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GUIDED WAVES ALONG A METAL GRATING ON THE SURFACE OF A GROUNDED DIELECTRIC SLAB

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Abstract

The propagation of guided waves along a metal grating lying on a grounded dielectric substrate is studied. Hitherto, investigation of the properties of such waves has been restricted to directions of propagation perpendicular to or nearly parallel to the strips of grating. Averaged boundary conditions for the fields at the grating are used here to simplify the analysis, and are expected to yield accurate results for grating periods that are sufficiently small compared to a wavelength. Comparisons made with more exact computations in the literature are shown to be good. The results have potential application for microwave and millimeter wave waveguides, slotted microstrip antennas and circuit elements.

1 Introduction

Guided waves propagating over an anisotropic structure (a periodically grated or corrugated surface, for example) have been of interest for many years in the study of leaky-waves and traveling-wave antennas, traveling-wave amplifiers, bandpass filters and transmission lines at microwave and millimeter wave frequencies. A large literature exists for such structures, of which we cite here only a representative sample [1]-[31]. A general theoretical study of the existence, uniqueness and spectral properties of such waves has recently appeared [32].

We will study the properties of guided waves propagating along a metal grating on the surface of a grounded dielectric slab as shown in Fig. 1. This structure has been analyzed under various assumptions by a number of authors [19]-[31]. All this work has been restricted in terms of directions of propagation (perpendicular to or nearly parallel to the strips of the grating), and some have made additional restrictive assumptions about the parameters. In this report, a new method using equivalent boundary conditions to describe the effect of the strip grating is used to address the problem of propagation of guided waves in an arbitrary direction.

Many authors, beginning apparently with Kontorovich [33], have investigated the use of such approximate boundary conditions to model wire gratings of various types. For an array of parallel strips, Sakurai [34], and later Sivov [35] and Vainshtein [36] as consequences of a more general analysis, derived the equivalent boundary conditions for the average electromagnetic field when the permittivity and permeability on both sides of the grating are the same. This was later generalized [37] to the case where the material parameters on both sides of the grating are different. This general condition has been recently rederived using the homogenization method, which shows it to be a first-order approximation whose error is of order $(k_0p)^2$, where k_0 is the free-space wavenumber and p is the period of the grating [38]. A number of other papers have derived equivalent boundary conditions or equivalent circuits for this structure, going all the way back to the work of Lamb [39]-[53]. However, these are all restricted either by allowing no field variation along the direction of the strips [39]-[41],

[44]-[46], [49]-[53], or limiting validity to narrow strips [42] or narrow slots [48], by not permitting different media on opposite sides of the grating [43], [47], [48] or in some other way (in [47] and [52]-[53], however, the model is valid for higher frequencies than any of the other models cited here, including the Sakurai-Vainshtein-Sivov condition that will be used in this report).

2 Definitions and Assumptions

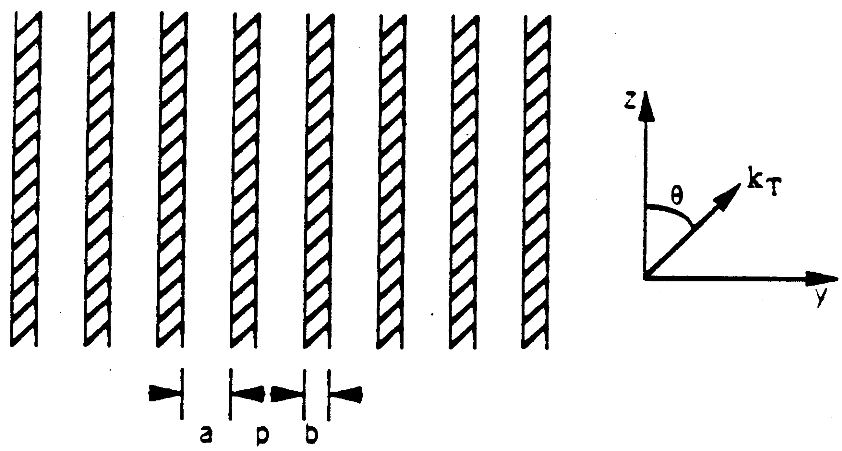
In this report, we are interested in a structure (see Fig. 1) consisting of a one dimensional periodic array of identical, equally spaced, thin metallic strips on a grounded dielectric substrate. As illustrated there, we choose z to be parallel to the axes of the strips, we take x upward perpendicular to the substrate surface, and y horizontal and transverse to the strips. The grounded lower surface of the substrate is taken to be the yz-plane. The thickness of the dielectric slab is d and the ground plane is located at $x = 0$.

An exact analysis of the electromagnetic field near this structure would be based on Floquet (or Bloch) waves of the form

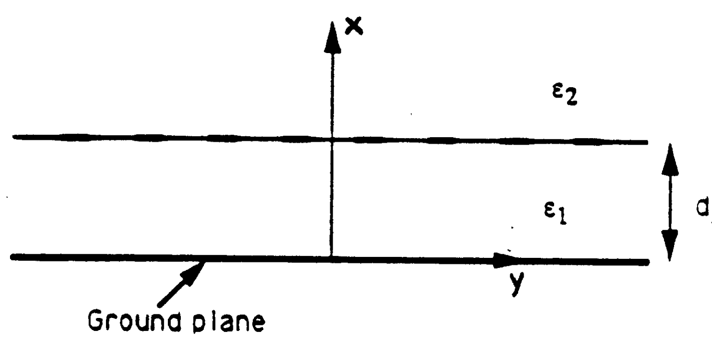
$$\vec{E} = \int \left[\sum_n a_n \vec{\mathcal{E}}_n(x, y; \gamma) \exp[-j(\beta_n z + \gamma y)] \right] d\gamma \tag{1}$$

and similarly for $\vec{H}$, where $\vec{\mathcal{E}}_n$ for each n is periodic in y with period p and $\beta_n(\gamma)$ is the propagation constant in the z-direction for a Floquet wave corresponding to a given value of the spectral variable γ. If $kp \ll 1$, then $|\vec{\mathcal{E}}_n| \to 0$ rapidly as x moves away from the plane of the strips (a distance more than $O(p)$), for all but one value of n , which corresponds to the fundamental Floquet mode. The use of average boundary conditions means we are explicitly constructing only that part of the field corresponding to this lowest Floquet mode. Our treatment of the problem is based on several important hypotheses:

1. The grating period p is small in comparison with the wavelength λ of the incident wave in such a way that the dimensionless parameter kp ($k = 2\pi/\lambda$) is small. This assumption implies that the grating can be represented as a semitransparent, infinitely thin film which both reflects and transmits the energy of the incident waves. Under this assumption, only the fundamental Floquet mode (the fundamental mode is defined to be the mode which propagates over the complete frequency range; in other words, there is no cutoff frequency) exists some distance away from the grating. These distances are such that fields of plane waves can be formed and they are greater than the order of a period p. The other higher-order Floquet waves are attenuated rapidly with distance from the grating. Based on these assumptions, the average currents and charges induced on the grating are governed by the average field of the main wave



(a)



(b)

Figure 1: Metal grating on a grounded dielectric slab: (a) cross-section; (b) top view.

and they are respectively proportional to the tangential components of the magnetic and the normal components of the electric field.

