Dynamic Returns and Volatility Spillovers Across the U.S. Stock Market, World Gold Market, and Chinese Stock Market: Insights for Hedging and Diversification Strategies

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Abstract

This paper implements a VAR-EGARCH model (Nelson, 1991; Koutmos, 1996) to explore the linkage between both the returns and volatility transmissions between the U.S. stock market, the world gold market, and the Chinese stock market over the period from January 15, 1996, through August 31, 2015. The exponential component of the model allows us to capture significant asymmetric effects across financial markets and confirms the necessity of a VAR-EGARCH model over a VAR-GARCH model. Also, we find that reciprocal volatility transmission existed between the U.S. stock market and the Chinese stock market over the period, while the transmissions from the U.S. stock market to the Chinese stock market are more significant. Moreover, the past U.S. stock market returns can be used to predict current returns of the Chinese stock market and the world gold market. This suggests the predominant status of the U.S. stock market in the world. This paper further analyzes the model's results by comparing dynamic hedging to portfolio diversification strategy. We show that diversification is far more effective in reducing risks than the dynamic hedging strategy. Moreover, the results of portfolio diversification suggest that passive investors should hold an equal weight portfolio that contain indices or commodities from these three markets, while active investors should re-balance their minimum variance line portfolio and hold more gold future contracts.

1 Introduction

The literature about stock markets and gold markets has gained significant attention from researchers and practitioners in recent years. The 2015 Chinese stock market crashed and decimated more than 7 trillion dollars worth of stock in six months, while creating strong spillover effects through global stock markets, especially in the United States. On August 24, 2015, the Standard and Poor's (S&P)500 plunged almost 4 percent, and the Dow Jones Industrial Average (DJI) momentarily slumped 1,089 points. These dramatic movements happened right after the benchmark Chinese stock index, the Shanghai Stock Market Composite declined, 8.5 percent. These observations reveal the spillover effects between financial markets. Moreover, these successive phenomena in the past twenty years have driven investors to consider alternative investment assets.

Consequently, examining both the dynamic returns, and volatility spillover effects across the three markets, will be compelling and meaningful. Moreover, a better understanding of financial market interdependencies with asymmetric effects will certainly improve the effectiveness of portfolio designs and diversification strategies. We use a multivariate vector autoregression-exponential generalized autoregressive conditional heteroskedasticity (VAR-EGARCH) model in order to consider and analyze the asymmetric effects of volatility spillovers across financial markets. Specifically, the VAR component examines how each market's current period of return is related with its own past, and with cross markets past period of returns. The GARCH component examines how volatility is transmitted within and across these three financial markets. The E component accommodates the potential asymmetric effects within and across financial markets. Subsequently, we compare the risk reduction of a dynamic hedging strategy to a static hedging strategy (portfolio diversification) by either finding an optimal hedging ratio, or by forming a mean-variance optimal portfolio that contains three assets (three-months gold futures and two stock indices). The significant level of contemporaneous covariances and correlations from the model's results provide useful insights to global investors, portfolio managers, and government agencies.

The objective of this paper is three-fold. We first examine the dynamic returns and volatility linkages between the U.S stock market, Chinese stock market, and world gold market, especially after accommodating the leverage effect[1] of financial markets (Black,1976) by capturing the asymmetric volatility spillovers across these three markets. Then,

[1] Negative returns drag down market value of firms which increase debt-to-equity ratio and associated with higher volatility

we implement the results of the model to discuss the dynamic hedging and static hedging (diversification) strategies. Third, we re-examine the results of the VAR-EGARCH model using data from only the past two major financial crises while testing the effectiveness and performances of both investment strategies in two major financial crises: the 1998 Asian Financial Crisis and 2008 Global Financial Crisis.

2 Literature Review

There is extensive literature that builds the background story for this paper. First, to understand the dynamic interactions between the gold market and stock markets, Baur and McDermott (2010) firstly utilize a univariate GARCH(1,1) model to examine the relationship between these two different financial markets in developed and developing countries, such as the U.S. and China, and further define the concepts of hedge[2] and safe haven.[3] Their research shows that gold serves as both a hedge and safe haven in the U.S. and China. However, a univariate GARCH(1,1) model does not take volatility spillover across financial markets into consideration. In addition, Hammoudeh et al.(2011) implements various univariate GARCH(1,1) models to compute the value at risk(VaR) of the daily returns of gold and further suggest that gold could serve as a hedge with other stocks markets during financial disturbances. However, univariate GARCH models do not test the significant level of correlation between these financial markets during financial disturbances and as a result, we must gather more evidence.

Moreover, Mensi et al.(2013) use a multivariate VAR(1)-GARCH(1,1) model to show that there is a significant correlation and volatility transmission between the S&P 500 and many commodities such as gold. They further suggest to portfolio managers and investors that to add commodities such as gold into stock portfolios will increase risk adjusted return. However, a better understanding of the mechanism(i.e., the leverage effect) of volatility spillovers across financial markets will certainly provide better suggestions. Similarly, Mohamed et al.(2015) use a bivariate VAR(1)-GARCH(1,1) model and find that gold assets can serve as a safe haven for stocks in the Chinese stock market, especially, past gold returns which can be used to predict current Chinese stock returns. Lai and Tseng (2010) use a bivariate distribution function to show that the Chinese stock market can serve both as a safe haven and a hedge for the U.S. stock market.

[2]Negatively correlated or uncorrelated with stock markets during certain periods

[3]A asset that holds it value or even reverse market condition during financial disturbances

Consequently, it is natural to consider these three markets together. However, the linkages between the U.S. stock market, Chinese stock market, and the world gold market have never been examined simultaneously. In terms of the gross domestic product (GDP), the United States and China are the first and the second largest economies in the world, respectively. In terms of the purchase power parity (PPP), according to the International Monetary Fund (IMF) report in 2014, China is the largest economy in the world. In either case, the insights provided by this research will be significant. Furthermore, according to the World Gold Council (WCC), China is the largest physical gold buyer. China is increasing its gold demands in terms of private jewelry, private gold investments (forward, future, and options), and government reserves. According to statistics from the Central Bank of the People's Republic of China, the Chinese government is increasing the purchase of gold reserves while decreasing holdings of foreign reserve-dollars. On the other side, the United States has the largest gold reserves in the world.

Building on the existing literature, this paper examines the spillover effects across all three markets by implementing a multivariate VAR-EGARCH model. Furthermore, if the multivariate GARCH model is successfully implemented, the multivariate distribution of the returns can be used directly to compute the optimal allocation of a portfolio that contains assets from all three markets (Bauwens et al. 2006). To the best of our knowledge, a multivariate VAR-EGARCH model has not been used to quantify the relationship between these three markets. Furthermore, the advantages of implementing a VAR-EGARCH model is nontrivial. Compared to univariate GARCH-based models, this multivariate VAR-EGARCH model investigates and analyzes three financial markets' interdependence in one step estimation, while capturing the asymmetric volatilities between positive and negative shocks within and cross financial markets.

The remainder of the paper is organized as follows: Section 3 introduces the empirical method. Section 4 presents the sample data and descriptive statistics. Section 5 discusses the empirical results with implications in portfolio management, and Section 6 provides some conclusions with remarks.

3 Empirical Method

There are many GARCH-based models that have been successfully implemented to address similar issues such as the Constant Conditional Correlation(CCC)-GARCH model of Bollerslev(1990); the Baba, Engle, Kraft, and Kroner(BEKK)-GARCH model of Engle and Kroner(1995); the Dynamic Constant Correlation(DCC)-GARCH model of Engle(2002); the VAR-GARCH model of Ling and McAleer(2003); the Vector autoregressive Moving Average(VARMA)-AGARCH model of McAleer, Hoti, and Chan(2009); and the EGARCH model of Nelson (1991).

