Dempster Shafer Theory Applications in Post-Seismic Structural Damage and Social Vulnerability Assessment

Wendy Ballent, Graduate Student Researcher, Dept. of Civil, Environmental, and Architectural Engineering, University of Colorado Boulder, Boulder, CO 80309 (email: wendy.ballent@colorado.edu)

Ross B Corotis, Professor, Dept. of Civil, Environmental, and Architectural Engineering, University of Colorado Boulder, Boulder, CO 80309 (email: ross.corotis@colorado.edu)

Cristina Torres-Machi, Assistant Professor, Dept. of Civil, Environmental, and Architectural Engineering, University of Colorado Boulder, Boulder, CO 80309 (email: cristina.torresmachi@colorado.edu)

ABSTRACT

The inherent uncertainty involved in assessments of both post-seismic structural damage and the social vulnerability of those affected motivate the search for a suitable analysis framework. As these risk assessments are subjective in nature and require expert opinion, many common methods such as probability may not provide the most natural mathematical structure. This paper explores how Dempster-Shafer Theory might be used in these applications, as this theory allows the combination of multiple expert beliefs while considering such uncertainties. A post-seismic structural damage assessment survey was used to evaluate combined belief outputs compared to probability outputs. The results computed using Dempster Shafer Theory were more consistent than probability when compared to the actual damage of the images shown in the survey. This suggests that Dempster Shafer Theory may be a suitable framework to represent ignorance and evidence-based assessments.

Introduction

Risk assessment is central to hazard mitigation, and by its very nature requires subjectivity (Elwood and Corotis, 2015; Ballent et al, 2018). The study of community risk and social vulnerability incorporates very different sources of uncertainty and yet is often evaluated using traditional probabilistic frameworks that are more naturally suited to objective-observed data (Armas and Gavris, 2013).

A variety of frameworks should be considered when handling such uncertainties. Probability often provides a reliable structure in such situations, where predictions of future performance can be made based on the outcome of previous similar occurrences and professional judgment. Powerful procedures include Klein's recognition-primed decision model and his later naturalistic decision making model (Klein, 2008). These models are based on extensions of Kahneman and Tversky (2000) heuristic discoveries, and point out the importance of pattern matching. These methods are based, however, on the traditional axioms that are the foundation of probability theory. Similarly Saaty's analytic hierarchy process combines expert judgement by examining the decision process as a series of nested steps (Saaty 2008), and is not reliant on traditional probability theory. There are many situations in risk assessment with respect to natural hazards in which the axioms of probability prove overly restrictive, even when incorporating subjective or Bayesian models (Cooke, 1991; Melchers, 1999; Vick, 2002; Corotis, 2015).

This paper explores two major applications of a method of decision making not constrained by probability axioms, but one based on monotone theory of generalized uncertainty. The motivation for new approaches is not intended to challenge the fundamentals of Probability Theory, but to present different mathematical models, which may be relevant in a variety of contexts. The results of these applications open the door for further exploration of the power of these non-probabilistic approaches. A review of past procedures will be briefly discussed, but further background is given in the recent paper by the authors (Ballent et al, 2018), published in this journal.

Background Review

Uncertainties and Evidence Theory

Dempster-Shafer theory of evidence (DST) was developed first by Glenn Shafer (1976), from work he performed with Arthur Dempster, his mentor. It has since been developed further and expanded by other researchers (see, e.g., Beynon et al., 2000; Yager and Liu, 2008). As Aven et al. (2014) state, "in looking for a general framework for treating uncertainties in risk assessment, we started with the probabilistic treatment of uncertainties, recognizing its merits and limitations, and then ventured beyond probability to describe uncertainties in a risk assessment context whose setting demands an extension of concepts and methods".

Generalized Information Theory

The area of study termed *Generalized Information Theory* (GIT) (Ayyub, 1998; Klir, 2006; Ross, 2010) expands probability theory by including non-additive probability measures and fuzzy sets (Klir, 2006; Klir and Smith, 2001; Wang and Klir, 2009). Dempster-Shafer theory falls under the rubric of GIT. The *additivity requirement*, which is a critical restriction of probability theory in terms of using expert opinions, describes the circumstance in which probability measures can be obtained from subsets of X if bound within the disjoint set as shown below Klir (2006):

$$P(A \cup B) = P(A) + P(B) \tag{1}$$

Equation (1) is commonly known as the third axiom of probability. Considering element B to be $\bar{\boldsymbol{A}}$ (the complement of A) requires that any information provided about element 'A' also provides contrary evidence about event $\bar{\boldsymbol{A}}$, since their union is one. In Probability Theory, uncertainty is represented by this single probability measure. If either the probability of an event or the probability of its complement is known, the additivity requirement guarantees that the probabilities of both are known.

Possibility/Necessity Measures

Possibility Theory provides a mathematical framework to explicitly represent ignorance by removing the additivity requirement (Ross, 2010). Possibility Theory differs from Probability Theory in that it explicitly recognizes the case when evidence or judgments support the possibility of one event, but does not necessarily implicate evidence regarding the contrary event (Dubois, 2006; Dubois and Prade, 1988). To fully characterize the uncertainty of an event A, uncertainty is represented by dual measures, termed possibility and necessity measures (Ayyub and Klir, 2006). These measures are detailed by Ballent et al. (2018), and are central to the concepts of expert beliefs and the combinations of such beliefs. They are supported by the fundamental precepts of Dempster-Shafer theory, for which a brief background is provided in Appendix I.

