A NEW EM SENSOR FOR THE DETECTION OF ROOF THICKNESS IN A COAL MINE

ABSTRACT

The use of an annular, coaxially-driven slot antenna for probing of the roof thickness in a coal mine is discussed. For the case when the thickness of the roof and the slot width are both small compared with free-space wavelength, the inversion from the measured data on radiated power to roof thickness proves to be a very simple process. It appears that such a sensor not only can be rugged in construction, simple to install, but also has the additional advantage of being less sensitive to scatterings from surrounding environment.

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In the development of automated mining machines, it is sometimes important to have sensors which are capable of detecting the thickness of a roof structure in a coal mine. Typically, we are talking about the sensing of a thin slab of coal, say ten centimeters in thickness, backed by a highly-reflecting overburden material such as draw slate. An idea sensor in this case is one that not only is rugged in construction, and simple to install, but also one that yields measurements easy to interpret. Two types of non-contact sensors have been reported previously. The first is a two-loop method based upon the mutual coupling of two small co-planar or perpendicular loops placed in front of the coal surface. It is basically a low frequency technique (below 1 MHz) which measures the contrast in conductivity between coal and slate [Ralston and Wait, 1977]. The second one, on the other hand, is a high frequency technique (say, 300 MHz) which utilizes the resonant property of a horizontal loop antenna and measures basically the contrast in permittivity between coal and slate [Chang and Wait, 1977]. Both methods, however, employ loop antennas whose electric properties are assumed to be influenced only by the roof structure with no significant disturbance due to scattering from near-by mining and measuring equipment. Thus in order to meet this requirement, these sensors ideally should be mounted onto a non-metallic boom structure which can extend out alone to the coal surface. Such an arrangement of course would complicate the mechanical design of these sensors in a typical mining environment.

One antenna structure which does not appear to have such a disadvantage is that of an annular slot antenna driven by a coaxial line. Specifically, it is made of a rigid coaxial, cylindrical transmission line with the open-end flush-mounted at right angle onto a large metallic ground plane.

