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A Perturbational Solution Methodology for Solving Integral Equations of Microstrip Circuit Discontinuities

by

John C. Moore

Department of Electrical and Computer Engineering
University of Colorado
Boulder, Colorado 80309-0425

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This thesis presents a generalized technique for solving large operator equation problems by first constructing a zeroth order solution and correcting that solution with a perturbational functional via the adjoint operator. The hope of this approach is that the computationally expensive process of inverting the original operator can be avoided when constructing a zeroth order solution and that the perturbational correction will provide sufficient accuracy in the final result.

Specifically, this perturbational approach is applied to the problem of finding S-parameter characterizations for general planar microstrip circuits. The operator equation is a Mixed Potential Integral Equation for the current distribution on the circuit metalization. Three different zeroth order models to this MPIE were developed. These models ignore parasitic coupling mechanisms so that the perturbational correction provides a means for efficiently accounting for the parasitic coupling effects within microstrip circuits.

The implementation of the perturbational analysis provides substantial computational savings over the exact MPIE analysis for large circuits. The perturbational analysis also provides accurate results for circuits for which parasitic coupling effects are no more than moderately important in the circuit performance.
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CHAPTER I

Introduction

1.1 Research Goals

The goal of this research is to produce a very general approach to efficiently analyze parasitic coupling effects within MMIC circuits. This becomes increasingly important as the MMIC technology base grows and MMIC circuits become more complicated and operate at higher frequencies. Currently, there are two basic approaches to modeling MMIC structures within a CAD environment. There are extremely efficient models (often based on network theory) that generally ignore parasitic coupling mechanisms or only crudely model these effects which become increasingly important at higher frequencies and for more densely constructed MMICs. There are also extremely accurate full wave analysis models that become prohibitively inefficient for larger and more complex MMIC structures. For many circuits, the parasitic coupling effects are small and using a full wave analysis approach throws too much effort to model these small effects. Some work has been attempted to include coupling models into network theory based algorithms [22] with good success. However, this thesis will approach the problem from the other side; how can full wave analysis algorithms be made more efficient when modeling parasitic coupling effects?

Very often, full wave analysis algorithms reduce to the problem of solving a set of linear equations. This system of equations is usually very large and/or very dense so that solving this system becomes extremely expensive for large and complex circuits. The approach of this thesis will be to construct an
approximate solution to this system of equations and then correct these results using a perturbational analysis. The approximate solution will use the power of the full wave analysis to model the more significant coupling mechanisms, while the perturbational analysis will account for the small parasitic coupling effects. The time savings of the perturbational approach will be the result of replacing the matrix inversion process of the full wave analysis with a more efficient approximate matrix inversion and a few additional matrix multiplications to realize the perturbational correction.

The full wave analysis algorithm to which we will apply this perturbational formulation is the PMESH model for solving single layer microstrip circuits ([29]-[30]). This choice was based on the access to the analysis and support software as well as the intimate understanding of the inner workings of the PMESH model that exist here at the University of Colorado at Boulder. However, the basic approach and results of the perturbational formulation are not unique to the PMESH model and can be implemented (with appropriate modifications) to a wide range of full wave analysis packages.

1.2 Thesis Format

The remainder of this thesis consist of eight chapters. Chapter 2 presents the Perturbational Solution Methodology in its general form. Chapter 3 presents a brief overview of the PMESH model. Further details of the PMESH implementation are given in the Appendicies.

Chapter 4 represents a slight departure from the Perturbational Solution Methodology. This chapter presents the Equivalent Circuit De-embedding technique as an alternative to the Three Point De-embedding scheme employed in the unmodified PMESH model. Chapter 5 returns to the Perturbational Solution Methodology by describing in a little more detail how the Perturbational
Solution Methodology applies to both the Equivalent Circuit De-embedding and the Three Point De-embedding techniques.

Chapters 6 through 8 present three possible means of implementing the Perturbational Solution Methodology by providing three different models for constructing zeroth order (approximate) solutions to the PMESH model. The zeroth order solution of the Isolated Subcircuit Method presented in Chapter 6 ignores the coupling between distinct subcircuits within a given circuit. The Network Connection Method of Chapter 7 produces a zeroth order model that is very analogous to network theory models. Chapter 8 presents the Circle of Influence Method which constructs a zeroth order model by ignoring couplings between sufficiently distant current basis functions.

Finally, Chapter 9 attempts to reiterate the successes and limitations of the research conducted, as well as provide direction for extensions on the work done in this thesis.

It should be pointed out that throughout this thesis, the perturbationally corrected results are compared to the unmodified PMESH results. The Perturbational Solution Methodology is inherently an approximate technique. Therefore, the results can be only as accurate as the unmodified full wave analysis results. Any improvement in accuracy due to the Perturbational Solution Methodology is purely coincidental. The PMESH model has shown itself to produce accurate results when it is applied appropriately. However, the burden of proof of this statement lies elsewhere [30].
CHAPTER II

The Perturbational Solution Methodology

This chapter develops a general methodology for approximately solving problems that can be cast in terms of linear functionals and linear operators. The merits and pitfalls of this methodology are discussed and an attempt is made to describe the types of problems for which this methodology is appropriate. In later chapters, this methodology will be applied to the analysis of large microstrip circuits.

2.1 Linear Operators and Functionals

Let us start by considering the linear operator equation

\[ f = \mathcal{L}\phi, \]  

(2.1)

where we take \( \phi \) to be the unknown function of interest, \( f \) is a known function that will be referred to as the forcing function, and \( \mathcal{L} \) is a linear operator which operates on the function \( \phi \). The linearity of \( \mathcal{L} \) means that \( \mathcal{L} \) satisfies the property

\[ \mathcal{L}(\alpha_1\phi_1 + \alpha_2\phi_2) = \alpha_1\mathcal{L}\phi_1 + \alpha_2\mathcal{L}\phi_2 \quad \forall \text{ functions } \phi_1, \phi_2 \text{ and scalars } \alpha_1, \alpha_2. \]  

(2.2)

Any linear problem can be cast in this form. For example, \( \mathcal{L} \) can be the operator \((\nabla^2 + k^2)\) in which case (2.1) becomes Helmholtz's equation, \( \nabla^2\phi + k^2\phi = f \).

In a large class of problems, solving for \( \phi \) is merely an intermediate step in the entire solution process. First, the function \( \phi \) is found and the ultimate scalar parameter of interest is then extracted from \( \phi \). Often, the parameter of
interest can be expressed in terms of linear functionals. Let us define a general linear functional $F_s(\phi)$ to be the inner product of two functions $\phi$ and $g$ as

$$F_s(\phi) = \langle g, \phi \rangle = \int g \cdot \phi \, ds.$$  \hspace{1cm} (2.3)

The integration in (2.3) is performed over the entire domain of the functions $g$ and $\phi$. The functional $F_s(\phi)$ describes some salient feature of the function $\phi$. An appropriate choice of $g$ allows us to describe a variety of different properties of $\phi$. For instance, if $g = \phi$, then $\sqrt{F_s(\phi)} = \sqrt{\langle \phi, \phi \rangle} = \| \phi \|_2$ is the $\ell_2$ norm of $\phi$.

There is an important distinction between linear operators and linear functionals. The result of the operator $L$ operating on a function $\phi$ is itself another function $f = L\phi$. The result of the functional $F_s$ operating on a function $\phi$ is a scalar $F_s(\phi)$.

2.2 Perturbational Expressions

In (2.1), $\phi$ is the unknown function of interest. Suppose we were to approximate $\phi$ by $\phi_0$, where the two functions differ by a 'small' function $\phi_s$ where $\phi = \phi_0 + \phi_s$. We will call $\phi_0$ the zeroth order approximation to $\phi$ and $\phi_s$ the first order correction function. The function $\phi_0$ is taken to be the solution to a zeroth order linear operator equation $f = L_0\phi_0$, where $L_0$ is a zeroth order approximation to $L$. There is also a first order correction operator, $L_1$, where $L = L_0 + L_1$. We could also expand the forcing function $f$ into zeroth and first order terms. However, we will be considering physical systems that are excited by a known function $f$ that will be the same in the zeroth order system as the exact system.

---

1 The size of a function is defined by some norm, say the $\ell_2$ norm. By small, we mean $\| \phi_s \| \ll \| \phi_0 \| \sim \| \phi \|$. 


Starting with (2.1) and expanding the operator $\mathcal{L}$ and the unknown function $\phi$ into zeroth and first order terms yields

$$f = \mathcal{L}\phi$$

$$= (\mathcal{L}_0 + \mathcal{L}_s)(\phi_0 + \phi_s)$$

$$= \mathcal{L}_0\phi_0 + \mathcal{L}_0\phi_s + \mathcal{L}_s\phi_0 + \mathcal{L}_s\phi_s$$

(2.4)

However, $f = \mathcal{L}_0\phi_0$, so that

$$\mathcal{L}_0\phi_s = -\mathcal{L}_s\phi_0 - \mathcal{L}_s\phi_s.$$  

(2.5)

Now let $\psi_0$ be the solution to the zeroth order adjoint operator equation

$$g = \mathcal{L}_0^*\psi_0$$  

(2.6)

where $g$ is the function that defines the functional of interest, $\mathcal{F}_s(\phi)$, and $\mathcal{L}_0^*$ is the adjoint operator associated with $\mathcal{L}_0$. The adjoint pair of operators $\mathcal{L}_0^*$ and $\mathcal{L}_0$ satisfy the condition

$$\langle \mathcal{L}_0^*\phi_1, \phi_2 \rangle = \langle \phi_1, \mathcal{L}_0\phi_2 \rangle \quad \forall \text{ functions } \phi_1, \phi_2.$$  

(2.7)

Now we can expand $\mathcal{F}_s(\phi)$ into zeroth and first order terms to get

$$\mathcal{F}_s(\phi) = \langle g, \phi \rangle = \langle g, \phi_0 + \phi_s \rangle = \langle g, \phi_0 \rangle + \langle g, \phi_s \rangle$$

$$= \mathcal{F}_s(\phi_0) + \langle g, \phi_s \rangle$$

(2.8)

From (2.6) and (2.7) we get

$$\mathcal{F}_s(\phi) = \mathcal{F}_s(\phi_0) + \langle \mathcal{L}_0^*\psi_0, \phi_s \rangle$$

$$= \mathcal{F}_s(\phi_0) + \langle \psi_0, \mathcal{L}_0\phi_s \rangle$$

(2.9)

Equation (2.5) gives

$$\mathcal{F}_s(\phi) = \mathcal{F}_s(\phi_0) - \langle \psi_0, \mathcal{L}_s\phi_0 + \mathcal{L}_s\phi_s \rangle$$

$$= \mathcal{F}_s(\phi_0) - \langle \psi_0, \mathcal{L}_s\phi_0 \rangle - \langle \psi_0, \mathcal{L}_s\phi_s \rangle$$

(2.10)

The definition of $\mathcal{L}_s = \mathcal{L} - \mathcal{L}_0$ gives

$$\mathcal{F}_s(\phi) = \mathcal{F}_s(\phi_0) - \langle \psi_0, [\mathcal{L} - \mathcal{L}_0]\phi_0 \rangle - \langle \psi_0, \mathcal{L}_s\phi_s \rangle$$

$$= \mathcal{F}_s(\phi_0) - \langle \psi_0, \mathcal{L}\phi_0 \rangle + \langle \psi_0, \mathcal{L}_0\phi_s \rangle - \langle \psi_0, \mathcal{L}_s\phi_s \rangle$$

(2.11)
Once again we use (2.7) and (2.6)
\[\mathcal{F}_s(\phi) = \mathcal{F}_s(\phi_0) - (\psi_0, L\phi_0) + (L_0^1 \psi_0, \phi_0) - (\psi_0, L_s \phi_s) \]
\[= \mathcal{F}_s(\phi_0) - (\psi_0, L\phi_0) + (g, \phi_0) - (\psi_0, L_s \phi_s) \]
\[= 2\mathcal{F}_s(\phi_0) - (\psi_0, L\phi_0) - (\psi_0, L_s \phi_s) \]
\[
(2.12)
\]
Since \(-(\psi_0, L_s \phi_s)\) involves the product of two first order correction terms, it is considered to be a second order correction to \(\mathcal{F}_s(\phi)\) and we have
\[\mathcal{F}_s(\phi) \approx 2\mathcal{F}_s(\phi_0) - (\psi_0, L\phi_0) \]
\[= 2\mathcal{F}_s(\phi_0) - \mathcal{F}_{\psi_0}(L\phi_0) \]
\[
(2.13)
\]
Equation (2.13) is correct to first order. Notice that in order to calculate (2.13), we only need to know \(g, L\) and the solutions to the adjoint pair of zeroth order equations,
\[f = L_0 \phi_0 \quad g = L_0^1 \psi_0. \]
\[
(2.14)
\]
Equation (2.13) has the property of being variational [1]. That is, (2.13) contains terms that are accurate only to zeroth order, but is itself accurate to first order. Implicit in this derivation is the assumption that the first order errors are small so that the second order term, \(-(\psi_0, L_s \phi_s)\), is even smaller and can be neglected. However, if either \(L_s\) or \(\phi_s\) is not small, the second order term might itself be large and it should not be neglected.

### 2.3 Perturbational Solution Methodology

The preceding development is the basis for a methodology for finding approximate solutions to problems that can be formulated as:

Find \(\mathcal{F}_s(\phi)\) where \(L\phi = f\).

The basic steps in this methodology are

**Step 1:** Formulate the problem.

**Step 2:** Identify \(L_0\) and \(L_0^1\).

**Step 3:** Solve equation (2.14) for \(\phi_0\) and \(\psi_0\).

**Step 4:** Use equation (2.13) to approximate \(\mathcal{F}_s(\phi)\).
Notice that Step 2 and Step 3 are closely related. If $\phi_0$ and $\psi_0$ can be found without identifying $L_0$ and $L_0'$ (say by inspection), then Step 2 can be skipped. We will refer to the combination of Step 2 and Step 3 as finding the approximate inverse of $L$. If the problem can be solved exactly, then these two steps would be replaced by the single step of solving equation (2.1), which involves finding (explicitly or otherwise) $L^{-1}$, the actual inverse of $L$.

This solution methodology will be most appropriate for problems in which finding the approximate inverse of $L$ is easier and/or faster than finding the actual inverse, $L^{-1}$. It is hoped that the use of the variational expression of Step 4 will recover enough accuracy so that, combined with the time and/or effort savings of finding the approximate inverse, the entire process provides an attractive alternative to finding an exact solution.

One such class of problems would be those in which $L^{-1}$ cannot be found. For instance, $L$ might be some complicated integral-differential operator for which there exists no analytic expression for $L^{-1}$. As long as a good enough guess for $\phi_0$ and $\psi_0$ can be found, this methodology provides a means for finding an analytic approximate solution to the problem. Another general class of problems would be those for which $L^{-1}$ can be found, (perhaps by some numerical method), but is very computationally expensive to do so. If a approximate inverse can be found that is less computationally involved, the solution method may provide a more efficient alternative to solving the problem. A third class of problems where this methodology could shine is in sensitivity analysis problems. Sensitivity analysis involves solving a set of equations $\{f = L_i \phi_i\}$ for the set of functions $\{\phi_i\}$. The operators $\{L_i\}$ are all found by changing some parameter(s) slightly from $L_0$, the operator of the ‘central’ problem of the sensitivity analysis. Instead of computing all the $\{L_i^{-1}\}$, only $\phi_0$ and $\psi_0$ need to be found exactly and the perturbational methodology can be applied with $L = L_i$. This thesis will
primarily address problems in the second class, complicated operator equations that are discretized and solved numerically. In particular, this thesis will focus on solving a Mixed Potential Integral Equation for the current distribution on microstrip circuits. Since we will be dealing with discretized systems, it is most natural to consider the linear operator equation and linear functional in the discretized domain.

2.4 Discretized Problems

One common approach towards solving complex problems is to discretize the problem and find a numerical solution on a computer. Such discretization converts the linear operator equation of (2.1) into the matrix equation

\[ [f] = [L][\phi], \quad (2.15) \]

where \([f]\) and \([\phi]\) are vectors and \([L]\) is a matrix. The linear functional of (2.3) also becomes

\[ \mathcal{F}_s(\phi) = \langle [g], [\phi] \rangle = [g]'[\phi]. \quad (2.16) \]

The corresponding transformations for equations (2.13) and (2.14) are

\[ \mathcal{F}_s(\phi) \approx 2[g]'[\phi_0] - [\psi_0]'[L][\phi_0]. \quad (2.17) \]

and

\[ [f] = [L_0][\phi_0] \quad [g] = [L_0]'[\psi_0] \quad (2.18) \]

The ‘t’ in equations (2.16), (2.17), and (2.18) denotes the vector and matrix transpose.

If the matrix \([L]\) is square and non-singular, then (2.15) can always be solved by finding \([L]^{-1}\) or some equivalent decomposition. If \([L]\) is dense, that is all or nearly all the elements \(L_{i,k}\) of \([L]\) are non-zero, then finding an equivalent decomposition for \([L]^{-1}\), (say an LU decomposition) takes roughly
$N^3$ operations, where $N$ is the size of the matrix $[L]$. For very large problems this becomes very computationally expensive. In contrast, evaluating equation (2.17) takes roughly $O(N^2)$ operations (the inner-product calculations are dominated by matrix-vector products which require $N^2$ operations). It is clear that for very large dense systems, the Perturbational Solution Methodology will be computationally less expensive as long as the approximate inversion of $[L]$ is less expensive than the LU decomposition.
CHAPTER III

PMESH Background

This chapter presents the background for understanding PMESH [30], a computer program that determines the S-parameter network characterization for arbitrarily shaped passive microstrip circuits. Nearly all of the computational effort of PMESH is involved in setting up and solving a linear operator equation like equation (2.1). In later chapters, the Perturbational Solution Methodology developed in Chapter 2 will be applied to this operator equation. However, an understanding of the underlying operator equation and its discretization is necessary in order to develop a approximate inversion algorithm that is efficient, versatile and sufficiently accurate.

3.1 The Integral Equation

The geometry of a microstrip circuit is given in Figure 3.1. The circuit is defined by metalization that exists along the dielectric/air interface. For simplicity, we consider the dielectric slab and ground plane to extend off to infinity. Let the surface $S$ be the surface of the metalization. In the presence of some impressed electric field $\vec{E}(\vec{r})$, a current density, $\vec{J}(\vec{r})$ with $\vec{r} \in S$, will be induced on the metalization. The total electric field will be the sum of the impressed field and the scattered field $\vec{E}_s(\vec{r})$, where $\vec{E}_s(\vec{r})$ can be expressed in

\[\vec{E}_s(\vec{r}) = \int_{S} \frac{\vec{J}(\vec{r}')}{2\pi |\vec{r} - \vec{r}'|} dS'\]

\[\vec{E}(\vec{r}) = \vec{E}_i(\vec{r}) + \vec{E}_s(\vec{r})\]

\[\vec{E}_i(\vec{r}) = \vec{E}_0 e^{j\omega t}\]

Throughout this thesis, we will drop the implied $e^{j\omega t}$ time dependency of the functional values.
terms of the induced currents by

$$\tilde{\mathcal{E}}^s(\tilde{r}) = \int_{S'} \tilde{G}(\tilde{r}, \tilde{r}') \cdot \tilde{J}(\tilde{r}') d\tilde{s}'$$  \hspace{1cm} (3.1)$$

where $\tilde{G}(\tilde{r}, \tilde{r}')$ is a dyadic Green Function. An integral equation for the unknown current density can be established by enforcing an appropriate boundary condition on the tangential electric field on the metalization surface $S$.

For perfectly conducting metalization, the total tangential electric field vanishes on $S$ so that we get

$$\tilde{\mathcal{E}}^t(\tilde{r}) + \tilde{\mathcal{E}}^s(\tilde{r}) = 0; \quad \tilde{r} \in S.$$ \hspace{1cm} (3.2)$$

The increasing use of microstrip circuits at higher and higher frequencies precludes the use of the perfectly conducting boundary condition if we want accurate circuit modeling. The effect of losses can be approximately accounted for by incorporating an equivalent surface impedance, $Z_s(\tilde{r})$, on the metalization surface which would modify (3.2) to become

$$\tilde{\mathcal{E}}^t(\tilde{r}) + \tilde{\mathcal{E}}^s(\tilde{r}) = Z_s(\tilde{r}) \tilde{J}(\tilde{r}); \quad \tilde{r} \in S.$$ \hspace{1cm} (3.3)$$

Finding the appropriate equivalent surface impedance to account for all positional and metalization edge shape effects is the topic of other research [10];
however, ignoring these geometrical effects, a simplified model given by Horton
[11] is

\[
Z_s(\vec{r}; t) = \sqrt{\frac{\mu_0}{\varepsilon_0 - j\sigma/\omega}} \{ \coth(\gamma k_c t) + \text{csch}(\gamma k_c t) \}
\]  

(3.4)

where \( \sigma \) is the conductivity of the metal, \( t \) is the metalization thickness and
\( k_c = \sqrt{\omega^2 \mu_0 \varepsilon_0 - j\omega \sigma \mu_0} \) is the propagation constant in the conductor.

The planar nature of the microstrip geometry allows the dyadic Green
Function, \( \tilde{G}(\vec{r}, \vec{r}') \) when \( \vec{r}, \vec{r}' \in S \), to be split into two scalar Green Functions
if we let the metalization thickness \( t \) approach zero. Since for most microstrip
circuits, \( t \) is very small (\( t \ll h, t \ll \lambda_c \)), modeling the metalization to be infinitely
thin is a valid approximation. Taking the limit \( t \to 0 \), we notice that \( S \) should
be taken to have two sides, both a top and a bottom side. Equation (3.3) now
becomes

\[
\begin{align*}
\vec{E}_i^+(\vec{r}) + \vec{E}_i^+(\vec{r}) &= Z_s(\vec{r}; t)\vec{J}^+(\vec{r}); \quad \vec{r} \in S^+ \\
\vec{E}_i^-(\vec{r}) + \vec{E}_i^- (\vec{r}) &= Z_s(\vec{r}; t)\vec{J}^-(\vec{r}); \quad \vec{r} \in S^-
\end{align*}
\]  

(3.5)

We can avoid this complication by rewriting this as

\[
\vec{E}_i^+(\vec{r}) + \vec{E}_i^-(\vec{r}) = \frac{1}{2} Z_s(\vec{r}; t)\vec{J}(\vec{r}); \quad \vec{r} \in S,
\]  

(3.6)

where now \( \vec{E}_i^+(\vec{r}) \) and \( \vec{E}_i^-(\vec{r}) \) denote the average scattered and impressed tan-
gential electric fields on \( S \), respectively. The factor of \( \frac{1}{2} \) comes about since
\( \vec{J}(\vec{r}) = \vec{J}^+(\vec{r}) + \vec{J}^-(\vec{r}) \) is now the total current density on \( S \).

The reduction of \( \tilde{G}(\vec{r}, \vec{r}') \) into two scalar Green Functions is presented
elsewhere [29], [4]; doing so reduces (3.6) to the Mixed Potential Integral Equa-
tion (MPIE)

\[
\vec{E}_i^+(\vec{r}) = \frac{j \omega \mu_0}{4\pi} \int_{S'} [G_m(\rho)\vec{J}(\vec{r}') + \frac{1}{k_0^2} \nabla G_e(\rho) \nabla' \cdot \vec{J}^+(\vec{r}')] \, ds' + \frac{1}{2} Z_s(\vec{r}; t)\vec{J}(\vec{r}); \vec{r} \in S,
\]  

(3.7)
where \( G_m(\rho) \) and \( G_s(\rho) \) are scalar Green Functions for a horizontal dipole along the dielectric interface and are given by

\[
\begin{align*}
G_s(\rho) &= \int_0^\infty J_0(\xi \rho) \frac{2\xi[u_0 + \mu_r u_1 \tanh(u_1 h)]}{D_{se} D_{sm}} d\xi, \\
G_m(\rho) &= \int_0^\infty J_0(\xi \rho) \frac{2\mu_r \xi}{D_{te}} d\xi,
\end{align*}
\]

(3.8)

where

\[
D_{se} = \mu_r u_0 + u_1 \coth(u_1 h) \quad D_{sm} = \epsilon_r u_0 + u_1 \tanh(u_1 h)
\]

(3.9)

and

\[
\rho = |\bar{r} - \bar{r}'| \quad u_1 = \sqrt{\xi^2 - \epsilon_k k^2} \quad \text{Re}(u_1) \leq 0, \text{Im}(u_1) \leq 0.
\]

(3.10)

3.2 The Method of Moments

The Method of Moments (MoM) is a powerful technique which can be used to find a numerical approximation to the solution of integral equations such as equation (3.7). Start with the integral equation

\[
\tilde{\mathcal{E}}_i'(\bar{r}) = \mathcal{L} \tilde{\mathcal{J}}(\bar{r}); \quad \bar{r} \in S,
\]

(3.11)

where \( \mathcal{L} \) is the integral-differential operator of equation (3.7). We then expand the current into a set of independent basis functions \( \{\tilde{B}_n(\bar{r})\} \) defined on \( S \) as

\[
\tilde{\mathcal{J}}(\bar{r}) \approx \sum_{n=1}^N J_n \tilde{B}_n(\bar{r}),
\]

(3.12)

where the expansion would be exact if we were to use a complete set of basis functions. Since this would require an infinite number of basis functions, we truncate the expansion to \( N \) terms. Inserting (3.12) into (3.11) and noting the linearity of \( \mathcal{L} \) yields

\[
\tilde{\mathcal{E}}_i'(\bar{r}) \approx \sum_{n=1}^N J_n \mathcal{L} \tilde{B}_n(\bar{r}); \quad \bar{r} \in S,
\]

(3.13)

where we now have \( N \) unknown coefficients, \( \{J_n\} \), associated with the basis functions \( \{\tilde{B}_n(\bar{r})\} \) to solve for. Once these coefficients are determined, (3.12)
can be used to obtain the approximate current distribution on the microstrip circuit. In order to obtain $N$ conditions to determine the $N$ current amplitudes, we can multiply both sides of equation (3.13) by a set of $N$ independent testing functions \( \{\bar{T}_m(\vec{r})\} \) and integrate over $S$, which yields

\[
\int_S \bar{T}_m(\vec{r}) \cdot \vec{E}_i(\vec{r}) \, ds = \sum_{n=1}^N J_n \int_S \bar{T}_m(\vec{r}) \cdot \vec{L} \vec{B}_n(\vec{r}) \, ds \quad m \in \{1, 2, \ldots, N\}. \tag{3.14}
\]

Equation (3.14) defines a set of linear equations which can be written as the matrix equation

\[
[ZC][J] = [V] \tag{3.15}
\]

where $[ZC]$ is the $N \times N$ impedance coupling matrix with

\[
ZC_{m,n} = \int_S \bar{T}_m(\vec{r}) \cdot \vec{L} \vec{B}_n(\vec{r}) \, ds \quad V_m = \int_S \bar{T}_m(\vec{r}) \cdot \vec{E}_i(\vec{r}) \, ds \quad n, m \in \{1, 2, \ldots, N\}. \tag{3.16}
\]

which can be rewritten using the inner product notation

\[
ZC_{m,n} = \langle \bar{T}_m(\vec{r}), \vec{L} \vec{B}_n(\vec{r}) \rangle \quad V_m = \langle \bar{T}_m(\vec{r}), \vec{E}_i(\vec{r}) \rangle \quad n, m \in \{1, 2, \ldots, N\}. \tag{3.17}
\]

The coefficients \( \{J_n\} \) can be found by solving equation (3.15). This would involve inverting the matrix $[ZC]$ using some standard linear system solving routine.