2. The thickness of the substrate is electrically small ($k_0 d \ll 1$, where $k_0 = 2\pi f\sqrt{\mu_0\epsilon_0}$, f is the frequency, ϵ_0 is the free space permittivity and μ_0 is the free space permeability). Although this assumption is somewhat restrictive, it is true for a number of practical designs.

3. Both the ground plane and the strips are assumed to be perfectly conducting metal.

4. The thickness of the strips is assumed to be infinitesimal. This assumption is used for most theories applicable to microstrip antennas.

5. The substrate and the ground plane are assumed to be of infinite extent. The characteristics of a structure of finite extent (such as resonant frequency, input impedance, radiation pattern, etc.) are essentially dependent on the shape and dimensions of the finite structure. However, the properties of the guided waves that exist on infinite structures can give us crucial insight and semi-quantitative models for the behavior of finite structures through the use of approximate transverse resonance techniques.

Our theory includes the case where the dielectric is lossy (the permittivity is taken to be $\epsilon_0\epsilon_r(1-j\tan\delta)$ where ϵ_r is the relative permittivity, and $\tan\delta$ is the loss tangent). It also includes the possibility of a lossy magnetic substrate. In most practical applications, the substrate is nonmagnetic and free of magnetic losses; therefore, in this report the permeability will be taken to be that of the free space.

In MKS units and time dependence $e^{j\omega t}$, the generalized Sakurai-Vainshtein-Sivov boundary conditions for the average fields (i. e., the average fields of the fundamental Floquet mode) at a strip grating lying in the plane interface between two different materials are [38] that E_y and E_z (hence also B_x) are continuous, while:

$$E_z = \frac{l_e}{2}[j\omega\frac{2\mu_1\mu_2}{\mu_1+\mu_2}(H_{y2}-H_{y1}) + \frac{2}{\epsilon_1+\epsilon_2}\frac{\partial}{\partial z}(\epsilon_2 E_{x2} - \epsilon_1 E_{x1})] \qquad (2)$$

$$H_{z2} - H_{z1} = 2l_h[-j\omega(\frac{\epsilon_1+\epsilon_2}{2})E_y + (\frac{\mu_1+\mu_2}{2\mu_1\mu_2})\frac{\partial B_x}{\partial z}] \qquad (3)$$

where

$$\begin{aligned} l_h &= \frac{p}{\pi}\ln\sec\frac{\pi b}{2p} \\ &= \frac{p}{\pi}\ln\csc\frac{\pi a}{2p} \end{aligned} \qquad (4)$$

$$
\begin{aligned}
l_e &= \frac{p}{\pi} \ln \csc \frac{\pi b}{2p} \\
&= \frac{p}{\pi} \ln \sec \frac{\pi a}{2p}
\end{aligned} \tag{5}
$$

and a is the width of a slot in the grating while b is the width of one of the strips. In the next section, we make use of (2) and (3) to study the properties of guided waves propagating along a metal grating on the surface of a grounded dielectric slab.

3 Derivation of the Eigenvalue Equation for the Guided Waves' Propagation Constants

The dielectric slab covered by a periodically slotted conducting plane supports guided modes. In this section, we will derive the eigenvalue equation for these modes using the equivalent boundary conditions presented in the previous section. From the eigenvalue equation, the normalized propagation constant (with respect to the free space propagation constant k_0) can be determined as a function of the direction of propagation of the mode. This same method has been used previously to obtain the eigenvalue equation for a rectangular metallic waveguide filled with a layered dielectric on top of which lies a periodic metallic grating [17]. It can be verified that if metallic side walls perpendicular to the y-axis are inserted into our structure, and a transverse resonance method applied in the y direction, the eigenvalue equation of [17] is reproduced for this case, as must happen since the same methods have been used in the analysis.

In our derivation, a spatial dependence of $e^{-j(\beta z+\gamma y)}$ will be understood to multiply all field quantities: $\vec{E}(x,y,z) = \vec{\mathcal{E}}(x)e^{-j(\beta z+\gamma y)}$, etc., where β and γ are respectively the propagation constants in the z-direction (along the strips) and y-direction (transverse to the strips). We introduce the quantity k_T as the propagation constant of a mode propagating at an angle θ with respect to the strips; β and γ are related to k_T as follows:

$$\beta = k_T \cos\theta \tag{6}$$

$$\gamma = k_T \sin\theta \tag{7}$$

so that

$$k_T^2 = \beta^2 + \gamma^2 \tag{8}$$

The mode fields satisfy the source-free version of Maxwell's equations, namely

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \tag{9}$$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} \tag{10}$$

in both regions I and II which are respectively the dielectric slab and the half space above the slab. The parameters of I are $\epsilon_1 = \epsilon_{r1}\epsilon_0$ and μ_0 and those of

II are $\epsilon_2 = \epsilon_0 \epsilon_{r2}$ and μ_0. From the y and z dependences assumed above, we have

$$\begin{aligned} \frac{\partial \vec{E}}{\partial y} &= -j\gamma \vec{E} \\ \frac{\partial \vec{H}}{\partial y} &= -j\gamma \vec{H} \\ \frac{\partial \vec{E}}{\partial z} &= -j\beta \vec{E} \\ \frac{\partial \vec{H}}{\partial z} &= -j\beta \vec{H} \end{aligned} \tag{11}$$