Imprecise Probabilities

While several frameworks have surfaced as possible alternatives to probability in cases with uncertainty, including Dempster Shafer Theory, there have also been adaptations of probability itself. Walley (1991) introduced the idea of imprecise probabilities as a generalization of Probability Theory which follows the principles of Probability Theory but does not require precise probability assignments (Ayyub and Klir, 2006). As discussed previously, a main goal of this theory is to determine how to best represent uncertainty in risk assessments. Providing upper and lower bound probabilities is one way to demonstrate some uncertainty of a model output in a quantifiable way. The lower bound probabilities are related to the Mobius measures introduced in Appendix I by the following (Ayyub and Klir, 2006):

$$m(A) = \sum_{all\ B\ such\ that\ B \subseteq A} (-1)^{|A-B|} \underline{P}(B) \tag{2}$$

$$\underline{P}(B) = \sum_{all\ B\ such\ that\ B \subseteq A} m(A) \tag{3}$$

where $\underline{P}$ is lower bound probability. Upper bound probabilities can be calculated from lower bound probabilities with the following (Ayyub and Klir, 2006):

$$\overline{P}(A) = 1 - \underline{P}(\overline{A}) \tag{4}$$

where $\overline{P}$ is upper bound probability of A. Probability bounds are used in line with Dempster Shafer Theory to analyze data from the survey instrument used in this paper.

Social Vulnerability

Since Dempster-Shafer Theory is evaluated in multiple capacities in this paper, including as an analysis tool for social vulnerability, a short review of this topic is necessary as well. A community's vulnerability to a hazard is often thought of in physical terms; their infrastructure, environmental surroundings/global location, etc. Social vulnerability is the inter-personal counterpart – the vulnerability one might experience due to factors such as income disparity, class, gender, age, disability, health, living situation, income, or race/ethnicity (Thomas et al., 2013). This type of vulnerability plays a significant role in how well someone is able to recover after experiencing a disaster, such as an earthquake, hurricane, or heat wave.

One current method of determining a community's social vulnerability is the Social Vulnerability Index (SVI), and is dependent on a set of 15 census variables grouped into four themes (Flanagan et al., 2011). Within a Socioeconomic Status theme, the considered variables are percentage of individuals below poverty, percentage of civilians unemployed, per capita income, and percentage of persons with no high school diploma. A Household Composition/Disability theme considers variables percentage of persons 65 years of age or older, percentage of persons 17 years of age or younger, percentage of persons more than 5 years old with a disability, and percentage of single parent with child under 18 years old. The Minority Status/Language theme variables are percentage of minority, and percentage of persons 5 years or older who speak English less than "well". Finally, the Housing/Transportation theme considers variables percentage of multi-unit structures (10 or more units in structure), percentage of mobile homes, and percentage of crowding (more people than rooms at household level), percentage of with no vehicle available, and percentage of persons in group quarters (nursing homes, dorms, and military quarters).

A Social Vulnerability Index for Disaster Management outlines how these variables are analyzed: "To construct the SVI, each of the 15 census variables, except per capita income, was ranked from highest to lowest across all census tracts in the United States with a non-zero population. Per capita income was ranked from lowest to highest because, unlike the other variables, a higher value indicates less vulnerability" (Flanagan et al., 2011). If the percentile is 90% or higher (inverse for "Per capita income"), the town is flagged. These percentiles are also summed within each of the four themes. If the group percentile is 90% or higher, this earns another flag. Finally, the percentiles for all 15 variables are summed, and if the overall percentile is 90% or higher, the town receives yet another flag. The number of total flags is reviewed on a variable level, theme level, and overall level. The number of received flags indicates the level of social vulnerability within the town (Flanagan et al., 2011).

Due to the fairly limited scope, this method might not reflect all the factors that influence one's true social vulnerability. Alternative methods have been proposed, such as the Baseline Resilience Index for Communities (SoVI) by Cutter et al. (2003; 2014) in which 42 independent variables were used to compile 11 factors: personal wealth, age, density of built environment, single-sector economic dependence, housing stock and tenancy, race (African American, Hispanic, Native American, Asian), occupation, and infrastructure dependence. Another proposed method by Cutter and colleagues, the Baseline Resilience Index for Communities (BRIC), involves combining the SVI with hazard event frequency and economic loss data to examine 36 factors that influence large dollar losses, and hence the ability of a community to recover from a disaster (Cutter et al., 2010). These are grouped into five major categories of resilience: Social Resilience, Economic Resilience, Institutional Resilience, Infrastructure Resilience, and Community Capital. Still another approach has been taken by Peacock et al (2010), which focuses on resilience of human social systems in the context of disaster. His Community Disaster Resilience Index (CDRI) is based on 75 measures of social welfare, and their groupings into four major indicator categories: Social Capital, Economic Capital, Physical Capital and Human Capital.

Each of these methods has strengths and weaknesses when attempting to relate social vulnerability to community resilience. BRIC and CDRI have only been calibrated for limited parts of the United States, and SoVI depends on calibrating to neighboring communities. SVI, used in this paper, is relatively comprehensive and computable, with its major drawback being the use of the flag system, which this current study addresses. As Cutter et al. (2003) once stated (and it is still true), "there is no consensus within the social science community about social vulnerability or its correlates".