It turns out that the AR(1)-DCC-GARCH model and AR(1)-CCC-GARCH model cannot capture the simultaneous return and volatility spillovers between financial markets. Also, in three markets (variables) set ups, a full BEKK-GARCH model might encounter a convergence problem due to parameter proliferation (Mohamed et al. 2015). Furthermore, Hansen and Lunde (2005) found that GARCH(1,1) outperforms all other exacting GARCH-based models, except in the presence of the leverage effect due to the volatility of financial returns changing over time. As a result, this paper considers an EGARCH–based model, a multivariate VAR-EGARCH model that successfully accommodates the leverage effect (Nelson 1991) and also allows simultaneous volatility transmission across these three markets. Overall, this paper uses a tri-variate VAR-EGARCH model similar to Koutmos's (1996) approach:

$$R_{i,t} = \beta_{i,0} + \sum_{i=1}^{3} \beta_{i,j} R_{j,t-1} + \varepsilon_{i,t} \tag{1}$$

$$ln(\sigma_{i,t}^2) = \alpha_i + \sum_{i=1}^{3} \gamma_{i,j} f\left(v_{j,t-1}\right) + \tau_i ln\left(\sigma_{i,t-1}^2\right) \tag{2}$$

$$f\left(v_{j,t-1}\right) = \left(|v_{j,t-1}| - E\left|v_{j,t-1}\right| + \delta_j v_{j,t-1}\right), \tag{3}$$

$$\sigma_{i,j,t} = \rho_{i,j}\sigma_{i,t}\sigma_{j,t} \tag{4}$$

where $i, j = 1, 2, 3$, and $R_{i,t}$ are the percentage returns for the Chinese stock market, the U.S. stock market, and world gold market, respectively. The conditional mean and conditional variance are represented by $\mu_{i,t}$ and $\sigma_{i,t}^2$, respectively. The innovation at time t is defined by ($\varepsilon_{i,t} = R_{j,t-1} - \mu_{i,t}$) and standardized innovation is defined by ($v_{j,t}=\frac{\varepsilon_{i,t}}{\sigma_{i,t}}$). The derivation from Nelson's univariate EGARCH model to Koutmos's multivariate EGARCH model is in the Appendix I.

Eq.(1) denotes the vector auto-regression of the adjusted returns for the three financial markets, where the conditional mean in each market is a function of its own past returns and cross-market past returns. A significant $\beta_{i,j}$ measures the direct effect of the past period returns in market j toward current period returns in market i.

Eq.(2) represents the EGARCH part(the variance of $\varepsilon_{i,t}$) ; where the natural logarithm function of the conditional variance of each market's returns is equal to its past own and cross-market standardized innovations and past own conditional variance. The volatility spillovers across markets are captured by $\gamma_{i,j}$. The persistence in volatility is measured by τ_i.

Eq.(3) is the asymmetric function of past standardized innovations. $|v_{j,t-1}| - E\,|v_{j,t-1}|$ measures the magnitude effect of an innovation. For example, if $\gamma_{i,j}$ is positive, then $|v_{j,t-1}| - E\,|v_{j,t-1}| > 0$ implies that the impact of $v_{j,t-1}$ on the conditional variance ($\sigma^2_{i,t}$) is positive. Moreover, the term $\delta_j v_{j,t-1}$ measures the sign effect, where δ_j measures the asymmetric impact on the volatility of market i to itself or j, and it can either reinforce or partially offset the magnitude effect. For example, a negative δ_j and a negative $v_{j,t-1}$ will reinforce the magnitude effect, while a positive $v_{j,t-1}$ will partially offset it. We assume the residuals($\varepsilon_{i,t}$) of each function in Eq.(1) are normally distributed, instead of a generalized error distribution(GED) by Nelson(1991). Consequently, the distribution difference leads to $\alpha_i(\Pi/2)^{1/2}$ difference in Eq(3) which we already incorporated in the estimation. Overall, the asymmetric function will be present in the following partial derivative from Eq.(3) with respect to $v_{j,t}$:

$$\frac{\partial f\,(v_{j,t})}{\partial v_{j,t}} = \begin{cases} 1+\delta_j, \text{ if } v_{j,t} > 0 \\ \delta_j - 1, \text{ if } v_{j,t} < 0 \end{cases}$$

If δ_j=0, then a positive shock and a negative shock have the same effect with the same magnitude; if δ_j is between -1 and 0, then a negative shock increases volatility more than a positive shock; if δ_j smaller than -1, a negative shock increases volatility, while a positive shock reduces volatility. A 1% positive innovation, measured by $\gamma_{i,j}(1+\delta_j)$, indicates that the effects from a 1% positive innovation in market j to the volatility of market i. Similarly, a 1% negative innovation will be measured by $\gamma_{i,j}|-1+\delta_j|$. Consequently, the ratio of relative importance of asymmetric will be measured by $\frac{|-1+\delta_j|}{(1+\delta_j)}$. Overall, the magnitude and sign effects measure whether or not the volatility spillovers within and across market are asymmetric, and Black(1976) refers to it as the leverage effect.

Eq.(4) provides the constant conditional correlations between the returns of three markets. The coefficient $\rho_{i,j}$ is the cross-market correlation coefficient of the standardized residuals between two markets. The significance of $\rho_{i,j}$ indicates that time-varying volatilities across markets i and j are correlated over time, for i≠j. This assumption largely simplifies the estimation of the model (Bollerslev, Chou, & Kroner, 1992). Overall, this trivariate EGARCH requires generating 33 parameters under one step estimation. The log likelihood function for this multivariate EGARCH model is (Koutmos, 1996):

$$L(\theta) = -0.5\,(NT)\,ln\,(2\pi) - 0.5\sum\left(ln\,|S_t| + \varepsilon_t^{'}S_t^{-1}\varepsilon_t\right) \tag{5}$$

where N is the number of equations (three in this cases); T is the number of observations; θ is the 33×1 parameter vector to be estimated; $\varepsilon_t^{'}$=$[\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}]$ is the 1×3 vector of innovations at time t; and S_t is the 3×3 time-varying conditional variance-covariance matrix with the diagonal elements given in $Eq.\,(2)$ for $i = 1, 2, 3$, and cross diagonal elements given in $Eq.\,(4)$ for $i, j = 1, 2, 3$, and i≠ j. The log likelihood function is highly nonlinear in θ and as a result, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is used to estimate L(θ) .

4 Data and Descriptive Statistics

This paper extracts 3 pieces of data from the Bloomberg database from January 15, 1996 to August 31, 2015. The first is the daily closing price of S&P 500 index (SPX) which represents the U.S. stock market. The second is the daily closing price of the Shanghai Stock Exchange Composite Index (SHCOMP) , which is the benchmark stock index in China. The third is the daily closing price of three months of gold futures traded in New york (COMEX), which is the global benchmark price for gold trading. By taking the difference of logarithm of price, the return series are calculated as follows:

$$R_{i,t} = 100 * (LogP_t - LogP_{t-1}) \tag{6}$$

where P_t and P_{t-1} are daily closing prices at time t and t-1, respectively.

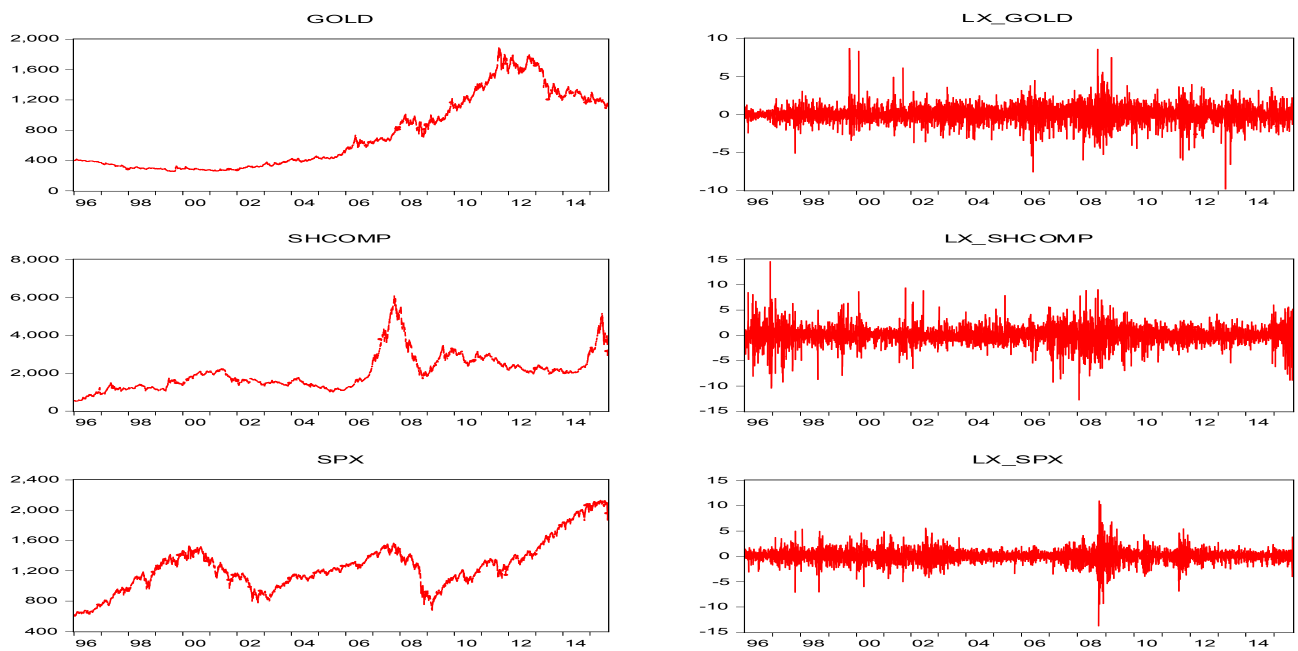

Figure 1: Log of Return Series and Daily Closing Price Chart.