From (9)-(10) and (11) we observe that all field components can be expressed in terms of $\mathcal{E}_x$ and $\mathcal{H}_x$ only. The fields tangential to the yz-plane, $\vec{\mathcal{E}}_T$ and $\vec{\mathcal{H}}_T$, are given by:

$$\begin{aligned} \vec{\mathcal{E}}_T &= \frac{1}{k_T^2}\left\{\omega\mu(\gamma\vec{a}_z - \beta\vec{a}_y)\mathcal{H}_x - j(\gamma\vec{a}_y + \beta\vec{a}_z)\frac{d\mathcal{E}_x}{dx}\right\} \\ \vec{\mathcal{H}}_T &= \frac{1}{k_T^2}\left\{-\omega\epsilon(\gamma\vec{a}_z - \beta\vec{a}_y)\mathcal{E}_x - j(\gamma\vec{a}_y + \beta\vec{a}_z)\frac{d\mathcal{H}_x}{dx}\right\} \end{aligned} \tag{12}$$

where $\mathcal{E}_x$ and $\mathcal{H}_x$ satisfy

$$\left(\frac{d^2}{dx^2} + h^2\right)\begin{pmatrix} \mathcal{E}_x \\ \mathcal{H}_x \end{pmatrix} = 0 \tag{13}$$

in region I ($0 \leq x \leq d$), and

$$\left(\frac{d^2}{dx^2} - q^2\right)\begin{pmatrix} \mathcal{E}_x \\ \mathcal{H}_x \end{pmatrix} = 0 \tag{14}$$

in region II ($x \geq d$), where

$$\begin{aligned} h^2 &= \omega^2\mu_0\epsilon_1 - k_T^2 \\ q^2 &= k_T^2 - \omega^2\mu_0\epsilon_2 \end{aligned} \tag{15}$$

In general, q and h can be taken to be complex; q must have positive real part to correspond to a proper guided mode.

The forms of the solutions for Eqs. (13)-(14) are:

$$\begin{aligned} \mathcal{E}_x &= E_1\frac{\cos hx}{\cos hd} \\ \mathcal{H}_x &= H_1\frac{\sin hx}{\sin hd} \end{aligned} \tag{16}$$

in region I, and

$$\begin{pmatrix} \mathcal{E}_x \\ \mathcal{H}_x \end{pmatrix} = \begin{pmatrix} E_2 \\ H_2 \end{pmatrix} e^{-q(x-d)} \tag{17}$$

in region II. We have already taken into account that the solutions we look for are guided modes, and required the fields to decay exponentially in the x-direction.[1] We have also enforced the condition that at $x = 0$ (on the ground plane), the tangential electric field components must be equal to zero:

$$\begin{aligned} \mathcal{E}_z &= 0 \\ \mathcal{E}_y &= 0 \end{aligned}$$

Now, since at $x = d$, the tangential electric field component must be continuous, we get from (12), (16) and (17) that

$$\begin{aligned} \vec{\mathcal{E}}_T\Big|_{x=d} = \frac{jq}{k_T^2}(\beta\vec{a}_z + \gamma\vec{a}_y)E_2 - \frac{k_0\eta_0}{k_T^2}(\beta\vec{a}_y - \gamma\vec{a}_z)H_2 = \\ \frac{jh}{k_T^2}(\beta\vec{a}_z + \gamma\vec{a}_y)E_1 \tan hd - \frac{k_0\eta_0}{k_T^2}(\beta\vec{a}_y - \gamma\vec{a}_z)H_1 \end{aligned} \tag{18}$$

where $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$ is the free space wave impedance. Taking the dot product of (18) respectively with $\beta\vec{a}_z + \gamma\vec{a}_y$ and $\beta\vec{a}_y - \gamma\vec{a}_z$, we get

$$hE_1 \tan hd = qE_2 \tag{19}$$

$$H_1 = H_2 \tag{20}$$

From the equivalent boundary condition (2) on $\mathcal{E}_z$ at $x = d$, one obtains

$$\mathcal{E}_z = \frac{l_e}{2}[jk_0\eta_0(\mathcal{H}_{y2} - \mathcal{H}_{y1}) - \frac{2j\beta}{\epsilon_1 + \epsilon_2}(\epsilon_2 E_2 - \epsilon_1 E_1)] \tag{21}$$

Substituting for $\mathcal{E}_z$ and $\mathcal{H}_y$ from (12), one gets

$$\begin{aligned} \frac{\gamma k_0\eta_0}{k_T^2}H_2 + \frac{jq\beta}{k_T^2}E_2 = \frac{l_e}{2}\Big[jk_0\eta_0[\frac{jq\gamma}{k_T^2}H_2 + \frac{k_0\epsilon_{r2}\beta}{k_T^2\eta_0}E_2 + \frac{jh\gamma}{k_T^2}H_1 \cot hd \\ - \frac{k_0\epsilon_{r1}\beta}{k_T^2\eta_0}E_1] - \frac{2j\beta}{\epsilon_1 + \epsilon_2}(\epsilon_2 E_2 - \epsilon_1 E_1)\Big] \end{aligned} \tag{22}$$

From (19), (20), and (22) one gets

$$\begin{aligned} k_0\eta_0\gamma H_1[1 + \frac{l_e}{2}(q + h\cot hd)] = \\ -j\beta E_1 \tan hd[h + \frac{l_e}{2}(k_0^2 - \frac{2k_T^2}{\epsilon_{r1} + \epsilon_{r2}})(\epsilon_{r1}\cot hd - \epsilon_{r2}\frac{h}{q})] \end{aligned} \tag{23}$$

[1] *Improper* modes—ones for which q has a negative real part—may exist under certain conditions. In some circumstances, such modes may be *leaky waves*, and contribute a significant amount to the total field of a source-excited structure. To find such modes, we will simply reverse the sign of q wherever it appears in our final result.

Finally, using the equivalent boundary condition (3) on $\mathcal{H}_z$ at $x = d$,

$$\mathcal{H}_{z2} - \mathcal{H}_{z1} = 2l_h \left[-j\frac{k_0}{\eta_0} \left(\frac{\epsilon_{r1} + \epsilon_{r2}}{2} \right) \mathcal{E}_y - j\beta H_1 \right] \tag{24}$$

and substituting for $\mathcal{H}_z$ and $\mathcal{E}_y$ from (12), one.obtains

$$\frac{jq}{k_T^2}\beta H_2 - \frac{k_0\epsilon_{r2}\gamma}{k_T^2\eta_0}E_2 + \frac{jh}{k_T^2}\beta H_1 \cot hd + \frac{k_0\epsilon_{r1}\gamma}{k_T^2\eta_0}E_1 =$$
$$2l_h[-j\frac{k_0}{\eta_0}\left(\frac{\epsilon_{r1}+\epsilon_{r2}}{2}\right)\left(\frac{jh\gamma}{k_T^2}E_1 \tan hd - \frac{k_0\eta_0\beta}{k_T^2}H_1\right) - j\beta H_1] \tag{25}$$

From (19), (20), and (25), one gets

$$\frac{k_0\gamma}{\eta_0}E_1 \tan hd[\epsilon_{r1}\cot hd - \epsilon_{r2}\frac{h}{q} - l_h(\epsilon_{r1}+\epsilon_{r2})h] =$$
$$-j\beta H_1\{q + h\cot hd + l_h[2k_T^2 - k_0^2(\epsilon_{r1}+\epsilon_{r2})]\} \tag{26}$$

