A Systematic Approach To Portfolio Creation In Restructured Electricity Markets

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Abstract. This paper determines and compares the efficacy of systematic approaches to portfolio construction using hourly assets from the PJM (Pennsylvania-New Jersey-Maryland) Independent System Operator (PJMISO). The two models addressed herein are an unconditional mean-covariance model using ordinary least squares (OLS) estimation and a lagged *seemingly unrelated regression* (SUR) model using feasible generalized least squares (FGLS) estimation. Despite the abundance of quantitative research on electricity markets, market participants are known for doing anecdotal analysis on the qualitative features of the market, e.g. weather patterns, transmission and generation outages, gas prices, etc. to construct daily portfolios and manage risk. Further, there is a lack of academic research that takes a practical approach to daily, portfolio risk in these markets. This paper bridges the gap between academic and industry analysis by incorporating quantitative analysis into a practical, systematic approach to portfolio construction. To test for and compare the efficacy of optimized daily portfolios, this paper will simulate portfolios using realized returns from the PJMISO market.

1   Introduction

Portfolio risk management is of principle concern to participants in financial markets, especially in whole-sale power markets whose volatility far exceeds that of traditional financial markets such as the Standard & Poor's (S&P) 500.[1] Such volatility is typically associated with opportunities for speculators to reap large returns at the risk of potential, large losses. The market itself benefits from speculator participation in the form of increased liquidity and theoretical market convergence. However, if market participants are unable to manage the risk of their portfolio, they face the possibility of sustaining losses that force them out of the market, reducing market liquidity. This paper evaluates the profitability and accuracy of different methods of systematically managing daily portfolio risk.

One of the prevailing approaches to risk management that participants in electricity markets take is evaluating physical phenomena to re-balance a daily portfolio. These qualitative factors include weather patterns, transmission and generation outages, gas prices, load forecasts, and electricity flow modeling. However, these techniques are largely used to determine which locations are ripe for financial speculation or risk hedging and do little to determine asset allocations. Further, some of those factors are localized whereas others are not. For instance, a low voltage transmission line outage in Boston is likely to have only have a localized effect whereas excess natural gas supply in the US is likely to have a market-wide effect. Market participants would therefore benefit from supplementing causal asset determination with a quantitative method of asset allocation that takes into account the covariation across location and time of day.

The most well-known procedure for this form of portfolio risk management is that of the modern portfolio theory (MPT) (Markowitz, 1952). Despite the myriad of modifications that have been made to advance this seminal theory, MPT still stands as a useful exercise in portfolio construction and optimization, especially in relatively young markets such as electricity.[2] The theoretical foundation of MPT is that assets in the same market sometimes tend to move together and sometimes tend to move apart. By including the covariance of assets in the process of portfolio creation, an investor can better manage their

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[1]To demonstrate the volatility of these markets see Figure 2 which compares the S&P 500 to the average of the assets used in this analysis.

[2]Restructured electricity markets still lack the theoretical and practical foundations that other, more mature financial markets have: indices, a risk-free asset, and the like.

risk; a portfolio composed of assets that perfectly move together is exposed to significant, centralized risk.

Limited work has been done to incorporate MPT to restructured electricity markets. In particular, Liu and Wu (2007) examined the trade off between allocation in local bilateral, non-local bilateral and spot energy markets in PJM (Pennsylvania-New Jersey-Maryland) Independent System Operator (PJMISO) from the perspective of a hypothetical generation-owning market participant. Ghorbaniparvar and Ghorbaniparvar (2013) take a similar approach to the MERALCO market in the Philippines, including a univariate multiple linear regression to predict prices. Both papers focus on the maximization of a defined utility function, constraining on assets weights, from the perspective of a hypothetical risk-averse company that owns generation.

This paper expands on such research by instead minimizing portfolio variance, constraining on both asset weights and desired return, from the perspective of a financial speculator. Further, this analysis takes the more practical approach of hourly contracts for energy as opposed to daily aggregates. The analysis also implements two models of returns to conduct the daily portfolio optimizations: an unconditional mean-covariance model using ordinary least squares (OLS) estimates and a lagged *seemingly unrelated regression* (SUR) model using feasible generalized least squares (FGLS) estimation. The latter, lagged, modeling approach is appealing due to electricity prices' well-documented autoregressive tendencies (Aggarwal et al., 2009).

The objectives of this analysis are to determine: (1) can daily portfolios constructed via portfolio optimization of pre-selected assets from PJMISO, on average, produce positive returns? (2) Is the computationally costly extension from the unconditional mean-covariance model to the lagged SUR model with FGLS estimates informative? (3) Which of the two models creates optimized portfolios that produce the desired portfolio return more accurately? (4) What, if any, differences exist between the series of simulated returns from the optimized portfolios produced by each model?

Section 2 will address important characteristics of electricity markets, section 3 will detail the research methodology, section 4 will describe data, section 5 will report results, and section 6 will conclude.

2 Institutional Characteristics of Electricity Markets

In the United States, the conventional model of electricity historically involved a vertically integrated natural monopoly in both generation and transmission, that served all the consumers of electricity in a given area. The characteristics of this conventional market structure are price stability at the cost of high barriers to entry and limited innovation.

However, in the late 1990s, certain areas of the United States decided to break from the conventional model by creating largely deregulated markets for electricity. Instead of a single producer setting electricity prices, these new markets would set price according to the supply and demand for electricity. This paper draws electricity assets from PJMISO, one of those particular markets, which is the largest and oldest independent system operator in the United States with hourly spot prices for thousands of settlement locations across the Mid-Atlantic.

In theory, the market would see gains in both efficiency and innovation as market forces took the place of regulation following the transition ot the deregulated marker structure. However, the regime shift towards increased competition also exemplified the non-conventional characteristics of electricity as a market commodity: non-storable, short-term supply shortage intolerance, perfectly inelastic demand and inelastic supply in the spot market (Aggarwal et al., 2009). The short-term settlement basis, price inflexibility, and frequent, binding transmission constraints cause returns in restructured electricity markets to be highly volatile (Weron, 2006).

PJMISO operates on a two-market system: the forward or day-ahead (DA) market and the spot or real-time (RT) market. The DA market is comprised of a uniform price auction that PJMISO executes, closing at noon Eastern Time each day (Weron, 2006). In preparation for the DA market, participants leverage forecasts of market factors to arrive at a prediction of prices for the next market day. Market participants submit bids (long positions) and offers (short positions) in price-quantity pairs for a given hour of operation the next day at a given settlement location.[3] These bids and offers are aggregated to form the demand and supply curves for the DA market (Cheung et al., 2010). The equilibrium price of these two curves will dictate the forward price (marginal clearing price or MCP) of electricity at that given settlement location for that given hour (Weron, 2006). This paper will refer to that price as the

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[3] The pricing units for these contracts are in $/megawatt ($/MW) and quantity units are in megawatts (MWs).

DA price.

Participants who submitted a price-quantity bid that is greater than the MCP will 'receive' those MWs at the chosen hour the next day and pay the MCP for each, hence the uniformity of auction price. Similarly, a participant who submitted a price-quantity offer that is lower than the MCP will 'produce' those MWs at the chosen hour the next day and receive the MCP for each contracted MW (Weron, 2006). This paper will strictly focuses on speculative market participants who do not have the ability to deliver on awarded contracts and execute purely financial contracts. All positions of this nature settle the next day at the end of the chosen hour by definition, prohibiting a 'buy and hold' strategy; market participants must successfully bid or offer into the market prior to each day they intend to participate.[4]

In contrast, the RT market does not operate based on an auction, rather operating on short-term (hourly) supply and demand which are notably volatile. Conventional commodities are storable and transportable allowing for arbitrage across time and space. Significant amounts of electricity, however, cannot be stored in an economically viable manner or transported to a specific place; once a generator supplies power to the grid it instantaneously travels in unknown proportions along the paths of least resistance (Cheung et al., 2010). Not only that, but once electricity has been supplied to the grid, it must be consumed to avoid the risk of damaging infrastructure in accordance with the Law of Conversation of Energy. Further, the prevailing aversion to power shortages in the US causes short-term demand inelasticity (Weron, 2006). Therefore, PJMISO must quickly accommodate a short-term change in demand or supply by ramping generators up and down.[5] Since the objective of the ISO is to supply power in the least-cost manner, the marginal generator will definitionally be more expensive than the last. Additionally, unlike other financial markets, electricity prices can go negative. One cause for negative prices is production subsidies, typically for wind and hydroelectric generators, that allow generators to make positive revenues even when paying consumers to use power. Another cause for negative prices is inflexibility of certain types of units which have a preference for continual, full production, such as

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[4]Note: speculative participants do not actually receive or generate MWs of power. These contracts are the equivalent of a commodity contract that is already 'zeroed-out'. For each long position, the ISO will ensure that there will be sufficient generation the next day and vice versa for each short position. The function of speculative trading is to increase liquidity in the market to ensure price convergence and diminish regional market power of individual generators and load-serving entities.

[5]Ramping here refers to changing the current rate of production on a generator; ramping up implies and increase in production and ramping down implies a decrease.

nuclear and brown coal (Staff, 2013).[6] If market participants inadequately forecast RT prices during the DA auction and fail to regionally diversify their portfolio they may be exposed to extreme returns as the RT and DA prices diverge.

There are various contracts available in PJMISO with which financial speculators can seek profits. This paper focuses on bids and offers for virtual contracts for a given settlement location i at a specified hour ending (HE) for market day $t+1$. These contracts involve the market participant (MP) submitting a price-quantity bid in the DA market for a chosen number of megawatts (MWs), paying the DA clearing price ($DA_{i,t}$) per MW. The contract settles the next day at the RT clearing price ($RT_{i,t+1}$) for the same settlement location and HE, receiving that amount per MW. Such a contract will profit if $RT_{i,t+1} > DA_{i,t}$ and will result in a loss if $RT_{i,t+1} < DA_{i,t}$.[7] Offers have the opposite properties: the ISO pays MPs the DA clearing price and gets paid the RT clearing price the next day. Return and log return are defined as follows:

$$R_{i,t} = \frac{(RT_{i,t+1} - DA_{i,t})}{DA_{i,t}} \tag{1}$$

$$r_{i,t} = \ln(\frac{RT_{i,t+1}}{DA_{i,t}}) = \ln(R_{i,t} + 1) \tag{2}$$

$$\{r_{i,t}\} \begin{array}{cc} N & T \\ i=1 & t=1 \end{array}$$

However, since the DA market closes as noon Eastern Time each day, $RT_{i,t}$ for hours 13-24 that day are not known by auction close. This means $r_{i,t-1}$ cannot be observed at time t for contracts on those hours when constructing a portfolio for the auction at hand. For consistency across hours, for any given time t the first observable return is $r_{i,t-2}$ for all hours.

