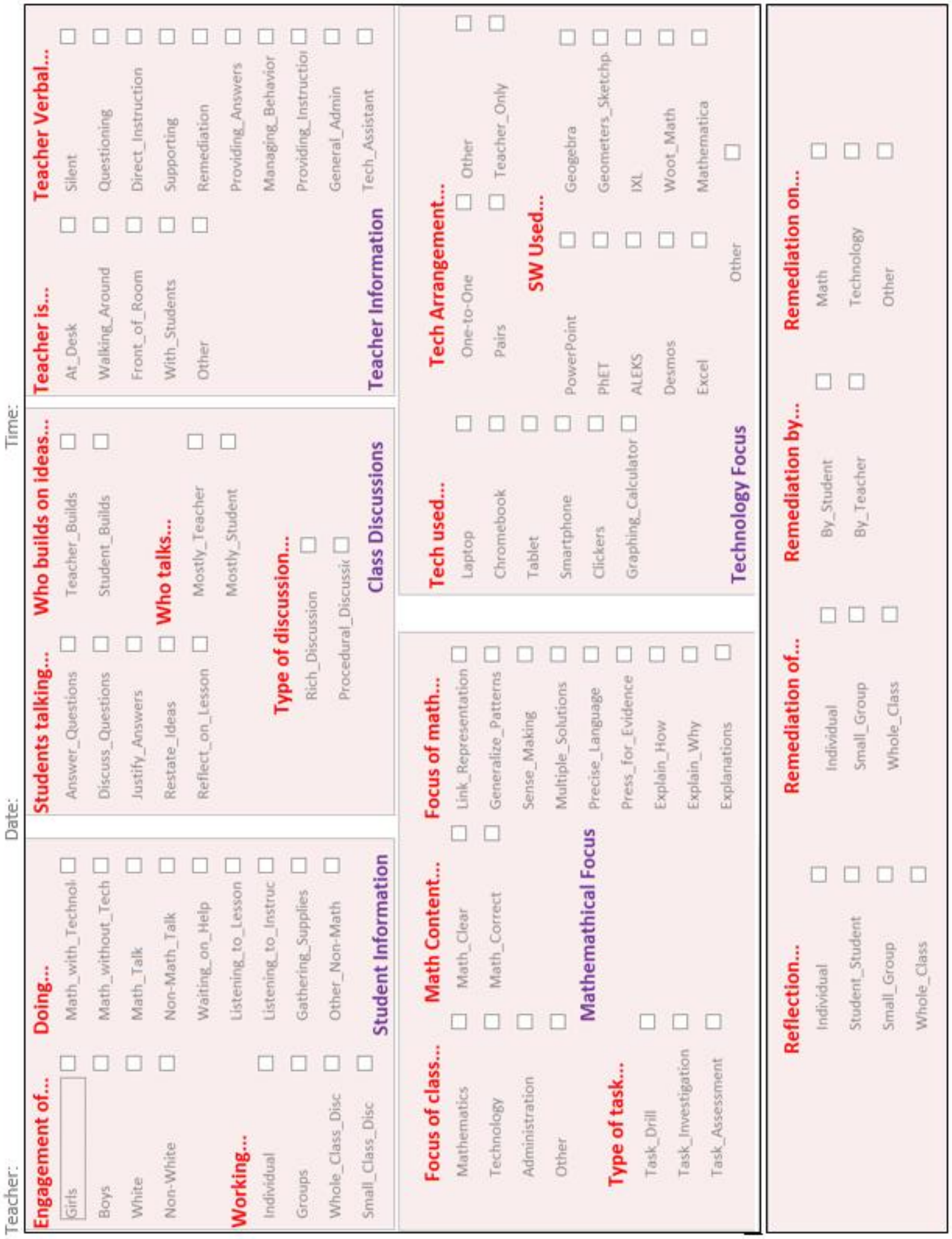
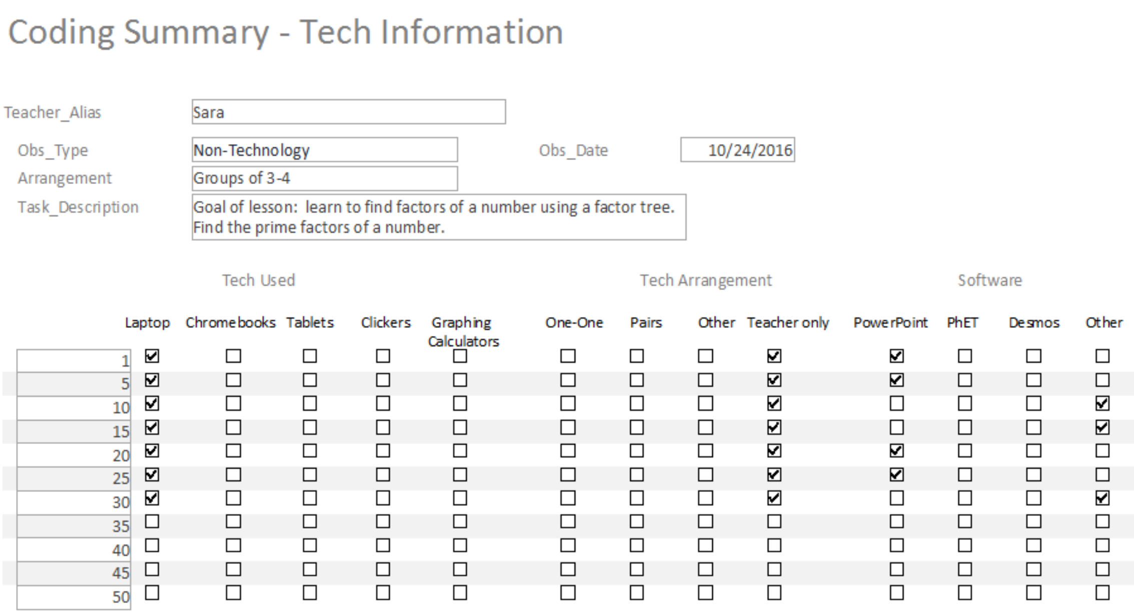


Figure 4-2: Summary of Observation Coding Structure

Once the codes were entered into the database, I was then able to view reports that sorted the data by the time field, enabling me to see how the lesson progressed. Three reports were generated for each observation capturing the instructional moves of the teacher, as well as the actions of the student, and the setup of the technology. Examples of these three reports are shown below in Figure 4-3 (Technology information), Figure 4-4 (Teacher Information), and Figure 4-5 (Student Information).



Coding Summary - Tech Information

Teacher_Alias: Sara

Obs_Type: Non-Technology   Obs_Date: 10/24/2016

Arrangement: Groups of 3-4

Task_Description: Goal of lesson: learn to find factors of a number using a factor tree. Find the prime factors of a number.

	Tech Used					Tech Arrangement				Software			
	Laptop	Chromebooks	Tablets	Clickers	Graphing Calculators	One-One	Pairs	Other	Teacher only	PowerPoint	PhET	Desmos	Other
1	☑	☐	☐	☐	☐	☐	☐	☐	☑	☑	☐	☐	☐
5	☑	☐	☐	☐	☐	☐	☐	☐	☑	☑	☐	☐	☐
10	☑	☐	☐	☐	☐	☐	☐	☐	☑	☐	☐	☐	☑
15	☑	☐	☐	☐	☐	☐	☐	☐	☑	☐	☐	☐	☑
20	☑	☐	☐	☐	☐	☐	☐	☐	☑	☑	☐	☐	☐
25	☑	☐	☐	☐	☐	☐	☐	☐	☑	☑	☐	☐	☐
30	☑	☐	☐	☐	☐	☐	☐	☐	☑	☐	☐	☐	☑
35	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
40	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
50	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

Figure 4-3 Sample Coding from Observation - Technology Constructs

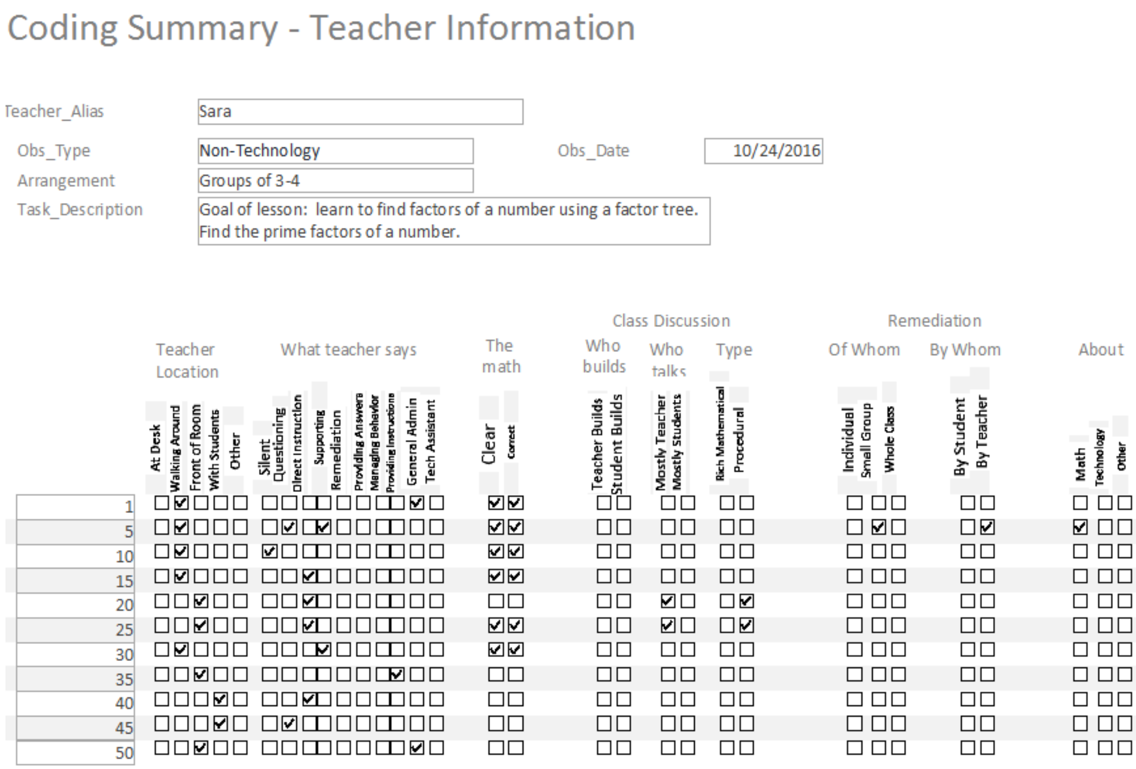


Figure 4-4 Sample Coding from Observation - Teacher Constructs

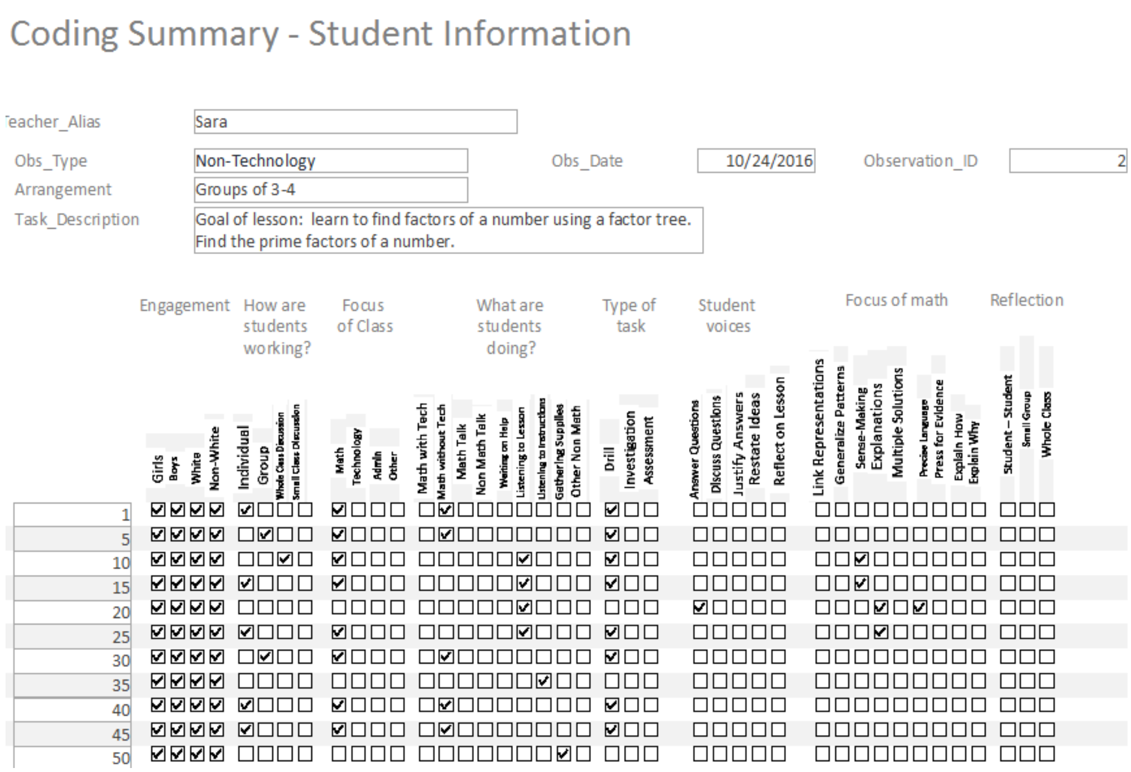


Figure 4-5 Sample Coding from Observation - Student Constructs

With this information all coded for each of the twenty-two observations, I was then able to view reports that showed the summary of observations for each of the four teachers. This enabled me to easily capture changes in instructional practices from non-technology to technology based mathematical activities.

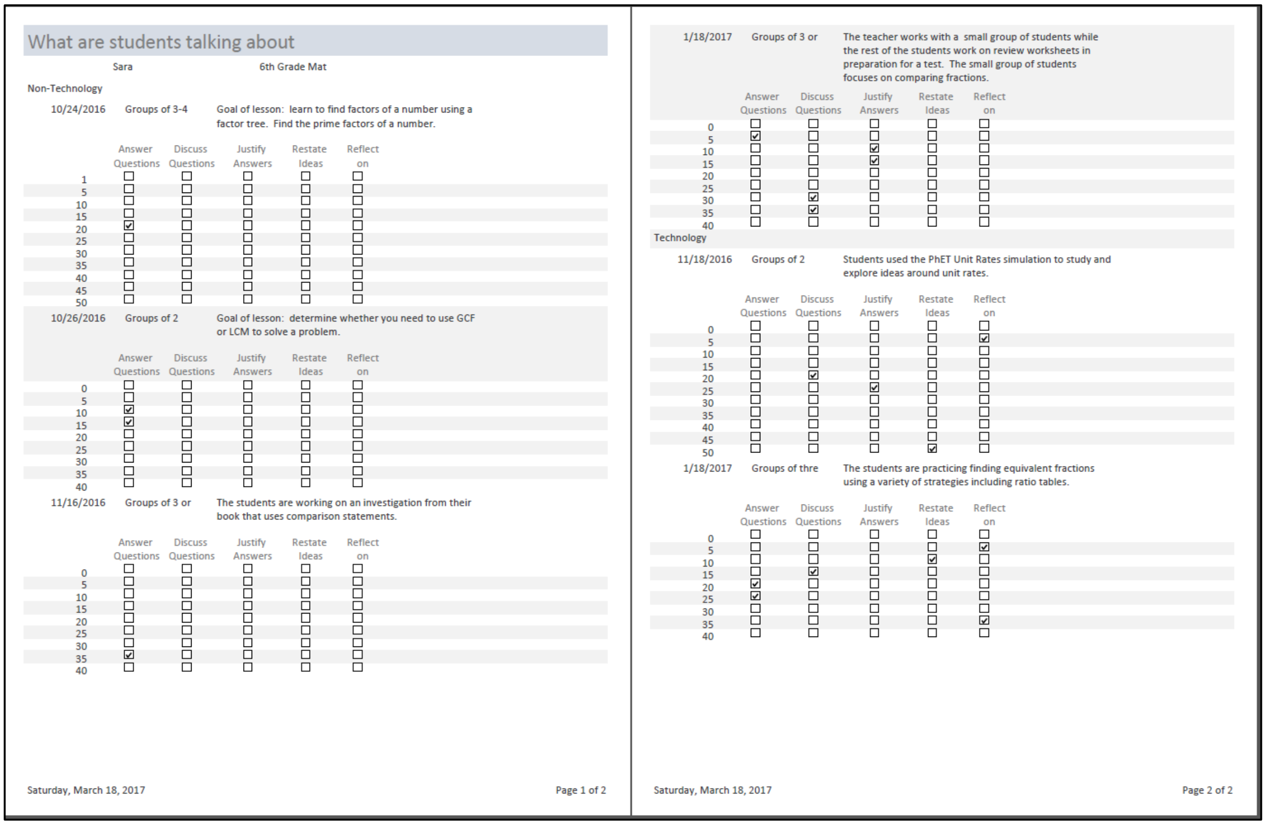


Figure 4-6 Sample Coding across observations - Class Discussions

Each of these examples are specific to one teacher, Sara. In Figure 4-5, you can find Student Voices, with the only identified time of students talking as part of a class discussion when students were answering questions at the 20 minute mark of that class. In Figure 4-6, above, you can see this same notation of students talking at the 20 minute mark of the first, non-technology based lesson in the summary. However, now I could also see all of the six observations together at one time, enabling me to then look for changes in patterns from non-technology to technology based lessons.

4.5.2.8 Inter-Rater Reliability

Using the codebook and an hour long training where six five-minute segments were shown, discussed and coded, two 20 minute observation segments were coded by another education researcher. The inter-rater reliability was found to be 87%.

4.5.3 Coding and Analysis of Interviews and Reflections

This section describes the process I used to systematically analyze the interviews and reflections. This analysis was significantly informed by the work of Miles, Huberman & Saldaña (Miles, Huberman, & Saldaña, 2013; Saldaña, 2012). The first analysis completed was to look for initial patterns. I read through all of the notes from each interview and reflection by teacher, to get a sense of what was shared by each teacher. An initial round of coding was completed using deductive codes. To accomplish this, sections of the interviews were coded with holistic codes, developed both as provisional codes based on the literature review, and also with codes based on the hypotheses I had made as to what one might expect to see during non-technology and technology based lessons. The next step was coding using inductively created codes. To accomplish this, I reread through these documents, adding descriptive codes (words and phrases that assisted me in categorizing parts of the conversations) such as those seen under Technology >>Richer Math, in Figure 4-7. I also used "in-vivo" codes (actual quotes that were often shared between teachers), such as "use what I know," "fit it in," and "try it out." Finally, I used codes that gave me some insight into the teachers' values, attitudes and beliefs about teaching in general, and using technology in classrooms more specifically. A diagram of the codes used is provided in Figure 4-7.

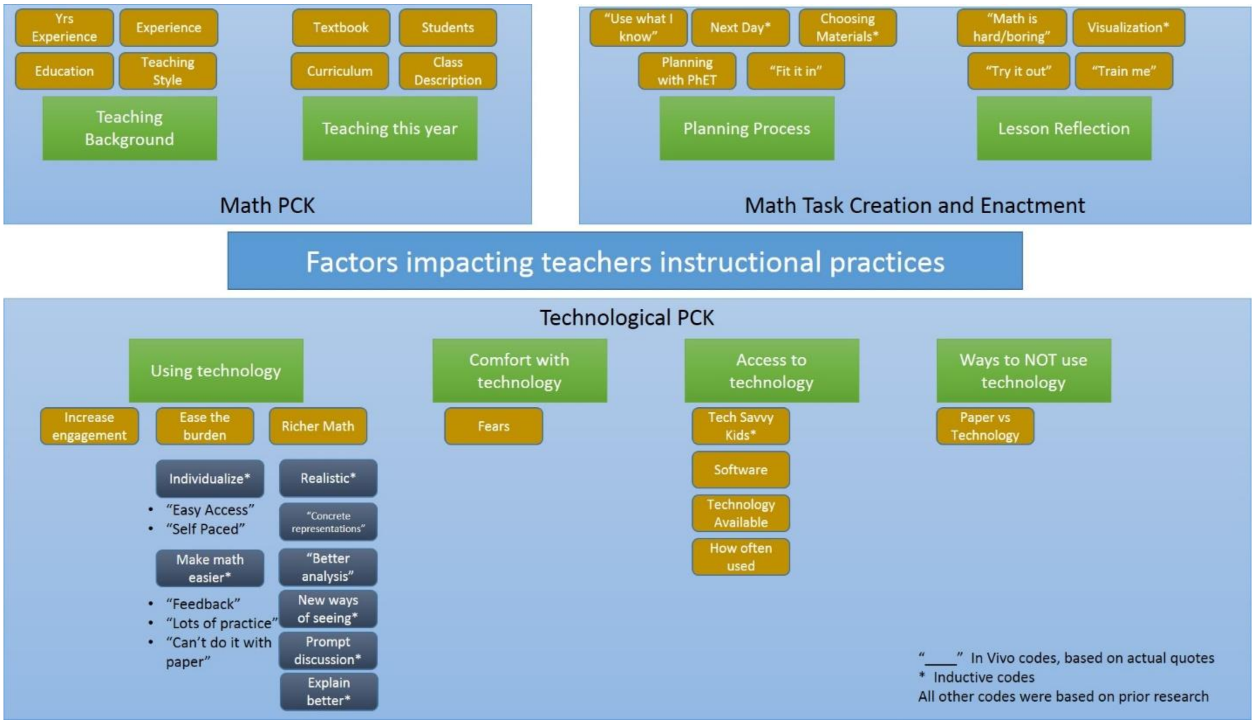


Figure 4-7 Code Summary for Interviews and Reflections

This process was repeated for each interview, and later, for each reflection. After each round of coding was completed, I reviewed the summary report of the coded sections to determine if sub-codes were needed. As new codes were determined to be necessary, I returned to prior coded documents to ensure that this new code was accurately reapplied, as needed. All of this coding was completed in MAX-QDA, a Computer-Assisted Qualitative Data Analysis Software (CAQDAS) program. A copy of the final code book used is located in Appendix H.

Once the coding was completed, analytic memos were written to summarize and highlight commonalities between and among observations, and to think through common themes that were appearing through coding.

Using Capspecken's (1996) ideas of analyzing data in a thorough, systematic way in order to make claims with evidence, observations were grouped (non-technology based, for example) and were analyzed first for possible meaning fields, in that I was looking for ways in which the data *could* be interpreted. In doing this, I considered what norms, rules or underlying assumptions could be in line with the interaction. To consider a variety of possible viewpoints, data were interpreted from the perspectives of both the student and the teacher. I also looked at data from both an emic (insider) and etic (outsider) perspective, looking at how each group (an insider to the school, and an outsider) might view these observed interactions. I considered ways in which interactions were similar to other activities within the classroom, and ways in which these interactions were part of a larger set of activities. From there, I considered the "pragmatic horizons" or the background against which meaning is creating for these interactions. An analogy to this process is to consider chess. Individual moves are meaningful as part of a specific strategy, yet must also be viewed in the larger context of the chess game itself. In the same way, understanding of the specific interactions was done using the school and classroom as the framework in which to base these conclusions.

4.5.4 Summary

As described above, both deductively and inductively created codes were used to document the observations at five minute intervals. Additionally, each five-minute segment was summarized, capturing the activities of the session and key points of conversations. Interviews and reflections were also coded, enabling me to view themes across and between conversations. Memoing, described by Miles et al (2013) as writing narratives that documents one's thinking and reflections about the data, was used in order to fully see how the pieces of information, categorized by these codes, were working together to form a picture of the classroom. These narratives provided a broader view of the processes that combined to form the enactment of tasks in the classroom.

4.6 VALIDITY ISSUES

There are three types of validity issues: descriptive validity, interpretive validity and theoretical validity.

Descriptive validity, or the accuracy of my findings, can be an issue as there are always areas of interpretation and focus which can bias the collection of data. To minimize this concern, interviews were recorded and transcribed, and observations were video recorded, with associated field notes compiled. Technology and non-technology observations were fully transcribed as well, to enable me to focus on verbal actions more clearly. Descriptive validity was maximized by videotaping each observation. Threats to interpretive validity, or the possibility that the participants may not agree with my interpretations, were minimized to the extent possible by discussing my findings with the teacher, to ensure I maintain a broad perspective, and accurately report my findings. For example, I would often check in with the teacher at the end of the lesson to confirm aspects from the observation coding.

Finally, theoretical validity, or the ability to generalize my findings, is certainly the largest concern, especially in light of the small size and lack of diversity of the cohort. While these findings are not fully generalizable to the teaching population as a whole, they do provide insight into specific differences in how technology-based activities are enacted in the classroom, furthering the discussion of how to implement technology effectively within a mathematics classroom as well as provide a sound reasoning for continued research with a larger and more diverse cohort.

5 CHAPTER 5: RESULTS

The intent of this research was to study two questions.

1. What observed instructional moves by teachers in math classrooms are unique to technology-based mathematical activities created with student-oriented simulations?

2. In what ways do specific design features of student-oriented simulations impact instructional moves in math classrooms

This study draws upon the my conceptual framework that suggests that teachers' instructional practices are determined by teachers' mathematical pedagogical content knowledge as well as the developed and enacted task, and are mediated by the use of technology.

Therefore, I analyzed the data in light of those questions. For the first research question, concerning the changes in instructional practices for teachers when moving from non-technology based activities to technology based activities, I start with a description of the participants, and their school contexts. Then, using data from interviews of each teacher, a case study has been written to gain understanding of her education and background, and dispositions towards mathematics and the use of technology.

Next, I summarize the observed lessons, both non-technology based and technology based so that the reader has a sense of the teaching practices seen in each case. I also provide a description of the lessons planned by each teacher, and an analysis of how that lesson was eventually enacted.

Finally, I provide a summary of the differences in each teacher's instructional practices when moving from non-technology based lessons to technology based lessons.

For the second research question, concerning ways in which the features of the simulations might impact instructional practices, I use data from the Theory of Change document, teachers' observations, and teachers' reflections.

The synthesis of this data into three key themes that appeared regularly across the four teachers produced findings that are not only relevant, but provide the basis for continued research.

5.1 SITES/PARTICIPANTS

Four teachers participated in this research. Three of the teachers, Sara, Pat and Christine, are middle school math teachers while the fourth, Lily, teaches in a high school. Pat, Christine and Lily have all taught more than 8 years, with Pat and Lily having more than 20 years' experience. While Sara has taught for 8 years, only two years have been in a middle school math classroom. All four teachers taught a mixed set of courses spanning over several grades. As mentioned earlier, this selection fills a gap in the literature, by placing the focus on middle and early high school teachers.

While their schools had similar percentages of girls/boys, there were distinct differences in racial makeup of the students, as well as percentage of Free/Reduced Lunch (and indicator of SES), and English Language Learners. A full comparison of the three schools is shown in the Table 5-1. Through a PhET grant, subject teachers received a $500 stipend for their participation as participation involved extra time in lesson planning and reflection, as well as a small stipend for their summer work creating lessons.

Christine had participated in other research projects before, and was very open to having me in her classroom. Sara is the youngest of the teachers. For her, participating in research was a new venture, but she, too, was very open to having me come to observe her whenever I wanted. Both Pat and Lily were more reluctant about participating in research and often mentioned their discomfort when I was there to do observations. Both were a bit concerned about whether this would be evaluative, in that they worried I would be determining if their teaching was 'good' or not. I reassured both teachers often that I was not there to evaluate their teaching, but rather to note differences in their teaching styles. Both welcomed me into their classrooms, but both teachers were more specific about which days I should come and observe.

Table 5-1 Comparison of the four schools

Category	School A Christine	School B Sara	School C Pat	School D Lily
Female	47.8%	51.0%	45.4%	48.9%
Male	52.2%	49.0%	54.6%	51.1%
African-American	1.5%	1.3%	1.0%	2.0%
American Indian	0.2%	0.9%	0.5%	1.0%
Asian	3.1%	6.3%	13.4%	3.6%
White	52.4%	63.7%	72.2%	39%
Hispanic	39.5%	23.4%	7.2%	55%
Multi-Racial	3.3%	4.4%	5.7%	Unk.[4]
Free Lunch	37.7%	26.0%	4.2%	49%
Reduced Lunch	3.0%	3.1%	1.4%	3%
English Language Learners	23.4%	13.4%	8.7%	21%
Special Education	12.7%	16.9%	6.8%	Unk.
504	2.8%	6.3%	2.7%	Unk.
TAG	18.8%	20.3%	11.9%	Unk.
Out of District	2.6%	2.9%	2.6%	Unk.

Sara and Pat both taught middle school classes in a traditional 45-50 minute class period. Christine has three 50 minute periods, and one 90 minute block period. Lily, the high school teacher, taught a block-style class, with 90 minute periods, every other day. A summary of these differences is provided below in Table 5-2, Teacher Summary.

[4] Some values were not available due to differences in reporting requirements.

Table 5-2 Teacher Summary

Teacher	Grades Taught	% Free/ Reduced lunch	% Non white	Years Experience	Class style
Sara	Middle	29%	36%	2 (+ 6 yrs elem.)	52 min daily
Christine	Middle	40%	48%	8	50 min/1 block
Pat	Middle	5.6%	28%	10 (+12 yrs HS)	45 min daily
Lily	High	51%	61%	19	90 min block

While the cohort is small, it provided diversity in terms of experience, grade level, school schedule, and student population.

5.2 CASE STUDIES

In the following section, case studies are provided for each teacher. The information in these case studies was gathered from the initial planning meetings, which were held over the summer, the interviews of the teachers, and the post lesson reflections. These findings provide insight into teachers' background and experience, lesson planning process, classroom design and norms, and dispositions towards mathematics more generally, and the use of technology in math courses more specifically. This is a central data source to support answering the question of how teachers' instructional practices change when moving from non-technology based lessons to technology based lessons.

5.2.1 Pat

5.2.1.1 Background

Pat, a white female teacher with more than 20 years of teaching experience (10 years in middle school, and 12 years in high school), has a degree in secondary math education, as well as a masters in Multicultural education, which serves to add a linguistically diverse endorsement to her secondary math licensure. During this study, she taught two eighth grade math classes, three advanced algebra classes, and one advanced geometry class. In the observed class, there are twenty-four students. The teacher described her class as being representative of the overall school, where the students are 73% white, with 5.6% receiving free/reduced lunch[5].

5.2.1.2 Classroom Design and Norms

Pat's classroom is arranged in groups of three to four desks, with two facing the front of the room, and the other two facing in toward each other, allowing all students to see the board as needed, while still working together in groups. The tables are spaced sufficiently apart such that Pat can easily walk around the room and talk with students. In most groups there is an open seat, enabling Pat to sit down and join students during their work time, either to give help or to listen to their conversation. The classroom itself is well-organized, with many posters around the room, most of which were made by students in her classes. This suggests that Pat organizes her classroom to support student interactions and the mathematical tasks are designed to promote both interactions within the group, as well as whole class discussions.

Pat's textbooks also support student sharing of different solution strategies. The book series, Connected Math, is designed around a "Launch – Explore – Summarize" pattern where a group of students are given a problem to explore and solve, and then apply those findings to a broader range of problems. Students spend a significant amount of class time working on problem solving tasks, and group work is emphasized. All lessons include class discussion as a component. As Pat works in a large district that adopts a textbook for all schools, this may reflect district choices as much as her own feelings on good instructional practices, yet I would argue that since she has the opportunity to use other materials yet still regularly uses the textbook, she also sees value in this. This is noted in her interview as well, as she describes her textbook:

[5] See section 4.1 for a full description of the school demographics.

> *"I like that it's investigative and that the kids are doing things. I'm not up here showing. They're doing." (08/11/2016)*

Pat's beliefs on creating an environment that supports students comes through most clearly as Pat describes what a great lesson might look like:

> *"When you have great student discourse. When kids are engaged. […] You just want to hear a lot of student engagement. […] I remember, we were doing this Pythagorean thing. I don't know what. I wish I could replicate it, but it was just that group of kids and it was making these squares and figuring out, given the diagonal, what's the length of a side, and all that. They were up, and they were comparing, and talking." (Personal Interview 08/11/2016)*

Despite the fact that she works to create an environment that supports students, Pat also recognizes that the students themselves play a role in the classroom environment. As she finishes talking about that day working on the Pythagorean Theorem, she noted that while she's repeated the lesson, she never had the same reaction. "Those kids," she said, "for some reason that day were just on fire." For Pat, the success of that lesson could not be solely attributable to the lesson itself, but also to the engagement of that particular group of students.

5.2.1.3 Curriculum and Resources

From her interview (08/11/16), I learned that when designing classes, Pat draws primarily from the text books selected by the district. For eighth grade math, the district uses the Connected Math 3 series (Lappan, Fey, Phillips, & Friel, 2014). Advanced algebra uses Intermediate Algebra: Concepts and Applications 9th edition, by Prentice Hall (Charles et al., 2011), while the geometry class uses Discovering Geometry by Michael Serra (2002). She described herself as being very comfortable with the subject matter, and only hesitates a bit when discussing eighth grade math, as she reported that the book has only been used for a couple of years. The Connected Math series is based on a constructivist view of mathematics, in that the students are given investigations such that they make sense of and build their own mathematical knowledge. On the other hand, the algebra and geometry books are more traditional text books, with a few example problems, and then a series of problems for students to work through independently. Beyond these resources, Pat brings in worksheets she has found online.

5.2.1.4 Lesson Planning with PhET Simulations

As part of this research, the teachers were asked to design a lesson that they would use in their own classroom, but to write the lesson with enough clarity that someone else could modify it if desired for a different class. When planning the lessons over the summer, Pat mentioned several times that she found the process to be uncomfortable. She noted that she felt intimidated, and was not sure that the lessons she was planning would work for other teachers. She says,

> *Well, I look at everybody else's and I feel a little intimidated. I've not written formal lessons for someone else, where you're trying to do all this stuff. I don't know. I don't know how I feel. I know I felt intimidated through the whole process. (Personal Interview, 08/11/16)*

When I probed her more about that, noting that she's the most experienced teacher in the group, she explained that the process of writing formal lessons was not something she would regularly do.

> *When I'm [planning a lesson], I try to put myself in a classroom, but there's so many things I do now just on the fly, or in response to what's happening, and don't anticipate all of that necessarily. I've always been just a general outline person, and so I felt it was frustrating for me and harder for me to think ... Like when people say, "Well put some more [questions] here," and I'm just like, "Okay. What? What do I put?" (Personal Interview, 08/11/16)*

Pat went on to tell me that she used to write a full lesson plan when she first started teaching, but now, it feels unnecessary as she knows generally what she plans to cover, and has taught the material so long that she knows where students might get stuck. Instead, she plans in units, considering her big ideas, and then she used a calendar where she kept notes about what she wanted to teach each day. As a result, this process of writing out a full lesson plan felt alien to Pat.

Ultimately, Pat wrote two lessons. One, with the Graphing Lines simulation was focused on the equations of parallel and perpendicular lines. The second, written for the Expression Exchange simulation, focused on comparing equivalent expressions. However, the lesson she chose to teach using a simulation for this research was actually a lesson designed by another teacher who also worked with the team over the summer.

5.2.1.5 Dispositions towards mathematics

On multiple occasions Pat would talk about how math was "boring" and "very procedural," and therefore had to be spiced up to get the kids to want to participate. She also had a perception that students did not like the work of mathematics, such as graphing lines, making tables, etc. She describes this when talking about the Connected Math book:

> *I just feel like it's not even their reading level. I feel like kids get turned off by [the book]. Oh my gosh. It's just [like] The Britannica [because it is so wordy]." [Furthermore,]some kids have trouble tracking from the book, to the paper, to the book.*
>
> *That's why I'm like, "Okay. I'm going to give them [resources]. Here's your investigation sheet. Answer question one here. Answer question two here. Here's a grid for question three, because they want you to graph it. I've already given you the grid. I've already set up a T table here because you're going to make a table." Filling something out makes it a little bit more friendly [sic], because some kids will be like… they don't even want to [make that effort]... "I have to make a table?" Just a T [style table], but if it's already made, [kids feel like] "Okay. I can fill in the numbers."* (Personal Interview, 08/11/2016)

There seems to be an underlying belief here that math is troublesome to students and students will avoid the work if possible; instead if she does all of the work to set up each problem, then the students will comply and complete the remaining analysis. This desire to minimize the strain on students to do the more tedious aspects of the math problems then directly impacts her planning of each day's tasks. As she describes her planning process, she again talks about ways in which she provides additional resources to support her students.

> *I don't want the students sitting and just watching me. They usually have something they're filling out or using for the investigation, because the other thing is, with the CMP, if they have to make a graph and they have to actually make the X and Y ... I mean, some kids, that will take them fifteen minutes. I usually give them an investigative sheet, already with a graph. Maybe they have to figure out the scale or something, but I don't want them to be struggling with that, because honestly, when they start doing graphing, they're going to use spreadsheets and things. I'm thinking that the skill of actually drawing that is not necessary, and with a 52 minute period, it takes a lot of time. (Personal Interview, 08/11/16)*

In this segment, not only is Pat referring to students' dislike of these activities, but also the time it takes the students to complete the graphs. Instead, Pat indicates that since technology will be minimizing the need to complete such tasks in the future (i.e. using a graphing tool rather than graphing by hand), her providing premade graphs is a reasonable support for students.

Even with the support of technology, there remains for Pat a desire to ensure that all students remain engaged in the activity and have a reason to figure out the math on their own.

> *When I do the sims, I really want each kid to have his own [device]. If worse came to worse, I would put two on a computer. I even saw with the GeoGebra, kids wanted to be able to just sit back and watch the other one. No. Everybody needs their own computer. It's easy to watch someone, and then they don't have to figure it out. (Personal Interview, 08/11/16)*

Later in the interview, she reemphasizes this point, of adjusting the lessons to ensure that all students are engaged, active participants in the classroom:

> *If I put [graphed lines on the board], I bet two kids paying attention. You know how that is. When I'm doing stuff [on the board], I do not feel like I have them all. When they're all doing it, I feel like that's where technology really helps. They can all graph their own lines on their own computer. They're more engaged, because they have their own thing and they're putting it in. (Personal Interview, 08/11/16)*

Later, she notes:

> *I use clickers almost every day with algebra, because that can be so dry and procedural I want them engaged. I don't want the students sitting and just watching me. (Personal Interview, 08/11/16)*

Pat appears to believe that students must be engaged in the learning process and that this engagement comes from making the activity fun and relatively painless (in that the tedious work is minimized). This appears to be an essential precursor to the other aspects of learning. This is to say, before a student can construct his/her own knowledge, she must first design a task that is sufficiently engaging and easy that the student is willing to complete the task. This strikes me as related in some ways to Lotan's ideas of 'group-worthy' tasks (Lotan, 2003) in that in order to have students work together, the tasks must be worthy of their collective engagement. Pat's view of this, however, takes this concept to a different level, suggesting that the tedium of the mathematics must be alleviated in order for the task to be group-worthy. This highlights some of the ideas by other researchers that teachers' own beliefs about mathematics will significantly impact how tasks and activities are enacted in the classroom (Froman & Ansell, 2001).

5.2.1.6 Use of technology in mathematics classrooms

Especially in the area of technology in the classroom, Pat clearly describes the student learning as a process of constructing knowledge and making sense of the mathematics.

> *That's why I do the MARS activities. Anything where they're having to engage with the material, with the concepts, and do some analysis. The more they construct their own learning, that's what they're going to remember. (Personal Interview, 08/11/16)*

In describing the need for students to make sense of the mathematics, Pat identified two additional key aspects of technology: the dynamic responsiveness of the tool (the fact that a student can easily try something, and undo that action if needed, based on instant feedback) as well as the ability of the technology to eliminate the tedious aspects of math, freeing students up to do more analysis, saying,

> *I love that when you graph on Desmos, if your equation is wrong and you graph it, and you get this weird graph, you can quickly redo it. If you graph it wrong on paper, you've done all that work. It's just so tedious. I think with some of the technology, you can get higher level thinking. Instead of all your time spent graphing it, you can start analyzing different graphs. (Personal Interview, 08/11/16)*

For Pat, technology supports students' construction of knowledge through two features. First, technology provides students with immediate feedback, something that is not possible with pencil/paper activities. Second, technology takes on some of the heavy lifting of the task, which enables students to engage more deeply with the mathematics.

5.2.2 Lily

5.2.2.1 Background

Lily is a high school teacher, and has taught high school through most of her nearly 20 year teaching career. She teaches a wide range of classes, from Algebra 1 through pre-calculus. This year, she has four sections of Algebra 1, and two sections of Pre-Calculus. The class I observed was the Algebra 1class she taught during the first period of the day. All but two of the students are Hispanic, and Lily shared that a handful of her students have IEPs and roughly a third are ELLs. The classes are taught in a block schedule, with each class period lasting 90 minutes, and taught every other day.

5.2.2.2 Classroom Design and Norms

Students are arranged in groups of two, and the room is crowded with desks. On the left side of the room is a set of windows, with a white board in the front of the classroom. There are some posters around the classroom. On the right side is another whiteboard where the objectives for the class are listed. The class begins with school announcements and the Pledge of Allegiance, with some students still not yet seated until math instruction begins. As the class begins, there is often a warm-up activity. Students are very quiet, and most do not even chat during the warm-up. In general, the class is very quiet with little interaction among students. Lily notes that this is likely due to the early start time[6].

Lily chooses a student to complete the warm-up on the board, to share their work with the other students. Students do not ask questions during this time, and they speak only to answer direct questions asked by the teacher. Once the warm-up is completed, the students are directed to the activity of the day. Because the class is a block style class, meaning it is a double period taught every other day, Lily typically had more than one activity planned for each day. These activities varied from whole class activities to stations, where students rotated through a series of activities, to direct instruction.

The end of class typically had some sort of exit ticket that the students needed to complete as a form of assessment before class was dismissed. However, exit tickets were not completed by most students due to lack of time. Furthermore, any students who were late to class had to meet with the teacher at the end of class. Due to the relatively high number of late students each day, the teacher was not free to follow up with students completing the exit ticket or other work before they left the classroom.

5.2.2.3 Curriculum and Resources

Lily has a classroom set of Algebra 1 books, by Prentice Hall (Charles et al., 2011), but rarely if ever uses them. Instead, she plans all of her lessons individually, using resources from prior years. She describes her planning in this way:

> *I am a textbook rebel as far as algebra one goes. I barely open it up. I [used the book faithfully] in the beginning. Then, in the 13 years I just know the*

[6] I did observe one of her classes that she teaches later in the day, and found the students to be far more talkative.

> *natural progression and I am constantly checking that curriculum map to make sure I'm getting everything. I do it on Google Calendar and I write so I can save because you can have it repeat every year. Then, I write the objective and important terms on there that the kids need to know. Then, I write my agenda for what I'm going to do. With algebra one, I will make a presentation sometimes. I know generally what the objective is. From that objective, I will look at what I did the previous year and modify it or change it. (Personal Interview, 08/23/16)*

She goes on to note that using the book feels "like a crutch" and "takes away from my creativity" (Personal Interview, 08/23/16). Instead, she uses the book as a check to see how the material is being covered, and whether she's considered all of the different variations of a problem. For her, the textbook does not engage the students sufficiently. Lily also believes that math should be engaging for students, and that the way to do that is to make the tasks fun and appealing.

> *If the textbook were to have interactive activities, like let's say the textbook was online and it had a SIM that went with it, I would probably use it more. I mean we're adopting all new textbooks next year. It'll be interesting because I know they're compatible with the iPads, but I don't know how much more is on them that's going to make them more appealing for in class activities.* (Personal Interview, 08/23/16)

The school just moved to a one-to-one technology plan, and all students now have the use of an iPad. The teachers are encouraged to make use of the iPads, and therefore, in this class nearly all work is done with that device. Each unit, students create a portfolio that includes notes, classwork and homework. Much of this is done through Schoology, a learning management system that allows teachers and students to create, manage, and share content and resources, and through apps such as Notability, which enables students to import and annotate the documents.

5.2.2.4 Lesson planning with PhET

As mentioned earlier, Lily describes herself earlier as a textbook rebel, so rather than moving through her textbook one lesson, one day at a time, Lily instead creates her own unit plans through a Google calendar, which she revisits each year. As a result, for Lily, the process of writing out full lesson plans was contrary to her normal routine. She noted that it if were up to her, she would be more likely to just try out a simulation in a classroom without a full lesson plan, just to see if the lesson worked (Field notes, 08/11/16).

As part of the planning process, Lily developed one two-day lesson using the Function Builder simulation. In the first day of the lesson, students would explore the first screen, which contains only images, without numbers. On the second day, students would move to numbers and explore how the functions worked. Lily taught both of those lessons as part of this research. In addition, she used another PhET simulation, Graphing Lines, in exactly the way she had described earlier. She created stations where students rotated through a series of activities design to support exploration of linear equations. While no formal lesson plan was created, Lily did write a set of instructions for the students.

5.2.2.5 Use of technology in mathematics classes

Since technology was always a part of the classroom, with each student using his/her iPad regularly, I considered those classes to be 'business as usual', and compared those against the classes when PhET simulations were used. This is a bit different from the other teachers, as technology was not regularly a part of the business as usual class.

Lily sees value in the technology, describing as an example how Desmos (an online graphing tool) can help students:

> *I definitely think there are other types of activities where the technology gives them the freedom to play around with different things so that they have a better understanding. For example, using sliders on Desmos and looking at your leading coefficient, just doing that on the computer and having that freedom to move it around so easily and see the immediate consequences allows them to have that deeper understanding.*
>
> *I think it enhances them in the sense that there's more opportunity for self-play and discovery. I mean it's just so much quicker and easier as far as showing, like I said, with the slider. Because of that, I think it also allows more opportunity for enrichment and extensions because there will be those kids*

who just get it right away, get the concept. Then, they can move on to something more challenging. (Personal Interview, 08/23/16)

For Lily, she saw benefits in a dynamically responsive system that enables students to see changes instantly. This type of responsiveness could lead to students better able to generalize patterns and make deep connections between representations. Furthermore, Lily saw technology as a way to make connections more quickly, allowing students to progress mathematically as appropriate.

