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PHYSICAL CRYSTALLOGRAPHY

# THE OPTICAL PROPERTIES OF CRYSTALS

# THE OPTICAL PROPERTIES
OF
# CRYSTALS

WITH A GENERAL INTRODUCTION TO THEIR PHYSICAL PROPERTIES

BEING SELECTED PARTS OF THE
PHYSICAL CRYSTALLOGRAPHY

BY
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GEOLOGY

TRANSLATED (WITH THE AUTHOR'S PERMISSION) FROM THE FOURTH, REVISED AND AUGMENTED GERMAN EDITION

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With 121 Figures in the Text and Two Colored Plates

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FIRST THOUSAND

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GEOLOGY

# PREFACE

UNTIL recently, in the higher institutions of learning, crystallography has largely been taught only in connection with mineralogy, as an aid to the characterization of minerals and therefore in a purely descriptive manner. But this does not accord with the present state of the science. Haüy, the founder of crystallography, had already made an attempt to *explain* the forms of crystals, while the investigations of Brewster and the later ones of Senarmont and Grailich, together with those conducted recently by Mallard and others, have given us a detailed knowledge of the regular connection between the physical properties of crystals and the crystal form. As a consequence of these discoveries the conviction has gradually made its way that the form of a crystal is solely a consequence of its *interior structure*, — of its make-up from the smallest crystal particles, which act on one another with definite forces depending regularly on the crystallographic direction, — and is therefore a *physical property* of the substance in question. Hessel, and later Bravais and Gadolin, independently, succeeded in determining the entire number of possible crystal forms by purely geometrical methods; while reasoning based on the physical properties of crystals leads to exactly the same results. For the conclusions as to the interior structure of crystallized media — as set forth in the theories of Bravais, Sohncke, Fedorow, Schönfliess, and others — that necessarily follow on this basis, point to the existence of exactly the same kinds of symmetry. The consequence is that crystallography

must be regarded as a part of molecular physics; and, since Voigt has made it seem probable that the so-called "amorphous" bodies are to be conceived of as aggregates of very small crystalline particles, the science may be designated quite generally as the "molecular physics of solids".

A scientific treatment of this subject, then, can proceed only hand in hand with the entire physics of crystals. And in its theoretical aspect the edifice of crystal lore stands at the present day as one of the best established in the whole realm of physics, — of fundamental importance for an understanding of the material world.

But not only in theoretical but also in practical respects has the treatment of crystal science by physical methods attained a constantly increasing importance. Our complete knowledge of the regular relation between the optical properties and the symmetry of crystals has given us the means to carry out, by optical methods, the determination of crystallized substances in microscopic preparations with a certainty which, only a few decades ago, no one would have believed possible. It is commonly known what a revolution petrography has experienced in consequence of this, and how important an aid to the chemist "crystal-analysis" has become through the work of O. Lehmann, C. Haushofer, and others; it is well known, too, how fruitful have been the methods of crystal optics in botanical and histologic-zoölogical investigations.

Under these circumstances students of natural science, especially those devoting themselves to chemistry, mineralogy, and geology, can no longer be permitted to neglect the study of crystallography in the sense indicated. The present text-book presumes an acquaintance with general experimental physics and chemistry, but with no mathematics beyond what is afforded by secondary schools; it aims not only to lead the student to an understanding of the laws to which crystallized substances are subject, but also to enable him to turn the methods of the science to their practical application. No

difficulties of any sort should be met with, especially by those who have attended a lecture on mineralogy and gained thereby an idea of the most common crystal forms.

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In the fourth edition of "Physikalische Krystallographie", in describing the properties of crystallized bodies in general, a system has been introduced which makes it possible to place clearly before beginners the laws governing the dependence of the crystal properties on the crystallographic direction, in a steady advance from simple to complex. While in respect of their optical, thermal, electrical, and magnetic behavior the totality of crystals fall into only *five* groups, this number is increased by the properties of cohesion and elasticity to *seven* and *nine* respectively; and the behavior of crystals with regard to solution and growth gives us, finally, all the possible symmetry classes of crystals, — numbering *thirty-two*. The most important part of the subject from a practical standpoint is that concerned with optics. Here, in consequence of Lord Kelvin's researches and Fletcher's lucid exposition, the untenable "elasticity" of the ether had already been laid aside in the third edition. The present treatment of crystal optics is based on its purely geometrical aspect, suggested by the latter author. This method makes it possible, yet without mathematical theory, to gain a correct insight into even the most complicated phenomena (conical refraction, for example), something which is indispensable for microscopical studies. Other changes are limited essentially to particular amplifications — an example of which occurs in the discussion of total reflection — and to making the wording more precise. — *Excerpts (with slight adaptations by the translator) from the prefaces to the third* (1895) *and fourth* (1905) *German editions.*

# TRANSLATOR'S NOTE

THIS partial translation of the fourth edition of Groth's "Physikalische Krystallographie" is made up chiefly of matter contained in Part I of the original work, — on "The Properties of Crystals"; besides embracing the general introduction and all that falls under the heading "Optical Properties" in this part, it includes also whatever may be found there on the influence of other properties on the optical properties. Short extracts from Parts II ("Systematic Description of Crystals") and III ("The Methods of Crystal Investigation") have been introduced, on occasion, for illustration and example.

The translation was undertaken at the instance of Professor Russell D. George, of the University of Colorado; the translator is indebted to him for his kindly interest and frequent advice, as well as to Professor Oliver C. Lester for several important suggestions.

While the author has elsewhere been closely followed, it was found expedient, in discussing the influence of other properties on the optical properties, to select the parts to be translated and sometimes to condense them, as well as to make considerable changes in their arrangement and classification. The confusing, tautological expression "direction of" an optic axis, common in both languages, has been avoided. Additional references and cross-references have sometimes been given, and a complete table of abbreviations is appended; any further amplifications by the translator will be found enclosed in brackets or otherwise indicated.

BOULDER, COLORADO,
*March*, 1910.

# ABBREVIATIONS

*Abh. Ges. d. Wissensch. Göttingen.* — Abhandlungen der k. Gesellschaft der Wissenschaften zu Göttingen. Berlin.

*Bull. soc. franç. de minéral.* — Bulletin de la Société française de minéralogie. Paris.

*Carls Repert. f. Exper.-Physik.* (Carls) Repertorium für Experimental-Physik. Leipzig.

*Centralbl. f. Min.* — Centralblatt für Mineralogie. Stuttgart.

*Leiss: Die op. Instrum.* — C. Leiss: "Die optischen Instrumente der Firma Fuess." Leipzig, 1899.

*Min. Mag.* — Mineralogical Magazine (and Journal of the Mineralogical Society). London.

*Nachr. Ges. d. Wissensch. Göttingen.* — Nachrichten der k. Gesellschaft der Wissenschaften zu Göttingen. Göttingen.

*Phil. Mag.* — London, Edinburgh, and Dublin Philosophical Magazine. London.

*Phys. Kryst.* — P. Groth: "Physikalische Krystallographie." Leipzig; 3rd ed. 1895, 4th ed. 1905.

*Pogg. Ann. d. Physik.* — (Poggendorff's) Annalen der Physik. Leipzig.

*Proceed. Phys. Soc.* — Proceedings of the Physical Society. London.

*Sitzungsber. Akad. d. Wissensch. Berlin.* — Sitzungsberichte der k. p. Akademie der Wissenschaften zu Berlin. Berlin.

*Tscher. min. u. petrog. Mitteil.* — (Tschermak's) mineralogische und petrographische Mitteilungen. Vienna.

*Zeitschr. f. Kryst.* — Zeitschrift für Krystallographie. Leipzig.

*Zeitschr. f. Kryst. u. Min.* — (Groth's) Zeitschrift für Krystallographie und Mineralogie. Leipzig.

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# GENERAL INTRODUCTION TO THE PROPERTIES OF CRYSTALS

ERRATA

P. 5, line six from bottom of the page, for "composition" read "constitution".

P. 264, next to the last line in the footnote, for "negatively" read "positively".

# GENERAL INTRODUCTION

## TO THE PROPERTIES OF CRYSTALS

WHEN a body has the same constitution at all points, so that any two equal, similar, and similarly oriented parts of it are undistinguishable from each other by any difference in quality, that body is said to be **homogeneous.**

Homogeneous bodies fall into two classes: —

1. Bodies in which not only all points, but also *all directions, are equivalent; i.e.* in which the different directions are undistinguishable from one another by any physical property of the body. These bodies are spoken of as **amorphous**, because they have no peculiar shape, or as **isotropic**, because they transmit every kind of motion in the same way in all directions. Here belong all gases and vapors, and nearly all liquids; also a number of so-called "solids", as colloids, resins, glasses. But the latter bodies are not sharply separated from the liquids; for example, with an increase of temperature they pass through the softened, or viscous, state gradually over into the liquid.

2. Homogeneous bodies whose properties depend on the direction, so that the value of any one property attains in certain directions a maximum, in others a minimum. (This may be the case only for particular physical properties, while for others there may exist, grounded in the nature of the property in question, an equality of the value for all directions.) Bodies of this kind are capable of crystallization, — *i.e.* of assuming a regular form which is peculiar to the body and which stands in a regular relation to the non-equivalence of the directions within it — and are therefore called **crystallized** or **crystalline** bodies.

The physical properties are distinguished into **scalar** and **vector.**

The scalar properties are properties represented by a single quantity independent of the direction, as temperature, density, specific heat, etc.

The vector properties are such as are defined by a numerical value *and* a direction. When the numerical value is necessarily the same in the opposite direction, *i.e.* when the two directions that pass out from any point and belong to the same straight line are always absolutely equivalent in respect of a property, then that property is designated as **bi-vector.***

From the foregoing definitions it follows that the amorphous bodies can possess only scalar properties, while the crystallized, on the other hand, have, besides these, vector and bi-vector properties.

To set forth the regular relations that exist among the crystal properties depending on the direction — and with those properties belongs the geometric form — is the object of **physical crystallography.** A plausible explanation of the laws of this branch of science is supplied us by the molecular hypothesis, if we assume that within crystals, while the molecules are indeed in motion, yet their motion consists of vibrations about certain intermediate loci, and that these loci are *regularly* arranged in space.† Then the manner of this arrangement, which is known

* Instead of the usual designation "bi-vector", Voigt has proposed the use of "tensor"; but for the purposes of crystallography the former term seems the more representative.

† In contradistinction from this, for the amorphous bodies there must be assumed an *irregular* distribution of the molecules in space, something which for gases and liquids is at once clear. An amorphous body, according to this second assumption, is only apparently homogeneous. That is, its unhomogeneousnesses, because of their too rapid succession within the smallest compass, are no longer accessible to physical examination; and the latter therefore yields for every property only a mean value, which, naturally, is found to be the same for all directions. For this reason crystals have even been designated as the only really homogeneous bodies, and the terms "crystallized" and "homogeneous" as equivalent.

as **crystal structure**, corresponds to a state of stable equilibrium among the interior forces. But, since this equilibrium is influenced by the vibrations depending on the heat content of the body, the arrangement corresponding to the more stable equilibrium may for other temperatures be different. In this case the crystallization under other conditions of temperature and pressure results in an arrangement other than the first; and if a certain critical temperature is transgressed there will come to pass a transformation of the first crystallized body into a second, chemically the same as the first but physically different, just as at the so-called melting, or freezing, point a transformation takes place from the crystalline state into the amorphous, or vice versa. Precisely as the melting, or freezing, point can be overstepped without transformation, — the state of the body then becoming labile, — so too does the same occur in the transformation of the different crystalline states, or "modifications", into one another. The property of a substance to present itself in several modifications is termed **polymorphism** (dimorphism, trimorphism, etc.), or, to distinguish it from the chemical isomerism, **physical isomerism.** The differences among the polymorphous modifications of a body exist only in the crystalline state: the transition into the amorphous state (by fusion or vaporization, and also by solution) necessarily does away with them.

Since, according to the foregoing assumptions, the crystal structure of a body depends on the nature of its molecules, there must exist a regular relation between the crystal structure and the chemical composition of the body; to set forth this relation is the object of chemical crystallography.* This study teaches that two chemically related bodies can present themselves in crystalline modifications whose respective structures stand in close relationship to one another, so that it is possible to determine the variation in crystal structure that is produced by

* A condensed statement of the subject is given by the author in his "Introduction to Chemical Crystallography". Authorized translation by Hugh Marshall. Edinburgh and New York, 1906.

a change in the composition of the molecule; such relationships are designated as **morphotropic.** They are most intimate (always supposing that corresponding modifications of the different substances are taken for the comparison) when it is a matter of two bodies whose respective molecules differ only in this: that into the place of an atom in the molecule of the first body there has entered, in the case of the second, an atom of different element, which has the same valence as the first element and is very closely related to it. In such cases the crystal structure of the two bodies is so similar that their crystal form is very nearly the same, under some circumstances even absolutely identical; such crystallized substances are therefore said to be **isomorphic.** Isomorphic bodies are capable of crystallizing together; *i.e.* of mixing together in various proportions to form crystals (so-called isomorphic mixtures) which behave, physically, like homogeneous bodies and whose properties vary continuously with the composition.

In order to present the regular connection among the crystal properties depending on the direction, these properties must be brought into a system from which the individual regularities follow as a matter of necessity. The basis of this system is **symmetry.** When a body is so constituted that the two opposite directions passing out from any point are absolutely equivalent, we say it has a "center of symmetry"; therefore the bi-vector properties are designated also as "centrally symmetrical", and, in contradistinction from them, the vector as "acentric".

The bi-vector properties are further distinguished into those of **higher symmetry** and those of **lower symmetry.**

The former have in common that for any and every direction their numerical value is determined by three, at the most, numerical quantities, which refer respectively to three definite, mutually perpendicular directions; and the numerical value of a definite property of this kind, for any direction, is proportional to the radius vector corresponding to that direction, of a triaxial ellipsoid whose three semi-axes are proportional to the three

numerical quantities holding good with the property. The "bi-vector properties of higher symmetry" are therefore called also **ellipsoidal properties.** In the special case where the three principal axes of the ellipsoid are of equal length, the ellipsoid passes over into a sphere; that is, the numerical value of the property in question is independent of the direction, like that of a scalar property. For this reason it is with the ellipsoidal properties that the relations are simplest, wherefore the presentation of the subject begins most properly with them; and, of them, with the optical properties. These, on account of their practical importance, will not only be developed from the fundamental ideas, but also treated so far in detail as is demanded by the practical application of optical methods to the determination of crystalline forms in chemical crystallography, mineralogy, petrography, and other natural sciences. The remaining ellipsoidal properties, *i.e.* the thermal, the electrical, and the magnetic, are so absolute analogous to the optical that they need be presented only in brief.*

With the properties of elasticity and cohesion of crystals the relations are more complex: the dependence of these properties on the direction cannot be represented by a surface of such simple form as an ellipsoid or a sphere, but only by one of more complicated form and of an in general lesser degree of symmetry. These properties shall therefore be distinguished from those of higher symmetry, the ellipsoidal properties, as properties of lower symmetry.

The lowest degree of symmetry, finally, is possessed by the vector properties, the properties in respect of which even the two opposite directions pertaining to the same straight line are not necessarily equivalent. Among these crystal properties belong those of solution and growth; the latter are the properties in virtue whereof the crystal assumes its definite geometric form.

Those bi-vector and those vector properties with which it is

* [This refers of course to the original work, as the present translation deals only with the optical properties.]

a matter of the action of mechanical forces on the crystal, are of especially high theoretical significance for the reason that, with the crystal under such conditions, the forces must be overcome that are exerted by its smallest particles on one another. The consideration of these properties therefore leads to that of the causes of crystal structure, and thus to the consideration of these particles and forces themselves. For treating the theories of crystal structure that come into consideration there are requisite certain conceptions from geometry*; and these conceptions are of additional importance for the reason that, being applicable likewise to the theories respecting the configuration of the atoms in the molecules, they may be considered as the basis of stereochemical views. These conceptions shall therefore first of all be elucidated, and then, with their aid, all the laws deduced that govern the geometric form of crystals; while the particulars of the several classes of symmetry, which follow from all those general laws, and the description of a number of crystallized bodies forming especially important or interesting examples of the several classes, are reserved for Part II.†

* According to this, geometry is an auxiliary science to crystallography, not the latter a part of the former; for geometry deals with the form, crystallography with the contents of the same, — *i.e.* with the material of the crystal as carrier of the properties, among which the form belongs.

† "Systematische Beschreibung der Krystalle." (Cf. footnote on p. 7.)

# THE OPTICAL PROPERTIES OF CRYSTALS

# THE OPTICAL PROPERTIES OF CRYSTALS

## THE NATURE OF LIGHT

To explain the properties of light we assume it is a periodic motion of the smallest particles of the **luminiferous ether**, a form of matter which pervades universal space and likewise all bodies, but which within the latter, under the influence of ponderable matter, takes on certain peculiarities.

If we imagine the ether at rest, *i.e.* the forces acting among its particles in equilibrium, and if by an impulse a particle be removed from the position corresponding to this equilibrium, if it thus be given a certain velocity in any direction, then according to this theory a force is awakened; this force is such that it drives the particle back toward its original position, thus diminishing its velocity, and that in exact accordance with the same law after which gravity acts on a pendulum set in motion by a sudden push.* The ether particle will therefore at a certain distance from its original position acquire the velocity zero, then

* After Huygens, the originator of the theory of the luminiferous ether, this force is commonly designated as elastic, and therefore we speak of the "elasticity of the ether"; yet it must be remarked that to explain the phenomena of light no further assumption is requisite than the quite general one mentioned above — that by a displacement of the ether particle a force is awakened which produces a motion corresponding to the laws of the pendulum. For this property of the ether, on which the transmission of light depends, Fletcher ("The Optical Indicatrix and the Transmission of Light in Crystals." London, 1892, p. 97) proposes the name "resilience". If one imagines in parallel position a row of pendulums among which there exists a connection representing that force, and imparts a motion to one of them, one obtains the complete analogue of the motion of a row of ether particles, described in the following.

return, under the influence of that force, with increased velocity, and reach the position of equilibrium with the same velocity it received from the initial impulse, but in the opposite direction; so that from the original position it now moves in the same way, but in the other direction, with diminishing velocity, in consequence of the force mentioned, to the same distance as before, and thence returns with increasing velocity. When finally the ether particle has again arrived at the point where its motion began, it is moving in the same direction as in the beginning, with exactly the same velocity; it therefore begins a second **vibration**, as such a to-and-fro motion is called. The extent of the vibration, the distance the particle travels during the same, is called its maximum amplitude, or briefly its **amplitude** *; the time required for the execution of one whole vibration, *i.e.* up to the next return of the same state of vibration, is called the **time of vibration**, or the **period.** The amplitude of the vibration depends on the velocity with which the particle is moved from the position of rest, and is proportional to that velocity; for example, if the initial impulse is such that it imparts double the initial velocity, then in the same time the particle travels double the distance and so attains double the maximum amplitude, while the period remains constant.

If the ether is a homogeneous medium in the most proper sense (cf. footnote †, p. 4), then on a straight line its particles must stand at equal distance from one another, and this distance must correspond to equilibrium of the forces (of attraction and repulsion) which act between each particle and the surrounding particles. Hence, if we imagine one of the particles of such an equidistant row, *e.g.* $\alpha$, Fig. 1, as set into the described vibratory motion, for example, if $\alpha$ be given a velocity in the direction of $\alpha'$, its distance from $\beta$ is increased,

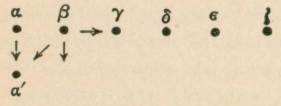

Fig. 1.

* [Amplitude is more commonly defined as the extent of the excursion of the particle on either side of its position of equilibrium.]

and in consequence of the force thereby awakened the particles $\alpha$ and $\beta$ must try to approach each other: not only is $\alpha$, in its new position $\alpha'$, drawn back by $\beta$, but also $\beta$ is attracted toward $\alpha'$, and attracted the more, the greater that force awakened in the ether. But in virtue of the same force the motion of $\beta$ toward $\alpha'$ is resisted by the attraction of $\gamma$; so that $\beta$, in consequence of these two attractions of $\alpha'$ and $\gamma$ respectively, takes an intermediate direction, — namely, that parallel to $\alpha\alpha'$.* In the same way the particle $\gamma$ is hereupon caused by the motion of $\beta$ to move in the same direction; and so on, all the following particles. When the motion has been transmitted as far as a certain particle, $\nu$ (Fig. 2), just beginning its motion, the row

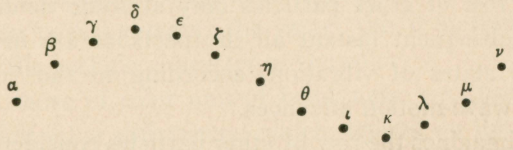


Fig. 2.

of points, previously straight, at this instant forms a **wave**, consisting of a crest and a trough of equal length. Such a motion is called a **wave-motion**; the distance $\alpha\nu$, the distance the wave-motion was transmitted while the first particle carried out one vibration, is called the **wave length** and is denoted by $\lambda$. Within one wave length are present, *side by side*, all states of vibration that are possessed *in succession*, by one and the same particle, during the time of one vibration. Every two particles whose distance apart is $\frac{1}{2}\lambda$ are in the opposite state of vibration. The quantity $\lambda$ is obviously proportional both to the vibration period and to the velocity with which the motion is transmitted. For if a wave-motion had the same transmission velocity, but double the vibration period, it would advance double the distance while the first particle executed one vibration; but likewise would $\lambda$ become twice as great if, with the same vibration

* Since the displacement $\alpha\alpha'$ is infinitesimally small, the angle between $\alpha'\beta$ and $\beta\gamma$ is to be regarded as infinitesimally less than 180°.

period, the transmission of the motion took place with double the speed.

Supposing the motion to have been transmitted through a longer row of points, these now form a **wave system**, which is divided into a number of wave lengths, all equal if the relations on the whole row of points, and accordingly also the transmission velocity of the motion, remain the same. All the particles of the wave system that stand apart from one another by $\lambda$ or by a whole multiple of $\lambda$ are in the same state of vibration; on the other hand, all those whose distance apart amounts to $\frac{1}{2}\lambda$ or to any uneven multiple of $\frac{1}{2}\lambda$ are in the opposite state of vibration. But of course such a wave system represents the state of a row of ether particles only at some definite instant: at every subsequent instant all the particles are in one of the succeeding states of vibration, according to the velocity with which the wave-motion advances.

If one imagines the row of ether particles considered on pages 12 and 13 as continued in the same way on the left of $\alpha$, then the same consideration must apply to all the particles here following $\alpha$; that is to say, if at any point of such a row a light-motion is excited, it must be transmitted out from that point in the two opposite directions in the same way. On this depends the membership of the optical properties among the bi-vector and the circumstance that every optical construction applies in the reverse sense.

A **light ray**, according to the above theory, the so-called "Undulatory Theory of Light", is thus a wave system; *i.e.* a straight row of ether particles along which a wave-motion is transmitted with a velocity that, although finite, is very great. The amplitude of the vibration determines the **brightness** of the ray, while its **color** is conditioned by the period. The transmission velocity of light in empty space is for all colors the same (about 186,000 miles per second); and, since the wave length $\lambda$ is the distance the wave-motion is transmitted during the time of one vibration, the wave length of light for different colors

must be exactly proportional to the vibration period. This latter is greatest for red rays; less for yellow, green, blue; and least for violet light, which, accordingly, carries out its vibrations the most rapidly. The wave length of red light in empty space amounts to about 0.000760 mm. and that of violet to about 0.000400 mm.; it follows that a red ray must execute about 390 bill on single vibrations in one second, a violet on the other hand about 750 billion.

As mentioned, the ether pervading ponderable bodies takes on, under their influence, other properties, in virtue whereof it transmits a light-motion with another velocity (usually less than in empty space). Consequently THE WAVE LENGTH OF LIGHT OF THE SAME COLOR IS DIFFERENT IN DIFFERENT BODIES; only the vibration period remains the same; this it is, therefore, that determines the specific character of the color.

Besides those of color and of brightness, we observe among light rays still other differences, which depend on the manner of the vibratory motion. If this takes place as in the example assumed on page 12, along a definite direction perpendicular to the ray, in which case it is said to be "transverse", the ray in question cannot behave alike in all directions about the line along which it is transmitted: it must, since all the vibrations take place in one definite plane, exhibit a certain one-sidedness (polarity); it is therefore designated as a **plane-polarized** light ray. This one-sidedness must obviously be symmetrical with reference both to the plane in which the transverse vibrations take place and to the plane that intersects this plane perpendicularly in the transmission line of the ray; we shall call the former plane the **transverse plane** of the ray, the latter its **plane of polarization** (briefly: "polarization plane"). Such a ray can only then behave alike in all directions, when the azimuth of its transverse, or "vibration", plane rotates very rapidly about the line of transmission; that is to say, when the successive vibrations always take place in a plane that, as compared with the plane of the immediately preceding vibration, is rotated to the amount of a small angle, so

that after a certain time the total rotation amounts to 180°: during this time, then, the vibrations have taken place in all directions about the ray. Now with the enormous number of light-vibrations during one second (see above), this time is far shorter than is required to apprehend an impression of light. During the latter interval, therefore, the vibration plane will have rotated many times about the line of transmission, and the light ray must accordingly produce the impression of identical behavior in all transverse directions. So-called common, or **ordinary** (not polarized), light consists of rays whose vibratory motions are of the kind just described.

Besides the rectilinear vibrations there are, however, light rays of other kinds, in which the ether particles move in circles or ellipses and which consist accordingly of *circular* or *elliptical* vibrations.

## COMBINATION (INTERFERENCE) OF PLANE-POLARIZED LIGHT

If an ether particle is caught up at once by two light-motions having the same vibration period, it arrives after a certain time at the point to which the single motions would have carried it, had they acted just as long in succession; the two motions thus combine to form one resultant. The simplest case of this phenomenon is that where the combining wave-motions are transmitted along the same direction and where their transverse vibrations take place in the same plane, the two motions differing therefore only in state of vibration and in amplitude; in this case the two partial motions are said to **interfere**, and their combining is called **interference.**

If from a source of light there proceed simultaneously light rays having parallel vibration direction and the same state of vibration, and if two of them come together in the same line of transmission after they have passed over a different length of path, then the state of vibration with which they come to inter-

fere at a point of their now common path is in general unlike. The difference between their respective states of vibration follows from the difference between the lengths of path they have travelled, expressed in wave lengths. This difference is called their **difference of path** * (briefly: "path difference"). If it equals $n\lambda$ (where $n$ is an integer), then at every point of their now common path the two wave systems interfere with the same state of vibration. Supposing *I* (Fig. 3) to represent the state in which a row of ether particles would be at a certain instant if only the first of the interfering wave-motions existed,

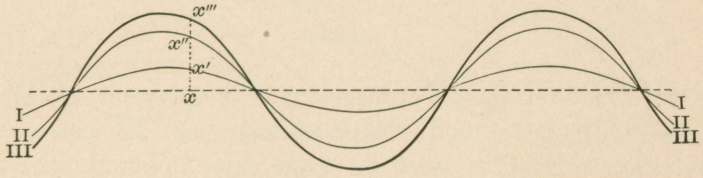


Fig. 3.

and *II* their state at that instant if only the second acted, then is *III* the state of the ether particles in consequence of the wave-motion resulting from the two. For any point, $x$, would in virtue of the first partial wave-motion have been moved to $x'$, in virtue of the second to $x''$; so it must now be at a distance $xx'''$ from the position of rest, this distance being the sum of the displacements $xx'$ and $xx''$, produced by the single motions. Accordingly, there results from the interference a wave-motion which has the same state of vibration as the combining motions and whose amplitude is the sum of their amplitudes.

When two interfering light rays of the given kind come together in such a way that their difference of path amounts to

* [Commonly designated by crystallographers as "difference in phase". But on account of the frequent misapplication of the word 'phase' the author, in his latest edition, has discarded it; and accordingly has made no service of the term "Phasendifferenz", using instead the obviously more representative "Gangunterschied".]

$\frac{1}{2}\lambda$ or to an uneven multiple of this quantity, then, on every ether particle caught up by them simultaneously, they act in the opposite sense. In Fig. 4, for example, the point $y$ would by the motion $I$ alone have been driven upward as far as $y'$, but downward to $y''$ if only $II$ had acted; and consequently, after the action of the partial motions its distance, $y'y'''$, from the position of rest must be the difference between the two distances. Thus, from the interference of two wave-motions having $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, or $\frac{5}{2}\lambda$ (expressed generally: $[n + \frac{1}{2}]\lambda$) difference of path there results a single wave-motion (*III* in Fig. 4) whose

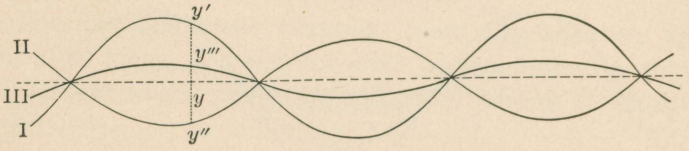


Fig. 4.

state of vibration is the same as that of the stronger of the two combining wave-motions, and whose amplitude is equal to the difference between their amplitudes. In the special case where the two interfering light rays have equal amplitude, *i.e.* equal brightness, that of the resultant is zero; in other words, the two motions completely annihilate each other.

If the path difference has another value than in the two cases considered above, there results from the interference a wave-motion of another state of vibration and of another amplitude.

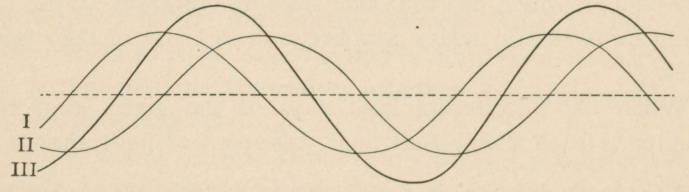


Fig. 5.

For example, if the path difference amounts to $\frac{1}{4}\lambda$ or $\frac{5}{4}\lambda$ (expressed generally: $[n + \frac{1}{4}]\lambda$), the resulting motion is shifted, with re-

spect to the two wave systems (provided they have equal amplitude), by $\frac{1}{8}\lambda$, as may easily be seen from Fig. 5. Shifted just as much in the opposite direction is the wave arising by interference, if the two wave systems have a path difference of $\frac{3}{4}\lambda$, $\frac{7}{4}\lambda$, . . . (expressed generally: $[n + \frac{3}{4}]\lambda$). See Fig. 6.

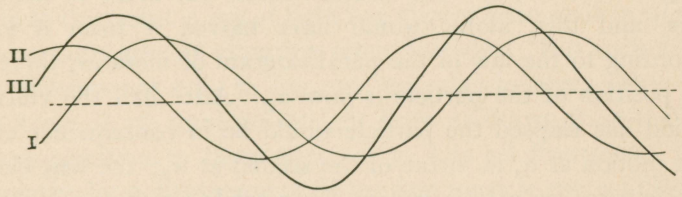


Fig. 6.

So the intensity of the vibration that is produced by the interference of two plane-polarized light rays with parallel vibration direction, may have the most different values lying between zero and the sum of the intensities of the single rays, according to their difference of path; but the vibration plane of the resulting light ray is always the same as that of the two interfering rays.

The combination of two plane-polarized rays transmitted along the same line leads to a different result, on the other hand, if their vibration planes form an angle. Let us consider, for example, two vibrations of equal amplitude whose vibration directions form a right angle with each other; and let their path difference be $\frac{1}{4}\lambda$, or expressed generally, $(n+\frac{1}{4})\lambda$. Then, if the two rays be perpendicular to the plane of Fig. 7 at the point $\alpha$, an ether particle at $\alpha$ will in virtue of the one motion be moved in the direction $\alpha\alpha_1$; in virtue of the other, toward $\alpha_3$. At the instant when, of the former motion, one-fourth of its vibration period has elapsed, if this motion alone had acted the particle would be at $\alpha_1$; in virtue of the second motion, which is retarded

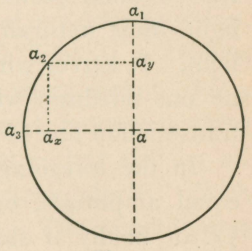


Fig. 7.

$\frac{1}{4}\lambda$ as compared with the first, the particle would be just about to leave its position of rest,—*i.e.* its displacement in the direction $\alpha\alpha_3$ would be zero. Accordingly $\alpha_1$ is the position of the particle at that instant. At a somewhat later instant the particle would in virtue of the first motion have been moved a certain distance, $\alpha_1\alpha_y$, downward; but the second motion has acted at the same time, and this, alone, would have moved it from $\alpha$ to $\alpha_x$; according to the law of the parallelogram of motions, therefore, the position of the particle is now $\alpha_2$. After half the vibration period has elapsed the particle would be in consequence of the first motion at $\alpha$, in virtue of the second at $\alpha_3$; the first elongation is zero; so at this instant the particle reaches $\alpha_3$. And so on. The path of the ether particle will in the case before us be exactly that described by the end of a pendulum, if by a sudden push one moves it from its position of rest and then, at the instant when it attains its greatest distance (at $\alpha_1$ in Fig. 7) from that position, gives it a push of equal strength in the direction $\alpha\alpha_3$, *i.e.* tangentially: its path is a *circle.* Thus, by the interference of two light rays plane-polarized perpendicularly to each other there arises light vibrating in a *circle;* and this light has the same vibration period as had the two single rays, since a quarter of the circular path has been passed over in a quarter of the vibration period. Such light, known as **circularly polarized** light, must naturally possess different properties from ordinary light. It agrees with ordinary light only in this: that it exhibits no one-sidedness with respect to a plane passing through the transmission line of the ray.

In the foregoing case, if the two interfering rays have not equal amplitude, or if their difference of path is other than $\frac{1}{4}\lambda$, or finally if the angle between their respective vibration directions is not 90°, then in general there results a motion of the ether particles in an ellipse; and the ray arising by the combination is said to be **elliptically polarized.** In different planes through its line of transmission such a ray must behave differently.

## OPTICALLY ISOTROPIC (SINGLY REFRACTING) BODIES

### PROPAGATION OF LIGHT

Up to this point a ray of light has always been spoken of only as a straight row of vibrating ether particles. But such a particle belongs not only to *one* row, but also to every other that we obtain if we imagine straight lines as drawn from the particle in question to those adjacent to it in all directions. Consequently, by a displacement of the first particle the equilibrium must be disturbed and a force awakened in all these rows; that is, the light-motion will be propagated in all directions in the ether.

The manner of this propagation will depend on whether vibratory motions of any one kind are transmitted *along different directions* with equal, or with different, velocity; in other words, it will depend on whether the ether is an isotropic medium (see p. 3) or one that is anisotropic (heterotropic) — one in which the transmission velocity of a wave-motion in different directions is different.

The ether is isotropic in empty space, in isotropic (amorphous) bodies (see p. 3), and in the crystals of one definite group. In such homogeneous bodies as these the light of a definite color is accordingly transmitted not only on every section of one of the above-mentioned straight lines, — and moreover along the two opposite directions of every section, — but also on all straight lines, of whatever direction, with equal velocity. Such bodies are said to be **optically isotropic.** The luminiferous ether contained in them differs from the ether of space in this particular: it transmits light of a different color with a different velocity, but likewise in all directions with equal velocity. So the velocity of light in optically isotropic bodies depends not only on the nature of the body, but also on the period of the light-vibrations.

If at any point in such a medium there begins light-motion of a definite vibration period, then, since the motion is transmitted in all directions equally fast, at the end of one whole vibration period, $T$, all points standing at a distance of one wave length $\lambda$ from the first point, *i.e.* all points on a spherical surface of the radius $\lambda$, will begin their motion simultaneously. After the time $2T$ the same will be done by all the points of a spherical surface having the radius $2\lambda$, while at the same instant those of the surface first mentioned will begin their second vibration. And so on. Just as a row of ether particles is divided by the vibratory motion into a number of equal wave lengths, so does the ether surrounding the starting-point of the light-motion become divided into a number of spherical shells standing at the distance $\lambda$ from one another; and in these shells all points lying at equal distance from the boundary of two shells, on the same side, are in the same state of vibration. If from the starting-point of the motion we lay off in all directions lengths proportional to the wave length, their extremities form a surface called the **wave-surface** of the motion emanating from the first point. This surface contains all ether particles that begin their motion simultaneously. Thus, in an isotropic body, as glass, a light-motion which has emanated from the point $C$ (Fig. 8) will have arrived after the definite time $T$ at the surface of a sphere, $SS'$. At that instant every point of this surface, as $P$, begins its motion; and since $P$, as also the point $A$, lies at once on all possible rows of points, *i.e.* rows of ether particles, diverging from it, it must disturb the equilibrium in all these rows; in other words, a like wave-motion must proceed from the disturbed point of $SS'$ in all

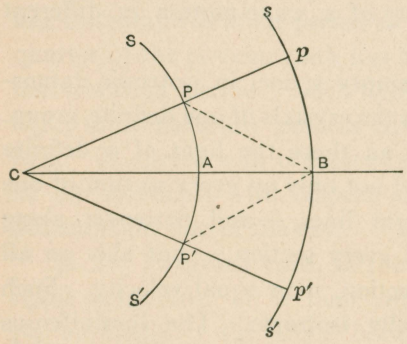


Fig. 8.

directions. Since, therefore, every disturbed point of such a system itself thus becomes the center of a new wave-surface, then at the point $B$ of a spherical surface $ss'$, whose points later begin their motion, there will arrive light-motion not only from $A$, but also from every other point of the first spherical surface $SS'$. In order to judge of the effect that all these motions produce on the motion of the point $B$, one must take into account the distance, from $B$, of the points whence the motions proceed. If we observe the sphere $SS'$ from $B$ and imagine circles of various diameters as drawn on its surface about the point $A$ (like parallels of latitude about the north or south pole of the earth), then obviously all the points of such a circle stand equally distant from $B$ [$B$ is supposed to lie on $CA$ prolonged]; the several circles, on the other hand, lie at different distances from $B$. If in addition to every circle we further imagine that one as constructed whose distance from $B$ is greater by exactly $\frac{1}{2}\lambda$, the two motions that, emanating respectively from a point of the one circle and from the corresponding point of the other, come together at $B$ will by their interference totally annihilate each other, and produce no motion at all at $B$. Now if one compares the effect on the point $B$ of all these circular zones of the spherical surface $SS'$, taking their area into account, it is found, as the aggregate result, that the motions coming to $B$ from all the different parts of the surface are entirely destroyed by the motions coming from other parts; with the exception of the motion emanating from $A$. Therefore $B$ is reached only by that vibratory motion that emanated from $A$; this, alone, sets $B$ in motion. Since the same consideration applies to every point, the result is this: of the motion that, emanating from $A$, must in a certain time have been transmitted as far as the spherical surface $ss'$, only one portion can exercise an effect,—namely, the portion that comes to $B$; and in like manner, from $P$ motion is communicated only to $p$ [on $CP$ prolonged], instead of in all directions, from $P'$ only to $p'$, and so on.

The points at which the motion from $C$ gradually arrives thus lie on a *straight line;* that is, the propagation of light as a wave-motion of the ether must be *rectilinear.*

From the surface $SS'$, to which the light has been propagated in a certain time, we obtain for a later instant the corresponding surface, $ss'$, if about the single points of the former surface we construct wave-surfaces with the radius corresponding to the intervening time, and find the surface that envelopes all these wave-surfaces. (See Fig. 9.) If we connect the point of tangency of each of these wave-surfaces and the enveloping surface with its center, *e.g.* the point of tangency $p$ with $P$, then for each single surface we obtain the corresponding **ray.** This construction (called, after its inventor, **Huygens's construction**) will farther on be repeatedly employed to find the wave-surfaces, and thereby the positions, of rays — the straight lines ($AB$, $Pp$, $P'p'$, in Fig. 9) connecting the centers of the several wave-surfaces with the points at which these surfaces are touched by the enveloping surface. The surface whose radii are the several rays is therefore called also the **ray-surface.**

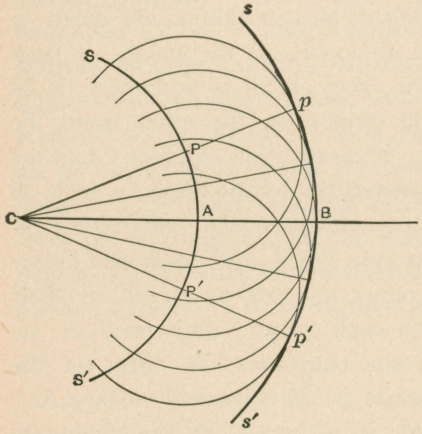

Fig. 9.

Now if we consider a light ray as a row of transversely vibrating ether particles, the vibrations for any ray $Cp$ obviously take place in the plane (the so-called "wave-plane") tangent to the ray-surface at the point $p$; further, if we imagine a bundle of parallel rays as simultaneously transmitted along the direction $Cp$, then at a certain instant the tangential plane to the ray-surface at the point $p$ forms their "front", where-

fore it is spoken of also as the **ray-front.** The ray that belongs to a given ray-front is according to this the straight line between the center of the ray-surface and the point at which the ray-front touches the ray-surface. In the most simple case, the one before us, in which the latter has the form of a sphere, the ray obviously coincides with the normal to the ray-front (designated more briefly as **front-normal**).

If a medium is *not homogeneous* (**heterogeneous**), the ether in different parts of it will be differently constituted, and a light ray transmitted in the medium will therefore pass, as it were, through divers media; but we may regard such a body as composed of divers homogeneous parts, and reduce this case to that of the homogeneous body if we know what change a wave-motion suffers in passing from one medium into another.

If we imagine an ether particle at the boundary of two bodies in which the ether is differently constituted and in which therefore the transmission velocity of light is different, this particle belongs both to the ether of the one body and to that of the other body; consequently, if a light-motion reaches such a particle and sets it in vibration, the equilibrium must thereby be disturbed in all the rows of ether particles that connect the particle in question with the adjacent ones of the first medium, — and disturbed likewise in all the rows of the second medium to which the considered ether particle at the boundary belongs. Light-motion must therefore be propagated backward from that particle into the first medium, but at the same time also penetrate the second.* A ray of light arriving at the boundary of two

* That the motion penetrates, appears to be the case only with transparent bodies; yet, since likewise the apparently opaque exhibit a weakening of the reflected light, with them too a part of the incident light must have penetrated. In fact, the difference consists only in this: that in transparent bodies the light rays can penetrate to a greater depth without becoming perceptibly weakened (absorbed), but with the so-called " opaque " bodies only to a slight depth. This weakening (absorption) of the light consists in a transference of the motion from the ether particles to the smallest parts of the body itself, and therefore in an apparent loss of the motion, as light.

media must, according to this, become divided into a ray that is thrown back (**reflected**) and one that penetrates the second medium. As will be found farther on, the latter ray is deflected (**refracted**) from its original direction.

### REFLECTION OF LIGHT

The law according to which a ray of light reflected at the boundary of two media is transmitted backward into the first, can easily be deduced by means of Huygens's construction, explained in the last section. This, in the following, will again be done first for the simplest case, that of two *optically isotropic* media. If upon a boundary-surface of the same, and at any angle, there fall parallel rays, *i.e.* rays coming from a source of light so distant that a number of mutually adjacent ones may be regarded as exactly parallel and the corresponding area of the ray-surface (a sphere with infinite radius) as absolutely plane, then the area of the separating surface on which they impinge, since it is exceedingly small, must also be regarded as a plane. If $MN$ (Fig. 10) be the intersection of this plane (perpen-

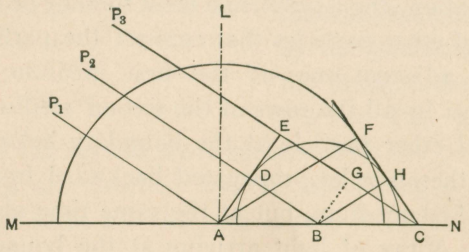


Fig. 10.

dicular to that of the figure) with the plane of the figure, and $P_1, P_2, P_3$ three rays lying equidistant from one another in the plane of the figure, then the plane perpendicular to these rays through $ADE$ is their ray-front at the instant when the first ray strikes the boundary-surface. During the time that elapses

before also the last, $P_3$, has reached the boundary-surface (at $C$), the reflected first ray is transmitted from $A$ just as far into the upper medium; *i.e.* to some point on the upper half of the sphere we have to construct, as its ray-surface, about $A$ with the radius $EC$. The ray $P_2$, after half the time interval has elapsed, strikes the boundary-surface at $B$, and during the second half of this time it is transmitted backward into the upper medium to some point of the sphere to be constructed about $B$ with the radius $GC = \frac{1}{2} EC$. Finally, the ray $P_3$ reaches the boundary-surface exactly at the end of the same time interval; so the ray-surface of the reflected ray belonging to it is a sphere constructed about $C$ with the radius zero, — in other words, the point $C$ itself. The common ray front after the reflection is the plane tangent to the several ray-surfaces; and since obviously by this plane all spheres whose centers lie on the straight line $MN$ must be touched at points falling in the plane of the figure, we obtain the intersection of the reflected ray-front with the plane of the figure if we draw from $C$ the tangent $CHF$. The reflected rays are thus $AF$ (the reflected ray $P_1$) and $BH$ (the reflected ray $P_2$). Now since according to construction $AF = EC$, and since the two right-angled triangles $ACF$ and $ACE$ have their hypothenuse in common, the angle $CAF$, which the reflected ray forms with $MN$, is equal to the angle $ACE$, — *i.e.* to the angle included between $MN$ and the incident light rays. The same applies also to the ray $BH$. In other words, the rays, parallel before the reflection, are parallel likewise after the reflection; and their common ray-front, $CHF$, forms the same angle with $MN$ as did the original ray-front, $ADE$, but lies reversed in its relation to the normal, $AL$, to the boundary-surface. If we call this normal line the **axis of incidence** and the angle that any incident ray forms with the same its **angle of incidence** ($i$), designating further the plane through the incident ray and the axis of incidence (*i.e.* the plane of the figure in Fig. 10) as the **plane of incidence,** and finally the angle included by the reflected ray

with the axis of incidence as the **angle of reflection**, — then the law of the reflection of light reads:

" The reflected ray lies in the plane of incidence, and the angle of reflection is equal to the angle of incidence."

If we suppose a reflecting plane $MN$ (Fig. 11) to be struck not by parallel but by diverging light rays, which proceed from a luminous point, as $A$, situated at only a short distance from it, then, reflected according to the law just stated, the rays will diverge exactly as though they came from a point $A_1$, lying on the normal to $MN$, or to $MN$ prolonged, just as far on the other side of $MN$ as $A$ on this side. The same applies to every other point of a luminous body. Since our vision is so organized that we locate the image of an object wherever the divergent rays entering our eye converge or seem to converge, an eye looking toward the reflecting surface in the direction from $E$ will see $A$ in the position $A_1$, $B$ in the position $B_1$, and intermediate points of the object $AB$ between $A_1$ and $B_1$ — thus an image of the object at $A_1B_1$. The plane $MN$ is a " mirror-plane ", or **plane of symmetry**, of the construction shown in Fig. 11; that is, the image $A_1B_1$ lies equal and opposite to $AB$ with reference to that plane; we therefore say that $AB$ and $A_1B_1$ are " symmetrical with reference to $MN$ ".

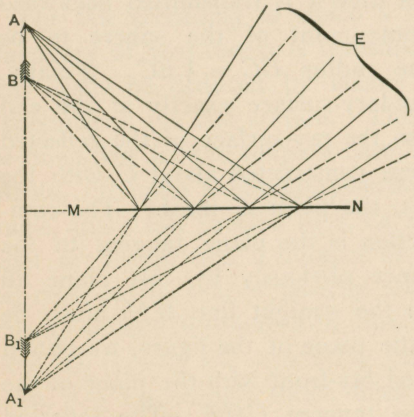

Fig. 11.

In crystallography the law of the reflection of light finds an important application; namely, for determining the angles which the plane faces of crystals form with one another. The instrument employed for this purpose is called the **reflection goniometer,** and its principle is as follows: —

The graduated circle $C$ (Fig. 12) has a rotatable axis rigidly connected with the index or vernier $V$, so that by means of $V$ one may read off on the graduations of the circle an angle through which the axis has been rotated. The axis projects in front of the circle, where it carries the crystal, this being so

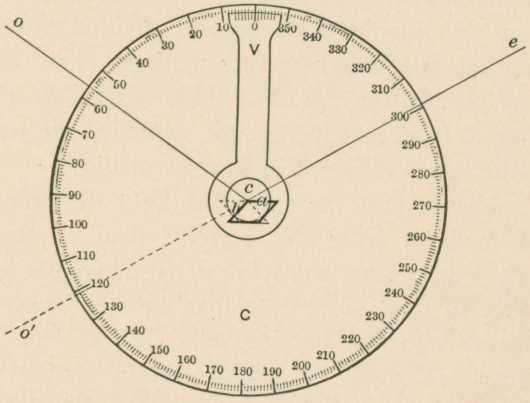


Fig. 12.

fastened that the intersection-edge of the two faces whose inclination is to be measured is **centered** (*i.e.* in its prolongation strikes the center of the circle) and **adjusted** (*i.e.* stands normal to the plane of the circle). If of the two crystal faces designated by $a$ and $b$ the former be then in such a position that to the eye at $e$ the virtual image of an object lying in the direction $co$ appears in the direction $eco'$, and if we subsequently rotate the axis, and with it the crystal, — whereat the edge $a/b$ of course remains motionless, — until $b$ has arrived at a position parallel to that previously occupied by $a$, the crystal will now have the position represented by the dotted outline, and we shall now see the reflected image of $o$, thrown back from the second face in the same direction, $ce$, in which it was previously reflected from the first. Hence it is seen from Fig. 12 that the angle we must rotate is the angle at which the two

planes $a$ and $b$ intersect each other, and that accordingly the exterior angle * of the crystal is determined when this rotation angle is read off on the circle.†

## REFRACTION OF LIGHT

As was mentioned on page 25, a wave-motion arriving at the boundary of two media is in general divided there into a reflected motion and a penetrating motion. The direction of the wave-ray penetrating the second medium is, as will now be shown, different from that in the first, provided the transmission velocity of the ray is different in the two media.

Let us consider first the case of a light ray transmitted in the second medium with less velocity than in the first. If $P_1, P_2, P_3, P_4$ (Fig. 13) be rays lying so close together and derived from a source of light so remote that we may regard them as exactly parallel and the portion, $CD$, of the ray-front lying between them as a plane, then at a certain instant the ray $P_1$ will strike the boundary-plane, $MN$, at $D$. From thence onward the penetrating wave-motion is transmitted in the second medium, but with less velocity. If this velocity be, for example, only half that in the first medium, then after the time $t$ has elapsed, which the ray $P_4$ requires to travel the distance

* This is called the "normal angle" (because it is equal to that included between the normals to the faces $a$ and $b$), or simply the "angle", of the faces $a$ and $b$. Its supplement is the interior, or actual "interfacial", angle — the angle the two faces $a$ and $b$ include within the crystal.

† [Particulars as to the arrangement and use of the reflection goniometer, as also of other instruments, apparatus, and various appliances employed in the investigation of crystals, will be found, illustrated with numerous figures, in Part III of the author's "Physikalische Krystallographie". Leipzig; 3rd ed. 1895, 4th ed. 1905. See also the trade catalogues (cf. Appendix) and C. Leiss: "Die optischen Instrumente der Firma Fuess". Leipzig, 1899. In English many of the same are more briefly described in various works, especially in E. S. Dana: "A Text-book of Mineralogy", New York, 1904, and in A. J. Moses: "The Characters of Crystals", New York, 1899. Cf. also footnote †, p. 80. A very comprehensive and profusely illustrated French work is that of Duparc and Pearce (L. Duparc et F. Pearce: "Traité de technique minéralogique et petrographique. 1re partie: Les méthodes optiques". Leipzig, 1907. Additional literature is mentioned ibid.]

$CA$, $P_1$ will have penetrated into the second medium a distance amounting only to the half of $CA$. $P_1$ will then have reached some point of the hemisphere described about $D$ with the radius

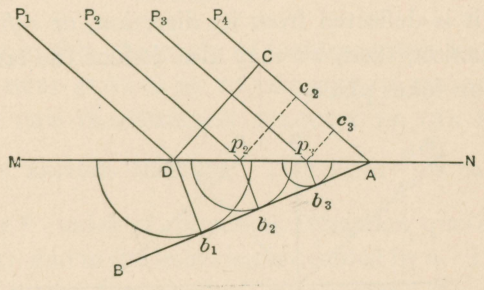


Fig. 13.

$Db_1 = \frac{1}{2} CA$. The second ray, $P_2$, strikes the boundary-surface $MN$ somewhat later, namely, at the instant when $P_4$ has arrived at the point $c_2$; up to the end of the time $t$ the ray $P_4$ moves in the first medium a distance $c_2A$ forward; the ray $P_2$, during the same time in the second medium, can travel only half that distance. This ray $P_2$, therefore, at the end of the time $t$, will be at a point of the hemisphere constructed about $p_2$ with the radius $p_2b_2 = \frac{1}{2} c_2A$. In the same way, after the time $t$ has elapsed the ray-surface of the ray transmitted from $p_3$ into the second medium will be a hemisphere about $p_3$ with the radius $p_3b_3 = \frac{1}{2} c_3A$. Finally, that of the ray $P_4$ at the same instant will be the point $A$ itself. The common front of all rays lying between $P_1$ and $P_4$ in the second medium is the surface tangent to all their respective ray-surfaces in that medium; namely, the plane passing through $AB$ perpendicular to the plane of Fig. 13. We should also have found this plane had we constructed the ray-surface only of one ray, as $P_1$, and drawn the tangent to it from $A$. So the plane ray-front $CD$ in the first medium is in the second still plane, but has a different direction. The latter is naturally true also of the rays—the straight lines between the points $D$, $p_2$, $p_3$, etc., and the points of tangency $b_1$, $b_2$, $b_3$, etc.

From Huygens's construction we accordingly see in the first place that, although the wave-motion penetrating the second medium remains in the plane containing the incident ray and the axis of incidence (*GF*, Fig. 14), *i.e.* in the plane of incidence, yet it is deflected from its direction, or *refracted.* From this construction, then, we may also deduce the law after which the refraction takes place.

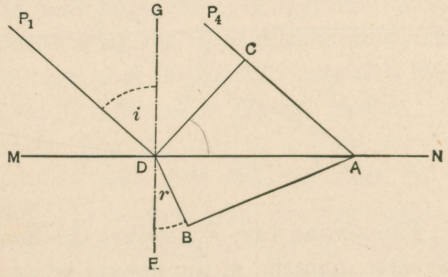


Fig. 14.

According to construction the lengths $CA$ and $DB$ are to each other as the transmission velocity of light in the first medium is to that in the second medium (in our example as 2 : 1); if we call the former velocity $v$ and the latter $v'$, then

$$\frac{AC}{BD} = \frac{v}{v'}.$$

But the angle of incidence $i = P_1DG = ADC$, and the angle $BDF$, which the refracted ray forms with the axis of incidence and which is called the **angle of refraction** ($r$), $= \angle DAB$. In the two right-angled triangles $ACD$ and $ABD$ we have

$$\frac{AC}{AD} = \sin ADC = \sin i,$$

$$\frac{BD}{AD} = \sin BAD = \sin r.$$

Dividing the first expression by the second gives

$$\frac{AC}{BD} = \frac{\sin i}{\sin r},$$

and since $AC$ and $BD$ are proportional to the transmission velocities,

$$\frac{\sin i}{\sin r} = \frac{v}{v'}.$$

According to this, the angle of incidence may have any size, but ITS SINE MUST ALWAYS BE TO THE SINE OF THE ANGLE OF REFRACTION AS THE TRANSMISSION VELOCITY IN THE FIRST IS TO THAT IN THE SECOND MEDIUM. This ratio $\frac{v}{v'}$ is called the **refractive index** * (index of refraction, refraction quotient, refraction exponent), and in the following is denoted by $n$. If, therefore, by determining the direction of any incident ray and of the corresponding refracted ray, $n$ has once been found for two optically isotropic media, then for every ray of light falling in a different direction upon the boundary-surface of the same two media it permits the direction of the corresponding refracted ray to be calculated.

In the cases hitherto considered the lesser light velocity was that in the second medium: $v$ was greater than $v'$ ($v > v'$) and accordingly the refractive index $\frac{v}{v'} > 1$. In this case, for every angle of incidence, the angle of refraction is smaller than the angle of incidence: the ray is refracted *toward* the axis of incidence.

When on the other hand the light ray is transmitted in the second medium with the greater velocity, *i.e.* when $v' > v$ and consequently $n < 1$, then, as follows from the construction in

* The term " refractive index" of an optically isotropic solid usually refers to the refraction from air into the solid, to the ratio therefore of the light velocity in air to that in the solid; and this ratio, since in nearly all solid bodies light travels more slowly than in air, is in most cases greater than unity. Since for light rays of any one color the definite numerical value of $n$ is always the same with one and the same substance, then, under circumstances that do not admit of the determination of other properties (*e.g.* in the case of gems), this value may be employed for identifying the substance.

Fig. 15, — which, being wholly analogous to those preceding, needs no special explanation, — the ray is refracted *away* from the axis of incidence. Since accordingly the angle of refraction is here greater than the angle of incidence, there must be a value of $i$ for which $r = 90°$. This is the case when $\sin i = n$; for $\sin r$ must then be unity. That ray, therefore, whose angle of incidence is of such a size that its sine is exactly equal to the refraction exponent, is so refracted that it is transmitted *parallel to the boundary-surface*, and thus it does not penetrate the second medium. Just as little, according to the law of refraction, can a ray penetrate whose angle of incidence is still greater; for then $\frac{\sin i}{n} > 1$. This value $\frac{\sin i}{n}$ is the value for the sine of the angle of refraction; but no angle exists whose sine is greater than unity; so in this case also there is no angle of refraction.

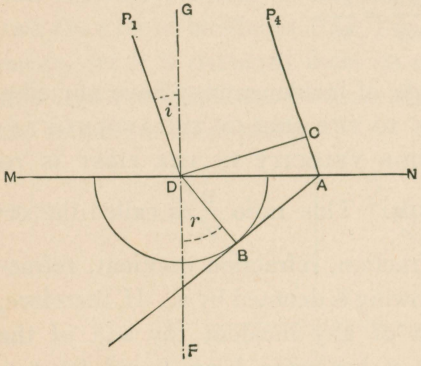

Fig. 15.

Figure 15 represents the refraction at the boundary of one medium with a second (the lower), in which the velocity is double that in the upper medium, so that the refractive index is $\frac{1}{2}$. The ray-front in the second medium is found, in a simplified manner, as the tangential plane $AB$ to the wave-surface of the ray $P_1$ in the lower medium, this ray-surface being described about $D$ with the radius $DB = 2\,AC$. Accordingly $DB$ is the direction of the refracted ray. In Fig. 16 the angle $i$ is such that $\sin i$, *i.e.* $\sin CDA \left( = \frac{AC}{AD} \right)$, has the value $n = \frac{1}{2}$; so that $AD = 2\,AC$. The wave-surface of the ray $P_1$ must be constructed about $D$ with the radius $DA$, and thus it passes through $A$; the

tangent to the circle from the point $A$, *i.e.* the front of the refracted ray, has the direction $AB$; consequently the ray itself,

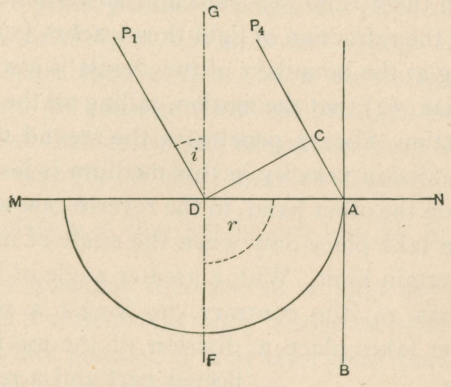


Fig. 16.

the straight line from the point of tangency to the center, lies in $MN$: the angle of refraction is 90°, as was already taught by the above calculation. Figure 17, finally, shows the refraction

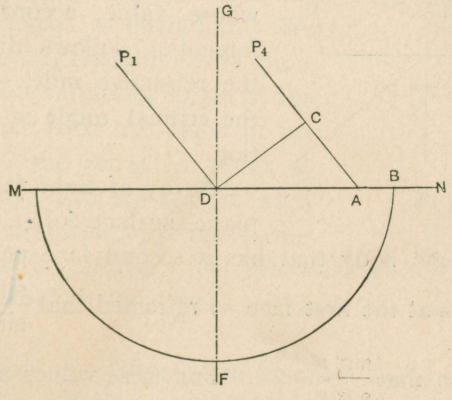


Fig. 17.

in the case of a still greater angle of incidence, under otherwise the same conditions. The wave-surface of the ray $P_1$ in the second medium is the spherical surface described about $D$ with

the radius $DB = 2\,AC$; to this surface no tangential plane may be passed from $A$, because $A$ lies within it, and therefore in the second medium there is no ray-front and no refracted ray.

The law of the refraction of light thus teaches (1) that a light-motion arriving at the boundary of two media is always deflected from its direction; (2) that the motion, falling on the boundary in whatever direction, always penetrates the second medium, provided its transmission velocity in this medium is less than in the first; (3) that, on the other hand, in the reverse case an entrance of the motion can take place only when the angle of incidence does not exceed a certain limit. With a greater angle of incidence the incident ray has, on the contrary, no longer a refracted ray: there no longer takes place a division of the motion into a reflected part and a refracted part: the whole motion is reflected. Therefore this phenomenon is called the **total reflection** of light; and the smallest angle of incidence with which it occurs — whose value, according to the foregoing, follows directly from the refractive index — is termed the **critical angle** of total reflection.

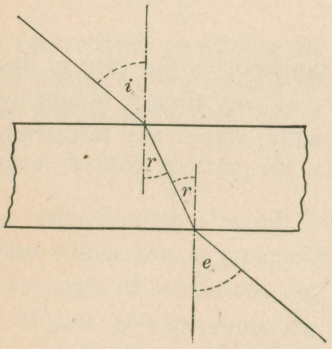

Fig. 18.

A ray of light falling on the plane surface of a transparent optically isotropic body that has a second face parallel to the first (Fig. 18) is at the first face so refracted that $\frac{\sin i}{\sin r} = \frac{v}{v'}$, and at the second so that $\frac{\sin r}{\sin e} = \frac{v'}{v}$. For these values are the ratios of the transmission velocity of the light in the medium from which it arrives at the second boundary, to that in the medium into which it emerges. But thence follows

$$\sin e = \sin i, \quad e = i;$$

that is, the ray continues its way on the other side of the second boundary-surface along the same direction in which it fell upon the first. So a **plane-parallel** transparent plate introduced into the path of parallel light rays does not change their direction. It is at the same time evident, from the foregoing, that the refractive index for passage from one medium into a second is the reciprocal of that in the case of passage from the second into the first; for example, if the refractive index of a ray passing from air into glass is $n$, that for the exit into air is $\frac{1}{n}$.

Now if we consider the case of a light ray entering and emerging from a transparent body whose faces of entrance and emergence are *not* parallel, it is at once clear that the emergent ray cannot then be parallel to the incident ray. If Fig. 19 represent the cross-section of such a body, called a **prism**, bounded by the two planes $MN$ and $MO$, the construction shows that a light ray, $LL'$, passing through the prism experiences a **deviation.** We may calculate this deviation if we know $\alpha$, the so-called **refracting angle** of the prism, together with the angle of incidence $i$ and the refractive index $n$. But conversely, with the aid of such a prism we are able, by measuring the deviation it produces, to *determine the refractive index n of a light ray passing from air into the substance of the prism.*

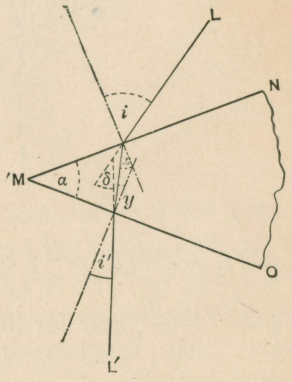

Fig. 19.

The solution of this problem assumes an especially simple form when the entering and the emerging ray include equal angles with the two faces of the prism respectively, in other words, when $i = i'$ and $x = y$; because then (as may be proved in various ways) the deviation, $\delta$, produced by the prism assumes its *minimum value.*

This case is represented in Fig. 20, where $ANOC$ indicates the path of the ray. Since here $\angle MNO = \angle MON$, the direction, $NO$, of the ray within the prism is perpendicular to the bisector of the prism-angle; consequently $\angle ONL' = \frac{\alpha}{2}$. If $Nm$ and $Om$ be the prolongations of the two directions of the ray outside the prism, and if $no$ be parallel to $NO$, then $\angle Omo = \angle Nmn = \frac{\delta}{2}$, where $\delta$ stands for the total deviation produced by the prism; *i.e.* for the supplement of the angle $AmC$. If we prolong $ON$ to $B$, the angle of incidence $i = ANL = ANB + BNL$. But $ANB = ONm = Nmn = \frac{\delta}{2}$, and

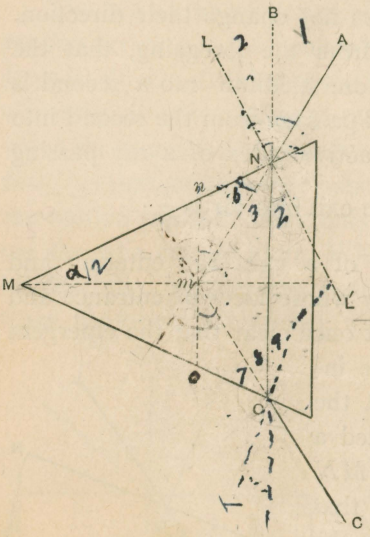

Fig. 20.