3   Econometric Methodology

This analysis includes daily simulation of portfolios constructed via constrained minimization of portfolio variance. Daily portfolio optimization of this nature requires forecasts for expected returns and a variance-covariance matrix of the selected assets. This analysis uses two models of log returns to obtain those

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[6]For example, some unites are 'must run' units. They are defined as such because ramping them can be costly due to repairs and the fixed cost of start up (Staff, 2012). Therefore, it can sometimes be economical for a generator of this nature to 'pay' consumers to use energy when their is excess supply rather than ramping down their unit. Since the market does not price these costs, prices can and do go negative. It is interesting to note that this eliminates the assumption of zero-bound losses for long positions that other markets possess.

[7]Since prices can be negative in these markets, a negative DA clearing price implies that a speculator is paid to take a long position and pays to take a short position. Similarly, a negative RT price implies that a speculator pays to hold a long position and is paid to hold a short position.

forecasts:

1. Unconditional mean-covariance model using OLS estimates

2. Lagged SUR model using FGLS estimation

Model 2 should prove more robust than Model 1, but requires more computational demand. The two models will be compared by jointly testing the coefficients that are present in Model 2 that are not present in Model 1. Post-simulation comparisons are also made to determine the relative efficacy of the model estimates.

3.1 Models of Returns

3.1.1 Unconditional Mean-Covariance Model

The unconditional mean-covariance model reflects the base case of the SUR model. This model has the benefit of parsimony and limited computational demand but lacks complexity and a conditional expectation. The model specification is as follows:

$$r_{i,t} = \mu_i + \varepsilon_{i,t} \tag{3}$$

(3) in its equivalent compact notation is:

$$r_i = R_i \mu_i + \varepsilon_i$$

$$R_i = \underbrace{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}}_{Tx1} ; \varepsilon_i = \underbrace{\begin{bmatrix} \varepsilon_{i,1} \\ \vdots \\ \varepsilon_{i,T} \end{bmatrix}}_{Tx1}$$

$$r = R\mu + \varepsilon \tag{4}$$

$$R = \underbrace{\begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N \end{bmatrix}}_{TNxN} ; \mu = \underbrace{\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix}}_{Nx1} ; \varepsilon = \underbrace{\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{bmatrix}}_{TNx1}$$

Where

1. $E(\varepsilon_i) = 0$

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2. $E(\varepsilon_{i,t}\varepsilon_{j,t}) = \sigma_{ij}$

3. $E(\varepsilon_{i,t}\varepsilon_{j,s}) = 0$

The conditional expected return[8] for a given asset i at time t is easily obtained as follows:

$$E(r_i) = E(R_i\mu_i + \varepsilon_i) \ = \mu_i \tag{5}$$

The OLS estimates for μ are then:

$$\tilde{\mu}_i = (R_i^T R_i)^{-1} R_i^T r_i \tag{6}$$

Assuming $\tilde{\mu}_i \xrightarrow{P} \mu_i$, $\tilde{\mu}_i$ is the estimated conditional expected value of r_i. Similarly, the conditional covariance is obtained as follows:

$$\sigma_{ij} = E[(r_i - \mu_i)(r_j - \mu_j)] = E(\varepsilon_i\varepsilon_j) = \frac{\varepsilon_i^T \varepsilon_j}{T} \tag{7}$$

$$\tilde{\sigma}_{ij} = \frac{\tilde{\varepsilon}_i^T \tilde{\varepsilon}_j}{T} \tag{8}$$

Assuming $\tilde{\sigma}_{ij} \xrightarrow{P} \sigma_{ij}$, $\tilde{\sigma}_{ij}$ is the estimated conditional covariance of r_i and r_j. Given $\tilde{\mu}_i$ and $\tilde{\sigma}_{ij}$, notate $\tilde{\mu}$ as a Nx1 vector of estimated conditional expected returns with $\tilde{\mu}_i$ in its ith element and $\tilde{\Omega}$ as a NxN estimated skedasticity matrix with $\tilde{\sigma}_{ij}$ elements.

$$\tilde{\mu} = \begin{bmatrix} \tilde{\mu}_1 \\ \vdots \\ \tilde{\mu}_N \end{bmatrix} \tag{9}$$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\sigma}_{11} & \cdots & \tilde{\sigma}_{1N} \\ \vdots & \ddots & \vdots \\ \tilde{\sigma}_{N1} & \cdots & \tilde{\sigma}_{NN} \end{bmatrix} \tag{10}$$

3.1.2 Lagged SUR Model with FGLS Estimates

As noted above, researchers have empirically shown that returns in electricity markets have autoregressive tendencies. When the series of returns in this analysis are individually modeled on lags 1-10, lags 1, 2,

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[8]Note that in this case the conditional expected value of r_i is equivalent to the unconditional expected value of r_i since R is a constant.

and 7 are most commonly significant. However, since $r_{i,t-1}$ is unobservable at time t, this modeling approach only implements lags 2 and 7. The model also takes into account day of the week fixed effects to control for intraweek seasonality. The model specification is as follows:[9]

$$r_{i,t} = \beta_{0,i} + \beta_{1,i}r_{i,t-2} + \beta_{2,i}r_{i,t-7} + \sum_{d=1}^{6} \gamma_{i,d}D_{t,d} + \varepsilon_{i,t} \tag{11}$$

$$D_{t,d} = \begin{cases} 1, & \text{if } t \text{ corresponds to } d \\ 0, & \text{otherwise} \end{cases}$$

The compact notation for (11) is then:

$$r_i = R_i\beta_i + \varepsilon_i$$

$$R_i = \underbrace{\begin{bmatrix} 1 & r_{i,-1} & r_{i,-6} & D_{1,1} & \cdots & D_{1,6} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & r_{i,T-2} & r_{i,T-7} & D_{T,1} & \cdots & D_{T,6} \end{bmatrix}}_{TxK} ; \beta_i = \underbrace{\begin{bmatrix} \beta_{0,i} \\ \beta_{1,i} \\ \beta_{2,i} \\ \gamma_{1,i} \\ \vdots \\ \gamma_{6,i} \end{bmatrix}}_{Kx1} ; \varepsilon_i = \underbrace{\begin{bmatrix} \varepsilon_{i,1} \\ \vdots \\ \varepsilon_{i,T} \end{bmatrix}}_{Tx1}$$

$$r = R\beta + \varepsilon \tag{12}$$

$$R = \underbrace{\begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N \end{bmatrix}}_{TNxKN} ; \beta = \underbrace{\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_N \end{bmatrix}}_{KNx1} ; \varepsilon = \underbrace{\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{bmatrix}}_{TNx1}$$

Where

1. $E(\varepsilon_i \mid R) = 0$

2. $E(\varepsilon_{i,t}\varepsilon_{j,t} \mid R) = \sigma_{ij}$

3. $E(\varepsilon_{i,t}\varepsilon_{j,s} \mid R) = 0$

4. d = 1,...,6 representing the day of the week from Monday to Saturday

5. $r_{i,t}$ is weakly stationary

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[9]See Appendix for the MatLab code used to estimate Model 2.

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6. $K = 9$

The conditional expectation of r_i and the conditional covariance of r_i and r_j are:

$$\mu_i = E(r_i \mid R) = E(R_i\mu_i + \varepsilon_i \mid R) \ = R_i\beta_i \tag{13}$$

$$\sigma_{ij} = E[(r_i - \mu_i \mid R)(r_j - \mu_j \mid R)] = E(\varepsilon_i\varepsilon_j) = \frac{\varepsilon_i^T\varepsilon_j}{T} \tag{14}$$

Without proof, GLS estimates have greater efficiency than OLS estimates. However, obtaining GLS estimates requires a known variance-covariance matrix of the stacked residuals. One method for dealing with the lack of a such a matrix is to conduct FGLS estimation following the methodology of Zellner (1962). First, the OLS estimates for β_i are:

$$\tilde{\beta}_i = (R_i^T R_i)^{-1} R_i^T r_i \tag{15}$$

Given $\tilde{\beta}_i$, the estimated conditional expectation of r_i and estimated residuals are:

$$\tilde{\mu}_i = R_i\tilde{\beta}_i \tag{16}$$

$$\tilde{\varepsilon}_i = r_i - R_i\tilde{\beta}_i = r_i - \tilde{\mu}_i \tag{17}$$

Assuming $\tilde{\mu}_i \xrightarrow{P} \mu_i$, $\tilde{\mu}_i$ is the estimated conditional expected value of r_i. Following from (17), the estimated covariance of r_i and r_j is:

$$\tilde{\sigma}_{ij} = \frac{\tilde{\varepsilon}_i^T\tilde{\varepsilon}_j}{T} \tag{18}$$

Assuming $\tilde{\sigma}_{ij} \xrightarrow{P} \sigma_{ij}$, $\tilde{\sigma}_{ij}$ is the estimated conditional covariance of r_i and r_j. The NxN estimated skedasticity matrix $\tilde{\Omega}$ is therefore comprised of $\tilde{\sigma}_{ij}$ in its (i,j)th elements:

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\sigma}_{11} & \cdots & \tilde{\sigma}_{1N} \\ \vdots & \ddots & \vdots \\ \tilde{\sigma}_{N1} & \cdots & \tilde{\sigma}_{NN} \end{bmatrix} \tag{19}$$

Given $\tilde{\Omega}$, an estimated variance-covariance matrix of the stacked residuals can be found as follows:

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$$\tilde{\Sigma} = \tilde{\Omega} \otimes I_T \tag{20}$$

The *KNx1* vector of stacked FGLS estimates is now given by:

$$\hat{\beta} = (R^T \tilde{\Sigma}^{-1} R)^{-1} R^T \tilde{\Sigma}^{-1} r \tag{21}$$

