

**An Exploration of the Relationship between Construction Cost and
Duration in Highway Projects**

By

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The final copy of this thesis has been examined by the signatories, and we Find that both the content and the form meet acceptable presentation standards of scholarly work in the above-mentioned discipline.

Abstract

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An exploration of the relationship between construction cost and duration in highway project

Thesis directed by Professor Keith Molenaar and co-directed by Professors Paul Goodrum and Matthew Hallowell

Understanding and quantifying the relationship between construction cost and duration has become very important for both state highway agencies (SHAs) and contractors. Developing accurate cost-duration models can help SHAs to calculate reasonable incentive/disincentive (I/D) contracts parameters and contractors to identify the optimum price-time combination under the price-time bi-parameter procurement method. Available literature on this topic has presented the relationship between construction cost and duration as a part of bigger optimization approaches, and no papers were published to discuss the variation of this relationship from one project to another. This research studies the effects of important project characteristics on the relationship between construction cost and duration in highway projects. The findings of this research showed that the cost-duration relationship can vary depending on the project highway agency, project type and contract type. Finally, these findings were applied to create more accurate cost-duration models for Florida's road maintenance projects.

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Chapter 1: Introduction

To limit the problem of time delays, state highway agencies (SHAs) started to utilize new measures to ensure on time project completion. These measures include the use of new contractual provisions, such I/D contracts, the use of new procurement methods, such as the price time bi-parameter method, or a combination of both methods (Ellis and Thomas 2002; Herbsman et al. 1995).

The increasing use of the I/D contract provisions, and the price-time bi-parameter procurement method in highway construction has introduced both SHAs and contractors with new challenges. SHAs are presented with the problem of calculating reasonable incentive amounts when the I/D contract provisions are used. Underestimating these amounts may reduce the effectiveness of the I/D contracts by not providing sufficient motivation for the contractor to finish the project earlier. Overestimating these amounts may also result in wasting public money (Shr and Chen 2004). Similarly, contractors are faced with the problem of choosing the best price-time combination to maximize their chances of winning new projects under the price-time procurement method.

To address these challenges, researchers have studied the relationship between construction cost and duration in highway projects. With knowledge of this relationship, they developed optimization approaches to help both SHAs and contractors to adapt to the use of these new procurement methods and contractual provisions (Shen et al. 1999; Shr and Chen 2004; Shr et al. 2004 and Wu and Lo 2009). SHAs and contractors who aim to use these optimization approaches effectively, should first be able to understand and quantify the

relationship between construction cost and duration in their projects. This task is usually complicated since the relationship between construction cost and duration can significantly vary from one project to another.

This research studies the effects of important project characteristics on the relationship between construction cost and duration in highway projects. Previous researches on this topic have only focused on using the functional relationship between construction cost and duration as a part of bigger optimization approaches to find the optimum price -time combination or to calculate the I/D contract parameter, and no researches have studied the variation of this relationship from one project to another. The findings of this research will help SHAs and contractors to better understand the variations in the construction cost-duration relationship, and thus allow them to develop more accurate empirical construction cost duration models by selecting the most appropriate historical data to build these models.

1.1 Literature Review:

The functional relationship between construction cost and duration can take the form of a second-degree polynomial with a general equation of:

$$C = a_2(D^2) + a_1(D) + a_0 \quad (1)$$

where C is construction cost, D is construction duration and a_2 , a_1 and a_0 are all constants.

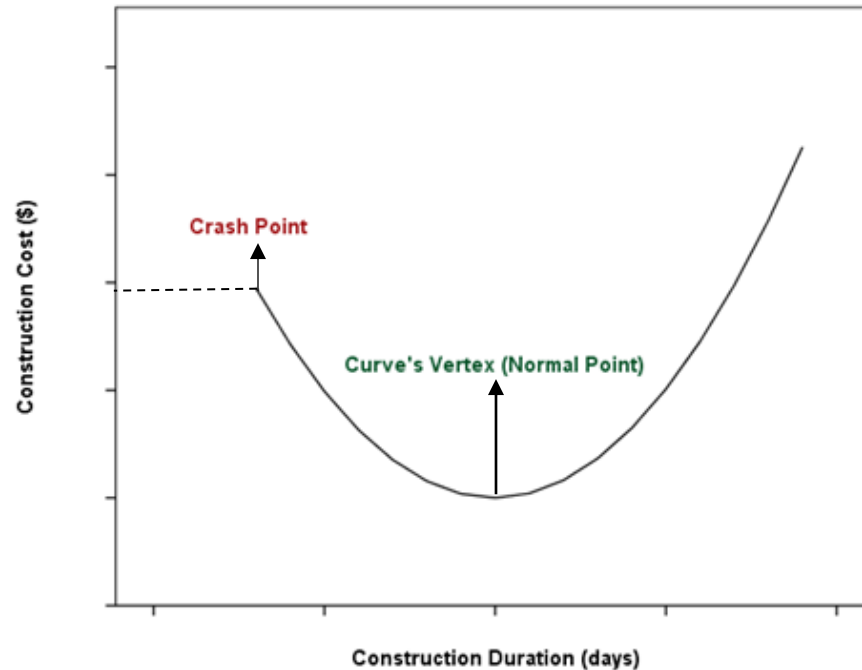


Figure 1: Construction cost - duration curve as it appears in the literature (Callahan et al. 1992; Cusack 1992; Munzer 1998; Shr and Chan 2004)

Figure 1 shows the general graph of the construction cost-duration curve. The vertex of this curve is usually called the normal point and it represents the construction duration (normal duration) where the construction cost for a certain project is minimized (normal cost). Cutting project schedule behind the normal duration will usually increase the construction cost by adding acceleration costs. These acceleration costs are usually born by the contractor and they include the costs of adding more resources, laborers and technology to finish the project earlier. On the other hand, delays in project schedule beyond the normal point will increase the project construction costs because contractors will be paying unnecessary fixed or indirect costs. The crash duration on figure 1 represents the minimum construction time in which the construction process can be accomplished and its corresponding construction cost is also called the crash cost.

Shen et al. (1999) studied the functional relationship between construction cost and duration in highway projects. They used this relationship to develop an optimization bidding model to help contractors find the optimum price-time combination under the price-time bi-parameter procurement method (see appendix A for more details). To use this approach, contractors should establish their own construction cost-duration model for the project under consideration. To establish the functional relationship between construction cost and duration as it appears in equation 1, Shen et al. suggested that contractors should use their own estimations to come up with at least three price-time combinations. These three combinations were suggested as the shortest time combination, lowest price combination and most likely combination. Using these three combinations, contractors can then use polynomial regression to find the values of a_0 , a_1 and a_2 in equation 1 above.

A similar approach was also presented by Wu and Lo in 2009 to establish an optimization model for the price time bi-parameter bidding method. However, instead of using only three price-time combinations as suggested by Shen et al., Wu and Lo suggested that more price time combinations should be used to create more accurate construction cost – duration models. Moreover, they also suggested that since it may be hard or impractical for the contractor to estimate the shortest time combination, the lowest price combination and the most likely combination, contractors could instead use more reasonable estimations by taking general short time, low cost and likely price-time combinations.

Shr and Chen (2004) have also studied the relationship between construction cost and duration in Florida highway projects. They used this relationship to develop systematic

procedures and guidelines to help Florida department of transportation (FDOT) in calculating the values of important I/D contracts parameters, such as the maximum incentive amount and the minimum contract duration. Their overall approach followed the same steps as suggested by Shen et al. in 1999. However, to develop their own construction cost duration model, Shr and Chen followed an empirical approach in which historical data from 15 projects completed by Florida department of transportation (FDOT) were used to fit a general construction cost-duration model. For each one of the used projects, it was assumed that project initial cost and duration are located on the normal point, and the cost growth ratio (Y) and time growth ratio (X) were calculated using equations 2 and 3 below:

$$Y = \frac{C - C_0}{C_0} \quad (2)$$

$$X = \frac{D - D_0}{D_0} \quad (3)$$

where:

-C: actual construction cost

-D: actual construction duration

-C₀: initial or planned construction cost

-D₀= initial or planned construction duration

Because it is known that any variation in time from the normal point will result in a corresponding increase in construction cost, multiple linear regression was used to establish the relationship between cost growth ratio (Y) and time growth ratio (X). The resulting equation was mathematically rearranged to obtain the functional relationship between construction cost and duration as appears in equation 4 below:

$$C = 0.466C_0\left(\frac{D - D_0}{D_0}\right)^2 + 0.105C_0\left(\frac{D - D_0}{D_0}\right) + 0.032C_0 \quad (4) \text{ (Shr and Chen 2004)}$$

Finally, equation 4 was mathematically shifted to ensure that C_o and D_o are located on the normal point. The model's final equation is given by equation 5 below:

$$C = 0.466C_o\left(\frac{D-1.113D_o}{D_o}\right)^2 + 0.105C_o\left(\frac{D-1.113D_o}{D_o}\right) + 1.006C_o \quad (5) \text{ (Shr and Chen 2004)}$$

Given that both the planned construction cost (C_o) and the planned construction duration (D_o) should be known at the beginning of the construction process, their values can be considered as constants for any given project. Hence, equation (4) above can be used to describe the functional relationship between the actual construction cost (C) and the actual construction duration (D).

1.2 Point of Departure:

Despite the importance of understanding and quantifying the relationship between construction cost and duration, no literature exists regarding the major factors that may influence this relationship. While previous research on this topic has mentioned that the relationship between cost and duration can vary depending on several factors, such as project highway agency, project type, contract type, contractor's managerial practices and construction techniques (Shen et al. 1999; Shr and Chen 2004), the effects of these factors on the relationship were never investigated or measured. To address this gap in literature, this research focuses on studying the effects of important project characteristics on the relationship between construction cost and duration. These characteristics include project highway agency, project type, contract type and delivery method.

1.3 Research Methods:

This research follows an empirical approach to investigate the effects of important project characteristics on the relationship between construction cost and duration. The research data was obtained from the post construction records of all highway projects performed by Florida department of transportation (FDOT) and Ohio department of transportation (ODOT) between the years 1999 and 2014. After cleaning the data, projects were divided according to their highway agency, project type, contract type and delivery method as shown on table 1. Finally, the research hypotheses were tested using appropriate hypothesis testing procedures.

Table 1: Data breakdown according to the studied project characteristics

| Project Highway Agency | n |
|-------------------------------|----------------------------------|
| FDOT | 1936 |
| ODOT | 1180 |
| Project Type | n |
| Road maintenance | 2454 |
| Bridge maintenance | 593 |
| New construction | 69 |
| Contract Type | n (Florida projects only) |
| I/D provisions used | 1646 |
| I/D provisions not used | 290 |
| Delivery Method | n (Florida projects only) |
| Design bid build (DBB) | 1842 |
| Design build (DB) | 94 |

Based on the findings of this research, the important factors that can affect the relationship between construction cost and duration were considered and used to select the

best clusters of data to fit new construction cost-duration models. To fit the new models, a similar but improved method to that used by Shr and Chen in 2004 was used. The relationship between cost growth ratio and time growth ratio was first established using multiple linear regression. The resulted regression model was then mathematically rearranged to obtain the functional relationship between construction cost and duration. The performance of the new models was evaluated on separate testing sets by calculating the mean percent error (PE) value for each model. Finally, the performance of the new models was also compared with the performance of the original model created by Shr and Chen in 2004 to measure the effects of the introduced modifications.