Technology is seen as both a blessing and a curse in the classroom. Lily likes that students have ready access to the internet to use tools such as Desmos and the PhET simulations, but notes that it is very difficult to see whether students are on task. In the observations I conducted, students did the assigned work, but were quick to switch between the classwork and other interests on the internet such as surfing Facebook or watching a basketball game. IPads appeared to be particularly difficult to monitor for two reasons – first, the screen is not propped up and viewable as it might be on a laptop or Chrome Book. Many students instead held the iPads in their hands near their laps, or placed them flat on the tables. Second, the ability to dim the screen made it nearly impossible to walk by and see what students were seeing unless the angle was exactly correct. Both of these issues were regularly addressed in Lily's class, without substantial changes.

Talking with Lily at the beginning of the school year, just before students started working regularly with iPads, she also expressed other concerns with the technology.

Actually when you asked that question how comfortable are you, I'm comfortable with what I know. Now that we're getting the iPads there's a lot of things out there that I'm not comfortable with like Near Pod and all these different apps that I haven't used hardly at all. That does scare me. I'm kind of scared.

I guess that I'm going to put all this time into planning something and then it's just going to fail. Then, I won't have anything. I'll be relying on it. Also, I get

> *frustrated as a learner if somebody is battling with technology. I just feel like it's a waste of time. (Personal Interview, 08/23/16)*

For Lily, technology could provide many benefits for students, but when she, as a teacher, is trying to learn how to use the technology at the same time she is teaching the lesson, the technology challenge just becomes too great. For Lily, she worries that problems with technology will lead to frustration (both for her and her students) and that overall, will not give her the results she wants.

5.2.3 Sara

5.2.3.1 Background

Sara is a young white female teacher with a combined 9 years of teaching experience, including two years as a third grade teacher, five years as a fifth grade teacher, and two years as a middle school math teacher. While still relatively new to middle school, Sara has taught sixth grade math content for four years, as she taught an accelerated fifth grade math class, which covered sixth grade content. She has an undergraduate degree in Psychology with a master's degree in Curriculum and Instruction with an emphasis in math and science education. Besides the sixth grade class I observed, Sara also teaches two eighth grade classes, two sixth grade classes, and a support class for sixth grade math.

In the observed class, there were twenty-four students. The school is described on the district website as having a student population that is predominantly white (64%). For Sara's class, she describes only three of her students as students of color. She also does not have any English Language Learners in the observed class, although the school demographics lists 13% of the student population as ELL students. Approximately 30% of the school population receives free/reduced lunches. The daily duration of the class is 52 – 54 minutes.

5.2.3.2 Classroom Design and Norms

Sara's classroom is extremely colorful, with posters covering the walls. Some of these posters are premade, with quick motivational quotes such as 'We all make mistakes!' but most of the posters are handmade by Sara to remind students of key skills. Sara mentions several times over the year that she uses Pinterest® as a source of inspiration, and the posters may be evidence of this, as they are written in flowing handwriting using a variety of markers, and often including kid-friendly mnemonics to help students remember steps.

There are desks for each student in the room, which have both the chair and the table attached. These are easily moved by the students, a skill they have apparently practiced often as Sara quickly counts down from twenty when she wants students to arrange themselves for a specific task. Most often, the desks were arranged in groups of two or three, although students had great flexibility to rearrange their desks if students felt it helped them work better together. There is also a rug at the front of the room, more reminiscent of a lower grade classroom. Students are regularly called up to the rug, either as a way to move the conversation closer to the board, or to work with a small group one on one. Both classroom design features, having desks that are easily rearranged and a rug for smaller group conversations suggests that Sara values student interaction. During observations of her class, I also saw that there were multiple opportunities for student interactions in their prearranged groups, in their own self-designed groups, and in small groups on the rug.

5.2.3.3 Curriculum and Resources

Sara described her curriculum as Common Core, which is made more specific first by the state, and later by the district (Personal Interview, 031617). She uses the Connected Math Series for her 6th grade class, which she describes as "an exploratory curriculum with great real world examples" yet goes on to note that it is supplemented roughly 25% of the time "in order for students to be able to access the problems." She gave an example of what she meant by this, saying,

> *Our last unit ... was fractions. I mean it was operations with fractions, like division with fractions and it jumps students into their example first. So, to an application problem first. And we really wanted our students to be really solid with common denominator strategy for dividing fractions and we've found that there was no practice for them to be able to really know how to do the process before applying it. Which, I mean, you could also say that you have this process and you're learning about the strategy through the process, but we've found that our students are having a lot of trouble even approaching the problems when they don't know how to solve them. And we don't do that in every case but that was a time when we felt like we wanted to teach them how to do it and then, so that they could complete the problems. (Personal Interview 030617)*

So while the book is designed to support students in constructing their own strategies prior to developing traditional algorithms, the book is often supplemented prior to these lessons to build skills first, and then conceptual understanding later. This is seen as a way for students to be able to more easily access challenging problems. This supports the wide diversity of learners typically found in the sixth grade classrooms.

> *I think that the interesting thing about the sixth grade math classes at our school is that they are so diverse. So we have kids who are, I mean in that class in particular, we have kids who are working at a fourth grade level. Actually in that class, we have kids at a third grade level, who are receiving intervention either through their special ed class or through our math clinic, all the way through kids who are probably at a seventh grade math level. (Personal Interview 030617)*

It is this diversity of learners and their prior knowledge in mathematics that seems to push Sara to look closely at her lessons and consider what materials best serve the students.

5.2.3.4 Planning with PhET simulations

In general, Sara develops full lesson plans for each lesson, and actively uses those plans as she teaches. She shared her planning process with me, saying, *So, I still have a lesson plan for all of my lessons and the reason that I do that is because if I'm going to put that much work into it at one point, I want to be able to reference it later on and I do try and plan ahead. So, I just have a Google doc with my unit plan and then I have each of my lesson plans linked to it and mine are just mostly like a piece of paper that have my timeline. So it'll have what I want to do- It has my objective, and then materials. And then it'll have kind of a table with my timeline, so what I'm doing for my warm-up that day, if there's any direct instruction, what I want to make sure to cover while students are working, kind of during their independent work time and then a summary question.*

I have that [lesson plan] out when I teach because I suffer from 'out of sight out of mind,' so, I still write lesson plans for all of my lessons. And I don't think that that'll change the longer that I teach, it's just my teaching style, cause I don't want to forget what I'm doing. (Personal Interview, 030617)

Sara did the least amount of planning with the other teachers, as she was newly married that summer. She did attend two meetings, allowing her to get to know the other teachers, and to get an overview of the PhET simulations. In the second meeting she attended, she reviewed lessons designed by the other teachers, and shared her ideas of how she could use the simulations in her classroom.

Sara described herself as always looking for new ideas for her classroom. This was evident in our planning process as well. As she talked about possible simulation use in her classroom, she also talked about the lessons other teachers had prepared.

Then, I was looking through, and I saw that there were two lessons using the expression exchange. One of them had to do with combining like terms. I was thinking about maybe [writing another lesson] that built on that, because I felt like [combining like terms] was something that was so hard for my kids. I wanted to look at that [other lesson] a little bit more, too. (Planning meeting, 07/24/16)

Sara considered other lessons, and looked for ways to use or build on these lessons within her own classroom. Her openness to new ideas also keeps her open to changing her current lessons. She notes that she often makes changes from year to year, and even from class to class.

> *Between [class] periods, I'll sometimes change the problem that I asked, or an equation maybe, because of the way that kids talked about it. Maybe there was a point that I was trying to get across that didn't come across, so I have to change the way I ask my question, or maybe the numbers in the problems, so that they do get the point of what I'm trying to get across without me telling them. (Personal Interview, 03/07/17)*

This openness to trying new ideas, and then modifying these lessons to better fit her class suggests that she sees lessons as malleable. These changes are then captured on her lesson plan, so that she can make changes when she teaches the unit again, next year.

> *My unit plan is on Google docs and so I'll just go in and maybe make a comment, like "this lesson needs two days next year." Or I'll highlight the lesson and be like, "use this activity instead." Or "use this activity with this lesson," so that when I go back and am planning next year, I see all those notes that I made for myself. (Personal Interview, 03/07/17)*

Overall, these quotes suggest that Sara takes lesson planning seriously, and sees her overall yearly plans as a living document that can be changed as needed to support her students.

Sara wrote two lessons using PhET simulations both of which I observed. The first lesson involved the Unit Rates simulation, and was designed to support students' conceptual understanding of using a double number line to find a unit rate. The second lesson involved the Fraction Matcher, which was used as a means of building fluency with equivalent fractions.

5.2.3.5 Use of technology in mathematics classes

Sara has a wide range of technology available in her classroom including a projector and smart board, iPad, class calculators, and a document camera. She notes that while she doesn't have clickers, she has a program that she uses that can make either computers or phones into polling devices, so it's not something she misses. She also has access to a cart of Chromebooks that is typically stored in her room. She was instrumental in getting those computers, saying,

> *So the Chromebooks that we have in my room are the result of a grant that I wrote last year. We got 30 Chromebooks through that grant and the purpose*

was really for the math clinic class to help those students. But also in the grant I just talked about all the amazing things that kids can do to get really great feedback and how they can share the ideas of others, and so [we received] a [set of Chromebooks for the] math department that's housed in my classroom. (Personal Interview 03/06/17)

Sara is enthusiastic about bringing technology into her classroom as is evidenced by the above quote. She talks about the "amazing things" that kids can do with technology, and the ability for technology to provide feedback to her students. She also notes another benefit to technology and that is the ability for students to more easily share their work.

Sara sees value in bringing technology into her classroom, and uses it as both an organizational tool as well as an instructional tool. As an organizational tool, Sara uses technology to keep herself and her students on track. Sara maintains a website for her class that includes daily objectives, and links to homework. The website also includes videos of Sara teaching topics for students to review on their own as needed. She also uses her SmartBoard® regularly, indicating that while she could probably do the same thing with PowerPoint®, the SmartBoard software enables her to attach documents directly in the files, making it more efficient. In addition to her math website, she also maintains a Google Classroom, a learning management system for K-12 classrooms that supports creation, sharing and grading of documents, and uses that to organize her daily activities.

As an instructional tool, Sara mentions using both Desmos, an online program that supports graphing activities, as well as GeoGebra, a tool designed to support geometry and algebra activities. Desmos is primarily used as a tool, for graphing activities, although, as Sara notes, they have "come up with some really cool activities lately." GeoGebra is used predominantly for their geometry unit, where she uses premade activities that align with her textbook.

While she enjoys using technology in the classroom, Sara notes that these tasks are typically a full class activity, so she has to plan accordingly. For Sara, that means adjusting the rest of the class to enable enough time to support the use of technology and all that it entails.

> *Because it does take a lot more time to get the devices out and to get kids going and then if it's an activity that's really worth it, I want kids spending a lot of time on it. So there's some other things that might go [be eliminated] that day, like we might not have a warm up that day. We might have to change the time to talk about homework from the previous night if there were lots of questions about something. So I think it's like more time is devoted to the activity because if I'm going to use computers, I want it to be worth it. I want them to be able to really meet an objective from that. So the majority of the class period is just taken up by that activity. (Personal Interview, 03/07/17)*

This suggests that Sara wants to have technology that is focused or targeted to a specific curricular objective, and in order to support that objective, she needs to ensure that students have an adequate time to use the technology as intended. Timing often came up in our conversations, with Sara talking about how she ensures that she has enough instructional time.

> *When I think about any kind of application I want them to use, or any kind of manipulative, I would say that I don't often get out physical manipulatives. A lot of things we just do on some kind of applet that they have and that makes it a lesson go much faster cause they can have those materials and have something that is really tailored to what we're doing for the lesson and people have things out there that are free and you can find them and that's really helpful to my planning process and my purpose and to meet my objectives with students. (Personal Interview, 03/07/17)*

As a result, Sara is drawn to technology where it helps her increase her instructional time and not waste time. As I mentioned earlier, she shared that she often uses Pinterest as a resource for new ideas, as well as talking with other teachers, and going to conferences. When she finds an interesting activity, she adds it to her yearly plan so that she can consider the activity when planning that unit next year.

5.2.4 Christine

5.2.4.1 Background

Christine is in her 7th year teaching, with most of her experience in middle school after one year teaching Algebra 1 in high school in another state. All of her middle school experience has been in her current school, where she teaches classes from sixth to eighth graders. She has a Bachelor's of Science in Mathematical and Computational Science, is licensed to teach secondary mathematics, and describes herself as "very comfortable" with the content. Christine regularly attends a variety of conferences and classes in an effort to continually learn and strengthen her skills as a teacher. She also regularly presents at conferences, sharing activities from her own classroom.

She describes her class as very diverse, considering both demographics and mathematical ability. She notes that her class is representative of the overall school, which is described as having a population that is 48% non-white, 40% free/reduced lunch and 23% ELLs. Then she describes the heterogeneity of the class when considering mathematical ability.

> *In all of eighth grade math classes, we group the kids heterogeneously, so in terms of ability level, you'll see kids in who are working at the advanced level some days and you will have kids who are far below grade level, and they're all in the same room. We don't do any kind of pullout. We do a fully inclusive [classroom], in terms of students with English learning needs, students with special education needs and then just every other kind of learning quality that students have. Yeah, if you're in eighth grade math, you have an equally likely chance of being in any classroom as any other kid, which I think is something that our department really values and really believes in. (Personal Interview, 08/17/16)*

For Christine, this diversity in the classroom is something she clearly values as well, and as I'll describe later, her lesson planning builds in opportunities for that diversity to broaden student thinking. This also means that as she plans lessons, she must take this diversity into account, finding activities that are easily accessible yet provide a high level of challenge when needed. Christine also speaks a significant amount of Spanish, which is the predominant language for her ELL students. As a result, it is common to hear her speak to an individual student in both Spanish and English, and she encourages her students to answer in Spanish if that is more comfortable.

5.2.4.2 Classroom Design and Norms

Christine's classroom is large, with angled tables that easily connect to make groups of two, three or four students. The angles on the tables support students working together as well as class discussions as the desks are turned slightly toward one another, and yet every student has a clear view to the front of the room. Most of the time, the desks are arranged in groups for two or three students. Along the left side of the room, there is a large bank of windows, while the right side houses a set of cabinets with counter space. The front wall has a large white board, and the back wall is covered with posters made mostly by students as part of class projects. The posters detail big ideas such as connections between tables, graphs and equations, and parent graphs of linear, quadratic and cubic functions.

Class discussions are a regular part of Christine's classroom norms, with students talking either to one another, or in a whole class discussion. Students are also encouraged to work with one another on their investigations. Christine notes this when she describes what a great class might look like,

> *I would want to see every kid talking and every kid sharing their ideas. In my head right now, I'm envisioning there are groups of four and they're all leaning in and their butts are up in the air, and I come by and a group has a question. […] Then, they're like, "Oh, we need to talk about it." So my favorite thing is when I walk away from that group because one kid had a question and they didn't check in with their group about it, that they ask their question to the group and another kid is like, "Oh, well you just do this, this and this." They can answer the question for the kid, so it's like they don't need me. They can learn from each other. (Personal Interview, 08/17/16)*

Christine does not see herself as the answer-keeper in the classroom, but rather sees each student as a strong contributing member of the team. Student contributions are welcomed and encouraged, something she describes as taking significant effort from herself and the other teachers she works with. Students appear to be very comfortable with Christine as a teacher, and they have a great rapport with her. Many chat before and after class, and I often saw a few students who could come join her in her classroom for lunch.

5.2.4.3 Curriculum and Resources

Christine's school uses a curriculum strongly based on the Common Core Standards, listing a financial literacy standard as the only addition for 8th grade students. In her district, all teachers use the same textbooks for the same course, with the ability to supplement. She notes,

> *It's sort of this hierarchy. Most of your time you should be teaching from the main set of materials, and then you can use the [approved] supplemental materials, for example, to teach a support class, and then you can use whatever else you want to just enhance those [materials]. (Personal Interview, 08/17/16)*

Christine was on the textbook committee when the district selected the Connected Math series. She describes the series as having a "really rich set and sequence of problems that guide students toward actually understanding concepts" and goes on to say, "I really believe in the materials." Yet despite the value she sees in the textbook, she does feel like there is an important curricular piece that is missing.

> *I don't think those materials themselves provide enough practice for kids, so they develop a really good conceptual understanding, but then when we actually want them to develop the skills that go along with that, we often have to supplement. (Personal Interview, 08/17/16)*

This suggests that Christine sees value in both rich mathematical problems, which develop conceptual understanding, as well as skill-oriented problems that develop fluency for students. As a result, Christine shares that most of her supplementation is around skills practice, which she feels is good, since she says that it's much easier to find skills back supplemental material rather than good rich problems for students.

Christine says that she mostly uses the Connected Math investigations in her lessons, which was consistent with what I observed. I also saw some supplementation with worksheets to support a review of topics in preparation for an upcoming test. Overall, Christine tells me that she's "definitely using the same activities from year to year, but the way I'm approaching them changes from year to year." For Christine, this suggests that there isn't as much need to find new resources, but rather to ensure that these resources are correctly adapted to the specific group of students in her classroom. She reinforces that a bit later, as she describes the types of notes she might leave herself for the next year, when she reuses the activity:

> *Every year, it's interesting to see, I make notes to myself about, "Wow, that took 40 minutes. I thought it was only going to take 10 minutes." That's part of knowing 13-year-olds, but then there's still these little things where it's like, "But these specific 13-year-olds, they may need something really different." (Personal Interview, 08/17/16)*

Christine again references this attention to diversity in her classroom, and uses this diversity as a driving force towards modifying lessons. She regularly attends to 'these specific 13 year olds,' ensuring that the intended lesson fits their particular needs.

5.2.4.4 Planning with PhET simulations

When thinking about planning her lessons, Christine begins first with what she calls the "big ideas of math," which for her 8^{th} grade class would be functions. "Algebra really is the study of patterns and functions, you know? It's generalizing the real world using expressions." With this focus on functions, then, Christine plans her units, and then later, her lessons. Lesson planning is an important part of teaching for Christine, and she writes full lesson plans for each class period. She described her organization of the plans in this way:

> *At the top, I list the date, the general idea for the lesson, and my learning goals. Then, I have a list of materials, and things I need to do before the lesson. My actual plan has timing on one column, the actual activity that students are working on in my middle column, and notes to myself in the far right column. That's often the place where I'll say how I want to group kids or ways that I want to elicit questions or answers from students, or it'll be where I write down questions that I have that I want to ask students. It's also where I'm really trying to highlight for myself which math practices I want to make sure students are thinking about in each of those activities. Finally, I list the homework assignment for that night at the end of the lesson plan. (Personal Interview, 08/17/16)*

These are very detailed lessons describing not only the timing and actions of the students, but also the instructional moves she plans to enact within her classroom. The fact that she includes her instructional moves in the lesson suggests that she sees these practices as essential for learning in her classroom. As mentioned earlier, the diversity of her classroom is also an important consideration in Christine's planning process. She noted that one of her goals is to create a mathematical community, so she considers that as she plans her lesson.

> *How are we going to really show that having different ways of thinking is a good thing? How are we really going to emphasize having different perspectives on the exact same drawing or the exact same set-up is a really good thing? It definitely makes it a challenge, but I […] really feel like it's a worthy challenge. (Personal Interview, 08/17/16)*

Therefore, Christine considers not only the tasks in which students will engage, but also the instructional practices that will support a community of learners such that students will similarly learn to value the diversity of thought within mathematics.

Throughout the summer process of lesson writing, Christine was by far the most active participant. She reviewed all of the other lessons, and provided comments to many teachers on their lessons. Often, her comments were geared towards pushing teachers to think through ways to engage their students in conversations. For her own class, Christine wrote two lessons using PhET simulations. The first lesson involved the Function Builder simulation, and was designed to support students' conceptual understanding of connections in representations of a function. The second lesson involved the Proportion Playground simulation, which was designed to support student understanding of slope of a line.

5.2.4.5 Use of technology in mathematics classes

Christine has a variety of technology tools at her disposal including an iPad, a small set of tablets, a document camera, and a projector. She also has access to a cart of Chromebooks for classroom use. This year, her school has just moved the eighth graders to a one-to-one device program, so each 8^{th} grader has been assigned a Chromebook for school and home use. For Christine's class, which is an 8^{th} grade math class, with 10 seventh graders assigned to it, this means that she only needs Chromebooks for a portion of her class. Finally, she has a full set of graphing calculators in her classroom. Christine also talks about the educational software that she uses regularly with her class which includes Desmos, the online graphing tool, GeoGebra, an online tool designed to support reasoning with algebra and geometry, and the National Library of Virtual Manipulatives (NLVM) which provides applets of commonly used manipulatives for math classrooms. She is quick to note, however, that NLVM is not compatible with Chromebooks and therefore is not often used anymore.

Christine sees technology as a way to make mathematics more accessible for students. She says,

> *I think a lot about my kids who are really high-performing thinkers, but who really struggle with even just the physical act of writing or who really struggle with drawing a straight line on a graph, even when they have a ruler, like how challenging that is, and then how they miss out on really interesting learning because this other thing that got in the way of them actually thinking about a problem. (Personal Interview, 08/17/16)*

So for Christine, technology opens up access to the mathematics and perhaps even levels the playing field a bit for students who are able to think deeply but can't articulate that knowledge in the same way as others can because they struggle with the mechanics of writing and drawing the math.

Christine also notes that the use of images instead of numbers can also help students visualize and make comparisons in mathematics with technology that may not be considered otherwise. As an example of this, she talks about PhET's Function builder simulation.

> *Right, that's what I was thinking, especially with the shape screen [on Function Builder] where you put a shape in and maybe it reflects it but it looks the same. This is a really interesting idea, because geometrically it's interesting, but it's also like, what happens when you have a function where you put in the number one, you get out the number one? It definitely squared it, but it doesn't look any different. (Personal Interview, 08/17/16)*

This ties back to Christine's notion of looking at mathematics as a series of big ideas that need to be connected. For her, technology is one way that students can start making these connections more visible that otherwise may remain hidden.

Technology is also used as a tool, supporting her community of mathematicians.

> *One of the coolest things, I think, is when students are working on a task and there are multiple solutions or solution strategies. I often use my iPad to take pictures of [their work] and just save it to my Google Drive, and then we can immediately just project [their solutions on the board]. I snap pictures of it and then have other students, explain what that strategy is doing or how it's different from [their own strategy]. (Personal Interview, 08/17/16)*

As Christine later notes as we are talking, she used to share student work using her document camera, and now, using the iPad for photos enables her to share the work more quickly, and minimize student distractions. In this use, the technology is replacing activities that can be done without technology.

Despite Christine's recognition of the value of technology, her students do not often use it in her class. Christine tells me that the class probably uses technology five to six times a month. This is likely due to Christine's perceptions that technology should only replace hands-on activities when those activities aren't possible or are too difficult to do in class. Many times, Christine notes that hands-on activities bring a sense of touch that is important and memorable for students.

5.2.4.6 Dispositions towards Mathematics

Christine sees mathematics as a series of connected ideas. Her role is to ensure that units and lessons are planned in such a way that students grow to see and understand those connections. She does this through a community of learners where students are expected to work with one another and learn from one another. Christine notes,

> *For me, I don't see my role as the explainer, right? That, to me, it feels like then I know everything and you know nothing, and so my job is to help you learn so that you know more, and by telling you, right? I just really don't see that as my job. (Personal Interview, 08/17/16)*

Instead, Christine seems to see her role as the builder of community and norms such that students are able to work together and learn from one another. This social aspect of learning is seen as vitally important to her classroom. Later, Christine talked about how some teachers are trying out a flipped classroom. In a flipped classroom, students watch videos of the lesson at home for homework, and the use classroom to complete the typical homework, so that the teacher is right there if students need help. Christine was discussing why she struggles with this idea, and relayed the following story.

> *I was chatting with a former student this summer and she said, "Oh, I really like the flipped classroom." I said, "That's really great. I'm glad that you like that teaching style. Is there anything that you miss?" She said, "I felt like I went through the whole year and I learned a lot of math, but I never even learned half the names of the kids in my class." I was thinking yeah, not only did you not learn your names, but you didn't learn from them. That's too bad, you are in this social environment and yet there was no social aspect at all. (Personal Interview, 08/17/16)*

Again, for Christine, a math classroom should be a social environment, where students are able to engage in the mathematics and are supported by strategic moments of student collaboration and shared thinking. So her role, as a teacher, is to create this environment.

She notes that she wants to create opportunities for active learning in the classroom, and in doing so, brings on a totally different dynamic.

> *"For some people, it looks really chaotic, to have 30 kids talking at the same time, and to have kids moving around the room and doing different tasks, and they're like, "How do you monitor their learning?" Well, in some ways, I'm not. [The students] are responsible for monitoring their learning, and it's when we come back together that I can get more of a sense of where people are." (Personal Interview, 08/11/16)*

Therefore, as her role moves to one that creates and supports such an environment that allows for active learning, and rich discussions, students' roles change as well, shifting to students' being responsible for their own learning.

5.3 THEMES

Three general themes were evident from the four cases studied. These themes include the benefits of simulations, the deterrents to using simulations, and the challenges in choosing the best mathematical task.

5.3.1 Simulations provide important benefits to student learning

All four teachers shared a belief that technology in general, and simulations specifically provided significant benefits for students' conceptions of mathematical ideas. These benefits were discussed and supported through both their interviews, and later, their reflections. Teachers identified three key attributes of simulations that were important reasons for bringing them into the classroom: a richer mathematical experience through real world contexts and dynamic responsiveness of the tool which helped facilitate student analysis and student to student interactions, through the creation of concrete representation for abstract ideas; increased student engagement; and the reduction of some of the more tedious aspects of mathematics to spend more time building conceptual understanding.

5.3.1.1 Rich Mathematics

For the first of these ideas, teachers described several features of simulations that could facilitate a rich mathematical experience: connections to real-world contexts, features which enabled students to quickly see changes in representations, generalize patterns, and increase student to student interaction.

5.3.1.1.1 Realistic worlds

Teachers welcomed the realistic settings such as the function tube in Function Builder, the necklace and pool representations in Proportion Playgrounds, and the fruit and vegetables in Unit Rates. Sara noted that "The way that you can model real life situations with the math that's involved and I feel that that's truly math, rather than just solving some problems. So I feel like [this simulation] gives kids a real life application for where they might actually see things happening" (Personal Interview, 03/06/17). Pat expressed similar support saying, "The sims I like, because they bring a real world [environment] - a realistic world, not a real world. They're not doing financial whatever, but a realistic world where they can do things" (Personal Interview, 08/17/16).

Pat also later reflected that simulations provided a more realistic environment in that using a simulation with premade graphs and tables was more similar to the online tests that students take as part of the district assessments (Lesson Reflection, 02/21/2017).

5.3.1.1.2 Concrete Representations of Abstract Ideas

This realistic environment was also seen as a benefit because it provided the opportunity to provide a "concrete representation for abstract ideas" as Christine described it in her interview (08/11/16). She continued on thinking about this, saying, *Like the sim that I wrote one of my lessons for [Proportion Playground], which was all the necklace patterns, and then the color splash, and all of those different things; it's a really interesting way of thinking about like, "Well, what is a ratio? How can you tell when ratios are equivalent? What does that mean? The numbers look totally different; how could those be equivalent?" (Personal Interview, 08/11/16)*

Christine appeared to see the idea of ratios as very abstract to students. However, through the use of the concrete representations, not just the realistic pictures of the necklace and pool tables, for example, but also of the other representations, which facilitated student understanding about whether the two ratios are equal (See Figure 5-1).

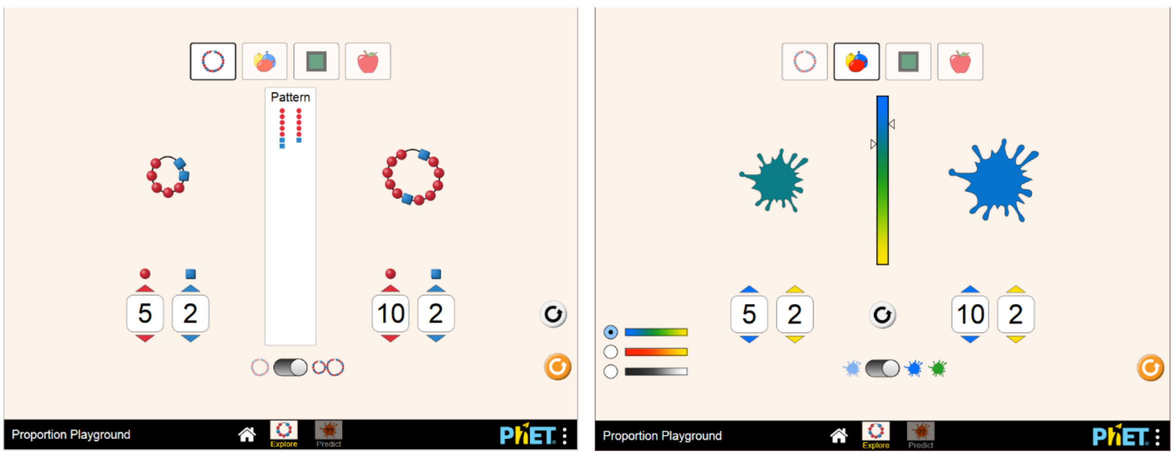


Figure 5-1 A screen shot of Proportion Playground. On the left, note the difference in the pattern display as well as the differences in the picture of the necklace. On the right, the same two ratios displayed with the paint splashes.

Both Pat and Lily identify this in similar ways. Pat notes that the simulation offers students the ability to physically interact with the mathematics, saying, "I can have a worksheet where they're building things, but it's different than moving things around a screen. It's the concrete aspect, even though it's in a digital environment" (Personal Interview, 08/17/2016). Lily brings up this feature as well, although in her case, she uses Desmos as an example, rather than one of the simulations, saying,

"I definitely think there are other types of activities where the technology gives them the freedom to play around with different things so that they have a better understanding. For example, using sliders on Desmos and looking at your leading coefficient, just doing that on the computer and having that freedom to

move it around so easily and see the immediate consequences allows them to have that deeper understanding" (Personal Interview 08/23/2016).

While this idea of a concrete representation is similar to the first notion of a realistic world, I assert for the teachers that these are two distinct features. The first is using a context that is realistic and memorable to the students, while the second (a concrete representation of an abstract idea) is more linked to the students being able to visualize the math using multiple representations.

5.3.1.1.3 Promotion of Student Discussions

Having a context that was realistic for students was seen as important feature that might prompt student to student interactions. Two teachers mentioned that simulations offered opportunities for rich discussions. Christine posed this as simulations provide a "natural reason to have a conversation" (Christine, Lesson Reflection, Tech Observation 2), while Sara noted that, "[working with sims] can also increase [students] interaction with each other because they have something to talk about and a problem to solve" (Personal Interview, 3/6/17).

While teachers all expressed their ideas in slightly different ways, it seems that all four of these teachers saw technology in general, and simulations more specifically, as offering a rich mathematical environment for their students. Simulations offered a realistic environment, where students could interact with the mathematics, facilitating ways to analyze, or consider more deeply, patterns being observed. The simulation therefore played a role in their mathematizing, as it provided a concrete representation of an abstract idea, as Christine put it. By opening up spaces for this mathematizing, teachers seemed to indicate that this would result in more student to student interactions, and facilitate mathematical discussions.

Not only did the premade graphs help provide for connections to their testing environment, Pat also felt that having these computer generated graphs enabled students to start thinking more deeply about the mathematics. "I think with some of the technology, you can get higher level thinking. Instead of all your time spent graphing, you can start analyzing different graphs" (Personal Interview 08/17/2016).

5.3.1.2 Increased Student Engagement

The second benefit seen by the teachers was that the use of technology, generally, and simulations more specifically, would increase student engagement. All of the teachers raised similar ideas that student engagement would increase through the use of simulations, but Pat articulated this best. She described it this way:

> *"I'm doing lines right away. That's why I'm thinking about it. You quickly have three lines graphed and then be able to do the comparison in a 52 minute period. If I put it up here, I bet two kids paying attention. You know how that is. When I'm doing stuff up here, I do not feel like I have them all. When they're all doing it, I feel like that's where technology really helps. They can all graph their own lines on their own computer. They're more engaged, because they have their own thing and they're putting it in. […] I'll have a PowerPoint. I'll just give them some information, write down whatever. I'll give a question right after that and half the class misses it. I know that they're tuning out. I've got to have them doing something." (Personal Interview, 08/11/2016)*

In every reflection of their lessons, teachers consistently mentioned student engagement as one of their first comments as well. Student engagement was considered, evaluated and discussed in three ways: were the students on task, were they having fun, and were they engaging in the mathematics. As I describe how teachers talked about student engagement, I have chosen to include quotes from Christine's reflection to help tell this story as she talks about engagement that is consistent with the other teachers' account, and clearly touches on all three ideas about engagement. In Christine's lesson, students used Proportion Playground to create an interesting ratio that was acceptable to the whole group. Later, the students had to create a picture with an equivalent ratio. As the lesson came to a close, Christine's reflection focused on all three aspects of student engagement. First, she made a determination of whether students were on task, doing the assigned activity, and using the simulation.

> *I was just like, "Oh my God, these girls are, they're the ones who almost always get the Chromebooks, and they immediately go to like [social media sites], and so this time, I went over, and I was like, "God, they're so engaged, they're probably not engaged in the right thing," and I was like, "Oh my God, they're actually arguing about the ratios." (Lesson Reflection, 11/18/16)*

The second determination was whether the students were having fun. In other words, she was looking for a sense of enjoyment by the students while completing the activity.

> *Well, and I don't know if you captured this one, but you know how challenging it is to get [student] to do anything, and way he chose the Paint Splash first, and he was like, "Now, see where I put this, this is awesome," and I was like, "Okay, note to self: Give his group Paint Splash," and then, when he changed to the pool table, he's like, he was even more excited about it, and I was like, "Okay, note to self: Give his group the paint table" (Lesson Reflection, 11/18/16)*

The final sense of student engagement was the engagement of the students with the mathematics of the activity. This was discussed in terms of children "getting it" and were they engaging with the mathematics.

> *"I mean, you carrying the camera. I don't know how many conversations you could capture, but even though [I felt] like, "Oh, man, I don't know if I'm doing what I'm supposed to be doing," the kids would be like, "No, it's not 10-4, it has to be… you have to make sure you do 8-4, because you went up one here, so you have to go up two," and it was like, they're actually into reading it, you know? (Lesson Reflection, 11/18/16)*

These three checks into student engagement – were the students on task, having fun, and mathematizing – were consistently considered by each teacher in their reflections. Teachers also commented on each during their reflections. In each of their lessons using PhET simulations, comments on how engaged the students occurred regularly, and typically used phrases such as 'boy, they were having fun' and 'the kids really liked that'. While the teachers came into the project assuming that the simulations would promote student engagement to varying degrees, I believe that this was solidified through their own personal experiences. In the initial interviews, most of which were conducted at the very start of the project, teachers mentioned student engagement as a reason to use technology. However, it wasn't until the observations that I saw the ideas of student engagement being categorized in these three ways.

5.3.1.3 Accessibility

The third benefit to technology generally, and simulations more specifically, was the accessibility of the simulations. When talking about accessibility, teachers used this term in several different contexts. First, accessibility was used to talk about ways in which the simulation could do the 'heavy lifting' for the students, so that the students could concentrate on mathematizing. In other words, the simulation reduced the amount of time spent on the more tedious aspects of the problem, such as graphing the lines, creating the table, drawing out the picture, in order to preserve time for student analysis of the patterns in order to be able to gain more conceptual understanding. This made the learning more accessible for students. Second, teachers talked about ways in which the content of the simulation was made accessible to students. That is to say, the ways in which students could engage with the mathematizing whether they were a novice or more advanced in their understandings.

Pat first talked about how simulations could reduce unnecessary work early on in the project, when she talked about how she used her textbook.

> *Well, for each lesson I make a lab sheet. And I get much more student participation than if they just opened the book, get out paper, they have to draw their own graph, like, if I provide a grid ... And any testing that they do, I have yet to see statewide where they have to draw the X and Y axis. Usually there's something provided. They don't have to make a table, there's a table there to fill out. (Personal Interview, 08/11/2016)*

In this segment, she noted that for each lesson, she provided the basic starting points so that students don't have to create their own graphs or tables. This is done for two reasons, first, because she gets more student participation (engagement) and second because this support matches what is expected on the state tests taken by the class. This perception, then, that students will be more engaged and get moving more quickly in class is similarly solidified when she uses technology. As she reflects about a future lesson with a simulation and why this is the best choice for the students, she argues,

> *"Especially when you're analyzing graphs and stuff, I think it's better to use the technology because they get bogged down." (Pat, reflection, 02/21/2017)*

For Pat, she saw the technology as a way to ease up on the burden of graphing so that students could focus on the analysis of the graphs. Sara sees it similarly, as she argues that reducing this effort (having the software take on that heavy lifting) is also an issue of accessibility.

> *So I think about how taxing it would be for a kid who struggles to graph a line and then to not be accurate and have to keep graphing it and keep graphing it and keep graphing it and for them- they could easily do that, they could try something ten times pretty easily because there's not as much effort that's going into doing that. (Sara, personal interview, 03/06/17)*

Christine takes on this idea as well, although she modifies the idea just slightly. To her, accessibility is also looking at how students can access the information, and how the material is accessible for each student. She says,

> *Well, what I love is the accessibility piece. I'm thinking about my classes and how different each kid is. This is true. Even if someone said, "Oh, this class is not heterogeneous," kids still are different and they think in different ways and they have different skills. I always think about what can I do so that all kids can access this task, and then so that all kids can feel challenged in this task. (Personal Interview, 08/17/2016)*

This idea of an easy entrance point with sufficient challenge is often referred to as a low floor, high ceiling. This concept is used by computer science education when describing tools that are designed to have easy access points (low floor), making the tasks accessible for all students, yet the tool is sufficiently powerful to sustain the needs of more advanced users (high floor) (Grover & Pea, 2013). For Christine, the varied features of the simulation provided easy entry into the task, making the simulation accessible for all students, yet also gave students somewhere to grow, providing a challenge.

Lily has a similar take on this idea of accessibility referring again to these ideas of a low floor, high ceiling, saying,

> *"I think [technology] enhances [my lessons] in the sense that there's more opportunity for self-play and discovery. I mean it's just so much quicker and easier as far as showing, like I said, with the slider [referring back to using Desmos where students can use a slider to change a coefficient and quickly see the impact on a line]. Because of that, I think it also allows more opportunity for enrichment and extensions because there will be those kids who just get it right away, get the concept. Then, they can move on to something more challenging." (Personal Interview, 08/23/16)*

For Lily, she also appears to see value in the features of the simulation design that support students being able to quickly use the simulation irrespective of the incoming level of understanding. Students can start with a basic feature (such as the adjustment of a slider) to begin to generalize patterns, and later explore more challenging figures to generalize even more.

Accessibility, therefore, was a common theme discussed by all four teachers, albeit in slightly different ways. Pat and Sara saw accessibility as a means to have students get past the more tedious aspects of the math such as graphing lines to focus on the analysis of their work. For Christine and Lily, accessibility included not only this 'low floor' where students could easily jump into the simulation, but also the richness of the features that enabled more advanced students to challenge themselves.

5.3.1.4 Dynamic Responsiveness

The fourth feature talked about by the teachers is what I'm calling the dynamic responsiveness of the software. As mentioned earlier, teachers noted features that enabled students to quickly graph lines, and later change their lines to begin to make sense of linear and non-linear equations. This was true within the PhET simulations as well. All four of the teachers noted that students could easily change between representations and see instant changes to those representations, supporting students to see patterns. Dynamic responsiveness was also noted in terms of the system providing instant feedback. Teachers saw this process of students being able to change something, and then receive instant feedback as a positive step, often noting that the teachers couldn't provide as much feedback as quickly as the sim did.

> *"I think the really great thing about technology is that cycle of predict and confirm with math is immediate, so they get immediate feedback on their work." (Sara, Personal Interview, 3/06/17)*

Yet Sara also mentioned another feature of using the sims that was unique. She talked about the ways in which students could use the simulations (and other technology) as a means of practice, such that students could work at their own pace and work through a large number of problems with regular feedback.

When I asked Sara about her thoughts that using technology might reduce student-teacher interactions, she stated,

> *"But I also think that that's a good thing because a lot of the time, technology allows students to be self paced. So it does decrease student to teacher interaction, but that means that the kids are working at their own pace. It's not that they're off task doing their own thing, they're just working on what they need to be doing." (Personal Interview 03/06/17)*

This is consistent with one of her lessons where the students used Fraction Matcher (See Figure 5.2) to practice finding equivalent fractions. Here, students worked side by side, but the students were all working on their own levels of the game.

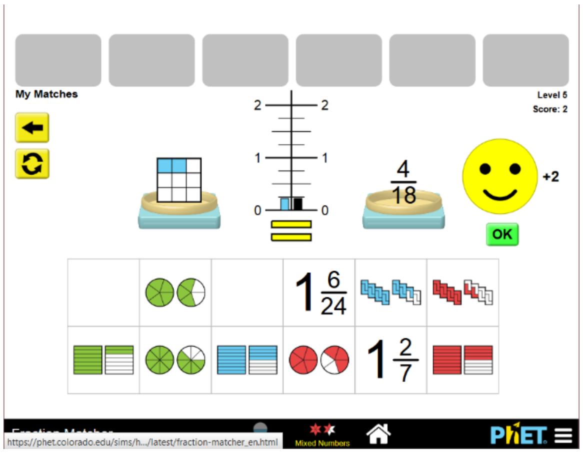


Figure 5-2: Screen shot of Fraction Matcher

> *Sara: What I loved is that the first couple are easy and then you get to the other ones and I was even doing them and I was like, this takes a little bit of brain power.*
>
> *Researcher: Yep. Well that's good because they can kind of find a level that seemed to work for them.*
>
> *Sara: Totally. The other thing for me is I look around and the girls over here were stuck on level four but these guys were on level six so I'm like, okay I'm going to help [the girls], these guys are doing okay, I can come back to them. That was really good for me to see so ... yeah I liked this sim a lot.*
>
> (Technology Lesson, 01/18/17)

For Sara, part of the dynamic responsiveness was having the features in the sim that enabled students to make their own progress and gain that instant feedback. As she noted, this enabled her to then focus on students who might need more help while students who were more advanced could continue to proceed ahead.

This idea, then, of dynamic responsiveness is a two-fold consideration. First, the responsiveness of the simulation features enabled students to make sense of what they saw. Second, the instantaneous feedback supported students as the utilized the simulation.

5.3.2 Technology is not always the best answer

Despite the teachers' articulations about the many benefits of the technology and simulations in classrooms, none of the teachers felt that technology was always an appropriate part of math lessons. Teachers were specifically asked when does technology *not* support student learning. Several themes were consistently raised and each warranted discussion.

5.3.2.1 Technology can be too easy

For Christine, she saw pencil and paper activities as being beneficial in a couple of different ways. First, she felt that pencil and paper satisfied a tactile need for students to actually touch and make contact with the math (Personal Interview 08/17/2016). In discussing this, Christine cited a recent article by Jo Boaler and colleagues (Boaler, Chen, Williams, & Cordero, 2016) that argued that students need to count on their fingers, and that the use of tablets and touch screens is actually doing a disservice for students as they have less recognition of their individual fingers.

Christine similarly felt that pencil and paper activities could create a permanent memory for students, saying,

> *I feel like some of the investigations are really nice to do hands-on because it does prove that things in our world sometimes are obviously linear; even if they're not perfectly linear, they're linearly related and some are quadratic, and some are inverse, and some have some crazy relationship you can't even describe with words.*
>
> *I think in that regard, the actual experimentation is really cool to do hands-on, even though maybe it takes maybe double or triple the amount of time, but I think it creates a permanent memory in their head, so like as we went on through the year, we could always go back and say like, "Is this linear, or is this non-linear? What were situations we had where we saw linear things, and what were things that were non-linear?" It's like they have these memories*

> *they've actually created, because they're related to an experience.* (Personal Interview, 08/17/2016).

For Christine, while technology offers many benefits for students, an excellent hands on, pencil/paper activity can offer significant benefits as well, including most importantly, perhaps, a belief that this memory will be sustained by the students, resulting in a significant learning moment.

> *I think we were talking about this last week, where I was saying that there's some investigations that are really cool to do where kids are actually physically building things. There's something about it being in the physical environment that's really unique. Unique that you can't get from it being on a device. I think we were talking about bridge breaking and actually constructing the bridges and understanding what multiple layers on a bridge would look like and making predictions, versus using that simulated investigation on the [textbook] website. Just in terms of engagement, in terms of remembering that activity and remembering what happened, I think there's a difference there. That being said, there are some investigations you just can't do physically. Whether it's because of space or because it's just going to be too dangerous or because it's going to be too wild. There are some really nice things that technology can afford that sort of eliminate those issues.* (Personal Interview 08/17/2016)

Therefore, the decision of technology versus non-technology activity is based on what is possible as a hands-on activity, and then whether she has time and space to make it happen. If she feels that the activity is too difficult to do by hand, then a technology-based option is considered.

Later in the school year, Pat reflected on a lesson in ways that were very reminiscent of Christine's views of when a hands-on activity can be better than a technology-based lesson.

> *"It's interesting because since I had the flu [at the start of second semester], and when I came back in January, usually we would do our whole transformation unit on GeoGebra, and I wasn't set up for it [so we did it by hand]. And then I saw them do it with paper and pencil and then I'm like, well, maybe they did get more out of it, because for some of them [the transformations] just happen."* (Lesson reflection 02/21/2017)

Here, Pat similarly saw a distinct advantage to non-technology based lessons. In this case, it was the idea that by having to solve the problems (transformations of shapes) that the act of actually drawing the movement of the shapes may be valuable to the students. Again, it seemed that both teachers felt that a hands-on activity may be more memorable and impactful for students than what could be obtained from a dynamically responsive system with automatic results.