The first available forecast period is *t+2*. Since $E(\varepsilon_{i,t}\varepsilon_{j,t} \mid R) = \sigma_{ij}$ is time invariant, Ω is assumed to stay constant. However, the estimated conditional expectation can be forecasted as follows:

$$r_{i,t+2} = \hat{\beta}_{0,i} + \hat{\beta}_{1,i} r_{i,t} + \hat{\beta}_{2,i} r_{i,t-5} + \sum_{d=1}^{6} \hat{\gamma}_{i,d} D_{t+2,d} + \varepsilon_{i,t+2} \tag{22}$$

$$r_i = R_i \hat{\beta}_i + \varepsilon_i \tag{23}$$

$$R_i = \begin{bmatrix} 1 & r_{i,1} & r_{i,-4} & D_{3,1} & \cdots & D_{3,6} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & r_{i,T} & r_{i,T-5} & D_{T+2,1} & \cdots & D_{T+2,6} \end{bmatrix}; \beta_i = \begin{bmatrix} \hat{\beta}_{0,i} \\ \hat{\beta}_{1,i} \\ \hat{\beta}_{2,i} \\ \hat{\gamma}_{1,i} \\ \vdots \\ \hat{\gamma}_{6,i} \end{bmatrix}; \varepsilon_i = \begin{bmatrix} \varepsilon_{i,3} \\ \vdots \\ \varepsilon_{i,T+2} \end{bmatrix}$$

$$\hat{\mu}_i = R_i \hat{\beta}_i \tag{24}$$

$$\hat{\mu} = \begin{bmatrix} \hat{\mu}_1 \\ \vdots \\ \hat{\mu}_N \end{bmatrix} \tag{25}$$

To conduct hypothesis tests on the FGLS estimates, asymptotic t-tests are used. For sufficiently large N, the estimates are asymptotically normal such that:

$$\sqrt{N}(\beta_{FGLS} - \beta) \xrightarrow{d} N[0, (\plim_{N \to \infty} \frac{R^T \tilde{\Sigma}^{-1} R}{N})^{-1}] \tag{26}$$

The null and alternative hypotheses of interest are:

$$H_0 : \beta_i = 0 \tag{27}$$

$$H_A : \beta_i \neq 0 \tag{28}$$

Under this framework, the relevant t-statistic for each coefficient can be obtained via:

$$\frac{\hat{\beta} - 0}{\sqrt{\sigma_{\hat{\beta}}^2}} \overset{A}{\sim} N(0, 1) \tag{29}$$

Where $\sigma_{\hat{\beta}}^2$ represents the diagonal elements of $(R^T \tilde{\Sigma}^{-1} R)^{-1}$. Since $\hat{\beta}$ is a *KNx1* and each *K=1* element is an estimate for $\beta_{0,i}$, the hypotheses for joint significance are specified as follows:

$$H_0 : \beta_{res} = 0 \equiv R\beta = \gamma \tag{30}$$

$$H_A : \beta_{res} \neq 0 \equiv R\beta \neq \gamma \tag{31}$$

Where $\hat{\beta}_{res}$ is a *(K-1)Nx1* vector of coefficients representing all the elements of $\hat{\beta}$ except $\beta_{0,i}$, R is a *(K-1)NxKN* which represents a *KNxKN* identity matrix that is missing each $K_i = 1$ row, and γ is a *(K-1)Nx1* vector of zeroes.

$$W = T(R\hat{\beta} - \gamma)^T [R(R^T \Sigma^{-1} R)^{-1} R^T]^{-1} (R\hat{\beta} - \gamma) \overset{d}{\sim} \chi_{(K-1)N}^2 \tag{32}$$

3.2   Portfolio Optimization

Given μ and Ω, it is now possible to optimize a daily portfolio. Model 1 is extended by using $\tilde{\mu}$, the OLS estimate for conditional expected return, and $\tilde{\Omega}$, the OLS estimate for conditional covariance. Model 2 is extended by using $\hat{\mu}$, the FGLS estimate for conditional expected return, and $\tilde{\Omega}$, the OLS estimate for conditional covariance. The optimization technique follows Markowitz (1952) aside from using conditional estimates of μ and Ω, rather than an unconditional estimates, for simplicity. The constrained optimization problem is as follows:

$$\min_{w_k} w_k^T \Omega w_k \ s.t. \begin{cases} \mu_{p,k} = w_k^T \mu_k \\ 1 = w_k^T 1_N \end{cases} \tag{33}$$

Where w_k represents a *Nx1* vector of asset allocation weights and $\mu_{p,k}$ is the desired portfolio return for day *k*. The above constraints require that the expected portfolio return must equal the desired portfolio return and that the sum of all assets weights must equal 1. The relevant Lagrangian for this maximization is:

$$L(w, \lambda_1, \lambda_2) = w_k^T \Omega w_k + \lambda_1(\mu_{p,k} - w_k^T \mu_k) + \lambda_2(w^T 1_N - 1) \tag{34}$$

The algebraic solution to the above Lagrangian is then (Ruppert, 2004):[10]

$$\overset{*}{w}_k = g + \mu_{p,k} h \tag{35}$$

$$g = \frac{(B\Omega^{-1} 1_N - A\Omega^{-1} \mu_k)}{D} \tag{36}$$

$$h = \frac{(C\Omega^{-1} \mu_k - A\Omega^{-1} 1_N)}{D} \tag{37}$$

$$A = 1_N^T \Omega^{-1} \mu_k; \ B = \mu_k^T \Omega^{-1} \mu_k; \ C = (1_N^T \Omega^{-1} 1_N; \ D = BC - A^2 \tag{38}$$

There are many approaches to choosing a desired portfolio return, the final input to the optimization. Common approaches include choosing the desired portfolio return from the minimum variance portfolio, the desired portfolio return from the portfolio that maximizes the Sharpe ratio, or simply a fixed value (Treynor and Black, 1973). Since the aforementioned optimization allows for short-selling, the portfolio return is unbounded; the optimization could allocate an infinite amount in any given asset and short-sell others to satisfy the constraint that the sum of all asset weights must equal unity. Further, the efficient frontier is not necessarily the same for each iteration of the optimization rendering a fixed value unsuitable. Therefore, this analysis defines the desired portfolio return as:

$$\mu_{p,k} = \frac{(\mu_{max} + \mu_{min})}{2} \tag{39}$$

Where μ_{max} is equal to the expected return of the asset with the greatest expected return and μ_{min} is equal to the expected return on the minimum variance portfolio:

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[10]See Appendix for the MatLab code used to execute the optimization.

$$\mu_{max} = \max(\mu_k) \tag{40}$$

$$\mu_{min} = -\frac{g^T \Omega h}{h^T \Omega h} \tag{41}$$

There is also the added condition that if $\mu_{p,k}$ is negative, the optimization does not occur and w_k becomes an Nx1 vector of zeros, representing no trading activity the following day. This specification ensures that the desired portfolio return is on the efficient frontier, is finite, and falls directly between the highest expected asset return and the return on the minimum variance portfolio. The optimization then produces a portfolio return and standard deviation of:

$$\overset{*}{r}_{p,k} = r_k^T \overset{*}{w}_k \tag{42}$$

$$r_{i,k} = \ln(R_{i,k} + 1); \; r_k = \begin{bmatrix} r_{i,k} \\ \vdots \\ r_{N,k} \end{bmatrix}; \; \{r_{i,k}\} \; \underset{i=1}{\overset{N}{}} \; \underset{k=1}{\overset{H}{}}$$

$$\overset{*}{\sigma}_{p,k} = \sqrt{\overset{*}{w}_k^T \Omega \overset{*}{w}_k} \tag{43}$$

3.3 Simulation

This analysis continues by conducting the aforementioned optimization for a given day k to evaluate the performance of the optimized portfolio. As a measure of the accuracy of the optimization, the root mean squared error (RMSE) between the realized log return and the desired portfolio return can be calculated as follows:

$$RMSE = (\frac{1}{H} \sum_{k=1}^{H} (\overset{*}{r}_{p,k} - \mu_{p,k})^2)^{\frac{1}{2}} \tag{44}$$

Coefficient of variation of the RMSE or CV(RMSE) is also calculated to account for the difference in $\mu_{p,k}$ between the two models:

$$CV(RMSE) = \frac{RMSE}{\bar{\mu}_{p,k}} \tag{45}$$

To determine the performance of a given daily optimization the return on asset i on day k is

$$R_{i,k} = \frac{(RT_{i,k+1} - DA_{i,k})}{DA_{i,k}}; \ R_k = \begin{bmatrix} R_{i,k} \\ \vdots \\ R_{N,k} \end{bmatrix} \tag{46}$$

The realized return of the portfolio of assets is then

$$R_{p,k} = R_k^T w_k \tag{47}$$

The portfolio return can then be aggregated across the holding period H ($k=1, 2,...,H$) to evaluate the performance over time.

$$R_p = \begin{bmatrix} R_{p,k} \\ \vdots \\ R_{p,H} \end{bmatrix} \tag{48}$$

Various evaluative metric can then be calculated, including the mean, the 1st, 5th, and 10th percentile, standard deviation, and Sharpe ratio absent a risk-free return. For demonstrative purposes, a Rosenblatt-Parzen density estimation for each model is also included as follows:

$$\hat{f}_h(R_p) = \frac{1}{Hh} \sum_{k=1}^{H} K(X_k) \tag{49}$$

$$h = cH^{-0.2} \sigma_{R,p} \tag{50}$$

Where $c = 1$ and:

$$X_k = \frac{R_p - R_{p,k}}{h} \tag{51}$$

With a Gaussian kernel:

$$K(X) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}X^2) \tag{52}$$

14

4 Data

The data available for analysis is vast; PJMISO has more than ten thousand settlement locations that separately settle on an hourly basis every day of the year.[11] To reduce complexity and computational demand, this analysis focuses on five of the predetermined load zones within the market as settlement locations. These include (1) American Electric Power (AEP), (2) Commonwealth Edison (COMED), (3) Dominion Virginia Power (DOM), (4) Allegheny Power Systems (APS), and (5) Public Service Electric and Gas Company of New Jersey (PSEG).[12]

Each zone represents a weighted average of prices for the settlement locations that make up that zone and has 24 individual hourly settlements for forward and spot prices every day. To address the intraday variations in price, each of the 120 zone-hour combination is modeled as an individual asset ($i = 1, 2, \ldots,$ N; N=120). This analysis uses a rolling lookback period of 365 days for daily simulations ($t = 1, 2, \ldots,$ T; T=365) for a six year period ($k = 1, 2, \ldots, H$; $H = 2191$) starting on January 1st, 2009.

Using log returns is standard in financial econometrics, largely due to ease of interpretation, log-normality, and approximation of return. Unfortunately, due to the nature of power markets, negative and zero-value pricing can occur. When $x \leq 0$, $ln(x)$ is undefined or does not exist rendering certain observations unusable which may introduce bias into the models. Further, if either of the models includes an observation for $r_{i,t}$, $r_{i,t-2}$, $r_{i,t-7}$, $r_{i,k}$ or $R_{i,k}$ during the lookback period that is missing, the relevant asset i and/or day t must be removed. Therefore, not all model estimations and/or simulations include all N ($N < 120$) and T ($T < 365$).