1.4 Thesis Format:

This thesis follows the journal paper format. The first chapter is a general introduction that addresses the research problem, the significance of the research topic, the literature review and the research methods. The second chapter is a stand-alone paper that has its own abstract, introduction and conclusion. The final chapter is a general conclusion that discusses the research contributions, limitations and suggested future research.

Chapter 2: An Exploration of the Relationship between Construction Cost and Duration in Highway Projects

2.1 Abstract

Understanding and quantifying the relationship between construction cost and duration has become very important for both state highway agencies (SHAs) and contractors. Developing accurate construction cost-duration models can help SHAs to calculate reasonable incentive/disincentive (I/D) contracts parameters such as the maximum incentive amount and the minimum contract duration. For contractors, accurate cost-duration models can help in identifying the optimum price-time combination under the price-time procurement method and thus increase their chances to win new projects. Available literature on this topic has always presented the relationship between construction cost and duration as a part of bigger optimization approaches and no papers were published to discuss the variation of this relationship from one project to another. This research utilizes an empirical approach to study the effects of important project characteristics on the relationship between construction cost and duration in highway projects. The findings of this research showed that the cost-duration relationship can vary depending on the project highway agency, project type and contract type and thus confirmed that this relationship is project dependent. Finally, these findings were applied to create more accurate cost-duration models for Florida's road maintenance projects.

2.2 Introduction:

To limit the problem of time delays, state highway agencies (SHAs) started to utilize new measures to ensure on time project completion. These measures include the use of new contractual provisions, such I/D contracts, the use of new procurement methods, such as the price time bi-parameter method, or a combination of both methods (Ellis and Thomas 2002; Herbsman et al. 1995).

Although the use of these alternative procurement methods and contractual provisions can effectively reduce time delays in highway projects (Ellis and Thomas 2002; Herbsman 1995), they also introduce new challenges to both SHAs and contractors. In the case of I/D contracts, SHAs are also faced with the problem of selecting reasonable and realistic values for the main I/D contract parameters, such as the daily incentive amount, the daily disincentive amount, the maximum incentive amount and the maximum acceleration period. In case of the price-time bi-parameter method, contractors who are accustomed to the traditional low bid procurement method, in which tender price is the only dominant selection criterion, must develop new bidding strategies to account for project duration as another important selection criterion.

To address these challenges, researchers have studied the relationship between construction cost and duration in highway projects. With knowledge of this relationship, they developed optimization approaches to help both SHAs and contractors to adapt to these new contractual provisions and procurement methods (Shen et al. 1999; Shr and Chen 2004; Shr et al. 2004 and Wu and Lo 2009). To use these optimization approaches effectively, SHAs and contractors should follow these main steps:

- Establish the functional relationship between construction cost and duration for the project under consideration (build the construction cost-duration model)
- Establish the time value equation for the project under consideration
- Combine both equations above to obtain the total construction cost equation

The obtained total construction cost equation (also known as the total combined bid equation) can then be minimized to obtain the optimum price-time combination and the minimum contract period for the project under consideration (Shen et al. 1999; Shr and Chen 2004). This equation can also be used to identify the actual cost savings/overruns that may result from cutting/extending the project schedule and thus allow SHAs to fairly calculate the important I/D amounts.

To maximize the earned benefits from using the available optimization approaches, contractors and SHAs should first be able to build accurate construction cost-duration models. This task is usually complicated because the relationship between construction cost and duration can significantly vary from one project to another.

This research studies the effects of important project characteristics on the relationship between construction cost and duration in highway projects. Previous researches on this topic have only focused on using the functional relationship between construction cost and duration as a part of bigger optimization approaches, and no researches have studied the variation of this relationship from one project to another. The findings of this research will help SHAs and contractors to better understand the variations in the construction cost-duration relationship, and thus allow them to develop more accurate empirical construction cost duration models by selecting the most appropriate historical data to build these models.

2.3 Background:

The functional relationship between construction cost and duration can take the form of a second-degree polynomial with a general equation of:

$$C = a_2(D^2) + a_1(D) + a_0 \quad (4)$$

where C is construction cost, D is construction duration and a_2 , a_1 and a_0 are all constants.

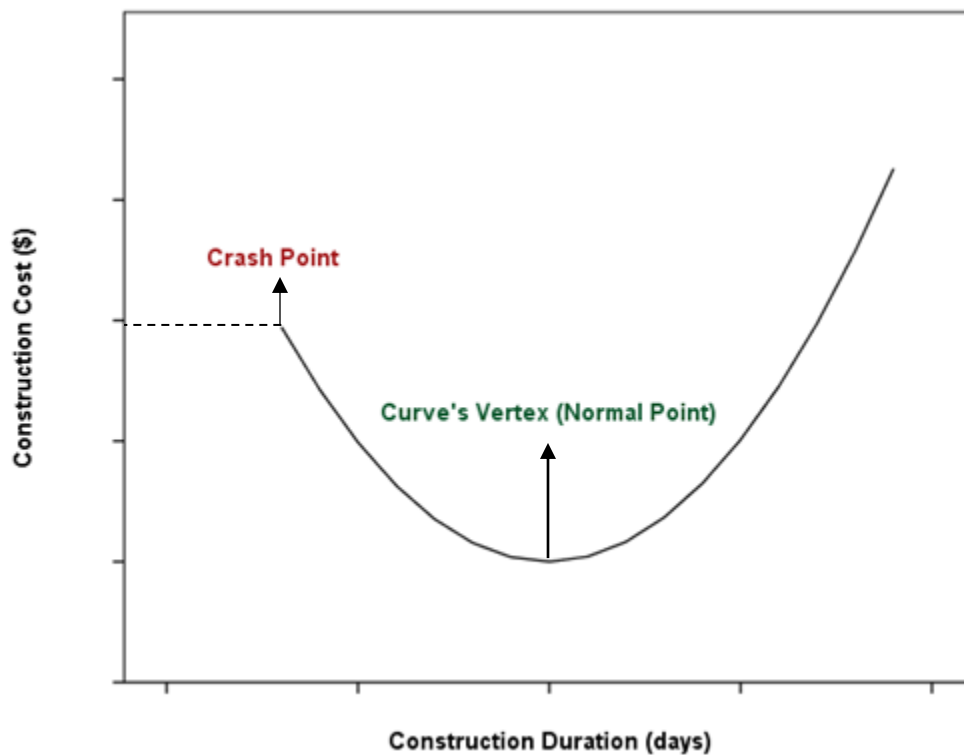


Figure 2: Construction cost - duration curve as it appears in the literature (Callahan et al. 1992; Cusack 1992; Munzer 1998; Shr and Chan 2004)

Figure 2 shows the general graph of the construction cost-duration curve. The vertex of this curve is usually called the normal point and it represents the construction duration (normal

duration) where the construction cost for a certain project is minimized (normal cost). Cutting project schedule behind the normal duration will usually increase the construction cost by adding acceleration costs. These acceleration costs are usually born by the contractor and they include the costs of adding more resources, laborers and technology to finish the project earlier. On the other hand, delays in project schedule beyond the normal point will increase the project construction costs because contractors will be paying unnecessary fixed or indirect costs. The crash duration on figure 2 represents the minimum construction time in which the construction process can be accomplished and its corresponding construction cost is also called the crash cost.

Shen et al. (1999) presented a method to quantify the functional relationship between construction cost and duration. They suggested that contractors should use their own estimations to come up with at least three price-time combinations for the project under consideration. These three combinations were suggested as the shortest time combination, lowest price combination and most likely combination. Using these three combinations, contractors can then use polynomial regression to find the values of a_0 , a_1 and a_2 in equation 4 above. Since the relationship between construction cost and duration is determined by several factors such as contractors' managerial practices and construction techniques, Shen et al. suggested that different contractors can have different construction cost-duration curves for the same project.

Another method to establish the functional relationship between construction cost and duration was also presented by Shr and Chen in 2004. The new method followed an empirical

approach in which historical data from 15 projects completed by Florida department of transportation (FDOT) were used to fit a general construction cost-duration model. For each one of the used projects, it was assumed that project initial cost and duration are located on the normal point, and the cost growth ratio (Y) and time growth ratio (X) were calculated using equations 5 and 6 below:

$$Y = \frac{C-C_0}{C_0} \quad (5)$$

$$X = \frac{D-D_0}{D_0} \quad (6)$$

where:

-C: actual construction cost

-D: actual construction duration

-C₀: initial or planned construction cost

-D₀= initial or planned construction duration

Because it is known that any variation in time from the normal point will result in a corresponding increase in construction cost, multiple linear regression was used to establish the relationship between cost growth ratio (Y) and time growth ratio (X). The resulting equation was mathematically rearranged to obtain the functional relationship between construction cost and duration as appears in equation 7 below:

$$C = 0.466C_0\left(\frac{D-D_0}{D_0}\right)^2 + 0.105C_0\left(\frac{D-D_0}{D_0}\right) + 0.032C_0 \quad (7) \text{ (Shr and Chen 2004)}$$

Finally, equation 7 was mathematically shifted to ensure that C₀ and D₀ are located on the normal point. The model's final equation is given by equation 8 below:

$$C = 0.466C_0\left(\frac{D-1.113D_0}{D_0}\right)^2 + 0.105C_0\left(\frac{D-1.113D_0}{D_0}\right) + 1.006C_0 \quad (8) \text{ (Shr and Chen 2004)}$$

2.4 Research Needs and Objectives:

Despite the importance of understanding and quantifying the relationship between construction cost and duration, no literature exists regarding the major factors that may influence this relationship. While previous research on this topic has mentioned that the relationship between construction cost and duration can vary depending on several factors, such as project highway agency, project type, contract type, contractor's managerial practices and construction techniques, the effects of these factors on the relationship were never investigated or measured.

The main objective of this research is to study the effects of project highway agency, project type, contract type and delivery method on the relationship between construction cost and duration. To achieve this objective, the following four hypotheses will be tested:

- Ho: Project highway agency has no significant effect on the relationship between construction cost and duration in highway projects;
- Ho: Project type has no significant effect on the relationship between construction cost and duration in highway projects;
- Ho: Project contract type has no significant effect on the relationship between construction cost and duration in highway projects and,
- Ho: Project delivery method has no significant effect on the relationship between construction cost and duration in highway projects.

2.5 Methodology:

This research was conducted through three main methodological steps. First, the properties of the cost-duration relationship curve were studied in details and a metric to measure the changes in this relationship was defined and quantified. Second, data from projects that were completed by Florida department of transportation (FDOT) and Ohio department of transportation (ODOT) was obtained and analyzed to investigate the effects of important project characteristics on the relationship between construction cost and duration. Finally, the findings of this research were applied to develop new construction cost-duration models for Florida's road maintenance projects.

Since this was the first study to investigate the effects of project characteristics on the relationship between construction cost and duration, it was necessary to study the relationship's properties and to define a metric to measure the changes in this relationship. The defined metric from this step will serve as the dependent variable through the data analysis part of the study. Future research that aims to study the effects of other project characteristics on the cost-duration relationship can use the same metric defined in this study.

The research data was obtained from the post construction records of all highway projects performed by FDOT and ODOT between the years 1999 and 2014. After cleaning the data, projects were divided according to their highway agency, project type, contract type and delivery method as shown on table 2. The research hypotheses were then tested using appropriate hypothesis testing methods.