3.3 PMESH

Zheng [30] has developed a computer algorithm called PMESH that solves for the S-parameter characterization for microstrip structures of arbitrary geometry based on the MPIE of (3.7) which is discretized by the Method of Moments. The first step is to describe the geometry of a microstrip circuit, including the location of the circuit ports. The circuit geometry is then subdivided into rectangular and triangular sub-regions. A length of microstrip transmission line is then extended outward from each of the ports and is divided into rectangular sub-regions, see Figure 3.2. Once the geometry is described,
the coupling matrix \([ZC]\) of equation (3.15) can be constructed by applying the Method of Moments to the right hand side of (3.7) with basis and testing functions that are defined on the rectangular and triangular sub-domains of the circuit geometry. The last cells of the port extensions are taken to be voltage gap sources. PMESH then constructs \(P\) linearly independent source distributions (where \(P\) is the number of ports) by applying the Method of Moments to the left hand side of (3.7). Each source distribution defines a matrix equation of the form of (3.15), each of which is inverted by performing an LU decomposition and subsequent backsubstitution on the LU decomposition of \([ZC]\). PMESH then extracts (de-embeds) the \(S\)-parameters from the \(P\) current distributions.

3.3.1 Basis Functions

To solve equation (3.7), PMESH employs the Method of Moments with testing functions equal to the basis functions, \(\tilde{T}_n(\vec{r}) = \tilde{B}_n(\vec{r})\), a technique commonly referred to as Galerkin's Method. PMESH uses sub-domain basis functions; each \(\tilde{B}_n(\vec{r})\) is defined to exist only on a subdomain of \(S\). This gives PMESH the flexibility to model arbitrarily shaped circuits. The surface \(S\) is
approximated by a grid of rectangular and triangular cells. A basis function is then associated with each interior cell edge. The $n^{th}$ interior edge is common to two cells and it is over this two cell subdomain, $S_n$, that the basis function $\hat{B}_n(\vec{r})$ is defined, see Figure 3.3.

On the periphery of this sub-domain, the normal (to the periphery) component of the basis function is zero $(\hat{B}_n(\vec{r}) \cdot \hat{b} = 0$, where $\hat{b}$ is normal to the boundary of $S_n$). Along the $n^{th}$ side, $|\hat{B}_n(\vec{r}) \cdot \hat{n}| = 1$, where $\hat{n}$ is normal to side $n$. This ensures that the normal component of the current constructed by (3.12) is continuous across all cell boundaries. Within a rectangular half-domain, $\hat{B}_n(\vec{r}) \times \hat{n} = 0$, it has no transverse (to $\hat{n}$) component and varies linearly to zero at the side opposite to side $n$. The basis function within a triangular half-domain also varies linearly to zero at the vertex opposite to side $n$ but has a non-zero transverse (to $\hat{n}$) component. Essentially, the basis function $\hat{B}_n(\vec{r})$ describes a unit amplitude normal current density flowing across the $n^{th}$ interior side.
By progressively decreasing the coarseness of the gridding, the error introduced by truncating the expansion in Equation (3.12) would be decreased. However, this would produce a large (approaching infinite) system of linear equations to solve. Also, since the problem is being solved on a computer, finite precision errors will become a problem for very fine griddings. In the interest of computational efficiency, $S$ should be gridded as coarsely as possible. However, too coarse of a gridding will sacrifice accuracy. Numerical experimentation has shown that as long as the cell dimensions remain less than $\lambda_g/20$, where $\lambda_g$ is the guided wavelength for the microstrip fundamental mode, then these basis functions give reasonably accurate results.

3.3.2 Three Point De-embedding

In order to determine the S-parameter characterization of the microstrip circuit, PMESH extends a length of microstrip transmission line off each of the $P$ ports of the circuit. The port extensions are subdivided into rectangular cells that are typically $\lambda_g/20$ in length. The circuit is excited by considering the cells furthest from the circuit ports to contain voltage gap sources. Within these source cells, the impressed electric field, $\bar{E}_i(\bar{r})$, is taken to be constant and directed along the axis on the port extensions. ($\bar{E}_i(\bar{r}) = 0$ elsewhere on $S$, see Figure 3.4). Let $[E]$ be a matrix where $E_{ik}$ is the amplitude of the impressed electric field within the $i^{th}$ gap source for the $k^{th}$ excitation distribution. A system of linearly independent source excitations is constructed, say $[E] = [I]$, the identity matrix. Once equation (3.15) is solved for a given source distribution, (one of the columns of $[E]$), the current density distribution within the port extensions can be constructed from (3.12). Within these transmission line regions, the total longitudinal current distribution can be expanded into the forward- and backward-propagating fundamental modes as well as higher order evanescent modes of the microstrip line. These higher order modes are a result of the
Figure 3.4 PMESH Voltage Gap Source.

circuit and source discontinuities.

We can construct a local coordinate system on each of the extensions, with \( \hat{z}_i \) along the \( i^{th} \) extension and directed towards the circuit. Let us place the \( z_i = 0 \) plane near the center of the extension. If the extension is long enough, then \( z_i = 0 \) is far from any discontinuities and we can assume the higher order mode distribution to be negligible in the vicinity of \( z_i = 0 \). Under this assumption, we can write the total longitudinal current in the \( i^{th} \) extension arm due to the \( k^{th} \) excitation as a function of position by

\[
I_k(z_i) = \tilde{a}_{ik} e^{-\gamma_i z_i} - \tilde{b}_{ik} e^{\gamma_i z_i}, \quad z_i \sim 0 \tag{3.18}
\]

where \( \gamma_i \) is the complex propagation constant of the fundamental mode on the \( i^{th} \) extension arm and \( \tilde{a}_{ik} = a_{ik}/\sqrt{Z_0} \) and \( \tilde{b}_{ik} = b_{ik}/\sqrt{Z_0} \) are the forward and backward propagating wave variables normalized by the square root of the line's characteristic impedance. Equation (3.18) contains three parameters of interest, \( \tilde{a}_{ik}, \tilde{b}_{ik} \) and \( \gamma_i \). If the total currents in the \( i^{th} \) port extension at three adjacent points equispaced by a distance \( d \),

\[
I_{k0} = I_k(-d) \quad I_{k1} = I_k(0) \quad I_{ke} = I_k(d), \tag{3.19}
\]

\[
I_{k0} = I_k(-d) \quad I_{k1} = I_k(0) \quad I_{ke} = I_k(d), \tag{3.19}
\]
are inserted into equation (3.18), then we can find these parameters by

\[
\begin{align*}
\gamma_i &= \frac{1}{d} \cosh^{-1}\left(\frac{I_{ke} + I_{ka}}{2I_{k}}\right) \\
\tilde{a}_{ik} &= \frac{I_{ke}e^{\gamma_id} - I_{ke}e^{-\gamma_id}}{2\sinh 2\gamma_id} \\
\tilde{b}_{ik} &= \frac{I_{ke}e^{-\gamma_id} - I_{ke}e^{\gamma_id}}{2\sinh 2\gamma_id}
\end{align*}
\]

(3.20)

where the reference plane for \( \tilde{a}_{ik} \) and \( \tilde{b}_{ik} \) is the \( z_i = 0 \) plane. The definition of \( \gamma_id \) is unique if we restrict \( |\gamma_id| < \pi/2 \). If we take \( d \) to be the length of the cells within the port extensions, then \( d \leq \lambda_s/20 \) which implies \( |\gamma_id| \leq \pi/10 < \pi/2 \). Also note the \( 1/2I_k \) term in (3.20). If a standing wave null happens to lie near \( z_i = 0 \), then equation (3.20) can become numerically unstable. This condition is easily detectable and if it occurs, then the arbitrary choice of position for \( z_i = 0 \) can be shifted slightly.

From the \( P \) linearly independent voltage excitations (for an \( \times \) port structure) the S-parameters can be determined by solving the equation of \( \times \) \( \times \) matrices

\[
[\tilde{A}][\tilde{S}] = [\tilde{B}],
\]

(3.21)

where

\[
\begin{align*}
\tilde{A}_{i,k} &= \tilde{a}_{ik} \\
\tilde{B}_{i,k} &= \tilde{b}_{ik}
\end{align*}
\]

(3.22)

Since \( \tilde{S}_{i,k} = S_{i,k}\sqrt{\frac{Z_{ik}}{Z_{i}}} \), we can find the S-parameter matrix for reciprocal networks by letting

\[
S_{i,k} = S_{k,i} = \sqrt{\tilde{S}_{i,k}\tilde{S}_{k,i}}.
\]

(3.23)

To de-embed non-reciprocal networks, the characteristic impedances for each port would also need to be determined.

The S-parameters, [S], will be referenced to the \( z_i = 0 \) planes. Since \( \gamma_i \) is also de-embedded, the S-parameters can be shifted to any desired set of
reference planes by multiplying $S_{1,4}$ by the factor $e^{-\pi \omega L_{1,4}}$. This technique of de-embedding gives S-parameters that are normalized by the characteristic impedances, $Z_\alpha$, of the $P$ extended microstrip transmission lines even though these $Z_\alpha$ are never calculated. A knowledge of these $Z_\alpha$ is not strictly necessary: the S-parameter characterization is complete without them. However, in many practical applications a different characterization for the circuit is desired (Z-parameters, Y-parameters, S-parameters normalized to a reference characteristic impedance, etc.), in which case $Z_\alpha$ is a very crucial parameter that needs to be calculated accurately.

Experience has shown that this Three Point De-embedding scheme works well when the port extensions are taken to be $0.7\lambda$, in length and gridded into rectangular cells $\lambda/20$ long. If the extensions are much shorter, the higher order mode distributions tend to corrupt the assumed distribution of (3.18). Longer extensions add to the size of the problem, increasing the computational time without significantly increasing accuracy.
CHAPTER IV

Equivalent Circuit De-embedding

This chapter develops an alternative de-embedding scheme to the Three Point scheme discussed in section 3.3.2. This new scheme, Equivalent Circuit De-embedding, is based on an equivalent circuit model for a voltage gap source in an infinite microstrip line. It has the advantage over the Three Point scheme by being able to determine $Z_{ei}$ for the port extension lines as well as being more efficient when used with the Perturbational Solution Methodology. However, the Three Point scheme is more accurate and is also more efficient when implemented in the exact formulation.

4.1 Equivalent Circuit Model

As an alternative to the port extensions described in Section 3.3.2, consider a microstrip structure excited by voltage gap sources where each source is located between a port and a loaded end discontinuity. We make the extension long enough so that the higher order modes created by the loaded end and the microstrip circuit discontinuities can be considered negligible at the sources, see Figure 4.1. An equivalent circuit for this structure is shown in Figure 4.2.

The equivalent circuit of the $i^{th}$ source is an ideal $\delta$-gap voltage source in parallel with a shunt admittance $Y_{ei}$, which accounts for the higher order modes as well as the radiation modes that would be generated by the gap source on an infinite length of transmission line [16]. This ideal $\delta$-gap voltage source has zero dimension along the direction of the microstrip extension. The source as realized by PMESH basis functions has a finite dimension (the length of
Figure 4.1 Gridded Circuit for Equivalent Circuit De-embedding.

Figure 4.2 Circuit Model for Equivalent Circuit De-embedding.

the source cell) as well as a linear current distribution within the source. In order for the δ-gap source model to be accurate, the source cells must be kept short, \( l \ll \lambda \) and \( l \ll w \) where \( l \) is the length of the source cell and \( w \) is the microstrip line width. Also, we will consider the ideal δ-gap source to be located in the center of the actual source cells. The equivalent circuit of the source determines the definition of the characteristic impedance, \( Z_c \), of the line that is most convenient to deal with. In this case, a power-current definition most
naturally fits the geometry of the delta gap voltage source.

The circuit along with the port extensions out to the sources can be characterized by the impedance matrix $[Z_\ast]$ or the equivalent S-parameter matrix $[S_\ast]$ which are related by

$$
[S_\ast] = ([z] - [I]) ([z] + [I])^{-1}
$$

$$
z = [Z_c]^{-1/2} [Z_\ast] [Z_c]^{-1/2}
$$

(4.1)

where $[Z_c]$ is a diagonal matrix with $Z_{si,i} = Z_{ii}$ and $[I]$ is the identity matrix.

Once $[S_\ast]$ is found, the S-parameter matrix of the circuit $[S]$ can be found by translating the reference planes of $[S_\ast]$ back to the network interface. It is straightforward to show that $[Z_\ast]$ is expressible as

$$
[Z_\ast] = ([Y_\ast] - [Y_s])^{-1} - [Z_L],
$$

(4.2)

where $[Y_s]$ and $[Z_L]$ are diagonal matrices with $Y_{si,i} = Y_{ii}$ and $Z_{Li,i} = Z_{Li}$, where $Z_{Li}$ is the load presented by the $i^{th}$ loaded end discontinuity. $Z_{Li}$ is expressible in terms of $\Gamma_{Li}$, the reflection coefficient of the $i^{th}$ loaded end, referenced at the end of the extension as:

$$
Z_{Li} = Z_{ii} \left\{ \frac{1 + \Gamma_{Li} + (1 - \Gamma_{Li}) \tanh(\gamma_i d_i)}{1 - \Gamma_{Li} + (1 + \Gamma_{Li}) \tanh(\gamma_i d_i)} \right\},
$$

(4.3)

where $d_i$ is the distance between the center of the $i^{th}$ source and the extension end.

In equation (4.2), the mutual admittance $Y_{ini,k}$ is the total current at the midpoint of the $i^{th}$ source due to a unit impressed voltage on the $k^{th}$ source.

From Figure 4.3 we see that

$$
Y_{ini,k} = \left. \frac{I_i}{V_k} \right|_{V_{ini} = 0} \right.
$$

$$
Z_{si,k} = \left. \frac{V_i}{I_{ki}} \right|_{I_{ini} = 0}
$$

$$
Z_{Li} = \frac{V_{Li}}{I_{Li}}
$$

(4.4)
Figure 4.3 Voltage and Current Definitions of the \( i^{th} \) Source.

\([Y_{11}]\) can be found by exciting the microstrip structure with one source on at a time and measuring the total current through all the sources. To find \([Z_s]\) (and also \([S_s]\)), we would need to know \([Z_e]\), \([Y_s]\), and \([Z_L]\) which from equation (4.3) we see that we need to know \(Z_{ei}, Y_{si}, \Gamma_{Li}, \gamma_i\) and \(d_{Li}\). However, these are all parameters that characterize the individual port extensions and sources and can be pre-determined.

4.2 Determining Source and Feedline Parameters

The source and feedline parameters can be determined by analyzing a simple structure as in Figure 4.4. This secondary structure is a length of transmission line with loaded ends and two identical sources in the middle. The loaded ends, sources and the microstrip line are all the same as those in one of the circuit port extensions. A different secondary structure needs to be constructed for each distinct port.

The secondary structure can be analyzed by the PMESH algorithm. Exciting the two sources, A and B, separately will produce two current density distributions for the secondary structure. If the secondary structure is long enough, then the sources will be sufficiently separated from the loaded end
discontinuities so that the Three Point De-embedding scheme of section 3.3.2 can be employed on either of these current distributions. This gives a means for finding $\gamma_i$ and $\Gamma_Li$ since we can consider the loaded ends as one-port circuits and $\Gamma_Li = S_{11}$ referenced at the end of the strip.

Once $\gamma_i$ and $\Gamma_Li$ have been determined, $Y_i$ and $Z_{ci}$ can be determined from the two independent current distributions on the secondary structure. If we let $Y_{in,A}$ and $Y_{in,B}$ be the input admittances seen by the sources A and B respectively, then we can get $Y_i$ and $Z_{ci}$ by

$$Y_i = \left\{ \frac{y_B Y_{in,A} - y_A Y_{in,B}}{y_B - y_A} \right\}$$
$$Z_{ci} = \frac{y_A - y_B}{Y_{in,A} - Y_{in,B}}$$

(4.5)

where $y_A$ and $y_B$ are the normalized admittances due to the transmission line mode presented to the sources by the two lengths of lines terminated with loaded ends. $Y_{in,A}$ and $Y_{in,B}$ are found by dividing the total current at the middle of the sources by the source voltages. From circuit theory,

$$y_A = (z_{L,A} + z_{R,A})^{-1} \quad y_B = (z_{L,B} + z_{R,B})^{-1}$$

(4.6)
where \( z_{L,A} \) is the normalized impedance of a line of length \( L_{L,A} \) terminated by a loaded end. Transmission line theory allows \( z_{L,A} \) to be expressed in terms of parameter already calculated by

\[
z_{L,A} = \frac{1 + \Gamma_{Li} + (1 - \Gamma_{Li}) \tanh(\gamma_i d_{L,A})}{1 - \Gamma_{Li} + (1 + \Gamma_{Li}) \tanh(\gamma_i d_{L,A})}.
\] (4.7)

Expressions for \( z_{R,A}, z_{L,B}, \) and \( z_{R,B} \) can be obtained from equation (4.7) by replacing \( d_{L,A} \) with \( d_{R,A}, d_{L,B} \) and \( d_{R,B} \) respectively.

4.3 Other Considerations

In this derivation, we have used one secondary structure with two independent sources. Instead, we could have used two independent secondary structures with a single source on each. The derivation and results would be essentially identical. However, creating two secondary structures would require formulating and solving two secondary Method of Moments problems. By using only one secondary structure, only one secondary Method of Moments problem needs to be formulated. Also, nearly all algorithms for solving systems of linear equations share the characteristic that solving for two different excitations is only slightly more computationally expensive than solving for only one excitation status. Therefore, using only one secondary structure with two source excitations is more efficient then creating two secondary structures.

Constructing the coupling matrix for the secondary problem can be more efficiently done if the secondary structure is taken to be geometrically equivalent to the port extension to which it corresponds. If this is the case, then the quadruple surface integrals (see equation (3.14)) needed to find the elements of the coupling matrix will be the same for both the secondary structure and the port extension. In fact, except for the additional modeling of the loaded ends (see the next section), the coupling matrices \([ZC_n]\) for the secondary structures are submatrices of the coupling matrix \([ZC]\) for the entire structure. Since the
quadruple integrals take considerable time to compute, not having to repeat these computations represents a large time savings.

Some care must be taken in creating the secondary structure. If \( d_{L,A} = d_{R,B} \), then the two sources are symmetrically located and the two current distributions are not independent. When this happens, \( y_A = y_B \), \( Y_{in,A} = Y_{in,B} \) and equation (4.5) is indeterminate. Also, if

\[
\gamma_i d_{L,A} = \gamma_i d_{R,B} + 2n\pi j \tag{4.8}
\]

for any integer \( n \), then \( y_A = y_B \) and \( Y_{in,A} = Y_{in,B} \). equation (4.8) will not be exactly satisfied for \( n \neq 0 \) if the port extensions are lossy. However, even if (4.8) is close to being satisfied, \( y_A \approx y_B \), \( Y_{in,A} \approx Y_{in,B} \) and equation (4.5) is numerically unstable.

It is somewhat objectionable to use Three Point De-embedding procedure to determine \( \gamma_i \) and \( \Gamma_{Li} \). In principle, the four unknown parameters \( (Z_{ei}, Y_{si}, \gamma_i, \text{ and } \Gamma_{Li}) \) can be determined from four independent structures (or four independent excitations on the same structure). However, this leads to a system of four nonlinear equations that must be solved for \( Z_{ei}, Y_{si}, \gamma_i, \text{ and } \Gamma_{Li} \). This nonlinear system is very unstable in determining \( Z_{ei} \). If only \( \gamma_i \) is determined using the Three Point scheme, then the remaining parameters can be expressed in terms of three nonlinear equations. The remaining system of three non-linear equations in \( Z_{ei}, Y_{si}, \text{ and } \Gamma_{Li} \) can be reduced to two linear equations and a quadratic equation in \( Z_{ei} \). One root of this quadratic equation is \( Z_{ei} \), the other is a spurious root. Two problems can arise when the spurious root is close in magnitude to the physical root. First, it is difficult to distinguish between the two roots. Secondly, the results are unacceptably unstable although the system of three equations is more stable in determining \( Z_{ei} \) than the system of four equations.
4.4 Surface Losses for Nearly Matched Loads

It is possible to let the port extensions in Figure 4.1 extend to a truncated microstrip discontinuity. Then $Z_{Li}$ would be an open circuit load and $|\Gamma_{Li}| \approx 1$. However, such a highly reflective load tends to cause the Equivalent Circuit De-embedding to be numerically unstable to variations in the length of the port extensions. This occurs primarily because the secondary structure with open circuit loads is a high $Q$ resonator. When the secondary structure is near a resonant length, the large amplitude current distribution causes numerical instabilities in equation (4.5). To avoid this problem, we would like to terminate the extensions with matched loads. Even reasonably well matched loads, say $\Gamma_{Li} < 0.2$, eliminate the numerical instability exhibited by the highly reflective loads. However, we do not want to realize a perfectly matched load in the secondary structure. If the loads are perfectly matched, then $y_A = y_B$ and (4.5) becomes indeterminate.

The ability for PMESH to model a surface impedance provides a possible means of creating an artificial load. We can model a thin-film resistor load by allowing the surface impedance for the cells at the end of the port extensions to be highly lossy. The surface impedance and the length of the lossy region can be chosen to create a resistor that best matches the characteristic impedance of the line. In fact, since the loaded end is not modeling an actual thin-film resistor, the surface impedance of the loaded end need not be constrained by equation (3.4).

The loaded end can be modeled as a length of lossy transmission line whose distributed model is shown in Figure 4.5. Here, $z_s$, $l_s$ and $c_s$ are the surface impedance, series inductance and shunt capacitance per unit length of the equivalent transmission line. The characteristic impedance $Z_c$ and propagation
constant $\gamma$ of this lossy line are given by

\[
Z_c = \sqrt{\frac{z_s + j\omega \ell_s}{j\omega c_s}}, \\
\gamma = \sqrt{(z_s + j\omega \ell_s) j\omega c_s}
\]  
(4.9)

If we consider the lossy line to be an impedance transformer between a perfect open circuit and the port extension transmission line, then we can find the optimal surface impedance, $z_s$, by minimizing $|\Gamma|$ where $\Gamma$ is the reflection coefficient at the lossy line interface and is given approximately by

\[
\Gamma \approx \frac{Z_c \coth \gamma d - Z_{ci}}{Z_c \coth \gamma d + Z_{ci}},
\]  
(4.10)

where $d$ is the length of the lossy line. Equation (4.10) is derived by assuming that the truncated lossy line presents a perfect open circuit load. To simplify the optimization of $|\Gamma|$, it is convenient to keep $d$ as a fixed parameter (say roughly a quarter wavelength) and only minimize $|\Gamma|$ as a function of the complex variable $z_s$ as determined by equations (4.9) and (4.10). Once the optimal $z_s$ is determined, the lossy line load can be modeled by letting $Z_s(\tilde{r}; t) = z_s w$ in the cells of the lossy line where $w$ is the width of the microstrip line.
In order to apply equations (4.9) and (4.10) correctly, we would need to know the extension port transmission line characteristics $Z_{ei}$ and $\gamma_i$ before finding the optimal surface impedance $z_s$. However, $Z_{ei}$ and $\gamma_i$ are only found after $z_s$ is chosen. We can obtain reasonably accurate approximations for $Z_{ei}$ and $\gamma_i$ from a variety of design formulas as a function of microstrip width, substrate height and relative permittivity as well as possibly metalization thickness, conductivity and frequency. We can then find a best value of $z_s$ (and $Z_s$) by minimizing $|\Gamma|$ as previously discussed using these approximations for $Z_{ei}$ and $\gamma_i$ in (4.9) and (4.10). The secondary structure can be constructed with this value of surface impedance in the lossy line. From the current distributions on the secondary structure, the Three Point scheme and equation (4.5) give more accurate values for $Z_{ei}$ and $\gamma_i$. If our interest is to create the best matched load possible, then we would use these values in (4.9) and (4.10) and iterate the procedure. However, since we are only interested in creating a load that will remove the numerical instabilities of highly reflective loads, we really only need to repeat this procedure if the de-embedded load reflection is too large, say $|\Gamma| > 0.2$.

4.5 Example Results

Figure 4.6 shows the magnitude of the de-embedded load reflection, $|\Gamma_L|$, for a microstrip line with a width of 75 microns, a thickness of 3 microns and a conductivity of $3.33 \times 10^7 \Omega^{-1} \text{m}^{-1}$ on a GaAs substrate ($\epsilon_r = 12.9$) that is 100 microns high. The magnitude of the reflection coefficient is given as a function of frequency and the coarseness of the gridding in the transverse (to the direction of propagation) dimension. The secondary structure used in finding these values was roughly $1.4 \lambda_s$ long and the lossy lines were roughly $0.15 \lambda_s$ long.