Given the many uncertainties and nuances involved in social vulnerability, using a belief-based analysis tool like Dempster-Shafer Theory has the potential to provide a more comprehensive evaluation of social vulnerability. While the variables above may indicate that vulnerability is more or less likely to be present, the ability of a community to recover after a disaster is influenced by much more. For example, Pfefferbaum et al. (2008) discuss the importance of community bonds such as those established by town-wide participation in common activities: church, school, self-help groups, or neighborhood watch committees. These types of values cannot be quantified in the current indexing method of social vulnerability, but may be represented by a more subjective framework like Dempster-Shafer Theory.

Literature Review Conclusion

Aven et al. (2014) write that "Evidence Theory [Dempster Shafer Theory] provides an alternative to the traditional manner in which Probability Theory is used to represent uncertainty by means of the specification of two degrees of likelihood, belief and plausibility, for each event under consideration." This paper aims to analyze the applications of Dempster Shafer Theory in the field of civil engineering and hazard mitigation, specifically in post-seismic structural damage assessments and social vulnerability, to determine how such uncertainties can be incorporated.

Application I: Dempster-Shafer Theory for Post-Seismic Damage Assessment

A structural damage assessment survey was constructed to test one of the real-life applications of this theory. This survey was intended not as a statistically significant guideline for seismic damage assessment, but as an illustration of how evidence theory can be used to obtain information from multiple experts, and combined to give richer assessments that differ from those based on traditional probability theory. The survey was part of a larger project conducted by Stanford University under the direction of Professor Anne Kiremidjian, and with the assistance of Dr. David Lallemant, seeking to determine the extent to which aerial photography could be used for early estimates of neighborhood seismic damage. They graciously made the images available to our research team, and our objective was to incorporate evidence theory questions in the survey, and then compare the estimated damages with the actual ground-verified inspections, as well as the results based purely on probabilistic methods.

The survey includes 5 different aerial images of Port-au-Prince, Haiti, taken shortly after the 2010 earthquake there. A separate damage scale was provided, giving examples of images that have damage ranges of 0 - 20%, 20 - 40%, 40 - 60%, 60 - 80%, and 80 - 100%. These damage levels were determined from actual ground surveys following the earthquake, and the images were selected just to provide reference guidance to the survey takers, not to be evaluated (these five different levels were chosen to give good discrimination for calibration). The participants were then presented with five different images, asked to evaluate each image, and to assign their belief that the image has a damage in the ranges of 0 - 33%, 34 - 66%, and 67 - 100%. Just three ranges were used because it was decided that any finer resolution would be too difficult to estimate. They are also asked to assign their belief that the damage is within the ranges of 0-66% and 34 – 100%, with the provided explanation that they may have more confidence in the larger ranges than simply the sum of the smaller ranges. The participants were not asked for their belief in [0 - 33% + 67 - 100%] (the combination of the two outer ranges), as this is not a commonsense question in terms of damage assessment. The belief in this combined event is necessary to calculate the total combined belief since values for all members of the power set are required. Therefore, the missing value was calculated using the provided beliefs. The individual beliefs in 0 - 33% and 67 - 100% were summed along with half of the extra belief in 0 - 66% and 34 - 100%. Without additional information, this is the only unbiased way to allocate the the missing information that the damage might be 0 - 33% + 67 - 100% (Ballent et al, 2018). One of the survey questions is provided in Appendix II for reference. It is noted that the survey takers were explicitly asked for their belief (which was explained as the degree of confidence they felt by the evidence provided). It was carefully explained that this is different from probability, and thus the beliefs in the larger ranges did not have to be completely distributed to the smaller ranges.

The survey was delivered by hand to selected structural-focused engineering classes at both the undergraduate and graduate level. Professors with a similar focus were also offered the survey either in person or via email. The survey results were recorded and each participant's beliefs were combined to achieve combined damage beliefs in each of the five images. The objective was to test Dempster Shafer Theory in a real world application and analyze the results to determine if (a) the participants were willing to learn and utilize a new framework, (b) the combined belief results were logical, and (c) this framework is able to provide helpful outputs while acknowledging the uncertainty of the contributors.

Survey Results

A total of 46 surveys were filled out and returned. If any questions were filled out not in accordance to the specified rules, namely if the provided belief exceeded 100%, the individual question was not considered in the results. Each of the five survey questions produced at least 40 correctly provided beliefs. The vast majority of the completed surveys came from undergraduates in an upper division level structural analysis course and an upper division probability course, both in civil engineering. The participants were not given detailed definitions of damage (other than an explanation from a sample set of images), and were certainly not field-trained experts in damage assessment. Nevertheless, they were reasonably informed members of society.

Evaluating Survey Results Using Dempster Shafer Theory

The results for each question were calculated in two different ways to examine how the results varied with the number of experts. This investigation was based in part on the mathematical results of the companion paper by Ballent et al (2018), which derived convergence behavior as a function of the number of experts. The first combines five groups of eight experts each, with the idea that many real life damage assessments will not have 40 available experts to provide their beliefs. The second combines all 40 available surveys to analyze the result of large quantities of opinions. The results can be seen in Table 1.

Two observations are evident when examining the results above. One; when all 40 experts are combined, the beliefs either reach 0% or 100%. Regardless of the individual beliefs, there is enough provided evidence among the 40 experts to fully support one single event. Second, the smaller groups of experts provide combined beliefs that vary heavily based on the individual beliefs. For example, consider the results of Question 1. Groups 1, 2, 4, and 5 strongly back 0-33% damage. The Group 5 experts supported this damage range so strongly that just combining those 8 experts' beliefs produced

100% belief in 0-33% damage. Group 3, on the other hand, had enough varied individual beliefs that the combined belief was still fairly scattered. Regardless, when combining all 5 groups of experts, Group 3's split belief became negligible and the total combined belief supported 0-33% damage.

