Clearance Matching

Tadayoshi Kohno Hal Gabow, Advisor

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1 Introduction and Definitions

Consider a graph $G = (V, E)$ and a set $M \subseteq E$. Let us define the function $d(v, M)$ as the number of edges in M incident on a vertex $v \in V$. Let $d(v) = d(v, E)$ be the degree of a vertex v. Furthermore, let the *degree constraint* function u be a mapping from the vertices in V to the set of non-negative integers.

A clearance matching (CM) instance $C = (G, u, c)$ consists of a bipartite graph $G = (V, E)$, a degree constraint function u, and a positive integer clearance c. Let $V = I \cup S$ where I and S are disjoint and all edges in E are between I and S. Consider I to be a set of "items" and S to be an *ordered* set of "slots".

We can now define the terms *perfect clearance matching* and *complete clearance matching*:

Definition 1.1 *A set $M \subseteq E$ is a complete clearance matching on $C = (G, u, c)$ if (1) $d(i, M) = u(i)$ and $d(s, M) \leq u(s)$ for all $i \in I$, $s \in S$, and (2) (i, s), $(i, s') \in M$ implies that $|s - s'| \geq c$.*

A complete clearance matching is *perfect* if $d(s, M) = u(s)$ for every $s \in S$.

When the first criteria of the above definitions are met, M is said to satisfy C's degree constraints. When the second criteria is met, M is said to satisfy C's *clearance condition.* A set $M \subseteq E$ that satisfies the C's clearance condition is also said to be *conflict-free.*

Notice that in the above definitions, M is *not* a matching in the traditional sense of the word. That is, a vertex v may be incident to more than one edge in M. For abrievety, we may sometimes refer to a clearance matching M as a *c-matching.*

Definition 1.2 *In the clearance matchings problems, we are given a CM instance $C = (G, u, c)$ and wish to determine if there exists a complete clearance matching M on C.*

The clearance matching problem is motivated by the real-life problem of scheduling doctors at Denver General Hospital. At the Denver General Hospital, each doctor is available is capable of working certain months of the year. Furthermore, each doctor must work a certain number of months a year, and each month must have at least a certain number of doctors on duty. A doctor cannot work two consecutive months.

We can model the Doctor's Scheduling problem as a clearance matching problem with the set of items corresponding to the doctors and the set of slots corresponding to the months. The degree constraint on each doctor is the number of months the doctor must work. The degree constraint on each month is the number of doctors that must be on duty that month. A doctor and month are adjacent in the clearance matching graph if the doctor can work that month. By setting $c = 2$ and finding a clearance matching on the above graph, we can find a scheduling of doctors that meets the requirements of the Denver General Hospital.

2 NP-Completeness

Theorem 2.1 *The perfect clearance matching problem with $c = 2$ is NP-complete.*

Proof: We base our proof on the Exact Cover by 3-Sets (X3C) problem. In the X3C problem, we are give a set R of $3q$ elements and a collection X of three-element subsets of R and wish to determine if X contains an exact cover for R. That is we wish to determine if there is a pairwise disjoint subcollection $X' \subseteq X$ such that $\bigcup X' = R$. This problem is NP-complete [6].

We reduce X3C to perfect clearance matching with clearance 2. Consider an instance of X3C specified by set R and collection X. The perfect clearance matching problem is specified by $C = (G, u, c)$ where G is a bipartite graph with vertex set $V = V_1 \cup V_2$ and edge set E. We take $V_1 = \{b\} \cup \{r : r \in R\} \cup \{s_i, d_i : S_i \in X, \}$ and $V_2 = \{l_{f(i,j)} : S_i \in X,\ 1 \le j \le 8\}$ where $f(i,j) = 8(i-1)+j$. The edge set is

$$\begin{aligned} E &= \{(d_i, l_{f(i,j)}) : S_i \in X,\ j = 1, 8\} \\ &\cup\ \{(s_i, l_{f(i,j)}) : S_i \in X,\ 1 \le j \le 8\} \\ &\cup\ \{(b, l_{f(i,j)}) : S_i \in X,\ j = 3, 5, 7\} \\ &\cup\ \{(x, l_{f(i,2)}), (y, l_{f(i,4)}), (z, l_{f(i,6)}) : S_i = \{x, y, z\} \in X\} \end{aligned}$$

We take $c = 2$ and specify the degree constraints as follows: $u(b) = 3|X| - |R|$, $u(r) = 1$ for all $r \in R$, and $u(s_i) = 4$, $u(d_i) = 1$, $u(l_{f(i,j)}) = 1$ for all $S_i \in X$ and $1 \le j \le 8$. Figure 1 gives an example of the component in G for each collection $S_i \in C$.

We claim that X contains an exact cover for R if and only if there is an perfect clearance matching on C.

Suppose that $X' \subseteq X$ is an exact cover on R. Then, as illustrated in Figure 2,

$$\begin{aligned} M &= \{(s_i, l_{f(i,j)}) : S_i \in X',\ j = 1, 3, 5, 7\} \\ &\cup\ \{(d_i, l_{f(i,8)}) : S_i \in X'\} \\ &\cup\ \{(x, l_{f(i,2)}), (y, l_{f(i,4)}), (z, l_{f(i,6)}) : S_i = \{x, y, z\} \in X'\} \\ &\cup\ \{(s_i, l_{f(i,j)}) : S_i \notin X',\ j = 2, 4, 6, 8\} \\ &\cup\ \{(d_i, l_{f(i,1)}) : S_i \notin X'\} \\ &\cup\ \{(b, l_{f(i,j)}) : S_i \notin X',\ j = 3, 5, 7\} \end{aligned}$$

is a subset of E such that $d(v, M) = u(v)$ for all $v \in V^1$ and $|i - j| \ge c$ for all pairs of edges (v, i), $(v, j) \in M$, $v \in V_1$, $i, j \in V_2$. Thus, M is an perfect clearance matching on C.

Next suppose C has an perfect clearance matching M. Since $u(d_i) = 1$, $d_i \in V_1$, exactly one of $(d_i, l_{f(i,1)})$ or $(d_i, l_{f(i,8)})$ is in M. If $(d_i, l_{f(i,8)}) \in M$ then $(s_i, l_{f(i,j)}) \in M$, $j = 1, 3, 5, 7$, and $(x, l_{f(i,2)})$, $(y, l_{f(i,4)})$, $(z, l_{f(i,6)}) \in M$. See Figure 2a. Conversely, if $(d_i, l_{f(i,1)}) \in M$ then $(s_i, l_{f(i,j)}) \in M$, $j = 2, 4, 6, 8$, and $(b, l_{f(i,j)}) \in M$, $j = 3, 5, 7$. See Figure 2b. Let $X' = \{S_i : (d_i, l_{f(i,8)}) \in M\}$.

[1] $u(b, M) = 3|X| - |R|$ because $|X - X'| = |X| - |R|/3$.

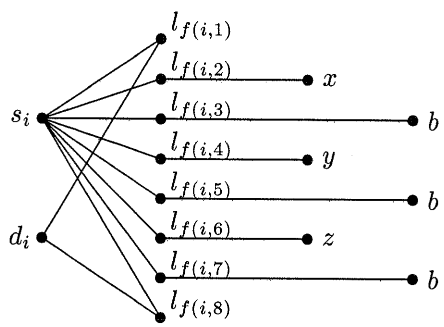


Figure 1: Component in G corresponding to $S_i = \{x, y, z\} \in X$. Nodes $l_{f(i,3)}$, $l_{f(i,5)}$, $l_{f(i,7)}$ adjacent to b.

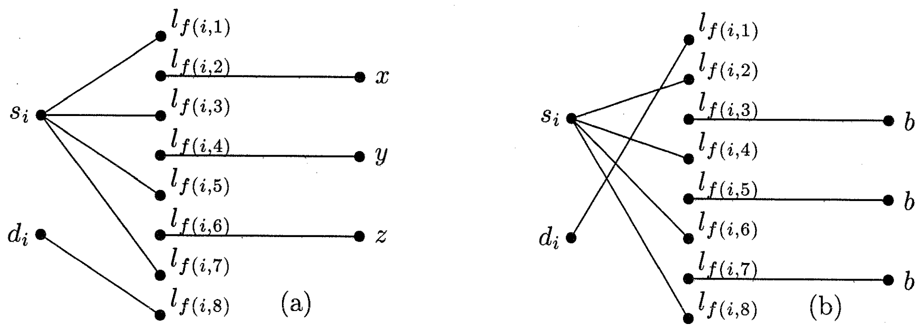


Figure 2: Matched edges in G corresponding to the (a) inclusion and (b) exclusion of $S_i = \{x, y, z\}$ in exact cover X'.

Notice that $d(r, M) = u(r) = 1$ for all $r \in R$. Also notice that $(r, l_{f(i,j)}) \in M$ if and only if $(d_i, l_{f(i,8)}) \in M$, $j \in 2, 4, 6$, $r \in S_i = x, y, z$. Namely, either $(x, l_{f(i,2)}), (y, l_{f(i,4)}), (z, l_{f(i,6)}), \in M$ or $(x, l_{f(i,2)}), (y, l_{f(i,4)}), (z, l_{f(i,6)}), \notin M$ for any given $S_i \in X$. Consequently, $\bigcup X' = R$ and all sets in X' are pairwise disjoint. X' is therefore an exact cover for R. □

Theorem 2.2 *The perfect clearance matching problem with $c \geq 2$ is NP-complete.*

Proof: X3C can be polynomially time reduced to perfect clearance matching with clearance greater than or equal to 2. This follows naturally from the construction of Theorem 2.1.

For each set S_i in collection X, consider a component expansion similar to Figure 1 except with $c - 2$ additional slots with degree constraints 0 inserted sequentially after $l_{f(i,2)}$, $l_{f(i,4)}$, and $l_{f(i,6)}$. The proof is then similar to the proof of Theorem 2.1. □

The complete clearance matching problem is also NP-complete. This should be clear from the definitions; i.e., the complete clearance matching problem deals with a of superset of the perfect clearance matching problems.