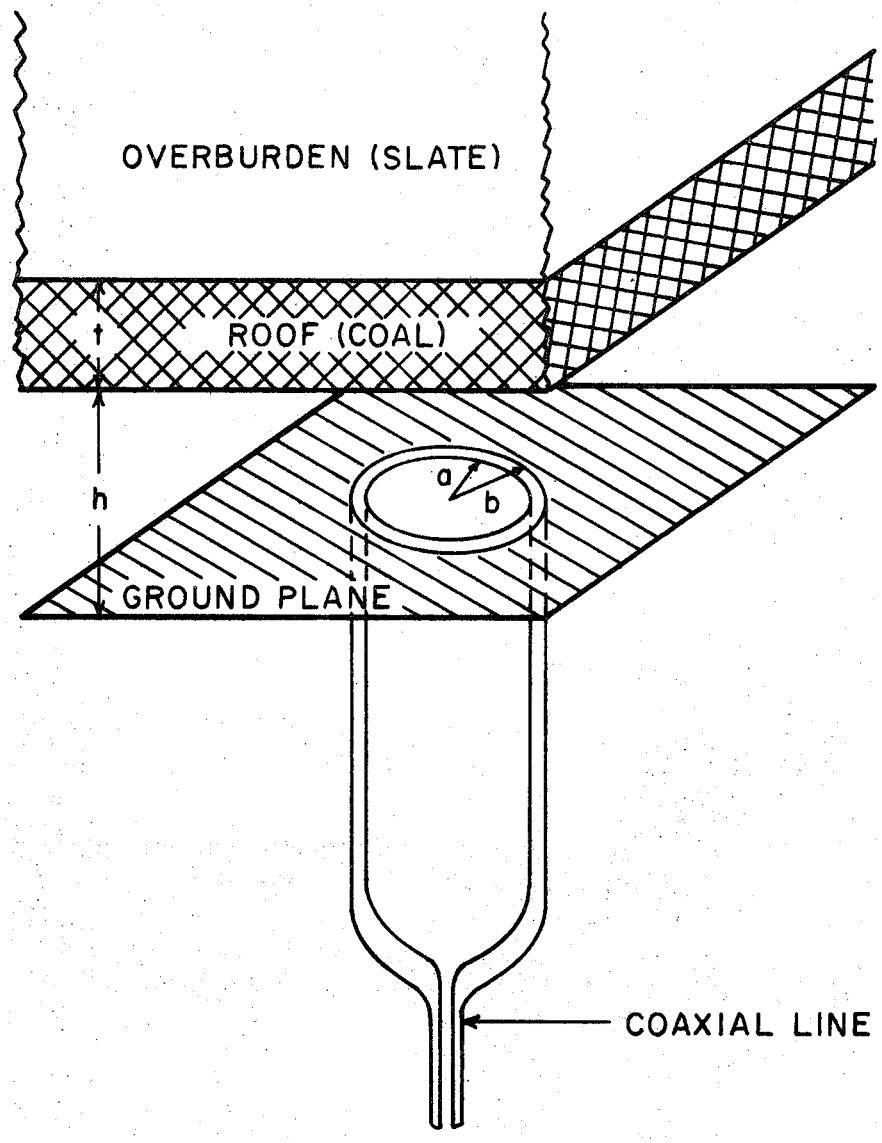

Figure 1

Electromagnetic interference from surrounding objects is minimized in this case because of the isolation inherently provided by the ground plane. If it operates at a frequency above a few MHz, the small physical size of such a sensor may even allow its installation onto the surface of the rotating drum of a mining machine. The question is then whether this type of antenna can be used effectively as a sensor for the detection of the roof thickness.

To answer this question, we have included in Appendix A an analysis of a narrow, coaxial-driven, annular slot antenna with an infinite ground plane in the presence of a lossless coal slab, together with a perfectly-reflecting overburden. The radiated power measured in the coaxial line is found in (A.11) and (A.12)of the Appendix to be

$$G = \frac{2k_o a}{\zeta_o} \left\{ \frac{\pi}{2} - \frac{1}{2} \operatorname{Re} \int_{-\infty}^{\infty} [1 - i\pi a R(\alpha) J_1(\alpha a) H_1^{(2)}(\alpha a)] d\alpha / \xi_o \right\} , \qquad (1)$$

where

$$R(\alpha) = [\varepsilon_o \xi_1 \tan(\xi_o h) \tan(\xi_1 t) - \varepsilon_1 \xi_o] / [\varepsilon_o \xi_1 \tan(\xi_1 t) + \xi_1 \xi_o \tan(\xi_o h)] \qquad (2)$$

$\xi_o = (k_o^2 - \alpha^2)^{\frac{1}{2}}$, $\xi_1 = (k_1^2 - \alpha^2)^{\frac{1}{2}}$ with $\operatorname{Im}(\xi_o)$, $\operatorname{Im}(\xi_1) \leq 0$,

$k_o = \omega(\mu_o \varepsilon_o)^{\frac{1}{2}}$; $k_1 = \omega(\mu_o \varepsilon_1)^{\frac{1}{2}}$,

$\omega = 2\pi f$ is the angular operating frequency,

ε_o and ε_1 is, respectively, the permittivity of air and coal; $\varepsilon_1 = \varepsilon_r \varepsilon_o$,

μ_o is the permeability of a non-magnetic material,

$\zeta_o = (\mu_o/\varepsilon_o)^{\frac{1}{2}} = 120\pi$ ohm, is the intrinsic impedance of air,

h is the antenna height as measured from the coal surface,

t is the thickness of the coal slab (roof),

(a,b) is the inner, outer radius of the coaxial-line.

We note that explicit dependance on the outer radius in (1) disappears as a result of the narrow gap assumption that $(b-a) \ll a$. Thus, one can see that

the information regarding roof thickness in principle can be retrieved by comparing graphically the measured data with theoretical curves for different thickness, in very much the same way as the two loop methods. As shown in the Appendix A, however, substantial simplication of the result can be achieved if one employs an operating frequency low enough so that both the antenna height and roof thickness are small with respect to the effective wavelength. (Typically, this means the operating frequency should be in the neighborhood of 100MHz for a roof thickness of 10 centimeters or less.) In that case, the expression for the radiated power reduces to (A.18) and (A.19) which are repeated here.