Eliminating H_1 and $E_1 \tan hd$ from (23) and (26), and using (6) and (7), we get

$$-\cos^2\theta\frac{q + h\cot hd + l_h[2k_T^2 - k_0^2(\epsilon_{r1}+\epsilon_{r2})]}{1 + \frac{l_e}{2}(q + h\cot hd)} =$$
$$k_0^2\sin^2\theta\frac{\epsilon_{r1}\cot hd - \epsilon_{r2}\frac{h}{q} - l_h(\epsilon_{r1}+\epsilon_{r2})h}{h + \frac{l_e}{2}(k_0^2 - \frac{2k_T^2}{\epsilon_{r1}+\epsilon_{r2}})(\epsilon_{r1}\cot hd - \epsilon_{r2}\frac{h}{q})} \tag{27}$$

which when solved will yield the allowed values of propagation constant k_T for the guided waves of the structure. Let the normalized propagation constant with respect to the free space propagation constant be denoted by χ:

$$\chi = k_T/k_0 \tag{28}$$

Let also $\epsilon_{r2} = 1$ and $\epsilon_{r1} \equiv \epsilon_r$, and

$$E_s = \sqrt{\epsilon_r - \chi^2} = \frac{h}{k_0} \tag{29}$$
$$E_t = \sqrt{\chi^2 - 1} = \frac{q}{k_0} \tag{30}$$

The eigenvalue equation then becomes

$$\cos^2\theta\frac{F_{TE}(\chi) + k_0 l_h(2\chi^2 - \epsilon_r - 1)\tan(E_s k_0 d)}{\tan(E_s k_0 d) + \frac{k_0 l_e}{2}F_{TE}(\chi)}$$
$$+\sin^2\theta\frac{F_{TM}(\chi) - k_0 l_h E_s(\epsilon_r + 1)\tan(E_s k_0 d)}{E_s\tan(E_s k_0 d) + \frac{k_0 l_e}{2}(1 - \frac{2\chi^2}{\epsilon_r+1})F_{TM}(\chi)} = 0 \tag{31}$$

where

$$F_{TE}(\chi) = E_t \tan(E_s k_0 d) + E_s \tag{32}$$

$$F_{TM}(\chi) = \epsilon_r - \frac{E_s}{E_t}\tan(E_s k_0 d) \tag{33}$$

are functions whose zeroes are the normalized propagation constants for the TE and TM modes respectively of the grounded slab with no grating present.

3.1 Limiting Cases

Equation (31) can be rewritten as

$$\begin{aligned}
0 = & [\cos^2\theta F_{TE}(\chi)E_s + \sin^2\theta F_{TM}(\chi)]\tan(E_s k_0 d) \\
+ & \left(1 - \frac{2\chi^2\cos^2\theta}{\epsilon_r + 1}\right)\left[\frac{k_0 l_e}{2} F_{TE}(\chi)F_{TM}(\chi) - k_0 l_h E_s(\epsilon_r + 1)\tan^2(E_s k_0 d)\right] \\
- & \frac{k_0^2 l_e l_h}{2}(\epsilon_r + 1)\tan(E_s k_0 d) \\
\times & \left[\cos^2\theta\left(1 - \frac{2\chi^2}{\epsilon_r + 1}\right)^2 F_{TM}(\chi) + E_s F_{TE}(\chi)\sin^2\theta\right]
\end{aligned} \tag{34}$$

In this form we can examine the limiting cases in which either the strips vanish or the slots close up. If the slots close up ($a \to 0$), then $l_h \to \infty$, $l_e \to 0$, and $l_e l_h \to 0$, so that (34) reduces to

$$\tan(E_s k_0 d) = 0 \tag{35}$$

whose solutions are the well-known ones for the perfectly conducting parallel-plate waveguide. If the strips disappear ($b \to 0$), then $l_h \to 0$, $l_e \to \infty$ and $l_h l_e \to 0$, and (34) shows that we must have either

$$F_{TM}(\chi) = 0$$

which is the well-known equation for the TM-mode surface wave guided by a dielectric slab on a perfectly conducting slab on a perfectly conducting ground plane, or

$$F_{TE}(\chi) = 0$$

which is the equation for an TE-mode surface wave on the same grounded dielectric slab. The solutions of these equations are well known, and discussed, for example, in Walter [5].

Another limiting case of interest is when the period of the grating goes to zero while its density remains fixed: $p \to 0$ while a/p ($\neq 0$ or 1) remains constant. In this limit, l_e and l_h both go to zero in the averaged boundary conditions (2) and (3), and we reduce to the traditional unidirectional conductor boundary conditions [54], [55] used, e. g., in [7]-[9], [12] in the analysis of various guided wave structures. The eigenvalue equation (34) in this limit becomes then

$$0 = [\cos^2\theta F_{TE}(\chi)E_s + \sin^2\theta F_{TM}(\chi)] \tag{36}$$

If $\epsilon_r \longrightarrow 1$, (36) reduces to a result given in [12], as well as a limiting case of the result of [8] (which contains many misprints in the relevant equations). For $\theta \longrightarrow 90°$, a special case of the result of [9] is duplicated, while for $d \longrightarrow \infty$, then the result of [7] is obtained.

Finally, the cases when θ becomes 0° (propagation parallel to the strips) or 90° (propagation perpendicular to the strips) will be of particular interest later on. In both cases, our solutions become either pure TE or TM modes with respect to the direction of propagation (that is, E_1 or H_1 respectively goes to zero). The relevant limits of (34) are:

$$\begin{aligned}
F_{TE}(\chi) + k_0 l_h(2\chi^2 - \epsilon_r - 1)\tan E_s k_0 d &= 0 \quad ; \quad \theta = 0°, \quad \text{TE} \quad (37)\\
E_s \tan E_s k_o d + \frac{k_0 l_e}{2}\left(1 - \frac{2\chi^2}{\epsilon_r + 1}\right) F_{TM}(\chi) &= 0 \quad ; \quad \theta = 0°, \quad \text{TM} \quad (38)\\
\tan E_s k_o d + \frac{k_0 l_e}{2} F_{TE}(\chi) &= 0 \quad ; \quad \theta = 90°, \quad \text{TE} \quad (39)\\
F_{TM}(\chi) - k_0 l_h(\epsilon_r + 1)E_s \tan E_s k_o d &= 0 \quad ; \quad \theta = 90°, \quad \text{TM} \quad (40)
\end{aligned}$$