Table 2: Projects breakdown according to the studied project characteristics

| Project Highway Agency | n |
|--|----------------------------------|
| FDOT | 1936 |
| ODOT | 1180 |
| Total | 3116 |
| Project Type | n |
| Road maintenance projects: including all roads reconstruction, resurfacing, maintenance, lane adding and widening project | 2454 |
| Bridge maintenance projects: including all bridges reconstruction, replacement, maintenance, widening and enhancement projects | 593 |
| New construction projects: including all new roads and bridges construction projects | 69 |
| Total | 3116 |
| Contract Type | n (Florida projects only) |
| I/D provisions used | 290 |
| I/D provisions not used | 1646 |
| Total | 1936 |
| Delivery Method | n (Florida projects only) |
| Design bid build (DBB) | 1842 |
| Design Build (DB) | 94 |
| Total | 1936 |

Finally, the findings of this research were applied to select the best clusters of data to fit new construction cost-duration models. To fit the new models, a similar but improved method to that used by Shr and Chen in 2004 was used. The relationship between cost growth ratio and time growth ratio was first established using multiple linear regression. The resulted regression model was then mathematically rearranged to obtain the functional relationship between construction cost and duration. The performance of the new models was evaluated on separate testing sets by calculating the mean percent error (PE) value for each model. Finally, the

performance of the new models was also compared with the performance of the original model created by Shr and Chen in 2004 to measure the effects of the introduced modifications.

2.6 Construction Cost – Duration Curve Properties:

Since it is generally accepted to graph the relationship between construction cost and duration in the form of a quadratic parabola (Callahan et al. 1992; Cusack 1992; Munzer 1998; Shr and Chan 2004), the properties of the resulting curves will also be the same as the general quadratic parabola properties. These properties can play a paramount role in defining the way project cost and duration are related to each other in any given project. These properties include: the direction of the curve's opening, the location of the x and y intercepts, the location of the curve's vertex and the steepness of the curve.

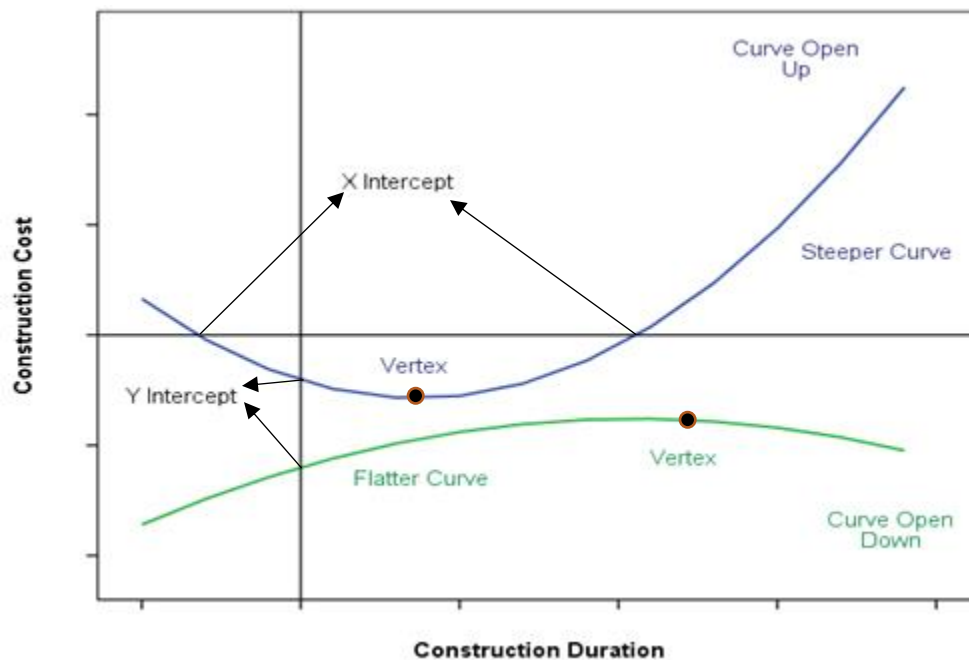


Figure 3: Properties of the second-degree polynomial curve

2.6.1 The Direction of the Curve's Opening:

For any construction project, it is assumed that schedule delays or savings from the normal duration will result in cost overruns (Figure 3). Moving either to the right or to the left from the curve's vertex will always increase the project construction cost. As a result, the value of the constant a_2 in equation 1 should always be greater than zero and all of the resulting construction cost – duration curves will always open up.

2.6.2 The X and Y Intercepts:

Since no project can have negative or zero cost or duration values, the construction cost – duration parabola will always be located in the first quarter of any Cartesian coordinate system. The curve will not cross the X axis at any point, and thus there will be no X intercepts. Similarly, the Y intercept will have no practical meaning since no project can be finished in zero time.

2.6.3 The Location of the Vertex:

To establish accurate construction cost – duration models using historical data, it is always assumed that cost overruns from the initial or planned project cost (C_0) are related to schedule delays or cuts from the initial or planned project duration (D_0). To account for this assumption, the resulting construction cost duration models should always be shifted to guarantee that the project's initial cost and time estimates are located on the normal point (the vertex of the curve). As a result, the location of the curve's vertex will be different from project to project depending on the values of D_0 and C_0 . While the location of the vertex remains an important curve property, it still has no direct effect on the behavior of the construction cost – duration relationship.

2.6.4 The Steepness of the Curve (The Sensitivity of the Relationship):

The steepness of the construction cost – duration curve is the most important curve property. It indicates the sensitivity of the relationship between construction cost and duration, and can vary from project to project. Figure 4 shows the effects of curve steepness on the relationship between construction cost and duration. When the curve is steeper (as in curve A), the relationship between construction cost and duration tends to be more sensitive. Thus, schedule delays or cuts from the normal duration are expected to be accompanied with higher cost overruns. On the other hand, when the construction cost – duration curve is flatter (as in curve B), the relationship tends to be less sensitive, and the same schedule delays or cuts are expected to be accompanied with less cost overruns.

The sensitivity of the construction cost – duration relationship is measured by the curve steepness and can have a direct effect on contractors' bidding decisions when the price – time bi-parameter procurement method is used. When the relationship is more sensitive (the relationship's curve is steeper), contractors should expect higher acceleration costs because of cutting the project schedule. This will limit the contractors' ability to reduce project duration and the resulting total optimum project duration will be closer to the construction normal duration. Moreover, when the relationship is more sensitive, contractors should pay special attention to ensure on time project completion and thus avoid the unnecessary cost overruns that are associated with schedule delays.

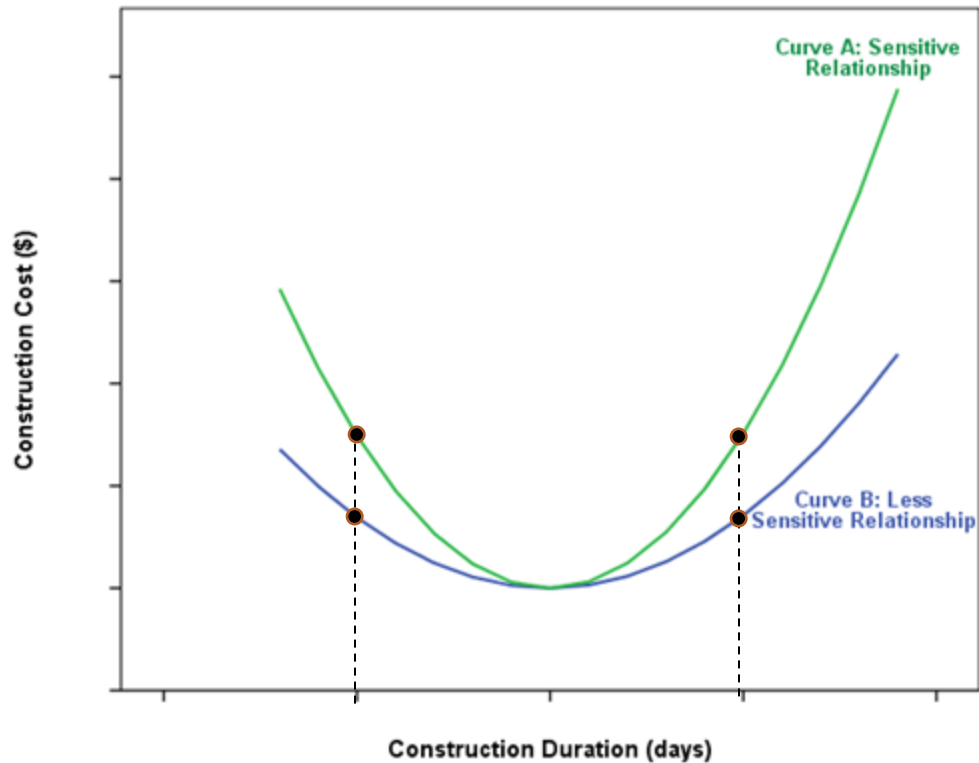


Figure 4: Construction cost and duration relationship sensitivity

Similarly, the sensitivity of the construction cost – duration relationship can affect the values of the main I/D contract parameters. When the relationship is more sensitive, the dollar daily incentive amount and the overall maximum incentive amount should be increased to fairly compensate the contractor for the higher acceleration costs required to cut the project schedule. Neglecting the acceleration costs when calculating the values of the daily incentive amount and the maximum incentive amount will result in underestimating these values and thus reduce the effectiveness of I/D contracts in achieving their main goal of reducing the construction time.

As mentioned before, the sensitivity of the construction cost – duration relationship is indicated by the steepness of the relationship's curve, and can vary from one project to

another. In an available construction cost – duration model, the easiest way to measure the steepness of the curve is to consider the value of the constant a_2 in equation 1. A higher absolute value of a_2 indicates a steeper curve and thus a more sensitive cost – duration relationship. However, since it is very important to choose appropriate historical data to develop accurate construction cost – duration models, SHAs and contractors should also be able to assess the steepness of construction cost – duration curves for a group of projects before using them to build the model. In this case, the value of the constant a_2 will not be available, and another method to assess the steepness of the curve should be used.

An alternative method to assess the steepness of the construction cost – duration curves for a group of completed projects is to consider the absolute value of cost change ratio (Y) over time change ratio (X). When the curve is steeper and the relationship is more sensitive, lower absolute values of time change ratio (X) are expected to be accompanied with higher values of cost growth ratio (Y). Hence, projects with higher absolute values of Y/X are expected to have steeper construction cost – duration curves and more sensitive relationships. The main purpose of using cost and schedule growth ratios is to account for the fact that different projects may have different costs and durations. Thus, using cost or duration growth by itself will be misleading in this case.

The absolute value of (Y/X) can clearly indicate the most important property of the construction cost – duration curve and it will be used as a metric to measure the changes in the relationship from one project to another. As a result, the absolute value of (Y/X) will serve as the dependent variable in this research.

2.7 Data Collection and Analysis:

The research data was obtained from FDOT and ODOT. Post construction records of all highway projects performed by FDOT and ODOT between the years 1999 and 2014 were combined in an Excel data sheet. Obtained data included information about project ID, highway agency, project type, project delivery method, initial or planned construction cost, initial or planned construction duration, final construction cost, final construction duration and the I/D amounts.

The obtained data was first cleaned by removing those projects in which one or more variables were missing. Projects with invalid data such as zero or negative initial or final construction costs or durations were also removed. Moreover, since the obtained data set represents the post construction records of all highway projects performed by FDOT and ODOT, this data set contained data from small highway projects that may differ in characteristics than normal or large projects. Thus, it was necessary to control for projects initial costs and durations, and only projects with initial construction costs that are greater than or equal to \$500,000 and with initial construction durations that are greater than or equal to 60 days were included in the analysis.

Cost growth ratio (Y) and time growth ratio (X) were then calculated for each project using equations 5 and 6 respectively. Since this research is focusing on studying the relationship between construction cost and duration by exploring how schedule delays or savings can be associated with cost overruns, projects with negligible time growth ratios for which the absolute values of schedule growth (X) were less than 0.01 were not included in this analysis.