$$G \simeq \pi^2(\zeta_o S)^{-1}(\alpha_1 a)J_1^2(\alpha_1 a), \tag{2}$$

where

$$\alpha_1 = k_o[(h+t)/(h+\varepsilon_r^{-1}t)]^{\frac{1}{2}}, \quad S = [(h+t)(h+\varepsilon_r^{-1}t)]^{\frac{1}{2}}/a \tag{3}$$

Here, α_1 actually corresponds to the propagation constant of the dominant TM_{oo} - mode in the composite waveguide consisting of the air and coal slab located between the ground plane and the reflecting overburden material. In Fig. 2, the normalized radiated power, $g = (\zeta_o G)\pi^{-2}$, is plotted as a function of $\alpha_1 a$ with S as a parameter. Since only α_1 varies explicitly as a function of frequency, we can first measure g by sweeping the operating frequency and superpose the plot of g with frequency onto the precalculated theoretical curves in Fig. 2. The best match of the measured result would then yield the values for $(\alpha_1 f^{-1})$ and S. Now since the produce of these two numbers depends only upon the ratio $(h+t)/a$, we have from (3),

$$h+t = 4.78 \times 10^7 \, Sa(\alpha_1/f) \quad , \tag{4}$$

so that the roof thickness t can be immediately determined with a prior

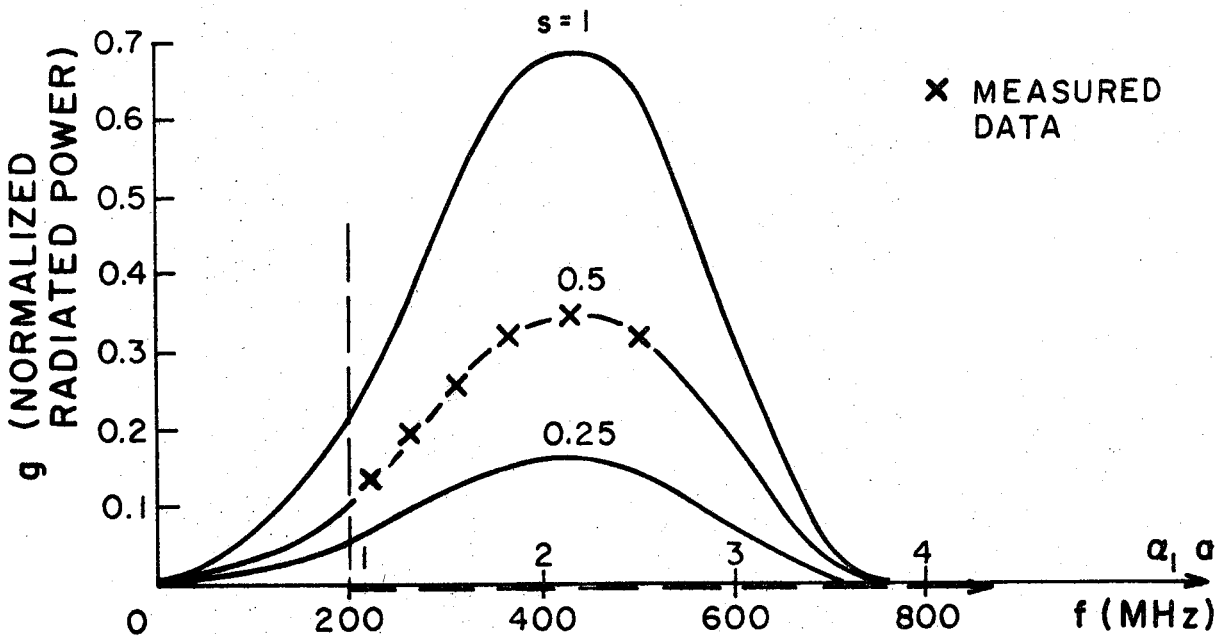
s = 1
0.5
0.25
× MEASURED DATA
g (NORMALIZED RADIATED POWER)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
200
400
600
800
f (MHz)
1
2
3
4
$\alpha_1 a$

Figure 2

knowledge of the height and radius of the slot antenna. For instance, if the measured data matches the theoretical curve for $s = 0.5$ as postulated in Fig. 2, we have from (4) the thickness of 7.3 cm for a slot antenna located at a distance of 4.7 cm from the coal surface. Furthermore, one can show from (3) that $\varepsilon_r = 3$ if such a measured data is obtained for a slot sensor of radius $a = 18.4$ cm.