We note that when $\epsilon_r = 1$ and $a \ll p$, (39) and (40) reduce to results previously obtained by Li and Oliner [6] for leaky waves on a slotted parallel plate waveguide. Eqn. (39) for $\epsilon_r = 1$ but without the restriction on a/p had also been found earlier by Honey [3], and is found in the form given here for $\epsilon_r \neq 1$ in Walter's book ([5], pp. 246-250). Walter ([5], pp. 250-253) also gives the result (37), which is attributed to Honey in an unpublished report. Sigelmann and Ishimaru [20], [22] obtained an approximate version of (40) valid if $\ln(p/a) \gg 1$ (we let $k_0 p \ll 1$ in their results to compare with ours). These authors detected "improper" (on a "wrong" Riemann sheet of the complex k_T-plane) roots of this equation for sufficiently high frequencies, but their locations were such that they would not be true leaky waves, as they would never contribute to the steepest descent evaluation of the far field of the structure. Nevertheless, we are led to anticipate that in at least some parameter ranges there will be leaky wave solutions to our eigenvalue equation as well.

4 Numerical Results

In the first part of this section, numerical results are presented and discussed. In the succeeding subsections, comparison is made to the results obtained by other authors.

In our numerical search for roots of the eigenvalue equation, we always found two proper modes when the electrical substrate thickness $k_0 d$ was small enough. One has a normalized propagation constant k_T/k_0 that is near $\sqrt{\epsilon_r}$ at $\theta = 0°$ and increases without bound as $\theta \to 90°$.[2] The fields of such a mode become highly localized to the neighborhood of the grating when k_T/k_0 exceeds $\sqrt{\epsilon_r}$ by a large enough margin. We thus denote this mode as a "grating" mode. The other mode has k_T/k_0 between 1 and $\sqrt{\epsilon_r}$, and shares many of the characteristics of an ordinary surface wave on a dielectric slab, so we will call it by that name.

In Figures 2 and 3, we show the dependences of the normalized propagation constants on θ for several values of the parameters a/p and p/d, at the relatively small electrical substrate thickness of $k_0 d = 0.2$. We see that the surface wave is very weakly bound to the substrate, as k_T is quite close to k_0. Neither mode is much dependent on a/p or p/d at this low frequency. This implies that they would be well described by the unidirectionally conducting limit (36) of the eigenvalue equation.

Figure 4 shows a case with a thicker substrate where leaky modes are present (i. e., the real part of q is negative and the real part of k_T/k_0 is less than 1). We recall that leaky waves are not proper modes, but represent the cumulative effect of a collection of continuous spectrum modes, which gradually shed energy by radiation into the space above the grating [5]. This energy loss accounts for the imaginary part of the propagation constant.

4.1 Comparison with Weiss's Method

Weiss [26] and Crampagne *et al.* [28] investigated the microstrip meander line structure to estimate the performance of a vacuum-tube crossed-field amplifier (CFA): power distribution, coupling between RF and an electron beam, interaction impedance, etc. The basis of their work was an analysis of the propagating waves of the grating structure studied in this report, but with consideration limited to the case when the direction of propagation is nearly parallel to the strips. Their method is quite different than ours but as will be seen below, good agreement is found. They assumed that frequency is low enough that waves may be taken to have a quasi-TEM character relative to the axes of the strips. In accordance with usual practice, the propagation constant in these conditions is denoted by $k_z = k_0\sqrt{K_{eff}}$, where K_{eff} is the effective dielectric constant of the mode of propagation being considered and k_0 is the propagation constant of the waves in free space. The velocity of propagation v is related to these

[2] Our results based on the averaged boundary conditions predict that $k_T/k_0 \to \infty$ as $\theta \to 90°$. This, however, would contradict the assumptions underlying the derivation of (2) and (3), namely that the average fields vary slowly over distances of order p in the y-direction: $\gamma p = k_T p \sin\theta \ll 1$. We can reasonably expect that some mode will exist for θ near 90° with k_T/k_0 merely very large, but not necessarily accurately predicted by our theory.

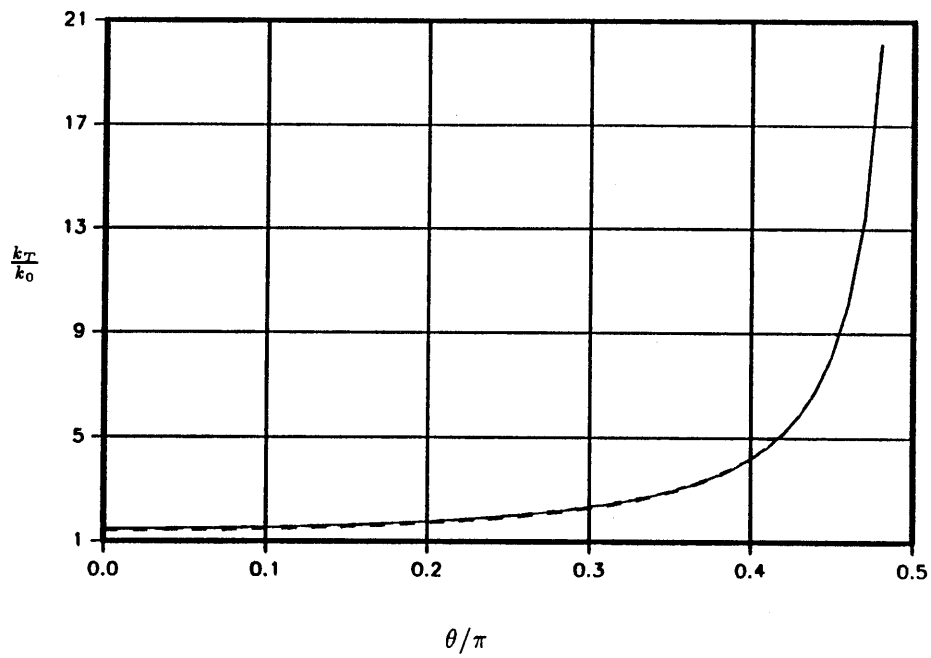


Figure 2: Normalized dispersion curves for the grating mode ($k_0d = 0.2$, $\epsilon_r = 2.2$): ————, $a/p = 0.01$, $p/d = 1.0$; — — — —, $a/p = 0.99$, $p/d = 1.0$; - - - - - - - - - - -, $a/p = 0.01$, $p/d = 0.2$; — - — - — - —, $a/p = 0.99$, $p/d = 0.2$.