The price movements followed by gold future prices, the S&P 500, and the SHCOMP are depicted in Fig. 1. The Chinese stock market modestly increased from 1996 to 2005, and started to steeply rise until the end of 2007; then it dramatically slumped between 2008 and mid-2009, due to the contagion effects of the U.S. sub-prime crisis. Afterwards, it started to recover and was followed by another major rise between mid-2014 to Aug-2015. Also, the S&P 500 sustained an upward trend over the whole period while declining due to the collapse of the 2001 Internet

Bubble and the 2008 Subprime Mortgage Crisis. On the other hand, gold future changes showed a consistent upward trend between 2000 and mid-2012 and started to decline afterwards. Overall, this charts partially confirm some practitioners' opinion that gold price changes in an opposite direction to stock market movements during financial disturbances.

Table 1 presents selected descriptive statistics for the return series. The gold market provided 0.3 and 1.6 basis points lower daily returns over the whole period compared to the U.S. stock market and Chinese stock market, respectively; however, the unconditional volatility (a measure as standard deviations) is substantially lower than both stock markets. The Chinese stock market has a 1.3 basis point higher daily return than the U.S. stock market but has the highest unconditional volatility, 1.632. As a result, gold futures provided a high sharpe ratio considering its risk-adjusted returns. Furthermore, the low unconditional correlations of return series between the gold market and the Chinese stock market, and gold market and the U.S. stock market, imply that gold is an potential hedge for portfolio management with stock assets by diversifying the unsystematic risks.

Moreover, negative skewness and high positive kurtosis coefficients address the asymmetric distribution of returns in the gold market, the U.S. stock market, and the Chinese stock market. It implies that these three markets had a large volume of negative returns in the left skew. Moreover, the JarqueBera(JB) test shows strong evidence of rejecting normality for all three return series. This evidence implies that an EGARCH-based model is well-suited to implement in this context(Nelson, 1991). Also, the results from the Ljung-Box(Q) tests indicate evidence of autocorrelation in three return series and squared return series. It implies that linear and nonlinear autocorrelated innovations exist. The ARCH(p) test provides strong evidence of conditional heteroscedasticity in all three return series, that further confirms the usefulness of an EGARCH-based model in this particular context. Finally, the Augmented Dickey-Fuller(ADF) test and Phillips-Perron(PP) test address the stationarity property of the return series, which implies we can utilize these data for further analysis directly.

Table 1 Selected descriptive statistics of daily return series for three financial markets.

	Gold	SHCOMP	S&P 500
Panel A. Summary Statistics			
Mean(%)	0.023	0.039	0.026
Maximum(%)	8.716	14.602	10.96
Minimum(%)	-9.837	-12.764	-13.777
Standard Deviations	1.131	1.783	1.277
Skewness	-0.033	-0.255	-0.370
Kurtosis	9.880	8.679	12.700
Jarque-Bera	9034.81***	6183.09***	18053.73***
$Q(12)$for $R_{i,t}$	1148.60***	1184.50***	1195.90***
$Q^2(12)$for $R_{i,t}$	1211.00***	1661.40***	5088.10***
ARCH(p)	28.82***	32.36***	32.67***
ADF	-67.283***	-14.92***	-12.64***
PP	-67.289***	-67.229***	-71.50***
Observations	4581	4581	4581
Panel B. Unconditional Correlations			
Gold	1.00	0.045	-0.001
SHCOMP		1.00	0.043
S&P500			1.00

Notes: The table reports the selected descriptive statistics of three return series by testing each residue of a first order autoregressive model(AR(1)). The JarqueBera test is a test for normality based on skewness and excess kurtosis. The Ljung-Box(Q) tests are tests for autocorrelations, in this paper we choose the lag of 12 for both return series and square return series. ARCH(p) refers to a test for conditional heteroscedasticity of lag up to order 1. Augmented Dickey-Fuller(ADF) and Phillips-Perron(PP) are two unit root tests for testing whether the return series are stationary. Estimated unconditional correlations are based on three return series between -1 and 1. *** indicates the rejection of the null hypotheses of normality, absent of autocorrelation, absent of ARCH effects, and the unit root at the 1% levels.

5 Empirical Results with Implications

This section reports the estimated results from a trivariate VAR-EGARCH model and discusses its portfolio implications with relevant hedging and diversified strategies.

5.1 Financial markets interdependence

To examine the lead/lag relationships[4], as well as volatility spillovers across financial markets, we estimate the tri-variate VAR-EGARCH model first. The significant level of 33 parameters and its relevant diagnostic tests are reported in Panel A of Table 2.

First, the statistical significance of $\beta_{1,2}$ and $\beta_{3,2}$(two coefficients are in the VAR equation from the U.S. stock market to the Chinese stock market and the gold future market, respectively) imply that past returns of the U.S. stock market can be used to predict the current period return for both the Chinese stock market and the world gold future market. Also, the insignificant of $\beta_{1,3}$, $\beta_{3,1}$ show that the past returns of the gold future market can not use to predict the Chinese stock market, and vise versa. This results do not agreed with the Mohamed et al.(2015) results, as they consider the Chinese stock market and gold future market only. Furthermore, the insignificance of $\beta_{1,1}$, $\beta_{2,2}$ and $\beta_{3,3}$ mean that the past return of each market is not related with its current period of return. This results are consistent with the random walk hypothesis in finance literature that past movements of its own market cannot be use to predict the future movement. Overall, it suggests the significance of lead/lag relationships between the U.S. stock market and the other two financial markets.

Second, the volatility spillover (the second moment of market's interdependence) effects are measured by the coefficient $\gamma_{i,j}$. The conditional variance in each market is affected by its own past innovations. (i.e.,$\gamma_{1,1} = 0.21, \gamma_{2,2} = 0.11, \gamma_{3,3} = 0.12$). Furthermore, the conditional variance of the Chinese stock market is also significantly affected by the world gold market's past innovations ($\gamma_{1,3} = 0.02$) and the U.S. stock market's past innovations ($\gamma_{1,2} = 0.01$).

[4]The past value of one leading variable is cross-correlated with the current values of another(lagging) variable

Similarly, the conditional variance of the U.S. stock markets is affected by the other two financial markets past standardize innovations($\gamma_{2,1} = 0.03$,$\gamma_{2,3} = 0.03$). Nevertheless, the conditional variance of the world gold market is not significantly affected by the past standardized innovations of Chinese stock markets. Overall, we conclude that reciprocal volatility spillovers exist between the U.S. stock market the Chinese Stock market, and the U.S. stock market and the world gold market, while we only observe one-sided transmission from the world gold market to the Chinese stock market. This means the U.S. stock market incorporates risks (information) from other financial markets and its market movements reflect global investors' expectations. It confirms the important status of the U.S. stock markets to other financial markets.

Third, volatility persistence is denoted by τ_i. We observe significant volatility carry over from its past periods and the magnitudes are almost unity for the U.S. stock market ($\tau_2 = 0.98$), the Chinese stock market ($\tau_1 = 0.98$), and the world gold future market ($\tau_1 = 0.99$). Moreover, the contemporaneous conditional correlation is denoted by $\rho_{i,j}$. We notice that significant contemporaneous conditional correlation exist between the U.S. stock market and Chinese stock market, and the Chinese stock market and the world gold future market, but not between the U.S. stock market and the world gold future market, in the innovations of returns. These results indicate that the time-varying volatilities of the returns across these two pairs of markets are correlated over time.

Furthermore, the asymmetric effect exists when δ_j is negative and statistically significant. Indeed, it is negative and statistically significant in both the Chinese stock market and the U.S stock market, but not the gold future market. Moreover, we conclude that a negative shock from the Chinese stock markets provides 1.172 times more volatility than a positive shock ($\delta_1 = -0.079$) to its own and other two financial markets, while a negative shock from the U.S. stock market provides 17.13 times more volatility to its own and the other two financial markets but a positive shock reduce the volatility ($\delta_2 = -1.124$) transmission by a much smaller amount. This results first show that the leverage effects exist in and across stock markets and especially in the U.S. stock market. Also, the results confirm the necessity of implementing a VAR-EGARCH model than a VAR-GARCH model.