From the above discussion, it is seen that the annular slot sensor not only can minimize the electromagnetic interference from near-by equipment, but also can provide measured results that are easy to interpret, which by and large, is not true for most other high frequency antenna structures. While this analysis necessarily needs to be modified when applied to a realistic environment because of the non-perfect reflection at the overburden, the finite size of the ground plane, and the dissipation inherent in the coal medium, it does help to point out a new type of sensor which in our opinion, warrants a closer and more comprehensive examination.

Appendix A

A.1 Formulation of the problem:

Consider the structure of an annular slot antenna as consisting of a semi-infinitely long, perfectly conducting coaxial line of inner and outer radii of a,b with its open-end flush-mounted onto a perfectly conducting ground screen of infinite extent as shown in Figure 1. The antenna is then placed at a distance h away from the coal slab which has a permittivity ε_1, thickness t and is backed up by a highly-reflecting slate of complex permittivity $\hat{\varepsilon}_2$. A cylindrical coordinate system (ρ,θ,z) is chosen so that the z-axis coincides with the axis of the annular slot and the plane $z=0$ with the ground plane. A current wave of the form $\exp(i[\omega t-k_o z])$ on the inner conductor of the coaxial line is assumed to be incident from below. Here, ω is the angular frequency and k_o, the free-space wave number. We seek in particular the dependance of the reflected current wave, and hence the apparent input admittance of the slot antenna, on the coal seam thickness.

Solution of the problem can be obtained by first solving for the tangential electric field distribution at the aperture. As shown in Appendix B, the transverse magnetic field component H_ϕ in air is related to the electric field E_ρ by the following expression,

$$H_\phi(\rho,z) = i\omega\varepsilon_o \int_a^b E_\rho(\rho',0)G_o(\rho,\rho';z,o)\rho' d\rho' \quad 0 < z < h \text{ and } \rho > 0' \qquad \text{(A1)}$$

where the kernel $G_o(\rho,\rho';z,0)$ of the integral is given by

$$G_o(\rho,\rho',z,0) = \int_0^\infty \frac{\alpha d\alpha}{\xi_o} [-\sin\xi_o z + R(\alpha)\cos\xi_o z]\, J_o(\alpha\rho')J_o(\alpha\rho) \qquad \text{(A2)}$$

$$R(\alpha) = \frac{\varepsilon_o\xi_1\tan\xi_o h - Y_2(\alpha)\xi_o\varepsilon_1}{\varepsilon_o\xi_1 + \xi_o\varepsilon_1 Y_2(\alpha)\tan\xi_o h} \; ; \; Y_2(\alpha) = \frac{i\xi_1\varepsilon_2 - \varepsilon_1\xi_2\tan\xi_1 t}{\varepsilon_1\xi_2 + i\xi_1\varepsilon_2\tan\xi_1 t} \tag{A3}$$

and $\xi_j = (k_j^2-\alpha^2)^{\frac{1}{2}}$, $\mathrm{Im}(\xi_i)\leq 0$ and $k_j = \omega(\mu\varepsilon_j)^{\frac{1}{2}}$ for $j = 0, 1, 2$; J_o is the Bessel function of order zero. We note that because of the symmetry inherent in the present problem, only the field components E_ρ, E_z, H_ϕ exist and they all have no angular dependence. A similar expression for the transverse magnetic field component in the coaxial region is known to be of the following form [Chang, 1970].

$$H_\phi(\rho,z) = -i\omega\varepsilon_o \int_a^b E_\rho(p',0)G_c(\rho,\rho';z,0)\rho' d\rho' + (\pi\rho)^{-1}\cos k_o z$$

$$a \leq \rho \leq b \; , \; z \leq 0 \tag{A4}$$

with the kernel $G_c(\rho,\rho';z,0)$ given as

$$G_c(\rho,\rho';z,0) = -i \int_{\infty}^{\infty} d\alpha \, \exp(i\alpha z)M(\alpha,\rho_>,b)M(\alpha,\rho_<,a)\Delta^{-1}(\alpha;b,a) \tag{A5}$$