5.3.2.2 Technology can be distracting

While Pat had talked earlier about ways in which technology could support students by reducing tedious tasks so that students could focus on analysis, she also raised concerns about technology being more of a distraction than a support (Personal Interview, 08/11/2016). She noted that students are sometimes playing games rather than attending to the task at hand. “I know on study hall days, of course, they're playing games. You can tell they're playing games, because as soon as you come by they're like hitting the button to make it go away.” For Pat, this was often a concern, that students would not be engaged. Pat went on to note that this concern for distractions was less of an issue when students were working on a specific activity. “When we did GeoGebra and stuff, and I had them all on Chromebooks, I didn't really have a problem with that, because there is a specific activity.” She also noted that, “I'm pretty welcoming of technology [but] I don't want to do technology for the sake of technology.” When pressed on what ‘technology for the sake of technology’ might look like, she first shared a reflection on another teacher in her building.

> *We have a teacher here who does most of her unit summative assessments using PowerPoint or a cartoon. They can use all these different things. She gives them two workdays. I don't know if that detracts or not.* (Personal Interview, 08/11/16)

Pat’s concern here seems to be both the types of activities (PowerPoint or cartoon) as well as the time spent completing that activity (two workdays). Pat goes on to say that she definitely wants her use of technology to have a learning goal. I think that Pat believed that having a specific learning goal would reduce the likelihood of technology being a distraction; this comes through as she reflects on how the high schools use technology.

> *The high school teachers talk about they don't always feel like [technology] is the best. They went to one-to-one at the high school. Every student has to have a [device] or they can check one out. There's been some talk of maybe it's not been the best move. I don't know. So far, I feel like whenever I use technology, I have a very specific purpose. (Personal Interview, 08/11/16)*

This reflection shows her continued questioning about how and when to use technology. It seems that Pat feels strongly that technology can reduce the burden on students having to create their own graphs and tables, yet this is balanced by a concern that technology can be more of a distraction than a support for learning.

Christine also noted problems with technology being too distracting, and like Pat, felt that the response to this was an engaging task.

> *The two games the kids loved last year, I just could not believe it, they're so silly, but one is this game where there are these dots that you click on and they get to be different sizes and they move at different rates. It was just like, "Really? You'd rather do this right now?" Then the other one was Slither. It's just a little snake going around the screen. Oh my gosh, it was like if you went over behind someone and they were off task, it was one of those two games. I think [that with technology] they can be sneakier with it. Again, I think if the task has purpose and it's engaging, I really don't think that [the off-task behavior] particularly worse. It's doesn't necessarily take away from learning any more than anything else. I think that's on the teacher to really establish an environment that is conducive to learning, whatever you're using to do that.* (Reflection after lesson, 09/27/16)

Christine sees the potential for distractions in technology, but does not believe that these distractions are made worse as a result of technology, but rather that students are simply finding different ways to distract themselves. Thus, for Christine, she places the impetus on teachers to design and enact engaging tasks as a way to fight off distractions.

Lily also raised concerns about distractions. Her school had just become a one-to-one school, meaning that each student had been given their own iPad. Teachers were encouraged to make use of the iPads as much as possible, but that meant that students could have constant access to technology. Lily noted

> *My co-teacher and I right now we're debating should we have [the students] take notes by hand or should we order a bunch of styluses and have them do it on the iPads? Taking notes is taking notes. On the other hand, if they do it in their notebooks, they lose their notebooks or they don't have their notebook. It's this debate. We're not really sure. We're going to try and have [the students] do everything online, but we're not sure how beneficial that will be because they'll have the temptation to be distracted by other things on their iPads.* (Personal Interview, 08/23/2016):

There is again this concern that access to technology without a specific, engaging, activity will result in students using the technology as a distraction, rather than as a learning tool. For Lily, there is a second tension as she talks about how technology can be used for 'procedural things' such as note taking and filing of papers using Notability, and how that does not contribute specifically students' learning; yet, she felt pressured to use the iPads as much as possible as the tables were a large investment by the school.

Finally, Sara shared these same concerns about student distractions when using technology. She notes during a reflection after a lesson

> *There are these extensions on Chromebooks that they can make the page snow or be on fire or have little hamsters come up, just things like that that are real distracting. [The students] can message [each other] and they can talk to each other things that are not [on task] - I mean, [devices] could be used to be really positive, but they're not. So those kind of distractions are really hard.* (Lesson Reflection, 10/26/16)

So for Sara, these same concerns come to the forefront as she considers the use of technology. How will she ensure that students remain engaged, particularly when they have access to a device that provides a ready distraction with just a click of a button?

5.3.2.3 Technology can be an added challenge

Every teacher except Lily (whose students each have their own iPad) spoke to the challenges of getting access to technology. All three of the other schools had similar systems whereby the teachers had to reserve the Chromebooks for their classes. Christine's process was slightly easier, as her class was a blend of 7th and 8th graders, and all 8th graders had their own device. As she noted, it was much easier to get a small number of Chromebooks for those who didn't have one, than to get a full classroom. Sara raised this as an issue too, although as she noted, the Chromebooks were stored in her classroom, so at times, she could use the devices more freely as they were readily available. For Pat, this was a large challenge, as the devices were not stored in her room, and therefore, she had to make arrangements every time she wanted to use them in her classroom. As we sat in her classroom one afternoon, talking about the possibility of a technology-based lesson, Pat muttered to herself as she checked her computer,

> *We can definitely do this. And it's great because we won't have talked about no solution, or infinitely many solutions. So let's say ... So we've got 1.3 ... I'm gonna do the test, 1.1, 1.2 ... Why don't we do it on the seventh? I'll do it before I quiz. Let me make sure I can get the computers that day. Nobody has it, let me get it. Hopefully it'll all work out, I don't think we've started PARC testing yet.* (Personal Interview, 02/21/17)

You could hear the stress in her voice as she worked to ensure that the right lesson would fit in the right spot of her unit, *and* that the computers would also be available that day.

Even when the technology was present, it didn't mean that teachers were able to fully utilize it. Lily and Christine (the two teachers who had students with their own devices) ran into problems where the devices weren't charged. Pat had issues where students needed a 'digital license' to use their technology. "We have at our school where kids have to have a digital driver's license or they can't be on the computer. Then, I have kids say, "Oh I don't have my license. I can't be on it today[7]" (Personal Interview 08/11/2016).

Teachers also raised concerns when the technology simply didn't work. Sara addressed this, saying:

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[7] Soon after, the policy changed such that when technology was being used as part of a class, students were all able to participate even if they didn't have their license.

> *I think accessibility, like getting technology in the room, is sometimes hard. And then there are enough glitches that happen when you just want it to work. That sometimes it's frustrating. I think about- I have this app called Splashtop that's really cool, that I can control my computer from an iPad, but it doesn't work half the time and I don't have time to figure it out. You always know a pencil and paper's going to work, but you kind of have your fingers crossed every time you do something with technology. Hopefully the website's not down, or the wireless isn't down, or kids can find the website or it's working like you had planned.* (Personal Interview, 03/06/17)

For Sara, trying to deal with glitches when using technology becomes a frustration, and she cannot stop and take time out of class to figure it out. As she notes, with pencil and paper, she knows the task will work; with technology, she is always left hoping that there will be no problems.

In the following exchange, during her interview (08/23/16), Lily expressed this same fear of a technology based activity not working as expected.

Researcher: What's your worst fear with it?

Lily: I guess that I'm going to put all this time into planning something and then it's just going to fail. Then, I won't have anything. I'll be relying on it.

Researcher: The no backup-what do I do if it doesn't work?

Lily: Yeah, I get frustrated as a learner if somebody is battling with technology. I just feel like it's a waste of time.

Furthermore, with many devices no longer able to run Java-based code, tools that had been readily available no longer worked in their classroom. Pat talk about the impact of that, saying,

> *I used to have a set of twelve laptops in the back of the classroom. And so, I could, at a moment's notice, say, "Okay, we're all going to go on", and I would use the virtual manipulatives a lot. Virtual manipulatives don't talk to the Chromebooks. Which is unfortunate. I used to use those pretty consistently. Now, with Chromebooks, they don't work.* (Personal Interview 08/11/2016)

5.3.3 Choosing resources is a complex problem

All four teachers talked extensively about the planning process, and how materials were chosen for their class. This is a complex problem for teachers as there were many variables that needed to be considered. First, as noted in the previous section, one of the key considerations is the availability of resources. As teachers noted, access to technology is not always available, particularly in the second semester, when many classes are taking online assessment tests. Therefore, there can be a disconnect between when teachers might want to use technology such as a PhET simulation and whether technology is available during that time period. Additionally, even outside of that testing window, most teachers are sharing devices among a number of classes, therefore access may be limited even outside of the testing window.

Furthermore, as discussed earlier, teachers are looking for activities that are memorable and engaging. As activities are considered, and either used or discarded, part of that consideration is the experience of the student and the teacher. Christine remarked on this concern, saying,

> *I was just thinking, it'd be so much more fun to start units with [engaging activities] and then just have their unit become richer. I just think for me, as a teacher, I think it'll be way more interesting, and I think for the kids, it'll be way more interesting.* (Personal Interview, 08/11/17).

Teachers wanted activities that would engage the students, and through this engagement with the mathematics, make the activity richer in that students would engage more deeply with the mathematics. The benefit is that not only would students have a stronger mathematical experience, both the students and their teachers would also have a more enjoyable experience.

Yet, even beyond the issues of access to technology and engagement are other key concerns raised across the teachers when discussing choosing and planning with new resources. The first one is around efficacy and efficiency (will it meet the needs of my students in the timeframe I have available), and the second is in comparing, choosing and planning new activities. Both of these concerns are discussed below.

5.3.3.1 Efficacy and Efficiency

When choosing and planning with new resources, teachers' first concern was whether the task adequately met the needs of the students. As I discuss this, I give specific examples from Sara as she articulated the concerns well, but these tensions were shared among all four teachers.

The teachers all mentioned the match between the task, the goal, and the students as a key consideration when choosing any task, and this was true of technology based activities as well. Teachers described the process as finding the best possible task for these particular students, and wanting to fill a need that they saw with a specific group of students.

The second part of efficacy was whether or not the task met the needs of the students in the most effective way possible. This is mentioned most succinctly by Sara who says,

> *I guess I'm just looking for the most efficient way to teach something and the most efficient way for kids to have a deep understanding of what's going on. So if it's using technology, then I'll do it. If it's giving direct instruction, I'll do that. If it's an exploratory activity, then I'll do that. So I'm always looking for how can I teach this the best way possible in the shortest amount of time.* (Personal Interview, 03/06/17)

This idea of finding the best possible mathematical task that creates the desired results in the shortest time was echoed by all of the teachers. This was particularly true when considering adding technology to the lesson. Sara noted this when explaining why her technology lessons were almost always planned to fill an entire lesson, rather than being only part of a lesson.

> *Because it does take a lot more time to get the devices out and to get kids going and then if it's an activity that's really worth it, I want kids spending a lot of time on it.* (Personal Interview, 03/06/17)

In this quote, Sara touched on both parts of efficacy and efficiency. First, the task had to be one that was "really worth it," meaning that it provided a rich mathematical environment that met the needs of her particular students. Second, if the task was indeed "worth it," then Sara wanted to get the most out of it in the time she had allotted to the task.

5.3.3.2 Comparing, Choosing and Planning New Activities

Teachers have access to a huge number of mathematical activities through their existing curriculum, supplemental materials provided by the district, online resources (both hands-on and technology based) and resources found through professional development and conferences. Sorting through all of these resources would require significant effort, and therefore, it becomes incumbent upon the teacher to wade through and choose the best and most appropriate resources for her students. Sara described the process she and the other teachers in her school go through when looking for activities:

> *[We] will look at the book and see what the book has and then if the book is missing it, then we'll go [look at] Pinterest resources that I've collected and I'll be like, "Oh, that's a great one." Or [I know that] Desmos always has new things, or it could even be that somebody went to a conference and they're like, "There's this really great activity that I'll make sure to implement."* (Personal Interview, 03/06/17)

This challenge of finding the right activity is compounded by the fact that most math textbooks are linear in design, and do not have open days for labs or similar activities as might be found in a science book, for example. As a result, not only do teachers need to select the activity, ensure that it meets the needs of the students and a timeframe that is acceptable, the task must have an appropriate place in the overall unit plan.

Here is the challenge for teachers: how to select the best possible activity for their students that meets all of the above constraints. Add to that, the challenge of access to technology for any technology based activities and it becomes easy to understand why this theme cut across all four teachers and became a major point of tension. As Pat asked herself in one reflection,

> *Sometimes, I feel overwhelmed with the amount of stuff I've accumulated. Am I going to do the PhET activity? Am I going to do the MARS activity? What am I going to do?* (Personal Interview, 08/11/16)

For teachers, this can be a large, even overwhelming problem, trying to whittle down all of the possible resources to the ones that best meet the needs of the classroom. Following on to that, as I mentioned earlier, is figuring out where the activity will fit best in the existing curriculum adds an additional layer of concern. As Pat goes on to say,

Even with [all the great resources out there], it's very frustrating. I'd love to have curriculum materials that did all of that, and then it was aligned, because that's the thing. You do this one thing and you feel like, "Well I really need to teach them scientific notation before I do that." You have to figure out the alignment instead of it all just being there. (Personal Interview, 08/11/16)

All of the teachers talked about the need to adapt any task to their specific class, but also noted that it was helpful to start with some basic activity. The teachers largely found it challenging to create their own lessons from scratch. This was in part because of the design of the simulations to be so open, enabling use in a variety of contexts, and also because lesson writing required a certain amount of creativity, or as Sara called it, "inspiration."

> *Sometimes it just happens, and you have this brainstorm and you can plan several lessons around something, but sometimes it's a little bit harder to do. [...] If I have an inspiration, then it's much easier*
>
> *When I look at Function Builder, the first [screen] was patterns. I'm like, I wish I had this last year for all the transformations and all that. Then, I was looking the other [scenes with numbers and equations], and it wasn't as immediately obvious to me [how I'd incorporate it into a lesson]. I'd have to play around with it a little bit more. I don't know, I think we probably all have*

these familiarities with certain topics, concepts, tools, or models that we bring into lesson planning, and sometimes it clicks right away, and sometimes it doesn't, because you haven't thought as much about some of those things. (Summer Lesson Planning Session, 07/24/16)

Ultimately, for teachers, it seemed that the choice of activities was an extremely complex decision that had significant impacts on the classroom. Trying to find a rich activity that met the needs of the students in the allotted timeframe with the necessary resources was only part of the challenge. When using the PhET simulations, teachers also had to determine how best to use the tool and create a lesson that would meet that goal.

5.4 INSTRUCTIONAL PRACTICES

In this section, I describe each teachers' typical non-technology based lessons as well as their technology based lessons. In doing so, I focus on the teachers' practices, through descriptions as well as short excerpts from their class, to highlight particular moves. In this section, I focus specifically on teachers' instructional practices, differences in the practices enacted in non-technology and technology based mathematical activities to help inform Research Question 1. In the next section (5.5 Analysis of the Simulation Features), I'll use the information gleaned from these summaries to focus more specifically on the features of the simulation that impact teachers' practices to inform Research Question 2.

After the descriptions of their lessons, the changes in the instructional moves for each teacher from non-technology classes to technology classes are highlighted. Each of these lessons were coded, as described in Chapter 4. Recall that there will six main dimensions around which instructional practices were described; Teacher information, Student information, Class Discussions, Mathematical Focus, Technology Focus, and Reflection/Remediation. The coding was completed in 5 minute segments and represented significant moments within the classes (See Appendices I-L for the full summary of the lessons).

5.4.1 Pat

5.4.1.1 Classroom Practices – Non Technology Based Lessons

In Pat's classroom, each lesson began with a warm-up activity, typically a small set of drill-based problems for the students to complete. One day, the students are given a series of problems to solve such as 4x4x4. Later, students are asked to find the cubed root of 64. In another class, students are asked to find the new ordered pairs when a figure is translated (shifted) five units up and six units left. Each of these activities were specifically tailored to that day's lesson. For example, when students were given the points to translate, the task later required students to find the ordered pairs when a shape was rotated or turned, rather than translated, or shifted. Students appear to expect a warm-up each day, and begin to work on it right away without a lot of prompting.

The problems were not designed as group worthy tasks (Lotan, 2003) but rather as skill builders. These types of drill-based problems would appear to meet what Pat views as a need for reiteration and practice. Generally, students worked silently on the warm-ups, and any talking was usually non-math related. These type of skill based problems would appear to meet an unmet need that Pat sees.

> *I was just at a training with [another teacher], talking about how to teach solving an equation. It's like they [the CMP book] have one investigation and then that's it. It's like, well, that's not enough for [the students] to really be fluent in solving two step equations or whatever. I think [that's what I don't like:] the lack of skill practice.* (Personal Interview 081116)

Pat is looking for ways to support students in gaining fluency in their skills. Pat shared that she never used to do a warm-up, but her partner teacher always had one in her class, so now Pat has started implementing similar ones in her own class (Lesson Reflection, 02/21/17). Once the students have had five to ten minutes to work on the warm-up, Pat asks students to come to the board to show their solution. There is usually not a lot of discussion about their solution, other than for Pat to ask students if they have any questions.

After the warm-up, Pat introduced that day's activity. She said that she uses the textbook relatively often for the activities, and then supplements with other materials to provide additional practice or to provide a more engaging activity. Of the four non-technology lessons I observed, all four were centered on some type of investigation: exploring cubed roots; finding the points on a rotated shape; generalizing the rule to find the number of solutions of an equation; and discovering the rules for multiplying and dividing exponents. These activities had a predominant mathematical focus on generalizing patterns; succinctly, the goal was to explore a number of cases to formulate a generalized rule. These activities were designed to support students working together, although several portions of the lesson became more directed, resulting in students working more individually, than in groups. For example, when working to find the rules for multiplying and dividing exponents, Pat first asks the students to fill in a chart listing the powers of each number from one to 10 (e.g., 2^1, 2^2, 2^3, 2^4, etc., up to 2^{10}, and so on up to 10^{10}). She would throw out questions to the class at times, such as "For the powers of 10, do you need to put them in your calculator?" or, "Could I look at a number and know that it's not a power of 5?" She also walked around the classroom, usually confirming answers (Yes, that's right) or showing students how to do the problem correctly. As Pat walked around the classroom, she was usually doing one of three verbal moves: restating the directions for the task, providing answers for students, or doing small sessions of direct instruction for students who were confused.

Despite the fact that the activities were all designed to support student interaction, Pat often structured her class such that students did not need to struggle through the activity with one another. Perhaps this was done as a means of minimizing confusion, which would enable students to get to the essential question of the day. For example, the original activity for the rotation of shapes activity came from the CMP-3 book that Pat uses. The activity asked students to rotate a shape 90 degrees about the origin and find the new coordinates. The original task did not provide directions for the students on how to find the new coordinates of the shape, but rather presumed that students would work together to find a strategy. Pat modified the activity, such that students were shown how to rotate the paper ninety degrees to the right. Students were then told to rename the x and y axes, based on the new orientation of the graph, and then use the original coordinate points to redraw the shape. Once Pat confirmed that everyone had the shape correctly drawn, her students were instructed to turn the paper back to its original location, and find the new coordinates using the original x and y axes. By choosing to decrease the cognitive load of the task by providing additional scaffolding, the student to student interactions were minimized, and the activities became more focused on building skills rather than gaining conceptual knowledge.

When students were working on the investigation, Pat walked around and answered questions. Nearly all of the questions were answered in small groups, even when multiple groups would ask the same question.

In this exchange, which was typical for her classes, students are rotating points 90 degrees counter clockwise around the origin. The teacher has had the students turn their paper 90 degrees to determine the new location of the object. Many students are confused. As she walks around, I record this exchange.

Student 1:	*Wait, I don't get it. I'm switching the X and Y?*
Pat:	*You are switching the X and the Y. When you turn it, now you're pretending this is your Y [points to the rotated x axis], and this is your X [points to the rotated y axis]. To get to B, I would go negative four, positive two. That's why B is -4, +2. How would I get to E?*
Student 1:	*It's six…*
Pat:	*Negative six.*
Student 1:	*And then three over.*
Pat:	*Positive three.*

Student 1: *Yeah.*
Pat: *Okay?*
Student 1: *All right.*
Pat: *Who else is stuck? Yep?*
Student 2: *I'm still stuck.*
Student 2: *I'm doing negative four, minus four right, because you said this is the X and the Z. You go…*
Pat: *Up two.*
Student 2: *Up two.*
(Non Tech Observation 3, 012317)

Once students had worked for a while (usually 10-15 minutes) on the investigation, Pat would use her document camera to project the questions on the board. She would ask students direct questions to fill in missing answers, and when answered, would move on to the next step in the process.

Students typically finished their work prior to the end of the class, and were allowed to begin work on their homework. For most classes, homework consisted of a worksheet, often from Pizazz Algebra, where students are given problems and the answers in random order. The answers spell out some sort of funny statement. Occasionally, students are asked to finish the day's activity at night for homework. Students also had assignments online using MathXL.

On average, the students in Pat's classes spend slightly more than a third of their time engaged in math talk, either as part of a whole class discussion, such as when Pat talks through the answers with the students, or in short bursts when students are asked to turn and talk with their neighbor about some question (such as the brief questions as to how one would know that a number is not a power of five). Most of the time is spent answering direct questions, rather than in discussion of the questions. Roughly eighty five percent of their time is spent engaging with mathematics, with the remaining time spent gathering supplies, and listening to a lesson or directions. Despite choosing student-centered activities, Pat's enactment of these activities results in students working independently 65% of the time, with only 20% of their time spent working in groups. Overall, the classes appeared very engaging with students engaged nearly 85% of the time.

5.4.1.2 Classroom Practices –Technology Based Lessons

Pat completed one Technology Based lesson during this research project. The lesson used the PhET simulation Graphing Lines to explore points of intersection. In this lesson, students graphed two different equations to determine if they shared one point of intersection, no points of intersection or "infinitely many points of intersection," as Pat described it to the class (Observation, 030917). In this lesson, as in her non-technology based lessons, Pat began with a warm-up activity. Unlike her other lessons, however, this warm-up was not geared towards practice of a specific skill, but rather on starting students thinking about the topic the class will tackle within their lesson. Students were given a scenario describing two arcades, each with different entrance fees and different costs per game. Students were challenged to determine which activity you would "encourage your sporty crush to attend with you this weekend" (Observation, 03/09/17). Students were encouraged to work independently and silently. When some appear stuck, Pat encouraged the students to consider how much the arcade would cost if they played 10 games.

Shortly after, she asked students to share their thoughts with their table partners. Students had an active conversation as Pat walked around the room listening to each conversation. She encouraged one group in particular to share their thoughts. This group had a full discussion of this trip to the arcade being an actual date (since the trip was with your favorite crush), so therefore dinner was likely part of the night. As a result, the students felt that it was unrealistic to consider ten games as Pat had suggested, since the students felt there just simply wouldn't be enough time!

At this point, unlike in her non-technology classes, Pat decided not to have students come to the board and share their work. Instead, she told the class to set the problem aside and continue thinking about it. She then had a second warm-up that is more consistent with her non-technology classes – three problems to solve. It turns out that these three problems match the equations in the upcoming activity. Students in an earlier class struggled with these problems, so Pat decided to add them as a warm-up to minimize mistakes. For these three problems, she had students come to the board and solve the problems like she did with non-technology lesson warm-ups.

Students then were asked to get their Chromebooks from the back of the room. Despite students being able to bring their own device, all but two students get a Chromebook from the cart. Once everyone had a device, Pat directed the students to the PhET website to open the Graphing Lines simulation. There was quite a bit of confusion in just opening the simulation as there is both a ‘play button’ on the image of the simulation, as well as a button nearby that invites users to download the simulation (See Figure 5-3). Students asked multiple times if they should click on the download button.

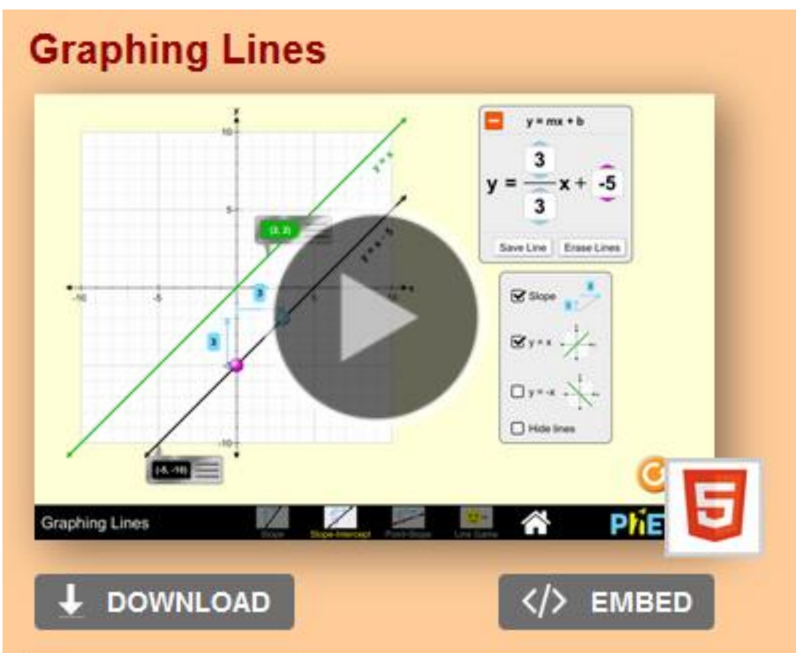


Figure 5-3 Screenshot of Graphing Lines simulation.

Once the students have access the simulation, Pat draws students’ attention to the front of the room. Displaying the simulation on the board, Pat explained that students should complete the lab sheet she just passed out. She asked the students to graph the lines using the simulation, sketch the lines on the worksheet, and then identify whether the two equations have one solution, no solutions or infinitely many solutions (See Figure 5-4).

1. For each row of the table, **graph** the system of equations on a clean coordinate plane.

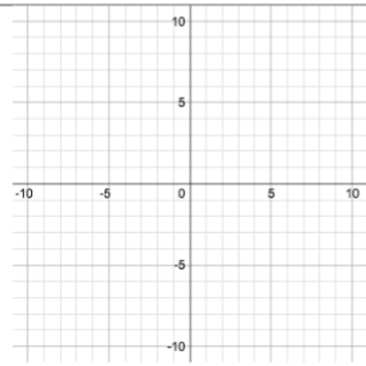
System	Graph	Solutions
$y = \frac{2}{3}x + 3$ $y = \frac{4}{6}x - 5$		☐ One solution (,) ☐ No solutions Ø ☐ Infinitely many solutions

Figure 5-4 Sample problem from the student activity sheet.

On the back of the worksheet (Figure 5-5), students were tasked with completing the same analysis without first graphing the lines (although Pat noted that it was okay for students to graph the lines if they wanted to check their work).

$y = \frac{4}{5}x + 2$ $y = \frac{4}{5}x - 6$	Try this without graphing. How do you know how many solutions will there be?	☐ One solution (,) ☐ No solutions Ø ☐ Infinitely many solutions

Figure 5-5 Sample problem from the back of the worksheet

The process took nearly ten minutes for the students to get their devices, access the simulation, listen to the instructions and begin the activity.

As in her non-technology lessons, as students began to work, Pat walked around to offer assistance. At first, Pat's interactions were focused almost entirely on technical support (coded as acting as a technical assistant) as she restated the directions to access the simulation to a number of students, and provided (or restated) the instructions for the task. Very quickly, however, the interactions began to shift from technical support and instructions to asking questions about what students are doing. This was a typical exchange:

Pat:	*So do you think there's going to be one solution, no solution, or infinitely many?*
Student1 :	*None.*
Pat:	*And why do you think there's no solution?*
Student 1:	*Because they don't cross.*
Pat:	*And why do you think they'll never cross? What kind of lines are those?*
Student 1:	*They're going to go straight.*
Pat:	*Mm-hmm (affirmative), do you know what those called?*
Student 1:	*Parallel?*
Pat:	*Yeah, parallel. Are these ever going to cross?*
Student 2:	*No*

(Observation, Technology Lesson, 03/09/17)

There were more conversations about the slope and the intercept, and the impact of those two values on the determination of the number of solutions. While some of these discussions were more robust than just basic procedural discussions, Pat does remain consistent in her patterns of interactions with students, providing answers at times, and stepping students through their thinking with step by step questions. In one problem, students needed to solve an equation where the coefficient of x is 14/15. This causes confusion for many groups. In this conversation, Pat was more directed and procedural in her approach with students.

Pat:	*So fourteen fifteenths, yeah. So then what does x equal?*
Student 3:	*Is it fourteen fifteenths?*
Pat:	*Well if I multiply by fourteen fifteenths I'm gonna get one ninety-six over two twenty five. I want to get one here [pointing to the x]. So what would I multiply by?*
Student 3:	*Fifteen?*
Pat:	*Over?*
Student 3:	*Fifteen. Is it fifteen fifteenths?*
Pat:	*What would you multiply fourteen fifteenths by to get one?*
Student 3:	*Umm.*
Pat:	*What would you multiply one third by to get one?*
Student 3:	*Oh, three.*

Pat: *Over?*

Student 3: *One.*

Pat: *So what am I going to multiply fourteen fifths by to get one?*

Student 3: *Fifteen over fourteen.*

(Observation, Technology Lesson, 03/09/17)

As students continued to work, Pat moved from group to group, continuing to have similar conversations, sometimes focused on conceptual understanding, and others focused more on procedural compliance. The students worked on the activity for a total of eighteen minutes. Pat then drew the students' attention back to her. She passed out a final short assessment for the students to complete (See Figure 5-6).

Solutions of a System of Linear Equations

If the system of linear equations has...	Sample system of equations	What is true about the slopes and y-intercepts in the equations?	What do the graphs look like?
One solution			
No solutions ∅			
Infinitely many solutions			

Figure 5-6 Example of assessment problem.

The final assignment included a question that asked how knowing a lines' slope and intercept would help you decide the number of solutions. The room is silent as students work. Several students appear to be looking back at the simulation as they answer the question. While the students completed their assessment, Pat asked small groups of students to bring their Chromebooks back to the cart. She advised the students to take their time answering the questions.

“I want detailed answers. Give me good answers. Don’t rush it,” she tells them (Observation, Technology, 03/09/17). As students finish the assessment, Pat asked the students to turn and talk with one another, “What if you have different slopes but the same y-intercept. Pause, and talk with your group.” This time, everyone is talking. Pat summarizes what she’s heard, noting that if the lines share the same y intercept, that is the point of intersection.

After the students completed the PhET activity, Pat directed the students to return to the original warm-up activity. “Now, what do you think?” she asked them. Students were given time to discuss their thoughts a second time. Students talked about their answers, but the class was ending so that teacher asked students, “If I play ten games, where should I go?” Students answered, the homework was quickly announced and the class ended.

On average, in the technology-based lesson, the students spent just slightly more than a quarter of their time engaged in math talk, with most of this discussion time at the start, and end, of the lesson. More of the time was spent discussing the questions, rather than just providing direct answers. Students had an opportunity to discuss the warm-up problem, and later, their thoughts on the number of solutions for a line with the same y-intercept and different slopes.

In the activity itself, the mathematical focus was on connecting representations as students make connections between the equation of the line and the resulting graph. Moreover, the task was enacted in a way that supported students working through the problems without significant scaffolding, resulting in less time spent on drill-like activities and more time spent on conceptual understanding. Just over ninety percent of their time was spent engaging with mathematics, with the remaining time spent gathering supplies, and listening to directions. In this lesson, as students each had their own device, roughly 64% of the class time was spent on individual work, and 18% of the time was spent working in groups. Engagement was very strong (over 90%).

5.4.2 Summary of Changes between Non-Technology and Technology based lessons

Below, I share the highlights of differences in Pat's lessons as she moved from non-technology to technology based lessons. These differences are summarized in Table 5-3 and discussed below. A full listing of the comparison can be found in Appendix I.

Table 5-3 Summary of Differences from Non-Technology Based Lessons to Technology Based Lessons for Pat

PAT: Differences from non-technology to technology-based lessons	
Discussions	Decreased discussion time, but more time spent discussing answers rather than just providing direct answers. Decreased student-student math focused interactions
Group work	Similar levels of individual and group work
Mathematical Focus	Broadened focus from generalizing patterns to connecting representations
Lesson Reflection	Increased time spent on student reflection
Engagement	Increased student engagement
Teacher talk	More time spent on providing instructions and asking questions. Decreased direct instruction
Mathematical Task Type	More time spent on investigational activity

Teacher Information

In terms of Pat's verbal utterances, there were two key differences from non-technology lessons to technology based lessons. First, in the technology based lesson, the teacher spent slightly less time on direct instruction, and no time providing answers to the students. On the other hand, Pat spent significantly more time (50% vs. 18% of the class) providing instructions for students. It is important to note that much of this time in the technology class was spent restating instructions (telling students what to do) rather than a whole class delivery of instructions.

Student Information

Students spent comparable time working on math in both non-technology and technology classes, and nearly identical amounts of time working individually and in groups. The big differences in technology based classes for these students was that while students spent more time in a class discussion than in non-technology classes on average, they spent less time overall talking about mathematics. This was likely influenced by the teacher's choice to assign an individual device for each student, as well as the students spending less time answering direct questions. On the other hand, students spent 10% more time discussing questions in their class discussions.

Overall, while both non-technology and technology-based lessons were very engaging, students experienced slightly higher levels of engagement in their technology based lesson (84% to 91%).

Mathematical Focus

In this area, the focus of the class changed significantly. In all of the non-technology classes, the focus was predominantly on generalizing patterns. This was not the case for the technology-based class. While students were still generalizing a pattern to decide the number of points of intersection, the focus of the class shifted to connecting representations between the equation and the representation of the graph. In non-technology classes, the teacher's choices to provide additional scaffolding resulted in lessons being more drill-like, with less emphasis on conceptual understanding. This additional scaffolding was not present in the technology based lesson, creating a shift to a more mathematical investigation.

Remediation and Reflection

In this class, there were very few instances of either remediation or reflection in both non-technology and technology classes. In the non-technology classes, there were short instances of remediation, focused on mathematics, whereas in the technology class, there were no instances. However, in the technology class, there was a small segment of reflection at the end of the class, whereas there had been no cases of reflection in the non-technology class.

5.4.3 Lily

In Lily's classroom, I regularly observed her first period Algebra 1 class, which started very early for most high school students, at 7:30 am. We chose that class as it is the only Algebra 1 class that Lily teaches on her own. The other sections are team taught to support high needs students in those sections. Lily's school is termed a 'one-to-one' school, meaning that every student is assigned his/her own device, in this case, an iPad. As a result, teachers are encouraged to make full use of the technology so readily available. As a result, in Lily's class, students use the iPad for all note taking activities within the class. I observed Lily for a total of five block periods, with two focused exclusively on non-technology based lessons, and three focused on technology based activities. I also observed a co-taught block period later in the day, which was not recorded, but helped support my understanding of her teaching routine.

5.4.3.1 Classroom Practices – Non Technology Based Lessons

Both of Lily's non technology classes started with a warm-up activity. As students enter the room, they are typically very quiet. The students immediately take their seats and are reminded to get started on the warm-up. As mentioned above teachers are encouraged to make as much use of the iPads as possible, so Lily puts nearly all of her assignments, including the warm-up activity on Schoology, the online classroom website. Students then download assignments, import the worksheets into Notability (an app that facilitates note taking through the use of a stylus, and enables students to organize and keep track of all their papers). This process of opening Schoology, finding the correct assignment, importing it into Notability, and then 'filing' the assignment in the right section of Notability is challenging for students, and generally takes the first five minutes or so of class.

This class period is interrupted by school announcements each morning, five to ten minutes after class starts. For that reason, Lily does not typically say much about the warm-up (other than to prompt students to open it and begin working on the warm-up) until after announcements are completed.

In the observed lessons, one warm-up involved a set of lines to graph (again doing the work with a stylus on a digital worksheet stored in Notability); on the other day, the warm-up was a set of sentence prompts that referred to equations she had written on the white board. Students were asked to identify possible patterns such as “I think the binomials in the polynomials are…”, or “All of the non-polynomials have…” Along with the warm-up activity, students also kept track of the date and the goal for the day, which is posted on the wall.

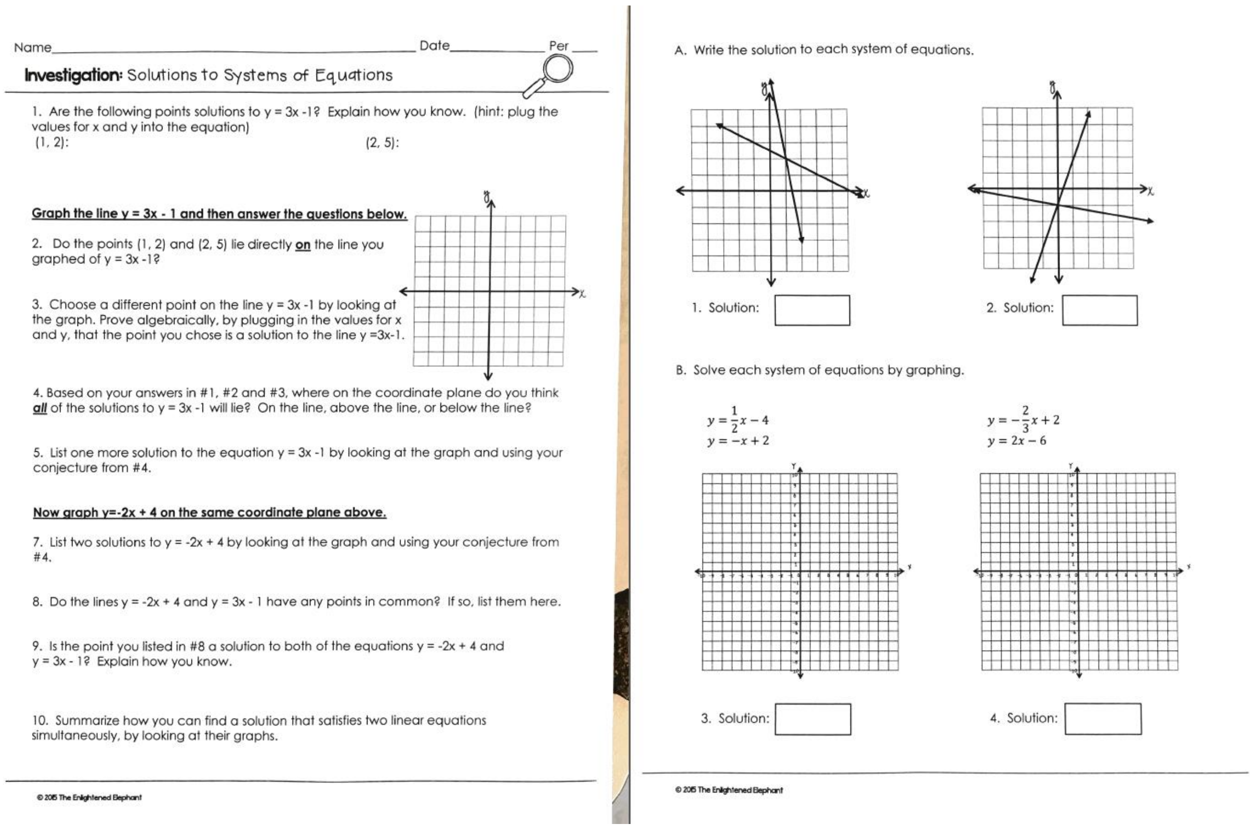
As students worked on the warm-up, Lily walked around checking in with each student. Most of the comments at this time were directed at classroom management (student behavior) or technology (accessing and filing the day’s worksheet). For the drill-based warm-up (graph two different lines, one in slope-intercept form and another in standard form), Lily asked two students to come to the board and show their work. When one struggled to complete the task, Lily stepped in and told the student which steps to take. Lily then presented the students’ work, asking direct questions to the students as she did so, such as this exchange:

> *Lily:* *“What does he need to plug in, to find the x-intercept? .*
> *Jorge[8]:* *Zero*
> *Lily:* *Zero for what? Here he plugged in zero for x. Now what does he plug in zero for?*
> *Jorge:* *Y?*
> *Lily:* *For Y, yeah. Good. So, instead of changing X to zero, you change Y to zero. You see that, Mario? So, instead of changing this [pointing to the x] to zero, we change this to zero instead. Okay? And this just cancels out. It just becomes zero.*
>
> (Observation, Non-Technology, 01/23/17)

This is a typical exchange for Lily. She often asks a lot of questions of her class, but these questions are predominantly questions that have a right or wrong answer. Rarely does she ask students to explain or justify their thinking. The iPads also become a form of distraction for the students, preventing students from engaging in the class activity. As a result, Lily often asks students to close their devices at times to better focus in class.

[8] All names are pseudonyms

After the warm-up, the class moves to the activity. Because of the length of the class, Lily works to break up the time by working on several different skills in each class. In one, the students worked on an investigation studying systems of equations as can be seen in Figure 5-7.


Name________________________ Date__________ Per ____

Investigation: Solutions to Systems of Equations

1. Are the following points solutions to y = 3x -1? Explain how you know. (hint: plug the values for x and y into the equation)
(1, 2): (2, 5):

Graph the line y = 3x - 1 and then answer the questions below.

2. Do the points (1, 2) and (2, 5) lie directly **on** the line you graphed of y = 3x -1?

3. Choose a different point on the line y = 3x -1 by looking at the graph. Prove algebraically, by plugging in the values for x and y, that the point you chose is a solution to the line y =3x-1.

4. Based on your answers in #1, #2 and #3, where on the coordinate plane do you think ***all*** of the solutions to y = 3x -1 will lie? On the line, above the line, or below the line?

5. List one more solution to the equation y = 3x -1 by looking at the graph and using your conjecture from #4.

Now graph y=-2x + 4 on the same coordinate plane above.

7. List two solutions to y = -2x + 4 by looking at the graph and using your conjecture from #4.

8. Do the lines y = -2x + 4 and y = 3x - 1 have any points in common? If so, list them here.

9. Is the point you listed in #8 a solution to both of the equations y = -2x + 4 and y = 3x - 1? Explain how you know.

10. Summarize how you can find a solution that satisfies two linear equations simultaneously, by looking at their graphs.

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A. Write the solution to each system of equations.

1. Solution:

2. Solution:

B. Solve each system of equations by graphing.

$y = \frac{1}{2}x - 4$
$y = -x + 2$

$y = -\frac{2}{3}x + 2$
$y = 2x - 6$

3. Solution:

4. Solution:

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Figure 5-7 Systems of Equations Investigation

Later, in that class, students worked on posters for an integrated project where they would manufacture and sell a product to maximize profit. In the other class, the lesson was mostly taught through direct instruction, where the goal was being able to name and then later add and subtract polynomials. In this activity, students took notes, watched and copied examples, and later completed sample problems on their own. In the latter part of this lesson, there was a planned activity where students would solve problems posted around the room. The answers each came with a set of letters that, when correct, would spell out a funny phrase about their teachers. Unfortunately, the initial lesson ran far longer than Lily expected it to, so the students simply used their iPads to take pictures of the problems and were assigned that task as homework.

Lily also moves the class between more individualized work, and group work. When students are working on investigations, such as the task where they are finding points of intersection, students are grouped in pairs and work together. In other lessons, students sit in pairs but the work is done mostly on their own.

As students are working on their mathematical tasks, Lily varies between walking around checking in with students, and sitting with a small group of students. During check-ins, Lily primarily looks at what students are doing, and asks students how they are doing. If students answer, "good," she moves on to the next set of tables. Otherwise, she'll stop to provide assistance with the assignment. This assistance is often in the form of direct instruction, where the teacher asks questions that lead to the correct answer. This is a typical exchange of that type.

Lily:	*Look, you're almost done with the front page. So, you need to copy what he has here. So, if I were to ask you, "How do you know if you have a solution to two equations?" how do you know if it's a solution to both? Where do you find that solution to both equations?*
Student:	*Right here.*
Lily:	*And how do you describe that?*
Student:	*I don't know.*
Lily:	*Like, you're there. Where would I find the solution? It's the point where what?*
Student:	*Where it crosses.*
Lily:	*Yeah. So, a solution to two equations is the point where it crosses. That's what you need to write here. Okay? You're almost done with this, [Carlos]. Keep going. And then you can write that. This is a lot faster. You just graph. All you had to do is tell me what is the solution. Write it here.*

(Observation, Non-Technology Lesson, 01/23/17)

In this exchange, the student is unsure of his answer, yet has correctly drawn the graph of the two equations. Lily does not just tell the student to write the point of intersection as the answer, but rather walks the student through the thinking process to get to the correct answer.

This insecurity by the students in their own mathematical answers is often evident in the classroom, and may also play a role in Lily reducing the cognitive load of the tasks. By that, I mean that she often will sit down with a group of students who struggle and essentially do the work for the students, while talking them through the process. For example, in this exchange, she sits next to a student and explains how to solve the problem. As she does this, she is doing all of the writing. The student sits next to her and watches.

Lily:	*So, both of these are in Y = MX + B. So, Dante, what's your B for this equation? What's the number by itself?*
Dante:	*-4.*
Lily:	*So that's where you begin. At -4. And where do you move? What's your slope?*
Dante:	*[appears to point to the paper]*
Lily:	*Up one, over one? And you go up because it's positive, right?*
Dante:	*[Dante nods]*
Lily:	*Okay. Where's the ruler? [Lily draws the line] So, [pointing to the second equation, y = -x+2] what's the number by itself in this equation? 2?*
Dante:	*[Dante nods]*
Lily:	*And where do you move from there? Instead of up, where do you move? Down one, over one. And so on. You can see where it crosses. So, you're gonna graph them. And then find the solution. What's the solution?*
Dante:	*[appears to point to the paper]*
Lily:	*Yeah, good. So try it with this one and you're done. Put your names on it and hand it in. You got an assignment done, Dante. That's good.*

(Observation, Non-Technology Lesson, 01/23/17)

You can hear at the end of the discussion, that Lily has praised Dante for completing the assignment. This is a common for Lily to provide encouragement for students. Furthermore, during her reflection, Lily notes that this class is two units behind her other section of Algebra 1. It appears that Lily's intent was to ensure that students are making progress, but was concerned about the timing of her lessons.

Both lessons ended fairly abruptly, as the class spent more time on the activity than was expected. At the end of both lessons, Lily made a quick announcement to ensure that all students knew what homework was assigned, and then dismissed the students.