---

1 The parameters $\ell_s$ and $c_s$ can be found as $\ell_s = \frac{2 \lambda_s}{j \omega}$ and $c_s = \frac{\lambda_s}{j \omega Z_{ei}}$. 
First we must note that the PMESH basis functions on a one cell transverse gridding models a current distribution that has no transverse component and no variation in the transverse direction. A gridding for multiple cells in the transverse dimension does model the transverse current components and the transverse distribution of an actual microstrip line. Therefore, it is not surprising that the one cell transverse gridding gives markedly different results from the other gridding results. The formulas used to approximately determine $Z_{el}$ and $\gamma_1$ in (4.9) and (4.10) were quasi-static approximations that do not account for any frequency dispersion. These approximations should become more valid at lower frequencies. In addition, the assumption that the open circuit load presented to the lossy line is a perfect open circuit (unit reflection with no phase) also becomes more valid at lower frequencies. We can therefore expect to be able
to create more perfectly matched loads at lower frequencies. This is observed for the multiple cell transverse griddings. Since $|\Gamma_L| < 0.1$ for the entire range of frequency and transverse griddings, the procedure of finding an optimal load does not need to be iterated to produce numerically stable values for $Z_{e1}$ and $\gamma_1$ given these microstrip parameters.

Figure 4.7 gives the characteristic impedance of this line also as a function of frequency and transverse gridding. The line is lossy so that the characteristic impedance has a non-zero imaginary part. The line has a characteristic impedance of roughly 49$\Omega$ as determined by a common design formula due to Wheeler [26]. We notice that the one cell transverse gridding gives values roughly 4% larger than the seven cell gridding for $Re(Z_e)$. At the lower frequencies, we see an increase in magnitude for both the real and imaginary parts of $Z_e$. Such an effect is observed to a lesser extent in other full wave analysis models [9], [23]. There are two explanations for this effect. First, the fields begin to significantly penetrate into the lossy conductor due to the increase skin depth at lower frequencies. This increases the internal inductance of the microstrip transmission line which increases $Re(Z_e)$. The increased field penetration also increases $Im(Z_e)$ at the lower frequencies. Secondly, part of the increase in $Re(Z_e)$ and $Im(Z_e)$ seems to be due to numerical error. At frequencies below 10 GHz, the aspect ratio of the extension arm cells (cell length to substrate height) begins to become quite large (since the extension arm cells maintain a length of roughly $\lambda_e/20$). This taxes the assumption that the Green Functions can be approximated by a low order polynomial curve fit in the quadruple surface integration calculations. Additional numerical error is introduced since we are attempting to model the field penetration effects with a surface impedance. Such a simplification may not be accurate enough when the skin depth becomes comparable to the metalization thickness which also occurs at low frequencies.
Figure 4.7 Characteristic Impedance $Z_c$. 
The shunt admittance of the sources used to excite the secondary structure is given in Figure 4.8. We would expect $\Re(Y_s)$ to increase with frequency since the gap source should excite the radiative modes more strongly at higher frequencies. However, $\Re(Y_s)$ is a small quantity compared to $|Y_e|$ and $|Z_e|$ and is therefore very difficult to accurately determine using this procedure. Since $\Im(Y_s)$ is an order of magnitude larger than $\Re(Y_s)$, and $\Im(Y_s)$ varies roughly linearly with frequency, we could do a reasonably good job modeling $Y_s$ as a shunt capacitor. However, such a model will not account for the radiative modes and becomes more inaccurate at higher frequencies. The variation of $Y_s$ with respect to transverse gridding shows about the same 4% variation seen in $Z_e$. In fact, the product $Z_eY_s$ shows relatively little variation with transverse gridding.

A comparison of the Equivalent Circuit and Three Point De-embedding schemes is given in Figure 4.9 for a truncated microstrip discontinuity. The reference plane is at the truncation. Displayed are the results for the five cell transverse gridding. The values for the other griddings lie very close to the displayed results, except for Equivalent Circuit de-embedding with only one cell gridding. The values of $Z_{ei}$ and $Y_{ei}$ used in the Equivalent Circuit de-embedding must be calculated from a secondary structure with the same transverse gridding as the port extension arms. In other words, the values of $Z_{ei}$ and $Y_{ei}$ obtained from the seven cell transverse gridding cannot be used when de-embedding a circuit with extension arms gridded with five cells in the transverse direction. While these two de-embedding techniques give results that differ by less than 2%, the Three Point De-embedding clearly gives more physically desirable results. Since the Equivalent Circuit results depend on results found from the Three Point scheme ($\Gamma_{L1}$ and $\gamma_1$), it is not surprising that the Equivalent Circuit results are no more accurate than the Three Point results. We also note that the Equivalent Circuit S-parameters are calculated at the sources and then transformed to the reference
Figure 4.8 Shunt Admittance $Y_s$. 
Figure 4.9 Comparison of De-embedding Techniques for a Truncated Microstrip Discontinuity.
planes. The Three Point S-parameters are found between the source and discontinuity and therefore (usually) have to be transformed a shorter distance to the reference planes. For this reason, the Three Point S-parameters are less sensitive to errors in determining $\gamma$ than are the Equivalent Circuit S-parameters.

Because of the longer extension arms necessary for the Equivalent Circuit de-embedding scheme, it is computationally less efficient than the Three Point scheme. Add to that the observation that the Three Point scheme seems to be more accurate than the Equivalent Circuit scheme and it is clear that for the PMESH algorithm the Three Point method is the preferable de-embedding technique. However, it is more efficient to implement the Perturbational Solution Methodology using the Equivalent Circuit scheme than the Three Point scheme, as will be shown in the next chapter. Whether the savings while using the perturbational methodology make up for the extra costs associated with the longer port extensions and the secondary structure without much sacrifice in accuracy will depend on the structure being analyzed. Regardless of the de-embedding technique the process of finding the characteristic impedance of the port extensions from a secondary structure is a valid approach. This gives a method for finding the characteristic impedance to which the de-embedded S-parameters are referenced.
CHAPTER V

Perturbational PMESH

In this chapter, the appropriate linear functionals of the Perturbational Solution Methodology for the Equivalent Circuit and Three Point de-embedding techniques are developed. The merits of both these de-embedding methods within the Perturbational Solution Methodology framework are discussed. Also, other de-embedding techniques and their applicability with the Perturbational Solution Methodology are briefly presented.

5.1 Equivalent Circuit De-embedding

Let us consider the inner product

$$\langle \tilde{E}^e_i(\tilde{r}), \tilde{J}_k(\tilde{r}) \rangle = \int_S \tilde{E}^e_i(\tilde{r}) \cdot \tilde{J}_k(\tilde{r}) \, ds,$$  \hspace{1cm} (5.1)

where $\tilde{E}^e_i(\tilde{r})$ is the excitation tangential electric field of the $i^{th}$ voltage gap source and $\tilde{J}_k(\tilde{r})$ is the current density distribution due to the $k^{th}$ source. Ideally, the source is a $\delta$-gap source, but we take the source to be a constant electric field in the direction of the port extensions within the source cells. If we consider the contribution to (5.1) due to the excitation field within a single rectangular cell $\alpha_i$, where the $i^{th}$ source consists of the $L_i$ cells $\alpha_i$ all having the same length $d$, then we have

$$\langle \tilde{E}^e_i(\tilde{r}), \tilde{J}_k(\tilde{r}) \rangle_{\alpha_i} = \int_{\alpha_i} \tilde{E}^e_i(\tilde{r}) \cdot \tilde{J}_k(\tilde{r}) \, ds,$$  \hspace{1cm} (5.2)

---

1 The port extensions are automatically gridded with rectangular cells so that the source cells will always be rectangular.
Figure 5.1 Equivalent Circuit Extension Arm Local Coordinate System.

and

$$\langle \hat{E}_{\alpha}^t(\vec{r}), \hat{J}_z(\vec{r}) \rangle = \sum_{t=1}^{L_t} \langle \hat{E}_{\alpha}^t(\vec{r}), \hat{J}_z(\vec{r}) \rangle_{\alpha_t}. \quad (5.3)$$

Without loss of generality, we can define a local coordinate system as in Figure 5.1 so that $\hat{E}_{\alpha}^t(\vec{r}) = \mathcal{E}_{\alpha} \hat{a}_z$ in cell $\alpha_t$. This leads to

$$\langle \hat{E}_{\alpha}^t(\vec{r}), \hat{J}_z(\vec{r}) \rangle_{\alpha_t} = \begin{cases} \int_{-d/2}^{d/2} \int_{-w/2}^{w/2} \mathcal{E}_{\alpha} \hat{a}_z \cdot \hat{J}_z(\vec{r}) \, dx_t \, dz_i \\ \int_{-d/2}^{d/2} \int_{-w/2}^{w/2} \mathcal{E}_{\alpha} \hat{J}_z(\vec{r}) \, dx_t \, dz_i \end{cases}. \quad (5.4)$$

Since the current is expanded in terms of the PMESH basis functions, we note that within the rectangular cell $\alpha_t$, $\hat{J}_z(\vec{r})$ has no $x_t$ variation and varies linearly with $z_t$. Therefore, we can expand the $\hat{z}$ directed current within cell $\alpha_t$ as

$$\hat{J}_z(\vec{r}) = \hat{J}_{0z} + z_t \hat{J}_{1z}, \quad \vec{r} \in \alpha_t. \quad (5.5)$$

This gives

$$\langle \hat{E}_{\alpha}^t(\vec{r}), \hat{J}_z(\vec{r}) \rangle_{\alpha_t} = \begin{cases} \int_{-d/2}^{d/2} \int_{-w/2}^{w/2} \mathcal{E}_{\alpha} (\hat{J}_{0z} + z_t \hat{J}_{1z}) \, dx_t \, dz_i \\ \mathcal{J}_{0z} w \mathcal{E}_{\alpha} d + \mathcal{J}_{1z} \mathcal{E}_{\alpha} w \int_{-d/2}^{d/2} z_t \, dz_i \end{cases}. \quad (5.6)$$
We notice that \( E_{\alpha i} d = v_i \) is the excitation voltage across cell \( \alpha \) (which is the voltage of the \( i^{th} \) source) and that \( J_{\alpha i} w = I_{k, \alpha i}(z_i = 0) \) is the total \( z_i \) directed current due to the \( k^{th} \) source at \( z_i = 0 \) within cell \( \alpha \). Therefore,

\[
(\hat{E}_{hi}(\bar{r}), \hat{J}_k(\bar{r}))_{\alpha i} = v_i I_{k, \alpha i}(z_i = 0).
\]  

(5.7)

If we ensure that all the source cells within the \( i^{th} \) arm extend from \( z_i = -d/2 \) to \( z_i = d/2 \), then we notice that

\[
(\hat{E}_{hi}(\bar{r}), \hat{J}_k(\bar{r})) = v_i I_k(z_i = 0)
\]  

(5.8)

where \( I_k(z_i = 0) = \sum_{l} I_{\alpha l}(z_i = 0) \) is the total longitudinal current at \( z_i = 0 \) due to the \( k^{th} \) source. Since we define the mutual admittance \( Y_{lk,i} \) to be

\[
Y_{lk,i} = \frac{I_k(z_i = 0)}{v_k}
\]  

where \( v_k \) is the voltage of the \( k^{th} \) source, we have

\[
Y_{lk,i} = \frac{(\hat{E}_{hi}(\bar{r}), \hat{J}_k(\bar{r}))}{v_k v_i}.
\]  

(5.9)

After the Moment Method is applied to the integral equation, we can calculate \((\hat{E}_{hi}(\bar{r}), \hat{J}_k(\bar{r}))\) from its discretized analog which is given by \( ([V]_i, [J]_k) \). Therefore, the values for the input admittance matrix of the Equivalent Circuit de-embedding can be calculated from the discrete functional

\[
Y_{lk,i} = \frac{[V]_i [J]_k}{v_i v_k}.
\]  

(5.10)

To completely fill the \( P \times P \) input admittance matrix, the exact formulation requires that we solve for the \( P \) current distributions \( \hat{J}_k(\bar{r}) \) which are the solutions to the integral equations

\[
\hat{E}_{hi}(\bar{r}) = L \hat{J}_k(\bar{r}) \quad \bar{r} \in S \quad k \in \{1, \ldots, P\}.
\]  

(5.11)

---

2 The gap sources of PMESH are realized by a unit electric field across the gap. Due to normalizations in the PMESH code, the voltage \( v_i \) has a magnitude of \( 1000k_0d \) where \( d \) is the length of the gap cell.
After the Method of Moments discretization, this requires that we find \([J_k]\), which are the solutions to the matrix equations

\[
[ZC][J_k] = [V], \quad k \in \{1, \ldots, P\}.
\]  
(5.12)

If we instead wish to apply the Perturbational Solution Methodology described in Chapter 2 to the discretized problem, then we can approximate the input admittance matrix elements by

\[
Y_{lmk,i} \approx 2 \frac{[V]_l^[i][J_0]_k}{v_l v_k} - \frac{[J_0]_l^[i][ZC][J_0]_k}{v_l v_k},
\]  
(5.13)

where \([J_0]_i\) and \([J_0]_k\) are formally the solutions to the equations

\[
[ZC_0]'[J_0]_i = [V]_i; \quad [ZC_0][J_0]_i = [V]_i \quad i \in \{1, \ldots, P\}.
\]  
(5.14)

This normally would require solving for \(2P\) zeroth order current distributions. However, whenever the zero order coupling matrix \([ZC_0]\) is symmetric, then \([ZC_0]' = [ZC_0]\) and \([J_0]_i = [J_0]_k\). Therefore, (5.13) becomes

\[
Y_{lmk,i} \approx 2 \frac{[V]_l^[i][J_0]_k}{v_l v_k} - \frac{[J_0]_l^[i][ZC][J_0]_k}{v_l v_k},
\]  
(5.15)

and we will only need to solve for \(P\) zeroth order current distributions. Whether we use the exact formulation or the perturbational one, we would still need to find the source and feedline parameters by constructing secondary structures for each distinct port. Since the accuracy of the de-embedding depends on the accuracy of the source and feedline parameters, these calculations should be done as accurately as possible. Therefore, the current densities on the secondary structures should be determined using the exact formulation. This will not cause a major execution time penalty since the secondary structures are usually small. Also the source and feedline parameters can be pre-calculated and tabulated for commonly used feedline dimensions.
5.2 Three Point De-embedding

If we wish to employ the Three Point de-embedding scheme, then we will consider local coordinate systems on each of the port extensions that are slightly different from the local coordinate systems constructed for the Equivalent Circuit de-embedding scheme, see Figure 5.2. Here the \( z_i = 0 \) planes lie between the sources and the circuit discontinuities. Any metalization 'beyond' the sources is considered to be 'far' from actual circuit and should have negligible effect on the circuit current distributions. Therefore, we can perform the Three Point de-embedding using the Equivalent Circuit port extensions. However, in the interest of computational efficiency we do not continue the Three Point extensions beyond the source cells.

The Three Point de-embedding scheme requires the sampling of the total longitudinal currents flowing through three constant \( z_i \) planes in the port extensions. The current due to the \( k^{th} \) source excitation sampled at \( z_i = z_0 \) can be expressed in terms of the functional

\[
I_k(z_i = z_0) = \langle \tilde{g}_{z_i = z_0}(\vec{r}), \tilde{J}_k(\vec{r}) \rangle,
\]

(5.16)
where
\[ \bar{g}_{z_i=z_0}(\bar{r}) = \delta(z_i - z_0)P_i(x_i)\hat{a}_i. \]

(5.17)

Here \( \delta(z_i - z_0) \) is a Dirac delta function centered at \( z_0 \) and \( P_i(x_i) \) is a unit pulse function defined by
\[ P_i(x_i) = \begin{cases} 
1, & \text{if } |x_i| \leq w_i/2; \\
0, & \text{if } |x_i| > w_i/2.
\end{cases} \]

(5.18)

and \( w_i \) is the width of the \( i^{th} \) port extension. Equation (5.16) can easily be shown to be valid since
\[
\begin{align*}
(\bar{g}_{z_i=z_0}(\bar{r}), \bar{J}_k(\bar{r})) & = \int_{\bar{g}} \delta(z_i - z_0)P_i(x_i)\hat{a}_i \cdot \bar{J}_k(\bar{r}) \, ds \\
& = \int_{-w_i/2}^{w_i/2} J_{nk}(x_i, z_i = z_0) \, dx_i \\
& = I_k(z_i = z_0)
\end{align*}
\]

(5.19)

After the application of the Method of Moments, we have the discretized analog of (5.16) given by
\[ I_k(z_i = z_0) = [g]_{z_i=z_0}^i[J]_k. \]

(5.20)

Therefore, the Three Point scheme can be employed with the exact formulation by solving for the \( P \) current distributions given by the matrix equations (5.12) and the \( 3P \) functionals of (5.20) with \( k \in \{1, \ldots, P\} \) and \( z_0 = 0, \pm d \). Equations (3.20)-(3.23) can then be used to find the \( S \)-parameters.

To apply the Perturbational Solution Methodology, we approximate the functionals of (5.20) by
\[ I_k(z_i = z_0) \approx 2[g]_{z_i=z_0}^i[J_0]_k - [\tilde{J}_0]_{z_i=z_0}^i[\mathbf{ZC}][J_0]_k. \]

(5.21)

Here \([\tilde{J}_0]_{z_i=z_0}^i\) and \([J_0]_k\) are formally the solutions to the matrix equations
\[ [\mathbf{ZC}_0][\tilde{J}_0]_{z_i=z_0}^i = [g]_{z_i=z_0}^i \quad [\mathbf{ZC}_0][J_0]_k = [V]_k \quad i \in \{1, \ldots, P\}, z_0 = 0, \pm d, \]

(5.22)

which corresponds to \( 4P \) separate matrix equations to be solved for \( 4P \) zeroth order current distributions. We can reduce this to \( 3P \) matrix equations and current distributions by using a slight modification to the Three Point de-embedding scheme which we will call the Two Point scheme.
5.2.1 Two Point De-embedding

If we have some way of determining \( \gamma_i \), then we can find \( \tilde{a}_{ik} \) and \( \tilde{b}_{ik} \) referenced at the \( z_i = 0 \) planes from only two current samples, \( I_{ik} = I_k(z_i = -d) \) and \( I_{ik} = I_k(z_i = 0) \) in a manner analogous to (3.20). In terms of these two currents and \( \gamma_i \) we have

\[
\begin{align*}
\tilde{a}_{ik} &= \frac{I_{ik} - I_{ik} e^{-\gamma_i d}}{2 \sinh \gamma_i d} \\
\tilde{b}_{ik} &= \frac{I_{ik} - I_{ik} e^{\gamma_i d}}{2 \sinh \gamma_i d}
\end{align*}
\]  

(5.23)

Therefore, we can find \( \tilde{A}_{i,k} \) and \( \tilde{B}_{i,k} \) using the functionals of (5.21) as

\[
\begin{align*}
\tilde{A}_{i,k} &= \frac{I_k(z_i = -d) - I_k(z_i = 0)e^{-\gamma_i d}}{2 \sinh \gamma_i d} \\
\tilde{B}_{i,k} &= \frac{I_k(z_i = -d) - I_k(z_i = 0)e^{\gamma_i d}}{2 \sinh \gamma_i d}
\end{align*}
\]  

(5.24)

The methods for finding the zeroth order current distributions \([J_0]_i\) that will be discussed in later chapters will share a common feature. These zeroth order current distributions will be considered accurate enough so that \( \gamma_i \) can be determined from the standing wave pattern on the port extensions. \(^3\)

Therefore, we can find the propagation constants \( \gamma_i \) from the zeroth order current distributions in much the same way as from the exact currents by

\[
\gamma_i = \frac{1}{d} \cosh^{-1} \left( \frac{[g]_{i,z=d}[J_0]_k + [g]_{i,z=-d}[J_0]_k}{[g]_{i,z=0}[J_0]_k} \right).
\]  

(5.25)

Equations (5.24) and (5.25) require solving the matrix equations

\[
[ZC_0][\dot{J}_0]_{i,z=0} = [g]_{i,z=0} \quad [ZC_0][J_0]_k = [V]_i, \quad i \in \{1, \ldots, P\}, \quad z_0 = 0, -d,
\]  

(5.26)

\(^3\) The fundamental modes excited on the zeroth order structures will have the same propagation constant as the fully modeled structures. It is only the amplitudes of these modes (the values of \( \tilde{a}_{ik} \) and \( \tilde{b}_{ik} \)) that will be determined less accurately by the zeroth order current distributions.
which corresponds to solving only $3P$ matrix equations for the $3P$ zeroth order current distributions.

5.3 De-embedding Comparison

The primary difference (as far as the Perturbational Methodology is concerned) between the Equivalent Circuit De-embedding and Three Point De-embedding schemes is highlighted by comparing (5.13) and (5.21). The two functionals for $Y_{\text{int},i}$ and $I_i(z_k = z_0)$ have nearly identical form. However, finding the input admittance matrix elements $Y_{\text{int},i}$ requires normalization by the voltage of the excitation sources $v_i$ and $v_k$. Since the Three Point de-embedding obtains the $S$-parameters from the ratio of normalized wave variables (see (3.21)) whose amplitudes are proportional to the amplitudes of $v_i$ and $v_k$, the $I_i(z_k = z_0)$ need not be normalized by the source excitation strengths. However, since the Equivalent Circuit de-embedding scheme requires the characterization of the gap sources, we need to account for the source strengths in this scheme. The longer port extensions of the Equivalent Circuit de-embedding are another result of needing to characterize the gap sources.

An efficiency comparison can be made by examining (5.14) and (5.22) and noticing that the Equivalent Circuit de-embedding requires solving for $2P$ current distributions ($P$ distributions if $[ZC_0]$ is symmetric) while the Two Point de-embedding requires solving for $3P$ current distributions (whether $[ZC_0]$ is symmetric or not) when used in conjunction with the Perturbational Solution Methodology. If the current distributions were equally expensive to compute, then the Equivalent Circuit de-embedding would be clearly more efficient to implement within the perturbational framework. However, the longer port extensions of the Equivalent Circuit de-embedding increases the number of basis functions needed to expand the total current distribution and the size of the
coupling matrix. Also, we need to compute up to $2P$ additional current distributions on the secondary structures used to find the feedline parameters. For relatively small circuits with many ports, the extra computations associated with these longer port extensions and the secondary structures can exceed the apparent savings of the Equivalent Circuit method. However, for very large circuits, particularly those with few ports, the number of port extension basis functions becomes an increasingly small fraction of the total number of basis functions for both types of port extensions. Also, the Perturbational Solution Methodology is most appropriate for large circuits. Therefore, the Equivalent Circuit scheme is generally more efficient for large circuits when used in conjunction with the Perturbational Method. For smaller circuits where an exact methodology is more appropriate, then the Three Point scheme is the more efficient de-embedding technique as well as being more accurate.

The greater efficiency of the Equivalent Circuit scheme for large circuits is traded off against the greater accuracy of the Three (and Two) Point techniques. However, the de-embedding (with either technique) is computationally quick compared to the process of finding the coupling matrix $[ZC]$ and approximate inverse operators $[ZC_0]^{-1}$ and $[ZC_0]^{-1}$. Therefore, the savings of the Equivalent Circuit technique are a very small percentage of the overall computational time. Since the Two Point technique gives more accurate results with generally a small computational time penalty, it is usually more appropriate to use the Two Point de-embedding scheme. The results presented in later chapters are all obtained using the Two Point scheme (or the Three Point scheme where appropriate).

5.4 Other De-embedding Techniques

In principle, the Perturbational Methodology can be applied using
any linear de-embedding technique. While this chapter addresses only two de-embedding techniques, other methods of de-embedding have been proposed in the literature. In order to apply the Perturbational Methodology to these de-embedding techniques, an appropriate linear functional needs to be found. Then the Perturbational Methodology can be applied to find functional parameters corrected to first order. The desired de-embedding method can then be carried out in terms of these corrected parameters. To obtain optimal efficiency, the linear functional which forms the basis of the Perturbational Methodology needs to be carefully chosen to take advantage of any distinguishing features of the de-embedding technique. Therefore, an intimate knowledge of the de-embedding technique is required in order to take full advantage of the Perturbational Methodology. However, even without such in-depth background, the basic advantages and disadvantages of applying the Perturbational Methodology to other de-embedding techniques can be briefly examined.

Rautio [23] employs a “double delay” de-embedding technique. While developed to de-embed S-parameters for shielded circuits (circuits enclosed in a box) the circuit model on which this algorithm is based is not much different (indeed is simpler) than the circuit model of the Equivalent Circuit de-embedding. Therefore, the “double delay” de-embedding can be formulated in an analogous manner to the Equivalent Circuit de-embedding and should share the same basic computational advantages. Without actually implementing this de-embedding scheme, it is difficult to determine the accuracy and the computational penalty of the calibration standards (secondary structures).

Another common procedure ([8], [17]) involves curve fitting the current distribution on a port extension of a standing wave pattern. This is a generalization of the Two Point de-embedding algorithm where the standing wave pattern is fit to only two current samples. Another approach is to try to extract
the higher order mode distributions in addition to the fundamental mode distributions. Some success has been found using Prony's method as well as the "matrix pencil" method [12]. Both of these approaches require currents sampled at 'many' points. Functionals of the form of (5.21) can be used to obtain corrected values for these current samples. The attempt to more accurately find the standing wave parameters using these techniques (either by minimizing the higher order mode 'errors' or by modeling these modes directly) cause greater computational cost. For each additional current sample there is an additional inner product like equation (5.21) to evaluate as well as an additional current distribution $[\tilde{J}_i,_{\omega_0}$ to be found from equation (5.22). Further study is required to determine if the increased accuracy of the techniques justify their increased computational costs.