Computing Probabilities from Combined Beliefs

The combined beliefs from the survey instrument were used to compute probabilities, including upper and lower bound probabilities. Only the smaller groups of experts were used to calculate probabilities, as the full groups of 40 experts produced "all or nothing" results that would produce identical probabilities. It is also important to note the difference in how probability and Dempster Shafer Theory handle multiple events. In Dempster Shafer Theory, experts are able to have belief in a combined event without being forced to allocate all of that belief to the individual events comprising it. Probability simply sums the individual event probabilities to obtain the probability in disjoint combined events. The results for the first survey question are outlined in Table 2.

There are several notable observations to be made from the results above. First, examine Group 5. As expected, the calculated probabilities are identical to the combined beliefs since there was enough combined belief from the experts in Group 5 to produce either 100% belief or none at all. Groups 1, 2, and 4 all produced very similar results; the calculated probability for each event in these groups was nearly identical to the calculated beliefs. Since almost all of the combined belief was in the 0% - 33% damage range, the probability bounds are very tight. There is less than one percent difference between the upper and lower bounds for all damage ranges in these three groups. Group 3 probabilities display slightly larger upper and lower bounds of up to 3% in range. Larger bounds are expected with this group as the combined belief is more diverse.

To further examine how combined belief affects probability bounds, the 20 surveys with the largest extra belief in combined events were analyzed and the probability bounds were determined. As expected, the bounds for this group of respondents were significantly wider with an average bounds range of 3.8%, compared to an average range of 0.5% for the results in Table 2. Since more belief was allocated to a range of events, rather than one single event, the probability of single events is less absolute. In a few cases for the results above, the lower bounds exceed the middle probability or the upper bounds undercut the middle probability by a very small percentage (0.1%). Since the bounds in these cases are so tight, this is apparently due to rounding error.

Further Evaluation of Survey Results by Averaging

For further comparison, the individual beliefs provided by the surveys were combined to approximate probability, rather than using Eqs. I.6 and I.7. For each of the small damage ranges seen in Table 1, the additional belief from the associated larger range(s) was allocated according to the fraction of belief for the smaller ranges. The results were normalized to ensure the results summed to unity. Although the survey respondents were not asked to think in terms of probability, this method provides a way to logically convert their individual beliefs consistent with the laws of probability. The results of this can be seen in Table 3.

The difference between these results and the results using Dempster Shafer Theory are significant. Most noticeable is the effect of combining many experts. When combining more and more expert opinions, Dempster Shafer Theory will continuously weight single events until they reach 100% or 0%, as shown by the results of combining all 40 expert beliefs using Dempster Shafer Theory in Table 1. Outliers and small amounts of contradicting opinion are eventually considered negligible as more and more beliefs are combined. Averaging, however, will continue to incorporate all individual responses. While the results above do have favored damage ranges for each question, the highest combined belief is 64.5%. Even though 40 individual beliefs were combined to obtain these values, the simple averaging of probability results are varied and arguably inconclusive. The most strongly supported damage range in questions 1, 2, and 4 is 34-66% damage, while the results for those three questions in Table 1 strongly support the 0-33% range. Although these results are not strictly probability-based, it is clear that the influence of ignorance and conflicting opinion in Dempster Shafer Theory is significant.

Actual Damage Results

The actual damage state for each image on the survey is provided in Table 4. Whichever range the majority of buildings in that image fell into is the range shown. The most strongly supported damage range calculated from the survey results using both Dempster Shafer Theory and averaging/Probability are also shown for comparison.

Table 4 provides valuable insight on how a different framework can produce significantly different results from the same data. The Dempster-Shafer results correctly matched four of the five actual damage ranges, while the results from averaging correctly matched one. This not only reinforces the idea that a framework that allows the user to have some uncertainties changes the output, but it suggests that this framework could produce a more accurate output.

Application II: Dempster Shafer Theory for Social Vulnerability

Rethinking Social Vulnerability Indexing

Social vulnerability is an aspect of hazard management that is often hard to quantify. As previously mentioned, one current method of analyzing social vulnerability relies on indexing 15 different census variables. There are several potential issues with this method. The most basic one is that being in or out of the 90+ percentile does not necessarily mean the group within a particular census tract is or is not socially vulnerable. It assigns an “all or nothing” ranking – those with an 89th percentile ranking would not receive a flag but are nearly just as vulnerable as those in the 90th percentile. Along the same lines, ranking in the 90th percentile or above may not actually indicate a vulnerability. For example, those living in group quarters such as a dorm might experience a benefit of having close-knit groups, or they may have predetermined recovery plans laid out by the school.

Another aspect to consider is that the 15 variables are not split evenly among the four themes. Since the groups have the potential to be assigned one flag, and there are differing numbers of variables within each group, then each variable does not carry equal weight. For example, not having access to a vehicle is in a group with four other variables, while being under the age of 17 is in a group with three other variables. This means that being under the age of 17 carries more weight than not having a vehicle. The ability to escape or recover from a disaster may or may not rely more on a method of travel than age. Finally, consider the result if one group is in the 90th+ percentile in a few variables but ranked very low in all other categories. This group would register as two or three flags. In comparisons to a group that is ranked in the 60th or 70th percentile in nearly every category, the latter would receive zero flags, and be ranked as less vulnerable than the first group.