5 Results

Unsurprisingly the simulations of the optimized portfolios from Model 1 and Model 2 differed significantly. For instance, on the vast majority of simulation days, Model 1 produced daily average asset allocations that varied between -1 and 1 for each of the five settlement locations.[13] In contrast, Model 2 produced average daily asset allocations that varied between -4 and 4 for each settlement location.[14] This indicates

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[11]The data in this analysis is publicly available at pjm.com but was compiled by Yes Energy. For more information, please contact Nick@YesEnergy.com.

[12]See Figure 1 for the area each covers. These five load zones were chosen based on age, data availability, and regional diversity.

[13]See Figure 3.

[14]See Figure 4.

that Model 2 had a greater tendency for portfolio leverage than Model 1. Further, the average daily asset allocations for the five settlement locations produced by Model 1 appear to have much more persistence than those produced by Model 2.

Similarly, the desired portfolio return for Model 1 was more consistent than Model 2.[15] The disparity between the two models is likely due to the difference in model conditionality; since the desired portfolio return is a function of the inputted estimate for expected returns and Model 1 relies on an unconditional estimate for expected returns, the desired portfolio return for Model 1 is unlikely to make short-term changes.

Interestingly, there was also a distinct difference between whether or not the models traded on a given day. Overall, Model 1 traded on 68% of the simulation days whereas Model 2 traded on 80% of the simulation days. Further, Model 1 had a significant drop off in trading in 2013 (23%) and 2014 (0%).[16] These non-trade days reflect a negative desired portfolio return and must have had a negative μ_{min} and/or a negative μ_{max}. Since Model 1 is based on time-invariant estimates of expected returns from up to the past 365 days, individual observations can have a persistent effect on the estimates. Therefore, sufficient negative values in Model 1 for μ_{min} and/or μ_{max} in 2013 and 2014 may explain the drop off in trading during those years.

The first research objective was to determine whether daily portfolios constructed via portfolio optimization of pre-selected assets from PJMISO can, on average, produce positive returns. Model 2 was successful in doing so with an average daily return of 1.17 or 117%, whereas Model 1 was not successful with an average daily return of -16.23 or -1623%. The sum of return over the six year simulation, equivalent to the total return from investing \$1 each day, for Model 2 was \$2,559.04 compared to -\$35,551.34 for Model 1.[17]

The second research objective was to determine whether the computationally costly extension to the lagged SUR model with FGLS estimates is informative. To answer that objective, this analysis relies on a joint significance test on all the coefficients included in Model 2 that were not included in Model 1. The test statistic was greater than the critical value ($\alpha = 0.05$) for each day during the six year

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[15]See Figures 5 and 4.

[16]See Figure 7.

[17]The geometric mean was unobtainable since some daily returns were less than -1 due to negative pricing and short-selling.

	RMSE	CV(RMSE)
Model 1	0.19	3.68
Model 2	0.30	3.39

Table 1: Error Comparison Between Model 1 and Model 2

simulation. This indicates that the additional independent variables have a jointly significant impact on log returns and non-inclusion of those variables is expected to introduce bias to the remaining coefficients. Individually, the coefficients had mixed values and significance on average across hours.[18]

The third objective was to determine which of the two models produces the desired daily return more accurately. Overall, Model 1 had a lower RMSE than Model 2 while Model 2 had a lower CV(RMSE) than model 1, as detailed in Table 1. This result indicates that, while Model 1 had a tendency to produce portfolio returns that were closer to the desired portfolio return than Model 2, Model 2 had less of a deviation between portfolio return and desired portfolio return per unit of average desired portfolio return.

The fourth objective was to determine what, if any, differences exist between the series of simulated returns from the optimized portfolios produced by each model. This analysis addresses that objective by comparing a myriad of performance metrics.[19] Overall Model 2 outperformed Model 1 across all the chosen metrics. This difference is likely due to the large losses for Model 1 and large gains for Model 2 in 2011. Interestingly 2011 was Model 1's worst year whereas 2011 was Model 2's best year. The time series of return from each model also had different distributional qualities. Although Model 1 suffered significant losses in 2011, it had a better 5% expected shortfall in all other years besides 2009.[20] Due to the extreme values for both models, kernel density estimations on the full series are difficult to analyze. However, when the 1% and 99% tails are removed, the image becomes more clear. Although Model 2 appears to have more upside, Model 1 appears to have less variance.[21]

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[18]See Table 2 and Figures 9 and 10 for numerical and graphical representations of the average coefficient values and significance percentage.

[19]Yearly and overall performance metrics by model can be found in Table 3.

[20]See Table 4 for a comparison of several distributional qualities by year and model.

[21]See Figure 8. Those results are also reflected in Table 4 that shows Model 1 has better 1, 5, and 10 percentile values than Model 2 in almost all cases.

6   Concluding Remarks

This paper demonstrates the implementation of MPT to restructured electricity markets using OLS estimates from an unconditional mean-covariance model and FGLS estimates from a lagged SUR model. The simulation results indicate that Model 2 was more accurate in producing the desired portfolio return after controlling for average desired portfolio return and also induced a more desirable series of returns.

However, the inadmissibility of certain observations due to the use of log returns likely biased the results of the modeling approaches and portfolio optimizations. On average, Model 1 was able to evaluate 120 assets ($N{=}120$) and 268 days ($T{=}268$) whereas Model 2 was able to evaluate 118 assets ($N{=}118$) and 177 days ($T{=}177$), although it is unclear whether those combinations create a sufficient number of observations for asymptotic properties to be satisfied.[22]

This analysis also demonstrates the staggering volatility of restructured electricity markets. When the simulation results from Model 1 and Model 2 are compared to the unmodeled average of the 120 series, it is clear that the modeling and optimization techniques in this analysis actually increase volatility.[23] This increase in volatility is likely due to the ability to short-sell assets since the unmodeled average of the 120 series is equivalent to taking a long position on each asset equal to $\frac{1}{120}$ of total capital.

Another drawback to this approach is operating on the assumption of capital constraints rather than credit constraints. There may exist a market day where all assets are expected to yield negative returns. According to the optimization technique in this analysis, no trading would occur that day as the desired portfolio return would be zero. Further, the constraint that the sum of all asset weights must equal unity requires the investor to take long positions that may be unprofitable. Constraining instead on a fixed credit amount would alleviate both issues. However, this involves forecasting the credit exposure, a function of random variables, that PJMISO will assign to each asset the next market day.

Finally, given the significant downside *and* upside in these markets, taking a Post Modern Portfolio Theory (PMPT) approach and minimizing downside volatility rather than total volatility may prove beneficial.

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[22]See Table 5 to see the average number of assets and days used year and model.
[23]See figure 13.

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Appendix

MatLab Code for Model 2 Estimation and Forecast

```
%% Model 2

    % Establish rt as the dependent variable

    Y = r(:,2:end);

    % Establish rt-2 as the first independent variable

    X1 = r_2(:,2:end);

    % Establish rt-7 as the second independent variable

    X2 = r_7(:,2:end);

    % Create a Tx1 vector of dow indicators

    dow = weekday(r(:,1),'long');

    % Combine, find NaNs, remove them

    comb = [Y X1 X2];

    nonan = ~any(isnan(comb)')';

    Y = Y(nonan,:);

    X1 = X1(nonan,:);

    X2 = X2(nonan,:);

    dow = dow(nonan,:);

    % Reestablish T

    T = size(X1,1);

    % Create a Tx1 vector of ones to represent the constant

    bfone = ones(T,1);

    % Preallocate space

    Reg = cell(1,N);

    % Turn the Tx1 vector of day of week indicators into a Tx7

    % matrix of binaries (categorical variable)

    dow = dummyvar(dow);
```

%Remove Sunday

dow = dow(:,2:end);

% Create N TxK (in this case K = 9) matrices of regressors

for i = 1:N

Reg{i} = [bfone X1(:,i) X2(:,i) dow];

end

% Stack Y into a TNx1 vector

Y = reshape(Y,[],1);

% Make Reg a block diagonal matrix with the N components in

% the diagonal

Reg = blkdiag(Reg{:});

% Invert Reg'*Reg

[U, S, V] = svd(Reg'*Reg);

s= diag(S); g= sum(s¿ 1e-9);

reginv = (U(:, 1: g)* diag(1./ s(1: g))* V(:, 1: g)')';

% Obtain OLS estimates for Beta

beta = reginv*(Reg'*Y);

% Find estimated errors

e_est = (Y-Reg*beta);

% Reshape errors into TxN matrix

e_est = reshape(e_est,[T,N]);

% Obtain the error covariance matrix omega

omega = (1/T)*(e_est'*e_est);

% Invert omega

[U, S, V] = svd(omega);

s= diag(S); g= sum(s¿ 1e-9);

omegainv = (U(:, 1: g)* diag(1./ s(1: g))* V(:, 1: g)')';

% Create an inverted TxT identity matrix

```matlab
I_t_inv = inv(eye(T));

% Find the inverted variance-covariance matrix of the stacked residuals

invSigma = kron(omegainv,I_t_inv);

% Invert Reg' * invSigma * Reg to find coefficient covariance

% matrix

[U, S, V]= svd(Reg' * invSigma * Reg);

s= diag(S); g= sum(s¿ 1e-9);

var_beta = (U(:, 1: g)* diag(1./ s(1: g))* V(:, 1: g)')';

% Calculate the FGLS estimates

beta_gls = var_beta * Reg' * invSigma * Y;

% Find the difference between the t_stat for each estimated

% coefficient and the critical value for a normal

% distribution

t_stat = abs(beta_gls./sqrt(diag(var_beta)))-norminv(0.95,0,1);

% Mark with a 1 if it is positive, 0 otherwise

t_stat = t_stat>=0;

% Report in a daily table

daily_tstat(i3+1,1:K*N) = t_stat;

% Determine the percentage rejection and report in a

% daily table

daily_sig(i3+1,1:K) = mean(reshape(t_stat,K,N),2)';

% Create a KNxKN matrix with ones in the diagonal

R = ones(K*N,1);

R = diag(R);

% Remove the rows that represent the constant term (the K =

% 1 component of each i-¿N group)

R(1:K:K*N,:)=[];

% Create a (K-1)Nx1 vector of zeros for hypothesis
```

```
g_ =zeros((K-1)*N,1);

% Take several steps to calculate the t_stat for the joint

% significant test

[U, S, V]= svd(R*var_beta*R');

s= diag(S); g= sum(s¿ 1e-9);

aux_= (U(:, 1: g)* diag(1./ s(1: g))* V(:, 1: g)')';

aux_ = aux_*(R*beta_gls-g_);

% Finish calculation and compare the absolute value to the

% critical value from a chi-squared distribution with

% (K-1)*N restrictions

t_statj=abs(T*(R*beta_gls-g_)'*aux_)-chi2inv(0.95,(K-1)*N);

% Mark with a 1 if it is positive, 0 otherwise

t_statj = t_statj>=0;

% Report in a daily table

daily_tstat(i3+1,end) = t_statj;

% Report again for consistency with individual t-tests

daily_sig(i3+1,end) = t_statj;

% Rename beta for consistency

beta = beta_gls;

% Report betas in a daily table

daily_beta(i3+1,:) = beta';

% Create a scalar 1 to represent the constant term

bfone = 1;

% Preallocate space

Reg = cell(1,N);

% Create a 1x6 vector of DOW indicators for the market day

% (absent sunday)

dow_t = zeros(1,7);
```

```
dow_t(weekday(ret_t(:,1),'long')) = 1;

dow_t = dow_t(2:end);

% Create N TxK (in this case K = 9) matrices of regressors

% for the market day

for i = 1:N

Reg{i} = [bfone r_2_t(i+1) r_7_t(i+1) dow_t];

end

% Make Reg a block diagonal matrix with the N components in

% the diagonal

Reg = blkdiag(Reg{:});

% Determine if any of the rows have missing values

admis = ~any(isnan(Reg)')';

% Determine the row number of admissible rows

admis_locat = find(admis==1);

% Remove rows and columns from omega and omega inverse that were inadmissible

omega = omega(admis,admis);

omegainv = omegainv(admis,admis);

% Preallocate space

beta_admis = cell(N,1);

% Make a TNx1 vector of logicals for admissible betas

for i = 1:N

beta_admis{i} = repmat(admis(i),K,1);

end

beta_admis = cell2mat(beta_admis);

% Remove inadmissible rows from beta

beta = beta(beta_admis,:);

% Remove inadmissible rows and columns from Reg

Reg = Reg(admis,beta_admis);
```