$BNL = ONL' = \frac{\alpha}{2}$; consequently the whole angle of incidence $i = \frac{\alpha + \delta}{2}$, and for the refraction from air into the prism the angle of refraction $r = ONL' = \frac{\alpha}{2}$. Therefore the refractive index

$$n = \frac{\sin i}{\sin r} = \frac{\sin \frac{\alpha + \delta}{2}}{\sin \frac{\alpha}{2}}.$$

Now this last equation makes possible the most usual method of determining the refractive index of a light ray passing from air into a transparent solid. For this purpose one has only to determine the refracting angle $\alpha$ of a prism composed of

the solid and the **minimum deviation** of a light ray passing through the prism. The former of these quantities is determined by means of the reflection goniometer (see p. 29); the latter likewise by means of a graduated circle at whose center the prism is so adjusted as to be rotatable, but on which circle one may read off, besides the direction of the ray emerging from the prism, that of the entering ray as well. The diagram Fig. 21

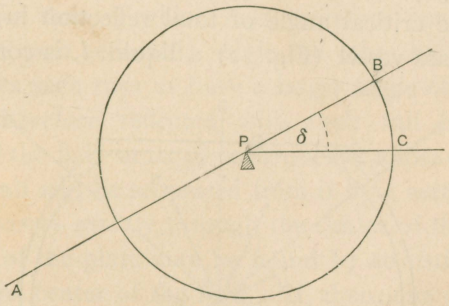


Fig. 21.

may serve to illustrate this. Here $P$ is the center of the circle, in front of which is the refracting edge of the prism, and $AP \parallel$ to $BP$ is the direction of the incident light, $PC$ that of the refracted ray; the angle $BPC$, to be read on the graduated circle, is then the deviation $\delta$. One rotates the prism about its refracting edge, *i.e.* about the axis of the circle, until $\delta$ has its minimum value —until, with further rotation, the ray $PC$ would move farther away from $PB$. When this value, as well as that of the refracting angle $\alpha$, has been determined, the refractive index sought is found directly from the equation

$$\frac{\sin\frac{\alpha+\delta}{2}}{\sin\frac{\alpha}{2}} = n.*$$

* For details of the practical manipulations in such a measurement cf. footnote †, p. 30.

The foregoing method of determining the refractive index is the most accurate, but requires a carefully ground prism of a certain size. However, it is also possible, with the aid of a small plate of a transparent substance, to determine the refractive index of the same; and this may be done by two different methods.

The first of these methods depends on the law of total reflection (see p. 36), and as employed by Kohlrausch consists in measuring the critical angle of total reflection in the following way: In a glass vessel (Fig. 22) a liquid, *l*, is contained whose

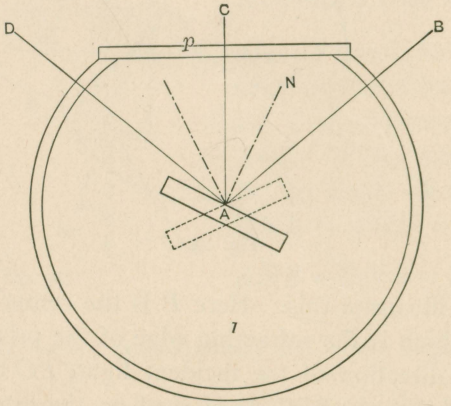


Fig. 22.

refractive index must be larger than that of the substance to be investigated. (In order to render this method applicable to the largest possible number of substances a liquid having very great "refringency", or power to refract, as bromnaphthalin, is chosen.) In front the otherwise cylindrical vessel consists of a plane-parallel glass plate, *p*, before which and directed perpendicular to which is a small telescope focused on an object at infinite distance, hence for parallel rays. On a rotatable support plunging into the liquid from above, the plate of the substance to be investigated is so fastened that the rotation axis falls in its (plane) front face, which is illuminated from the side

by diffused light. Among the light rays falling on this face at various inclinations are such as have a direction $BA$; — and let it be these that form with the normal, $AN$, an angle $i$, the critical angle of total reflection. If one rotates the plate to be investigated into exactly the position indicated in Fig. 22, this position being such that the rays reflected parallel to $AC$ ($CAN = BAN = i$) converge at the center of the field of the telescope, then the one-half of the field must receive rays falling on the plate at angles smaller than $i$, which rays therefore penetrate in part, being only in part reflected; while the other half, on the other hand, receives such rays as have a larger angle of incidence and which are therefore reflected with their full intensity. The latter half of the field must in consequence appear brighter than the former half and be separated from it by a vertical boundary-line, which passes exactly through the center of the field; hence this position of the plate may be found by adjusting the boundary-line on the center of the field. If, then, one causes diffused light to fall from the other side of the vessel and rotates the plate until that boundary-line again appears at the center of the field, the plate now has the position indicated in dots, and the direction $AB$ has taken the position of $AC$, while the rays corresponding to the critical angle of total reflection fall in the direction $DA$. The angle the plate must be rotated is obviously $2i$; and hence, by reading off this rotation on a graduated circle and dividing by two, one obtains the critical angle of total reflection. But according to page 33, the sine of this angle is equal to the refractive index — the ratio of the light velocity in the liquid, $L$, to the light velocity in the plate, $v$; therefore we have

$$\sin i = \frac{L}{v}, \text{ whence } v = \frac{L}{\sin i}.$$

If we divide each member of the last equation into the value of the light velocity in air — let it be $V$ — we obtain

$$\frac{V}{v} = \frac{V}{L} \sin i.$$

But $\frac{V}{L}$, the ratio between light velocity in air and in liquid, is the refractive index of the latter, and this index can be found with the aid of a hollow prism filled up with the liquid, by the method first given, of minimum deviation. If it be determined once for all, for the liquid in question, and be denoted by $\mu$, then

$$\mu \sin i = \frac{V}{v} \cdot$$

But the quotient $\frac{V}{v}$, the ratio of the light velocity in air to that in the plate, is the required refractive index of the latter, which accordingly is found by multiplying the sine of the measured critical angle of total reflection by the refractive index of the liquid employed.

Instead of a liquid we may use a transparent optically isotropic solid having higher refringency than the one to be investigated (Wollaston's method); and for accuracy of the determination this method has a great advantage. For the refractive index of every substance varies with the temperature, and in the case of liquids this variation is very considerable, wherefore in using the method just described an inavoidable rise in temperature perceptibly influences the result; while in the case of solid bodies the variations are much less. To which must be added that it is possible to manufacture homogeneous glasses whose refractive index amounts to 1.92, thus being greater than that of most other substances. By reason of these and other advantages the refractometer constructed by Abbe and Czapski is especially adapted for determining the refractive indices of crystals; the instrument named is based on the following principle: —

The plate, $P$ (Fig. 23), of the body to be investigated is laid with its plane face on the plane surface of a hemisphere composed of the highly refractive glass, and by means of a very thin plane-parallel layer of a highly refractive liquid the two planes are brought into perfect contact. Let $ON$ be the normal

to the separating surface, and $NOJ$ (or $NOJ'$) the critical angle of total reflection. Then, if light rays fall from below on the right, as indicated by the radial lines in the figure, falling in part at larger, in part at smaller angles, the former rays are totally reflected, while those falling at the smaller angles are only partially reflected. (In the figure this is indicated by the lines being dotted.) Consequently, in a telescope focused for infinite distance, whose axis is brought into the direction $OJ$, the left half of the field will appear bright but the right half less bright; that is, a vertical boundary will pass through the center of the field. Since of rays that are parallel to a radius, which must accordingly have passed through the spherical surface perpendicularly, no deviation takes place, the reading of the position of the telescope on a graduated circle whose zero point corresponds to the radius $ON$ gives directly the critical angle of total reflection.

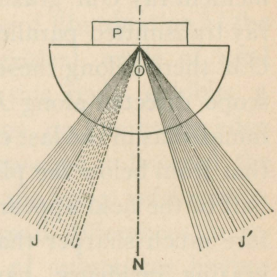

Fig. 23.

Still more accurate is the adjustment of the boundary if one employs a modified procedure, for which it is requisite that the plate laid upon the hemisphere permit the lateral entrance of light; the best results are obtained when the plate is circular and bounded by a polished cylindrical surface, so that its cross-section has the form shown in Fig. 24. (The upper side may be rough-ground or otherwise constituted.) Should one, then, cause light rays to fall upon the hemisphere from below on the left in directions adjacent to $JO$, the rays that fall exactly parallel to $JO$ would be transmitted in the boundary plane,—parallel to $OM$ therefore,—while those falling within the angle $JON$ (in the figure these are given in dots) would leave the plate through the side face. Hence, if we imagine this optical

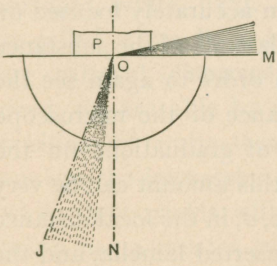

Fig. 24.

construction as reversed (see p. 14), *i.e.* the rays as entering through the cylinder face and on the right from above, then to the incident ray $OM$ grazing the boundary-surface there belongs the ray transmitted parallel to $OJ$; while to the rays following above $OM$ there belong those represented in dots, wherefore in the telescope directed along $JO$ the right half of the field appears illuminated as in the last case. (See p. 43.) Now if it is so arranged that from below the plane $OM$ not the least light can fall, the left half of the field appears absolutely dark and the boundary therefore much sharper than before. This procedure, called that of **grazing incidence**, can of course be employed also with the Kohlrausch total-reflectometer.

Finally, the *microscope* too may be used for the determination of refractive indices. Such a method was suggested as early as the eighteenth century by the Duke of Chaulnes, and depends on the principle that the focal distance of a microscope changes when a transparent plane-parallel plate is inserted between the objective and the focus. As shown in Fig. 25, the rays appear to come from a point $a'$, while in reality they proceed from $a$. So if the microscope has been accurately focused on any object, and if a lamella of the substance is now inserted above the same, then in order to again see the object distinctly one must alter the distance of the microscope from it a certain amount; by means of graduations on the fine-adjustment screw of the microscope this amount can be very accurately measured. Among the change, $d$, in the focal distance of the objective, the thickness, $e$, of the inserted lamella, and the refractive index $n$ of the latter there exists the relation

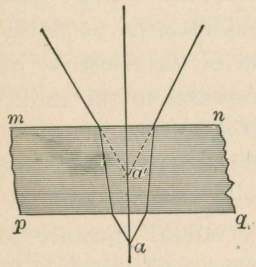

Fig. 25.

$$n = \frac{e}{e - d};$$

from this equation, after $e$ and $d$ have been measured, the refractive index may be calculated.

Another method depending on the total reflection of light is due to Becke. If in the field of the microscope two transparent bodies lie contiguous to each other, the refringency of one of them being known, and if the cone of illumination of the microscope is narrowed down to the critical angle of total reflection of the more refractive body, this body exhibits a bright border if we adjust the microscope on the upper edge of the boundary-surface, while if the microscope is lowered the bright border appears in the medium of the lesser refringency. One can thus demonstrate even very small differences in refringency, and, with the aid of a scale of substances having known refractive indices, determine those of a body to be investigated.*

If in one of the ways mentioned the refractive index of an optically isotropic body is determined for a light ray of a certain color, as well as for a ray of another color, the two values are found to differ. In accord with this, it is seen that by a prism a ray of white light is split up into rays of different color, which in the prism have all experienced unequal deviation. Since these colors, as also their sequence in deviation, are in general the same with prisms of different substances, they cannot have been created by the prism, but must have preëxisted in the incident white light. It must accordingly be assumed that so-called white light is composed of the different colors which are scattered (**dispersed**) by the refraction into a **spectrum.** Indeed, we are able by means of a similar prism in the reverse position to unite these rays, and they then again produce in our eye the impression of white. The color spectrum which the white light yields with a prism has a greater extent when the prism has a greater refracting angle, because with an increase of the latter there is an increase in the deviation of every color and consequently in the distance of the least refrangible ray in the spectrum from the ray most strongly refracted. But the length

* Concerning more detailed information on all the before-mentioned methods and apparatus see footnote †, p. 30.

of the spectrum depends, besides, on the **dispersive power** of the substance composing the prism; that is, on the power the substance of the prism possesses to scatter the different colors.

Empirically, therefore, we learn that the refractive index varies not only with the substance, but also with the color of the light; *i.e.* with the vibration period and hence with the wave length. Accurate theoretical deduction of the transmission velocity of any wave-motion does indeed reveal a dependence of the same on the wave length of the motion. This dependence is expressed, for the refractive index, in close approximation by the formula (Cauchy's)

$$n = A + \frac{B}{\lambda^2},$$

in which $A$ and $B$ are constants for one and the same substance while $\lambda$ stands for the wave length. Accordingly, when for two colors whose respective wave lengths are $\lambda_1$ and $\lambda_2$ the refractive index $n$ has been determined, the constants $A$ and $B$ may be calculated (by substituting these known values in the above formula and solving the two equations for $A$ and $B$), and one then knows the $n$ proper to any color having another $\lambda$. From this dispersion formula it is seen that the refractive index, and therefore also the deviation, is the less, the greater the wave length of the light refracted. Since according to page 15 the red, of all the colors of the spectrum, has the greatest wave length, it is refracted the least; orange, yellow, green, blue are more strongly refracted, and the color deflected the farthest of the whole spectrum is violet. So this latter color consists of vibrations of the least wave length that can be perceived as light. If we look at such a spectrum (*e.g.* one produced by a prism of glass — see Plate I, Fig. 1), we see that each of the above-named colors still occupies a certain portion of the length, still contains rays therefore whose deviation, and thus whose wave length, is different within certain limits. This being the case, an accurate determination of the refractive index for any color, as yellow in general, can not be made, but, of all the rays belonging to

the yellow, only for those of one definite wave length, which accordingly appear only in one definite position in the yellow of the spectrum. Such light, which consists only of rays of one and the same wave length, is spoken of as homogeneous or as **monochromatic** light. We can obtain such light in various ways:—

1. The light emitted by the incandescent vapor of certain metals is monochromatic. For example, the sulphates of lithium, sodium, or thallium, fused on a thin platinum wire and vaporized in the non-luminous flame of a Bunsen burner, emit monochromatic light: in the case of lithium, red light with a wave length (in air) of 0.000670 mm.; in that of sodium, yellow light ($\lambda =$ 0.000589 mm.); in that of thallium, green ($\lambda$ = 0.000535 mm.). The light of such colored flames is the most convenient aid to determining the refractive index of a substance for different colors.

2. If one causes electricity to be discharged in a so-called Geissler's tube filled with hydrogen, one obtains light which is a mixture of only three homogeneous colors ($\lambda =$ 0.000656, 0.000486, 0.000434 mm.); and on prismatic decomposition such light accordingly exhibits no continuous color spectrum, but only three bright lines, one red and two blue, which may be used for accurate adjustments. This so-called "line spectrum" of hydrogen may therefore be employed with advantage, in determining the refractive index of a prism, when the electric current from an induction-coil is available.

3. Sunlight does not contain all the colors between the extreme red and the extreme violet as they are exhibited, for example, by the light of a white flame when decomposed by a prism, but in their passage through the sun's outer atmosphere certain kinds of light are annihilated. If the bright spectrum is suitably produced there accordingly appear, in the positions corresponding to these kinds of light, exceedingly narrow dark breaks (the so-called Fraunhofer's lines), whereby certain positions in the spectrum are sharply defined; and for these positions the devia-

tion, and thus the refractive index of the prism, can then be very accurately measured. The most important of these lines are inserted in Fig. 1, Plate I, with their customary designation by letters and specification of their wave length (in millionths of a millimeter — $\mu\mu$). Several of them lie in the positions occupied by the bright lines of the emission spectrum of incandescent gases and vapors. (Sodium corresponds to the line *D*; hydrogen to *C*, *F*, *G*.)

4. Light is approximately monochromatic when it has passed through certain colored bodies; *e.g.* through red glass, which annihilates all kinds of light except the red. Yet the transmitted rays still have perceptibly different wave lengths, so that in the spectrum they appear not in one single position but within a transverse strip of some breadth, and therefore are not adapted for accurate measurements of the deviation.

By determining the refractive index of transparent solids at their boundary with air it is found, as already mentioned, that the light velocity in them is in general less than in air; that accordingly the refraction quotient for passage from air into such a body is greater than unity. With the large majority it lies between 1.4 and 1.7, but there are also substances whose refractive index is considerably higher.*

* For example, the refractive index of the diamond is —

$n = 2.4135$ for red,
$= 2.4195$ for yellow,
$= 2.4278$ for green.

So for this body the refractive index for light emerging from it into air is about $\frac{1}{2\frac{1}{4}}$, or approximately the sine of 25°. Therefore a light ray transmitted within a diamond and striking a boundary-face at an angle of incidence greater than 25° can no longer have a refracted ray; such a ray is accordingly *totally reflected*, and, because of the high refractive index, the critical angle of total reflection is so small that within a diamond bounded by many plane facets a ray of light has usually to suffer many total reflections. This is the cause of the numerous reflections of light within a cut diamond.

## POLARIZATION OF LIGHT BY REFLECTION AND REFRACTION

In the *reflection* of light a change in its constitution takes place in so far as, of a ray of ordinary light, — which according to page 16 contains vibrations in all possible azimuths, — it is chiefly the vibrations perpendicular to the plane of incidence that are thrown back. This occurs especially at a certain angle of incidence, wherefore by reflection at this angle, the so-called " polarizing angle ", one obtains light the greater part of which is plane-polarized. The light can be polarized more completely if we cause it to be reflected from a number of thin glass plates laid one upon the other, because then at each single plate there takes place reflection of the light that has penetrated through those lying above it, with polarization of the same. Such a bundle of thin glass plates therefore presents a very convenient means of producing from ordinary light, by reflection at the polarizing angle, light that is plane-polarized. The plane of polarization (see p. 15) of the reflected light is parallel to the plane of incidence.

In *refraction*, likewise, the ordinary light suffers polarization in a slight degree; here, however, it is not the rays whose vibrations take place perpendicular to the plane of incidence, but those vibrating in it, that are specially favored.

In the special case of *total reflection* the two phenomena are united, and therefore polarization of the ordinary light does not occur. A plane-polarized light ray, on the other hand, is resolved into two mutually perpendicular vibrations, of which one lies in the plane of incidence and the other in the plane perpendicular to this plane; at the same time the two vibrations acquire a difference of path, which depends on the nature of the medium and on the angle at which the rays fall upon the totally reflecting surface; the result is that after the total reflection the previously plane-polarized light in general exhibits elliptical polarization. If the vibration plane of the incident light ray forms 45° with the plane of incidence, the two vibrations aris-

ing on the resolution have equal amplitude; and if, of those two vibrations, the path difference arising on the total reflection is then $\frac{1}{4}\lambda$, their combination must according to page 20 result in a circular vibration. On this principle depends the use of the so-called Fresnel's rhomb, *abcd* (Fig. 26), for the production of circularly and of elliptically polarized light. The rhomb (really a parallelopiped) consists of a kind of glass for which the difference of path with total reflection at a certain angle corresponds to $\frac{1}{8}\lambda$. Therefore, if a light ray $PP'$ is caused so to fall, as represented in the figure, that a total reflection at that angle twice takes place, the two arising vibrations acquire a path difference of $\frac{1}{4}\lambda$. When the incident light is plane-polarized, then if its vibration direction includes 45° with the plane of incidence a circularly polarized ray emerges from the Fresnel's prism; but if the vibration plane of the entering light has a different direction, the two arising vibrations are unequal in amplitude and yield an elliptically polarized ray.

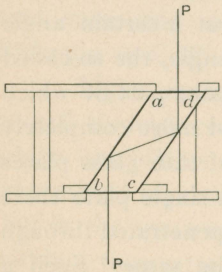

Fig. 26.

## DOUBLE REFRACTION OF LIGHT

A ray of ordinary or of plane-polarized light falling perpendicularly on the surface of an optically isotropic body, a body such as those forming the subject of the considerations up to this point, suffers no refraction ($i = 0$; so $r = 0$), nor does it if it leaves the body through a face that is parallel to the first face. Now if we investigate the character of the ray after its exit, we find in the first case that it consists, as before, of ordinary light; in the second, that it has remained plane-polarized and has suffered no change in its vibration direction. This is the natural consequence of the isotropy of the luminiferous ether in such a body, as is really understood from the following. Let *o* (Fig. 27) be the point at which a ray of ordinary light falls perpen-

dicularly on the surface of incidence (represented by the plane of the figure) of the optically isotropic body. If at a certain instant an ether particle at $o$ is displaced in vibrations along the direction $aa'$, then with every succeeding vibration the vibration direction will be rotated a very small angle; for example, at a certain instant it will lie parallel to $cc'$. An impulse moving the ether particle $o$ to the point $c$ has just the same effect as two impulses, exerted simultaneously toward $a$ and $b$, that by themselves would have moved the particle to $\alpha$ and $\beta$ respectively. But in the medium in question vibrations are transmitted equally fast, whatever the direction in which they take place; consequently, at every particle following beyond $o$ in the line of transmission, that is caught up by the motion, the two impulses toward $a$ and $b$ will arrive simultaneously and so will combine to form a single motion to $c$. The same applies to every vibratory motion; and accordingly, a motion that takes place in all azimuths in rapid succession must, after penetrating a medium that fulfills the above-stated conditions, keep up the same change of its vibration direction. Thus is it explained that in such a body a ray of ordinary light is transmitted as ordinary light; that a plane-polarized light ray of any vibration direction whatsoever will in such a medium be transmitted with unchanged vibration direction, follows as a matter of course.

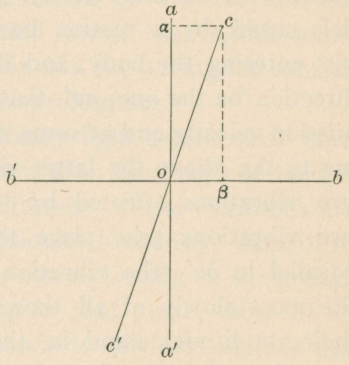

Fig. 27.

A light ray must behave differently, however, when it enters an **optically anisotropic** body — a body whose ether is an anisotropic medium and in which therefore differently directed vibrations are transmitted with different velocity. Now it is learned empirically that, in all homogeneous media in which the ether behaves differently in different directions, among all

the vibrations that take place in any one plane those having a certain direction are transmitted the most rapidly, while those vibrations whose direction is *perpendicular* to this direction are transmitted the most slowly. Hence, if $o$ again be an ether particle struck by a motion having the nature of ordinary light, just entering the body, and if at a certain instant the vibration direction be the one, $aa'$, that corresponds to the greatest transmission velocity and at some subsequent instant $cc'$, then according to the above the latter vibration must be equivalent to the two vibrations effected by two simultaneous impulses; these two vibrations take place the one parallel to $aa'$, the other parallel to $bb'$ (the vibration direction of the rays transmitted the most slowly of all those vibrating in the plane $ab$), and their amplitudes stand in the ratio of the lengths $o\alpha$ and $o\beta$. But these two vibrations are transmitted with different velocity, and therefore at one of the ether particles later set in motion they arrive at different times; there, accordingly, they can no longer combine to form one resultant vibration $cc'$, but will continue their way separately. At a still later instant the particle $o$ will be moved in a direction forming a still greater angle with $aa'$ than does the direction $cc'$; so this motion likewise will be resolved into two, parallel to $aa'$ and $bb'$ respectively; but in this case the component parallel to $bb'$ has a greater value than before. Thus the entire motion of a ray of ordinary light, whose vibration plane very rapidly changes, will be resolved into two light rays whose intensity varies with the azimuth of the entering light. One of these rays vibrates parallel to $aa'$, and at the instant when the entering light vibrates parallel to this direction the intensity of this ray equals the full intensity of that light; the intensity then diminishes, becomes zero as soon as the entering vibration takes place parallel to $bb'$, then increases again, and so on. But this variation of intensity occurs so rapidly that the light ray produces the impression of a ray of constant, intermediate intensity, — of a ray therefore whose intensity is half that of the entering ray. As for the second ray

arising by the resolution, whose vibrations take place parallel to $bb'$, this ray, at the same instants, has the intensity zero when that of the first is maximum, increases in intensity as the first diminishes, reaches the same maximum as did the first ray, when that ray has zero intensity, and so on; that is, this second ray likewise produces the impression of a ray having the constant brightness equal to half that of the entering ray.

When, therefore, a ray of ordinary light enters such ether, anisotropic ether, there arise from it two light rays of unequal transmission velocity but equal brightness, whose vibrations take place in the plane standing perpendicular to the ray; those of the one ray take place in the direction corresponding to the greatest light velocity of the rays vibrating in that plane, those of the other in the direction that corresponds to the least of these light velocities. Thus we obtain two plane-polarized rays polarized perpendicularly to each other, one of which suffers a certain retardation as compared with the other so that the two rays, when they emerge, have a certain difference of path, according to the difference between their transmission velocities in the optically anisotropic body and to the length of their path in it. The same is the case also if the entering light was plane-polarized; except that then the brightness of the two rays is in general not equal, but depends on the angle formed by the vibration plane of the entering light with the two directions parallel to which the vibrations of those rays take place that are transmitted with the greatest and the least velocity.

If the face through which the light emerges from the body is not parallel to the face of entrance, and the light consequently refracted, then, since the amount of refraction depends on the light velocity in the body, the deviation of the two rays will be different; so the light ray, previously single, will in such a medium be " doubly refracted " into two rays of different direction. Therefore the optically anisotropic substances are designated also as **doubly refracting** or as **birefringent,** the optically isotropic as **singly refracting.**

ACCORDING TO THEIR BEHAVIOR TOWARD LIGHT RAYS CRYSTALS FALL INTO TWO CLASSES, THE SINGLY REFRACTING AND THE DOUBLY REFRACTING. The former, like amorphous bodies, have for the rays of a definite color the same refractive index, in whatever direction the rays pass through the crystal. Optically, therefore, the singly refracting crystals behave wholly as do amorphous bodies, and to these crystals applies all that has been said up to the present, while the doubly refracting shall form the subject of what now follows.

*To distinguish between singly refracting and doubly refracting crystals* observation of the different deviation of the two light rays arising in the latter can, in general, not be availed of, because the difference is often so slight that with moderately small crystals it escapes perception. It is possible, however, even with crystals of the smallest dimensions and of very feeble double refraction, to distinguish the two kinds from each other by the circumstance that the two polarized rays arising in a crystal of the latter kind emerge from it with a difference of path, although a very slight one. In consequence of this the two rays may be caused to *interfere* (see p. 16), provided it is possible to make them vibrate not in two mutually perpendicular planes, but in the same plane. This is accomplished by introducing into the path of the two rays emerging from the crystal a medium which permits the passage only of rays having one definite vibration direction. The same purpose may be served by reflection at the polarizing angle from a glass plate or, better, from a bundle of thin glass plates. (See p. 49.) But it is more effective to pass the light through a doubly refracting crystal in which one of the rays arising therein is eliminated, so that only plane-polarized light of a definite vibration direction is allowed to pass through. This is the case, for example, with certain varieties of the mineral *tourmaline*, and in a still higher degree with the crystals of iodo-quinine sulphate (the so-called *herapathite*), by which one of the two rays arising in consequence of the double refraction is so strongly absorbed

that for this ray crystals of only a slight thickness are quite opaque. But, since at the same time the other ray is considerably weakened and colored, a uniformly polarized ray is produced to greater advantage by using a colorless crystal that has strong double refraction; that is to say, in which the transmission velocities of the two vibrations taking place perpendicularly to each other are so different that with a certain arrangement one vibration is totally reflected within the crystal, and only the other transmitted. Such a contrivance, which in a later section will be described in more detail (see p. 99), was constructed first by Nicol, from calcite, and after him it is called a **Nicol prism** or briefly a **nicol**.* Since with its aid one can transform incident ordinary light into plane-polarized light of a definite vibration direction, such a contrivance is spoken of also as a " polarizer ". (Cf. p. 58.)

If, then, we bring a nicol into the path of the two light rays emerging from a doubly refracting crystal, each ray is resolved in the polarizer into two rays of which only one, the one falling to the definite vibration plane of the polarizer, passes through; so the two vibrations, previously perpendicular to each other, are brought back, as it were, to one vibration plane.

But the production of interference phenomena by this means depends on still a second condition, as will be seen from the following: —

In Fig. 28 (p. 56) let $PP'$ be the vibration direction of a plane-polarized ray entering a doubly refracting crystal in the direction perpendicular to the plane of the figure and there resolved into two rays having the vibration directions $RR'$ and $SS'$. Supposing the crystal to be of exactly such a thickness that the two rays transmitted in it with unequal speed have on their exit a path difference of one whole wave length, or of a multiple of one wave length, of the monochromatic (homogeneous) light employed, then as soon as the light has left the crystal an ether

* [The calcite crystals employed for these polarizing prisms (as well as for similar optical purposes) are of the large, transparent variety known as *Iceland spar*.]

particle lying in its path will in virtue of one of the two arising vibratory motions always move from $O$ to $r$ at the same instant when the other of the two arising vibrations would carry it from $O$ to $s$. If by means of a nicol transmitting only such vibrations as are parallel to $PP'$ the two rays are now brought back to *one* polarization plane, then of each motion only that component passes through that is parallel to $PP'$; namely, $O\rho$ in the one case, $O\sigma$ in the other. These two partial motions, since they take place simultaneously in the same direction, must simply combine to form the sum of their respective amplitudes; must interfere therefore with *like* state of vibration, — that is, with the *same* path difference they acquired in the crystal. When on the other hand the thickness of the crystal is such that the retardation one of the two rays suffers as compared with the other amounts to $\frac{1}{2}\lambda$ or to an uneven multiple of $\frac{1}{2}\lambda$, the motion of the ether particle $O$, at the instant when in virtue of the first vibration it is moved to $r$, takes place in virtue of the second to $s'$. The two motions can be made to interfere by reducing them to the same vibration plane with the aid of a nicol. If, as before, this so stands that it transmits only the vibrations parallel to $PP'$, the interfering motions take place simultaneously in opposite directions; namely, along $O\rho$ and $O\sigma'$. So they interfere with opposite state of vibration; that is, as before, with the same path difference with which they emerged from the crystal. Hence it follows that, stated generally, TWO RAYS ARISING FROM ONE PLANE-POLARIZED RAY BY DOUBLE REFRACTION INTERFERE WITH THE SAME PATH DIFFERENCE THEY ACQUIRED FROM THE DOUBLY REFRACTING CRYSTAL, PROVIDED THEY ARE BROUGHT BACK TO A VIBRATION PLANE PARALLEL TO THAT OF THE FIRST RAY.

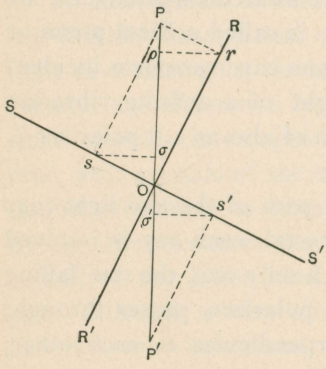


Fig. 28.

Let us now consider the opposite case, in which the vibration plane of the light ray entering the crystal stands perpendicular to the plane to which the Nicol prism reduces the two rays. Again supposing $PP'$ (Fig. 29) parallel to the vibration plane of the entering light, and supposing the crystal to be of such a thickness that the two rays arising therein have on emerging $\lambda$ or a multiple of $\lambda$ difference of path, then by the one of the rays the ether particle $O$ will be moved to $r$, by the other simultaneously to $s$. But in the case now before us, where the nicol is rotated 90° from its former position, its vibration plane (that of the light it transmits) thus becoming parallel to $QQ'$, the two interfering components are $O\rho$ and $O\sigma'$; these lie in opposite directions; so the interference takes place with opposite state of vibration, although the two rays had emerged from the crystal with like state of vibration. When on the other hand the thickness of the crystal is such that the two rays on emerging are mutually opposite in their state of vibration, the simultaneous magnitudes of their motions (see Fig. 29) are $Or$ and $Os'$; after these latter motions are reduced to the single vibration plane $QQ'$ their mutually interfering, equal components, $O\rho$ and $O\sigma$, lie in the same direction; so the interference takes place with like state of vibration. Stated generally: TWO RAYS ARISING FROM ONE PLANE-POLARIZED RAY BY DOUBLE REFRACTION INTERFERE WITH THE OPPOSITE DIFFERENCE OF PATH FROM THAT WITH WHICH THEY EMERGED FROM THE DOUBLY REFRACTING CRYSTAL, IF THE VIBRATION PLANE TO WHICH THEY ARE REDUCED STANDS PERPENDICULAR TO THE ORIGINAL VIBRATION DIRECTION.

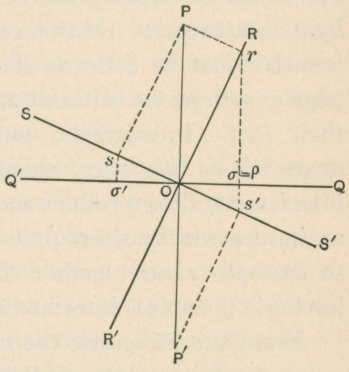

Fig. 29.

If we now imagine, instead of a plane-polarized ray, a ray of ordinary light entering the crystal, then at the instant when

the vibration plane of this entering ray is parallel to that of the reducing nicol the interference takes place with the same path difference with which the two arising rays emerge from the crystal; while after an exceedingly short space of time, during which the vibration plane of the ordinary light has rotated 90°, the two rays interfere with the opposite path difference, — *i.e.* with their path difference at emergence + or $-\frac{1}{2}\lambda$. Consequently, in the path of the light the maxima and minima of light intensity arising by the interference must follow one another just as rapidly as the polarization plane of the ordinary light rotates 90°. Of this change of intensity we shall therefore perceive just as little as if we viewed the rays directly, as on page 52 *et seq.*; *i.e.* without passing the rays through a nicol after their exit. Interference phenomena can, accordingly, not be observed in this way, since, although at a definite point the interference does produce momentary darkness, it produces after an immeasurably short time just so much the greater brightness, so that all taken together there results the impression of a ray having a constant, intermediate intensity.

From the foregoing the conditions necessary for the interference of plane-polarized light are now fully understood; they read: TWO PLANE-POLARIZED RAYS INTERFERE ONLY WHEN THEY HAVE ARISEN FROM A SINGLE PLANE-POLARIZED RAY BY DOUBLE REFRACTION AND ARE REDUCED TO THE SAME POLARIZATION PLANE BY MEANS OF A NICOL PRISM.

In order, then, to investigate a crystal for its double refraction by means of the interference resulting from the difference in velocity of the doubly refracted rays, we must employ *two* nicols. One of these serves to transform the ordinary light, before it enters the crystal to be investigated, into plane-polarized light, wherefore this first nicol is called the **polarizer** in the narrower sense of the word; the other permits the light emerging from the crystal to be analyzed, and is therefore designated as the **analyzer.** [Such a combination of two nicols or other polarizing device, used for investigating a substance in polarized

light, is generally called a **polariscope**, sometimes a "polarization-instrument". (See p. 74 *et seq.*)]

If we look through two nicols placed one behind the other in parallel position, the light rays emerging from the one will pass through the second with the same vibration direction, and the brightness of the light will accordingly experience no change. But if we rotate the second nicol about the direction of the light rays as an axis, only that component will be transmitted that corresponds to the vibration plane of this nicol; with further rotation of the nicol this component always becomes smaller, and finally, after 90° rotation, equal to zero, because then the light entering the second nicol has exactly the vibration direction that in the nicol is totally reflected. So when two nicols are "crossed" in this way they transmit no light.

Inserting a singly refracting crystal between the two crossed nicols can produce no change, since the light-vibrations in such a body, in whatever direction they take place, are always transmitted in the same way, without resolution into components. BETWEEN CROSSED NICOLS, THEREFORE, A SINGLY REFRACTING CRYSTAL APPEARS DARK IN EVERY POSITION. Even crystals of microscopic dimensions can be subjected to this experiment, by placing a polarizer beneath the stage of a microscope and mounting an analyzer on the eyepiece. If one rotates the analyzer 90° from the position in which it is parallel to the polarizer, the analyzer cuts off all the polarized light entering the instrument from below; the field of the microscope becomes dark, as do all singly refracting crystals that may be in it; and moreover, in the darkness of these crystals no change can take place if we rotate them in their own plane (for which purpose the preparation to be investigated must be on a *rotatable stage* fitted to the instrument).

Quite different is the behavior of doubly refracting crystals.

If, as represented by their cross-sections in Fig. 30 (p. 60), two crossed nicols one of which transmits the vibrations that are parallel to $PP'$, the other those taking place parallel to $QQ'$,

are inserted one behind the other (in the figure, for the sake of clearness, they are placed side by side) in the path of the light rays, and if a thin doubly refracting crystal, $K$, is brought between the two in such a way that the vibrations, $RR'$ and $SS'$, of the two rays arising in it are parallel respectively to the vibrations transmitted by the two nicols, — then complete extinction of the light does indeed occur, as with a singly refracting crystal; for the vibration taking place parallel to $PP'$, which enters $K$ from the first nicol, is then exactly parallel to $RR'$, wherefore the whole vibration, since the component falling to the direction $SS'$ is zero, passes through the crystal unaltered

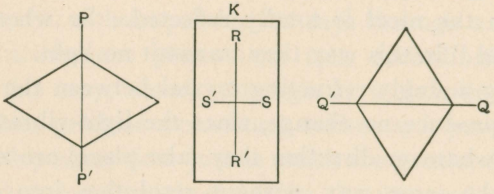


Fig. 30.

and in the second nicol is totally reflected. This is no longer the case, however, so soon as by rotating the crystal $K$ in its own plane we bring it from the above position into another.

Assuming the faces of entrance and emergence of the light to be so close together, *i.e.* the crystal to have the form of a plane-parallel plate so thin, that on emerging from it the two rays of a definite color polarized perpendicularly to each other have acquired a path difference of exactly a half wave length, then, when the plate $K$ has been rotated an angle $POR$ (Fig. 31), there arise in it two vibrations which on emerging take place simultaneously to $r$ and $s'$ and of which the components that pass through the second nicol, $O\rho$ and $O\sigma$, are added to each other. Consequently the plate now appears bright, and its brightness depends on the magnitude of the components $O\rho$ and $O\sigma$. From a comparison of Fig's 31, 32, and 33 it follows that these two components, which are always of equal magni-

tude, attain their maximum, and that accordingly their sum is greatest, when the rotation angle $POR$ is 45°. (Fig. 32.) After a rotation of 90° they both become zero again; we then have the case represented in Fig. 30, except that the directions $RR'$ and $SS'$ have exchanged places. So the crystal plate illuminated by light rays of the color in question becomes dark only when its vibration directions, $RR'$ and $SS'$, coincide with those, $PP'$ and $QQ'$, of the nicols; and this is obviously the case four times during one complete rotation; in every other position, on the other

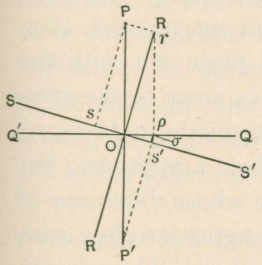


Fig. 31.

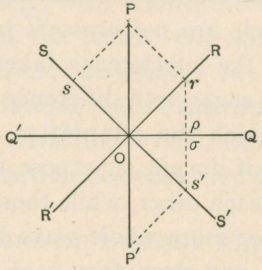


Fig. 32.

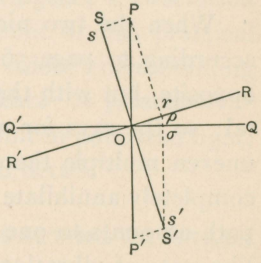


Fig. 33.

hand, it will appear bright, bright with the color of the light used, and the brightness will reach a maximum whenever the directions $RR'$ and $SS'$ form 45° with $PP'$ and $QQ'$.

Let us now suppose the crystal to be investigated not in monochromatic, but in white light. The same retardation of one of the doubly refracted rays as compared with the other will now, for the vibrations of those colors that have a greater wave length, amount to less than $\frac{1}{2}\lambda$ and for the colors having a lesser wave length to more than $\frac{1}{2}\lambda$; in both cases, therefore, the resultant from the interference of the two components parallel to $PP'$ and $QQ'$ is not equal to their sum, but less. (Cf. p. 19.) Consequently the different colors, after they have passed through the second nicol, will no longer have the same relative intensities as previously, in the white light; they will then no longer produce in our eye the impression of white, but a color impression in which that color predominates that on the

interference is most favored in intensity. Since this favoring reaches its maximum with 45° rotation, it is in this position of the plate that the color in question appears relatively purest.

If the plate has a different thickness, the same will apply to a different position in the spectrum, and the plate will accordingly be lighted up with a different color. Stated generally: —

A THIN DOUBLY REFRACTING CRYSTAL ROTATED BETWEEN CROSSED NICOLS APPEARS DARK FOUR TIMES; IN THE INTERMEDIATE POSITIONS BRIGHT WITH A DEFINITE COLOR, WHICH IS PUREST AND BRIGHTEST IN THE FOUR DIAGONAL POSITIONS.

When the two nicols are not crossed, but set parallel, then according to page 56 the interference takes place not with the opposite, but with the same path difference as arose in the crystal; so the rays for which the retardation amounts to $\frac{1}{2}\lambda$ or an uneven multiple thereof do not superpose their amplitudes, but completely annihilate each other, while those whose difference of path amounts to one or more whole wave lengths interfere with like state of vibration, and strengthen each other. Thus, in the case of illumination with white light, between parallel nicols those colors are destroyed that most predominate when the nicols are crossed, and those colors predominate that with crossed nicols have the least intensity. The composite color arising in this way is called the **complementary color** of the other.

That the colors exhibited in polarized light by thin plates of doubly refracting crystals are really composite colors arising in the manner described, may be demonstrated by decomposing them into a spectrum with a prism. In the place of the color that the interference destroys, one then sees a dark stripe; and this fades out on both sides, because after the interference there remains, of the adjacent colors as well, only a slight intensity. If we rotate one of the nicols 90°, the greatest brightness is now seen where previously the dark band appeared, with diminishing intensity of the light on either side.

But the appearance of a color when a crystal is rotated between crossed nicols not only in general distinguishes that crys-

tal as doubly refracting, but the definite shade of color observed indicates directly what path difference the two rays have acquired as compared with each other.* Now we say of a crystal plate that it has " stronger double refraction ", or " higher birefringence " (cf. p. 101), than another, if with the same thickness it produces a greater difference of path of the two rays vibrating perpendicularly to each other. Hence it is manifest that with known plate thickness the color shade observed permits a conclusion to be drawn as to the strength of the double refraction, or conversely, with known birefringence a conclusion as to the thickness of the plate. On account of this practical importance in optical crystallography the colors corresponding to the different retardations [called **interference-** or **polarization-colors**], which doubly refracting crystals exhibit in polarized light, will in the following section be considered in detail.

## POLARIZATION-COLORS OF DOUBLY REFRACTING CRYSTALS

The color shades appearing in polarized light as the result of interference are represented on Plate I, in Fig. 2, where each tint corresponds to that value of $\Delta$, the path difference of the two interfering rays, that is specified below it in $\mu\mu$ (millionths of a millimeter).

* The dependence in which the magnitude of this path difference stands to the thickness of the plate and to the difference between the refractive indices of the two rays, is very easily understood. If we denote the thicknesses by $e$, then for the ray having the wave length $\lambda_1$ the number of vibrations in this distance $e$ is $\frac{e}{\lambda_1}$, and for the ray having the wave length $\lambda_2$ it is $\frac{e}{\lambda_2}$. Therefore the difference in the number of the two kinds of vibrations is $\frac{e}{\lambda_1} - \frac{e}{\lambda_2}$; and the path difference ($\Delta$) referred to the wave length in air ($\lambda_0$) is

$$\Delta = \left(\frac{e}{\lambda_1} - \frac{e}{\lambda_2}\right)\lambda_0.$$

But, if the refractive indices of the two rays (for the crystal as compared with air) are denoted by $n_1$ and $n_2$, we have $n_1 = \frac{\lambda_0}{\lambda_1}$, $n_2 = \frac{\lambda_0}{\lambda_2}$; and thence follows, on substituting above,

$$\Delta = e\,(n_1 - n_2).$$

The wave lengths of the different colors of the visible spectrum (see Fig. 1 on the same plate) lie between the limits

0.000380 and 0.000775 mm.,

since the lines $H$, which lie inside the most refrangible violet, have the wave lengths 0.000393 and 0.000397 mm., while for the line $A$, lying inside the least refrangible red, $\lambda = 0.000760$ mm.

Hence, if we imagine a doubly refracting crystal plate so thin or of a substance having such low birefringence that the two rays arising in it acquire a path difference amounting to only a very small fraction of the wave length of the extreme red, this path difference is likewise, although not so slight a one, yet a very small fraction of the wave length of the extreme violet; so the rays of all colors emerge from the crystal vibrating in nearly the same state of vibration, and between crossed nicols (cf. p. 57) are almost completely annihilated. Consequently such a crystal plate will exhibit only a very slight brightening, a shade of gray, which with increasing retardation naturally becomes lighter. Not until the retardation amounts to about 0.000100 mm. does a pale violet tint (lavender-gray) appear. For this quantity is about $\frac{1}{8}\lambda$ of the extreme red and about $\frac{1}{4}\lambda$ of the extreme violet; so that for the latter color the difference of path approaches considerably closer to the amount of a half wave length, with which value of the same the interference between crossed nicols takes place with like state of vibration, giving maximum brightness. Relatively, therefore, the violet must have somewhat more brightness than the remaining colors of the white; but, since its absolute intensity in the spectrum is very slight, the aggregate color impression produced differs so little from a white of slight intensity, *i.e.* from pure light gray, that in the figure the difference can hardly be reproduced. With increasing amount of the retardation the same applies to colors having greater wave length. Farther on therefore blue, then green, then yellow, predominate somewhat more in the color shade arising by the mixing but, since all the

colors now naturally acquire greater intensity and since the less refrangible are in general the brighter, at first only in a slight degree; so that the composite color, continuously becoming brighter, still differs but little from white,* until finally, with a path difference of about 0.000300 mm., a vivid yellow becomes visible. This quantity, as may be learned from the solar spectrum, Fig. 1, is equal to $\frac{1}{2}\lambda$ of a very intense yellow, and so between crossed nicols this yellow interferes with like state of vibration; the extreme violet, since the quantity mentioned amounts to almost one whole wave length of this color, is nearly annihilated, while green and red suffer by the interference about an equal weakening; but the mixing of these latter, so-called complementary colors gives white, so that by them the yellow coloring of the aggregate impression is influenced only in so far as a lighter yellow results. With increasing thickness or rising birefringence of the crystal plate the yellow passes over into orange and red; and the red is most vivid when the retardation of the one ray as compared with the other amounts to about 0.000530 mm. For, this quantity being the wave length of green near the line *E*, between crossed nicols the green is with this retardation totally annihilated, while the red rays interfere with about $\frac{3}{4}\lambda$ and the violet with almost $\frac{3}{2}\lambda$, both the red and the violet therefore with nearly that path difference that yields the sum of the intensities of the two combining rays; so that, since into the composition of white light the red rays enter with a much greater intensity than do the violet, the aggregate impression is an intense red, whose prismatic decomposition yields a spectrum having a dark stripe in the green.†

* In passing over from bluish to yellowish white it appears even pure white, owing to the intensities of the component colors.

† With parallel nicols a vivid green appears, as complementary color, whose spectrum naturally exhibits a light maximum in the green ($\lambda = 530$), the brightness diminishing on both sides. With the nicols in this position, however, no dark stripe is visible, because complete annihilation would take place only for $\lambda = 265$ ($= \frac{1}{2} \times 530$) and for $\lambda = 795$ ($= \frac{3}{2} \times 530$), and such wave lengths as these fall outside the visible spectrum.

Farther on, this red passes over into violet, the violet being most vivid when the retardation amounts to 0.000575 mm., because this is nearly equal to $\frac{3}{2}\lambda$ of the brightest violet in the spectrum and because it is precisely then that the most intense yellow (near the line *D*) is annihilated. The interference-color corresponding to this retardation is termed **sensitive violet,** because a small variation of the path difference in either sense produces a comparatively great change in the color, toward red or blue. Since from now on the same colors in a certain measure repeat themselves, the colors hitherto considered are designated as **colors of the first order;** these accordingly are gray, lavender-gray, bluish gray, greenish white, yellowish white, yellow of the first order, orange of the first order, red of the first order, and sensitive violet of the first order.

As may be seen from the colored plate, the shades change, in proportion to the retardation, far the most rapidly in the entire last part of the colors of the first order; therefore, if a thin doubly refracting crystal plate giving the red or violet of the first order is inserted in the path of the light rays passing through a microscope provided with two crossed nicols, any crystals of very low birefringence that may be present in the field of the microscope will appear distinctly in a different color from the rest of the field, because the addition or subtraction of a small path difference effected by them at once produces a perceptible change in the tint. On this principle is based the employment of thin crystal plates of the kind mentioned (*e.g.* of gypsum), for recognizing exceedingly feeble double refraction. (Particulars in a later section, on the Determination of the Character of the Double Refraction.)

With still increasing thickness of the considered crystal plate the violet passes gradually over into blue, which is purest with a path difference of 0.000660 mm.; for this value is equal to $\frac{3}{2}\lambda$ of the brightest blue in the spectrum but equal also to $\lambda$ of the brightest orange, so that after the interference the blue is at a maximum, while orange and yellow, which would cause the

color mixture to appear green, are at a minimum. Not until the retardation amounts to about 0.000800 mm. does a vivid green appear; for this value corresponds to $\frac{3}{2}\lambda$ of the brightest green (at *E*), as well as nearly to one whole wave length of the extreme red and to twice that of the violet, so that by the interference the colors lying at the two ends of the spectrum are totally annihilated. Between 850 and 900 $\mu\mu$ the maximum of intensity moves to yellow and orange, wherefore these colors predominate; and farther on the latter passes over more and more into red; until finally, with a retardation of 0.001060 mm. (*i.e.* 2 × 0.000530 mm., the retardation giving the red of the first order), the so-called **red of the second order** appears in its purest shade. For the value given is equal to $2\lambda$ for green near the line *E*, this color being therefore completely annihilated, and equal also to $\frac{5}{2}\lambda$ for indigo of the wave length 424 and to $\frac{3}{2}\lambda$ for red with the wave length 700; so that, since the latter color is considerably more luminous than indigo, a comparatively pure red appears. Decomposed into a spectrum, this color, like the red of the first order, yields, as is manifest from the foregoing, only a dark stripe in the green; with parallel nicols, on the other hand, — and the maximum brightness then lies in the green, — two dark stripes appear, one in the indigo and the other in the red, so that both ends of the spectrum appear darkened, something which is not the case with the red of the first order. (See footnote †, p. 65.)

As may readily be understood, with further increase of the retardation the red passes over into violet, and this, from the path difference 0.001130 mm. on, into blue (the transition color in question is called "sensitive violet No. II"); so that here there begins a new series of the colors blue, green, yellow, red, and violet, which are known as **colors of the third order.** The red of this order corresponds to a retardation of 0.001590 mm.; *i.e.* again to a whole multiple of $\lambda$ of the green at *E*, namely, to 3 × 0.000530; accordingly, with this double refraction also, the color named is annihilated by the interference. But so too is a

part of the red and of the violet; for the difference of path is now, also, nearly equal to $2\lambda$ of the extreme red and equal to $4\lambda$ of violet. Therefore the colors having their maximum intensity after the interference are those with $\frac{5}{2}\lambda$ and $\frac{7}{2}\lambda$ difference of path, namely, orange-red ($\lambda = 636$) and indigo-blue ($\lambda = 454$); these mix to form a red that is much less pure and also less intense, because at the same time very considerable portions of the spectrum are obliterated. (The spectrum exhibits a dark stripe in the green, a second in the violet, and a shortening of the red end; when the nicols are set parallel, two dark stripes naturally appear in the orange-red and the indigo-blue respectively.)

In the **fourth order**, now following, the purity and the vividness of the colors diminish in a much higher degree; for the red of this order, for example, corresponds to a path difference of 0.002120 mm., and this, while equal to $4 \times 530$, is equal also to $3 \times 707$ and to $5 \times 424$ $\mu\mu$, the former of these values being the wave length of a very vivid red and the latter that of indigo near the line *G*. The spectrum of the red of this order accordingly yields, besides the dark stripe in the green, yet two more, one in the red and the other in the indigo; so that the mixing results only in a pale red strongly tinged with yellow. In just the same way, the complementary color, arising with parallel nicols, is a very pale and impure green, whose prismatic decomposition exhibits maxima at the same three positions in the spectrum where crossed nicols gave a dark stripe, while a dark stripe now appears in both the blue and the yellow.

The colors of the first, second, third, and fourth orders represented in Fig. 2 of Plate I can be best studied with the aid of a thin wedge of the form shown in Fig. 101, page 202, cut from a doubly refracting crystal, as quartz *; one pushes the

* Quartz wedges of excellent workmanship, which exhibit the colors of the first four orders, are supplied by the firm of Steeg and Reuter in Homburg. (See Appendix.) But, in such wedges the colors of the first order begin with the light gray corresponding to a path difference of about 100 $\mu\mu$, because it is not possible to grind the quartz thinner.

wedge between two crossed nicols in such wise that its long edge forms an angle of 45° with their vibration directions. But it must here be remarked that the foregoing explanation of the interference-colors of thin doubly refracting lamellæ presupposes that the difference of path produced by such lamellæ be the same for all colors contained in the white light. This presupposition, as will be shown farther on, does in the case of no doubly refracting body prove to be exactly correct, the retardation for the rays of a different vibration period being a little greater or less. Variations are thereby produced in the resulting composite color, — variations so slight, however, in the majority of cases, that in a reproduction such as is given in Fig. 2 of Plate I they can hardly be represented. Since the strength of the double refraction, and accordingly also the difference of path arising with a definite thickness of plate, does with certain substances increase with increasing wave length of the light, but with others diminish, this figure may be said to represent the normal case; which is the more closely approximated by the colors of the first, second, third, and fourth orders actually observed in crystals, the less the birefringence of the crystals differs for different colors.

The greater the thickness becomes, of the crystal plate viewed in the polarized light, or the higher its birefringence, — the greater therefore the retardation of the two rays arising in it, as compared with each other, — the less may the colors of the fifth, sixth, and higher orders resulting from the interference differ from white, as is taught by a simple consideration. Suppose, for example, the difference of path to be 0.011475 mm. This quantity is equal to $15 \times 765\ \mu\mu$, *i.e.* to $15\lambda$ of the extreme red beyond the line *A*; but likewise equal to the following:

$16 \times 717\ \mu\mu$, or $16\ \lambda$ of the red at the line *a*
$17 \times 675\ \mu\mu$, or $17\ \lambda$ of the red between *B* and *C*
$18 \times 637\ \mu\mu$, or $18\ \lambda$ of orange
$19 \times 604\ \mu\mu$, or $19\ \lambda$ of the yellow between line *D* and orange
$20 \times 574\ \mu\mu$, or $20\ \lambda$ of the yellow on the opposite side of *D*

21 × 547 $\mu\mu$, or 21 $\lambda$ of yellowish green
22 × 522 $\mu\mu$, or 22 $\lambda$ of the green between *b* and *E*
23 × 499 $\mu\mu$, or 23 $\lambda$ of bluish green
24 × 478 $\mu\mu$, or 24 $\lambda$ of the blue near *F*
25 × 459 $\mu\mu$, or 25 $\lambda$ of the most vivid blue
26 × 441 $\mu\mu$, or 26 $\lambda$ of the blue near the line *G*
27 × 425 $\mu\mu$, or 27 $\lambda$ of indigo
28 × 410 $\mu\mu$, or 28 $\lambda$ of the transition to violet
29 × 395 $\mu\mu$, or 29 $\lambda$ of the violet at the line *H*

If a crystal plate of this kind is illuminated with white light, and if after the interference the emerging rays are decomposed by prism, then in their spectrum there accordingly appear fifteen dark stripes, while, of the colors lying between, those for which the path difference amounts to an uneven multiple of a half wave length have their maximum brightness. Therefore the arising mixture contains all colors of the spectrum, containing them, too, in nearly the same relative intensities as did previously the white light. The consequence is that in our eye this mixture also will produce the impression of white, — or, to be more correct, of light gray, since by the interference extinction has taken place for portions of all the colors, the total intensity thus naturally becoming less considerable. This interference-color is called **white of a higher order.** If we rotate the nicols into parallel position, then in the spectrum dark stripes appear at all points where with crossed nicols there were maxima of light, and bright ones where previously the light was minimum; but the arising interference-color, the complementary color of the last, can, of course, without spectral decomposition, just as little as that last color be distinguished from white. The thicker a plate that exhibits the white of a higher order, the more numerous are the positions in the spectrum for which, by interference, the annihilation takes place, wherefore the more exactly do the remaining colors possess the same relative intensities as in the white light and the more exactly are their intensities equal to one-half what they would be without the interference.

Hence it follows that, from a certain thickness on, a doubly refracting crystal plate rotated between crossed nicols no longer exhibits a color but simply becomes light and dark; the greatest brightening with white light takes place when the vibration directions of the plate are in diagonal position to those of the nicols. The thickness with which the white of a higher order appears depends on the birefringence of the crystallized substance in question, *i.e.* on the specific strength of its double refraction; for, obviously, the greater this is, with so much the less thickness of a plate composed of the substance is the requisite difference of path attained.

As may be understood from the foregoing directly, the color observed in thin crystal plates suffers a perceptible change when there is a small variation in the thickness; consequently such a plate, if at different points it has not exactly the same thickness, does not exhibit everywhere the same interference-color. Therefore the appearance of the same color shade throughout the whole extent of the plate is a very delicate means of checking the uniformity of its thickness.

But different polarization-colors at different points are exhibited also by a plate whose thickness is everywhere the same, if it is cut not from a homogeneous crystal but, for example, from an aggregate of crystals of the same kind intergrown with one another in different orientations; because, as will be shown in the following, in all doubly refracting crystals the strength of the double refraction varies with the direction. Now many so-called "compact" minerals are **crystalline aggregates**, *i.e.* consist of very small crystalline particles oriented irregularly with respect to one another; and hence, if these particles are doubly refractive, a very thin plate prepared from the aggregate, viewed between two crossed Nicol prisms, exhibits **aggregate polarization**: the single particles differ from one another in their coloring and in their position of extinction on rotation of the plate. Thereby the texture of such an aggregate is brought into bold relief, while without the use of polarized light the ag-

gregate may possibly appear quite uniform. We have to do here with a body that is *not homogeneous*; and this is sometimes the case also with freely developed crystals. By disturbances in the growth of one of these it may happen that single parts of it have been deflected, as it were, from their normal orientation. Since the vibration directions of light in doubly refracting crystalline bodies stand in a definite relation to the arrangement of the smallest particles of the same, the deflection of any part of such a body must be accompanied by a variation of these vibration directions, and between crossed nicols the parts in question do not then extinguish in the same positions as does the rest of the crystal. Again, if during the growth of a crystal a gradual change in the chemical composition has taken place, so that in their nature the outer zones of the crystal differ the most from the inner, and if to this difference there corresponds at the same time a difference in the orientation of the vibration directions, then the crystal does not appear dark in its entire extent at once, but on rotation the extinction travels over it.

In such cases it is a matter of aggregates of crystals not parallel, or else of conglomerates of crystals of different kinds. It not infrequently happens in the solidification of molten metals, in the slow congelation both of artificial vitreous fluxes and of natural eruptive rocks, and finally sometimes also during the crystallization from solutions, that crystal aggregations of spherical shape are formed which have, within, a radiating-fibrous structure such that the single fibers meeting at the center of the sphere each consist of one crystal. Now if from such a mass, a so-called **spherulite**, one cuts a very thin central plate, and if the vibration directions of the radial fibers composing the plate are parallel and perpendicular respectively to the long direction of these fibers, then in parallel light between crossed nicols such a plate exhibits a dark cross, whose arms are parallel to the principal sections of the nicols. The explanation of this phenomenon is very simple: The fibers whose

long direction passes parallel to the polarization plane of either nicol must, since their vibration directions are parallel and perpendicular to the long direction, obviously appear dark; and the immediately adjacent fibers nearly dark, since their vibration directions diverge but little from those of the nicols; yet, the greater the angle that a fiber makes with these latter directions, the more is it lighted up by the polarized light, and this is the case in the highest degree with the fibers standing at 45° to the nicols. Consequently, from the center of the radiating-fibrous mass there must pass out four black arms, becoming broader outward; and these arms divide the surface into four quadrants, whose brightness is maximum on the intermediate radius, diminishing toward the dark arms. Since spherulites are often so small that even with microscopical observation of the sections their fibrous structure can not be perceived, the described phenomenon, which with such a body always appears when the nicols are crossed, may serve to distinguish the radiating-fibrous character of a spherulite. This interference phenomenon may be very easily imitated if a crystal extinguishing parallel to its long direction is cemented over a radial opening cut in an opaque disk: when the disk is rapidly rotated between two crossed nicols the crystal takes up in quick succession the different positions of the single fibers of a radiating-fibrous plate.

In explaining the above-described phenomenon it has been supposed that we have to do with a plate passing through the center of the spherulite, and so thin that it consists only of a layer of radial fibers lying parallel to its plane. If the plate is thicker, and if therefore only one or neither of its two faces passes through the center of the spherulite, then at its center the preparation consists of fibers standing perpendicular, while around this central axis it consists of inclined fibers, whose inclination increases with their distance from the center. In these inclined fibers therefore the rays experience a different double refraction, and consequently from the center of the

plate out to the edge there appears a succession of different interference-colors. When the vibration directions of the fibers composing the spherulite are oblique to their long direction, the arising cross, also, lies oblique to the vibration directions of the nicols.

As Bütschli has recently shown, a similar interference phenomenon (a cross parallel to the vibration directions of the nicols) arises in amorphous bodies,—*e.g.* in flakes of resin — by reason of concentric, slanting cracks, and in consequence of the polarization effected at these cracks by total reflection. (Cf. p. 49.)

### POLARIZATION APPARATUS

For observing the phenomena brought about by the double refraction of light, so far as they have yet been considered, service is made of the polarization apparatus shown in vertical section in Fig. 34. This consists of a mirror, *m*, so inclined that by it the light coming from a bright part of the sky is reflected vertically upward; and of a polarizing nicol, *p*, which with advantage is put in a tube, between two glass lenses, *l* and *l*, in such a way that the common focus of these lenses falls in the center of the nicol. Thus the rays of light, falling parallel upon the first lens, must all pass through the nicol drawn together into the form of a double cone, and hence emerge, again parallel, from the second lens. The instrument consists further of the stage, *s*, with a horizontal, rotatable glass plate on which is laid the crystal to be investigated, *k*; and finally of the analyzing nicol, *a*, which brings the doubly refracted vibrations back to one plane of polarization. Both nicols, as well as the stage, are rotatable about the vertical axis of the apparatus. The polarizer may be replaced by substituting for the reflecting mirror a bundle of thin glass plates, which according to page 49 gives to the reflected light a certain degree of polarization, although not a quite complete one. But the use of such a polarizer is frequent, because it is far cheaper to manufacture than

is a Nicol prism of so large a size as is desirable for the illuminating power of the instrument. Finally, the simplest — and, to be sure, also the least perfect — form of polarization apparatus is the so-called **tourmaline tongs**, consisting of two tourmaline plates (cf. p. 54) so mounted as to be rotatable with reference to each other; the crystal to be investigated is simply wedged in between the two plates and observed by directing the little instrument toward the bright sky.

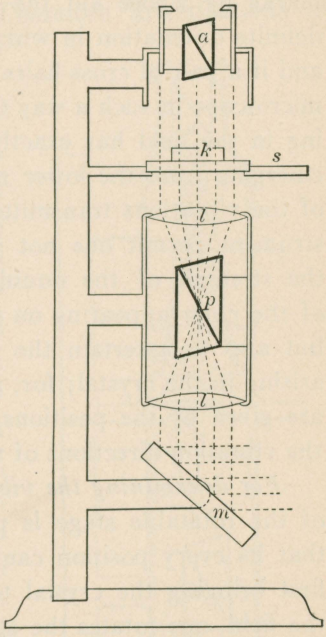

Fig. 34.

The polarization apparatus pictured in Fig. 34 is called also an **orthoscope**, because, with it, only those light rays that pass through the crystal plate *at right angles* are investigated for the alteration they have suffered in the crystal. When the crystal is not visible to the naked eye, *i.e.* if to observe it a *microscope* is requisite, the microscope is converted into an orthoscope by inserting a Nicol prism below the stage — which for such investigations must of course be rotatable — and mounting a second nicol on the eyepiece or putting it between the eyepiece and the objective, within the microscope itself. [The instrument is then known as a **polarization-microscope.**] Now if we look into such an instrument when the upper nicol is crossed as regards the lower, the field of view appears dark, and so do all singly refracting bodies that may be in it. But if we place on the stage a transparent preparation containing doubly refractive crystals, *e.g.* a so-called " thin section " of a rock (*i.e.* a plate ground extremely thin), then, when the stage is rotated in its own plane these crystals must pass through the change

from dark to light or colored elucidated in the foregoing sections. If the mounting of the rotatable nicols is provided with marks by whose aid the nicols may at any time be given a definite orientation in which they are at the same time crossed, and if a pair of cross-hairs is brought into the focal plane of the microscope in such a way that one of the two hairs then appearing in the field has exactly the vibration direction of the light emerging from the lower nicol and the other hair the direction of the vibrations transmitted by the analyzer, then does the instrument permit one not only to determine the existence and the strength of the double refraction of a crystal, by means of the color appearing on rotation, according to page 63 *et seq.*, but also to ascertain the vibration directions of the two rays arising in the crystal; for, according to page 61, these directions are given by the positions of darkness of the crystal, provided the vibration directions of the nicols are known.