It should be made clear that the objective of applying Dempster-Shafer theory to social vulnerability is not to dismiss other aspects that predict the robustness and resilience of communities to sustain earthquakes. Issues of risk perception and affect heuristics are important for any disaster response, including previous experience, anchoring and adjustment. Many papers discuss these aspects (Hurley and Corotis, 2014; Kahneman and Tversky, 2000; Lechowska, 2018; Slovic, 2000; Slovic et al, 2002;). The extensions presented here are intended to demonstrate the improvement of social vulnerability indexing by the incorporation of Dempster-Shafer concepts.

Dempster-Shafer Theory in Social Vulnerability

Using Dempster-Shafer Theory offers a new perspective on social vulnerability. While there are several ways DST could be used in this capacity, the method analyzed in this paper involves combining the provided census data using DST rather than summing and ranking. To do this, the census percentage for each variable is assigned to C) very vulnerable. The rest of the population is assigned to A) not vulnerable and B) moderately vulnerable. For example, if 30% of a town is below poverty, that category is marked as 30% very vulnerable, and the remaining 70% is assigned to not vulnerable and/or moderately vulnerable. How this 70% is split between A and B can be determined by a standard rule or by more subjective means depending on the analyst. These values can then be combined with the other variables within their theme, and with the other 14 categories to determine a combined percentage for "not vulnerable", "moderately vulnerable", and "very vulnerable". It should be mentioned that theories for the combinations of expert opinions are based on the assumption that the experts are independent of each other. There was no attempt here to determine the effect of possible correlation among the social vulnerability variables.

Testing Dempster-Shafer Theory

This method was tested using the available 2016 census data for three census tracts of varying social vulnerability (1, 4, and 9 flags using the existing methodology) in Denver, Colorado. The census percentage for each of the 15 variables was assigned to C (very vulnerable). The rest of the population was assigned to AB (not vulnerable and/or moderately vulnerable). The individual values for A and B were varied between 0% and 25% (at 5% increments) of the remaining population that was not assigned to C. The combined values for AC and BC were the sum of the individual percentages for A and C, and B and C, respectively. The results using values of A and B as 15% of the population that was not assigned to C are outlined below in Table 5. The 15% value was chosen for two reasons. First, since the census data pertain to the "very vulnerable" population of each variable, it is difficult to confidently assign the rest of the population to one or the other. Second, this method was tried with values of 0%, 5%, 10%, 15%, 20%, and 25%. The effect on the percentage of "very vulnerable" from changing the percentage assigned to A and B was small or nonexistent. Since the 15% value is somewhat arbitrary at this point (ideally it would be based on more in-depth census data), the observations made from these results should only be considered satisfactory for this level of analysis. Note that the "very vulnerable" percentages in white font (black boxes) indicate that this variable was flagged using the 90th percentile rule. Also note that the Per Capita Income was calculated based on inverse percentile (the lowest per capita income was in the highest percentile so that it counted as the most vulnerable).

There are several observations to be made about the results above:

- The total vulnerability of all 15 variables is identical for each tract; nearly 50% not vulnerable, 50% moderately vulnerable, and 0% very vulnerable. There are several reasons for this. First, consider that the most number of flags a tract could receive is 15 (one for each variable). The most vulnerable tract observed here has 9, which is 60% of the maximum. The overall low vulnerability of these tracts correlates to relatively low percentages of "very vulnerable" population. As the variable percentages are combined using DST, the low percentage of vulnerable population is damped out by the overwhelming evidence that most of the population is "not/moderately vulnerable". Second, since the values for A and B were determined by taking an identical specified percentage of the population (15% of the remaining population that was not assigned to C), it makes sense that their values are equal, and nearly exhaustive. Note that they do not add to 100%, which DST allows as the percentages assigned to A, B, and C for each variable do not include the entire tract population. The practical implications of this result deserve some consideration. On one hand, the third tract evaluated in this paper is one of the most vulnerable tracts in the Denver area even though it is only 60% of the maximum vulnerability on the currently used scale, so if this small group is surrounded by largely "not vulnerable" tracts with resources, then perhaps it really is not very vulnerable. On the other hand, it is unreasonable to look at the total combined vulnerability and consider all three tracts to have the same amount of vulnerability. While this proposed method provides valuable insight within the four themes, it foregoes insight when all 15 variables are combined into a final vulnerability ranking. Using more in-depth census data to appropriately assign values to A and B may affect the final combined results, but further studies will need to be done to evaluate how analyzing these 15 individual variables as a whole relates to the actual vulnerability experienced by a census tract, and how the surrounding tracts may influence this value.

- It is instructive to compare the number of overall flags each tract received to the vulnerability ranking for each theme. The sum of the "very vulnerable" percentage for the 4 themes is 0.17%, 19.8%, and 81.5% for tracts 1, 2, and 3, respectively. This corresponds positively with the number of flags (being 1, 4, and 9 for tracts 1, 2, and 3, respectively). While the ranking is slightly different (81.5% very vulnerable compared to 60% of the possible flags), it might provide different insight on the type and range of vulnerability experienced by each tract, as explained in the next bullet point.
- Insight can be gained by noting which variables were flagged compared to which ones were not, and which of those contributed to the high vulnerability calculated by DST.