% Obtain the expected value of rt market day

mu_ = Reg*beta;

% Establish new N

N = length(mu_)

MatLab Code for Optimization

%% Optimization

  % Decomposition of algebraic solution to optimization

  one = ones(N,1);

  A=one'*omegainv*mu_;

  B=mu_'*omegainv*mu_;

  C=one'*omegainv*one;

  D=B*C-(A.^2);

  g=(B*omegainv*one-A*omegainv*mu_)/(D);

  h=(C*omegainv*mu_-A*omegainv*one)/(D);

  hh = h'*omega*h;

  gh = g'*omega*h;

  mumin = -gh/hh;

  % Establish desired portfolio return as the average between the

  % maximum possible return and the return on the minimum

  % variance portfolio

  retp = (nanmax(mu_)+mumin)/2;

  % Establish protocol which makes w an Nx1 vector of zeros if

  % the desired portfolio return happens to be negative,

  % otherwise calculate w based on the solution to the

  % optimization

  if retp¿0

  w=g+retp*h;

```
            notrade(i3+1,1) = 0;

        else

            retp = NaN;

            w = zeros(N,1);

            notrade(i3+1,1) = 1;

        end
```

Figures
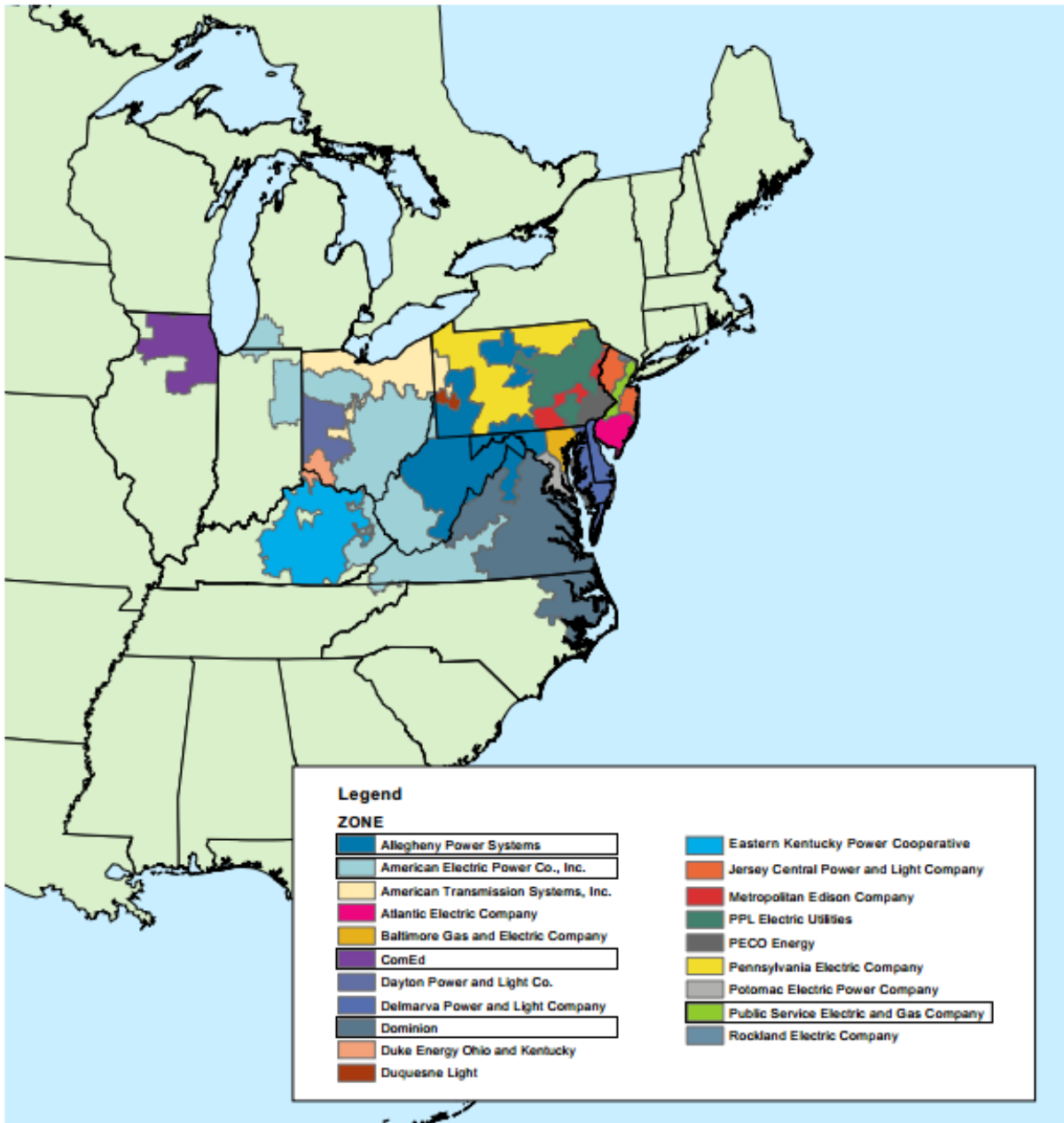
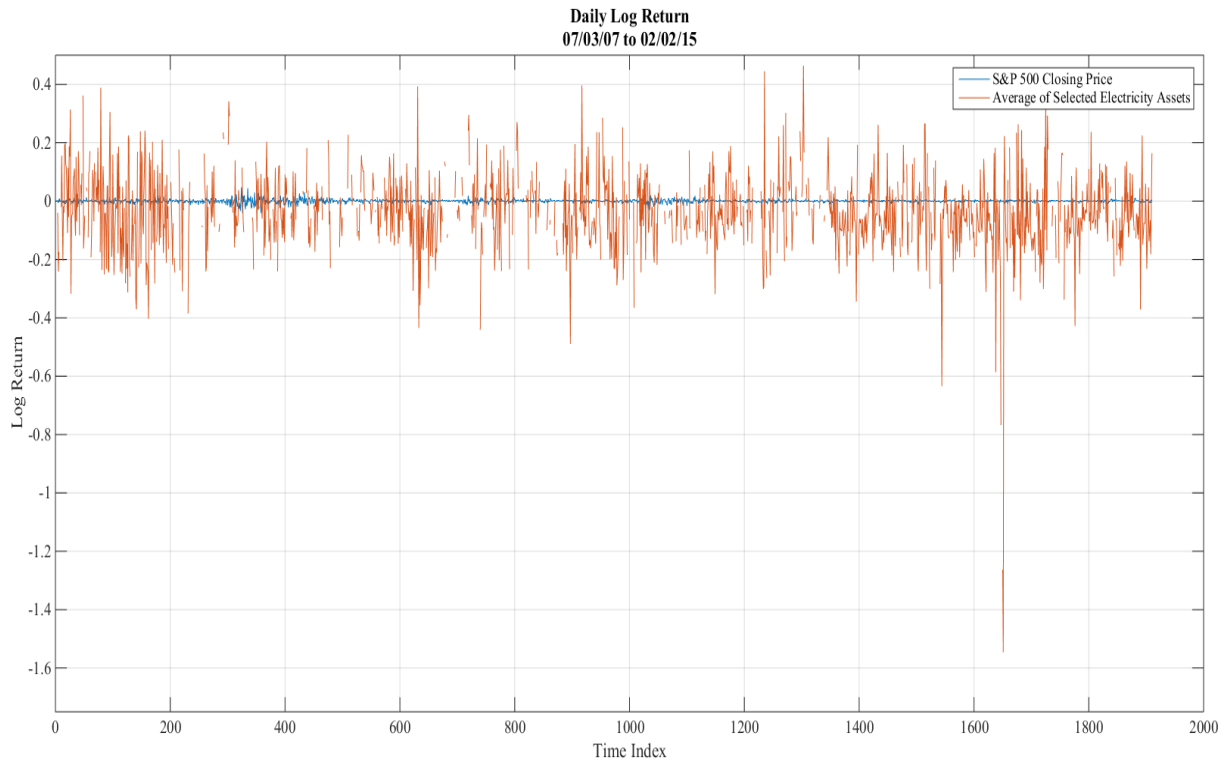Figure 1: Map of the load zones in PJMISO