Table 2. Panel A. Maximum Likelihood Estimates of the VAR-EGARCH: 1= SHCOMP, 2= S&P 500, 3=Gold

SHCOMP		S&P 500		Gold	
$\beta_{1,0}$	0.008(0.019)	$\beta_{2,0}$	0.025*(0.013)	$\beta_{3,0}$	0.007(0.014)
$\beta_{1,1}$	0.007(0.015)	$\beta_{2,1}$	-0.002(0.006)	$\beta_{3,1}$	0.007(0.007)
$\beta_{1,2}$	0.107***(0.018)	$\beta_{2,2}$	-0.021(0.017)	$\beta_{3,2}$	0.031***(0.012)
$\beta_{1,3}$	0.012(0.018)	$\beta_{2,3}$	-0.010(0.013)	$\beta_{3,3}$	−0.003(0.016)
α_1	0.033***(0.003)	α_2	0.009***(0.002)	α_3	0.010***(0.001)
$\gamma_{1,1}$	0.207***(0.009)	$\gamma_{2,1}$	0.026***(0.007)	$\gamma_{3,1}$	−0.007(0.007)
$\gamma_{1,2}$	0.007*(0.004)	$\gamma_{2,2}$	0.106***(0.008)	$\gamma_{3,2}$	0.028***(0.003)
$\gamma_{1,3}$	0.02***(0.009)	$\gamma_{2,3}$	0.032***(0.006)	$\gamma_{3,3}$	0.119***(0.004)
τ_1	0.980***(0.002)	τ_2	0.982***(0.002)	τ_3	0.990***(0.001)
δ_1	−0.079***(0.026)	δ_2	−1.124***(0.110)	δ_3	0.197***(0.024)
$\rho_{1,2}$	−0.043***(0.013)	$\rho_{2,3}$	0.001(0.013)	$\rho_{1,3}$	0.046***(0.013)

Panel B. Model Diagnostics	SHCOMP	S&P 500	Gold
Mean(%)	0.001	-0.004	0.013
Standard Deviations	1.000	1.000	1.000
Skewness	-0.136	-0.441	-0.008
Kurtosis	6.076	4.982	8.769
$Q(12)$	31.80***	17.38	10.28
$Q^2(12)$	7.09	19.77*	31.95***
BDS^6	-0.315	−2.522**	−2.027***
Observations	4581	4581	4581

Note: Table 2 illustrate the diagnostics tests and conditional correlation matrix for Standardized Innovation $\left(v_{j,t} = \frac{\varepsilon_{i,t}}{\sigma_{i,t}}\right)$. BDS independent, which test up to 6 correlation dimensions, is a nonlinear structure test with a null hypothesis of residue independent identified distributed(i.i.d). Standard errors are in parenthesis. ***, **, * denote the significant level at the 10%, 5% and 1% level, respectively.

In Panel B, we summarize the selective statistics of standardized innovations for the three markets. The L-B(Q) test shows that for each market's standardized innovation, their still exists significant linear dependence on SHCOMP but the value is small(i.e., Auto-Correlations=0.03) ; while $S\&P500$ and Gold's standardize innovations show significant non-linear time dependence, but the values are small as well. We further address the issue by employing the BDS independent test(Brock, Dechert, et al. 1996), which has a null hypothesis of i.i.d.(whiteness).The results suggest that we confirmed the standardized innovation is white linear dependence for the SHCOMP and are non-white & non-linear dependence for the S&P 500 and Gold, after considering both L-B(Q) tests' results. Overall, this model well explain the dynamic of the return series, because these statistics tests show improvements in terms of autocorrelation between the three markets as compared to the summary statistics in table 1.

5.2 Dynamic hedging and diversification

The implications of a tri-variate VAR-EGARCH model will be discussed in the following.

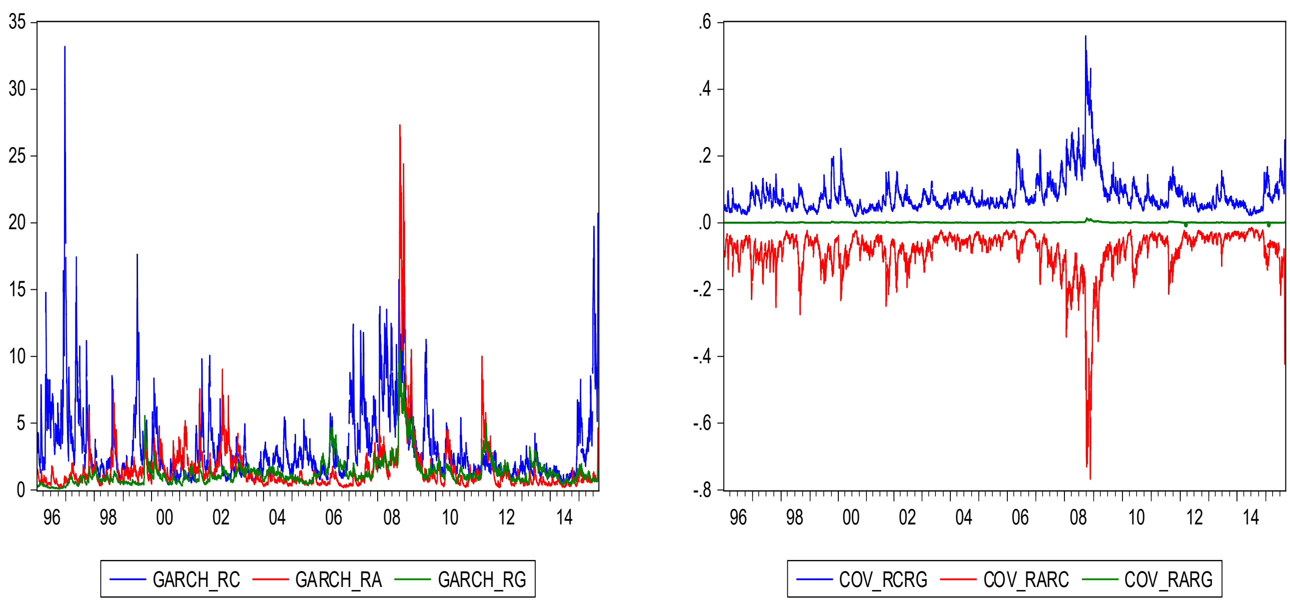

Figure 2: Graphs of conditioanl volatility and covariance volatiltiy between financial markets

In section 4.1 and Figure.2, we observe significant time-varying conditional correlation between the U.S. stock market and Chinese stock market, the Chinese stock market and the gold future market. Moreover, the conditional variance of gold futures in the whole study period is the lowest of the three financial markets. Consequently, we expect gold future contracts to be an ideal instrument in terms of dynamic hedging and static hedging (portfolio diversification).

From the figure.2 in the left hand side, we observe the strongest conditional volatility for the Chinese stock market between June 1997 and July 1998 (during Asian Financial Crisis), and the strongest conditional volatility for the U.S. Stock market between April 2007 and October 2008 (Global Financial Crisis). As a result, we not only examine these two strategies over the whole study period, but also examine their performances during these two major financial disturbance periods.

First, we implement the dynamic hedging strategy by following Kroner and Sultan's (1993) approach to finding the optimal hedging ratio :

$$\beta_t^{gc} = \frac{h_t^{gc}}{h_t^{g}}, \beta_t^{ga} = \frac{h_t^{ga}}{h_t^{g}} \tag{6}$$

where h_t^{gc} and h_t^{ga}represent the conditional covariance between gold future market and the Chinese stock market, gold future market and the U.S. stock market, respectively. h_t^{g} is the conditional variance of gold future market. β_t^{gc} and β_t^{ga} are the optimal hedging ratios. The purpose of dynamic hedging is to help investors to maintain the expected return while minimizing the risk by taking appropriate positions on the gold future contracts. For U.S. investors, they can take a long position of one dollar on the U.S. stock market and hedged by a short position of β_t^{ga}, whereas Chinese investors can take a long position of one dollar on the Chinese stock market and hedged by a short position of β_t^{gc}. We report the optimal hedging ratio during the overall study period and two major financial crisis in Table 3.