5.5 Comparison With Regular PMESH

The justification for using the Perturbational Solution Methodology within the PMESH algorithm is the hope to save time when analyzing large problems. The main PMESH algorithm consists of three major steps, evaluating the coupling matrix elements $ZC_{i,k}$ (which consist of two main steps, evaluating the Green Functions and performing the quadruple surface integrals), solving the linear systems of equations

$$[ZC][J]_i = [V]_i, \quad i \in 1, \ldots, P$$  \hspace{1cm} (5.27)

for the current basis function amplitudes $[J]_i$, and finally de-embedding the S-parameters from these current distributions. This chapter has investigated the relative benefits and costs of two de-embedding schemes. However, de-embedding itself takes up a very small fraction of the total computational time of a PMESH run. Nearly all of the computational effort lies in the first two steps, the matrix fill and the matrix 'inversion'. Since the coupling matrix $[ZC]$
is complex and symmetric (not Hermitian), the most straightforward way to 'find' \([ZC]^{-1}\) is to perform an LU decomposition with partial pivoting. This takes \(O(N^3/3)\) operations \([6]\) where \(N\) is the size of the system \(([ZC] \text{ is an } N \times N \text{ matrix})\). From the equivalent LU decomposition, the \(P\) current amplitude vectors \([J]\), can be found from \(P\) back-substitutions. This is an \(O(PN^2)\) process. Since in general \(3P \ll N\), we can consider the entire process of finding the \([J]\), from (5.27) to take \(O(N^3/3)\) operations.

The matrix fill is dominated by the quadruple surface integrals and takes \(O(\alpha_f N^2)\) operations where \(\alpha_f\) is a function of both frequency and the gridded circuit geometry (see Appendix A). Strictly speaking, the number of operations is a function of the number of cells \(N_c\), not \(N\), the number of interior sides. However, \(1 \sim N/N_c \leq 2\) and \(N/N_c\) is a function of the gridded geometry and can be absorbed into \(\alpha_f\). Typically \(\alpha_f \sim 500\), although it can be in the range 200-1000. Therefore, for small to moderately sized circuits, the matrix fill takes the majority of the computational effort of PMESH. However, for very large circuits \((N > 3\alpha_f)\) the matrix 'inversion' becomes dominant. If we examine both (5.13) and (5.21) we notice that both require the entire 'exact' coupling matrix \([ZC]\). Therefore, the Perturbational Solution Methodology does not reduce the matrix fill time and can only be more efficient than the regular PMESH by reducing the matrix 'inversion' time. Since the matrix fill time is the same for both the regular and perturbational PMESH algorithms, the only valid comparison between these two approaches is the computational time for finding and solving the zeroth order systems of (5.14) and (5.22) (as well as any additional de-embedding overhead) compared to the matrix 'inversion' computational time. Any total execution time savings due to the perturbational approach will be most dramatic for simulations of very large circuits.
CHAPTER VI

The Isolated Subcircuits Method

This chapter discusses the Perturbational Solution Methodology as applied to microstrip circuits consisting of distinct (separated) subcircuits. The execution of this Isolated Subcircuit Method is compared to the exact formulation, comparing both execution time and accuracy. Also, the structure of this class of problems can be exploited to produce additional efficiencies in the evaluation of the linear functionals as well as providing a framework for implementing an efficient sensitivity analysis on the relative positioning of the subcircuits.

6.1 The Zeroth Order Operator

Consider a microstrip circuit which consist of two distinct subcircuits, A and B, each with at least one port as is shown in Figure 6.1. If the two subcircuits are not too close, then the coupling effects between them will be small compared to the self interactions of the subcircuits. Therefore, we can obtain a zeroth order current distribution on the two subcircuits by analyzing them in isolation. Let us label the basis functions and the ports of the gridded subcircuits such that the basis functions 1 → $N_A$ and the ports 1 → $P_A$ are defined on subcircuit A while the basis functions and ports on subcircuit B are numbered $N_A + 1 → N = N_A + N_B$ and $P_A + 1 → P = P_A + P_B$ respectively.

---

1 We will consider primarily the (somewhat) special case of only two subcircuits; the results are easily extended to the more general case of multiple subcircuits.
The matrix equation of (3.15) can be written as

\[
\begin{bmatrix}
Z_{C_{1,1}} & \ldots & Z_{C_{1,N_A}} & Z_{C_{1,N_A+1}} & \ldots & Z_{C_{1,N}} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
Z_{C_{N_A,1}} & \ldots & Z_{C_{N_A,N_A}} & Z_{C_{N_A,N_A+1}} & \ldots & Z_{C_{N_A,N}} \\
Z_{C_{N_A+1,1}} & \ldots & Z_{C_{N_A+1,N_A}} & Z_{C_{N_A+1,N_A+1}} & \ldots & Z_{C_{N_A+1,N}} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
Z_{C_{N,1}} & \ldots & Z_{C_{N,N_A}} & Z_{C_{N,N_A+1}} & \ldots & Z_{C_{N,N}}
\end{bmatrix}
\begin{bmatrix}
J_1 \\
\vdots \\
J_{N_A} \\
\vdots \\
J_{N_A+1}
\end{bmatrix}
= \begin{bmatrix}
V_1 \\
\vdots \\
V_{N_A} \\
V_{N_A+1} \\
\vdots \\
V_N
\end{bmatrix}
\quad i \in \{1, \ldots, P\}
\]

or more compactly as

\[
\begin{bmatrix}
[ZC_A] & [ZC_{AB}] \\
[ZC_{BA}] & [ZC_B]
\end{bmatrix}
\begin{bmatrix}
[J_A] \\
[J_B]
\end{bmatrix}
= \begin{bmatrix}
[V_A] \\
[V_B]
\end{bmatrix}
\quad i \in \{1, \ldots, P\}. \tag{6.1}
\]

The submatrices \([ZC_A]\) and \([ZC_B]\) account for the couplings within the subcircuits while \([ZC_{AB}]\) and \([ZC_{BA}] = [ZC_{AB}]^t\) account for the couplings between the subcircuits. Clearly, if we were to analyze the two subcircuits in isolation, then \([ZC_{AB}] = [0]\) and \([ZC_{BA}] = [0]\). Therefore, we can explicitly write the zeroth order coupling matrix as

\[
[ZC_0] = \begin{bmatrix}
[ZC_A] & 0 \\
0 & [ZC_B]
\end{bmatrix}. \tag{6.2}
\]
Notice that \([ZC_0] = [ZC_0]^t\) since \([ZC_A] = [ZC_A]^t\) and \([ZC_B] = [ZC_B]^t\). Since \([ZC_0]\) is block diagonal, we can find \([ZC_0]^{-1}\) simply by
\[
[ZC_0]^{-1} = [ZC_0]^{-t} = \begin{bmatrix} [ZC_A]^{-1} & 0 \\ 0 & [ZC_B]^{-1} \end{bmatrix}.
\] (6.3)

Since we are considering dense coupling matrices, we can effectively 'invert' the matrices finding the LU decomposition of the matrices which takes \(O(N^3/3)\) operations for an \(N \times N\) matrix. Because of the block diagonal nature of \([ZC_0]\), we can find its LU decomposition directly from the LU decompositions of \([ZC_A]\) and \([ZC_B]\) which requires \(O\left(\frac{N_A^2 + N_B^2}{3}\right)\) operations while finding the LU decomposition of \([ZC]\) requires \(O\left(\frac{(N_A + N_B)^3}{3}\right)\) operations. Therefore, there is a savings of \(O(N_A^2 N_B + N_A N_B^2)\) operations when finding the zeroth order LU decomposition compared to the exact LU decomposition. The largest percentage savings occurs when \(N_A = N_B\) where finding the LU decomposition of \([ZC_0]\) takes roughly 1/4 the computations of finding the LU decomposition of \([ZC]\). The savings become even more pronounced if the circuit consists of more than two subcircuits. If there are \(M\) equally sized (same number of basis functions) subcircuits, then finding the zeroth order LU decomposition will take about \(1/M^2\) the computations of finding the exact LU decomposition.

### 6.2 Efficiencies in the Inner Products

The special structure of the zeroth order coupling matrix can be exploited to obtain an additional (although less significant) reduction in the overall computational time by reducing the number of necessary operations to calculate the linear functionals of equation (5.21). Although we are free to choose any system of \(P\) independent source excitations to find the zeroth order current distributions of equation (5.22), if we let the excitations fields \(\tilde{E}_i(\hat{r})\) of the \(i^{th}\) excitation be non-zero only in the source cells of the \(i^{th}\) extension, then we notice
that the zeroth order currents will take the form of

\[
[J_0]_i = \begin{cases} 
\begin{bmatrix} [J_{DA}]_i \\ 0 \end{bmatrix}, & \text{if } i \in \{1, \ldots, P_A\}; \\
\begin{bmatrix} 0 \\ [J_{DB}]_i \end{bmatrix}, & \text{if } i \in \{P_A + 1, \ldots, P\}.
\end{cases}
\]  

(6.4)

Furthermore, we note that \([J_{DA}]_i\) and \([J_{DB}]_i\) are the solutions to the equations

\[
[ZC_D][J_{DA}]_i = [V_A]_i; \quad i \in \{1, \ldots, P_A\}
\]

\[
[ZC_B][J_{DB}]_i = [V_B]_i; \quad i \in \{P_A + 1, \ldots, P\}
\]

That is, \([J_{DA}]_i\) and \([J_{DB}]_i\) are the current densities for the isolated subcircuits.

Also, since \([\mathcal g]_t=\mathcal z_0\) corresponds to an equivalent source that is non-zero only on the \(i\th\) extension arm, the 'currents' \([\tilde J_0]_{t=\mathcal z_0}\) take on the form

\[
[\tilde J_0]_{t=\mathcal z_0} = \begin{cases} 
\begin{bmatrix} [\tilde J_{DA}]_{t=\mathcal z_0} \\ 0 \end{bmatrix}, & \text{if } i \in \{1, \ldots, P_A\}; \\
\begin{bmatrix} 0 \\ [\tilde J_{DB}]_{t=\mathcal z_0} \end{bmatrix}, & \text{if } i \in \{P_A + 1, \ldots, P\}.
\end{cases}
\]  

(6.6)

and are the solutions to the equations

\[
[ZC_D]'[\tilde J_{DA}]_{t=\mathcal z_0} = [\mathcal g_A]_{t=\mathcal z_0}; \quad i \in \{1, \ldots, P_A\}
\]

\[
[ZC_B]'[\tilde J_{DB}]_{t=\mathcal z_0} = [\mathcal g_B]_{t=\mathcal z_0}; \quad i \in \{P_A + 1, \ldots, P\}
\]

\[z_0 = 0, d.\]  

(6.7)

When vectors of the form of (6.4) and (6.6) are used in equation (5.21), two things happen. First, it is trivial to show that \([\mathcal g]_{t=\mathcal z_0}[J_0]_k = 0\) for \(i \in \{1, \ldots, P_A\}\) and \(k \in \{P_A + 1, \ldots, P\}\) as well as for \(k \in \{1, \ldots, P_A\}\) and \(i \in \{P_A + 1, \ldots, P\}\). That is, to zeroth order, an excitation on one subcircuit does not excite currents on the other subcircuit. Secondly, we notice that the matrix inner product \([\tilde J_0]_{t=\mathcal z_0}[ZC][J_0]_k\) reduces to

\[
[\tilde J_0]_{t=\mathcal z_0}^\dagger[ZC][J_0]_k = \begin{cases} 
[\tilde J_{DA}]_{t=\mathcal z_0}^\dagger[ZC_D][J_{DA}]_k & \text{if } i \in \{1, \ldots, P_A\}; \\
[\tilde J_{DB}]_{t=\mathcal z_0}^\dagger[ZC_B][J_{DB}]_k & \text{if } i \in \{P_A + 1, \ldots, P\}; \\
[\tilde J_{DA}]_{t=\mathcal z_0}^\dagger[ZC_{AB}][J_{DB}]_k & \text{if } i \in \{1, \ldots, P_A\}, \\
& \text{if } k \in \{P_A + 1, \ldots, P\}; \\
[\tilde J_{DB}]_{t=\mathcal z_0}^\dagger[ZC_{BA}][J_{DA}]_k & \text{if } i \in \{P_A + 1, \ldots, P\}, \\
& \text{if } k \in \{1, \ldots, P_A\}.
\end{cases}
\]  

(6.8)
Furthermore, for $i, k \in \{1, \ldots, P_A\}$, $[g]_{i, z_0}^t [J_0]_k = [g_A]_{i, z_0}^t [J_{0A}]_k$ and from (6.7) we get $[\hat{J}_{0A}]_{i, z_0}^t [Z_{CA}] = [g_A]_{i, z_0}^t$ so that for $i, k \in \{1, \ldots, P_A\}$

$$I_k(z_i = z_0) = 2[g]_{i, z_0}^t [J_0]_k - [\hat{J}_0]_{i, z_0}^t [Z_{CA}] [J_0]_k$$

$$= [g_A]_{i, z_0}^t [J_{0A}]_k + [g_A]_{i, z_0}^t [J_{0A}]_k - [\hat{J}_{0A}]_{i, z_0}^t [Z_{CA}] [J_{0A}]_k$$

$$= [g_A]_{i, z_0}^t [J_{0A}]_k + [g_A]_{i, z_0}^t [J_{0A}]_k - [g_A]_{i, z_0}^t [J_{0A}]_k$$

$$= [g_A]_{i, z_0}^t [J_{0A}]_k$$

(6.9)

Similarly,

$$I_k(z_i = z_0) = [g_B]_{i, z_0}^t [J_{0B}]_k \quad i, k \in \{P_A + 1, \ldots, P\},$$

(6.10)

which leads to

$$I_k(z_i = z_0) \approx \begin{cases} [g_A]_{i, z_0}^t [J_{0A}]_k, & i, k \in \{1, \ldots, P_A\}; \\ [g_B]_{i, z_0}^t [J_{0B}]_k, & i, k \in \{P_A + 1, \ldots, P\}; \end{cases}$$

(6.11)

$$\begin{cases} [\hat{J}_{0A}]_{i, z_0}^t [Z_{CA}] [J_{0B}]_k & i \in \{1, \ldots, P_A\} \\ k \in \{P_A + 1, \ldots, P\}; \end{cases}$$

$$\begin{cases} [\hat{J}_{0B}]_{i, z_0}^t [Z_{CA}] [J_{0A}]_k & i \in \{P_A + 1, \ldots, P\} \\ k \in \{1, \ldots, P_A\}. \end{cases}$$

Evaluating equation (6.11) is much more efficient than (5.21). First, the matrix inner products of (6.11) involve smaller matrices ($N_A \times N_B$ and $N_B \times N_A$ instead of $N \times N$). Also, the inner products only need to be calculated for the cross-coupling currents. Since a matrix inner product takes $O(N_1 N_2)$ operations for an $N_1 \times N_2$ matrix and since (5.24) requires $2P^2$ current samples, (the factor of 2 comes from the 2 current samples needed in the Two Point de-embedding scheme) equation (5.21) will produce $O(2P^2 N^2)$ operations. However, (6.11) will produce only $O(2P_A P_B N_A N_B)$ operations. In the case where the two subcircuits are identical, this corresponds to a factor of sixteen savings. More

2 Since the vectors $[g]_{i, z_0}$ are non-zero only for a few elements, calculating the vector inner products like $[g]_{i, z_0}^t [J_0]_k$ takes essentially no time.
generally, equation (6.11) requires $O(2P_A P_B \ldots P_n N_A N_B \ldots N_n)$ operations for a circuit consisting of the distinct subcircuits $A, B, \ldots, \Omega$. For a circuit with $M$ identically sized ($N_A = N_B = \ldots = N_n$ and $P_A = P_B = \ldots = P_n$) subcircuits, equation (6.11) corresponds to a factor of $M^4$ savings over equation (5.21). While this is a greater percentage time savings than is obtained by 'inverting' smaller submatrices, 'inverting' the submatrices is a more computationally expensive step so that the overall time savings is primarily due to savings described in Section 6.2.

6.3 Results

To demonstrate the power of the Isolated Subcircuit Method, let us consider microstrip circuits like the coupled meander line circuit shown in Figure 6.2. This is a four port structure that consists of two isolated meander lines. This structure was created to conveniently demonstrate many of the merits and shortcomings of the different methods presented in this thesis. First, due to its symmetry, the meander line circuit can be characterized by only four independent $S$-parameters, $S_{11} (= S_{22} = S_{33} = S_{44})$, $S_{12} (= S_{21} = S_{34} = S_{43})$, $S_{13} (= S_{31} = S_{24} = S_{42})$ and $S_{14} (= S_{41} = S_{32} = S_{23})$. Secondly, since this circuit consists of two isolated circuits, it can be analyzed using the Isolated Subcircuit Method as well as the methods presented in later chapters which allows for direct comparisons between these methods. Also, this is a rather large circuit; it contains 514 sub-domains (unknowns) which makes the $O(N^3/3)$ growth of the exact LU decomposition a significant factor. Lastly, larger (and smaller) circuits with similar geometries can easily be constructed by increasing (or decreasing) the total length of the lines by adding (or subtracting) meanders.

Table 6.1 shows the time savings exhibited by the Isolated Subcircuit Method compared to the exact PMESH analysis on four coupled meander line circuits similar to the circuit shown in Figure 6.2. These circuits differ in the
Substrate thickness = 100 micron
Relative permittivity = 12.9
Metalization Thickness = 3 micron,
Conductivity = 4.55 \times 10^7 \text{ (ohm m)}

Figure 6.2 Coupled Meander Line Circuit.

total number of meanders in the lines (and therefore unknowns) but otherwise have the same basic geometry. Since in all four cases, the entire circuit consists of two identically sized subcircuits (the individual meander lines), we would ideally expect a Time Fraction (defined as \( t_{approx}/t_{exact} \) where \( t_{approx} \) and \( t_{exact} \) are the approximate inversion and exact inversion CPU times respectively) of 0.25. This would be the case if the inner products of (6.8) took no time to calculate. Since the inner products grow as the square of the problem size and the inversions grow as the cube, the Time Fraction will approach 0.25 in the limit of \( N \to \infty \) which is observed.
Table 6.1 Isolated Subcircuit Method Approximate Inversion and Overall Time Savings

<table>
<thead>
<tr>
<th>Number of Unknowns</th>
<th>Inversion Time Exact*</th>
<th>Inversion Time ISM*</th>
<th>Time Fraction</th>
<th>Overall Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>194</td>
<td>0.94</td>
<td>0.28</td>
<td>0.298</td>
<td>3 to 6%</td>
</tr>
<tr>
<td>354</td>
<td>5.70</td>
<td>1.5</td>
<td>0.275</td>
<td>6 to 13%</td>
</tr>
<tr>
<td>514</td>
<td>17.23</td>
<td>4.64</td>
<td>0.269</td>
<td>7 to 20%</td>
</tr>
<tr>
<td>674</td>
<td>39.02</td>
<td>10.30</td>
<td>0.264</td>
<td>11 to 26%</td>
</tr>
</tbody>
</table>

* (CPU minutes on a HP375 workstation)

Although the approximate inversion represents a considerable time savings over the exact inversion (nearly a factor of 4 for two identical subcircuits), the overall time savings is less dramatic due to the large matrix fill time. For example, the matrix fill for the smallest circuit (194 unknowns) takes 9.87 - 19.78 minutes in the 10 GHz - 40 GHz range. Therefore, the overall times savings \(\left(\frac{\text{exact}}{\text{ISM}}\right)\) for these four structures ranges from 3% to 26%. The overall time savings does increase for larger circuits. However, the largest circuit (674 unknowns) is about at the practical limit of the HP375 workstation. To observe the more dramatic time savings of larger circuits, the analysis needs to be carried out on a more powerful computer.

Figures 6.3 and 6.4 show that the Isolated Subcircuit Method gives accurate results as well as a significant time savings. The results shown are for the coupled meander line circuit shown in Figure 6.2 and has 514 unknowns. All the curves agree quite well with the exact PMESH results. The agreement does start to break down in the higher frequencies when the coupling between the two lines becomes strong. There is also an discrepancy in the low frequency range of Figure 6.4. However, \(S_{14}\) is small to begin with so that the 1 to 2 dB
Figure 6.3 Corrected S-parameters of the Coupled Meander Line Circuit from the Isolated Subcircuit Method.
Figure 6.4 Corrected S-parameters of the Coupled Meander Line Circuit from the Isolated Subcircuit Method.
difference at -38 dB is a very small absolute error.

The zeroth order analysis of the Isolated Subcircuit Method is equivalent to analyzing the two meander lines in the absence of the other line. Such an approximation predicts $S_{13}$ and $S_{14}$ to both be zero. Clearly, the variational nature of Equation (5.21) is able to greatly improve on the zeroth order cross-coupling parameters. However, the direct parameters ($S_{11}$ and $S_{12}$) are not greatly affected by the presence of the second line. This is consistent with the assumption that the parasitic coupling is a small effect.

The coupled meander line circuit has a very interesting frequency response. Since both $S_{11}$ and $S_{12}$ are essentially the same for an single meander line, they can be explained in terms of a meander line circuit. Ideally, one would hope that a meander line would behave the same as a straight length of line with the same path length. However, two features of the frequency response differ considerably from the response of a straight line. First, there is a ripple with a period of about 2.5 GHz which is more pronounced in $S_{11}$. The explanation of this ripple is that the meanders of the line cause the line to have an effective characteristic impedance and propagation constant of the line that differs from that of a straight line of the same width. Since the port extensions are straight lines of roughly 50 Ω, the effective characteristic impedance of the meander line presents an impedance mismatch to the port extensions, thus the ripple. There is also a major dip in the $S_{12}$ response (and a corresponding rise in the $S_{11}$ response) from 26 to 32 GHz. At 29 GHz, a single meander is approximately one wavelength long. Therefore, the back scattering off the bends are likely constructively reinforced, decreasing the transmission through the line in this frequency range.

The cross-coupling response is more difficult to explain since the meanders are so long (electrically), especially at the higher frequencies. However,
a few features of the cross-coupling response are worth noting. First, below 25 GHz, both \( S_{13} \) and \( S_{14} \) are small and \( S_{14} \) has a surprisingly flat response. Since the cross coupling is small in this range, the perturbational analysis is very appropriate. Above 30 GHz, the cross coupling is no longer very small and the perturbational approach becomes less accurate. The transition range between large and small cross coupling is the same range in which the resonance of the individual meanders strongly effects both \( S_{11} \) and \( S_{12} \). It therefore appears that the coupled meander line circuit behaves somewhat like straight coupled lines (in terms of cross coupling parameters) below the single meander resonance frequency. Above this frequency, the cross coupling process becomes much more complicated.

6.4 Sensitivity

If we were to completely ignore the cross-coupling between the subcircuits in (6.11), we would obtain as a zeroth order approximation

\[
I_{0k}(z_i = z_0) = \begin{cases} 
[g_A]_{i,z_0} [J_{0A}]_k, & i, k \in \{1, \ldots, P_A\}; \\
[g_B]_{i,z_0} [J_{0B}]_k, & i, k \in \{P_A + 1, \ldots, P\}; \\
0 & i \in \{1, \ldots, P_A\} \\
& k \in \{P_A + 1, \ldots, P\}; \\
0 & i \in \{P_A + 1, \ldots, P\} \\
& k \in \{1, \ldots, P_A\}.
\end{cases}
\] (6.12)

Comparing (6.11) and (6.12), we notice that they differ only for the cross-coupling current samples. Therefore, the perturbational approach will not be accurate when the presence of a separate, nonexcited subcircuit strongly effects the current distribution on an excited subcircuit. As long as this condition is met, we can take advantage of the invariance of the directly coupled currents to perform a sensitivity analysis on the relative positioning of the subcircuits. We notice that as the relative position between subcircuits \( A \) and \( B \) changes, the only quantity in (6.11) that needs to be recomputed is \([ZC_{AB}]([ZC_{BA}] = [ZC_{AB}]^t)\).
There is no need to perform any additional matrix 'inversions' for each new relative positionings. Recalculating $[ZC_{AB}]$ will take $O(\alpha_f N_A N_B)$ operations. However, since the elements of $[ZC_{AB}]$ only involves coupling between 'distant' basis functions, the factor $\alpha_f$ is smaller than the factor for a complete coupling matrix. For $N_A = N_B$, $N_A N_B = \frac{1}{4} N^2$ and the reduction of $\alpha_f$ means that finding $[ZC_{AB}]$ can easily be an order of magnitude faster than finding $[ZC]$. Add to that the time savings of avoiding doing additional matrix 'inversions' for each relative position and it is clear that such an approach can be much more efficient than analyzing each relative positioning as a separate problem.

6.5 Weaknesses

Although applying the Isolated Subcircuit Method to isolated subcircuits leads to an efficient formulation for the linear functionals as well as providing an easily implemented sensitivity analysis to the relative positioning of the subcircuits, such a formulation is not without shortcomings. Most obviously, such an approach is not applicable to the many complex structures that do not consist of distinct subcircuits. Also, this formulation fails to take advantage of the Perturbational Solution Methodology to account for all parasitic coupling within the circuit. Two sections of a subcircuit might be 'distant' from each other (although connected by some series of transmission line segments and discontinuities) and therefore interact parasitically. The zeroth order current distributions on the isolated subcircuits account for these interactions exactly. It is only the cross-coupling (between distinct subcircuits) that are accounted for perturbationally. If the self-circuit parasitic couplings could be accounted

---

3 The Green functions for 'distant' basis functions can be fit to a lower order polynomial thereby reducing the number of terms in the quadruple surface integrals, see Appendix A.
for perturbationally, then the Perturbational Solution Methodology can provide an even greater time savings compared to the exact PMESH formulation as well as being applicable to a larger class of problems than the approach presented in this chapter. Another disadvantage is that the Isolated Subcircuit Method reduces the inversion time only by a constant factor. The approximate inversion still grows as the cube of the total number of unknowns. For example, the approximate inversion for two identical subcircuits will require $O(N^3/12)$ operations. Therefore, this only slightly increases the size of problems that can be solved practically.