5.4.3.2 Classroom Practices –Technology Based Lessons

Lily taught three classes using PhET simulations. The first two used the lessons she created as part of the summer lesson planning project, while the third lesson she enacted without any written lesson plan. I'll describe each lesson separately, and then describe the similarities and differences seen.

Lesson 1

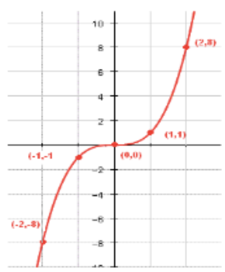
In the first lesson, part one of a two day activity focused on exploring functions, the class was structured very similarly to the non-technology based lessons. For the first lesson, the activity is broken into three parts. First, as a warm-up, students are given a table, graph and an equation and asked to first identify each items as a graphic, algebraic or real life example, and then to determine what all three have in common (see Figure 5-8).

<u>Function Fun Day 1</u>

<u>Part A: Warm-up/Introduction:</u> What do you think it means for something to be a function? Here are three examples of functions presented using different representations.

A. $y = 3x - 12$

B.



C. The number of people living in your town is a function of the number of jobs available.

1. Label each example as either ***graphic***, ***algebraic***, or ***real-life*** representations.
2. What do these examples have in common? What do you think makes them all functions?

Figure 5-8 Technology based lesson 1 - Warm up

As has been true for the non-technology lessons, these worksheets are all digital forms and were completed on the iPads using the Notability app as their note taking tool. The teacher then stopped the students after five minutes, asking students to discuss their answers with one another. There is not much talking so the teacher then called on specific students, who provided some answers to the questions. Lily then asked the students to open the PhET link which is posted on

Schoology. They were given fifteen minutes to explore the simulation. The worksheet gives the students prompts to answer several questions as shown in Figure 5-9.

Part B: Patterns Open Play

Take the time to get to know, explore and play with the Patterns activity of Function Builder. Make sure you try all of the "buttons" and write down the following:

What do you find interesting?	What questions do you have?
• • •	• • •

Figure 5-9 Open Play Portion of Lesson

As students explore the simulation, the teacher walks around asking students what they notice and pointing out specific features of the simulation, asking students if they know what the feature does. Students are highly engaged at this point, all using the simulation. She brings the class together for a brief discussion of what the students noticed. For the most part, students are reluctant to talk, and therefore the teacher asks the students to send their answers back to her in Schoology, and then move to the next part of the activity.

For this portion of the lesson, students are grouped as pairs. Each student still has an iPad, but one iPad displays the simulation, while the other displays the worksheet. First, as shown in Figure 5-10, students are asked to label key parts of the simulation.

Part C: Exploring Patterns

1. Go to the Patterns activity of the applet and hit refresh. Spend some time discussing with a partner what happens when you try different things. See if you can agree on what's happening (the "function rule") to the input to get the output values. Then, see if you can agree on what to label the different "buttons" on the applet with the following terms:

- **Function Rules**
- **Domain** (Input Values) (*x*)
- **Range** (Output Values) (*f(x) or y*)
- **Function**

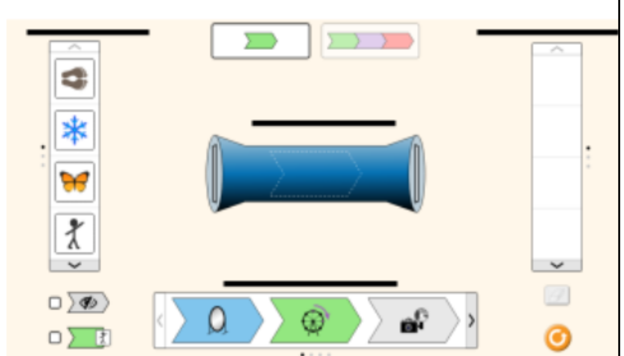


Figure 5-10 Exploring Patterns portion of lesson

In this portion of the activity, students are asked to label the buttons on the screen (using a screen shot) as Function, Domain, Range and Function Rule. Next, as shown in Figure 5-11, students are given images from Function Builder and asked to build these equations on their screens. Then students draw or describe the output and describe the rule.

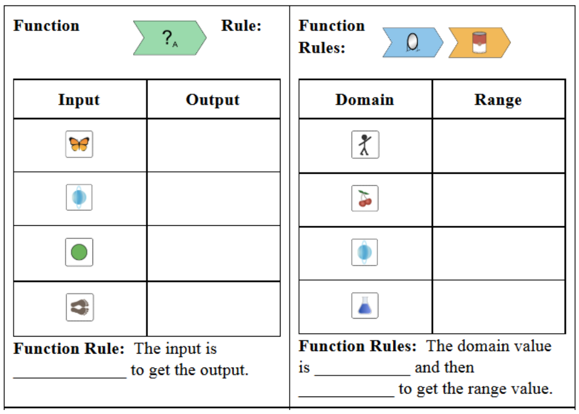


Figure 5-11 Sample activities for Lesson Part I

As the students worked, the teacher again walked around the classroom talking with students. She had planned to ask the following questions during the lesson: What happened to the input when . . . ?? Does this rule work for any domain value you try? How did you find this rule? Do you ever get the same range value for different domain values? The conversations were more open ended, in that rather than Lily providing step-by-step instructions to complete the task, I heard more questions such as, "what happens when you put this image through the tube?" There were also moments when the teacher focuses explicitly on generalizing patterns as students try, in particular, to figure out the challenge (first box in Figure 5-11). In this case, the mystery function was rotating the shapes, but some shapes have rotational symmetry, so the rule was not obvious to students at first glance. However, the questions were not as specific as she planned, nor does she use the academic language she had prepared. This appears to be another example of how Lily lowered the cognitive demand for the task.

After nearly twenty-five minutes, Lily paused the students and asked several volunteers to put their answers on the board. She intended to display the students' work using AirPlay®, which automatically projects the iPad on the board, but the devices were not connecting. Lily told the students to submit their work to her so she can see it. She then asked questions of the students to focus their attention on the patterns they saw. She asks students prompts such as "What did you notice was happening. What did you come to expect? Did different things happened with the rules? Would it change, or would the same thing happen to it? Eventually, students seem to recognize that the rule always produces the same output for each input, which was an explicit goal from her lesson.

The class ended with a brief exit ticket, where students are given two different input/output tables and asked which is a function and which is not. Students are then asked to explain their answers.

Lesson 2

For the second lesson, taught during the next block session two days later, the activity was again broken into three parts. This was the second part of the lesson Lily designed over the summer, but was somewhat changed in the enactment. The teacher began by reviewing the homework, which was to define function, range and domain using a graphic organizer. She asked students to compare their answers to hers. Then the students watched a short 10 minute video teaching about functions. This video was a cute story about a chicken nugget factory, where chickens enter, and nuggets come out. Lily led a brief class discussion following the video, summarizing the key points. In her reflection after the lesson, the teacher noted that this was a film that she really enjoyed showing to students, which is why she added it to this lesson. She then projects the graphic organizer again, and goes through each portion of the organizer on the board so students can complete their own version. Then students were instructed to open the PhET simulation on their iPads and a worksheet in Notability.

In this activity, students used the numbers tab instead of the pictures tab (See Figure 5-12), but the activity was similarly structured.

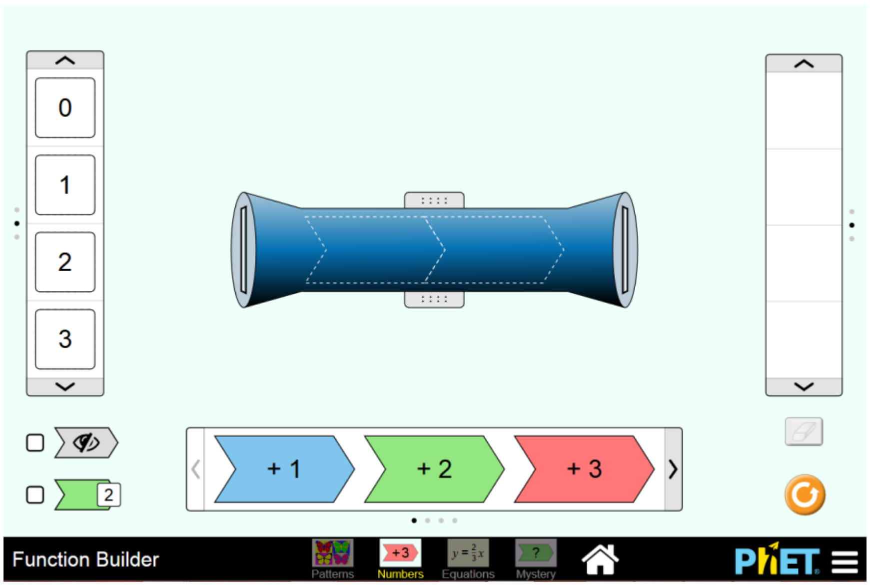


Figure 5-12 Screenshot of the numbers scene in Function Builder

The students first explored the simulation, and then built the functions on the worksheet (See Figure 5-13), listing the output for each input.

Function Fun Day 2

Part A: Warm-up/Introduction:

1. Go to the Numbers activity, +3, of the Function Builder sim. Complete the missing information in the following tables and try to write the equation that represents the function rule(s) for what is happening to the domain value to get the corresponding range value. Be prepared to present one to the class.

Function Rule: - 3

Domain (x)	Range (y)
-2	
-1	
0	
1	
2	
3	

Equation:
y =

Function Rules: - 2 then x 2

Domain (x)	Range f(x)
-2	
-1	
0	
1	
2	
3	

Equation:
y =

Function Rules: ÷2 then + 3

Domain (x)	Range (y)
-2	
-1	
0	
1	
2	
3	

Equation:
y =

Function Rules: (you pick)

Domain (x)	Range f(x)
-2	
-1	
0	
1	
2	
3	

Equation:
y =

Figure 5-13 Activity Part A

Lily explained that the students should use -3 as the first function. Then she asked the students to create a table of input/output values, and see if they can find the equation. It is important to note at this point that the simulation does both of these tasks for the student (See Figure 5-14 as an example of what students could see through the existing features of the simulation).

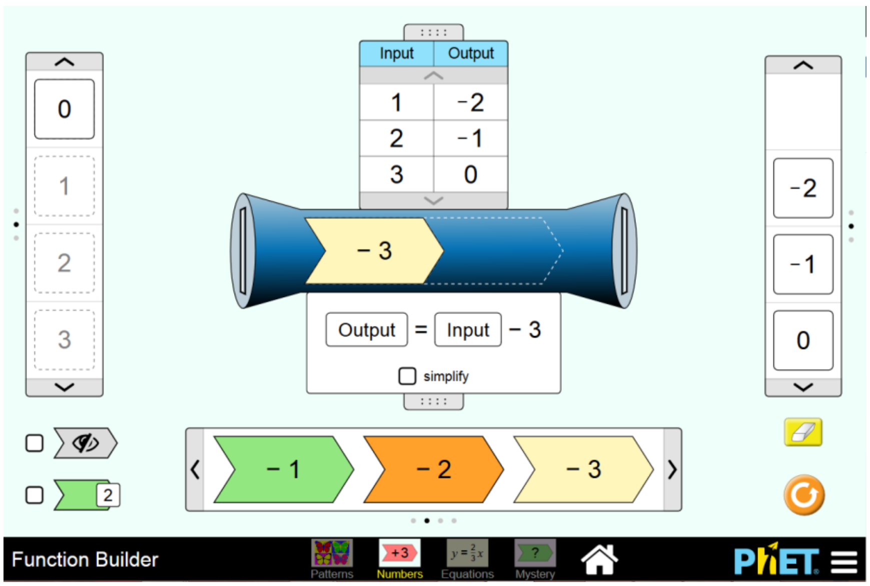


Figure 5-14 Screenshot of Function Building showing the table and equation

There were many student questions at this point, mostly about the instructions of the task. Lily stopped at several desks and restated the instructions. There were also questions about what she meant by equation, however the students were able to clearly articulate what is happening to the numbers that were sent through the function tube. When Lily realized that one student does not have his iPad, she instructed him to pair up with another student like they did in the last lesson. Students work on this portion of the lesson for fifteen minutes. Lily then asks students to put their answers on the board. Each problem is discussed, with Lily doing most of the talking, and students only speaking when answering specific questions. Lily explicitly made connections between the functions and the final equation. She also made explicit connections between the equation, and the simplified version of the equation (see Figure 5-15 for an example of the simplified equation).

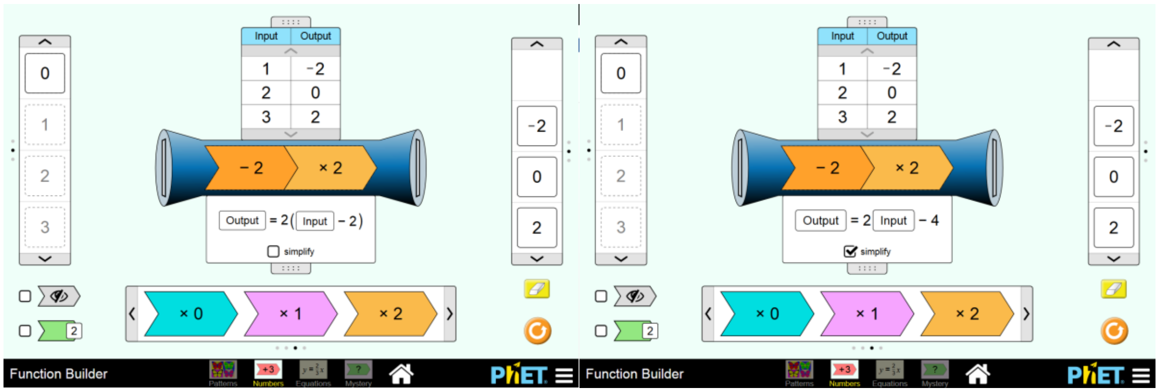


Figure 5-15 Function Builder with un-simplified equation (left) and simplified version (right)

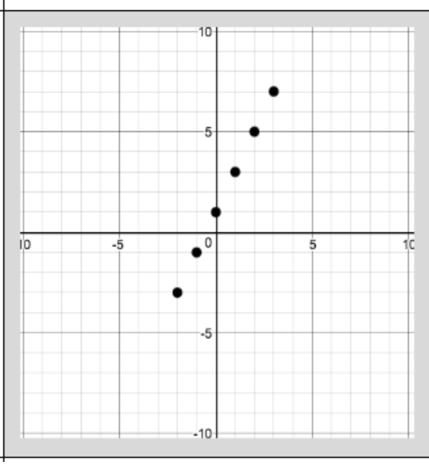
Students were then prompted to move on to the next portion of the lesson. The final part of the activity was designed to support students as they link between representations. Students were given one piece (table, graph or equation) and asked to find the two other pieces (see Figure 5-16 for the activity).

Part C: Equations Exploration

1. After seeing the relationship between table, equation, and graph, see if you can find the missing representations of a function if you are only given one of the three. Use the applet to check your answer.

Table	**Graph**	**Equation** **y =** **f(x) =**

x	f(x)
-2	
-1	
0	
1	
2	
3	



What happens to x?

So, y =

or , f(x) =

Figure 5-16 Part C of the lesson

The teacher notes that the graph will look differently in the simulation than in the worksheet due to the different scales. Students are told to complete this portion of the task as their exit ticket for the class. Lily then quickly revised her plan and did the first problem of the activity as a group, projected on the board with students following along. Lily asked students a series of questions, using the graph to find the ordered pairs, and then wrote the ordered pairs on the table. Students noted that the outputs are growing by 2, so Lily switched back to the simulation and enters the function x2. "Does that work?" Another student suggested adding a one to the function. Lily changed the function to read (times 2,plus 1). The table matched the table in their worksheet. At this point, the bell rang and students were dismissed.

Lesson 3

In the third lesson, Lily decided to use the PhET simulation, Graphing Lines, as one of four stations through which students would cycle to complete a review of linear equations. The remaining three activities included a premade Desmos activity where students created equations to draw lines that matched a picture with overhead power lines, a short interactive video lesson that students watched and then paused while answering questions, and a matching card game where students had to find the two matching representations between tables, graphs, equations and descriptions. Graphing lines, as shown in Figure 5-17, is designed to support students as they explore the connections between an equation and a graph. Using this simulation, students can manipulate the equation and change the graph, or manipulate the graph to change the equation.

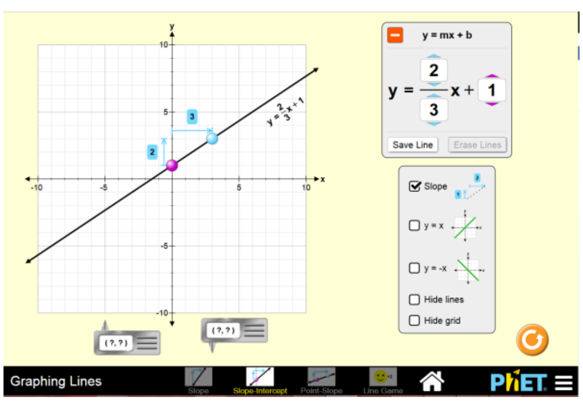


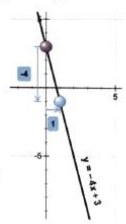
Figure 5-17 Graphing Lines Simulation

In implementing this lesson as part of the stations, Lily chose not to write a formal lesson plan; instead, she chose to simply write the instructions for the students as shown in Figure 5-18.

STATION 3:
Find That Line!

In this station you will go to the linked website and do the following:

1. Take 5 minutes to explore what the "Slope-Intercept" simulation can do. Talk about it with your group members.
2. Refresh everything on the page.
3. Starting with equation A below (y = -4x + 3), identify the b-value (y-intercept) and slide the purple point. So for equation A you would put the purple dot on (0, 3).
4. Then identify the m-value (slope) of the equation and use it to plot the other blue point. So, for equation A that would be -4, so you would move down 4 and right 1 to plot the blue point. Then, check to see if the equation in the upper right corner actually matches the equation for A. If so, you know you graphed it correctly using the y-intercept and slope.
5. Do steps 2 and 3 again for the other equations. Pay attention to the slope arrows as you graph them.



A. $y = -4x + 3$
B. $y = 4x - 3$
C. $y = \frac{1}{4}x + 2$
D. $y = -\frac{1}{3}x - 1$
E. $y = -5x$
F. $y = \frac{5}{3}x - 2$
G. $y = -\frac{3}{5}x + 3$
H. $y = -5$
I. $y = 3$
J. $x = 3$

Figure 5-18 Student instructions for PhET activity

This was the only lesson that did not begin with a formal warm-up activity. Instead, a quick review was provided through a class discussion as the teacher asked students what they remembered about graphing lines and the equation y = mx + b. Then Lily explained each of the stations to the students, students were put in groups and allowed to quickly start at their stations.

It takes nearly five minutes for the students to get themselves started at each station. In general, the teacher worked predominantly with one group who appeared to need more support, but occasionally walked around answering questions for the other groups.

I focused my attention at the PhET table to see how students would interact with the task. Students quickly open the simulation using the link on Schoology. As directed in the lesson, students first begin to explore the simulation. The teacher, in the meantime, has checked in with each of the other four groups, but does not stop at the PhET table. As the students read the directions, they appear confused. Another student also reads the directions. The first student uses the arrow buttons to change the equation, thus changing the line. I point out that the blue and purple buttons on the line also move the line. The student moves the line, and then calls over the teacher, asking, "How do we do this." The teacher reads the directions aloud to the students, demonstrating each step on the simulation. When the teacher walks away, the students continue to play with the simulation, but there is little evidence that they are completing the assigned task. The students spend nearly twenty minutes at this station, before moving to the next station.

With each subsequent group, the teacher comes over and has the students read the directions out loud. She demonstrates the first problem with the students, and then leaves the students to work on their own. Again, students are using the arrow keys to change the equation, rather than changing the line and checking the equation.

By the time the last group reaches the simulation station, the amount of engagement is decreased. Students only have 15 minutes at this station, but most of the time is spent just playing with the simulation and changing the equation. The teacher comes by and talks with this last group twice, each time restating the directions. No other student interactions are noted.

5.4.4 Summary of Changes between Non-Technology and Technology based lessons

Below, I share the highlights of differences in Lily's lessons as she moved from non-technology to technology based lessons. These differences are summarized in Table 5-4 and discussed below. A full listing of the comparison can be found in Appendix J.

Table 5-4 Summary of Differences from Non-Technology Based Lessons to Technology Based Lessons for Lily

LILY:	Differences from non-technology to technology based lessons
Student/Student and Student/Teacher Interactions	Increased student-student math focused interactions
Group work	More time spent in groups
Mathematical Focus	Move from generalizing patterns to connecting representations
Engagement	Increased student engagement with targeted technology use
Teacher talk	More time spent on providing instructions and asking questions. Decreased direct instruction
Mathematical Task Type	More time spent on investigational activity

Teacher Information

In terms of the teachers' verbal utterances, Lily spent significantly more time posing questions to students in the technology lessons than in the non-technology lessons, as well as approximately 10% more time providing instructions. She spent significantly less time on direct instruction in non-technology lessons, moving from 56% of the time to 10% of the time. The exception to this was the third lesson where the PhET simulation was used as a station. In this lesson, the teacher only provided directions, and did not use any other questioning techniques with the students. She did spend some time as a technical assistant, although more was spent during the first lesson, than on the second. Furthermore, this time was comparable to the time spent on technology issues in her non-technology classes when she was just addressing issues with the iPads. Lily also spent less slightly time walking around as well as less time with students when using the technology based lessons. However, this appears to be more reflective of the fact that she does not spend as much time proving direct instruction to small groups. In non-technology lessons, there is a pretty even mix of teacher use and student use of technology due to the prevalence of the iPads in the classroom (52%-48%, teacher: student); alternatively, technology was much more student centered during technology based lessons (35%:70% teacher to student).

Student Information

Students spent less time working on math (engaged in mathematical tasks) and slightly more time in engaged in mathematical discussions in the technology classes. Students also spend less time listening to a lesson (🡇7%), but more time listening to instructions (🡅9%). Student engagement was higher for the majority of the class in the technology-based lessons, moving from 75% of the time to nearly 90% of the class period.

Mathematical Focus

The focus on the mathematics shifted dramatically with the use of technology from a focus on generalizing patterns (such as the lesson in the categorization of polynomials) to connecting representations. There was also a shift to more time being spent on conceptual understanding through investigations rather than drill-like activities that build fluency. In technology based lessons, there was a small decrease in the amount of time the class was focused on mathematics, with a slight increase in the time spent focusing on technology. This appears to be related to the need to introduce and spend time allowing the students to explore simulations prior to using them. This time decreased significantly with each additional technology class.

5.4.5 Sara

5.4.5.1 Classroom Practices – Non Technology Based Lessons

Each lesson in Sara's classroom begins with a warm-up projected on the board. Students generally come right in to the classroom and start working on the task. Usually, the work is done with dry-erase markers and written right on their desks. Sara often engages in a bit of small talk as students enter, but for the most part, the room quickly quiets as students begin to work. In each of the four observed classes, the warm-up activity was directly related to the planned task prepared for later in the classroom. For example, in a lesson focused on least common multiples and greatest common factors, students had to find the factors of a number in one problem by finding all of the possible rectangles one could make with that number, and the multiples of another number in a separate problem. While students work on their warm-up activity, Sara walks around the classroom and checks each student's homework. She tells me that she is checking a specific problem or two, and often stops to talk with a student in more detail about their work. As she walks around, she often makes note of strategies she sees for their warm-up work, and later calls on students to share their particular solution. As students share their work, other students are called on to share whether they agree or disagree, and to offer a restatement of what they heard.

Once the warm-up is completed, the lesson progresses to the activity of the day. In three of the four observed classes, the activity is student-centered, in that there is little or no direct instruction, and students are working in groups to make sense of some problem. For example, in one lesson, the students work together to solve a problem involving rates and ratios; in another lesson, students worked together to figure out when two people will be at the same point in the Ferris Wheel. In the fourth class, there was nearly 20 minutes of direct instruction, where Sara taught students how to find the prime factors of a number using factor trees, before students started their group activity. In three of the four classes, the activity was designed to help students to generalize patterns, while the fourth class had a set of activities designed for students to review the topics from the entire unit. As students worked together, Sara walked around the room, stopping regularly to check in with students and to ask probing questions, focused on determining if students understand the mathematics rather than on basic recall of facts or information.

The following segment is characteristic of her interactions with the groups. In this segment, the students have watched a video of two Ferris Wheels moving in a circle, one larger than the other. The question asks when the two riders (Debra and Jeremy) will both be at the lowest point (bottom of the Ferris Wheel).

Sara:	*I want to know if you think it's least common multiple or greatest common factor.*
Student 1:	*I think we're looking for the greatest common factor because as this small Ferris Wheel goes around in a shorter amount of time and the bigger Ferris Wheel takes more so that means that they have to find how many times they will go around together. I don't know where this is going [pointing to the Ferris Wheel image].*
Student 2:	*All the way around.*
Student 1:	*I know but…we're looking for the greatest common factor because if they both have the same factor then ... wait.*
Sara:	*What would you be finding factors of? What would your factors be in this situation?*
Student 2:	*It would be how many times they go around until they are at the same ... does that mean that Debra would go around more times than Jeremy?*
Sara:	*Now if you're looking at the number of times something happens, is that a factor, or is that a multiple?*
Student 1:	*A multiple?*
Sara:	*How do you know?*

(Non Tech Observation 2, 102616)

This type of conversation is repeated with each group. Sara asked questions of the students, and the students were all quick to offer their thoughts, even when they are unsure of their answers.

She almost never made a declarative statement, but rather continued to pose question after question. Sara continued to move from group to group while the students were working together.

With five to ten minutes left of class, Sara brought the class together for a discussion that builds on the activity, or summarizes the salient points of the lesson. She then reviewed the assigned homework and dismissed her class.

5.4.5.2 Classroom Practices –Technology Based Lessons

I observed two technology classes with Sara. One was with the usual 6th grade class that I had been observing. In this class, Sara used the Unit Rates simulation as shown in Figure 5-19 to build on their conceptual understanding of double number lines (Field notes, 11/18/16).

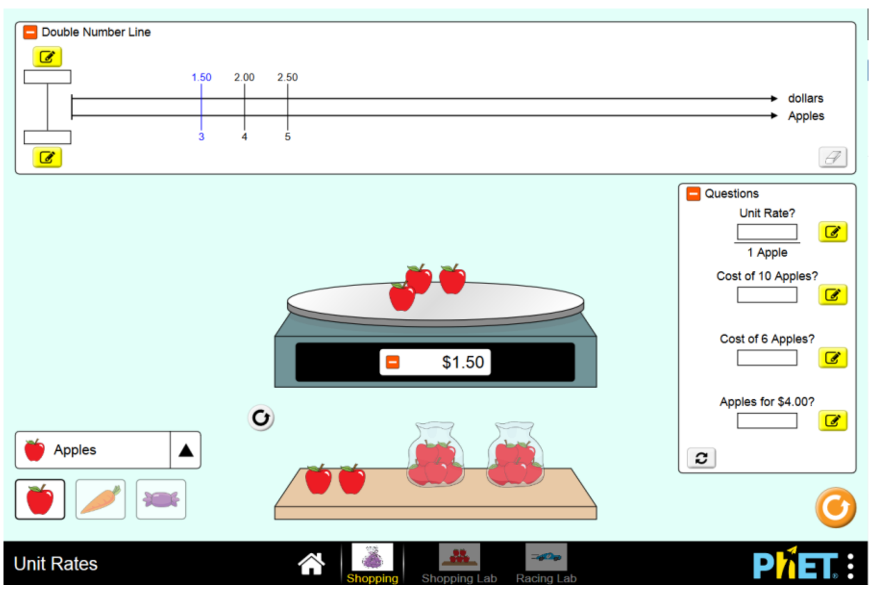


Figure 5-19 PhET Unit Rates Simulation

The other was with her math clinic, which has many of the same students as her 6th grade class. In this class, students used the Fraction Matcher simulation as shown in Figure 5-20 to build fluency in finding equivalent fractions (Field notes, 011816).

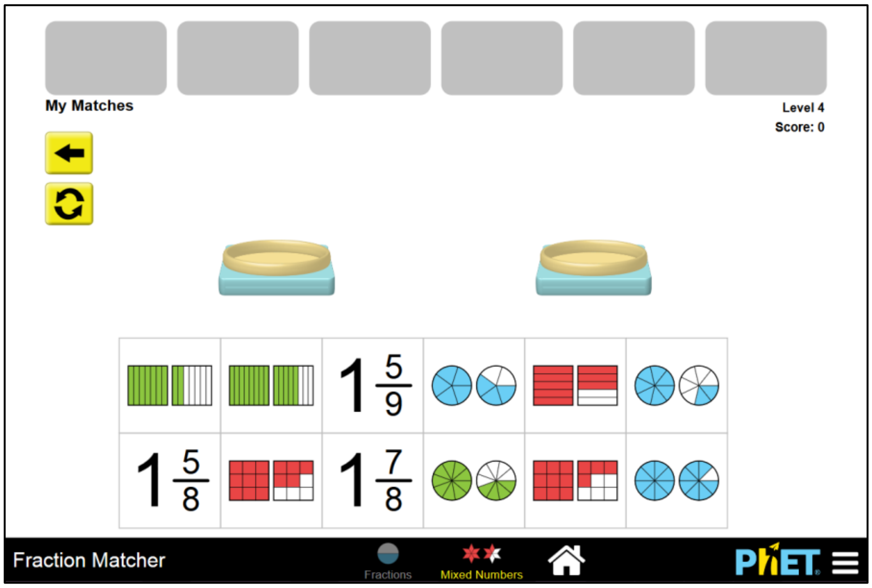


Figure 5-20 PhET Fraction Matcher Simulation

Lesson 1

As noted earlier, Sara's technology classes take a slightly different format due to time constraints as students all have to get logged in to their Chromebooks and access the internet. Therefore, Sara did not start with a warm-up activity when using technology. Instead, students came in and immediately got their devices and accessed the internet. For the 6th grade class, Sara then asked students to put their Chromebook lids at a 45 degree angle (to prevent distraction) and talk with their partner about a ratio table. In the math clinic class, students similarly discussed a common fraction (3/4) and shared everything they know about it, which led to a discussion of equivalent fractions. Therefore, despite not having a more typical warm-up where students complete some type of math drill, there was still a defined lead-in for the activity. Students in both classes were then invited to open the PhET website and access the simulation.

The ways in which the activities were structured then varied based on the class. For the traditional sixth grade class, students also opened a digital worksheet which provided step by step directions to follow, prompting students to complete various tasks while using the simulation. In this activity, students were placing items on a scale to create a double number line. The double number line automatically updated with changes to the number of items. See Figure 5-21 for a screen shot from the simulation.

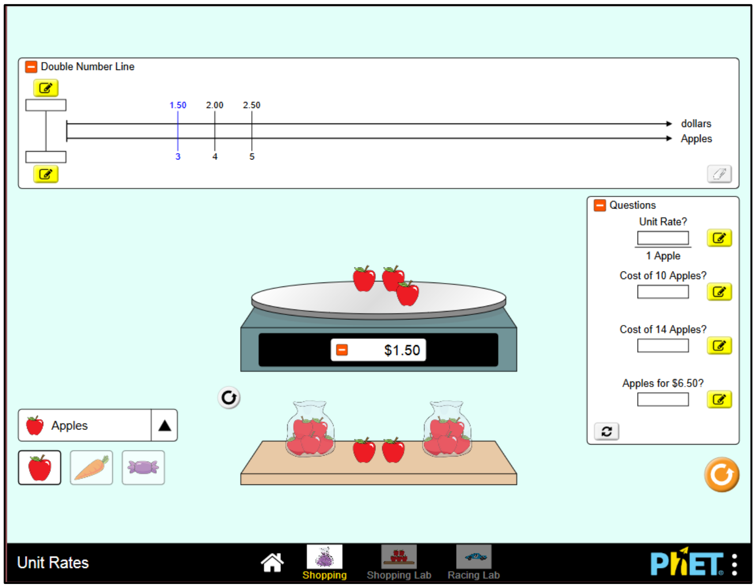


Figure 5-21 PhET Unit Rate Simulation Screen Shot. Students place a bag of apples on the scale. As they remove apples, the double number line automatically updates.

Once students were allowed to work on their own, Sara walked around the room, with her eyes oriented to student screens. For the first 5 minutes of the 6th grade class, all of Sara's comments were technology oriented, helping students open the simulation, find the activity sheet and explain how to share the Google document. Later, she began to ask students questions about the mathematics, in ways that were similar to her non-technology based lesson. The students spend a total of 35 minutes on the activity, which was paused after the first 10 minutes, for a brief five minute class discussion to share what they noticed. The teacher took time in this discussion to prompt students to make connections between the number of apples on the scale and the prices listed on the double number line. At the end of the 35 minutes, the teacher again prompts a class discussion, this time focus is on unit rates, and how students could see the unit rate on a double number line. The class wraps up with a brief discussion about unit rates in the real world, and an assignment to look for and take pictures of unit rates over their break.

Lesson 2

In the math clinic, the goals of the simulation were different (focused on fluency more than conceptual understanding), and perhaps as a result, took on a slightly different structure. For the math clinic class, Sara opened the simulation on her computer and projected it to the class. With the class watching, she explained the features of the simulation and modeled how to play the matching game.

Significantly more time with spent on the initial discussion (15 minutes as compared to less than 5 minutes) and significantly less time was spent using the simulation (13 minutes, vs. 35 minutes). In this simulation, there is an activity where students can play a game to match a series of fractions, starting with level 1 which is very easy, and moving up to level 8. Once students choose two representations, the students can click on the box labeled "Check" to see if their answers are correct. The simulation provides feedback in three different ways. First, the simulation shows a comparison sign (equal to, less than or greater than) to help students know if they are correct, and to scaffold their understanding. Second, the simulation provides a "smiley face" when students find a match. Finally, there is a sound component, as the simulation makes a chime, or "bing" sound, letting them know their answer is correct. See Figure 5-22 for an example of Level 4, and the associated supports. As students begin, Sara reminds them to be "metacognitive", thinking about why the two fractions match, and to look specifically at the number line as part of their process.

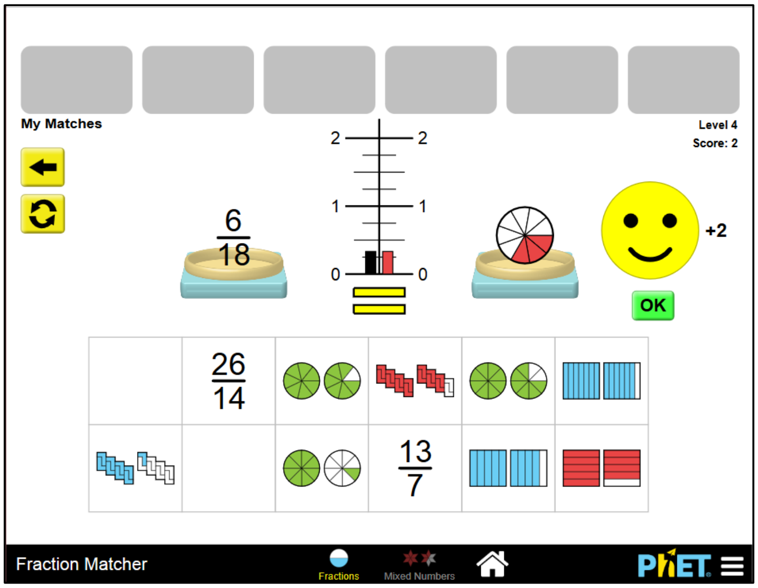
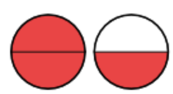


Figure 5-22 PhET Fraction Matcher Simulation Level 4. Students place equivalent fractions on both scales and check to see if they match. A correct match results in a chime sound, a smiley face, and matching values on the number line.

Again, as students work, Sara moves around the class talking with students. There are few comments around the technology this time. Instead, Sara is asking students to justify whether the two fractions are alike, and working one on one with students who need more support. In the excerpt below from her second technology based lesson (01/18/17), Sara takes some time to talk with a student who is having trouble seeing mixed numbers in the images.

Student 1: *So, what about this?*

Sara: *Well, let's look at this before we just guess. So, how many pieces do you have shaded in there?*



Student 1: *Three. [Looking at this image]*

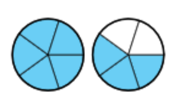
Sara: *Mm-hmm (affirmative). So, our whole has two pieces in it, but we have three pieces that are the size of a half in here.*

Student 1: *So, it would be three.*

Sara: *It would be that one. Yeah, and look, it's in between, on our number line, between one and two, okay? I want you to look at this one next. So, you're moving really quickly through these. What I want you to really think about is why they are matching, okay?*

Student 1: *Okay.*

Sara: *So, what fraction does this represent right here?*



Student 1: *8/10ths. [Looking at this image]*

Sara: *Okay. So, let's look at this. We have our part and our whole, so your fingers off your keyboard for just a minute. What is the size of your whole, how many pieces are in your whole?*
Student 1: *Ten.*
Sara: *So, one of these circles is your whole.*
Student 1: *Oh. So, five?*
Sara: *How many parts do you have shaded in?*
Student 1: *I have eight.*
Sara: *Eight. All right, so you have eight pieces that are the size of a fifth. So if we wanted to talk about some things we've been doing in class, we could write this as the mixed number. Do you want to write it as a mixed number?*
Student 1: *Yeah.*
Sara: *What would it be?*
Student 1: *So, it would be ... it would be one, 1 and 3/5ths.*
Sara: *Awesome. How is 1 and 3/5ths represented in that picture there?*
Student 1: *Because there's one whole and three out of five, so one -*
Sara: *Yeah, yeah.*

This pattern of discussion continues until there are twelve minutes left of class, and the students are asked to put away their Chromebooks. Students are then given a small final assessment, as shown in Figure 5-23.

FRACTION MATCHING - Exit Ticket

What advice do you have for the student who made the mistake below?

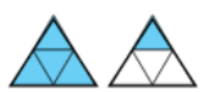

$= \frac{5}{8}$

Figure 5-23 Exit Ticket

The students are asked to advise the student, based on the work shown. This work was done on their desks, using expo markers. As students leave, the work remains on the desks, and Sara again wanders the room. Later, Sara tells me that she will use what she saw to begin the next days' class discussion (Field notes, 01/18/17).

5.4.6 Summary of Changes between Non-Technology and Technology based lessons

Below, I share the highlights of differences in Sara's lessons as she moved from non-technology to technology based lessons. These differences are summarized in Table 5-5 and discussed below. A full listing of the comparison can be found in Appendix K.

Table 5-5 Summary of Differences from Non-Technology Based Lessons to Technology Based Lessons for Sara

SARA: Differences from non-technology to technology based lessons	
Student/Student and Student/Teacher Interactions	Increased student-student math focused interactions
Group work	More time spent in groups
Mathematical Focus	Move from sense-making and explanations to connecting representations
Teacher talk	More time devoted to student questioning. More diversity in types of questions asked, answer, discuss, restate and reflect
Technology focus shifts	Shifts towards being more student centered

Teacher Information

In terms of the teachers' verbal utterances, Sara spent significantly more time posing questions to students in the technology lessons (50% of time) than in the non-technology lessons (30% of time), as well as more time providing instructions, which increased 12% from non-technology to technology lessons. However, it is important to note that this time focused on instructions decreased significantly the second time she taught using PhET simulations, to a level that was comparable to non-technology lessons. Sara on average, spent about 15% of her non-technology based lessons doing some type of direct instruction; in technology based lessons, she used no direct instruction. She did spend some time as a technical assistant, although more was spent during the first lesson (30%), than on the second (20%). Sara also spent less slightly time walking around talking with students.

Student Information

Students spent more time working on mathematical tasks and in mathematical discussions in the technology classes. Student engagement, while still high, was slightly lower in the technology lessons (85% vs. 94% in non-technology classes). In the typical non-technology class, the time spent working in groups versus working individually was pretty evenly split. In technology based classes, this was very dependent on the setup of the computers. Where students worked one-on-one with the computers (Fraction Matcher simulation), most of the work was individual work. In contrast, when students shared a computer, the work was predominantly completed with partners.

Mathematical Focus

The focus on the mathematics shifted dramatically with the use of technology as there was a large increase in connecting multiple representations. Sense making as a mathematical focus remained constant. There were no significant shifts in the types of activities nor in amount of time the class was focused on mathematics.

Class Discussions

Between non-technology based lessons and technology based lessons, the amount of time students spent talking was not significantly different, but what did differ was the broadness of student responses, in that students were asked questions that expected justification of answers, discussion with partners, etc. With non-technology classes, students typically provided short answers without any further elaboration; although a few classes did have brief moments of discussing or justifying their answers. In contrast, in technology-based lessons, students engaged in all five types of discussions (answering questions, discussing questions, justifying answers, restating ideas, and reflecting on the lesson). Class discussion had more student participation with technology based lessons, as well as more instances of both the teacher and the students building on ideas being discussed. Finally, the conversations with more likely to be described as rich mathematical discussions than procedural discussions.

Remediation and Reflection

In this class, there were no instances of either remediation or reflection in both non-technology lessons. There were a few instances of both reflection and remediation in the technology lessons.

5.4.7 Christine

5.4.7.1 Classroom Practices – Non Technology Based Lessons

I observed four non-technology based classes with Christine. In all four, the pattern was very consistent. Class started with a warm-up activity projected on the front board. Typically, the warm-up was a small set of problems which started easy and got progressively more challenging. Students were encouraged to pick the right starting point for themselves. As students worked through the warm-up, Christine walked around the class checking on homework. Usually, several students had either forgotten their homework or had not completed it. For those students, Christine gently reminded them to get it done that night. For students who had completed their homework, Christine took note of their work and added a grade to her grade sheet. As she walked around, Christine also looked at their warm-up work, commenting on what students have completed, and often encouraging students to try the next challenging part of the problem. Rarely did she give the students the answer – instead, she often asked more questions, or pointed out differences with a partners' work and asked the partners to work together to see what's happening. As she walked around, she also selected students who would come up and show their work on the board.

Once the problems were answered on the board, Christine asked a few select students about their solution, which had differed from the one presented. A class discussion ensued each time, with students chiming in to answer her questions.

Christine: *Oh, you were so close though, you were so close. Oh, nice. So Mary, it should go up, it goes all the way up there. Okay, so there's two clues I see Laura knows one of the clues, Laura do you want to give us one of the clues?*

Laura: *Which number is the grow by numbers?*

Christine: *And how did you find that? Either from the table or the graph or what did you do?*

Laura: *Well, something we had to put attention to was that the x, in the table, there wasn't a zero and there wasn't a two, so you would probably get confused if you didn't know that. And then, from negative one to zero is negative one.*

Christine: *Okay.*

Student: *And from one to two and the -*

Christine: *Up here.*

Student: *It's, what's it called, [crosstalk 00:18:11] it's three.*

Christine: *It's three, so how much did that go up? My markers up there.*

Student: *It went up two.*

Christine: *It went up two, so you're saying it's gotta grow by two every time?*

Student: *Yeah.*

Christine: *Yeah, so I like that you're saying you gotta be really careful on a table because if it skips, you gotta watch out for that. Sam, are you with us here? Okay. So, if we look at the graph, what's our strategy for finding grow-by on a graph? I hear 'drawing the slope triangles.' Good. So it doesn't really matter where you draw it, I might just go from an easy point to another easy point but over one up two. So I still get a grow-by of two.*

(Technology Lesson 11/18/16)

Christine actively seeks out students with ideas and strategies and highlights those in her discussions. Christine also moved fluently between asking individual students for ideas, and having students turn and share their ideas with their table partners.

Once the warm-up activity is complete (typically 20 minutes of the class period), Christine then moves students on to the activity for the day. While she tells me that she often brings in her own activities to supplement the book, on all the days I observed, the students worked on investigations from the CMP book. Students typically work in small groups to complete the investigations, while Christine varies between walking around asking questions, and working with students in small groups for some one-on-one instruction. This is a typical exchange as she walks around (all names are pseudonyms):

Christine: *[Talking to a pair of students] So you both made similar mistakes on this one. So whenever we have our rule, we have something x plus something and this number is our grow-by number, okay? And this other number is our figure zero. Now you both put two as your grow-by number and I*

	thought that was really interesting so if we look at our table, one, three, five, 85, 110, 135, how much does this grow-by on the top?
Student:	*Two.*
Christine:	*Okay, so the top grows by two, so Maria do you want to add that and say plus two plus two?*
Student:	*Mm-hmm (affirmative).*
Christine:	*Okay, go add that. But the down here we don't add four, four divided by two would give us a grow-by of two every time. How much do we add every time?*
Student:	*25?*
Christine:	*Okay, so then do you guys remember back when Sharon said in the warm-up where if we skip some in the x then we have to actually divide because we're missing one in the middle there, right? So if we go forward two, we grow by 25 so you can say grow by 25 there. So that means if we only went up one, we have to divide, 25 divided by two, how much is 25 divided by two?*
Student:	*12.5.*
Christine:	*And that's my grow-by number.*

(Observation Non Technology 2, 092716)

At the end of the activity, the class is often brought back for a brief discussion where the teacher summarizes the key points from the lesson. Then, students are asked to write in their Learning Log, a journal where students write an answer to their essential question of the day. For example, on the same day as the exchange above, students were asked to answer the following question in their learning log: "How can we create an equation from a graph or from a table? What are you looking for in your graph or in your table to help you write the equation?"

5.4.7.2 Classroom Practices –Technology Based Lessons

Christine taught two technology based lessons as part of this research project. The first lesson was a lesson that she designed over the summer months with the other teachers. In this lesson, students used the Function Builder simulation in a game that she called Function Sleuths. Students created a function and listed either a table or an equation. This information was recorded in a Google Document and shared with another team who had to use the information to find the missing pieces (table, equation or graph). An example of this task is shown in Figure 5-24 below.