*For determining the vibration directions of a crystal* the edge of the rotatable stage is provided with a graduated circle, so that its every position can be read off at a fixed mark. After first bringing the crystal to be investigated into the middle of the field, one rotates the stage until a straight boundary of the crystal touches or is parallel to one of the cross-hairs throughout its length, and reads this position of the stage on the graduated circle. In general neither vibration direction will be parallel to this boundary of the crystal plate, and the plate will therefore not appear dark; but if one rotates the stage, and with it the crystal, from the position first read to that where extinction of the light occurs (in which position the vibration directions of the crystal coincide with those of the nicols), then the rotation, read off on the graduated circle, gives the **extinction angle,** — *i.e.* the angle formed by one of these vibration directions with the previously adjusted crystal boundary.*

Both with the polariscope (cf. p. 59) described on page 74

* For particulars as to the arrangement and use of polarization-microscopes cf. footnote †, p. 30.

and with the microscope arranged for polarization, we can, in general, investigate only those optical phenomena that the crystal exhibits in *one direction.* Therefore investigations of this kind are termed **investigations in parallel polarized light.**

But very frequently one aims to observe at once the alterations which the polarized light suffers in passing through a crystal along directions as different as possible. This purpose is served by the polariscope for **observations in convergent light**, called also a **conoscope.** This instrument, represented in Fig. 35 in vertical section through the axis, consists of a mirror, *m*, and of the polarizer, *p*, which is enclosed between two lenses, *l* and *l*, exactly as with the orthoscope. Behind the upper lens *l* is a screen (diaphragm) of blackened metal having in it a circular opening of the diameter *de*. This opening is illuminated from the bright sky through the medium of the mirror *m*, the nicol *p*, and the lenses *l*; every point of it by a cone of rays whose fulcrum is that point itself and whose base is the lower lens *l*. In the figure these ray cones are given for the two points *d* and *e*, at the edge of the diaphragm, and for its center, *c*; but below the upper lens *l* the cones are continued only in dots, because, on account of the refraction in the lenses, the true path of the rays is other than that indicated: their path is, indeed, such that all rays falling parallel on the lower lens are again parallel after their exit from the upper, but naturally

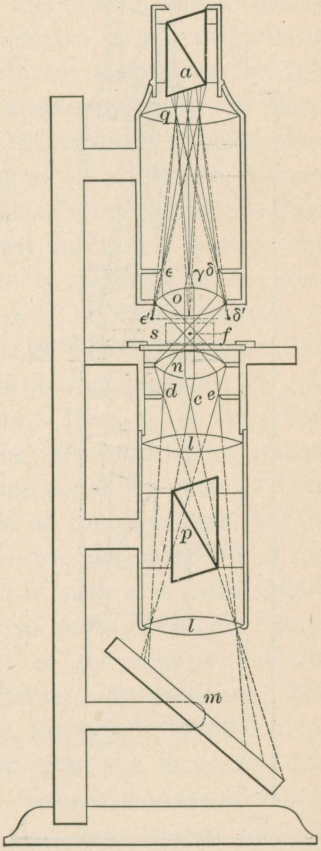


Fig. 35.

then become components not of the same cone but of different ones of the cones of light falling respectively on *c*, *d*, *e*, etc. Hence, it is the bright opening *de*, illuminated from below by plane-polarized light, that we look at through this instrument. We may therefore consider every point of this opening — *e.g.* the point *d* — as a point from which diverging light rays proceed; these rays do not, however, proceed in all directions, as from a self-luminous point, but only in such directions as lie within the conical angle of the cone whose fulcrum is *d* and whose base is the lower lens *l*. The rays proceeding from *d* are rectilinear prolongations of those of the cone in question. If we follow their path upward, we see these diverging rays impinge on a very convex lens, *n*; this stands, from the diaphragm *de*, at exactly the distance of its focal length, so that in consequence all the rays diverging from a point of the focal plane *de* are transformed by *n* into parallel rays. Above *n* there is a lens *o*, of the same size and convexity, mounted at the lower end of a vertically movable tube, which contains also the eye-piece, *q*; this tube is depressed until the focus of *o* and that of *n*, which has the same focal length as *o*, coincide at *f*. The ray cone proceeding from *d* is refracted at the left side of *n* and transformed thereby into a ray cylinder; this cylinder passes through the right side of *o* and, since the rays are parallel, is again concentrated in the focal plane of *o* at the point

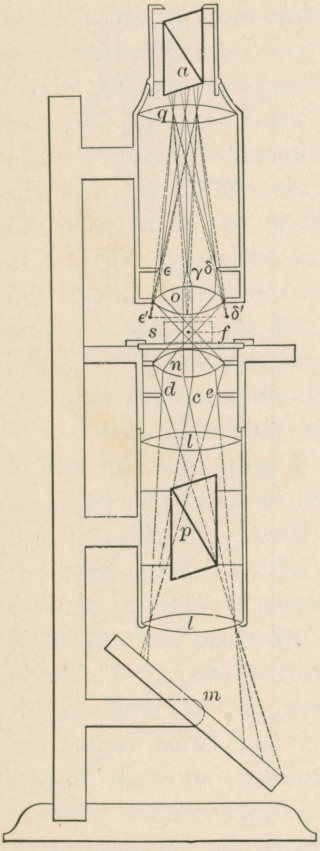


Fig. 35.

$\delta$, corresponding to $d$. In the figure the same construction is carried out for the rays that proceed from the center $c$ of the illuminated opening $de$, which must converge at $\gamma$; also for those that, coming from $e$, converge at $\varepsilon$. Since the same applies to all points of the opening $de$, an image of this opening must arise in the plane $\delta\varepsilon$. This image, then, we observe with a magnifying glass of very low power; namely, with the eyepiece $q$, through which we see a so-called virtual image approximately in the plane $\delta'\varepsilon'$. The rays that seem to come from this image pass, before they arrive in our eye, through the analyzing nicol, $a$. Hence, if upon the stage, $s$, we lay a plane-parallel plate of a crystallized medium (represented in the figure by the dotted outline) in such a way that $f$ falls within the plate, the plate is traversed by ray systems of very diverse direction; all rays of the same direction — which in a homogeneous crystal must of course experience the same alterations — converge at a single point of the image $\delta'\varepsilon'$, all those of other directions at other points. Thus, in the image $\delta'\varepsilon'$ we are able to view at a glance all the interference phenomena suffered within the crystal to be investigated by rays of very many diverse directions, — namely, of all the directions that lie within the cone projected from $f$ upon the circumference of the lens $n$, — and we then see in the field of view a so-called **interference-figure** * of the crystal, different parts of which figure, corresponding to different directions of the rays within the crystal, exhibit different color and different light intensity. The shorter the focal distance of $n$ and $o$, the greater is the conical angle of the cone projected on them from the focus, and the larger is the **field** of the instrument. Since it is often necessary to concentrate within the field rays that have passed through the crystal in very divergent directions, Norrenberg replaced each of the two lenses $n$ and $o$ with a system of several plano-

* As follows from Fig. 35, the rays inclined to the right converge on the right side of the field; so the interference-figure exhibits the phenomena on the same side to which the rays are inclined in the crystal, while the image of the crystal itself, which one sees on removing the tube, is reversed.

convex lenses almost touching one another, —these lenses having, together, the effect of a lens of very short focus. Constructed in this form, which is now in common use, the apparatus is therefore often designated as " Norrenberg's polariscope ".

The apparatus just described serves for investigating fairly large crystal plates in convergent light, exactly as the one pictured in Fig. 34 serves for investigating them in parallel light. But just as a *microscope* with rotatable stage can, through the addition of two Nicol prisms, be made available for the orthoscopic-optical investigation of small crystals, so also may it be transformed into a *conoscope*, with which even a crystal of microscopic smallness can be investigated in convergent polarized light. To this end it is necessary, besides adding the two nicols, only to insert a condensing lens into the opening in the stage, directly under the crystal, —which lens projects strongly convergent light rays through the crystal lying at the center of the field,— and to adjust the optical parts of the microscope proper no longer on the crystal, but on the interference-figure arising in another plane. The latter purpose is accomplished either by removing the eyepiece, after which one sees the very small interference-figure at a definite place in the in tube; or by inserting an auxiliary eyepiece, which corrects the adjustment and at the same time magnifies the interference-figure, so that in the field one then sees not the crystal, but the interference-figure produced by it. * †

* For the more detailed description of conoscopes and the methods of using them cf. footnote †, p. 30.

† [As regards application of the polarization-microscope to petrographical investigations see especially the following: Joseph P. Iddings: "Rock Minerals", New York, 1904; Rosenbusch-Iddings: "Microscopic Physiography of the Rock-making Minerals", New York, 1888; N. H. and Alexander Winchell: "Elements of Optical Mineralogy", New York, 1909; L. M. I. Luquer: "Minerals in Rock Section", New York, 1908.]

## OPTICALLY UNIAXIAL CRYSTALS

### DOUBLE REFRACTION OF LIGHT IN CALCITE

The relations in which, in a doubly refracting crystal, the vibration directions and transmission velocities of the two rays arising by the double refraction stand to the form of the crystal, were perceived first in calcite (crystallized calcium carbonate), and this mineral thereby became the point of departure in the investigation of the laws of double refraction. Calcite cleaves very perfectly along three directions forming with one another

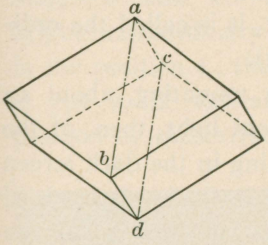


Fig. 36.

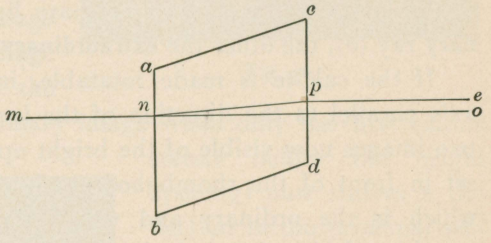


Fig. 37.

angles of 74° 56′, and since it occurs in nature (especially in Iceland) in very large crystals of watery clearness, large, transparent cleavage blocks can be easily produced which have the form of a so-called *rhombohedron.* (See Fig. 36.) These blocks are highly suitable for the study of double-refraction phenomena, especially because calcite has the property of deviating the two rays transmitted in it an amount whose difference for the two rays is very considerable. Let Fig. 37 represent a so-called **principal section** of the rhombohedron; *i.e.* a section through the four solid angles, *a*, *b*, *c*, *d* (Fig. 36), so that *ab* and *cd* are the short diagonals of the rhombohedron faces. The plane of such a principal section divides the rhombohedron *symmetrically, i.e.* into two equal and opposite halves; it is therefore said to be a **plane of symmetry** of the rhombohedron. Now if upon one of the rhombohedron faces, as *ab* in Fig. 37, we cause to fall perpendicularly a bundle of rays of ordinary light that are parallel to

the line *mn* lying in the principal section, for example, if we look through the two parallel rhombohedron faces toward a small, round, illuminated opening in a dark screen, we see two equally bright images of that opening. Whence it follows that a light ray entering the calcite along the line *mn* is split up into two rays, of which one, *no*, like a ray of ordinary light with perpendicular incidence, passes through the calcite unrefracted, while the second, deflected in the plane of the principal section, moves in the crystal from *n* to *p* and there suffers the opposite refraction to that at *n*, emerging therefore along the direction *pe*. Since, of these two rays, only the first, *no*, conforms to the law of refraction for ordinary light, it is called the **ordinary** ray (*o*), the other the **extraordinary** (*e*).

If the calcite is made rotatable, in a mounting, about an axis parallel to the direction of the incident light, then, of the two images now visible of the bright opening in the dark screen set in front of the rhombohedron, it can easily be determined which is the ordinary and which the extraordinary; for on rotation of the rhombohedron the ordinary image, not being deflected, remains stationary, while the extraordinary, deflected in the plane of the principal section, moves around the ordinary image in a circle. If into absolutely parallel position before this calcite we now bring a second, of equal size and mounted in just the same way, then on looking through both calcites toward the bright opening we see, first, that the ordinary image *o* emerging from the first calcite passes through calcite *II* *wholly* as an ordinary image, *without being again doubly refracted;* and second, that just as little doubly refracted is the extraordinary image *e*, which, unlike *o*, passes through calcite *II* as an extraordinary image, since it suffers a further deflection of the same amount as in *I*: we see, as before, two equally bright images, one above the other, but at double the distance. If we now rotate the foremost calcite to the right or left in its mounting, there appear *four* images of the opening, but of *different brightness;* and from the deflections it is easily under-

stood that each of the two previous images *o* and *e* has been resolved into an ordinary and an extraordinary. Figure 38 shows the phenomenon observed when the principal section of calcite *II*, that next to the eye, has been rotated a small angle to the right. While with this calcite in parallel position to the first, *I*, there were visible only an ordinary image *o* and an equally bright extraordinary image in the position $e_2$, the image $e_1$ (by $e_1$ is indicated the position of the extraordinary image when only the first calcite stands before the bright opening) now suffers a deflection in the principal section of the second calcite, appearing therefore at $\varepsilon_e$, but with somewhat less brightness; and at $e_1$, *i.e.* without deflection, a second faint image appears: on the other hand, *o* too has diminished in brightness, and there now appears, deflected in the principal section of the second calcite,—with extraordinary refraction, therefore —an image $\varepsilon_o$, of the same slight brightness as the now ordinary $e_1$. The farther we rotate, the more does the brightness diminish of the ordinary image *o*, formed from the first ordinary, and of the extraordinary image $\varepsilon_e$, arising from the extraordinary, while the brightness of the other two images continuously increases; with 45° rotation all four images are equally bright and are in the position represented in Fig. 39.

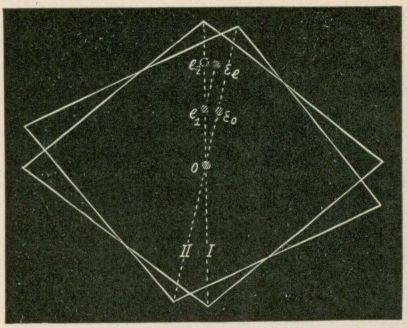


Fig. 38.

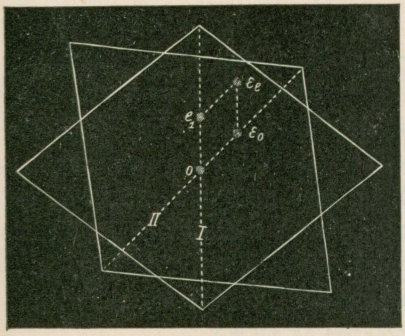


Fig. 39.

When the principal sections of the two calcites stand mutually perpendicular the ordinary image of *o* and the extraordinary of *e* entirely vanish: there appear only two images, $\varepsilon_0$ and $e_1$, — the deflected, hence extraordinary, of *o*, and the not further refracted, hence ordinary, of *e*. (See Fig. 40.) After a rotation of 180° there appears only a single image, since in the second calcite *e* is deflected the same amount as in the first but in the opposite direction, consequently being after its exit identical with the image *o*. To sum up: During one whole rotation of one of the two calcites through 360° there are, as observation shows, only four positions (viz. when the principal sections of the two calcites intersect at 45°), when the rays *o* and *e* emerging from the first rhombohedron behave like rays of ordinary light, *i.e.* each become resolved into two rays of equal brightness: in all other positions there arise from each of them two rays of unequal intensity; except, finally, with the principal sections mutually parallel or perpendicular, when the two rays are not doubly retracted at all.* †

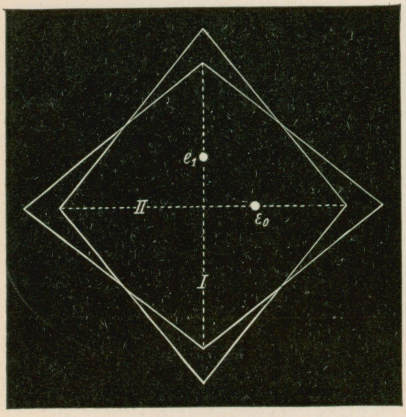


Fig. 40.

From this behavior, therefore, it is seen that each of the two rays (really bundles of rays) emerging from the first calcite exhibits a complete symmetrical *one-sidedness* with reference to

* These phenomena are suitable for distinguishing plane-polarized light from ordinary light, something of which the eye is incapable.

† [For an interesting discussion of the appearance of a bright spot viewed through calcite (also through other uniaxial, as well as through singly refracting and biaxial crystals), showing how these phenomena are modified (the images distorted, etc.), see a posthumous article by the late H. C. Sorby: "The Optical Properties of Crystals", *Min. Mag.* 1909, **15**, 189–215.]

one of the diagonals of the rhombus that represents the surfaces of entrance and emergence of the light, — to that diagonal, namely, that corresponds to the principal section. If, therefore, we would explain these phenomena by the arising in the crystal of two rays polarized perpendicularly to each other, we can do so in no other way than by assuming that the vibration direction of one of these rays coincides with the diagonal mentioned, and that the vibration direction of the other ray coincides with the second diagonal (the longer one of the rhombus), perpendicular to the diagonal first mentioned.

Assumed: that the ordinary ray *o* emerging from the calcite consists only of rectilinear vibrations parallel to the longer diagonal of the rhombohedron face, the extraordinary *e* only of such as are parallel to the shorter diagonal. If *I* and *II* in Fig. 41

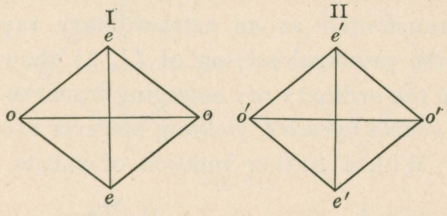


Fig. 41.

might represent, *side by side*, the two calcite rhombohedrons through which the light is to pass, — which therefore are to be imagined as lying one *before* the other, — then would *oo* and *o'o'* be the vibration directions in them of the ordinary ray whose transmission direction stands perpendicular to the plane of the figure, *ee* and *e'e'* those of the extraordinary ray of that direction. The ordinary ray emerging from *I* with the vibration direction *oo* arrives in *II*, when *I* and *II* are parallel, as Fig. 41 represents, with the vibration direction *o'o'*, and the motion is here resolved into one along *o'o'* and one along *e'e'*; the portion falling to the latter direction is zero, however, while the component parallel to *o'o'* corresponds to the total intensity of the ordinary ray entering *II*; so *in the second calcite* this ray is

*not doubly refracted*, but passes through wholly as an ordinary ray. If we now imagine *II* as rotated a small angle (Fig. 42), the ordinary ray from *I* enters *II* with the vibration direction $\omega\omega$, being here resolved into two rays which vibrate parallel to $o'o'$ and $e'e'$, and whose amplitudes are proportional to the lengths *CO* and *CE*. In other words, the ray does not

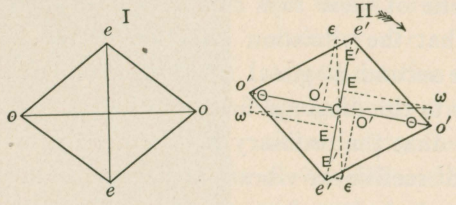


Fig. 42.

pass through with its full intensity as an ordinary ray: the remainder is transmitted as an extraordinary ray, — deflected, therefore, in the principal section of *II*, as shown in Fig. 38. So in this case the ordinary ray emerging from the first calcite is in the second *doubly refracted*, yielding however two rays of *different intensity*. With a further rotation of calcite *II* (Fig. 43),

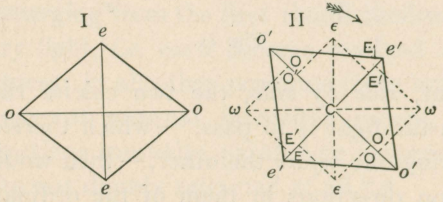


Fig. 43.

namely, when it forms 45° with *I*, the ordinary ray is by *II* resolved into two components of equal magnitude, thus yielding two rays, an ordinary and an extraordinary of equal brightness; accordingly it passes through the second calcite only to the half part as an ordinary ray. This is exactly the phenomenon shown in Fig. 39. Now the farther one rotates *II*, the smaller is the portion of the horizontal ether vibrations emerging from *I* that passes through *II* as an ordinary ray; and when finally

the calcites have the position represented in Fig. 44, in which position *I* and *II* are crossed at right angles, the ordinary ray coming out of *I* enters *II* with the vibration direction $e'e'$, being resolved therefore into two mutually perpendicular components of which the one, parallel to $o'o'$, is zero, while the other, parallel to $e'e'$, is equal to the whole amplitude of the entering light. So with this position of the calcites the ray *o* can *nowise* pass through the second calcite as an ordinary ray: it must go through wholly as an extraordinary. (Cf. Fig. 40.) In part again as an ordinary ray does the ray *o* pass through *II* on further rotation of the latter, wholly as such when the total rotation amounts to 180°; — and it is evident that entirely the same would occur

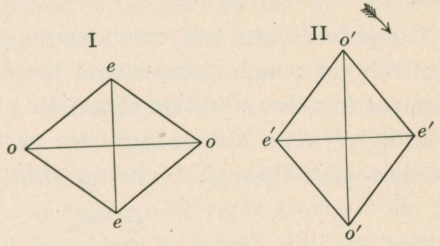


Fig. 44.

if the rotation, starting out from the position in Fig. 41, should take place to the left, instead of to the right. The fundamental assumption as to the nature of the light that has passed through the first calcite accordingly teaches, just as do the observed and previously described phenomena, that the principal section of the calcite rhombohedron is a *distinctive character* of the ordinary ray emerging from the first rhombohedron, inasmuch as this ray passes through *II wholly* as an ordinary ray when the principal section of *II* is *parallel* to that of *I*, *the less* as an ordinary ray *the greater the angle* included between the two principal sections, and finally *not at all* as such when the principal sections stand mutually *perpendicular*. So this plane which is thus characteristic of the ray *o* is the plane that stands perpendicular to the vibration direction assumed for this ray; in other words, it is the plane designated on page 15 as the *plane of*

*polarization* of a plane-polarized light ray. Therefore we say the ordinary ray is *polarized in the principal section* of the crystal.

Let us now, under the same assumptions, consider the second, the extraordinary, light ray emerging from calcite *I*. Figure 41 or Fig. 45 shows that this ray, entering *II* with the vibration

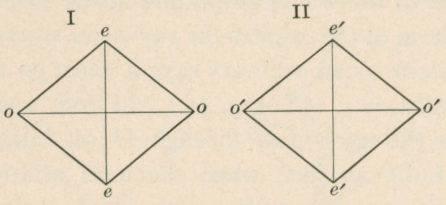


Fig. 45.

direction $e'e'$, is resolved into two components parallel to $o'o'$ and $e'e'$ respectively, of which components the former is zero; but this component has the vibration direction of the ordinary ray; and accordingly, when the calcites are parallel, the ray *e* from *I* can *nowise* pass through *II as an ordinary ray*, as is

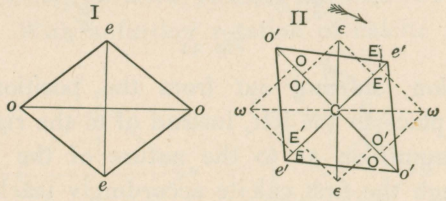


Fig. 46.

taught in fact by observation. (Cf. p. 82.) If we rotate *II* somewhat (see Fig. 46), then *e* enters with the vibration direction $\varepsilon\varepsilon$, and is resolved into the two components $CO'$ and $CE'$. Thus the extraordinary ray from the first calcite now passes *in part* as an ordinary through the second; and moreover in greater part, the farther we rotate. When finally, as in Fig. 47, the plane perpendicular to the principal section of the second calcite — which plane is determined by $o'o'$ and the transmission line of the ray — stands parallel to the principal section of the first calcite, the extraordinary ray emerging from this latter

must pass through *II* wholly as an ordinary ray (as is, according to Fig. 40, actually the case); on still further rotation, again only in part as an ordinary ray; and so on. In order, therefore, that the ray *e* may pass through *II* wholly as an ordinary ray, the plane perpendicular to the principal section of the second calcite must be parallel to the principal section of the first; the greater the angle included between these two planes, the smaller is the part of the ray *e* that can pass through *II* as an ordinary ray. So the plane standing perpendicular to the principal section, *i.e.* the transverse plane (see p.15) of the ordinary ray, is a character of the extraordinary ray in wholly the same way as the principal section itself is a character of the ordinary; it is accordingly the *polarization plane* of the extraordinary ray: this ray is *polarized at right angles to the principal section.*

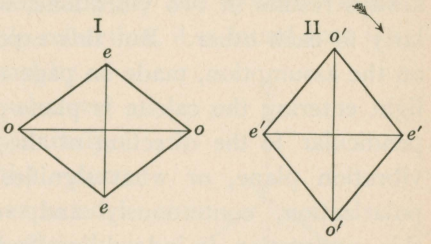

Fig. 47.

From the foregoing it is seen that the assumption that the ordinary ray vibrates parallel to the longer diagonal of the rhombohedron face and the extraordinary parallel to the shorter, affords a perfect explanation of the observed phenomena.*

* The same is the case also with the converse assumption — that the extraordinary ray carry out its vibrations parallel to the longer diagonal, the ordinary parallel to the shorter. For if in Figs. 41 to 47 the letters *o* and *e* are substituted for each other throughout, exactly the same deductions result; for example, from Fig. 41 or Fig. 45 it then follows that the ordinary ray emerging from the first calcite passes wholly as an ordinary through the second, and so on.

These two, sole possible assumptions differ from each other in this: that with the first it is supposed that a plane-polarized light ray vibrate *perpendicular to its plane of polarization*, with the second that it vibrate *parallel to that plane.* As hitherto, so also in the following, it will always be assumed that the vibration plane of a plane-polarized light ray stands perpendicular to its plane of polarization. According to Maxwell's electro-magnetic theory of light this holds true, properly speaking, only of the electric vibrations, while the magnetic take place in the polarization plane; but Wiener has demonstrated that for the optical phenomena it is the electric vibrations that are the criterion.

The phenomena in question are further consistent with the exposition given on page 52, according to which the resolution always results in two vibrations, which take place perpendicularly to each other. But this exposition, for its part, is based on the assumption, made on page 16, that likewise the *ordinary* light entering the calcite is plane-polarized light vibrating perpendicular to the direction of the ray — light, however, whose vibration plane, or what signifies the same, whose plane of polarization, continuously and very rapidly changes. That this assumption is indeed justified is proved by the following experiment, suggested by Dove. When a calcite rhombohedron upon one of whose faces a ray of ordinary light falls at right angles — in consequence whereof the arising ordinary ray must emerge, at the opposite face, along the same straight line — is caused to rotate very rapidly about this line as an axis, the emerging ordinary ray no longer exhibits toward a second calcite the behavior of a polarized ray, but of a ray of ordinary light. Here, unquestionably, we have to do with a plane-polarized ray, only its plane of polarization (the principal section of the calcite) is caused by the rotation of the latter to continuously and very rapidly change its direction.

A calcite rhombohedron exhibits the behavior described on page 81 *et seq.*, whichever of its three pairs of faces is employed as the faces of entrance and emergence of the light. So the rhombohedron has three equivalent principal sections, and these intersect one another along the direction that in Fig. 48 is placed vertical; with this direction the three upper, and consequently also the three lower, faces of the rhombohedron form equal angles. If we call the direction $AA$* the **axis** of the rhombohedron, we may now define its **principal optic section** for a ray falling perpendicularly on a rhombohedron face as the plane that contains the axis of incidence (*i.e.* the normal to the rhombohedron face) and the axis of the rhombohedron. Now if one

* If we think of it as a line connecting two opposite corners of the rhombohedron, then at each of its extremities, *a*, there intersect three *equiangular* edges.

causes light rays to fall in such a principal section, not perpendicular however to the rhombohedron face, but inclined at different angles, and determines for each ray the *refractive index* of the two rays arising by double refraction, it is found that the index *of the ordinary ray* (denoted by $\omega$) *always remains the same*, at whatever angle the light may fall; having, namely, for a certain medium color the value 1.6583. This was indeed to be expected, on account of the fact that, for all inclinations of the incident light that lie in the principal section, this ray has always the *same vibration direction;* namely, according to our assumption, the normal to the principal section. If under the same conditions one determines the refractive index of the *extraordinary ray*, this index ($\varepsilon$) is found to *vary with the direction in which the ray moves in the calcite* (as also was to be expected, since the vibration direction, falling in the principal section and normal to the transmission direction, varies with the inclination of the incident ray to the axis). This refractive index is exactly equal to that of the ordinary ray (1.6583), when the extraordinary is transmitted parallel to the axis; it is the smaller, the greater the angle between the ray direction and the axis; and has its least value, namely 1.4864, when the ray is perpendicular to the axis.

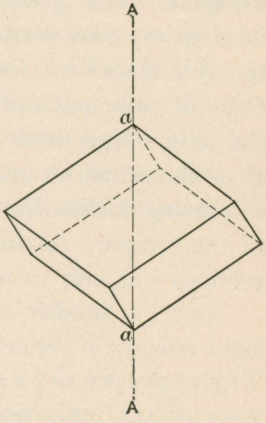

Fig. 48.

If instead of one of the three above-defined principal sections we now choose any plane that, like those three, passes through the axis, and if perpendicular to this chosen plane we cut on the calcite a plane face through which the light enters, then we observe that the refractive index of every ray falling at whatever inclination to the axis within the chosen plane has exactly the same value as was found for the ray inclined at that same angle within a principal section. Thus,

for this plane chosen at random the optical relations of the crystal are wholly the same as for the three above-defined principal sections. Consequently this random plane likewise may be spoken of as a principal section; and hence we may designate quite generally, as the *principal optic section* for an incident ray, *that plane that passes through the ray and the axis of the rhombohedron.* Calcite, accordingly, is a crystal having an infinity of principal optic sections, all intersecting one another in the axis. And these sections are all equivalent; for in each of them the refractive index of the extraordinary ray diminishes with increasing inclination to the axis in the same way, while that of the ordinary retains the same value not only in all principal sections but also in all directions within every one of them.

Since a smaller refractive index corresponds to a greater light velocity in the crystal, there follows from these observations: THE ORDINARY RAY *o* arising in the calcite IS TRANSMITTED EQUALLY FAST IN ALL DIRECTIONS; THE EXTRAORDINARY *e* IN THE AXIAL DIRECTION WITH THIS SAME VELOCITY, IN EVERY OTHER DIRECTION WITH A GREATER VELOCITY. Since the two rays have the same velocity when they move parallel to the axis, then parallel to this direction there can (see p. 51) occur no double refraction. A direction in which an otherwise doubly refracting crystal behaves like a singly refracting one, *i.e.* a direction in which its birefringence is zero, is called an **optic axis.*** Accordingly, ordinary light is transmitted through the calcite *as* ordinary light when it goes through along the optic axis; *i.e.* parallel to the axis of the rhombohedron (*AA* in Fig. 48).

Quite as with a ray of ordinary light in a singly refracting medium is the behavior of the ordinary ray, and this without exception; so the direction of this ray, for every case, is to be determined by Huygens's construction: ITS RAY-SURFACE IS A

* [It is to be borne in mind that an "optic axis" is nowise a line, but merely a *direction;* thus too with a "principal optic section": it is not a definite plane, but *any one of an infinity* of *parallel* planes. The same applies to still other axes, sections, etc., employed in crystal optics; for it is obvious that in a crystal all parallel lines, as well as all parallel planes, are *optically equivalent.*]

SPHERE. As for the extraordinary ray, Huygens * demonstrated that the different velocities of the extraordinary rays lying within a principal section but differently inclined to the axis are to one another as the radii vectores, in the same direction, of an ellipse, whose minor axis (*i.e.* its length) is the velocity of the extraordinary ray parallel to the optic axis and whose major axis is the velocity of that ray perpendicular to the axis. Let $Y$ (Fig. 49) be a direction perpendicular to the optic axis, $X$ a direction parallel to that axis, the plane $XY$ therefore a principal optic section of the calcite; and let further $ox : oy$ be the ratio between the light velocities of the ray $e$ parallel and perpendicular to the optic axis. Then the velocity of any extraordinary ray, $oR$, forming an angle $\phi$ with the optic axis is $or$; *i.e.* the length of the radius vector of the ellipse in the same direction. Since it is known empirically that the light velocity of the extraordinary ray is the same for all directions forming equal angles with the axis, the above applies to all the innumerable principal sections that may be imagined round about the axis. The length $or$ is the transmission velocity for all rays inclined at the same angle $\phi$ —rays obtained if one imagines Fig. 49 to be rotated 360° about $ox$ as an axis. When thus rotated the whole ellipse yields a **spheroid,** or **ellipsoid of rotation,** whose radius vector in any direction is a measure of the velocity of $e$ in that direction. If, therefore, from any point $o$ within the calcite there emanates a light-motion, the extraordi-

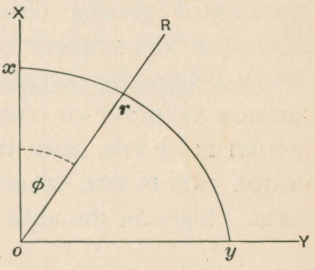

Fig. 49.

* Christian Huygens: ["Traité de la lumière. Ou sont expliquées les causes de ce qui lui arrive dans la reflection, et dans la refraction, et particulièrement dans l'étrange refraction du cristal d'Island." Leyden, 1690. A recent (1885) edition is still in print; and an English translation of the first three chapters is included in "The Wave Theory of Light", edited by Henry Crew. New York, 1900. (No. X of the Scientific Memoirs ed. by J. S. Ames.) There is also a German edition of the entire treatise:] "Abhandlung über das Licht", etc., ed. by E. Lommel. Leipzig, 1890. (Ostwald's Klassiker der exacten Wissenschaften Nr. 20.)

nary part of it, although it will be transmitted in all directions, will not be transmitted in all directions with the same velocity: after a certain time it will have been transmitted as follows: Along the axis as far as $x$; perpendicular to the axis, in all directions the same distance, namely, to $y$; along the random direction $oR$, as far as $r$; and so on — in a word, as far as the ellipsoid surface generated by the rotation of the ellipse $xry$ about $ox$. Since the wave length of light increases in proportion to the transmission velocity, the radii of this ellipsoid must be proportional to the several wave lengths of the light of a definite vibration period, the light of which several wave lengths is transmitted parallel respectively to the different radii. THE WAVE- OR RAY-SURFACE OF THE EXTRAORDINARY RAY IS ACCORDINGLY A ROTATION ELLIPSOID, WHOSE MINOR AXIS IS THE TRANSMISSION VELOCITY OF THE RAY PARALLEL TO THE OPTIC AXIS — WHICH IS AT THE SAME TIME THE ROTATION AXIS — AND WHOSE MAJOR AXIS IS THE VELOCITY OF $e$ PERPENDICULAR TO THE OPTIC AXIS. Since in the axial direction, $oX$, the respective transmission velocities of the ordinary and the extraordinary ray are equal, then in the direction $oX$ the ray-surface of the former must have the same diameter, $ox$, as that of the latter; we accordingly obtain the complete ray-surface of the light in the calcite if we cause the two curves represented in Fig. 50 to rotate about the vertical axis. If, therefore, a light-motion begins at any point within the calcite and can advance unhindered in all directions, then after a certain time it has arrived at a double surface which consists of a rotation ellipsoid and of a sphere enveloped by that ellipsoid, the latter touching the sphere at the extremities of the rotation axis (the minor axis of the generating ellipse).

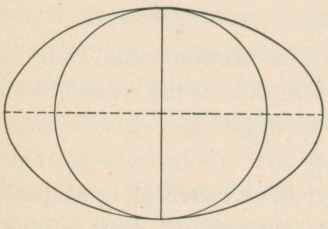

Fig. 50.

By our knowledge of the complete wave-surface we are

provided, then, in Huygens's construction, with a means of determining, for a light ray entering the calcite in any direction, the direction of the extraordinary ray just as well as of the ordinary.

For example: If *abcd* (Fig. 51) be a principal section of a calcite rhombohedron (*ab* and *cd* short diagonals, *ac* and *bd* obtuse edges), then is the direction *aA* the optic axis; hence if, in the same way as when (see Fig. 37) for the first time we observed the phenomena of double refraction in calcite, there fall upon *ab* rays of ordinary light with perpendicular incidence, their ray-front is parallel to *ab*; so they will all begin simultaneously, starting from the points $m, m_1, m_2, \ldots$ respectively, to be transmitted in the calcite. Proceeding from these points, the arising ordinary ray arrives in a certain time at the surfaces of the spheres $o, o_1, o_2, \ldots$; according to a previous page its new ray-front is the plane tangent in common to all these several wave-surfaces, namely *OO*; and thus the (common) direction of $m\omega, m_1\omega_1$, etc., is the direction of the arising ordinary rays. The second, the extraordinary, ray has in the same time advanced from $m, m_1, m_2, \ldots$ to the surfaces of the rotation ellipsoids $e, e_1, e_2, \ldots$, whose rotation axis is parallel to *aA*; consequently its ray-front is the tangential plane — *EE* in our section — common to all these wave-surfaces; so the direction, $m\varepsilon, m_1\varepsilon_1$, etc., of the extraordinary rays is given by the straight lines connecting $m, m_1, \ldots$ with the points of the rotation ellipsoids at which these ellipsoids are touched by *EE*. The

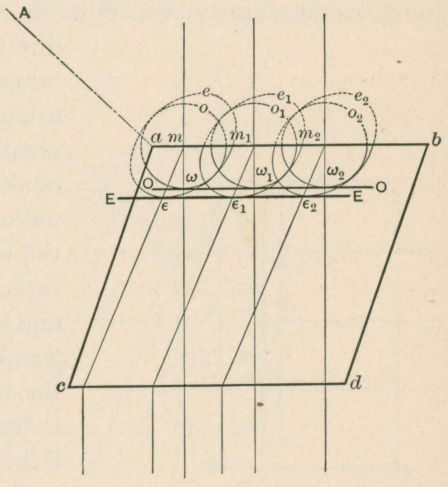


Fig. 51.

extraordinary ray must, accordingly, experience a deviation in the principal section, quite as we have previously observed; and one sees, besides, that with this ray the ray-front and the ray direction do not, as with the ordinary light ray, stand perpendicular to each other.

This is due to the circumstance that in general the tangent to an ellipse — and likewise the tangential plane to a spheroid — does not stand normal to the radius vector at whose extremity it touches the ellipse or the ellipsoid. The obliqueness of the two directions (*i.e.* radius vector and tangent), and thus also the deviation of the extraordinary ray, is greatest for a certain inclination of the ray to the axis, this inclination depending on the form of the ellipse; that is, on its axial ratio, the ratio between major and minor axis. Only at the extremities of these two axes does the tangent stand perpendicular to the radius vector. Therefore, if the face of entrance *ab* (Fig. 51) were perpendicular to the optic axis of the calcite, the direction of the light rays falling normal to *ab* would coincide with *Aa*, *i.e.* with the common axis of both skins of the several wave-surfaces; the ray-fronts tangent to these skins respectively would then be identical, as would likewise the directions of the two rays; so there would then be only one ray-front and only one ray; in other words, there would occur no double refraction. If on the other hand the face of entrance were *parallel* to the axis (see Fig. 52), then the two ray-fronts, *OO* and *EE*, touching the several wave-surfaces would stand at their greatest distance from each other, this corresponding to the greatest difference between the transmission velocity of the ordinary and of the extraordinary ray; but, since

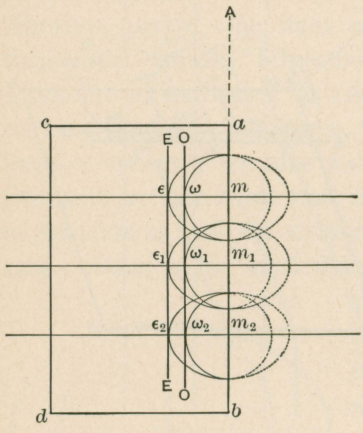


Fig. 52.

*EE* touches the ellipses at the end of an axis, then $m\varepsilon$, $m_1\varepsilon_1$, etc., are perpendicular to *EE*; consequently, in this case also, the two rays arising by double refraction would, in the crystal, both have the same transmission direction. Hence, if we should imagine plane-parallel pairs of faces of various inclinations to the axis as ground on a piece of calcite and should look through each pair toward a small bright opening, this opening would appear single when we used the pair of faces perpendicular to the axis (*i.e.* when the rays passed through the calcite parallel to the

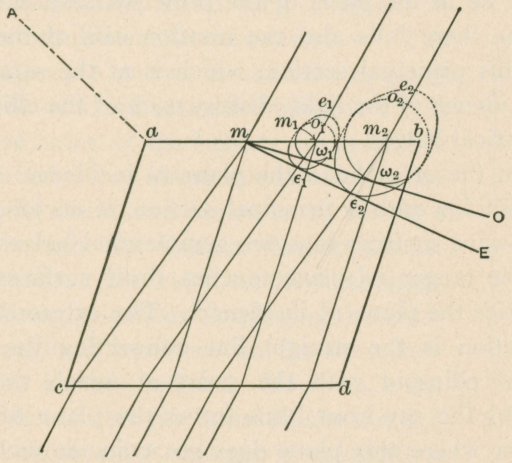


Fig. 53.

axis); when the pairs of faces having slight inclination to the axis were used two images would appear, whose distance apart would increase with diminishing obliqueness of the faces to the axis, reach its maximum when the face pair through which the light passed formed a certain angle with the axis, and then diminish again; until finally, on our looking through two faces parallel to the axis, the two images would be seen to coincide.

For light rays falling not with perpendicular, but with oblique incidence on a plane face of the calcite the simplest case is that represented in Fig. 53, where the principal section is at the same time the plane of incidence; in other words, where optic axis,

normal to the face of entrance, and ray direction all lie in one plane. Here the points $m$, $m_1$, $m_2$ of the face of entrance are reached by the wave-motion at different instants; the front of the resulting ordinary ray is then the tangential plane through $mO$ perpendicular to the principal section, and that of the extraordinary the corresponding plane through $mE$; so the ordinary wave-motion is transmitted along the direction $m_1\omega_1$, $m_2\omega_2$, the extraordinary parallel to $m_1\varepsilon_1$, $m_2\varepsilon_2$. The ray-front $mE$ touches the several ellipsoids $e_1$, $e_2$ at the points $\varepsilon_1$, $\varepsilon_2$; these points must lie in the plane of the principal section *abcd*, since in this plane there falls also the rotation axis of the ellipsoids and since this principal section, which is at the same time the plane of incidence of the light, divides each of the ellipsoids into two symmetrical halves.

When on the other hand the plane of incidence of the light does not coincide with a principal section, it no longer divides the several wave-surfaces into two symmetrical halves, and consequently the tangential plane touches those surfaces at points that lie *outside* the plane of incidence. The extraordinary ray, whose direction is the straight line connecting the center of the rotation ellipsoid with the point at which the ellipsoid is touched by the ray-front, thus *leaves* the plane of incidence in every case where this plane does not coincide with principal section; the ordinary, of course, always remaining in the plane of incidence, since its wave-surface, like that of ordinary light, is a sphere. But in all cases, for any direction of the incident light, if we know the position of the calcite face struck by the light we can, from the position and form of the wave-surface, determine the direction not only of the arising ordinary ray but also of the extraordinary.

Now that the transmission of light in calcite is fully understood, we may explain the contrivance for the polarization of light mentioned on page 55, the so-called Nicol prism.* This is made as follows: From calcite (cf. footnote *l.c.*) a parallelo-

* As to the more recent improvements in the Nicol prism cf. footnote †, p. 30.

piped is split out which consists of four larger and two smaller faces and whose principal section has the form of *ABCD* in Fig. 54. After the upper and lower end faces have been ground down so that they form an angle of 68° (instead of 71°) with the vertical edges, the parallelopiped is sawn through along *BC*, at right angles to the principal section; the two cut surfaces are then made perfectly even and polished, and finally the two halves cemented together again in their original position with Canada balsam. A light ray (*r* in Fig. 54) falling on such a prism parallel to its long direction is split up into two differently refracted rays. The extraordinary ray *e* moves, in the calcite, along a direction in which its refractive index is 1.536; the refractive index of the balsam (since this is an isotropic body) has about this same value for all kinds of vibrations, and consequently the ray *e* will pass through the Canada balsam almost without deviation. The ray *o*, on the other hand, is far more strongly deflected, impinging therefore at a larger angle of incidence on the boundary of calcite and Canada balsam. Now, since the refractive index of this ray in the calcite is 1.658 and in the balsam 1.536, this ray is transmitted in the first medium with less velocity than in the second; and, according to page 34, in such a case the light can pass from the first medium into the second only when the angle of incidence does not exceed a certain limit. In the case described this limit is exceeded, in consequence whereof the ray *o* is totally reflected; thereby it is thrown upon the lateral faces of the prism and absorbed by the blackened mounting of the same. Entirely through the nicol, therefore, there passes only the extraordinary ray, vibrating in the plane of the principal section and polarized perpendicular to it.

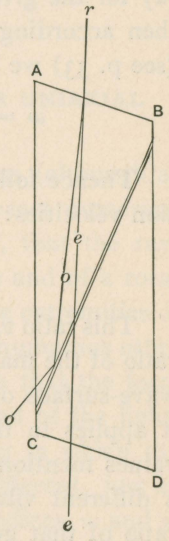

Fig. 54.

On page 91 it was stated that in calcite, for the light of a certain medium color, the refractive index $\omega$ of the ordinary ray

has the constant value 1.6583; and that the index $\varepsilon$ of the extraordinary ray, when the transmission direction is perpendicular to the axis, has that one of its values that differs the most from $\omega$, — namely, 1.4864. These two quantities are called the **principal refractive indices** of calcite for the color in question. If we denote by $v$ the transmission velocity of the light of this color in air, by $v_o$ that of the ordinary ray of the same vibration period, and finally by $v_e$ the velocity of the extraordinary light ray for the given crystallographic direction and the same color, then according to the general definition of the refractive index (see p. 33) we have

$$\omega = 1.6583 = \frac{v}{v_o}, \qquad \varepsilon = 1.4864 = \frac{v}{v_e}.$$

Thence follows the ratio between the two principal transmission velocities:

$$v_e : v_o = \omega : \varepsilon = \frac{1.6583}{1.4864} = 1.1157.$$

This ratio $v_e : v_o$, according to page 93, is at the same time the ratio of the major to the minor axis of the ellipse generating the wave-surface of the extraordinary ray; and the above value of it applies to the line $D$ in the spectrum, since the numerical values mentioned for $\omega$ and $\varepsilon$ refer to this line. Light rays of a different vibration period yield a somewhat different axial ratio of that generating ellipse. If we determine the values of the two principal refractive indices in calcite for light of less refrangibility, *e.g.* for the line $A$ in the red, we find $\omega = 1.6499$, $\varepsilon = 1.4826$; and therefore the ratio

$$v_e : v_o = \omega : \varepsilon = \frac{1.6499}{1.4826} = 1.1128.$$

This value is somewhat smaller than the last. Conversely, for light of greater refrangibility a higher value results; *e.g.* for the line $H$ in the violet $\omega = 1.6832$ and $\varepsilon = 1.4977$, so that the ratio

$$v_e : v_o = \omega : \varepsilon = \frac{1.6832}{1.4977} = 1.1238.$$

Hence it follows that the form of the ray-surface varies with the color of the light, and that the double refraction becomes stronger with diminishing wave length.

It is customary to designate as **strength of double refraction,** or as **birefringence,** the difference between the two principal refractive indices; this difference has in calcite, for the three colors named, the following values:

Line $A$ : $\omega - \varepsilon = 0.1673$
Line $B$ : $\omega - \varepsilon = 0.1719$
Line $C$ : $\omega - \varepsilon = 0.1855$

## DOUBLE REFRACTION OF LIGHT IN OTHER UNIAXIAL CRYSTALS

The fact established first for calcite, by Huygens (although at that time — in the year 1678 — not with such exact numerical values as those given above), the fact, namely, that the ray-surface of light in the crystal consists of a sphere and of a rotation ellipsoid, the latter touching the former at the extremities of its rotation axis, has later been demonstrated for numerous other crystals. These all have in common with calcite that the form of the ray-surface depends on the vibration period of the light; but for each of these kinds of crystals the manner in which the form of the ray-surface varies with the color is different, just as also the values of the two principal refractive indices, $\omega$ and $\varepsilon$, are different for the same color with different crystallized substances of this kind. Thus, among them are crystals in which $\omega$ and $\varepsilon$ have higher values than in calcite; others, with lower refringency; further, crystals with higher (among these belongs calcite) and with lower birefringence, in which latter therefore $\omega$ and $\varepsilon$ differ but little. (*E.g.* in penninite $\omega - \varepsilon = 0.001$.)

But in each of these crystals, as in calcite, the direction of the rotation axis of the spheroid of the ray-surface, *i.e.* the direction without double refraction, is *the same for all colors.* These crystals, therefore, have only *one optic axis,* and for that reason are designated as **optically uniaxial** crystals.

In one respect, however, do only a part of the optically uniaxial crystals agree with calcite; namely, in that with them

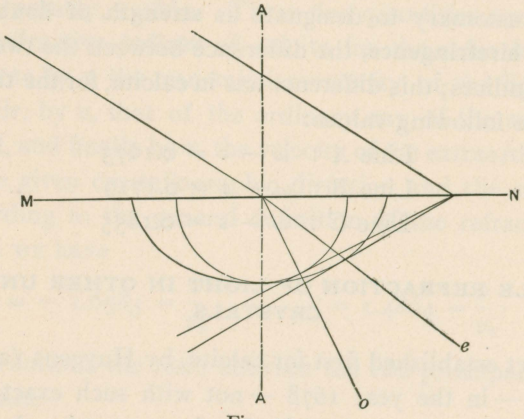


Fig. 55.

the transmission velocity of the extraordinary light ray is greater than that of the ordinary, while with the rest the contrary is the case. The optically uniaxial crystals accordingly fall into *two divisions*.

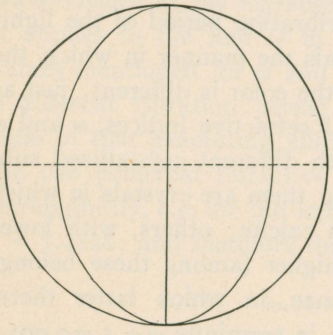

Fig. 56.

The crystals of the first division, among which calcite belongs, are those whose wave-surface, bisected along the principal section, has the form shown in Fig. 50 on page 94,—except that with the majority of them the ellipse diverges far less from the circle than is the case with calcite.* As is manifest from Fig. 55, in these crystals the extraordinary ray *e* is, as reckoned from the ordinary *o*, always refracted *away from the*

* With calcite, also, the difference between major and minor axis of the ellipse is not so considerable as it has, for the sake of distinctness, been assumed in the figure.

*optic axis AA;* they are for this reason called repulsive, or **negative** crystals.

In a crystal of the second division, where, for example, quartz belongs, the wave-surface has, in the principal section, the form shown in Fig. 56 on the opposite page: in the axial direction, only, the transmission velocity of the extraordinary ray is equal to that of the ordinary; in every other direction it is less, and the refractive index therefore greater, than that of the ordinary. From Fig. 57, then, may be gathered that here, contrary to

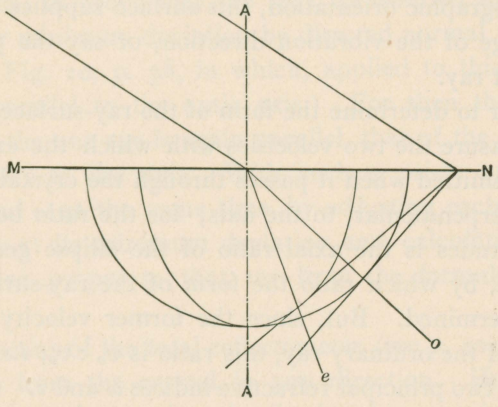


Fig. 57.

what is the case with the negative crystals, the extraordinary ray *e*, as reckoned from the ordinary, is refracted *toward* the optic axis. These crystals are therefore designated as optically **positive**, or attractive.

To whichever of the two divisions an optically uniaxial crystal may belong, from its ray-surface the transmission direction of any ray can be found by Huygens's construction, in exactly the same way as was shown for calcite on page 95 *et seq.* With regard also to the vibration directions of the two rays arising by the double refraction, all uniaxial crystals conform to the same laws: in them a vibration taking place in any direction is

always resolved into two; viz. (1) an ordinary vibration, polarized in the plane of the principal section and therefore, according to our previous assumption, directed perpendicular to it, and (2) an extraordinary vibration, whose polarization plane is the plane that passes through the ray perpendicular to the principal section,—this latter vibration, therefore, according to the same assumption, taking place in the plane of the principal section and perpendicular to the ray. Since the principal section is always defined by the ray direction and the optic axis, but since both these directions follow from the form of the ray-surface and its crystallographic orientation, this surface supplies us also with a knowledge of the vibration direction, or say the polarization, of any light ray.

In order to determine the form of the ray-surface, it is necessary to measure the two velocities with which the extraordinary ray is transmitted when it passes through the crystal once parallel, once perpendicular, to the axis; for the ratio between these two magnitudes is the axial ratio of the ellipse generating the ray-surface, by which ratio the form of the ray-surface is completely determined. But, since the former velocity is identical with that of the ordinary ray, this ratio is $v_o : v_e$; *i.e.* the inverse ratio of the two principal refractive indices $\omega$ and $\varepsilon$. (Cf. p. 100.) Hence one needs only to measure the refractive index of the ordinary ray for any direction, and that of the extraordinary when it is transmitted perpendicular to the axis; *i.e.*, according to our assumption, the refractive index $\omega$ of light-vibrations that take place perpendicular to the axis, and the index $\varepsilon$ of rays vibrating parallel to the axis. This may be done in various ways: —

1. By means of a prism whose refracting edge is parallel to the optic axis. In such a prism, when the method set forth on page 39 is employed, the plane of incidence of the light stands perpendicular to the optic axis; therefore an incident ray is resolved into two rays of which the one vibrates parallel to the optic axis, the other perpendicular to this axis; substituted in the formula

given *l. c.*, the minimum deviation of the former ray supplies $\varepsilon$, that of the latter ray $\omega$. The two rays can easily be distinguished from each other with the aid of a Nicol prism held before the eye; for when the principal section of the nicol is parallel to the refracting edge of the prism, the extraordinary ray only is transmitted, the ordinary on the other hand extinguished.

2. By means of a prism whose refracting edge is perpendicular to the optic axis. But here this further condition must be fulfilled: that the two faces of the prism include equal angles with the axis, in order that the rays transmitted in the prism may in their minimum deviation be directed normal to the optic axis. (Cf. Fig. 20, p. 38, in which, applied to this case, *Mm* would be parallel to the optic axis.) For then the vibration direction of the one ray is again parallel, that of the other again perpendicular, to the axis. With such a prism one can determine $\varepsilon$ and $\omega$ at the same time, by adjusting each of the two rays, singly, at its minimum deviation and calculating the refractive index proper to that ray from the formula given on page 38.

3. By means of the total-reflectometer (see p. 40 *et seq.*) and a plate cut from the crystal in any direction. Herewith one obtains two boundaries of total reflection, because the ray transmitted in the boundary-layer of the plate is resolved into two rays which vibrate perpendicularly to each other and which have, in correspondence to the difference in their velocity, different critical angles of total reflection. Whatever be the crystallographic orientation of the plate, there always exists in its plane a direction forming 90° with the optic axis; hence, if one so fastens the plate in the total-reflectometer that this direction is horizontal, *i.e.* parallel to the plane of incidence of the light, it is along this direction that the light is transmitted when total reflection occurs. But a ray transmitted in the crystal perpendicular to the axis is resolved into two rays, of which one vibrates parallel, the other perpendicular, to the axis. There-

fore the adjusting of the two boundaries of total reflection supplies $\omega$ and $\varepsilon$.

If the determination of the two principal refractive indices is carried out by one of these methods for two homogeneous colors, as lithium-red and sodium-yellow, then from the values obtained we may calculate the refractive indices for all the remaining colors, according to the dispersion formula given on page 46. But, since the two rays vibrating perpendicularly to each other are differently refracted and also, as is learned empirically, always unequally dispersed, the constants $A$ and $B$ of the dispersion formula are, of course, not the same for $\omega$ as for $\varepsilon$.

Besides the determination of the form of the ray-surface for one or more colors there is requisite, for the complete optical knowledge of a crystal, that the orientation of its optic axis be ascertained. The quest of this direction takes place with the aid of the interference phenomena in convergent polarized light, wherefore these phenomena shall be treated in the following section, in conjunction with those observed in parallel light.

## BEHAVIOR OF UNIAXIAL CRYSTALS IN THE POLARIZATION APPARATUS

*a.* In Parallel Light (orthoscope).— If into horizontal position upon the stage of the orthoscope (see Fig. 34, p. 75) we bring a crystal plate bounded by two natural or artificial plane surfaces that are parallel to each other and *perpendicular to the optic axis*, then through the plate there pass only such rays as are vertical in direction and therefore parallel to the optic axis. But this is the direction in which there is no double refraction,— the direction in which a uniaxial crystal behaves like a singly refracting body. Accordingly the plate does not in the least degree alter the polarization of the rays passing through it: it appears just as dark as the rest of the field when the nicols are crossed at right angles, just as bright with the nicols parallel. But in the first case the plate naturally appears absolutely dark

only when no rays pass through other than such as are exactly parallel to the optic axis. This, strictly speaking, is the case only with microscopic crystals, because through larger plates rays still find their way that are more or less inclined to the axis; these rays experience double refraction and consequently a brightening.

When the faces of incidence and emergence are *not perpendicular* to the optic axis, double refraction takes place also for rays that fall normal to the plate, since these rays now form with the optic axis an angle other than zero. Every incident ray is resolved on the one hand into an ordinary ray, vibrating at right angles to the principal section and passing through without deviation; and on the other hand into an extraordinary ray, which, vibrating in the principal section and deviated in this same plane, is, on emerging from the plane-parallel plate, again refracted so that it becomes once more parallel to its previous direction, — *i.e.* to the normal to the plane of the plate. If we imagine the whole crystal plate as illuminated by rays falling perpendicular to it, then from every point of the face of emergence there will be transmitted normal to the plate two light rays; namely, an ordinary, which has passed through the plate at right angles, and an extraordinary, which within the crystal moved oblique to the faces of entrance and emergence and hence must have been derived from another one of the incident rays than was the ordinary emerging at that same point. The two rays, transmitted in the air along the same path, vibrate perpendicularly to each other and, because of their difference in velocity within the crystal, have a difference of path; so with crossed nicols they must exhibit the interference phenomena treated in detail on pages 64–70. The strength of the double refraction in the crystal plate depends on the inclination of the normal of the plate to the optic axis: when that inclination is small, so also is the double refraction, and the colors of low order appear; with equal plate thickness the order of the color rises with increasing amount of that inclination, and is highest in the case of

a plate parallel to the axis; for in such a plate the two rays are transmitted perpendicular to the axis and therefore acquire the greatest difference of path.* If we investigate plates of different thickness, the path difference naturally increases, and with it the order of the color, — proportionally to the thickness if the plates have equal inclination to the optic axis, — until at last the white of a higher order appears. But the plate thickness with which this occurs is obviously the greater, the smaller the angle formed by the light rays with the optic axis, *i.e.* the less the double refraction; therefore, of all the plates that can be imagined as cut from a crystal in different directions, those cut parallel to the axis exhibit the white of a higher order with relatively the least thickness.

If we compare crystal plates of different substances with one another, then, even with equal thickness and equal inclination to the optic axis, these plates exhibit a different interference-color; for naturally the color exhibited depends moreover on the birefringence, or specific double refraction, of each substance, — *i.e.* on the relative distance of the two skins of its ray-surface from the center. When this distance is slight, there is requisite either a great thickness of the crystal plate or else a considerable inclination of its normal to the optic axis, in order to produce the white of a higher order; on the other hand, a crystal of high birefringence exhibits this white, even with slight thickness or when there is only a small angle between the axis and the normal to the plate.

Finally, in addition to depending on the conditions enumerated, the color phenomena that a uniaxial crystal presents in polarized light are influenced by the circumstance that the

* Therefore, if in a very thin plane-parallel plate made from a rock — a so-called "thin section" of the rock — there lie sections of a number of diversely oriented crystals of an optically uniaxial mineral, these crystal sections, in spite of their having equal thickness, must according to the above exhibit different interference-colors; and those crystals that the cut happens to have intersected parallel to the axis will exhibit the highest in order of the interference-colors described on p. 64 *et seq.*

birefringence is not exactly equal for the different colors (*i.e.* that the ordinary and extraordinary rays of white light experience unequal dispersion), thin plates of the crystal therefore exhibiting smaller or larger deviations from the normal interference-colors. (Cf. p. 69.) This circumstance becomes of importance in cases where the birefringence is very low; and here it can even lead to phenomena quite different from those otherwise observed. For example, if with increase in the refrangibility of the rays the birefringence become greater, but still be so slight that for the least refrangible colors — thus for red — it is zero, then a very thin plate of such a crystal will not exhibit the gray of the first order, corresponding to the very slight birefringence proper to it, but a very vivid color; so that from the appearance it looks as though the substance in question had very strong double refraction. The explanation is as follows: Since the crystal plate is singly refracting for red, it remains for this color, between crossed nicols, dark in every position, the red rays being totally annihilated; further, for orange and yellow the plate exhibits only a slight brightening; somewhat more light passes through from the green, while for blue and violet the difference of path is so great that even after the interference these colors still have a certain intensity; the latter colors must therefore greatly predominate in the color impression. The arising interference-color must be still more vivid when the birefringence is so low that even for a medium color, *e.g.* yellow, it is zero; the crystal accordingly then has, for the colors lying at opposite ends of the spectrum, *double refraction of opposite character*: for the colors of one end it is positively uniaxial, for those of the other end negatively uniaxial. Hence, if we bring a thin plate of such a crystal between crossed nicols and turn it so that its vibration directions form 45° with those of the nicols, the yellow rays, the brightest rays of the spectrum, are totally extinguished, because for them the crystal is singly refracting; for the adjacent colors, orange and green, it is but feebly birefringent and so for these exhibits only a

slight brightening; while for the outermost parts of the spectrum — red on the one hand, blue on the other — the double refraction is relatively strongest, and the rays of these colors accordingly acquire a path difference amounting to a perceptible fraction of a half wave length. The consequence is that between crossed nicols a very vivid violet interference-color appears. (An excellent example of this phenomenon is exhibited by certain varieties of the mineral gehlenite.)

*b.* In Convergent Light (conoscope). — As in parallel polarized light, so also in convergent, the behavior of a uniaxial crystal will be considered first for a plate cut *perpendicular to the optic axis.* If we lay such a crystal plate upon the stage of the conoscope (cf. Fig. 35, p. 78) in such a way that the optic axis is parallel to the vertical axis of the whole apparatus, and that the point *f* lies within the crystal, then an infinity of ray bundles, each bundle consisting of mutually parallel rays, pass through the crystal in all possible directions within the cone between *f* and the so-called **condensing lens** *n.* One of these ray cylinders — namely, the one proceeding from the point *c* and therefore appearing in the image exactly at the center of the field — passes through the

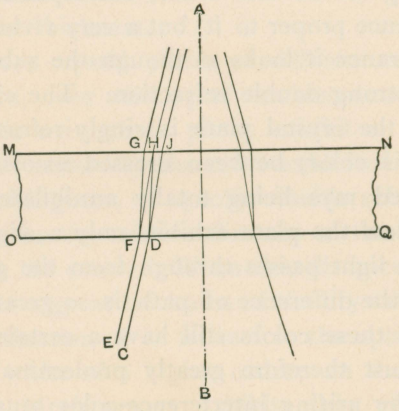


Fig. 58.

crystal parallel to its optic axis. In order to trace the changes experienced within the plate by all these differently directed ray bundles, let us consider first the rays that lie in *one vertical plane* passing through the optic axis of the crystal. Such a plane we called a *principal section* of the uniaxial crystal, and in Fig. 58 this shall be represented by *MNOQ*. Let us then suppose, in addition, that the two nicols of the instrument are crossed, that their polarization planes form 45° with the principal section *MNOQ*, and that monochromatic light is passing through the apparatus. The light rays parallel to the optic axis of the crystal, the direction *AB*, are not doubly refracted, but pass through the crystal unaltered; so the central part of the image appears dark, exactly as would the whole field, with crossed nicols, if no crystal plate were present. But if we now consider the ray cylinders that have a slight inclination to the optic axis, then among these cylinders there will be one — the one to which, for example, the ray *CD* belongs — to which the following applies: The ray *CD* is split up in the crystal into two rays, *DH* and *DJ*, of different velocity, one of these rays vibrating in the principal section *MNOQ*, the other perpendicular to it; just so does another ray *EF*, of the same bundle and therefore parallel to *CD*, split up into an ordinary and an extraordinary ray, *FG* and *FH*. Thus, from *H* there proceed farther, along the same path, two rays which both came from the same source of light, a source emitting plane-polarized light, — namely, from the point in question of the bright opening *de* in Fig. 35, — but which are polarized perpendicularly to each other. Of each of these vibrations, only that component is transmitted through the upper nicol, that falls to its vibration plane; so we have the case represented in Fig. 32, page 61. And if the inclination of the considered rays (*CD* and *EF*) to the optic axis of the crystal is such that the path difference of the arising light rays *DH* and *FH* amounts to exactly a half wave length of the color with which the instrument is illuminated, then, according to page 60, these rays *DH* and *FH* will combine by the interference to

form the sum of their transmitted components. Now among the parallel rays that traverse the crystal in the same direction as the two just considered, $CD$ and $EF$, there must exist for every ray of them a ray that stands to this first ray in the same relation as the relation between $CD$ and $EF$; that is, the ordinary ray arising from the one interferes with the extraordinary arising from the other in the same way as do $DH$ and $FH$. Hence, in the image visible in the conoscope, at the point where all

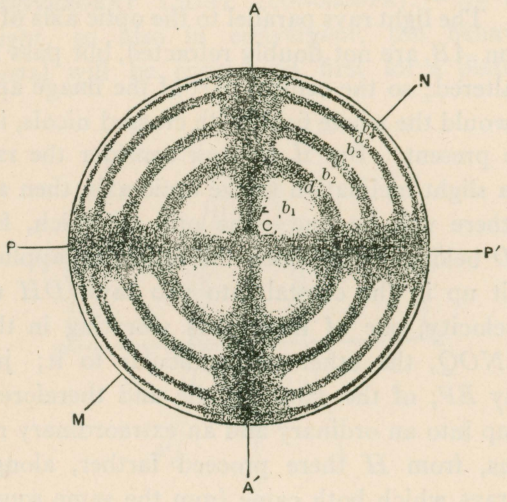


Fig. 59.

these rays converge — a point somewhat removed from the center of the field — there will appear the brightness corresponding to the sum of the transmitted components of all these pairs of interfering rays. If Fig. 59 represent the image seen in the polarization apparatus, and if $AA'$ and $PP'$ be the vibration directions of the two crossed nicols, $MN$ the direction of the principal section $MNOQ$ of the last figure, then is $C$ the dark center of the image and $b_1$ the bright place at which those rays converge that were last discussed. The positions between $C$ and $b_1$ represent the points of convergence for ray cylinders that

have a smaller inclination to the optic axis, so that for these the path difference acquired in the crystal is less than $\frac{1}{2}\lambda$. Such rays, on the interference, will combine to form a wave-motion whose state of vibration is different from that of either component, and whose intensity must accordingly be less than the sum of the intensities of the single rays (see p. 19); the resulting intensity approaches the more closely to this latter value, however, the closer we come to $b_1$, — that is, the less the difference of path differs from $\frac{1}{2}\lambda$. So from $b_1$ to the center, where it is zero, the brightness must *gradually* diminish. With rays in the principal section $MO$ that are inclined to the optic axis of the crystal at a greater angle than the rays converging at $b_1$, the difference in the transmission velocity of the two arising vibrations is greater than with the latter rays, since the velocity of the extraordinary ray differs the more from that of the ordinary, the more the ray is inclined to the axis. Therefore at a point $d_1$, on the straight line $MN$ but farther removed from the center, there will converge all rays whose direction in the crystal was such that the ordinary and the extraordinary arising from each acquired a path difference of $\lambda$. But, according to page 57, on the interference between crossed nicols each of these two arising vibrations and the vibration with which it interferes, which arose from a different ray of the same direction, — thus all the rays, pairwise, that arise from rays of this direction, — must completely annihilate each other. Therefore the point $d_1$ will appear just as dark as does the center of the image; and the brightness will gradually diminish from $b_1$ to $d_1$; but on the other side of $d_1$ it will increase, since the point $b_2$ corresponds to the convergence of those rays that passed through the crystal so inclined that the path difference of the two rays arising by the double refraction is $\frac{3}{2}\lambda$, and that accordingly on the interference a superposing of the two components again takes place. The same is the case at $b_3$, where the path difference is $\frac{5}{2}\lambda$. And so on. If, therefore, proceeding outward from the center of the image, we consider the intensity of the light on the line $MN$,

we observe a continuous alternation of bright and dark. And therewith, as the distance from the center increases, the distance between the light minima and light maxima always becomes less; because with greater obliqueness of the rays to the optic axis there is an increase not only in the difference in velocity of the two interfering rays but also in the length of path they must travel in the crystal, so that, in the case of the rays that form larger angles with the optic axis, the same difference in the inclination corresponds to a greater difference in the retardation than in the case of those standing at smaller angles to the axis.