The next two chapters will present two different approaches to accounting for parasitic couplings within a directly connected circuit. While these techniques can be incorporated in conjunction with the isolated subcircuit formulation, both will be presented independently of the derivations of this chapter.
CHAPTER VII

The Network Connection Method

This chapter presents a method (the Network Connection Method) of constructing zeroth order current distributions for microstrip circuits that is based on segmenting (or diakopting) the circuit into smaller circuit elements. These elements are analyzed in isolation and combined in a manner analogous to standard Network Theory. While this method can provide an extremely efficient means of creating zeroth order currents, these currents are sometimes too inaccurate for the perturbational approach.

7.1 Finding the Zeroth Order Currents

The motivation for a second method of finding zeroth order current distributions comes from the success of transmission line network theory in modeling MMIC circuits when parasitic couplings are small. A complicated circuit can be considered to consist of interconnected transmission line segments and discontinuities. The circuit is modeled using standard network theory by cascading some appropriate network parameterization of the circuit elements to find the parameterization of the entire circuit. The general approach of breaking a large problem into many smaller problems and then recombining the constituent problem results is often referred to as segmentation [3], [7] or sometimes diakoptics [18]. Network theory and the PMESH algorithm can be merged to create a simple algorithm that uses the power of PMESH to accurately analyze circuit elements and Network theory to segment these circuit elements.
7.1.1 Block Level Description

The first step in creating a Network Connection Model is to consider the microstrip circuit on a three level organizational hierarchy. The highest level is the complete circuit, including the de-embedding port extensions. The lowest level consists of the gridded cell description that determines the basis and testing function subdomains. The middle level consists of a unit description that we will refer to as blocks.\(^1\) A block consists of a small (say < 30) number of cells that describe a single transmission line segment or circuit discontinuity with sufficient transmission line extentsions such that essentially only the fundamental mode exists at the block interfaces. Each block corresponds to a circuit element of the standard transmission line network model. Just as the blocks consists of connected cells, the entire circuit consists of interconnected blocks. The blocks are connected together at common cell sides called internal port connections. Figure 7.1 gives an example of a simple single stub structure described on all three levels. Notice that this circuit consist of four blocks with three internal port connections.

One way of producing a block level description is to construct the circuit from the bottom-up out of blocks. This is the approach taken by MBUILD [25], an interactive graphical interface developed to create geometry file descriptions for PMESH and related circuit analysis programs. An alternative to the bottom-up process would be a top-down process where the block description is derived after the entire circuit is gridded by grouping cells into blocks. This would require a rather sophisticated algorithm for automatically grouping the

\(^1\) If we were to include the isolated subcircuit analysis to the Network Connection Model, then we would include the subcircuit description as a fourth level between the block and complete circuit levels.
7.1.2 Analysis of the Constituent Blocks

Once the block level description is complete, each block $\alpha$ can be considered a complete circuit with $P_\alpha$ ports and analyzed individually as separate PMESH problems. These $P_\alpha$ ports consist not only of the (external) ports of the complete circuit (if block $\alpha$ consists of the cells at an external port), but also the internal ports where the block connects to another block. The internal port extensions need not be the same length as the extensions for the external ports. The sources at the ends of the external port extensions are taken to have the same amplitude that they would have in the regular PMESH model, while those at the ends of the internal ports will be considered to have variable magnitude and phase. By exciting each of the $P_\alpha$ sources separately with a unit amplitude, we can obtain $P_\alpha$ normalized current distributions

$$[J_\ell]_\ell \in \{1, \ldots, P_\alpha\}$$  \hspace{1cm} (7.1)
on block $\alpha$ as well as the normalized wave variables $[\tilde{A}_\alpha]$ and $[\tilde{B}_\alpha]$ referenced to the internal port connection planes where $[\tilde{A}_\alpha]$ and $[\tilde{B}_\alpha]$ are $P_\alpha \times P_\alpha$ matrices. ($[\tilde{A}_\alpha]$ and $[\tilde{B}_\alpha]$ are similar to $[\tilde{A}]$ and $[\tilde{B}]$ given in equation (3.21) except that they are referenced to the internal port connection planes instead of halfway down the port extensions.) If we let $[E_{\alpha}]$ be a vector of dimension $P_\alpha$ where $E_{\alpha \ell}$ is the amplitude of the $\ell^{th}$ source, then we can express the current distributions on block $\alpha$ by

$$E_{\alpha \ell} [J_{\alpha}]_\ell \quad \ell \in \{1, \ldots, P_\alpha\}.$$ (7.2)

While strictly speaking $[J_{\alpha}]_\ell$ is a vector of $N_\alpha + N_{\text{int}, \alpha}$ current density amplitudes, where $N_{\text{int}, \alpha}$ is the number of unknowns in the internal port extensions (excluding the unknowns at the internal port connections), we will not need to retain the currents on the internal port extensions and therefore will only need to store $[J_{\alpha}]_\ell$ as vector of $N_\alpha$ elements.

Let us emphasize here that the current distributions $[J_{\alpha}]_\ell$ are found by exciting the $\ell^{th}$ source with a unit magnitude while the distribution $E_{\alpha \ell} [J_{\alpha}]_\ell$ is the distribution that would be obtained by exciting the $\ell^{th}$ source with an amplitude of $E_{\alpha \ell}$. At this point, the only known values of $E_{\alpha \ell}$ correspond to the external port excitations. The amplitudes of the internal port excitations will be adjusted in order to maintain continuity of wave variables in the zeroth order current distributions.

### 7.1.3 Enforcing Continuity of Wave Variables

Let us consider two blocks, $A$ and $B$, and the connection between the $i^{th}$ internal port of block $A$ and the $k^{th}$ internal port of block $B$ as in Figure 7.2. From the analysis of the individual blocks, we have the wave variables $[\tilde{A}_A]$, $[\tilde{A}_B]$, $[\tilde{B}_A]$ and $[\tilde{B}_B]$ due to all the internal and external port excitations of the two blocks. We know that the wave variables must be continuous along
any transmission line-like section between the two blocks. We can therefore enforce continuity of the wave variables at the common connection plane. This is accomplished by the condition that the sum of the wave variables going into block A at the connection equals the sum of the wave variables exiting block B at the connection. We also have the similar condition on the waves exiting A and entering B. This is given more precisely as

$$\begin{align*}
\sum_{t=1}^{P_A} E_{At} \tilde{A}_{At,t} = & \sum_{t=1}^{P_B} E_{Bl} \tilde{B}_{Bl,t} \\
\sum_{t=1}^{P_A} E_{At} \tilde{B}_{At,t} = & \sum_{t=1}^{P_B} E_{Bl} \tilde{A}_{Bl,t}
\end{align*}$$

(7.3)

This gives two conditions involving the \((P_A + P_B)\) source amplitudes \([E_A]\) and \([E_B]\) associated with the two blocks. We can obtain two additional conditions at each additional internal port connection. Since there are \(P_{int}\) internal block connections, we obtain a total of \(2P_{int}\) conditions on internal port source amplitudes. This gives sufficient conditions to determine the amplitudes of the \(2P_{int}\) internal port sources. These conditions involve the \(2P_{int}\) variable source amplitudes as well as the \(P\) known external source amplitudes. The
entire set of constraining conditions can be rewritten as the $2P_{\text{int}} \times 2P_{\text{int}}$ matrix equation

$$[M][E_{\text{int}}]_i = [W]_i \quad i \in \{1, \ldots, P\}. \quad (7.4)$$

Here $[E_{\text{int}}]_i$ is a vector of the $2P_{\text{int}}$ variable internal port source magnitudes, $[W]_i$ contains the wave variables at the internal port connections due to the external port excitations, where the known values of $E_{ae}$ for the external excitations are absorbed in $[W]_i$, and the matrix $[M]$ contains the wave variables at the internal port connections due to the unit amplitude internal port excitations. Since the basic PMESH algorithm requires solving for current distributions due to $P$ independent external port excitations, equation (7.4) involves $P$ systems of equations for the $P$ forcing vectors $[W]_i$.

The choice to use wave variables as the vehicle with which to construct the constraining conditions of Equation (7.4) was made primarily to simplify the overall algorithm implementation. As an alternative to matching the wave variables at the internal port connections, other appropriate parameters could be matched at (or around) the internal port connections as long as two constraining conditions per connection are constructed. For example, the current and voltage, or nearly equivalently the current and charge, at the internal port connections could be matched. However, the voltage to the ground plane is not an easy parameter to deal with within the PMESH formulation. Also, the internal port extension process becomes more complicated since connecting blocks would have to have at least one common cell (instead of only a common side) to implement the other constraints.

7.1.4 Current Construction

Once equation (7.4) is solved for $[E_{\text{int}}]_i$, we can construct $[J_0]_i$ from $[J_a]_i$ and $[E_{\text{int}}]_i$. For each external port excitation status, we obtain $[J_0]_i$ by
adding the block current distributions due to the external sources with the block
distributions due to the internal port sources scaled by the internal port source
magnitudes of $[E_{ini}]$. Very crudely, this would correspond to the operation

$$[J_0]_i = \sum_\alpha \sum_\ell E_{\alpha \ell} [J_0]_{\ell}.\quad (7.5)$$

The only difficulty of this operation is maintaining the proper mapping between
the unknowns on the block level to the unknown numberings of the complete
circuit level. Somewhat hidden in this mapping is the fact that at each internal
port connection side, two block level unknowns map to the same circuit level
unknown. Some care must be exercised so that these currents values are not
added to give twice the current magnitudes at the internal port sides. Construc-
tion of $[\hat{J}_0]_{z_i=ze}$ can be accomplished in an analogous manner.\footnote{This corresponds to assuming the equivalent operation of finding $[J_0]$.} Since the
'sources' of $[g]_{z_i=ze}$ exist only on the external port extensions arms, these sources
will produce an additional $2P$ currents $[\hat{J}_0]_{z_i=ze}$ analogous to the $[J_0]_{\ell}$ due to the
external sources for each block $\alpha$. Also we can get $2P$ vectors $[\hat{W}]_{z_i=ze}$ of wave
variables analogous to $[W]_i$. This produces the matrix equations

$$[M][\hat{E}_{ini}]_{z_i=ze} = [\hat{W}]_{z_i=ze} \quad i \in \{1, \ldots, P\} \quad z_0 = 0, -d.\quad (7.6)$$

Notice that $[M]$ is the same as in equation (7.4) so that no new LU decompo-
sions are needed. Therefore, finding $[\hat{J}_0]_{z_i=ze}$ merely triples the backsubstitution
and current construction time.

7.2 Efficiency of the Network Connection Method

The approximate inversion process of the Network Connection Method
consists of three steps, analyzing the constituent blocks, solving the wave vari-
able continuity equations, and constructing the zeroth order current distribu-
tions. Although it is the sum of these steps that should be compared to the

\(O(N^3/3)\) operations required of the exact inversion, individual comparisons demonstrate the potential for considerable time savings over the exact formulation when using the Network Connection Method.

7.2.1 Wave Variable Continuity Equations Time

From equation (7.3) we notice that each row of \([M]\) contains at most \(2P_{\text{max}}\) nonzero entries where \(P_{\text{max}}\) is the maximum number of ports per block (usually, \(P_{\text{max}} \leq 4\)). Since for large circuits (circuits consisting of many blocks), \(2P_{\text{max}} \ll 2P_{\text{int}}\), \([M]\) is a very sparse matrix. This arises since the constraining conditions used to construct \([M]\) only involve blocks that are directly connected. Also, since most blocks consist of more than one cell, \(2P_{\text{int}} < N\), where \(N\) is the size of the dense coupling matrix. Comparing (7.4) with (3.15), we notice that \([M]\) is smaller than \([ZC]\) so that from this fact alone, solving (7.4) will be more efficient than solving (3.15). The time savings of (7.4) can be greatly increased if we take advantage of the sparsity of \([M]\). Unfortunately, in general the matrix \([M]\) has no special sparsity pattern (bandedness, block structure).

However, algorithms exist (like F01BRF from the NAG FORTRAN library [19]) which can find the equivalent LU decomposition of a randomly sparse matrix in less than \(O(N^3/3)\) operations. The routine F01BRF is able to find an equivalent LU decomposition of \([M]\) in \(O(\frac{5}{2}\tau^2/P_{\text{int}})\) operations where \(\tau\) is then number of nonzero elements in the LU factorization of \([M]\) [19]. Since \([M]\) has at most \(4P_{\text{max}}P_{\text{int}}\) nonzero elements, we can find an equivalent LU decomposition of \([M]\) in \(O(40P_{\text{max}}^2P_{\text{int}})\) operations if no partial pivoting were required.\(^3\) This will almost always be less than \(O(640P_{\text{int}})\) operations. Actually, this is a rather

\(^3\) If there is no pivoting, then the number of nonzero elements in the LU decomposition is the same as the number of nonzero elements in the original matrix so that \(\tau \leq 4P_{\text{max}}P_{\text{int}}\).
conservative estimate. More typically, a decomposition without partial pivoting will take roughly \(O(200P_{\text{int}})\) operations which is very much less than \(O(N^3/3)\) operations for any reasonably sized problems. Notice also that the sparse decomposition time increases linearly with \(P_{\text{int}}\), which means that finding the 'inverse' of \([M]\) will take time that would grow roughly linearly with the number of blocks in the circuit. Even with partial pivoting, (which is necessary to maintain accuracy in the 'inversion') the LU decomposition of the sparse matrix usually increases only by a factor of five or so and still takes considerably less than \(O(N^3/3)\) operations.

### 7.3 Current Construction Time

Another significant part of the approximate inversion involves constructing the zeroth order current distributions \([J_0]\) (and \([\tilde{J}_0]_{\alpha=\varepsilon}\)) from the block current distributions \([J_\alpha]\) (and \([\tilde{J}_\alpha]_{\alpha=\varepsilon}\)). This is, however, at most an \(O(3PP_{\text{max}}N)\) operation. The factor \(P_{\text{max}}N\) comes from having to add up to \(P_{\text{max}}\) current distributions, \([J_\alpha]\), per block for all \(N\) elements of the zeroth order current distributions. The factor of \(3P\) comes from the \(P\) external excitations of \([J_0]\), and the \(2P\) external excitations of \([\tilde{J}_0]_{\alpha=\varepsilon}\). Since \(P\) and \(P_{\text{max}}\) are usually small \((P, P_{\text{max}} \leq 4)\), this current construction time is also much smaller than the \(O(N^3/3)\) operations of the LU decomposition of the exact \([ZC]\). Also note that the current construction time grows linearly with the number of blocks in the circuit.

#### 7.3.1 Block Analysis Overhead

The most problematic step in finding the approximate inverse comes from finding the individual block current distributions \([J_\alpha]\). Finding \([J_\alpha]\) is akin to solving a separate PMESH problem of block \(\alpha\) and therefore consists of two computational parts, the matrix fill and the matrix 'inversion'. If the
wave variables for the blocks could be found without the need for constructing internal port extensions, then the Network Connection Method would enjoy all the computational advantages of replacing the exact coupling matrix [ZC] with multiple smaller matrices that is observed in the Isolated Subcircuit Method. The matrix fill for each block would take $O(\alpha_f N_\alpha^3/3)$ operations where $N_\alpha$ is the number of basis functions in block $\alpha$. However, all the coupling elements of block $\alpha$ are already computed to find [ZC] so that it takes essentially no extra time to fill the individual block matrices once [ZC] is computed. If the circuit consisted of $N_{blk}$ similarly sized blocks, then the inversion time would take roughly $O(N_{ave}^2 N/3)$ operations where $N_{ave} = N/N_{blk}$ is the average number of unknowns in the blocks. For circuits constructed with a large number of blocks, this represents a very large time savings. However, the need for creating internal port extensions creates a significant computational overhead.

With the internal port extensions, block $\alpha$ consists of $N_\alpha + N_{\alpha, int}$ unknowns. The extra $N_{\alpha, int}$ basis functions of the internal port extensions create an additional $N_{\alpha, int}^2 + 2N_\alpha N_{\alpha, int}$ quadruple surface integrals in the matrix fill. Also the LU decomposition for block $\alpha$ is increased by $O(N_{\alpha, int}^3/3 + N_{\alpha, int}^2 N_\alpha + N_{\alpha, int} N_\alpha^2)$ operations. Therefore, the total overhead for block $\alpha$ due to the internal port extensions is $O(N_{\alpha, int}^3/3 + (\alpha_f + N_\alpha) N_{\alpha, int}^2 + (2\alpha_f + N_\alpha) N_{\alpha, int} N_\alpha)$ operations. We have noted previously that $\alpha_f$ is large (especially since the blocks are small so that most of the polynomial curve fits in the quadruple integrals are to maximum order) and $N_\alpha$ is kept small. Also, because $N_{\alpha, int}$ is typically 10 - 40, which is about the same magnitude as $N_\alpha$, we notice that $\alpha_f \gg N_\alpha, N_{\alpha, int}$ and the internal port extension overhead is dominated by the matrix fill time and is roughly $O(\alpha_f N_{\alpha, int}^2 + 2\alpha_f N_\alpha N_{\alpha, int})$.

In order to keep the internal port extension overhead down, we would like to keep $N_{\alpha, int}$ small. However, there is a minimum allowable size to the
internal port extensions. The extensions must be long enough so that the Three Point De-embedding scheme can be employed to find the wave variables at the internal port connection planes. Since the MBUILD blocks are constructed such that the internal port connection planes are separated from the block discontinuities by a short transmission line segment, the internal port extensions can be slightly shorter than the standard port extensions of PMESH. Another way to keep down the internal port extension overhead would be to keep $N_\alpha$ small. However, decreasing $N_\alpha$ would mean an increase in the total number of blocks necessary to construct the entire circuit. This tends to increase the total overhead.

### 7.3.2 Block Redundancies

One effective way to reduce the overhead associated with analyzing the individual blocks is to note that many MMIC circuits exhibit a great deal of redundancy on the block level. That is, although a large circuit will consist of many blocks, it may be possible to construct the circuit from a much smaller set of distinct blocks. Two blocks $\alpha$ and $\beta$ are considered redundant if

$$[J_\alpha]_\ell = [J_\beta]_\ell \quad \ell \in \{1, \ldots, P_\alpha\} \quad P_\alpha = P_\beta.$$ (7.7)

For example, the circuit in Figure 7.3 consists of ten blocks. However, there are only seven distinct blocks if we account for translational redundancies and only four distinct blocks if we consider rotational redundancies as well. Of course, the more blocks in a circuit the greater the possibility of redundant blocks. This means that block redundancy is more likely if the circuit is constructed from smaller blocks. This gives an additional reason for using small blocks. Since we do not need to recalculate $[J_\alpha]_\ell$ for redundant blocks, the total block overhead depends on the number of distinct blocks in the circuit. Therefore, greater block redundancy will reduce the significance of the block overhead. Since the optimal
block size is such a complicated function of circuit and block geometries, choosing an optimal block size for all cases is impractical. The actual block sizes were chosen more to provide easy and versatile circuit construction than for efficient circuit simulation.

7.4 Observed Time Savings

The possible time savings of the Network Connection Method are demonstrated in Table 7.1. The top half of the table are the four coupled meander line circuits (see Figure 6.2) analyzed using the Isolated Subcircuit Method in Table 6.1. The bottom half of the table are the results of analyzing a single meander line of the coupled meander line circuits. Therefore, each of these circuits are exactly half the size of a corresponding circuit from the top half of the table. For each of these cases, the individual block overhead was reduced by accounting for translational, but not rotational, redundancies. The circuits were constructed such that the coupled line circuits consist of nine translationally independent blocks while there are seven translationally independent blocks in the single meander line circuits.\footnote{The extra two non-redundant blocks are associated with the two additional ports of the coupled line circuits. While these blocks are rotationally redundant, the data of Table 7.1 reflect only translational redundancy savings.} Although the approximate inversion time for the Network Connection Method is a function of frequency due to the frequency dependence of the individual block overhead (matrix fill time), the dependence is small due to the small size of the constituent blocks. Once again, the overall time savings is a function of frequency due to the strong frequency dependence of the large complete matrix fill time. The Overall Savings results of Table 7.1 are for a frequency range of 10 - 40 GHz.
Table 7.1 Network Connection Method Approximate Inversion and Overall Time Savings

<table>
<thead>
<tr>
<th>Number of Unknowns</th>
<th>Inversion Time Exact*</th>
<th>Inversion Time NCM*</th>
<th>Time Fraction</th>
<th>Overall Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>194</td>
<td>0.94</td>
<td>3.04</td>
<td>3.23</td>
<td>-10 to -20%</td>
</tr>
<tr>
<td>354</td>
<td>5.70</td>
<td>3.46</td>
<td>0.61</td>
<td>3 to 7%</td>
</tr>
<tr>
<td>514</td>
<td>17.23</td>
<td>4.12</td>
<td>0.24</td>
<td>9 to 22%</td>
</tr>
<tr>
<td>674</td>
<td>39.02</td>
<td>5.00</td>
<td>0.13</td>
<td>13 to 32%</td>
</tr>
<tr>
<td>97</td>
<td>0.12</td>
<td>2.18</td>
<td>18.2</td>
<td>-34 to -49%</td>
</tr>
<tr>
<td>177</td>
<td>0.72</td>
<td>2.25</td>
<td>3.12</td>
<td>-8 to -15%</td>
</tr>
<tr>
<td>257</td>
<td>2.17</td>
<td>2.30</td>
<td>1.09</td>
<td>0 to -1%</td>
</tr>
<tr>
<td>337</td>
<td>4.91</td>
<td>2.51</td>
<td>0.51</td>
<td>4 to 9%</td>
</tr>
</tbody>
</table>

* (CPU minutes on a HP375 workstation)

The first thing to note is that for small structures, the Network Connection Method takes more time than the exact PMESH analysis (the Time Fraction is greater than one and the Overall Savings is negative). This is due to the overhead of analyzing the independent blocks which takes roughly 3 minutes for the coupled line circuits and 2 minutes for the single meander line circuits. In addition to this overhead, the approximate inversion of the Network Connection Method is dominated by the current construction time. In fact, for all of these cases, the LU decomposition of \([M]\) in equation (7.4) took less than 2 seconds. For the larger circuits, the time savings of the Network Connection Method exceed the time savings of the Isolated Subcircuit Method given in Table 6.1. The time savings of the largest circuit analyzed is 32% which is the result of reducing the inversion time by a factor of \(1/0.13 = 7.7\). More importantly, since the approximate inversion time of the Network Connection Method is dominated
by the current construction time for highly redundant circuits, it grows as \( N \) instead of \( N^3 \) for the exact formulation so that even greater time savings can be realized for larger circuits. In fact, since the matrix fill time of [ZC] grows as \( N^2 \), the total execution time of the Network Connection Method analyzing circuits with highly redundant block structures will always be dominated by the matrix fill time.

7.5 When It Works

Since the conditions embedded in (7.4) are equivalent to the conditions obtained when cascading network S-parameters, we would expect that the zeroth order current distributions obtained from the Network Connection Method would be very analogous to the current distributions that would be obtained from many circuit simulation algorithms. If we were to de-embed S-parameters from these zeroth order currents without taking advantage of the variational expressions of Equation (5.21), then these zeroth order S-parameters should be comparable to the S-parameters obtained by analyzing the circuit using any circuit simulator that ignores parasitic couplings. In cases where the parasitic coupling effects are small, the variational nature of (5.21) should give corrected S-parameters that are much more accurate than the zeroth order S-parameters.

An example where the Network Connection Method accurately models the parasitic couplings of a circuit is the double stub filter shown in Figure 7.3. This structure has become somewhat of a benchmark for parasitic coupling models [5]. At the frequency which the stubs are a quarter wavelength long (10 GHz for the circuit in Figure 7.3), the open circuit load of the stub end is transferred back to the through line as a short circuit. For a single stub filter, the short circuit presented to the through line causes a deep null in the transmission parameter \((S_{12})\) of the line. Basic network theory would predict a deepening of the null due to the second stub. This is indeed observed in the zeroth order S-parameters
(a) Double Stub Filter Geometry

Port 1

Substrate Thickness = 127 microns
Relative Permittivity = 9.9
Metalization Thickness = 3 microns
Conductivity = $4.55 \times 10^7$ (ohms m)$^{-1}$

Port 2

636 microns
1641 microns
2921 microns

(b) Double Stub Filter $|S_{12}|$

![Graph of |S_{12}| vs Frequency (GHz) showing null at 10 GHz with PMESH, Zeroth Order, and First Order approximations.]

Figure 7.3 Network Connection Results for a Double Stub Filter Circuit.
as displayed in Figure 7.3. However, the zeroth order parameters do not account for parasitic couplings between the stubs. This coupling causes a split in the effective lengths of the two stub system that manifests itself as a 'double dip' in the frequency response of $S_{12}$ for the circuit. Figure 7.3 shows that these coupling effects are predicted quite accurately by the corrected S-parameters of the Network Connection Method. Although the Network Connection Method gives very accurate results for this circuit, it does not do so efficiently. Since the double stub filter is a small circuit (it is constructed with 67 basis functions), the internal port extensions overhead causes the Network Connection Method to take longer to analyze the circuit than the unmodified PMESH algorithm. However, for larger circuits, the Network Connection Method is capable of producing accurate results with a considerable time savings compared to the unmodified PMESH algorithm.

7.6 Weaknesses

The biggest weakness of the Network Connection Method is that the transmission line-like zeroth order current distribution will not always be sufficiently accurate to ensure a small second order error in (5.21). Essentially, this means that the 'parasitic' coupling of the circuit is not truly parasitic, but is an important effect in the circuit performance that needs to be modeled accurately. Unfortunately, it is difficult to predict a priori whether the unmodeled couplings of the zeroth order current distributions in the Network Connection Method are indeed parasitic or if they are an important coupling mechanism. Even more importantly, if the Network Connection Method fails because of these coupling effects, there is no straightforward means of improving the zeroth order current distributions so that the second order errors of the variational expression of Equation (5.21) become small. A very strong example of a circuit where the
Network Connection Method fails is the coupled meander line circuit shown in Figure 6.2.

The failure of the Network Connection Method on the coupled meander line circuit is primarily the result of poor zeroth order current distributions. Figure 7.4 gives the S-parameters de-embedded directly from the zeroth order current distributions (without taking advantage of (5.21)) compared to the unmodified (exact) PMESH analysis results. It should be noted that the zeroth order analysis predicts no currents excited on the second meander line when a source is excited on the first meander line. This is because each of the two meander lines is constructed from a separate set of connected blocks. Had the circuit been constructed such that the lines shared at least one common block (say a coupled line block), then currents would be excited on the entire circuit due to a source on any of the meander line inputs. Because of this construction, the zeroth order analysis of the Network Connection Method predicts no cross coupling so that \( S_{13} = S_{14} = 0 \).