Initially, we might think of proving this in several ways. For example, the discard node b in the proof of Theorem 2.1 seems to provide no functionality other than to ensure that $d(v_2, M) = u(v_2)$ for all $v_2 \in V_2$. It would seem, then, that we can drop b from our construction and have a complete clearance matching problem whose solution is equivalent to that of the initial X3C instance.

Unfortunately, however, if we drop b, nothing prevents all d_i's from matching with their respective $l_{f(i,8)}$'s. That is, while each component will still be in one of two matched configurations, nothing prevents all components from being in the included configuration. If all components are in the included configuration, all items $r \in R$ can legally match with any adjacent slot.

We must thus provide some additional background before we can claim that the complete clearance matching problem in NP-complete.

It should first be pointed out that if a CM instance C has an perfect clearance matching, then it also has a complete clearance matching. This follows naturally from the definitions.

Furthermore, if $C = (G, u, c)$ has an perfect clearance matching and $G = (V, E)$ where $V = V_1 \cup V_2$, then the equality

$$\sum_{v_1 \in V_1} u(v_1) = \sum_{v_2 \in V_2} u(v_2) \tag{1}$$

must hold. If the equality does not hold, then C cannot have a perfect clearance matching. To formalize this for future use:

Lemma 2.3 *If $\sum_{v_1 \in V_1} u(v_1) \neq \sum_{v_2 \in V_2} u(v_2)$, then $C = (G, u, c)$, $G = (V_1 \cup V_2, E)$, does not have a perfect clearance matching.*

We now provide a lemma that shows that a complete clearance matching solver can be used to solve the perfect clearance matching problem.

Lemma 2.4 *If $C = (G, u, c)$ has a complete clearance matching M and Equation 1 holds, then M is also a perfect c-matching on C.*

Proof: Assume that M is not a perfect c-matching on C. Then it must violate C's clearance condition and/or C's degree constraints.

Because the clearance condition for complete and perfect c-matchings are the same, M cannot violate C's clearance condition.

This means that M must violate C's perfect degree constraints. We know that $d(v_1, M) = u(v_1)$ for all $v_1 \in V_1$ by the definition of complete clearance matching. This means that there must exist at least one slot $s \in V_2$ such that $d(s, M) \neq u(s)$; i.e., such that $d(s, M) < u(s)$. Consequently,

$$\sum_{v_2 \in V_2} d(v_2, M) < \sum_{v_2 \in V_2} u(v_2)$$

But, since

$$\sum_{v_2 \in V_2} u(v_2) = \sum_{v_1 \in V_1} u(v_1) = \sum_{v_1 \in V_1} d(v_1, M)$$

we have that

$$\sum_{v_2 \in V_2} d(v_2, M) < \sum_{v_1 \in V_1} d(v_1, M)$$

But this is impossible since all edges in M go between V_1 and V_2. We have thus established a contradiction and our proof is complete. □

We can now prove the following theorem:

Theorem 2.5 *The complete clearance matching problem with $c \geq 2$ is NP-complete.*

Proof: This proof extends naturally from Lemma 2.4. Namely, we provide a polynomial time reduction from the *perfect* clearance matching problem to the *complete* clearance matching problem.

Let $C = (G, u, c)$ be an instance of the perfect clearance matching problem that satisfies Equation 1. Let $C' = C$ be the corresponding complete clearance matching instance. (From Lemma 2.3 we already know that if Equation 1 is not satisfied, then C cannot have a perfect clearance matching.)

C has an perfect clearance matching solution if and only if C' has a complete clearance matching solution. We have already seen that by definition if C has an perfect c-matching, then C' has a complete c-matching. Conversely, Lemma 2.4 tells us that if C' has a complete clearance matching, then C has an perfect clearance matching. □

Because we can consider the perfect clearance matching problem to be a special case of the complete clearance matching problem, we shall often times refer to the complete clearance matching problem simply as the *clearance matching* problem:

Definition 2.1 *We may refer to a complete clearance matching problem simply as a clearance matching (CM) problem.*

It should be pointed out that if M is a clearance matching (either perfect or complete) on $C = (G, u, c)$ whre $G = (V_1 \cup V_2, E)$, then $\sum_{v_1 \in V_1} u(v_1) = \sum_{v_2 \in V_2} d(v_2, M) = |M|$.

3 Reductions

In this section we provide additional polynomial time reductions from known NP-complete problems to the clearance matching problem.

We consider these reductions because a great deal of energy has been spent in analyzing these problems and creating useful data sets on which to run tests. In particular, the Second DIMACS Implementation Challenge focused on cliques, coloring, and satisfiability [8].

3.1 Independent Set

In the Independent Set (IS) problem we are given a graph $G_{IS} = (V_{IS}, E_{IS})$ and an integer k and wish to decide if there are k vertices in V such that no two are adjacent in G. This problem is NP-complete [6].

Given an instance $G_{IS} = (V_{IS}, E_{IS})$ of IS, we construct a CM instance $C = (G_{CM}, u, c)$ with $c = 2$ and $G_{CM} = (V_{CM}, E_{CM})$ specified by:

$$
\begin{aligned}
V_{CM} &= V_1 \cup V_2 \\
V_1 &= \{i\} \cup \{e\} \cup \{b\} \cup \{v_r, v_d : v \in V_{IS}\} \\
V_2 &= \{v_j : v \in V_{IS},\ 1 \leq j \leq 4d(v) + 2\} \\
E_{CM} &= \{(i, v_1), (e, v_{4d(v)+2}) : v \in V_{IS}\} \\
&\cup\ \{(v_d, v_j) : v \in V_{IS},\ 1 \leq j \leq 4d(v) + 2\} \\
&\cup\ \{(v_r, v_{2(d(v)+j)}) : v \in V_{IS},\ 1 \leq j \leq d(v)\} \\
&\cup\ \{(v_r, w_{2b(w,v)} : v \in V_{IS},\ (v, w) \in E\} \\
&\cup\ \{(b, v_{2j}) : v \in V_{IS},\ 1 \leq j \leq d(v)\} \\
&\cup\ \{(b, v_{2j+1}) : v \in V_{IS},\ 1 \leq j \leq 2d(v)\}
\end{aligned}
$$

and the degree constraint function u defined as:

$$
\begin{aligned}
u(i) &= k \\
u(e) &= |V_{IS}| - k \\
u(b) &= 2|E_{IS}| \\
u(v_r) &= d(v),\ v \in V_{IS} \\
u(v_d) &= 2d(v) + 1,\ v \in V_{IS} \\
u(v_j) &= 1,\ v \in V_{IS},\ 1 \leq j \leq 4d(v) + 2
\end{aligned}
$$

where v is w's $b(w, v)$th adjacency: i.e., if w's adjacency list was sorted, v would be the $b(w, v)$th one in the list. See Figure 3(a).

G_{IS} has an independent set of size k if and only if C has a clearance matching.

The component replacement for each vertex v in V_{IS} can be in only one of two matched configurations: $(i, v_1) \in M$ *or* $(e, v_{4d(v)+2}) \in M$. See Figures 3(b,c). Suppose $V'_{IS} \subseteq V_{IS}$ is a set of k independent vertices in G_{IS}, then a clearance 2 matching M for G_{CM} exists such that $(i, v_1) \in M$ if $v \in V'_{IS}$ and $(e, v_{4d(v)+2}) \in$

M if $v \notin V'_{IS}$. Conversely, if M is a clearance 2 matching on G_{CM}, then $V'_{IS} = \{v : (i, v_1) \in M\}$ is an independent set on G_{IS}.

When convenient, we may refer to i, e, b, v_r, and v_d as the *includer*, *excluder*, *discard*, *driver*, and *decision* nodes, respectively.

$|V_{CM}| = 4|V_{IS}| + 8|E_{IS}| + 3$ and $|E_{CM}| = 4|V_{IS}| + 18|E_{IS}|$.

3.2 Clique

In the Clique problem we are given a graph $G_{CL} = (V_{CL}, E_{CL})$ and an integer k and wish to decide if V_{CL} contains a subset of k mutually adjacent vertices. The Clique problem is NP-complete [6].

Since the Clique problem is equivalent to the IS problem on the complement graph, the reduction from Clique to CM is straight forward; the "inclusion" of one vertex excludes non-adjacent (rather than adjacent) vertices.

3.3 3-CNF-SAT

In the 3-CNF-SAT problem we are given a boolean equation ϕ in conjunctive normal form such that each clause c is a disjunction of three distinct literals (a variable or its compliment). We wish to determine if ϕ is satisfiable. This problem is NP-complete [6].

Given an instance ϕ of 3-CNF-SAT, we construct a CM instance $C = (G, u, c)$ with $c = 2$ as follows.

Let $\mathcal{C}$ be the set of clauses in ϕ and let X be the set of variables in ϕ. Let $D(x)$ be the number of clauses containing x or $\neg x$. Let A be the number of clauses containing both a variable and its compliment. $G = (V_1 \cup V_2, E)$ is then specified by:

$$
\begin{aligned}
V_1 &= \{x_a, x_r : x \in X\} \cup \{c : c \in \mathcal{C}\} \cup \{b\} \\
V_2 &= \{T_{x,a}, F_{x,a} : x \in X\} \\
&\cup \ \{T_{x,c}, F_{x,c} : x \in X,\ c \in \mathcal{C},\ x \text{ or } \neg x \in c\} \\
E &= \{(x_r, T_{x,a}), (x_r, F_{x,a}) : x \in X\} \\
&\cup \ \{(x_r, T_{x,c}), (x_r, F_{x,c}) : x \in X,\ c \in \mathcal{C},\ x \text{ or } \neg x \in c\} \\
&\cup \ \{(x_a, T_{x,a}), (x_a, F_{x,a}) : x \in X\} \\
&\cup \ \{(c, T_{x,c}) : x \in X,\ c \in \mathcal{C},\ x \in c\} \\
&\cup \ \{(c, F_{x,c}) : x \in X,\ c \in \mathcal{C},\ \neg x \in c\} \\
&\cup \ \{(b, T_{x,c}), (b, F_{x,c}) : x \in X,\ c \in \mathcal{C},\ x \text{ or } \neg x \in c\}
\end{aligned}
$$

The degree constraint function u is specified by:

$$
\begin{aligned}
u(x_a) &= 1,\ x \in X \\
u(x_r) &= D(x) + 1,\ x \in X \\
u(c) &= 1,\ c \in \mathcal{C} \\
u(b) &= 2|\mathcal{C}| - A
\end{aligned}
$$

$$
\begin{aligned}
u(T_{x,a}) &= 1, \ x \in X \\
u(F_{x,a}) &= 1, \ x \in X \\
u(T_{x,c}) &= 1, \ x \in X, \ c \in \mathcal{C}, \ x \text{ or } \neg x \in c \\
u(F_{x,c}) &= 1, \ x \in X, \ c \in \mathcal{C}, \ x \text{ or } \neg x \in c
\end{aligned}
$$

The slots in V_2 are ordered $F_{x,a}, T_{x,c_i}, F_{x,c_i}, \ldots, T_{x,c_j}, F_{x,c_j}, T_{x,a}$ with respect to any $x \in X$. See Figure 4(a).