Since optically uniaxial crystals behave absolutely the same toward all light rays that include equal angles with the optic axis, then in the image the same maxima and minima of light must in all directions be present at the same distance from the center; for what applies to the principal section $MN$, Fig. 59, must apply in wholly the same way to every other principal section, standing at any other angle to $PP'$. The center of the field must accordingly be surrounded by bright and dark rings of exactly circular form. The bright rings can, however, not everywhere have the same intensity, because their intensity corresponds to the sum of two variable amplitudes; these amplitudes, as follows from Fig's 31–33 on page 61, are at their maximum for the principal sections diagonal to $AA'$ and $PP'$, diminishing on either side; and for the two principal sections parallel to $AA'$ and $PP'$ they become zero. In consequence of this a dark cross must appear, with its arms parallel to $AA'$ and $PP'$; — and this same conclusion also follows directly from the consideration that, for any inclination within one of these two planes, only one ray is formed (because the component perpendicular to it is zero), and this ray in one of the two nicols totally reflected. All phenomena observed on a circle described about the center correspond, as it were, to those that, in parallel polarized light, a plate of the same thickness and inclination to the optic axis exhibits in succession when it is rotated 360°; the

places where the circle intersects the arms of the dark cross correspond exactly to the four positions of darkness of such a plate. Figure 59 accordingly represents the interference-figure that we observe with crossed nicols when the instrument is illuminated with *monochromatic light.*

If, however, for the illumination we choose light of another color, *e.g.* a color having greater wave length, then, obviously, if the birefringence for this color is just about as high as for the last, a greater inclination than before will be necessary, of the rays to the optic axis, in order to give the rays a path difference of one whole — now greater — wave length; so the distance of the first dark ring from the center, and likewise that of the succeeding rings from the first, will be greater than with the previous color. The less the wave length of the light used for illuminating, the narrower will be the rings we see in the polariscope; and on the other hand, the greater the wave length, the wider the rings.

Hence if, instead of monochromatic, we use *white* light, by causing the light from a brightly illuminated part of the sky to be reflected from the mirror of the conoscope into interior of the instrument, those rays that traverse the crystal at a certain inclination to its axis will so interfere that for one definite color the difference of path amounts exactly to $\lambda$; consequently, between crossed nicols this color is extinguished, while the others are weakened the less, the more their wave length differs from that of the color in question. After the annihilation of a certain color, therefore, the light appearing at the point in question, of the image, will no longer exhibit white, but a composite color. This color will be the same for all rays having the same inclination to the optic axis of the crystal; accordingly, all points of the interference-figure that are equally distant from the center will exhibit the same color, but all those of different distance different color. Thus, with crossed nicols there appear colored rings intersected by a black cross. (See Plate II, Fig. 1.)

When the *nicols* are *parallel*, the rays interfere with the opposite difference of path, *i.e.* the same with which they emerge

from the crystal; consequently, instead of a black, a white cross appears, and at every distance from the center exactly that color is extinguished that with crossed nicols has its maximum intensity; accordingly, the colored rings of the interference-figure with the black cross are colored exactly in complement to those of the same diameter in the figure with the white cross.

The curved lines of an interference-figure, which exhibit the same color at all points, are called **isochromatic curves**; these curves for an optically uniaxial plate perpendicular to the axis are thus exact *circles*, whose common center corresponds to the axis. By our seeing through two parallel faces of a crystal the circular isochromatic curves with the (with $\perp$ nicols) dark cross we not only — since only the optically uniaxial exhibit this phenomenon — recognize the crystal as optically uniaxial, but, in addition, the direction of its optic axis is determined as normal to the pair of faces looked through.

Since, in the behavior of the crystal toward rays of light, all principal sections intersecting one another in the optic axis are absolutely equivalent, the interference-figure yielded in the conoscope by such a plate must remain absolutely unaltered when the plate is rotated in its own plane. Only the rotation of a nicol can alter it; and this rotation, when it amounts to 90°, transforms the previous interference-figure into the complementary one — that with the white cross.

The colors exhibited by the interference-figure represented in Fig. 1, Plate II, are obviously the same * as were considered in detail on page 63 *et seq.*; for if we advance from the center of the figure to the edge, we arrive at the points of convergence of rays that experience in the crystal an amount of double refraction beginning with zero (at the center) and continuously increasing; in other words, we observe side by side the same phenomena that a series of plates of the crystal would exhibit to us in succession, should we begin with a plate cut perpendicular

* With those differences, usually small, that result from the inequality of the birefringence for different colors. (Cf. p. 69 and pp. 108–109.)

to the axis and then follow with such as had an increasing inclination of their normal, to the axis, combined with an increase in thickness corresponding to this inclination. If within the field of the conoscope there fall rays that in the crystal were retarded a larger number of wave lengths as compared with each other, the white of a higher order naturally appears. Thus, while in monochromatic light a crystal plate of high birefringence exhibits the interference-figure represented in Fig. 60, *i.e.* a series of bright and dark rings which become narrower and narrower from the center out to the edge of the field or until they are so fine that they can no longer be distinguished, the same plate in white light yields only a limited number of rings, since those having the colors of even the fifth and sixth orders differ very little from white, and since beyond these rings a gradual transition takes place into the uniform light gray of the field. Here, therefore, a circle described about the center corresponds, as it were, to the rotation in its own plane of a plate having such high birefringence that between the four positions of darkness it exhibits only a simple lightening, without color.

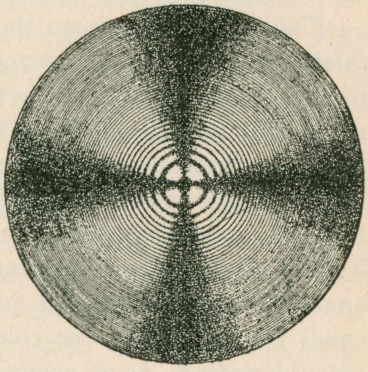

Fig. 60.

Up to the present one circumstance has been disregarded; namely, the thickness of the crystal plate. If instead of the plate hitherto considered, which we always supposed of equal thickness, we take another plate of the same optically uniaxial substance, likewise perpendicular to the axis but only half as thick, the two rays arising from one in this plate by the double refraction will, when they have the same inclination to the axis as before, travel only half as far in the crystal, and accordingly the relative retardation will be only half as great. The *same*

path difference, therefore, which requires the same interference, can arise only when we come to rays having a far greater inclination to the axis; thus, for example, in monochromatic light the first dark ring of the interference-figure can be formed only at a much greater distance from the center; and this applies in the case of white light to each of the rings of color. The isochromatic curves exhibited by a uniaxial crystal will accordingly stand the farther apart, the thinner the plate investigated; the closer together, the thicker it is chosen.

Finally, if we compare the color rings exhibited by *equally thick* plates of *different substances*, we find that, in consequence of the inequality of the birefringence of different bodies, the rings differ in their width. The explanation is that in a crystal of lower birefringence — in which the difference between the velocity of the ordinary and of the extraordinary ray is slight — the rays, in order to each acquire a path difference of one wave length, will have to be inclined to the optic axis at a greater angle than in a crystal of higher birefringence; so that with equal plate thickness the color rings of the interference-figure must be *wider*.* As an example of a substance having very high birefringence, whose plates accordingly, unless very thin, always exhibit narrow rings, calcite may be mentioned. When the double refraction of an optically uniaxial substance is very weak, and at the same time the inequality of its strength for the different colors great enough to be noticeable (as is the case *e.g.* with the mineral apophyllite), there results considerable deviation of the colors in the rings of the interference-figure from the normal scale of the colors of the first, second, third, etc., orders. For, if the first dark ring for red appears at a certain distance from the center, and if that for blue — which by reason of the lesser wave length would, with equal birefringence, be formed nearer the center — appears, in consequence of the

* Hence it is seen that the width of the rings may serve to determine the refractive index of the extraordinary ray, if that of the ordinary is known. (Concerning this method see *Zeitschr. f. Kryst.* **7**, 394.)

birefringence for this color being lower and the rings therefore wider, *at the same place* as that for red, then here red *and* blue must be annihilated, and yellow therefore arise; while somewhat farther toward the center, where otherwise red is extinguished and blue appears, the red will now have an intensity such that the mixing here of the red and the blue gives violet. Thus color rings arise in which the red is in most cases wholly lacking; and in certain crystals of the mineral named the dark rings for all the colors so nearly coincide that to all appearance only white and black rings arise (wherefore this variety has been named *leucocyclite*). Some crystals of apophyllite belong with the substances already mentioned, on page 109, as having such slight birefringence that for the colors forming one end of the spectrum they are *positively* doubly refracting, for those of the other end *negatively* doubly refracting: with the former colors the ordinary ray, with the latter the extraordinary, is transmitted the faster, so that there is an intermediate color for which the crystals are singly refracting (without for that reason belonging with the optically isotropic crystals, since these are singly refracting for *all* colors, but the crystals in question, for all the remaining colors, really uniaxial).

*Plates parallel or oblique to the optic axis*, when they are investigated in convergent light, can, if this is white, according to the exposition on page 70 yield color phenomena only in case their thickness is very slight. Any such plate will produce at the center of the field the same color that one observes through it in parallel light. The rays that on the other hand pass through the plate in an inclined direction, although they will indeed all travel an increasing *length* of path in the crystal, will not acquire a *difference* of path increasing in the same way; since in certain directions, namely, where they approach the optic axis, the difference in the velocity of the two rays continuously diminishes, wherefore in spite of the increasing thickness traversed their path difference also becomes less. In directions perpendicular to these inclined directions, when the plate is cut parallel

to the optic axis, the difference in velocity will not vary; so with growing inclination, and consequent increase of the thickness traversed, the difference of path also will increase. In intermediate directions the two effects will neutralize each other, and the path difference will remain the same for all inclinations of the rays to the normal of the plate. In homogeneous light, systems of dark and bright stripes thus arise, which have in general hyperbolic form; in white light the stripes are colored, but are visible only when the plate is very thin, since with greater thickness the white of a higher order results. Only that phenomenon is of special practical interest in crystallography, that is exhibited by a plate inclined at no great angle (not over 35° or 40°) to the plane standing normal to the optic axis; for with such a plate, in convergent light and in a polariscope having a large field, rays still converge within the field that pass through the crystal along the axis. Near the edge of the field, in some direction, the observer will accordingly behold the interference-figure of the axis — the black cross with the color rings (although the latter are no longer exactly circular). In this direction therefore, inclined at an acute angle to the normal of the plate, there lies the optic axis, for finding which the phenomenon in question may serve.

The conversion of the microscope into a conoscope (see p. 80) is of special advantage for determining an optically uniaxial crystal as such when, for example, in a rock section (see footnote, p. 108) there are visible a number of differently oriented sections of a mineral most of which appear doubly refracting, while on the rotation between crossed nicols some remain dark. If the sections in question belong to an optically uniaxial mineral, those last-mentioned must be such whose optic axis happens to be directed perpendicular to the plane of the rock section, and must therefore, when convergent light is employed, exhibit the above-described axial figure, the colored rings with the black cross.

## OPTICALLY BIAXIAL CRYSTALS

### DEDUCTION OF THE OPTICAL PROPERTIES OF CRYSTALS FROM A SURFACE OF REFERENCE (OPTICAL INDEX-SURFACE OR INDICATRIX)

On page 95 *et seq.* the optical properties of the uniaxial crystals were deduced from their ray-surface; *i.e.* from the double surface whose radii vectores correspond to the velocities of the rays transmitted parallel to these radii. Except in the optic-axial direction each radius vector intersects the surface twice; that is to say, along the same path, but with different velocity, two rays are transmitted whose ray-fronts are respectively the tangential planes of the double surface at the two points of intersection. As already mentioned on page 96, and as may be seen from Fig. 51 *et seq.*, the front of the ordinary ray always stands perpendicular to the ray; that of the extraordinary, however, in general not perpendicular, since the tangent of an ellipse is normal to the radius vector only when this coincides with one of the principal axes of the ellipse. While, therefore, for the ordinary ray the front-normal (called also the "wave-normal") coincides with the ray, with the extraordinary such is in general not the case.

In Fig. 61 (p. 122), then, let there be represented the principal section of the ray-surface of a uniaxial crystal. If $Or$ be any radius vector of the wave-surface of the extraordinary ray, $rV$ the tangent to the ellipse at the point $r$, and finally $OR$ the radius vector parallel to this tangent, then the tangent to the ellipse at $R$, namely $RV$, is parallel to $Or$; $Or$ and $OR$ are then two so-called "conjugate radii" of the ellipse, for which the rule holds good, that the area of the parallelogram $ORVr$ is constant and equal to that of the parallelogram $OX \cdot OZ$, —*i.e.* equal to the product of the two semi-axes of the ellipse. If we draw $RN$ perpendicular to $Or$, then, as is easily seen, the area of $ORVr$ is equal also to the rectangle we obtain if from $O$ and $r$ we let fall perpendiculars to the tangent $RV$, —*i.e.* equal to

the right-angled parallelogram $Or \cdot RN$. Thus, for every direction of the ray, the area, $Or \cdot RN$ is equal to the constant product $OX \cdot OZ$; consequently

$$Or = \frac{OX \cdot OZ}{RN} = \frac{1}{RN}$$

So the transmission velocity of any extraordinary ray, *i.e.* the length $Or$, is proportional to the reciprocal length of the straight line that stands perpendicular both to the ellipse (consequently also to the spheroid) at the point $R$, and to the ray itself. But this line $RN$, according to our previous assumption (see footnote, p. 89), is at the same time the *vibration direction* of the ray $Or$; for the polarization plane of this ray is the plane passed normal to $RN$ through the transmission line $Or$ of the ray and perpendicular to the principal section.

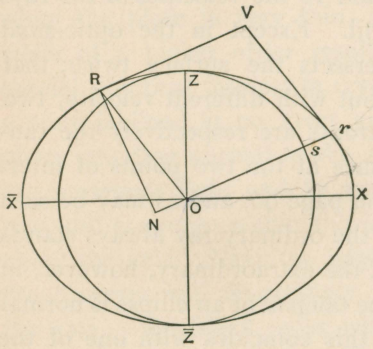

Fig. 61.

THE POINT $R$ ON THE ROTATION ELLIPSOID THUS DETERMINES ABSOLUTELY THE TRANSMISSION DIRECTION, THE VELOCITY, AND THE POLARIZATION OF AN EXTRAORDINARY RAY $Or$. One has only to draw the normal to the surface at the point in question, *i.e.* the normal to the tangential plane at this point, and then obtains the direction of the ray proper to the point by dropping a perpendicular from the center to this normal. These two lines define all the three above-named elements, the ray characters, marking the properties of the ray.

But, for any radius vector $Or$ of a rotation ellipsoid there are *two* normals to the surface that stand at the same time perpendicular to the radius vector. One of them, of course, is $RN$. The other is the perpendicular erected at $O$ to the principal section represented in Fig. 61; this perpendicular always lies in the equatorial plane of the ellipsoid and therefore has the con-

stant length $OX$, however $Or$ be inclined in the principal section. Now this second normal obviously bears the same relation to the ordinary ray $Os$ as does $RN$ to the extraordinary $Or$; for according to our assumption it is the vibration direction of the former ray; and since its length is $OX$, but the product $OX \cdot OZ$ constant (or $= 1$) and $OZ$ therefore proportional to $\frac{1}{OX}$, the velocity of the ray proper to this second normal is inversely proportional to its length $OX$.

THE CHARACTERS OF THE ORDINARY, AS OF THE EXTRAORDINARY, RAY OF ANY DIRECTION ARE DEDUCIBLE THEREFORE FROM A SINGLE SURFACE, A SPHEROID THE LENGTH OF WHOSE ROTATION AXIS STANDS TO THE DIAMETER OF ITS EQUATOR, OR CIRCULAR SECTION,* IN THE RATIO OF THE RECIPROCAL VALUES OF THE TRANSMISSION VELOCITIES OF LIGHT-VIBRATIONS THAT TAKE PLACE PARALLEL AND PERPENDICULAR TO THE AXIS; *i.e.* (according to p. 100) IN THE RATIO OF THE TWO PRINCIPAL REFRACTIVE INDICES $\varepsilon$ AND $\omega$. THIS SURFACE SHALL THEREFORE BE CALLED THE **OPTICAL INDEX-SURFACE** OR "INDICATRIX". IT HAS FOR A NEGATIVE UNIAXIAL CRYSTAL THE FORM OF A SPHEROID WHOSE ROTATION AXIS CORRESPONDS TO THE SHORTEST DIAMETER; FOR A CRYSTAL WITH POSITIVE DOUBLE REFRACTION, ON THE OTHER HAND, *i.e.* WHEN $\varepsilon > \omega$, IT HAS THE FORM OF THE SURFACE THAT IS GENERATED BY THE ROTATION OF AN ELLIPSE ABOUT ITS MAJOR AXIS.

From this surface, which accordingly is defined by the two principal refractive indices of the crystal and therefore has for a different color a different axial ratio, the optical properties of the crystal in any direction are deduced as follows: —

To the diameter parallel to the direction in question there in general correspond *two* points on the surface, the normal to the surface at these points standing at the same time perpendicular to that diameter; along the direction of the latter, therefore, two rays are transmitted whose vibration directions are

* Thus will be designated the section through the center perpendicular to the rotation axis: this section (like every one parallel to it) obviously has the form of a circle.

those two normals and whose velocities are to each other inversely as the lengths the ray intercepts on the two normals. Only one diameter, the rotation (*i.e.* optic) axis, is different in this respect; to it there correspond an infinity of points on the index-surface; for the normal to the surface at every point of the equatorial plane stands perpendicular to this diameter. Parallel to the axis, therefore, rays are transmitted whose vibrations take place in all possible directions perpendicular to the axis. In a word: to every diameter (ray) there correspond *two* vibration directions; to the axis, only, an *infinity* of vibration directions.

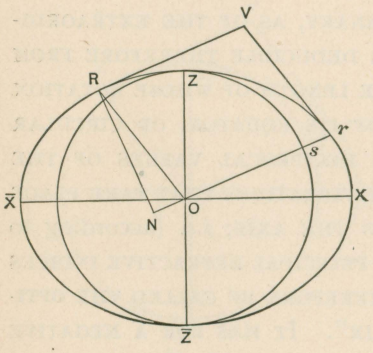

Fig. 61.

Conversely, to every point of the index-surface of a uniaxial crystal there in general corresponds *one ray*. The direction of this ray is that of the diameter intersecting at right angles the normal erected on the surface at the point in question; the transmission velocity is inversely proportional to the intercept on this normal between the surface and the ray; the polarization plane stands perpendicular to this same normal, the normal being the vibration direction of the ray. If the given point lies neither in the equatorial plane of the index-surface nor at one end of the optic axis, the corresponding vibration direction lies in the plane passing through axis and ray, *i.e.* in the principal section; so the ray is *extraordinary*. If the point lies at one end of the axis, *e.g.* if in Fig. 61 *R* coincides with *Z*, then the direction of the ray proper to it is indefinite, because the normal to the surface then passes through the center, wherefore the points *N* and *O* likewise coincide; to the point *Z*, therefore, there correspond all the *extraordinary* rays transmitted in the equatorial plane, which all have the vibration direction *OZ* and the velocity *OX*. If on the other hand the given point lies on the " cir-

cular section " of the index-surface, the normal erected here on the surface likewise passes through the center; so that, in this case also, the direction of the ray is indefinite. That is to say, to such a point there correspond an infinity of rays which are transmitted in the plane that stands perpendicular to the normal erected at the point; the rays all vibrate normal to this plane, and therefore they have the constant velocity $OZ$; for this quantity is inversely proportional to the length $OX$ of the normals. These rays, since their vibration direction is perpendicular to the principal section, are all *ordinary*. While, therefore, to the two points at the ends of the axis there correspond an infinity of extraordinary rays transmitted in the equatorial section, there belong to every point of the equatorial section an infinity of ordinary rays transmitted in one principal section; and since there are an infinity of points of the latter kind, the index-surface has an infinity of equivalent principal sections corresponding to them, which all intersect one another in the axis.

THE ANALOGOUS OPTICAL SURFACE OF REFERENCE FOR SINGLY REFRACTING BODIES IS A SPHERE. For with such bodies $\omega = \varepsilon$; in other words, the refractive index is the same in all directions. The normal erected on the index-surface at its every point then passes through the center, and the direction of the ray is indefinite; that is to say, all rays of the vibration direction in question, however they are transmitted in the plane perpendicular to that direction, have the same velocity and hence are ordinary. But since the normals at all points of the surface have equal length, the ray velocity is equal in all planes of the crystal, wherefore in such a crystal there can be only ordinary rays.

Just as the index-surface of the singly refracting crystals presents that special case of the surface for uniaxial crystals * in

* As was mentioned on p. 109 *et seq.*, there are some uniaxial crystals that for a certain color are singly refracting. Here, then, we have this special case; in other words, the index-surface for light of the color in question is a sphere, but for the remaining colors a rotation ellipsoid, in part prolate (with negative double refraction), in part oblate (with negative double refraction). But with the majority of optically uniaxial crystals the index-surfaces for the different colors are of one and the same kind, differing only in their axial ratio.

which the diameter parallel to the axis is equal to the diameter perpendicular to the axis, so again might the uniaxial index-surface be conceived of as a special case of one that is still more general; namely, of an index-surface whose diameter is different in all three dimensions of space. This surface were then a so-called **triaxial ellipsoid**, with three mutually perpendicular, unequal principal axes; and the lengths of the three principal axes must be proportional to the refractive indices of the rays that correspond to their extremities. In such an ellipsoid, as is pictured in Fig. 62 on the opposite page, the three principal axes stand perpendicular to the surface at their extremities; so the normals to the surface at these six points (and at no others) pass through the center of the ellipsoid. To such a point, therefore, exactly as to a point on the equatorial circle of the uniaxial index-surface, there correspond an infinity of ordinary rays lying in the principal section that stands perpendicular to the normal erected at the point, and having the same velocity and vibration direction. But, while with the uniaxial index-surface there exist an infinity of such principal sections, here only three are possible, the three that stand perpendicular to the three principal axes of the ellipsoid; and, since the transmission velocity of the ordinary rays within such a principal section is inversely proportional to the semi-axis standing perpendicular to it, but the length of that semi-axis always different, these rays have in each of the three principal sections a different refractive index. These three refractive indices, which accordingly determine the form of the index-surface, are called the three **principal refractive indices**; the respective vibration directions of these three kinds of ordinary rays, which therefore are nothing else than the directions of the three principal axes of the ellipsoid, are known as the **principal vibration directions.**

The three principal refractive indices will be denoted by $\alpha$, $\beta$, $\gamma$; and of these $\alpha$ shall be the smallest index, $\beta$ the so-called intermediate (it may lie the closer either to $\alpha$ or to $\gamma$), $\gamma$ the largest. If with its semi-axes of the lengths $OX = \alpha$,

$OY = \beta$, $OZ = \gamma$ we construct the triaxial ellipsoid represented in Fig. 62, then by each of the three principal optic sections $XZ\overline{XZ}$, $XY\overline{XY}$, $YZ\overline{YZ}$ this surface will be intersected in a different ellipse; and each of those three sections divides the whole form into two equal and opposite halves. They are "planes of symmetry" (see p. 28) of the surface, something which applies to no further planes. A random section through the center has in general likewise the form of an ellipse, but of one whose axial ratio varies with orientation of the intersecting plane; two planes only, besides the three principal sections, are of special importance, they being so for the reason that they intersect the surface not in an ellipse but in a circle. That such is the case is brought out by a consideration of the following: The principal section $XY\overline{XY}$ of the triaxial ellipsoid Fig. 62 is an ellipse whose major and minor axes are to each other as the intermediate to the smallest refractive index; a section taken through $Y\overline{Y}$ but inclined to the plane $XY\overline{XY}$ is obviously (because of the symmetry of the form) likewise an ellipse, whose one axis is $Y\overline{Y}$ and whose other axis is a radius vector of the principal-section ellipse $XZ\overline{XZ}$; but these radii vectores have, according to the inclination, all possible values between $OX$ (= the smallest refractive index $\alpha$) and $OZ$ (= the largest $\gamma$); consequently, somewhere between, there must be a radius vector, $OC_1$, that is equal to the intermediate refractive index $\beta$; the intersection curve corresponding to this radius vector, namely $YC_1\overline{YC}_1$, has equal axes, or in other words is a circle. This form is approached by the ellipses for lesser inclinations as their inclination increases, the minor axis always becoming more like the major, the more it approaches the direction $OC_1$;

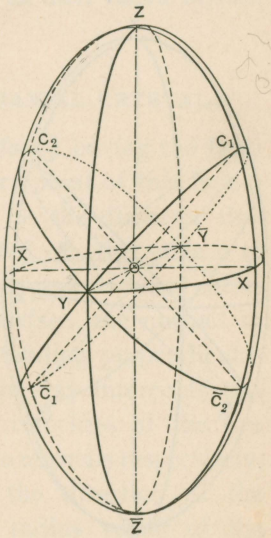

Fig. 62.

while for greater inclinations the major axis is the one lying in the plane $XZ\overline{X}\overline{Z}$, and $Y\overline{Y}$ now becomes the minor. Finally, after a rotation of 90°, *i.e.* for the section $YZ\overline{Y}\overline{Z}$, the variable axis has its greatest length. This axis then grows smaller again, on the other side of $YZ\overline{Y}\overline{Z}$; and in the direction $OC_2$, which — because of the symmetry of the triaxial ellipsoid with reference to the three principal sections — forms with $OZ$ the same angle as does $OC_1$, it attains again the value $OY$. Accordingly, the triaxial ellipsoid has two **circular sections,** $YC_1\overline{Y}\overline{C}_1$ and $YC_2\overline{Y}\overline{C}_2$, which intersect each other in the intermediate axis at an angle that is bisected by the two principal sections passing through this axis. The sections through $X\overline{X}$ are all ellipses whose minor axis is $\alpha$ and whose major varies from $\beta$ to $\gamma$, while the sections through $Z\overline{Z}$ are ellipses having the major axis $\gamma$ and a minor axis varying from $\alpha$ to $\beta$; in neither case, therefore, can the ellipses ever assume the form of a circle.

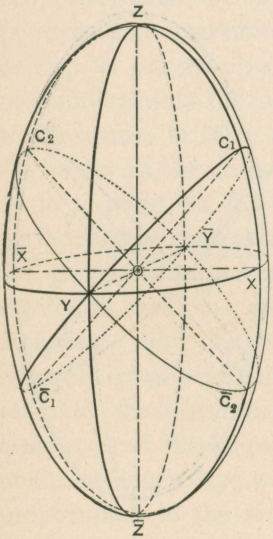

Fig. 62.

Now a triaxial ellipsoid, whose properties are described in the foregoing, would present the most general case of an optical index-surface. For the ellipsoid would pass over into the index-surface of a uniaxial crystal, *i.e.* into a rotation ellipsoid, if two of its principal axes became of equal length; into that of a singly refracting crystal, *i.e.* a sphere, if all three of its principal axes assumed the same value. In fact, it turns out that if, instead of with the two special cases last mentioned, one starts out with such an ellipsoid having three unequal axes, as the surface of reference, and from this ellipsoid derives the ray-surface in the same way as was done with the uniaxial crystals, then for this ray-surface there results a form corresponding to the

optical properties of all the crystals that are neither singly refracting nor optically uniaxial. Since the index-surface of such crystals has two circular sections, the directions normal to these sections are in a certain sense analogous to the optic axis of the uniaxial crystals (this being the normal to the single circular section of the latter crystals); therefore these directions also are spoken of as optic axes,* and the crystals whose ray-surface is derived from a triaxial ellipsoid as their index-surface are known as **optically biaxial** crystals.

### RAY-SURFACE OF THE OPTICALLY BIAXIAL CRYSTALS

The *ray-surface* proper to an *index-surface* † having the form of a triaxial ellipsoid is obtained if for every point of this index-surface we find the corresponding ray; *i.e.* the diameter that stands perpendicular to the normal erected on the surface at the point in question: to this ray there in general then corresponds yet a second point on the index-surface, the normal to that surface at this second point likewise standing perpendicular to the ray; and on these two normals the ray intercepts two lengths whose reciprocal values are the velocities of the two light-motions that vibrate parallel to the two normals respectively, these motions being transmitted along the direction of the diameter in question. Parallel to every radius vector of the index-surface we thus obtain two rays, and ultimately, for all directions, a double surface; of this surface, the form of its intersection with the three principal sections of the index-surface shall first be considered.

* [Sometimes designated more specifically as **primary optic axes.** (Cf. pp. 132 and 142.)]

† This surface was called by Cauchy the "ellipsoid of polarization"; by Billet, Verdet, and others the "inverse ellipsoid (of elasticity, of velocity)"; by MacCullagh the "ellipsoid of indices"; by Stefan the "ellipsoid of equal work"; by Kirchoff the "ellipsoid of elasticity"; finally, by Fletcher, whom we follow here in essentials, the "indicatrix". The name "index-surface" has, besides, been applied by some authors to another surface of reference, of which we have here no need.

Let us begin with the principal section $XZ\overline{XZ}$, Fig. 62. In this section there are transmitted an infinity of ordinary rays, which correspond to the point $Y$ of the index-surface; for the normal to the latter at $Y$, it being the normal at the extremity of one of the principal axes, passes through the center, wherefore the direction of the rays proper to it is indefinite. These rays have the vibration direction $OY$ and therefore are transmitted with the velocity proportional to $1/\beta$. So if about $O$ we describe a circle with the radius $OB$, proportional to this quantity, we obtain the locus at which all ordinary rays from $O$ have arrived in the same time; and this circle constitutes one of the curves in which the ray-surface is intersected by the principal section $XZ\overline{XZ}$. (Cf. Fig. 64.) To obtain the other intersection, which corresponds to the extraordinary rays transmitted in the same plane, we begin with the point $X$ of the index surface. To this point, since it likewise is an extremity of a principal axis, there correspond an infinity of rays in the plane perpendicular to that axis, the plane $YZ\overline{YZ}$; these rays all have the vibration direction $OX$ and therefore the velocity $1/\alpha$, which quantity, it being the reciprocal of the smallest index, is the greatest light velocity in the crystal in question. In the plane $YZ\overline{YZ}$ the ray-surface accordingly has a section of circular form with the radius $OA = 1/\alpha$. (Cf. Fig. 65.) Among these rays belongs the ray that is parallel to $OZ$; so in this direction we obtain a second point of the ray-surface if in Fig. 64 we lay off a length $OA$, proportional to the greatest light velocity. The two lengths $OA$ and $OB$ have the ratio, therefore, of the intermediate to the smallest principal refractive index. Hence, if on the ellipse $XZ$ of the index-surface we pass

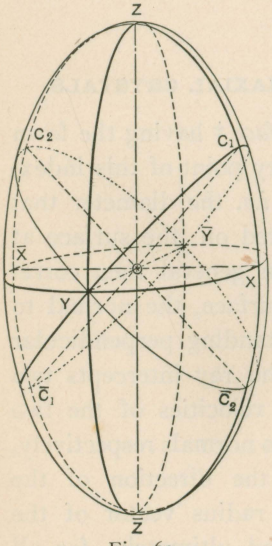

Fig. 62.

out from $X$ over to adjacent points, we obtain *e.g.* for the point $R$, Fig. 63, the corresponding ray, $Or$, in the usual way; this ray has the vibration direction $RN$, and its velocity is inversely proportional to the length $RN$. But $RN$, the distance of the tangent to the ellipse at $R$ from the radius vector $Or$ parallel to it, is obviously greater than $\alpha = OX$; for this latter length too is the distance of a tangent (at $X$) from the radius vector parallel to it

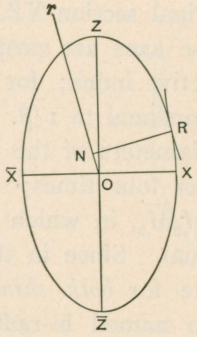


Fig. 63.

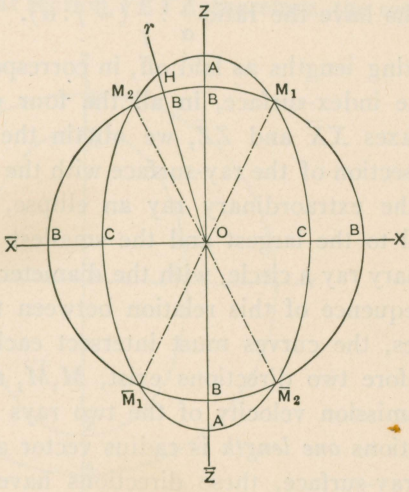


Fig. 64.

$(OZ)$, and altogether the least that exists. Consequently the reciprocal value $\frac{1}{RN}$ is less than the length $OA$ in Fig. 64, since this was made proportional to $1/\alpha$. So in the direction parallel to $Or$ we have to lay off a length $OH$ (see Fig. 64), correspondingly shorter than $OA$. But in the same direction, as we saw on page 130, there is transmitted a second ray, with the velocity proportional to $1/\beta$, and this ray corresponds to the point $Y$ on the index-surface; accordingly the point $H$ must be nearer to the point $B$ on $OH$ than is $A$ to the point $B$ lying on $OA$. Hence, the farther $R$ (Fig. 63) lies from $X$, the shorter does the corresponding radius of the ray-surface (Fig. 64) become; and

when $R$ coincides with $Z$, so too does $N$ coincide with $O$, and $Or$ with $O\overline{X}$. That is to say, the ray transmitted along the direction $OX$, which vibrates parallel to $OZ$, has the velocity proportional to $1/\gamma$ — the least in the crystal. Consequently, for the extraordinary rays vibrating in the principal section $XZ\overline{X}\overline{Z}$, when the transmission direction varies from $OZ$ to $OX$ the velocity varies from $OA$ to $OC$ (Fig. 64); these two lengths have the ratio $\frac{1}{\alpha} : \frac{1}{\gamma} (= \gamma : \alpha)$. If, then, we imagine the resulting lengths as laid off, in correspondence to the symmetry of the index-surface, in all the four quadrants lying between the axes $X\overline{X}$ and $Z\overline{Z}$, we obtain the following curves as the intersection of the ray-surface with the principal section $XZ\overline{X}\overline{Z}$: for the extraordinary ray an ellipse, whose axes are proportional to the largest and the smallest refractive index; for the ordinary ray a circle, with the diameter proportional to $1/\beta$. In consequence of this relation between the diameters of the two curves, the curves must intersect each other four times; and therefore two directions exist, $M_1\overline{M}_1$ and $M_2\overline{M}_2$, in which the transmission velocity of the two rays is equal. Since in these directions *one length* is radius vector at once for *both skins* of the ray-surface, these directions have been named **bi-radials**; they are known besides as **ray-axes**, or as **secondary optic axes** in distinction from the two directions mentioned at the end of the last section, the so-called "*primary* optic axes", which lie in the same plane and likewise present certain analogies with the optic axis of uniaxial crystals.

Now as for the intersection of the ray-surface with the second principal section, $YZ\overline{Y}\overline{Z}$, it has already been shown that one of its curves is a circle, with the radius proportional to the greatest light velocity. The second curve has a variable radius vector. For if we start out from the point $Y$ of the index-surface, there corresponds to this point in the direction $OZ$ an extraordinary ray with the vibration direction $OY$ and therefore with the velocity proportional to $1/\beta = OB$ (Fig. 65); to points

lying between $Y$ and $Z$ on the same principal section of the index-surface there corresponds greater length of the normals (since, of all the normals in this principal section, that at $Y$ is the shortest) and therefore less velocity of the rays proper to them; at $Z$, finally, the normal to the surface is longest, so the rays vibrating parallel to $OZ$ and transmitted along the direction $OY$ have the least velocity $OC$. Of the light rays propagated in the principal section $YZ\overline{Y}\overline{Z}$, therefore, the extraor-

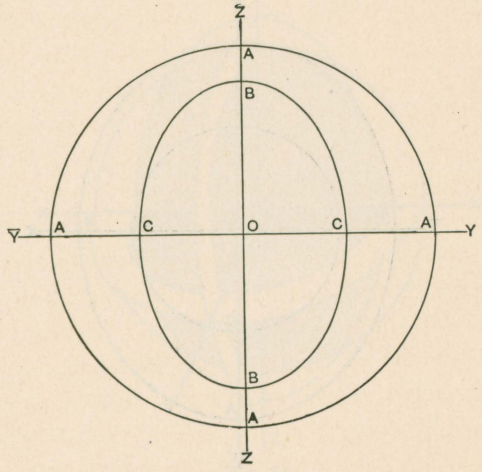


Fig. 65.

dinary advance in a certain time as far as an ellipse whose axes are to each other as the intermediate to the least light velocity, *i.e.* as the smallest refractive index to the intermediate; while the ordinary, in that same time, have been propagated as far as a circle with the radius proportional to the greatest light velocity, — proportional therefore to the reciprocal of the smallest refractive index. So the circle surrounds the ellipse without touching it.

The curves corresponding to the third principal section, $XY\overline{X}\overline{Y}$, are shown in Fig. 66 on the next page. One of them is a circle with the radius $OC$; it contains all the rays that cor-

respond to the point $Z$ on the index-surface, these rays all having the vibration direction $OZ$ and therefore the constant transmission velocity $OC$ proportional to $\frac{1}{\gamma}$. The extraordinary ray transmitted along the same direction as any one of these ordinary rays always has a greater velocity, since the point corresponding to it on the index-surface lies in the principal section $XY\bar{X}\bar{Y}$, wherefore the normal erected at that point is shorter than $OZ$

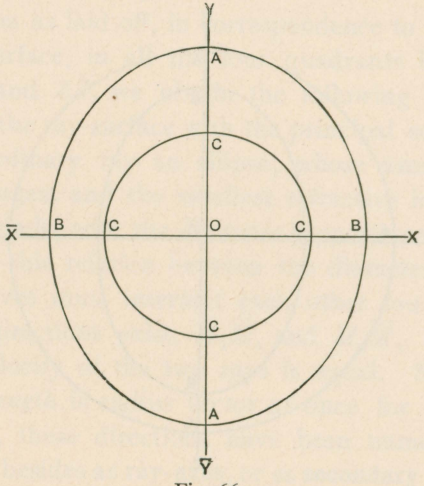


Fig. 66.

(this being altogether the longest that exists). The transmission velocity of the extraordinary rays transmitted in this principal section reaches its maximum, $OA$, for the ray vibrating parallel to $OX$ and hence having the direction $OY$; its minimum, $OB$, for the vibration parallel to $OY$, —*i.e.* for the ray direction $OX$. Of the ray-surface intersections, accordingly, the curve corresponding to the ordinary rays lies wholly within the ellipse whose radii vectores represent the transmission velocities of the extraordinary rays.

So far as is possible without a model, the perspective view in Fig. 67 may serve to give a more connected idea of the form of

the entire ray-surface derived from the index-surface; it is to this surface, therefore, that a light-motion of a definite color, beginning at the center, has been transmitted after a definite time. In the figure the space within the inner skin of the surface is shaded, while that included between the two skins is white.

This double surface was derived first by Fresnel, although in a somewhat different way; he called it the " wave-surface ", and showed that through it the optical properties of the biaxial

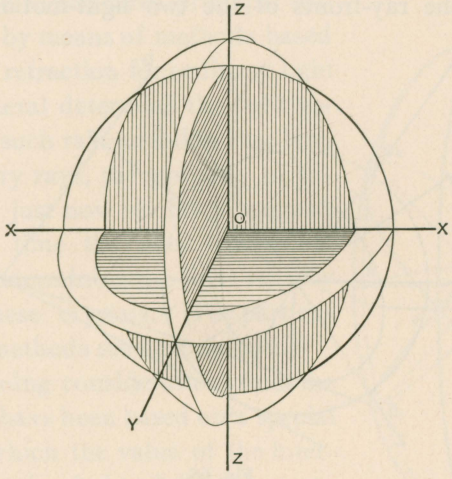


Fig. 67.

crystals could be completely explained. It is for this reason often designated also as " Fresnel's surface ".*

As follows from the foregoing, the form of this surface is completely determined when the greatest, the intermediate, and the least light velocity, with their respective directions in the crystal, are known. Supposing these data to have been determined (by methods to be discussed in the next section), then

* Models of Fresnel's wave-surface in gypsum can be obtained from M. Schilling in Leipzig; in brass wire, after specifications of the author, from Böhm and Wiedemann in Munich. (See Appendix.) Polished wooden models of the corresponding index-surfaces, which can be taken apart, are supplied by G. J. Pabst of Nuremberg. (See ibid.)

for every ray entering the crystal in any direction we may determine, by means of Huygens's construction, the direction in which it is refracted, just as was done on page 95 *et seq.* for the uniaxial crystals. If we carry out the construction for a ray whose plane of incidence coincides with one of the three principal sections, *e.g.* for the parallel rays *DB* to *CO*, Fig. 68, — whose common ray-front is *OE* and whose common plane of incidence is parallel to *YOZ*, — then at the instant when *DB* enters the crystal the ray-fronts of the two light-motions arising by

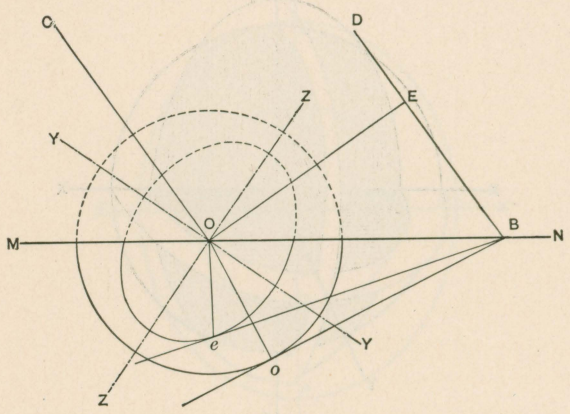


Fig. 68.

double refraction will be the tangential planes from *B* to the two skins of the ray-surface; and one easily sees that owing to the symmetry of the latter with reference to the principal section *YOZ* the two points of tangency, *o* and *e*, lie in the same principal section, and accordingly also that although the two rays *Oo* and *Oe* are indeed deflected they do not leave the principal section *YOZ*. If, however, the plane of incidence is parallel to *none* of the three principal sections, then, with a construction analogous to the last, the ray-surface intersection parallel to the plane of incidence will divide that surface into unequal halves, so that the halves lying respectively before and behind the plane of the figure do not lie symmetrically with ref-

erence to this plane. The points where the tangential planes representing the refracted ray-fronts touch the two skins of the ray-surface will then lie no longer in the plane of the figure, but *before* or *behind* it. Accordingly, the refracted rays are both deflected from the plane of incidence; that is to say, neither of them any longer follows the law of refraction for ordinary light: both are *extraordinary*. So an ordinary ray (in addition to an extraordinary) is obtained only when the plane of incidence is parallel to one of the three principal sections. Since by means of methods based on the law of refraction for ordinary light we can in general determine the light velocity only of such rays as follow that law, *i.e.* of ordinary rays, the property of biaxial crystals just now set forth suggests at the same time the most convenient methods for determining the light velocity in one of these crystals. (For particulars of these methods see next section.)

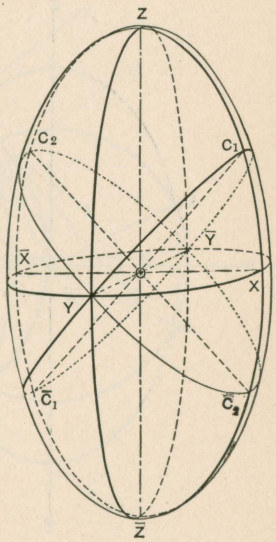

Fig. 69.

The foregoing considerations and the figures 62–66 have been based on a special example, in which the value of the intermediate refractive index $\beta$ lies closer to that of the smallest $\alpha$ than to that of the largest $\gamma$. The form of the index-surface in Fig. 69 therefore approximates — and if the difference between $\beta$ and $\alpha$ were still less this would be the case in a still higher degree — to that of a spheroid having $OZ$ as rotation axis; and it passes over into such a spheroid when $\alpha = \beta$, — *i.e.* when $OX = OY$. Since this extreme case corresponds to a *positive* optically uniaxial crystal, those biaxial crystals also are designated as **positive** in which $\beta$ lies closer to $\alpha$ than to $\gamma$. But the less the lengths $OX$ and $OY$ differ from each other, the smaller is the angle included between $OX$ and the radius vector $OC_1$, equal in length to $OY$; and consequently the

smaller also is the angle that the two optic axes — standing, as they do, perpendicular respectively to the circular sections $C_1\overline{C}_1$ and $C_2\overline{C}_2$ — form with $OZ$. *Optically positive biaxial crystals*, therefore, are those in which the *vibration direction of the rays transmitted the most slowly* (those with the largest refractive index) is the so-called **acute bisectrix** (first mean line) of the optic axes; *i.e.* bisects their acute angle.

Conversely, when $\beta$ is the closer in value not to $\alpha$ but to $\gamma$, *i.e.* $OY$ but little different from $OZ$, the form of the index-surface approximates to a spheroid with the rotation axis $OX$; in other words, to the index-surface of a *negatively* uniaxial crystal. The value $\beta$ for a radius vector of the index-surface between $OX$ and $OZ$ is then attained only near the latter; so the two circular sections form an acute angle with $OZ$, and the optic axes normal to them an obtuse angle. Such crystals are said to be **negatively** biaxial, and are characterized by the fact that the *vibration direction of the rays transmitted the most slowly* is the **obtuse bisectrix*** (second mean line) of the optic axes; *i.e.* bisects their obtuse angle. In correspondence to this, in a crystal of the latter sort the bi-radials, $M_1\overline{M}_1$ and $M_2\overline{M}_2$, lie as represented in Fig. 70.

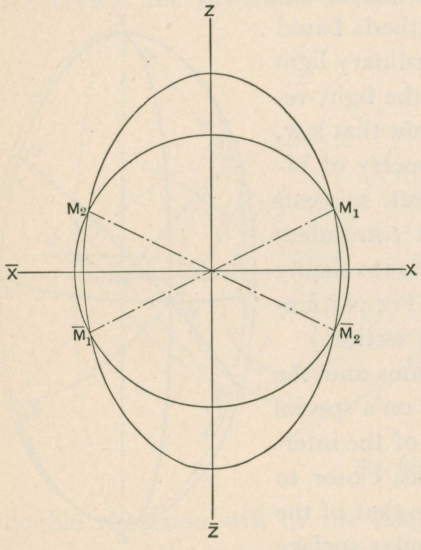

Fig. 70.

Since the quantities $\alpha$, $\beta$, $\gamma$ are different for different colors, there are biaxial crystals, also, that for certain colors are posi-

*[The term " bisectrix" when used alone, without special qualification, refers to the *acute* bisectrix.]

tive and for others negative, the angle between the axes amounting, for example, for one color to 89° and for another to 91°; there is then a color for which the crystal is exactly intermediate between positive and negative, thus being neither the one nor the other.

Now it yet remains to deduce from the ray-surface the relations in which the two so-called primary optic axes stand to the

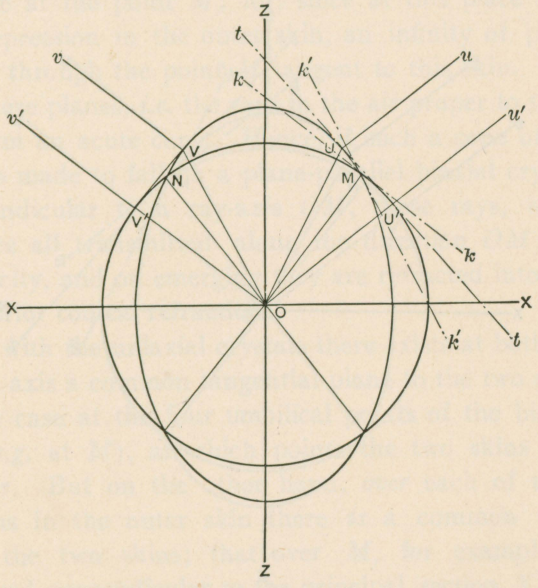


Fig. 71.

secondary, the ray-axes, and to learn in what way these directions depend on the values of the three principal refractive indices.

To this end let us first consider somewhat more closely the properties of the rays transmitted parallel to the two kinds of optic axes.

As the form of the ray-surface teaches, the two rays arising in any direction are in general transmitted with unequal velocity; only in the direction of a so-called bi-radial, or ray-axis, as *OM* in Fig. 71, are these two rays transmitted equally fast.

Such a direction differs essentially, however, from the optic axis of a uniaxial crystal, in that although the two rays in question have indeed the same transmission velocity, their respective ray-fronts have not the same direction. For the front of the ordinary ray — the plane perpendicular to the principal section $XZ$ through the tangent, $kk$, to the circular intersection of the ray-surface at $M$ — stands perpendicular to $OM$, while on the

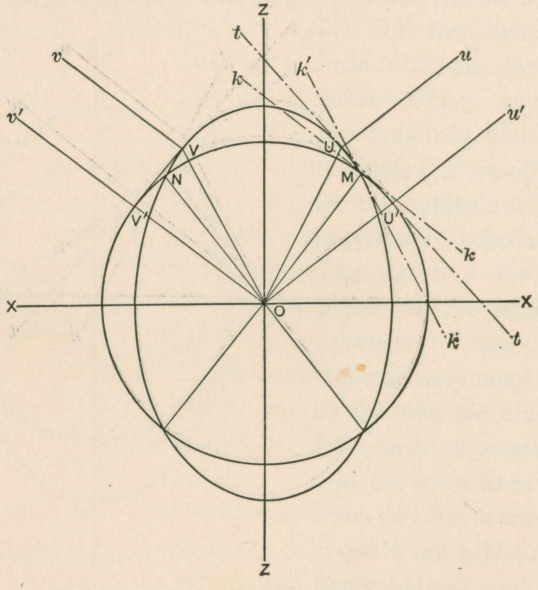


Fig. 71.

other hand the front of the extraordinary ray of the same transmission direction stands oblique to $OM$, it being the plane perpendicular to the principal section $XZ$ through $k'k'$, the tangent at $M$ to the elliptical intersection of the ray-surface. The distance of these two ray-fronts from the point whence the light-motion proceeds, *i.e.* the length of the normals let fall from the center $O$ upon the two planes, is consequently different; in other words, the transmission velocities of these two wave-motions, as measured along their front-normals, are unequal.

*But the velocities so measured come into consideration only in* the case of optically isotropic media, where ray and front-normal are identical; and hence it follows that the two rays passing through the crystal in the direction $OM$ with equal velocity are, in correspondence to the difference in their ray-fronts, on emerging into the air differently, *i.e.* doubly, refracted. Now these two fronts are not the only tangential planes to the ray-surface at the point $M$; for, since at this place there is a conical depression in the outer skin, an infinity of planes can be passed through the point $M$ tangent to this skin. The normals to these planes, *i.e.* the rays in the air proper to the several fronts, form an acute cone. Hence, if such a cone of converging rays is made to fall on a plane-parallel biaxial crystal plate cut perpendicular to a ray-axis $OM$, these rays, within the crystal, are all transmitted along the direction $OM$ and with equal velocity, and on emerging they are refracted into a similar cone (**exterior conical refraction**).

While with the uniaxial crystals there exists at both extremities of the axis a common tangential plane to the two skins, this is not the case at the four umbilical points of the biaxial ray-surface (*e.g.* at $M$), at which points the two skins penetrate each other. But on the other hand, *over* each of these four depressions in the outer skin there *is* a common tangential plane to the two skins; that over $M$, for example, is the plane passed perpendicular to the principal section $XZ$ through the straight line $tt$, which is tangent to the ellipse at $U$ and to the circle at $U'$. This plane touches the ray-surface around the point $M$ along a closed curve, to which the points $U$ and $U'$ belong; and from the geometrical properties of the ray-surface it follows that this curve is a *circle*. Hence, if we imagine radii vectores as drawn to all points of this circle (visible in Fig. 71 are the two radii, $OU$ and $OU'$, that lie in the plane of the figure) we obtain a cone of rays all having one common ray-front; namely, the plane passed through $tt$ perpendicular to the principal section $XZ$. Consequently, also the normal to this

plane, the front-normal of these rays, is common to them all; for example, the front-normal $Uu$ of the ray $OU$ is parallel to the front-normal $U'u'$ of the ray $OU'$; and so likewise are the front-normals $Vv$ and $V'v'$ of the rays $OV$ and $OV'$ parallel to each other. Now these two directions $Uu$ and $Vv$ (or $OU'$ and $OV'$) are — as follows from the equation of the index-surface — nothing else than the so-called primary optic axes; *i.e.* the normals to the circular sections of the index-surface from which the ray-surface is derived. So the behavior of all rays transmitted parallel to a primary optic axis, whose ray-front therefore is the plane of the circular section, is analogous to that of the rays transmitted parallel to the optic axis of a uniaxial crystal, inasmuch as their vibrations in this plane can have any azimuth. The primary optic axes accordingly exhibit a still closer analogy with the optic axis of uniaxial crystals than do the ray-axes, and will therefore in the following be designated briefly as **optic axes.*** If parallel to a circular section of the index-surface, *i.e.* perpendicular to an optic axis, we imagine a plane face as ground on the crystal, and cause parallel light rays to fall perpendicularly on this face, then all the rays that belong to a circular cylinder will, after their entry into the crystal, form a cone with the conical angle $UOU'$ (**interior conical refraction**). Conversely, all the diverging rays transmitted within a biaxial crystal along the convex surface of this cone will, on emerging through the plane face in question, become parallel and be transmitted in the air as a ray cylinder.

Now the aperture of this cone of interior conical refraction is in reality very much smaller than represented in Fig. 71 (seldom over 2°), because the difference among the principal refractive indices, and therefore the diversity in the transmission velocities of rays vibrating in different directions, is never so great as was,

* Since $OU'$ (Fig. 71) is perpendicular to $tt$, it is parallel to such an optic axis, and the same line $OU'$ is the normal both for the ray-front at the point $U'$ and for the ray-front at the point $U$. Fletcher proposed for the two optic axes the name **bi-normals,** for the reason that here it is a matter of directions in which one line is in a double sense the normal to a ray-front.

for the sake of distinctness, assumed in Fig's 62–71. The same applies also to the cone of exterior conical refraction.

Let the angle between the two optic axes (**optic axial angle**), *i.e.* the angle $U'OV'$ in Fig. 71, be represented by $2V$: let $V$ ($= U'OZ$ in Fig. 71) thus be the angle included between either optic axis and the vibration direction of the rays transmitted in the crystal the most slowly. From the equation of the index-surface there follows for this angle

$$\text{tang } V = \sqrt{\frac{\frac{1}{\alpha^2} - \frac{1}{\beta^2}}{\frac{1}{\beta^2} - \frac{1}{\gamma^2}}}.$$

If $\beta$ so lies between $\alpha$ and $\gamma$ that $\frac{1}{\alpha^2} - \frac{1}{\beta^2} = \frac{1}{\beta^2} - \frac{1}{\gamma^2}$, then is tang $V = 1$, and $V$ therefore $= 45°$. If $\beta$ is more nearly equal to

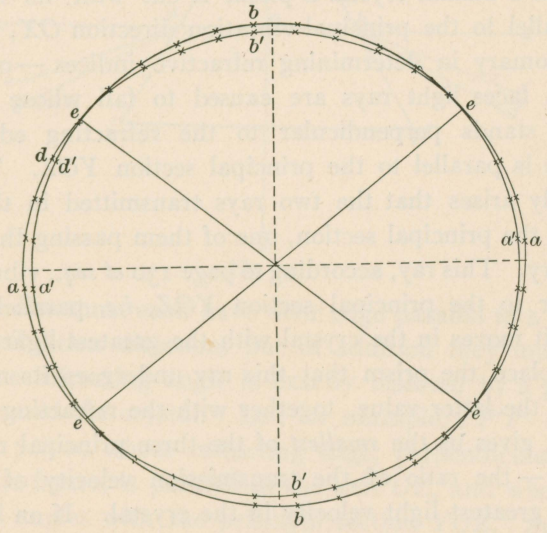


Fig. 72.

$\alpha$ than to $\gamma$, and in consequence the numerator of the fraction under the square-root sign less than the denominator, then $V$ is less than 45°; that is, we have to do with a *positively* biaxial crystal, in

which the vibration direction of the least light velocity bisects the smaller (the acute) angle of the optic axes. In the reverse case — *i.e.* when $\beta$ is the nearer to $\gamma$, and consequently the numerator greater than the denominator — $V$ is greater than 45°, and we have before us a *negative* crystal, in which the vibration direction of the least light velocity bisects the larger (the obtuse) angle of the optic axes.

REMARK. — Figure 64 *et seq.* are of course much exaggerated. In order to give an idea of the actual form of the wave-surface of a biaxial crystal, of one even with very strong double refraction, the intersection (according to W. Kohlrausch's observations — see p. 150) of the wave-surface of tartaric acid (for sodium light $\alpha = 1.4961$, $\beta = 1.5359$, $\gamma = 1.6055$) with the optic axial plane (corresponding therefore to Fig. 64) is represented in Fig. 72.

## DETERMINATION OF THE PRINCIPAL REFRACTIVE INDICES OF BIAXIAL CRYSTALS

If from a biaxial crystal a prism is cut with its refracting edge parallel to the principal vibration direction $OX$, and if — as is customary in determining refractive indices — on one of the prism faces light rays are caused to fall whose plane of incidence stands perpendicular to the refracting edge, then this plane is parallel to the principal section $YOZ$. The case accordingly arises that the two rays transmitted in the prism remain in the principal section, one of them passing through as an ordinary. This ray, according to page 130 *et seq.*, vibrates perpendicular to the principal section $YOZ$, *i.e.* parallel to $OX$; therefore it moves in the crystal with the greatest light velocity. If we so place the prism that this ray undergoes its minimum deviation, the latter value, together with the refracting angle of the prism, gives us the *smallest* of the three principal refractive indices $\alpha$ — the ratio of the transmission velocity of light in air to the greatest light velocity in the crystal. If on the other hand we cut a prism whose refracting edge is parallel to the axis $OY$ of the index-surface, this prism in like manner will supply an ordinary ray vibrating parallel to its refracting edge; this ray therefore is transmitted through the prism with the *inter-*

*mediate* velocity, and so the minimum deviation of this ray yields, by the usual formula, the principal refractive index $\beta$. Finally, a third prism, with its edge parallel to the axis $OZ$ of the index-surface, gives us the refractive index $\gamma$ of the ordinary ray having the vibration direction $OZ$; *i.e.* the *largest* of the refraction exponents of the crystal.

The determination of the three principal refractive indices is possible also, however, with the aid of only *two* prisms; these

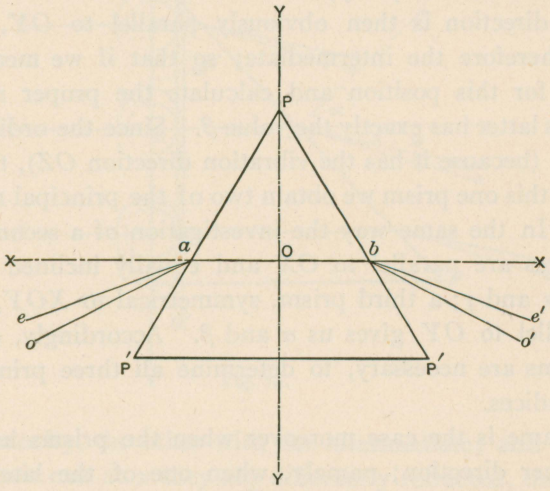


Fig. 73.

prisms likewise must each have their edge parallel to a different principal vibration direction, but in addition they must be so cut that the refracting angle is exactly bisected by a principal optic section of the crystal. Let, for example, $PP'P'$ (Fig. 73) be such a prism, whose refracting edge, perpendicular to the plane of the figure, is parallel to the axis $OZ$, and whose sides form equal angles with the principal section $YOZ$. Supposing the plane of incidence of the light to be the plane of the figure, it is consequently the principal section $XOY$; and therefore the two doubly refracted rays are transmitted in the same plane. The ordinary ray, which vibrates parallel to $OZ$ and accordingly

perpendicular to this plane, experiences the minimum deviation when it passes in the prism from $a$ to $b$; it then enters the prism in the direction $oa$ and emerges in the direction $bo'$. If we so turn the prism, or if we shift the source of light in such a way, that the extraordinary ray on its part suffers the minimum deviation (which in this case is less than with the ordinary ray), it enters the prism in the direction $ea$ and emerges along $be'$, thus passing through the prism in the same direction $ab$ as did previously the ordinary ray; *i.e.* in the direction $XX$. But its vibration direction is then obviously parallel to $OY$, and its velocity therefore the intermediate; so that if we measure the deviation for this position and calculate the proper refractive index, this latter has exactly the value $\beta$. Since the ordinary ray gives us $\gamma$ (because it has the vibration direction $OZ$), then with the aid of this one prism we obtain two of the principal refractive indices. In the same way the investigation of a second prism, whose faces are parallel to $OX$ and equally inclined to $XOZ$, supplies $\alpha$ and $\gamma$; a third prism, symmetrical to $XOY$, with its edge parallel to $OY$, gives us $\alpha$ and $\beta$. Accordingly, only *two* such prisms are necessary, to determine all three principal refractive indices.

The same is the case moreover when the prisms are cut in still another direction; namely, when one of the lateral faces coincides with a principal optic section of the crystal. Let $PP'P'$ (Fig. 74) be such a prism with the refracting angle $w$, whose edge is parallel to $OZ$ and whose left face is parallel to the principal section $YOZ$. If light rays $l$, $l'$, $l''$, etc., are caused to fall perpendicularly on this face and therefore parallel to $XX$, the two ray-fronts transmitted in the crystal are the tangential planes, $tt$ and $t't'$, to the ray-surfaces to be constructed, all of equal dimensions, from each point of entrance, — so constructed because the incident plane front strikes all the points of entrance at the same time. From the construction, and from the symmetry of the wave-surface with reference to the principal sections, it follows directly that the two tangential planes are exactly per-

pendicular to the plane of the figure and exactly parallel to each other, as also to the face of entrance of the light. So the two rays experience no deviation whatever, but are both transmitted parallel to $OX$, as in the prism of Fig. 73; the one therefore with

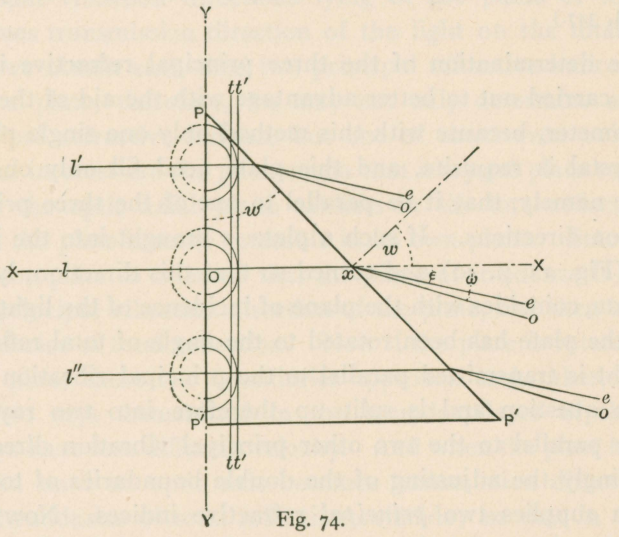


Fig. 74.

the least velocity, the other with the intermediate; and accordingly, on their exit at $x$ they are differently refracted, the one to $o$, the other to $e$. If we determine the deviation of each ray from its original direction and denote these deviations by $\omega$ and $\varepsilon$ respectively, the former is the angle $Xxo$, the latter $Xxe$. As is manifest from Fig. 74 directly, the refractive indices of the two rays are then

$$\gamma = \frac{\sin(\omega + w)}{\sin w},$$

$$\beta = \frac{\sin(\varepsilon + w)}{\sin w}.$$

By means of such a prism, therefore, two principal refractive indices may be determined; and thus, by means of two differ-

ent prisms, each of which has one side parallel to a different principal section, we can determine all three.