Challenge Creators:	
Function Sleuths:	

The challenge:

Function 1		
Table	**Graph**	**y= Function Rule**

Input (x)	**Output (y)**

Recursive Sentence:	
Explicit Sentence:	

Figure 5-24 Function Sleuths Worksheet

The second lesson was an adaptation of the lesson she had written with the PhET simulation Proportion Playground as shown in Figure 5-25. In the original lesson, Christine asked students to connect a unit rate to the slope of a line. However, in the final enactment of the lesson, the students instead created a ratio using an image in Proportion Playground, and then all four members of their team had to find another image with an equivalent ratio.

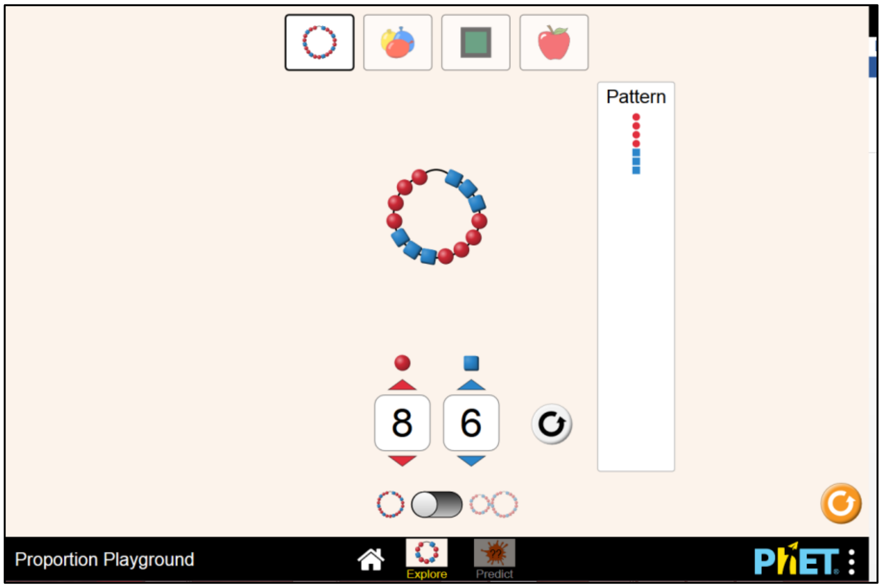


Figure 5-25 PhET Proportion Playground Simulation

The structure of the lessons was nearly identical to that of the non-technology based lessons. Both lessons began with a warm-up activity on the board. Both warm-ups are associated with their respective lesson. For the Function Builder lesson, the warm-up is related to using a graph and a table to find an equation. For the Proportion Playground lesson, the warm-up is associated with simplifying a ratio and solving a problem using a ratio. Students completed the task on their own, and then discussed their answers as a class.

Christine then introduced the simulation in both lessons and provided 5-8 minutes for the students to explore the simulation and discover all of its features. As students explored the simulation, Christine walked around pointing out key features and asking students what happened when they clicked on specific buttons. After this exploration time, there was a very brief discussion to highlight the features of the simulation.

Then, Christine gave students the instructions for the activity. In both cases, the instructions were posted online to her website, so students had multiple tabs open in their browsers. Once the instructions were completed, students began to work in groups and Christine again walked around answering questions. Unlike with her non-technology based lessons, however, the interactions with students were far more technology-oriented, as she restated the directions and explained technical steps to the students as both activities required students to take screenshots of various steps in the activity.

This was a typical exchange:

Christine:	*So did you click "copy to clipboard"? You need to screenshot one pool table.*
Student:	*Oh.*
Christine:	*It's okay, you can just screenshot one of them and then copy to clipboard.*
Student:	*Do we have to screenshot the thing?*
Christine:	*Not yet. You just do your original, right there.* *All right, if you have your screenshot done please follow your instructions and continue moving through this. I'm going to help groups who need help with screenshots.* *I need you guys to try to focus on this, okay. Raise your hand if you're done with your screenshot.*
Student:	*I just took it, I'm going to fix it.*
Christine:	*Great. So you guys keep moving through your task card. Keep moving through, you're going to be doing some more designing. I'm going to help the group that needs their first screenshot done and then I can help you with the next steps.*

(Observation, Technology Lesson 2, 11/18/16)

Whereas in her non-technology classes, the focus of questioning appeared to be on the mathematics, in her technology class, the focus moved to handling the technical aspects of the class. She even goes so far at the end of that excerpt to say, “I’m going to help the group that needs their first screenshot done, and then I can help you with the next steps.” She seems to feel pressed on time, and is rushing to ensure everyone has the technical skills needed to complete the task.

In both of the lessons, students are grouped for the activity. In the Function Sleuths, students create a function and then another team tries to find their function. This leads to a significant amount of talking among students, but there are very few moments where students are directed to pursue specific conversations. In other words, students are not paused, for example, to consider a specific aspect of the activity. In fact, when Christine does pause the class, it is to remind the students of the technical aspects of ensuring that they take a screen shot of their work.

> *Every single person needs to be doing some kind of a screenshot where they put it into their own box. So our blue person needs a screenshot here, our yellow person's screenshots here, our green person's screenshot is here. And then you need to write your ratios. Clear? Okay so go for it, you need to get your screenshots in, you need to get your ratios in. You have two more minutes to get your screenshots in and your ratios in.*
>
> (Observation, Technology Lesson 2, 11/18/16)

At the end of both activities, she again had the students answer a Learning Log question, but did not do any type of full class discussion as there was no available time.

5.4.8 Summary of Changes between Non-Technology and Technology based lessons

Below, I share the highlights of differences in Christine's lessons as she moved from non-technology to technology based lessons. These differences are summarized in Table 5-6 and discussed below. A full listing of the comparison can be found in Appendix L.

Table 5-6 Summary of Differences from Non-Technology Based Lessons to Technology Based Lessons for Christine

CHRISTINE:	Differences from non-technology to technology based lessons
Class Interactions	Decreased discussion time
Student/Student and Student/Teacher Interactions	Decreased student-teacher math focused interactions
Group work	Similar patterns of working in groups
Teacher location	More time spent walking around talking with students
Mathematical Focus	Move from generalizing patterns to connecting representations

Lesson Reflection	Decreased time spent on lesson reflection
Teacher talk	More time spent on providing instructions and tech support
Mathematical Task Type	More time spent on investigational activity
Technology focus shifts	Shifts towards being more student centered

Teacher Information

In terms of the teachers' verbal utterances, there were significant issues in the first lesson, particularly looking at the amount of time spent by Christine providing instructions (60% of the segments had significant moments of statement or restatement of instructions, as compared with33% in a non-technology class) and providing technical assistance (80% of the segments). In the second lesson with technology, the amount of time spent in those two activities became comparable to non-technology based lessons, however, the time she spend questioning students in the second lesson significantly decreased as well. In non-technology lessons, Christine seemed to use students' answers as a way to determine which questions were appropriate for each child. When all of the students were working on different problems in the non-technology lessons, Christine's use of questioning was far more limited. In terms of Christine's movements around the classroom, her first lesson was comparable to her non-technology lessons, with roughly 30% of the segments finding her walking around, 40-50% with her in the front of the room, and 15% of the segments working with a small group of students. In the second lesson, however, Christine spent far more time walking around and checking in briefly with students, and no segments with significant moments with small groups of students.

Student Information

Students spent comparable time working on math in both non-technology and technology classes, although again, there were significant decreases in time working on mathematics due to the increased time listening to instructions with the first technology lesson. Engagement was generally very high in all classes, but slightly lower for the second lesson, with Proportion Playground. Despite being paired up for both activities, students worked more individually in the first activity, and did more group work in the second lesson. This is likely a result of the lesson design. In the first lesson, each student had a specific task, and group work was not needed to complete their portion of the work. On the other hand, in the second lesson where all students needed to find a comparable ratio, it was essential for students to work together.

Mathematical Focus

In this area, the focus of the class did not change significantly. Christine typically varied the mathematical focus of her class, particularly including generalizing patterns and making connections between representations. The technology lessons were similarly varied as well, although the second technology-based lesson had a stronger focus on connecting representations than did any other lesson. Again, due to the strong emphasis placed on instructions for the first technology lesson, a higher proportion of the class was focused on technology, reducing the time spent on mathematics.

Between non-technology based lessons and technology based lessons, the most significant difference was the lack of justification when using technology. In non-technology lessons, students spend roughly fifteen minutes of each class justifying their answers. In the two technology classes, there were no instances of justifying answers.

Finally, in Christine's classes, there were very few instances of either remediation or reflection in either non-technology or technology classes. In the non-technology classes, there were short instances of remediation, focused on mathematics, whereas in the technology class, there were no instances. Conversely, in the technology class, there was a small segment of reflection at the end of the class, whereas there had been only one brief case of it in the non-technology class.

5.4.9 Intersection of teachers' conceptions of technology with instructional practices

At this point, it is important to consider how each teacher's conceptions of both mathematics and technology may play a role in their respective instructional practices.

5.4.9.1 Pat

Pat was the most vocal in raising concerns about using a new task such as PhET simulations without training. She noted that she would have liked to explore the simulations more fully and use them as students would. She also shared that writing and sharing lessons was uncomfortable for her. This is notable as Pat ultimately chose not to use one of her own lessons that she designed over the summer; instead, she chose to use a lesson designed by another teacher, and worked her way through the lesson before she used it with the students.

Pat, as described in Section 5.2.1, designed a classroom that supported student-student interactions. In line with her focus on interactions, Pat did create spaces at the beginning and end of the technology-based lesson for student discussions and reflection. On the other hand, Pat noted many times that she preferred technology to be used one-to-one with students, so that each student remained fully engaged. Therefore, it is not surprising that Pat chose to individualize the assignment of devices to students and enact a lesson where students worked more individually on the technology, thus limiting student-student interactions during much of the lesson.

Pat's instructional practices around student-teacher interactions also varied for her technology based lesson. As she did in her non-technology lessons, Pat walked around the room, pausing to check in with students. However, the interactions changed with the technology-based lesson. Since the Graphing Lines simulation is designed to be open-ended, meaning that students can create a line for any equation, Pat could not check for specific answers as she walked around while students were working. This seems to have changed the interactions with students. As Pat walked around, she had fewer significant moments of direct instruction (as those were typically for students with incorrect answers in her non-technology based lessons) to more moments of questioning, where she asked students what they noticed, and how they would explain their results.

5.4.9.2 Lily

Lily's main concern with using technology was that students had quick and immediate access to the internet, providing them the opportunity to disengage from class. Therefore, it is not surprising that when designing a technology-based task, Lily chose to assign devices to partners, rather than individuals. This resulted in significantly more student-student interactions than were typical in her non-technology based lessons.

Additionally, Lily was somewhat uncomfortable with the research process. She noted that the process of designing lessons was difficult and she often worried about how I was perceiving her teaching. As a result, I believe that she tried extra-hard to have a well-planed lesson in place each time I observed her class. I say this because Lily would often comment on the time she had spent reviewing the lesson before I got there (or apologize because she had not had as much time as she wished she did). This is important to note as some of the changes in her lessons (between non-technology and technology based tasks) may in part be due to the lesson planning process itself. While it appeared that she took extra care in both types of lessons, it was still only the technology-based lessons that were fully planned.

Furthermore, Lily described herself as a "textbook rebel" and felt that the process of lesson writing when using new simulations was inconsistent with her own practices; instead, she shared that she would be far more likely to "just try out a lesson" (Summer lesson planning, 08/17/16). I encouraged her to do what was comfortable. As a result, she was the only teacher who chose to teach a PhET lesson without writing the full lesson. This was possible not only due to her own conceptions about lesson planning, but also because her students all had their own devices, making it easy to quickly use the simulations without the need for preplanning access of the technology. This lesson did differ from the other two technology-based lessons in two ways. First, she used stations for the students, meaning that the technology-based simulation task was a small part of the overall class period. Second, Lily used it as part of a set of review activities, and spent most of her time working with a smaller group of students who were struggling with the basics of graphing. As a result, she interacted minimally with the students using the simulation, and most of those interactions were restating instructions and demonstrating the use of the simulation. Therefore, it seems likely that the existence (or lack) of a fully designed lesson plan may directly impact Lily's instructional practices.

5.4.9.3 Sara

In choosing the lessons to use with her classroom, some of Sara's beliefs about technology and mathematics were evident. Sara talked often about how she used Pinterest® and similar resources to find the best activities, which would be both efficient and efficacious. As a result, Sara placed an emphasis on considering how the lesson would meet her goals for the class. That may be reflected in the fact that her two lessons (Unit rates and Fraction Matcher) had very distinct goals, with the first lesson focused on connecting representations, and the second lesson focused on fluency.

Furthermore, while Sara was able to clearly articulate benefits of using simulations, she recognized the challenges of bringing them into the classroom, including a lack of access to technology, as well as trying to fit them into the school's curriculum. Therefore, it is not surprising that while both lessons were originally designed to be used for her observed class, only one was used in that manner; the second lesson was instead used in her math-clinic class. In this class, Sara could choose her own curriculum to support the needs of the students assigned to the class, with no predetermined pacing required by the district.

For Sara, her non-technology-based lessons and technology based lessons were more similar than the other three teachers. That is to say that her teaching style seemed not to significantly vary in many areas between the two types of lessons. There are a couple of reasons why that may be true. Sara noted that she deliberately plans each lesson, writing it up and storing it on her computer. Therefore, this process of designing and later enacting the lesson did not vary as much from her typical patterns. As noted earlier, Sara is also new to teaching middle school math, and therefore is often looking for new activities for her students. This is perhaps why Sara seemed quick to jump in and use the PhET simulations in a lesson.

What did differ between non-technology and technology based lessons for Sara, however, was the types of questions she asked the students. Sara shared that when designed a lesson, she wrote a set of question prompts, which she attached to her clipboard, referring to them as she walked around talking to students. While she may have had a list of questions in mind to ask students in non-technology based lessons, Sara often spent a significant amount of time in direct instruction, working one-on-one with students who had incorrect answers, rather than asking questions. Furthermore, Sara had mentioned the time it took for students to learn how to use technology a number of times over the course of the year. Her feeling was that students had to be taught how to use the software. This is reflected in her use of the PhET simulations in her classroom, as she took time in both lessons to demonstrate how to use the software. As a result, once students were released to work independently, her initial questions all concerned discussing features of the simulations. Later, since the PhET simulations gave students different questions, the focus for Sara seemed to shift from first checking answers, and then asking questions if there was time (in non-technology based lessons),to asking richer mathematical questions that prompted students to discuss, justify, restate ideas, and reflect on what they were seeing.

5.4.9.4 Christine

From the interview, I learned that Christine sees mathematics as a series of big ideas to be connected. This belief is clearly seen in the design of her technology-based lessons as the lessons were both structured to support students in making connections between different representations. In the first lesson, with Function Builder, students were using one representation to find the other two representations. In the second lesson, with Proportion Playground, students had to find a proportion that was equivalent to the proportion chosen by the team of students, using a variety of representations to know if they were correct. In that respect, these two lessons mimicked the type of warm-ups often seen in her classroom where students had to match or compare representations.

Another area where Christine's beliefs were evident in the non-technology lessons was in the support of student interactions. Christine valued discussions among students, as was evident in her non-technology classes where she regularly asked students to "turn and talk to your partner," discuss their ideas, and justify their answers as part of her lessons (Non-tech lesson, 09/26/16, 09/27/16). Therefore, it is not surprising that each student had access to his/her own device, Christine chose design a lesson where students were paired up in both tasks. This afforded the students the opportunity to discuss the questions from the lesson, and aligned with her sense of mathematizing as being a social process.

However, Christine's beliefs about technology and mathematics did not always match the design and enactment of her technology-based tasks. For example, Christine saw her role to support students through her interactions (and regularly spent significant time as she walked around asking questions of her students designed to elicit their reasoning). Therefore, it was surprising to see that Christine spent a significant amount of time during technology-based lessons providing directions rather than engaging students in mathematical discussions. To understand why this may have happened, it is important to consider how Christine typically uses technology in her classroom. She often uses the document camera to show work using the projector, and has an iPad that is used to check students' homework. The students have access to Chromebooks, on which two sets of tools are predominantly used: Google documents and Desmos. While Desmos has a suite of pre-created student activities, these are not typically used by Christine's class. Instead, she has the students use Desmos as a tool to graph lines. Therefore, for Christine, the use of technology is most often a replacement of pencil/paper activities. Moreover, using student centered simulations is a relatively new experience and may have brought some concerns about how students would take that up.

5.4.9.5 Summary

These four teachers provide interesting case studies due both to their commonalities as well as their differences. Both Lily and Christine have similar years of experience, but their levels of experience in middle school are vastly different. Christine and Sara both typically write lesson plans, while Pat and Lily do not. Lily's class has one-to-one devices, while Christine and Sara have more ready access to technology; alternatively, Pat has to schedule her access to technology. Yet, despite these differences, there are many similarities in their articulation of the affordances and constraints of using simulations in their classrooms. These similarities and differences facilitate this research as exploratory cases to consider the richness of differences in instructional practices, and it seems likely that some instructional practices in both the design and enactment of tasks may be tied to teachers' conceptions of mathematics and teaching. In the next section, I explore the ways in which instructional practices can also be ascribed to designed features of the simulations.

5.5 ANALYSIS OF THE SIMULATION FEATURES

Research question 2 focuses on the design features of the simulation and the impact those features had on teachers' instructional practices. In analyzing these features, I considered three key sources: 1) the projects' Theory of Change diagram and associated narrative, 2) teacher interviews, and 3) classroom observations. In this section, I discuss the salient design features articulated in the project's Theory of Change that were intended to influence teachers' instructional practices (i.e., the designer's perspective). Then, I elucidate ways in which these features were articulated in the teacher interviews and exemplified in classroom observations.

5.5.1 Theory of Change

In the first step of this analysis, the PhET projects' Theory of Change (Singleton & Shear, 2017) was reviewed for relationships between design characteristics and goals of PhET mathematical simulations and intended teacher roles and outcomes. The Theory of Change diagram (Figure 5-26) was reviewed to highlight the features specific to teacher practices and look for connections between these features and the teachers' process of choosing, designing and enacting the lesson. Once the features that specifically related to teachers' instructional practices were identified, the narrative document describing the design team's Theory of Change was studied to fully understand the goal behind each feature.

The first focus in this case was on the Sim Design Characteristics, which are described as "design characteristics [which drive] the development of each sim, with the goal of producing sims that are grounded in a coherent vision for how to support math learning" (Singleton & Shear, 2017, p. 5) Most notable in this grouping is the intent to incorporate supports for teacher facilitation, and the ability of the simulation to display multiple representations, both of which might prompt teachers' instructional practices.

These Design Characteristics are intended to support the Design Goals, of which two are specifically focused on the following instructional practices: Provide opportunities around rich discussion, and support differentiation for students and teachers.

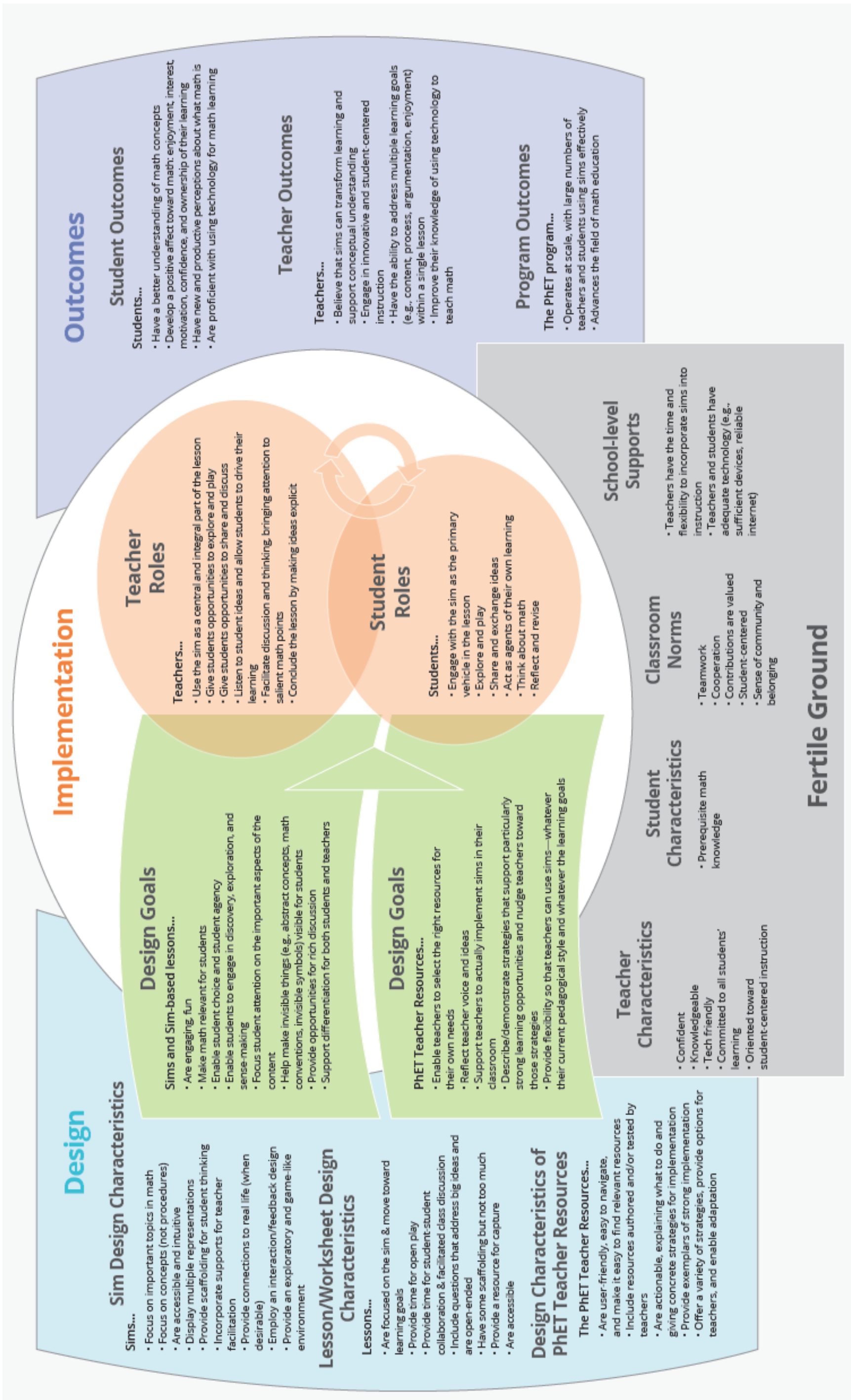


Figure 5-26 PhET Theory of Change Diagram

The second key focus area concerned the teacher roles. Here, the intended role of the teacher is to use the simulation as a central and integral part of the lesson, with five potential activities as part of this process: explore the simulation, share ideas, support student learning, facilitate student discussion and provide a strong conclusion that supports reflection.

The final area for focus centered on the intended teacher outcomes; which included: believing that sims can transform learning and support conceptual understanding; engaging in student-centered instruction; addressing multiple learning goals in one lesson; and improving their knowledge of using technology to support math.

Recalling that a Theory of Change is just a theory of how the simulations would or could impact classrooms (and in this focused study, the instructional practices of the teacher), these design characteristics, design goals, teacher roles and teacher outcomes were then reflected against what was visualized during the observation process.

5.5.2 Interviews and Observations

As mentioned earlier, many of the design characteristics and goals, as well as the teacher roles and outcomes identified through the Theory of Change narrative became elucidated through the interviews and observations with the teachers. Below, I will highlight the most notable aspects of this enactment, using the structure of the Theory of Change narrative: Design Characteristics, Design Goals, Teacher Roles and Teacher Outcomes as they were interpreted through interviews and observations.

5.5.2.1 Physical Design Features

PhET simulations are designed with a number of features that are common to all simulations. Look below at Figures 5-27 and 5-28. It is important to note some of the characteristics of the simulations specifically designed to support students and teachers.

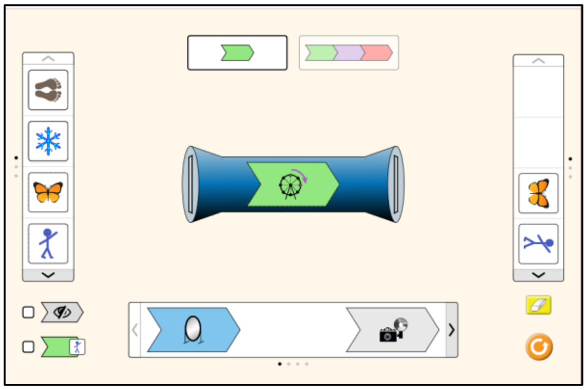


Figure 5-27 Function Builder

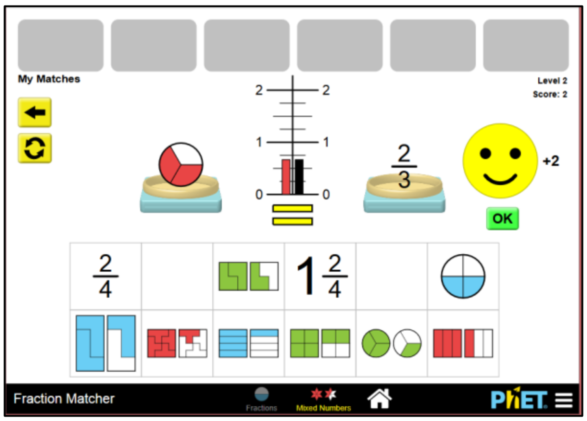


Figure 5-28 Fraction Matcher

Note the design similarities in both Function Builder (Figure 5-27) and Fraction Matcher (Figure 5-28). Neither simulation uses many words, yet both are colorful and designed to elicit a positive response from the viewer. Both of these simulations use realistic objects that students can move and manipulate. Both provide feedback, either (in the case of Function Builder) in being able to see the 'before and after' for the tiles as the move through the function tube, or in the case of Fraction Matcher, in the form of the number line, the equals sign, and the smiley face. Fraction Matcher also uses sound as a means of supporting students, in that it emits a chime or "bing" sound to let students know they are correct.

Consider two more images from simulations in Figures 5-29 and 5-30. In these two images, the ability of the simulation to immediately display and update multiple representations enables students to make connections between those representations.

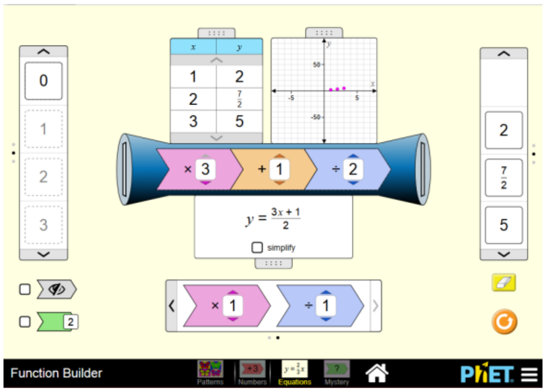


Figure 5-29 Function Builder - Connecting Representations

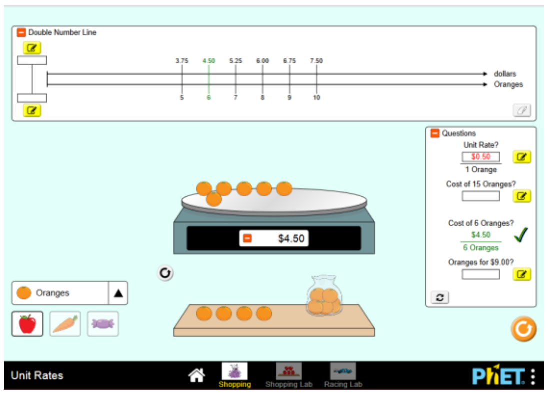


Figure 5-30 Unit Rates Simulation

In Function Builder (Figure 5-29), students see the input/output table, the graph with points from that table, and the matching equation. To the left is the range, to the right is the domain of the function. There are additional boxes that can be checked/unchecked to further student understanding. A gray box with an eye crossed out indicates that information will be hidden from view. This enables teachers to hide some information during a class discussion, for example. There is another feature in this simulation  that is designed to help students see 'behind the scenes' to understand what is happening as the number travels down the tube. Iconic representations, such as the eraser and the curved arrow are consistent in all simulations to facilitate ease of use when working between simulations.

In the Unit Rates Simulation (Figure 5-30 above), the same features can be seen. In this simulation, additional feedback features are available to support students. The incorrect answer (the unit rate of the oranges) is red, while the correct answer (cost of 6 oranges) is in green, and has a check mark next to it.

Each of these features provide some support for teachers to elicit specific instructional moves. For example, when Sara used the Fraction Matcher in her classroom, the 'bing' sounds coming from the devices supported her in knowing who was able to quickly find matches, and who was not. Therefore, it appeared that Sara was drawn to those students whose devices emitted fewer sounds. However, it is important to note that this feature of sound could also be misguided in that students who quickly clicked through a variety of choices could subsequently find matches (and have lots of 'bings' as a result), even if they did not fully understand why the two representations matched.

5.5.2.2 Design Characteristics.

The PhET simulations also have a set of simulation-specific design characteristics, which the Theory of Change document clarifies in the Explanation and Rationale statement. I have listed each of these characteristics, along with an explanation detailing the ways in which this characteristic played out through interviews and observations. In this section, I consider each of the design characteristics identified in the Theory of Change. While not all are specifically focused on teachers' instructional moves, these design characteristics play a critical role in the evaluation process teachers use to select activities for their classrooms.

5.5.2.2.1 Focus on important topics in math

The range of possible topics for middle grades mathematics is vast. PhET sims focus on topics that are central to the middle grades standards and curricula, connect with big ideas in *mathematics, and provide an important foundation for building future math skills. In contrast, the sims do not focus on obscure concepts, factoids, or math trivia.* (2017, p. 3)

Teachers did note in the summer planning lessons that these simulations addressed specific topics covered in their curriculum. The current simulations available (Function Builder, Fraction Matcher, Unit Rates, Proportion Playground and Expression Exchange) all focus on substantive topics covered in middle school in preparation for Algebra. Teachers did ask whether other simulations were available; the two topics most requested were ones to support an area model of multiplication and non-linear equations.

5.5.2.2.2 Focus on concepts (not procedures)

PhET sims are designed to highlight mathematical concepts. As such, visual set-up of the sims, the activities therein, are all geared toward emphasizing the target concept. Students do not engage in "drill and kill" or repeated procedural solutions to problems. (2017, p. 3)

PhET simulations were seen largely as supporting conceptual understanding; however, as most of the simulations had some sort of game or activity, there was also a way for students to engage in fluency-building activities as well. For example, Sara's lesson with Fraction Matcher was a skill-building activity, which lead to more conceptual thinking as the problems became more difficult. At lower levels, the problems were easy to solve, and students could quickly work through the game, becoming more fluent. As the level increased, the problems became more challenging, requiring students to not only match different representations, but also to find equivalent fractions, increasing the cognitive demand of the task.

In both Pat's and Lily's lessons using Graphing Lines, students were prompted to "try the game" if they finished early. See Figure 5-31 for a screenshot from this game.

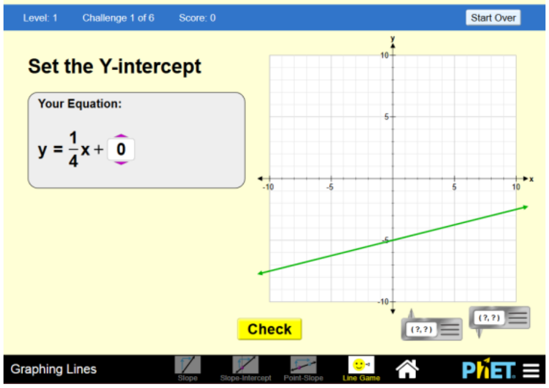


Figure 5-31 Graphing Lines Game

This game is designed to build fluency, having students find the equation of a line to support students' need for additional practice, beyond conceptual understanding.

<u>Be accessible and intuitive</u>

PhET sims are designed so that users can jump right in: the sims do not include written instructions within the sim, or indeed much writing at all. Instead, the layout and images are designed so that users can quickly grasp how the sim works and begin engaging with the sim right away. (2017, p. 3)

The ease of use and accessibility of the simulations was seen and described by all of the teachers. Students were able to easily access the simulation and begin using it. As noted in the explanation above, the intent is that students can readily use the simulation with minimal instructions. Despite that, teachers still spent a significant amount of time teaching students how to use the simulation. Teachers spent from ten to nearly thirty minutes describing to students how to use the simulation, often demonstrating its use in advance of students trying it on their own. Teachers also mentioned the need for students to learn how to use the simulation. Sara addresses this very specifically in this excerpt from her interview:

> *I feel like PhET lessons could last days, which is really cool because there are so many things that students can get out of them and you can use them a lot once they learn how to use the tool. Which I think is sometimes a hard part of technology is that they have to learn how to use the tool in order to be able to*

> *use it. And I think that [the PhET simulations are] pretty straight forward but once they learn the tool, then they can apply it so many different ways and they can build on the understanding that they've done in previous days, which is cool. (03/06/17)*

Despite the usability of the simulations, there remains a perception that students must learn the simulation, perhaps leading teachers to see the activity as taking longer to accommodate that need. Pat mentioned during one of her interviews (08/11/17) that she would have liked to use the simulation "as a student would." Adding a professional development component where teachers were able to try out the simulation might lead to teachers feeling more comfortable to having their students figure out the simulation, rather than having to be taught how to use it.

5.5.2.2.3 Display multiple representations

PhET sims include multiple representations, including numeric, algebraic, and pictographic representations; graphs; charts; diagrams; pictures, etc. The purpose is both to help students learn different mathematical representations individually and, equally important, to help students make conceptual connections between them. (2017, p. 3)

The PhET simulations do include multiple representations, which shifted the mathematical focus in many lessons to making mathematical connections. On the other hand, in Lily's lesson on Graphing Lines, where students used the simulation in a station, the dynamic responsiveness between the representations (meaning that students could move the line and look at the equation, or similarly move the equation and look at the line) possibly resulted in minimizing the connections made by students (See Figure 5-32 for an image of Graphing Lines). By that I mean that students were able to simply match the line to the worksheet, without the need to determine where the line should be on the graph.

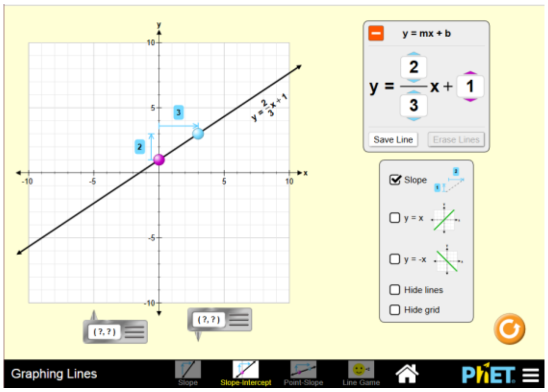


Figure 5-32 Graphing Lines

The dots on the line are color coded to match the toggle switches on the numbers in the equations to help support students conceptions of both representations. Yet, as one item moves, the corresponding item moves, reducing the need for students to figure out how the two colored objects connect.

In similar fashion, in one exchange by Sara and a student, described above in Section 5.4.5.2, Sara is talking with a student as she tries to find matches of fractions. Sara notes that the student is going quickly.

> *It would be that one. Yeah, and look, it's in between, on our number line, between one and two, okay? I want you to look at this one next. So, you're moving really quickly through these. What I want you to really think about is why they are matching, okay?*
>
> (Observation, Technology, 01/18/16)

When Sara says, "you're really moving quickly through these," the student is just clicking on different matches until one works and she sees a smiley face on the screen. It is Sara's instructional move that points out to the student that she needs to slow down and think about the connections.

5.5.2.2.4 Provide scaffolding for student thinking

PhET sims include supports for students to help them make sense of what they are seeing and doing. For example, the sims might include pictographic representations to help students understand an algebraic concept. (2017, p. 3)

The scaffolding for student thinking was only identified in one particular lesson: Christine's lesson using Function Builder. As a reminder, Christine had students create a function and then later share a portion of the function with another team. Using this 'hint,' the other team had to discover the original function. In other words, the first team creates a function like (times 2, subtract 3). The students then create an input/output table and send it to their partner team. Using only the input/output table, the new team must discover the original equation. While this could be easy using connections students have already made between tables and equations, the simulation gave the task a new, added challenge that Christine noted.

> *Yesterday, what I was noticing was the kids were using the function builder, creating these functions, then they were sending them off with certain clues. So either they send the table or they send the rule to another person, and what I was noticing yesterday was a lot of the more advanced kids, they would look at the table and they'd be like "Oh, I know the rule already." But when it came to actually building the rule in the function builder, it was trickier because, for example, some people came up with the rule Y = 4x + 0 which you can't build in the function builder unless you do x 2 x 2 + 0, or unless you do something more tricky. (Lesson Reflection, 11/19/16)*

*In Function Builder, toggle switches are used to enable students to increase or decrease the values (e.*g. students can toggle between -3,-2, -1, 0, 1, 2, and 3). The switches do limit the students, however, to values of -3 to 3. As a result, to multiply by a number greater than 3, the student would have to break the operation into two pieces as is shown in Figure 5-33. This demands a higher level of thinking by students.

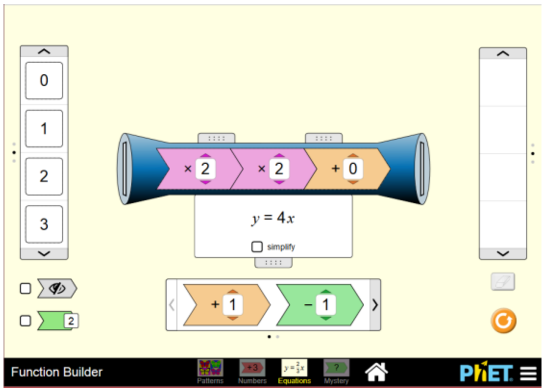


Figure 5-33 Function Builder - Multiply by 4

5.5.2.2.5 Incorporate supports for teacher facilitation

No examples noted.

No examples were provided by the PhET team for this design characteristic. However, there are some available supports online, providing teachers with guidelines of best practices when using simulations. These links were shared with teachers, and the key points were shared during our first meeting. Some of the simulations also include features that assist teachers in knowing students' progress without monitoring each students' screen. An example of this is the Fraction Matcher which provides a "bing" sound every time students answer a problem correctly.

No features that fit this category were mentioned by teachers as part of this research. Teachers did not refer back to, or discuss any of the support materials when discussing how they planned their lessons. When considering the sound feature, in Sara's lesson, which used the Fraction Matcher simulation, she did not ask students to turn down their sound, so to some degree this feature appeared to have supported Sara; yet again, it could also misdirect the teacher since students could achieve success by simply clicking on items until they found a match.

On the other hand, Lily does mention how another software product (Desmos) does support her, saying,

> *You just can't be everywhere at the same time. Ideally, having access to what every kid is doing is nice and that's one thing the Desmos Classroom does. You can see [what the students are doing]. That's nice.*
>
> (Personal Interview, 08/23/16)

Therefore, it may be that teachers have an idea of what teacher supports would look like, and did not see features that they felt supported their teaching choices. On the other hand, teachers did mention that they would have liked training on the simulations, and a chance to use these simulations in a manner that was similar to their students.

5.5.2.2.6 Provide connections to real life (when desirable)

PhET sims often use real-world contexts to situate the sim. These scenarios help make the math more concrete. They can also make content more relevant for students, in turn supporting student interest and motivation. (2017, p. 3)

This is a feature that was often mentioned by the teachers. Having a real-world context (such as the fruit on the scale in Unit Rates, the necklace in Proportion Playground, and the pie pieces in Fraction Matcher) was an important consideration when choosing an activity, and everyone noted the contexts available in PhET simulations.

One important additional consideration that was raised but not specifically identified in the Theory of Change document was the notion of using the simulation itself becoming a shared memory for the class, which could later be referenced. Sara brings this to light this during her reflection of her Fraction Matcher lesson, when I asked if she ever referred back to a simulation after they used it.

> *Yes. Especially in our math clinic class, when we were talking about fractions and we were talking about parts versus whole and pictorial models, definitely was something that we referred back to cause we all had that shared understanding of it. So yeah we did and it's something we've all done together and we've experienced it. (Lesson Reflection, 03/06/17)*

Not only are their connections to real-world contexts within the simulation, but it is also a shared real-world experience in using the simulation itself, which prompts teachers to reference it in later lessons.

5.5.2.2.7 Employ an interaction/feedback design

> *PhET sims are designed to be interactive: when a user does something within the sim (manipulates something, changes a parameter, moves an object, etc.) the sim responds accordingly. Through this interaction and feedback, users can explore the sim and develop an increasingly deeper understanding of the mathematics at work.* (2017, p. 4)

Each of the simulations has some means of providing feedback to the student. In Function Builder, students see the immediate consequence of their choice of function. Note in Figure 5-34, if students change a function without clearing their inputs/outputs, the outputs automatically change.

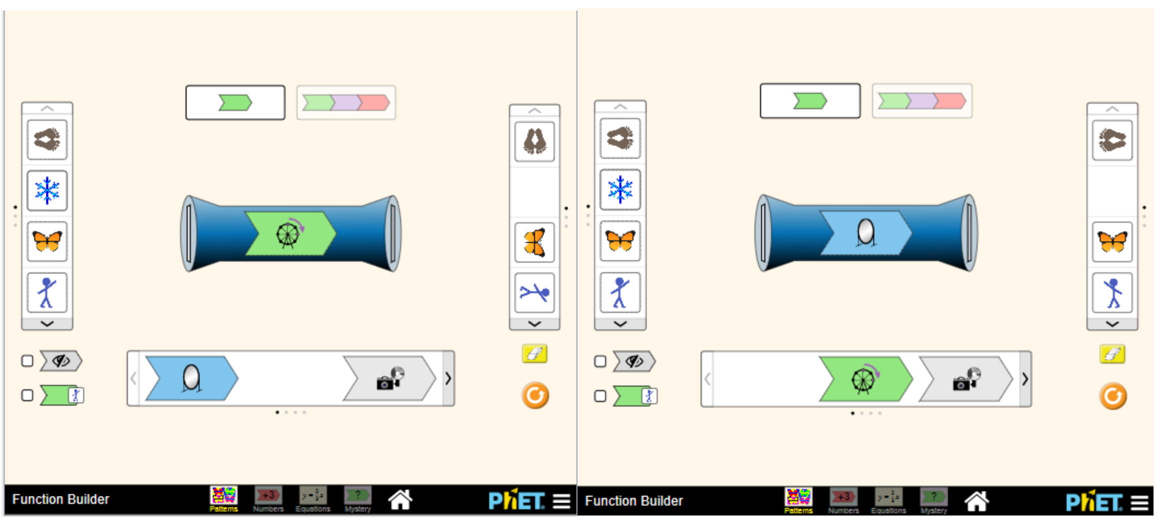


Figure 5-34 Function Builder - A change in function automatically changes the output.

Other feedback features, as noted earlier, include the ability of the simulation to identify that an answer is correct. Some simulations, such as Fraction Matcher, do this through sounds, or visual rewards such as a chime sound or a smiley face. In other cases, such as Proportion

Playground (see Figure 5-35 Proportion Playground Feedback Clues), the feedback in given in terms of arrows on a number line.

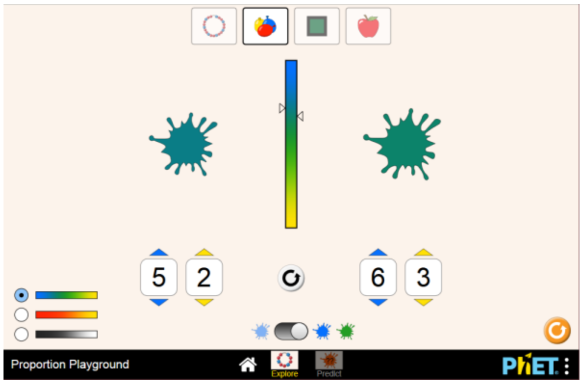


Figure 5-35 Proportion Playground Feedback Clues

In this case, the proportions are not the same. Students have two ways to recognize their mistake. First, the paint splotches are different colors. Second, the arrows on both sides of the color lines are at different points.

5.5.2.2.8 Provide an exploratory and game-like environment

PhET sims include activities that encourage exploration; many have built-in challenges to motivate the user to continue exploring or to complete certain tasks. (2017, p. 4)

While teachers did not explicitly refer to these features either in interviews or reflections, they were seen during observations. Each of the simulations offered features that 'rewarded' students for correct answers (either through green check marks, smiley faces, or sounds). Students were often observed playing with these features, even when it went beyond their assignment. While this observation does not specifically address teachers' instructional moves in line with the research questions, it does play into teachers' selection of materials in that teachers were looking for activities that were fun and engaging (See section 5.3.1.2 for more discussion on this.).

5.5.2.3 Design Goals

The PhET Theory of Change document suggests that the above Design Characteristics will result in the following Design Goals:

> *Simulations…*
>
> - *Are engaging, fun*
> - *Make math relevant for students*
> - *Enable student choice and student agency*
> - *Enable students to engage in discovery, exploration, and sense-making*
> - *Focus student attention on the important aspects of the content*
> - *Help make invisible things (e.g., abstract concepts, math conventions, invisible symbols) visible for students*
> - *Provide opportunities for rich discussion*
> - *Support differentiation for both students and teachers* (Singleton & Shear, 2017, p. 5)

Many of these goals are focused on the students, themselves and do not play a role in teachers' instructional practices. The last two, however, provide significant opportunities to direct teachers' instructional practices and are therefore discussed below.

5.5.2.3.1 Provide opportunities for rich discussion

One of the specific goals of PhET simulations is to provide opportunities for rich discussion. The enactment of this was very mixed with the teachers in this study. For Sara, who regularly opened spaces for discussion in her classroom in non-technology lessons, creating spaces for discussion seemed to be similarly enacted in her technology lessons, and resulted in slightly more time spent asking questions. As a result, there were smaller changes in the level and content of her discussions. For Lily and Pat, using a PhET simulation appeared to move the focus of discussion from procedural steps to conceptual understanding. This may be in part because there were not shared problems to be solved, and therefore, the conversations had to move to focus on what students were seeing, rather than a determination of whether an answer was correct. Christine was perhaps the outlier in this goal, as she regularly provided spaces for students to have rich discussions in her non-technology lessons; yet, her focus on providing instructions left her little time to enact these same types of discussion techniques in her technology based lessons.