As with the X3C and IS reductions, a component for $x \in X$ can be matched in one of two configurations. See Figures 4(b,c). It is easy to see that ϕ is satisfiable if and only if G has a clearance 2 matching. Namely, if ϕ is satisfiable, then G has a c-matching M such that $(x_a, T_{x,a}) \in M$ if x is true, and $(x_a, F_{x,a}) \in M$ otherwise. Conversely, if G has a c-matching M, then variable x is true if $(x_a, T_{x,a}) \in M$, and x is false otherwise.

We may refer to any node x_a as an *assignment* node, x_r as a *driver* node, and c as a *clause* node. We may refer to b as the *discard* node.

$|V_1| = 2|X| + |\mathcal{C}| + 1$, $|V_2| = 2|X| + 6|\mathcal{C}| - 2A$, and $|E| = 4|X| + 15|\mathcal{C}| - 4A$.

3.4 CNF-SAT

In actuality, any CNF-SAT instance can be reduced to CM with clearance 2. The formulas for V and E are the same as in Section 3.3 except that the degree constraint on item b is different.

3.5 Discard Node

It should be pointed out that the discard node b is not needed for the independent set, clique, and CNF reductions to be correct. That is, unlike with the X3C to CM reduction of Section 2, the discard node b can be dropped from the reduction.

When b is include, clearance matchings on CM instances will be perfect. When b is excluded, clearance matchings will be complete.

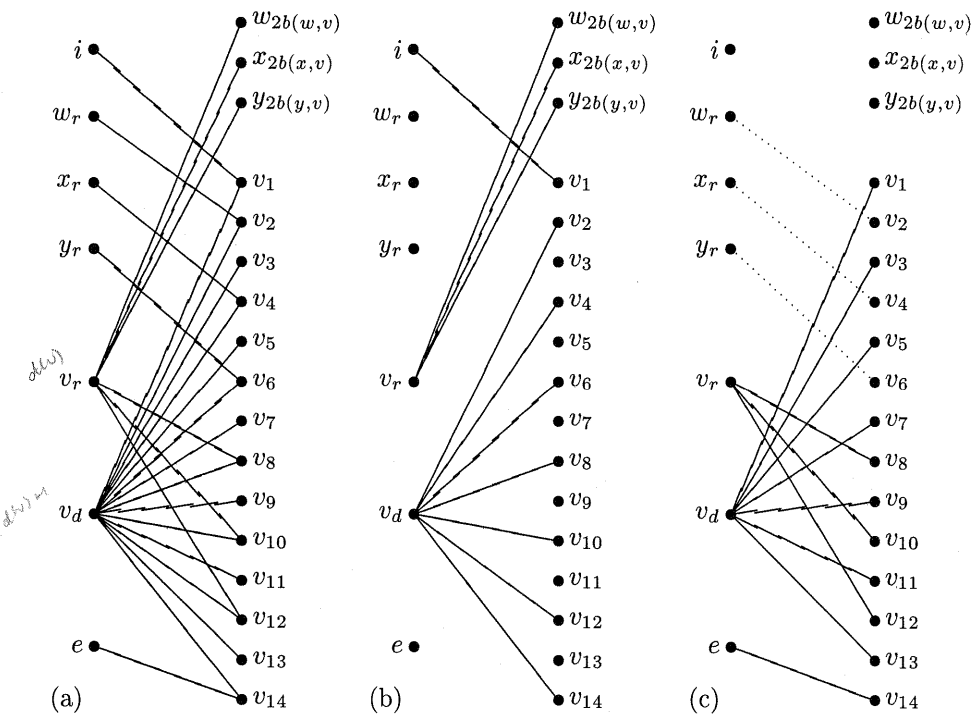


Figure 3: IS to CM reduction. (a) component in G_{CM} corresponding to $v \in V_{IS}$ of degree 3; (b), (c) possible c-matching for vertex v corresponding to $(i, v_1) \in M$ and $(e, v_j) \in M$, $j = 4d(v) + 2$, respectively. Note that discard node b matches to unmatched slots.

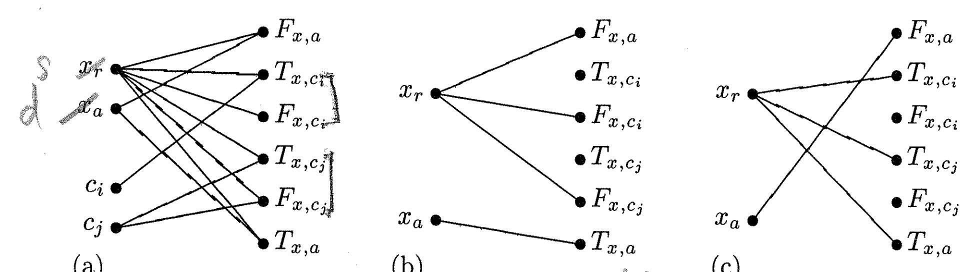


Figure 4: 3-CNF-SAT to CM reduction. (a) component in G corresponding to $x \in X$ where $c_i = (x \vee \cdot \vee \cdot)$ and $c_j = (x \vee \neg x \vee \cdot)$; (b), (c) possible assignments for $x \in X$ corresponding to $x = T$ and $x = F$, respectively.

4 Complete Bipartite Instances

In this section we would like to consider instances of the clearance matching problem consisting of complete bipartite graphs. In doing so, we hope to illuminate some underlying properties about generic clearance matching instances.

For review, a complete bipartite graph $K_{p,q} = (I \cup S, E)$ is a bipartite graph with disjoint vertex sets I and S where $p = |I|$ and $q = |S|$. E is the set of all possible edges between I and S; $m = |E| = pq$.

We first start with a few general observations about matchings on complete bipartite instances.

Theorem 4.1 *Take $C = (K_{p,q}, u, c)$, $K_{p,q} = (I \cup S, E)$, such that $u(i) = u_i$ and $u(s) = u_s$ are constants for all $i \in I$ and $s \in S$. If there is a perfect clearance matching for C, then the equality $|I|/|S| = u_s/u_i$ must hold.*

Proof: Let M be a perfect clearance matching on C. Then $|M| = \sum_{i \in I} u(i) = \sum_{s \in S} u(s)$. Since $u(i)$ and $u(s)$ are both constants, we have $|I|u_i = |S|u_s$. Therefore $|I|/|S| = u_s/u_i$. □

Theorem 4.2 *Take $C = (K_{p,q}, u, c)$, $K_{p,q} = (I \cup S, E)$, such that $u(i) = u_i$ and $u(s) = u_s$ are constants for all $i \in I$ and $s \in S$. If there is a complete clearance matching for C, then the equality $|I|/|S| \leq u_s/u_i$ must hold.*

Proof: Let M be a complete clearance matching on C. Then $|M| = \sum_{i \in I} u_i \leq \sum_{s \in S} u_s$. Therefore $|I|/|S| = u_s/u_i$. □

Consequently, given a complete bipartite clearance matching instance with degree constraints u_i and u_s over the items and slots, respectively, we may be able to easily determine if the instance does not have a clearance matching.

The following gives an upper bound on the degree constraints of the items. The results of this proof apply to any bipartite graph (not just complete bipartite graphs).

Theorem 4.3 *Let $C = (K_{p,q}, u, c)$ where $K_{p,q} = (I \cup S, E)$. If there exists an $i \in I$ such that $u(i) > \lceil |S|/c \rceil$, then there does not exist a perfect or complete clearance matching solution for C.*

Proof: Assume there does exist a clearance matching solution M for C. Linearly group the slots in S into $\lceil |S|/c \rceil$ groups such that each group except the last contains c slots. The last group contains less than or equal to c slots. (Figuratively, put the first c slots in the first group, the second c slots in the second group, and so on).

Because of the clearance condition, an item can match with only one slot in each group. Let i be an item such that $u(i) > \lceil |S|/c \rceil$. Since there are only $\lceil |S|/c \rceil$ groups, the pigeon hole principle requires i to be matched with two slots in the same group. This is, however, impossible. □

Theorem 4.4 *Take $C = (K_{p,q}, u, c)$, $K_{p,q} = (I \cup S, E)$, such that $u(i) = u_i$ and $u(s) = u_s$ are constants for all $i \in I$ and $s \in S$. If $|I|$ and $|S|$ are relatively prime, and $|S| > 1$, then there does not exist a clearance matching solution with $c \geq 2$.*

Proof: If $|I|$ and $|S|$ are relatively prime, then $u_s = k|I|$ and $u_i = k|S|$ where k an integer ≥ 1. But from the above, theorem, if $u_i > \lceil |S|/c \rceil$, there is no solution for C. □

We now propose an algorithm that may be able to solve the clearance matching problem on $c = 2$ complete bipartite instances in which the degree constraints of all the items are the same and the degree constraints of all the slots are the same. It remains to be proven that the algorithm finds a clearance matching if and only if the complete bipartite instance has a clearance matching. Let $C = (K_{p,q}, u, 2)$ be a clearance matching instance with items I and slots S. Let u_i be the degree constraint of the items and u_s the degree constraint of the slots.