For example, look at the Socioeconomic Status Theme for the second tract. 29.8% of this tract does not have a high school diploma, which received a flag for ranking in the 90th percentile or higher of all tracts in this variable. However, this tract is also in the 79.9th percentile for low per capita income, and did not receive a flag for this variable. This means that this tract was flagged as “vulnerable” for about 30% of the population not having a high school diploma but was not flagged for having a lower per capita income than almost 80% of all other tracts, and since these variables are in the same theme, they count for the same weight. By using DST, both of these factors influence the vulnerability output of this tract with the amount of influence being respective of their weight, not as a yes-or-no flag.

- The Housing/Transportation theme has a combined percent of 0 for “very vulnerable” for each tract. It is noted that none of the population in any of these tracts resides in mobile homes. Due to this factor, and the low vulnerable percentages in the other variables in this theme, there is a 0% very vulnerable ranking for this group in all 3 tracts.
- The most vulnerable output is the Minority/Language Theme for the third tract. This makes sense, as there are only two variables in this theme, and the minority percentage is comparatively very high (67%). Even though the minority percentage for the second tract is even higher at 68%, the percentage of people who speak English “less than well” is only at 8.3% for the second tract, compared to 14.8% for the third.

Social Vulnerability Observations

It is clear that using Dempster Shafer Theory to analyze social vulnerability produces different results when compared to the method of indexing, and also compared to standard probability. Rather than ranking each group and taking the top 10%, or averaging each variable, DST analyzes the evidence for or against each vulnerability ranking. If not all of the population has been assigned to a specific vulnerability ranking, then the output will have some ignorance factor.

There are several ways to evaluate social vulnerability using DST. For example, the per capita income, age, disability type, and language variables could all be ranked on a scale rather than as a “yes or no” output. An alternate method would involve basing social vulnerability on the beliefs and familiarity knowledge of local analysts rather than just the percentage of the population under or over a certain standard. These types of data would provide a far more comprehensive insight into the vulnerability of individual tracts and their overall communities/towns. While Dempster Shafer Theory can offer solutions to some of the problems with the current indexing method, it also has some limitations. The results of combining all 15 variables using DST clearly presented an issue when the results put all three tracts at the same vulnerability ranking. On a theme level, however, the results offer more detail and insight into the type and range of vulnerability when compared to the current indexing method. By gaining a more comprehensive understanding of the vulnerability scale, it is possible to prepare or mitigate hazards in areas that currently do not register as vulnerable.

Conclusion

A primary goal of this paper has been to further understand the role of uncertainty in civil engineering and analyze potential frameworks with which this uncertainty might be captured, specifically in post-seismic structural analysis and social vulnerability capacities. While various frameworks have been presented in previous publications, the real-world applications in this paper now demonstrate the advantages Dempster Shafer Theory has to offer. Dempster Shafer Theory provides significantly different results in subjective cases when compared to the alternative of probability. Such results often provide a much more definitive and involved joint belief that takes into account aspects such as the confidence levels the experts have, any extra belief there may be in a wider range of events, and how conflicting the contributing beliefs are. Using a method that contains these nuances could yield significantly different results in damage assessments when compared to probability. In such cases of post-seismic structural analysis, a limited number of experts may be available to visit the site and provide an evaluation. Combining these valuable subjective individual beliefs to obtain a result requires consideration of those factors that make this assessment subjective, such as ignorance or confidence in a wider range of damage as opposed to a more specific range. Further, as each expert provides more or less evidence (or belief) of an event, the combined belief will increase or decrease support for one event, rather than averaging each added belief. The rules of probability, namely additivity, handle a potential doubt, or lack of belief, in an event as evidence to its contrary. By using a framework that acknowledges such a lack of belief as ignorance, rather than belief of the contrary, it is possible to achieve more meaningful results. The results discussed in this paper suggest that Dempster Shafer Theory is a viable, if not preferential, treatment of post-seismic structural assessments.

While the survey instrument used in this paper yielded notable results, it is worth acknowledging that the participants were mostly students. Structural engineers in real post-seismic analysis scenarios might have different confidence trends when evaluating damage. Due to these reasons, further testing is recommended using structural engineers in the process of post-seismic structural analysis.

A second goal of this paper was to investigate the applications of Dempster-Shafer Theory in social vulnerability ranking. While one of the current indexing methods provides an objective analysis using readily available data for every census tract in the United States, the output ignores many key factors that play into a community's ability to deal with a disaster. Dempster-Shafer Theory was tested using the same census data to determine if this framework would provide a more in-depth analysis. This theme-level output offered more detail of a tract's vulnerability, but combining all 15 variables still presents shortcomings. Another method for using DST in this capacity could be based on ranking tracts or communities on a belief basis, rather than relying solely on census data, and further testing is recommended to pursue this possibility.

The main applications of Dempster Shafer Theory explored in this paper are post-seismic structural damage analysis and social vulnerability indexing, but the possibilities extend far beyond that. Subjective investigations and assessments are unavoidable in many civil engineering and social science operations, as no two locations, projects, communities, and environments are exactly the same. The fields of civil engineering and social science are challenged with recognizing and accounting for these uncertainties, even with the use of subjective probabilities. A mathematical framework such as Dempster Shafer Theory, which allows for this uncertainty has the potential to change the outcome of infrastructure and hazard management decisions on a large scale.

Acknowledgments

The authors would like to thank Professor Anne Kiremidjian of Stanford University and Dr. David Lallemant (now of Nanyang University in Singapore) for creating the original survey instrument and adding the modifications we provided. The authors gratefully acknowledge the support of the National Science Foundation; grant number CMMI 1552855. The opinions expressed in this paper are those of the authors, and do not necessarily reflect the views or policies of NSF.