On the other hand, Christine, Sara, and Lily all chose to write at least one lesson that promoted group work. As a result, students had to work together and talk throughout the activity. As a result, for these teachers, student to student interactions were strong.

5.5.2.3.2 Support differentiation for both students and teachers

The second goal in the PhET simulations directly related to teachers' instructional practices is the ability of the simulation to support differentiation. This is discussed earlier in Section 5.3.1.3, where teachers noted the benefits of simulations having a "low floor, high ceiling." Simulations supported students at a variety of levels, and provided additional challenges for those who were ready.

While this was seen as a benefit by teachers, observations did not reveal any instructional moves by teachers that were designed to make use of this feature. For example, while teachers might have decided to prompt specific students to try more challenging aspects of the simulation as a means of differentiation, this was not noted in any observation.

5.5.2.4 Teacher Roles

The Theory of Change suggests that teachers must enact the following roles for simulations to be most effective and to drive student roles; however, it notes that, "Teachers and students in real classrooms may or may not bear out these roles; oftentimes they may fulfill some roles but not others" (2017, p. 7). These roles include:

- *Use the sim as a central and integral part of the lesson*
- *Give students opportunities to explore and play*
- *Give students opportunities to share and discuss*
- *Listen to student ideas and allow students to drive their learning*
- *Facilitate discussion and thinking, bringing attention to salient math points*

- *Conclude the lesson by making ideas explicit*

As expected, teachers took on these roles to various degrees. Each role is discussed in more detail below.

5.5.2.4.1 Use the sim as a central and integral part of the lesson

As noted earlier, in Section 5.3.2, technology can prove challenging in the mathematics classroom, particularly when teachers and students to not have ready access to devices. As a result, many of the teachers noted that a lesson had to be sufficiently supportive of student learning to make the use of technology worthwhile. Furthermore, most of the teachers noted that due to the time involved in getting access to the technology, they usually planned for technology lessons to use the full class period. As Pat noted, when discussing why she uses the technology for the whole lesson,

> *You know the time it takes to get [the Chromebooks] out, and to get logged in, and to get everybody [ready to go], and then somebody can't log in. It usually ends up taking the most of the period. Then, getting them put back in the cart right, and making sure I have them all. I would think usually it would be the whole period. To use [the Chromebooks] for 10 or 15 minutes would only be if every kid just came in with their device, and they were logged in immediately. The in and out of the cart and all that takes time.* (Personal Interview, 08/11/16)

This likely explains why Lily is the only one who used the PhET sim as a small portion of the lesson (in her stations) rather than the main lesson, as she is the only teacher who has ready, available access to student devices.

As a result, while the rest of the lessons did centralize the PhET simulation, this is likely an artificial artifact that results from teachers' desire to make the most out of using technology.

5.5.2.4.2 Give students opportunities to explore and play

Each of the teachers built in some amount of time to explore with the PhET simulations, although all but Lily provided some direct instruction as to how to use the simulation. This again, may be more related to the use of technology than an intentional teacher practice. Lily's students are older (being high school freshmen) and have their own device. Furthermore, Lily has block periods, providing her with a perception of more lesson time.

5.5.2.4.3 Give students opportunities to share and discuss

Again, this enactment varied by teacher. Christine did this minimally in terms of a focused class discussion, but did provide significant opportunities for student to student discussions through the design of the lesson. Sara provided multiple opportunities for students to discuss the lesson as a class, but did not have many opportunities for students to work in groups. Lily and Pat were somewhere between these two, opening up spaces for discussions both between students and as a class.

5.5.2.4.4 Listen to student ideas and allow students to drive their learning

When teachers brought simulations into the classroom as technology based mathematical activities, one of the notable teacher practices was engaging in discussion. For classes that paired students on devices, this discussion was between students, and less observable to the teacher. On the other hand, there was significant evidence of teachers engaging with the students, asking questions about what the students noticed, and then engaging further to draw out their conceptions.

Furthermore, the use of simulations shifted the class to be more student-centered, enabling students to bring in their own prior knowledge and ideas to influence the direction of the task. Sara noted this in one of her lesson reflections:

> *I also think that reflecting back on it, when I used [the simulation] with one of my math clinic classes, I had a definite plan with it. But the way that my students took it was totally different [than what I planned], which was awesome. And I didn't realize that they had so many skills in some areas, but yet they lacked skills in other areas. So that was really cool for me.* (Lesson Reflection, 01/18/16)

For Sara, she saw this shift to student-driven learning as students worked with the simulation in ways that were different than what Sara had planned. Therefore, the simulations open spaces for students to significantly impact the enactment of the lesson.

5.5.2.4.5 Bringing attention to salient points and making ideas explicit.

The final two teacher roles, Facilitate discussion and thinking, bringing attention to salient math points, and Conclude the lesson by making ideas explicit, are both ways that teachers assist students in focusing on the most important parts of the lesson. Teachers accomplished both of these goals (bringing attention to salient points, and making ideas explicit) in a variety of ways. In Christine's lesson, she used a quick reflection where students answered a question in their learning logs. In Pat's lesson, she used a brief end of class discussion to ensure students understood the critical features of equations that helped determine the number of points of intersection. Lily and Sara used a review of the worksheet answers at the end of one of their lessons, and an exit ticket for the other lesson. Therefore, all of the teachers made some efforts to take on this role of making ideas explicit.

5.5.2.5 Teacher Outcomes

The final set of criteria to consider are the intended teacher outcomes. These are not specifically related to instructional moves, but may prompt teachers to enact more specific instructional practices as a result of these outcomes. It is difficult to make any assertions at this point as to teacher outcomes as first, only one or two simulation based lessons were observed, and second, many of the outcomes reference the teachers using PhET designed lessons, which was not the case in this study. It is impossible to observe a teachers' beliefs, yet it may be possible to gain some insight to teachers' beliefs when drawing from the teachers' statements in their interviews. For each teacher outcome, I have included the explanation and rationale directly from the PhET Theory of Change narrative, and then provided existing supporting evidence.

5.5.2.5.1 Believe that sims can transform learning and support conceptual understanding

> *PhET believes that their materials, combined with the experience of seeing PhET sims at work in their own classrooms, will show teachers the value of sims for teaching and learning. Seeing is believing.* (2017, p. 9)

This outcome is difficult to see after just a few technology based lessons. Teachers have indicated through their reflections that using simulations was enjoyable for themselves and their students, and they would consider using these simulations again in the future. Teachers also specifically mentioned that they liked the simulations because the simulations supported conceptual understanding.

5.5.2.5.2 Engage in innovative and student-centered instruction

> *As described above, the PhET teacher resources and sims can support varied teaching styles. That said, the resources highlight student-centered pedagogies, and the sims and sim-based lessons are conducive to those same approaches. PhET hopes that teachers who use PhET resources and sims will find themselves engaging in innovative student-centered instruction.* (2017, p. 9)

In working together to design lessons over the summer, the teachers did work to design lessons that were student-centered, which was seen in the observations of their lessons as well.in other lessons.

5.5.2.5.3 Have the ability to address multiple learning goals (e.g., content, process, argumentation, enjoyment) within a single lesson

> *The PhET sims provide opportunities for teachers to address different kinds of goals within a single lesson. For example, the focus on conceptual understanding and the interaction-feedback design provide inherent opportunities for teachers to question students about what they think is going on and why. Teachers who skillfully capitalize on these learning opportunities can integrate multiple learning goals into a single lesson.* (2017, p. 10)

This outcome matches the overall goals that teachers hoped to achieve in their lesson selection process. As explained in Section 5.3.3.2, teachers were looking for activities that met the needs of their students, effectively and efficiently, and in a way that supported student engagement. Therefore, while it would be hard to argue that using the PhET simulations produced this outcome, this outcome is in line with teachers' goals more generally, and would be supported by the teachers in this study.

5.5.2.5.4 Improve their knowledge of using technology to teach math

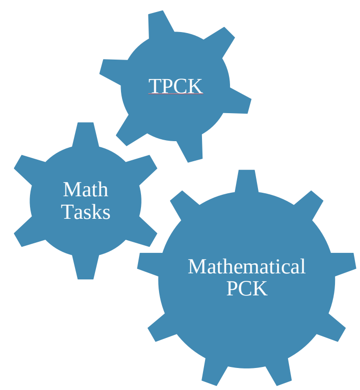
Both the guidance from the teacher resources and the experience & practice of teaching with sims will help teachers develop their skills for using technology to teach math. (2017, p. 10)

This last outcome is difficult to assess in a relatively small study. These types of outcomes would be expected after a longer use of PhET simulations than have been enacted here. However, there is at least a small piece of evidence that indicates that teachers' practices will change with more experience. Christine's first lesson with a PhET simulation had her spending a large amount of time focusing on giving students instructions. While watching her second lesson, the time spent giving instructions decreased almost by half. More study would be needed to see if these changes continued over time, but it is possible that experience did make a difference in her enactment of lessons.

6 CHAPTER 6: CONCLUSION

In this final section, I draw conclusions using the results from this study to answer the two research questions:

- What observed instructional moves by teachers in math classrooms are unique to technology-based activities created with student-oriented simulations?
- In what ways do specific design features of student-oriented simulations impact instructional moves in math classrooms?



As noted earlier in the discussion of the conceptual framework used to support this research, my study brings three components into play asserting that teachers' instructional practices are determined by teachers' mathematical pedagogical content knowledge as well as the developed and enacted task, and are mediated by the use of technology. As such, I considered the ways the tasks were enacted by teachers, and how those enactments were impacted by the beliefs and practices of those teachers, as well as by the design features of the simulations. Later, I discuss the implications of this research for teacher education, and provide recommendations for further study.

6.1 SUMMARY OF RESULTS

6.1.1 Research Question 1

Results for Research Question 1, concerning the ways in which teacher practices changed with the inclusion of technology based mathematical activities found some commonalities among all four teachers studied. From a task selection standpoint, teachers understood and articulated the benefits of using technology, and specifically PhET simulations in their classrooms, including the ability for the simulations to support rich mathematical ideas, use realistic worlds to promote engagement, offer flexibility to make the simulations accessible for a range of students, and use dynamic representations to support a concrete representation of abstract ideas. In recognizing these benefits, there were clear connections later to the resulting instructional practices.

For most (although not all) teachers, the amount of time spent in rich mathematical discussions increased significantly, prompting students to move from answering direct questions requiring simple responses to higher levels of thinking and reasoning. The exception to this was Christine, who spent a much longer time than other teachers providing instructions on how to use the technology (not only the simulation, but also using Google Docs to create screen shots, which was a new skill for the students). These changes in instructional practices directly relate to teachers' conceptions of the simulation as a means for promoting discussion. Furthermore, teachers' inclination that these tasks would support connecting representations was accurate as well, as the use of simulations resulted in a significant shift in the area of mathematical focus, moving from generalization of patterns, to linking representations.

When considering teachers' mathematical knowledge of teaching, and pedagogical content knowledge, teachers looked for activities that supported their students, were memorable and engaging, and taught in the most efficient and efficacious manner. When using PhET simulations in the classroom, most of the teachers spent less time in direct instruction, spent more time focused on the mathematics, spent more time on investigations rather than drill-oriented tasks and had classrooms demonstrating increased student engagement.

The areas where teachers struggled to support students centered on their technological pedagogical content knowledge. As mentioned earlier, the use of technology increased the amount of time Christine spent giving (and restating) the instructions. For her class, this attention to instructions resulted in less mathematical talk, and a larger focus on procedural steps. Furthermore, the very nature of students working independently with their own devices in several lessons meant that student-student interactions significantly decreased. Therefore, if teachers believe that rich mathematical discussions will promote student understanding, then teachers must orchestrate moments for those discussions within their lessons.

6.1.2 Research Question 2

Research Question 2 then considered the ways in which the simulation impacted teachers' instructional practices. Of the stated design goals from the Theory of Change document, two of goals significantly came to light in the use of the simulations in classrooms: *Provides opportunities for discussions*; and *Supports Differentiation*. These goals were made visible through specific instructional practices that were observable in their classrooms. Teachers used the simulations to prompt discussions and discuss reasoning. Furthermore, the openness of the simulations facilitated differentiation by students.

These design goals led to the design characteristics which were similarly visible in the classroom, including:

- Focus on important topics in math
- Focus on concepts (not procedures)
- Be accessible and intuitive
- Display multiple representations
- Provide scaffolding for student thinking
- Provide connections to real life (when desirable)
- Employ an interaction/feedback design
- Provide an exploratory and game-like environment.

The one feature that was not as evident was: *Incorporate supports for teacher facilitation*. Recall Sara's comments about trying to create a new lesson.

> *Sometimes it just happens, and you have this brainstorm and you can plan several lessons around something, but sometimes it's a little bit harder to do. […] If I have an inspiration, then it's much easier*
>
> *When I look at Function Builder, the first [screen] was patterns. I'm like, I wish I had this last year for all the transformations and all that. Then, I was looking the other [scenes with numbers and equations], and it wasn't as immediately obvious to me [how I'd incorporate it into a lesson]. I'd have to*

> *play around with it a little bit more.* (Summer Lesson Planning Session, 07/24/16)

Teachers felt that the simulations are so open and broad that without some "inspiration," it was challenging to know where to begin. Furthermore, teachers also had to determine where the lesson would best fit in their curriculum. Both of these challenges caused teachers to reconsider their use. This challenge is likely in part because the simulations are new, and there are limited resources available for teachers.

The Theory of Change document suggests that teachers will take on the following roles when using simulations:

- Use the sim as a central and integral part of the lesson
- Give students opportunities to explore and play
- Give students opportunities to share and discuss
- Listen to student ideas and allow students to drive their learning
- Facilitate discussion and thinking, bringing attention to salient math points
- Conclude the lesson by making ideas explicit.

In the lessons observed as part of this research project, only the first role, *Use the sim as a central and integral part of the lesson,* was evident with all four teachers, although it was not clear that the teachers were using the simulation as a central and integral part of the lesson due to the value of the simulation, as much as due to their concerns about the time and effort involved in using technology, and wanting to maximize students' time.

The second role, "*Give students opportunities to explore and play*" was not taken up fully by the teachers. Only two teachers gave students time to explore with the simulation (Lily and Christine). Sara demonstrated the simulation using her projector, while Pat had her students jump right in to the activity. It is important to note that both Lily and Christine had a majority of students with their own devices, limiting the amount of time needed to gain access to the technology. This may account for the difference in lesson styles.

The remaining four roles were all associated with supporting rich mathematical discussions. While the amount of time spent in mathematical talk (either as a whole class, or student-student) did increase for most classrooms, teachers did not take up this role as significantly as was conjectured.

Finally, considering teacher outcomes, the project's Theory of Change noted that the use of simulations was theorized to create the following outcomes for teachers: *Believe that sims can transform learning and support conceptual understanding*; *Engage in innovative and student-centered instruction*; *Have the ability to address multiple learning goals (e.g., content, process, argumentation, enjoyment) within a single lesson*; and *Improve their knowledge of using technology to teach math*. I argue that some of these outcomes were clearly visible. Teachers definitely noted that the simulations supported conceptual understanding. For the teachers, that was a key reason for using the simulations in the first place. On the other hand, there remained this tension about the value of new activities when compared against those activities that teachers had already used. As a result, teachers did not talk about the process of using simulations as being transformational in their classrooms.

The second outcome, a move towards student centered instruction was evident in all four classrooms. This was a direct result of the design of the simulation which placed the student central to the activity. Teachers also felt that using the simulation met all of their goals mentioned earlier, most notably a memorable and engaging activity that would meet the needs of their students in a way that was both efficacious and efficient.

It is difficult to make any assertions that teachers improve their knowledge of using technology to teach math due to the small number of participants and the limited use of simulations. However, Christine's first lesson with simulations had many discrepancies in her instructional practices when compared to her non-technology based lessons, especially in the amount of time she spent giving instructions. By the second time she had used a simulation, these differences were significantly smaller. This may indicate that this outcome is realistic for Christine, but more study would be needed.

6.1.3 Unintended Consequences of Design Features

When considering the goals and intended outcomes of the simulation, it is important to also consider unintended consequences of these design decisions.

6.1.3.1 Tool vs. Activity

For the first example, I would argue that the very nature of simulations being designed to support conceptual knowledge rather than act as a tool that replaces pencil and paper could result in a lower use by teachers. Technology can play a large role in the classroom, and all of the teachers talked about both the value of using technology, as well as the types of technology found in their classrooms. Yet despite the value teachers saw in using technology, there was a tension that impacted how the simulation was used. It is easier to bring a tool into a classroom to support an existing curriculum, than to bring in a new activity, and determine where it will best fit to meet the needs of the students. As Lily noted, “If the textbook were to have interactive activities, like let's say the textbook was online and it had a simulation that went with it, I would probably use it more” Personal Interview (08/17/16). Teachers were also aware of the potential pitfalls of using technology, from the lack of access to the possibilities that students may be distracted with ready access to the internet. Several teachers also mentioned that technology was sometimes simply a replacement for pencil and paper work.

This vision of technology as a tool, rather than an activity, was supported by teachers when the tool reduced the tediousness of setting up problems such as graphing points or creating a table of data. Several teachers noted that the available simulations (Function Builder, Proportion Playground and Unit Rates) would fit into the curriculum in a number of places, and could be used for a variety of lessons, rather than just one. This indicated that the teachers saw simulations as an activity, with value in supporting the curricular goals. The tension arose, then, when teachers had to determine exactly where these simulations would fit into their curriculum, and how they would be structured. This is very different from many of the other software tools used by teachers in their classrooms. Desmos, for example, has standalone activities, but these activities were used less than simply using Desmos as a tool for graphing. When teachers used GeoGebra (in particular, Pat and Christine), both noted that GeoGebra was used for a large portion of one unit. When asked more about it, I learned that the district had created lesson plans that directly correlated with the unit of the book. In both of these situations, the use of these tools differs from simulations in that these tools were used to support existing activities rather than used to create new activities.

Restated differently, PhET simulations are explicitly designed to be open ended, and with the right teacher supports, simulations can enhance students' conceptual understanding of a mathematical concept. Yet the concept of interest is at the discretion of the teacher. Take Function Builder as an example. A teacher could choose to use the first screen both as an introduction to functions, or as a tool to discussion transformations of shapes (i.e. rotations, reflections and dilations). Students who simply explore the tool may get some understanding on one or both of these objectives, but it is through the design of specific activities, discussion points, and questioning that strong conceptual understanding is supported.

6.1.3.2 Barriers to Technology

As a second concern, I argue that despite seeing value in using simulations, the online nature of the simulations makes teachers reluctant to use them. This study confirmed some of the barriers to using technology found in prior research, such as insufficient access to technology (either because it was not available, or was poorly organized); lack of confidence; insufficient support for planning (both in access to materials, as well as time); and the potential for technical issues causing teacher fear (Jones, 2004). As noted by other researchers (Ertmer et al., 1999), the shared nature of technology resources such as carts of laptops, Chromebooks or tablets means that teachers struggle not only to determine where and when to include a new resources such as a PhET simulation into the curriculum, but also the teacher must make arrangements to gain access to the technology. For the four teachers studied, the access to technology varied greatly. For Lily, each student had been given an iPad, since they were enrolled in a one-to-one school. Despite this ready access, other problems came into play such as a student forgetting his/her iPad or not having it charged. Christine's class was a blend of access in that all eighth graders had their own Chromebook, but seventh graders did not. For Christine, that meant that she still needed to coordinate access, albeit needing fewer devices. Both Sara and Pat had to gain access to a Chromebook cart. For Sara, it was a bit easier in that the cart resided in her room, enabling her to use the cart on occasions even when she had not signed up for it. Pat's cart did not reside in her classroom, so she had to make arrangements to both sign up for the cart, and send students to bring it to the classroom.

Additionally, while it was never an issue when I observed, all of the teachers noted the possibly of the internet going down and the problems that would cause. For PhET simulations, an app is available to put all of the simulations on the iPad. In the case of three of the four teachers, this was not a viable solution since iPads were not used in their classrooms. Moreover, since the app costs $0.99 per download, this may be cost prohibitive for some teachers depending on the number of tablets. When I raised purchasing the app as a possibility with Lily, she thought that the cost was reasonable and something that could get funded through the school or district. She noted that having the app would reduce the concern about internet accessibility and would be more likely to use the simulations in class. Even so, as that only provides a solution to schools that have chosen iPads over Chromebooks, it is the very accessibility of the simulations that similarly caused angst for the teachers in this study. Therefore, despite different ways and means of access to computers, all teachers found that the access to the technology regularly caused some problems. Every teacher worried about what she would do if the technology did not work as expected. As a result, teachers gave serious consideration to the ways in which technology was be a significant improvement over comparable hands-on activities prior to implementing the simulations.

6.1.3.3 Game versus Rich Mathematical Task

The third possible unintentional consequence to the design is the decision to make the simulation visually engaging and fun. Each of the simulations are wonderfully colorful, with designs intended to catch students' attention. Yet it may be this very design that 'colors' teachers' impressions of simulations as well. Because these simulations are colorful and exciting to look at, they take on a feeling of being more game-like. The simulations are intentionally designed to be seen as fun. As a result, teachers, and subsequently students, talk about the simulations in terms of the word "game." Teachers made comments as they introduced the simulation to the students, describing the simulation as "It's kind of a game, just explore" (Christine, Technical Observation 1, 10/06/16) or when describing the activity to me, saying, "It's totally a game to them" (Sara, Lesson Reflection, 11/18/16).

Why is this an issue? Shouldn't we want these types of programs to be visually engaging and entertaining? Of course. However, here is the concern. Of the seven technology lessons taught to students, four of the lessons were taught on the day before a break (e.g., the day before Thanksgiving break). As noted by Larry Cuban, years ago, television and films were used by teachers as a way to break up the day, or deal with students' energy loss.

> *They need a breather for a short time, in order to launch another lesson; or they need filler material that will plug gaps in text; where they simply need interesting programs that will entertain for a few minutes. (1986, p. 69)*

If teachers perceive simulations in a similar manner, being a fun way to break up the routine of math classrooms, then they may miss out of the full value of what simulations have to offer. Taken even further, what does that mean for how students perceive the use of simulations? Going back to the movie analogy, a movie was a reward, which was perceived by students as something special that replaced the real work of school, even if the movie was directly related to what was being taught. Simulations could take on that same role for students, as a fun way to 'escape' the real work of mathematics.

6.1.3.4 Summary

These three concerns (i.e., that teachers may be more reluctant to use the simulation because it requires a full lesson, rather that acting as a tool that supports an existing lesson; that the online nature of the tool makes it widely available yet brings a host of other concerns for the teacher; and that the view of the simulations as games may relegate its use in ways that minimize its value) are not intended to dispute the value of simulations. Instead, these issues are elevated as concerns because they may unintentionally limit student access. Based on the other benefits of simulations seen in this study (the increased time for discussion, and the move towards connecting representations), more research is needed to determine if there are ways to minimize these potential concerns.

6.2 Revised conceptual model

As additional technology tools are designed and brought to the market, more attention is needed, focusing on how these tools will be used in classrooms. Stein et al (1998) bring important structures of mathematical tasks to the forefront, recognizing that tasks differ from what was designed to what was enacted as a result of both teacher and student actions.

Yet, their diagram does not fully capture the complexity of the issue when the mathematical task includes a technology component. When technology is being added as part (or all) of the mathematical task, there are additional considerations that play into the design and enactment of the task. The original diagram from Stein et al (1998) has been modified as shown in Figure 6-1 to include extra factors which influence technology-based mathematical tasks.

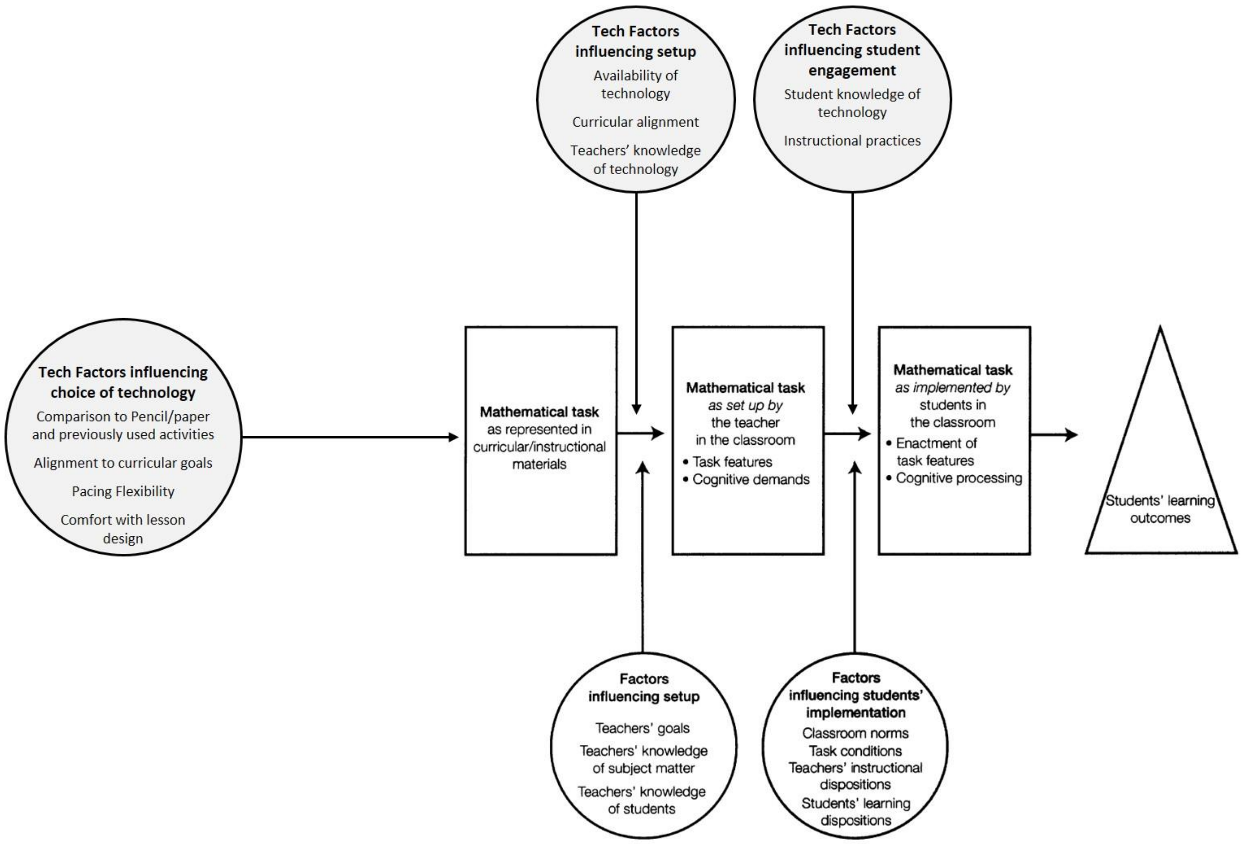


Figure 6-1 Revised diagram reflecting factors which influence the design and enactment of tech-based mathematical tasks

6.2.1.1 Tech Factors influencing the choice of technology

The original model begins at the point where teachers have a task in hand, either one that has been found or one they created. For technology-based lessons, there is an earlier step that must be considered: the ways in which teachers choose to use (or forgo) technology. From the four teachers studied, I found when teachers learned of any new activity, there was always a comparison to tasks that had previously been used. For technology-based lessons, this comparison was even more explicit. Teachers considered whether this new activity would provide benefits over traditional pencil/paper methods. Teachers also noted that they compared the proposed activity to their prior tasks, asking themselves if this new activity was indeed better (more engaging, more efficient, and more efficacious) than the prior (typically non-technology) task. The next step was a consideration of how this activity or tool would fit into the classroom curriculum, and align with the pacing guide. For technology-based lessons using simulations, teachers determined where the simulation would support the curricular goals, and identified the existing task which would then be replaced with the simulation. This last step is very important – due to the high number of standards to be taught, teachers see their time teaching as a zero-sum game. If the teachers add in a new activity, they must subsequently remove another task.

Ultimately, if teachers find that the new activity, such as a simulation, is appropriate for the classroom and their students and can fit into the curriculum, the final step in the process is determining how the technology will be used. As noted earlier, when technology is used as a tool to replace pencil and paper (what Pea (1985) refers to as an amplifier), it is relatively easy to fit this into existing curricular plans, without the need to totally revamp the tasks. On the other hand, when technology is used as a rich, mathematical task (or a reorganizer (Pea, 1985)), lesson plans providing significant opportunities for students to engage with and discuss mathematics must be developed. Therefore, when considering a technology-based lesson, teachers must also consider the time and effort needed to create a specific task that meets their curricular goals and fits into their available timing.

6.2.1.2 Tech Factors influencing setup

There are also some ways in which the use of technology can influence the setup of the task that differ from non-technology based lessons. As noted earlier, one of the most limiting conditions is the availability of the technology itself. Teachers must ensure that the needed technology will be available on the day that the lesson is most appropriately covered. As much state testing has now moved to online testing, availability of computers provides an added challenge. For example, topics covered in late March, or April are likely to be done using more traditional pencil/paper activities since most school computers are assigned to students completing the annual tests. Furthermore, many classrooms still share technology, meaning that teachers must schedule access on specific days, limiting the number of students who have opportunities to use these tasks. As a result, this interplay between when technology is available, and the alignment of the lesson within the curricular plan influences the ways in which teachers are able to insert a new lesson into their plans.

Furthermore, because the new technology-based lesson may not directly align with the replaced lesson (that is to say, the simulation lesson may cover important concepts but not fully cover everything that would have been taught during the replaced lesson in the book) teachers also must determine how future lessons will be adjusted to accommodate the new task.

Teachers' own familiarity with technology also significantly impacts the set-up of tasks in the classroom. As seen through these four case studies, teachers' concerns about learning the software led them to provide significant directions on its use. Furthermore, teachers' lack of comfort in using the technology may limit students' opportunities to explore without constraints.

6.2.1.3 Tech Factors influencing student use

The final part of the diagram to be considered reflects the technology factors that influence student use of the mathematical task. Here, factors include student knowledge of the technology, instructional practices of the teacher, and they ways that these two components impact student engagement with the technology. Students' knowledge of technology, generally how savvy they are with technology, and more explicitly, their comfort with the specific technology being used, can significantly impact the lesson. Many teachers felt that despite the features designed to support student use of the simulations, it was still important to teach students how to use the simulation. As students became more familiar with the simulations (such as Lily's class who used the same simulation twice), the need to 'teach' the simulation subsequently declined. Yet the time spent ensuring students know how to use the simulations can significantly impact the time students spend engaging with the mathematics.

Furthermore, teachers' own instructional practices, the ways in which they were able to promote discussions, focus students' attention on salient features, and support students in exploring the mathematics could also significantly impact the ways in which students engage with the lessons. As noted earlier in the summaries of the lessons, the simulations provided students with a variety of different problems, enabling the teacher to move away from focusing on whether the student found the correct answer, and instead move to richer discussions about the mathematics. By pairing students up, and giving tasks that required interactions between students, teachers were also able to promote more student-student mathematical discussions. As a result, students' own comfort with the technology, along with teachers' instructional practices to support student interaction combine to support student engagement with the task.

6.2.1.4 Summary

I assert that there are significant differences between using non-technology based mathematical tasks and technology-based mathematical tasks in the classroom. The current model proposed by Stein et al captures important reasons why tasks vary from what was designed to what was enacted, but fails to capture the additional idiosyncrasies that come from adding technology to mathematical tasks. This revised model provides some insight into the ways in which the use of technology can influence the design and enactment of the tasks, and should be considered in future research endeavors.

6.3 IMPLICATIONS FOR TEACHER EDUCATION

This study has significant implications for teacher education as Schools of Education work to find ways to support preservice and in-service teachers' use of technology in mathematics classrooms. As I theorized early on in the paper, the use of technology in mathematics classrooms is not simply a function of knowing the available tools and how to use them. It is also incumbent on us to ensure that these teachers know more generally how technology can be used to support students' conceptions of mathematical ideas. Going back to the original technology framework that considers the degree to which technology impacts the classroom, this research informs educators about ways that we can support teachers in moving from *using* technology to *infusing* it in their classroom.

While the sample size of teachers is small, several consistent trends were noticed that could be addressed with future teachers. We need to find ways to support teachers in not only teaching their current curriculum, but in finding ways to open that curriculum to support supplemental materials as they are discovered. The teachers in this study struggled to determine where and when simulations would best fit in their lessons.

This is particularly interesting as the simulations used are part of a whole suite of simulations designed by the PhET team. Most of these simulations are science-based, and are used millions of times each year by teachers all around the world. The PhET website has "has recorded over 90 million simulation runs to date with over 35 million simulations run in 2012" (Hensberry, Moore, Paul, Podolefsky, & Perkins, 2013, p. 149). So this leads to the question: why are simulations widely used by the science community and yet found to be challenging by math teachers.

From this research, I believe that curricular materials play a large role in this difference. Science textbooks typically open spaces for teachers to add lab activities, or other investigative tasks as part of the unit. Math textbooks, on the other hand, typically work on a day-to-day structure, where each day is assigned its own lesson. This is problematic in that simulations are intentionally designed to be used broadly, covering a number of standards based on the task's focus. Therefore, when teachers choose to bring in a simulation, it must replace an existing lesson. Furthermore, since the simulations are designed to support conceptual understanding, but not designed to replace specific lessons, teachers must determine which aspects of a lesson will be fulfilled by the simulation, and which must be covered using the original lesson. As teachers felt that technology was best used as a whole lesson experience (rather than using it for 15-20 minutes and then returning to the lesson), this puts teachers 'off-cycle' the next day. By that, I mean that teachers skip a lesson to use a simulation, and the next day, rather than moving right on to the next day's lesson, they must first 'catch-up' on any missed activity from the skipped lesson as those skills are likely needed for the next day's lesson. It becomes this whirlwind of where do I fit this in, how do I do this, which lesson will I skip, what day do I teach this and how do I get the devices?

As a result, perhaps it is time we rethink our teaching of mathematics and the development of our textbooks, moving away from a daily lesson, to a more cohesive set of "big picture" ideas that allows for the inclusion of multiple activities to support conceptual knowledge, dynamically responsive connections between representations, and rich, engaging mathematical discussions. This may mean redesigning textbooks to support conceptual understanding of key ideas as well as practice for fluency, while opening spaces for students to take on more investigative experiences.

Additionally, even without this major textbook overhaul, teachers need additional support to find ways to open spaces for discussions without feeling rushed to finish the activity. Again, this is part of the move from *using* technology to *infusing* it in the classroom. Math textbooks currently provide a detailed plan for the teacher, noting the day's task, as well as suggesting stopping points and questions to ask the students. For example, in a typical lesson from the Connected Math series used by Christine, Sara and Pat, the teacher's manual provides a set of questions to pose to the students as well as details stopping points throughout the lesson to host these discussions. Conversely, when using student-centered technology-based tasks such as PhET simulations, teachers have no choice but to create lessons and add in their own stopping points and questions.

This skill, of designing and writing lessons, is challenging; for teachers may be a daunting task. If teachers don't have open days in their curriculum to easily fit in new activities, then instead, they must be supported in designing their own lessons. This means learning to create technology-based tasks in ways that support student engagement. Going even further, well-planned curricular materials, aligned to current pacing plans, with designed stopping points for discussion would be an excellent way to address this concern.

6.4 IMPLICATIONS FOR FUTURE RESEARCH

This study was an exploratory study to better understand the range of teachers' practices and the potential areas for exploration. There is strong evidence that the use of simulations as a form of technology-based mathematical activities can lead to classrooms supporting rich mathematical discussions and student centered investigations focused particularly on making connections between representations. More research is needed, however, to understand how to better support teachers in their practices. Most notably, a study with an increasingly diverse cohort of teachers would help to determine if these same instructional practices would be evident in a broader context of school and teachers.

6.4.1 The role of curriculum in technology-based lessons

Several potential areas of interest were also raised throughout this study. Despite the evidence that simulations could support students in a variety of ways within the classroom, there were some potential reasons why teachers might still limit the use of simulations. Teachers noted that using simulations was challenging in that there was clarity as to when and how the simulation would fit into the overall curriculum. With the strong concerns raised as to the development and inclusion of curricular materials, further research that features premade curricular materials could determine ways in these resources might provide support to teachers to more fully embrace the use of simulations in classrooms.

6.4.2 The role of simulations in the classroom

Other areas of interest are centered on supporting teachers' Technological Pedagogical Content Knowledge. Investigating further how and when teachers use simulations, especially in light of their view of simulations as games is an important area to explore. As mentioned earlier, seeing simulations as games is not necessarily problematic, however, it may limit the use of simulations, restricting opportunities for students. It also appeared that teachers were more likely to use technology seen as tools, rather than activities. This was evident not only in their discussions about the simulations, but also in the ways that the teachers described their use of other technology-based products such as Desmos. More research is needed to understand how the design of simulations as either tools or activities impacts their use within the classroom.

6.4.3 The role of professional development

Since teachers often used activities they learned while attending a conference, professional development, or similar event, I would argue that additional training on using rich student-centered technology lessons such as simulations could support teachers in two ways. First, spending time with the simulation, and using the simulation in ways that would mimic the students' experience may increase confidence by the teachers and may also minimize teachers' concerns about its use in the classroom. Furthermore, professional development that supports teachers in finding spaces to open up discussions may be helpful as well. More research is needed to understand the impact of providing additional training and support to teachers prior to using simulations in the classroom.

6.4.4 New methodological approach

From a methodological standpoint, this research has brought to light a means of coding technology-based lessons that maintains the focus on the richness of the mathematics, while accounting for the use of technology in ways that have not previously been accomplished. This methodological approach is teacher-centered, and focuses on differences in practices, rather than an evaluative assessment of the teacher. In this research, the methodological approach provided insight into teachers' practices, but was limited by differences in each teachers' classrooms; however, this same approach could be even more effective for a larger scale study where curricular choices were more controlled. For example, as noted earlier, it appears that providing additional professional development, as well as clear curricular supports such that teachers know when and how to replace an existing lesson with a simulation-based lesson would support teachers in bringing in technology to the classroom. Using this new methodological approach, would provide a systematic means of identifying changes within the classrooms in a scalable manner.

6.5 FINAL COMMENTS

This study contributes to research on ways in which teachers' instructional practices can change when moving from non-technology based lesson to technology based mathematical activities. Using the original framework proposed early on in this study, I asserted that teachers' instructional practices were determined by teachers' mathematical pedagogical content knowledge as well as the developed and enacted task, and were mediated by the use of technology. Teachers have a wealth of activities from which to choose, and use their mathematical pedagogical content knowledge to consider any potential new activity in light of the goals of the classroom and the needs of the student. There appears to be an unspoken belief by the teachers that just because technology may be perceived as being 'cool, flashy and exciting' by parents, students and even society, it doesn't mean that using technology is the best solution for their curricular needs. Furthermore, in light of the challenges brought by technology, technology based activities may be less effective than the previously used hands-on activities. Therefore, teachers look critically at newly available tasks to determine their appropriateness.

Additionally, the enactment of that task, once chosen, was influenced most by teachers' instructional moves towards opening spaces for discussions and reflection. As teachers work to override the challenges of using technology in their classrooms, they strive to make every minute count in their lessons. This leaves some teachers to perhaps spend more time than is necessary in setting up a lesson. Yet the very nature of the technology offering different problems to different students removes the tendency to check for answers, and instead invites a deeper questioning to take its place.

Teachers saw significant benefits in using the simulations, as the design features of the simulation supported conceptual understanding, and increased student engagement. Furthermore, simulations provided that 'low floor, high ceiling' needed to support a diverse classroom of students. Some of the features of the simulation left teachers a bit on the fence. Teachers saw real benefits to the dynamic responsiveness of the simulations and the ways these simulations made connections between representations. On the other hand, teachers raised concerns that some features of the simulation could do the math for the students. Furthermore, the perception of simulations as being a game may impact how and when they are used.

Additional studies will be needed to better understand teachers' instructional moves when using technology in mathematics classrooms. As was seen in this study, teachers struggled with how and when to use a simulation. It will be important to consider if simulation use changes if teachers are provided with pre-made curricular materials, already aligned with their curriculum, as well as whether premade curricular materials can prompt teachers to continue to open up more spaces for discussion and sharing.

If teachers' instructional practices are determined by teachers' mathematical pedagogical content knowledge as well as the developed and enacted task, and are mediated by the use of technology, then it is the development of instructional practices that complement the use of technology, which will support teachers as they move from *technology use* to *technology infusion*. Bringing simulations in the classroom is certainly one piece of the puzzle. With simulations, the technology provided a means of supporting conceptual understanding, providing an experience that would be difficult, if not impossible without the tool. As Sara noted, the simulation also becomes a shared experience, one that can persist in the classroom, even after the activity is completed. Additional training and curricular materials may be the supports teachers need to continue to build practices that support rich mathematical discussions.

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8 APPENDIX A: SCREEN SHOTS OF PhET SIMULATIONS

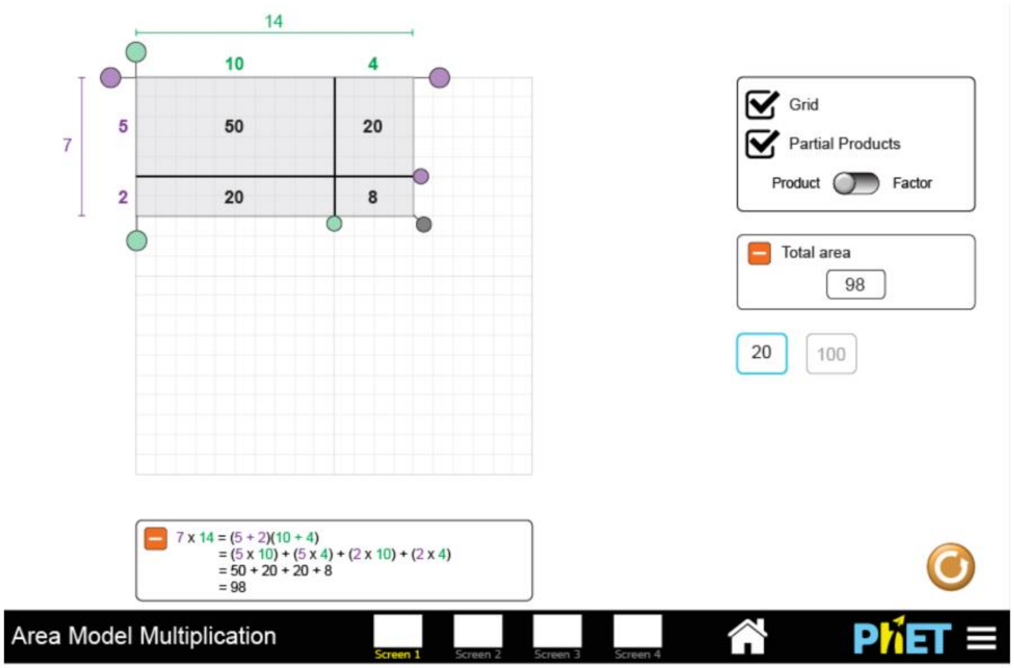


Figure 8-1 Area Model Simulation: Students can learn to multiply through the use of partial products in preparation for multiplying binomials in algebra.