REMARK. — When the optic axial angle is known, the measurement or calculation of the three principal refractive indices may be accomplished also with a prism of different orientation. (See, for example, Hutchinson, *Zeitschr. f. Kryst.* 1901, **34**, 347.)

The determination of the three principal refractive indices can be carried out to better advantage with the aid of the total-reflectometer, because with this method only one single plate of the crystal is requisite, and this plate need fill only one condition; namely, that it be parallel to one of the three principal vibration directions. If such a plate is brought into the instrument (Fig. 22, p. 40) and turned so that this direction, lying in the plate, coincides with the plane of incidence of the light, then, when the plate has been rotated to the angle of total reflection, the light is transmitted parallel to the principal vibration direction in question and is split up therefore into two rays that vibrate parallel to the two other principal vibration directions; accordingly the adjusting of the double boundaries of total reflection supplies two *principal* refractive indices. Now if we rotate the plate 90° in its own plane and fasten it in this position to the rotation axis of the instrument (the latter is then parallel to the principal vibration direction falling in the plane of the plate), then on the total reflection the transmission direction of the light, although in general it will be parallel to no principal vibration direction, will nevertheless, since it is perpendicular to one of these, lie in a principal section; therefore one of the two rays, the ordinary, arising by double refraction vibrates parallel to the principal vibration direction that in the first case was transmission direction, and consequently the adjustment of the critical angle peculiar to this ray gives the third principal refractive index. In order to distinguish the two boundaries from each other, one brings before the telescope of the total-reflectometer a Nicol prism with its principal section vertical. This transmits only the ray vibrating vertically, and therefore only

that boundary of total reflection appears in the field, that supplies the third refractive index.* It is easy to see that, with the use of a plate parallel to a principal optic section, if the plate is so oriented that first the one and then the other of the two principal vibration directions lying in the plane of the plate becomes transmission direction of the light on the total reflection, we obtain each time two principal refractive indices; with such a plate, therefore, we may not only determine all three principal refractive indices, but one of them even twice. Of special interest are the phenomena that a crystal plate cut parallel to the optic axial plane exhibits in the total-reflectometer, when the plate is made rotatable in its own plane. Starting out with the plate so oriented that the plane of incidence is parallel to a principal vibration direction of the plate, one then sees two boundaries of total reflection. Now if the plate is rotated in its own plane these two boundaries approach each other, because the two rays transmitted in the boundary-layer of the plate always have less different velocity. And when the plate has been rotated until an optic axis becomes transmission direction, the two boundaries of total reflection coincide; so that in the field of view, besides the vertical boundary, corresponding to the ordinary ray, there appears the second boundary of total reflection, crossing the former at the center of the field at an acute angle. Provided this angle is not too sharp, that is to say, provided the transmission velocity of the extraordinary ray varies rapidly with the direction (*i.e.* the crystal has high birefringence), a rotation of the plate through 360° in its own plane accordingly brings into view, little by little, the whole intersection of the ray-surface with the principal section *XZ*. (See Fig. 64, in which the ratio of the greatest to the least light velocity is of course much exaggerated; see also Fig. 72, p. 143; and cf. Fig. 75, p. 150, with the explanation.) With this method as a

* It is hardly necessary, after the previous expositions, to remark that in every determination of the refractive indices of a doubly refracting body the vibration directions of the several rays must be determined, by means of a nicol.

basis W. Kohlrausch measured the transmission velocity of light in numerous directions within the three principal optic sections of tartaric acid (cf. remark on p. 144), and found it wholly in keeping with Fresnel's surface.*

Finally, the three principal refractive indices can be determined also through total reflection from a biaxial crystal plate of any orientation, by measuring the maxima and minima of the two curves that such a plate exhibits [in Abbe's refractometer. (See pp. 42–44.)]

Should we here employ a cylindrical plate (see Fig. 24 *l. c.*) and illuminate the cylindrical surface from all sides, then on a white screen placed beneath the glass sphere we should see the closed boundary-curves of total reflection.† While in the case of a uniaxial crystal plate parallel to the axis these curves have a form similar to that of Fig. 50 (p. 94) or of Fig. 56 (p. 102), and in the case of a biaxial crystal parallel to the axial plane a form as in Fig. 72 (p. 143), with a biaxial plate oriented at random there appear in general two curves as in Fig. 75, where the radius vector of each curve has a maximum and a minimum value. Now with Abbe's crystal refractometer the four values of the light velocity corresponding to these maxima and minima can be very accurately determined, by measuring, during a rotation of 360°, the refractive indices in a considerable number of azimuths; and, since the rotation angles may be read off on the horizontal graduated circle, we can also ascertain the orientation of the radii in question. Hence one of these radii, $\alpha$, corresponds to the absolute minimum, and therefore to the smallest refractive index of the crystal; the greatest, $\gamma$, to the absolute maximum, and accordingly to the largest refractive index; while to the intermediate refractive index of the crystal there corresponds either the maximum $\beta$ or the minimum $\beta'$.‡

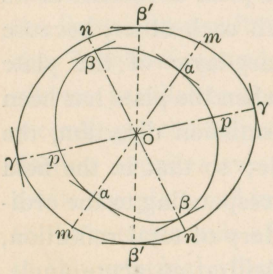

Fig. 75.

* For the details of such measurements as the foregoing and that next following cf. footnote †, p. 30.

† Apparatus for projecting and photographing the boundary-curves have been constructed by Leiss (see *Zeitschr. f. Kryst.* 1898, **30**, 357) and are supplied by the firm of Fuess. (See also Die op. Instrum., 49–52.)

‡ For particulars see Viola, *Zeitschr. f. Kryst.* 1899, **31**, 40 *et seq.*; 1900, **32**, 311 *et seq.*, and **33**, 30 *et seq.*; 1902, **36**, 245 *et seq.*; 1904, **39**, 179 *et seq.*

Supposing $\alpha$, $\beta$, and $\gamma$ to have been determined by one of the methods just discussed, then from them the axial angle $2V$ may be calculated, according to the formula given on page 143. On the other hand, if $V$ itself has been measured (by a method to be discussed farther on), and, besides this quantity, only two of the three principal refractive indices (when, for example, the development of the crystal permits the preparation of prisms only in *one* direction), then by means of the same formula we may calculate the third principal refractive index.

The numbers by which the optical properties of a crystal are completely defined we call its **optical constants;** with an optically biaxial crystal these are the *crystallographic orientations of the three principal vibration directions* and the *values of the three principal refractive indices.* These quantities refer in general only to one definite color, and therefore all that has been said up to the present concerning the index-surface and the ray-surface applies only to light of one definite vibration period. If for determining the three principal refractive indices we employ light of another color, we find other values for them; and what is more, with each index the variation with the vibration period is different; that is to say, for $\alpha$, $\beta$, and $\gamma$ the constants of Cauchy's dispersion formula (see p. 46) have different values. In consequence of this, for a different color the values of the greatest, the intermediate, and the least light velocity stand in different ratios; in other words, the ray-surface (like the index-surface) for the latter color has a different shape, and the optic axes form a different angle with each other; this angle increasing with one substance with the wave length of the light, with another diminishing. Therefore the refractive indices $\alpha$, $\beta$, $\gamma$ must always be measured for several colors, when the optical properties of a crystal are to be determined. As for the positions of the three *principal vibration directions* in the crystal, these for different colors may be the same or different. They are determined with the aid of the *interference phenomena* that

biaxial crystal plates exhibit in certain directions, wherefore these phenomena must now be discussed first; and here, too, the consideration shall always be begun with the more simple case — that of monochromatic light.

### INTERFERENCE PHENOMENA OF BIAXIAL CRYSTALS IN PARALLEL POLARIZED LIGHT

Light rays of a definite color falling perpendicularly on a plate cut in any direction from a biaxial crystal are in general each split up into two (extraordinary, according to p. 137) rays; these rays, after they emerge from the crystal plate, vibrate at right angles to each other and have a difference of path, in virtue whereof, between crossed nicols, interference takes place, exactly as with the uniaxial crystals. (See p. 107 *et seq.*) The incident rays have in this case a common ray-front, which is parallel to the plane of the crystal plate. Now within the crystal there belong to this, as to every other, ray-front two light rays of different transmission direction; for if two parallel planes are tangent to the two skins, respectively, of the ray-surface of a biaxial crystal, the two points of tangency do not lie on the same radius vector. The transmission and vibration directions of the two rays having parallel ray-fronts, *i.e.* the same front-normal, are obtained in a more simple way from the index-surface, as follows: Through the center of the index-surface a plane is passed parallel to the front of the incident rays, parallel therefore to the crystal plate; in general this plane intersects the index-surface in an ellipse. (Fig. 76.) The extremities, $R$ and $T$, of the major and the minor axis of this ellipse are obviously those points of the ellipse that stand respectively at the greatest and the least distance, measured along the normals to the ellipsoid ($RN$ and $TN'$ in this case), from the rays proper to the points; and the transmission direction of these rays is obtained if from $O$ we let fall the normals to $RN$ and $TN'$ ($ON$ and $ON'$ respectively). Thus $RN$ and $TN'$ are the vibration directions of the vibrations transmitted the slowest (inversely proportional to the length $RN$) and the

fastest proportional to $(1/TN')$ of all the vibrations that correspond to the original wave-plane $ORT$; *i.e.* to the plane of the crystal plate. So any vibration taking place in this plane is on entering the crystal plate always resolved into two vibrations $RN$ and $TN'$, which are transmitted along the directions $Or$ and $Ot$ respectively. In Fig. 76 the vibration planes of the two rays $Or$ and $Ot$ are hatched parallel to the vibration directions of these rays. These two vibration *directions*, $RN$ and $TN'$, are not perpendicular to each other; the two vibration *planes*, however, intersect in $Of$ at right angles. Hence, since the plane face through

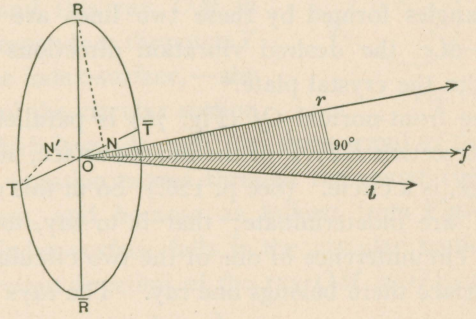


Fig. 76.

which the light emerges from the crystal plate is perpendicular to $Of$, and since on emerging the two rays are so refracted that they again, as before entering, assume the direction parallel to $Of$ (because in the air front-normal and ray are identical), then after the exit $RO$ and $TO$ are the two vibration directions. In other words, there are then transmitted in the air two *mutually perpendicular* vibrations; the directions, $RO$ and $TO$, of these vibrations are called the **vibration directions of the crystal plate** in question. These directions are defined, for every plane of a biaxial crystal, by the form of the index-surface; and conversely, with a crystal that is optically known their orientation may be employed for determining the orientation of

the crystal plate. They can also be found by the following construction: —

Through the normal to the crystal plate (*i.e.* the front-normal of the incident rays) and one of the two optic axes one passes a plane, and a second plane is passed through the same normal and the other optic axis. These two planes intersect the plane of the crystal plate in two straight lines, which are obviously normal respectively to the intersections of the latter plane with the planes of the two circular sections. Consequently the two diameters of the ellipse (Fig. 76) that are parallel to the lines in question are of equal length; and therefore the bisectors of the two angles formed by these two lines are the axes of the ellipse,— *i.e.* the desired vibration directions of the rays emerging from the crystal plate.*

When the front-normal *Of* (Fig. 76) is parallel to an optic axis, the intersection $RT\overline{R}\overline{T}$ of the index-surface, according to a previous page, is a circle. (See p. 128.) So in this case the axes $R\overline{R}$ and $T\overline{T}$ are indeterminate; that is to say, to every point lying on the circumference of one of the two circular sections of the index-surface there belongs one ray. The rays belonging to

* To find these directions graphically it is best to make use of a model, such as that supplied by Böhm and Wiedemann, in Munich. (See Appendix.) This model consists of a horizontal slate tablet representing the plane of the crystal plate, and at the center of the tablet the normal is represented by a metal bar. Rotatable in all directions about the foot of the latter, and movable at will with reference to each other, are fitted two metal bars which represent the two optic axes. Suppose, then, it be desired to determine the two vibration directions for a plate cut from a crystal of known axial angle $2V$ in a direction that is given by the inclination of the plate to the two optic axes. With the aid of a protractor (supplied with the model) one sets the two axes at the correct angle $2V$ to each other, and each, with reference to the horizontal plane, in the direction that corresponds to the orientation of the plate; one then lays the protractor against the normal and one of the axes, with its edge resting on the slate, and draws the corresponding line on the latter. In the same way the trace on the slate is found of the plane passed through the normal and the second optic axis; and it is then further necessary only to bisect the angle these two traces form with each other. If the outline of the crystal in the plane in question has been sketched on the plate, then with the protractor one can now read directly the angles the vibration directions form with the edges of the plate.

all the points of such a circle, which form the "bi-normal cone", or "cone of interior conical refraction" (see p. 142), all have the same direction and position of their ray-front, this for each being parallel to the circular section; their several vibration directions therefore, the normals to the index-surface at all points of the circular section, can have any azimuth. As may be seen from Fig. 71, p. 140, one of these rays coincides with the optic axis itself; and this ray, $OM$ in Fig. 77, corresponds to the points $Y$ and $\bar{Y}$ on the index-surface,—the only points of the circular section $CY\bar{C}\bar{Y}$ of which it can be said that the normal to this surface at the point in question passes through the center (because $Y\bar{Y}$ is one of its axes, and because, as follows from Fig. 62, no other axis of the index-surface falls in the circular section). Therefore the vibration direction of the ray $OM$ is $OY$, and its velocity is proportional to $\frac{1}{OY} = \frac{1}{\beta}$; in other words, A LIGHT RAY TRANSMITTED ALONG AN OPTIC AXIS HAS THE INTERMEDIATE REFRACTIVE INDEX $\beta$.

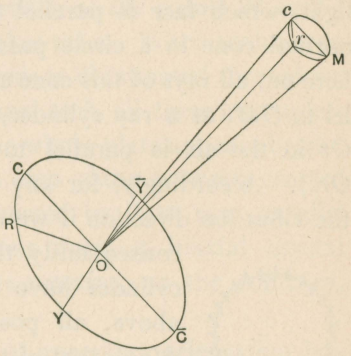

Fig. 77.

To the points $C,\bar{C}$ of the circular section, which lie in the principal section, there corresponds the ray $Oc$, which stands perpendicular to the normals erected on the index-surface at $C$ and $\bar{C}$; of this ray, therefore, the vibration direction within the crystal lies in the principal section in question, and after emerging through that boundary-plane of the crystal that is parallel to the circular section (whereat the bi-normal cone is transformed into a cylinder) the ray vibrates parallel to $Mc$, perpendicular therefore to the vibration direction of the ray $OM$. To any and every point lying between $Y$ and $C$ on the circular section, *e.g.* $R$, there corresponds a ray $Or$ of the bi-normal cone; this ray is

perpendicular to the normal erected on the index-surface at *R*, and therefore its vibrations take place in the plane *ROM* passing through the front-normal. Since the face of emergence of the light, which face is parallel to the ray-front, intersects the binormal cone in a circle passing through *Mrc*, and since, from here on, all rays of this cone are transmitted in the air and parallel to *OM*, as a ray cylinder, the vibration direction of the ray *Or* in the air is parallel to *Mr* (its polarization plane being *Orc*). Accordingly, for any ray *Or* of the cylinder we obtain the vibration direction if we connect *r* and *M* by a straight line; consequently the rays on the periphery of the cylinder have indeed, as was already mentioned above, all possible vibration azimuths between *Mc* — that of the ray *Oc* — and the azimuth perpendicular to *Mc* — that of the ray *OM*. (These vibration directions for the rays corresponding to different points on the circle *Mcr* are sketched in Fig. 78, with a ⟷.)

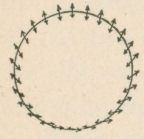

Fig. 78.

The last-mentioned fact has, as its consequence, that between crossed nicols a biaxial crystal plate cut perpendicular to one of the two optic axes behaves in an essentially different manner from a plate cut from a uniaxial crystal perpendicular to its axis. For a thin bundle of parallel rays falling on the former plate is transformed within the crystal into a cone of diverging rays, and this cone, on emerging, into a circular cylinder; so that, instead of a small bright spot, a ring of light* must appear, and on the circumference of this ring there must exist all possible vibration azimuths. With the analyzing nicol one can extinguish only those rays of the ring of light that vibrate perpendicular to the principal section of the nicol, of all the rest only the corresponding component. Between crossed nicols

* As to how this ring of light arising from the image of a small aperture may be made visible under the microscope, see Kalkowsky, *Zeitschr. f. Kryst. u. Min.* 1884, **9**, 486; further, Liebisch, *Nachr. Ges. d. Wissensch. Göttingen*, 1888, Nr. 5. The contrivance suggested by the latter is supplied by the optical works of R. Fuess, in Steglitz *bei* Berlin.

every infinitely thin ray bundle that has passed through a biaxial crystal parallel to an optic axis yields a ring of light, and this ring (when the vibration directions of the nicols are parallel and perpendicular to *Mc*, Fig. 77) is broken at the points *M* and *c*, because there complete extinction takes place. When such a crystal plate perpendicular to an optic axis is illuminated throughout its whole extent by rays falling normal to it, all these incomplete light rings cover one another, and the whole plate appears bright. While, therefore, on rotation in its own plane between crossed nicols a uniaxial crystal plate traversed by light rays parallel to the optic axis is in all positions *dark*, a biaxial crystal plate under the same circumstances remains, on the contrary, *bright*.

On the other hand, every plate of any other orientation has two mutually perpendicular vibration directions — to be found by the construction given on page 154 — and therefore between crossed nicols appears dark so often as these directions are parallel to the principal sections of the two nicols. In the special case where the plane of the biaxial crystal plate coincides with one of the three principal optic sections, the above-mentioned construction shows in a very simple manner that, as also follows directly from a consideration of the index-surface, the vibration directions of the plate are the two axes of the index-surface lying parallel to the plate; in other words, the two principal vibration directions lying in the plane of the plate. When the plate is cut parallel to only one axis of the index-surface, but inclined to the others, the direction of rays falling normal to it lies in a principal optic section; and here also, from considerations (see p. 130 *et seq.*) even previous to the construction described on page 154, it is manifest that one of the vibration directions of the plate is the axis of the index-surface (hence a principal vibration direction) to which the plate is parallel, the other the straight line drawn in the plane of the plate normal to that axis.

Consequently, if into the orthoscope we bring any plate of a biaxial crystal that is not cut perpendicular to one of the optic

axes, and rotate it between crossed nicols in its own plane, it becomes dark four times and in the intermediate positions bright, thus exhibiting the interference phenomena of monochromatic light, considered on page 60. The phenomena to be observed in white light depend, in the first place, on whether the vibration directions of the rays of different color emerging from the plate are the same or different. When the crystallographic orientation of the axes of the index-surface is for all colors the same, it follows directly that the vibration directions also, of a plate parallel to one or to two of the axes of the index-surface, coincide for all colors. A plate oriented parallel to none of the three axes of the index-surface exhibits, on the other hand, a dispersion of the vibration directions; for in this case the construction mentioned on page 154 yields, because of the inequality of the optic axial angles for different colors, for light of another wave length diverging vibration directions as well. Since, however, the difference among the axial angles seldom exceeds a few degrees, the angles between the vibration directions of such a crystal plate, for differently colored light, are usually very small.

But, when the crystallographic orientation of the principal vibration directions for different colors is itself different, then the vibration directions diverge, even of such plates as are parallel to one or to two of the axes of the index-surface for a certain color. Accordingly, if we desire here to determine the vibration directions by means of the positions of darkness in the orthoscope, in the way mentioned on page 76, this must be done in monochromatic light, and specially for each color. A more exact method of determination is that of the so-called **stauroscope.**

This method depends on the principle that, when the crystal to be investigated has the orientation corresponding to the maximum extinction of the light, it effects no resolution of the light passing through it and therefore does not disturb the formation of an interference-figure produced by another crystal, inserted in the path of the light rays and illuminated by convergent

light. If the latter crystal is suitably chosen, then, when the vibration directions of the crystal to be investigated diverge from those of the nicols even very slightly, there results a perceptible deformation of the observed interference-figure; and hence the adjustment can be made exceedingly sensitive.*

A biaxial crystal plate from which the light rays of all colors emerge with the same, or at least very nearly the same, vibration directions will obviously, even with the use of white light, during one complete rotation in the orthoscope appear dark four times, but in the intermediate positions colored, if it is thin; for the considerations on page 107 as to the phenomena with a uniaxial crystal plate cut oblique to the optic axis manifestly apply to every doubly refracting body. The arising color must vary with the thickness of the plate, and with a certain thickness of the same pass over into the white of a higher order. Plates of a different orientation exhibit, in general, different interference-colors even with equal thickness; and hence, with them also, the phenomenon of the white of a higher order appears with different plate thickness. It appears with the least thickness in the case of a plate parallel to the optic axial plane; for in such a plate the light is split up into two rays of which one has the greatest, the other the least transmission velocity, the double refraction (corresponding to the difference between the principal refractive indices $\gamma$ and $\alpha$) thus being altogether the strongest in the crystal. The double refraction is weaker in plates parallel to the two other principal sections, and its strength is different for the two orientations, according to the magnitude of the differences $\gamma - \beta$ and $\beta - \alpha$. While with uniaxial crystals all plates forming equal angles with the optic axis have the same birefringence, and consequently with equal thickness exhibit the same color, there exists no such uniformity around an optic axis of a biaxial crystal; for if, starting out from a circular section of the index-surface as the plane of the plate,

* For the description of such instruments and the procedure in stauroscopic measurements, cf. footnote †, p. 30.

we approach the one or the other of the principal sections, we always find different ellipses as intersections of the index-surface, and hence, also, different strength of the double refraction.

In case the vibration directions of a biaxial crystal plate for different colors diverge considerably, then in white light the plate appears in no position absolutely dark; for when it has been rotated in its own plane to the position of extinction for a definite color, the remaining colors still pass through — with an intensity that is the greater, the greater the dispersion of the vibration directions — and therefore make the plate appear bright, bright with a composite color. With exceptional size of the angle between the vibration directions of different colors there appears, on rotating the plate in the orthoscope, a continuous change of very vivid colors.

A certain similarity to the phenomena resulting from the dispersion of the vibration directions is possessed by those exhibited in white light by a biaxial crystal plate cut perpendicular to an optic axis. Here, for that color for which the plate stands exactly perpendicular to the optic axis, there exist, in consequence of the interior conical refraction (as we saw on p. 156), all possible vibration azimuths; on rotation, therefore, brightness is observed in every position. For all the remaining colors the optic axis stands more or less oblique to the plate, so that for them a resolution takes place into two mutually perpendicular vibrations; but, for the different colors, these vibrations are differently oriented. Therefore in white light, also, the plate can in no position exhibit darkness.

In the microscopical investigation of rocks this last-mentioned behavior of biaxial crystals often permits a mineral appearing in a thin rock section in numerous diversely oriented sections of the mineral to be recognized as biaxial. When these mineral sections exhibit very different birefringence (those with the highest, according to p. 159, are approximately parallel to the optic axial plane), then, if among them there chance to be some that on the rotation between crossed nicols always remain

bright and at the most exhibit changing colors, these sections, in the cutting, must have been struck perpendicular to an optic axis. Absolute certainty that one has to do with this case and not with that, previously considered, of a very great dispersion of the vibration directions, is obtained if one brings such a section into the center of the field and makes in the microscope the changes through which convergent light is produced. Through the crystal section one then beholds those phenomena of a plate cut perpendicular to an optic axis that are to be discussed in the next section.

## INTERFERENCE PHENOMENA OF BIAXIAL CRYSTALS IN CONVERGENT POLARIZED LIGHT

Like those cut oblique to the axis from uniaxial crystals, thin plates of biaxial crystals exhibit in monochromatic light dark and bright curves; the points of such a curve correspond to the rays that in the crystal have acquired the same difference of path. When the plate is cut parallel to the optic axial plane, these curves have the form of hyperbolas, as with a uniaxial plate parallel to the optic axis. (See p. 120.) In white light there appear analogous curves, each one of a uniform color (isochromatic curves); with excess of a certain thickness, however, only the white of a higher order is visible, unless there fall within the field of the conoscope rays that have passed through the crystal parallel to an optic axis.

To elucidate the phenomena arising in the case last mentioned, let us consider first the behavior in monochromatic light of a crystal plate cut perpendicular to an optic axis (for the color in question); in such a plate the rays that have passed through parallel to this axis converge at the center of the field. If one turns the plate in its own plane so that its optic axial plane is parallel to the vibration direction of one of the two crossed nicols, then, in the field of view, parallel to this principal section there must appear a dark bar, since in the principal section there now take place only such vibrations as in the second nicol

are totally annihilated. Points on the two sides of the dark bar correspond to rays whose vibration directions are more and more inclined to the principal section; consequently, as we pass out from the middle line of the bar there gradually takes place a brightening. And this brightening goes on the most rapidly near the center of the field; because here, since this point corresponds to the optic axis, the vibration direction varies the most rapidly, and because in immediate proximity to the axial point there come in addition the rays of the conical refraction, with their vibrations taking place in all azimuths. So the bar traversing the field of view is narrowest at the center. At a certain distance from the center of the field the rays will converge that in the crystal have acquired a path difference amounting to one wave length of the color in question; at this distance there must occur annihilation of the light, and therefore the center must be surrounded by a closed curve of darkness. Unlike the dark rings of the uniaxial crystals (see p. 112 *et seq.*), however, this curve cannot have the form of a circle. For in the direction parallel to the dark bar the path difference increases from the center, where it is zero, on the two sides of the center unequally (in the one case the birefringence approaching the value $\gamma - \beta$, in the other $\beta - \alpha$), and on the two sides of the black bar the arrangement, although symmetrical with reference to the bar, is different in every azimuth. The dark ring therefore resembles an ellipse in form and is symmetrically bisected by the dark bar. The same applies to the second ring surrounding the center, which corresponds to the path difference $2\lambda$. And so on. When the plate is cut oblique to an optic axis, the center of the bright and the dark rings does not appear at the center of the field, and on account of the refraction the rings are somewhat different in form.

Under some circumstances the ring systems even of both optic axes may fall in the field of the conoscope; — and of practical importance, here, are especially those interference phenomena that are exhibited by a plate whose plane stands per-

pendicular to the acute bisectrix (the bisector of the acute angle of the optic axes — see p. 138). If with crossed nicols we view such a plate in monochromatic light and in a position where its optic axial plane is parallel to the polarization plane of one of the two nicols, we behold the following phenomenon (Fig. 79): Through the center of the field there passes a black cross, whose two opposite arms lying parallel to the axial plane appear far narrower and more sharply defined than do the two, of less regular form, that stand perpendicular to them. The two points — they lie equidistant from the center of the field on the two sides — at each of which the rays passing through the crystal along an optic axis converge, are surrounded by dark and by bright oval rings; and two of these rings, of a certain distance from the

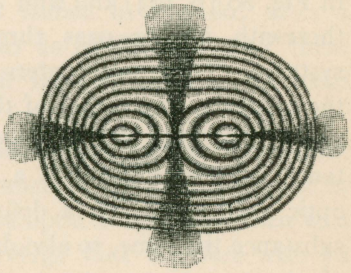

Fig. 79.

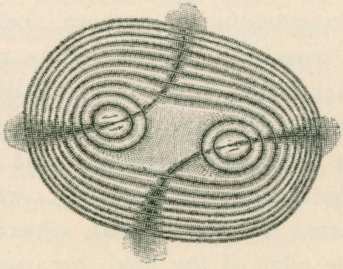

Fig. 80.

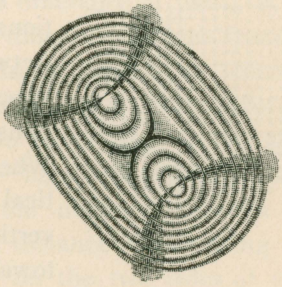

Fig. 81.

two centers respectively, have together the form $\infty$, while those of greater distance are shaped like the outermost curves represented in Fig. 79. These curved lines, in the case before us lines of equal brightness, are called **lemniscates.** If without altering the crossed position of the nicols one rotates the crystal plate in its own plane, the rings do not change in the

least, but simply rotate with the plate; the arms of the cross, on the other hand, previously straight, become transformed into two hyperbolas, which with slight rotation of the plate appear as in Fig. 80 (p. 163) and with 45° rotation as in Fig. 81, but which therewith always pass through the two centers of the ring systems. These two centers, the so-called "axial points", are the nearer to the center of the field, the smaller the optic axial angle; the closer to the edge, the larger that angle: their distance from each other is a measure of that angle. Since the optic axial angle (for a definite color) is with all crystals of a substance the same, so also does the distance between the centers of the two ring systems remain the same, whether the plate be thick or thin, provided only that it consists of the same material.

The above-described phenomena are explained as follows: —

In Fig. 82* are sketched with crosses, for numerous points in the field of view, the two vibration directions that are obtained, by the construction described on page 154, for the transmission directions corresponding to these points. Between crossed nicols all points must appear dark at which the vibration directions coincide with those of the nicols. When the latter are horizontal and vertical a black cross must appear, whose vertical arms are very broad, especially toward the edge of the field, because on the two sides of the vertical middle line the obliqueness of the vibration directions grows only very gradually; the same is the case also for the extreme parts of the horizontal arms, while above and below the axial points the obliqueness of the vibration directions increases very rapidly. So the horizontal dark bar must be narrowest near the axial points, and in both directions from the same increase in breadth. Hence, if we imagine

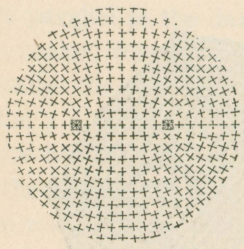

Fig. 82.

* Copy after E. G. A. ten Siethoff, *Centralbl. f. Min.* 1890, 268.

Fig. 82 as rotated * a small angle to the right (*i.e.* clockwise), and connect the points at which the vibration directions now stand horizontal and vertical, thus coinciding with those of the nicols, we obtain the dark curves shown in Fig. 80; after a rotation of 45°, the two hyperbolas as in Fig. 81; and these curves must necessarily have their least breadth at the axial points.

If from these axial points, at which there is no double refraction and consequently no difference of path, we pass out in a direction that is not parallel to the optic axial plane, then at a certain distance those rays will converge that interfere with $\frac{1}{2}\lambda$ difference of path; at a greater distance, those interfering with $\lambda$ difference of path; and so on. Advancing in this direction, one must, in the interference-figure, come alternately upon minima and maxima of brightness. But if we vary the direction in which we proceed from the center, then at the same distance there is a variation not only in the difference in velocity of the two arising rays, but also in the distance they travel in the crystal; we thus obtain the same difference of path, *i.e.* the same minimum and maximum, at a different distance from the center. Therefore, while with a uniaxial crystal the points of equal brightness lie on *circles*, — because the variation of the double refraction with the inclination takes place in the same way in all directions about the axis, — here oval curves of equal brightness must arise; but, since the distance between the two skins of the ray-surface — this distance determining the birefringence — varies symmetrically on the two sides of each principal section of the crystal, these ovals, too, must be symmetrically bisected by the optic axial plane and by the principal section standing perpendicular to that plane; *i.e.* by the two black bars. As a matter of fact it follows from the theory, agreeably to observation, that the dark and the bright curves have the form of

* Instead of by rotating the crystal plate, one can of course produce the same effect by rotating the two crossed nicols together while the plate retains its original orientation.

so-called *lemniscates*, which fulfill the above-stated conditions. If, then, we rotate the plate in its own plane, the lemniscate systems likewise must rotate, since their formation is, as we know, closely associated with definite directions in the crystal and since the line connecting their centers must always remain parallel to the optic axial plane. But the points of the interference-figure at which the rays converge whose vibration directions are parallel to the polarization plane of a nicol — these rays therefore being totally annihilated — now lie no longer on two straight lines crossing each other at right angles, but on two hyperbolas, and naturally these hyperbolas must each pass through an axial point.

If with unchanged color of the light used we employ a thicker plate of the same substance, the black cross or the hyperbolas, as also the distance between the two axial points, must, according to what has been said up to the present, remain wholly unaltered. And to be sure, at any given distance from an axial point the difference in velocity of the two doubly refracted rays will indeed be the same as before. But, since at that distance the path of these two rays in the crystal is longer, their " path difference " at that distance must be greater; and therefore, in the position where in the case of the thinner plate

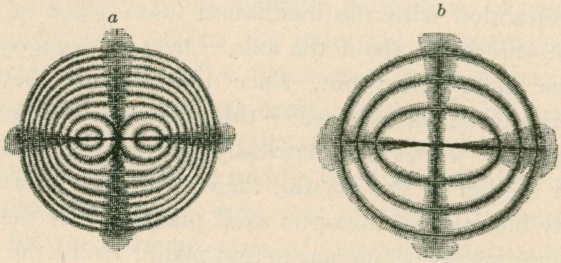


Fig. 83.

the first dark ring was seen, there appears with the thicker plate even the second ring or the third. Thus, the thicker the plate, the less will be the width of the rings; the thinner the plate, the wider the rings. Accordingly, with a certain slight thickness of

a plate having a small optic axial angle, the case will arise that even the innermost lemniscate no longer consists of two separate ovals, but of an ellipse-like form surrounding both centers, like one of the outermost lemniscates appearing with a thicker plate. Figure 83 *b* represents the interference-figure of a plate

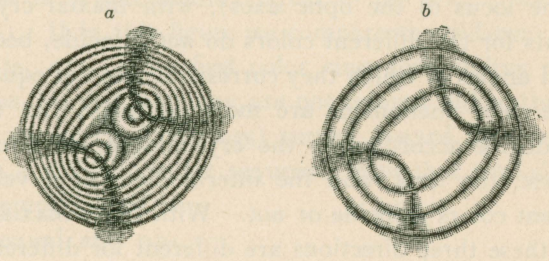


Fig. 84.

having such a thinness, compared with that, Fig. 83 *a*, of a thicker plate of the same substance, both under parallelism of the axial plane with the principal section of a nicol; while in Fig's 84 *a* and *b* is given the interference-figure of the same plates after a rotation of 45°.

Since the width of the rings depends on the difference in the velocity with which the two rays arising by the double refraction are transmitted in the crystal, their width, just as in the case of optically uniaxial crystals, is different with different substances, even when the plate thickness remains the same. In other words, the width of the rings depends on the *birefringence* (the difference among the three principal refractive indices $\alpha$, $\beta$, $\gamma$): with a plate that consists of a substance having slight birefringence the rings are wider than are exhibited by an equally thick plate of a body whose double refraction is stronger.

Finally, as follows from their origin, the width of the rings depends further on the wave length of the light used: when this is greater the rings stand farther apart from one another, and vice versa. So the dark rings for the different colors fall in different positions; and consequently, if the plate is investi-

gated in *white* light, *colored* rings will arise, the explanation of which is wholly the same as with the uniaxial crystals. But, while with the latter crystals the dark rings for the different colors simply overlap one another as concentric circles, the isochromatic curves thus being again circles having the same center (the locus of the optic axes), with biaxial crystals the axial points for the different colors do *not* coincide, because the optic axial angles to which they correspond are not equal. As a result, the color phenomena are more complex; but this in a different degree according as the crystallographic directions of the greatest, the least, and the intermediate light velocity for the different colors coincide or not. With those biaxial crystals in which these three directions are different for different colors, the same is accordingly true also of the axial planes. Therefore a plate cut from such a crystal can stand perpendicular to the acute bisectrix only for *one* color, and consequently when only *this* is used the lemniscate system will have for its center the exact center of the field; but if we illuminate with a different color, the rings will not only have a different width and their centers be a different distance apart, but the whole figure will be shifted in the field. In white light, therefore, color curves will arise that are, to be sure, similar to the lemniscates when the divergence of the bisectrices for different colors amounts at the most to a few degrees, — as is usually the case, — but in which the color sequence is in detail unsymmetrical, so that the right side is not symmetrical with the left, nor the upper side with the lower.

Turning our attention first of all to the simplest case, in which the three principal vibration directions, and consequently also the optic axial plane, have for all colors the same crystallographic orientation, the interference-figure for each single color must be symmetrically bisected both by the straight line connecting the two axial points and by the straight line intersecting this first line perpendicularly at the center of the field; *i.e.* by the two black bars that appear when the axial plane is

parallel to the principal section of a nicol. Since these lines of symmetry coincide for all colors, there appears in white light an interference-figure that likewise is symmetrically bisected by those same straight lines; in other words, whose upper half is exactly equal to the lower (in the reverse position) and whose right side is in like manner equal to the left. (See Fig. 3 of Plate II.) If in this figure we compare the different parts of a lemniscate, *e.g.* of the first color ring surrounding one of the optic axes, we see that they are not colored alike: that the side (also of the succeeding rings) turned toward the center of the field has a different color sequence from that turned outward, while the upper half is exactly equal and opposite to the lower. This is explained simply by the circumstance that the centers of the ring systems for the different colors do not coincide. Let, for example, *AA* (Fig. 85) be the direction of the axial plane (for all colors), *BB* that of the principal section standing perpendicular to it; and let *r*,*r* represent the two axial points for red, *b*,*b* those for blue, the distance *rb* being of course the same on both sides, because the bisectrices for the two colors coincide at the center of the figure. Then may the curves drawn full be the dark lemniscates for red, the dotted curves those for blue. Hence, as the figure shows, if we pass out from the center of one of the two ring systems, when we move toward the center of the figure blue first is annihilated, red only at a greater distance; but when we advance outward the extinction takes place in the reverse order. Thus, in white light and with such a crystal, the color sequence of the inner-

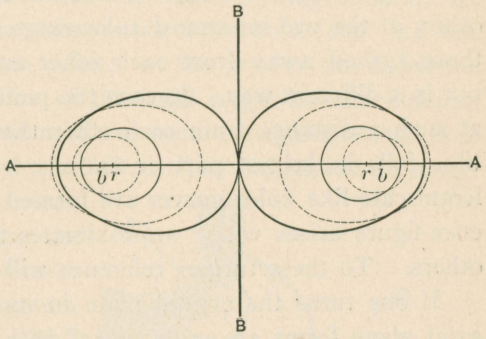

Fig. 85.

most ring must in these two directions be exactly the opposite; if the distance between (dispersion of) the optic axes, *rb*, is not so great, this color sequence in the two directions must at least be different. Figure 85 shows further that the upper half of the color rings must be exactly equal and opposite to the lower, because the shifting takes place exactly along the straight line *AA*; and that the right-hand ring system must likewise be equal and opposite to the left, because the opposite shifting of the rings on the two sides of the line *BB* must always proceed in the same way, so that this line exactly bisects the systems for all colors. The phenomena must accordingly appear as represented by Fig. 3, Plate II, in which the sides turned toward each other, of the two innermost color rings, are colored alike, while those turned away from each other are likewise colored alike but in a different way. In case the points *r* and *b* (Fig. 85) are at such a distance from each other that the first dark ring for blue falls in largest part or entirely outside that for red, no lemniscate-like color curves are formed at all, but an interference-figure arises which approximates to that of brookite and others. To these further reference will be made on page 172.

If one turns the crystal plate in its own plane so that the axial plane forms an angle of 45° with the nicols, then, as we know, there appear the black hyperbolic brushes passing through the axial points. As in Fig. 85, so also in Fig. 86, which corresponds to this position, let *AA* be the axial plane, *r*,*r* and *b*,*b* the axial points for red and blue, the full and the dotted curves the dark lemniscates for these colors; then it is seen that the color rings arising in white light must be wholly the same as arose in the former position of the plate. But if we consider the dark hyperbolas, we see that the curves for the different colors cannot coincide, since the two arising for each color must pass through the axial points. In the figure the two hyperbolas appearing in red light are hatched vertically, those for blue horizontally; and therefrom it is manifest that the curves for the different colors (since for the remaining colors they lie between

those mentioned) partially overlap one another, overlapping the more, the less the distance $rb$, — *i.e.* the less the optic axial angles for the different colors differ from one another. At the points where the component hyperbolas for all colors fall one upon the other, *i.e.* along the middle of the hyperbolic stripes, total darkness will arise; not so, however, at the two edges, where the extinction takes place only for a part of the colors. Therefore the borders must be edged with color, as may be seen from Fig. 4 of Plate II, while the black bars appearing with the first position of the crystal plate (Fig. 3, ibid.) can exhibit nothing of the kind. The color edging of the dark hyperbolas is the broader and the more vividly colored, the fewer the number of the component hyperbolas that fall one upon the other at any one point; *i.e.* the greater the **dispersion of the axes** — the variation of the axial angle for the different colors. When the dispersion attains such a magnitude that even along the middle of the hyperbolic stripes the curves no longer overlie one another for all colors, no black appears even there: the hyperbolas consist only of color stripes, in a definite sequence from within outward. This sequence must at the same time show the **sense of the dispersion** of the axes; that is to say, whether the axial angle for the rays at the red end of the spectrum is smaller than for those of the violet part (briefly expressed: $\rho < \upsilon$), or vice versa ($\rho > \upsilon$). In Fig. 86, and in Fig. 4 of Plate II, is represented the interference-figure of a crystal of the former kind. In this figure, on the two hyperbolic curves passing through the points $r,r$, the red light is totally extinguished, and so too are the parts of the spectrum adjacent to the red (or at least

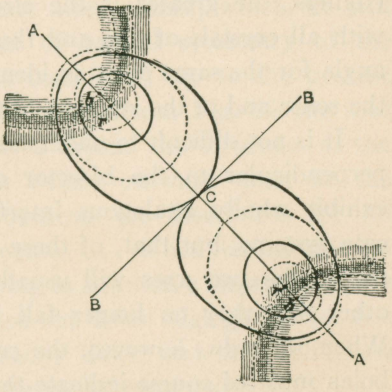


Fig. 86.

much weakened in intensity); but not the colors of the other end, namely blue and violet, which are extinguished only when we reach the points designated by *b*,*b*. Consequently these two colors will appear as an edging of the two hyperbolas on the side turned toward the line *BB*, while on the concave side, turned outward, a red edging will appear, because here the blue rays are totally annihilated. These color edgings always stand out the most distinctly on those parts of the hyperbolas that lie within the innermost color ring, where the hyperbolas are most sharply bounded. If one there observes that the inner side (*i.e.* the side turned toward the center of the field) of the hyperbolas is colored blue, the outer side red (see Fig. 4, Plate II), one has to do with a crystal whose optic axial angle for red is smaller than for blue (sense of the dispersion, $\rho < \upsilon$); when on the other hand the inner side is red and the outer blue, the sense of the dispersion is $\rho > \upsilon$ — the axial angle for red larger than for blue. The broader and the more vivid the color edgings, the greater is the strength of the dispersion. Since with all crystals of one and the same substance the optic axial angle for the same color is identical, this holds true also both of the sense and of the strength of the dispersion.

It is not difficult to understand that a plate whose faces stand perpendicular to the bisector of the obtuse axial angle must exhibit wholly analogous interference phenomena, *i.e.* lemniscate systems, but that, of these systems, the centers corresponding to the two axes will usually stand so far apart from each other that they no longer fall in the field of the instrument. When they do, however, the color edgings of the dark hyperbolas must of course indicate the opposite sense of dispersion to that with the plate perpendicular to the acute bisectrix.

In some substances, as brookite ($TiO_2$), ammonium mellitate, and others, the three principal refractive indices vary with the color so diversely that, for example, the index that for the colors lying at one end of the spectrum is the smallest becomes, for a certain color, equal to the intermediate, and for a vibration period that is

still more different these two indices exchange rôles. In crystals having such strong dispersion the axial plane for one part of the spectrum must be one principal section, and that for another part one of the other two, the latter axial plane thus standing perpendicular to the former; and for a certain color lying intermediate in the spectrum the crystal must be *uniaxial.** When the acute bisectrix coincides for all colors, then, if one brings into convergent polarized light a plate cut perpendicular to this bisectrix, one sees in red light an ordinary interference-figure, and in blue light likewise such a figure but with the axial plane oriented perpendicular to that of the former figure; in white light, on the other hand, an absolutely different color image appears, which is represented in Fig. 5 on Plate II.

In the crystals hitherto considered the three principal sections of the index-surface, or of the ray-surface, for all the different colors coincide; the optical properties, therefore, although they do vary for every color in a different way, always vary symmetrically with respect to these three planes, which we will accordingly designate as **planes of optical symmetry** of the crystal. So the biaxial crystals considered up to the present have *three planes of optical symmetry.* From Fig. 86 one sees directly that in the case of such a crystal the vividness of the colors and their sequence from within outward must be the same with both hyperbolas, because the bisectrices for all colors absolutely coincide at the center *C*. When this latter is not the case, however, the crystal having in addition a **dispersion of the principal vibration directions,** then the color edgings of the two hyperbolas can no longer be exactly alike; and this unlikeness itself presents the most delicate means of recognizing such a dispersion, — *i.e.* the existence of a lower grade of optical symmetry.

The difference in the orientation of the principal vibration

* In consequence of two principal refractive indices being equal for this color the index-surface for it is a rotation ellipsoid. Here, therefore, arises the special case mentioned on p. 126, but only for the light of one definite vibration period, while in the case of really optically uniaxial crystals the index-surface is a spheroid for *all* colors.

directions for different colors may be either complete or partial. That is to say, one of these vibration directions may be for all colors the same, while the other two are dispersed in the plane standing perpendicular to it (this plane is then a common principal optic section for all colors, and the crystal has only this *one plane of optical symmetry*); or else all three principal vibration directions may vary their crystallographic orientation with the vibration period of the light (the crystal then having *no plane of optical symmetry*).* In the former of these two cases, as will be seen from the following, the interference phenomena assume different forms, according as the common principal vibration direction for all colors is that of the intermediate light velocity or of one of the other two.†

When the vibration direction of the light rays having the intermediate refractive index $\beta$ has the same crystallographic orientation for all colors, this holds true also of the plane perpendicular to that direction; *i.e.* of the principal section $XZ$ of the ray-surface (Fig. 64). The optic axes for all colors then lie in one and the same plane, but their bisectors are dispersed. Hence, if from such a crystal a plate is made that exhibits in convergent light the interference-figure of both axes, such a plate, although it may indeed be cut perpendicular to the axial plane for all colors, can be normal to the acute bisectrix only for one single color; *e.g.* for an intermediate color lying in the brightest part of the spectrum. Then does the acute bisectrix for the colors of lesser refrangibility stand inclined to the plane

* The coinciding of *two* principal vibration directions for all colors leads naturally to the coincidence of the *third*, perpendicular to those two; wherefore, besides the cases treated in the last section, only the two mentioned above are possible.

† For illustrating the optical relations that result from this, service may well be made either of the dispersion models supplied by Böhm and Wiedemann of Munich (see Appendix), in which the ray-surface for each of three different colors is represented in a manner similar to that mentioned on p. 135; or of the little glass models with threads stretched through which are made for a small price (five or six marks) by C. Mohn, an employee (*Diener*) in the Mineralogical Institute of Rothstock University in Mecklenburg.

of the plate in one direction, the bisectrix for those of greater refrangibility in the other direction; and since the rays parallel to the several bisectrices must, on emerging, still suffer a refraction, their divergence is, in appearance, still increased. With the majority of crystallized substances this divergence amounts moreover only to one or two degrees, frequently still less, seldom considerably more. If the point in the conoscope at which all the rays converge that traverse the plate perpendicularly, *i.e.* the center of the field, be *C* (Fig. 87), and if the direction of

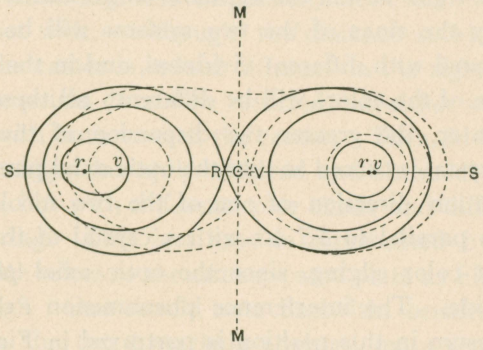


Fig. 87.

these rays may correspond to the acute bisectrix for an intermediate color, then is the bisectrix for red inclined in one direction and that for violet in the other direction, but all three lie in the plane indicated by the straight line *SS*. Hence, if *R* be the point in the field where all the rays converge that pass through the plate parallel to the acute bisectrix for red, and *r*,*r* the axial points for the same color, then the lemniscates drawn full represent those that appear when the instrument is illuminated with light of this color. Similarly, if *V* is the point of convergence of the rays that pass through the crystal parallel to the acute bisectrix for violet, and *v*,*v* the axial points for this same color, whose axial angle is of course different (in the figure $\rho > \upsilon$ is assumed), then are the dotted lemniscates those appearing in homogeneous violet light. If, then, we observe the

interference-figure in *white* light, there follows from Fig. 87 directly that, although this figure must be symmetrical to the line *SS*, and thus identical above and below, yet in no case will it be symmetrical to the line *MM*. While the axial figure of a crystal having parallel principal vibration directions for all colors (cf. Fig. 85, p. 169) is symmetrical right and left also, the latter identity here comes to an end, in consequence of the dispersion of the bisectrices: on the right and on the left the curves overlap one another in different ways, and so the color distribution in the right- and in the left-hand rings cannot be the same; consequently the rings of the two systems will be of different size and colored with different vividness, and in the two systems the sequence of the colors will be different; all these differences are the greater, the greater the dispersion of the bisectrices. When the crystal is turned so that the optic axial plane is parallel to the vibration direction of one of the two nicols, the black bar appears parallel to *SS*, as with a crystal of the first kind; and without color edging, since the optic axial *planes* for all colors coincide. The interference phenomenon exhibited by a plate of gypsum in this position is portrayed in Fig. 6 of Plate II; and here the difference that stands out between the two systems is to be distinctly perceived, especially for the innermost rings. If one rotates the plate 45°, so that the dark hyperbolas appear, the difference stands out still more; for, since on account of the dispersion of the bisectrices the points *r* and *v* (Fig. 87) do not stand at the same distance from the center on the right as on the left, the color edging exhibited by the two hyperbolas on their two sides — and this edging, as we know, is most distinct within the first ring — must, with the two hyperbolas, be of different width and different vividness. If the dispersion of the bisectrices is great and that of the axes likewise great (*i.e.* the axial angle very different for red and for violet), which case is realized in Fig. 87, then on one side of the center the axial point *r* for red lies inward, *v* that for violet outward, and on the other side *r* outward, *v* inward; since, further, the distance

of both *r* and *v* from the center is very different on the right and on the left, the left hyperbola would appear edged inwardly with red, outwardly with blue, and very broadly, while the right would exhibit only narrow color edgings, the inner one blue and the outer red. Were the dispersion of the bisectrices somewhat less, then on one side the axial points for the different colors would almost entirely coincide; in the interference-figure there would then be seen, instead of two reversely colored hyperbolas, one with distinct color edgings and the other without them. With still less dispersion, however, — as is more frequently met with, — although the distance at which *r* and *v* stand from the center is indeed different on the right and on the left, yet these points do not lie reversed, but either with both *v*'s outward or with both toward the center. Then do both hyperbolas appear edged inwardly with red and outwardly with blue, or vice versa, but with different color shade and different vividness of coloring. This last case is represented by Fig. 7 of Plate II, which refers to the same gypsum plate as Fig. 6, after a rotation of 45°. One should note herewith, besides the difference between the two hyperbola edgings, more especially that between the inner side of the first ring on the right and on the left. Since this kind of dispersion of the colors of the axial figure is due to an unequal *inclination* of the principal vibration directions within the same plane, it is called **inclined dispersion** (Des Cloiseaux's " dispersion inclinée ").

When on the other hand the principal vibration direction common to all colors is that of the greatest or of the least light velocity, *i.e.* one of the two mean lines of the optic axes, it is clear that the plane of the latter must now have for different colors a different orientation. For if the bisectors of the obtuse axial angle for all colors have the same direction, those of the acute angle, and consequently also the axial planes of the different colors, are, as it were, rotated about that direction; while if the bisectrix coinciding for all colors is the acute, the bisectors of the obtuse angle, and therefore also the axial planes, diverge about the acute bisectrix.

If from a crystal corresponding to the first of the two cases just mentioned we cut a plate perpendicular to the acute bisectrix for an intermediate color, the optic axial planes for less refrangible light rays are oblique to the plate in one direction, those for more refrangible in the other direction. But the bisectrices for all colors lie in a plane perpendicular to the plate; namely, in that principal section that stands perpendicular to the common principal vibration direction (the obtuse bisectrix). Let this plane (the plane of optical symmetry) be indicated in Fig. 88 by the straight line *SS*, and let *C* again, as in the last

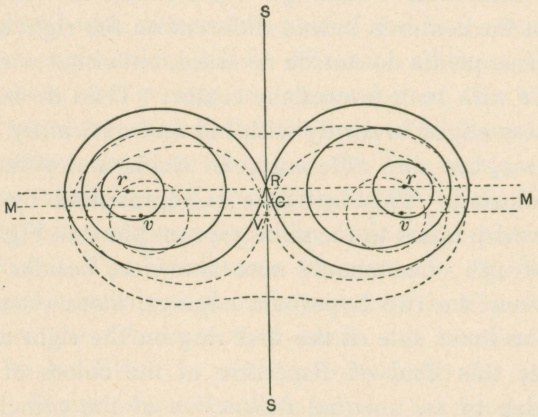


Fig. 88.

figure, be the center of the field in the conoscope, *i.e.* the point of convergence of all rays that pass through the crystal plate perpendicularly; then the direction of these rays is identical with the bisectrix only for an intermediate wave length, while for other colors the rays lie inclined in part upward, in part downward, in the plane *SS*. Hence, if *R* be the point where all rays converge that in the crystal move parallel to the bisectrix for red, and if *r*,*r* be the axial points for this color, then the line *rr* connecting these two points must be both normal to *SS* and bisected by it, since *MM* is the obtuse bisectrix. For violet the point at which the rays parallel to the bisectrix converge must lie on the

other side of $C$, — about at $V$; and the axial points $v,v$ likewise must lie symmetrically on the right and left of $SS$, but at another distance, since the axial angle for this color is other than for red. From Fig. 88, then, it is easy to understand how the interference-figure must look in white light; it will be absolutely symmetrical with reference to the line $SS$, because the right and left sides will have entirely equal and opposite color distribution. Not so the upper and lower halves, since the rings for the different colors do not lie equally on the two sides of $MM$; at their upper and lower sides the color rings will thus exhibit different colors. When the optic axial planes of the crystal are parallel to the principal section of one of the two crossed nicols of the apparatus, their difference in position for the different colors will manifest itself most distinctly at the dark middle bar, which gives the position of the axial plane and by reason of the variation of the same for the different colors exhibits a colored edging above and below; either blue above (because the black bar for red light lies there) and red below, as in the case represented in Fig. 88, or vice versa. This dispersion, because with it the color distribution varies on the different horizontal lines, has been named **horizontal dispersion** (Des Cloiseaux's "dispersion horizontale"). Figure 8 of Plate II represents the interference-figure of a feldspar (sanidine) crystal in the position where its axial planes are parallel to the principal section of a nicol, in which position therefore the colored edgings are to be seen; Fig. 9 shows the interference-figure of the same plate when its axial plane forms 45° with the nicols. In the latter figure the difference between the color curves on the two sides of the line connecting the axial points can be seen even better than in Fig. 8.

The second of the two cases mentioned on page 177 is that in which the *acute* bisectrix is identical for all colors. A plate cut perpendicular to this direction stands normal at once to the optic axial planes for all colors; but, since these planes are rotated about the common principal vibration direction (*i.e.* the normal to the plate), they intersect the plane of the plate in

different directions. Let *C* (Fig. 89) again be the center of the field, corresponding to the acute bisectrix for all colors; if further *r*,*r* be the axial points for red and the curves drawn full the lemniscates for this color, then does the straight line *rr* indicate the position of the axial plane for red. For another color, as violet, although the acute bisectrix is indeed the same, yet in the principal section perpendicular to it the obtuse

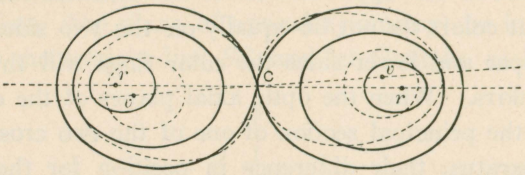


Fig. 89.

takes another position; consequently the axial plane for this color is different, being rotated about *C* by a certain angle. Let its position be represented by the straight line *vv*; and for the same color let *v*,*v* be the two axial points, the dotted curves the lemniscates. The axial planes for the intermediate colors naturally lie between *rr* and *vv*. Hence it follows that only in monochromatic light can the axial figure exhibit symmetry with reference to two mutually perpendicular straight lines; in white light, on the other hand, with reference to none: rather is the color distribution different on the right and on the left, likewise above and below; but it must be identical on the right below and on the left above, as well as on the right above and on the left below. In other words, the interference-figure must be the same in all respects when it is rotated 180° about the normal to the plate at *C*. Thus it exhibits symmetry only with reference to the center, which may therefore be designated as the " center of symmetry " of the figure. Since the axial planes for the different colors are all rotated about *C*, the black middle bar appearing in the case of parallel position with a nicol must in this case too be edged with color, but differently on the right and on the left; so either red on the right above and on the

left below, and blue on the left above and on the right below, or vice versa. It is by this cross-wise agreement of the coloring of the middle bar, and of the inner rings, that this dispersion, which is known as **crossed dispersion** (Des Cloiseaux's " dispersion croisée ou tournante ") is most easily recognized. The interference phenomenon produced by a plate of borax in this position is shown in Fig. 10 of Plate II, while Fig. 11 represents the one exhibited after the plate has been rotated 45°.

Were the optic axial angle of a crystal so little different from 90°, and its refractive indices so small, that the axial figures could be seen not only through a plate perpendicular to the acute, but also through one perpendicular to the obtuse bisectrix, then it is clear that one of these plates would exhibit the phenomenon of horizontal, the other that of crossed dispersion, — always supposing their axial planes to stand perpendicular to the common principal section for all colors. In reality, therefore, the two last-mentioned kinds of dispersion are in such crystal always combined, except that usually only one of them can be observed, because of the obtuse axial angle's being too large.

The crystals considered on pages 174–181 may accordingly be designated as those whose optical phenomena exhibit symmetry with respect to a plane and to a " binary axis " normal to that plane, — a " binary axis " being a line such that the crystal, when rotated about it, exhibits identical behavior in two positions differing from each other by a rotation of 180°.

There yet remains, finally, only that case to consider in which all three principal vibration directions for the different colors of the spectrum have a different orientation in the crystal. Here, therefore, when we pass over from one color to another, there is a change not only in the form of the index-surface, — this being defined by the ratios among the three principal refractive indices, — but also in the directions of its three mutually perpendicular axes. If from such a crystal a plate is cut perpendicular to the acute bisectrix for an intermediate color, as yellow, then for every other color its plane is oblique not only to

the bisector of the optic axes, but also to their plane. In Fig. 90 therefore, if $C$ is again the center of the field, and if $y,y$ are the points where the rays corresponding to the two optic axes for yellow converge in the field of view, the plane of the optic axes for yellow thus intersecting that of the plate in the line $yy$, then the points and lines that correspond respectively to the bisectrices and axial planes for the remaining colors are all other than these. For example, let $R$ be the point in the field where those rays converge that passed through the crystal parallel to the acute bisectrix for red, and let $r,r$ be the two axial points for this color, while $V$, $v$, and $v$ represent the analogous

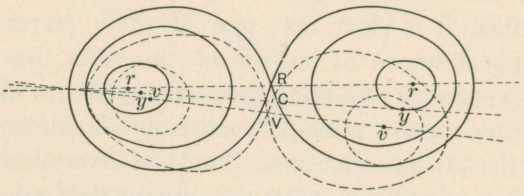


Fig. 90.

points for violet. From the lemniscates drawn here, for these colors, in the same way as in the preceding figures, it is at once understood that the arising interference-figure must have a color distribution that is wholly unsymmetrical; because neither right and left nor above and below can there be any identity, while just as little can the figure be the same after a half rotation about its center. As an interference-figure corresponding to this relative position of the principal vibration directions, that of copper sulphate, a bluish-colored salt, is shown in Fig's 12 and 13 on Plate II.

In case the angles between the optic axial planes for different colors are greater than with this salt, then, in white light, color curves of lemniscate-like shape are formed no longer, but an interference-figure arises similar to the one represented in Fig. 5, Plate II; differing from that figure, however, by the circumstance that the color distribution varies in all directions.

The dispersion phenomena described in the foregoing, which offer such a sensitive means of recognizing an unlike orientation of the principal vibration directions in a crystal, are of special practical importance for this reason: the coincidence or divergence of these directions for the different colors stands in a regular relation to the symmetry of the form of the crystals in question; so that from the interference-figure observed in a particular case, or from the manner of the color distribution in it, a conclusion may be drawn as to the symmetry of the crystal investigated.

### DETERMINATION OF THE OPTIC AXES IN BIAXIAL CRYSTALS AND MEASUREMENT OF THEIR ANGLE

Now the phenomena considered in the foregoing section, the interference-figures of biaxial crystals, to be observed in the conoscope, present the means of finding the position of the two optic axes and thereby of ascertaining the crystallographic orientation of the principal vibration directions, which, according to page 151, is requisite for the determination of the optical constants of the crystal. For, if a plane face of the crystal gives egress to rays transmitted parallel to an optic axis, *i.e.* if in the field of the conoscope we see the ring system intersected by one dark bar, which surrounds the axial point, we are able, by methods to be discussed farther on, to measure the angle the optic axis in question forms with the normal to this face. If in addition the same rays are caused to emerge through another crystal face, whose angle with the former one is known, and if we here too determine their inclination to the normal to the face of emergence, then from these data the crystallographic orientation of the optic axis may be calculated. When the same procedure is feasible also for the second optic axis, the orientation is thereby given of the three principal vibration directions —of the acute and the obtuse bisectrix and the normal to the optic axial plane. On account of the dispersion of these directions the measurements in question must of course

be carried out specially for each color. The problem assumes its simplest form when, as occurs with many biaxial crystals having certain kinds of symmetry, one natural face of the crystal exhibits the two axial figures symmetrically in the field of the conoscope. For then the normal to the plate is the acute bisectrix, and the line parallel to the plate and connecting the two axial points the obtuse bisectrix, while the normal common to these two directions is the vibration direction of intermediate light velocity.

Supposing that in this way the crystallographic orientation of the three principal vibration directions has been found, the three principal refractive indices can now be determined by one of the methods given on pages 144–150. From these indices one may calculate the optic axial angle, according to the formula given on page 143; the same can also be determined directly, however, by methods that, on the exit from the crystal of the rays corresponding to the optic axes, prevent these rays suffering a deviation. This would of course also be the case if the plane at which emerge the rays that correspond to the first axis were exactly perpendicular to that axis, and the face of emergence of the rays passing through parallel to the second axis, exactly perpendicular to this axis. But without knowing the axial angle, the determination of which is primarily the object of the method, such plane surfaces cannot be produced. On the other hand, the rays in question emerge wholly undeviated, from the crystal, if this is ground down to the form of a sphere, or of a cylinder whose axis is perpendicular to the optic axial plane. Those rays, $AA$ and $A'A'$ (Fig. 91), that pass exactly through the center of the sphere or cylinder, and parallel to the two axes, always strike the surface at a point where the surface is perpendicular to the axis, and consequently are not refracted. If, therefore, the sphere or cylinder were rotatable about an axis passing through its center and perpendicular to the plane of Fig. 91, then by adjusting the axial figure of $A$ and that of $A'$ in a fixed polarization apparatus we

could measure the angle $ACA'$, in virtue of the rotation requisite for these adjustments. But, owing to the difficulty of preparing so perfect a sphere or such a cylinder, it is preferable to determine the axial angle in some other way; *e.g.* by immersing the crystal in a liquid whose refractive index is approximately equal to $\beta$ of the crystal; for then, at the boundary of crystal and liquid, no deviation of the light rays occurs, wherefore the crystal may be of any shape. On this principle is based the rotation apparatus constructed by C. Klein,* by means of which, even when the refringency of a crystallized substance is known only approximately, one is able with a crystal, or a fragment of one, of any shape to make an approximate determination of the orientation of the two optic axes.

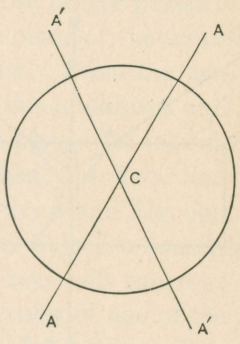

Fig. 91.

For exact determination of the optic axial angle, service is made of a plane-parallel plate cut perpendicular to the bisector of the acute axial angle. When it is a matter of a crystal without dispersion of the principal vibration directions, the plate mentioned is perpendicular to the acute bisectrix for all colors; otherwise, only for a definite one. Yet, since the dispersion of the bisectrices is in most cases but slight, a plate cut normal to the bisectrix for yellow commonly stands very nearly perpendicular to that for the remaining colors as well, and may therefore, unless the greatest accuracy is required, serve also to determine the axial angle for red, blue, etc.

In Fig. 92 (next page) let $PPP'P'$ be the intersection of such a plate with the plane of the optic axes, and let the normal to the plate, $MM'$, be the bisector of their acute angle; then the ray sys-

* Described and illustrated by figures in Phys. Kryst. 4th ed. 782–783; 3rd ed. 749–750. See also, *Sitzungsber. d. Akad. d. Wissensch. Berlin*, 1895, 91; Leiss: Die op. Instrum., 232.

tems that are parallel to the two axes will strike the surface at equal but opposite angles and therefore suffer an equal refraction in the opposite direction. While these ray systems form within the crystal the angle $ACA'$, the *true* optic axial angle, they include after their exit a greater one, the so-called **apparent axial angle,** $BDB'$; and this angle, precisely as $\angle ACA'$, is bisected by $MN'$. The apparent axial angle may be measured in the following way: —

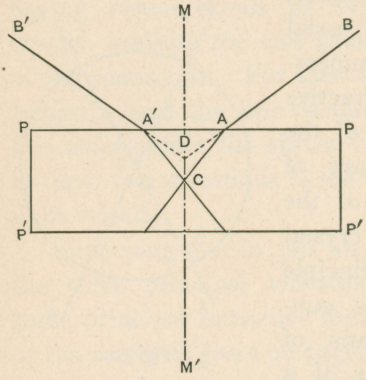

Fig. 92.