References

Armas, I., and Gavris, A. (2013). "Social vulnerability assessment using spatial multi-criteria analysis (SEVI model) and the Social Vulnerability Index (SoVI model) – a case study for Bucharest, Romania." *Natural Hazards and Earth System Sciences*, **13**(6), 1481–1499.

Aven, T., Baraldi, P., Flage, R., and Zio, E. (2014). *Uncertainty in Risk Assessment: The Representation and Treatment of Uncertainties by Probabilistic and Non-Probabilistic Methods*. The Atrium, Southern Gate, Chichester, West Sussex, United Kingdom: John Wiley & Sons, Ltd.

Ayyub, B. M. (1998). *Uncertainty Modeling and Analysis in Civil Engineering*. Boca Raton, Florida: CRC Press LLC.

Ayyub, B. M. (2001). *Elicitation of Expert Opinions for Uncertainty and Risks*. Boca Raton, Florida: CRC Press LLC.

Ayyub, B. M., and Klir, G. J. (2006). *Uncertainty Modeling and Analysis in Engineering and the Sciences*. Boca Raton, Florida: Chapman & Hall/CRC.

Ballent, W., Corotis, R. B., and Torres-Machi, C. (2018). "Representing Uncertainty in Natural Hazard Risk Assessment with Dempster Shafer (Evidence) Theory." *Sustainable and Resilient* Infrastructure, in press.

Beynon, M., Curry, B., and Morgan, P. (2000). "The Dempster-Shafer theory of evidence: an alternative approach to multicriteria decision modelling." *The International Journal of Management Science*, **28**(1), 37–50.

Cooke, R. M. (1991). *Experts in Uncertainty: Opinion and Subjective Probability in Science*. Oxford, United Kingdom: Oxford University Press.

Corotis, R. B. (2015). "An Overview of Uncertainty Concepts Related to Mechanical and Civil Engineering." *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering*, **1**(4), 12.

Cutter, S., Ash, K. and Emrich, C. (2014). "The Geographies of Community Disaster Resilience." *Global Environmental Change*, November, 29, 65-77.

Cutter, S., Boruff, B., and Shirley, W. L. (2003). "Social Vulnerability to Environmental Hazards." *Social Science Quarterly*, **84**(2), 242–261.

Cutter, S., Burton, C. and Emrich, C. (2010). "Disaster Resilience Indicators for Benchmarking Baseline Conditions." *Journal of Homeland Security and Emergency Management*, 7(1).

Dubois, D. (2006). "Possibility Theory and Statistical Reasoning." *Computational Statistics & Data Analysis*, **51**(1), 47–69.

Dubois, D., and Prade, H. (1988). *Possibility Theory: An Approach to Computerized Processing of Uncertainty* (1st ed.). New York: Springer US.

Elwood, E., and Corotis, R. B. (2015). "Application of Fuzzy Pattern Recognition of Seismic Damage to Concrete Structures." *ASCE ASME Journal of Risk and Uncertainty in Engineering Systems Part A Civil Engineering*, **1**(4).

Flanagan, B. E., Gregory, E. W., Hallisey, E. J., Heitgerd, J. L., and Lewis, B. (2011). "A Social Vulnerability Index for Disaster Management." *Journal of Homeland Security and Emergency Management*, **8**(1), 22.

Hurley, M. and Corotis, R. (2014). "Perception of Risk of Natural Hazards: a Hazard Mitigation Framework." *International Journal of Risk Assessment and Management*, 17(3), 188-211.

Kahneman, D. and Tversky, A. (2000). *Choices, Values, and Frames*, Russell Sage Foundation, Cambridge University Press, Cambridge, UK.

Klein, Gary (2008). Naturalistic Decision Making, *Human Factors*, 50(3), June, 456-460.

Klir, G. J. (2006). *Uncertainty and Information Foundations of Generalized Information Theory*. Wiley IEEE. Hoboken, New Jersey: John Wiley & Sons, Inc.

Klir, G. J., and Smith, R. M. (2001). "On measuring uncertainty and uncertainty-based information: Recent developments." *Annals of Mathematics and Artificial Intelligence*, **32**(1–4), 5–33.

Lechowska, E. (2018). "What Determines Flood Risk Perception? A Review of Factors of Flood Risk Perception and Relations Between its Basic Elements." *Natural Hazards*, 94, 1341-1366.

Peacock, W., Gillis, W. and Mayunga, J. (2010). *Advancing the Resilience of Coastal Localities: Developing, Implementing and Sustaining the Use of Coastal Resilience Indicators: A Final Report*, Texas Hazard Reduction and Recovery Center, College of Architecture, Texas A&M University, College Station, Texas.

Pfefferbaum, B., Wyche, K., Pfefferbaum, R., Norris, F., and Stevens, S. (2008). "Community Resilience as a Metaphor, Theory, Set of Capacities, and Strategy for Disaster Readiness." *Springer Science+Business Media, LLC*, **41**(1–2), 127–150.

Ross, T. J. (2010). *Fuzzy Logic with Engineering Applications* (3rd ed.). West Sussex, England: John Wiley & Sons.

Saaty, T. (2008). "The Analytic Hierarchy and Analytic Network Measurement Processes: Applications to Decisions under Risk." *European Journal of Pure and Applied Mathematics*, 1(1), 122-196.