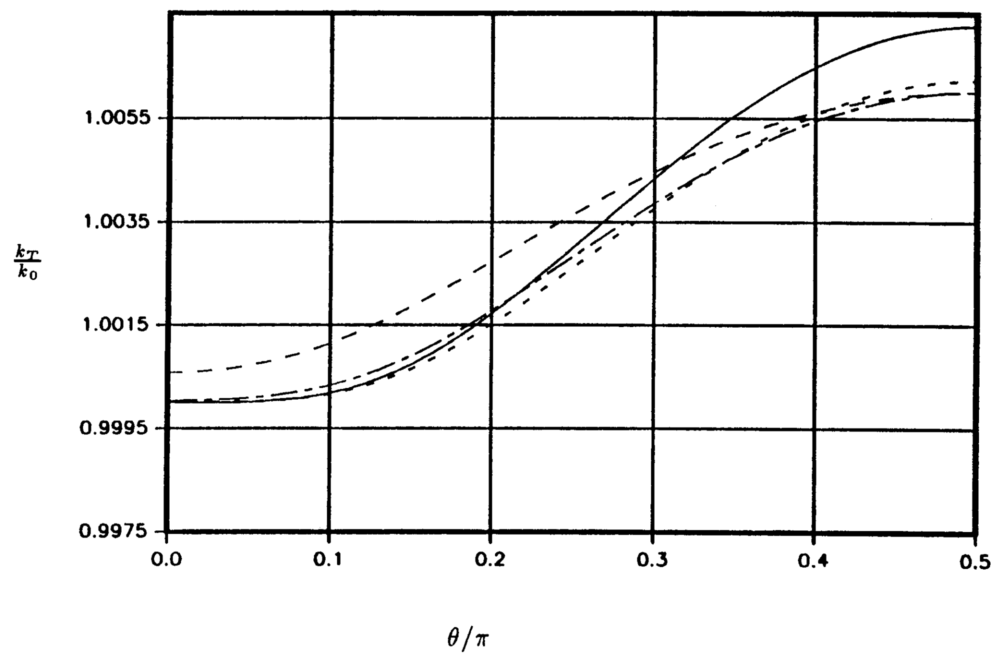


Figure 3: Normalized dispersion curves for the surface wave mode (same parameters and legend as in Fig. 2).

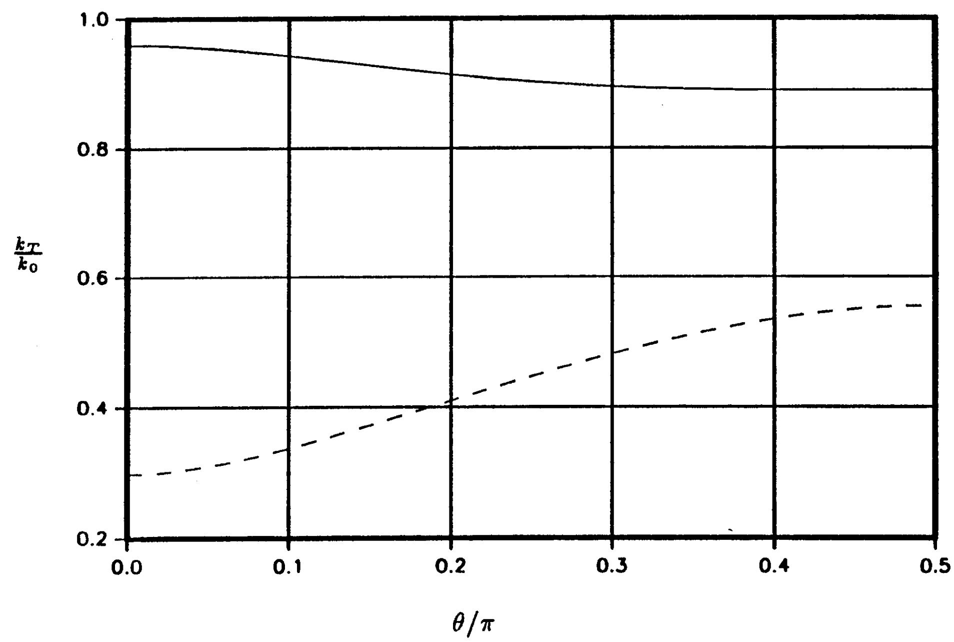


Figure 4: Real (————) and imaginary (— — — —) parts of k_T/k_0 for a leaky wave mode ($k_0 d = 0.7$, $a/p = 0.1$, $p/d = 1.0$, $\epsilon_r = 2.2$).

quantities by

$$\frac{c}{v} = \sqrt{K_{eff}} = \frac{k_z}{k_0} \tag{41}$$

where c is the velocity of light in free space (approximately 3×10^8 m/s); the ratio in (41) is the normalized propagation constant parallel to the strips. This quasi-TEM approximation, together with a Green's function method, allows the normal modes of propagation parallel to the strips to be determined.

In [26] and [28], the periodic structure is broken up into unit cells containing two strips each; between successive cells there is assumed to be a given phase shift of φ, but within each cell (for each value of φ) the voltage on the two strips can be either identical or 180° out of phase. Thus there can exist two linearly independent modes, one even and the other one odd. When comparing their results with ours, we are only interested in the even mode. This is so because the odd mode will have a severe phase change from one conductor to the next, that would correspond to zero average field, and the use of an average boundary condition at the grid such as that upon which our results are based precludes a mode like that from appearing.

To compare our results with those of Weiss and Crampagne *et al.*, we must first relate their expressions to ours. Their quasi-TEM mode is our grating mode for θ close to 0°. From eqn. (6), in our notation:

$$k_z = k_T \cos\theta \quad k_y = k_T \sin\theta \tag{42}$$

where k_T is the propagation constant of the surface wave in the strip plane and θ is the angle with respect to z-direction at which the surface waves propagate. Because of the quasi-TEM assumption (and this applies to the results of [19] as well), θ is restricted to values very close to zero; otherwise substantial components of $\vec{E}$ or $\vec{H}$ would appear in the z-direction.

Now, eqn. (41) can be rewritten as

$$\frac{k_T}{k_0} = \frac{\sqrt{K_{eff}}}{\cos\theta} \tag{43}$$

On the other hand, the phase shift φ per distance $2p$ in the y-direction must be

$$\varphi = 2k_y p = (2k_0 p)\frac{k_T}{k_0}\sin\theta \tag{44}$$

But the quasistatic and quasi-TEM assumptions underlying the analysis of Weiss imply that both $k_0 p$ and $\sin\theta$ are small, so that it is appropriate to take $\varphi = 0$. Thus, we have

$$\frac{k_T}{k_0} = \frac{\sqrt{K_{eff}|_{\varphi=0}}}{\cos\theta} \tag{45}$$

We can thus read values of K_{eff} at $\varphi = 0$ from the data for the even mode in [26] or [28], choose a small value of $k_0 p$, and obtain the corresponding value of k_T/k_0 from (45) for comparison with our theory.