Keeping these projects would have adversely affected the findings of this research since very small X values will usually result in unrealistically high absolute values of (Y/X). The absolute value of Y/X was then calculated and reported for each project. Table 3 shows the descriptive statistics of the research final data.

Table 3: Descriptive statistics for the final data according to the studied characteristics.

| Highway agency | n | Cost growth ratio (Y) | | Time growth ratio (X) | | Absolute value of (Y/X) | |
|--------------------------------|------|-----------------------|----------------|-----------------------|----------------|-------------------------|----------------|
| | | Mean | Std. deviation | Mean | Std. deviation | Mean | Std. deviation |
| FDOT | 1936 | 0.021 | 0.087 | 0.239 | 0.240 | 0.534 | 0.832 |
| ODOT | 1180 | 0.021 | 0.185 | 0.046 | 0.649 | 1.138 | 2.578 |
| Project type | n | Cost growth ratio (Y) | | Time growth ratio (X) | | Absolute value of (Y/X) | |
| | | Mean | Std. deviation | Mean | Std. deviation | Mean | Std. deviation |
| Road maintenance | 2454 | 0.017 | 0.117 | 0.163 | 0.400 | 0.651 | 1.253 |
| Bridge maintenance | 593 | 0.028 | 0.175 | 0.171 | 0.592 | 1.164 | 2.768 |
| New construction | 69 | 0.104 | 0.205 | 0.223 | 0.701 | 1.298 | 3.661 |
| Contract type (Florida only) | n | Cost growth ratio (Y) | | Time growth ratio (X) | | Absolute value of (Y/X) | |
| | | Mean | Std. deviation | Mean | Std. deviation | Mean | Std. deviation |
| I/D provisions not used | 1646 | 0.012 | 0.085 | 0.245 | 0.233 | 0.471 | 0.745 |
| I/D provisions used | 290 | 0.068 | 0.088 | 0.203 | 0.273 | 0.894 | 1.148 |
| Delivery Method (Florida only) | n | Cost growth ratio (Y) | | Time growth ratio (X) | | Absolute value of (Y/X) | |
| | | Mean | Std. deviation | Mean | Std. deviation | Mean | Std. deviation |
| DBB | 1842 | 0.020 | 0.087 | 0.243 | 0.240 | 0.540 | 0.841 |
| DB | 94 | 0.037 | 0.092 | 0.146 | 0.219 | 0.416 | 0.605 |

Hypothesis testing, including the independent samples T-test, also known as Welch T-test or unequal variances T-test, and the Welch – ANOVA test were then used to determine if

each one of the independent variables has a significant effect on the dependent variable (the absolute value of Y/X). All hypothesis testing procedures were conducted with a significant level (α) of 0.95. Hence, the results of the tests were perceived as statistically significant, and the null hypotheses were rejected at a P-value of 0.05 or lower.

Because of the large sample sizes (n is always greater than 30), the conditions of the central limit theorem were assumed to be met, and normality tests were not conducted in this analysis. Levine's test was used to verify the assumption of equal variances in the data sets. In most cases, the test concluded that the assumption of the homogeneity of variances was violated. To minimize the effects of type one error that may be increased by the unequal sample sizes and unequal variances, the results of the Welch – ANOVA test were reported instead of the results of the classical ANOVA test. The Welch – ANOVA test is a reliable alternative for the classical ANOVA test even if the assumptions of equal variances and equal sample sizes are violated (Tomarken and Serlin 1986) (see appendix B for Welch-ANOVA test procedures). The independent samples T-test (also known as Welch T-test) is also known to give robust results even if the assumptions of homogeneity of variances and equal sample sizes were violated (Welch 1947) (see appendix B for Welch T-test test procedures).

2.8 Research Findings:

2.8.1 Highway Agency:

Table 4: T-test results for project highway agency

| Independent Samples T-test | | | | | | |
|-----------------------------------|----------|---------|-----------------|-----------------------|---|--------|
| Measurement: Absolute value (Y/X) | | | | | | |
| T | df | P-value | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference | |
| | | | | | Lower | Upper |
| -7.802 | 1330.089 | 0.000 | 0.604 | 0.077 | -0.756 | -0.452 |

Table 4 shows the results of the independent samples T-test for project highway agency. The results of the T-test showed that there is a significant statistical difference between means depending on project highway agency (P-value ≤ 0.001). Hence, it can be concluded that Ohio highway projects (mean absolute (Y/X) of 1.138) are expected to have a more sensitive relationship between construction cost and duration than Florida highway projects (mean absolute (Y/X) of 0.512). The results of the T-test (given in table 4) have provided enough evidence to reject the null hypothesis that project location has no significant effect on the relationship between construction cost and duration in highway projects.

2.8.2 Project Type:

Table 5: ANOVA - Welch test results for project type (Florida projects)

| Measurement: Absolute value (Y/X) – Florida | | | | | | | | | |
|---|----------------|------|-------------|-------|---------|------------|-----|--------|---------|
| ANOVA | | | | | | Welch | | | |
| | Sum of squares | df | Mean Square | F | P-value | Statistics | df1 | df2 | P-value |
| Between Groups | 9.042 | 2 | 4.521 | 6.572 | 0.001 | 4.365 | 2 | 99.683 | 0.015 |
| Within Groups | 1329.645 | 1933 | 0.688 | | | | | | |
| Total | 1338.687 | 1935 | | | | | | | |

Table 6: ANOVA - Welch test results for project type (Ohio projects)

| Measurement: Absolute value (Y/X) - Ohio | | | | | | | | | |
|--|----------------|------|-------------|-------|---------|------------|-----|--------|---------|
| ANOVA | | | | | | Welch | | | |
| | Sum of squares | df | Mean Square | F | P-value | Statistics | df1 | df2 | P-value |
| Between Groups | 129.114 | 2 | 64.557 | 9.859 | 0.000 | 5.240 | 2 | 65.229 | 0.008 |
| Within Groups | 7706.734 | 1177 | 6.548 | | | | | | |
| Total | 7835.848 | 1179 | | | | | | | |

To neutralize the effect of project location, the effect of project type on the relationship between construction cost and duration was studied in each state separately. A Welch ANOVA test with a significance level (α) of 0.95 was conducted to determine if the differences between means were statistically significant. The results of the test (provided in table 5 and table 6) showed that there is significant difference between the means depending on project type in both Florida and Ohio (P-value of 0.004 in Florida and 0.008 in Ohio). As a result, the null

hypothesis that project type has no significant effect on the relationship between construction cost and duration in highway projects can be rejected.

Post hoc comparison using Tamhane's T2 test was used to further understand the results of the Welch ANOVA test. Tamhane's post hoc test does not assume equal variances and thus can give robust results even in the case of unequal variances. Table 7 and table 8 provide a summary of Tamhane's T2 post hoc test results in Florida and Ohio respectively. The results of the test showed that the relationship between construction cost and duration tends to be more sensitive in bridge maintenance projects than in road maintenance projects in the two states. However, there was not enough evidence to say that new construction projects have a more sensitive relationship between construction cost and duration than road maintenance or bridge maintenance projects.

Table 7: Tamhane's T2 post hoc test results (Florida Projects)

| Multiple Comparisons – Florida Projects | | | | | | |
|--|--------------------|-----------------------|------------|-------|-------------------------|-------------|
| Dependent Variable: ABS (Y/X) | | | | | | |
| Tamhane's T2 Post Hoc Test | | | | | | |
| (I) Type | (J) Type | Mean Difference (I-J) | Std. Error | Sig. | 95% Confidence Interval | |
| | | | | | Lower Bound | Upper Bound |
| Road Maintenance | Bridge Maintenance | -0.182 | 0.067 | 0.021 | -0.343 | -0.021 |
| | New Construction | -0.198 | 0.154 | 0.498 | -0.581 | 0.185 |
| Bridge Maintenance | Road Maintenance | 0.182 | 0.067 | 0.021 | 0.021 | 0.343 |
| | New Construction | -0.016 | 0.166 | 1.000 | -0.424 | 0.392 |
| New Construction | Road Maintenance | 0.198 | 0.154 | 0.498 | -0.185 | 0.581 |
| | Bridge Maintenance | 0.016 | 0.166 | 1.000 | -0.392 | 0.424 |

Table 8: Tamhane's post hoc test results (Ohio projects)

| Multiple Comparisons – Ohio Projects | | | | | | |
|---|--------------------|-----------------------|------------|-------|-------------------------|-------------|
| Dependent Variable: ABS (Y/X) | | | | | | |
| Tamhane's T2 Post Hoc Test | | | | | | |
| (I) Type | (J) Type | Mean Difference (I-J) | Std. Error | Sig. | 95% Confidence Interval | |
| | | | | | Lower Bound | Upper Bound |
| Road Maintenance | Bridge Maintenance | -0.648 | 0.213 | 0.007 | -1.158 | -0.139 |
| | New Construction | -1.291 | 1.091 | 0.574 | -4.073 | 1.492 |
| Bridge Maintenance | Road Maintenance | 0.648 | 0.213 | 0.007 | 0.139 | 1.158 |
| | New Construction | -0.643 | 1.108 | 0.919 | -3.457 | 2.172 |
| New Construction | Road Maintenance | 1.291 | 1.091 | 0.574 | -1.492 | 4.073 |
| | Bridge Maintenance | 0.643 | 1.108 | 0.919 | -2.172 | 3.457 |

2.8.3 Contract Type:

Table 9: T-test results for project contract type

| Independent Samples T-test | | | | | | |
|--|---------|---------|-----------------|-----------------------|---|--------|
| Measurement: Absolute value (Y/X) | | | | | | |
| T | df | P-value | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference | |
| | | | | | Lower | Upper |
| -6.057 | 333.214 | 0.000 | 0.423 | 0.070 | -0.561 | -0.286 |

The effects of contract type in terms of using I/D contract provisions was only studied in Florida projects because of the limited availability of contract types information in Ohio projects. The results of the independent samples T-test are shown on table 9. The test's reported P-value is less than 0.001 and thus the null hypothesis that project contract type has no significant effect on the relationship between construction cost and duration in highway projects can be rejected.

2.8.4 Delivery Method:

Table 10: T-test results for project delivery method

| Independent Samples T-test | | | | | | |
|-----------------------------------|------|---------|-----------------|-----------------------|---|-------|
| Measurement: Absolute value (Y/X) | | | | | | |
| T | df | P-value | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference | |
| | | | | | Lower | Upper |
| 1.412 | 1934 | 0.158 | 0.124 | 0.088 | -0.048 | 0.297 |

Due to the limited number of DB projects in Ohio data base, the effect of delivery method on the relationship between construction cost and duration was also studied in Florida projects only. Table 10 shows the result of the independent samples T-test for project delivery method. The results of the T-test show that the variation between means in DBB and DB projects is not statistically significant (P-value of 0.613). As a result, there is not enough evidence to reject the null hypothesis that project delivery method has no significant effect on the relationship between construction cost and duration in highway projects.

2.9 Applying the Research Findings to Build New Cost-Duration Models:

Based on the findings of this research, project highway agency, project type and contract type can affect the sensitivity of the construction cost – duration relationship. Thus, SHAs and contractors should always consider these factors when developing new construction cost – duration models. In this section, new construction cost – duration models that account for these important factors will be developed using data from FDOT. The performance of the new models will then be compared to the performance of one general model developed by Shr and Chen for FDOT in 2004.