Table 3. Optimal hedging Ratio

Estimated from the VAE-EGARCH Model		
Panel A. Overall Period		
The U.S. Stock Market with Gold future	h_t^{ga}	0.001
The Chinese Stock Market with Gold future	h_t^{gc}	0.080
Panel B. AFC Period		
The U.S. Stock Market with Gold future	h_t^{ga}	0.094
The Chinese Stock Market with Gold future	h_t^{gc}	0.014
Panel C. GFC Period		
The U.S. Stock Market with Gold future	h_t^{ga}	0.054
The Chinese Stock Market with Gold future	h_t^{gc}	0.101

Note: This table reports the optimal hedge ratio of gold future contracts for U.S. investors and Chinese investors. The overall study period from January 15, 1996 to August 31, 2015; The AFC period from July 2, 1997 to December 31, 1998; The GFC period from November 30,2007 to June 30, 2008.

According to table 3, in the overall study period, Chinese investors should long(buy) one dollar of Chinese stock while shorting 8 cent of gold future contracts to minimize its risk position. On the other side, the hedging relationship between the U.S. stock market and the gold future market are not obvious , since the average hedging ratio is 0.1 cent(0.001) for each dollar of U.S. stock investment.

However, during financial disturbances, such as the AFC and GFC period, U.S. investors should increase their hedging ratio of gold future contracts from 0.1 cent to 9 cents during the AFC period and reduce modestly to 5 cents during the GFC period. It shows that U.S. investors should only consider shorting gold futures during financial disturbances. We further test the effectiveness in the next subsection. On the other side, Chinese investors should reduce their optimal hedging with respect to gold future contracts during its own AFC period (0.014), while increasing it during the GFC period(0.101). This is an interesting phenomena for both the U.S stock market and the Chinese stock market. One potential explanation of this phenomena is that in the AFC period, successive negative shocks would transmit higher volatility from Asian Stock markets, such as the Chinese stock market, to the U.S. stock market. Consequently, U.S. investors increase their hedging ratio of gold future contracts during the AFC period, whereas

Chinese investors increase their hedging ratio of gold future contracts dring CFC period. This explanation also supports the merits of a VAR-EGARCH model. As we already confirmed that negative shocks from the U.S. stock market and the Chinese stock market will transmit higher volatility to other financial markets.

Second, a static hedging strategy(i.e., portfolio diversification) is discussed. We follow Markowitz's (1987) mean-variance approach to determine the optimal holding weight of three assets:

$$\mu_v = \vec{m}\vec{w^T} \tag{7}$$

$$\sigma_v^2 = \vec{w}C\vec{w^T} \tag{8}$$

$$W^i = \frac{\vec{u}C^{-1}}{\vec{u}C^{-1}\vec{u^T}} \tag{9}$$

where μ_v denoted the expected return of the portfolio, $\vec{m}$ denoted a row vector of three daily return assets, $\vec{w^T}$ is a transpose of vector that contains three assets' weight. σ_v^2 denoted the conditional volatility of return of the portfolio. $\vec{u}$ is a row vector of 1s. C is a 3 by 3 conditional variance-covariance matrix of the U.S. Stock market, Chinese stock market, and world gold market at time t. C^{-1} is its inverse matrix. The global minimum portfolio will be measured in Eq.(9) and a proof will be given in Appendix II. The main purpose of forming this portfolio is to minimize the risk(conditional variance) of investments. These series are estimated from the tri-variate VAR-EGARCH model in table 3, Panel A. We also compare the performance of this mean-variance portfolio to an equal weight portfolio (i.e., $W_n = \frac{1}{n}$).

However, one potential disadvantage of a global minimum portfolio is that the expected return of an optimal portfolio will potentially be negative since we only seek for the smallest conditional variance. As a result, we consider another optimal portfolio named the minimum variance line. The purpose of forming this portfolio is to minimize the conditional variance while holding the same expected return as compared to investing in one asset only. In this section, we assume a position of U.S. investors during the overall study period and the GFC period, comparing with a Chinese investor position during the AFC period. Our goal is to keep our portfolio returns above the average daily return of S&P 500(0.026%) for both U.S. and Chinese investors. In this set up, short selling is allowed. The minimum variance line portfolio are the following:

$$W^i = \frac{A\vec{u}C^{-1} + B\vec{m}C^{-1}}{C} \tag{10}$$

where

$$A = \begin{bmatrix} 1 & \vec{u}C^{-1}\vec{m^T} \\ \mu_v & \vec{m}C^{-1}\vec{m^T} \end{bmatrix} B = \begin{bmatrix} \vec{u}C^{-1}\vec{u^T} & 1 \\ \vec{m}C^{-1}\vec{u^T} & \mu_v \end{bmatrix} C = \begin{bmatrix} \vec{u}C^{-1}\vec{u^T} & \vec{m}C^{-1}\vec{u^T} \\ \vec{m}C^{-1}\vec{u^T} & \vec{m}C^{-1}\vec{m^T} \end{bmatrix}$$

where A, B, and C denoted 2 by 2 matrix. Given investors' expected return of a portfolio μ_v, we can find the smallest conditional variance via Eq.(10).

In table 3, we notice that in the whole study period, in order to minimize the risks(conditional variance), a U.S. investor should hold a portfolio that contains 43.03% of gold, 38.69% of the U.S. stocks, and 18.28% of Chinese stocks. By comparison, we provide an equal weight portfolio and minimum variance line when we assume expected daily returns will be 0.03%, which is slightly above the daily mean returns of S&P500 in the past 20 years.

Similarly, during the AFC period, seeking a global minimum variance of the portfolio, Chinese investors should hold less Chinese and U.S. stocks (16.22% and 24.63%) due to financial disturbances with high volatility, while increasing their portfolio weights of gold futures from 40% to 60%. Alternatively, for Chinese investors who first consider their priority to their daily returns, they should hold 52.65% of U.S. stocks while decreasing the portfolio weight of its Chinese stock market to 12.46%. The portfolio weight of gold should increase from 24.85% to 34.89%. The change of portfolio weights of gold futures show that gold potentially serve as a hedge and even a safe haven during financial disturbance for the Chinese investor.

Table 4. Optimal diversified portfolio

Estimated from the VAE-EGARCH Model						
Panel A. Overall Period						
Global Minimum Portfolio	W_t^c	18.28%	W_t^a	38.69%	W_t^g	43.03%
Equal Weight Portfolio	W_t^c	33.33%	W_t^a	33.33%	W_t^g	33.33%
Minimum Variance Line	W_t^c	38.47%	W_t^a	36.68%	W_t^g	24.85%
Panel B. AFC Period						
Global Minimum Portfolio	W_t^c	16.22%	W_t^a	24.18%	W_t^g	59.60%
Equal Weight Portfolio	W_t^c	33.33%	W_t^a	33.33%	W_t^g	33.33%
Minimum Variance Line	W_t^c	12.46%	W_t^a	52.65%	W_t^g	34.89%
Panel C. GFC Period						
Global Minimum Portfolio	W_t^c	24.63%	W_t^a	28.57%	W_t^g	46.81%
Equal Weight Portfolio	W_t^c	33.33%	W_t^a	33.33%	W_t^g	33.33%
Minimum Variance Line	W_t^c	-0.524%	W_t^a	5.268%	W_t^g	95.256%

Note: this table reports three different portfolio designs during three periods. The numerical numbers are taken from the mean(average) of the estimated model and utilize Eq.(9) and Eq.(10) to find the optimal weights of the Chinese stock index, the U.S stock index, and the 3-month gold future contracts, respectively.

In addition, during the GFC period, U.S. investors should hold more Chinese stocks (24.63%), while reducing its U.S. stock to 28.57%. The portfolio weight of gold should roughly be the same as compared to the whole study period. These results shows that to minimize the risks, U.S. investors should hold about 45%-55% of gold during both financial disturbances and the overall period. Moreover, for U.S. investors who want to maintain a 0.03% daily returns during the GFC period, they should go short 0.5% of Chinese stocks and holding 5.27% of U.S. stock, and holding 95.26% of gold futures. These portfolio designs show that gold assets potentially serve as a hedge and even a save haven for the U.S. investors. We further confirm these conclusions by testing effectiveness of dynamic hedging and diversification in the following subsection.