One inserts the plate (let $PPP'P'$, Fig. 93, be its section, as above) between condensing lens and objective of the conoscope in such a way that it is rotatable about an axis passing

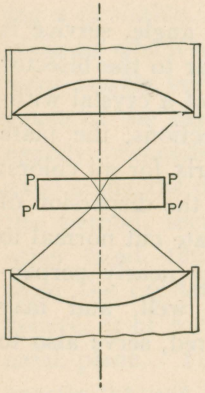

Fig. 93.

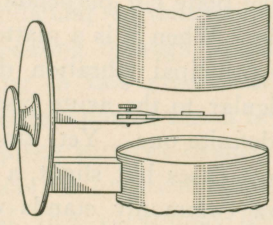
Fig. 94.

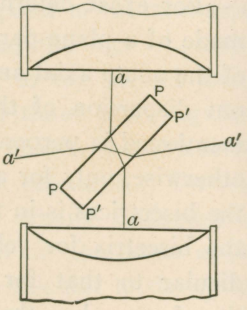

Fig. 95.

exactly perpendicular to the optic axial plane and approximately through the center of the plate. In Fig. 93 are given, in vertical section, the near-by parts only, of the instrument; and the plate is in the position where it exhibits the interference-figure symmetrical, since the acute bisectrix coincides with the

axis of the instrument. Now the above-mentioned rotation is realized by fastening rigidly to the instrument, above the plane of the figure, a graduated circle through whose center passes a rotatable axis; this axis stands normal to the plane of the figure and, terminating in a pincette, carries the plate. From the perspective view in Fig. 94 the arrangement of this apparatus and the possibility of measuring a rotation of the plate with its aid, will be understood directly. Supposing, then, the axis, and thus the crystal plate, to be rotated until the rays (*aa* in Fig. 95), that traversed it along an optic axis enter the objective exactly parallel to the axis of the instrument, these rays will converge at the center of the field; consequently the center of one of the ring systems will appear exactly at the center of the field. This point may be marked by a pair of cross-hairs in the focal plane of the conoscope; and the adjustment of the axial figure on the intersection of the cross-hairs is accomplished with special accuracy if we employ the interference-figure with the hyperbolas, —*i.e.* if we cause the crossed nicols of the instrument to form 45° with the axial plane of the plate. Since always the adjustment of an optic axis can be made only for one definite color, the apparatus must of course be illuminated with homogeneous light, — a sodium flame, for example. The interference figure then appears as portrayed in Fig. 96, where *CC* and *C'C'* represent the cross-hairs of the telescope, *NN* and *N'N'* the vibration directions of the two nicols. When the plate has been rotated until the middle of the dark hyperbola and the vertical cross-hair absolutely coincide, as shown in the figure, it has exactly the position indicated in Fig. 95. So if we rotate back to the original position and just as far in the other direction, until the

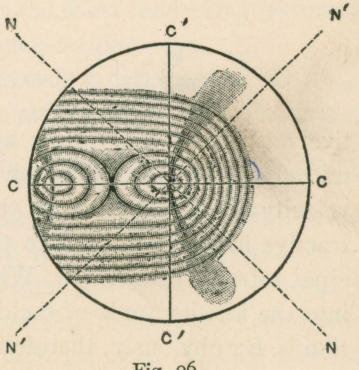

Fig. 96.

second axial figure is in the middle of the field in exactly the same way, *i.e.* until $a'a'$ of Fig. 95 coincides with the axis of the instrument, we must obviously, between these two adjustments of the one and the other optic axis respectively, on the center, have had to rotate to the amount of the axial angle after exit into air. Therefore the rotation to be read off on the circle gives directly the apparent axial angle for the color used. If the instrument is illuminated with light of another color, we obtain, because of the dispersion of the axes, other readings for both adjustments and thus a larger or a smaller axial angle.

There has now to be determined the relation in which the apparent axial angle stands to the true. If $PPP'P'$ (Fig. 97) again be the intersection of the crystal plate with the axial plane, $MM'$ the acute bisectrix and at the same time the normal to the plate, and $AA'$ an optic axis, then obviously $V_a = A'AM'$ is half the true axial angle and $E = BAM$ is half the apparent, so that the true (interior) and the apparent (exterior) angles are respectively $2V_a$ and $2E$. Now according to page 155 a ray transmitted along the direction $AA'$ has the intermediate light velocity; it is therefore at the point $A$ so refracted that its refractive index from air into the crystal is the intermediate principal refractive index $\beta$. When the ray $AA'$ emerges, refracted, into the air, its angle of incidence is $V_a$ and its angle of refraction is $E$; obviously, therefore,

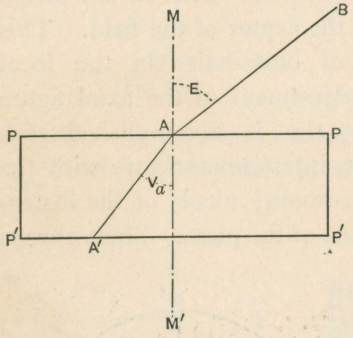

Fig. 97.

$$\frac{\sin E}{\sin V_a} = \beta, \text{ and consequently } \sin E = \beta \cdot \sin V_a.$$

By means of this equation we determine the ratio between the true axial angle and the apparent axial angle in air. If, therefore,

one has measured all three principal refractive indices for a definite color and from them deduced the true axial angle for the same color, the apparent follows from the above equation. Hence, if the apparent angle is determined directly, in the manner described, a comparison of it with the values calculated from the refractive indices, only, supplies a standard for judging of the accuracy with which the latter have been determined; and this so much the more, as in most cases — namely, when for the preparation of prisms and plates one has only small crystals at one's disposal — the measurement of the axial angle is more accurate than of the refractive indices. But still more important is the determination of the apparent axial angle in those cases where the development of the crystal permits only in *one direction* the preparation of prisms large enough for accurate determination of the refractive indices, in consequence whereof only two, at the most, of the principal refraction quotients can be determined. Provided these two are not $\alpha$ and $\gamma$, but either $\alpha$ and $\beta$ or $\beta$ and $\gamma$, then with the aid of $\beta$ we may from the apparent axial angle calculate the true; and from this angle and the two measured refractive indices the third index may be found by solving for it in the equation given on page 143.

If the angle of the optic axes exceeds a certain size, the rays parallel to the axes can no longer emerge into air. For when $V_a$ is such that $\sin V_a = \frac{1}{\beta}$, then $\sin E$ is unity and the apparent axial angle therefore 180°; so that, beginning with this value for $V_a$, a value which depends on the intermediate refractive index of light passing from the crystal into air, total reflection of those rays occurs. Were the crystal to be surrounded, instead of with air, with a fluid in which the light velocity differed less from that in the crystal, then at the boundary-surface of the two media the rays corresponding to the optic axes would be less deviated; and if in this fluid the transmission velocity of light were even less than in the crystal itself, these rays would be refracted toward the axis of incidence. That is to say, in

this medium the apparent optic axes would include an angle smaller than the true. Let $PPP'P'$ (Fig. 98) be the crystal plate and $HH$ the boundary-surface, parallel to the plate, between the surrounding medium and the air; then, if $MM'$ be the axis of incidence, a ray $AA'$ parallel to an optic axis will be refracted at $A$. If further we place $\angle\ A'AM'$ equal to $V_a$ (as heretofore, half the axial angle) and $\angle\ MAB$ equal to $H_a$ (since $MM'$ is the bisectrix, this is half the apparent axial angle in the surrounding medium), letting the velocity of light in the crystal be $v_b$, in the surrounding medium $v_h$, and in the air $v$, we have

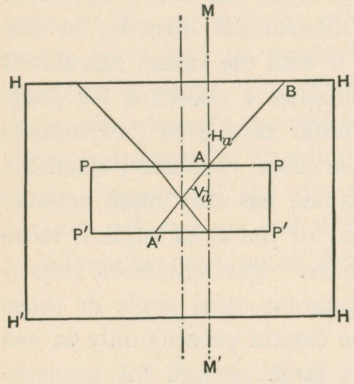

Fig. 98.

$$\frac{\sin V_a}{\sin H_a} = \frac{v_b}{v_h} = \frac{v_b}{v} \cdot \frac{v}{v_h}.$$

From this equation, since $\frac{v}{v_b}$ is the intermediate refractive index $\beta$ and $\frac{v}{v_h}$ the refractive index from air into the enveloping medium, which we shall call $n$, there follows

$$\frac{\sin V_a}{\sin H_a} = \frac{1}{\beta} \cdot n, \quad \text{or} \quad \sin V_a = \frac{n}{\beta} \cdot \sin H_a.^{*} \qquad (1)$$

According to this the true axial angle may be calculated even in a case such that the axes no longer emerge into air, if we surround the crystal, whose intermediate refractive index we know, with a highly refractive medium whose refractive index $n$ for the color used is likewise known and determine the angle $2H_a$

* Since $\sin E = \beta \cdot V_a$, then if in the above equation we substitute for $\sin V_a$ its value $\frac{\sin E}{\beta}$, we obtain $n = \frac{\sin E}{\sin H_a}$. According to this we can, by measuring the apparent axial angle in air and in the fluid above-mentioned, determine the refractive index $n$ of that fluid by means of one and the same crystal plate.

included between the axes in that medium. This latter is done in the following way: —

One surrounds the crystal plate with a vessel $HH'H'H$ (Fig. 99), whose front and whose back wall, $HH$ and $H'H'$, each consist of a plane-parallel glass plate, and fills the vessel up with bromnaphthalin — whose refractive index is known — so that the plate is entirely covered, it being at the same time connected with the apparatus for measuring the axial angle in wholly the same way as though the apparent angle in air were to be determined. If, then, one rotates the plate until those rays, $AB$ (the notation is entirely the same as in the last figure), that in the crystal move along an optic axis are parallel to the axis of the polariscope, they suffer a deviation neither at the boundary of the oil with the enclosing glass plate nor because of the plate itself, since this stands perpendicular to the axis of the instrument; consequently this position is to be found in wholly the same way as in measuring the apparent axial angle in air — by adjusting the dark hyperbola on the intersection of the cross-hairs in the field of the polariscope. Hence, if one rotates back, and in the opposite direction until the second axial figure appears centered in the field in the same way, the entire rotation necessary for this is obviously $2H_a$; i.e. *the apparent axial angle in the bromnaphthalin.* Therefore, $\beta$ and $n$ being known, the *true* axial angle follows from the $H_a$ thus measured, according to the equation

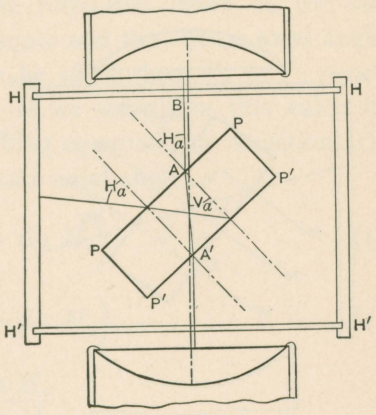

Fig. 99.

$$\sin V_a = \frac{n}{\beta} \cdot \sin H_a,$$

derived on the opposite page.

The same determination, finally, can be made even without knowing $\beta$ and $n$; namely, with the aid of a second crystal plate, whose faces are cut perpendicular to the bisector of the obtuse axial angle, the so-called obtuse bisectrix. From such a plate the axial rays will in general no longer emerge into air; but into bromnaphthalin they will, even when the obtuse axial angle is very large, provided only the refractive index of the liquid is at least as large as of the crystal. In Fig. 100 let there be represented such a plate in the vessel of bromnaphthalin, this plate likewise rotatable about the normal to the optic axial plane; let also $A'A$ be a ray that in the crystal moves parallel to an optic axis, being transmitted in the bromnaphthalin along the direction $AB$. Then, if $MM'$ be the normal to the plate, *i.e.* the obtuse bisectrix, the angle $A'AM'$ is $V_o$, or half the true obtuse axial angle, and $\angle MAB$ is $H_o$, or half the apparent obtuse axial angle in bromnaphthalin. Hence the measurement of this latter angle is made entirely as with the last plate — by rotation, with successive adjustment of the two axial figures. If for the light velocity and the refractive indices we retain the same notation as above with the acute axial angle, then here, in wholly the same way as there, we obtain

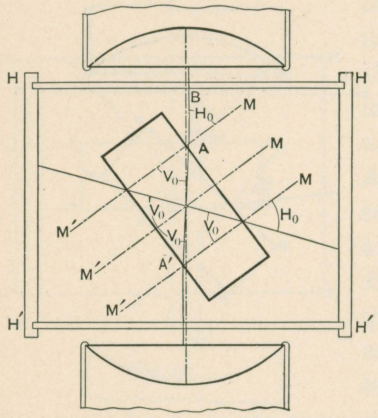

Fig. 100.

$$\frac{\sin V_o}{\sin H_o} = \frac{v_b}{v_h} = \frac{1}{\beta} \cdot n,$$

$$\sin V_o = \frac{n}{\beta} \cdot \sin H_o. \qquad (2)$$

By means of this equation, therefore, when the apparent *obtuse* axial angle in bromnaphthalin has been determined, the true may

be calculated from it, just as, by means of the previously developed equation, from the apparent *acute*. But both calculations presuppose a knowledge of the intermediate refractive index of the crystal and of the refractive index of the oil. Since, however, the sum of the acute and the obtuse axial angle for the same color must always be 180°, then $V_a + V_o = 90°$ and therefore $\sin V_o = \cos V_a$. If we substitute this value in equation (2) and divide the resulting equation into equation (1), developed on page 190 for the acute axial angle,

$$\left(\sin V_a = \frac{n}{\beta} \cdot \sin H_a\right) \qquad (1)$$

$$\left(\cos V_a = \frac{n}{\beta} \cdot \sin H_o\right) \qquad (2)$$

we obtain

$$\text{tang } V_a = \frac{\sin H_a}{\sin H_o}.$$

That is, we can determine the *true* optic axial angle of a crystal without knowing any refractive index; for, if we cut from it two plates, one of them perpendicular to the acute, the other to the obtuse bisectrix, and determine with both plates in the manner described the apparent axial angle in bromnaphthalin, the quotient of the sines of these angles is the tangent of half the interior axial angle sought. This method of determining the angle is of special importance because, for the preparation of prisms with which the refractive indices can be very accurately measured, transparent crystals are required of a size that, by far, is not to be had with all substances; while the plane-parallel plates for this method may be of almost any smallness* and, moreover, can be more easily prepared in sufficient exactness than can correctly oriented prisms. When, therefore, only very small crystals may be had for determination of the refractive indices, one must be content with measuring the true axial angle, by the

* Even for determining the refractive indices with the total-reflectometer the crystals must not be too very small, because otherwise the light reflected from the plate is so faint that the boundaries of total reflection can no longer be discerned.

method described; and, as for the rest, the intermediate refractive index $\beta$ is obtained if by means of the plate perpendicular to the acute bisectrix one determines the apparent axial angle $2E$ in air, according to the equation (see p. 188)

$$\sin E = \beta \cdot \sin V_a, \quad \text{or} \quad \beta = \frac{\sin E}{\sin V_a}.$$

But if $\beta$ is known, then, by measuring (with the aid of the polarization-colors) the birefringence of two plates of any smallness that are parallel to $XY$ and $YZ$ respectively, one can determine in addition the differences $\gamma - \beta$ and $\beta - \alpha$, and thereby $\gamma$ and $\alpha$.*

Instead of a liquid of strong refraction, for measuring the acute and the obtuse apparent axial angle, we may use a highly refractive solid body. On this principle is based the measurement of axial angles by means of W. G. Adams's apparatus.† With this method the plane-parallel crystal plate lies between the plane faces, which stand vis-à-vis, of two lenses of highly refractive glass; the other two lens faces, after the lenses are put together, form a sphere; and this sphere is rotatable separately from the rest of the apparatus. Rays that have passed

* With the aid of the quantities $V$ and $\beta$ we can furthermore determine the complete optical constants of a crystal by measuring the interval between the lemniscates that are exhibited by a plate of known thickness cut perpendicular to the acute bisectrix. For when this distance is known it is possible (see A. Müttrich, *Pogg. Ann. d. Physik*, 1864, **121**, 206 *et seq.*) to calculate the value $\frac{1}{\alpha^2} - \frac{1}{\gamma^2}$; so that, if this value is substituted in the equation

$$\cos V = \sqrt{\frac{\frac{1}{\beta^2} - \frac{1}{\gamma^2}}{\frac{1}{\alpha^2} - \frac{1}{\gamma^2}}},$$

which holds good for the optic axial angle just as does that on p. 143, there remains only one unknown quantity, $\gamma$; and after the latter has been calculated, $\alpha$ follows directly from the same formula.

† Described and illustrated by figures in Phys. Kryst. 4th ed. 753–759; 3rd ed. 723–729. See also the following literature: W. G. Adams, *Proceed. Phys. Soc.* **1**, 152; *Phil. Mag.* 1875, 50, and 1879, [5] **8**, 275; and in *Zeitschr. f. Kryst.* **5**, 381; further, E. Schneider, *Carls Repert. f. Exper.-Physik*, **15**, 774 (1879); F. Becke, *Tscher. min. u. petrogr. Mitteil.* 1879, **2**, 430. And see in addition Leiss: Die op. Instrum., 166 *et seq.*

through the crystal plate lying at the center of the sphere experience, on emerging from the latter, no further deviation; and consequently, by adjusting the two axial figures a measurement is made of the acute or of the obtuse apparent axial angle in the kind of glass in question (instead of in bromnaphthalin).

When the optic axial angles of a biaxial crystal are determined for different colors, whether by complete measurement of the optical constants (of the three principal refractive indices) or by their direct determination, they are found to differ; and moreover, the size of the axial angle rises or falls steadily with the wave length of the light to which the axes refer. Since each of the three principal refractive indices varies with the color approximately after the same law, stated on page 46 as Cauchy's dispersion formula for singly refracting media, except that naturally with each index the constants of this formula have a different value, the natural conjecture is that the axial angles, too, vary with the color according to a similar law. As a matter of fact, the axial angles of those crystals in which the angle increases with the wave length (sense of the dispersion $\rho > \upsilon$) correspond with remarkable approximation to the formula

$$V_a = A - \frac{B}{\lambda^2},$$

and the angles of those whose axial angle diminishes with greater wave length of the light ($\rho < \upsilon$), to the formula

$$V_a = A + \frac{B}{\lambda^2}.$$

When, therefore, one has determined the true axial angle $2V_a$ of a substance for two colors of known wave length, then by substituting the values found, in the proper one of these two equations, one can deduce the constants $A$ and $B$ for the body, and from them the axial angle for every other wave length. It must be remarked, however, that some few substances (*e.g.* gypsum) present exceptions to this rule, having an abnormal dispersion of the optic axes, such that their angle exhibits a maximum or a minimum for a definite intermediate color.

## RECAPITULATION: CLASSIFICATION OF CRYSTALS ACCORDING TO THEIR OPTICAL PROPERTIES

As follows from what has preceded, the relation in which the transmission of light in a crystal stands to the crystallographic direction can in general be represented by a triaxial ellipsoid, wherefore the optical properties of crystals belong with the ellipsoidal properties. (See p. 7.) The most general case is that treated on page 181 *et seq.*, in which the index-surface has for the different colors not only a different form, but also a different crystallographic orientation of its three principal axes. A special case is presented by those crystals (see p. 174 *et seq.*) whose index-surfaces for the different colors have one axis in common; an additional one by those in which all three principal axes of all the ellipsoids corresponding to the different colors are oriented alike in the crystal. Still more special is the case in which two principal axes of the index-surfaces become of equal length, the surfaces passing over into ellipsoids of rotation. And finally the last, most special case lies before us when all three principal axes of these surfaces become equal; *i.e.* when the index-surfaces for all colors assume the same (spherical) form.

Thus, for the totality of crystals, there results a division according to their optical properties into the following five groups:—

1. *Biaxial crystals with no plane of optical symmetry*,—in which crystals no two straight lines of different orientation are optically equivalent, two different directions being equivalent only when they are opposite, lying therefore in the same straight line. (That two such directions are equivalent is because the optical properties belong with the bi-vector —see p. 14.)

2. *Biaxial crystals having one plane of optical symmetry.* In these crystals there exists for every direction a direction that

in optical respects is equivalent to it, the latter direction lying symmetrical with the former, with reference to that plane of symmetry; here, therefore, counting the two directions opposite to those mentioned, there are always four optically equivalent directions, and these lie in two straight lines whose angle is bisected by the plane of symmetry.

3. *Biaxial crystals having three mutually perpendicular planes of optical symmetry.* To a direction of any orientation there here belong, as equivalent, yet seven others; these lie in four straight lines, which always have, pairwise, equal and opposite inclination to the three planes of symmetry.

4. *Uniaxial crystals.* These have an infinity of planes of optical symmetry, which all intersect in the axis. Crystals of this kind exhibit the same behavior in all directions (infinite in number) that include the same angle with the axis.

5. *Singly refracting crystals.* Here all directions (an infinity therefore in a higher sense than in the last case) are optically equivalent.*

Each of the first four groups embraces crystals with positive double refraction and crystals with negative double refraction; yet this difference among crystals is not essential theoretically, since for different colors the same crystal can have double refraction of opposite character, nor is the distinction requisite for the systematic classification. Practically, however, for the determination of crystallized bodies by their optical properties, it is important to tell the positively and the negatively doubly refracting apart. The methods adapted for this purpose are elucidated in the following section.

* [Group 1 comprises the triclinic crystal system; 2, the monoclinic; 3, the orthorhombic; 4, the hexagonal, the trigonal (Dana's trigonal division of the hexagonal system), and the tetragonal; 5, the cubic, or isometric.]

## COMBINATIONS OF DOUBLY REFRACTING CRYSTALS

### DETERMINATION OF THE CHARACTER OF THE DOUBLE REFRACTION OF UNIAXIAL AND BIAXIAL CRYSTALS BY COMBINATION WITH OTHER DOUBLY REFRACTING CRYSTALS

As was shown on page 63 *et seq.*, it is possible to determine the birefringence of a thin crystal plate, *i.e.* the difference of path of the two light rays transmitted in such a plate, by means of the arising interference-color. This determination is, to be sure, only approximate, since the color is further influenced by the diversity of the double refraction for different colors; there is found only a mean value, as it were, for the retardation that has taken place in the crystal. From this retardation and the plate thickness we can find the difference between the refractive indices for the two interfering rays by the formula given in the footnote on page 63. But herewith it remains undetermined which of the two rays has the greater, which the lesser velocity; in other words, what is the character of the double refraction of the crystal — whether positive or negative. To determine this it is necessary to combine the plate with a second crystal plate, the positive or negative character of whose double refraction is known. When it is a matter of very thin crystal sections, such as usually form the subject of microscopical investigations and which, being so thin, exhibit colors only of the lower orders, the most suitable plate for determining the sign of the double refraction is a thin plate of gypsum, a plate that between crossed nicols exhibits the red of the first order (see p. 66); in cases of very low birefringence we may use instead a so-called **quarter-undulation mica plate**, — *i.e.* a cleavage-lamella of mica so thin that the two vibrations arising from a ray entering it at right angles are retarded, the one as compared with the other, only $\frac{1}{4}\lambda$.* Mica is negatively biaxial, that is to say, the vibration

* Strictly speaking, this holds true only for one definite color; but, for the

direction of its greatest light velocity bisects the acute angle of the axes; and the so extremely perfect cleavage of mica is almost exactly perpendicular to that vibration direction. If the sheet of mica is cut into a rectangular form so that the long direction corresponds to the optic axial plane, its longer edges are then parallel to the vibration direction of the least light velocity, the shorter to that of the intermediate; and these directions are at the same time the two vibration directions of rays emerging perpendicular to the plate (hence parallel to the acute bisectrix).

When such a mica plate is so introduced into the path of the light rays, before they enter the analyzer, that its long direction bisects the angle formed by the vibration directions of the two crossed nicols (for which purpose the microscope must be fitted with a suitable slit), the field is lighted up with a blue-gray color, corresponding to the $\frac{1}{4}\lambda$ for medium colors,—to a path difference, therefore, amounting to about 140 $\mu\mu$. (Cf. p. 64.) Now supposing there is present in the field of view a very thin crystal of extremely low birefringence, in the position where its vibration directions coincide with those of the nicols, it can produce no change and therefore appears lighted up just like the rest of the field. But if one rotates the crystal 45° in its own plane, there takes place in it a resolution of the light into two vibrations of very small difference of path. If the sense of the rotation was such that the direction of the vibration that in the crystal is transmitted with the greater velocity coincides with the vibration direction of the ray that has the greater velocity in the mica plate (in which case therefore the two vibration directions of the rays transmitted the slower in the two crystals, respectively, also become parallel), then to the path difference arising in the crystal there will be added that ($\frac{1}{4}\lambda$) arising in the mica; con-

methods treated in the following, it is sufficient if the retardation amounts to $\frac{1}{4}\lambda$ of a medium color, since for the remaining colors it is then but little different from $\frac{1}{4}\lambda$. "Achromatic" retardation plates, which yield exactly $\frac{1}{4}\lambda$ difference of path for all colors, can be produced by combining different plates. (See *Zeitschr. f. Kryst.* 1903, **37**, 292.)

sequently the blue-gray color passes over into a brighter, greenish gray, or, when the crystal is somewhat more birefringent, as far as into yellowish white. Even when the double refraction of the crystal is extremely slight, it can be perceived by this method, owing to the greater brightness on rotation. If, however, one rotates the crystal 45° in the opposite sense, then does the vibration direction of the ray transmitted the faster in the crystal coincide with the direction of the vibration that in the mica advances the slower, and vice versa. The total difference of path becomes, in consequence, just as much smaller as it previously became larger; so the order of the interference-color diminishes; that is, the crystal becomes darker, and lavender-gray. But, through observation of the crystal to be investigated, in both positions, the problem before us is solved; for in the crystal THAT VIBRATION IS ALWAYS TRANSMITTED THE SLOWER THAT IS PARALLEL TO THE OPTIC AXIAL PLANE OF THE MICA WHEN THE CRYSTAL APPEARS BRIGHTER THAN THE FIELD. By this procedure, therefore, one is able not only to recognize double refraction in a crystal, even when it is so slight that without addition of the mica plate the brightening on rotation between crossed nicols would escape observation, but also to determine the character of the double refraction.

The same is the case when the crystal is combined with a **gypsum plate** exhibiting with crossed nicols the *red of the first order*. If we insert this plate into the orthoscope in wholly the same way as we inserted the mica, the whole field appears in the color named; and so likewise do any doubly refracting crystals present in it, provided their vibration directions coincide with those of the nicols. But if we rotate the stage of the instrument until the vibration directions of a crystal to be investigated include 45° with those of the nicols, then in the crystal there arises a difference of path, which is added to or subtracted from that effected in the gypsum plate, according as the two vibration directions of the greater of the two light velocities in crystal and gypsum plate respectively are parallel or crossed. Conse-

quently the crystal, as compared with the rest of the field, exhibits a color of higher or of lower order, according as it has been rotated 45° in the one or in the opposite sense. When, therefore, by comparison with a crystal of known double refraction, one has ascertained for the rectangular gypsum plate whether the vibration parallel to its long direction is that transmitted the faster or the slower, it is obvious that through this change of color one can determine the unknown character of the double refraction of every crystal present in the field of the orthoscope, —unless the path difference produced by the crystal is so considerable that doubt can arise as to the order of the resulting color shade. Supposing this to be the case, the crystal when in diagonal position producing, of itself alone, a color of the third or of the fourth order, then by inserting the gypsum plate parallel to the one diagonal this color is transformed into a color whose order is higher by the path difference $\lambda$, *i.e.* by one whole order; while if we insert the gypsum plate along the other diagonal, or rotate the crystal 90°, the order of the color is just so much lowered. Since interference-colors differing by two whole orders can always be clearly distinguished, owing to the circumstance that the higher is less vivid than the lower and more nearly approximate to white, then with the aid of the sensitive gypsum plate the character of the double refraction can, in such cases also, be established beyond question.

Finally, if in consequence of considerable thickness or very high birefringence a crystal plate exhibits the white of a higher order, then, to produce a color that differs perceptibly from this white, the path difference must be diminished more than one wave length. For this purpose it is best to use a doubly refracting crystal ground to the form of a wedge; *e.g.* a **quartz wedge** one of whose faces is parallel to the optic axis, *AA*. (See Fig. 101, p. 202: *a*, front view; *b*, longitudinal section.) The wedge, in order to render the thinnest part less liable to break, is commonly cemented on a rectangular glass plate, *g*. Since the double

refraction of quartz is positive, then in such a wedge a ray falling perpendicularly at any point is split up into two rays of which the ordinary, vibrating parallel to $BB$, is transmitted the faster; the extraordinary, vibrating parallel to the axis $AA$, the slower. Now the difference of path with which the two rays emerge from the quartz obviously increases with the thickness of the latter; consequently, by shifting the wedge from right to left we can increase this difference of path, since a thicker part of the wedge then comes into play. Hence, if one places a

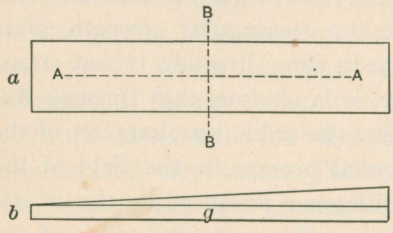


Fig. 101.

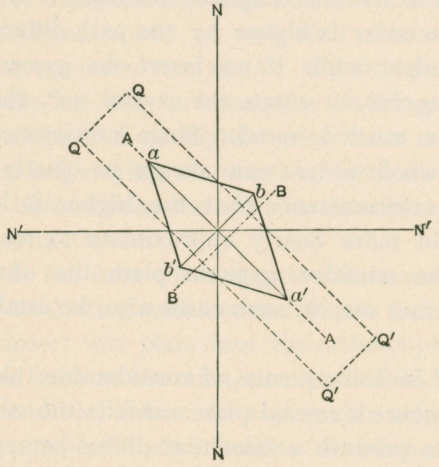


Fig. 102.

doubly refracting crystal plate, $aba'b'$ (Fig. 102), in the orthoscope so that its vibration directions form 45° with those of the two crossed nicols ($NN$ and $N'N'$), and if, of all the vibration directions lying in the plane $aba'b'$, $aa'$ be that of the greatest light velocity and $bb'$ that of the least, then in the crystal the

vibrations parallel to $aa'$ will be transmitted faster than those parallel to $bb'$; accordingly the two rays, when they emerge, will have a difference of path; — and let this difference, $n\lambda$, be of a magnitude such that the white of a higher order results. If, then, the quartz wedge, $QQQ'Q'$, is so inserted that its optic axis, $AA$, is parallel to the vibration direction $aa'$ of the crystal plate, each of the two rays emerging from the crystal will be transmitted in the quartz with unaltered vibration direction; but the vibration parallel to $aa'$ here advances the slower, the vibration parallel to $bb'$ the faster; consequently the path difference acquired in the quartz — let it be $n'\lambda$ — is of the opposite sense to that arising in the crystal, and after the two rays have passed through both crystal and quartz their path difference is thus $(n - n')\lambda$. The quantity $n - n'$ can be made as small as desired, provided the crystal plate is rather thin and the quartz wedge thick enough, by shifting the latter parallel to $AA$, so that a thicker part comes to the center. With this diminished path difference, then, the two rays enter the analyzer, and after the reduction there to one vibration plane they interfere in a manner corresponding to this path difference. When $n - n'$ is very small, wholly the same comes to pass as though the crystal plate were even extremely thin and no quartz wedge present; that is, vivid interference-colors appear.

On the other hand, should the quartz wedge have been so inserted in the polariscope that $AA$ were parallel to the vibration direction $bb'$, the same vibrations that, as compared with those perpendicular to them, were in the crystal retarded $n\lambda$ would then, in the quartz likewise, experience a relative retardation, of $n'\lambda$; in the end, therefore, they would have a path difference of $(n + n')\lambda$ and interfere correspondingly. So in this case the quartz wedge operates as though the crystal plate had become thicker, and consequently a more nearly perfect white of a higher order appears.

If the vibration direction of the greatest light velocity in the crystal plate were not $aa'$, as we have assumed, but $bb'$, and $aa'$

that of the least, then everything would be reversed; that is, to obtain the interference-colors we should have to so insert the quartz wedge that $AA$ were parallel to $bb'$.

This procedure for determining the character of the double refraction may be employed also in the investigation of a crystal in convergent light. Here, unless the interference-figure of an optic axis happens to appear in the field, a plate of some thickness exhibits no bright and dark curves as in monochromatic light, but merely the white of a higher order. If by rotating the stage of the conoscope the crystal plate is turned so that it exhibits the maximum brightness (its vibration directions then form 45° with those of the crossed nicols), and if between crystal and analyzer the quartz wedge is now inserted once with its long direction parallel to the one vibration direction of the crystal plate, and afterward parallel to the other, — then, of these two vibration directions, the one to which the long direction of the wedge is parallel when hyperbolic color curves appear in the center of the field is, of all vibration directions parallel to the plate, that of the *greatest* light velocity, the vibration direction standing perpendicular to it that of the *least* light velocity.

If we have at our disposal, for the determination of its optical character, a uniaxial crystal plate whose faces stand perpendicular to the optic axis, the determination can be made by laying the plate on the stage of the conoscope, and upon this plate a second, cut perpendicular to the axis from another uniaxial crystal, the sign of whose double refraction is known. When the plate to be investigated has the same optical character as the latter, the same ray (of the two vibrating perpendicularly to each other) that was retarded in the lower plate is retarded in the upper one as well; so this upper plate operates exactly as though the lower had become thicker. In other words, the circular color rings will become narrower than they appeared before the plate of known optical character was superposed. When on the other hand the crystal plate to be in-

vestigated is of opposite character to the known, the latter plate will operate as though the former had become thinner; that is, the color rings will become wider. This widening or narrowing of the color rings can very easily be made perceptible, by placing in the focal plane of the polariscope a glass plate having fine lines scratched upon it; these lines are then seen on the interference-figure, and we thus have a measure for determining the diameter of the color rings.

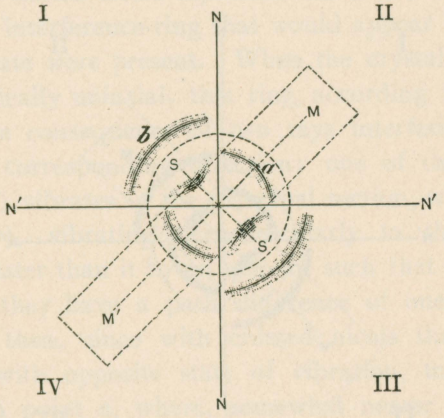


Fig. 103.

A further very convenient method — and, in fact, the one most frequently employed — of determining the optical character of a uniaxial plate cut perpendicular to the axis, consists in the use of the *quarter-undulation mica plate*, described on page 198. If this is introduced into the path of the light rays, between crystal plate and analyzer, then, in homogeneous light, instead of the circular dark rings with the black cross we obtain when the crystal is *positive* the interference-figure shown in Fig. 103, but when the crystal is *negative* the one shown in Fig. 104 (p. 206), — provided that in both cases the mica sheet has the diagonal position indicated by dots; *i.e.* that its long axis forms 45° with the nicols in the two quadrants marked *II* and *IV*. The

former interference-figure differs from the usual one in these particulars: the dark rings in quadrants *II* and *IV* are narrowed to the extent of about quarter the interval between two adjacent rings, while those in quadrants *I* and *III* are just so much widened, the result being that at the boundary of two quadrants a bright ring always abuts against a dark; instead of the black cross, two black spots appear, and the line connecting these spots stands perpendicular to the long direction of the mica. Should the mica sheet have been inserted perpendicular

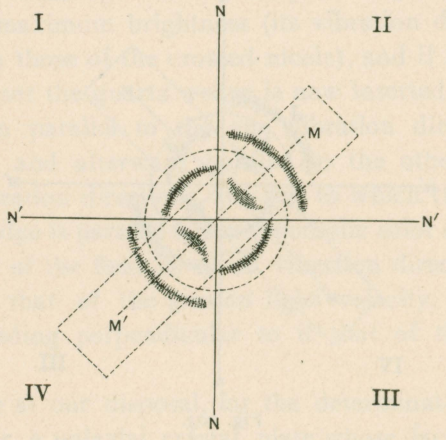


Fig. 104.

to the position indicated in the figure, so that its long direction had fallen in the middle of quadrants *I* and *III*, then would the rings of these quadrants be narrowed, those of *II* and *IV* widened; and in the latter quadrants the black spots would appear. In the case of Fig. 104, representing the interference-figure of a negative crystal, the same widening of the rings, together with the dark spots, lies in those quadrants that are *bisected* by the long axis of the mica, *i.e.* in *II* and *IV*, but the narrowing of the rings occurs in *I* and *III*. Should the mica sheet have been so inserted that it bisected quadrants *I* and *III*, then the widening of the rings, together with the dark spots,

would appear in these quadrants, the narrowing in *II* and *IV*. What takes place for the dark and bright rings in homogeneous light holds true also of the color rings appearing in white light, so that accordingly in the latter case the isochromatic rings are widened or narrowed in the same way; the black spots appear just the same as in homogeneous light. It is sufficient, therefore, to explain the phenomenon for monochromatic light.

As before, let $NN$, $N'N'$ (Fig. 103) be the vibration directions of the nicols, $MM'$ the long direction of the mica sheet; then may the dotted circle represent the locus, in the field, of the first dark interference-ring that would appear if no quarter-undulation plate were present. When the crystal to be investigated is optically uniaxial, this ring, according to a previous page, arises in consequence of two rays interfering with each other in the corresponding direction; one of these rays (the extraordinary) vibrates in the principal section, while the other (the ordinary), vibrating perpendicularly to the former, is transmitted faster than it by an amount such that when the two rays emerge they have a path difference of one whole wave length. For then, since with crossed nicols the interference takes place with opposite state of vibration, total extinction occurs. At a point $a$, where, somewhat nearer the center of the field, the black quarter circle is given, the one ray is retarded only $\frac{3}{4}\lambda$ as compared with the other. But if we suppose the two rays to enter the mica sheet, this retardation is increased an additional $\frac{1}{4}\lambda$; for the extraordinary ray, vibrating in the principal section of the crystal and therefore parallel to the axial plane $MM'$ of the mica, is in this latter, likewise, transmitted slower than the ray vibrating perpendicularly to it, since the direction $MM'$ is the vibration direction of least light velocity in the mica, and $SS'$ that of the intermediate. Thus, after addition of the quarter-undulation plate the two interfering rays have the path difference of one whole wave length at a smaller distance from the center of the field; so the first dark ring, which arises with this retardation, has become correspond-

ingly *narrower*. Let us now consider the point *b*, at which rays converge whose difference of path without the mica plate would be $\frac{5}{4}\lambda$. Here the extraordinary, the more slowly transmitted, ray vibrates parallel to *SS'* (in the principal section); the ordinary, the more rapidly transmitted, parallel to *MM'*. But in the mica the velocity of the latter ray is the lesser, that of the former the greater. Accordingly the path difference, $\frac{5}{4}\lambda$, that arose in the crystal plate is *diminished* $\frac{1}{4}\lambda$, in consequence whereof the first dark ring in quadrant *I* does not appear until, in passing out from the center, we reach the distance of the point *b*: it has become *wider* than before. In wholly the same way, for every succeeding ring in quadrants *II* and *IV* there is joined to the path difference of the crystal plate an additional $\frac{1}{4}\lambda$ on the part of the mica, and these rings all become narrower; while with all the rings in quadrants *I* and *III* the path difference arising in the plate is by the mica diminished $\frac{1}{4}\lambda$, so that the latter rings become wider. In the case of negative crystals the transmission velocity of the rays vibrating parallel to the axis is the greatest, and the extraordinary ray therefore the faster; consequently, with the mica in the same position, exactly the same must occur in quadrants *II* and *IV* as took place with the positive crystals in quadrants *I* and *III*, and vice versa.

In those quadrants in which the path difference acquired in the crystal is by the mica diminished $\frac{1}{4}\lambda$ (in which therefore the rings are widened), the darkness corresponding to the path difference zero must arise at that distance from the center at which the path difference without insertion of the mica plate would be $\frac{1}{4}\lambda$; consequently, in these quadrants dark spots must appear at the distance mentioned. The center of the figure, and likewise the points at which without insertion of the mica the arms of the black cross would appear, must be bright; because there the plate is traversed by only one vibration, and this is resolved in the mica into two equal components with $\frac{1}{4}\lambda$ difference of path. These latter then combine to form a circular vibration (see p. 20), which cannot be extinguished by the analyzer.

The practical procedure for determining the character of the double refraction of a uniaxial crystal plate cut perpendicular to the axis consists, accordingly, in inserting between it and the analyzer a quarter-undulation mica plate of the stated form in such a way that its long direction includes 45° with the arms of the black cross. There then appear, instead of the cross, two black spots; and if the line connecting these spots forms a cross (+) with the long direction of the mica, *i.e.* stands perpendicular to it, the crystal is *positive* (+), while if that connecting line is identical with the long direction (−) the uniaxial crystal is *negative* (−). In the former case the rings are widened in those quadrants through which the long direction of the mica does *not* pass; in the latter case, in those it *bisects*.

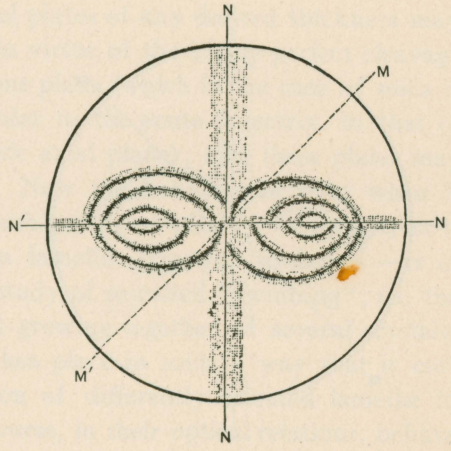

Fig. 105.

The same mica plate may serve also to determine the sign of the double refraction in the case of a *biaxial* crystal plate, if this is cut perpendicular to the acute bisectrix and therefore exhibits in convergent light the two axial figures. For this purpose the crystal plate is so placed in the instrument that its axial plane is parallel to the polarization plane of one of the nicols, and that accordingly the lemniscates are intersected by a black cross. Between crystal plate and analyzer one then inserts the mica plate; and if the crystal to be investigated is *positive*, *i.e.* the acute bisectrix the vibration direction of least light velocity, one now observes the interference-figure represented in Fig. 105, in which the color rings are widened in the

two quadrants through which the mica sheet — whose long direction is indicated by the line $MM'$ — does *not* pass. When on the other hand the crystal is *negative*, *i.e.* its acute bisectrix the vibration direction of greatest light velocity, then, with the same position of crystal plate and of mica, there appears the phenomenon Fig. 106, in which the rings of those two quadrants are widened that the long direction of the mica *bisects*. From this analogy of the phenomena with those of uniaxial crystals it is at once seen that likewise the explanation must be analogous, although with a biaxial crystal the deducing of it is far more complicated. Yet a simple reflection teaches that, by a quarter-undulation mica plate, the interference-figure of a crystal plate cut perpendicular to the *obtuse* bisectrix (to be observed only in a strongly refracting liquid, as a rule) must experience exactly the *opposite* alteration. Therefore the obtuse bisectrix of a positive crystal exhibits the phenomenon Fig. 106; that of a negative, Fig. 105.*

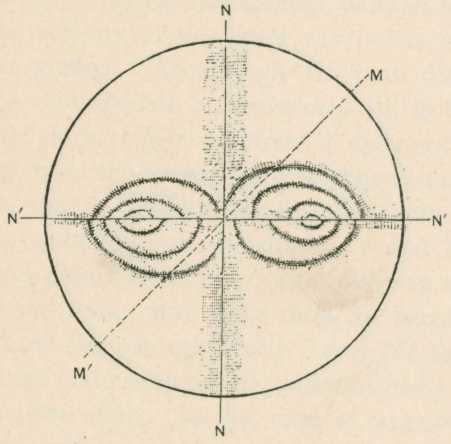

Fig. 106.

REMARK. — It must be remarked that by different optical firms (Steeg and Reuter, Fuess) gypsum lamellæ giving the red of the first order, quarter-undulation mica plates, etc., are now supplied in the reverse orientation, the longer side thus being parallel to the *greater* of the two light velocities. (See Leiss: Die op. Instrum., 210–212.) If desired such lamellæ, plates, etc., having the orientation formerly in common use, and on which page 198 *et seq.* are based, may of course still be obtained.

* For this reason a bisectrix that, when investigated with the quarter-undulation mica plate, exhibits the phenomenon Fig. 105 is pretty generally designated

## OPTICAL BEHAVIOR OF COMBINATIONS OF DOUBLY REFRACTING CRYSTALS OF THE SAME KIND

From the two optically biaxial minerals last mentioned, mica and gypsum, plane-parallel plates of any desired thickness may be very easily prepared, in virtue of the highly perfect cleavage of these minerals along one plane (which in the case of mica is almost exactly perpendicular to the acute bisectrix, in that of gypsum parallel to the optic axial plane); and these plates may be grouped in any way. Now the behavior of light when it passes through such a combination of plates — a packet, as it were, of mica or gypsum lamellæ arranged in layers — is of great importance for the study of so-called "twinning"; *i.e.* the regular (but not parallel) growing-together of several crystals, which not infrequently takes place in such a way that it corresponds to a superposition of differently oriented lamellæ in layers. Such masses, of course, in their optical relations, behave differently from simple crystals. Therefore the most important cases of such combinations of doubly refracting crystals shall now be treated; and of these combinations the greater number are such as may easily be realized by arranging mica or gypsum lamellæ one above the other in layers.

It is in general evident, in the first place, that by superposing two doubly refracting crystal plates in such a way that their vibration directions coincide one obtains a packet having the same vibration directions, — a packet therefore that in parallel polarized light becomes dark in the same positions as do the single plates. What phenomenon such a packet exhibits in

as *positive*, and one exhibiting the phenomenon Fig. 106, as *negative*. Yet this designation is not entirely correct, since, as follows from pp. 137–138, with biaxial crystals the distinction between "positive" and "negative" refers to the greater similarity of the form of the index-surface to that of a uniaxial positive or negative crystal; not, therefore, to a definite direction. If about a bisectrix one observes the phenomenon Fig. 105, and designates that bisectrix as "positive bisectrix" or as "bisectrix with positive double refraction", this signifies, strictly speaking, only that the bisectrix in question is the vibration direction of least light velocity, and therefore that the crystal *would* be positive *if* that bisectrix were the *acute*.

convergent light depends on the optical properties of the two plates and on their thickness. To begin with a simple example, let us suppose the plates to be uniaxial and cut oblique to the optic axis, yet so cut that the axial figure falls within the field of the conoscope. Then, if they are superposed in such a way that their principal sections are parallel to each other and to the principal section of one of the crossed nicols, but their optic axes inclined to the normal of the plate in opposite directions, one sees, equidistant from the center of the field, the black cross and color rings of each, and with rings of equal width if the two plates are of equal thickness; visible between these two axial figures are secondary color stripes, arising by the combined action of the two crystals.* If, instead of uniaxial, one takes for this experiment biaxial plates from which the rays parallel to an optic axis emerge obliquely, then, when the optic axial plane is parallel to the principal section of a nicol, the interference phenomenon in convergent light resembles that of a single plate cut perpendicular to the acute bisectrix; for in the field, symmetrically on the two sides, there appear two axial figures — lemniscates intersected by a dark bar. But this phenomenon, which not infrequently is to be observed in combinations of two crystals of certain substances grown together in nature, differs from the interference-figure of the single plate just referred to, by the presence of the above-mentioned secondary color stripes, — these stripes here taking the place of the second dark bar, which there passes through the center of the field at right angles to the optic axial plane. When two plates having

* These interference phenomena and a series of similar ones, produced by "twin plates" chiefly in homogeneous sodium-light, are contained in the collection of excellent photographs published by H. Hauswaldt ("Interferenzerscheinungen an doppeltbrechenden Krystallplatten in convergenten polarisierten Licht. Photographisch aufgenommen von Hans Hauswaldt in Magdeburg. Mit einem Vorwort von Th. Liebisch in Göttingen." Magdeburg, 1902). A fresh series, appearing in 1904, contains further examples and, in addition, photographs of interference phenomena of biaxial crystals having great dispersion and with the use of different wave lengths; also of anomalous crystals, absorption spectra, *et al.*

their plane perpendicular to the acute bisectrix —and for this purpose, as already mentioned, two cleavage-plates of mica are the most suitable — are so laid, one upon the other, that, while their vibration directions are indeed parallel, their optic axial planes are crossed at right angles, one sees in the field of the conoscope four axial figures, all equidistant from one another and from the center; between them appear hyperbolic color stripes, whose asymptotes form a black cross passing through the center; and when the two axial planes are in diagonal position to the nicols, the arms of this cross pass parallel to their principal sections. If several such pairs of crossed mica plates are arranged one above the other, these secondary interference-curves stand out still more, at the expense of the lemniscates; and with four or five pairs we obtain the interference phenomenon represented in Fig. 107: of the color curves surrounding the four axial points this phenomenon exhibits only remnants, and it closely resembles the interference-figure portrayed in Fig. 5 on Plate II — that of a crystal having crossed axial planes for different colors. The center of this interference-figure always remains dark, even when the combination is rotated in its own plane, provided the single plates are *of exactly equal thickness; otherwise there remains a path difference* even for the rays that have passed through perpendicularly, and on rotation a color arises at the center.

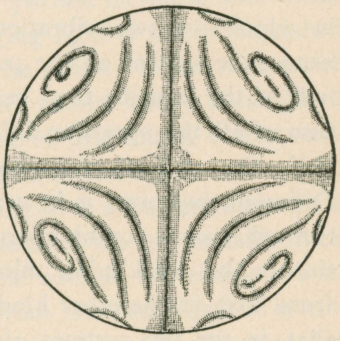

Fig. 107.

A quite *different interference* phenomenon (observed first by Norrenberg) is obtained, on the other hand, if the single mica lamellæ arranged crosswise in layers are so thin that in any one of them the two rays arising by the double refraction acquire less than one wave length difference of path. When the principal sections of the mica coincide respectively with the two

crossed nicols of the instrument, we observe an interference-figure which agrees absolutely with that of a plate cut perpendicular to the optic axis from a uniaxial crystal, consisting therefore of circular color rings intersected by a dark cross. That such a packet composed of mica lamellæ of exactly equal thinness must, as in the foregoing case, behave, for rays passing through perpendicularly, like a uniaxial plate cut normal to the optic axis (*i.e.* in parallel light appear singly refracting), is clear. For the small difference of path produced by the double refraction in one of the lamellæ is done away with by the next, in which the two vibration directions are interchanged; and hence, the sum of all the positive differences of path in the one half of the lamellæ thus being entirely counterbalanced by the sum of all the opposite differences of path arising in the other half, the rays of this direction suffer no double refraction. Different, however, is in general the behavior of rays that pass through such a combination obliquely: their transmission direction in the one kind of mica plates has a different orientation from that in the other kind, crossed as regards the former; so that, in the two systems of mica lamellæ, such rays experience a different double refraction and in the end emerge with a difference of path, this producing a definite color. Thus, in convergent light, isochromatic curves must appear, which in the case of a smaller number of plates being superposed have the form of hyperbolas, but which pass over into concentric circles, on the other hand, when the number of plates is very large and their thickness very slight. If the single lamellæ of the one system, while themselves all of equal thinness, are thicker or thinner than those of the other system, crossed as regards the former, then does the whole packet — and again the more perfectly, the larger the number and the less the thickness of the lamellæ — operate like a plate cut perpendicular to the acute bisectrix from a single biaxial crystal, a crystal however whose axial angle were the smaller, the less the lamellæ of the two systems differed in thickness.

These phenomena become more complicated when the vibration directions of the combined plates do not coincide; for example, when the two gypsum or mica plates of equal thickness are superposed not parallel or at right angles but crossed obliquely. From such a combination, supposing it to be illuminated with parallel polarized light, there in general then emerges light that is elliptically polarized; and, since a light ray of this kind (see p. 20) corresponds to two mutually perpendicular vibrations of unequal intensity, and since consequently on the resolution in a Nicol prism only a portion of these two vibrations can be annihilated, the emerging light is not extinguished by the analyzer in any position of the same. If one rotates the combination itself, between crossed nicols, then, although it does exhibit a decrease and increase of brightness, corresponding to the orientation of the major and of the minor axis of the elliptical path of the ether vibrations, yet it never becomes quite dark. In case the vibration directions of the two equally thick plates form 45° with each other, the axes of the ellipse are of equal length: the light emerging from the combination is circularly polarized (see p. 20) and therefore, on the rotation between crossed nicols, suffers no variation in intensity. As with the plates parallel and at right angles, so too when they are crossed obliquely do the phenomena in convergent light depend on the nature and thickness of the plates: provided their thickness is not too slight, and if from them there emerge rays parallel to the optic axis, one observes, as with plates combined at right angles, the isochromatic curves of both systems at once, in the relative orientation corresponding to the crossing-angle; and between these curves there appear color curves whose form, likewise, depends on the angle mentioned.

As in the case first considered, of the crystal plates being crossed at right angles, so too when they cross at any oblique angle does the optical behavior of the packet pass over into that of a homogeneous crystal if the lamellæ are taken thin enough and their number sufficiently large. When very thin

cleavage sheets of mica are arranged alternately at an oblique angle above one another, the resulting packet behaves, optically, like a single biaxial crystal whose optic axial plane bisects the acute angle formed by the common axial plane of the single lamellæ of the one system with that of the lamellæ of the other system, and whose optic axial angle is smaller than that of the mica used. Let, for example, in Fig. 108*a*, the long direction of the rectangular mica plates marked 1 and 2 be the trace of their optic axial plane; and let each of these two rectangles represent a number of parallel mica lamellæ lying one above the other, so that in the little triangle under the central hexagon, *R*, there lie the two kinds of lamellæ, crossed at 60°, one above the other in alternate layers. Investigated in parallel polarized light, this triangle then exhibits the exit of two plane-polarized rays whose vibration directions are oriented horizontal and vertical respectively, thus bisecting the acute and the obtuse angle of the two systems of mica lamellæ; if by rotating the combination these directions are brought into coincidence with the principal sections of the nicols, the triangle appears quite dark, like a homogeneous doubly refracting crystal. In the conoscope it exhibits the normal axial figure of a biaxial crystal having its optic axes oriented as mentioned at the top of the page.

A biaxial crystal plate that is cut perpendicular to the acute bisectrix, and in which the principal vibration directions for different colors are not dispersed, will obviously, when rotated 180° about the bisectrix, pass over into a position where it must exhibit exactly the same behavior toward light as before; because then, in consequence of the symmetry throughout with reference to the principal optic sections, equivalent directions must arrive at the same orientation. With mica these conditions are, to be sure, not fulfilled with absolute exactness, yet they are so nearly, that the small deviations have no essential influence on the resulting phenomena. Therefore, if upon a thin rectangular mica lamella whose long direction corresponds

to the optic axial plane one lays a second, at an acute angle dividing into 180° without a remainder (*e.g.* 60° or 45°), and if upon this one lays a third, rotated by that same angle relatively to the second, and so on, then in the first case (60°) the fourth lamella, in the second case (45°) the fifth lamella, is parallel to the first; in other words, after every three or every four plates the same optical orientation will recur; a column arises, built up of similar packets of three, four, etc., lamellæ. But, as is seen

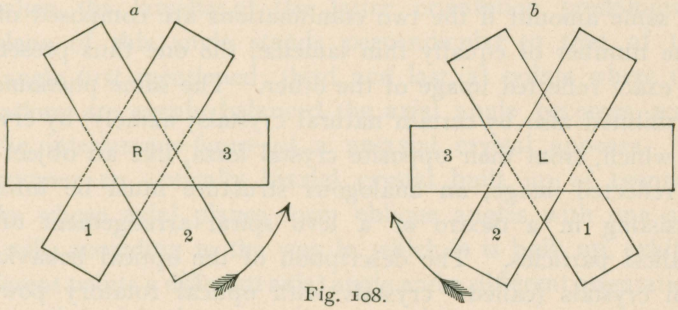


Fig. 108.

from a glance at the above figure, which is based on a rotation angle of 60°, such packets may be combined in two ways; viz. either so that the single lamellæ form winding stairs ascending from left to right, as represented in Fig. 108*a*, or so that they form stairs ascending from right to left, as shown in Fig. 108*b*. Now in the central hexagon (*R* and *L*), where the plates of all three orientations lie in equal number above one another, a column of such packets — called, after its inventor, Reusch's mica combination — behaves differently according to the sense in which the packets themselves are built up. Between crossed nicols, in parallel light, the hexagon appears bright in every position; but if the analyzer is rotated a certain angle we obtain complete extinction of the emerging light, provided it is monochromatic; if white light is employed, then, as the upper nicol of the orthoscope is rotated, changing colors appear. These phenomena are explained by the fact that, while rays passing

through the combination perpendicularly are indeed not doubly refracted, their vibration direction (or, what means the same, their polarization plane) on the other hand experiences a *rotation;* and that for different colors this rotation is unequal, so that in white light, by the analyzer in any single position, only one color can be extinguished. Now in a mica combination such as that of Fig. 108*a* this rotation of the polarization plane is to the *right*, *i.e.* clockwise ("right", or "dextro", rotation); in that of 108*b*, to the *left* ("left", or "levo", rotation), but of exactly the same amount if the two combinations are composed of the same number of equally thin lamellæ, the one thus presenting the exact reflected image of the other. The same phenomenon is exhibited also by certain natural crystals; namely, by crystals for which, from their opposite crystal form, like an object and its reflected image, an analogous structure must be inferred, consisting in a dextro or a levo spiral arrangement of the smallest particles. The description of the optical behavior of such crystals (called "crystals with optical rotatory power", or **rotatory crystals**) will form the subject of the next section. — In convergent light the Reusch mica combinations * exhibit in general the behavior of optically uniaxial crystal plates, except that the black cross traversing the color rings is lighted up at the center; if white light is employed, a color appears there, and on rotation of the analyzer this color changes. Therefore the phenomena described in the following section for the rotatory uniaxial crystals, especially for those to be observed in the combination of a right, or dextro (*i.e.* dextro-rotatory), with a left, or levo (*i.e.* levo-rotatory), crystal, may be produced likewise with dextro- and with levo-rotatory mica combinations.

From the behavior of the mica combinations mentioned on

* Reusch mica combinations of excellent workmanship are supplied by the firm of Steeg and Reuter in Homburg, being composed of thirty mica lamellæ so thin that singly they produce a path difference of only $\frac{1}{8}\lambda$. The rotatory power of such a preparation is equal to that of a quartz plate 8 mm. thick. (See following section.)

pages 213–214 the following conclusions may be drawn: First, a plate cut normal to the acute bisectrix from a " twin crystal " (see p. 211) that is built up of biaxial lamellæ crossed in mutually perpendicular orientation but having different size and thickness, must, at points where the light passes through layers only of one orientation, exhibit the biaxial interference-figure with normal axial angle; second, at points where between the layers of this orientation there are interlaid fine lamellæ of the other, the interference-figure corresponds to a smaller axial angle, and when the lamellæ of the latter orientation predominate the plane of this angle stands perpendicular to that of the axial angle first mentioned; third and last, at points where the orientations are evenly balanced the axial angle becomes zero, and the interference-figure of a uniaxial crystal appears. In like manner an optically biaxial crystal built up of twinned lamellæ whose axial planes form oblique angles with one another will, according to the way in which it is built up, exhibit at different points a different axial angle and a different orientation of the optic axial plane; under some circumstances even uniaxial phenomena may appear, combined with rotation of the polarization plane. Hence, if in such a composite crystal the lamellæ are so thin that they themselves escape observation in the microscope, then how it is built up can be concluded only from the aggregate effect of the lamellæ upon the polarized light passing through them: when the structure is exactly the same at all points, the optical behavior must be that, corresponding to this aggregate effect, of a homogeneous crystal; but when the different parts of the crystal differ in their structure in the manner just stated, the crystal appears heterogeneous, exhibiting at different points a different optical behavior. Such crystals are said to be **optically anomalous.** (Cf. pp. 279–282.) Properly speaking, however, it is here no question of an " anomaly "; for the behavior of such a composite mass must necessarily follow regularly from the nature of the component lamellæ. Mallard, starting out from the undulatory theory of

light, deduced quite generally the changes that a plane-polarized light ray must suffer in passing through packets of thin doubly refracting crystal lamellæ oriented at will, and thereby he supplied a complete theoretical explanation of all the phenomena considered in the foregoing. On the basis of this theory there follow, from the optical constants of the single lamellæ of such a packet, those of the packet itself; so that in cases where a substance occurs both homogeneous, *i.e.* in single crystals, and in crystals apparently homogeneous but really built up of fine twinned lamellæ (polysynthetic),* the optical constants of these lamellæ may be calculated from those of the homogeneous crystal.

## ROTATION OF THE POLARIZATION PLANE OF LIGHT IN CRYSTALS

Independently of whether they are doubly refracting or not, the crystals of a number of substances exhibit a property that belongs also to certain liquids, as oil of turpentine, sugar solution, and others, — the property, namely, of rotating the polarization plane of a plane-polarized light ray passing through it; the rotation amounts to a definite angle, this depending on the color of the light and on the nature of the crystal. Such substances are said to be **optically active.**

The cause of this phenomenon is a peculiar kind of double refraction, quite different from the ordinary, within these crystals: the entering plane-polarized, *i.e.* rectilinear, vibration is in such a crystal resolved into two circular vibrations, of which the one takes place clockwise, the other in the opposite sense; these two "circularly polarized" (see p. 20) rays of light are transmitted in the crystal with different velocity and therefore leave it with a difference of path, which depends on the thickness of

* Such substances, which appear to be differently crystallized, are designated as "polysymmetric". (See the author's "An Introduction to Chemical Crystallography". English by Hugh Marshall, Edinburgh and New York, 1906, p. 7.)

the crystal. Now the theory teaches that two circular vibrations, one retarded as compared with the other, must combine to form a rectilinear, *i.e.* plane-polarized, vibration which as compared with the original vibrations is rotated an angle depending on the amount of this retardation. Since, then, in such a crystal the arising path difference of the two circular rays is different, according to their vibration-period, —being the greater, the greater the refrangibility of the color in question,— the rotation of the vibration plane produced by a plate cut from such a crystal is least for red rays, greater for yellow, green, blue, and greatest for violet; so the several vibration directions, parallel before entrance, of the component rays of the incident white light are by such a crystal dispersed. This dispersion takes place after a law similar to that holding good with the dispersion produced by a refracting prism. (Cf. p. 46.) If $A$ and $B$ represent the constants that apply to a certain body, and $\alpha$ the rotation angle for a plate one millimeter thick, then according to Boltzmann the relation between rotation angle and wave length (in air) may be expressed as follows:

$$\alpha = \frac{A}{\lambda^2} + \frac{B}{\lambda^4} + \cdots$$

From the measured rotation of the polarization plane for at least two colors of known wave length we may therefore calculate that for the remaining colors, just as with the above-mentioned dispersion formula for refraction of light. Thus, for example, by plates of sodium chlorate (according to Guye) and quartz (according to Soret and Sarasin) 1 mm. thick the vibration plane of the light corresponding to the several Fraunhofer's lines indicated by the letters at the top of the following table is rotated the angles specified below the letters:

| | *B* | *C* | *D* | *E* | *F* | *G* | *H* |
|---|---|---|---|---|---|---|---|
| Sodium Chlorate : $\alpha =$ | 2°.27 | 2°.50 | 3°.13 | 3°.94 | 4°.67 | 6°.00 | 7°.17 |
| Quartz | 15°.75 | 17°.31 | 21°.71 | 27°.54 | 32°.76 | 42°.59 | 51°.19 |

As one sees from these examples, the **specific rotatory power**

of different substances is very different. Further, since the relative retardation of the two circular vibrations arising in such a crystal must be twice as great if the rays have twice the distance to travel in the crystal, the rotation must increase in proportion to the thickness of the plate; thus, for a plate of double the thickness the rotation angle is double the value given, and so on.

Now of each of the crystallized substances here coming into consideration there are two optically opposite modifications, with equal but opposite rotatory power, this being in consequence of the circumstance that in the crystal of the one modification the dextro-rotary (clockwise) vibration has the greater transmission velocity, in those of the other the levo-rotary; crystals of the first sort rotate the polarization plane from left to right, those of the second sort — in which, with the same plate thickness, the path difference is equal in amount but opposite in sense — just as far from right to left.