Weiss, apparently the first one to use the Green's function method with the quasi-TEM approximation in solving the normal modes of propagation along an infinite array of parallel strips, considers in his analysis a shielded grounded dielectric substrate. He took the upper ground plane position to be

$$\frac{H_2}{H_1} = 2.6 \tag{46}$$

where H_1 and H_2 are, respectively, the heights of the substrate surface and the upper shield plane above the bottom ground plane. It turns out when comparison is carried out for the parameters in his paper ($\epsilon_r = 6.5$, $k_0 d = 0.030637$, $a/d = 0.912$, $a/p_1 = 0.279412$), agreement with our results is only fair, presumably because the upper shield plane has a considerable effect on the grating mode near $\theta = 0°$. Crampagne *et al.*, who later used the same analysis as Weiss, consider the upper ground plane position to be

$$\frac{H_2}{H_1} = 100 \tag{47}$$

The upper shield plane is so far from the bottom ground plane that it has almost no effect. We compared our results with these for the following parameters: ($\epsilon_r = 9.6$, $k_0 d = 0.05376$, $k_0 p_1 = 0.05$, $a/d = 0.13$, $a/p_1 = 0.068893$). We found that the equivalent boundary condition method and the Green's function methods are in good agreement as illustrated in Fig. 5 as long as the direction of propagation is along the strips or makes a small angle with the z-direction (the z-direction is the direction of the strips).

4.2 Comparison with Zlunitsyna's Work

Zlunitsyna [23] analyzed a structure consisting of a metallic grid embedded in a screened dielectric, treated as a transmission line. The grid is formed of a one dimensional periodic array of identical, equally spaced thin metallic strips on a grounded dielectric substrate. The strips as well as the ground plane are assumed to be perfect conductors. Since propagation is here assumed perpendicular to the strip axes, it is sufficient to consider separately the cases of E-polarization and H-polarization. In both cases, the solutions are analogous, and as a result of the principle of duality, either one of these cases reduces to the other if the strips and the slots are interchanged. In her work, she analyzed the TM_m modes ($m \in \mathcal{N}$) of the structure. Her method is again quite different from ours but a good agreement is found considering that the long wavelength approximation ($p \ll \lambda$, where p is the grating period and λ is the wavelength of the incident field) inherent in our analysis is not satisfied in the results she presents.

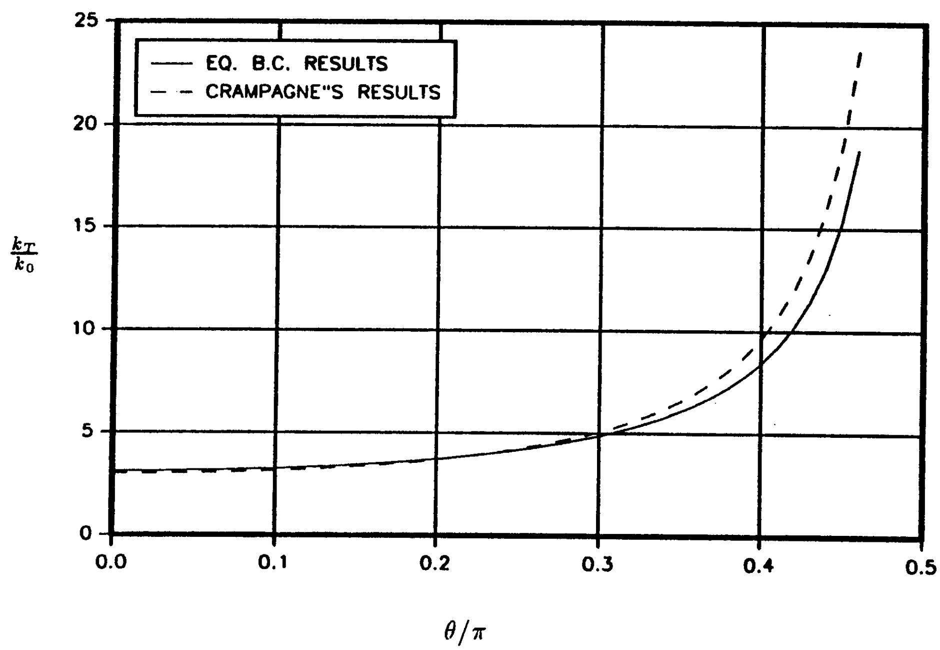


Figure 5: Comparison between Green's function method (Crampagne) and equivalent boundary conditions method ($\theta \rightarrow 0°$).

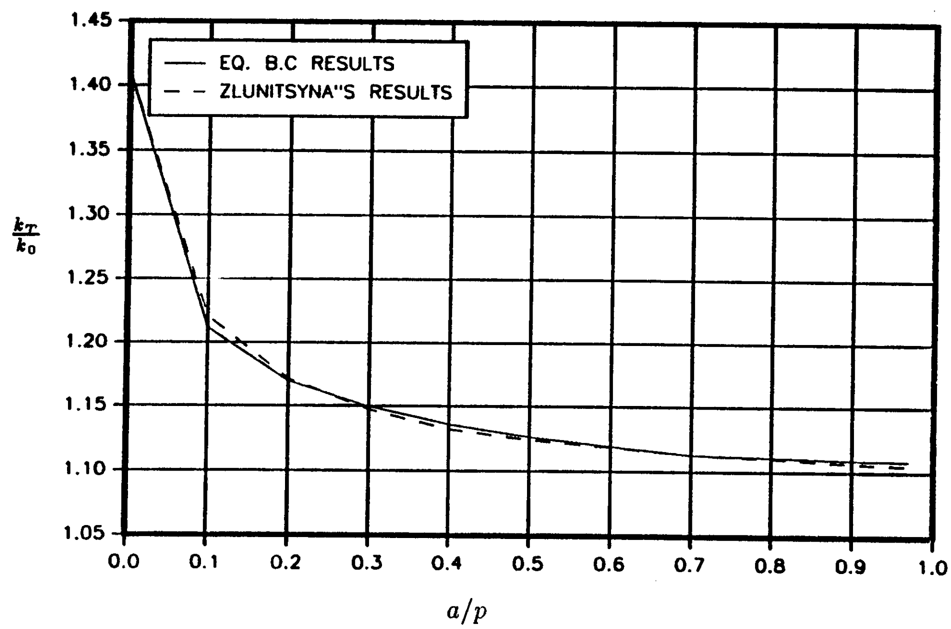


Figure 6: Comparison between Zlunitsyna's work and equivalent boundary condition method ($\theta = 90^0$).