2.9.1 New Cost – Duration Models for FDOT’s Road Maintenance projects:

To build the new construction cost – duration models, a similar approach to that used by Shr and Chen on 2004 will be followed. Empirical data from projects that were completed by FDOT between the year 1999 to 2014 were used to fit the new models. To account for the assumption that schedule delays and cuts from the normal point will always increase the cost of project construction, only projects with positive cost growth ratio (Y) were included in the model fitting process.

To account for the effects of project type, only projects in the road maintenance group were used to build the new models. These projects were later divided based on their contract types, and projects in each contract type group were further divided into an 80% training set and a 20% testing set. Projects in the training sets were used to build the new modified models, while projects in the testing sets were used to test the performance of the new models and compare it to the performance of the original model developed by Shr and Chen. Table 11 shows the breakdown of the 937 Florida road maintenance projects according to their contract type and the breakdown of projects in each group into the training sets and the testing sets.

Table 11: Breakdown of Florida road maintenance projects used in the model building process

| Contract Type | n | n training | n testing |
|------------------------|----------|-------------------|------------------|
| No I/D provisions used | 746 | 596 | 150 |
| I/D provisions used | 191 | 152 | 39 |
| Total | 937 | 748 | 189 |

Because projects in the training sets have different construction costs and durations, the relationship between cost growth ratio (Y), and time growth ratio (X) was first developed. Since the general form of the functional relationship between construction cost and duration is known to follow the general quadratic equation as in equation 1, the value of time growth ratio squared (X^2) was calculated for each project in the training sets, and multiple linear regression was then used to define the relationship between the dependent variable Y, and the independent variables X and X^2 . The resulting regression models followed the general form of equation 9 below:

$$Y = a_2X^2 + a_1X + a_0 \quad (9)$$

where Y is cost growth ratio, X is time growth ratio and a_2 , a_1 , and a_0 are all constants defined by regression coefficients.

Using equations 5 and 6, equation 9 can be rewritten as follow:

$$\frac{C-C_0}{C_0} = a_2\left(\frac{D-D_0}{D_0}\right)^2 + a_1\left(\frac{D-D_0}{D_0}\right) + a_0 \quad (10)$$

where C is the actual final construction cost, D is the actual final construction duration, C_0 is the initial or planned construction cost and D_0 is the initial or planned construction duration.

The values of C_0 and D_0 should be known at the beginning of the construction process. Thus, the values of C_0 and D_0 in equation (10) can be considered as constants for any given project and equation 6 can thus represent the relationship between the actual construction cost (C) and the actual construction duration (D).

Finally, since the new construction cost – duration models assume that cost growth from the initial or planned construction cost (C_0) are related to time growth from the initial or planned construction duration (D_0), equation 6 above was mathematically shifted for each one of the new models to ensure that both D_0 and C_0 will be located on the normal point (the vertex of the curve) for any given project. The following discussion explains the development of the new models.

Road Maintenance – No I/D Contracts Model:

Table 12 shows the results of the multiple linear regression analysis for the road maintenance projects that did not utilize any kind of I/D contracts. ANOVA procedure was used to determine if the regression model is statistically significant at a confidence level (α) of 0.95. The ANOVA P-value is reported to be less than 0.001, and thus it was concluded that at least one of the two independent variables (X and X^2) has a significant effect on the dependent variable (Y), and that the resulting model has a statistically significant explanatory power. The corresponding t statistics P-values indicated that both X^2 and the intercept have statistically significant predictive capability and that they should be included as predictors in the regression model (P-values of less than 0.001 for both X^2 and the intercept). In addition, the independent variable X (P-value = 0.549) was not removed from the regression analysis to maintain the general quadratic equation form of the model. Finally, the model's R-Square was reported to be 0.198 meaning that around 19.8% of the total variation about the mean is explained by the regression model. Given that the model's overall P-value is significant (less than 0.001), and given the number of projects used to fit the model ($n = 596$) the resulting R-squared value is

considered acceptable for the scope of this research. This value can be improved in the future by including more predictors and variables in the model fitting process.

Table 12: Multiple linear regression results for road maintenance model

| Model Summary | | | | | |
|----------------------|-----------------------------|---------------------|--------------------------------|-----------------------------|---------|
| R | R - Square | Adjusted R - Square | | Std. Error of the Estimates | |
| 0.445 | 0.198 | 0.195 | | 0.056 | |
| ANOVA | | | | | |
| | Sum of Squares | df | Mean Square | F | P-value |
| Regression | 0.457 | 2 | 0.228 | 73.154 | 0.000 |
| Residual | 1.852 | 593 | 0.003 | | |
| Total | 2.309 | 595 | | | |
| Coefficients | | | | | |
| | Unstandardized Coefficients | | Standardized Coefficients Beta | t | P-value |
| | B | Std. Error | | | |
| X | 0.022 | 0.019 | 0.046 | 0.600 | 0.549 |
| X ² | 0.099 | 0.019 | 0.404 | 5.309 | 0.000 |
| Constant | 0.048 | 0.004 | | 11.489 | 0.000 |

Based on the results of the regression analysis provided in table 8, the fitted model for road maintenance projects without I/D contracts is given by equation 11 below:

$$\frac{C-C_0}{C_0} = 0.099\left(\frac{D-D_0}{D_0}\right)^2 + 0.011\left(\frac{D-D_0}{D_0}\right) + 0.048 \quad (11)$$

Finally, the resulting equation must be mathematically adjusted to ensure that C_0 and D_0 are located on the normal point (The shifting process is explained in details in appendix B).

The final adjusted model is given by equation 12 below:

$$C = 0.099C_0\left(\frac{D-1.0556D_0}{D_0}\right)^2 + 0.011C_0\left(\frac{D-1.0556D_0}{D_0}\right) + 1.004C_0 \quad (12)$$

Road Maintenance – I/D Contracts Model:

Table 13 shows the results of the regression analysis for road maintenance projects that used I/D contracts. The ANOVA P-value was reported to be less than 0.001 and thus indicated that the model has a statistically significant prediction power. The t statistics P-values also indicated that both X^2 and the intercept have a significant predictive capability and that they should be included as predictors in the regression model (P-value of 0.018 for X^2 and P-value of less than 0.001 for the intercept). The model's R-Square was reported to be 0.377 meaning that the model can explain around 37.7% of the variations about the mean.

Table 13: Multiple linear regression results for bridge maintenance model

| Model Summary | | | | | |
|----------------------|-----------------------------|---------------------|--------------------------------|-----------------------------|---------|
| R | R - Square | Adjusted R - Square | | Std. Error of the Estimates | |
| 0.614 | 0.377 | 0.368 | | 0.066 | |
| ANOVA | | | | | |
| | Sum of Squares | df | Mean Square | F | P-value |
| Regression | 0.398 | 2 | 0.199 | 45.016 | 0.000 |
| Residual | 0.658 | 149 | 0.004 | | |
| Total | 1.056 | 151 | | | |
| Coefficients | | | | | |
| | Unstandardized Coefficients | | Standardized Coefficients Beta | t | P-value |
| | B | Std. Error | | | |
| X | 0.091 | 0.053 | 0.264 | 1.734 | 0.085 |
| X^2 | 0.129 | 0.054 | 0.364 | 2.393 | 0.018 |
| Constant | 0.061 | 0.008 | | 7.610 | 0.000 |

Based on the regression results in table 9, the resulting model is given by equation 13 below:

$$\frac{C-Co}{Co} = 0.129\left(\frac{D-Do}{Do}\right)^2 + 0.091\left(\frac{D-Do}{Do}\right) + 0.061 \quad (13)$$

The final adjusted model's equation after shifting the curve's vertex to ensure that C_0 and D_0 are always located on the normal point (refer to appendix B) is given by equation 14 below:

$$C = 0.129C_0\left(\frac{D-1.3527D_0}{D_0}\right)^2 + 0.091C_0\left(\frac{D-1.3527D_0}{D_0}\right) + 1.016C_0 \quad (14)$$

2.9.2 Testing and Validating the New Models:

Figure 5 shows the graph of the two new models along with the original model for a project with C_0 value of \$5,000,000 and D_0 value of 200 days. It is obvious from the graph that the existing model resembles a more sensitive relationship between construction cost and duration than the new models. In other words, the existing model expects that schedule delays and cuts will be accompanied with higher cost overruns while the new model expects these delays and cuts to be accompanied with less cost overruns.

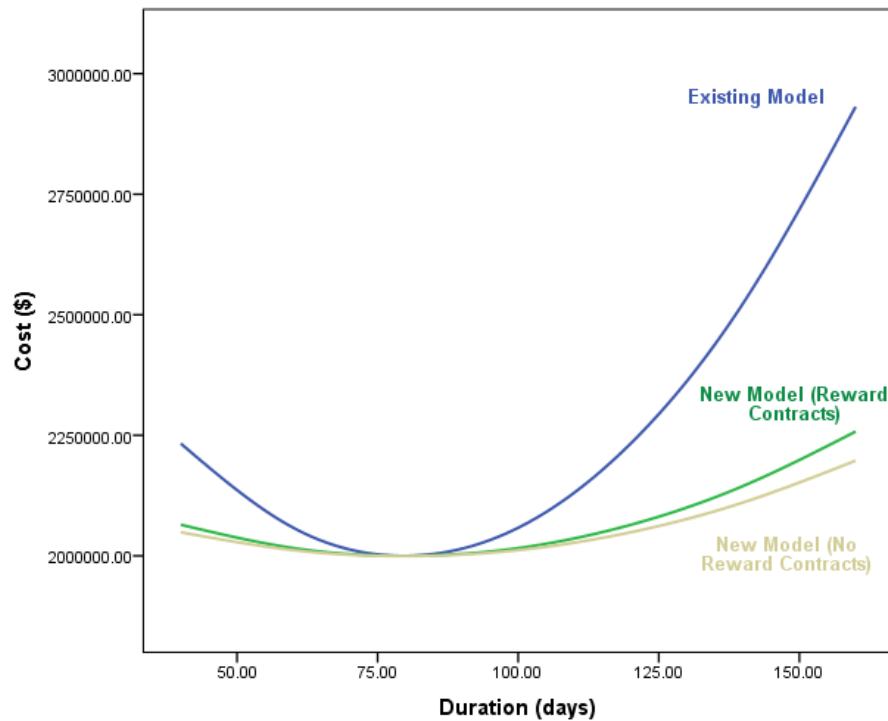


Figure 5: Graphs of the new models and the original model for a project with $C_0 = \$2,000,000$ and $D_0 = 80$ days

To test the performance of the new models, each model was used to predict the final construction costs of the highway projects in its own testing set. The generated predictions were then compared to the values of the observed or actual final construction costs and the values of the percent error (PE) were then calculated using equation 15 below:

$$PE = \left| \frac{\text{predicted final construction cost} - \text{actual final construction cost}}{\text{actual final construction cost}} \right| \times 100\% \quad (15)$$

The values of the PE can give a reliable indication about the accuracy of each one of the developed models even though different projects in the testing sets have different final construction costs. For example, a mean PE value of 5% means that the model tends to either underestimate or overestimate the actual final construction cost by an average of 5% and it should be interpreted in the same way regardless to the project's actual final construction cost. To the contrary, other methods like the root – mean – square error (RMSE) can give misleading results when a wide range of project costs is covered. For example, a model with an RMSE value of \$100,000 can be considered acceptable and even accurate for projects that cost more than \$10,000,000. However, the same model with the same RMSE value will not be even appropriate for projects that cost less than a million dollars.