Optical activity has been observed, up to the present time, in the following *singly refracting* bodies:

Sodium chlorate
Sodium bromate
Sodium sulphantimonate (Schlippe's salt)
Uranyl sodium acetate
Active amylamin alum
Conin alum

In these crystals the phenomena resulting from the rotation of the polarization plane are exhibited in the same way in every direction; that is, their specific rotatory power is equal for all directions, depending only on the color of the light. Plates of equal thickness all behave alike, in whatever direction they be cut from such a crystal. But with *optically uniaxial* crystals it is otherwise; of these a larger number, as follows, have been recognized as possessing optical rotatory power:

Quartz (silica, silicon dioxid)
Cinnabar (mercuric sulphid)
Potassium lithium sulphate and seleniate

Rubidium lithium sulphate
Ammonium lithium sulphate
Potassium lithium chromate sulphate
Potassium dithionate and the corresponding rubidium salt
Calcium dithionate
Strontium dithionate
Lead dithionate
Sodium periodate
Guanidin carbonate
Rubidium and cæsium tartrates
Ethylenediamin sulphate
Benzil (diphenyldiketone)
Di-acetylphenolphthalein
Common camphor and matico camphor
Strychnin sulphate
Sulphobenzoltrisulphid

The laws of circular double refraction and of the rotation of the polarization plane resulting from it were discovered by Arago first in quartz, the most important of the series of optically uniaxial crystals just enumerated, and more closely investigated chiefly by Biot and Fresnel; they are as follows: —

In the optic-axial direction, since there is here no double refraction of the ordinary kind, only those phenomena arise that are dependent solely on the circular double refraction. For example, a quartz plate cut perpendicular to the axis exhibits, when illuminated only by plane-polarized rays falling normal to it, the same behavior as a plate of corresponding thickness cut in any direction from a singly refracting rotatory crystal. But, so soon as the entering light rays are inclined to the optic axis, there is added to the path difference resulting from the circular double refraction that produced by the ordinary double refraction; and this latter path difference grows larger with increasing inclination of the rays to the axis. The behavior hereby effected, of an optically active uniaxial crystal toward light rays, follows from the theory, developed by Gouy and in a somewhat

different form by Wiener, of the joint action of circular and of linear (plane) double refraction; and this theory rests on the presupposition that the two differences of path, the one produced by the circular double refraction, the other by the ordinary, are simply superposed. Experimental investigations made with quartz have quite borne out the conclusions deduced from this theory. These investigations have proved especially that, just as in the optically active singly refracting crystals, with equal plate thickness and the same color of the light rays the former of these two differences of path retains the same value in all directions; but also that by reason of the ordinary double refraction the effect of the circular is, with increasing inclination to the axis, diminished more and more, until finally it is wholly annihilated. This diminishing effect of the circular double refraction is explained as follows: The two circular vibrations transmitted parallel to the axis are transformed, when the rays include a small angle with this direction, into two elliptical vibrations, whose respective paths are similarly shaped but differently oriented, and traveled by the ether particles in the opposite sense; and these two paths, which in the beginning have nearly equal axes, always assume a more elongated form as the inclination of the rays to the optic axis becomes greater. When this inclination attains a certain value the minor axis of the ellipse is infinitesimal as compared with the major, and the vibration of the ether particles no longer differs from a rectilinear, *i.e.* plane-polarized, vibration. A plane-polarized ray of light entering the crystal is then resolved into two rays, likewise plane-polarized, and vibrating perpendicularly to each other, whose polarization plane no longer exhibits rotation; in other words, the crystal behaves like an optically uniaxial crystal without optical rotatory power.

From the foregoing it follows then, what phenomena must be exhibited by a plate cut from quartz perpendicular to the axis. In the orthoscope, if illuminated with monochromatic light, between crossed nicols it appears bright, and in order to

annihilate the light the analyzer must be rotated a definite angle, — in the case of a dextro (*i.e.* dextro-rotatory) crystal clockwise, in the case of a levo-crystal in the opposite sense. This rotation angle proves to be the larger, the less the wave length of the light used, and it increases for the same color in proportion to the thickness of the plate. With a plate one millimeter thick the rotation angle for the different colors amounts, according to Biot, to the following values (the more exact values for the several Fraunhofer's lines have already been given on p. 221):

| | |
|---|---|
| Extreme red | 17°.5 |
| Medium red | 19°.0 |
| " orange | 21°.4 |
| " yellow | 24°.0 |
| " green | 27°.8 |
| " blue | 32°.3 |
| " indigo | 36°.1 |
| " violet | 40°.8 |
| Extreme violet | 44°.1 |

It is because of the great diversity in the amount of rotation which accordingly is experienced by the vibrations of different colors, that in white light we are unable with the analyzer in any position to make the plate dark: the rays of different colors vibrate in entirely different planes, and therefore the nicol cannot extinguish all of them at once. If $N'N'$ (Fig. 109, p. 226) be the vibration plane of the *parallel* rays of *white* light entering a *dextro-rotatory* quartz plate of about three millimeters thickness, the red rays contained therein will, in the quartz, suffer the least rotation; their vibration direction on emergence will be about $\rho\rho$, that of the yellow rays $\iota\iota$, of the green $\gamma\gamma$, of the blue $\beta\beta$, of the violet $\upsilon\upsilon$. Supposing the analyzer to stand perpendicular to the polarizer, the vibration plane of the light passing through the former thus being parallel to $N''N''$, then of the rays emerging from the quartz the analyzer will transmit unweakened only those that vibrate parallel to $N''N''$; these are the green. All rays of the other colors are resolved in the

nicol into two components, one of these being annihilated; and this latter component is the greater, the larger the angle its vibration direction forms with $N''N''$. Thus all colors except the green are weakened, and weakened the more, the larger the angle their vibration direction forms with that, $N''N''$, of the analyzer. The aggregate impression of all these colors after their reduction to one polarization plane can accordingly no longer be white, but must be a *composite* color, —one in which green predominates

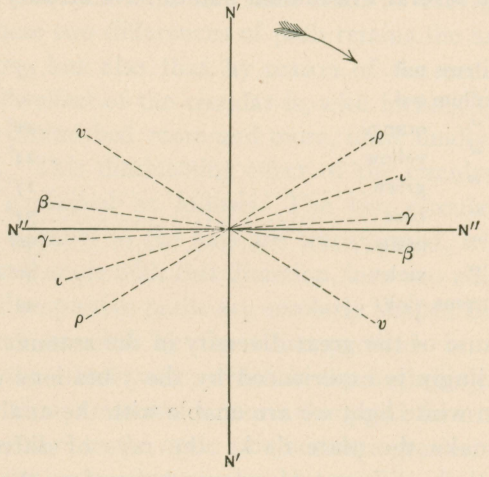


Fig. 109.

over the other colors. Observed in white light, between crossed nicols such a plate must therefore appear *green*. In this phenomenon, by a rotation of the plate in its own plane not the least alteration will be produced, since herewith, as we know, the direction in which the rays traverse the plate, and hence also the rotation of their polarization plane, always remains the same. But in the case of a *nicol* being rotated from its position relative to the other nicol it is otherwise. For example, if from the position $N''N''$ (Fig. 109) we rotate the analyzer from *left to right* (as indicated by the arrow in the figure), its vibration direction becomes parallel to that of the blue light; consequently, in the

arising mixture this color has its maximum intensity, the others being weakened, and the plate will appear *blue*. With further rotation in the same direction, $N''N''$ coincides with the vibration direction $\upsilon\upsilon$ of the violet light, and the plate appears *violet*. If we rotate beyond $N'N'$, until $N''N''$ becomes parallel to $\rho\rho$, the plate is colored with a composite color in which red predominates; so we see red. On further rotation yellow appears, then green, blue, violet. Accordingly, if we rotate the analyzer

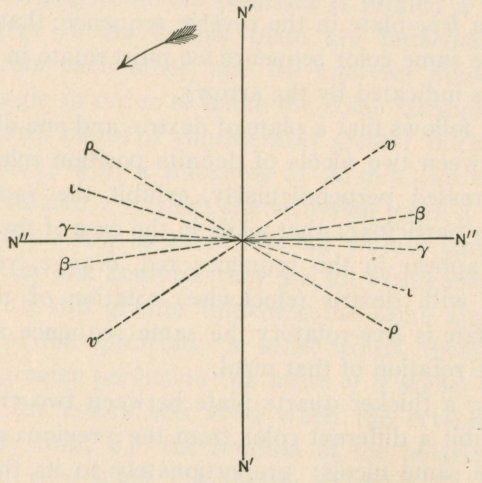


Fig. 110.

from *left to right* the plate always appears colored, but its coloring changes in such a way that the different colors of the spectrum appear in the *sequence of their refrangibility* (from the least refrangible, *i.e.* red, to the violet, which is the most strongly refracted). One easily sees that with reversed rotation the color sequence is the opposite.

Now if instead of the dextro-quartz plate we take an equally thick plate of *levo*-quartz, and if $N'N'$ (Fig. 110) again be the vibration direction of the *white* light entering the same, the vibration directions of the red, yellow, green, blue, and violet rays after their exit from the plate will here be $\rho\rho$, $\iota\iota$, $\gamma\gamma$,

$\beta\beta$, $\upsilon\upsilon$. As in the case of the dextro-quartz, therefore, here too there is no position of the nicols in which darkness can arise: the plate always exhibits a composite color. With the analyzer in crossed position this plate, like the previous one, when its vibration direction is $N''N''$, appears *green;* here, however, when the upper nicol is turned around from *left to right* the first color to appear is yellow, then red, and so on: with the same nicol rotation by which in using a dextro-quartz plate we obtained the colors in the sequence of their refrangibility, they appear in the case of a *levo*-plate in the reverse sequence; that is, in order to obtain the same color sequence we must rotate in the opposite direction (as indicated by the arrow).

Hence it follows that a plate of dextro- and one of levo-quartz brought between two nicols of definite position relative to each other, as crossed perpendicularly, exhibit the same color but differ in this particular: that while in the case of the *dextro*-plate the colors appear in the sequence red, orange, yellow, green, blue, violet with dextro (clockwise) rotation of the analyzer, when the plate is *levo*-rotatory the same sequence appears with the opposite rotation of that nicol.

If we lay a thicker quartz plate between two crossed nicols, it must exhibit a different color from the previous one observed between the same nicols: proportionately to its thickness, this plate produces more rotation; so that on emerging from it the vibration directions of the different colors have a different (more rotated) position, and in consequence the color extinguished by the analyzer is *not the same* as before. With a plate thickness of 3.75 mm., as may easily be seen from the numerical values given on page 225, the red vibrations are rotated more than 60°, the green over 100°, the violet about 180°; when the nicols are parallel such a plate appears purplish violet, and this color (*teinte sensible, teinte de passage*) goes pretty quickly over into red or blue if the analyzer is rotated in either direction. Such a plate, a so-called Biot's quartz plate, is sometimes employed in microscopical investigations instead of the gypsum plate exhibit-

ing the red of the first order (see p. 200); but the latter is to be preferred, on account of its being more sensitive. On the other hand, for measuring small rotations of the polarization plane a very suitable contrivance is a double plate consisting half of dextro- and half of levo-quartz, of 3.75 mm. thickness, which between two nicols exactly parallel or crossed exactly at right angles exhibits the same color shade in both halves: if the polarizer or, through inserting a feebly rotatory substance, the vibration emerging from the polarizer is rotated a small angle, the two halves of the double plate are immediately colored differently from each other, and the analyzer must be rotated that same angle in order to make the color identical in the two halves. Quartz plates of considerably greater thickness exhibit less vivid colors; with a thickness of 20 mm., for example, the red rays are rotated $2 \times 180°$, the yellowish green $3 \times 180°$, the indigo-blue $4 \times 180°$; so these rays all vibrate parallel to one another, and in the analyzer they are all extinguished simultaneously. With still greater thickness the same applies to still more colors of the spectrum; in other words, there appears in greater and greater perfection the *white of a higher order*. But likewise are the colors less vivid when the crystal plate employed is very thin; *i.e.* the rotation of the polarization plane effected by it very slight. A quartz plate of 0.1 mm. thickness, for example, rotates the plane of the vibrations for the outermost colors only 1°.7 and 4°.4 respectively; and these colors can therefore, by means of a small rotation of the analyzer, be extinguished almost simultaneously, while with the nicols crossed at exactly right angles the plate produces only a slight brightening.

Since rotatory power manifests itself only in the optic-axial direction and in directions immediately adjacent to the same, then, viewed in convergent light, a plate of an optically active uniaxial crystal cut perpendicular to the axis must likewise exhibit, when the nicols of the polariscope are crossed, the black cross with the circular color rings,—but with this exception: that part of the

field at which the rays traversing the crystal parallel to the axis converge, *i.e.* the center and its immediate vicinity, cannot appear dark, because it is of precisely these rays that the polarization plane is rotated. The center of the rings must therefore exhibit the same color * that the whole plate exhibits in parallel light; and accordingly, with different plate thickness, a different color. So too must this color change when the analyzer is rotated; and from the sequence of the arising colors and the sense of the rotation we shall be able, just as in parallel light, to determine whether the plate is dextro- or levorotatory.

If a dextro- and a levo-quartz plate are laid one upon the

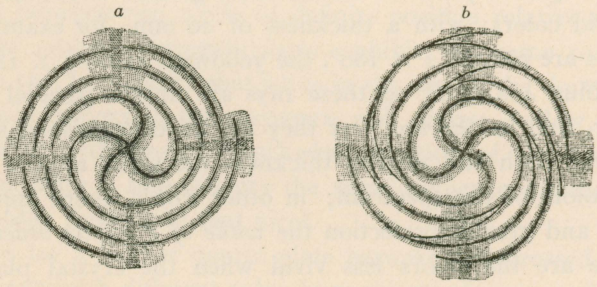


Fig. 111.

other and brought into the conoscope between crossed nicols, the center of the interference-figure must remain dark, because the rotations of the vibration directions by the two plates, respectively, exactly neutralize each other. From the dark center of the arising interference-figure there proceed, however, no black cross arms, but spirally wound curves, — called, after their discoverer, **Airy's spirals,** — the direction of whose winding indicates, at the same time, into which of the two oppositely rotatory crystals the light rays first enter. Figure 111*a* represents the phenomenon in the case where the light passes first

* Figure 2 of Plate II represents the interference-figure of a quartz plate cut perpendicular to the axis.

through the levo-plate, Fig. 111*b* that where it passes first through the dextro-plate.* Airy's spirals are likewise obtained if instead of the second quartz plate one employs a circular analyzer; *i.e.* the combination of a nicol with a quarter-undulation mica plate.

Now all that has been said applies also to the remaining substances whose crystals are optically active, but with this difference: their rotatory power is not the same as that of quartz, being with some higher but with the greater number lower.

In the *optically biaxial crystals* there is no direction without double refraction; for, although the rays parallel to the so-called optic axes have a common ray-front, yet there correspond to them an infinity of vibrations, with all possible azimuths. To demonstrate the occurrence of polarization-plane rotation, therefore, is here a matter surrounded by difficulties. Nevertheless, in a plate of cane sugar one centimeter thick Pocklington has recently been able to prove that rotation takes place at the center of the dark brush of each of the two axes; at the one center a levo rotation of 22°, at the other a dextro rotation of 64°. Again, Dufet, still more recently,† has shown that rotatory power is possessed by a more extensive series of biaxial crystals, and especially that it is of equal magnitude parallel to the two optic axes of a crystal if the axial angle is bisected by a plane of optical symmetry, else different. (As is seen from the example of cane sugar, the sense of the rotation can even be the opposite.) Moreover, it was shown earlier that quartz that by pressure has become biaxial still possesses rotatory power. (See p. 277, under "Influence of Other Properties on the Optical Properties".)

* Among the crystals of quartz occurring in nature are such as are composed of dextro- and of levo-quartz in layers, and this composition is then often to be recognized only by the appearance of Airy's spirals.

† In an article ("Recherches expérimentales sur l'existence de la polarisation rotatoire dans les cristaux biaxes." *Bull. soc. franç. de minéral.* 1904, **27**, 156 *et seq.*) that appeared while this work was in press.

## ABSORPTION OF LIGHT IN CRYSTALS

On its way through every body light suffers a weakening of its intensity, a partial absorption; when this is very slight, we say the body is **transparent;** when it is so pronounced that even after traveling only a short distance the light-motion is annihilated, we call the body **opaque.*** Secondly, this absorption lays hold of the differently colored light rays in a different degree; so that the light emerging from the body not only has no longer its former brightness, but also no longer its former color. If the diversity in the strength of the absorption for the different wave lengths in the light is but slight, the different wave lengths will combine to form a color impression so little different from that of the entering light that we are unable to distinguish the one from the other. We then speak of the body as **colorless.**† But an absolutely colorless substance exists just as little as one that is absolutely transparent. And even in bodies that to us seem altogether colorless the different colors composing the white light are not all absorbed in exactly the same degree; so that, if we but cause the white light to pass over a very long distance through the crystal, the rays that are the more strongly absorbed fall perceptibly back in intensity, relatively to the others, and the emerging rays then no longer produce the impression of white, but of a color. Very many substances, however, absorb the different kinds of light so differently that even in thin layers of them the white light becomes vividly colored; these it is, that are designated especially as **colored** substances. Many absorb light only of one or of several definite wave lengths, this absorption being complete if

* Hence it follows that the two words express only relative ideas, and that neither an absolutely *transparent* nor an absolutely *opaque* body exists.

† Such bodies can, however, exert a very strong absorption upon ether vibrations whose period is greater or less than those of the visible spectrum; *i.e.* on infra-red or ultra-violet vibrations.

the layer is of some thickness; so that the transmitted light, decomposed by a prism of high dispersive power, yields a spectrum that is broken by dark stripes (**absorption spectrum**). Other substances absorb the majority of colors far more strongly than they do one or several of them; so that these latter alone appear in the spectrum, as bright stripes or bands. But there is no body that would transmit the light only of one definite wave length and absorb all the rest in equal degree,— no body, therefore, that might serve to produce really *mono*chromatic light. With many red glasses, to be sure, the absorption of all the colors except red is as good as complete; yet, that the transmitted light consists of various, although not very many, wave lengths follows from the spectral decomposition, this always yielding a broad red band. Absolutely *homogeneous* light cannot be obtained through absorption, but only through emission from glowing bodies; *e.g.* from the incandescent metallic vapors already mentioned for that purpose.

The coloring of a crystal can be either **allochromatic,** *i.e.* not belonging to its own substance, but arising from the admixture of foreign matter; or **idiochromatic,** *i.e.* an essential attribute of its material nature. Among the latter colorings there belong the yellow and red colors of the chromic acid salts and of many organic nitro- and azo-compounds, also the blue and green colors of the copper salts. When the coloring of a crystal is allochromatic, the coloring body may be disseminated in small isolated particles, which can be recognized in the thin section of the enclosing crystal by means of the microscope (if these particles are regularly distributed, then in reflected light they give rise to a so-called "schiller", a bronze-like luster, in definite directions); or else the coloring body exists in the crystal in a dissolved (**dilute**) state, so to speak, this being true especially as regards many organic pigments. In the case of dilute coloring the crystal is apparently homogeneous, like the solution of a pigment in a liquid, wherefore it may then be designated as a "solid solution"; such a crystal, in those of its optical proper-

ties that depend on the color, behaves like a crystal whose coloring is idiochromatic.

The so-called **body color** * of the *homogeneously colored* crystals, *i.e.* the color these crystals impart to the white light rays passing through them, is only then the same in all directions, when the vibrations of all directions are transmitted in the same manner and hence equally fast; but, of all crystals, this is the case only with the singly refracting. With the crystals of all the four other groups (see p. 196), *i.e.* with the optically uniaxial and biaxial, the transmission of light in different directions is different, and, accordingly, so too are the kind and the magnitude of the absorption: with these crystals the light rays that have passed through an equal thickness but in a different direction possess different brightness and different color. The latter property, to exhibit a different color in different directions, — which property therefore can belong *only* to the doubly refracting crystals, — is termed **pleochroism**; of this property the laws have been investigated chiefly by Brewster and Haidinger.

Like its transmission, the absorption also of light is an ellipsoidal property of crystals; for the most proper treatment of it we take into account a quantity called the **coefficient of absorption** and denoted by $\alpha$. This quantity is specific to the absorbing substance, and its value, besides depending therefore on the nature of the substance, is further conditioned by the direction of the vibrations and by the color of the light. If we denote the intensity of the incident rays by $I_o$, that of the rays emerging from the absorbing body by $I$, and the thickness of the latter by $z$; and if $e$ be the base of the natural, or Naperian, logarithms, then the following law applies:

$$I = I_o\, e^{-\alpha z}.$$

According to this equation, therefore, $\alpha$ may be calculated from the loss in brightness suffered by rays of known intensity

* [Often designated simply as the color in transmitted light, while the "surface color" (see p. 249) is known also as the color in reflected light.]

in their passage through a crystal whose thickness is measured in the direction in question. Hence, if from a point within the crystal one imagines as laid off in every direction a length proportional to $\frac{1}{\sqrt{\alpha}}$, this length may be said to be a measure of the remaining brightness of a plane-polarized ray vibrating along the direction in question. If we connect the extremities of all these lengths with one another, we obtain a closed curved surface (in general a triaxial ellipsoid) which gives us, for a definite color, a mental picture of the brightness of the light rays vibrating in all possible crystallographic directions, just as does the index-surface a mental picture of the transmission of those rays. This surface, designated by Mallard as the " ellipsoide inverse d'absorption", we shall call briefly the **absorption-surface.** Like the index-surface, it has for a different color a different form, perhaps also a different crystallographic orientation.

*a.* Singly Refracting Crystals. — The absorption-surface has for every color the form of a sphere. Vibrations of a definite color, whatever their crystallographic orientation, suffer the same absorption; for a different color, however, the absorption-surface has a different diameter. In case the values of the coefficient of absorption for different colors are very unlike, the white light, even after it has passed through only a slight thickness of the crystal, will be vividly colored; but in all directions, when the light has traversed an equal thickness of the crystal, the color is exactly the same.

*b.* Optically Uniaxial Crystals. — The absorption-surface is a rotation ellipsoid, whose rotation axis is the optic axis of the crystal; in other words, only those rays undergo equal absorption whose vibration directions form equal angles with the optic axis, — only those rays, therefore, that in the crystal are transmitted with equal velocity. Hence, for the light of a definite color the absorption is greatest for vibrations parallel to the axis, diminishes with increasing inclination to the axis, and is least for all rays vibrating perpendicular to the axis; or else the reverse

takes place, the minimum absorption being suffered by the rays that vibrate parallel to the axis and the maximum by those vibrating perpendicular to it. In general the rule applies that was laid down by Babinet; namely, that the uniaxial crystals with positive double refraction present the former of the two cases distinguished, absorbing the extraordinary ray the more strongly, while in negative crystals it is the vibrations perpendicular to the axis, the ordinary rays therefore, that are weakened the most. To express this rule more briefly: the ray the more strongly refracted is also the more strongly absorbed.

For different colors the absorption-surface has not only a different size but also a different form; in other words, the coefficient of absorption is not only different in an absolute sense, but its value for vibrations parallel to the axis stands in a different ratio to its value for vibrations perpendicular to the axis. In consequence of this, while all rays whose vibration directions form the same angle with the optic axis will indeed (as in the singly refracting crystals) experience equal absorption for each individual color, yet with a different common inclination of the vibration directions to the axis the relative brightness of these colors will be different; therefore the arising composite color will be the same only for rays falling at equal inclination to the axis: with increasing or diminishing inclination, on the other hand, the color will vary, varying in the opposite sense when the vibration direction of the rays approaches and when it recedes from the optic axis. Let us denote by $a$ the body color of the crystal that arises with a definite thickness of the crystal when white light enters it parallel to the optic axis. Of these rays the vibration directions, although in all possible azimuths, are nevertheless always perpendicular to the axis; and these vibration directions are all equivalent, corresponding therefore to the same absorption. A light ray passing through an equal thickness of the same crystal but perpendicular to the axis is split up into two rays, of which one vibrates perpendicular, the other parallel, to the axis. The former ray, whatever

its direction otherwise may be, exhibits the absorption-color $\mathfrak{o}$; the latter a different color, which we will call $\mathfrak{e}$, and which according to the above is obviously, of all the absorption-colors of this crystal, the one differing the most from $\mathfrak{o}$. If we look at the crystal in such a way that the light falls through it in the direction perpendicular to the axis, then both of the polarized rays — that with the color $\mathfrak{o}$ and that with the color $\mathfrak{e}$ — reach our eye simultaneously; and we are unable to separate them from each other, but receive the resultant impression — of a color that we shall denote by $\mathfrak{o} + \mathfrak{e}$. Now the colors $\mathfrak{o}$ and $\mathfrak{o} + \mathfrak{e}$, *i.e.* the body colors of the crystal parallel and perpendicular to the axis, are obviously the more different, the more $\mathfrak{o}$ and $\mathfrak{e}$ themselves differ from each other. Hence, in the intermediate directions the body color of the crystal is intermediate, being the more like $\mathfrak{o}$, the more nearly the direction in which the light falls through the crystal approximates to the optic axis, and vice versa.* When the color $\mathfrak{o}$ differs but little from $\mathfrak{e}$, then from the composite color $\mathfrak{o} + \mathfrak{e}$ it differs still less; and in such cases as this the crystal, if observed without further aid, appears to have the same body color in all directions. More especially is this true of the so-called colorless substances; because, with them, even the sum total of the absorption is imperceptible, its difference in different directions accordingly escaping all observation. Therefore strong pleochroism, *i.e.* great variation of the body color with the direction, can be exhibited only by crystals having high "absorbency"; *i.e.* crystals that absorb the light very strongly, hence being vividly colored. Even among these there are many that have only a slight degree of pleochroism, the body colors they exhibit parallel and perpendicular to the axis thus being very similar.

In the cases last mentioned, in order to perceive the existence of pleochroism — whereby, at the same time, that of

* Therefore it is not entirely correct to call this phenomenon — as it frequently is called — by the name "dichroism"; because it is not a matter of *two* colors, but of a continuous *series* of colors *between two extremes*.

double refraction is verified — service must be made of a little apparatus constructed by Haidinger, known as the **dichroscope** or **dichroscopic lens.** This consists of a rhombohedron of calcite, whose section, *abcd*, is represented in Fig. 112 just as in Fig. 37, page 81; so that *ab* and *cd* are the short diagonals of two opposite rhombohedron faces. The rhomb is mounted in a brass tube, and both before and behind it, touching the calcite, there is a glass wedge (*g* and *g'*) so fitted that the faces of entrance and emergence of the light stand perpendicular to the rhombohedron edges *ac* and *bd*; at these faces, therefore, there is no refraction of the rays that pass through the instrument parallel to these edges. In front the mounting has a wide, round opening for looking inside, the eye being then at *E*; behind, on the other hand, there is a square opening, *o*, of only 2 or 3 mm. diameter, through which the light falls when this end is directed toward the bright sky or some other source of light. Immediately in front of the glass wedge *g* is a plano-convex lens, *l*, by means of which, if the calcite were not present, the eye at *E* would see at the distance of distinct vision a magnified virtual image of the bright opening *o*. By reason, however, of the double refraction in the calcite, two such images appear, and one of these, since the extraordinary is deflected in the principal section, appears exactly above the other. Now the length of the rhombohedron is so chosen that the two images do not partially coincide, but that the upper edge of the lower image just touches the lower edge of the upper.

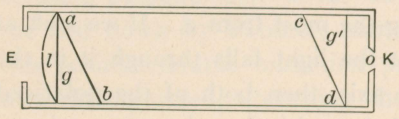

Fig. 112.

Hence, if a uniaxial crystal is held before the small opening*

* V. von Lang suggests that it is well to cover the end *K* of the tube (Fig. 112) with a rotatable metal cap having in it a circular opening; over this opening the crystal is fastened with wax and hence, by rotating the cap, can easily be brought into its correct position.

in such a way that the rays passing through parallel to the long axis of the dichroscope first traverse the crystal, in a direction perpendicular to its optic axis, and if in addition the crystal is so held that its axis is at the same time parallel to the principal section of the calcite, then the two rays arising in the crystal enter the calcite in such wise that the vibration direction of the ordinary ray is parallel to that of the ordinary ray in the calcite, and the vibrations of the extraordinary parallel to those of the extraordinary in the calcite. So neither of the two rays suffers a fresh splitting-up in the calcite; and thus, of the two images of the bright opening in the dichroscope, the one is formed only by the rays that on emerging from the crystal to be investigated were vibrating parallel to its axis, the second image only by those rays whose vibration direction in the same crystal was perpendicular to the axis. The color of the latter image, therefore, must be the one previously denoted by $\mathfrak{o}$; the color of the former, that denoted by $\mathfrak{e}$. This is readily seen from Fig. 113, where $k_1k_2k_3k_4$ is the outline of the crystal to be investigated, $AA'$ its optic-axial direction, and consequently $oo'$ the vibration direction of the ordinary ray emerging from it, $ee'$ that of the extraordinary; *abcd* is the cross-section of the calcite rhombohedron, whose principal section is parallel to that of the crystal; $\omega\omega'$ is the vibration direction of the light in the one image of the square opening covered by the crystal, $\varepsilon\varepsilon'$ the vibration direction in the other image. Hence, if one rotates the crystal about the axis of the dichroscope as an axis, $oo'$ and $ee'$ form an angle with $\omega\omega'$ and $\varepsilon\varepsilon'$, wherefore in the calcite each of the two rays is doubly refracted, and contributes to each of the two images; when the angle mentioned amounts to 45° the component contributed by each ray to each image is half its brightness, and so the two images are of the

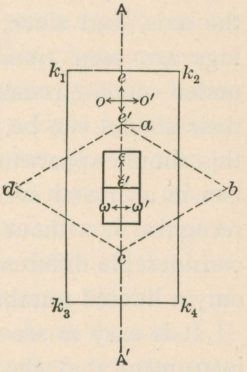

Fig. 113.

same color — of that color, $\omega + \varepsilon$, that the crystal exhibits in the direction in question to the unassisted eye. After 90° rotation the two images of the dichroscope again exhibit the greatest difference in coloring, but their colors have exchanged places. And so on.

Always, therefore, when the optic axis of the crystal to be investigated for pleochroism is parallel to the vibration direction of either of the two rays transmitted in the calcite, the one image exhibits the color $\omega$, the other the color $\varepsilon$. Since, then, these colors differ from each other more than do the colors $\omega$ and $\omega + \varepsilon$, visible in the crystal parallel and perpendicular to the axis, and since, besides, in this instrument the two colorings are seen *simultaneously* and *in contact, side by side*, — under which circumstances even very slight differences between their shades can be perceived, — it is clear that with the aid of this simple apparatus one can ascertain the existence of dichroism in a crystal even when it is rather weak, while in order to recognize it without the dichroscopic lens there must be a very considerable difference in the absorption, such as is exhibited by only a limited number of substances.

It is easy to see that, when the crystal is so held before the instrument that the rays traverse it oblique to its optic axis, the color of the one image will be $\omega$ and the color of the other a tint lying between $\omega$ and $\varepsilon$; finally, that when the light passes through the crystal parallel to the axis, however the crystal may be rotated in its own plane, both images must exhibit the same color, $\omega$.

As examples of very highly pleochroic uniaxial crystals, two minerals may be mentioned: First, *chlorite* (*penninite*), whose color as seen through a plate perpendicular to the axis is emerald-green, and as seen through a plate parallel to the axis brownish red; second, *tourmaline*, which occurs in very diverse colors, many varieties of the mineral presenting at the same time an example of dilute coloring by foreign pigments. In the latter crystals, as in those that are colored by their iron content

and therefore idiochromatically, the ordinary ray is so strongly absorbed that tourmaline plates cut parallel to the axis transmit almost no light except the extraordinary. (Cf. p. 54.)

In the case of certain crystals having strong pleochroism, by means of a plate cut perpendicular to the axis the difference in the absorption of rays passing through the plate parallel and inclined to the axis may be perceived directly, without instrumental aid. For example, if one holds near the eye a plate of magnesium platino-cyanide cut or split in the direction stated (the crystals of this salt cleave perfectly along a plane perpendicular to the axis), and looks through it at a white surface (best a uniformly white bank of clouds), one sees a circular violet spot upon a background of vermilion. This phenomenon is explained as follows: The magnesium platino-cyanide transmits blue rays only along the axis and at very slight inclinations to the axis, — and even here, only when the plate is very thin, — while in other directions it transmits only red rays; in consequence the light passing through along the axis becomes violet-colored, but with a certain inclination to this direction the blue is absorbed, both on account of the increasing thickness of crystal traversed and because of the deviation from the axis. If a nicol is held before or behind the plate, then, of the rays inclined to the axis in the vibration plane of the nicol, the ordinary — *i.e.* violet-colored — portion is extinguished, while of the rays inclined in a plane perpendicular to that plane the same portion is transmitted; parallel to the vibration direction of the nicol, upon the violet background, there then appear two red brushes of a shape similar to that portrayed in Fig. 114, page 246.

*c.* Optically Biaxial Crystals. — Here the absorption-surface for a definite color is in general a triaxial ellipsoid, the lengths of whose three axes, the so-called **absorption-axes** (*i.e.* the vibration directions of the greatest, the intermediate, and the least absorption), stand in ratios that are entirely independent of the ratios holding good with the index-surface; in direction, too, as

a general thing, these axes do not coincide with those of the index-surface. For a different color the absorption-surface has not only a different size but also a different form, — in other words, different ratios among its axes.

When, however, the crystal belongs to the group of biaxial crystals having the highest symmetry, *i.e.* when the three axes of the index-surface have the same crystallographic orientation for all colors (cf. p. 168), then do likewise the three absorption-axes for all colors coincide; they coincide with one another and also with the three mutually perpendicular axes of the index-surface. In this case not only the transmission, but also the absorption, of light is absolutely symmetrical to the three principal optic sections of the crystal. Let us consider this case first, it being the most simple. If a denote the color of the rays that have as their vibration direction in the crystal the $X$-axis of the index-surface, b the color of those vibrating parallel to the $Y$-axis, and c that of the rays vibrating parallel to the $Z$-axis, then in the dichroscope, on our looking through a plate cut perpendicular to the $Y$-axis, when $X$ and $Z$ are parallel to the vibration directions of the calcite we shall see in the one image the color a, in the other image the color c. But if we take a plate cut normal to $Z$, then, with the plate in the analogous position relative to the instrument, the one image exhibits a, the other b. Finally, a plate cut perpendicular to $X$ exhibits the colorings b and c separately, if it is so held before the opening of the dichroscopic lens that either $Y$ or $Z$ is parallel to the principal section of the calcite contained in the instrument Therefore, when the dichroscope is employed we need observe the crystal only in two of the directions named, in order to determine the three so-called **axial colors** — the colors arising by the absorption of white light rays vibrating parallel to the three absorption-axes.

Without the dichroscope, on the other hand, we are able to see none of these colors separately. For, when we look through a plate cut from the crystal normal to $X$, for example, our eye

receives simultaneously the rays vibrating parallel to $Y$, with the color $\mathfrak{b}$, and those vibrating parallel to $Z$, with the color $\mathfrak{c}$, so that we shall see a coloring $\mathfrak{b} + \mathfrak{c}$, composed of both these colors; while in just the same way a plate cut normal to $Y$ shows us a composite color $\mathfrak{a} + \mathfrak{c}$, and a plate whose faces stand perpendicular to the $Z$-axis $\mathfrak{a} + \mathfrak{b}$.* Hence, it is clear that the color impressions made up of two axial colors, viz. $\mathfrak{a} + \mathfrak{b}$, $\mathfrak{a} + \mathfrak{c}$, and $\mathfrak{b} + \mathfrak{c}$, will be less different from one another than are the axial colors themselves. Therefore, for the same reasons as were adduced on page 240 in discussing the uniaxial crystals, one can recognize with the dichroscope a far lower degree of pleochroism than without this instrument. With it, when there is high pleochroism, one can very easily distinguish a uniaxial crystal from a biaxial; because with a crystal of the latter kind there exists no direction in which, however the crystal be turned, the rays passing through it yield two images colored exactly alike, — as is the case with the rays parallel to the optic axis in crystals of the former kind.

As for the body colors of such biaxial crystals in other directions than along the three absorption-axes, these colors vary with the direction, and their variation is entirely symmetrical to the three principal optic sections. When the direction of the white light rays passing through the crystal lies within one of these principal sections, the color exhibited is a mixture of the following two: First, a shade lying between two of the axial colors, namely, between the two whose vibrations take place parallel to the principal section in question; and second, the third axial color, — *i.e.* the color of the ordinary rays vibrating perpendicular to that principal section. In a direction that falls in none of the three principal sections the crystal exhibits an

* This mixed color, which a crystal plate exhibits without the dichroscope, was called by Haidinger its **facial color** (Flächenfarbe). By the instrument named, therefore, the facial color of a plate cut parallel to a principal optic section is resolved into the axial colors; stated generally: the facial color of a plate having any chosen orientation is resolved into those two colors that correspond to the vibration directions of the plate.

absorption-color lying between those two of the three axial colors that present the greatest difference; there accordingly exist in the crystal all possible color shades between the two that are the most different; and therefore the word 'trichroism' — not infrequently employed — is just as little correct a name for the color phenomena of biaxial crystals as 'dichroism' is for those of the uniaxial.

In those optically biaxial crystals in which the principal vibration directions for the different colors do not coincide, the absorption-axes too, for the different colors, diverge both from these vibration directions and from one another. When a principal vibration direction has identical orientation for all colors, it is identical likewise with an absorption-axis, and the plane perpendicular to it is a plane of symmetry for the absorption, also, of light. When the crystal has no plane of symmetry for the transmission of light (Group 1, p. 196), the same applies to the absorption as well. Thus, for example, if in ordinary light falling on it at right angles we look at a plate of a highly absorbent crystal of this kind cut perpendicular to the acute bisectrix for any one color, it exhibits a definite absorption-color; but if we rotate it in either direction so that the direction of the rays passing through is approached first by the one and afterward by the other optic axis, then in the two directions the color in transmitted light does not vary in the same way; because the absorption is not the same symmetrically on opposite sides of the normal to the plate, the directions that correspond to the greatest, the least, and the intermediate absorption being oriented differently for every color but always one-sidedly oblique to the plate.

Thus, with reference to the crystallographic orientation of the absorption-axes the optically biaxial crystals fall into the same three groups as with reference to the principal axes of the index-ellipsoid.

As examples of optically biaxial bodies having especially high pleochroism the following may be mentioned: 1. *Cordier-*

*ite* (called also "dichroite", on account of this property), whose coloring must be attributed to a foreign body distributed in dilute form and which exhibits the following axial colors: a, light yellow to yellowish brown (corresponding to the vibrations parallel to the vibration direction of greatest light velocity); b, light blue (absorption-color of the vibrations having the intermediate light velocity); and c, dark blue (vibrations with the least velocity, *i.e.* of the greatest refraction, which therefore are the most strongly absorbed). 2. *Epidote*, which in the beautiful ferruginous variety from the Sulzbachthal exhibits a, yellow; b, brown; c, green. 3. *Glaucophane*, a mineral of the amphibole group, which has the axial colors a, light greenish yellow; b, violet; c, ultramarine-blue. The two latter examples refer to cases in which the absorption-axes and the principal vibration directions for different colors are dispersed.

Since the kind and the intensity of the color tones, besides depending on the direction of the vibrations, are conditioned also by the thickness of the plate, the colors have in the foregoing, as in the previous, examples been designated only in a quite general way. They could be specified more exactly by mentioning the number in Radde's International Color Scale * with which the color tone in question most nearly agrees; but an actual determination of the absorption-color would naturally require spectral decomposition of the light that has passed through the crystal, together with measurement of the intensity of the light in the several parts of the spectrum. In both cases it would be necessary to determine and state the thickness of the crystal plate investigated.

When a microscopic crystal — in the thin section of a rock,

* Since colors in transmitted light, colors such as we observe in pleochroic crystals, cannot very easily be compared with colors in reflected light, like those exhibited by Radde's scale, it would be desirable, for specifying the absorption-colors of crystals more precisely, to employ standard colors made from differently colored glasses; these glasses would have to be ground to the form of a sharp wedge, in order that the intensity of the color in question might be determined by stating the part of the glass, — *i.e.* its thickness.

for example — is to be examined for pleochroism, it is well to employ the method proposed by Tschermak; namely, to polarize the light entering the microscope, by means of a nicol: by so moving the rotatable stage, with the preparation, that first the one and then the other vibration direction of the crystal section in question coincides with the principal section of the polarizer, the observer can produce in the crystal, one after the other, the two absorption-colors that correspond to its vibration directions. In the dichroscope, it will be remembered, the same colors are observed side by side.

### BRUSH PHENOMENA

If one holds close to the eye a plate cut perpendicular to one of the two optic axes, either of Brazilian andalusite or of the epidote mentioned, and directs it toward the bright sky, one sees on a colored background two dark brushes* of the form shown in Fig. 114; toward the center these brushes can be recognized as traces of rings. The same phenomenon, less intense, may be observed also with various other pleochroic minerals; it is particularly beautiful, finally, with the so-called "Senarmont's salt",—*i.e.* strontium nitrate crystallized from an extract of logwood, which by its taking up the pigment of this solution in dilute form is colored red. If from a crystal of the latter kind one takes, instead of the plate cut perpendicular to an axis, a plate cut normal to the acute bisectrix, one sees two double brushes; and in the case of yttrium platino-cyanide, whose optic axial angle is very small, the brushes are so close together that they form four red sectors, between which the background, here violet-colored, appears in the form of a cross. The brushes always stand perpendicular to the optic axial plane, and their centers correspond to the two axes. The

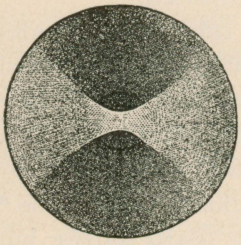

Fig. 114.

* [The so-called **polarization-brushes** (absorption-tufts, epoptic figures).]

phenomenon may accordingly be employed for finding the position of the latter without the aid of a polariscope. An explanation of all these phenomena was given by W. Voigt in his theory of the absorption of light in crystals.

A phenomenon similar to the brushes must be exhibited by radiating-fibrous aggregates of crystals, when these are distinctly pleochroic. If we imagine, for example, a plate composed of radially arranged uniaxial crystals that fulfill this condition and whose long direction is parallel to their optic axis (or if we cause a pleochroic crystal to rotate as described on page 73, so that it assumes in rapid succession the positions of the different crystals of such a plate), and if we view the mass through a nicol or, what signifies the same, cause it to be entered by plane-polarized light and observe it without the nicol, — then the following must take place: The light passing through the crystals that lie with their long direction in the principal section of the nicol, vibrates parallel to their optic axis; while, therefore, these crystals appear in the color of the extraordinary ray, and the adjacent crystals, diverging but little from them, in nearly that same color, those crystals that lie farther off in direction split up the incident light into an ordinary and an extraordinary ray and therefore exhibit a mixture of the colors of these rays; passing on, finally, to the crystals that stand perpendicular to the vibration direction of the entering light, the oscillations transmitted through these crystals take place perpendicular to the axis, so that the crystals last named appear in the color of the ordinary rays. Therefore, while with the preparation between crossed nicols we should see the black cross as in every radiating-fibrous mass, we behold with *one* nicol, when the pleochroism of the single crystals is sufficiently strong, what is, to be sure, likewise a cross, but a cross of which the two arms lying parallel to the vibration plane of the polarizer have the color of the extraordinary rays and the other two arms the color of the ordinary light-vibrations. On this property of pleochroic radiating-fibrous masses depend **Haidinger's polari-**

**zation brushes**, the production of which, according to Helmholtz, is explained as follows: In the so-called yellow spot, the most important part of the human retina, the radial nerve fibers, which elsewhere in the retina stand perpendicular to its surface, converge obliquely toward the center of the yellow spot; like the majority of organic fibers, these fibers are doubly refractive, and it must be assumed that of the light rays vibrating parallel and perpendicular to their axis they absorb the blue and the yellow unequally. In consequence, if one looks upon a uniformly illuminated white surface and at the same time holds a nicol before the eye in order that plane-polarized light may enter, one beholds, passing out from the point on which the eye is fixed, two brownish yellow brushes; and between these brushes, in the direction perpendicular to them, bluish light appears. The brushes always lie in the polarization plane of the nicol, thus standing at right angles to the vibration direction of the polarized rays, and therefore when the polarizer is rotated the brushes likewise must rotate; consequently this phenomenon may serve for recognizing plane-polarized light as such, directly, and for determining its vibration direction. The brushes are found with special facility if one directs the Haidinger dichroscope toward a white bank of clouds: they then appear in the two bright, square images, and in crossed position, as shown in Fig. 115, because the images are polarized perpendicularly to each other. (The arrows indicate the vibration directions of the light in the two fields.) It must be remarked that not all persons are able to perceive the phenomenon of Haidinger's brushes (it being very weak), while certain individuals can see the brushes even without a nicol, when their eye is directed toward the sky, owing to the partial polarization of the light reflected from the sky.

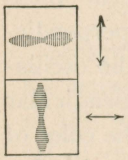

Fig. 115.

## SURFACE COLORS

The majority of bodies, no matter whether colored or colorless, do not alter the light reflected from them; so that white light falling on a plane surface of these bodies is also reflected as white light. Metals, as is well known, behave otherwise; for example, the image of a white body reflected from a polished copper plate appears not white, but red. Between the metals and the common transparent bodies there stand in intermediate position, as it were, those substances that are transparent for certain light rays and behave for others similarly to the metals, reflecting the light with metallic luster; such media have been designated as bodies with a **surface color** (" schiller-color "). Here belong a series of double cyanides of platinum with other metals; also a number of salts of organic bases, especially aniline pigments, and others. To explain the peculiar behavior of these bodies it must be assumed that the light reflected from them has first penetrated to a certain depth and the different colors therewith experienced unequal absorption; this absorption would then necessarily stand in a definite relationship to the absorption of the transmitted light. In fact, Haidinger found that the surface color of these media is in a certain sense complementary to their body color. Accordingly, since in doubly refracting crystals the latter color differs for differently directed vibrations, then in such crystals the surface color also must, on account of the relationship mentioned, vary with the direction of the vibrations. It is to be expected for example, that, though the surface color of a uniaxial crystal were the same on all faces lying parallel to the axis, it would be different on the faces that are inclined to the axis, and different again on the plane perpendicular to the axis. These suppositions are confirmed by the phenomena observed in such crystals; thus, for example, in carmine-red magnesium platino-cyanide a natural face, or an artificial one, that lies parallel to the axis exhibits a green surface

color,* a face perpendicular to the axis a violet one. If for observing the surface color one makes use of the dichroscope, then in looking upon faces perpendicular to the axis the two images are seen to be of the same color, while in viewing faces parallel to the axis the two images are of different color. It accordingly follows that the light observed in the surface color has experienced in the superficial layers of the crystal a change similar to that experienced by the transmitted light, — having been resolved into two rays polarized perpendicularly to each other (or at least behaving similarly to such rays) and unequally absorbed.†

The laws of metallic reflection were worked out theoretically by Cauchy and Voigt, and specially for the doubly refracting crystals by Drude, who also put the results to experimental test. These researches show that it is possible, through investigation of the light reflected from highly absorbent crystals, to calculate the refractive indices and absorption coefficients for differently directed vibrations of the light penetrating those crystals; but this may be done only when the reflecting surface is absolutely clean; and absolute cleanness is exhibited, essentially, only by fresh cleavage-surfaces. In this way very high refractive indices, with relatively low values for the absorbency, were found in the case of the singly refracting metals lead, platinum, and iron; the highest refringency was found in the case of galena (lead glance) and of the optically biaxial stibnite (antimony glance); while silver and gold, for example, have very low refractive indices (0.2 to 0.4, the velocity of light in

* The body color of this salt, which in a certain sense is complementary to the surface color, is of course pleochroic, being for the vibrations parallel to the axis a pure carmine-red and for the vibrations perpendicular to the axis a violet. This latter color is essentially identical with the one exhibited by a plate cut perpendicular to the axis, in the case of perpendicular reflection; so the reflection of vibrations taking place perpendicular to the axis is almost normal, the crystal behaving, for these vibrations, almost like a crystal without surface color.

† Particulars as to such phenomena will be found in B. Walter: "Die Oberflächen- oder Schillerfarben". Braunschweig, 1895. (Excerpt in *Zeitschr. f. Kryst.* **28,** 632.)

them being greater than in empty space) and a remarkably high absorbency. Phenomena of double refraction of the transmitted light were observed by Kundt in extremely thin metallic films produced by deposition through electric discharge from cathodes.

## FLUORESCENCE

When light is absorbed in a body the portion that appears to be lost is really converted into another kind of motion — into a motion of the particles of the body; that is, into heat. In certain bodies, however, this latter kind of motion excites vibrations again in the ether, these vibrations having a different period. In such cases, under the influence of an irradiation, the interior of the body in its turn emits light, but the emitted light is of a different color from that absorbed. This property is possessed in particular by the solutions of a series of organic substances; it appears likewise in fluor-spar (fluorite),— after which the property is named,—a mineral colored by certain organic substances distributed in dilute form; the same property is exhibited also by the so-called uranium glass. In the two latter cases, as in the first, it is a matter of a solution of the fluorescing substance, but in a solid body. Uranium glass and the crystals of fluor-spar are singly refracting media, hence emitting light whose color is independent of the direction of the vibrations. Doubly refracting fluorescent crystals must obviously behave otherwise; and here two cases must be distinguished: —

1. When the crystal is strongly pleochroic, the color must of course vary with the direction; therefore in the case of the optically uniaxial crystals it must vary with the angle that the vibration direction of the incident light excited by the fluorescence forms with the optic axis of the crystal. According to Lommel's observations a substance exhibiting this phenomenon particularly well is the already-mentioned magnesium platinocyanide. When sunlight that has passed through a blue or a violet glass is caused to fall on the crystal face that lies normal

to the axis, the face in question shines with scarlet-red fluorescent light, in whatever direction the entering ray be polarized; but if the light falls on a face parallel to the axis, this face exhibits "dichroic fluorescence", the light emitted being orange-yellow or scarlet-red, according as the vibration direction of the exciting light is parallel or perpendicular to the axis. The same two colors are observed, of course, if instead of polarizing the incident light we cause the light emitted from the crystal to pass through a nicol, and hold this with its vibration direction once parallel and then perpendicular to the principal section; the same colors are seen also if we observe the crystal through the Haidinger lens. In violet light the green surface color (see p. 249) disappears.

2. When the crystal is colorless and accordingly not dichroic, then, from a point of its interior at which fluorescent light is excited, there must be propagated each of two rays that on emerging have equal intensity. If the two rays exhibit a difference in intensity, this circumstance proves that the vibrations of the particles exciting the light are favored in one direction. Now a research by Sohncke shows that in calcite the fluorescing particles vibrate in the greater degree parallel to the axis; in apatite, perpendicular to the axis.

### PHOSPHORESCENCE

In the case of certain substances in which, under the influence of irradiation, an emission of light takes place, the latter continues even after the irradiation has ceased; in other words, such bodies still shine for a time in the dark. This phenomenon, called **phosphorescence,** may be called forth also by rubbing or by heating. Since phosphorescence occurs likewise with crystallized and doubly refracting substances, we must assume that it, too, is subject to the laws of pleochroism; as to this, however, no researches have yet been made; and in most cases, withal, the light emitted has only a very slight intensity.

## INFLUENCE OF OTHER PROPERTIES ON THE OPTICAL PROPERTIES OF CRYSTALS*

### THERMAL PROPERTIES

When solids are heated a change takes place in the distance between their smallest particles; therefore with a different temperature the forces which the particles exert on one another, and by whose equilibrium the distance between the particles is determined, must have become different. Since, then, the luminiferous ether of these bodies stands under the influence of the forces mentioned, this ether likewise must undergo a change by the heat. As a matter of fact, it is taught by observation that on a variation of the temperature of a solid body the transmission velocity of light in the body becomes different, different in such a way that in some solids the refractive index increases with the temperature, while in the majority of those investigated

* [For the purposes of this partial translation it has seemed inexpedient to retain the classification of crystal properties described in the Introduction (pp. 3–8), which is followed throughout Part I of the original work (in the fourth edition); this classification, so far as the present translation is concerned, would be as follows:

A. Bi-vector Properties of Higher Symmetry
(*Ellipsoidal Properties*)
1. Optical Properties
2. Thermal Properties (Influence of Heat)
3. Homogeneous Strains

B. Bi-vector Properties of Lower Symmetry
1. Elasticity-properties (Influence of Elastic and Other Strains, and Optically Anomalous Crystals)

C. Vector Properties
1. Polar Piezo-electricity
2. Gliding
3. Growth (Twinning)

But it is believed that, since the other crystal properties are here considered only with reference to their influence on the optical, the classification in the following pages, which is similar to that in the *third* German edition, will be more appropriate. The text, except for the necessary rearrangement, with the change in form thus entailed, is that of the fourth edition.]

(as in all the liquids) the contrary is the case.* Which of the two phenomena occurs, increase or decrease, depends, according to the electro-magnetic theory of light, on the manner of the absorption of light in the body: most substances possess in the ultra-violet a range of metallic reflection, and on heating the body this range is shifted toward the visible part of the spectrum; by reason of this shifting there is effected, at the time when the range of metallic reflection lies near the middle of the spectrum, an increase in the refraction and dispersion, while the refraction must, in itself, with rising temperature become less.†

Since the expansion of crystals by heat is different with each of the five groups mentioned on page 196,‡ these different groups must, with reference to the changes in their optical relations effected by the expansion, be treated separately, — as will be done in the following.

1. Singly Refracting Crystals. — These, as follows from the constancy of their angles for all temperatures, and as was proved directly by Fizeau's exact measurements,§ have the same coefficient of expansion in all directions; consequently, by heating such a crystal the transmission velocity of light in it is varied equally in all directions. So soon, therefore, as the crystal has assumed the higher temperature uniformly in all its parts, its refractive index, while smaller‖ than before, has the same value in all directions: THE CRYSTAL HAS REMAINED OPTICALLY ISOTROPIC AND REMAINS SO AT ALL TEMPERATURES.

When a singly refracting crystal exhibits rotation of the

* On account of this variation, in exact determinations of the refractive indices of a body its temperature during the measurement must always be stated.

† For particulars, see the inaugural dissertation by J. Königsberger: "Uber die Absorption des Lichtes in festen Körpern". Freiburg, 1900.

‡ Cf. Phys. Kryst. 4th ed. 181–189; 3rd ed. 169–179.

§ See Phys. Kryst. 4th ed. 190; 3rd ed. 181–182.

‖ At least, this is the case with the four substances investigated by Stefan, viz. potassium chloride, sodium chloride (of these the refractive index varies greatly with the temperature), fluor-spar (calcium fluorid), and alum; while the diamond (according to A. Sella) and the amorphous glass behave in the opposite way.

polarization plane, as is the case with sodium chlorate, for example (see p. 222), its rotatory power varies with the temperature, but equally in all directions. According to Sohncke, with rising temperature the rotatory power of the salt named exhibits a considerable increase.

2. Uniaxial Crystals. — Exact determinations of the variation of the refractive indices of uniaxial crystals by heat lie at hand only on quartz, calcite, beryl, and phenacite. As for the first of these crystals, Fizeau demonstrated that when its temperature is raised the refractive index both of the ordinary and of the extraordinary ray becomes smaller, the variation being very nearly equal with the two rays. For calcite, on the other hand, he found that both indices increase, the index of the ordinary ray very little, but the index of the extraordinary very considerably An increase of the refractive indices, but one that was greater for $\omega$ than for $\varepsilon$, followed from the measurements of Dufet and Offret on beryl (emerald). (This mineral takes an exceptional position with respect also to expansion by heat.)* In phenacite, finally, Offret found for $\omega$ and $\varepsilon$ an increase that was approximately the same for both indices. According to this, with rising temperature the double refraction becomes in the case of calcite considerably stronger; in that of quartz and of beryl, on the other hand, weaker.

From the behavior of the uniaxial crystals on heating, we know that in all directions forming equal angles with the optic axis they expand equally; it is therefore to be expected that in all such directions the variation of the light velocity by heat would be the same, whether that velocity increase or diminish with the temperature. Then A CRYSTAL THAT IS UNIAXIAL AT ONE TEMPERATURE MUST BE UNIAXIAL AT EVERY OTHER TEMPERATURE; and this is confirmed by the observations made on all the numerous crystals whose behavior toward light has been investigated. Since the expansion perpendicular to the optic axis has a value different from the value parallel to the axis, so also is

* Cf. Phys. Kryst. 4th ed. 181; 3rd ed. 169.

the variation that the optical behavior suffers in the former direction, by heat, more or less different from the variation it undergoes in the latter direction; in other words, with the crystal at a higher temperature the difference in its optical behavior for vibrations parallel and perpendicular to the axis, and thus the strength of its double refraction (see the examples on p. 255), is greater or less. If the latter is the case, and if the birefringence even at ordinary temperatures is very low, then there is a temperature at which, for *one* color, the ordinary and the extraordinary ray within the crystal have equal velocity; but this applies only to the light of a definite vibration period, and the crystal has not ceased by reason of the mentioned equality to be optically uniaxial. (Cf. footnote p. 125.)

Accordingly, in consequence of a uniform increase of temperature the interference phenomena of uniaxial crystals can experience no change other than such as results from a variation in the refractive indices and in the birefringence. So the color rings produced by a plate cut normal to the axis will become either narrower or wider, but will always retain their circular form.

Like the ordinary, so also does the circular double refraction experience a variation with the temperature. This fact stands in accord with the assumption, mentioned on page 218, that the rotatory power of the crystals in question depends on a spiral grouping of doubly refracting lamellæ; for, since the double refraction of these lamellæ is influenced by the temperature, the same must be the case with the rotation of the polarization plane resulting from this double refraction. It has been demonstrated in the case of quartz that its rotatory power increases with rising temperature, increasing faster than the temperature but, perceptibly, for all colors the same amount.

3. Biaxial Crystals. — In these crystals the expansion by heat is not equal in the vibration direction of the greatest, in that of the intermediate, and in that of the least light velocity; consequently, when the crystal is brought to a higher temperature the three principal refractive indices experience unequal

variation. With aragonite, where by Rudberg's measurements this fact was first demonstrated, there corresponds to the directions of the greatest, the intermediate, and the least expansion by heat the least, the greatest, and the intermediate decrease of the refractive indices; in the case of gypsum, according to Dufet's observations, to the directions of the greatest, the intermediate, and the least expansion there correspond the greatest, the intermediate, and the least decrease in the refraction of the rays vibrating parallel to those directions, except that here the directions of the maxima and minima for different colors do not exactly coincide, because gypsum belongs with the crystals that have their principal vibration directions dispersed. The three isomorphous minerals barite (heavy spar, barium sulphate), celestite (strontium sulphate), and anglesite (lead sulphate), according to the researches of Arzruni, all exhibit the most marked decrease for the largest refractive index $\gamma$, the least for $\alpha$; but there is no such agreement in the relative expansion by heat along the three principal vibration directions; for, while with the first of these bodies the direction of greatest expansion is that of the intermediate light velocity, with anglesite it is in the direction of the least light velocity that the crystal expands the most. For barite somewhat different values were found by Offret, who showed besides, in a very comprehensive and careful research, that in topaz, cordierite, sanidine, and oligoclase the three principal refractive indices do not diminish but increase. A decrease of the refractive indices was exhibited only by those substances that have very large coefficients of expansion. It may therefore be assumed that here the decrease is caused by the great change of volume, *i.e.* diminution of density, resulting from the rise in temperature, and that the refringency proper to the substance, referred to the same density, always increases with the temperature. Offret's research embraced also the variation of the birefringence and of the dispersion by heat, and led to the following result: With all the bodies investigated the dispersion increases with the temperature; the double re-

fraction, on the other hand, not only in different substances but also for the different principal refractive indices of one and the same substance, increases in part, in part grows less.

That with every biaxial crystal the variation, produced by heat, in the light velocities corresponding to the three principal vibration directions is different for each of these directions, may easily be proved in an indirect way; and for numerous substances this proof has already been supplied. For if the three principal refractive indices are varied unequally by heat, so too is there a change in tneir ratios to one another; but on these ratios depends the size of the optic axial angle, and accordingly this angle likewise must be a function of the temperature, becoming larger or smaller when the crystal is heated. To verify this, one must employ the method described on page 187, with a modification such that during the measurement of its axial angle the crystal is at a constant higher temperature. This latter is accomplished by inserting between condensing lens and objective of the horizontal polariscope a metal box projecting some distance on both sides; and in both the front and the back wall of this box there is fitted a plane-parallel glass plate, so that, as before, light may be caused to fall through the instrument. Hence, if between these two glass plates, inside the box, the crystal is so fastened that it is centered and rotatable, and if the air in the box is heated and maintained throughout a long time at constant temperature (with the aid of an inserted thermometer), — whereby the crystal also has assumed that same temperature in all its parts, — then the measurement, arranged just as at ordinary temperatures, gives us the size of the axial angle corresponding to that higher temperature.

Now numerous determinations of the optic axial angle at different temperatures, made — chiefly by Des Cloiseaux — with the aid of a heating apparatus of this kind,* have shown that as a matter of fact the angle in question varies with the tem-

* Cf. footnote †, p. 30.

perature in all the bodies investigated. In the case of some the variation is so slight that the difference could hardly be verified by the measurement; with the greater number the angle changed several degrees when the temperature was raised 100°; while there are also crystals, finally, whose optic axes vary their direction many degrees, even when heated but slightly. Among these latter crystals is included gypsum, for example, whose axial angle diminishes so fast when the crystal is heated that even at a temperature below 100° C. it is zero; thus, at a certain temperature the smallest refractive index becomes equal to the intermediate. The crystal is then temporarily uniaxial, but of course, on account of the dispersion of the axes, at *one temperature* it is uniaxial only for *one color*, not for the others (while a really, *i.e.* permanently, uniaxial crystal is, as we know, uniaxial for all colors and for all temperatures). If a gypsum crystal is still further heated, so that the previously smallest refractive index increases still more, thus becoming greater than the previously intermediate, the axial plane is now perpendicular to its former position (in other words, on further heating, the optic axes spread out in the plane normal to the former axial plane); and one easily sees that, for the same color for which their angle, as compared with their angle for the other colors, had previously the smallest value, the axes now include a greater angle than for any other color. The same phenomenon is exhibited by sanidine and by glauberite; with the latter mineral, whose crystals have a very small axial angle, the uniaxial state, arising for the different colors in succession, exists for violet, blue, and so on to red, light within the small range of temperature from 17° to 58° C.

The last-named substances belong, besides, to one of the groups of crystals whose principal vibration directions for different colors do not coincide, wherefore the principal thermic axes,* also, have an orientation different from that of the principal

* [These are defined by the author elsewhere as the only crystallographic directions that remain mutually perpendicular at all temperatures.]

vibration directions. In agreement with this, in these crystals the direction of the acute bisectrix, likewise, is dependent not only on the color but also on the temperature; that is, with rising temperature there is a change in the orientation of the bisectrix as referred to the crystal plate, and consequently in the field of view one observes an unequal angular movement of the two optic axes. For example, if we gradually heat a gypsum plate cut perpendicular to the acute bisectrix for intermediate colors, we see distinctly that the two axes approach each other with unequal velocity, until, when the temperature reaches a certain point, they coincide in a direction that stands perceptibly oblique to the plate; on further heating, the two axes recede from each other in the plane that stands perpendicular to the former axial plane, receding equally fast, — because in gypsum one principal optic section is common to all colors, wherefore with respect to this plane all optical phenomena take place with absolute symmetry,— but the position of the bisectrix is shifted farther in the same sense as before. In crystals where all three principal vibration directions for all colors have identical orientation, there exists, for all temperatures, complete symmetry of the optical properties with reference to the three mutually perpendicular principal sections (so far as the substance remains unaltered). Such a crystal, therefore, subjected to the same experiment, exhibits a mutual approach or recession of the optic axes that is absolutely equal in rate for the two axes; in other words, it exhibits constant orientation of the bisectrix.

Hence, from the phenomena just described it follows that, in consequence of the variation of the double refraction with the temperature, when the heat content of a biaxial crystal is different the index-surface of the crystal has a different form (*i.e.* different axial ratios); but that its crystallographic orientation is constant, provided there exists no dispersion of the axes for the different colors. Otherwise its orientation, also, depends on the temperature.

## ELASTIC STRAIN BY MECHANICAL FORCES

### Homogeneous Strain

The expansion of crystals by heat is to be regarded as a special case of "homogeneous strain", or "homogeneous deformation". By such a strain, or deformation, we understand a change in the shape of a homogeneous body by which the body does not cease to fulfill the conditions of homogeneity (see p. 3), and for which the following relations exist between the original body and the strained body: First, any straight line in the unstrained body is straight likewise after the strain, while by the strain the direction of that line in general suffers a variation; second, every two straight lines that are parallel in the original body are parallel also in the strained body, whatever change may have taken place in their direction.*

The change produced by a homogeneous deformation, in the linear dimensions of a crystal, necessitates a displacement of the crystal particles from the position where the forces acting among them are in equilibrium. This calls for a certain amount of work. In the case of thermic expansion this work is done by the heat added; in the other case it requires some external mechanical force. When the latter ceases to act the particles resume, in consequence of the above-mentioned internal forces of the crystal, the relative position that corresponds to equilibrium of those forces; that is, when the strain produced by the external force does not exceed a certain limit. This power of a solid body to resume its former shape is called "elasticity"; and accordingly those changes in a solid that, in consequence of its elasticity, disappear when the straining forces cease to act, are designated as "elastic strains". The value which the external force must not exceed if no permanent alteration in the shape of the body is to arise, is termed the "elastic limit". Within this limit the displacement of the particles, and therefore the deformation of the body, is proportional to the forces acting on the body.

* From these relations it follows directly that a plane appearing as a boundary-face of the original body is, after the strain of the body, still a plane, but one whose position, *i.e.* whose angle with other planes bounding the body, can have experienced a variation; and it follows, further, that every two parallel planes bounding the body are, after the strain, likewise parallel, in whatever manner or degree their position may have changed.

When the external force acting on a crystal is so great that the elastic limit is overstepped, deformations generally arise which are *not homogeneous;* and if the "limit of solidity" is exceeded, *i.e.* if the external force is so great as to completely separate the component particles, the crystal altogether ceases, as a unit, to exist.

Since the homogeneous strains produced by equal pressure or tension in all directions correspond absolutely to those effected by a uniform change in temperature, the exposition given on page 253 *et seq.* of the influence of the latter strains on the optical properties of crystals applies also to the influence of the former. The changes they produce in the velocity of light are accordingly *equal in all optically equivalent directions;* so that A SINGLY REFRACTING CRYSTAL IS STILL SINGLY REFRACTING EVEN AFTER THE STRAIN, A UNIAXIAL CRYSTAL STILL UNIAXIAL, AND SO ON.