where

$$M(\alpha,X,Y) = J_1(\xi_o X)H_o^{(2)}(\xi_o Y) - J_o(\xi_o Y)H_1^{(2)}(\xi X) \tag{A6}$$

$$\Delta(\alpha;b,a) = H_o^{(2)}(\xi_o b)J_o(\xi_o a) - H_o^{(2)}(\xi_o a)J_o(\xi_o b) \tag{A7}$$

and $H_o^{(2)}$ is the Hankel function of the second kind and order zero [Abramovitz and Stegun, 1965]. The term $(\pi\rho)^{-1}\cos k_o z$ in (A4) is explicitly related to the contribution of the incident current wave. Thus, an integral equation for the aperture electric field can be readily obtained from the application of continuity conditions at $z=0$ as

$$\int_a^b E_\rho(\rho';0)[G_o(\rho,\rho';0,0) + G_c(\rho,\rho';0,0)]\rho' d\rho' = -i\ 120(k_o\rho)^{-1},\ a\le\rho\le b \qquad \text{(A8)}$$

Once the aperture field $E_\rho(\rho',0)$ is known, the expression for the reflected current is then obtained directly from (A5) and the relationship $I(z) = 2\pi a H_\phi(a;z)$ in the coaxial line. As pointed out in Chang [1970], the solution of this seemingly very complicated integral equation indeed can be given analytically in closed form once we allow the assumption of a narrow gap, i.e., $(b-a) << a$. In that case, the kernal G_o is shown in Appendix C to be approximately

$$G_o(\rho,\rho';0,0) = -(\pi a)^{-1}[\ell n k_o|\rho-\rho'|/2 + \gamma + iC_o] , \qquad \text{(A9)}$$

where $\gamma = 0.577216$ is Euler's constant, and

$$C_o = \pi/2 - \int_0^\infty [1 - i\pi a\alpha R(\alpha)J_1^2(\alpha a)]d\alpha/\xi_o \qquad \text{(A10)}$$

Except for the expression of C_o , equation (A10) and hence the approximate form of the integral equation (A8) is identically the same as the one discussed in [Chang, 1970] for an annular slot antenna radiating into an unbounded free-space. Without much ado, we can write down the explicit expression of the input admittance as

$$Y_a = G + iB;$$

$$G = \frac{2k_o a}{\zeta_o}\mathrm{Re}(C_o) ,\ B = \frac{2k_o a}{\zeta_o}[-\gamma + 1 + \ell n\pi/2 + \ell n k_o(b-a)/2 + \mathrm{Im}(C_o)] \qquad \text{(A11)}$$

where $\zeta_o = 120\ \pi$ ohms is the free-space characteristic impedance. We note

that the input conductance is related directly to the radiated power of the antenna.

A.2 Evaluation of the Input Conductance G :

Utilizing the relationships that $2J_1(\alpha a) = H_1^{(2)}(\alpha a) + H_1^{(2)}(-\alpha a)$ and $J_1(\alpha a) = -J_1(-\alpha a)$, we can rewrite the expression for C_o as

$$C_o = \pi/2 - \frac{1}{2} \int_{-\infty}^{\infty} [1 - i\pi a\alpha R(\alpha) J_1(\alpha a) H_1^{(2)}(\alpha a)] d\alpha/\xi_o \tag{A12}$$

where the electric parameters of the coal seam and the slate are contained implicitly in the expression for $R(\alpha)$ given by (A4). Hence, the information regarding the thickness of the coal seam can be retrieved, at least in principle, from the measurement of the radiated power, and the comparison with theoretical results calculated from (A11) and (A12) for various thickness t . However, in order to further ease the computation, the slate is now assumed to be perfectly reflective so that $R(\alpha)$ reduces to

$$R(\alpha) = \frac{\varepsilon_o \xi_1 \tan(\xi_o h)\tan(\xi,t) - \varepsilon_1 \xi_o}{\varepsilon_o \xi_1 \tan(\xi_1 t) + \varepsilon_1 \xi_o \tan(\xi_o h)} \tag{A13}$$

and the second term in the integrand in (A12) contains only simple poles, while the first term has only a pair of branch cuts at $\alpha = \pm k_o$, in the complex α-plane. A subsequent deformation in the lower half-plane would then allow us to express C_o in the form of