The performance of each one of the developed models was then compared with the performance of an existing general model established by Shr and Chen in 2003 for FDOT. Both, the new models and the existing model were developed in a very similar way and share the same general equation (equation 6 above). Table 14 highlights the main differences and similarities between the new models and the existing model.

Table 14: new models VS existing model comparison

| Newly Developed Models | Shr and Chen Model (2003) |
|---|--|
| Account for project type and contract type | One general model (Do not account for different project types or contract types) |
| A total of 947 Florida highway projects were used to build the new models | Only 15 Florida highway projects were used to build the model |
| Multiple linear regression was used to define the relationship between cost growth ratio (Y) and time growth ratio (X) | Multiple linear regression was used to define the relationship between cost growth ratio (Y) and time growth ratio (X) |
| Time growth ratio (X) was calculated by taking the difference between final construction duration and planned construction duration | Time growth ratio (X) was calculated by taking the difference between final construction duration and adjusted or approved construction duration |
| Models were shifted to ensure that initial construction cost (Co) and duration (Do) are located on the normal point | Model was shifted to ensure that initial construction cost (Co) and duration (Do) are located on the normal point |
| Models can be used to calculate the final construction cost given that the final construction duration, planned construction cost and planned construction duration are all known | Model can be used to calculate the final construction cost given that the final construction duration, planned construction cost and planned construction duration are all known |

The new construction cost – duration models were not only modified in terms of accounting for the effects of project type and contract type, but also by including a larger number of projects to build the models. Moreover, the existing model has used two different definitions for time growth. During the model fitting process, time growth was defined as the difference between the final construction duration and the approved construction duration. The approved construction duration is the initial or planned construction duration adjusted for weather and additional works. However, in the final model's equation, time growth is calculated as the difference between the final and the initial construction durations since the approved or adjusted construction duration will not be available at the beginning of the

construction process. Since weather conditions and additional works can affect both cost and duration, adjusting only the initial duration without adjusting the initial cost during the model fitting process may affect the model's ability to reflect an accurate relationship sensitivity. The adjustment process will usually reduce the time growth ratio while keeping the cost growth ratio unchanged, leading to an overestimation of the real relationship sensitivity. The new models, on the other hand, have used one consistent definition of time growth, and it is always calculated as the difference between the final and the initial construction durations.

Road Maintenance – No I/D Contracts Model Performance:

The performance of the two models was evaluated by calculating the value of the PE for each project in the 150 projects testing set. The results of the comparison are given in table 15. Figure 6 shows the mean PE values along with the error bars for both the new and the existing model.

The results of the comparison showed that the new road maintenance model is superior in terms of performance and accuracy than the existing model. The mean PE value of the new model was 4.980% and it was less than the 6.422% mean PE value for the existing model. Even under a 95% confidence interval, figure 6 shows that the mean PE range for the new model is between 4.8% to 6.25% and it is less than mean range for the existing model which was between 5.8 to 8%. In addition, the new model has a much lower maximum PE value than the existing model (18.770% and 48.540% respectively).

Table 15: PE values comparison (no I/D model VS existing model)

| Descriptive Statistics – Percent Error (PE) | | | | | | |
|---|-----|---------|---------|--------|-------------------|----------------|
| | n | Minimum | Maximum | Mean | Std. Error (mean) | Std. Deviation |
| Existing Model | 150 | 0.000% | 48.540% | 6.422% | 0.651% | 7.977% |
| RM Model | 150 | 0.030% | 18.770% | 4.980% | 0.367% | 4.497% |

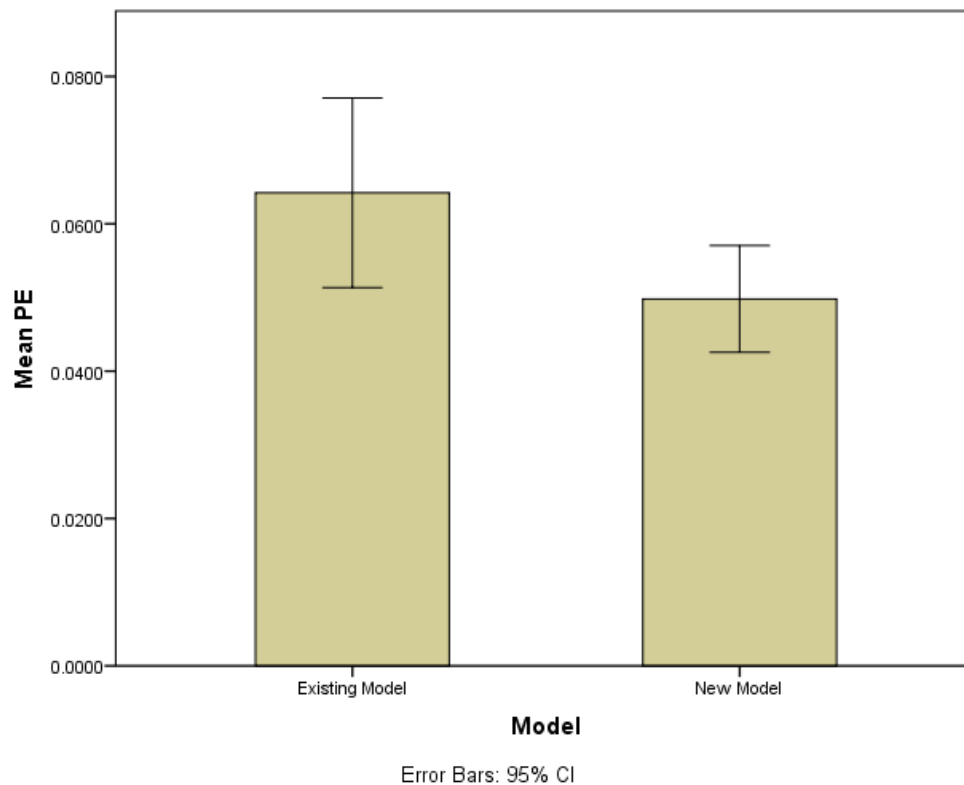


Figure 6: Mean PE values along with the error bars (95% CI) for the new road maintenance – no I/D contracts model and the existing model

Road Maintenance – No I/D Contracts Model Performance:

Table 16 shows the PE values for the new and existing model. The results of the comparison showed that the new model can give more accurate predictions than the existing model. The values of the mean and maximum PE were less for the new model (mean PE of

4.755% and maximum PE of 13.250%) than the existing model (mean PE of 6.384% and maximum PE of 28.410%). In addition, Figure 7 shows the mean PE values for both models along with the error bars. It shows that under a 95% confidence level, the new model still has a lower mean PE range than the existing model.

Table 16: PE values comparison (new I/D model VS existing model)

| Descriptive Statistics – Percent Error (PE) | | | | | | |
|---|----|---------|---------|--------|-------------------|----------------|
| | n | Minimum | Maximum | Mean | Std. Error (mean) | Std. Deviation |
| Existing Model | 39 | 0.040% | 28.410% | 6.384% | 0.934% | 5.831% |
| RM Model | 39 | 0.050% | 13.250% | 4.755% | 0.564% | 3.519% |

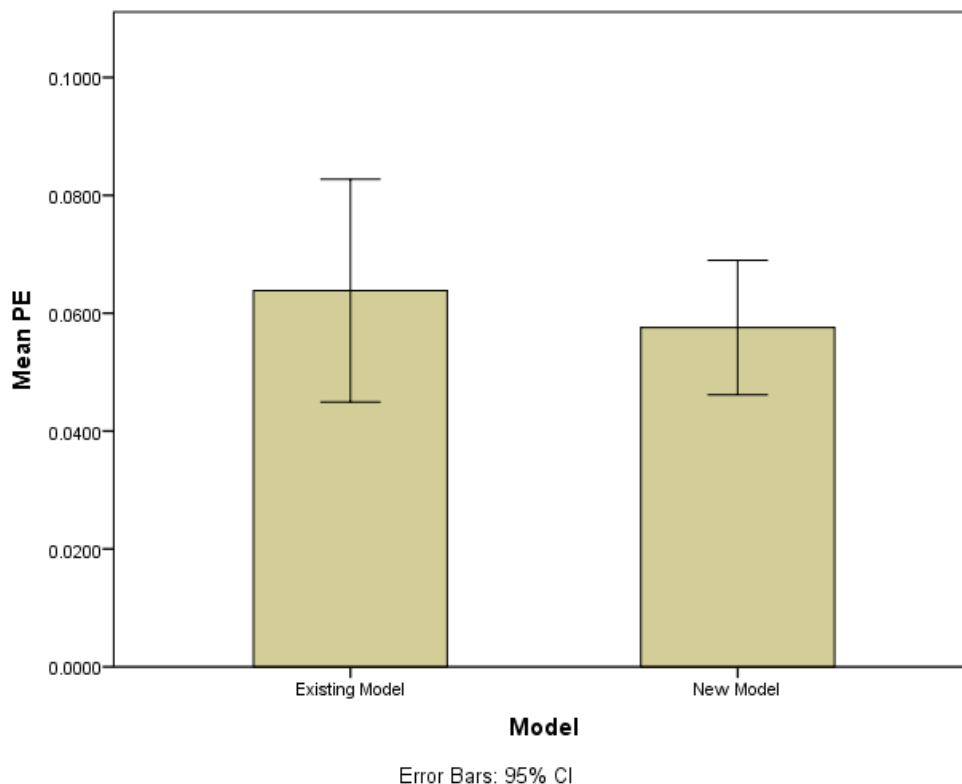


Figure 7: mean PE values along with the error bars (95% CI) for the new road maintenance – I/D contracts model and the existing model

2.10 Conclusion:

Understanding and quantifying the functional relationship between construction cost and duration has become very important for both SHAs and contractors. Building accurate construction cost-duration models can help SHAs to reasonably calculate some of the most important I/D contract parameters. Similarly, using accurate construction cost-duration models can help contractors to identify the optimum price-time combination and thus maximize their chances to win new projects under the price-time bi-parameter procurement method. Available literature about this important topic has always presented the relationship between construction cost and duration as a part of bigger optimization approaches, and no papers were dedicated to study this relationship in depth.

This paper focused on studying the relationship between construction cost and duration in highway projects by investigating the effects of important project characteristics on this relationship. The findings of this paper showed that project highway agency, project type and contract type can have significant effects on the relationship between construction cost and duration. These findings were then applied to develop more accurate construction cost-duration models for road maintenance project in Florida.

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Chapter 3: Conclusion:

Understanding and quantifying the functional relationship between construction cost and duration has become very important for both SHAs and contractors. Available literature about this important topic has always presented the relationship between construction cost and duration as a part of bigger optimization approaches, and no papers were dedicated to study the variation of this relationship from one project to another. This research focused on studying the effects of important project characteristics on the relationship between construction cost and duration. The findings of this paper showed that project highway agency, project type and contract type can have significant effects on the relationship between construction cost and duration. The following discussion provides an overview of the research contributions, limitations and suggested future research.

3.1 Contributions:

This research was the first to study the variations in the relationship between construction cost and duration. Previous researches on this topic, such as those conducted by Shen et al. in 1999 and Shr and Chen in 2004 mentioned that the relationship between construction cost and duration can vary from one project to another. This research provided a method to measure and quantify this variation. The final findings of this research also confirmed that the cost-duration relationship is project dependant.