The periodicity of the structure enables her to write the required fields (in our notation, the z-component of E) in the form of a Fourier series expansion, which corresponds to its representation in the form of an infinite set of spatial harmonics. Applying the boundary conditions for the slots and the strips (the tangential components of the electric field are zero at the strips, but at the slots the tangential electric and magnetic fields are continuous), she ended up with an infinite set of linear homogeneous algebraic equations. This set, after some transformations, can be reduced to a Riemann-Hilbert problem. The determinant of the reduced set of equations gives the dispersion equation for TM_m modes.

Comparison between our method and hers is carried out for the following parameters: a lossy dielectric of relative permittivity $2 + j4 \times 10^{-4}$, $k_0 p = 0.30\pi$ and $p/d = 1$. This value of $k_0 p$ is certainly not small compared to 1, and yet a comparison of our results with hers (plotted in Fig. 6) shows very good agreement for the lowest order surface wave mode.

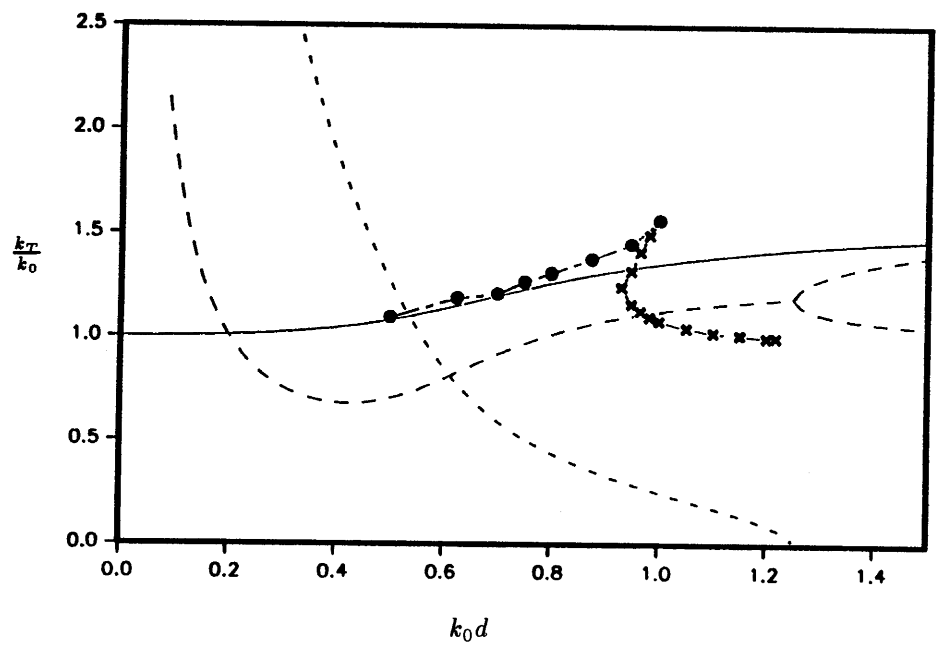


Figure 7: Comparison between Sigelmann's work and equivalent boundary condition method: ————, surface wave mode (present method); — — — —, improper mode, real part (present method); - - - - - - - - - - -, improper mode, imaginary part (present method); •— - — - —•, surface wave mode (Sigelmann); ×- - -×, improper mode, real part (Sigelmann).

4.3 Comparison with Sigelmann's Work

Finally, we compare our results with those of Sigelmann [22] for the surface wave mode as well as some improper modes when $\theta = 90°$ (Fig. 7). For this case, we take $\epsilon_r = 2.5$, $a/p = 0.3$, and $p/d = 2.0$. The normalized substrate thickness runs from $k_0 d = 0.0$ to 1.5, and clearly p will not be small with respect to either d or λ in this case. Nevertheless, we see very good agreement up to perhaps $k_0 p = 1.5$ for the surface wave mode. Moreover, we also predict the existence of a set of improper modes, which are leaky for some values of frequency. Although Sigelmann did not find any improper modes with complex propagation constants, he did find a pair of improper modes with real k_T/k_0, which bifurcate from a common value at about $k_0 d = 0.94$, one of which exists up until $k_0 d = 1.0$ and the other until somewhat higher frequencies. Our method, while not producing results that are numerically close to these, does predict the same qualitative behavior of bifurcation at a higher frequency, with normalized propagation constants of comparable magnitude, as seen in Fig. 7. Below the bifurcation frequency, a leaky wave behavior is exhibited, which should be expected to be quantitatively accurate below about $k_0 d = 0.75$ if we may judge from the agreement for the surface wave mode. We might note that near $k_0 d = 1.0$, the results of Sigelmann are approaching a stopband type behavior typical of Bloch waves on periodic structures when the period becomes comparable to a wavelength. Homogenization methods do not reproduce this behavior well, as is evident from this comparison. We can get an idea not only of the limitations of our approximation from this, but also of the qualitative behavior we may expect when our approximations no longer hold.

5 Conclusion

The method of equivalent boundary conditions has been used to study the properties of guided waves propagating along a metal grating on the surface of a grounded dielectric slab. Unlike previous analyses, this model can be used to analyze the propagation of surface waves in an arbitrary direction with respect to the axis of the grating. Comparisons of this model's results with a quasi-TEM Green's function method, with Zlunitsyna's work and with Sigelmann's work show the accuracy of the approach as well as its limitations for directions of propagation along or normal to the grating.

Two basic kinds of proper modes exist—a grating mode and a surface wave mode. The grating mode becomes a slow wave mode as the direction of propagation deviates from that of the slot axes. Other guided wave structures involving such gratings have previously been shown to possess similar slow-wave characteristics [11], [13], [15]-[16]. The structure has also been shown to support improper leaky wave modes when the substrate is electrically thick enough.

Future work will entail the use of the average boundary conditions employed here to model finite grated conducting structures (finely slotted microstrip patch antennas or resonant circuit elements for use in filter applications, for example) on a grounded substrate. There is some indication that unusual behavior can be expected as frequency is varied, and work is in progress to explore this more fully. It is possible that each slot or strip of the grating will tend to resonate at a slightly different frequency and that this staggering of nearby resonances could yield a wider intrinsic bandwidth for the patch compared to conventional unslotted ones. If this so, then the finely slotted microstrip patch antenna would be a new configuration to remedy the problem of narrow bandwidth from which most microstrip patch antennas suffer. This will be reported in future publications.

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