$$C_o = \lim_{M\to\infty}\{\pi^2 a \sum_{m=1}^{M} (\alpha_m/\xi_{om}) J_1(\alpha_m a) H_1^{(2)}(\alpha_m a) \lim_{\alpha\to\alpha_m} (\alpha-\alpha_m) R(\alpha)$$

$$-i \ \ell n\ 2|\alpha_M|/k_o\} \tag{A14}$$

where α_m is the mth root of the secular equation

$$\varepsilon_o \xi_{1m} \tan(\xi_{1m} t) + \varepsilon_1 \xi_{om} \tan(\xi_{om} h) = 0 \tag{A15}$$

and $\xi_{jm} = (k_j^2 - \alpha_m^2)^{\frac{1}{2}}$ for $j = 0,1$. We note that, provided the thickness t and height h are sufficiently small (more specifically, $k_o h < \pi/2$ and $k_1 t < \pi/2$), only one of these roots is located on the real axis between k_o and k_1 in the complex α-plane, while the rest are on the imaginary axis. With some manipulation it is then not difficult to show that all the terms except $m = 1$ in the summation in (A14) are purely imaginary, so that the input conductance (or the radiated power) as given by (A11) now takes the form

$$G = 2\pi^2 k_o a \, F_1/\zeta_o \; ; \tag{A16}$$

where

$$F_1 = \lim_{\alpha \to \alpha_1} (\alpha - \alpha_1) R(\alpha) \alpha_1 / \xi_{o1}$$

$$= \varepsilon_1 \xi_{11} \xi_{o1} \sec^2 \xi_{o1} h [\xi_{o1} \varepsilon_1 (\tan \xi_{11} t + \xi_{11} t \sec^2 \xi_{11} t)$$

$$+ \xi_{11} \varepsilon_1 (\tan \xi_{o1} h + \xi_{o1} h \sec^2 \xi_{o1} h)]^{-1} \tag{A17}$$

The dependence on the coal seam parameters is now given explicitly in the expression for F_1 since $\xi_{j1} = (k_j^2 - \alpha_1^2)^{\frac{1}{2}}$; $j = 0,1$ and $k_o = k_1 \varepsilon_1^{-\frac{1}{2}} = 2\pi f (\mu_o \varepsilon_o)^{\frac{1}{2}}$. Further simplification is still possible when both the thickness and height are small electrically, i.e., $k_o^2 h^2 << 1$ and $k_1^2 t^2 << 1$. In that case, a small argument expansion of the tangent functions in (A15) and (A17) immediately reduces to the following result:

$$\alpha_1 \simeq k_o [(h+t)/(h+\varepsilon_r^{-1} t)]^{\frac{1}{2}} \tag{A18}$$

and

$$G \simeq \pi^2(\zeta_o S)^{-1}(\alpha_1 a)J_1^2(\alpha_1 a); \quad S = [(h+t)(h\varepsilon_r^{-1}t)]^{\frac{1}{2}}/a \tag{A19}$$

where ε_r is the relative permittivity of coal defined as $\varepsilon_r = \varepsilon_1/\varepsilon_o$.

Appendix B

Derivation for the Transverse Magnetic Field

It is well known that for a structure with rotational symmetry, the transverse magnetic field H_ϕ satisfies the following wave equation:

$$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{\partial^2}{\partial z^2} + k_j^2 - \frac{1}{\rho^2}\right) H_\phi\ (\rho,z) = 0 \tag{B1}$$

in the jth region specified by a wave number $k_j = \omega(\mu_o\varepsilon_j)^{\frac{1}{2}}$ where μ_o is the permeability and ε_j the permittivity of that region. With the use of a Fourier-Bessel transform, one can therefore write down the spectrum representation of H_ϕ in each region as

$$H_\phi = \begin{cases} \int_0^\infty A(\alpha)\exp(-i\xi_2[z-h-t])\ J_1(\alpha\rho)\alpha d\alpha, & j > h+t \\ \int_0^\infty B(\alpha)[\sin\xi_1(z-h) + Y_2(\alpha)\cos\xi_1(z-h)]J_1(\alpha\rho)\alpha d\alpha, & h < z < h+t \\ \int_0^\infty C(\alpha)[\sin\xi_o z - R(\alpha)\cos\xi_o z]J_1(\alpha\rho)\alpha d\alpha, & 0 < z < h \end{cases} \tag{B2}$$