Figure 2: Daily Log Return for the S&P 500 Closing Price and Average of the Assets in this Analysis
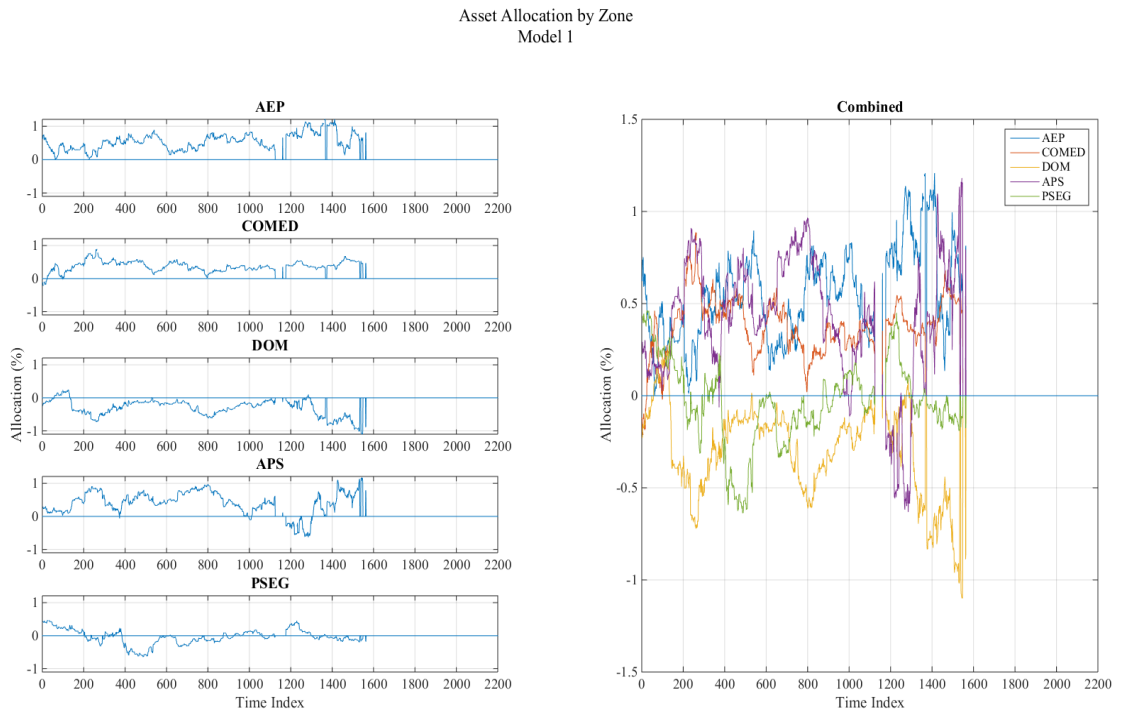

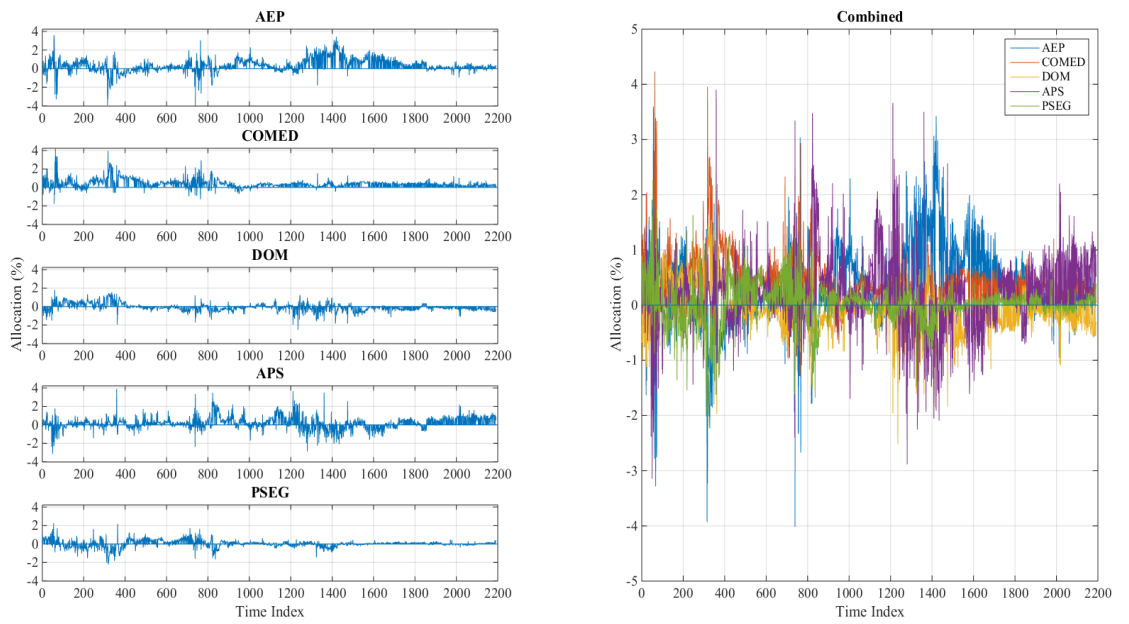Figure 3: Percentage Asset Allocation by Zone, Model 1 (gaps imply the simulation did not trade that day)

Figure 4: Percentage Asset Allocation by Zone, Model 2 (gaps imply the simulation did not trade that day)
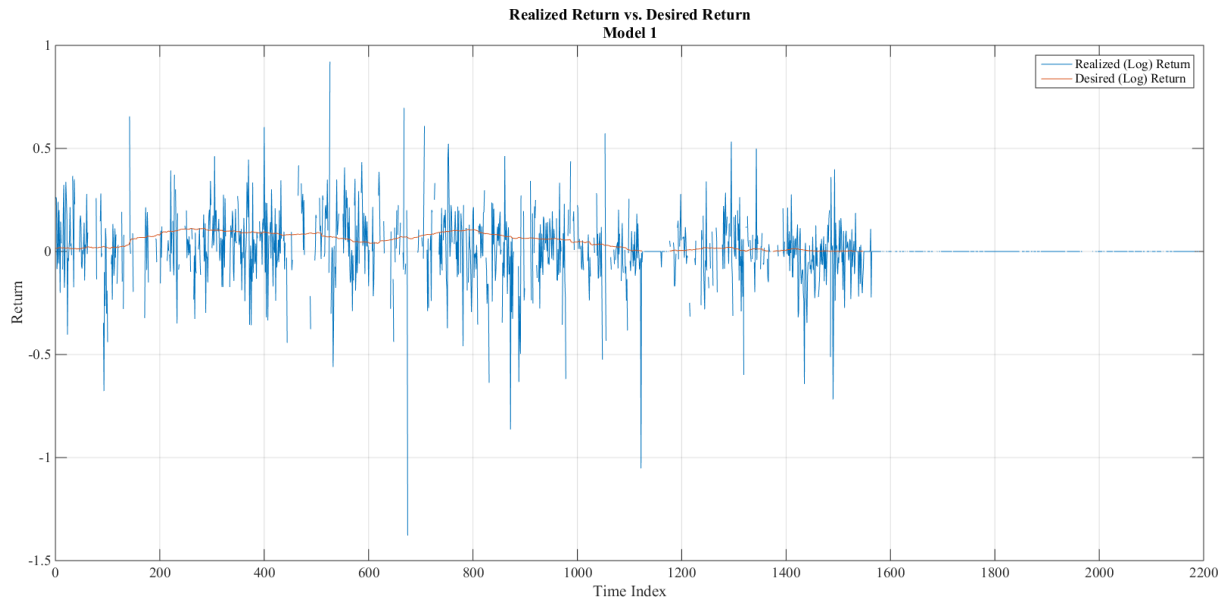
Figure 5: Realized Return vs. Desired Return, Model 1 (gaps imply the simulation did not trade that day)
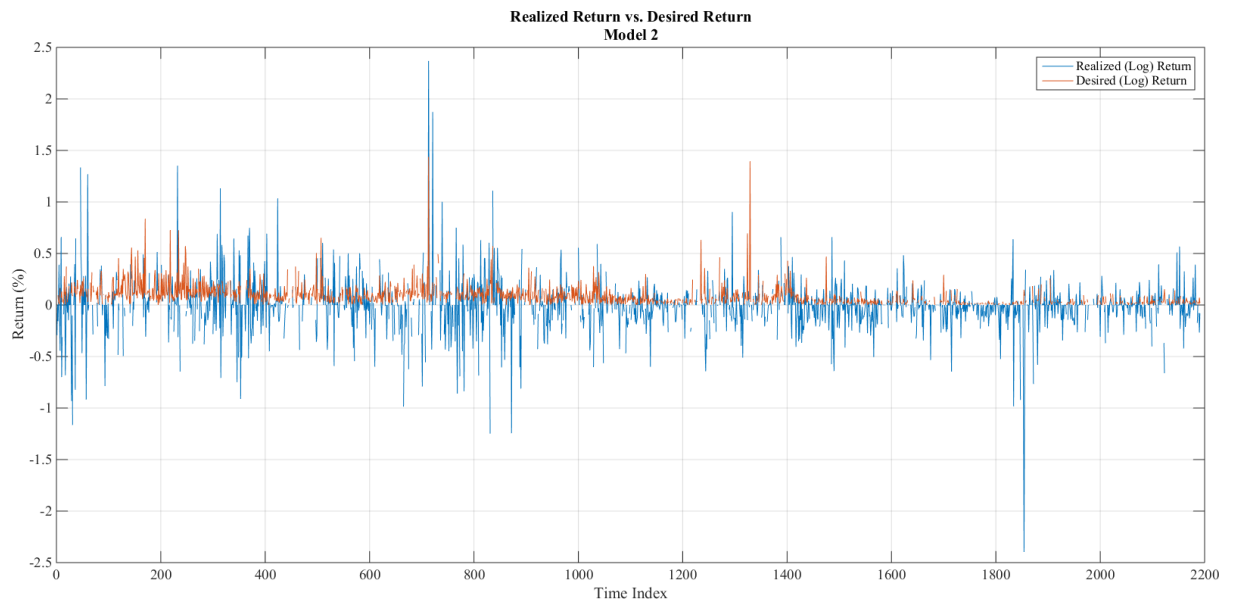
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Figure 6: Realized Return vs. Desired Return, Model 2 (gaps imply the simulation did not trade that day)
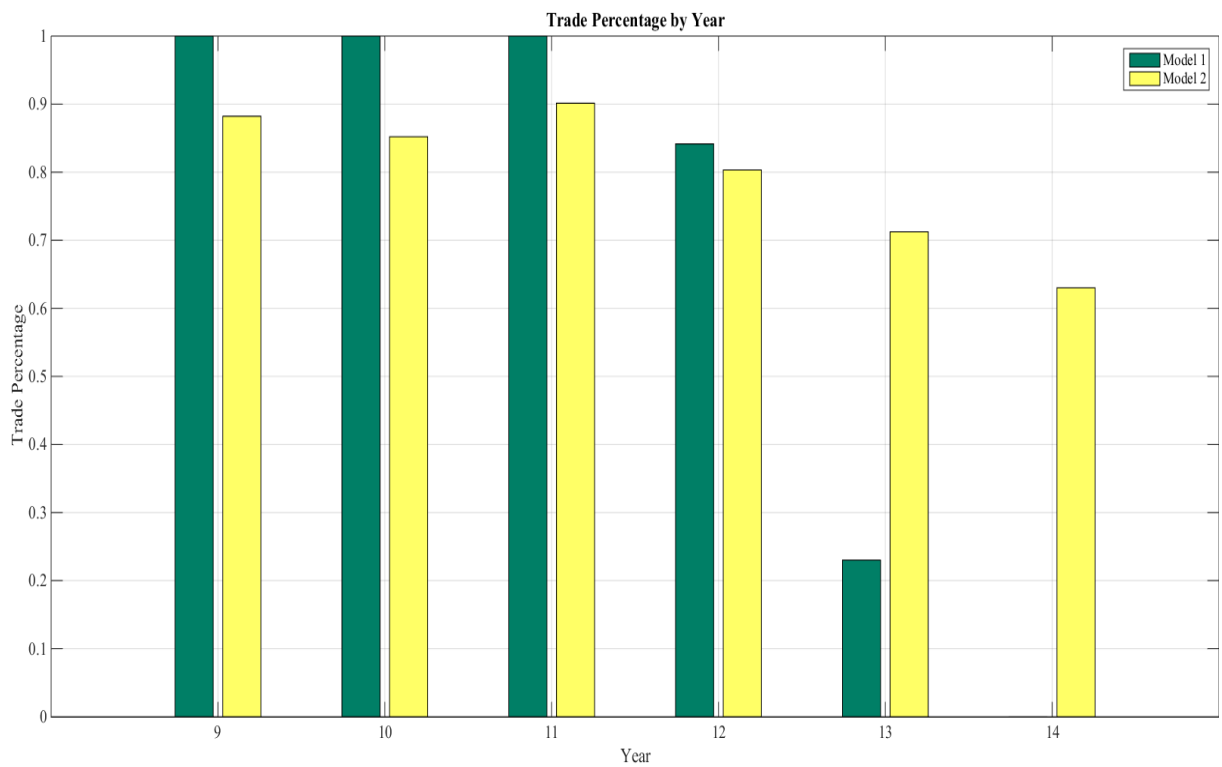