5.3 Dynamic hedging and diversification effectiveness

We test the effectiveness of our previous dynamic hedging and static hedging (diversification) by assuming different positions in different study periods and subsequently compare the daily returns of our portfolio designs to hold 100% of U.S. Stock index, Chinese stock index, or 3-month gold future contracts separately. We utilize realized hedging errors (HE) to conclude which strategy more effectively reduced the investors' risks (conditional variance) (Ku et al.,2007):

$$HE = \frac{Var_{unhedge} - Var_{hedge}}{Var_{unhedge}} \tag{11}$$

Intuitively speaking, the higher the HE the better the hedge effectiveness. In Table 5 Panel A, on average, the effectiveness of U.S. investors to take a short position of gold future to the U.S. stock market is zero, while reducing 0.21% of the risk for the Chinese investor. In Panel B, increased hedging for U.S investors will reduce risk about 0.31% but only about 0.01% for Chinese Investors. In Panel C, U.S. investors who take a short position of gold will reduce their risk about 0.21% and reduced 0.51% of Chinese investors.

Overall, although we notice that a dynamic hedging strategy with gold future contract will reduce the risks for both U.S. investors and Chinese investors, the H.E. ratio shows that the effectiveness for both markets in different periods are negligible. The transaction cost of dynamic hedging will most likely be larger than the benefits.

On the other side, we test the effectiveness of static hedging strategy. In Table 6 Panel A, we notice that the average daily returns of $S\&P500$ in the past twenty years is 0.026% with risk of 1.547 (unconditional variance). A global minimum portfolio will provide a 0.027% daily return while reducing 62.41% of risk. However, an equal weight portfolio provides better returns than the global minimum portfolio while comparactively reducing less risk (66.77%). In addition, a minimum variance line portfolio provides investors a 0.03% daily return (7.5% per year), which provide slightly better return than holding 100% $S\&P500$ index while reducing 61.42% risk.

Table 5. Effectiveness of dynamic hedging

	$\text{Var}_{unhedge}$	Var_{hedge}	H.E.
Panel A. Overall Period			
The U.S. Stock Market with Gold future	1.547	1.457	0.00%
The Chinese Stock Market with Gold future	3.163	3.157	0.21%
Panel B. AFC Period			
The U.S. Stock Market with Gold future	1.671	1.664	0.39%
The Chinese Stock Market with Gold future	2.234	2.233	0.01%
Panel C. GFC Period			
The U.S. Stock Market with Gold future	6.073	6.061	0.21%
The Chinese Stock Market with Gold future	6.851	6.817	0.51%

In Table 6 Panel B, during the AFC period, the Chinese stock market was affected by the contagious effects from other Asian countries, the average daily returns were negative. Assuming Chinese investors position, a global minimum portfolio can approximately maintain their stock value during that period(-0.009% daily return) while reducing risk 73.47%. However, an equal weight and minimum variance line portfolio can provide positive return and also reduce on average 73% of risks. In terms of static hedging strategy, it shows that gold not only serves as a hedge but also a safe haven for Chinese investors.

In table 6 Panel C, during the CFC period, both the U.S. stock market and the Chinese stock market were highly volatile and provided big negative returns. Global minimum portfolio mitigated the loss to -0.053% a day while reducing the risk about 71%. An equal weight portfolio mitigate slightly less than a global minimum portfolio and reduces less risks. However, if U.S. investors hold 95% of gold in minimum variance line, it even provides a 0.03% daily return. In terms of static hedging strategy, it shows that gold serves as both a hedge and safe haven for U.S. investors during financial crisis.

Table 6. Daily returns and static hedging effectiveness

	Daily return(%)	$\text{Var}_{unhedge}$	Var_{hedge}	H.E.
Panel A. Overall Period				
SHMOOP	0.039	3.163	-	-
S&P500	0.026	1.547	-	-
Gold	0.023	1.310	-	-
Global Minimum Portfolio	0.027	1.547	**0.581**	**62.41%**
Equal Weight Portfolio	**0.029**	1.547	**0.669**	**56.77%**
Minimum Variance Line	**0.030**	1.547	**0.597**	**61.42%**
Panel B. A.F.C. Period				
SHMOOP	-0.024	2.234	-	-
S&P500	0.091	1.670	-	-
Gold	-0.04	0.711	-	-
Global Minimum Portfolio	**-0.009**	2.234	**0.443**	**80.17%**
Equal Weight Portfolio	0.008	2.234	**0.585**	**73.82%**
Minimum Variance Line	**0.030**	2.234	**0.610**	**72.69%**
Panel C. G.F.C. Period				
SHMOOP	-0.141	6.851	-	-
S&P500	-0.126	6.073	-	-
Gold	0.04	3.402	-	-
Global Minimum Portfolio	**-0.053**	6.073	**1.737**	**71.40%**
Equal Weight Portfolio	**-0.077**	6.073	**1.850**	**69.53%**
Minimum Variance Line	**0.030**	6.073	**3.122**	**48.60%**

Note: Daily portfolio returns in each period are calculated by Eq.(7). During the overall study period, we assume a U.S. investor position, and all $Var_{unhedge}$ represents $S\&P500$. During the AFC Period, we assume a Chinese investor position and all $Var_{unhedge}$ represents SHMOOP variance. During the GFC period, we assume a U.S. investor position, and all $Var_{unhedge}$ represents $S\&P500$. Var_{hedge} are calculated by Eq.(8). H.E. ratio is calculated by Eq.(12).

Overall, for passive investors, we suggest that they should consider an equal weight portfolio that contains the SHMOOP, the $S\&P500$, and the gold future contracts in the long term, since it provides better daily returns with 56.77% less risk than merely investing $S\&P500$ index. During Asian financial disturbance, it provides 0.008% daily returns for Chinese investors. During global financial disturbance, it mitigates the loss from -0.126% to -0.077% per day. For active investors, they should re-balance their minimum variance line and hold more gold during financial disturbance because it can even provide positive returns.

6 Conclusion with Remarks

This paper analyzes three financial markets' interdependences, between January 15, 1996 to August 31, 2015. We use a multivariate VAR-EGARCH model to examine the first and second moment of interdependence, while capturing the significant asymmetric effects across these three financial markets during volatility transmissions.

The empirical finding and relevant hedging and portfolio implications are concluded as follows: (1) their existing significant lead/lag relationship and volatility transmissions between the U.S. stock market and the other two financial markets. We can use the past returns of the U.S stock market to predict the current returns of the Chinese stock market and gold future market. The spillover effects between the U.S. stock market and the other two financial markets are statistically significant at the 1 % level. (2) The leverage effect are significant within and across the U.S. stock market and Chinese stock market at the 1% level, we confirm that a multivariate VAR-EGARCH model is more necessary than a multivariate VAR-GARCH model. (3) A significant and persistent volatility τ_i carried from its past conditional variance tells us that when markets have a period of return that might have a tendency of going up, it will maintain the tendency until the end of the period, and vise visa. It suggests that investors should consider invest more when it has a successive period of positive return, and go short if it is the other way around. This is consistent with the momentum effects in finance literature. (4) Although dynamic hedging reduces investors' risk and provides insight for the position of gold future during financial disturbances, the magnitude is addressed to be trivial by H.E. ratio. On the other side, a static hedging that on average reduces 50% or more risks show that in practice this would be a better approach. (5) The significant time-varying conditional correlation between the U.S. stock market and Chinese stock market, the Chinese stock market and the world gold future markets provides excellent portfolio opportunity for investors to gain more risk-adjusted return in the long run and even during financial disturbances. Passive investors should consider an equal weight portfolio while active investors should re-balance their portfolio based on the minimum variance line. (6) A global minimum portfolio can reduce its risks to the minimum but provide negative daily returns during financial disturbances as compared to standalone investments. It suggests that investors should seek a portfolio that provides the highest risk-adjusted return, instead of merely seeking portfolio that provides the best hedge.

APPENDIX

I.