Although the zeroth order analysis crudely predicts the rough behavior of a single meander line, the agreement is quite poor. The minima of the zeroth order \( S_{11} \) response are offset from the exact analysis response, particularly in the lower frequency range. Also, the strong reflection band between 26 and 32 GHz is very poorly modeled by the zeroth order currents. For parts of the frequency range, the zeroth order analysis gives decent results. However, over most of the frequencies the error in the zeroth order analysis (first order error) is large. One of two things happens when the zeroth order currents for this circuit are used in the variational expression of (5.21). If the first order error is small, then the corrected parameters will be more accurate. However, if the first order error is large, then the 'corrected' parameters can be even less accurate than the zeroth order parameters. This is demonstrated in Figure 7.5. Notice that in certain
Figure 7.4 Network Connection Method Zeroth Order S-parameters for the Coupled Meander Line Circuit.
Figure 7.5 Corrected and Zeroth Order $S_{12}$ for the Coupled Meander Line Circuit.

frequency ranges, the corrected $S_{12}$ is more accurate than the zeroth order $S_{12}$ (for instance, from 35 to 40 GHz). However, the 'corrected' parameters behave erratically over much of the frequency range, even giving S-parameters that are greater than one. Notice that these regions of erratic behavior correspond to regions where the zeroth order $S_{12}$ (or $S_{11}$) are significantly in error.

For the coupled meander line circuit, the primary reason that the zeroth order analysis is so poor is the distributive nature of the unmodeled couplings between the constituent blocks. In the zeroth order analysis, each block is coupled to the other blocks only through fundamental mode coupling. However, consider any of the bends in the meanders, especially one that is near the other meander line. Such a bend is relatively close to three other bends and not
too much more distant from two more bends. While the coupling between these bends is small, there are many bends in the circuit so that their cumulative effect is significant. Another mechanism that can introduce error into the zeroth order current distributions is closely related to the distributive nature of the coupling. When each of the blocks are analyzed individually, there can be small errors in the calculated wave variables at the internal port connections. Indeed, a major cause of such errors can be the parasitic couplings that are not included in the individual block analysis. When these blocks are cascaded (by solving for the internal port excitation amplitudes in Equations (7.4) and (7.6)) these errors are compounded by the potentially large number of blocks in the circuit. To reduce this effect, larger blocks need to be analyzed. However, this would reduce the likelihood (and computational advantage) of redundant blocks and would complicate the circuit construction process. A possible compromise would be to consider another hierarchical level in which the individual blocks analysis is replaced by the analysis of small groups of blocks. This greatly increases the complexity of bookkeeping required by the Network Connection Method and hasn’t been pursued.

There is a subtle shortcoming of the Network Connection Method that can arise when analyzing a block that would have parallel port extensions. An example is the U-block shown in Figure 7.6. If the two port extensions are close, then they would form a coupled line system that has two fundamental modes (often designated as odd and even modes when the two lines are identical). However, the Three Point De-embedding procedure assumes that only one fundamental mode (in each direction) exists on the line. This causes the de-embedding to give erroneous results. One possible remedy for this is to add a triangular section to the port extensions as in Figure 7.6. While this keeps the port extensions from being parallel, there still exists two fundamental modes on
Figure 7.6 A U-block With a) Parallel Port Extension and
b) Nonparallel Port extensions.

the skewed line system (but it creates less of a problem than the parallel line
modes). Also, there is a slight reference plane ambiguity due to the triangular
cells and additional bend discontinuities that do not exist in the actual circuit.
Another possible remedy is to ‘zero out’ the elements in the block coupling ma-
trix that correspond to a basis function in one extension coupling to a basis
function in the other extension. This has the advantage of reducing the inter-
nal port extension overhead by reducing the number of elements in the block
coupling matrix that need to be calculated. However, since the basis functions
overlap, there is an ambiguity as to where the extension ends and the block
begins in terms of basis functions instead of the gridded cells. Also this math-
ematical ‘coupling shield’ creates a model that differs slightly from the physical
structure. However, the errors of both these remedies are rather small, especially
compared to the error of allowing the parallel port extensions to couple.

It should be emphasized that the coupled meander line circuit being
analyzed is a somewhat unfair benchmark for the Network Connection Method.
Most circuits do not conspire so greatly against the assumptions made when the
blocks are analyzed individually. The Network Connection Method is capable
of accurately analyzing a very broad range of circuits with a time savings over the unmodified PMESH algorithm. However, the coupled meander line circuit does point out that the Network Connection Method is not a completely robust algorithm. The next chapter presents a method for finding zeroth order current distributions that is more robust than the Network Connection Method. Another benefit to the new algorithm is that it is free of the internal port extension overhead that taints the Network Connection Method.
CHAPTER VIII

The Circle of Influence Method

This chapter presents yet another method for constructing zeroth order current distributions for use in the Perturbational Solution Methodology. This Circle of Influence Method bases its zeroth order model on the observation that the coupling matrix elements for distantly spaced basis functions are small and can be ignored. This method provides the most versatile method of the three presented in this thesis, primarily because the accuracy of the zeroth order model can be improved by simply changing one parameter in the analysis, the circle of influence radius.

8.1 The Zeroth Order Operator

A third method for finding zeroth order current distributions, the Circle of Influence Method, is based on work done by Wu [28]. The basic motivation of this method is the observation that the coupling element of \([ZC]\) between two basis functions decreases in magnitude as the distance between the two basis functions increases. For large separations, the coupling elements \(ZC_{i,k}\) decay roughly as \(1/\rho_{ik}\), where \(\rho_{ik}\) is the distance between basis functions \(i\) and \(k\). Presumably, if \(\rho_{ik}\) is large, \(ZC_{i,k}\) is small and setting \(ZC_{i,k} = 0\) would produce only a small change in \([ZC]\). This gives a basic method for creating a zeroth order coupling matrix \([ZC_0]\) by letting

\[
ZC_{0,i,k} = \begin{cases} 
ZC_{i,k}, & \text{if } \rho_{ik} < \rho_0; \\
0, & \text{if } \rho_{ik} > \rho_0.
\end{cases} \tag{8.1}
\]
for some $\rho_0$. If most of the coupling elements of $[ZC_0]$ are zero, then $[ZC_0]$ can be ‘inverted’ using a sparse algorithm in much less then the $O(N^3/3)$ operations required by the exact inversion.

In the original work of Wu, only $[ZC_0]$ is calculated and inverted. In other words, only a zeroth order solution is found. This gives the additional advantage of decreasing the matrix fill time since only the nonzero elements of $[ZC_0]$ need to be calculated. While this additional savings will not be evident once we employ the Perturbational Solution Methodology since we will have to calculate all the elements of $[ZC]$; hopefully the increase in accuracy due to the variational nature of (5.21) will make up for this loss of time savings.

The process we actually employ to ‘zero out’ the elements of $[ZC]$ to obtain $[ZC_0]$ is not quite as straightforward as Equation (8.1) describes. We again consider the circuit geometry as described by blocks. As with the Network Connection Method, MBUILD provides a convenient means for constructing such a hierarchical description. For each block, a ‘circle of influence’ is defined by some radius $\rho_0$ as is shown in Figure 8.1 where $\rho_0 > 0$. Although not a true circle, the ‘circle’ of influence describes the boundary of the points that lie within a distance of $\rho_0$ of a given block. Any other block that exists completely or partially within a distance $\rho_0$ of block $A$ is considered to lie within the circle of influence of block $A$.

When constructing $[ZC_0]$, we include $ZC_{0i,k} = ZC_{i,k}$ if the interior sides that define the basis functions $i$ and $k$ are part of blocks that lie within each other’s circle of influence. Otherwise, we let $ZC_{0i,k} = 0$. More specifically, block $B$ lies within the circle of influence of block $A$ if $\rho_{AB,min} \leq \rho_0$ where $\rho_{AB,min}$ is the minimum distance of cell vertices in block $A$ to cell vertices in block $B$. Notice that two connected blocks share at least two cell vertices; $\rho_{AB,min} = 0$ if blocks $A$ and $B$ are connected. Since $\rho_0 > 0$, $[ZC_0]$ always includes couplings
between basis functions that lie on directly connected blocks. It is convenient to keep the circle of influence concept reciprocal, that is if block $B$ lies within the circle of influence of block $A$, then block $A$ lies within block $B$'s circle of influence as well. This ensures that $ZC_{0,i,k} = ZC_{0,k,i}$, so that $[ZC_0]$ is symmetric. Defining a single circle of influence radius $\rho_0$ for the entire circuit ensures such a symmetric zeroth order coupling matrix.

By defining the circle of influence on the block level, we obtain a model that includes coupling between circuit elements instead of basis functions. This makes it somewhat easier to understand the parasitic coupling effects as a circuit level description. Also, because of the block level description, $[ZC_0]$ has a block sparsity pattern. For instance, consider the simple circuit of Figure 8.2 with $\rho_0$ defined such that the only block couplings are $AB$, $BC$, $CD$, $AE$, $BE$, and $CE$. The zeroth order coupling matrix $[ZC_0]$ for such a circuit will look something
\[
\begin{bmatrix}
[ZC_{AA}] & [ZC_{AB}] & 0 & 0 & [ZC_{AE}] \\
[ZC_{BA}] & [ZC_{BB}] & [ZC_{BC}] & 0 & [ZC_{BE}] \\
0 & [ZC_{CB}] & [ZC_{CC}] & [ZC_{CD}] & 0 \\
[ZC_{DA}] & [ZC_{DB}] & [ZC_{DC}] & [ZC_{DD}] & [ZC_{DE}] \\
0 & [ZC_{EB}] & 0 & [ZC_{ED}] & [ZC_{EE}]
\end{bmatrix}
\]

(8.2)

where \([ZC_{\alpha\beta}]\) consists of the coupling elements between blocks \(\alpha\) and \(\beta\). For simplicity, the port extensions are considered part of the block to which they are directly connected. In this example, the zeroth order coupling matrix will include coupling of port extension 1 to blocks \(A, B,\) and \(E\) as well as coupling of port extension 2 to blocks \(C\) and \(D\). In general, there is no special structure to \([ZC_0]\) beyond this block pattern (the matrix is not banded for instance) so that it is difficult to utilize the matrix structure in the inversion process. However, if there exists redundancies on the block level, then the matrix structure can be exploited to fill the matrix more efficiently. When this is the case, the block level description makes identifying the redundancies easier than if the circuit is subdivided only on the cell level.

Unfortunately, exploiting block level redundancies to economize the matrix fill time becomes a less significant savings once the Perturbational Solution Methodology is applied because of the need for the complete coupling matrix \([ZC]\). Most of the block redundancies exist for the self-block submatricies
\([ZC_{oo}]\), while the submatrices describing couplings between distinct blocks are less likely to be redundant. For example, for the circuit of Figure 7.3, \([ZC_{AA}]\) and \([ZC_{DD}]\) are identical (within a possible rotational factor) but \([ZC_{AE}] \neq [ZC_{DE}]\). Since \([ZC]\) contains all of these 'off diagonal' submatrices, a smaller percentage of the entire matrix can be filled taking advantage of redundant submatrices.

There exists essentially two methods for describing \(\rho_0\). Since parasitic coupling effects are a function of frequency, it would seem natural to let \(\rho_0\) be a function of frequency in order to obtain a more accurate zeroth order current distribution at higher frequencies where the parasitic coupling effects are more pronounced. Increasing \(\rho_0\) will increase the number of nonzero submatrices, \([ZC_{o\delta}]\), presumably increasing the accuracy of the zeroth order current distributions \([J_0]\). Indeed, it is the ability to increase the accuracy of the zeroth order solution by increasing \(\rho_0\) that makes the Circle of Influence Method more robust than either the Isolated Subcircuit Method or the Network Connection Method. As \(\rho_0 \rightarrow \infty\) (or at least the maximum extent of the circuit), the Circle of Influence Method should reproduce the unmodified PMESH results. However, increasing \(\rho_0\) creates two undesirable results. First, by decreasing the sparsity of \([ZC_0]\), the time required to find \([J_0]\), increases. Secondly, if \(\rho_0\) is a function of frequency, then the decision to include or not include \([ZC_{o\delta}]\) in \([ZC_0]\) must be made at every frequency. If \(\rho_0\) is considered to be a constant, then this decision only needs to be made once for a frequency sweep. Also, since the sparsity pattern does not change with frequency, a more efficient algorithm (F01BSF from the NAG library [19]) can be utilized to find the LU decomposition of \([ZC_0]\) for all frequencies except the first of a frequency sweep. Since parasitic coupling is also a function of the substrate height \(h\), it is convenient to describe \(\rho_0\) in terms of \(h\). A good rule of thumb is that most coupling effects are significant only to a distance of a few substrate thicknesses. Typically, the Circle of Influence
Method works well for $\rho_0$ in the range $3h$ to $7h$. However, for some circuits, $\rho_0$ may need to be even larger to give acceptable results.

8.2 Potential Time Savings

Most of the potential computational savings associated with the Circle of Influence Method is the result of the sparsity of $[ZC_0]$. The extent of the savings depends on the algorithms used to find the LU decomposition of $[ZC_0]$ and performing back substitutions to solve for $[J_o]_i$ and $[\dot{J}_o]_{i,m}$. The algorithms used in this thesis (F01BRF and F04AXF from the NAG library [19]) take $O(5\tau^2/N)$ and $O(2\tau)$ operations to find the LU decomposition of $[ZC_0]$ and perform one back substitution respectively. These must be compared to the $O(N^3/3)$ and $O(N^2)$ operations required for the associated dense matrix algorithms. Here $\tau$ is the number of nonzero elements in the LU decomposition of $[ZC_0]$, which is roughly the same as the number of nonzero elements in $[ZC_0]$. The first thing to note is that if we were to use the sparse routines on a dense matrix, then $\tau = N^2$ and these routines would take $O(5N^3)$ and $O(2N^2)$ operation respectively. Therefore, the matrix inversion would be more expensive with this inappropriate use of the sparse routines. However, if $[ZC_0]$ is sparse enough, then the matrix inversion will be computed more efficiently using the sparse algorithm. However, $\tau$ is not only a function of the sparsity of $[ZC_0]$, but also the number of partial pivots required to find the LU decomposition. It is therefore difficult to predict the level of sparsity in $[ZC_0]$ that represents the break-even point for the sparse algorithm or the overall time savings possible using the Circle of Influence Method without implementing the routines and doing a timing comparison.

There is a small cost computing which block submatrices are to be included in $[ZC_0]$, but this is very minor, particularly when $\rho_0$ is chosen to be frequency independent so that this cost occurs only once in a frequency sweep.
There is a possible matrix fill time savings associated with redundant blocks, but these savings can equally well be applied to the exact PMESH formulation and are not unique to the Circle of Influence Method.

8.2.1 Comparison to the Network Connection Method

The first computational advantage of the Circle of Influence Method over the Network Connection Method is that there is no internal port extension overhead. All of the elements of \([ZC_0]\) are calculated when \([ZC]\) is formed so there are no additional quadruple integrals required by the Circle of Influence Method. This is a very large time savings, especially for circuits with very little block redundancy.

If we compare the sparse matrix \([ZC_0]\) formed by the Circle of Influence Method with the sparse matrix \([M]\) of the Network Connection Method, we notice two things. First, since \(2P_{in} < N\), \([M]\) is a smaller matrix than \([ZC_0]\). Secondly, \([M]\) is much more sparse than \([ZC_0]\). Because we always choose \(\rho_0 > 0\), within the zeroth order model of the Circle of Influence Method, each basis function in block \(\alpha\) always couples to all the other basis functions in block \(\alpha\) as well as all the basis functions associated what blocks directly connected to block \(\alpha\). In addition, each basis function in block \(\alpha\) may couple to many other blocks besides those which are directly connected to block \(\alpha\). Therefore, each row (and column) of \([ZC_0]\) will have at least \(2N_{blk,\text{min}}\) nonzero elements where \(N_{blk,\text{min}}\) is the smallest number of unknowns in any block of the circuit.

This is a very conservative lower bound on the number of nonzero elements per row of \([ZC_0]\). Typically, \(\rho_0\) will be such that each block will couple to several other blocks, not just to itself and one other connected block. Nonetheless, since all MBUILD blocks have at least as many unknowns as internal ports, \(N_{blk,\text{min}} > P_{max}\) so that each row of \([ZC_0]\) has more nonzero elements than each row of \([M]\). Because \([ZC_0]\) is both larger and less sparse than \([M]\), finding
the LU decomposition of \([ZC_0]\) will always take longer than solving the system of equations of (7.4). However, since the Circle of Influence Method has no internal port extension overhead and no individual block analysis, the Circle of Influence Method can be more efficient than the Network Connection Method. In general, a zeroth order current distribution can be more efficiently obtained by the Network Connection Method for very large circuits with highly redundant block structures. For smaller circuits, or circuits without a great deal of block redundancies, the Circle of Influence Method will be more efficient, particularly when \(\rho_0\) is kept small.

8.3 Observed Efficiencies

Table 8.1 gives the Circle of Influence Method approximate inversion time for the same meander line and coupled meander line circuits previously analyzed. The execution time savings of the Circle of Influence Method is given for three circle of influence radii \(\rho_0\); three, seven and ten times the substrate height. Also given in 8.1 is the Sparse Ratio of \([ZC_0]\), which is defined as the number of nonzero elements in \([ZC_0]\) divided by \(N^2\), the total number of elements in \([ZC_0]\) and \([ZC]\). The Time Fraction and Overall Savings columns are defined as in Tables 6.1 and 7.1.

One of the first things to note from Table 8.1 is that the Circle of Influence Method does not always produce a time savings over the unmodified PMESH algorithm. This is the case for the smaller circuits, particularly for the larger values of \(\rho_0\). This is a reflection of the inefficiency of the sparse LU decomposition algorithm when \([ZC_0]\) is not a sparse matrix. Indeed, for all the cases in which the Circle of Influence Method is less efficient than the unmodified PMESH algorithm, the Sparse Ratio is at least 0.38. When the sparse ratio is large like this, it would be more efficient to use the dense matrix
Table 8.1 Circle of Influence Method Approximate Inversion and Overall Time Savings

<table>
<thead>
<tr>
<th>Number of Unknowns</th>
<th>Inversion Time Exact*</th>
<th>Circle Radius</th>
<th>Sparse Ratio</th>
<th>Inversion Time CIM*</th>
<th>Time Fraction</th>
<th>Overall Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>194</td>
<td>0.94</td>
<td>3h</td>
<td>0.20</td>
<td>0.40</td>
<td>0.43</td>
<td>2 to 5%</td>
</tr>
<tr>
<td>194</td>
<td>0.94</td>
<td>7h</td>
<td>0.40</td>
<td>0.92</td>
<td>0.98</td>
<td>0%</td>
</tr>
<tr>
<td>194</td>
<td>0.94</td>
<td>10h</td>
<td>0.53</td>
<td>1.29</td>
<td>1.4</td>
<td>-2 to -3%</td>
</tr>
<tr>
<td>354</td>
<td>5.70</td>
<td>3h</td>
<td>0.11</td>
<td>1.28</td>
<td>0.22</td>
<td>6 to 15%</td>
</tr>
<tr>
<td>354</td>
<td>5.70</td>
<td>7h</td>
<td>0.25</td>
<td>3.64</td>
<td>0.64</td>
<td>3 to 7%</td>
</tr>
<tr>
<td>354</td>
<td>5.70</td>
<td>10h</td>
<td>0.34</td>
<td>4.86</td>
<td>0.85</td>
<td>1 to 3%</td>
</tr>
<tr>
<td>514</td>
<td>17.23</td>
<td>3h</td>
<td>0.08</td>
<td>2.63</td>
<td>0.15</td>
<td>9 to 24%</td>
</tr>
<tr>
<td>514</td>
<td>17.23</td>
<td>7h</td>
<td>0.18</td>
<td>7.10</td>
<td>0.41</td>
<td>7 to 17%</td>
</tr>
<tr>
<td>514</td>
<td>17.23</td>
<td>10h</td>
<td>0.25</td>
<td>10.42</td>
<td>0.60</td>
<td>4 to 11%</td>
</tr>
<tr>
<td>674</td>
<td>39.02</td>
<td>3h</td>
<td>0.06</td>
<td>4.12</td>
<td>0.11</td>
<td>14 to 33%</td>
</tr>
<tr>
<td>674</td>
<td>39.02</td>
<td>7h</td>
<td>0.14</td>
<td>12.05</td>
<td>0.31</td>
<td>11 to 25%</td>
</tr>
<tr>
<td>674</td>
<td>39.02</td>
<td>10h</td>
<td>0.20</td>
<td>17.35</td>
<td>0.44</td>
<td>8 to 20%</td>
</tr>
<tr>
<td>97</td>
<td>0.12</td>
<td>3h</td>
<td>0.38</td>
<td>0.12</td>
<td>1.0</td>
<td>0%</td>
</tr>
<tr>
<td>97</td>
<td>0.12</td>
<td>7h</td>
<td>0.68</td>
<td>0.28</td>
<td>2.3</td>
<td>-3 to -4%</td>
</tr>
<tr>
<td>97</td>
<td>0.12</td>
<td>10h</td>
<td>0.90</td>
<td>0.45</td>
<td>3.7</td>
<td>-5 to -8%</td>
</tr>
<tr>
<td>177</td>
<td>0.72</td>
<td>3h</td>
<td>0.21</td>
<td>0.30</td>
<td>0.42</td>
<td>2 to 4%</td>
</tr>
<tr>
<td>177</td>
<td>0.72</td>
<td>7h</td>
<td>0.41</td>
<td>0.73</td>
<td>1.0</td>
<td>0%</td>
</tr>
<tr>
<td>177</td>
<td>0.72</td>
<td>10h</td>
<td>0.55</td>
<td>1.07</td>
<td>1.5</td>
<td>-2 to -4%</td>
</tr>
<tr>
<td>257</td>
<td>2.17</td>
<td>3h</td>
<td>0.15</td>
<td>0.49</td>
<td>0.23</td>
<td>4 to 10%</td>
</tr>
<tr>
<td>257</td>
<td>2.17</td>
<td>7h</td>
<td>0.29</td>
<td>1.20</td>
<td>0.55</td>
<td>2 to 6%</td>
</tr>
<tr>
<td>257</td>
<td>2.17</td>
<td>10h</td>
<td>0.40</td>
<td>1.74</td>
<td>0.80</td>
<td>1 to 3%</td>
</tr>
<tr>
<td>337</td>
<td>4.91</td>
<td>3h</td>
<td>0.11</td>
<td>0.70</td>
<td>0.14</td>
<td>6 to 15%</td>
</tr>
<tr>
<td>337</td>
<td>4.91</td>
<td>7h</td>
<td>0.22</td>
<td>1.64</td>
<td>0.33</td>
<td>5 to 12%</td>
</tr>
<tr>
<td>337</td>
<td>4.91</td>
<td>10h</td>
<td>0.31</td>
<td>2.43</td>
<td>0.49</td>
<td>4 to 9%</td>
</tr>
</tbody>
</table>

* (CPU minutes on a HP375 workstation)
LU decomposition and backsubstitution algorithms on \([ZC_0]\). However, for the larger circuits and smaller \(\rho_0\) combinations, the Circle of Influence Method is more efficient than the unmodified PMESH algorithm.

Comparing the time savings of the Circle of Influence Method to the Network Connection Method, we notice that the Circle of Influence Method is more efficient for all the structures analyzed when \(\rho_0 = 3h\). Also, the Circle of Influence Method is more efficient than the Network Connection Method for all the single meander line circuits regardless of the value of \(\rho_0\). However, the Network Connection Method is more efficient for the large coupled meander line circuits when \(\rho_0 = 7h\) or \(\rho_0 = 10h\). Also, it appears that the Network Connection Method would be more efficient than the Circle of Influence Method with \(\rho_0 = 3h\) for coupled meander line circuit larger than those analyzed for Table 8.1. Of course, this assumes that the larger coupled meander line circuits would include no additional block types. However, for the circuits analyzed, the Circle of Influence Method produced the greatest observed overall time savings, 33% for the largest coupled meander line circuit at 40 GHz with \(\rho_0 = 3h\).

Clearly the relative efficiencies of the Circle of Influence Method and the Network Connection Method depend on a number of parameters; circuit size, circuit geometry, \(\rho_0\), and the block redundancies are the most important parameters. In general, it is difficult to determine which Perturbational Solution Methodology will produce the most efficient algorithm for a given circuit.

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1 Actually, it would make no sense to use \([ZC_0]\) in these cases since the unmodified PMESH algorithm is both more efficient and more accurate. However, if on the zeroth order parameters are to be calculated as in the work of Wu, then the circle of influence approach is more efficient since only the nonzero elements of \([ZC_0]\) need be calculated.
Nonetheless, the Circle of Influence Method is roughly as efficient or more efficient than the Network Connection Method for a wide range of circuits for $\rho_0 = 3h$. Of course, it may be necessary to sacrifice efficiency by increasing $\rho_0$ in order to improve the accuracy of the analysis.