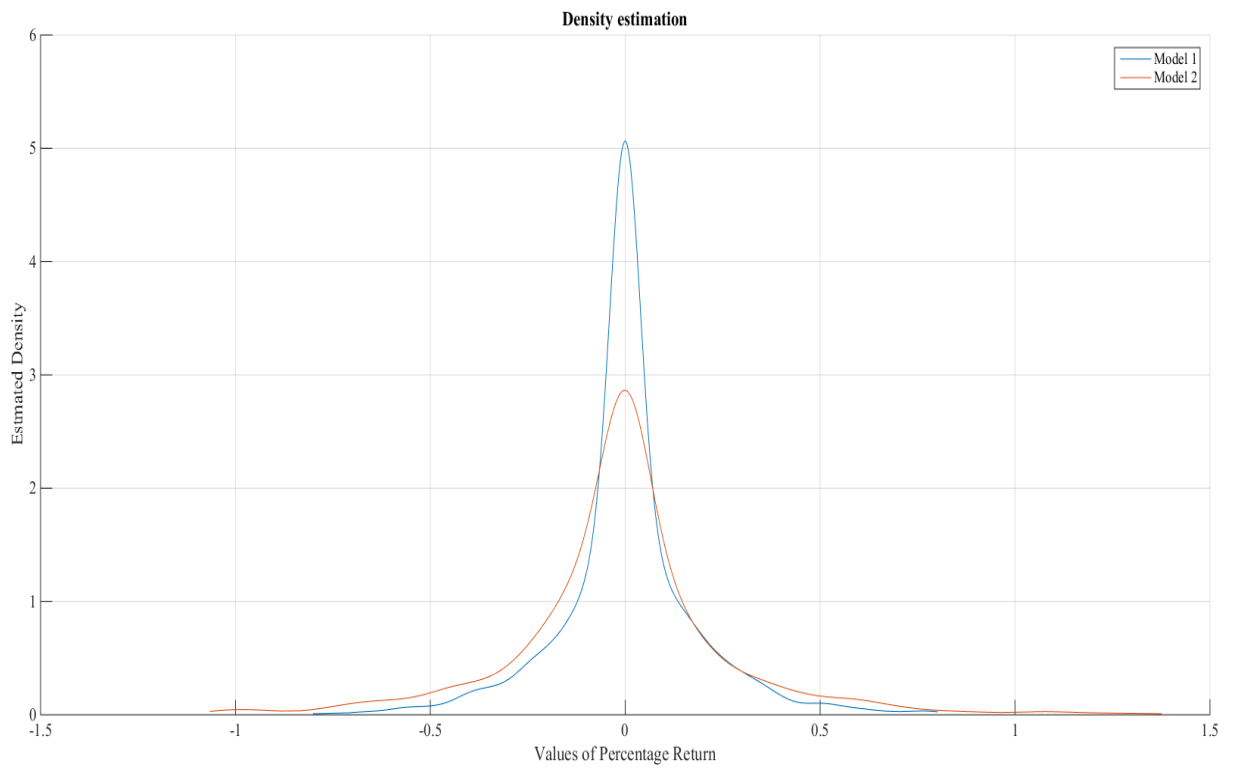Figure 7: Trade Percentage by Year for Model 1 and Model 2

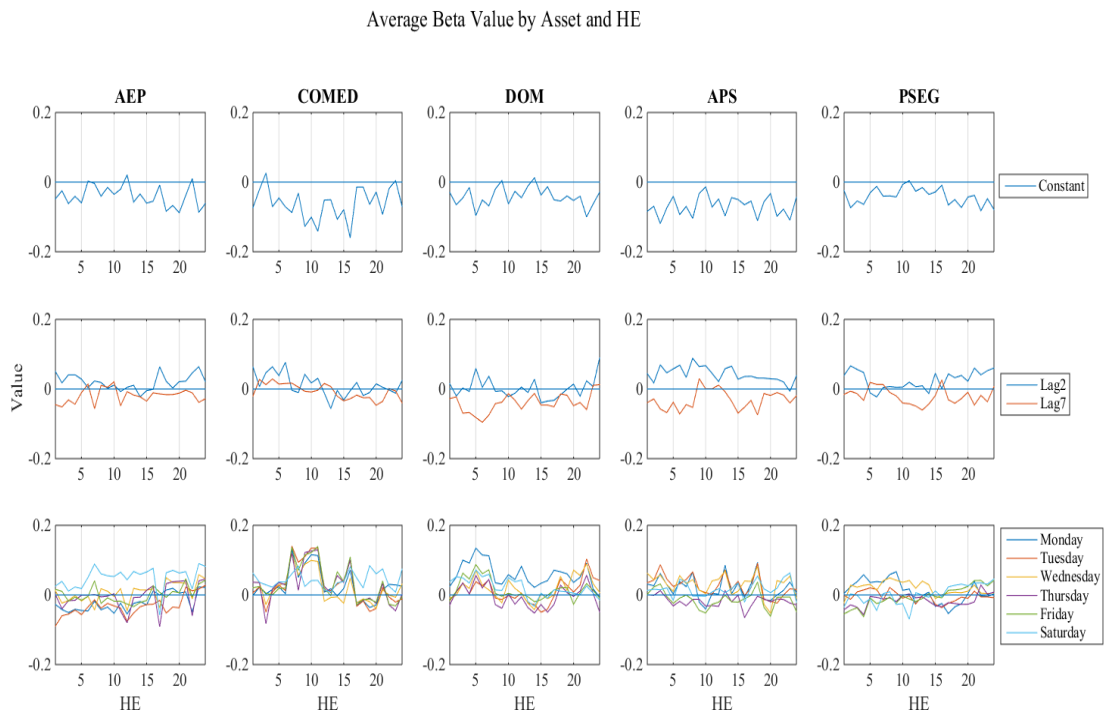Figure 8: Kernel Density Estimation for Model 1 and Model 2, 1% Tails Removed

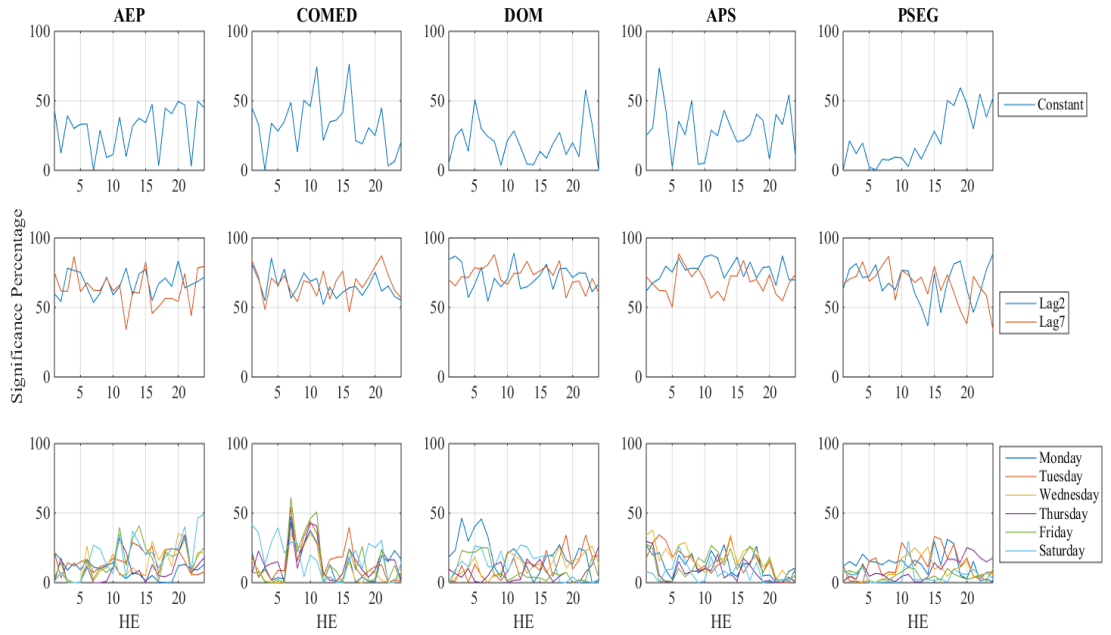Figure 9: Average Coefficient Values for Model 2 by Settlement Location and HE