According to Nelson(1991), Jane and Ding(2009) , a univariate EGARCH(p,q) model which capture asymmetric effect is presented as follow:

$$ln(\sigma_t^2) = \alpha_o + \frac{1 + \beta_1 B + ... + \beta_q B^q}{1 - \alpha_1 B - ... - \alpha_p B^p} g_{(v_{t-1})} \tag{1}$$

$$a_t = \sigma_t v_t, g_{(v_t)} = \delta z_t + \gamma[|v_t| - E(v_t)] \tag{2}$$

where a_t denote the innovation of return at time t. z_t denoted the standardized innovation. σ_t is the standard deviation of variance σ_t^2. α_o is a constant and B is a lag operator such that $B^i g_{(v_t)} = g(v_{t-i})$, both $1 - \sum_{i=1}^{p} \alpha_i B^i$ and $1 + \sum_{i=1}^{q} \beta_i B^i$ lie outside of unit circle and have no common root. $g_{(v_t)}$ is the asymmetric function, with an i.i.d. random sequence and mean zero. $\gamma[|v_t| - E(v_t)]$ measure the magnitude effect and γ denote past periods of the variance of standardized innovations symmetrically, whereas δv_t measure the side effect, and δ denote past periods of the variance of standardized innovations asymmetrically

According to Tasy(2005),

$$r_t = \mu_t + a_t \tag{3}$$

where μ_t=$E(r_t|\Omega_{t-1})$ is the conditional expectation of r_t given past information Ω_{t-1}. and we further assume μ_t to be a vector autoregressive moving average (VARMA) model:

$$\mu_t = \Phi_o + \sum_{i=1}^{p} \Phi_i r_{t-i} - \sum_{j=i}^{q} \Theta_j a_{t-j} \tag{4}$$

where Φ_o denote n x 1 vector intercepts. Φ_i and Θ_j denote n x n matrices of constant parameters. Eq.(4) is the mean equation of $r_t.H_t$ is a n x n positive-definitive matrix and denote as H_t=$Cov(a_t|\Omega_{t-1})$. Now, if we use the multivariate EGARCH (p,q) Model for return series r_t, then we can extend the univariate EGARCH model in the following:

$$ln(\sigma_t^2) = \alpha_o + \frac{I + \beta_1 B + ... + \beta_q B^q}{I - \alpha_1 B - ... - \alpha_p B^p} G_{(v_{t-1})} \tag{5}$$

$$a_t = H_t^{1/2} v_t, G_{(v_t)} = \delta z_t + \gamma[|v_t| - E(v_t)] \tag{6}$$

where $ln(\sigma_t^2)$ denote a vector of univariate $ln(\sigma_{i,t}^2)$. β_i and α_j denote n x n diagonal matrices. z_t denote a vector of $v_{i,t}$ and i,j=1,2,3...n. If (p,q)=(1,0) then

$$ln(\sigma_t^2) = \alpha_o + \frac{I}{I - \alpha_1 B} G_{(v_{t-1})} \tag{7}$$

So

$$ln(\sigma_t^2)(I - \alpha_1 B) = \alpha_o (I - \alpha_1 B) + I G_{(v_{t-1})} \tag{8}$$

We write Eq.(8) in a matrix form:

$$\begin{bmatrix} 1-\alpha_{11}B & 0 & \cdots & 0 \\ 0 & 1-\alpha_{22}B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1-\alpha_{nn}B \end{bmatrix} \begin{bmatrix} ln(\sigma_{1,t}^2) \\ ln(\sigma_{2,t}^2) \\ \vdots \\ ln(\sigma_{n,t}^2) \end{bmatrix} = \begin{bmatrix} 1-\alpha_{11}B & 0 & \cdots & 0 \\ 0 & 1-\alpha_{22}B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1-\alpha_{nn}B \end{bmatrix} \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \\ \vdots \\ \alpha_{n0} \end{bmatrix} + \begin{bmatrix} g_{1(v_{t-1})} \\ g_{2(v_{t-1})} \\ \vdots \\ g_{n(v_{t-1})} \end{bmatrix}$$

It turns out that

$$ln(\sigma_{i,t}^2) = (1 - \sigma_{i,i})\sigma_{i,o} + \sigma_{i,i} ln(\sigma_{i,t-1}^2) + g_{i,(v_{t-1})} \tag{9}$$

Since the intercept is not the focus of our equation, we denote $\sigma_{i,o}$=$(1-\sigma_{i,i})\sigma_{i,o}$.

Similarly, Eq.(6) can be extend to a matrix form,

$$\begin{bmatrix} g_{1(v_t)} \\ g_{2(v_t)} \\ \cdots \\ g_{n(v_t)} \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{nn} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ \vdots \\ z_{n,t} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix} \begin{bmatrix} |z_{1,t}| - E(|v_{1,t}|) \\ |z_{2,t}| - E(|v_{2,t}|) \\ \vdots \\ |z_{n,t}| - E(|v_{n,t}|) \end{bmatrix}$$

As a result,

$$g_{i(v_t)} = \sum_{j=1}^{n} \left(\delta_{i,j} v_{j,t} + \gamma_{i,j}[|v_{j,t}| - E(v_{j,t})]\right) \tag{10}$$

When we assume $\delta_{i,j} = \delta_{j,i}$, $i, j = 1, 2, 3$, and $i \neq j$. Then we finally obtain Koutmos(1996) Multivariate EGARCH model in the following:

$$R_{i,t} = \beta_{i,0} + \sum_{i=1}^{3} \beta_{i,j} R_{j,t-1} + \varepsilon_{i,t} \tag{11}$$

$$ln(\sigma_{i,t}^2) = \alpha_i + \sum_{i=1}^{3} \gamma_{i,j} f\left(v_{j,t-1}\right) + \tau_i ln\left(\sigma_{i,t-1}^2\right) \tag{12}$$

$$f\left(v_{j,t-1}\right) = \left(|v_{j,t-1}| - E\left|v_{j,t-1}\right| + \delta_j v_{j,t-1}\right), \tag{13}$$

$$\sigma_{i,j,t} = \rho_{i,j} \sigma_{i,t} \sigma_{j,t} \tag{14}$$

II.

We deride the minimum variance portfolio in the following(Capinski and Zastawniak 2011; Stutzer 2016).

$$\mu_v = E(K_{portolio}) = E\left(\sum_{i=1}^{n} w_i K_i\right) = \sum_{i=1}^{n} w_i \mu_i = \vec{m}\vec{w^T} \tag{1}$$

$$\sigma_v^2 = Var(\sum_{i=1}^{n} w_i K_i) = Cov(\sum_{i=1}^{n} w_i K_i, \sum_{j=1}^{n} w_j K_j) = \sum_{i,j=1}^{n} w_i w_j c_{ij} = \vec{w}C\vec{w^T} \tag{2}$$

$$\sum_{i}^{n} w_i = \vec{u}\vec{w^T} = 1 \tag{3}$$

$$W^i = \frac{\vec{u}C^{-1}}{\vec{u}C^{-1}\vec{u^T}} \tag{4}$$

where K_i denote the net return at period i and $\vec{u}$ is a row vector of 1s.

As a result, in order to obtain Eq.(4), we minimum the Eq.(2) subject to Eq.(4) using Lagrange multipliers method:

$$min_{\vec{w}} Var(K_{portfolio}) : F(\vec{w}, \lambda) = \vec{w}C\vec{w^T} - \lambda(\vec{u}w^T - 1) \tag{5}$$

$$\frac{\partial F(\vec{w}, \lambda)}{\partial \vec{w}} = 2\vec{w}C - \lambda\vec{u} = \vec{0} \tag{6}$$

$$\vec{w} = \frac{\lambda}{2}\vec{u}C^{-1} \tag{7}$$

$$\vec{w}u^T = 1 = \frac{\lambda}{2}\vec{u}C^{-1}u^T \tag{8}$$

$$\frac{\lambda}{2} = \frac{1}{\vec{u}C^{-1}\vec{u^T}} \tag{9}$$

combine Eq.(7) and Eq.(9), we obtain the result

$$W^i = \frac{\vec{u}C^{-1}}{\vec{u}C^{-1}\vec{u^T}} \tag{10}$$

III.

Similarly, we implement Lagrange multipliers method to deride minimize variance line portfolio in the following.

$$G(\vec{w}, \lambda, \mu) = \vec{w}C\vec{w_T} - \lambda(\vec{u}\vec{w^T}) - \mu(\vec{m}\vec{w^T}) \tag{1}$$

$$\sum_i^n w_i = \vec{u}\vec{w^T} = 1 \tag{2}$$

$$\mu_v = \sum_i^n w_i m = \vec{m}\vec{w^T} \tag{3}$$

$$\frac{\partial G(\vec{w}, \lambda, \mu)}{\partial \vec{w}} = 2\vec{w}C - \lambda\vec{u} - \mu_v\vec{m} = \vec{0} \tag{4}$$

where μ_v denote the portfolio expected return. $\vec{m}$ denote the expected return. Using constraint Eq.(2) and Eq.(3), we obtain:

$$\frac{\lambda}{2}\vec{u}C^{-1}\vec{u^T} + \frac{\mu}{2}\vec{u}C^{-1}\vec{u^T} = 1 \tag{5}$$

$$\frac{\lambda}{2}\vec{m}C^{-1}\vec{u^T} + \frac{\mu}{2}\vec{u}C^{-1}\vec{m^T} = \mu_v \tag{6}$$

we solve for $\frac{\lambda}{2}$ and $\frac{\mu}{2}$ and put it back to Eq.(4) to obtain the final result,

$$W^i = \frac{A\vec{u}C^{-1} + B\vec{m}C^{-1}}{C} \tag{7}$$

where

$$A = \begin{bmatrix} 1 & \vec{u}C^{-1}\vec{m^T} \\ \mu_v & \vec{m}C^{-1}\vec{m^T} \end{bmatrix} B = \begin{bmatrix} \vec{u}C^{-1}\vec{u^T} & 1 \\ \vec{m}C^{-1}\vec{u^T} & \mu_v \end{bmatrix} C = \begin{bmatrix} \vec{u}C^{-1}\vec{u^T} & \vec{m}C^{-1}\vec{u^T} \\ \vec{m}C^{-1}\vec{u^T} & \vec{m}C^{-1}\vec{m^T} \end{bmatrix}$$

IV.