$s \leftarrow 1$, $M \leftarrow \emptyset$
for *degree* $= 1$ to u_i **do**
  for $i = 1$ to $|I|$ **do**
    $M \leftarrow M \cup \{(i, s\}$, $s \leftarrow s + 1$
    if $s > |S|$ **then**
      $s \leftarrow 1$

Our proposition is that if the resulting set M is not a clearance matching on C, then C does not have a clearance matching.

5 Network Flow Solution

5.1 Network Flow Preliminaries

A *flow network* is a directed graph $G = (V, E)$ with source node s, sink node t, and capacity function $c(u,v) \geq 0$ for nodes $u, v \in V$. If edge $(u,v) \notin E$, we assume $c(u,v) = 0$.

A *flow* in G is a function $f : V \times V \to \mathbb{R}$ such that the following properties hold:

capacity constraint: For all $u, v \in V$, $f(u,v) \leq c(u,v)$.

skew symmetry: For all $u, v \in V$, $f(u,v) = -f(v,u)$.

flow conservation: For all $u \in V - \{s,t\}$, $f(u,V) = 0$.

For simplicity, we use the notation that $f(U,W) = \sum_{u \in U} \sum_{w \in W} f(u,w)$ for $U, W \subseteq V$. An edge (u,v) is *saturated* if $f(u,v) = c(u,v)$; and edge (u,v) is *void* if $f(u,v) = 0$.

The *value* of a flow is defined as

$$|f| = f(s,V) = f(V,t) \tag{2}$$

In the *maximum flow problem* we are given a flow network and wish to find a flow of maximum value.

A *cut* $(S, \overline{S})$ is a partition of V into two sets S and $\overline{S}$ such that $s \in S$ and $t \in \overline{S}$. The capacity of a cut $(S, \overline{S})$ is $c(S, \overline{S})$. The relation $|f| = f(S, \overline{S}) \leq c(S, \overline{S})$ holds for any cut $(S, \overline{S})$.

The *residual capacity* function r is defined as $r(u,v) = c(u,v) - f(u,v)$ for nodes $u, v \in V$; r specifies the amount of additional flow we can push from u to v in G without violating the capacity constraint. The *residual network* of G for flow f is $R = (V, E_r)$ where $E_r = \{(u,v) : r(u,v) > 0\}$. R is a flow network with capacity function r.

An *augmenting path* P is a simple (s,t)-path in R. The *residual capacity* of P is $r(P) = \min\{r(u,v) : (u,v) \in P\}$. To *augment* f *along* P, increase f by $r(P)$ units along P. This gives a valid flow f' in G with value $|f'| = |f| + r(P)$.

There has been much research in the field of network flows. We provide a few theoretical results here, the proofs of which can be found in [3, 5, 9].

Theorem 5.1 (Augmenting Path) *A flow is maximum if and only if it has no augmenting paths.*

Theorem 5.2 (Max-Flow Min-Cut) *The maximum value of a flow is equal to the minimum capacity of a cut.*

Theorem 5.3 (Integral Flow) *If the capacity function c is integral, then there is a maximum flow that is integral.*

In our discussions we shall consider the Ford-Fulkerson method of finding a maximum flow in a flow network:

while there is a path P from s to t in R **do**
augment $r(P)$ units of flow along P.

If the capacity function c is integral, the Ford-Fulkerson method halts with a maximum flow.

We refer the reader to [3, 5, 9, 1] for a more complete discussion of network flows.

5.2 The Network Flow Algorithm

We can now begin to describe our network-flow solution to the clearance matching problem. Consider a clearance matching instance $C = (G, u, c)$, $G = (I \cup L, E)$.

The basic idea of our algorithm is this. We find a set of edges $M \subseteq E$ that satisfies C's degree constraints. If the set M does not violate C's clearance condition, then we are done. If M does violate C's clearance condition, we create a subset $M_x \subset M$ of edges that does not violate the clearance condition. We call M_x the *fixed set.* We then induce a new instance C' such that a clearance matching M' on C' *extends* the set M_x. That is, we find a clearance matching M' on C' such that $M' \cup M_x$ is a clearance matching on C. If we cannot extend the set M_x, we remove an edge from M_x and try again. We must be careful about the order in which edges are inserted and removed from M_x. We shall discuss this momentarily.

We use network flows to find our edge sets $M \subseteq E$ that satisfy C's degree constraints. Namely, we make each item $i \in I$ a source node with supply $u(i)$. We make each slot $l \in L$ a sink node with demand $u(l)$. Each directed arc from I to L in G has capacity 1. Let f be a max flow in G. We can then define M_f as $M_f = \{(i, l) \in E : f(i, l) = 1\}$. i.e., M_f is the set of edges in G with residual capacity 0.

If $|f| = \sum_{i \in I} u(i)$, then M_f satisfies the degree constraints of C's items. If f is a maximum flow on G and $|f| < \sum_{i \in I} u(i)$, then C does not have a clearance matching. Remember, a flow is maximum if and only if it has no augmenting paths.

Before we describe our algorithm in detail, let us take a few moments to describe our data structures. Consider an item $i \in I$. The degree constraint of i is specified by $u[i]$. Notice that because our algorithm may temporarily modify the item i's degree constraint, we have chosen to use square brackets rather than parentheses when defining u. This is consistent with the notation used in [3].

Designate i's adjacency list by $i.adj$. Let $l = i.adj[j]$ be the jth slot in i's adjacency list. Let us define an array *chead* indexed by the items $i \in I$. *chead*$[i]$ is a pointer into i's adjacency list. We also define an array *mark* indexed by the items $i \in I$ and the slots $l \in L$. Each element in *mark* can take on two values: MARKED and UNMARKED.

We use the *chead* and *mark* arrays as a means to derive an induced instance C' from a clearance matching instance C in our recursive routines. That is, at any recursive stage, our algorithm will be operating on a clearance matching instance consisting of only the edges that are specified in $i.adj$ after and including the $chead[i]$th element. Furthermore, an augmenting path will never enter a marked node.

We now define, without explicit pseudo-code, the procedure FLOW-TO-FRONT. The FLOW-TO-FRONT procedure re-orders i's $chead[i]$th to last adjacency ($i.adj[chead[i]]$ to $i.adj[d(i)]$) such that all the edges with flow appear at the beginning of the list and all the edges without flow appear at the end of the list.

Algorithm 3 (FLOW-AUGMENT) provides the pseudo-code for augmenting flow from an item *root*.

Algorithms 2 and 1 (FLOW-SETUP and FLOW-FIND) form the heart of our algorithm. FLOW-SETUP finds an initial flow such that $f[i, L] = u[i]$ for all $i \in I$ except for $i = 1$. FLOW-SETUP then passes control over to FLOW-FIND with the first item as the *root*.

The recursive FLOW-FIND procedure assumes that upon every invocation the *root* item is deficient by one unit flow. That is, FLOW-FIND assumes that $f[root, L] = u[root] - 1$. The degree constraints for all the other items are assumed to be satisfied. Furthermore, the fixed set M_x consists of all the edges (i, l) with positive flow $f[i, l] = 1$ such that $i < root$ or $i = root$ and $l = root.adj[j]$ where $j < chead[root]$. The FLOW-FIND procedure will not modify the flow though any item before *root* or through any edge listed before $chead[root]$ in $root.adj$. In this way C can be considered an induced graph consisting of all the items below *root* and all the edges after and including *root*'s $chead[root]$th edge.

FLOW-FIND begins by first augmenting flow from *root* in C. If no augmenting path exists, then M_x cannot be extended and the algorithm returns. If augmentation was successful, then FLOW-FIND invokes FLOW-TO-FRONT. Subsequently, the algorithm scans the edges, adding each edge with flow to the set M_x until doing so will violate the clearance condition. If no conflict exists, then a clearance matching has been found.

If a conflict does exist, FLOW-FIND immediately stops adding edges to M_x. Let us assume that (i_b, l_b) is the edge that, if added to M_x, would have forced M_x to violate the clearance condition. Let j_b be the index into $i_b.adj$ such that $l_b = i_b.adj[j_b]$. We then remove the flow over (i_b, l_b) and recurse with $root' = i_b$ and $chead[root'] = j_b + 1$.

If the recursive call fails, we remove the last added edge from M_x and recurse again. We repeat this until either a clearance matching is found or we remove all the edges added to M_x at the current level of recursion.

The fine details of the algorithms are presented in Algorithms 1–3.

Algorithm 1 FLOW-FIND($C, M_x, f, root$)

```
if FLOW-AUGMENT(C, f, root) = AUGMENT-FAIL then
  sameflow ← TRUE
  return FIND-FAIL

FLOW-TO-FRONT(C, root)
for i = root ... |I| do {{ Extend M_x until find first conflict (i_b, l_b) }}
  for j = chead[i] ... d(i) do
    if f[i, i.adj[j]] = 1 and ∃ (i, l') ∈ M_x such that |i.adj[j] − l'| < c then
      i_b ← i, l_b ← i.adj[j], j_b ← j
      goto found-conflict
    else
      M_x ← M_x ∪ {(i, i.adj[j])}
sameflow ← false
return FIND-SUCCESS

found-conflict: {{ try to eliminate conflict }}
u[i_b] ← u[i_b] − (j_b − chead[i_b]) {{ Consider induced instance; reduced u[i] }}
loop
  f[i_b, l_b] ← 0 {{ remove flow over (i_b, l_b) }}
  savehead ← chead[i_b], chead[i_b] ← j_b + 1 {{ skip conflicting edge in C' }}
  mark[i = 1 ... i_b − 1] ← MARKED {{ don't consider any item before i_b }}
  mark[i = i_b ... |I|] ← UNMARKED, mark[s = 1 ... |S|] ← UNMARKED

  if FLOW-FIND(C, M_x, f, i_b) = FIND-SUCCESS then
    chead[i_b] ← savehead, u[i_b] ← u[i_b] + (j_b − savehead)
    return FIND-SUCCESS

  if sameflow = TRUE then
    f[i_b, l_b] ← 1 {{ restore flow over (i_b, l_b) }}

  M_x ← M_x − {(i_b, l_b)}
  if j_b > chead[i_b] then {{ backup (i_b, l_b) and try again: }}
    j_b ← j_b − 1, u[i_b] ← u[i_b] + 1
  else if i_b ≠ root then
    i_b ← i_b − 1, j_b ← chead[i_b] + u[i_b] − 1, u[i_b] ← 1
  else {{ i_b = root; cannot backup (i_b, l_b) further; cannot extend M_x }}
    sameflow ← FALSE
    return FIND-FAIL
  l_b ← i_b.adj[j_b]
```