Figure 10: Average Coefficient Significance ($\alpha = 0.05$) for Model 2 by Settlement Location and HE
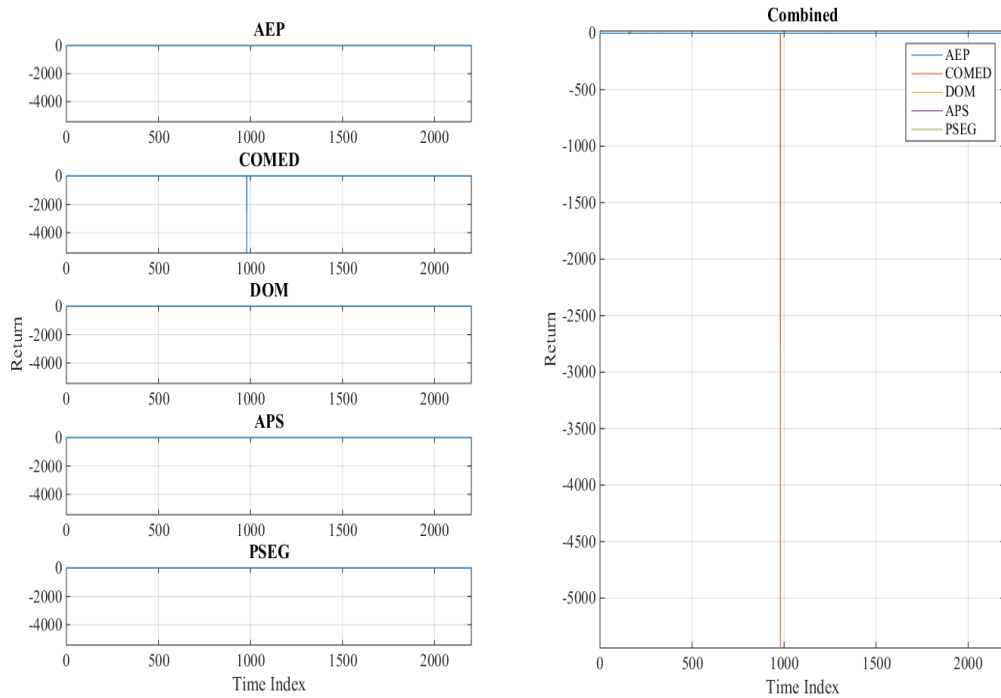

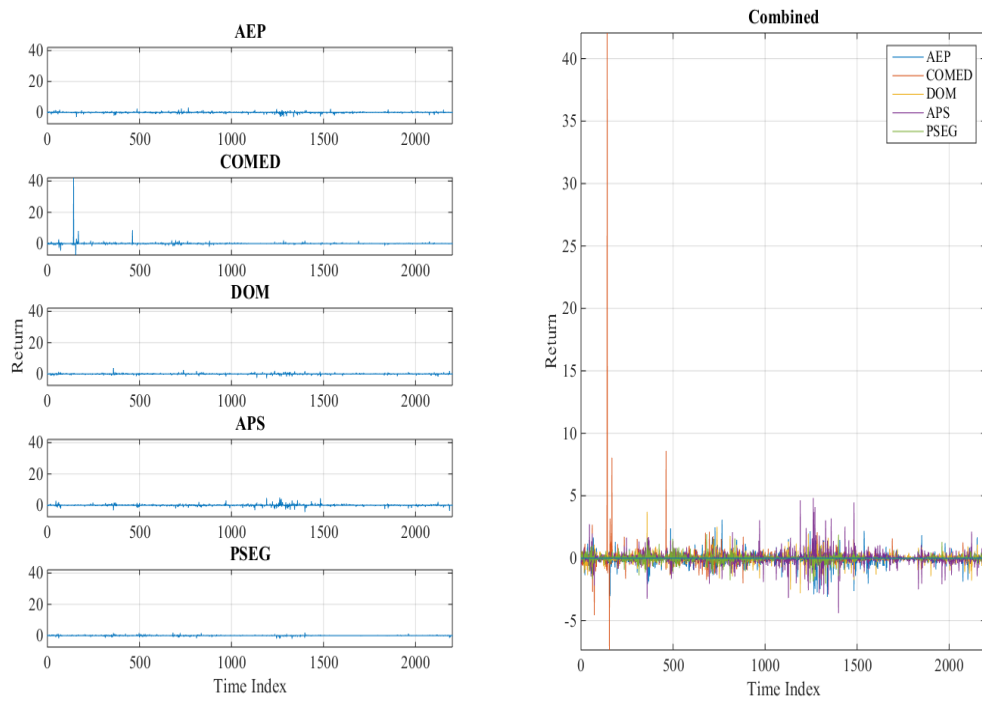Figure 11: Daily Return for Model 1 by Zone (gaps imply the simulation did not trade that day)

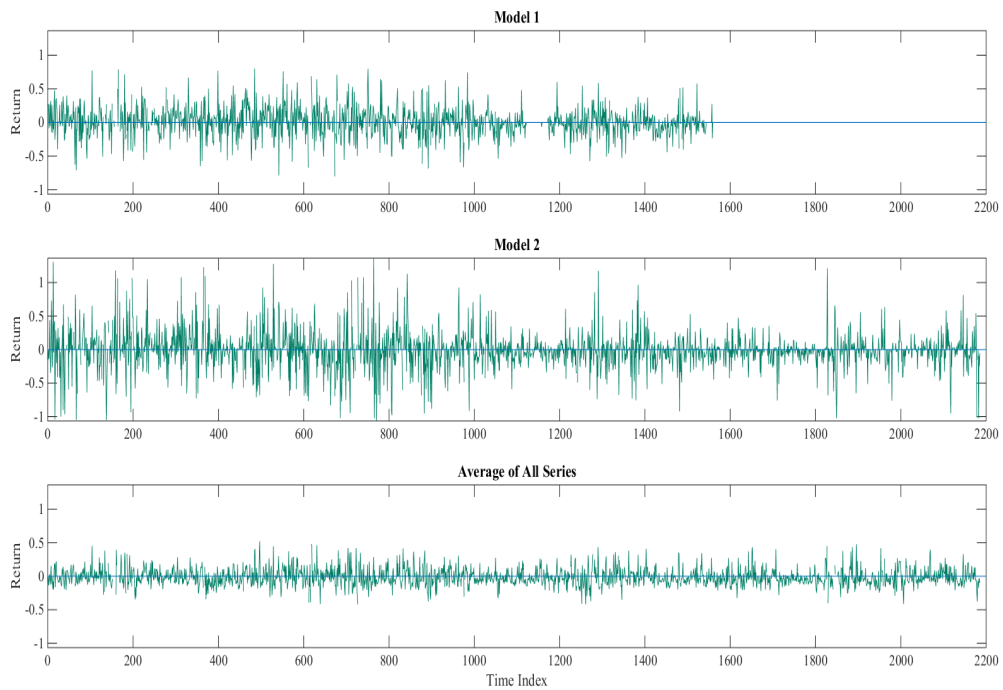Figure 12: Daily Return for Model 2 by Zone (gaps imply the simulation did not trade that day)

Figure 13: Daily Return for Model 1, Model 2 and the Average of the All Series, 1% Tails Removed

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Tables

	Constant	Lag 2	Lag 7	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
AEP	-0.040	0.020	-0.020	-0.021	-0.032	0.007	-0.004	0.001	0.050
	30.16%	67.51%	63.18%	9.23%	15.71%	12.93%	9.71%	15.88%	19.11%
COMED	-0.062	0.012	-0.007	0.028	0.029	0.022	0.029	0.034	0.044
	32.95%	65.55%	67.29%	13.19%	15.99%	9.47%	15.44%	14.79%	17.95%
DOM	-0.042	0.002	-0.040	0.058	0.011	0.019	-0.004	0.015	0.021
	19.96%	71.91%	72.61%	20.01%	11.8%	12.6%	7.48%	8.66%	12.22%
APS	-0.069	0.042	-0.031	0.020	0.027	0.030	-0.017	-0.013	0.017
	29.79%	76.93%	68.05%	12.94%	15.43%	17.89%	8.13%	12.74%	7.32%
PSEG	-0.041	0.026	-0.020	0.010	-0.004	0.019	-0.016	-0.005	0.002
	23.35%	67.85%	66.54%	13.54%	13.44%	9.03%	9.55%	5.3%	5.09%

Table 2: Average Coefficient Values and Significance Percentage by Zone ($\alpha = 0.05$)

	Year	Return (Average)	Return (Sum)	Desired Return (Average)	Return (Std. Dev.)	Sharpe Ratio
	9	0.190	69.300	0.064	2.623	0.072
	10	0.036	13.148	0.072	0.340	0.106
	11	-97.608	-35626.762	0.065	1604.572	-0.061
Model 1	12	-0.012	-4.290	0.009	0.189	-0.062
	13	-0.008	-2.755	0.002	0.104	-0.073
	14	0.000	0.000	0.000	0.000	0.000
	Overall	-16.226	-35551.357	0.051	655.178	-0.025
	9	0.148	54.083	0.148	2.333	0.064
	10	0.042	15.311	0.108	0.664	0.063
	11	6.911	2522.522	0.099	318.687	0.022
Model 2	12	-0.034	-12.590	0.075	0.291	-0.118
	13	-0.027	-9.762	0.033	0.182	-0.147
	14	-0.029	-10.515	0.036	0.276	-0.104
	Overall	1.168	2559.043	0.088	129.955	0.009

Table 3: Performance Metrics for Model 1 and Model 2 by Year

	Year	1st Percentile	5th Percentile	10th Percentile	Expected Shortfall (5%)
Model 1	9	-3.37	-0.51	-0.26	-2.31
	10	-0.83	-0.41	-0.32	-0.75
	11	-1.01	-0.39	-0.28	-1979.66
	12	-0.50	-0.28	-0.20	-0.44
	13	-0.38	-0.12	-0.03	-0.30
	14	0.00	0.00	0.00	0.00
	Overall	-0.63	-0.23	-0.14	-76.01
Model 2	9	-1.60	-0.71	-0.43	-1.71
	10	-1.15	-0.63	-0.46	-0.91
	11	-1.09	-0.62	-0.42	-159.14
	12	-1.21	-0.41	-0.27	-0.78
	13	-0.51	-0.28	-0.21	-0.45
	14	-1.00	-0.41	-0.25	-0.80
	Overall	-1.07	-0.52	-0.33	-26.85

Table 4: Distributional Qualities of Model 1 and Model 2 Returns by Year

	Year	N (Average)	T (Average)
Model 1	9	120.00	224.52
	10	120.00	255.30
	11	120.00	252.25
	12	120.00	269.23
	13	120.00	300.94
	14	120.00	317.61
	Overall	120.00	269.97
Model 2	9	117.33	114.43
	10	117.73	154.97
	11	118.19	144.03
	12	118.08	164.63
	13	119.55	224.17
	14	119.02	261.23
	Overall	118.32	177.24

Table 5: Average Observations and Trade Percentages for Model 1 and Model 2 by Year