The findings of this research were also applied to build more accurate construction cost-duration models for FDOT's road maintenance projects. The used model fitting process brought improvement over the one suggested by Shr and Chen in 2004 in certain aspects. First, the

quality of the used data was improved by not only accounting for the effects of project highway agency but also the effects of project type and contract type. Second, a significantly larger number of projects was used to fit the new models. Third, cross validation by taking 80% training sets to build the new models and 20% testing sets to evaluate their performance was utilized to ensure that the new models are not overfitted. Finally, unlike Shr and Chen method in which only the project initial duration but not the project initial cost is adjusted to reflect the effects of weather conditions and additional works, the new model fitting process defined time growth as the difference between the final construction duration and the initial or planned construction duration and used this definition consistently during the model fitting and application stages.

Although the new models were designed for road maintenance projects in Florida and cannot be generalized, the same procedures can be adopted and replicated by other SHAs and contractors. By understanding the concept of relationship sensitivity and how it can be measured, SHAs and contractors who aim to maximize their benefits from using the optimization approaches suggested by Shen et al. in 1999 and Shr and Chen in 2004, can use their own data bases to identify the important factors that affect the relationship between construction cost and duration in their projects. These factors can then be used as predictors in the model development process or can help contractors and SHAs to select the best clusters of historic data to build accurate construction cost – duration models.

3.2 Limitations and Future Research:

Due to data availability, this paper covered only some of the important project characteristics that can affect the relationship between construction cost and duration. Other factors, such as contractor's managerial practices, construction techniques, project complexity and average daily traffic values may also affect the relationship between construction cost and duration. Controlling for these variables could have improved the accuracy of this paper's findings by reducing the possible effect of confounding variables. Future research about the effects of these important factors on the construction cost-duration relationship can further help SHAs and contractors to use the best clusters of historic data to fit more accurate construction cost-duration models.

In addition, it should be clear that this paper only studied the correlation but not the causation between cost growths and time growths in highway projects. Although it was pointed out that when the relationship between construction cost and duration is more sensitive, time growths will generally be accompanied with higher cost growths, this study never claimed that time growths can be solely used to explained cost growths. Future researches and case studies about the cause and effect relationship between cost growth and time growth is still required to better understand the findings of this research.

Due to time availability, this research has not studied the direct effects of the relationship sensitivity on the paid I/D amounts or the selected price-time combinations. Future research on this topic may include studying the correlation between the relationship sensitivity (absolute value of Y/X) and the paid/deducted I/D amounts. This will allow for a better estimation of the

I/D amounts in the future and will help in evaluating the effectiveness of the I/D contracts in reducing the actual construction time.

Finally, because of data and time availability, this research has only considered certain aspects of project types, contract types and delivery methods. Future research that consider more aspects could be very useful in confirming and further explaining the findings of this study.

3.3 Final Thoughts:

Throughout this research efforts, I have developed a profound understanding of the relationship between cost and time in construction projects. At the beginning, my main purpose was to test and validate an existing construction cost-duration model and then use it as an optimization tool for calculating the I/D contract parameters. However, as I continued to make discoveries, it was obvious for me that the importance of studying the relationship between construction cost and duration is underestimated. As a result, I have decided to focus my research on studying the functional relationship between construction cost and duration in highway projects. While it is already known that understanding and quantifying the relationship between cost and duration can help SHAs and contractors to adapt with the use of the new contractual provisions and procurement methods, I still believe that establishing this relationship can also be helpful in many other areas like calculating more accurate contingencies for construction projects, calculating acceptable liquidated damage amounts and thus avoid unnecessary contractual disputes and identifying the optimum contract times for construction projects .

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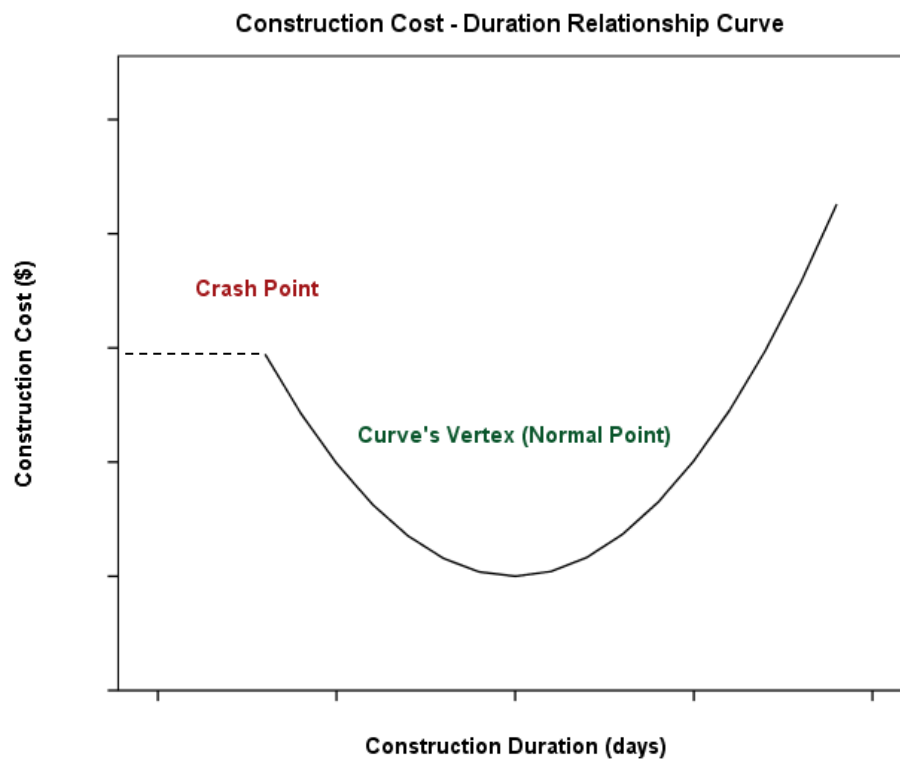
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Appendix A: Shen et al. (1999) Approach to Calculate the Optimum Price-Time Combination when Price-Time Bi-Parameter Procurement Method is Used

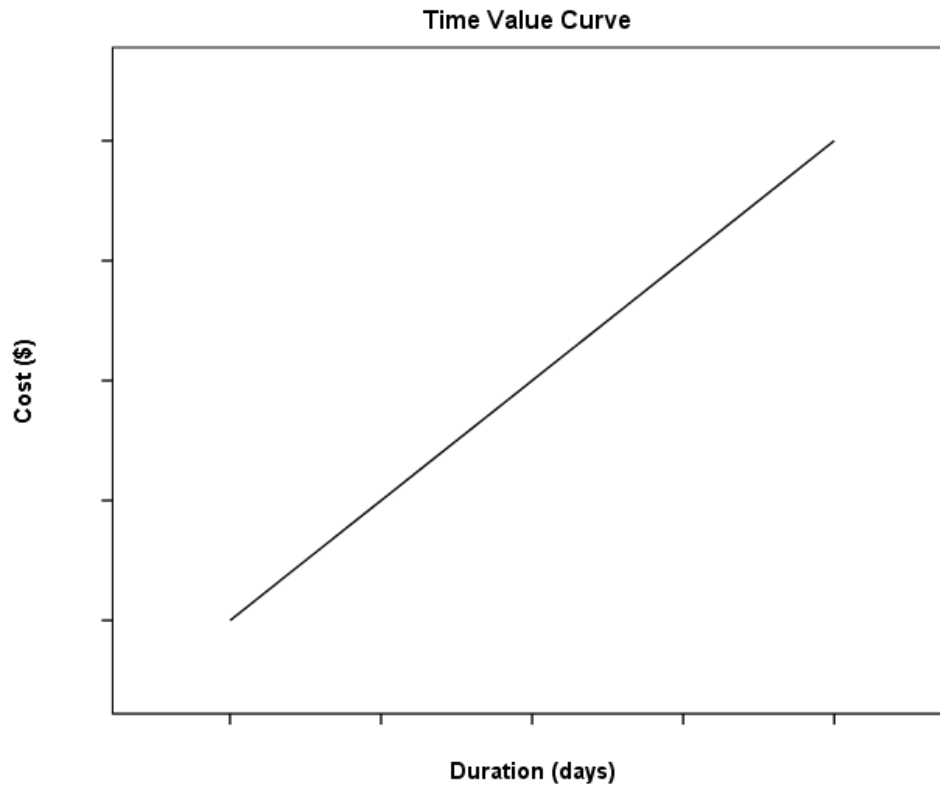
- 1- develop the functional relationship between construction cost and duration for the project under consideration:



Contractors should use their own experience to come up with at least three price-time combinations. These three combinations are suggested as the shortest time combination, lowest price combination and most likely combination. Using these price-time combinations, contractors can then use polynomial regression analysis to determine the values of the constants a_0 , a_1 and a_2 in equation 1 and thus define the functional relationship between construction cost and duration.

$$C = a_2(D^2) + a_1(D) + a_0$$

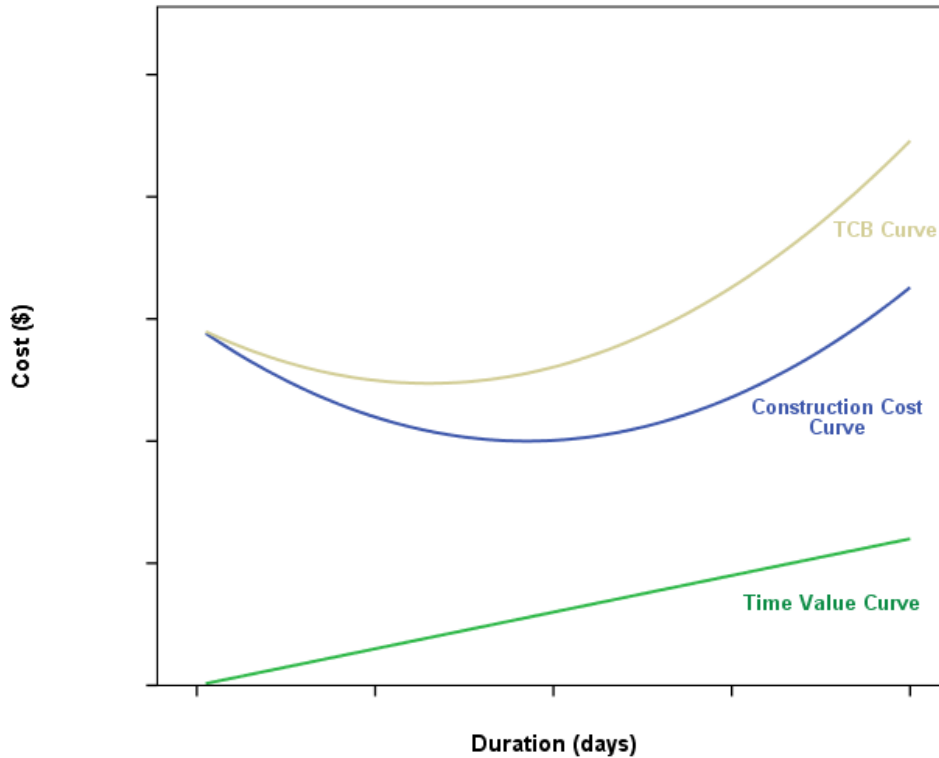
2- Develop the functional relationship for the time value (TV) curve:



The time value (TV) curve represents the indirect costs or the loss of money that may be caused by the construction process. Usually the time value equation is determined by multiplying the unit time value (UTV) by the construction duration as given in equation 14. The UTV is normally given in the contract appendix and is usually equals to the daily road user cost (DRUC) in highway projects.