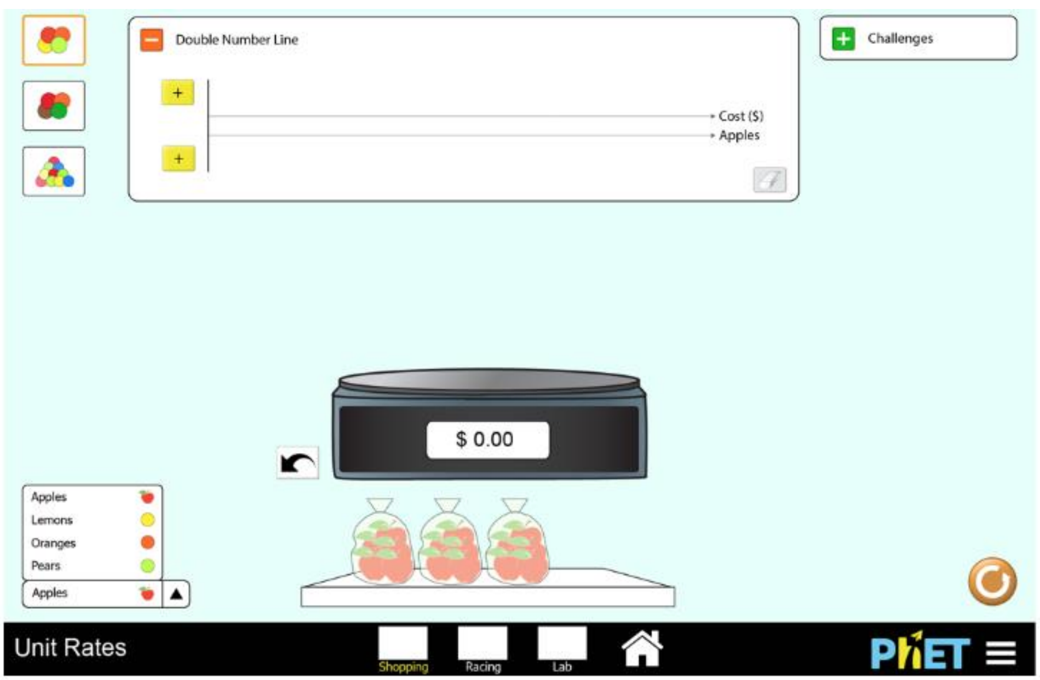


Figure 8-2 Unit Rate Simulation: Students learn to use a double number line to calculate unit rates of fruits and vegetables

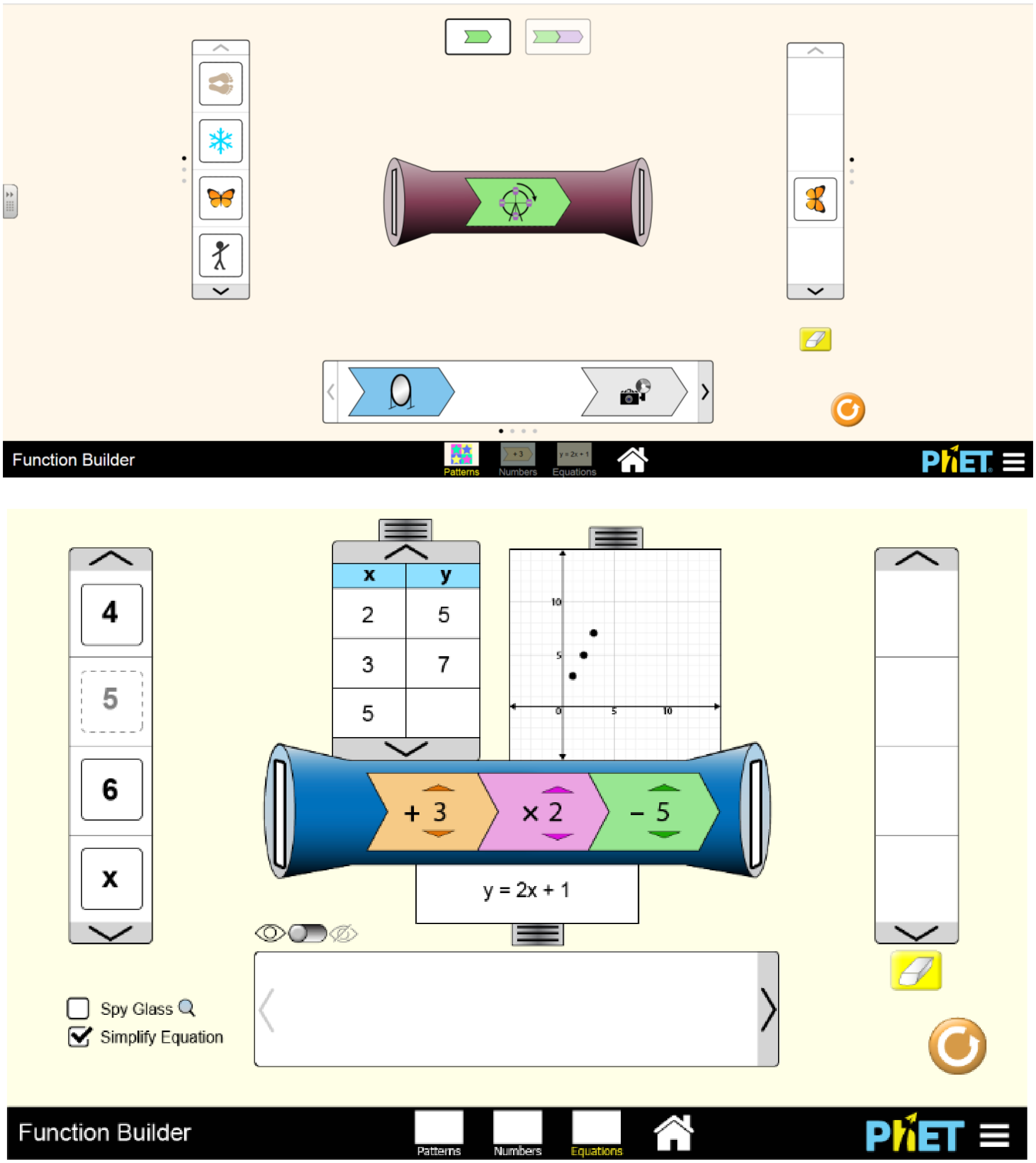


Figure 8-3 Function Builder Simulation: Students can 'see' inside a function builder machine, first with patterns, and later with equations

9 APPENDIX B: SCREEN SHOTS OF BENDING LIGHT PHET SIMULATION

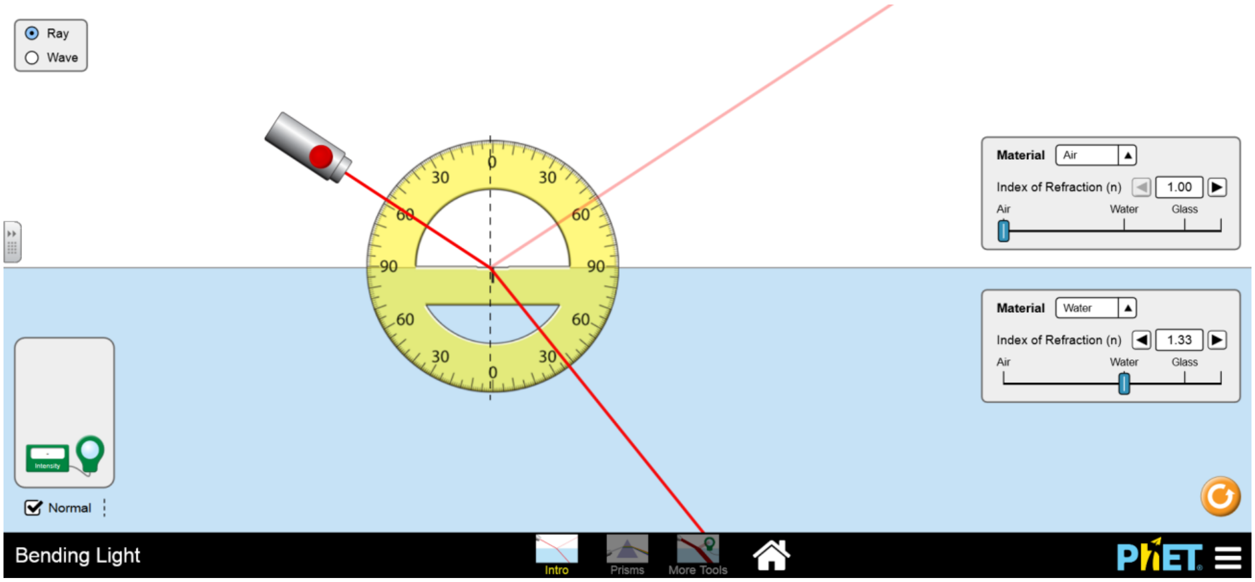


Figure 9-1 Bending Light Simulation: Note the use of the protractor

The full simulation can be viewed here:

http://phet.colorado.edu/sims/html/bending-light/latest/bending-light_en.html

10 APPENDIX C: INTERVIEW PROTOCOL

About the teacher:

- What degree do you have?
- How long have you been teaching?
- How long have you been teaching at this school?
- Tell me about your students in the class I'll be observing.
  - In what ways does student backgrounds (including gender, race, language) play a role in planning and enacting lessons
  - What do you know about how technically savvy they are?
- How long is each class period?
- How often do you meet with them?
- How is class placement determined for each student?

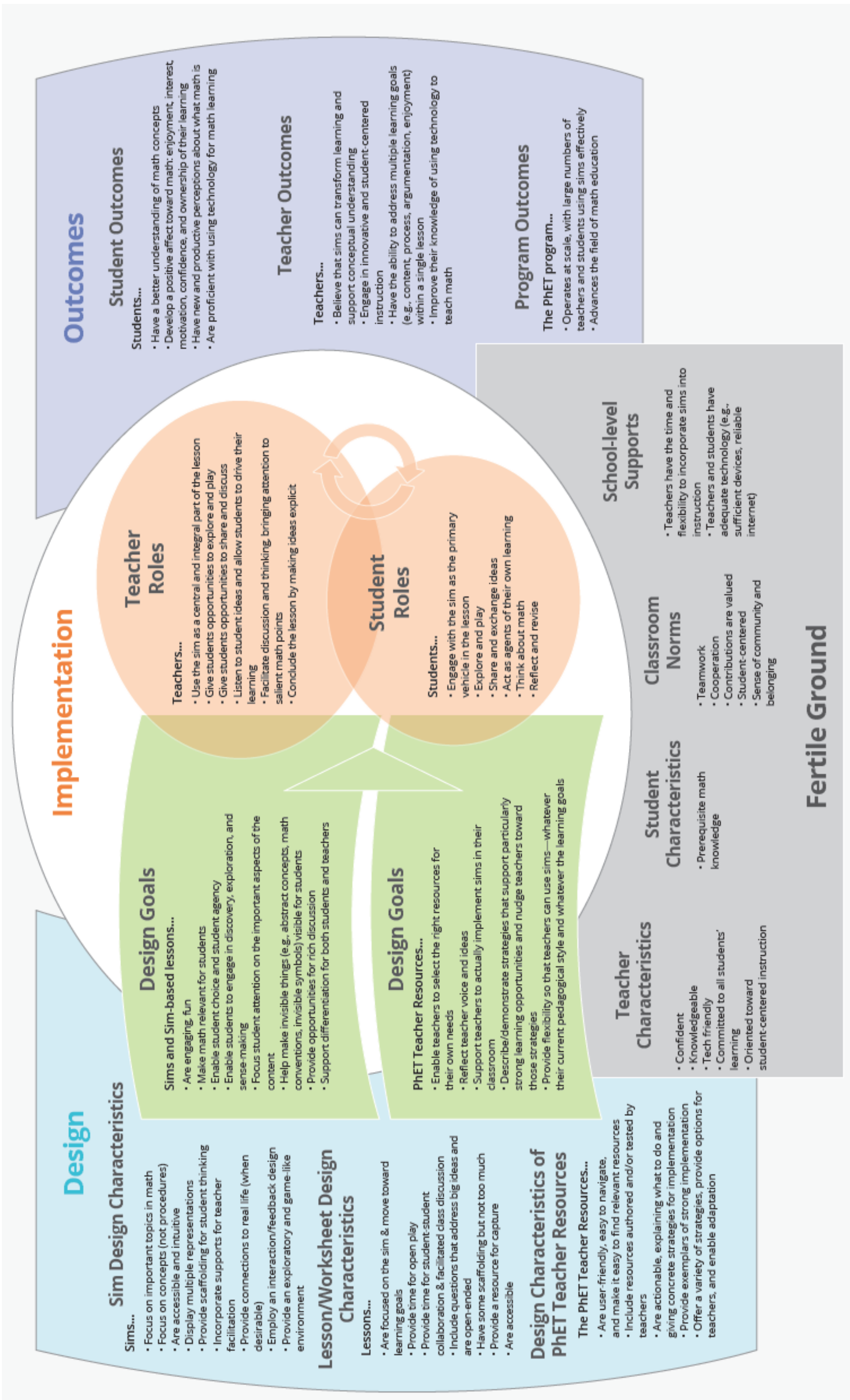
About the content:

- What classes are you teaching this year?
- How many times have you taught this class?
  - What is your comfort level with this material?
- What curriculum informs your planning? What textbook do you use?
- In what ways is it effective? (or not?)
- In what ways do you modify it?
- How do you plan for upcoming lessons?
- What are the most essential components for a good math class?
  - How do you promote those components?

About the technology:

- Can you give me examples of ways *you* have used technology in the past?
- Can you give me examples of ways *your students* used technology in your class in the past?
- In what ways does technology support student learning? Detract from student learning?
- What challenges have you had integrating technology into your classroom?
- What technology is available in your classroom?
- Is there other technology outside of the classroom available to you?
  - If so, what type(s)?
  - What is the procedure for accessing that technology?
- Does adding technology to a math lesson influence your teaching? In what ways?
  - Probe for details

11 APPENDIX D: THEORY OF CHANGE DIAGRAM



12 APPENDIX E: PhET THEORY OF CHANGE (DRAFT)

PhET Middle-Grades Math Sims

Narrative Description of the Theory of Change Model

February 2017

Introduction

During the latter half of 2016, the PhET team, in collaboration with SRI International, created a Theory of Change to describe the PhET middle-grades math sims and how they work to support math learning. This document accompanies the Theory of Change diagram. While the diagram can and does stand on its own, this document provides additional narrative to further describe and explain the model.

The purpose of the Theory of Change model and accompanying narrative is to thoroughly yet succinctly articulate what the middle grades math sims are, what they do, how they work and under what conditions, and what outcomes they aim to achieve. This kind of articulate vision serves many purposes:

- ***Shared vision:*** Having a clearly articulated model helps to ensure that everyone involved with the project is operating under the same assumptions and working toward the same ends. PhET may also choose to share the Theory of Change with external partners or funders, to help communicate their work.
- ***Guide for Development:*** The Theory of Change provides a framework for informing design decisions and checking products. For example, when selecting a topic for the next middle-grades math sim, developers should be sure that the topic is important within middle grades math curricula and connected to big ideas in mathematics. When working on sim development, designers can check back to the list of design characteristics and design goals to make sure their product meets those descriptions.
- ***Guide for Research:*** The Theory of Change tells us what is important to the math sims program and as such can focus and constrain any research on the sims. The model can be used to help identify salient research questions to study, components of implementation to explore more deeply, and outcomes of interest to measure.

The PhET team drew on their extensive experience with science sims and on research on teaching and learning to articulate this Theory of Change for their middle-grades math sims. The design characteristics and design goals aim to produce a powerful learning experience for students, characterized by guided exploration of key math concepts. The program offers teachers substantial autonomy in how to implement the sims and recognizes that the student experience is mediated by diverse contextual factors including the teacher. While keeping the door open for all kinds of teachers and students to use the sims, the PhET team also hopes to encourage and support teachers to use certain pedagogical strategies that they believe can maximize the potential of sims to produce the desired outcomes.

We note that the shift from science sims to math science is not inconsequential. PhET has extensive experience developing, teaching with, and researching science sims and their use in the classroom to support teaching and learning. While many of the same principles for developing and implementing science sims apply to math sims as well, there are some important distinctions. Notably, the shift from science to math represents a shift from concrete to abstract: while sims in both areas strive to make concepts visible for students, doing so within math sims requires an extra step to transform abstract concepts into something concrete that students can see and manipulate. This important distinction between science and math is one that the PhET team attends to closely in the design of their middle grades math sims.

Further, we note that the Theory of Change describes the vision for how the PhET middle grades math sims operate to support student learning. The linkages between design characteristics, design goals, implementation, and outcomes reflect research-based thinking and rationales—but they have not yet been rigorously tested in the context of PhET math sims specifically.

The Theory of Change model is divided into four main sections: Design, Implementation, Outcomes, and Fertile Ground. We "read" the diagram roughly from left to right, starting with design characteristics of each of the components of the PhET offering—and moving into implementation and outcomes. The "Fertile Ground" section anchors the diagram, enumerating contextual factors that provide an environment conducive to the effective use of sims. The diagram also shows Design Goals, which bridge the gap between Design Characteristics and Implementation. We will discuss each of these sections and sub-sections in turn.

Design

The Design section of the Theory of Change diagram describes the intended concrete design characteristics for each of the categories of PhET resources: the sims themselves, the lessons and lesson worksheets for teachers to use in their classrooms with students, and a broader selection of teacher resources designed to support teachers with using PhET sims as part of their instruction. The design characteristics refer to specific, tangible characteristics of each resource: these are traits or qualities that a designer would intentionally build into each resource—and that a user would see or experience directly. These intended design characteristics do not describe goals for what the resources will accomplish.

PhET Sims Design Characteristics

The centerpiece of any PhET sim lesson, of course, is the sim itself. A set of design characteristics drives the development of each sim, with the goal of producing sims that are grounded in a coherent vision for how to support math learning. The table below lists the design characteristics that sim developers hope will be evident in the sims they create, and provides an explanation and rationale for each.

Table 1: PhET Sims Design Characteristics

Sims are designed to…	**Explanation & Rationale**
Focus on important topics in math	The range of possible topics for middle grades mathematics is vast. PhET sims focus on topics that are central to the middle grades standards and curricula, connect with big ideas in mathematics, and provide an important foundation for building future math skills. In contrast, the sims do not focus on obscure concepts, factoids, or math trivia.
Focus on concepts (not procedures)	PhET sims are designed to highlight mathematical concepts. As such, visual set-up of the sims, the activities therein, are all geared toward emphasizing the target concept. Students do not engage in "drill and kill" or repeated procedural solutions to problems.
Be accessible and intuitive	PhET sims are designed so that users can jump right in: the sims do not include written instructions within the sim, or indeed much writing at all. Instead, the layout and images are designed so that users can quickly grasp how the sim works and begin engaging with the sim right away.
Display multiple representations	PhET sims include multiple representations, including numeric, algebraic, and pictographic representations; graphs; charts; diagrams; pictures, etc. The purpose is both to help students learn different mathematical representations individually and, equally important, to help students make conceptual connections between them.
Provide scaffolding for student thinking	PhET sims include supports for students to help them make sense of what they are seeing and doing. For example, the sims might include pictographic representations to help students understand an algebraic concept. [other examples of common types of scaffolding within sims?]

Incorporate supports for teacher facilitation	[Need example]
Provide connections to real life (when desirable)	PhET sims often use real-world contexts to situate the sim. These scenarios help make the math more concrete. They can also make content more relevant for students, in turn supporting student interest and motivation. [Would be helpful to add a note about when and why these connections might not be desired]
Employ an interaction/feedback design	PhET sims are designed to be interactive: when a user does something within the sim (manipulates something, changes a parameter, moves an object, etc.) the sim responds accordingly. Through this interaction and feedback, users can explore the sim and develop an increasingly deeper understanding of the mathematics at work.
Provide an exploratory and game-like environment	PhET sims include activities that encourage exploration; many have built-in challenges to motivate the user to continue exploring or to complete certain tasks.

Lesson & Worksheet Design Characteristics

Typically in the classroom, teachers use sims as part of a lesson, oftentimes in conjunction with a worksheet that guides students through their experience with the sim. Based on both experience and research, the PhET team has articulated a set of design characteristics for sim lessons/worksheets that support effective implementation of the sim.

We note that PhET encourages broad use of their sims: teachers can use PhET sims in a wide variety of ways, with or without accompanying lesson materials provided by PhET. The lesson/worksheet design characteristics described here refer first to the lessons/worksheets that PhET creates; and provide suggestions for others who are using sims or creating lesson materials.

Table 2: Sim Lesson/Worksheet Design Characteristics

Sim lessons/worksheets are designed to…	**Explanation & Rationale**
Focus on the sim and move toward learning goals	Using the sim as the centerpiece of the lesson, rather than a tangential or add-on experience, maximizes the opportunity for students to develop a deep understanding of the sim and the math that it explores.
Provide time for open play	Time for open-ended exploration with the sim, particularly at the start of the lesson, enables students to figure out how the sim works and begin making conjectures about the mathematics at play.
Provide time for student-student collaboration & facilitated discussion	Teachers should facilitate discussion to elicit student thinking and draw student attention to salient math points.
Include questions that address big ideas and are open-ended	Open-ended questions that address big ideas provide opportunities for students to reason about important math concepts, to build conjectures, make arguments about their thinking, etc.
Have some scaffolding but not too much	Lessons/worksheets should have enough scaffolding to keep students focused on the important aspects of the math, without eliminating their opportunities to think and reason on their own.
Provide a resource for capture	Having a place to record their work helps students think through the math and remember what they have done.
Be accessible	Students should be able to grasp what the worksheet is asking of them easily, so that they can focus their cognitive efforts on the mathematics.

Design Goals of Sims and Sim-based Lessons

While the design characteristics refer to concrete attributes that designers seek to build into the sims and sim-based lessons and worksheets, the design goals refer to the purpose of those characteristics. That is, the design goals answer the question: What kind of experience do the PhET sims and sim-based lessons aim to build for students? The design characteristics do not, for the most part, have a one-to-one relationship with the design goals. Instead, the design characteristics collectively lead into the design goals to support positive and productive experiences for students.

The Design Goals state that sims and sim-based lessons:

- Are engaging, fun
- Make math relevant for students
- Enable student choice and student agency
- Enable students to engage in discovery, exploration, and sense-making
- Focus student attention on the important aspects of the content
- Help make invisible things (e.g., abstract concepts, math conventions, invisible symbols) visible for students
- Provide opportunities for rich discussion
- Support differentiation for both students and teachers

Design Characteristics & Design Goals of PhET Teacher Resources

In addition to providing simulations and accompanying lesson worksheets, PhET also provides resources for teachers to support the use of PhET sims in the classroom. For example, PhET provides information both about strategies for using PhET sims in general and about integrating specific sims into their lessons. The resources include information, instructional videos, exemplar videos, etc. and are available on the PhET website.

The PhET teacher resources also adhere to a set of concrete design characteristics intended to ensure quality and intentionality across the resources. In the case of the teacher resources, the design characteristics (concrete attributes of the resources) and the design goals (the purpose of those attributes as far as supporting the user experience) do map directly one to the other.

Table 3: Mapping PhET Teacher Resources Design Characteristics to Goals

Design Characteristic		Design Goal
Are user-friendly, easy to navigate, and make it easy to find relevant resources	→	Enable teachers to select the right resources for their own needs
Include resources authored and/or tested by teachers	→	Reflect teacher voice and ideas
Are actionable, explaining what to do and giving concrete strategies for implementation	→	Support teachers to actually implement sims in their classroom
Provide exemplars of strong implementation	→	Describe/demonstrate strategies that support particularly strong learning opportunities and nudge teachers toward those strategies
Offer a variety of strategies, provide options for teachers, and enable adaptation	→	Provide flexibility so that teachers can use sims—whatever their current pedagogical style and whatever the learning goals

The first two design characteristics and design goals refer primarily to the PhET website. PhET works to optimize the website, for example, by tagging resources by subject, topic area, and grade-level, so that teachers can quickly find resources that meet their needs.

The last three design characteristics and associated design goals refer primarily to the individual online resources. These resources are intended to support teachers with integrating PhET sims effectively into their practice. PhET is sensitive about providing resources that open doors for as many teachers as possible, to make it easy and appealing for teachers of all types to make use of the sims. At the same time, PhET does have insights about pedagogical styles and practices that can maximize the potential of the sims for student learning. The resources are designed to delicately balance these two positions: they encourage uptake and enable adaptation so that teachers can use them within their own context and to meet their own needs; they also provide guidance and exemplars to share and gently encourage best practices.

Implementation

The PhET sims, sim lessons and worksheets, and the teacher resources are designed to work together to support effective implementation of the PhET sims in middle grades mathematics classrooms. While the sims and accompanying materials & resources can be used in many contexts and with a variety of pedagogical styles, the design of these products nonetheless lends itself to a particular model of implementation. The Theory of Change diagram describes this model of implementation, focusing on what it means for teacher and student roles in the classroom.

Teacher & Student Roles

The teacher and student roles during a PhET sim-based lesson derive from the design characteristics & goals of the materials, relate directly to one another, and drive toward the desired student learning outcomes. As was the case with the design components of the Theory of Change model, the implementation section similarly describes ideal teacher and student roles, as envisioned by the PhET team. Teachers and students in real classrooms may or may not bear out these roles; oftentimes they may fulfill some roles but not others.

Table 4: Teacher and Student Roles

Teacher Roles		Student Roles
Use the sim as a central and integral part of the lesson	→	Engage with the sim as the primary vehicle in the lesson
Give students opportunities to explore and play	→	Explore and play

Give students opportunities to share and discuss	Share and exchange ideas
Listen to student ideas and allow students to drive their learning	Act as agents of their own learning
Facilitate discussion and thinking, bringing attention to salient math points	Think about math
Conclude the lesson by making ideas explicit	Reflect and revise

Outcomes

The core purpose of the PhET middle grades math sims is to help students learn math, particularly to help students gain a deeper understanding of math concepts. Beyond this central goal, the PhET sims also aim to achieve other beneficial outcomes for students and teachers alike—and for the PhET program and the field of math education at large.

Student Outcomes

The PhET middle grades math sims aim to support a variety of academic and non-cognitive outcomes for students.

Table 5: Student Outcomes

Through using PhET sims, students…	**Explanation & Rationale**
Have a better understanding of math concepts	The fundamental purpose of PhET sims is to support student learning, particularly conceptual understanding

Develop a positive affect toward math: enjoyment, interest, motivation, confidence, and ownership of their learning	PhET understands that productive dispositions toward learning in general and toward mathematics in particular are valuable for supporting academic success. The PhET vision is that students need not have these dispositions already in order to be successful with the sims. Rather, the sims can help students develop or strengthen a positive affect toward mathematics. The sims do this through many of the qualities described in the design section, including their game-like environment, giving students some autonomy, scaffolding their understanding and success, etc.
Have a new and productive perception about what math is	PhET hopes that students will come to understand that math provides a whole new way of thinking about the world, and that engaging with math allows you to think, explore, and develop ideas about how the world works. The hope is that seeing math in these terms, instead of as a series of facts to be memorized, gives math more relevance and interest for students.
Are proficient with using technology for math learning	PhET sims are a technology-based resource for math learning and, as such, working with sims should help students develop skills with using technology to support their learning. These skills are valuable for success both in school and in the wider world.

Teacher Outcomes

PhET expects that teaching with PhET sims will have certain benefits for teachers as well as for students. Teachers can learn both from teacher resources and from the experience of teaching with PhET sims. The teacher resources are designed to provide direct information and guidance for teachers to help them understand how sims work and how to use them in the classroom. The experience of teaching with PhET sims will in turn give teachers new insights about whether and how sims can support student learning. The teacher outcomes articulated in the

Theory of Change represent the outcomes that PhET hopes will occur when teachers use their resources and teach with sims in the classroom.

Table 6: Teacher Outcomes

Through using PhET teacher resources and PhET sims, teachers….	**Explanation & Rationale**
Believe that sims can transform learning and support conceptual understanding	PhET believes that their materials, combined with the experience of seeing PhET sims at work in their own classrooms, will show teachers the value of sims for teaching and learning. Seeing is believing.
Engage in innovative and student-centered instruction	As described above, the PhET teacher resources and sims can support varied teaching styles. That said, the resources highlight student-centered pedagogies, and the sims and sim-based lessons are conducive to those same approaches. PhET hopes that teachers who use PhET resources and sims will find themselves engaging in innovative student-centered instruction.
Have the ability to address multiple learning goals (e.g., content, process, argumentation, enjoyment) within a single lesson	The PhET sims provide opportunities for teachers to address different kinds of goals within a single lesson. For example, the focus on conceptual understanding and the interaction-feedback design provide inherent opportunities for teachers to question students about what they think is going on and why. Teachers who skillfully capitalize on these learning opportunities can integrate multiple learning goals into a single lesson.
Improve their knowledge of using technology to teach math	Both the guidance from the teacher resources and the experience & practice of teaching with sims will help teachers develop their skills for using technology to teach math.

Program Outcomes

In addition to supporting positive outcomes for students and teachers, PhET cites two key goals for the PhET middle grades math program overall. First, PhET hopes to operate at scale, with large numbers of teachers and students using math sims effectively. PhET math sims are well positioned for large-scale use because the science sims already have a global audience of many thousands of users.

Secondly, PhET hopes to advance the field of math education. Research on PhET math sims can shed light on multiple topics that are broadly important to the field of math education at the present time. For example, research on PhET math sims could provide insights on questions of how best to use technology to support math learning for all students; relationships between pedagogy and learning; the value of open exploration and argumentation for math learning; the role of the teacher in making ideas explicit for students; and how teacher resources can support effective implementation of supplemental instructional resources.

Fertile Ground

The PhET middle grades math sims can be used in a wide variety of settings, with multiple teaching styles and students from any number of academic backgrounds. Nonetheless, certain contextual factors at the school along with certain teacher and student characteristics, can maximize the value and effectiveness of the sims. This section describes those contextual factors and participant characteristics.

Teacher Characteristics

PhET posits that the characteristics of effective teachers who use sims are essentially the same as the characteristics of effective teachers in general.

Table 7: Teacher Characteristics

Teachers should be…	**Explanation & Rationale**
Knowledgeable	Teachers should be knowledgeable about the content they are teaching—and also about how to teach and how students learn.
Confident	Teachers need confidence to be comfortable with their role in the classroom: comfortable managing students and technology, comfortable leading math-focused discussions, comfortable guiding students toward conceptual understanding.
Committed to all students' learning	Teachers should be committed to making sure that all of their students are learning; they cannot focus on some students to the detriment of others. Such a commitment requires aptitude with differentiated instruction, including an understanding of where students are and how to move them forward.
Oriented toward student-centered instruction	As discussed above, PhET strongly believes that teachers with various pedagogical orientations can make effective use of sims—though a student-centered approach can maximize the potential of sims to engage student thinking and develop an understanding of math.
Tech-friendly	Because sims run on technology, teachers must be open to using technology in their classrooms.

Student Characteristics

The PhET team maintains that the only condition for students is that they have a certain minimum level of pre-requisite math knowledge to engage with the particular sim they are using. The sims, like any other supplemental learning resource, do not teach all of mathematics from the ground up. Thus, in order to engage with, for example, a sim on fractions, students must have a foundational understanding of numbers, addition, subtraction, etc. Beyond this pre-requisite math knowledge, however, PhET does not believe any other student attributes are required for students to be able to successfully learn from sims. While positive affect (including interest, motivation, etc.) can certainly help students learn, PhET believes that students who do not start out with these attributes can not only still benefit from PhET sims, but will likely begin to develop some of those attributes through their engagement with the sims.

Classroom Norms

The classroom environment influences the teaching and learning that takes place therein. The classroom norms listed within the fertile ground section of the model support teachers and students in fulfilling their roles, as described in the implementation section of the model.

These classroom norms are:

- Teamwork
- Cooperation
- Contributions are valued
- Student-centered
- Sense of community and belonging

Again, PhET does not believe that every one of these norms must be solidly in place in order for teachers and students to benefit from the sims. Rather, these norms support the kinds of teaching and learning—and the kinds of interactions amongst teachers and students—that PhET finds most productive for learning and most conducive to optimizing the value of the sims.

School-level Supports

For teachers to be able to teach and teach well, they must have the time and resources they need to plan, prepare, and instruct. As would be the case for any supplemental resource, teachers who want to use sims need to have the time and flexibility to incorporate sims into their instruction. For sims, like any other technology-based instructional resource, teachers and students must also have adequate technology to enable sim-use. Specifically, teachers need to have a sufficient number of working devices (whether that's just a few and students rotate through or a classroom set where students can work in a one-to-one environment) and an internet connection to access and download the sims.

Conclusion

The Theory of Change model for PhET middle grades math sims describes the characteristics of PhET sims and accompanying resources, and how they operate to support teaching and learning within a school and classroom environment. The Theory of Change provides a thorough yet concise articulation of the vision for PhET sims that can be used to communicate both internally and externally. The Theory of Change can also guide product development (including the development of sims, lessons, and teacher resources) and research (including the definition of research questions and outcome metrics).

Two strands of research are indeed imminent. First, PhET is conducting a pilot study that will look at how math teachers use the middle grades math sims in their classrooms and what that experience is like for teachers and students. They also plan to examine measures of student learning. Second, SRI will conduct external evaluation research looking at how teachers find and make use of the PhET teacher resources and whether and how those resources support teachers to effectively incorporate sims into their classroom instruction. These two lines of research will yield insights that test the Theory of Change, about the extent to which the PhET middle grades math sims (and accompanying resources) are designed and operate as envisioned.

13 APPENDIX F: MQI CONSTRUCTS

Lesson Specific Codes

Classroom Work is Connected to Mathematics

Richness of the Mathematics

- Linking and Connections
- Explanations
- Mathematical Sense-Making
- Multiple Procedures or Solution Methods
- Patterns and Generalizations
- Mathematical Language
- Overall Richness of the Mathematics

Working with Students and Mathematics

- Remediation of Student Errors and Difficulties
- Teacher uses Student Mathematical Contributions
- Overall Working with Students and Mathematics

Errors and Imprecision

- Mathematical Content Errors
- Imprecision in Language or Notation
- Lack of Clarity in Presentation of Mathematical Content
- Overall Errors and Imprecision

Common Core Aligned Student Practices

- Students Provide Explanations
- Student Mathematical Questioning and Reasoning
- Students Communicate about the Mathematics of the Segment
- Task Cognitive Demand
- Students Work With Contextualized Problems
- Overall Common Core Aligned Student Practices

OVERALL LESSON CODES (5-Point Scale)

Whole Lesson codes

- Lesson Time is Used Efficiently
- Lesson is Mathematically Dense
- Students are Engaged
- Lesson Contains Rich Mathematics
- Teacher Attends to and Remediates Student Difficulty
- Teacher Uses Student Ideas
- Mathematics is Clear and not Distorted

- Tasks and Activities Develop Mathematics
- Lesson Contains Common Core Aligned Student Practices

Overall Mathematical Quality of Instruction

- Whole-Lesson Mathematical Quality of Instruction (MQI)

14 APPENDIX G: CODEBOOK FOR OBSERVATIONS

Codebook

Observations

3/5/2017

Code Category / Name		Description of Code
General School Information		
	Teacher Name	Name or Pseudonym of the teacher
	School	Name of the school
	Class Name	Name of the class, such as 2nd period Algebra 1
	Date	Date of observation
	Start time	Time the observation started
	End time	Time the observation ended
	# of girls	The number of girls present on the day of the observation
	# of boys	The number of boys present on the day of the observation
	Groups of	Indicate how many students are in the group
	Task Description	Describe the activity planned in the classroom.
Student Engagement		
	Engagement Girls	Select this is the MAJORITY of the girls are engaged in the task at hand during the five minute session.
	Engagement Boys	Select this is the MAJORITY of the boys are engaged in the task at hand during the five minute session.
	Engagement White	Select this is the MAJORITY of the white students are engaged in the task at hand during the five minute session.
	Engagement Non White	Select this is the MAJORITY of the non white students are engaged in the task at hand during the five minute session.
How are students working		
	Individual	Select this if the majority of the students are working independently
	Group	Select this if the majority of the students are working in groups
	Whole Class Discuss	Select this if the majority of the students are involved in a whole class discussion

	Small Class Discuss	Select this if the majority of the students are involved in small class discussions (multiple groups talking together)
What are students doing?		
	Math w/Tech	Select this if the majority of students are working on math while using technology
	Math w/o Tech	Select this if the majority of students are working on math NOT using technology
	Math talk	Select this if the majority of students are talking to those around them about math
	Non Math Talk	Select this if the majority of students are engaged in non-math discussions
	Listening to Lesson	Select this if the majority of students are listening to the direct instruction provided by the teacher
	Listening to Instructions	Select this if the majority of students are listening to the instructions provided by the teacher (e.g., "Next, we will be working on our Chromebooks. I want everyone to get their Chromebook, return to your desk, and go to our class website")
	Gathering Supplies	Select this if the majority of students are gathering supplies (such as worksheets, technology devices, glue, paper, etc.).
	Other Non-Math	Select this if the majority of students are working on a non-math related activity (e.g. free time, you can read a book)
	Waiting on help	Select this if the majority of students are waiting on help
Class Focus		
	Mathematics	Select this if the majority of the segment is focused on mathematics
	Technology	Select this if the majority of the segment is focused on technology
	Other	Select this if the majority of the segment is focused on non lesson related activities such as taking attendance, dealing with a call from the office, etc.
Math Content		
	Clear	Select this if the mathematics being discussed is clearly described
	Correct	Select this if the mathematics being discussed is correct

Class Discussion - Focus		
	Multiple representations	Select this if there are significant moments of a teacher or student making connections between multiple representations.
	Generalize (patterns)	Select this if there are significant moments of a teacher or student using data to generalize from a pattern
	Sensemaking	Select this if there are significant moments of a teacher or student focused on numeric sensemaking
	Press Evidence	Select this if there are significant moments of a teacher or student pressing a student for evidence of a statement.
	Multiple solutions	Select this if there are significant moments of a teacher or student discussing alternative solutions
	Precise language	Select this if there is evidence of students and/or teacher using precise mathematical language
	Explain (why)	Select this if the explanations predominantly answer the question WHY (focus on conceptual understanding)
	Explain (how)	Select this if the explanations predominantly answer the question HOW (focus on procedural understanding)
	Explanations	Select this if there are explanations that do not clearly fit into the how or why category
Mathematical Task		
	Drill	Select this if the task is primarily a drill-type task, practicing a skill
	Investigation	Select this if the task is an investigation, designed to help students construct knowledge
	Assessment	Select this if the task is an assessment of the students' knowledge
Students talking		
	Answer questions	Select this if there are significant moments where students are answering questions asked by the teacher
	Discuss questions	Select this if there are significant moments where students are discussing answers to questions with each other
	Justify answers	Select this if there are significant moments where students are justifying their answers

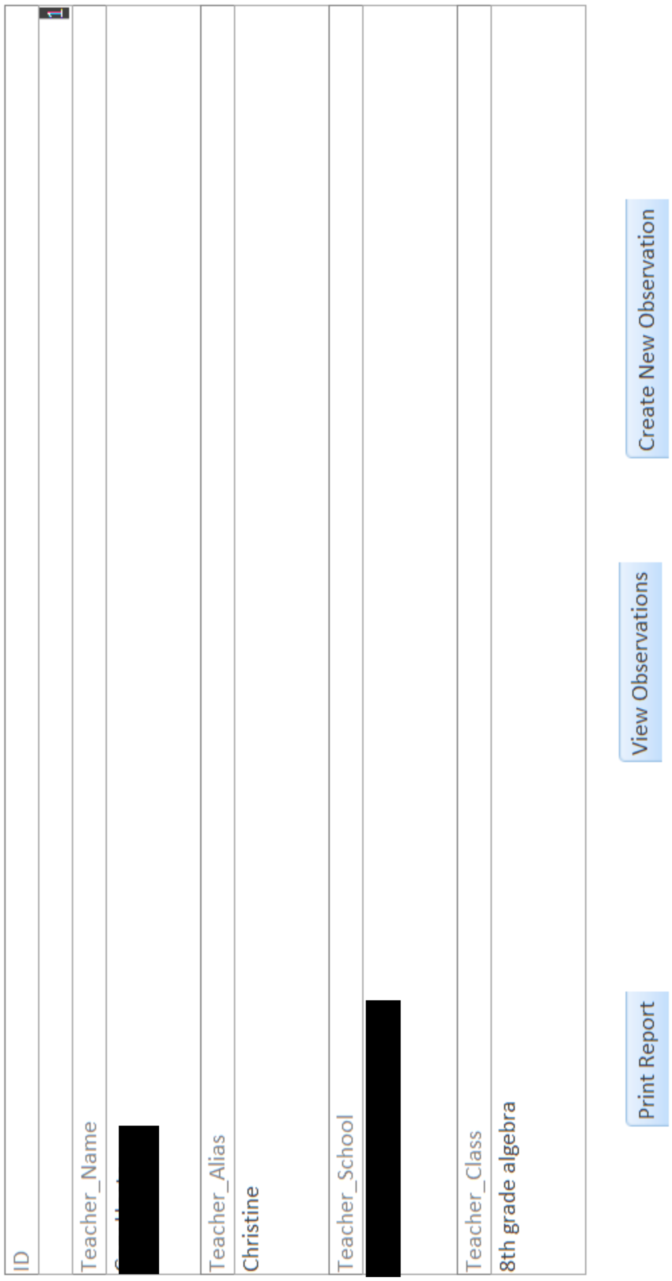
	Restate ideas	Select this if there are significant moments where students are restating ideas
	Reflect on lesson	Select this if there are significant moments where students are reflecting on the lesson
Class Discussion - Who Builds		
	Student builds	Select this is there are significant contributions of students building on other students' ideas
	Teacher builds	Select this is there are significant contributions of the teacher building on other students' ideas
Who talks		
	Mostly Teacher	The discussion is mostly the teacher talking
	Mostly Student	The discussion is mostly students talking
Type of Discussion		
	Rich mathematical	Select this if the discussion is a rich mathematical discussion
	Procedural	Select this if the discussion is a procedural discussion
Teacher Location		
	Front of the room	Select this if the teacher is positioned primarily at the front of the classroom during the five minute session
	Walking around	Select this if the teacher is primarily walking around the classroom during the five minute session
	At desk	Select this if the teacher is positioned primarily at his/her desk during the five minute session
	With students	Select this if the teacher is working with students during the five minute session
Teacher Verbal		
	Silent	Select this if, during the five minute session, the teacher is mostly silent
	Questioning	Select this if, during the five minute session, the teacher's verbal utterances are mostly questioning the students

Direct Instruction	Select this if, during the five minute segment, the teacher's verbal utterances are primarily giving direct instruction of a mathematical nature.
Supporting	Select this if, during the five minute session, the teacher's verbal utterances are mostly supporting students as they work
Remediation	Select this if, during the five minute session, the teacher's verbal utterances are mostly focused on providing remediation for a student, small group, or class
Providing Answers	Select this if, during the five minute session, the teacher's verbal utterances are mostly providing students with the correct answers (this could be noting correct or incorrect answers, leading the students to the answer (e.g. so when you multiply these two numbers you get…) or actually providing the answers)
General Admin	Select this if, during the five minute session, the teacher's verbal utterances are mostly administrative in nature, for example, providing details about an upcoming dance
Managing behavior	Select this if, during the five minute session, the teacher's verbal utterances are mostly behavior related
Providing Instructions	Select this if, during the five minute session, the teacher's verbal utterances are mostly providing instructions for the upcoming task (either on math or technology)
Tech Assistant	Select this if the teacher's interactions with the students focus on assisting with technology
Type of technology used	
Laptop	Indicate if laptops were used during this segment
Chrome books	Indicate if Chromebooks were used during this segment
Tablet	Indicate if tablets were used during this segment
Smart Phones	Indicate if smart phones were used during this segment
Clickers	Indicate if clickers were used during this segment
Graphing Calculators	Indicate if graphing calculators were used during this segment
Type of software used	
PowerPoint	Indicate if this software was used during this segment
PhET	Indicate if this software was used during this segment

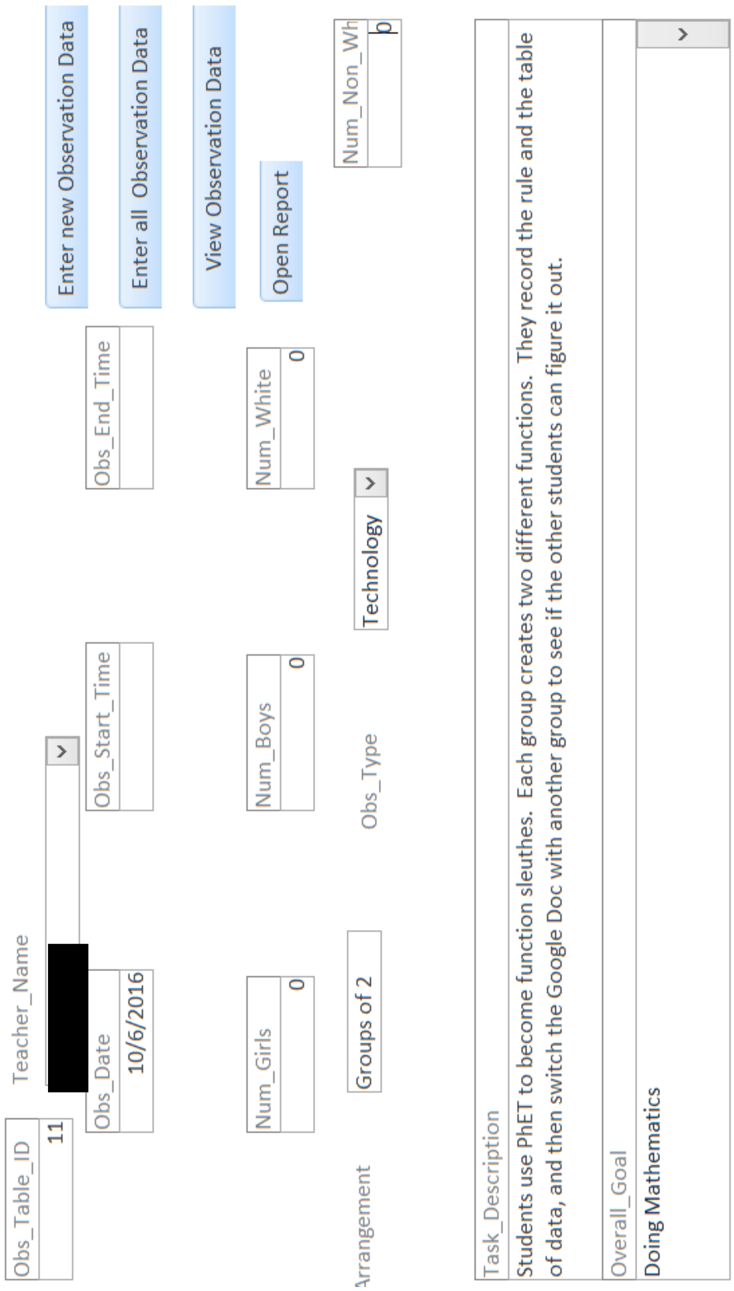
ALEKS	Indicate if this software was used during this segment
Desmos	Indicate if this software was used during this segment
Excel	Indicate if this software was used during this segment
GeoGebra	Indicate if this software was used during this segment
Geometers Sketchpad	Indicate if this software was used during this segment
IXL	Indicate if this software was used during this segment
Woot Math	Indicate if this software was used during this segment
Mathematica	Indicate if this software was used during this segment
Other	Indicate if a software other than one listed is used.
Tech Arrangement	
One to One	Each person has access to their own device. This should be selected if it represents the MAJORITY of the class
Pairs	Select this if two students share technology. This should be selected if it represents the MAJORITY of the class
Other	Select this if more than two students share technology. This should be selected if it represents the MAJORITY of the class
Teacher Only	Select this is only the teacher has access to the technology
Reflection - Who	
Individual	Select this if the majority of students are reflecting individually about their work (thinking or writing)
Student-student	Select this if the majority of students are reflecting with a partner about their work
Whole class	Select this if the majority of students are reflecting in a whole class discussion about their work
Small class	Select this if the majority of students are reflecting in groups about their work
Remediation - of whom	
Individual	Select this if the remediation (either math or technology) is predominantly addressed individually
Small Group	Select this if the remediation (either math or technology) is predominantly addressed in small groups

	Whole Class	Select this if the remediation (either math or technology) is predominantly addressed with the whole class
Remediation - by whom		
	By student	Select this if the remediation (either math or technology) is predominantly addressed by the student
	By teacher	Select this if the remediation (either math or technology) is predominantly addressed by the teacher
Remediation - about what		
	Math	Select this if the remediation is predominantly math related
	Technology	Select this if the remediation is predominantly technology related
	Other	Select this if the remediation is neither math nor technology related

Sample Teacher Listing



Sample Observation Form



Obs_Table_ID: 11

Teacher_Name: [redacted]

Obs_Date: 10/6/2016

Obs_Start_Time:

Obs_End_Time:

Enter new Observation Data

Enter all Observation Data

View Observation Data

Open Report

Num_Girls: 0

Num_Boys: 0

Num_White: 0

Num_Non_Wh: 0

Arrangement: Groups of 2

Obs_Type: Technology

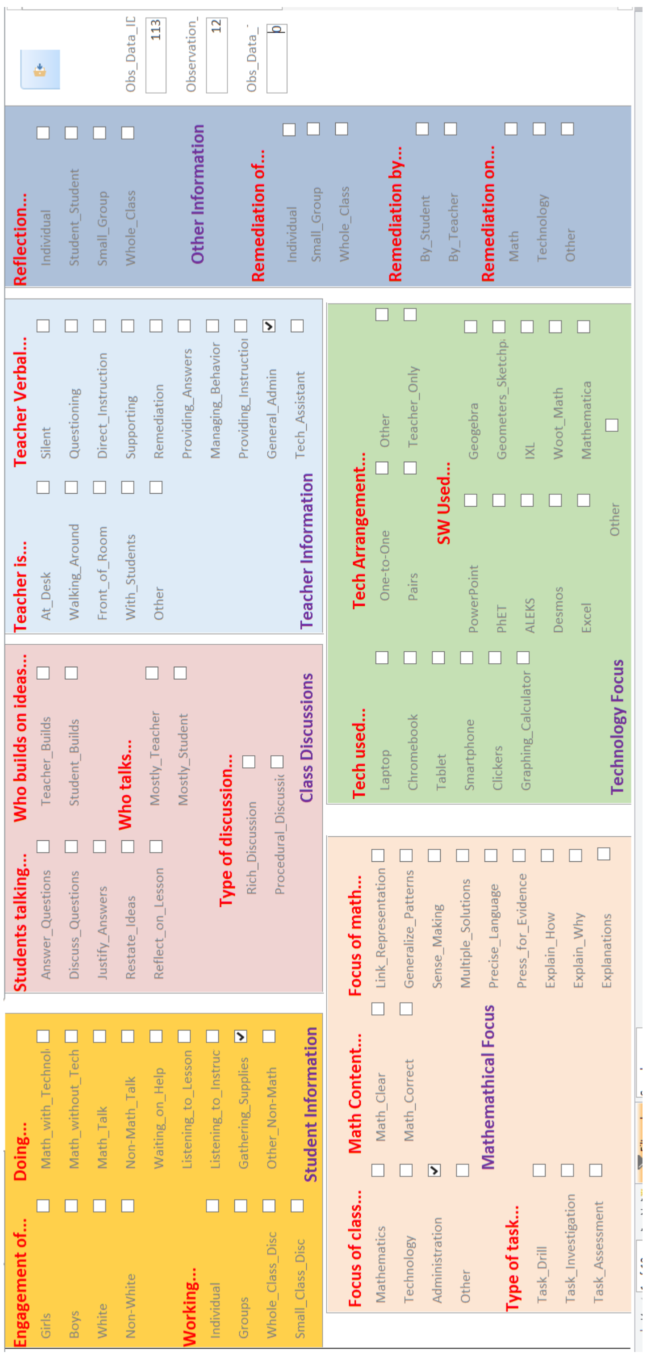
Task_Description

Students use PhET to become function sleuthes. Each group creates two different functions. They record the rule and the table of data, and then switch the Google Doc with another group to see if the other students can figure it out.