Shafer, G. (1976). *A Mathematical Theory of Evidence*. Princeton, New Jersey: Princeton University Press.

Shafer, G. (1987). "Belief Functions and Possibility Measures." In J. C. Bezdek (Ed.), *Analysis of Fuzzy Information* (Vol. Vol 1: Mathematics and Logic, pp. 51–84). Boca Raton, Florida: CRC Press.

Slovic, Paul and Elke U. Weber (2002). "Perception of Risk Posed by Extreme Events." Risk Management Strategies in an Uncertain World. Palisades. 1-21.

Slovic, Paul (2000). The Perception of Risk. London: Earthscan Publications Ltd.

Thomas, D. S. K., Phillips, B. D., Lovekamp, W. E., and Fothergill, A. (Eds.) (2013). *Social Vulnerability to Disasters* (Second.). Boca Raton, Florida: CRC Press: Taylor & Francis Group.

Vick, S. G. (2002). *Degrees of Belief Subjective Probability and Engineering Judgment*. Reston, Virginia: ASCE Press.

Walley, P. (1991). *Statistical Reasoning with Imprecise Probabilities*. Monographs on Statistics & Applied Probability (1st ed.). Chapman & Hall/CRC.

Wang, Z., and Klir, G. J. (2009). *Generalized Measure Theory*. IFSR International Series on Systems Science and Engineering (Vol. 25). New York, NY: Springer Science + Business Media, LLC.

Yager, R. R., and Liu, L. (2008). *Classic Works of the Dempster-Shafer Theory of Belief Functions*. (J. Kacprzyk, Ed.)Studies in Fuzziness and Soft Computing (1st ed., Vol. 219). Heidelberg: Springer.

APPENDIX I

Dempster Shafer Theory

Belief/Plausibility Measures

Dempster Shafer Theory, sometimes called evidence theory, is based on a measure of degree of belief, called a *belief measure*, Bel(A), which expresses the degree of belief that an occurrence belongs to the set A (the term A will now be interpreted as possibly consisting of a set, rather than being limited to a single element). A basic assignment or *Mobius Measure,* m(x), can then be uniquely calculated, providing "an assessment of the likelihood of each set in a family of sets identified by the analyst" (Ayyub and Klir, 2006). Therefore, Mobius Measures are the evidence that is compiled for each element or event. *Plausibility measures* represent the evidence that two sets have any possible overlap. Belief and plausibility measures are related to Mobius measures (Aven et al., 2014):

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{I.1}$$

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) \tag{I.2}$$

Equations (I.1) and (I.2) indicate that the belief in A is the sum of all Mobius measures relating to B in which B is fully contained within or equal to A. The plausibility measure is then the sum of all Mobius measures relating to B in which A and B have any potential commonality. The plausibility measure Pl(A) represents not only the evidence represented by the belief Bel(A), but also the evidence associated with any sets which overlap with A. The relationship between plausibilities and belief measures can be derived as follows (Ayyub and Klir, 2006):

$$Pl(\bar{A}) = 1 - Bel(A) \tag{I.3}$$

$$Pl(A) \geq Bel(A) \tag{I.4}$$

It is important to note that a degree of belief or evidential support of A, Bel(A), does not implicate disbelief of $\bar{A}$. Therefore, Dempster Shafer Theory differs from classical Probability Theory in that it provides a natural framework for modeling ignorance (Shafer, 1976), one minus the sum of the belief and the belief of the complement (Ross, 2010).

Belief Combination Using Dempster Shafer Theory

A powerful artifact of Dempster Shafer Theory is the ability to combine beliefs from multiple sources, yielding joint evidence (Shafer, 1987). Fundamental to the theory is that beliefs are combined through their associated Mobius measures, given by Eq.(I.5) below:

$$m_{1,2}(A) = \frac{\sum_{B \cap C = A} m_1(B) \cdot m_2(C)}{1-c} \tag{I.5}$$

Where the denominator is calculated using Eq.(I.6):

$$c = \sum_{B \cap C = \emptyset} m_1(B) \cdot m_2(C) \tag{I.6}$$

The numerator is determined by summing the products of the beliefs in every event or event combination in which the only commonality is the event being considered. The denominator is the sum of the products for every event or event combination that have nothing in common. As stated by Ayyub (2001) , “Probability Theory can be treated as a special case of the Theory of Evidence [Dempster Shafer Theory]. For cases in which all focal elements for a given basic assignment, *m* are singletons, the associated belief measure and plausibility measure collapse into a single measure, a classical probability measure”.

Combining judgment from multiple experts in a mathematically-founded framework is especially important in combining engineering judgment with both quantitatively- and qualitatively-based risk calculations. The examples in this paper combine field judgment in seismic damage assessment and building vulnerability, demonstrating the potential power of Dempster Shafer Theory for the combination of beliefs.

APPENDIX II

QUESTION 1 of 5

What is your belief that the damage in the image above is in the following ranges? You can have up to 100% in either range.

0%-66%? 34%-100%?

Using the amount of belief you assigned to the large ranges above, split that into the smaller ranges below. If you are not as confident in the smaller ranges, you can assign less belief, but you cannot have more confidence in the smaller ranges than you do in the larger ones (e.g., belief for 0%-33% plus belief for 34%-66% cannot be greater than the belief you assigned above for 0%-66%). **The sum of these 3 boxes cannot exceed 100%.**

0% to 33% 34% to 66% 67%-100%