where $\xi_j = (k_j^2 - \alpha^2)^{\frac{1}{2}}$, $\mathrm{Im}\,\xi_j \le 0$ and A,B,C,Y_2,R are some yet undetermined coefficients. The requirement that both H_ϕ and $E_\rho = -(i\omega\varepsilon)^{-1}\partial H_\phi/\partial z$ be continuous at the interfaces $z=h$ and $z=h+t$ would then allow us to eliminate four of the five coefficients to yield

$$H_\phi(\rho,z) = \int_0^\infty C(\alpha)[\sin\xi_o z - R(\alpha)\cos\xi_o z]\ J_1(\alpha\rho)\alpha d\alpha; \tag{B3}$$

and

$$R(\alpha) = \frac{\varepsilon_o \xi_1 \tan\xi_o h - Y_2(\alpha)\xi_o \varepsilon_1}{\varepsilon_o \xi_1 + \xi_o \varepsilon_1 Y_2(\alpha)\tan \xi_o h} \; ; \; Y_2(\alpha) = \frac{i\xi_1 \varepsilon_2 - \varepsilon_1 \xi_2 \tan\xi_1 t}{\varepsilon_1 \xi_2 + i\xi_1 \varepsilon_2 \tan\xi_1 t} \tag{B4}$$

At $z=0$ plane, the tangential electric field is then given by

$$E_\rho(\rho,0) = -(i\omega\varepsilon_o)^{-1} \int_0^\infty C(\alpha) J_1(\alpha\rho)\alpha d\alpha \tag{B5}$$

so that the expression for $C(\alpha)$ can be determined from the inverse Fourier-Bessel transform, together with the boundary condition that $E_\rho(\rho,0) = 0$ for $\rho > b$ or $\rho < a$ as

$$C(\rho) = -i\omega\varepsilon_o \int_a^b E_\rho(\rho',0)\, J_1\,(\alpha\rho')\rho' d\rho' \tag{B6}$$

Substitution of (B6) into (B3) would then yield the integral expression given in (A1).

Appendix C

Approximate Expression for the Kernel $G_o(\rho,\rho';0,0)$

In order to derive the approximate expression given in (A10) under the assumption of a small-gap, we first note that G_o has a logarithmic singularity at $\rho=\rho'$ and $z=z'=0$ since the integrand in (A2) decays only as fast as $-i(\pi a\alpha)^{-1}\cos\alpha(\rho-\rho')$ as $\rho\to\rho'$ and $z=0$. Thus if we first subtract out the leading term, we can evaluate the remainder term approximately by setting $\rho=\rho'=a$. This means

$$G_o(\rho,\rho';0,0) = -i(\pi a)^{-1}\int_0^\infty \cos\alpha(\rho-\rho')\,\frac{d\alpha}{\xi_o}$$

$$+\int_0^\infty [\alpha R(\alpha)J_o(\alpha\rho)J_o(\alpha\rho') + i(\pi a)^{-1}\cos\alpha(\rho-\rho')]\,\frac{d\alpha}{\xi_o}$$

$$\simeq -i(\pi a)^{-1}\int_0^\infty \cos\alpha(\rho-\rho')\,\frac{d\alpha}{\xi_o} + \int_0^\infty [\alpha R(\alpha)J_o^2(\alpha a) + i(\pi a)^{-1}]\,\frac{d\alpha}{\xi_o} \tag{C1}$$

The first integral is known exactly as $\frac{\pi}{2}H_o^{(2)}(k_o|\rho-\rho'|)$, which because of the small gap assumption, i.e., $k_o^2|\rho-\rho'|^2 << 1$ can be approximated by its small argument $\pi/2 - i(\ln k_o|\rho-\rho'|/2 + \gamma)$ and $\gamma = 0.577216$ is the Euler's constant [Abramovitz and Stegun, 1965]. Consequently, we have from (C1)

$$G_o = -(\pi a)^{-1}[\ln k_o|\rho-\rho'|/2 + \gamma + iC_o] \tag{C2}$$

where C_o is given in (A11).

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