Algorithm 2 FLOW-SETUP(C)

$u[1] \leftarrow u[1] - 1$
for $i = |I| \ldots 1$ **do** {{ find initial flow f }}
    for $j = 1 \ldots u[i]$ **do**
        if FLOW-AUGMENT(C, f, i) =AUGMENT-FAIL **then**
            return NO-C-MATCHING

$u[1] \leftarrow u[1] + 1$, $M_x \leftarrow \emptyset$
if FLOW-FIND($C, M_x, f, 1$) = FIND-FAIL **then**
    return NO-C-MATCHING
return YES-C-MATCHING {{ M_x is a c-matching on C }}

Algorithm 3 FLOW-AUGMENT($C, f, root$)

if $\exists$ an $(root, l)$-path P in the residual network for some $l \in L$ such that $f[I, l] < u[l]$ and P does not enter any MARKED nodes **then**
    for each edge $(v, w) \in P$ **do**
        $f[v, w] \leftarrow f[v, w] + 1$
    return AUGMENT-SUCCESS
else
    return AUGMENT-FAIL

6 Matroid Solution

6.1 Matroid Preliminaries

A *matroid* is a structure $\mathcal{M} = (S, \mathcal{I})$ where S is a finite, non-empty set and $\mathcal{I}$ is a non-empty family of subsets of S. Furthermore, a matroid $\mathcal{M}$ must satisfy the properties:

heredity If $B \in \mathcal{I}$ and $A \subseteq B$, then $A \in \mathcal{I}$.

exchange If $A \in \mathcal{I}$, $B \in \mathcal{I}$, and $|A| < |B|$, then there exists some element $x \in B - A$ such that $A \cup \{x\} \in \mathcal{I}$.

Matroid theory derives heavily from both linear algebra and graph theory. We say that $\mathcal{I}$ contains the *independent* subsets of S. We say that a subset $I \in \mathcal{I}$ is an *independent set* of the matroid $\mathcal{M}$; a subset of S which is not independent is *dependent.* A maximal independent set is a *base* of the matroid. All maximal independent sets of a matroid have the same cardinality.

There are various flavors of matroids. For example, a *matric matroid* $\mathcal{M} = (S, \mathcal{I})$ is a matroid in which S corresponds to the columns of a matrix A and $\mathcal{I}$ contains each set of linearly independent (in the algebraic sense) columns of the A. The *graphic matroid* of a graph $G = (V, E)$ is a matroid in which S corresponds to the edges E and $\mathcal{I}$ contains all cycle-free subsets of E.

A *partition matroid* $\mathcal{M} = (S, \mathcal{I})$ is a matroid such that the ground S is broken into m pairwise disjoint *blocks* S_i, $i = 1, 2, \ldots, m$ and $\mathcal{I} = \{I \subseteq S : |I \cap S_i| \leq u_i \text{ for all } i = 1, 2, \ldots, m\}$. u_i is the upper bound of the cardinality of the intersection between an independent set I and block S_i, $i = 1, 2, \ldots, m$.

A *hierarchical partition matroid* is a matroid in which each block is broken into multiple pairwise disjoint sub-blocks, each with its own upper bound u. A *two-level* hierarchical partition matroid is a matroid in which the ground set is only broken into blocks and sub-blocks (no sub-sub-blocks). Formally, in a two-level hierarchical matroid, the ground set S is broken into m disjoint blocks S_i, $i = 1, 2, \ldots, m$. Each block S_i is broken into m_i disjoint sub-blocks $S_{i,j}$, $j = 1, 2, \ldots, m_i$. A set I is independent if $|I \cap S_i| \leq u_i$ and $|I \cap S_{i,j}| \leq u_{i,j}$ for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, m_i$.

In the maximum-cardinality matroid intersection problem we are given two matroids $\mathcal{M}_1 = (S, \mathcal{I}_1)$ and $\mathcal{M}_2 = (S, \mathcal{I}_2)$ over the same ground set S and wish to find an independent set $I \in \mathcal{I}_1 \cap \mathcal{I}_2$ such that no other set in $\mathcal{I}_1 \cap \mathcal{I}_2$ has cardinality greater than $|I|$.

6.2 The Matroid Algorithm

Consider a clearance matching instance $C = (G, u, c)$ where $G = (V, E)$ and $V = I \cup L$. Let us define a two-level hierarchical partition matroid $\mathcal{M}_1 = (E, \mathcal{I}_1)$ and partition matroid $\mathcal{M}_2 = (E, \mathcal{I}_2)$. Notice that for both matroids the ground set is the set of edges in G. For clarity, let us refer to the ground set E of $\mathcal{M}_1$ and $\mathcal{M}_2$ as S and T, respectively.

Partition the ground set S of $\mathcal{M}_1$ into $|I|$ blocks such that each block S_i consists of the edges incident on item i in G ($S_i = \{(i,l) \in E\}$). Further partition each block into m_i sub-blocks $S_{i,j}$ such that there does *not* exist two edges (i,l_1), $(i,l_2) \in S_{i,j}$ where $|l_1 - l_2| \geq c$. i.e., if (i,l_1), $(i,l_2) \in S_{i,j}$, then $|l_1 - l_2| < c$. Namely, a valid c-matching can contain at most one edge in each $S_{i,j}$. A possible construction for the sub-blocks of block S_i is:

```
j ← 1, S_{i,j} ← ∅ for all j, L_i ← sorted list of slots adjacent to i in G
first ← L_i[1] {{ first slot in S_{i,j} }}
for cnt = 1 to d(i) do
  l ← L_i[cnt]
  if l ≥ first + c then
    j ← j + 1, first ← l
  S_{i,j} ← S_{i,j} ∪ {(i,l)}
m_i ← j
```

The upper bound for each S_i is the degree constraint $u(i)$ of item $i \in I$. The upper bound for each sub-block $S_{i,j}$ is 1.

Partition the ground set T of $\mathcal{M}_2$ into $|L|$ blocks such that each block T_l contains the edges incident to slot l in G. The upper bound for each T_l is the degree constraint $u(l)$ of slot $l \in L$.

A set $M \in \mathcal{I}_1 \cap \mathcal{I}_2$ is a set $M \subseteq E$ such that $d(i,M) \leq u(i)$ and $d(l,M) \leq u(l)$ for all $i \in I$ and $l \in L$. Furthermore M contains at most one edge from any sub-block $S_{i,j}$. The cardinality of any independent set $M \in \mathcal{I}_1 \cap \mathcal{I}_2$ can be at most $\sum_{i \in I} u(i)$. If there does not exist any set $M \in \mathcal{I}_1 \cap \mathcal{I}_2$ such that $|M| = \sum_{i \in I} u(i)$, then C does not have a clearance matching.

Our matroid algorithm is an extension of the network flow algorithm of Section 5. As with the FLOW-FIND algorithm, we first find a set $M \subseteq E$ that satisfies some of the conditions of a clearance matching. We then find the first edge that violates the clearance and recurse on an induced subgraph. In the matroid algorithm we only consider sets M that are independent sets in $\mathcal{I}_1 \cap \mathcal{I}_2$ and that have cardinality $|M| = \sum_{i \in I} u(i)$. The algorithm uses the upper bounds on the sub-blocks $S_{i,j}$ to prune extraneous searches.

As with the previous algorithm, we use network flows to find our independent sets M. Consider a clearance matching instance $C = (G, u, c)$, $G = (I \cup L, E)$. We can construct a corresponding directed flow network $G_f = (V_f, E_f)$ as shown in Figure 5. Formally, we define the vertex set V_f and the edge set E_f as

$$V_f = I \cup L \cup \{a_{i,j}, b_{i,j} : i \in I,\ 1 \leq j \leq m_i\}$$

and

$$E_f = \{(i, a_{i,j}), (a_{i,j}, b_{i,j}), (b_{i,j}, l) : i \in I,\ 1 \leq j \leq m_i,\ (i,l) \in S_{i,j}\}$$

The capacity of each arc is 1. Given a flow f on the network G_f, we can define the corresponding independent set M_f as:

$$M_f = \{(i,l) : f(b_{i,j}, l) = 1,\ (i,l) \in S_{i,j}\}$$

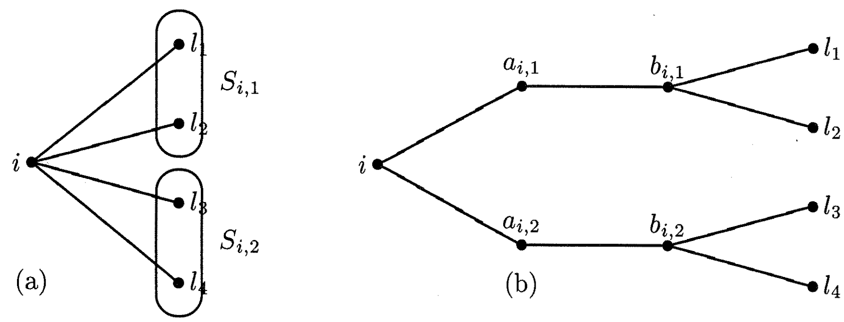


Figure 5: (a) clearance matching instance with $S_{i,1} = \{(i, l_1), (i, l_2)\}$ and $S_{i,2} = \{(i, l_3), (i, l_4)\}$ and (b) the corresponding flow network. All edges in (b) have unit capacity and are directed from left to right.