Overall_Goal

Doing Mathematics

Sample Observation Data Recording Sheet – Data recorded every 5 minutes



Sample Teacher Observation Notes

10/24/2016–Technology Observation 1 Notes.

Time	Activity
0:00	The teacher notes that this class has many late students. I do not have permission forms for many students, so this video cannot be shared. I explain to the class what I'll be doing and why I am in the classroom. The teacher is mixing up the groups for students. [Corrine joins me at this observation]. The teacher also has the students sign up for a text message service to get reminders.
0:05	The teacher continues with some general administration topics. The class pauses for the daily announcements. Then the students are asked to create their portfolios on the iPad.
0:10	Students are asked to go into Schoology. There is a new unit added. The teacher tries to pull up Schoology on her screen. Technical issues prevent it from coming up. Most students have their iPad up and running, although they are allowed to have the screens dimmed so it is difficult to see what they are doing. They are to click on Algebra 1, then open the new unit (Unit 2). There are three folders: activities and lesson, homework assignments, portfolio. Open portfolio (Unit 2 functions portfolio) in Notability. Save in Algebra 1 and call it Unit 2 Functions Portfolio. She projects another students' iPad to show them what they should do.
0:15	The teacher continues to explain how to create a folder in Notability and import the files. They open up a document that enables them to add a definition. There are warm-up sheets and definition sheets. They will define three words for homework tonight: Function, Domain and Range
0:20	Goal: identify characteristics and properties of functions. Who has heard of that word? (Knows the word, but doesn't know what it means). Work two things on your own, and then two with partners. The first two parts will be handed in with Schoology. Go into Unit 2 in Schoology. Under activities and lessons, see day 1 and 2, function fun. There are two assignments – first on your own, second with a partner. The first one, we'll open together. Open the activity. Complete part A. Think write, pair and share. Open in notability because you are going to write on it. What do you think it means for something to be a function. Here are some examples; there is an equation, a graph and a description. Think. Now answer these two questions.
0:25	Students are working on their iPads. The teacher walks around. She stops to talk with a student sitting at the wrong desk. She talks with another student about what he should be doing. Another student is asked to put his phone away. (He immediately takes it out again). Another student is prompted to answer the questions. "You are answering these two questions" "Answer these two questions. Which one do you think…"Students are quiet and working on their iPads, but there are minimal signs of

	engagement. Now, talk in your groups. How did you answer number 1? Go ahead and talk to each other.
0:30	The room is silent. No one talks. The teacher sits with one group. She again tells students to talk. There is just one or two students who talk. The teacher calls on specific kids. What did you say? Students give very cursory answers. Now go to Schoology, and open the link. It is a PhET link. You will get 10 minutes to play around with the activity (10-15 minutes). Stay in the patterns part. Are you all here? Fill in the questions for part B. Explore and list three things you discovered, and three questions you have. I'll start the whiteboard marker off with two people, write on the board and then pass the marker off. Then submit this form back to me. Notes that some kids are already playing with it. There is plenty to explore.

15 APPENDIX H: CODEBOOK FOR INTERVIEWS AND REFLECTIONS

Codebook

Interviews and Reflections

3/5/2017

1 Research process
Discussions about the research process

> *Ex: Just a second. Can I just get a general outline of what we're going to be doing? How many times are you going to observe me?*

1.1 Research process\Research discomfort
Teachers' discomfort with the research process. This may be with being observed, the process of critiquing lessons, or with what happens with the data after it is collected.

> *Ex: I'm fine. If something crashes and burns, I'm somewhat secure. It will be on tape.*

2 Lesson Reflection
This code is used generally to capture when teachers are reflecting on the lesson.

> *Ex: We even talked about that with the different sims, and it's like, "Oh, now I saw how that worked. I would do it differently the next time." You know*

2.1 Lesson Reflection\Visualization
This code is used to capture moments when teachers are thinking about how students visualize mathematics.

> *Ex: Yeah, I think that the Paint Splash one is interesting, because it's just another version of the Mix-IT problems they do in seventh grade when they're like, "Which drink is more orange-y?-*

2.2 Lesson Reflection\Just try it out
Teachers talk about wanting to just 'try it out' when thinking about how sims are used in the classroom. Try it out seems to be a replacement for lesson planning at times. At other times, it's a way to test a lesson prior to considering it to be done.

> *Ex: And that's why when we talk as a group it's like, some of it, to really learn it is you just take a day and throw one out there and see what the kids do.*

2.3 Lesson Reflection\Use what I know
When considering when to use sims in the classroom (or any new activity), teachers often talk about what else to use. Often, it is captured by this idea of 'using what I already know' or what I already have prepared.

> *Ex: Well, I guess I have three preps, so I just feel stretched, many times, and it's easier to draw what I've done, then to figure out something new.*

2.4 Lesson Reflection\Train me
Teachers talk about ways in which training impacts their choice of activities.

> *Ex: If you're gonna do a summertime prep with [the teachers], is they need to come in and be the kid.*

2.5 Lesson Reflection\Fit it in
Teachers struggle with when and where a lesson can be fit into the existing curriculum

Ex: The problem is, is that they say, "Okay, you can use this in this way; here's one lesson." It's like, "Oh, but I feel like there could be a different way to use it, and I just don't have the brainpower to think about that."

2.6 Lesson Reflection\Math is hard and boring

Teachers make value judgments about how certain activities remove either the difficulty or boredom from doing math. These can also be positive statements about how a certain activity promotes ease or engagement

Ex: Some kids have trouble tracking from the book, to the paper, to the book. That's why I'm like, "Okay. I'm going to give them, here's your investigation sheet. Answer question one here. Answer question two here. Here's a grid for question three, because they want you to graph it. I've already given you the grid. I've already set up a T table here because you're going to make a table." Filling something out makes it a little bit more friendly, because some kids will be like, they don't even want to ... "I have to make a table?" Just a T, but if it's already made, "Okay. I can fill in the numbers."

3 Technology

This parent code category captures teacher thinking about the use of technology in the classroom.

3.1 Technology\Tech Savvy Kids

This code captures thoughts about student use of technology related to their own abilities to figure things out.

Ex: They're getting more and more tech savvy. I would say for as long as I've been teaching, kids have had cell phones, but I feel like in the last couple of years, that's just blown up. It's very rare, by eighth grade, for kids to not have cell phones. I think there were three kids that I knew of last year who didn't have cell phones. I think that, in that regard, I think the exposure to technology is there.

3.2 Technology\Paper or Technology

This code captures the choice between using paper activities and using a technology tool. For example, do we draw graphs by hand or use Desmos.

Ex: I think that just brings up some really interesting conversations. Like that, I'm not sure it would be necessary to do it by hand because it's not the ... to me, I think it's that there are so many different things happening in the app that kids can sort of explore the same idea in so many different ways.

3.3 Technology\Make life easier

This code captures teachers' thoughts on how technology can make life easier for the student.

Ex: So, no. Especially when you're analyzing graphs and stuff, I think it's better to use the technology because they get bogged down.

3.4 Technology\Engagement

This code captures teachers' ideas on the interplay between technology and student engagement

Ex: This is so cool that they're realizing that 10-5 and 8-4 are the same level of scale. I was just like, "Oh my God, these girls are, they're the ones who almost always get the Chromebooks, and they immediately go to like Facebook and so *this time, I went over, and I was like, "God, they're so engaged, they're probably not engaged in the right thing," and I was like, "Oh my God, they're actually arguing about the ratios."*

3.5 Technology\Getting access to technology
This code captures teacher talk about how teachers gain access to technology, as well as times when access plays a role in decisions of using technology.

Ex: No. Well, the hard thing is, is I don't have my own computers in the classroom every day.

3.6 Technology\Software Used
This code captures which software products are used by the teachers.

Ex: There's an app that I've used. I've used Socrative Learning. That's just quick quizzes to see if kids are understanding. I have used an application. It's an app where you can record motion and capture data, Video Physics. Google Classroom I use a lot so kids can collaborate with projects. I can't think of anything else.

3.7 Technology\Comfort
Sections identified that included indicators of ways in which teachers were comfortable with technology

Ex: I'm pretty comfortable. I don't know. The first thing is, "Have you turned it off and on?" I think I'm okay. I've reprogrammed my clickers. We're learning Schoology today. I don't know.

3.7.1 Technology\Comfort\Fears
Sections identified that included indicators of ways in which teachers were NOT comfortable with technology. This was often expressed as a 'fear' when using technology.

Ex: My major was a blend of math and computer science, but I really don't think that that's ... People are always like, "You studied computer science. How can you be struggling so much with your smartphone." I think it's totally different, using technology versus writing code or piecing together a problem that you're going to solve with computer science, I think is really different. I would not say that just because I have a really good understanding of what happens behind the scenes in a computer doesn't make me an expert at teaching with technology.

3.8 Technology\How often technology
These sections were the times when teachers talked about how often technology was included in their lessons.

Ex: Yeah. In Eighth grade math, I worked with my student teacher a couple of years ago to totally rewrite one of the units from CMP so that it was in GeoGebra. It's all the same investigations, but based off of using GeoGebra, which is so cool. Yeah. The kids use Desmos and GeoGebra a lot in eighth grade math, and then in algebra, we use Desmos every single unit, a ton.

3.9 Technology\How not to use technology
This code was used for comments made about when technology does not support learning in the classroom.

> *Ex: Yeah. In Eighth grade math, I worked with my student teacher a couple of years ago to totally rewrite one of the units from CMP so that it was in GeoGebra. It's all the same investigations, but based off of using GeoGebra, which is so cool. Yeah. The kids use Desmos and GeoGebra a lot in eighth grade math, and then in algebra, we use Desmos every single unit, a ton.*

3.10 Technology\How to use Technology
This code was used for comments made about when technology does support learning in the classroom.

> *Ex: I definitely think there are other types of activities where the technology gives them the freedom to play around with different things so that they have a better understanding. For example, using sliders on Desmos and looking at your leading coefficient, just doing that on the computer and having that freedom to move it around so easily and see the immediate consequences allows them to have that deeper understanding.*

3.10.1 Technology\How to use Technology\Richer Mathematics
Reflective of comments teachers made about the use of technology to promote a stronger math conceptualization
3.10.1.1 Technology\How to use Technology\Richer Mathematics\Realistic
Teachers talk about ways in which using realistic features supports rich mathematics

> *Ex: I think that that at one point could have been true but I think that the discussions that result from it are better than any activity you could ever be able to do. The way that you can model real life situations with the math that's involved and I feel that that's truly math, rather than just solving some problems. So I feel like it gives kids a real life application for where they might actually see things happening.*

3.10.1.2 Technology\How to use Technology\Richer Mathematics\Concrete representation of Abstract idea
Teachers talk about how simulations provide a concrete representation of an abstract idea.

> *Ex: I think the strength of the sims is that it provides a really concrete representation of a really abstract idea.*

3.10.1.3 Technology\How to use Technology\Richer Mathematics\Better analysis
Teachers suggest that using simulations results in a better or more thorough analysis

> *Ex: I think with some of the technology, you can get higher level thinking. Instead of all your time spent graphing it, you can start analyzing different graphs.*

3.10.1.4 Technology\How to use Technology\Richer Mathematics\New ways of seeing
Teachers talk about how using simulations provide students with a new way of 'seeing' the math (technology vs paper and pencil)

Ex: and then when you use technology it does all that, so I feel like they've done paper, pencil, all that before

3.10.1.5 Technology\How to use Technology\Richer Mathematics\Prompts Discussion
Used when teachers noted ways in which simulations prompted discussions

Ex: But it can also increase their interaction with each other because they have something to talk about and a problem to solve.

3.10.1.6 Technology\How to use Technology\Richer Mathematics\To explain better
Used to discuss ways in which technology helped students explain better

Ex: there's so many cool ways to do that with Chrome Books, especially that enable kids who struggle with writing or who struggle with drawing, that they can create really cool diagrams or explanations on the Chrome Book, and then they can still record their voice

3.10.2.1 Technology\How to use Technology\Ease the burden\Individualized
Ways in which technology is individualized
3.10.2.1.1 Technology\How to use Technology\Ease the burden\Individualized\Easy access to the information
Teachers talk about the accessibility of the students in using the simulation at all levels

Ex: That they can just have, get to practice something a lot of times and if they're doing a math program it can be at their level.

3.10.2.1.2 Technology\How to use Technology\Ease the burden\Individualized\Self Paced
Technology allows students to work at their own pace.

Ex: But I also think that that's a good thing because a lot of the time, technology allows students to be self paced.

3.10.2.2 Technology\How to use Technology\Ease the burden\To make math easier
Technology makes math easier

Ex: I also think that it makes them less nervous to make a mistake, so I think about how taxing it would be for a kid who struggles to graph a line and then to not be accurate and have to keep graphing it and keep graphing it and keep graphing it and for them- they could easily do that, they could try something ten times pretty easily because there's not as much effort that's going into doing that.

3.10.2.2.1 Technology\How to use Technology\Ease the burden\To make math easier\Instant feedback
Technology provides instant feedback

Ex: I think the really great thing about technology is that cycle of predict and confirm with math is immediate, so they get immediate feedback on their work.

3.10.2.2.2 Technology\How to use Technology\Ease the burden\To make math easier\Lots of practice
Technology provides students with a lot of practice

Ex: That they can just have, get to practice something a lot of times and if they're doing a math program it can be at their level.

3.10.2.2.3 Technology\How to use Technology\Ease the burden\To make math easier\Can't do it with pencil and paper

Technology enables students to do work in ways that are different than pencil and paper

Ex: That being said, there are some investigations you just can't do physically. Whether it's because of space or because it's just going to be too dangerous or because it's going to be too wild. There are some really nice things that technology can afford that sort of eliminate those issues.

3.10.3 Technology\How to use Technology\To increase student engagement.

Technology increases student engagement

Ex: I guess I would use it I guess where I felt it's appropriate or to increase their engagement or their understanding.

3.11 Technology\Technology Available

This code was used the portion of the interviews when teachers listed the technology available in their classroom or in their school.

Ex: Technology in my classroom, I use clickers, graphing calculators. We have one set of Chromebooks for the math department and that's five teachers. We have five teachers. Well, one teacher teaches one section of math and the other teacher teaches two. Then, there's three full time math teachers. Document, camera, projector, laptop. I've used Desmos software. We do a unit using GeoGebra. I've used some of the illumination apps. We're going to use an app when we do the Beanie Baby bungee jump project. I use those apps. Google, of course, drive, documents, PowerPoints. That's all I can think of.

4 Planning Process

General overview of the planning process for the teacher.

4.1 Planning Process\Choosing materials

How teachers chose the materials they would use for a unit. Related to "using what I know".

Ex: Really, the bottom line was I wanted them to relay what a ratio was, and I think that's actually gonna be a lot easier to talk about in a week when we can say, "Remember when you made those ratios? Remember when you made the pool tables?" And then, it's not gonna be this, "Do you remember back to sixth grade math?-

4.2 Planning Process\Rollover to the next day

Code created based on hypothesis that teachers would continue to refer to the sim after the lesson.

Ex: So that's, you know, when you do your unit planning ... That's part of my unit planning, is I'm deciding how many days I have, what's my most important learning activity ... And I'll give this assessment just because I want to see.

Honestly, there should be another day, probably, to discuss what we just did. I don't have the time because I have to get the district algebra readiness test.

4.3 Planning Process\Planning with PhET
All moments when teachers reflected back on the lesson planning process.

Ex: Okay. Yeah, I was going to say I think my lessons are pretty set, and then I'll probably just make some adjustments on there once I know more specifically what the kids have done immediately prior to a lesson. For now, I feel like I'm at a point where I'm ... You know. I can plan ahead as much as I can, and then at a certain point, it's tailored to students.

5 Teaching this year

General category about teaching this year. Was too broad so I created subcodes and recoded this.
5.1 Teaching this year\Best Class Ever
Teachers were asked to describe their best class ever.

Ex: My principal said, "She said she'd never seen such an engaged math class." I've done that investigation since then, and it's never had that much. Those kids, for some reason, that day were just on fire. That makes me think of where you want this class, where they're just really into it.

5.2 Teaching this year\Curriculum
All comments related to the curriculum and the use of materials to support the curriculum

Ex: In terms of standards, we are totally common core aligned, and there's a lot of conversation about the math practice standards, which I think is the best part of the common core. I think the actual standards themselves, the content standards, it's just any set of content standards could be good or could be bad, but the cool thing about the Common Core is the emphasis on the practice of math, which I don't know if they have an equivalent of that in the English version of the math standards, the language arts ones. I think that's pretty special to have that for math. I think that's really cool.

5.3 Teaching this year\Textbook
All comments related to the textbook available to the teacher.

Ex: For eighth grade math, we use the Connected Math Program, which is Pearson. For my algebra class, we use, also Pearson, and I believe it's just called Algebra 1, or Common Core Algebra One.

5.4 Teaching this year\Class
Description of the classes being taught this year, as well as the specific class I observed.

Ex: I'll have two eighth grade math, that's just course three of middle school, and three advanced algebra, and one advanced geometry.

5.5 Teaching this year\Students
Description of the typical students in the teachers' classes.

Ex: Maybe a fifth of the class is kind of high achieving. Then, I've got the middle. Eighth grade year is when some of those kids decide that they want to take advanced algebra next year, so they might turn it on. I feel like most ... Well, I don't know. I'm trying to think about what percentage ends up going into the advanced algebra. Advanced algebra at the high school is different than advanced algebra at the middle school, obviously.

5.6 Teaching this year\Experience_Teaching_This_Class
The teachers' descriptions of how experienced they were teaching this years' classes.

Ex: Very. We got the new Connected Math materials I think two years ago, or maybe three years ago. I feel like we're now figuring out, "Okay. We're going to do this with Connected Math. We're going to do this as a supplement," so very comfortable.

6 Background

Main category to describe the teachers' background and experience as a teacher.
6.1 Background\Description of teaching
Description of the teachers' teaching experiences.

Ex: Altogether, this will be my seventh school year. I taught for one year at a high school in California, and then I taught, I just finished my fifth year here at <school>. Since I've been at <school>, obviously, it's middle school.

6.2 Background\Years_Experience
The number of years the teacher has taught.

Ex: Let's see. I actually didn't get my teaching degree in undergrad. I worked for NOAA, the environmental science place. Then, I went into the Peace Corps. That was my first teaching experience and that was in, let's see, 1998, 18 years. Holy cow, that's amazing. Then, I came back and I got my teaching degree as I was teaching in the classroom. I was under emergency license.

6.3 Background\Education
The educational background of the teacher.

Ex: My undergrad degree is in math and I double majored in math and environmental studies.

16 APPENDIX I: TEACHER LESSON CODING SUMMARY FOR PAT

Teacher Information Summary

What is the teacher saying?	Silent	Questioning	Direct Instruction	Supporting	Remediation	Providing Answers	Managing Behavior	Providing Instructions	General Admin	Technical Assistant
Non Tech	0%	36%	0%	9%	0%	27%	0%	18%	36%	18%
Non Tech	33%	50%	17%	0%	0%	0%	0%	17%	0%	0%
Non Tech	36%	27%	18%	0%	0%	18%	0%	18%	9%	0%
Non Tech	0%	70%	20%	0%	0%	0%	0%	20%	10%	0%
Average Non Tech	15%	43%	13%	3%	0%	13%	0%	18%	15%	5%
Tech	20%	40%	10%	0%	0%	0%	0%	50%	0%	0%

Where is teacher located?	At des	Walking around	Front of room	With Students	Other
Non Tech	0%	73%	9%	0%	0%
Non Tech	0%	67%	67%	0%	0%
Non Tech	45%	27%	9%	9%	0%
Non Tech	0%	50%	40%	10%	0%
Average Non Tech	13%	53%	26%	5%	0%
Tech	0%	64%	9%	0%	9%

Who controls the technology	Teacher	Student
Non Tech	55%	0%
Non Tech	100%	0%
Non Tech	45%	36%
Non Tech	50%	0%
Average Non Tech	58%	11%
Tech	18%	55%

Student Information Summary

What are students doing?

	Math w/Tech	Math w/o Tech	Math Talk	Non Math Talk	Listening to lesson	Listening to instructions	Waiting for help	Gather Supplies	Non Math
Non Tech	9%	45%	27%	0%	0%	0%	0%	18%	0%
Non Tech	0%	67%	33%	0%	0%	17%	0%	0%	0%
Non Tech	36%	36%	9%	0%	0%	0%	0%	9%	0%
Non Tech	0%	60%	30%	0%	10%	0%	0%	20%	0%
Average Non Tech	13%	50%	24%	0%	3%	3%	0%	13%	0%
Tech	55%	18%	18%	0%	0%	9%	0%	9%	0%

How are students working?

	Individually	Groups	Discussion Small class	Discussion Whole class
Non Tech	64%	0%	0%	9%
Non Tech	67%	50%	0%	0%
Non Tech	64%	18%	0%	0%
Non Tech	70%	20%	0%	0%
Average Non Tech	66%	18%	0%	3%
Tech	64%	18%	0%	9%

Student Engagement

	Girls	Boys	White	NonWhite
Non Tech	82%	82%	82%	82%
Non Tech	83%	83%	83%	83%
Non Tech	82%	82%	82%	82%
Non Tech	90%	90%	90%	90%
Average Non Tech	84%	84%	84%	84%
Tech	91%	91%	91%	91%

Mathematical Focus

What is the mathematical focus?

	Connecting Representat	Generalizi Patterns	Sense Making	Multiple Solutions	Precise Language	Press for Evidence	Explanatior	Explain How	Explai Why
Non Tech	0%	0%	9%	0%	9%	0%	0%	45%	0%
Non Tech	0%	83%	0%	0%	0%	0%	0%	0%	0%
Non Tech	0%	27%	0%	0%	0%	0%	0%	9%	0%
Non Tech	0%	70%	0%	0%	0%	0%	0%	0%	10%
Average Non Tech	0%	39%	3%	0%	3%	0%	0%	16%	3%
Tech	64%	0%	0%	0%	0%	0%	9%	0%	0%

What types of tasks are the students doing?

	Drill	Investigati	Assessment
Non Tech	55%	27%	0%
Non Tech	67%	50%	0%
Non Tech	45%	36%	0%
Non Tech	40%	50%	0%
Average Non Tech	50%	39%	0%
Tech	0%	73%	9%

		Focus of Class Math	Tech
55	Non Tech	73%	27%
30	Non Tech	117%	0%
55	Non Tech	82%	18%
50	Non Tech	90%	20%
47.5	Average Non Tech	87%	18%
55	Tech	91%	9%

Class Discussions

	What are students talking about?				
	Answer Questions	Discuss Questions	Justify Answers	Restate Ideas	Reflect on Lesson
Non Tech	36%	0%	0%	0%	0%
Non Tech	0%	50%	0%	0%	0%
Non Tech	27%	0%	0%	0%	0%
Non Tech	20%	0%	20%	0%	0%
Average Non Tech	24%	8%	5%	0%	0%
Tech	9%	18%	0%	0%	0%

	Class Discussion					
	Who talks		Who builds		Type of Discussion	
	Teacher	Student	Teacher	Student	Rich mathematical	Procedural
Non Tech	55%	0%	0%	0%	0%	55%
Non Tech	67%	33%	0%	0%	0%	67%
Non Tech	9%	0%	9%	0%	0%	9%
Non Tech	20%	10%	20%	0%	10%	10%
Average Non Tech	34%	8%	8%	0%	3%	32%
Tech	9%	9%	9%	0%	9%	0%

Remediation and Reflection

	Remediation							
	Of Whom			By Whom		Of What		
	Indiv.	Small Group	Whole Clas:	Student	Teacher	Math	Technolog	Other
Non Tech	9%	0%	0%	0%	9%	9%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Non Tech	0%	0%	18%	0%	18%	18%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Average Non Tech	3%	0%	5%	0%	8%	8%	0%	0%
Tech	0%	0%	0%	0%	0%	0%	0%	0%

	Lesson Reflection			
	Inidiv	Pairs	Small Group	Whole Class
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Average Non Tech	0%	0%	0%	0%
Tech	9%	0%	0%	0%

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Teacher Information Summary

What is the teacher saying?

	Silent	Questioning	Direct Instruction	Supporting	Remediation	Providing Answers	Managing Behavior	Providing Instructions	General Admin	Technical Assistant
Non Tech	7%	20%	73%	0%	7%	0%	0%	13%	7%	0%
Non Tech	17%	11%	39%	6%	0%	0%	6%	28%	0%	0%
Average Non Tech	12%	16%	56%	3%	3%	0%	3%	21%	3%	0%
Average Tech	8%	19%	10%	4%	0%	0%	2%	38%	6%	0%
Tech 1	0%	22%	6%	11%	0%	0%	6%	39%	11%	0%
Tech 2	12%	35%	24%	0%	0%	0%	0%	35%	0%	0%
Tech 3	11%	0%	0%	0%	0%	0%	0%	39%	6%	0%

Where is teacher located?

	At des	Walking around	Front of room	With Students	Other
Non Tech	0%	0%	20%	67%	0%
Non Tech	11%	28%	56%	6%	0%
Average Non Tech	6%	15%	39%	33%	0%
Average Tech	8%	10%	29%	19%	0%
Tech 1	11%	11%	17%	28%	0%
Tech 2	12%	18%	59%	12%	0%
Tech 3	0%	0%	11%	17%	0%

Who controls the technology

	Teacher	Student
Non Tech	13%	7%
Non Tech	83%	83%
Average Non Tech	52%	48%
Average Tech	35%	70%
Tech 1	17%	72%
Tech 2	88%	82%
Tech 3	0%	56%

Student Information Summary

What are students doing?

	Math w/Tech	Math w/o Tech	Math Talk	Non Math Talk	Listening to lesson	Listening to instructions	Waiting for help	Gather Supplies	Non Math
Non Tech	0%	67%	7%	7%	0%	7%	0%	7%	0%
Non Tech	0%	33%	11%	0%	28%	6%	0%	6%	11%
Average Non Tech	0%	48%	9%	3%	15%	6%	0%	6%	6%
Average Tech	26%	13%	13%	2%	8%	15%	0%	2%	8%
Tech 1	22%	11%	17%	6%	0%	17%	0%	0%	17%
Tech 2	24%	6%	24%	0%	24%	18%	0%	0%	6%
Tech 3	33%	22%	0%	0%	0%	11%	0%	6%	0%

How are students working?

	Individually	Groups	Discussion Small class	Discussion Whole class
Non Tech	53%	40%	0%	0%
Non Tech	56%	0%	0%	11%
Average Non Tech	55%	18%	0%	6%
Average Tech	39%	24%	2%	17%
Tech 1	56%	28%	0%	0%
Tech 2	29%	0%	6%	41%
Tech 3	33%	44%	0%	11%

Student Engagement

	Girls	Boys	White	NonWhite
Non Tech	80%	67%	73%	67%
Non Tech	78%	78%	78%	78%
Average Non Tech	79%	73%	76%	73%
Average Tech	89%	89%	89%	89%
Tech 1	72%	72%	72%	72%
Tech 2	100%	100%	100%	100%
Tech 3	94%	94%	94%	94%

Mathematical Focus

What is the mathematical focus?

	Connecting Representat	Generalizi Patterns	Sense Making	Multiple Solutions	Precise Language	Press for Evidence	Explanatior	Explain How	Explain Why
Non Tech	0%	60%	0%	0%	0%	0%	0%	7%	0%
Non Tech	0%	9%	0%	0%	0%	0%	0%	17%	0%
Average Non Tech	0%	32%	0%	0%	0%	0%	0%	12%	0%
Average Tech	40%	4%	0%	0%	0%	0%	2%	0%	0%
Tech 1	0%	11%	0%	0%	0%	0%	0%	0%	0%
Tech 2	41%	0%	0%	0%	0%	0%	6%	0%	0%
Tech 3	78%	0%	0%	0%	0%	0%	0%	0%	0%

What types of tasks are the students doing?

	Drill	Investigati	Assessment
Non Tech	13%	60%	0%
Non Tech	17%	33%	0%
Average Non Tech	15%	45%	0%
Average Tech	4%	56%	0%
Tech 1	0%	67%	0%
Tech 2	0%	41%	0%
Tech 3	11%	61%	0%

Focus of Class

	Math	Tech	Admin
Non Tech	87%	0%	13%
Non Tech	72%	6%	22%
Average Non Tech	79%	3%	18%
Average Tech	72%	17%	19%
Tech 1	56%	28%	22%
Tech 2	82%	18%	6%
Tech 3	78%	6%	28%

Class Discussions

	What are students talking about?				
	Answer Questions	Discuss Questions	Justify Answers	Restate Ideas	Reflect on Lesson
Non Tech	7%	0%	0%	0%	0%
Non Tech	11%	0%	0%	0%	0%
Average Non Tech	9%	0%	0%	0%	0%
Average Tech	11%	0%	0%	0%	0%
Tech 1	17%	0%	0%	0%	0%
Tech 2	18%	0%	0%	0%	0%
Tech 3	0%	0%	0%	0%	0%

	Class Discussion					
	Who talks		Who builds		Type of Discussion	
	Teacher	Student	Teacher	Student	Rich mathematical	Procedural
Non Tech	7%	0%	0%	0%	0%	7%
Non Tech	11%	0%	11%	0%	6%	6%
Average Non Tech	9%	0%	6%	0%	3%	6%
Average Tech	11%	0%	2%	0%	6%	4%
Tech 1	22%	0%	6%	0%	11%	0%
Tech 2	12%	0%	0%	0%	6%	12%
Tech 3	0%	0%	0%	0%	0%	0%

Remediation and Reflection

	Remediation							
	Of Whom			By Whom		Of What		
	Indiv.	Small Group	Whole Class	Student	Teacher	Math	Technolog	Other
Non Tech	7%	0%	0%	0%	7%	7%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Average Non Tech	3%	0%	0%	0%	3%	3%	0%	0%
Average Tech	0%	0%	0%	0%	0%	0%	0%	0%
Tech 1	0%	0%	0%	0%	0%	0%	0%	0%
Tech 2	0%	0%	0%	0%	0%	0%	0%	0%
Tech 3	0%	0%	0%	0%	0%	0%	0%	0%

	Lesson Reflection			
	Inidiv	Pairs	Small Group	Whole Class
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Average Non Tech	0%	0%	0%	0%
Average Tech	0%	0%	0%	0%
Tech 1	0%	0%	0%	0%
Tech 2	0%	0%	0%	0%
Tech 3	0%	0%	0%	0%

18 APPENDIX K: TEACHER LESSON CODING SUMMARY FOR SARA

Teacher Information Summary

What is the teacher saying?

	Silent	Questioning	Direct Instruction	Supporting	Remediation	Providing Answers	Managing Behavior	Providing Instructions	General Admin	Technical Assistant
Non Tech	10%	20%	40%	20%	0%	0%	0%	10%	20%	0%
Non Tech	0%	50%	17%	0%	0%	0%	0%	50%	33%	0%
Non Tech	0%	36%	9%	0%	0%	9%	0%	9%	18%	0%
Non Tech	0%	30%	0%	10%	0%	0%	0%	20%	30%	0%
Average Non Tech	3%	30%	15%	8%	0%	3%	0%	18%	23%	0%
Tech	0%	50%	0%	10%	0%	0%	0%	30%	0%	30%
Tech 2	10%	70%	0%	0%	0%	0%	0%	20%	0%	10%

Where is teacher located?

	At des	Walking around	Front of room	With Students	Other
Non Tech	0%	50%	40%	20%	0%
Non Tech	0%	38%	38%	25%	0%
Non Tech	0%	13%	13%	50%	0%
Non Tech	3%	38%	50%	25%	0%
Average Non Tech	1%	35%	35%	29%	0%
Tech	0%	20%	20%	30%	0%
Tech 2	13%	25%	38%	25%	0%

Who controls the technology

	Teacher	Student
Non Tech	70%	0%
Non Tech	75%	38%
Non Tech	50%	0%
Non Tech	75%	0%
Average Non Tech	68%	9%
Tech	20%	60%
Tech 2	25%	38%

Student Information Summary

What are students doing?

	Math w/Tech	Math w/o Tech	Math Talk	Non Math Talk	Listening to lesson	Listening to instructions	Waiting for help	Gather Supplies	Non Math
Non Tech	0%	50%	0%	0%	40%	10%	0%	10%	0%
Non Tech	25%	0%	25%	0%	0%	13%	0%	25%	0%
Non Tech	0%	50%	25%	0%	0%	13%	0%	38%	0%
Non Tech	0%	25%	38%	0%	0%	25%	0%	13%	0%
Average Non Tech	6%	32%	21%	0%	12%	15%	0%	21%	0%
Tech	50%	0%	30%	0%	0%	20%	0%	10%	0%
Tech 2	38%	13%	38%	0%	0%	13%	0%	25%	0%

How are students working?

	Individually	Groups	Discussion Small class	Whole class
Non Tech	50%	20%	0%	10%
Non Tech	25%	38%	0%	38%
Non Tech	13%	38%	0%	13%
Non Tech	38%	25%	0%	0%
Average Non Tech	32%	29%	0%	15%
Tech	0%	60%	0%	10%
Tech 2	63%	13%	0%	13%

Student Engagement

	Girls	Boys	White	NonWhite
Non Tech	100%	100%	100%	100%
Non Tech	100%	100%	100%	100%
Non Tech	88%	88%	88%	88%
Non Tech	88%	88%	88%	88%
Average Non Tech	94%	94%	94%	94%
Tech	80%	80%	80%	80%
Tech 2	88%	88%	88%	88%

Mathematical Focus

What is the mathematical focus?

	Connecting Representat	Generalizi Patterns	Sense Making	Multiple Solutions	Precise Language	Press for Evidence	Explanatior	Explain How	Explai Why
Non Tech	0%	0%	20%	0%	10%	0%	20%	0%	0%
Non Tech	0%	0%	13%	0%	0%	0%	0%	13%	0%
Non Tech	13%	0%	50%	0%	13%	0%	13%	13%	0%
Non Tech	25%	0%	38%	13%	0%	0%	0%	0%	0%
Average Non Tech	9%	0%	29%	3%	6%	0%	9%	6%	0%
Tech	60%	0%	30%	0%	10%	0%	0%	0%	0%
Tech 2	38%	0%	38%	0%	0%	0%	13%	0%	0%

What types of tasks are the students doing?

	Drill	Investigati	Assessment
Non Tech	80%	0%	0%
Non Tech	13%	38%	0%
Non Tech	0%	63%	0%
Non Tech	75%	0%	0%
Average Non Tech	44%	24%	0%
Tech	0%	60%	0%
Tech 2	38%	0%	13%

Focus of Class

		Math	Tech
50	Non Tech	80%	0%
40	Non Tech	75%	0%
40	Non Tech	75%	0%
40	Non Tech	75%	0%
42.5	Average Non Tech	76%	0%
50	Tech	80%	0%
40	Tech 2	75%	13%

Class Discussions

	What are students talking about?				
	Answer Questions	Discuss Questions	Justify Answers	Restate Ideas	Reflect on Lesson
Non Tech	10%	0%	0%	0%	0%
Non Tech	25%	0%	0%	0%	0%
Non Tech	13%	0%	0%	0%	0%
Non Tech	13%	25%	25%	0%	0%
Average Non Tech	15%	6%	6%	0%	0%
Tech	0%	10%	10%	10%	10%
Tech 2	25%	13%	0%	13%	25%

	Class Discussion					
	Who talks		Who builds		Type of Discussion	
	Teacher	Student	Teacher	Student	Rich mathematical	Procedural
Non Tech	20%	0%	0%	0%	0%	20%
Non Tech	25%	0%	0%	0%	0%	25%
Non Tech	13%	0%	0%	0%	13%	25%
Non Tech	25%	38%	13%	13%	50%	13%
Average Non Tech	21%	9%	3%	3%	15%	21%
Tech	10%	10%	30%	20%	20%	0%
Tech 2	0%	38%	0%	13%	63%	0%

Remediation and Reflection

	Remediation							
	Of Whom			By Whom		Of What		
	Indiv.	Small Group	Whole Class	Student	Teacher	Math	Technolog	Other
Non Tech	0%	10%	0%	0%	10%	10%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Average Non Tech	0%	3%	0%	0%	3%	3%	0%	0%
Tech	0%	10%	0%	0%	10%	10%	0%	0%
Tech 2	0%	0%	13%	13%	0%	0%	13%	0%

	Lesson Reflection			
	Inidiv	Pairs	Small Group	Whole Class
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Average Non Tech	0%	0%	0%	0%
Tech	0%	10%	0%	0%
Tech 2	13%	13%	0%	0%

19 APPENDIX L: TEACHER LESSON CODING SUMMARY FOR CHRISTINE

Teacher Information Summary

What is the teacher saying?

	Silent	Questioning	Direct Instruction	Supporting	Remediation	Providing Answers	Managing Behavior	Providing Instructions	General Admin	Technical Assistant
Non Tech	0%	44%	11%	0%	0%	22%	11%	33%	0%	0%
Non Tech	0%	33%	50%	0%	0%	0%	0%	50%	33%	0%
Non Tech	9%	55%	9%	0%	0%	0%	0%	36%	18%	0%
Non Tech	0%	40%	20%	0%	0%	10%	0%	30%	0%	0%
Average Non Tech	3%	40%	18%	0%	0%	8%	3%	33%	10%	0%
Tech	0%	50%	0%	10%	0%	0%	0%	60%	30%	80%
Tech 2	0%	10%	0%	0%	0%	0%	10%	30%	10%	30%

Where is teacher located?

	At des	Walking around	Front of room	With Students	Other
Non Tech	11%	33%	67%	0%	0%
Non Tech	0%	22%	56%	33%	0%
Non Tech	20%	30%	30%	10%	0%
Non Tech	11%	33%	44%	11%	0%
Average Non Tech	11%	30%	49%	14%	0%
Tech	0%	33%	39%	17%	0%
Tech 2	0%	80%	10%	0%	0%

Who controls the technology

	Teacher	Student
Non Tech	100%	0%
Non Tech	67%	0%
Non Tech	60%	40%
Non Tech	78%	0%
Average Non Tech	76%	11%
Tech	44%	67%
Tech 2	20%	60%

Student Information Summary

What are students doing?

	Math w/Tech	Math w/o Tech	Math Talk	Non Math Talk	Listening to lesson	Listening to instructions	Waiting for help	Gather Supplies	Non Math
Non Tech	0%	67%	11%	0%	11%	0%	0%	22%	0%
Non Tech	0%	67%	22%	0%	0%	22%	0%	11%	0%
Non Tech	30%	20%	30%	0%	20%	10%	0%	10%	0%
Non Tech	0%	67%	22%	0%	11%	11%	0%	11%	0%
Average Non Tech	8%	54%	22%	0%	11%	11%	0%	14%	0%
Tech	33%	11%	17%	0%	0%	22%	0%	6%	0%
Tech 2	50%	10%	0%	0%	0%	10%	0%	20%	0%

How are students working?

	Individually	Groups	Discussion Small class	Discussion Whole class
Non Tech	67%	22%	11%	0%
Non Tech	33%	44%	22%	0%
Non Tech	70%	0%	20%	0%
Non Tech	56%	22%	22%	0%
Average Non Tech	57%	22%	19%	0%
Tech	50%	17%	17%	0%
Tech 2	30%	40%	0%	0%

Student Engagement

	Girls	Boys	White	NonWhite
Non Tech	89%	89%	89%	78%
Non Tech	56%	89%	67%	56%
Non Tech	90%	90%	90%	90%
Non Tech	89%	89%	89%	89%
Average Non Tech	81%	89%	84%	78%
Tech	89%	89%	89%	83%
Tech 2	80%	80%	80%	80%

What is the mathematical focus?

	Connecting Representat	Generalizi Patterns	Sense Making	Multiple Solutions	Precise Language	Press for Evidence	Explanatior	Explain How	Explai Why
Non Tech	22%	0%	0%	0%	0%	11%	11%	0%	0%
Non Tech	44%	0%	0%	0%	0%	0%	0%	11%	0%
Non Tech	0%	60%	10%	0%	0%	0%	0%	20%	0%
Non Tech	22%	22%	11%	0%	0%	11%	0%	11%	0%
Average Non Tech	22%	22%	5%	0%	0%	5%	3%	11%	0%
Tech	28%	0%	0%	0%	6%	0%	0%	17%	0%
Tech 2	60%	10%	0%	0%	0%	0%	0%	10%	0%

What types of tasks are the students doing?

	Drill	Investigati	Assessment
Non Tech	33%	33%	0%
Non Tech	44%	33%	0%
Non Tech	30%	70%	0%
Non Tech	33%	56%	0%
Average Non Tech	35%	49%	0%
Tech	28%	33%	0%
Tech 2	20%	70%	0%

Focus of Class

		Math	Tech
45	Non Tech	89%	0%
45	Non Tech	89%	0%
50	Non Tech	90%	40%
45	Non Tech	89%	22%
46.25	Average Non Tech	89%	16%
90	Tech	44%	50%
50	Tech 2	60%	30%

Class Discussions

	What are students talking about?				
	Answer Questions	Discuss Questions	Justify Answers	Restate Ideas	Reflect on Lesson
Non Tech	22%	11%	11%	0%	0%
Non Tech	22%	0%	0%	0%	0%
Non Tech	10%	10%	30%	0%	10%
Non Tech	22%	0%	22%	0%	0%
Average Non Tech	19%	5%	16%	0%	3%
Tech	22%	0%	0%	0%	0%
Tech 2	0%	10%	0%	0%	0%

	Class Discussion					
	Who talks		Who builds		Type of Discussion	
	Teacher	Student	Teacher	Student	Rich mathematical	Procedural
Non Tech	11%	0%	11%	0%	11%	0%
Non Tech	22%	0%	11%	0%	11%	11%
Non Tech	10%	10%	0%	0%	10%	10%
Non Tech	11%	11%	0%	0%	11%	11%
Average Non Tech	14%	5%	5%	0%	11%	8%
Tech	11%	6%	0%	0%	0%	17%
Tech 2	0%	0%	0%	0%	0%	0%

Remediation and Reflection

	Remediation							
	Of Whom			By Whom		Of What		
	Indiv.	Small Group	Whole Clas:	Student	Teacher	Math	Technolog	Other
Non Tech	0%	0%	11%	0%	11%	11%	0%	0%
Non Tech	0%	33%	0%	0%	33%	33%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%	0%	0%	0%	0%
Average Non Tech	0%	8%	3%	0%	11%	11%	0%	0%
Tech	0%	0%	0%	0%	0%	0%	0%	0%
Tech 2	0%	0%	0%	0%	0%	0%	0%	0%

	Lesson Reflection			
	Inidiv	Pairs	Small Group	Whole Class
Non Tech	0%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Non Tech	10%	0%	0%	0%
Non Tech	0%	0%	0%	0%
Average Non Tech	3%	0%	0%	0%
Tech	0%	0%	0%	0%
Tech 2	10%	0%	0%	0%