$$TV = UTV \times D$$

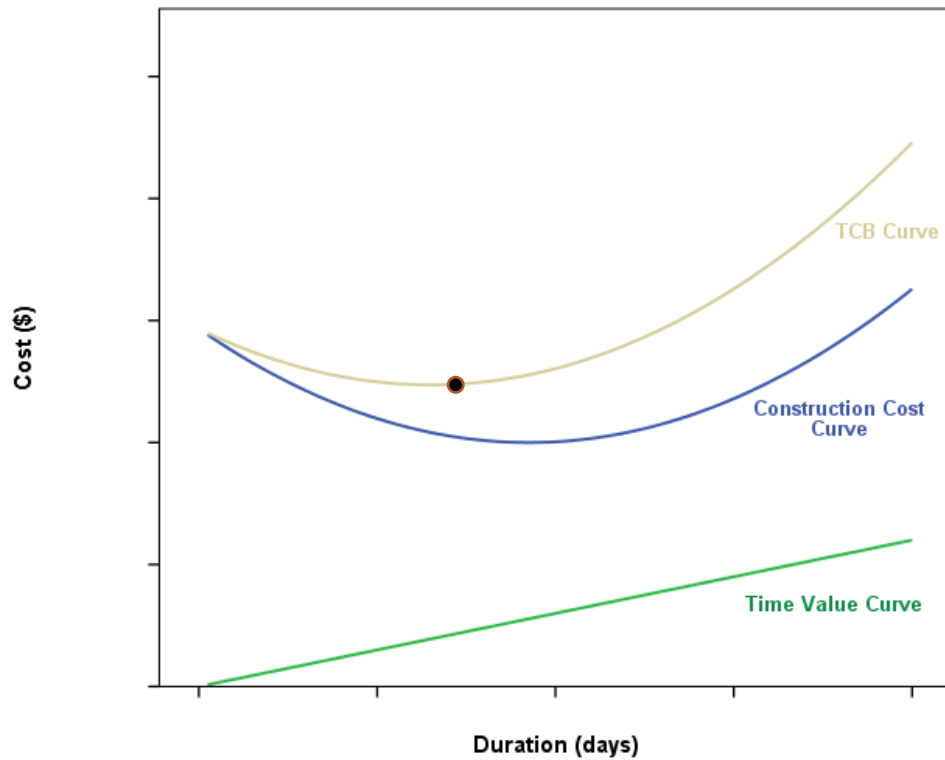
- 3- Add equation (1) to equation (14) to obtain the total construction cost or the total combined bid (TCB) equation:



The TCB equation represents the total cost of construction process from the public's point of view. Contractors should generally minimize the TCB value in their offers to increase their chances to win new projects. The TCB equation is obtained by adding the construction cost equation to the time value equation and is given in equation 15 below:

$$TCB = a_2(D^2) + a_1(D) + a_0 + (UTV \times D)$$

4- Use the TCB equation to identify the optimum price-time combination:



Ideally, the optimum price-time combination is the point where the TCB value is minimized. This point can be determined by taking the derivative of equation 15 or by looking directly on the TCB curve. Contractors could also use the TCB curve to look at different price-time combinations and then pick the most suitable one for them.

Appendix B: The Welch-ANOVA Test Procedures:

The Welch-ANOVA test is considered as a reliable alternative for the classical ANOVA test when the assumption of equality of variances is not met (Tomarken and Serlin 1986). The main difference between the Welch-ANOVA test and the classical ANOVA test is that The Welch-ANOVA test uses a weight (W) to reduce the effects of heterogeneity. This weight depends mainly on the sample size (n) and the observed variance (s) and can be calculated as follow:

$$W = \frac{n}{s}$$

Based on the calculated weight, the adjusted grand mean (\bar{Y}_{welch}) can be calculated as follow:

$$\bar{Y}_W = \frac{\sum_{i=1}^r w_i \cdot \bar{Y}_i}{\sum_{i=1}^r w_i}$$

where (\bar{Y}_i) is the sample mean for the (ith) group and $i = 1, \dots, r$.

After calculating the adjusted grand mean, the Treatment sum of squares (SSTR_w) and treatment mean squares (MSTR_w) can be calculated as follow:

$$SSTR_w = \sum_{i=1}^r w_i (\bar{Y}_i - \bar{Y}_w)^2$$

$$MSTR_w = \frac{SSTR_{\text{Welch}}}{r-1}$$

The final step before calculating the Welch-ANOVA F statistics is calculate a term called lambda (Λ), which is also based on weights:

$$\Lambda = \frac{3 \sum_{i=1}^r \frac{(1 - \frac{w_i}{\sum_{i=1}^r w_i})^2}{n_i - 1}}{r^2 - 1}$$

The test statistics can be calculated as follow:

$$F_w = \frac{MSTR_w}{1 + \frac{2\Lambda(r-2)}{3}}$$

Finally, the obtained F_w value will be compared with the critical F value (F_c) as follow:

If $F_w \leq F_c (1-\alpha; r-1, \frac{1}{\Lambda})$, then conclude the null hypothesis (H_0)

If $F_w > F_c (1-\alpha; r-1, \frac{1}{\Lambda})$, then conclude the alternative hypothesis (H_a)

Appendix C: Welch T-test Procedures:

The Welch t-test (also known as the independent samples T-test) is an adaption of the student's t-test. The Welch T-test is considered more reliable than the student's T-test when the two compared samples have unequal variances and sample sizes (Ruxton, G. D. 2006). Unlike the classic student's T-test, the Welch T-test does not use a common variance value when calculating the t statistics and degrees of freedom. Instead, the variance of each one of the compared two groups is used. The Welch t statistics (t_w) can be calculated as follow:

$$t_w = \frac{m_a - m_b}{\sqrt{\frac{S_a^2}{n_a} + \frac{S_b^2}{n_b}}}$$

Where:

m_a and m_b : sample means of the first and second compared groups respectively

S_a and S_b : sample variance of the first and second compared groups respectively

n_a and n_b : sample size of the first and second compared groups respectively

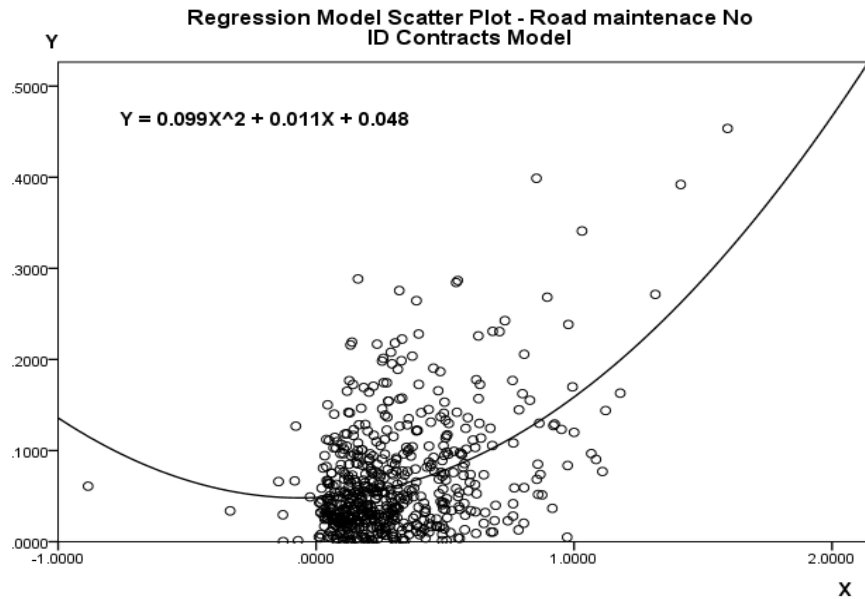
The degree of freedom can then be calculated according to following equation:

$$df = \frac{\left(\frac{S_a^2}{n_a} + \frac{S_b^2}{n_b}\right)^2}{\frac{S_a^4}{n_a^2(n_a - 1)} + \frac{S_b^4}{n_b^2(n_b - 1)}}$$

Once, the t statistics (t_w) and the degree of freedom (df) are calculated, the t statistics tables can be used along with the appropriate significance level (α) to test the null hypothesis under consideration.

Appendix D: Regression Scatter plots and Equations Shifting Procedures

Road Maintenance – No I/D Contracts Equation:



From the results of the regression analysis we have:

$$Y = 0.099X^2 + 0.011X + 0.048$$

$$Y' = 0.198X + 0.011$$

By Letting $Y' = 0$, we have $X(\min) = -0.0556$ and $Y(\min) = 0.0489$

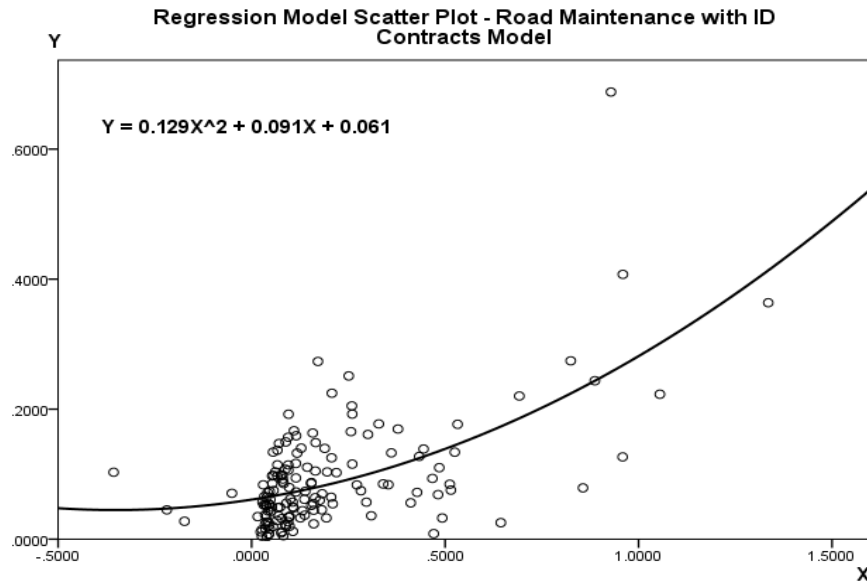
$$Y + 0.0489 = 0.099(X - 0.0556)^2 + 0.011(X - 0.0556) + 0.048$$

$$\text{But, } Y = \frac{C - C_0}{C_0} \quad \text{and} \quad X = \frac{D - D_0}{D_0}$$

$$\frac{C - C_0}{C_0} + 0.0489 = 0.099\left(\frac{D - D_0}{D_0} - 0.0556\right)^2 + 0.011C_0\left(\frac{D - D_0}{D_0} - 0.0556\right) + 0.048$$

$$C = 0.099C_0\left(\frac{D - 1.0556D_0}{D_0}\right)^2 + 0.011C_0\left(\frac{D - 1.0556D_0}{D_0}\right) + 1.004C_0$$

Road Maintenance – I/D Contracts Equation:



From the results of the regression analysis we have:

$$Y = 0.129X^2 + 0.091X + 0.061$$

$$Y' = 0.258X + 0.091$$

By Letting $Y' = 0$, we have $X(\min) = -0.3527$ and $Y(\min) = 0.0450$

$$Y + 0.0450 = 0.129(X - 0.3527)^2 + 0.091(X - 0.3527) + 0.061$$

$$\text{But, } Y = \frac{C - C_0}{C_0} \quad \text{and} \quad X = \frac{D - D_0}{D_0}$$

$$\frac{C - C_0}{C_0} + 0.0450 = 0.129\left(\frac{D - D_0}{D_0} - 0.3527\right)^2 + 0.091\left(\frac{D - D_0}{D_0} - 0.3527\right) + 0.061$$

$$C = 0.129C_0\left(\frac{D - 1.3527D_0}{D_0}\right)^2 + 0.091C_0\left(\frac{D - 1.3527D_0}{D_0}\right) + 1.016C_0$$