Conditional Variance of Chinese stock market	Var_C							
Conditional Variance of the U.S. stock market	Var_A							
Conditional Variance of Gold Futurestock market	Var_G							
Correlation between Chinese stock market and the U.S. stock makret	Corr_C_A							
Correlation between Chinese stock market and Gold future Market	Corr_C_G							
Correlation between the U.S. stock makret and Gold future Market	Corr_A_G							
Daily Return of Chinese stock Market	Mu_C							
Daily Return of the U.S. Stock Market	Mu_A							
Daily return of the Gold future Market	Mu_G							
Between 1996-2015								
Var_C	3.163241	SD_C	1.778550252	Mu_C	0.0386%	Corr_C_A	-0.043273	**H.E.**
Var_A	1.546828	SD_A	1.243715402	Mu_A	0.0259%	Corr_C_G	0.045872	
Var_G	1.310558	SD_G	1.144796052	Mu_G	0.0227%	Corr_A_G	0.000858	
C	3.163241	-0.095720323	0.093398938	C-1	0.317394757	0.019658801	-0.022637952	
	-0.095720323	1.546828	0.001221621		0.019658801	0.647702392	-0.002004763	
	0.093398938	0.001221621	1.310558		-0.022637952	-0.002004763	0.764648958	
uC-1	0.314415606	0.66535643	0.740006244		**Minimum Variance Line**			
mC-1	0.000122453	0.000174952	0.000164548		W_C	W_A	W_G	
mC-1mT	1.29995E-07				38.466%	36.686%	24.848%	
UC-1UT	1.71977828				Exp_Return		0.03%	
mC-1UT	0.000461952				Reduced S.D.		0.772515639	**37.89%**
UC-1mT	0.000461952							
					Global Minimun Portfolio			
					W_C	W_A	W_G	
					18.28%	38.69%	43.03%	
					Exp_Return		0.0269%	
					Reduced S.D.		0.762542002	**38.69%**
					Equal Weight Portfolio			
					W_C	W_A	W_G	
					33.33%	33.33%	33.33%	
					Exp_Return		0.0291%	
					Reduced S.D.		0.81774945	**34.25%**

Figure 3: Data of Table 4 and 6

Between 2007-2008

Var_C	6.851458	SD_C	2.617528987	Mu_C	-0.1412%	Corr_C_A	-0.060493	H.E.
Var_A	6.073494	SD_A	2.464445982	Mu_A	-0.1261%	Corr_C_G	0.071431	
Var_G	3.401831	SD_G	1.844405324	Mu_G	0.0377%	Corr_A_G	0.046292	

C	6.851458	-0.39022575	0.344853468	C-1	0.147306838	0.010003346	-0.01555167
	-0.39022575	6.073494	0.210417383		0.010003346	0.165682774	-0.01126224
	0.344853468	0.210417383	3.401831		-0.01555167	-0.01126224	0.296232475

uC-1	0.141758514	0.164423884	0.269418569
mC-1	-0.00022642	-0.00022734	0.000147817
mC-1mT	6.62078E-07		
UC-1UT	0.575600968		
mC-1UT	-0.00030594		
UC-1mT	-0.00030594		

Minimum Variance Line			
W_C	W_A	W_G	
-0.524%	5.268%	95.256%	
Exp_Return		0.03%	
Reduced S.D.		1.766818997	**28.31%**

Global Minimum Portfolio			
W_C	W_A	W_G	
17.86%	22.24%	59.90%	
Exp_Return		-0.0532%	
Reduced S.D.		1.318072	**46.52%**

Equal Weight Portfolio			
W_C	W_A	W_G	
33.33%	33.33%	33.33%	
Exp_Return		-0.00076533	
Reduced S.D.		1.360428	**44.80%**

Between 1997-1998

								H.E.
Var_C	2.234102	SD_C	1.494691272	Mu_C	-0.0240%	Corr_C_A	0.088707	
Var_A	1.67065	SD_A	1.292536266	Mu_A	0.0905%	Corr_C_G	0.007967	
Var_G	0.711389	SD_G	0.843438794	Mu_G	-0.0421%	Corr_A_G	0.062695	

C	2.234102	0.171376839	0.010043842	C-1	0.317394757	0.019658801	-0.02263795
	0.171376839	1.67065	0.068348536		0.019658801	0.647702392	-0.00200476
	0.010043842	0.068348536	0.711389		-0.022637952	-0.00200476	0.764648958

uC-1	0.40302765	0.501923047	1.351786924
mC-1	-0.00014939	0.000583697	-0.00064539
mC-1mT	8.35921E-07		
UC-1UT	2.256737622		
mC-1UT	-0.00021108		
UC-1mT	-0.00021108		

Minimum Variance Line			
W_C	W_A	W_G	
12.473%	52.649%	34.878%	
Exp_Return		0.03%	
Reduced Variance		0.795529611	**38.45%**

Global Minimum Portfolio			
W_C	W_A	W_G	
0.246279145	0.285656024	0.468064831	
Exp_Return		-0.0094%	
Reduced Variance		0.665671	**94.17%**

Equal Weight Portfolio			
W_C	W_A	W_G	
33.33%	33.33%	33.33%	
Exp_Return		0.0082%	
Reduced Variance		0.753928926	**41.67%**

Figure 4: Data of Table 4 and 6

V.

1996-2015									
sd_c	1.7786	3.1632	Return_c	0.0386%	Corr_c_a	-0.043		**beta_g**	**Beta_Ga**
sd_a	1.2437	1.5468	Return_a	0.0259%	Corr_c_g	0.0459		-0.08	-9E-04
sd_g	1.1448	1.3106	Return_g	0.0227%	Corr_a_g	0.0009			
	1								
	1	-0.08							
1	3.1632	0.0934				3.1632	-0.007		
-0.08	0.0934	1.3106				-0.007	0.0084		
			Var_After Hedg	**3.15668**					
	1		H.E	**0.21%**					
	1	-9E-04							
1	1.5468	0.0012				1.5468	-1E-06		
-9E-04	0.0012	1.3106				-1E-06	1E-06		
			Var_After Hedg	**1.54683**					
			H.E	**0.00%**					
1996-1997									
sd_c	1.4947	2.2341	Return_c	-0.0240%	Corr_c_a	0.0887		**beta_g**	**Beta_Ga**
sd_a	1.2925	1.6707	Return_a	0.0905%	Corr_c_g	0.008		-0.014	-0.094
sd_g	0.8434	0.7114	Return_g	-0.0421%	Corr_a_g	0.0627			
	1								
	1	-0.014							
1	2.2341	0.01				2.2341	-1E-04		
-0.014	0.01	0.7114				-1E-04	0.0001		
			Var_After Hedg	**2.23396**					
	1		H.E	**0.01%**					
	1	-0.094							
1	1.6707	0.0683				1.6707	-0.006		
-0.094	0.0683	0.7114				-0.006	0.0063		
			Var_After Hedg	**1.66409**					
			H.E	**0.39%**					
2007-2008									
sd_c	2.6175	6.8515	Return_c	-0.1412%	Corr_c_a	-0.06		**beta_g**	**Beta_Ga**
sd_a	2.4644	6.0735	Return_a	-0.1261%	Corr_c_g	0.0714		-0.101	-0.054
sd_g	1.8444	3.4018	Return_g	0.0377%	Corr_a_g	0.0463			
	1								
	1	-0.101							
1	6.8515	0.3449				6.8515	-0.035		
-0.101	0.3449	3.4018				-0.035	0.0346		
			Var_After Hedg	**6.8165**					
	1		H.E	**0.51%**					
	1	-0.054							
1	6.0735	0.2104				6.0735	-0.011		
-0.054	0.2104	3.4018				-0.011	0.0101		
			Var_After Hedg	**6.06066**					
			H.E	**0.21%**					

Figure 5: Data of Table 5

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