8.4 Results

Figures 8.3 and 8.4 show the zeroth order S-parameters of the 514 unknown coupled meander line circuit (see Figure 6.2) calculated using the Circle of Influence Method with $\rho_0 = 3h$ compared to the unmodified PMESH results. Since the two coupled meander lines are separated by a distance $(2.2h)$ which is less than $\rho_0$, the zeroth order model does include cross coupling between the meander lines. We can see that these zeroth order results are better overall than the zeroth order results obtained from the Network Connection Method (see Figure 7.4). The reason for this improvement is that in the Network Connection Method, each circuit element block (say a single bend in the meander lines) is analyzed only in the presence of the internal port extensions. However, in the Circle of Influence Method, the zeroth order model includes couplings of each circuit element block to some of the closer elements of the circuit. For instance, the coupled meander line circuit is constructed such that the U-bends of the meanders consist of two connected L-bends. The Circle of Influence Method will always include couplings between such connected blocks. Also, since the U-bends within a single meander line are less than $3h = 300$ microns apart, the zeroth order model includes couplings between adjacent bends within a single meander line as well as the coupling between the adjacent lengths of parallel lines. Furthermore, since the U-bends of the two meander lines are only $2.2h$ apart, some coupling between these discontinuities are also directly accounted for in the zeroth order model. The inclusion of all these couplings makes the Circle of Influence Method zeroth order model more accurate than the Network
Figure 8.3 Zeroth Order S-parameters for the Coupled Meander Line Circuit, $\rho_0 = 3h$. 
Figure 8.4 Zeroth Order S-parameters for the Coupled Meander Line Circuit, $\rho_0 = 3h$. 

(a) $|S_{13}|$

(b) $|S_{14}|$
Connection Method zeroth order model that includes only the fundamental mode couplings of directly connected blocks.

Applying Equation (5.21) to the zeroth order current distributions found from the Circle of Influence Method with \( \rho_0 = 3h \) produces the corrected S-parameters displayed in Figures 8.5 and 8.6. For the most part, the corrected S-parameters are more accurate than the zeroth order parameters and they are reasonably accurate compared to the unmodified PMESH results. However, part of the frequency range are poorly modeled by the Circle of Influence Method with \( \rho_0 = 3h \), particularly the 25 to 35 GHz range. Within this range is the frequency at which a single meander line is roughly a wavelength long. This produces a coupling effect that is no longer localized to the closely spaced discontinuities. A more distributive description of the couplings is necessary. Fortunately, the Circle of Influence Method can more fully account for the distributive nature of the couplings by increasing \( \rho_0 \).

Figures 8.7 and 8.8 give the corrected S-parameters for the coupled meander line circuit from the Circle of Influence Method for \( \rho_0 = 3h, 7h, \) and \( 10h \) within the 25 to 35 GHz range. The result for the rest of the frequency range are at least as accurate as the results in this range. Notice that increasing \( \rho_0 \) to \( 7h \) significantly increases the accuracy of the corrected parameters over the \( \rho_0 = 3h \) analysis. However, the Circle of Influence Method still gives questionably accurate results with \( \rho_0 = 7h \) in this range. When \( \rho_0 \) is increased to \( 10h = 1000 \) microns, the Circle of Influence Method gives very accurate corrected S-parameters for the entire frequency range. It should be noted that the coupled meander line circuit is constructed such that the longest length of connected straight line blocks is 1165.5 microns long, which is greater than \( 10h \). Therefore, even for the \( \rho_0 = 10h \) case, not all the couplings within a single meander are included. However, the Circle of Influence Method is still able to
Figure 8.5 First Order S-parameters for the Coupled Meander Line Circuit, $\rho_0 = 3h$. 
Figure 8.6 First Order S-parameters for the Coupled Meander Line Circuit, $\rho_0 = 3h$. 
Figure 8.7 First Order S-parameters for the Coupled Meander Line Circuit.
Figure 8.8 First Order S-parameters for the Coupled Meander Line Circuit.
accurately account for the distributive nature of the couplings associated with the resonance of a single meander.

The execution time cost of the increased accuracy for the \( \rho_0 = 10h \) model was an increase by a factor of four (2.63 minutes to 10.42 minutes) in the approximate inversion time. The \( \rho_0 = 10h \) model still produces a savings of 4 to 11% over the unmodified PMESH algorithm. Actually, since the increased accuracy of the \( \rho_0 = 10h \) model wasn't necessary over the entire frequency range, the time penalty over the \( \rho_0 = 3h \) (or \( \rho_0 = 7h \)) case could be reduced by increasing \( \rho_0 \) to \( 10h \) only when truly necessary. Also, the coupled meander line circuit is a harsh test of the accuracy of the Circle of Influence Method because the resonant nature of the circuit around 29 GHz. Near this frequency, the assumption that the unmodeled coupling effects are parasitic is strongly challenged. Therefore, it isn't surprising that the perturbationally based Circle of Influence Model struggles in this frequency range.

8.5 Weaknesses

The analysis of the coupled meander line circuit shows that the Circle of Influence Method is the most versatile method of the three methods discussed for obtaining zeroth order current distributions. However, comparison with the Isolated Subcircuit Method points out a major weakness. The accuracy of the Isolated Subcircuit Method results points out that the inclusion of cross coupling between basis functions on the different meander lines is not important in the zeroth order model. The cross coupling S-parameters (\( S_{13} \) and \( S_{14} \)) can be accurately modeled in the Perturbational Solution Methodology if the zeroth order current distributions of a single isolated meander line are accurate enough. The error in the corrected cross coupling S-parameters in the Circle of Influence Method are primarily a result of the error in the direct line S-parameters (\( S_{11} \) and \( S_{12} \)). Notice that in Equation (3.21), the S-parameters depend on all the wave
variables \( \hat{A}_{i,k} \) and \( \hat{B}_{i,k} \). Since for the coupled meander line circuit \( \hat{A}_{1,2} > \hat{A}_{1,3} \), and for much of the frequency range \( \hat{A}_{1,2} \gg \hat{A}_{1,3} \), a first order error in \( \hat{A}_{1,2} \) can be as large as \( \hat{A}_{1,3} \). Therefore, the error in \( S_{12} \) can swamp \( S_{13} \). Since it is not as important to model the couplings between blocks that are not part of the same subcircuit, a sparser \([ZC_0]\) can be constructed by specifying a smaller circle of influence radius for blocks that are not part of the same subcircuit. Although not done in this thesis, the framework required to specify two different circle of influence radii for blocks that are or are not part of the same subcircuit has been developed by Wu and can easily be implemented into the Circle of Influence Method.² The possible time savings of such an approach can be inferred from the Isolated Subcircuit Method results. Essentially, \([ZC_0]\) for the Isolated Subcircuit Method can be obtained from the Circle of Influence Method by letting \( \rho_0 = \infty \) for blocks on the same subcircuit and \( \rho_0 = 0 \) for blocks on different subcircuits. Presumably, using the sparse nature of the individual subcircuit submatrices for a finite \( \rho_0 \) would be more efficient than the dense submatrix routines used in the Isolated Subcircuit Method. For example, since the Circle of Influence Method produces a factor of 2 (Time Fraction = 0.49) savings in the approximate inversion of the 337 unknown single meander line circuit with \( \rho_0 = 10h \) (see Table 8.1), letting \( \rho_0 = 10h \) for blocks on the same subcircuit and \( \rho_0 = 0 \) for blocks on different subcircuits should produce a Time Fraction that is roughly 0.13 (half the Time Fraction of the Isolated Subcircuit Method) for the 674 unknown coupled meander line circuit without much loss in accuracy. This is a considerable improvement over the Time Fraction of 0.44 observed when \( \rho_0 = 10h \) for all the blocks.

² The designation of two circle radii for same and different subcircuit couplings is another bookkeeping task that is simplified by the block level description.
Another shortcoming of the Circle of Influence Method that would be relatively easy to overcome is the increased execution time over the unmodified PMESH algorithm for circuits in which \([ZC_0]\) is not very sparse. If the Sparse Ratio of \([ZC_0]\) is greater than some value, then it would be more efficient and more accurate to bypass the Circle of Influence Method and analyze the circuit using the unmodified PMESH algorithm. The cutoff value of the Sparse Ratio appears to be about 0.4 (from Table 8.1) but is a function of \(N\). Of course, switching to the unmodified PMESH algorithm will not create a major time penalty since the cost of finding the Sparse Ratio is small and \([ZC]\) is calculated in both the Circle of Influence Method and the unmodified PMESH algorithm.
CHAPTER IX

Tying it all Together

This chapter looks back on what was accomplished in this thesis and tries to highlight the successes and the limitations of the work presented in the preceding chapters. While looking back on the paths undertaken, it is prudent to look forward to new avenues through which the work started here can be expanded on and improved in the future.

9.1 The Equivalent Circuit De-embedding

The original intent of the Equivalent Circuit De-embedding scheme was to provide a de-embedding technique which would provide a more efficient means of implementing the Perturbational Solution Methodology. The basic mechanism for this hoped for efficiency is the need for only $P$ (the number of circuit ports) current distributions when the Perturbational Solution Methodology is applied using the self-adjoint operator of PMESH. This is contrasted with the $3P$ current distributions required when using the Three Point De-embedding scheme within the Perturbational Solution Methodology. Unfortunately, this perceived efficiency was lessened by the need for longer port extensions, the subsidiary structures needed to determine the source and transmission line parameters $Y_s$ and $Z_c$, as well as the need to develop a quasi-matched load for the port extensions. Since the Equivalent Circuit De-embedding demonstrated a reduction in accuracy determining the S-parameters of a circuit, the Equivalent Circuit De-embedding does not live up to its initial expectations. The Equivalent Circuit De-embedding does provide one important side benefit that gives
it validity for a small range of problems. Analysis of the subsidiary structure is able to determine a characteristic impedance for uniform transmission lines consistent with the planar framework of the PMESH model. However, to obtain accurate results for $Z_c$ requires a very fine gridding in the transverse dimension of the subsidiary structure which makes finding $Z_c$ a relatively time consuming process which is not justified if only the S-parameter characterization of a given circuit is of interest.

9.2 The Various Perturbational Solution Methodologies

9.2.1 The Isolated Subcircuit Method

The Isolated Subcircuit Method is a straightforward and relatively easy to understand means to implement the Perturbational Solution Methodology on a small class of circuits. It demonstrates that the Perturbational Solution Methodology can produce accurate S-parameter characterizations of circuits more efficiently than the unmodified PMESH algorithm. However, its limitation to circuits consisting of two or more distinct subcircuits precludes the use of this method as a general purpose algorithm. This method does have merit if it is incorporated within another Perturbational Solution Methodology or as the basis for sensitivity analysis on the relative positionings between distinct subcircuits. The only time savings of the Isolated Subcircuit Method within another Perturbational Solution Methodology would be in the inner product calculations since most of the savings in the matrix inversion process would presumably be due to the other Perturbational Solution Methodology implementation.

9.2.2 The Network Connection Method

The Network Connection Method provides a potentially very efficient method of implementing the Perturbational Solution Methodology, particularly for very large circuits with a high degree of block redundancy. The greatest
strengths of this method are the efficient utilization of block level redundancies and its close correspondence to network theory modeling. The Network Connection Method does suffer from two significant shortcomings. First is the computational overhead associated with the internal port extensions. Secondly, the zeroth order current distributions can be prone to be too inaccurate for the perturbational correction to be stable. Both of these fallbacks keep the Network Connection Method from being a good general purpose means of implementing the Perturbational Solution Methodology. The Network Connection Method is restricted to circuits with a high degree of block redundancy due to the overhead inefficiencies as well as to circuits with only weak to moderate parasitic coupling effects.

Two possible extensions to the Network Connection Method could be attempted to reduce the significance of the internal port extension overhead. First, the internal port extensions could be eliminated completely by constructing the zeroth order current distributions directly from an external network theory based analysis system such as those that exist in a number of commercial software packages. These packages could provide zeroth order current distributions extremely efficiently. Unfortunately, such an approach would tend to aggravate the accuracy shortfall of the Network Connection Method. The current distributions obtained from a standard network theory analysis package will tend to be less accurate than the distributions found in the Network Connection Method since they would (ordinarily) consist only of fundamental mode currents. Higher order mode current distributions localized to the circuit discontinuities are in general not obtainable through standard network theory techniques. A second possible modification to the Network Connection Method would be to develop another hierarchical descriptive level and analyze small groups of blocks as is discussed in Section 7.6. However, this would tend to
reduce the advantages gained from block redundancies and increases the individual structure analysis time. Essentially, this is merely an adjustment on the block size, a parameter which was not optimized for the Network Connection Method algorithm. On the up side, grouping the blocks would tend to increase the accuracy of the zeroth order current distributions.

9.2.3 The Circle of Influence Method

The Circle of Influence Method provides the best combination of accuracy and efficiency of all the implementations of the Perturbational Solution Methodologies discussed in this thesis. Also, the ability to vary $\rho_0$ makes this a very versatile algorithm; adjusting $\rho_0$ is a simple way of adjusting the accuracy/efficiency tradeoff inherent in the Circle of Influence Method. However, currently the process of adjusting $\rho_0$ is crude at best. A circuit can be analyzed with a given value of $\rho_0$ and if the results are determined to be too inaccurate, the circuit can be re-analyzed with a larger value of $\rho_0$. It would be convenient (though very difficult to implement) for the Circle of Influence Method to be able to automatically scale $\rho_0$ to provide a given level of accuracy. The biggest obstacle to such a procedure is that the accuracy of the corrected S-parameters is difficult to judge without knowing the exact S-parameter values. Other improvements to the Circle of Influence Method include those discussed in Section 8.5 such as designating two (or possibly more) different values of $\rho_0$ for blocks that are part of the same or different subcircuits. Another possible enhancement to the Circle of Influence Method that addresses the matrix fill time issue would be to apply the perturbational correction not with the exact matrix [ZC], but with an approximation [ZC₁] which is constructed in the same manner as [ZC₀] only with a different circle of influence radius $\rho_1 > \rho_0$. This would eliminate the computation of the extremely distant coupling elements.
9.3 The Perturbational Solution Methodology in General

In general, the Perturbational Solution Methodology has some weaknesses that should be pointed out. First, the Perturbational Solution Methodology only addresses the matrix inversion portion of the full wave analysis algorithm, it does not improve the matrix fill time. Improving the efficiency of the matrix fill time in the PMESH model would dramatically increase the significance of the time savings obtained from all three implementations of the Perturbational Solution Methodology. As an example, consider the 674 unknown, \( \rho_0 = 3h \) entry of Table 8.1. Although the matrix inversion time was reduced by 89\%, the overall execution time was reduced by as little as 14\% by the application of the Circle of Influence Method. This means that the matrix fill took as much as 84\% of the unmodified PMESH execution time for this circuit. If the matrix fill time were reduced by a factor of 2, then the overall savings range due to the Circle of Influence Method on this structure with \( \rho_0 = 3h \) would increase to 24 - 48\%. The improvements in the Network Connection Method would be even more significant because the internal port extension overhead would also decrease. The values presented in this thesis all reflect the time savings obtained by changing the Green Function curve fitting algorithm as discussed in Section A.5. However, further improvements in the matrix fill efficiency should be possible and definitely would be desirable.

Another weakness of all the Perturbational Solution Methodology implementations is that they all are based on the assumption that the unmodeled coupling effects are truly parasitic (almost by definition small) so that their effect can be accounted for using a simple perturbational model. The Perturbational Solution Methodology is bound to struggle when it tries to account for strong coupling mechanisms such as closely spaced discontinuities or couplings aggravated by resonant effects. In such cases a more exact formulation will be
Figure 9.1 A Coupled Resonant Line Circuit.

required to accurately describe the circuit performance. This requires some insight to the circuit performance and the zeroth order model on the part of a user of the Perturbational Solution Methodology algorithm and therefore keeps any Perturbational Solution Methodology based analysis from being truly general.

Besides the inability to accurately account for strong coupling mechanisms, the perturbational approach can fail for a general class of problems if the Perturbational Solution Methodology implementations are improperly applied. This shortcoming can be illustrated by considering a circuit such as the coupled resonant line of Figure 9.1. In the absence of block A, the circuit is a simple through line with $S_{11} = 0$ and $|S_{12}| \approx 1$ for all frequencies. However, the presence of the second line will produce dips in the $S_{12}$ frequency response for frequencies at which the second line resonates. However, the Isolated Subcircuit Method will fail to predict any decrease in $|S_{12}|$ due to block A if it is considered as a separate subcircuit. This is because there is no zeroth order excitation of this second subcircuit, it contains no port excitations. Similarly, the Network Connection Method will fail to predict any decrease in $|S_{12}|$ if blocks A and B are considered separate blocks as will the Circle of Influence Method if blocks A and B are separated by a distance greater than $\rho_0$. However, both these implementations will produce a decrease in the corrected $|S_{12}|$ if blocks A and B are
conceptually considered one block of if they are two blocks with a separation less than $\rho_0$ in the Circle of Influence Method.

Yet another weakness of both the Network Connection Method and the Circle of Influence Method is that the increased time savings of these techniques for larger sized problems are not realized if the problem size is increased by increasing the fineness of the block griddings. Both of these methods base their time savings on the block level structure of the circuit. The increased efficiency of these circuits for larger problems comes from the increased separation of the blocks (and therefore decreased significance of the coupling between these blocks) for spatially large circuits.

One obvious extension of the Perturbational Solution Methodology would be the inclusion of the Perturbational Solution Methodology within the framework of linear operator problems other than PMESH. The extension of the Perturbational Solution Methodology would be straightforward for other Method of Moment algorithms based on the PMESH model for coplanar stripline [14] and coplanar waveguide structures [20]. Furthermore, the Perturbational Solution Methodology can also be extended to other computationally extensive numerical algorithms as well, whether they be other Method of Moment based codes, finite element code, or whatever. However, doing so may require new implementations of the Perturbational Solution Methodology beyond the three presented here in order to take full advantage of unique characteristics of each particular algorithm.

Perhaps the class of problems for which the Perturbational Solution Methodology would prove most useful is sensitivity analysis. Presumably, the small changes in a circuit's performance due to manufacturing tolerances (line widths, substrate parameters, etc.) in MMIC circuits could be well modeled using a perturbational approach. The Perturbational Solution Methodology could
save considerable computational time by replacing the many matrix inversions (a change in any parameter would produce a separate linear system) of the sensitivity analysis with the perturbational functional. However, a truly efficient implementation of sensitivity would require considerable effort to produce a means to efficiently calculate the change in the impedance coupling matrix due to small changes in these parameters.

Finally, the Perturbational Solution Methodology can be extended into a much more powerful iterative technique (see Appendix C). The most likely implementation of this approach would be to use the Circle of Influence technique with a small circle of influence radius to find a zeroth order current distribution. This entire distribution can then be iteratively updated until the current samples needed for the Two Point deembedding converges. This provides a means for estimating the error of the approximation as well as increasing the accuracy of the approximations only when needed. These are the greatest drawbacks to the Perturbational Solution Methodology as presented in this thesis.
BIBLIOGRAPHY


APPENDIX A

PMESH Details

A.1 The Basis Functions

The basis function $\tilde{B}_n(\vec{r})$ for the $n^{th}$ interior side can be written as

$$\tilde{B}_n(\vec{r}) = \tilde{R}_{n,\alpha}(\vec{r}) - \tilde{R}_{n,\alpha'}(\vec{r}), \quad (A.1)$$

where $\tilde{R}_{n,\alpha}(\vec{r})$ is a rooftop function associated with side $n$ defined on cell $\alpha$ and cells $\alpha$ and $\alpha'$ share the common side $n$. The rooftop function $\tilde{R}_{n,\alpha}(\vec{r})$ is describable as

$$\tilde{R}_{n,\alpha}(\vec{r}) = \begin{cases} \frac{\ell}{2A}(\vec{r}_n - \vec{r}), & \vec{r} \in \text{triangle } \alpha; \\ \frac{\ell}{A}(\vec{r}_n - \vec{r}) \cdot \hat{n}, & \vec{r} \in \text{rectangle } \alpha. \end{cases} \quad (A.2)$$

where $\ell$ is the length of side $n$, $A$ is the area of cell $\alpha$, and $\hat{n}$ is the outward pointing normal along side $n$. If cell $\alpha$ is a triangle, $\vec{r}_n$ is the position vector of the vertex opposite side $n$. If cell $\alpha$ is a rectangle, $\vec{r}_n$ is any position vector along the side opposite side $n$, say one of the vertices opposite side $n$. See Figure A.1.

We will also need to know the divergence of the basis functions which is given by

$$\nabla \cdot \tilde{B}_n(\vec{r}) = \nabla \cdot \tilde{R}_{n,\alpha}(\vec{r}) - \nabla \cdot \tilde{R}_{n,\alpha'}(\vec{r}). \quad (A.3)$$

The divergence of the rooftop functions is given simply by

$$\nabla \cdot \tilde{R}_{n,\alpha}(\vec{r}) = \frac{\ell}{A} \quad \vec{r} \in \text{cell } \alpha \quad (A.4)$$

for both rectangular and triangular cells.
A.2 The Matrix Elements

If we expand $\mathcal{L}$ in equation (3.14) using (3.7) we get

$$ZC_{m,k} = (\bar{T}_m(\bar{r}), L\bar{B}_k(\bar{r})) = \frac{j\omega \mu_0}{4\pi k_0^2} \int_S \int_{S'} G_m(\rho) \bar{T}_m(\bar{r}) \cdot \bar{B}_k(\bar{r}') ds' ds$$

$$+ \frac{j\omega \mu_0}{4\pi k_0^2} \int_S \int_{S'} \bar{T}_m(\bar{r}) \cdot \nabla G_e(\rho') \nabla' \cdot \bar{B}_k(\bar{r}') ds' ds$$

$$+ \frac{1}{2} \int_S Z_s(\bar{r}; t) \bar{T}_m(\bar{r}) \cdot \bar{B}_k(\bar{r}) ds$$

(A.5)

The second integral on the right hand side can be integrated by parts

$$\frac{j\omega \mu_0}{4\pi k_0^2} \int_S \int_{S'} \bar{T}_m(\bar{r}) \cdot \nabla G_e(\rho) \nabla' \cdot \bar{B}_k(\bar{r}') ds' ds$$

$$= \frac{j\omega \mu_0}{4\pi k_0^2} \left[ \oint_C \int_{S'} G_e(\rho) \bar{T}_m(\bar{r}) \cdot \hat{n} \nabla' \cdot \bar{B}_k(\bar{r}') ds' d\ell \right]$$

$$- \int_S \int_{S'} G_e(\rho) \bar{\nabla} \cdot \bar{T}_m(\bar{r}) \nabla' \cdot \bar{B}_k(\bar{r}') ds' ds$$

(A.6)

where the line integral is performed counterclockwise along $C$, the outer boundary of $S$, and $\hat{n}$ is the outward pointing normal along $C$. We note that the
normal component of the current is zero on the boundary of $S$ (and the normal component of $\mathbf{T}_m(\mathbf{r})$ is zero on the boundary of $S_m$) so that this integral is zero.

This leaves

$$Z_{C_m,\epsilon} = (\mathcal{T}_m(\mathbf{r}), \mathcal{L}\mathbf{B}_k(\mathbf{r})) = \frac{j\omega \mu_0}{4\pi} \left\{ \int_S \int_{S'} G_m(\rho) \mathbf{T}_m(\mathbf{r}) \cdot \mathbf{B}_k(\mathbf{r}') \, ds' \, ds \\
+ \frac{j\omega \mu_0}{4\pi k_0^2} \int_S \int_{S'} G_e(\rho) \nabla \cdot \mathbf{T}_m(\mathbf{r}) \nabla' \cdot \mathbf{B}_k(\mathbf{r}') \, ds' \, ds \\
+ \frac{1}{2} \int_S Z_i(\mathbf{r}; t) \mathbf{T}_m(\mathbf{r}) \cdot \mathbf{B}_k(\mathbf{r}) \, ds \right\} \quad (A.7)$$

### A.3 The Green Functions

The scalar Green Functions $G_e(\rho)$ and $G_m(\rho)$ given by (3.8) have integrands that converge slowly. In order to speed the convergence of these integrals, we extract the analytic terms $\frac{2}{\epsilon_r + 1} \frac{1}{\rho}$ and $\frac{2\mu_r}{\mu_r + 1} \frac{1}{\rho}$ respectively from the integrals of (3.8). These analytic terms are found by taking the large argument ($\xi \rightarrow \infty$) expansion of the integrands of (3.8) and noting that

$$\int_0^\infty J_0(\xi \rho) \, d\xi = \frac{1}{\rho}. \quad (A.8)$$

Extracting this analytic term from $G_e(\rho)$ and $G_m(\rho)$ gives

$$G_e(\rho) = \frac{2}{\epsilon_r + 1} \frac{1}{\rho} + \int_0^\infty 2J_0(\xi \rho) \left\{ \frac{\xi[u_0 + \mu_r u_1 \tanh(u_1 \beta)]}{D_{te} D_{im}} - \frac{1}{\epsilon_r + 1} \right\} d\xi$$

$$G_m(\rho) = \frac{2\mu_r}{\mu_r + 1} \frac{1}{\rho} + \int_0^\infty 2J_0(\xi \rho) \left\{ \frac{\mu_r \xi}{D_{te}} - \frac{\mu_r}{\mu_r + 1} \right\} d\xi \quad (A.9)$$

Numerically evaluating the integrals of (A.9) is quicker and more stable than evaluating the integrals as given in (3.8). The integrals of (A.9) are evaluated for specific values of $\rho > 0$ by deforming the path of integration in the complex $\xi$ plane. The $J_0(\xi \rho)$ term in (A.9) can be replaced by $\frac{1}{2} H_0^2(\xi \rho)$ and the limits of integration changed from $[0, \infty)$ to $(-\infty, \infty)$. This gives

$$G_e(\rho) = \frac{2}{\epsilon_r + 1} \frac{1}{\rho} + \int_{-\infty}^\infty H_0^2(\xi \rho) \left\{ \frac{\xi[u_0 + \mu_r u_1 \tanh(u_1 \beta)]}{D_{te} D_{im}} - \frac{1}{\epsilon_r + 1} \right\} d\xi$$

$$G_m(\rho) = \frac{2\mu_r}{\mu_r + 1} \frac{1}{\rho} + \int_{-\infty}^\infty H_0^2(\xi \rho) \left\{ \frac{\mu_r \xi}{D_{te}} - \frac{\mu_r}{\mu_r + 1} \right\} d\xi \quad (A.10)$$
The path of integration is then deformed to both sides of the branch cut defined by $Re(u_o) = 0$ in the lower half plane. See Figure A.2. The contribution of the surface wave terms are calculated by taking the residue of the surface wave poles. For the sake of simplicity, the Green Function evaluations in PMESH assume a lossless ($Im(\varepsilon) = 0$), non-magnetic ($\mu = 1$) substrate. Also, only the first surface wave pole contribution is calculated. This limits the upper-frequency at which the model will be accurate to that which the second surface
wave mode of the substrate propagates. This upper frequency limit is given by

\[ f = \frac{75}{h\sqrt{\varepsilon_r - 1}} \]  

(A.11)

where \( h \) is the substrate height in millimeters and \( f \) is in Gigahertz [13].