Our algorithm is then analogous to FLOW-FIND with one notable exception. Namely, we must be careful to correctly ensure that no recursive call can modify the flow (and thus corresponding matching) for any item before i_b. Merely MARKing each item as we did in Line 21 of Algorithm 1 is insufficient because an augmenting path could visit some $b_{i,j}$ without visiting i itself. Thus, we also mark the $b_{i,j}$ nodes for the is before i_b and, if $i = i_b$; we must also mark the $b_{i,j}$ nodes corresponding to the slots before i_b's *chead*.

Please see Appendices A and **??** for a discussion of the our actual implementation of the matroid algorithm. Note that although our solution is based off the network flow description presented above and in Figure 5, we did not explicitly code our solution as a general network flow problem. Rather, we implemented a solution using our knowledge of the clearance matching problem and matroid solution (for example, we observed that the clearance matching problem is over a bipartite graph and, using Figure 5 as an example, only one edge in $\{(b_{i,1}, l_1),\ (b_{i,1}, l_2)\}$ may carry flow at any one time).

7 CNF Reduction Orderings

In this section we would like to discuss our observations related to the orderings of clearance matching instances reduced from CNF-SAT instances. More specifically, recall that our solvers considered the items I and slots S ordered and indexed from 1 to $|I|$ and $|S|$, respectively. Further recall that our CNF to CM reduction of Section 3 created items x_a and x_r for each variable x in ϕ and created items c for each clause in ϕ. The CNF to CM reduction did not, however, specify any ordering of these items in I.

Through empirical testing we observe the efficiency of the solvers from Sections 5 and 6 on different orderings of the items in the CNF reduction. Through these observations, we have found seemingly pathological orderings. Through testing, we have also been able to observe some differences between the flow and matroid solvers.

It should be pointed out that all tests were conducted on a Pentium 200MHz MMX computer with 32 megabytes of RAM running the Linux kernel 2.0.30. Actual source code for our solvers can be found in Appendices A and ??.

When timing our tests, we have tried to minimize the effects of background process that might have been running concurrently with our solvers (such as large `cron` jobs). Furthermore, our code reports the real time (wall clock time), user time (our code), and system time (time spent in the kernel) used in solving a clearance matching instance. Some statistics, such as the number of recursive calls made and the maximum depth of the recursive calls, are not dependent on time and were thus unaffected by background processes.

As one might imagine, there are many possible orderings of the items created during a CNF to CM reduction. We shall enumerate a few below. In considering these, remember that x_a is the *assignment* node for a CNF variable x and x_r is the *driver* node.

$x_a x_r$: For each variable x, the assignment node immediately precedes the driver node. The first x_a, x_r pair corresponds to the first variable in the list of variables (assuming the CNF variable list is ordered). The second x_a, x_r pair corresponds to the second variable in the CNF list of variables, and so on.

$x_a x_r d-$: Like the above except that the replacement for a variable x with a high degree (either measured by $d(x_r)$ or the number of clauses in which x appears) precedes the replacement for another variable x' with a lower degree.

$x_a x_r d+$: Like the above except that x_a, x_r pairs are sorted in non-decreasing order of the degree of x_r.

$x_a x_a x_r x_r$: All assignment nodes precede all driver nodes. An assignment node x_a precedes an assignment node x'_a if x precedes x' in the CNF list of variables. Similarly, a driver node x_r precedes a driver node x'_r if x precedes x' in the list of variables.

$x_a x_a x_r x_r d-$: Like the above, except the assignment nodes are sorted in non-increasing order based upon the number of clauses in which x occurs and the driver nodes are sorted in non-increasing order of $d(x_r)$

$x_a x_a x_r x_r d+$: Like the above, but sorting is non-decreasing

Similar orderings can be specified in which driver nodes precede assignment nodes: $x_r x_a$, $x_r x_a d-$, $x_r x_a d+$, $x_r x_r x_a x_a$, $x_r x_r x_a x_a d-$, and $x_r x_r x_a x_a d+$.

All the above orderings assume the presence of the discard node b. In order to be complete, we must also consider CNF reductions that do not use the discard node. We denote these reductions as above but with the added suffix "_nob" (meaning "no b"). For example, an ordering of a reduction without b in which a variable's x_a immediately precedes its x_r would be denoted as "$x_a x_r$_nob".

All these orderings place the *clause* nodes c and the discard node b (if there is a b) at the end of the items list. Our rational for this is that clearance condition conflicts will never occur with clause nodes and the discard node is just for book keeping with perfect clearance matching instances. The source code for our reductions appear in Appendices A and **??**.

In generating our CNF instances, we used the generators developed in conjunction with the Second DIMACS Implementation Challenge [8, 2]. In particular, the ftp cite `ftp://dimacs.rutgers.edu/pub/challenge` contained many useful CNF instance generators.

8 Observations about CNF Orderings

Because of the extensive observations made, the observations are presented here in somewhat hap-hazard order. After presenting our observations, we will make some attempt to analyze and understand their implications. Because of the large volume of data and statistics generated, we have chosen to present most of our figures in Appendix B.

In trying to understand how the network flow and matroid clearance matching solvers worked, and on what instances they performed well, our first reaction was to generate per-CNF-instance plots such as the one shown in Figure 6. Although these figures could not be used as conclusive proof that one ordering was significantly better than another, they did help provide us with an intuitive feel for what orderings might lend themselves to efficient solutions and what orders might not.

From those figures, one of our first observations was that the $x_a x_a x_r x_r$ ordering appeared to be the worst. This was supported by empirical testing as shown in Appendix B.1. The figures just listed show the ratio of the number of recursive calls and the amount of user time required to solve a CNF instance reduced to the $x_a x_a x_r x_r$ ordering over the same CNF instance reduced to other orderings (such as $x_r x_a$). The figures are broken down to display these ratios with respect to the flow and matroid solvers and with respect to both satisfiable and unsatisfiable CNF instances.

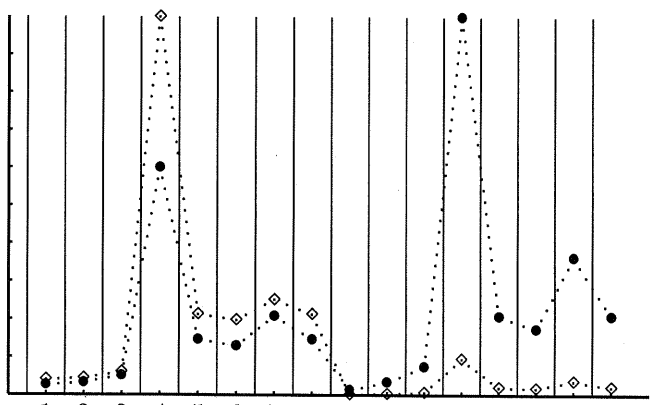


Figure 6: Plot of the user time ($\diamond$) and number of recursive calls ($\bullet$) the matroid algorithm used which solving clearance matching instances reduced from a satisfyable 3-CNF problem with 5 variables and 34 clauses. The maximum displayed user time is 54.24 seconds and the maximum number of recursive calls needed was 65439. The reductions, specified along the horizontal axis, are (1) $x_a x_r$, (2) $x_a x_r d+$, (3) $x_a x_r d-$, (4) $x_a x_a x_r x_r$, (5) $x_r x_a$, (6) $x_r x_a d+$, (7) $x_r x_a d-$, (8) $x_r x_r x_a x_a$, (9) $x_a x_r$_nob, (10) $x_a x_r d+$_nob, (11) $x_a x_r d-$_nob, (12) $x_a x_a x_r x_r$_nob, (13) $x_r x_a$_nob, (14) $x_r x_a d+$_nob, (15) $x_r x_a d-$_nob, (16) $x_r x_r x_a x_a$_nob.

Some initial per-instance graphs seemed to indicate that the $x_a x_r$ ordering was similarly problematic. Statistical testing over a fair number of instances indicated that this might be true but the results were not conclusive. See Appendix B.2.

Similarly, our per-instance graphs seemed to indicate that reduced CNF instances ordered in the $x_r x_r x_a x_a$ and $x_r x_a$ manners might be more efficiently solved than the same instance ordered in other manners. Figures showing the performance ratios of the $x_r x_r x_a x_a$ and $x_r x_a$ solvers over other solvers is show in in Appendices B.3 and B.4.

We also considered the effects of including the discard node b over excluding b. Our results are presented in Appendix B.5.

8.1 Detailed CNF Ordering Observations

With respect to the performance of the network flow and matroid algorithms on graphs with the discard node b and without the discard node b, we made the following observations. As mentioned above, the bases for these observations stem from the figures in Appendix B.5.

Before we delve too deeply into the discussion, let us first re-explain how the statistics in Appendix ?? were computed. Consider a CNF instance named `cnfinstance`. We reduced `cnfinstance` to two clearance matching instances: one with b and one without b. Each of these clearance matching instances could be ordered by any one of the above defined methods: $x_a x_r$, $x_r x_a$, etc. We measured several parameters about the network flow and matroid solvers when working on clearance matching instances. For example, we measured the user time and the number of recursive calls used by the algorithms.

Now consider two $x_a x_r$ orderings of `cnfinstance` — one with b and one without b. Let y be the number of recursive calls used by the generic flow algorithm when operating on the instance *with* b; let z by the number of recursive calls used by the generic flow algorithm when operating on the instance *without* b. y/z was then our random variable Our sample means ($\overline{x}$) with respect to each type of ordering are represented by the the circles in the figures in Appendix B.5. The vertical lines extending above and below the circles represent the 95% confidence intervals of the means computed using the equation $\overline{x} \pm z_{\alpha/2} \cdot s/\sqrt{n}$ where $z_{\alpha/2} = 1.96$ and s is the sample standard deviation [4].

On satisfiable CNF instances, the network flow solution appeared to take more time on instances with b than without b; i.e., the sample mean of the ratio was greater than 1. Unfortunately, the 95% confidence intervals for our means often included 1. So, with the exception of the $x_a x_a x_r x_r$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ orderings,[2] we cannot justifiably claim that the solutions without b were more quickly solved than solutions with b. The statistics do support the observation that the matroid algorithm takes less time on instances without b than the respective instances with b.