Those mechanical forces that effect homogeneous strains (equal pressure or tension from all sides) are the only ones with which, as with a uniform change in temperature, the *cause of the strain* is *scalar*, — *i.e.* independent of the direction. In such cases, that the effect produced in the crystal is different in different directions is only because the crystal is not constituted alike in different directions and consequently does not offer the same resistance to deformation. The relations are quite different, and considerably more complex, when the cause of the strain is a *vector* quantity; in other words, when a mechanical force acts in a crystallized body *in a certain direction.* The effect of definitely directed forces of equal magnitude, but of different orientation as referred to definite crystallographic directions, can no longer be represented in the simplest case (*i.e.* for a singly refracting crystal) by a sphere, nor in the most general case by a triaxial ellipsoid whose three axes determine the numerical value of the property in question for all the remaining directions in the crystal. The crystal properties of elasticity and cohesion, which belong under this head, are *never* equal in all directions. Moreover, in the case of such a deformation the crystal itself generally ceases to be homogeneous.

### Elastic Strain not Homogeneous

The differences produced in the constitution of a body in different directions by tension or pressure acting in a definite direction, are transferred to the constitution of the luminiferous ether standing under the influence of the particles of the body;

so that vibrations taking place in directions that previously were optically equivalent are now no longer transmitted by the ether with equal velocity. The laws after which these variations take place were investigated experimentally by F. Neumann, and explained theoretically on the basis of this assumption: that, when an amorphous body is extended or compressed uniformly at all points in a definite direction, it takes on the properties of a homogeneous optically biaxial crystal in which the axes of the index-surface coincide with those of the ellipsoid of strain.* In a body that is extended or compressed ununiformly, although the above must indeed be assumed for every individual point, yet the form and the orientation of the optical index-surface vary with the locality in the strained body. Since the behavior of crystals toward straining forces is different in different directions, the optical relations of crystals in the strained state, even when the crystals in question are singly refracting, are very much more complex than with amorphous bodies; let us therefore treat first the phenomena to be observed in the latter bodies.

Should we take a cylinder made from ordinary glass, and compress it perpendicularly to the two circular basal faces in such a way that its interior were strained uniformly at all points, it would take on exactly the same optical properties as are possessed by a negative uniaxial crystal whose optic axis lies in the direction of the pressure.† By *compression*, therefore, the glass

* In a crystal subjected to a homogeneous strain there always exist three mutually perpendicular directions whose position is not altered by the strain; and parallel to these directions there take place the greatest, the intermediate, and the least variation of the linear dimensions of the crystal. These directions are designated as the "principal axes of strain". The increase or decrease of the linear dimensions along these three principal axes is in general different for each axis, so that a sphere made from the crystal is transformed by the strain into a triaxial ellipsoid — the "ellipsoid of strain".

† This effect of pressure can be very beautifully demonstrated in soft gelatin, if a circular plate of the same is brought, between glass plates, upon the stage of the Norrenberg polariscope and the ocular-tube screwed down until the gelatin plate is compressed: the plate then exhibits the normal interference-figure

assumes negative double refraction. Precisely the same *extension*, in the same direction, would produce the opposite effect, and the glass would behave like a uniaxial crystal with positive double refraction; by such a strain, therefore, the glass becomes *positively* doubly refracting.

The fact that in glass *compression* in the direction of vibration of a ray does indeed call forth an *increase* of its transmission velocity, and *extension* a *decrease*, can be proved with the aid of the arising phenomena of double refraction, whose character may be determined by one of the methods described on page 198 *et seq.* These phenomena, and both of them (that produced by pressure and that produced by tension) simultaneously, moreover, can be brought into view most simply by means of the following experiment, suggested by F. Neumann: Two strips of plate-glass are taken, 2 or 3 decimeters in length, about ½ decimeter wide, and from 5 to 6 mm. thick, and between them is laid a metal wire; the glass strips are then gradually bent together, best by means of clamp-screws, until their ends touch, so that the concave sides of the strips are turned toward each other. (See Fig. 116, representing the glass strips as seen from the side and from above.)

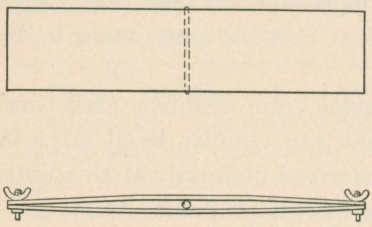

Fig. 116

In consequence of this bending the outer, or convex, faces of the plates have become longer than before, but the inner, or

of a uniaxial crystal. A mixture of wax and resin, pressed moderately between two glass plates, behaves in just the same way; and such a preparation exhibits the phenomenon even permanently. However, many authors assume that this phenomenon depends on the presence in such organic substances of very minute crystalline particles: that these particles lie irregularly distributed, so that the several doubly refracting effects which they exert on the light are mutually neutralized, while by pressure they become oriented parallel to one another and now exercise a common optical effect. Moreover, flint glass having a high lead content behaves differently from ordinary glass, in that by compression it becomes ~~negatively~~ positively doubly refracting.

concave, faces shorter; the former faces accordingly are stretched in their long direction, while the latter are compressed in that same direction. This is shown more clearly by Fig. 117, the amount of bending as here represented being far greater than is really possible with strips of glass. In the outermost layer of

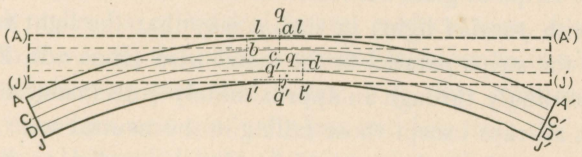


Fig. 117.

the glass, *e.g.* at *a*, the direction of the extension is parallel to *ll*; at this point therefore the least light velocity is that of the rays vibrating parallel to *ll*, and the greatest (namely, equal to the transmission velocity in the uncompressed glass) that of the rays vibrating parallel to *qq*. At a point *b*, lying farther inward in the strip, there is still a lengthening, although a less considerable one, in the direction *ll*; so here, too, the vibrations parallel to *ll* have the least transmission velocity and those parallel to *qq* the greatest, but the difference between the two velocities is smaller. Along the middle of the glass strip, *i.e.* at *c* and all points of the line *CC'*, which passes through *c*, the difference in question will be zero; this is the line that has the same length as with the glass strip in the unbent state, the line that has experienced neither a compressing nor a stretching. On the concave side, on the other hand, the glass strip, by reason of the bending, has been compressed in its long direction; and this compression is less considerable along the line *DD'* than along *JJ'*, the latter line having become shorter than any other in the strip. Both at *d* and at *j*, in consequence of the compression, the transmission velocity of the rays vibrating parallel to *l'l'* is the greatest, and that of the rays vibrating perpendicularly to them, or parallel to *q'q'*, the least; but at *j* the difference between the two velocities is at its maximum. Accordingly the glass strip must be doubly refractive throughout its whole

thickness as far as a zone in the middle; the double refraction is strongest in the outermost and the innermost zone, but in the one zone negative and in the other positive, while from these two extremes it diminishes in strength inwards to the neutral zone mentioned, where it is zero. Hence, if we bring the two bent strips of glass between two crossed nicols (in the polariscope with *parallel* light) in such a way that the light rays pass through the strips parallel to the metal wire (shown in Fig. 116) and accordingly through an approximately $\frac{1}{2}$ decimeter thickness of glass, all rays except those falling in the neutral zone are split up into two polarized rays, which vibrate parallel to *ll* and *qq* and which are transmitted with different velocity. With the dimensions given, a degree of bending even less than represented in Fig. 116 suffices, in the inner and the outer boundary-layer, to impart to the rays arising by the double refraction a difference of path that amounts to several wave lengths; in the zones lying nearer the center the path difference acquired is naturally smaller. At a certain distance outside the neutral zone the path difference will be exactly $\lambda$; at this distance therefore, with crossed nicols, there will occur total annihilation of the light, just as in the singly refracting zone itself; and the same will take place at the distances where the arising difference of path is $2\lambda$ etc. In case the glass strips are placed with their long direction parallel to one of the nicols they will naturally appear dark all over. But with the strips in any other position, when the light used is homogeneous, there must appear, between the zones of 0, $\lambda$, $2\lambda$, etc., difference of path, zones of brightness; and the brightness of these zones is greatest half-way between two dark ones, since on these intermediate lines the difference of path amounts to $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, etc. The greatest difference in intensity between the bright and the dark zones is found, of course, when the long direction of the glass strips forms 45° with the vibration directions of the nicols. Hence follows as a matter of course the phenomenon arising in white light: The neutral zone at the center of the glass strip is black throughout its whole length;

on each side of the neutral zone, parallel to it and to the outer faces of the glass strip, there appear stripes of color, and the colors of these stripes lie in exactly the same sequence as in the color rings (appearing in convergent light) of uniaxial crystals cut perpendicular to the axis.

If we take a circular plate of glass and wind a strong cord around it upon its cylindrical lateral surface, then, if we draw tightly on the cord, the plate is pressed together from all points of the periphery toward the center; it is thus compressed the most at the edges, and gradually less and less from the edges to the center. In parallel light all points of equal pressure, *i.e.* all points lying on the circumference of a circle concentric with the plate, exhibit the same color; there accordingly appear the circular isochromatic curves with the black cross, exactly as when uniaxial crystals are observed in convergent light.*

If we press together a square piece of glass, not uniformly from two opposite faces but only from two points, and bring it into parallel polarized light between crossed nicols in such a way that the straight line connecting these two points includes 45° with the nicols, the parts near the two points of pressure become lighted up, and with greater pressure a color arises. But, from the two points of pressure this lightening becomes less in all directions; that is to say, directly at these points the glass is under the greatest compression, and the compressing stress diminishes with increasing distance from them; the central portion is not under stress at all and hence remains dark. When the pressure exerted is greater, however, the adjacent particles of the glass will have assumed a so much greater degree of compression in the direction of pressure than parallel to it that the two wave systems arising by double refraction are displaced, as compared with each other, to the amount of several wave lengths;

* A small amount of double refraction is exhibited by lenses in optical instruments, if pressure is exerted upon them by their mounting. In a conoscope containing such lenses we can of course, between crossed nicols, obtain no uniformly dark field of view.

and accordingly, at the point of pressure there appears a color of the third or the fourth order. Figure 118 shows the phenomenon then observed in parallel light when the direction of pressure, *ab* (indicated by arrows), is parallel to the principal section of a nicol: since from *a* and *b* the compression diminishes in all directions, so also does the strength of the double refraction, and consequently at greater distances there appear other interference colors; these colors yield, together, a figure made up of isochromatic curves, which is very similar to the well-known lemniscate system of biaxial crystals.

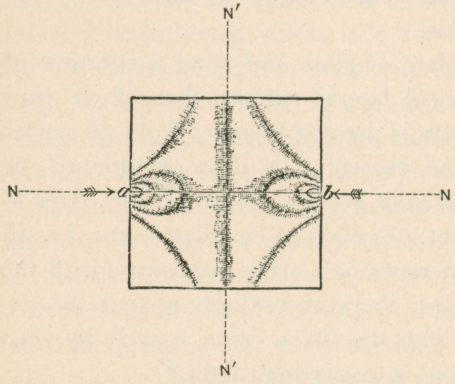

Fig. 118.

If a piece of glass is heated, but not uniformly, then owing to the unevenness of the expansion there arise in it extensions and compressions, and the corresponding phenomena of double refraction appear. These latter can be made *permanent*, moreover, by bringing the glass to a high temperature and then rapidly cooling it; we then obtain "cooled glasses", so called, which yield in polarized light, according to their shape, the most multifarious interference-figures.

Among the amorphous organic bodies there are many so-called colloid substances (as collodion, gelatin, etc.), and these have the property, in passing from the dissolved to the solid state, on removal of the solvent by drying, of exhibiting considerable contraction. If such bodies are caused to dry under circumstances such that the bodies cannot shrink uniformly in all directions, then in the directions in which the contraction was hindered they must be in a state of *strain*. If, for example, a strong solution of gelatin is poured while warm into a frame

lying on a glass plate (the frame should be provided with a handle), and if after the congelation the resulting colloid cake is lifted from the glass plate and allowed to dry freely in the frame, then the cake adheres to the frame all around and accordingly cannot shrink in its own plane. After the gelatin has hardened completely a plate thus formed is therefore in a state of strain in all directions parallel to its plane. Should we succeed in making this strain equal in all these directions, such a plate would necessarily exhibit in convergent light the interference-figure of a uniaxial crystal; since, however, the gelatin does not dry uniformly in all its parts, and many times even comes loose from the rim in places, it always happens, in fairly large plates, that only single parts have remained in a sufficiently uniform state of strain to exhibit the phenomenon mentioned. Commercial gelatine plates, such as are used by lithographers, sometimes exhibit the dark cross distinctly, and when several of them are arranged one above the other the color rings also appear; in these plates the double refraction is negative, whence it follows that tension has just the same influence in gelatin as in glass. But with plates congealed in a frame, on the other hand, the strain in different directions parallel to the plate is in general unequal; so the plate must assume the properties of a biaxial crystal. As a matter of fact, every part of a plate of this sort in which the strain remains nearly constant over a certain extent of the plate, exhibits in convergent light the interference-figure of the biaxial crystals; but the figures yielded by different parts of one and the same plate have not the same axial angle, nor their axial planes the same direction, because at the various points of such a plate there is a diversity in the relative amount of strain in different directions and in the orientation of the maximum strain. Finally, the distribution of the strain naturally depends on the form of the frame in which the congelation took place. In the case of thin plates, because of the double refraction being small in amount, the interference phenomenon becomes reduced to the appearance, on a light background, of

the dark cross or the dark hyperbolas (according as the optic axial plane at the point in question lies parallel to one of the nicols or oblique to both); therefore one observes only the central part, as it were, of the phenomenon represented in Fig's 83*b* and 84*b*. But with thicker plates the color rings, likewise, appear in the field of view.

Similar behavior is exhibited by horn, animal bladder (especially when in several layers), and other organic substances. Finally, also, in the above-mentioned cooled glasses there are places that give distinct axial figures; and where the axial angle is rather large these figures manifest a distinct dispersion of the axes, whence it must be concluded that by pressure the transmission velocity of the light of different colors is influenced unequally.

Like the transmission velocity, so too is the absorption of light influenced by strain. In consequence of this a pressure or tension that transforms an amorphous body into a doubly refracting one will, in case the body has a decided color, produce *dichroism* as well. In fact Kundt, with the aid of Haidinger's lens, demonstrated the existence of this " temporary dichroism", first in stretched plates of india-rubber and gutta-percha.

The fact that in crystals, just as in amorphous bodies, directed pressure or tension produces a change in the constitution of the ether, was observed by Brewster as early as the year 1815; yet it is only recently that F. Neumann's theory of the variation of the optical properties of amorphous bodies by elastic strains was applied to crystals. This was done by Pokels, who showed also in what way the variation of the optical index-surface is connected with the strain ellipsoid of the crystal.

[To explain this connection it is necessary to take into account the behavior of the crystal itself in respect of elastic strain; and let us first consider the relation of stress to strain in the most simple case.] If a homogeneous prism-shaped bar of square or rectangular cross-section is subjected to a compressive or tensive stress parallel to its long direction by means of a weight

$P$ resting on or hanging from one end, while the other end remains fixed and the bar in vertical position, the bar suffers a corresponding strain, denoted by $\lambda$; if, therefore, it had previously the length $L$, it now has the length $L \pm \lambda$. Observations made on a bar of rectangular cross-section whose breadth, $B$, and whose thickness $T$ — the cross-section thus having the area $BT$ — are slight as compared with its length, show that so long as the elastic limit is not transgressed the strain $\lambda$ is determined by the equation

$$\lambda = \frac{PL}{BT} \cdot E.$$

In other words, the lengthening or shortening of the bar is proportional to its length and to the stress producing the strain, as well as inversely proportional to the cross-section of the bar; but, in addition, it is proportional to $E$, a factor specific to the substance of the bar and independent of its dimensions. This quantity is termed the "coefficient of extension". If we place $P = 1$ (1 gram, for example), $L = 1$ (*e.g.* 1 meter), $B = T = 1$ (as 1 millimeter), then $\lambda = E$; that is, the coefficient of extension is the elongation experienced by a rectangular bar of unit length and unit cross-section when loaded with unit weight.

If by methods depending on the above we determine the coefficient of extension of an amorphous body, as glass, we find it always the same, in whatever direction a bar is cut from the body. Thus, for a given amorphous substance the coefficient has a constant value independent of the direction. *That this quantity E is independent of the direction applies, however, only to amorphous bodies.* If on the other hand we cut bars in different directions from a crystal, we find that, although bars whose long direction has the same crystallographic orientation have the same coefficient of extension, bars oriented differently from one another give different values for this quantity; that is to say, *in a crystal the coefficient of extension varies with the direction.*

The manner in which this coefficient varies with the direction depends on a series of quantities called the "constants of elasticity" of the crystallized substance in question; these constants number twenty-one in the most general case, while with the majority of crystals they are reduced in number, several of them becoming equal in accordance with definite laws. As for what concerns more especially the coefficient of extension, *i.e.* the strain in unit length along a direction in which there operates unit stress, a general view of its characteristic dependence on the crystallographic direction is obtained, if from a definite point one imagines as laid off in every direction the length proportional to the quantity $E$ for the crystallographic direction in question and thinks of the extremities of all these lengths as connected by a closed curved surface, — the so-called "surface of extension coefficients". This surface (which, according to what was stated above, is for the amor-

phous bodies, and them only, a sphere) has for different crystals a multifarious and, in part, most complicated form; but it is always centrally symmetrical, the two extremities of its every diameter always being equidistant from the center of the surface. According to which of the elasticity constants are equal, the surface calculated from the remaining ones assumes a more or less symmetrical form, which stands in a regular relation to the symmetry of the geometric form of the crystals in question, *i.e.* to the existence in those crystals of a larger or smaller number of equivalent directions; and these directions then prove to be equivalent in respect also of the elongation parallel to them. As is taught by theory and confirmed by experiment, there exist, with reference to the form of the surface of extension coefficients, *nine different groups of crystals*.

The first of these, designated as Group I, includes all the *singly refracting* crystals. For this group the surface in question has three equal diameters, which are mutually perpendicular and correspond either to the absolute maximum or to the absolute minimum of the extension coefficient; likewise equal are the four diameters that include equal angles with these three, the latter diameters being the shortest when the former are the longest, and vice versa; and each of the four planes passing through the center perpendicular to the latter directions intersects the surface in a circle.

The *optically uniaxial* crystals constitute Groups II, IIIa, IIIb, IVa, IVb (the "a" and the "b" groups corresponding to a higher and a lower symmetry of surfaces otherwise similar). The surface for Group II is a rotation figure produced by rotation, about the optic axis, of a curve symmetrical to that axis and having maxima and minima both in the optic-axial direction, in the direction perpendicular to it, and between these two directions. For Groups IIIa and IIIb the surface has only one circular section, which passes through the center perpendicular to the optic axis. For IVa and IVb the surface has no longer any circular section, has no plane, therefore, in which, under the same circumstances, the elongation is equal in all directions; but its diameter is still equal in two directions, these lying mutually perpendicular in the plane normal to the optic axis.

With the *optically biaxial* crystals, comprising Groups V, VI, and VII, the surface of extension coefficients not only has no circular section, but its symmetry, which for the preceding groups has become less and less, now almost entirely vanishes. For Group V, which includes only those crystals whose index-surface has identical orientation for all colors, the surface of extension coefficients is still symmetrical with reference to three mutually perpendicular planes; for VI, embracing the biaxial crystals in which only one axis of the index-surface is the same for all colors, the

extension-surface is symmetrical only to one plane; for VII, finally, or those biaxial crystals in which all the vibration directions are dispersed, the surface in question can be symmetrical only to its center. * †

Since, then, the behavior of a crystal with regard to compression and extension is different according as it belongs to one or another of the nine groups mentioned above, there is a corresponding difference in the variation of the optical properties.

1. SINGLY REFRACTING CRYSTALS, subjected to pressure (or tension) of whatever crystallographic orientation from one side, become doubly refracting; the optical change is proportional to the pressure, and for any direction of the same the form and orientation of the optical index-surface, generally biaxial, may be calculated, by means of the above-mentioned theory (which Pokels put to experimental test for several bodies belonging in this class), from the following data: First, the strain existing in the crystal, this being determined by the elasticity constants of the crystal and by the forces acting upon it; second, the optical constants of the crystal in the unaltered state; and third, certain quantities to be obtained by observation, which determine the variation of the optical properties. In order that a crystal of this group may become uniaxial, the direction of the pressure must be either normal to one of the four circular sections of the surface of extension coefficients, or else parallel to one of the three mutually perpendicular maximum or minimum diameters of that surface. (Cf. p. 272.)

2. OPTICALLY UNIAXIAL CRYSTALS when subjected to a pressure in the optic-axial direction become either stronger or weaker in their double refraction; but always remain optically uniaxial. For then, likewise, the direction of pressure corresponds to a circular section of the surface of extension coeffi-

* [The foregoing analysis is an abstract of that to be found in Phys. Kryst. 4th ed. 216–224; for additional particulars of the nine groups, and explanatory figures, see ibid. or in the 3rd ed. 201–208.]

† [Group I comprises the isometric crystal system, II the hexagonal, III a and b the trigonal (see footnote, p. 197), IV a and b the tetragonal, V the orthorhombic, VI the monoclinic, VII the triclinic.]

cients, or else to a direction perpendicular to which there exist two directions having the same extension coefficient. (Cf. on p. 272, Groups II–IVb.) But acting in any other direction, *i.e.* from one side, the pressure effects a strain that gives the crystal biaxial properties.

For observing the phenomena that appear in this case, service is made of the apparatus (pictured in Fig. 119) constructed by Bücking, which enables one at the same time to measure the intensity of the pressure exerted on the crystal. The apparatus consists in the first place of a brass disk, *b*, with a hole in its center, and this disk can be fastened to the Norrenberg polariscope in place of the rotatable stage for carrying the crystal.

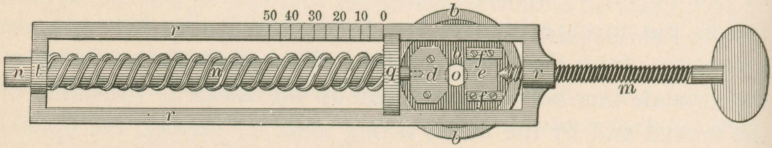


Fig. 119.

Upon the brass disk there is screwed firmly a steel plate, *d*, and on the opposite side of the opening, *o*, in the disk there is a second steel plate, *e*, movable between two guide-bars, *f*, *f*; the crystal plate to be investigated, resting over the opening, *o*, is laid against the first steel plate, *d*, and compressed through the medium of the second plate, *e*, by turning the screw, *m*. The latter passes through a strong brass frame, *r*, in the other end of which, at *t*, is drilled a round hole; movable lengthwise in this hole is a cylindrical brass rod, *n*, carrying at its end a disk, *q*; and the sides of the frame, *r*, are let into the edge of *q* in such a way that the latter can move only longitudinally, and not rotate. About the cylindrical rod, *n*, there is wound a strong spiral spring, which presses, when in the compressed state, at its one end against the cross-bar, *t*, of the frame and at its other end against the brass disk, *q*; at the center of the latter, on the side turned away from the rod, *n*, there is a pin, which projects into a depression in the plate, *d*.

If, then, after the crystal plate has been inserted the screw, *m*, is turned, the frame, *r*, is brought closer to the head of the screw and the spiral spring thereby compressed; the resisting stress in the spring acts through *q* upon the plate *d*, also through *m* upon the plate *e*, and thus upon the crystal as well, lying between those two plates. One of the longer sides of the frame is graduated, and the zero point of the scale corresponds to the position occupied by the disk *q* when the spring is under no strain; through this graduation the compression of the spring, and thereby the pressure exerted on the crystal, can be read off directly in kilograms.

With the aid of this apparatus it is easy to satisfy one's self as to the presence or absence of double refraction in a cube prepared from glass or a singly refracting crystal and subjected to pressure. To examine the behavior of a uniaxial crystal we cut from such a crystal a small cube having two of its pairs of faces parallel to the optic axis, the third pair perpendicular to the axis. If we bring this cube between the two steel plates *d* and *e* in such a way that two faces of the former kind lie against the plates, the optic axis of the crystal thus standing perpendicular to the plane of the figure, then on our turning the screw the crystal undergoes a compression whose direction is perpendicular to the optic axis. If, then, the compression apparatus is joined to the polariscope (for convergent light), one beholds in the instrument, so long as the plate is subjected to no pressure, the normal interference-figure of the uniaxial crystal. But if one turns the screw, the circular color rings begin to elongate into ellipses; and when the direction of this elongation lies diagonal to the principal sections of the nicols of the instrument, the arms of the cross separate at the center and become transformed into hyperbolas. In short, there arises the interference-figure of a biaxial crystal with small axial angle, the size of this angle increasing, however, with the pressure. As regards the plane of the two optic axes now existent, its direction depends on whether the double refraction of the uniaxial crystal in question

was positive or negative. If, as in glass, (see p. 264) by the pressure the velocity of light rays vibrating in the direction of pressure is increased, then the result as regards the position of the axial plane must, with the two kinds of uniaxial crystals, be exactly the opposite, — as is readily seen. For, since with *positive* crystals the rays vibrating parallel to the axis have the least velocity and those vibrating perpendicular to it the greatest, then, if for the latter rays the velocity is by the pressure increased, the vibration direction of these rays is still that of the greatest light velocity, and the vibration direction lying perpendicular to this direction in the same plane becomes that of the intermediate light velocity, while the direction that was the optic axis remains the vibration direction of the least light velocity; and since the optic axes of a biaxial body always lie in the plane of the first- and the third-named of these three vibration directions, the mutual recession of the two axes in consequence of the pressure occurs in a plane that is *parallel to the direction of pressure*. Since the axial angle resulting from pressure is always small, what was previously the optic axis always remains the acute bisectrix; and thus the biaxial crystal arising from a positive uniaxial one is likewise positive. In the case of the *negative* crystals the plane perpendicular to the optic axis contains all the vibration directions of the least light velocity; this light velocity is increased in the direction of pressure, in consequence whereof the direction named becomes vibration direction of the now intermediate light velocity and the vibration direction lying perpendicular to it in the same plane that of the least, while the direction that was previously the optic axis remains vibration direction of the greatest light velocity; accordingly, the mutual recession of the optic axes takes place in a plane standing *perpendicular to the direction of pressure*, and the arising biaxial crystal is negative. Now the above-described behavior is exhibited in fact by all the uniaxial crystals investigated up to this time; moreover, Bücking's experiments have shown that in a uniaxial crystal a relatively slight pressure suffices to call

forth a small axial angle, but that in order to make the angle larger a very much greater pressure is requisite, — proving that the variation of the axial angle is not proportional to the pressure.

Particularly interesting results are yielded by the application of pressure to such uniaxial crystals as rotate the polarization plane of light; *e.g.* to quartz. This crystal is positive in its double refraction and therefore acquires, when compressed at right angles to the axis, two axes in the plane parallel to the direction of pressure. In the arising axial figure, at the centers of the rings surrounding the two axes, a coloring then appears which on rotation of the analyzer varies, — just as such a coloring appears at the center of the interference-figure of an uncompressed quartz plate (see p. 230); so it follows that in the two optic-axial directions as well there must occur a rotation of the polarization plane of light. The experiments conducted by Mach and Merten have shown, first, that in each of these directions there are transmitted, not two circularly vibrating rays, but two rays vibrating in ellipses; second, that in each direction the two vibrations take place in the opposite sense and on emerging combine to form one single vibration, whose path is also an ellipse; third, that the major axis of this elliptical path is rotated in the sense of the ray having the greater transmission velocity; fourth and finally, that therewith the difference of path, as compared with that of the two circular rays in normal quartz, has, by the pressure, even been augmented. Taking into account that, according to Gouy's theory (see p. 223), the path difference produced by the ordinary double refraction is superposed on that called forth by the circular, Beaulard carried out a research by investigating the ellipticity of light rays that after having been plane-polarized were caused to pass through compressed quartz in various directions; his conclusions are that by pressure the rotatory power of quartz suffers no change, only the double refraction being altered, and that in each of the two optic-axial directions (forming in the case in question an angle of 13°) there exists normal circular double refraction.

3. Optically biaxial crystals, subjected to pressure parallel to one of their three principal vibration directions, must experience a variation of their axial angle, since the pressure effects a change in the ratios among the axial lengths of the optical index-surface, and on these ratios depends the size of that angle. The biaxial crystals as yet investigated with Bücking's appliance have shown that with them, too, the pressure effects an increase in the velocity of rays vibrating in the direction of pressure. Suppose, for example, that we have a cube cut from a *negative* crystal, with its sides parallel to the three principal optic sections, and let this cube be so placed in the instrument that the acute bisectrix stands vertical, — so placed therefore that in the instrument we see the interference-figure with the lemniscates. Then, if the crystal is compressed in the direction of the $Y$-axis of the index-surface, *i.e. perpendicular to the axial plane*, the intermediate light velocity becomes greater; it thus approaches the greatest light velocity, and, since this latter is the velocity parallel to the acute bisectrix, the axial angle must *increase*. Now if we rotate the crystal 90° about the acute bisectrix and subject it to pressure from the other two sides, *i.e.* parallel to the vibration direction of least light velocity, this light velocity must in its turn become greater, thus approaching the intermediate; in consequence the lengths of the two axes of the index-surface that, standing perpendicular to the acute bisectrix, lie horizontal in the instrument are then less different from each other, and the crystal, in its optical properties, approaches one in which these two directions are equivalent; in other words, it grows more like a uniaxial crystal, and thus the axial angle must become *smaller*. It is plain that in the case of a small axial angle, *i.e.* when the principal refractive indices $\beta$ and $\gamma$ differ but little even at the outset, a certain pressure will suffice to make the two quite equal; the crystal then becomes *negatively uniaxial*. If the pressure is brought still higher, then what previously was $\gamma$ becomes smaller than $\beta$, the $Z$-axis of the index-surface thus becoming the intermediate

axis; so the optic axes must recede from each other, but in a plane standing perpendicular to their former plane. That the crystal is uniaxial under the pressure with which $\beta = \gamma$ applies, however, only to one single color, since the change that pressure produces in the transmission of light is unequal for different colors; therefore, if we employ a pressure that makes the crystal uniaxial for medium colors, the optic axes for the one part of the spectrum still lie in the same plane as before, while for the other part they have already spread out in the new axial plane; such a plate accordingly behaves, when its dispersion is great enough, like the substances having crossed axial planes, described on pages 172–173. With *positive* crystals, as is taught by an analogous consideration, a pressure acting perpendicular to the plane of the axes will *diminish* their angle (transforming the crystal, when it has only a small angle to start with, into one that is positively uniaxial), while a pressure applied parallel to the axial plane will make the axial angle *larger.* It is evident, finally, that, when the direction of pressure is parallel to *none* of the three principal vibration directions, but oriented in any other way, there must occur a variation not only in the form, but also in the crystallographic orientation, of the optical index-surface.

## Optically Anomalous Crystals

We have learned from the foregoing that definitely directed pressure or tension in a crystal has, as its effect, that the crystal takes on properties different from those pertaining regularly to the group of crystals to which the crystal in question normally belongs. When the forces in question do not act uniformly on all parts of the crystal, the arising phenomena as well, just as in the strained amorphous bodies described on page 264 *et seq.*, vary with the *locality* in the crystal; the crystal, too, has then ceased to be homogeneous. Now supposing that, in a crystal, from any cause the deviations of its optical properties from those in the normal state of the crystal have become permanent,

the crystal is said to be **optically anomalous.** (Cf. p. 219.) In the diamond, for example, a singly refracting crystal, by reason of inclusion during the formation of the crystal there not infrequently exist foreign bodies (crystals of other minerals), and these are surrounded by a zone in which the diamond exhibits distinct double refraction. The cause of this phenomenon is the inequality between the coefficients of expansion of the two bodies, whose crystallization undoubtedly took place at a very high temperature: in consequence, while cooling to the present temperature the two bodies experienced a different degree of contraction; so that now the parts where they border on each other, whatever be the direction of the normal to the boundary-surface, must be in a state of compression or of extension. In colored crystals, since in colored bodies pressure influences also the absorption of light (see p. 270), when the crystal includes smaller ones of another substance there may arise also **pleochroic halos**, in the manner described.

Crystals having great plasticity (see p. 283), which accordingly very easily experience permanent strains, frequently suffer local compressions, and in consequence there arise optical anomalies. Thus the singly refracting halite (rock-salt) exhibits after compression, or indeed even after breaking it roughly, the stripes characteristic of double refraction.

Many other crystals that, to judge from their other properties, belong like those mentioned above to the singly refracting group, but which contain no recognizable inclusions and have been subjected to no strain subsequent to their formation, exhibit nevertheless throughout their whole extent distinct phenomena of double refraction; these phenomena are such that the crystals appear to be composed of regularly arranged parts in which the vibration directions are differently oriented. There are likewise crystals that, although optically uniaxial in form, are really biaxial, having moreover in different parts axial angles of different size and axial planes of different orientation. These optical anomalies (which, like the amorphous bodies described

on page 264 *et seq.*, were observed first by Brewster), Reusch sought to explain by the assumption that during their crystallization the substances in question, similarly to solidifying colloidal bodies, experience a certain contraction, and that in consequence there arise in the crystal a permanent strain. In crystals of alum, for example, the double refraction looks as though the substance of the crystal were strained within certain planes and as though, while the crystal was being built up, the superposition in layers had taken place parallel to these planes. The distribution of the anomalous state in question would then of necessity stand in a certain relationship to the outward form of the crystal; and such is indeed the case with substances of the kind now considered. For this reason, in connection with the above explanation of the phenomenon, the experiments are of interest that have been carried on with amorphous bodies to which a shape similar to that of the crystals in question was given artificially. To this end Ben Saude prepared molds of crystal models and filled them with gelatin, allowing the latter to dry two or three days before removal from the mold; he then cut plates from the gelatin in definite directions, and in order to prevent further drying out the plates were packed in Canada balsam. Now, as a matter of fact, these gelatin plates did indeed exhibit double-refraction phenomena analogous to those that are exhibited by optically anomalous crystals having the same shape as the gelatin models; in particular, they manifested a division into sectors extinguishing in different directions, the position of the sector boundaries depending on the shape of the model.

But there is no way of proving Reusch's assumption—that during the process of crystallization a crystallized substance, like a colloid passing gradually over into the solid state, suffers a change in volume. Accordingly, as an explanation opposed to the foregoing, it was suggested by Mallard that the crystals in question might be built up of interpenetrating parts, these being very thin lamellæ of the crystal substance, normally constituted

but differently oriented. That the crystals must then exhibit the phenomena mentioned has already been set forth on pages 217–219. And in fact, for very many crystals that formerly were termed optically anomalous, Mallard has demonstrated the correctness of this explanation. Since, then, with such composite masses, which in their optical relations behave differently at different points, it is a matter of crystals that are only *apparently* incomposite, the constitution of their differently oriented parts being optically *normal*, we may not properly speaking designate such a mass as an optically anomalous crystal, as is often done. The cases of true optical anomaly in a crystal (leaving out of the question the temporary state described on pp. 273–279) seem to reduce themselves to (1) those produced by inclusions (see p. 280) and, possibly, to (2) certain phenomena exhibited by so-called "isomorphous mixtures". (See p. 6.) In the bodies last-named, which are crystals built up of the smallest particles of two salts, the dimensions of the two kinds of component particles, although very similar, are not exactly the same; and accordingly it seems very possible that owing to this lack of molecular homogeneity there may arise permanent strains of the whole mass. This view is supported by the observations of Klocke and Brauns: according to these observations crystals of alum exhibit normal single refraction when they consist of the chemically pure salt, but are distinctly birefringent when they contain isomorphous admixtures; so that here we probably have to do with a true optical anomaly.

### ELASTIC STRAIN BY ELECTRICAL ACTION

[Elastic strain in crystals, and consequent variation of the optical properties, may be induced in still another way than by thermic expansion and by the action of mechanical forces on the crystal.]

Certain crystals that are non-conductors of electricity exhibit in definite mutually opposite directions a difference in their behavior, in that, when they are heated and then allowed to cool, they assume electric polarity; the polarity

is developed only on *variation* of the temperature: it disappears when the latter has become stationary. This property is termed " polar pyroelectricity ", and the directions in which the polarity is excited we call "electric axes". Since compression of a body operates like lowering its temperature, and extension like raising the temperature, it is to be expected that in polar pyroelectric crystals electropolarity would be excited in the electric-axial directions during compression or extension as well; and the electricity produced at one of the poles of such an axis by the diminution of pressure would necessarily be the opposite of that called forth at the same pole by the increase. This property of polar pyroelectric crystals bears the name " polar piezo-electricity".

Just as electricity is produced by elastic deformation of a polar dielectric crystal, so too does electrization of such a body effect a deformation of the body. While J. and P. Curie proved this by direct measurement, Kundt showed that the circular color rings exhibited by a quartz plate in convergent polarized light pass over, when viewed in an electrostatic field, into ellipses such as arise in consequence of pressure directed perpendicular to the optic axis.*

## PERMANENT STRAIN

### Plasticity

If a straining force acting on a crystal exceeds the elastic limit of the crystal, the latter suffers deformations that are lasting and generally unhomogeneous, [so that permanent variations in the optical properties must arise. (Cf. pp. 280 and 285.)] The property of a body to suffer, when subjected to external stresses exceeding its elastic limit, changes in shape which are permanent but also gradual and steady (without losing, through any sudden occurrence, the connection of its parts), is called "plasticity". Many crystals are so plastic that the slightest forces suffice to change their shape, and with some the elastic limit is so nearly zero that they behave like a liquid.

*[According to Pokels (F. Pokels, *Abh. Ges. d. Wissensch. Göttingen*, **39**, 1–204) the change is too great to be a consequence of the deformation alone, but must be due in part to a direct influence of the electric force on the light-vibrations within these crystals.]

According to Lehmann's researches an aggregate of such crystals — "flowing crystals", as it were — can be brought by pressure or tension into parallel orientation (Lehmann's "homeotropy"); and if the individual crystals are doubly refracting, the mass of course then exhibits parallel extinction. Furthermore, the surface tension can even exceed the elastic limit, and then each crystal assumes the form of a drop, just like a perfect liquid, whose elastic limit is zero (Lehmann's "liquid crystals").*

### Gliding

In addition to cleavage planes there exist in crystals planes distinguished by the fact that parallel to them, in a definite direction (but not necessarily in the opposite direction as well), there can take place with particular ease a slipping, or "gliding", of the crystal particles on one another. The result is that, when for example the crystal is pressed, its particles become displaced relatively to one another along such a plane, and sometimes even completely separate.

By Reusch, who was the first to point out their existence, these planes were called "gliding-planes" (Gleitflächen). In certain crystals they appear as planes of easy separation ("parting"), if the point of a steel cone (the center-punch used by metal-workers is particularly well adapted for this purpose) is set on the surface of the crystal and driven in by a sharp tap with a light hammer. The so-called "percussion-figure", obtained by this method, the "percussion-test", consists of rectilinear cracks radiating from the point of percussion in one or more directions; these cracks usually correspond to the gliding-planes, not to the cleavage planes, — as, for example, in crystals of halite. These crystals belong in the singly refracting class and therefore (see p. 272) possess three mutually perpendicular directions in which they behave alike with respect to the influence of external forces; parallel to these directions their strength is minimum, so that along planes perpendicular to these three directions they split very easily. Now on such a cleavage-face the percussion-figure appears as a four-rayed star, whose branches do not pass out

* O. Lehmann: "Flüssige Krystalle, sowie Plasticität von Krystallen in allgemeinen, molekulare Umlagerungen und Aggregatzustandsänderungen". With 483 figures in the text and 39 colored plates. Leipzig, 1904.

parallel to the two other cleavage planes, but bisect their angle. Since all three of the cleavage planes behave alike in this respect, there must exist in the halite six gliding-planes, which bisect the angles of a cube split from the crystal. If a prism bounded by the six cube faces is pressed together from the end faces and therefore in the direction of one of the edges, it becomes shorter and thicker in consequence of the particles slipping along the gliding-planes that form 45° with the direction of pressure; thus considerable deformations can be produced without fracturing the prism. In this case, just as when the crystal is broken, cut, etc., there always arise at the same time, owing to the plasticity of halite, local compressions, which are to be recognized by double refraction (brightening between crossed nicols). (See p. 280.) In fact, halite that is entirely homogeneous and free from doubly refracting areas is seldom found.

By far the most interesting and theoretically the most important phenomena of gliding are presented by calcite, in which

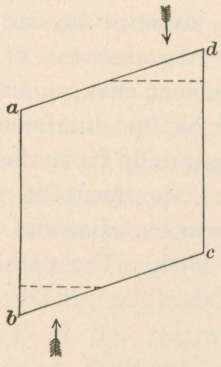


Fig. 120*a*.

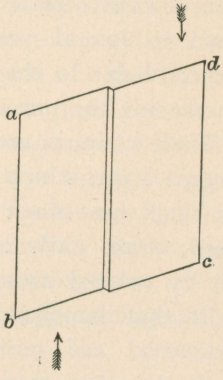


Fig. 120*b*.

mineral, likewise, the existence of gliding-planes was discovered by Reusch. On a cleavage-rhombohedron of calcite whose principal section is *abcd* in Fig. 120*a* (*ab* and *cd* being two opposite obtuse edges, *bc* and *ad* the diagonals of the end faces), let us imagine two horizontal planes to be cut as indicated by the dotted lines. Then, if pressure is exerted on the calcite in the direction of the arrows, the right half slips downward and the left half upward. This movement proceeds in such a way that

within the space between two planes parallel to the gliding-plane, which stand perpendicular to the plane of the figure and would therefore truncate the edges *ab* and *cd* evenly before and behind this plane, there takes place a rearrangement of the substance of the crystal into the symmetrically opposite position. The arising lamella (see Fig. 120*b*) appears only on those two sides of the cleavage-rhombohedron that, in Fig's 120*a* and *b*, are represented as the upper and the lower face, parallel to each other; it is seen as a very narrow, rectilinear stria, which runs exactly parallel to the longer diagonal of the rhombohedron face. Now it usually happens, in a rhomb of this kind, that a considerable number of such lamellæ arise, so that on two of its opposite faces the calcite is seen to be *striated* parallel to the longer diagonal.*

If, then, a calcite plate cut perpendicular to the optic axis is subjected to lateral pressure, it becomes biaxial. (Axial plane perpendicular to the direction of pressure — cf. p. 276.) But on increasing the pressure, permanent changes arise in the calcite. These changes are revealed by the interference-figure in convergent light, which differs essentially from the normal: the color rings are much narrower; the form of the cross is deranged; four dark spots appear in diagonal position, connected by colored arcs; and so forth. The cause of this behavior is that lamellæ arise in the plate in the manner described.

Now all the phenomena hitherto described, especially the striation parallel to the longer diagonal of the rhombohedron faces, are very often exhibited by calcite in the natural state, under the conditions of its occurrence. This leads to the assumption that here, too, a pressure exerted on the mineral has found relief in the formation of the lamellæ in question.

*The cause of the hollow passages within calcite crystals, which are often met with in nature and which can also be produced artificially, has been traced to these lamellæ — to their changing suddenly from one plane to another, ending abruptly within the crystal, or intersecting one another.

While according to Reusch's procedure there arise on pressing a calcite rhombohedron only a number of thin lamellæ,* Baumhauer has given us a method by which any part of a rhombohedron can be brought over into the symmetrically opposite position. Suppose a prism-shaped cleavage-fragment (Fig. 121) to be fixed (best in a groove cut to correspond in a wooden board) with one of its obtuse edges horizontal, and so that the long diagonal, *ce*, of its right end face, *cdef*, is likewise horizontal. Then, if at the point, *a*, the cutting-edge of a knife is set at right angles against the upper edge of the calcite and gradually pressed in, the part of the crystal that lies to the right of the knife becomes shifted in such a way that, when the cutting-edge of the knife has penetrated to the intermediate edges of the calcite (with large pieces this requires considerable pressure), it has assumed the exact form of a half-rhombohedron lying in the opposite position; *i.e.* the form of the mirror-image, with respect to the gliding-plane through *ce*, of the lower half-rhombohedron. The triangle *cef* has now taken the position *ceg*; the obtuse solid angle *f*, the so-called "pole-angle" (*a* in Fig. 48, p. 91),— at the junction of three similar, obtuse edges,—has become a "lateral angle", *i.e.* the meeting-point of two acute edges, *cg* and *eg*, and one obtuse edge, *fg*; consequently the left upper angle of the rearranged portion has become a pole-angle.

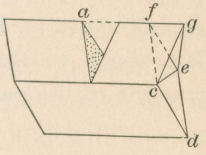

Fig. 121.

Now if we investigate the part of the crystal in the new position more closely, it proves to be, physically, entirely homogeneous; its properties no longer correspond to its previous form, however, but to its present form. For example, it is optically uniaxial uniformly at all points, and with the same

* It is sometimes possible, but only under very special circumstances, to apply the pressure on a rhombohedron like the one in Fig. 120 in such a way that the crystal substance is *all* rearranged; in other words, so that the obtuse angles now lie at *b* and *d*, while the acute are at *a* and *c*.

birefringence as before; but the optic axis, lying in the vertical principal section, does not incline from the right above to the left below, as in the part to the left of *a* and also in the lower part (cf., for example, Fig. 53 on p. 97, supposing the edge *ac* to be horizontal): it is now inclined, at the same angle, from the left above to the right below, in correspondence to the new position of the pole-angle.*†

## TWINNING

For the normal growth of a crystal, *i.e.* in order that a crystal may be homogeneous at all points, it is requisite that in being deposited the particles drawn to it from the mother-liquor shall be given, with respect to the particles of the previous layer, the same orientation as these were with respect to the next preceding. But in addition to this corresponding orientation of successive layers, there can exist a second orientation in which stable equilibrium exists among the forces acting among adja-

* This rotation of the optic axis amounts to $52\frac{1}{2}°$. Since the entire inner constitution of the crystal with reference to the new axis of the rhombohedron has become the same as it was previously with reference to the old axis, the smallest particles of the calcite must have experienced (in addition to the slipping) a rotation through this same angle. The rotation of the face *cef* (Fig. 121) into its new position *ceg* is considerably less — only 38°.

† Since calcite shifted along the gliding-plane has exactly the same properties as originally, although in different orientation, the arrangement and direction of its smallest parts must correspond in both cases to the same equilibrium of the internal forces. When, therefore, by the action of external forces such a shifting of the crytsal substance is effected, the accompanying rotation of the particles (see footnote *) must be such that, when the shifted part is exactly midway between its original and its final position, the smallest particles too have completed exactly half their rotation. Consequently these particles can be carried over, by the internal forces of the crystal, into the one position of equilibrium just as easily as into the other. In agreement with this (and as was found by Reusch) it may indeed be observed, when gliding-lamellæ are being produced in calcite, that a rearrangement which has proceeded less than half-way disappears completely when the pressure ceases to act; while when the labile state arising midway in the shifting has once been passed over, the lamella forms of itself, because, owing to the internal forces, the particles strive toward the second position of equilibrium.

cent particles.* Under such conditions, the particles attracted to the growing crystal being of course present in the solution in all possible positions, it will happen just as often that a particle has nearly the orientation corresponding to the first position of equilibrium as to the second. Since the particles must assume that one of the two orientations to attain which the less work (*i.e.* the less rotation) is necessary, they will be deposited just as often in the one orientation as in the other, and will now, in their turn likewise, become the cause of analogous deposition of additional particles. Thus, beginning immediately after the crystallization has begun, two distinct arrangements of the particles, differently oriented but otherwise similar, may grow on in the solution; and the result is a so-called "twin" crystal, whose two parts are in general bounded by like faces, since the conditions of their crystallization are essentially the same.

If twinning is repeated in such a way that with the second crystal there combines a third, after the same law, with this latter crystal a fourth, and so on, the resulting growth is termed a "trilling", a "fourling", etc. According to the law of the twinning, two cases may here arise: either the third crystal has a different position from the first, the number of the regularly combined crystals then naturally being limited; or else the third crystal is parallel to the first, the fourth crystal, combined with this third one, parallel to the second, and so forth. In the latter case, we have a fourling of crystals succeeding one another in alternate orientation with reference to the common twinning-plane. In such combinations the phenomenon, quite common in twin crystals, of especially marked development of the single crystals along the twinning-plane, appears in so high a degree that the single crystals usually have the form of thin lamellæ parallel to this plane. A mass consisting of many such lamellæ is said to be "polysynthetic"; an example would be the calcite described on pages 285–286.

* Such a case is explained in footnote † on the opposite page. In this case the two equilibria have equal stability.

The optical properties of such masses have already been treated in detail on pages 214–219. It was there shown that in case the twinned lamellæ are of sub-microscopic dimensions the combination must behave like a single crystal, whose properties depend on the character and orientation of the lamellæ, and shown further that when the lamellæ are unequal in distribution and thickness the behavior is optically anomalous. (See p. 219.)

# APPENDIX

## SUPPLY HOUSES

FOR

## APPARATUS, MODELS, CRYSTALS, AND PREPARATIONS

[The price lists in Professor von Groth's book have proved to be so generally useful that the translator deems it worth while to include similar notices here, no prices being given, however. The number of firms listed has been somewhat increased, but no mention made of any except German and American houses.]

# APPENDIX

## SUPPLY HOUSES

FOR

## APPARATUS, MODELS, CRYSTALS, AND PREPARATIONS

W. APEL (*Inhaber* DR. M. APEL), *Universitäts-Mechanikus*, Göttingen, Germany.

**Total-reflectometer** after F. Kohlrausch in three different styles, with various accessory apparatus. Also a **reflection goniometer** after Wollaston. Illustrated catalog.

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**Crystal refractometer** (Zeiss manufacture — see p. 297) specially adapted for crystallographical and mineralogical investigations.

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The **Dwight Petrotome,** a machine for cutting, trimming, and polishing rocks, minerals, etc., in rapidly adjustable positions and for the preparation of thin sections. Descriptive pamphlet.

EIMER & AMEND, 205–211 Third Ave., New York, N. Y., U. S. A.

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FOOTE MINERAL COMPANY, 107 North 19th street, Philadelphia, Pa., U. S. A.

**Specimens of minerals** and rocks. Decorative stones. Complete catalog and special lists.

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**Lapidary lathes** for slicing, grinding, and polishing minerals, etc., and for preparing thin sections. Diamond and emery **slicing disks; grinding** and **polishing laps. Supplies** for preparing and mounting thin sections.

DR. F. KRANTZ, *Rheinisches Mineralien-Contor u. Lehrmittelfabrik,* Herwarthstrasse 36, Bonn am Rhein, Germany.

Collection of four **glass models** illustrate the **dispersion** in orthorhombic and monoclinic crystals, with silk thread of different colors drawn through to represent the position of the optic axes and bisectrices. Rhombohedron made up of plate-glass to demonstrate the **double refraction in calcite,** as constructed by Prof. K. Busz. Glass model of the **Nicol prism,** as constructed by Prof's Busz and K. Vrba. Collections of models to illustrate **the optical relations of various crystals,** as constructed by Prof's Grubenmann and Weinschenck.

Six **gypsum models** of the **ray-surface** of crystals. Also six colored gypsum models of the ray-surface, as constructed by Prof. L. Duparc; can be taken apart along the three principal sections.

Three **wooden models** of the optical **index-surface** for uniaxial and biaxial crystals according to Prof. P. von Groth; made of hard wood and polished; can be taken apart. Also wooden models of the index-surface to explain the theory of double refraction and the optical properties of crystals; constructed according to Prof. Duparc. Are made of pear wood and mounted on three mutually perpendicular metal axes; are cut through along properly oriented planes as an aid to explaining the laws of light transmission.

Three **wire models** of the optical **index-surface** for uniaxial and biaxial crystals after Prof. K. Vrba; constructed in 3 mm. metal wire and lacquered, equivalent directions with the same color. Also four wire models of the **ray-surface** after Prof. Vrba, the surfaces for the different rays being in different colors.

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All kinds of crystallographical **models prepared** according to specification. **Stands** for models. Various catalogs.

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Eight different stands of **mineralogical microscopes;** various optical outfits with each stand. Also a mineralogical **demonstration microscope,** supplied with either of two outfits. Catalog in English. London office 9–16 Oxford street.

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**Index-ellipsoid models** in polished pear wood, which can be taken apart: The index-surface for **biaxial crystals,** having a circular and an oblique section, together with a wire model (see p. 153, Fig. 76); and the same surface for + and for − **uniaxial crystals,** with one oblique section. Each of the three models is made in a large and a small size. Catalog.

A. H. PETEREIT, 81–83 Fulton street, New York, N. Y., U. S. A.

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MARTIN SCHILLING, *Verlagsbuchhandlung,* Kantstrasse 12, Leipzig, Germany.

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**Cardboard models:** Two **triaxial ellipsoids** made up of 22 and 30 circles respectively, and flexible, so that the axial ratios can be varied at will. Illustrated catalog.

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**Microscopes** for petrographical and crystallographical investigations. Six stands, with different outfits, making eleven models. Catalog.

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**Various other apparatus and preparations,** as nicols and all kinds of polarization-prisms, quartz prisms with the refracting edge ∥ or ⊥ to the axis, calcite preparations, quartz or gypsum wedges, quarter-undulation mica plates, gypsum plates giving red of the 1st order, dextro and levo Reusch mica combinations, cooled glasses, crystal plates with absorption-stripes, pocket spectroscope, apparatus for observing conical refraction, apparatus and preparations for investigating piezo-electricity, etc., cutting and grinding machines in various sizes. Detailed catalog.

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**Collections of thin sections of minerals: Large educational collection** consisting of 115 thin sections of the petrographically most important minerals, the crystallographic orientation being arranged with special reference to deter-

**mination** of the crystal system; the material was chosen and classified by Prof. C. Klein, who also examined the collection employed as a model in preparing the others. **Smaller educational collection,** for beginners and schools: fifteen thin sections for the microscopical demonstration of important properties of crystals; arranged by Prof. F. Rinne. **Special collection** of twenty-five thin sections for demonstrating optical anomaly in some of the crystals of the isometric system; modeled after a collection arranged by Prof. C. Klein.

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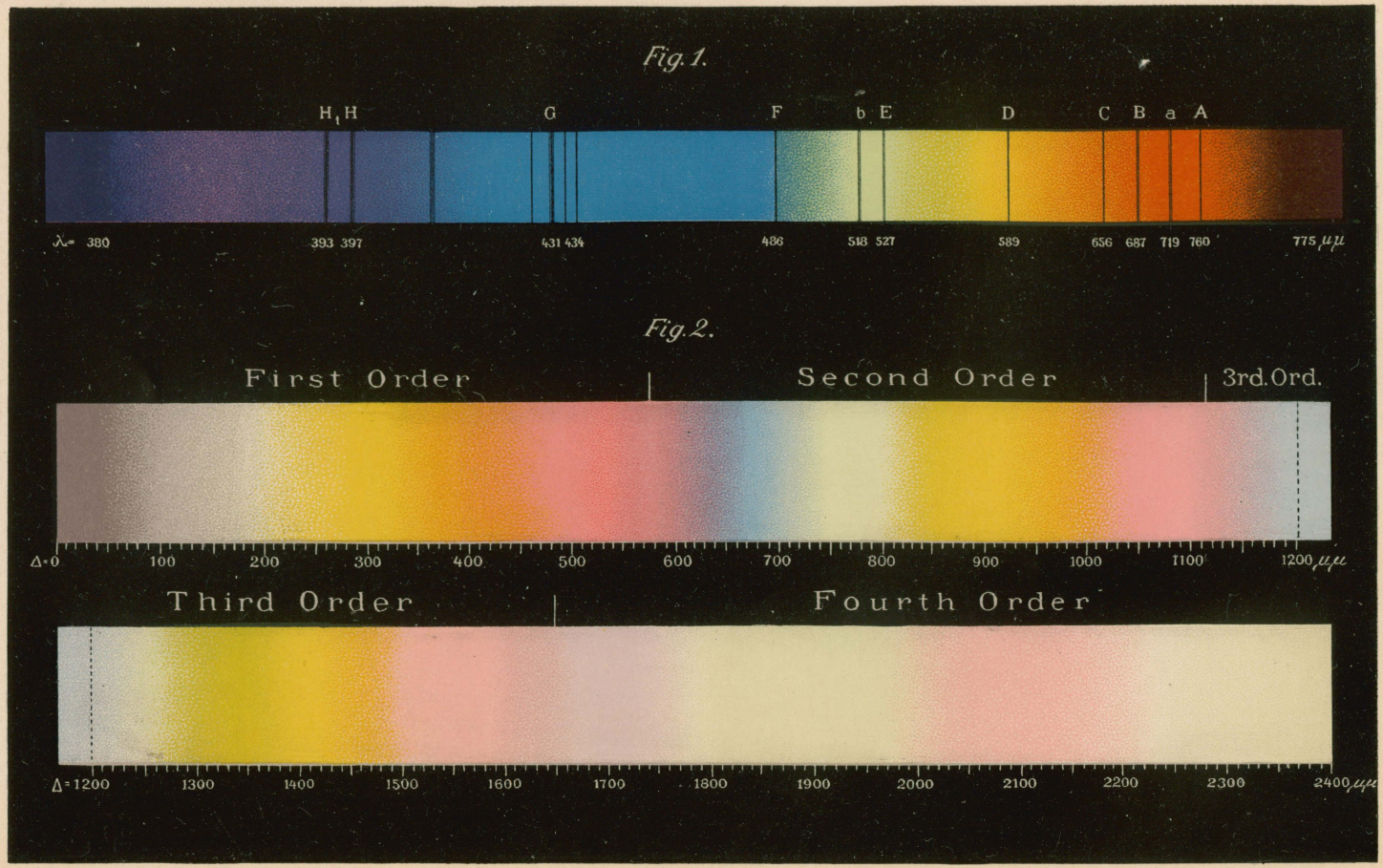


Fig. 1.
H1 H
G
F
b E
D
C B a A
λ= 380
393 397
431 434
486
518 527
589
656 687 719 760
775 μμ
Fig. 2.
First Order
Second Order
3rd. Ord.
Δ=0
100
200
300
400
500
600
700
800
900
1000
1100
1200 μμ
Third Order
Fourth Order
Δ=1200
1300
1400
1500
1600
1700
1800
1900
2000
2100
2200
2300
2400 μμ

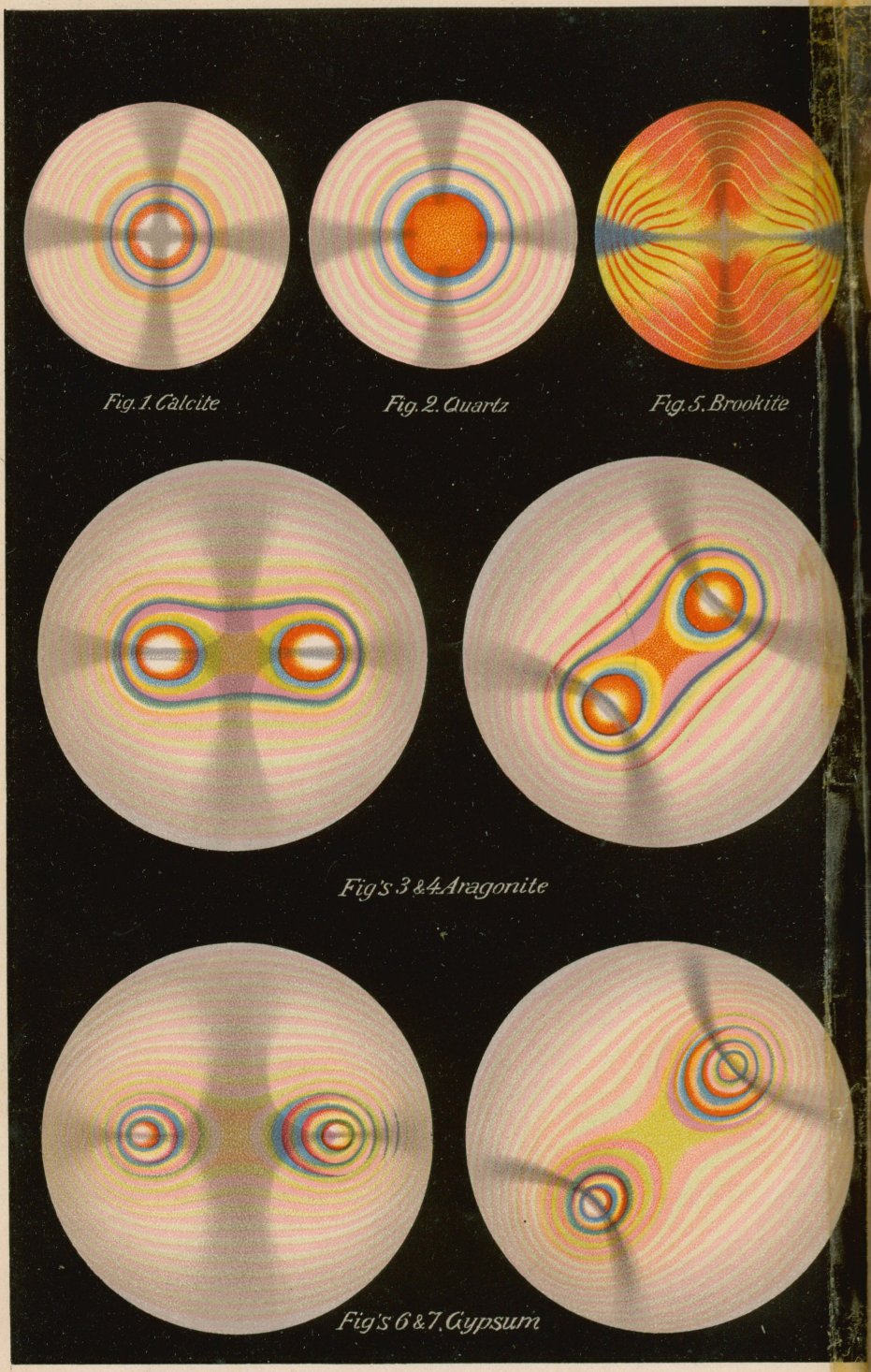


Fig. 1. Calcite

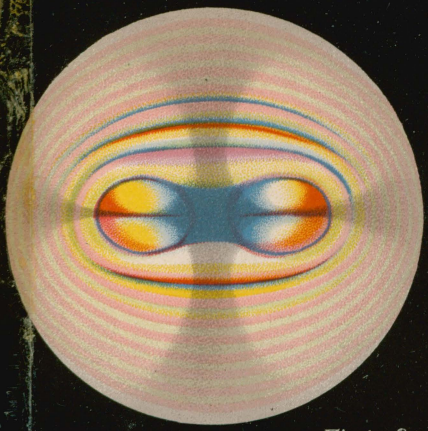
Fig. 2. Quartz

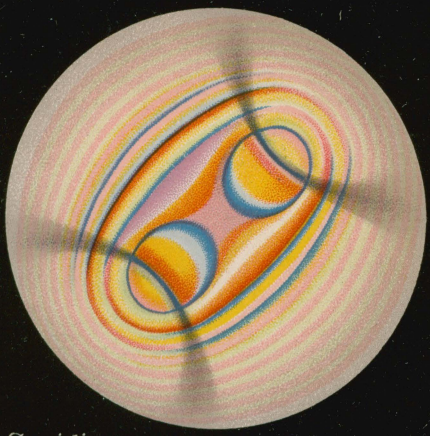
Fig. 5. Brookite

Fig's 3 & 4 Aragonite

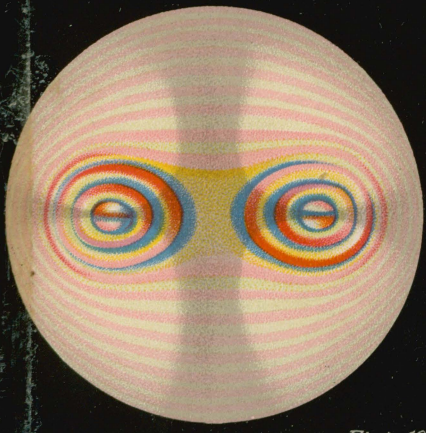
Fig's 6 & 7. Gypsum

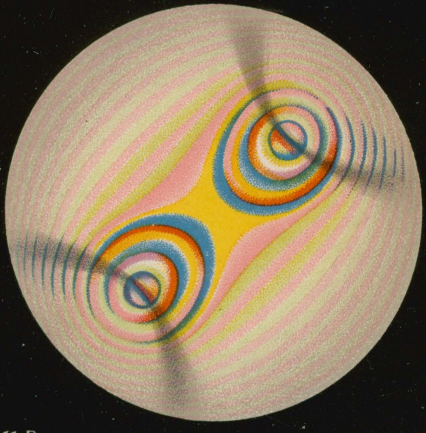
Verlag von Wilhelm Engelmann in Leipzig.



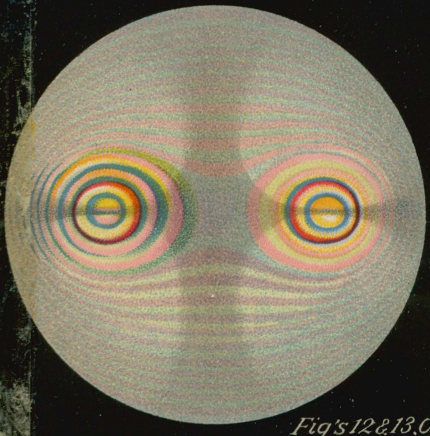


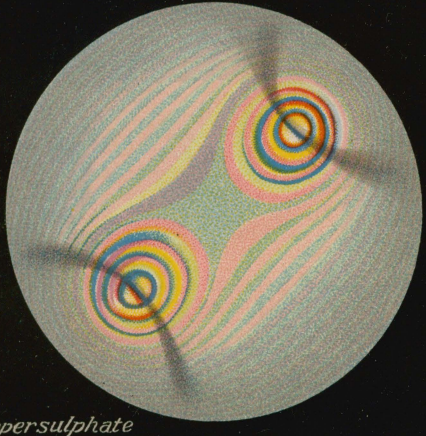
Fig's 8 & 9. Sanidine





Fig's 10 & 11. Borax





Fig's 12 & 13. Coppersulphate

H. F. Jütte, Kunstanstalt, Leipzig.

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