### A.4 The Quadruple Integrals

Computing the matrix elements in (A.7) requires the evaluation of pentuple integrals, the Green Function integrals are nested within quadruple surface integrals. To reduce this complexity, the Green Functions are evaluated numerically at discrete points which are then curve-fitted to a set of polynomials in \( \rho = |\vec{r} - \vec{r}'| \), \( \{G_{e,\ell}(\rho)\} \) and \( \{G_{m,\ell}(\rho)\} \) given by

\[
G_e(\rho) \approx G_{e,\ell}(\rho) = \sum_{\nu=m_{\ell}}^{M_{\ell}} C_{e,\ell,\nu} \rho^\nu \\
G_m(\rho) \approx G_{m,\ell}(\rho) = \sum_{\nu=m_{\ell}}^{M_{\ell}} C_{m,\ell,\nu} \rho^\nu
\]

(A.12)

When \( G_e(\rho) \) and \( G_m(\rho) \) are replaced by appropriate polynomials of (A.12), the quadruple integrals in (A.7) become the quadruple integrals

\[
\left\{ \begin{array}{l}
\sum_{\nu=m_{\ell}}^{M_{\ell}} C_{m,\ell,\nu} \int_{S_m} \int_{S_k} \rho^\nu \vec{T}_m(\vec{r}) \cdot \vec{B}_k(\vec{r}') \, ds \, ds' \\
\sum_{\nu=m_{\ell}}^{M_{\ell}} C_{e,\ell,\nu} \int_{S_m} \int_{S_k} \rho^\nu \nabla \cdot \vec{T}_m(\vec{r}) \nabla' \cdot \vec{B}_k(\vec{r}') \, ds \, ds'
\end{array} \right\}.
\]

(A.13)

Here the domains of integration are taken to be \( S_m \) and \( S_k \) since \( \vec{T}_m(\vec{r}) \) is only non-zero for \( \vec{r} \in S_m \) and \( \vec{B}_k(\vec{r}') \) is non-zero only for \( \vec{r}' \in S_k \). The integrals in (A.13) can be expressed in terms of integrals evaluated over the rectangular and triangular cells of the form

\[
\left\{ \begin{array}{l}
\int_{\alpha} \int_{\alpha'} \rho^\nu \vec{T}_m(\vec{r}) \cdot \vec{B}_k(\vec{r}') \, ds \, ds' \\
\int_{\alpha} \int_{\alpha'} \rho^\nu \nabla \cdot \vec{T}_m(\vec{r}) \nabla' \cdot \vec{B}_k(\vec{r}') \, ds \, ds'
\end{array} \right\}.
\]

(A.14)
Since the basis and testing functions are defined on subdomains that consist of two cells, the integrals of (A.13) consist of the summation of four integrals of the form of (A.14). The integrals of (A.14) can be evaluated analytically for the PMESH basis and testing functions and are given in [30] for $-1 \leq \nu \leq 2$.

### A.5 The Curve Fitting

To generate the set of curve fitted polynomials $\{G_{\ell}(\rho)\}$, the Green functions are evaluated numerically as described in section A.3 at a set of equally space points $\{\rho_n\}$, where

$$k_0\rho_n = \frac{0.06n}{\sqrt{\varepsilon_r + 1}}; \quad n \in \{1, \ldots, n_{\text{max}}\}$$  \hspace{1cm} (A.15)

for $n_{\text{max}} > \frac{k_0d_{\text{max}} \sqrt{\varepsilon_r + 1}}{0.06} > n_{\text{max}} - 1$. Here $d_{\text{max}}$ is the maximum vertex-to-vertex distance of the gridded microstrip geometry, including the port extensions. The $\ell^{th}$ polynomial is found by curve fitting the polynomial $\sum_{\nu=0}^{M_\ell} C_{\ell,\nu} \rho^\nu$ to the functional values $G(\rho_n)$ for $\ell \leq n \leq \ell + 2\ell_0$.\textsuperscript{1} To insure that each polynomial is valid over the two cell domain $\alpha \cup \alpha'$, $\ell_0$ is chosen such that

$$\ell_0 > k_0d_{\text{max}} \frac{\sqrt{\varepsilon_r + 1}}{0.06} > \ell_0 - 1$$  \hspace{1cm} (A.16)

where $d_{\text{max}}$ is the largest vertex-to-vertex distance within the largest cell. This insures that the $\ell^{th}$ polynomial is a valid approximation to the Green Functions in the range $\{\rho \mid \rho_t \leq \rho \leq \rho_t + 2d_{\text{max}}\}$. We can define $\rho_{\text{ave}}$ by

$$\rho_{\text{ave}} = \frac{\rho_\prec + \rho_\succ}{2},$$  \hspace{1cm} (A.17)

where $\rho_\prec$ and $\rho_\succ$ are the smallest and largest vertex-to-vertex distances between cells $\alpha$ and $\alpha'$. When calculating the integrals of (A.14), we choose the set of

\textsuperscript{1} If $m_\ell < 0$ then the polynomial $\rho^{-m_\ell} \sum_{\nu=0}^{M_\ell} C_{\ell,\nu} \rho^\nu$ is curve fitted to the values $\rho^{-m_\ell} G(\rho_n)$ for $\ell \leq n \leq \ell + 2\ell_0$ to avoid the singularity at $\rho = 0$. The curve fits are done by minimizing the error $\| \sum_{\nu=0}^{M_\ell} C_{\ell,\nu} \rho^\nu - G(\rho_n) \|_2$ for $\ell \leq n \leq \ell + 2\ell_0$. 

polynomial coefficients \( \{C_{t,w}\} \) which correspond to the \( \rho_t \) which gives the smallest value for \( |\rho_{ave} - \rho_t - d_{max}| \). This gives the polynomial that is best centered around \( \rho_{ave} \).

The order of the \( \ell^\text{th} \) polynomial \( G_t(\rho) \) (the values of \( m_t \) and \( M_t \)) depend on the value of \( \rho_t \). In its original implementation, PMESH always let \( m_t = -1 \) and assigned the value of \( M_t \) by

\[
M_t = \begin{cases} 
4, & \text{if } \rho_t \leq 2d_{max}; \\
3, & \text{if } 2d_{max} < \rho_t \leq 4d_{max}; \\
2, & \text{if } 4d_{max} < \rho_t \leq 6d_{max}; \\
1, & \text{if } 6d_{max} < \rho_t \leq 8d_{max}; \\
0, & \text{if } 8d_{max} < \rho_t. 
\end{cases} 
\]  
(A.18)

Since \( \rho_t \) is found approximately from \( \rho_t \approx \rho_{ave} - d_{max} \), (A.18) can be rewritten as

\[
M_t = \begin{cases} 
4, & \text{if } \rho_{ave} \leq 3d_{max}; \\
3, & \text{if } 3d_{max} < \rho_{ave} \leq 5d_{max}; \\
2, & \text{if } 5d_{max} < \rho_{ave} \leq 7d_{max}; \\
1, & \text{if } 7d_{max} < \rho_{ave} \leq 9d_{max}; \\
0, & \text{if } 9d_{max} < \rho_{ave}. 
\end{cases} 
\]  
(A.19)

Typically, the port extension arm cells are constructed to be \( \lambda_s/20 \) long in the port extension direction and are the largest cells in the structure.\(^2\) Therefore,

\(^2\) For reasonable accuracy, all cells should be no larger than \( \lambda_s/20 \) long.
$d_{max} \sim \lambda_z/20$ for most PMESH gridded structures and $d_{max}$ can be replaced by $\lambda_z/20$ in equation (A.18). Numerical experimentation has shown that the assignment of $M_z$ based on (A.18) is very conservative. If we replace $m_z = -1$ and (A.18) by the assignment

$$
(m_z, M_z) = \begin{cases} 
(-1, 4), & \text{if } \rho_{ave} \leq 2d_{max}; \\
(-1, 2), & \text{if } 2d_{max} < \rho_{ave} \leq 3d_{max}; \\
(0, 0), & \text{if } 3d_{max} < \rho_{ave}.
\end{cases}
$$

(A.20)

then the execution time of PMESH decreases with negligible loss of accuracy. The decrease in execution time is most pronounced for physically large structures at high frequencies. The rationale for (A.20) is that the integrals of (A.14) for odd powers of $\nu$ are computationally much more expensive than if $\nu$ is even.

Also, for large enough separations between cells $\alpha$ and $\alpha'$, the Green Functions can be approximated by their average value, $G(\rho_{ave})$ which is accomplished in (A.20) for $\rho_{ave} > 3d_{max}$. 

APPENDIX B

Adjoint Operators

B.1 The Adjoint Operator of the MPIE

If we replace $\tilde{f}_m(\bar{r})$ and $\tilde{B}_4(\bar{r})$ in (A.7) with $\tilde{J}_1(\bar{r})$ and $\tilde{J}_2(\bar{r})$, where $\tilde{J}_1(\bar{r})$ and $\tilde{J}_2(\bar{r})$ are defined over all of $S$ instead of just $S_m$ and $S_4$, we get

$$
\langle \tilde{J}_1(\bar{r}), \mathcal{L}\tilde{J}_2(\bar{r}) \rangle = \frac{j\omega \mu_0}{4\pi} \int_S \int_{S'} G_m(\rho) \tilde{J}_1(\bar{r}) \cdot \mathcal{J}_2(\bar{r}') \, ds \, ds' \left\{ \begin{array}{l}
+ \frac{j\omega \mu_0}{4\pi k_0^2} \int_S \int_{S'} G_s(\rho) \nabla \cdot \tilde{J}_1(\bar{r}) \nabla' \cdot \tilde{J}_2(\bar{r}') \, ds \, ds' \\
+ \frac{1}{2} \int_S Z_s(\bar{r}; t) \tilde{J}_1(\bar{r}) \cdot \tilde{J}_2(\bar{r}) \, ds \end{array} \right. \right. \quad (B.1)
$$

Make the change of variables $\bar{r} \to \bar{r}'$ and $\bar{r}' \to \bar{r}$ within the integrals and we get

$$
\langle \tilde{J}_1(\bar{r}), \mathcal{L}\tilde{J}_2(\bar{r}) \rangle = \frac{j\omega \mu_0}{4\pi} \int_{S'} \int_S G_m(\rho) \tilde{J}_1(\bar{r}') \cdot \tilde{J}_2(\bar{r}) \, ds \, ds' \left\{ \begin{array}{l}
+ \frac{j\omega \mu_0}{4\pi k_0^2} \int_{S'} \int_S G_s(\rho) \nabla' \cdot \tilde{J}_1(\bar{r}') \nabla \cdot \tilde{J}_2(\bar{r}) \, ds \, ds' \\
+ \frac{1}{2} \int_{S'} Z_s(\bar{r}'; t) \tilde{J}_1(\bar{r}') \cdot \tilde{J}_2(\bar{r}) \, ds' \end{array} \right. \right. \right. \quad (B.2)
$$

The order of integrations can be switched and the integrand rearranged so that

$$
\langle \tilde{J}_1(\bar{r}), \mathcal{L}\tilde{J}_2(\bar{r}) \rangle = \frac{j\omega \mu_0}{4\pi} \int_S \int_{S'} G_m(\rho) \tilde{J}_2(\bar{r}) \cdot \tilde{J}_1(\bar{r}') \, ds' \, ds \left\{ \begin{array}{l}
+ \frac{j\omega \mu_0}{4\pi k_0^2} \int_S \int_{S'} G_s(\rho) \nabla' \cdot \tilde{J}_2(\bar{r}) \nabla \cdot \tilde{J}_1(\bar{r}') \, ds \, ds' \\
+ \frac{1}{2} \int_S Z_s(\bar{r}; t) \tilde{J}_2(\bar{r}) \cdot \tilde{J}_1(\bar{r}) \, ds \end{array} \right. \right. \quad (B.3)
$$

or more simply

$$
\langle \tilde{J}_1(\bar{r}), \mathcal{L}\tilde{J}_2(\bar{r}) \rangle = \langle \mathcal{L}\tilde{J}_1(\bar{r}), \tilde{J}_2(\bar{r}) \rangle. \quad (B.4)
$$

Therefore, the integro-differential operator of (3.8) is self adjoint.
B.2 The Adjoint of a Matrix

The adjoint of the discretized operator described by the matrix [L] is given by [L]', the transpose of [L] where \( L_{i,k} = L_{k,i} \). To see this we note that for any vectors \([\phi_1]\) and \([\phi_2]\),

\[
\begin{align*}
\langle [L]'[\phi_1], [\phi_2] \rangle &= \langle [L]'[\phi_1] \rangle' [\phi_2] \\
&= [\phi_1]' \langle [L]' \rangle' [\phi_2] \\
&= [\phi_1]' [L] [\phi_2] \\
&= \langle [\phi_1], [L][\phi_2] \rangle
\end{align*}
\]

which satisfies the definition of the adjoint operator.

B.3 The Adjoint of the Zeroth Order Operators

The zeroth order operators from both the Isolated Subcircuit Method and the Circle of Influence Method are explicitly expressed as matrices. For both these methods, \([ZC_0]\) (the zeroth order matrix operator) is constructed from the symmetric impedance coupling matrix \([ZC]\) by setting \( ZC_{0i,k} \) equal to either \( ZC_{i,k} \) or 0. Furthermore, if we set \( ZC_{0i,k} = 0 \), then we also set \( ZC_{0k,i} = 0 \). Since \([ZC]\) is symmetric, this ensures that \([ZC_0]\) is also symmetric and is therefore self-adjoint. While it is possible to construct \([ZC_0]\) such that it is not symmetric (say by letting \( ZC_{0i,k} = 0 \) only if \( i < k \)), doing so would produce no advantage in the Perturbational Solution Methodology. Because solving the system \([ZC_0]'[\hat{J}_0]_{i,m} = [g]_{i,m} \) can be accomplished directly from the LU decomposition of \([ZC_0]\), having a zeroth order coupling matrix that is not symmetric would not directly increase the execution time. However, if we were to zero out only the elements in the upper triangular region of \([ZC_0]\), then \([ZC_0]\) would be less sparse than if the elements in the lower triangular region were allowed to be zero as well. This would increase the total execution time of the Isolated Subcircuit Method and the Circle of Influence Method.
The zeroth order operator equation constructed for the Network Connection Method is never explicitly expressed as a single matrix equation. Instead, the process of constructing zeroth order current distributions consists of three steps: finding the individual block current distributions and internal port wave variables, solving Equation (7.4) from the internal port source amplitudes that match the wave variables at the internal port connections, and constructing the circuit current distributions from the scaled block distributions. Trying to express these three steps in terms of a single equivalent matrix equation is a cumbersome endeavour that fails to give insight as to whether the matrix of this equivalent system is symmetric or not. The current distributions \( \tilde{J}_0 \) are constructed assuming that this equivalent matrix equation is in fact self-adjoint (symmetric). There are three basic reasons for this assumption. First, the zeroth order system is an approximation to a system that is self-adjoint and it mimics network theory modeling which is also a self-adjoint process (at least for passive circuits). It therefore seems reasonable to assume that the Network Connection Method is also a self-adjoint procedure. Secondly, assuming that the equivalent matrix equation to the Network Connection Method is self-adjoint saves considerable time. From the \( 3P \) current distributions calculated in the Network Connection Method, we could (in principle) construct \( 3P \) columns of \( [ZC_{eq}]^{-1} \), where \( [ZC_{eq}] \) is the equivalent matrix. In order to solve the adjoint problem, we would need to construct \( [ZC_{eq}]^{-1} \) or some equivalent decomposition (or at least a few columns of \( [ZC_{eq}]^{-1} \)). However, from the methods of current construction employed in the Network Connection Method, this would require finding \( N \) current distributions, which would increase the time required in solving Equation (7.4) and constructing the circuit current distributions from the block distributions by a factor of \( N/3P \) which is very substantial. Thirdly, if \( [ZC_{eq}] \) is not symmetric, then the expressions of Chapter 2 can be rederived
without the use of $L^1_0$. This will introduce a small correction term $\epsilon$ which we can hopefully consider insignificant. We begin modifying the derivations by replacing Equations (2.6) and (2.7) with
\[
g = L_0 \psi_0
\] (B.5)
and
\[
\langle L \phi_1, \phi_2 \rangle = \langle \phi_1, L_0 \phi_2 \rangle + \epsilon(\phi_1, \phi_2).
\] (B.6)

This gives us
\[
F_\delta(\phi) = (g, \phi) = (g, \phi_0 + \phi_\delta) = (g, \phi_0) + (g, \phi_\delta)
\]
\[
= F_\delta(\phi_0) + (g, \phi_\delta)
\]
\[
= F_\delta(\phi_0) + (L_0 \psi_0, \phi_\delta)
\]
\[
= F_\delta(\phi_0) + (\psi_0, L_0 \phi_\delta) + \epsilon(\psi_0, \phi_\delta)
\]
\[
= F_\delta(\phi_0) - (\psi_0, \{L - L_0\} \phi_0) - (\psi_0, L_0 \phi_\delta) + \epsilon(\psi_0, \phi_\delta)
\]
\[
= F_\delta(\phi_0) - (\psi_0, L \phi_\delta) + (\psi_0, \phi_0) - (\psi_0, L_0 \phi_\delta) + \epsilon(\psi_0, \phi_\delta)
\]
\[
= F_\delta(\phi_0) - (\psi_0, L \phi_0) + (\psi_0, \phi_0) - (\psi_0, L_0 \phi_\delta) + \epsilon(\psi_0, \phi_\delta)
\]
\[
= 2F_\delta(\phi_0) - (\psi_0, L \phi_0) - (\psi_0, L_0 \phi_\delta) - \epsilon(\psi_0, \phi_0) + \epsilon(\psi_0, \phi_\delta)
\] (B.7)

Presumably $\epsilon(\psi_0, \phi_\delta)$ is second order since both $\phi_\delta$ and $\epsilon$ are small quantities. Therefore, dropping the second order quantities $(\psi_0, L_0 \phi_\delta)$ and $\epsilon(\psi_0, \phi_\delta)$ gives
\[
F_\delta(\phi) \approx 2F_\delta(\phi_0) - (\psi_0, L \phi_0) - \epsilon(\psi_0, \phi_0).
\] (B.8)

We hope that $L_0$ is closer to being self-adjoint than it is to being $L$, that is $L_0 - L^1_0$ is at least an order of magnitude 'smaller' than $L_\delta$ so that $\epsilon(\psi_0, \phi_0)$ is a higher order error than first order in $\delta$ so that it can be dropped leaving
\[
F_\delta(\phi) \approx 2F_\delta(\phi_0) - (\psi_0, L \phi_0),
\] (B.9)

which is equivalent to Equation (2.13).
APPENDIX C

Iterative Refinement

The Perturbational Solution Methodology can be easily extended into an iterative technique by the use of Iterative Refinement [6], [21]. If we let \( \phi_0 \) be the solution to the zeroth order linear operator equation \( f = L_0\phi_0 \), then we can generate a set of approximate solutions to \( \phi = L^{-1}f \), \( \{\phi_i\} \), by the relationship

\[
\phi_i = \phi_{i-1} + \eta_i \quad i \in \{1, 2, \ldots\},
\]

where \( \eta_i \) is found by solving the system

\[
L_0\eta_i = f - L\phi_{i-1} = R_{i-1}.
\]

Notice that if \( \phi_{i-1} \) is replaced by the exact solution \( \phi \) in (C.2), then we would have the system

\[
L_0\eta_i = 0.
\]

Since \( L_0 \) is non-singular, this system has as its only solution \( \eta_i = 0 \) and the iteration should be terminated. After each iterative step of (C.1), we can compute the functional

\[
\mathcal{F}_i = (g, \phi_i)
\]

and continue the process until

\[
\mathcal{F}_i - \mathcal{F}_{i-1} \leq \epsilon\mathcal{F}_i
\]

for some small value of \( \epsilon \).
In order to compute $\mathcal{F}_i$, we need to find $\eta_i$ at each iterative step. From (C.2) we see that this would require one matrix multiplication ($L\phi_{i-1}$) and one matrix solve ($L_0\eta_i = \eta_{i-1}$) per iteration. We also need to solve the zeroth order system $L_0\phi_0 = f$. Since each matrix equation involves the same matrix $L_0$, these matrix solves will only require a backsubstitution operation once we obtain an equivalent decomposition of $L_0$. Overall, we note that computing the $i^{th}$ functional $\mathcal{F}_i$ will require $i+1$ backsubstitutions and $i$ matrix multiplications.

If we substitute (C.1), (C.2) and (2.6) into (C.4), then we obtain

$$\mathcal{F}_i = \langle g, \phi_i \rangle$$

$$= \langle g, \phi_{i-1} + L_0^{-1}(f - L\phi_{i-1}) \rangle$$

$$= \langle g, \phi_{i-1} \rangle + \langle g, L_0^{-1}f \rangle - \langle g, L_0^{-1}L\phi_{i-1} \rangle$$

$$= \mathcal{F}_{i-1} + \langle g, \phi_0 \rangle - \langle L_0^{-1}g, L\phi_{i-1} \rangle$$

$$= \mathcal{F}_{i-1} + \mathcal{F}_0 - \langle \psi_0, L\phi_{i-1} \rangle$$

(C.6)

If we now let $i = 1$ we see that we get

$$\mathcal{F}_1 = 2\mathcal{F}_0 - \langle \psi_0, L\phi_0 \rangle$$

(C.7)

which is equivalent to (2.13). Clearly, the Perturbational Solution Methodology gives results that are equivalent to the results that would be obtained from the first step of the Iterative Refinement technique. Interestingly, the adjoint problem (equation (2.6)) never needs to be explicitly solved in the Iterative Refinement formulation.

Of the three methods of generating approximate inverses $L_0^{-1}$ presented in this thesis, the Circle of Influence Method would be most appropriate to use with the Iterative Refinement technique. Computing $\eta_i$ at each iteration would be cumbersome in the Network Connection Method since $L_0$ is not explicitly determined with this method. Furthermore, the Isolated Subcircuits Method can be incorporated into the Circle of Influence Method by defining multiple
circle of influence radii. If incorporated into the Circle of Influence Method, each additional iteration of the Iterative Refinement technique would require $O(PN^2 + 2P\tau)$ operations where $P$ is the number of ports, $N$ is the size of the system, and $\tau$ is the number of nonzero elements in the decomposition of $L_0$. Since $\tau$ would increase with a larger circle of influence, each iteration would be more efficient for a smaller circle of influence size. However, more iterations would be required to reach a given level of accuracy in $F_i$. Since $\rho_0 = 3h$ gives reasonably accurate zeroth order results, the Iterative Refinement technique would probably work well with this as a starting point. Notice that since the Iterative Refinement formulation does not depend explicitly on the adjoint problem, computing all the functionals needed to perform the Two Point Deembedding would be more efficiently computed as in equations (C.1) and (C.2) than by (C.6). However, this savings is relatively small compared to the entire execution time.
APPENDIX D

The MIMICAD Blues

Lyrics by John Moore
Music by Committee

very freely

Woke up this morning, Rolled outta my bed.

Early class with Gupta, Nothin' more that I dread.

Went up to my office, The wind really blows.

Even in summer, So cold that it snows.
My bicycle's broken, It's back at my pad.

I guess I won't win, The tour MIMI-CAD.

I'm behind in my studies, Four problem sets due,

Here to tell ya buddy, Got them MIMICAD Blues.

Chorus

These MIMICAD Blues. These

MIMICAD Blues. Oh what can I do? Bout these
Last time only

MIMICAD Blues.

fade

hate these Blues. They'll get to you too. They

fade

tear me apart. These MIMICAD Blues. I

can't run away. They're here ev'ry day. Oh

where is my cure? For these MIMICAD Blues.

Additional Lyrics

The computers are down, and Janet is gone.
Lost all of my files, ain't got nothin' done.
My advisor is on me, I'm just spinning my gears,
At this rate of progress, I'll be here ten years.
Today DC caught me, playing golf at lunch.
Guess I get to stay here, an extra six months.
I wanna play football, it's all I can do.
To vent my frustrations, from these MIMICAD Blues.

Chorus
I do nothing but program, I'm down on my knees. How do I stop dreamin', in FORTRAN Cantonese? I'm working too hard, 'tis a bit scary. If the weather is good, I'll hit the library. I hate these computers, but there's nowhere to go, My retinas are burning, say no to day glow. MIMIGEEKS were awesome, but we lost to the Chews, Another sad case, of those MIMICAD Blues.

Chorus

At URSI in London, Cole we did mock, But he is the master, of the seven minute talk. Scott on the printer, his presence is felt, Wouldn't you know it, transparencies melt! And Chris over there, speaking Spanish all day, "Hola que tal. Bendecho, Chingate!" I need some blood sugar, the brownies are gone. Who ate them last night? It must be Rajan.

Chorus

An end to this lecture, for this I am rootin' Or at least some new info, it's painful, darn tootin'. We do nothing but simulate, 'tis our sad lot, We can do no measuring, students all we've got. In-house fabrication, that's where it's at, Need another wafer? Send Todd to Wright-Pat. A slim ray of hope, that's all that I had. Trying to figure, A fellow named Bad.

Chorus

Bob he has finished, now he's working for NIST, His favorite scab players? He's making a list. Pete quit racing, 'bout a month too late, A bike in the face, is something you hate. Now Kent off in DC, remember what's said, "To impress the Gen'rais, don't forget the bread." These have all finished, they payed all their dues. No need to be singin', any more MIMICAD Blues.

Chorus

Ed is an athlete, of that he can boast, But in MFL action, he'll always be toast. Also Pete has left, we spent some great noons, We'll miss all he gave us, especially cartoons. And goodbye to Henri, at this lively bash. As soon as he leaves, the computers will crash. More students are leaving, I'll be more alone. But the worst thing about it, it ain't me goin' home.

Chorus