For each ordering except $x_r x_a$ and $x_r x_r x_a x_a$, the flow algorithm appears to

[2] In these tests we did not consider the $x_a x_a x_r x_r d-$ and $x_a x_a x_r x_r d-$ orderings.

make *slightly* more recursive calls on instances with b than instances without b, For the $x_r x_a$ and $x_r x_r x_a x_a$ orderings, the means of the ratios of the number of recursive calls with b over without b are greater than 1, but the confidence intervals include 1. On instances with x_a first ($x_a x_r$, $x_a x_r d-$, $x_a x_r d+$, $x_a x_a x_r x_r$), the matroid algorithm appears to make roughly the same number of recursive calls on instances with b and instances without b. On instances with x_r first, however, it is quite clear that the matroid algorithm makes more recursive calls on instances with b than on instances without b.

For clearance matching instances reduced from unsatisfiable CNF instances in which the x_as are first ($x_a x_r$, $x_a x_r d-$, $x_a x_r d+$), the flow algorithm takes about the same amount of time on instances with b and without b (or perhaps even slightly less time). On instances in which x_r is first, the flow solver distinctly takes more time. On reductions from unsatisfiable CNF problems, the matroid algorithm takes more time on instances with b than without b. It should be pointed out that the confidence intervals of the mean of the ratio of the time used by the flow solver on instances with b over instances without b on unsatisfiable CNF problems appear to be much smaller than the same confidence intervals on satisfiable CNF problems.

Both the flow and matroid solver clearly make *less* recursive calls on unsatisfiable CNF instances reduced with b than on the same instance reduced without b. This is in contract to the more recursive calls made by the solvers on satisfiable CNF instances. It should be pointed out that the mean of the ratio are less for instances with x_a first than with x_r first.

Let us now consider the efficiency of the flow and matroid solvers with respect to the orderings $x_a x_a x_r x_r$, $x_a x_r$, $x_r x_r x_a x_a$, and $x_r x_a$, as mentioned above. In this discussion, remember that the $x_r x_a$ ordering can be constructed from the $x_a x_r$ ordering by swapping the first and second items, the third and fourth items, the fifth and sixth items, and so on. The $x_r x_a d+$ and $x_r x_a d-$ orderings can similarly be constructed from $x_a x_r d+$ and $x_a x_r d-$, respectively. Also remember that the $x_r x_r x_a x_a$ ordering can be constructed from the $x_r x_a$ ordering by putting all x_rs at the beginning of the items list and putting all x_as at the end of the items list without modifying the order of the x_rs with respect to themselves and the order of the x_as with respect to themselves.

Let us first look at the $x_a x_a x_r x_r$ ordering. As mentioned, Appendix B.1 gives the 95% confidence intervals for the ratio of the time and number of recursive calls used by the solvers on the $x_a x_a x_r x_r$ instances over the time and number of recursive calls used by the same solver on the other orderings.

Let us first look at the ratio of user time on clearance matching instances reduced from satisfiable CNF instances. From the figures, it appears that the flow solver takes more user time to find a solution for the $x_a x_a x_r x_r$ ordering than for nearly every other orderings. The means of the ratios were very large (indicating that it took roughly 20 to 70 times the amount of time to solve an $x_a x_a x_r x_r$ instance as any instance with another ordering). Our confidence intervals for those means are, however, also very large; the intervals for the ratio of the time taken by $x_a x_a x_r x_r$ over $x_r x_a d+$ and $x_r x_r x_a x_a d+$ include 1. The matroid solver clearly took more time on the $x_a x_a x_r x_r$ instances than any other instance (i.e., the confidence intervals of the ratios excluded 1).

On satisfiable instances, with the exception of the $x_a x_r d+$, $x_r x_a d+$, and $x_r x_r x_a x_a d+$ orderings, both the flow and the matroid solvers clearly made more recursive calls on the $x_a x_a x_r x_r$ orderings than on the other orderings. For the flow solver on satisfiable instances, the confidence intervals for the means of the ratios of $x_a x_a x_r x_r$ over $x_a x_r d+$, $x_r x_a d+$, and $x_r x_r x_a x_a d+$ were extremely large and included 1.

For unsatisfiable instances, the flow solver appeared to take more time and make more recursive calls on instances with the $x_a x_a x_r x_r$ orderings than on instances with other orderings. It should be pointed out, however, that the confidence intervals for the ratios with $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ were very large and included 1. The matroid solver clearly took the most time and the most recursive calls solving the unsatisfiable $x_a x_a x_r x_r$ instances.

At this point we can make an interesting observation. Namely, for unsatisfiable instances, the means and confidence intervals for ratios of $x_a x_a x_r x_r$ over $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$ are about the same with respect to a given parameter (user time or the number of recursive calls) and a given solver (flow or matroid). The same relation can be claimed about $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ and about $x_a x_r d+$, $x_r x_a d+$, and $x_r x_r x_a x_a d+$. Unfortunately, statistics were not gathered about the execution of our solvers on $x_a x_a x_r x_r d-$ and $x_a x_a x_r x_r d+$ instances.

For example, consider the matroid solver operating on unsatisfiable CNF instances. The means of the ratios of the number of recursive calls taken by the solver on $x_a x_a x_r x_r$ instances over the $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$ instances are 3.15, 3.08, and 3.11, respectively. The respective confidence intervals are $[2.82, 3.49]$, $[2.74, 3.42]$, $[2.76, 3.45]$. We shall return to this similarity later.

Let us now consider the ratio of the time taken by the various solvers on the $x_a x_r$ instances over the time taken by the other instances. On satisfiable CNF problems, the flow algorithm clearly took more time solving the $x_a x_r$ instances than the $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ instances (i.e., the confidence intervals for the means did not include 1). The confidence intervals given for the ratio of the time required to solve the $x_a x_r$ orderings with respect to the other orderings (except $x_a x_a x_r x_r$) included 1. The matroid algorithm took more time solving the $x_a x_r$ instances than all the other instances except $x_a x_a x_r x_r$. Similar observations can be made about the number of recursive calls.

For unsatisfiable CNF instances, we would be hard-pressed to conclusively say that the flow solver took more time on the $x_a x_r$ orderings than on the other orderings because the confidence intervals of the ratios were so large and so close to 1 (with many including 1). But it should be clear that the flow solver didn't take much less time on the $x_a x_r$ orderings than the other orderings. A similar thing can be said about the time required of the matroid solver by the other orderings with respect to the $x_a x_r$ ordering. The confidence intervals of the ratios of $x_a x_r$ over $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ were slightly above 1.

For unsatisfiable CNF instances, the confidence intervals for the number of recursive calls used by the flow and matroid solver on $x_a x_r$ over the other orderings (except $x_a x_a x_r x_r$) all included 1. It should also be pointed out that with unsatisfiable CNF instances the ratios of $x_a x_r$ over $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$ have similar sample means and confidence intervals. The same can be said about $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$, and about $x_a x_r d+$, $x_r x_a d+$, and $x_r x_r x_a x_a d+$.

The statistics of the ratios with $x_r x_r x_a x_a$ in the numerator (Appendix B.3) and with $x_r x_a$ in the numerator (Appendix B.4) appear to be very similar. In an order to conserve space, we shall consider them together.

According to the statistics presented in Appendices B.3 and B.4, on satisfiable instances the ratios of time taken by the flow solver with $x_r x_r x_a x_a$ and $x_r x_a$ as the numerator and $x_a x_a x_r x_r$, $x_r x_a d-$, $x_r x_a d+$, and $x_r x_r x_a x_a d+$ (and each other) as the denominator are very close to 1. The other ratios and their respective confidence intervals are greater than 1.

The ratios of the times taken by the matroid solver on satisfiable instances is peculiar. Namely, the mean of the ratio of $x_r x_r x_a x_a$ over $x_r x_a$ is slightly less that 1 with a 95% confidence interval that excludes 1. Similarly, the 95% confidence interval of the ratio of $x_r x_a$ over $x_r x_r x_a x_a$ is just under 1. In all other respects, the statistics for $x_r x_a$ and $x_r x_r x_a x_a$ are comparable and the ratios of $x_r x_a$ and $x_r x_r x_a x_a$ over all the other orderings (except $x_a x_a x_r x_r$) are greater than 1.

In regards to unsatisfiable CNF instances, the means of all the ratios are greater than one (except with $x_a x_a x_r x_r$ in the denominator), but the confidence intervals often include 1. It should again be pointed out the similarities of the ratios with $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$ as the denominators, with $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ as the denominators, and with $x_a x_r d+$, $x_r x_a d+$, and $x_r x_r x_a x_a d+$ as the denominators.

8.2 Discussion of CNF Observations

As we mentioned several times before, on *unsatisfiable* instances the means and confidence intervals of the ratios with $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$ in the denominators seemed to be almost identical. Similarly, the means and confidence intervals with $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ and with $x_a x_r d+$, $x_r x_a d+$, and $x_r x_r x_a x_a d+$ in the denominators seemed to be almost identical. Another way of saying the same things is that, for unsatisfiable CNF instances, the mean of the ratio of any one of the $x_a x_r$, $x_r x_a$, or $x_r x_r x_a x_a$ orderings over any other ordering from the set is approximately 1. The question is "Why?" If we can answer this for one "set" of denominators. (such as $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$), we can answer it for all three sets. Note that clearance matching instances reduced from unsatisfiable CNF instances do not have valid clearance matchings. This means that a clearance matching solver operating on an instance reduced from an unsatisfiable CNF instance must eliminate all possible subsets of edges.

Remember that an instance of the $x_r x_a$ ordering can be constructed from an instance of the $x_a x_r$ ordering by transposing every two items: i.e., swap the positions of the first and second items, the third and fourth items, and so on. Also remember that the x_rs in a $x_r x_r x_a x_a$ ordering appear in the same relateive order as the x_rs in the corresponding $x_r x_a$ instance. A possible intuitive explanation for why the ratios with $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$ in the denominators on unsatisfiable CNF instances are similar is because the x_as serve little functional purpose other than to ensure that the x_rs can match to slots in only one of two configurations. In other words, our cursory explanation is that the most important factor in determining the number of recursive calls and user time required of an algorithm is the order in which the driver nodes x_r appear in the list of items.

Such an explanation is not completely sufficient. For example, if the above were true, then we would expect that the ratios with $x_a x_a x_r x_r$ in the denominator and the ratios with $x_a x_r$, $x_r x_a$, and $x_r x_r x_a x_a$ in the denominators to have similar means and confidence intervals. But they do not. The reason for the difference is that in the $x_a x_r$ ordering, if a driver node x_r is un-c-matchable, we must only backup our fixed set to the item directly in front of the x_r (i.e., the x_a) and try again. In the $x_a x_a x_r x_r$ ordering, however, to toggle the matched edge for x_a for a given x_r, we must backup $|X|$ items from the x_r. This leads to a lot of wasted effort.

We can actually extend the above observation about the x_as to the $x_r x_r x_a x_a$ and $x_r x_a$ orderings. Namely, regardless of the satisfiability of the initial CNF instance, the mean of the ratio of the time and number of recursive calls required of the solvers on $x_r x_r x_a x_a$ instances over the respective time and number of recursive calls required of the same solver on the corresponding $x_r x_a$ instance is 1 (or *very* close to one). Our explanation for this is again because the x_as serve no functional purpose other than to ensure that the x_rs can match in only one of two configurations. Additionally, having the x_as after the x_rs means that the matching of the x_as are dependent on the matching of the x_rs and not visa-versa. Consequently, since an x_r appears before its corresponding x_a, the $x_r x_a$ and $x_r x_r x_a x_a$ orderings are similarly solved.

8.3 Concerns about CNF Observations

It should be pointed out that, as with all experimental studies, one must be careful to enumerate all the conditions that can effect the results of the experiments.

Along those lines, we must first point out that we used a limited set of CNF instance generators. Moreover, most of our unsatisfiable CNF instances were generated by the `nsat`[3] generator. Thus, when we considered unsatisfiable instances we were really considering `nsat` produced instances.

We used the `gensat`[4], `ysat`[5], and `dcnf`[6] programs to generate satisfiable instances (the `gensat` and `dcnf` generators *did*, although rarely, generate a few unsatisfiable instances). Unfortunately, because the generators we used could

[3] `ftp://dimacs.rutgers.edu/pub/challenge/sat/contributed/iwama.`
[4] `ftp://dimacs.rutgers.edu/pub/challenge/sat/contributed/dubois.`
[5] `ftp://dimacs.rutgers.edu/pub/challenge/sat/contributed/iwama.`
[6] `ftp://dimacs.rutgers.edu/pub/challenge/sat/contributed/pretolani.`

have produced instances with certain properties, our results may not accurately reflect the set of all CNF instances.

When computing our statistics, we were unable to include the results of all of our experiments. Consider our method for computing the sample means of the ratio of time required by the flow solver on the $x_a x_r$ instance over the $x_r x_a$ instances. If a particular $x_r x_a$ instance was solved in 0.00 seconds (which was not uncommon for small instances), we had to exclude the ratio of time for that particular CNF instance from the computation of our sample mean. Additionally, we had to exclude the ratio for the sample mean of a particular CNF instance if the solver did not terminate on its own when operating on either of the reduced $x_a x_r$ or $x_r x_a$ instances.[7]

We must also point out that there might be other (better) ways to order the items in CNF-reduced instances besides the obvious $x_a x_r$, $x_r x_a$, etc. orderings. The large confidence intervals in our figures indicate that the relationships between the various orderings might not be clearly definable.

[7]We used the `batch` program (Appendix A) to perform our tests. The `batch` program had a timeout. If a solver did not find a solution to a clearance matching instance within the timeout, the solver was be terminated. We cannot use any information about the execution of a terminated solver in our statistics.

9 Flow versus Matroid Algorithms on CNF Reduced Instances

In this section we would like to compare the performance of the clearance matching solver using network flows (Section 5) and the solver using matroids (Section 6). As anticipated, the matroid solver generally performed much better than the network flow solver on the same instance.

In Appendix B.6 We present statistics about the ratio of the amount of time and number of recursive calls required by the matroid algorithm over the respective parameter required by the flow algorithm on the same solution.

On clearance matching instances reduced from unsatisfiable CNF instances, the matroid algorithm performed exceedingly better than the generic network flow algorithm. The means of the ratios of the time required by the matroid solver over the time required by the flow solver were clearly less than 0.5 and, for instances without b, the ratios were around 0.10. The means of the ratios for the number of recursive calls used by the matroid solver over the flow solver were less than 0.10.

For satisfiable CNF instances the picture is a bit more obscure. It appears that the matroid algorithm does not use significantly less time than the network flow algorithm on instances with b. On instances without b and with the driver nodes first ($x_r x_a$_nob, $x_r x_a d-$_nob, $x_r x_a d+$_nob, $x_r x_r x_a x_a$_nob, $x_r x_r x_a x_a d-$_nob, $x_r x_r x_a x_a d+$_nob) the matroid algorithm appears to require much less time than the flow algorithm. On the $x_a x_r d-$_nob, $x_a x_r d+$_nob, and $x_a x_a x_r x_r$_nob instances it appears as though the matroid solver required less time than the network flow solver, but the confidence intervals were rather large.

The means of the ratios of the number of recursive calls required by the matroid algorithm over the network flow algorithm on satisfiable instances in almost all cases appears to be less than 1. The possible exceptions are with the $x_a x_a x_r x_r$, $x_a x_r$_nob, $x_a x_r d-$_nob, $x_a x_r d+$_nob, and $x_a x_a x_r x_r$_nob orderings in which the confidence intervals are larger and include or come close to including 1.

10 Random Clearance Matching Instances

As Asahiro, et al, point out in [2], "when evaluating the performance of algorithms *empirically*, the most important and controversial problem is how to generate test instances." In this section, we consider how to generate random clearance matching instances. As with Section 7, we can then compare the performance of different graph orderings and the performance of different solvers.

Before we begin our discussion in earnest, it should be pointed out that there is a difference between *artificial* (random) data, and *natural* data. Namely, random data does not always reflect the data that might occur in real instances of the clearance matching problem (such as with scheduling Doctors at the xxx Hospital (Section ??)).

10.1 General Constraints

When creating random clearance matching instances, we cannot simply create random bipartite graphs and call ourselves done. This is because clearance matching instances C involve not only a bipartite graph $G = (V, E)$, but a degree constraint function u and a clearance c.

In general, we could create a random graph G, a random mapping u, and a random clearance c, but doing so will tell us very little. In order for a clearance matching to even potentially have a valid solution, other constraints must be met. For example, given a graph $G = (V, E)$ with $V = I \cup S$, if there is an item $i \in I$ such that $d(i) < u(i)$, then there cannot exist a clearance matching on G. Similarly, there cannot exists a clearance matching on G if $\sum_{i \in I} u(i) > \sum_{s \in S} u(s)$. Such instances could be detected immediately by a clearance matching solver, and are thus of very little interest.

Therefore, in generating a clearance matching instance, we must first consider the degree constraints of the items and slots.

Our first generator, GEN-RANDOM1 (Algorithm 4, takes a very simplistic approach and takes the number of items to generate, n_i, the number of slots to generate, n_s, a lower bound on the number of edges to insert, n_e, and a degree constraint function u. Note that the second `for` loop in Algorithm 4 is not strictly necessary unless we are generating perfect clearance matchings instances. See Appendices A and ?? for a discussion of the actual code used to implement this algorithm.

Algorithm 4 GEN-RANDOM1(n_i, n_s, n_e, u, c)

$V \leftarrow \{i : 1 \leq i \leq n_i\} \cup \{s : 1 \leq s \leq n_s\}$
$E \leftarrow \emptyset$
for $i = 1$ to n_i **do** {{ Set $d(i, E) = u(i)$ for each item }}
  for $cnt = 1$ to $u(i)$ **do**
    $s \leftarrow$ random slot such that $(i, s) \notin E$
    $E \leftarrow E \cup \{(i, s)\}$
for $s = 1$ to n_s **do** {{ Ensure that $d(s, E) \geq u(s)$ for each slot }}
  for $cnt = d(s, E)$ to $u(s)$ **do**
    $i \leftarrow$ random item such that $(i, s) \notin E$
    $E \leftarrow E \cup \{(i, s)\}$
for $cnt = |E|$ to n_e **do** {{ Finally, ensure that $|E| \geq n_e$ }}
  $(i, s) \leftarrow$ random edge such that $(i, s) \notin E$
  $E \leftarrow E \cup \{(i, s)\}$

There are several properties about the graphs generated by this algorithm. First, there is no guarantee that the graphs generated by GEN-RANDOM1 will have a clearance matching. Moreover, as we shall see, many of the GEN-RANDOM1 generated instances that did not have c-matchings also failed to even have sets $M \subseteq E$ that satisfied the instances' degree constraints.

In general, this could be considered good because the graphs are truly "random." In order for us to better understand our algorithms for solving the clearance matching problem, however, we would like to be able to better categorize our clearance matching instances. We would thus like to be able to generate instances with know properties; i.e., we would like to be able to generate clearance matching instances that are known to have solutions, have no solutions, have only one solution, etc.

It should also be pointed out that there are many potential modifications to Algorithm 4. For example, in addition to (or instead of) specifying a minimum bound on the number of edges, n_e, we could specify the minimum bound on the degree of each node in the generated graph. We could also specify a maximum bound on the degree of each node.

10.2 Matchable Graphs

In this section we consider how to generate clearance matching instances that are guaranteed to have valid clearance matchings.

A bit of an impediment, however, is that given parameters n_i, n_s, and u, we may not be able to generate an instance that has a c-matching. Namely, as Section 4 points out, not all complete bipartite graphs have clearance matchings.

That aside, there are several possible ways of generating clearance matching instances known to have c-matchings. For example, given a desired number of items p, a desired number of slots q, a degree constraint mapping u, and a lower bound on the number of edges m, we could find a c-matching E, if one exists, on the complete bipartite graph $K_{p,q}$ that satisfies the appropriate degree constraints. We could then add additional randomly chosen edges to E until $|E| = m$. Our clearance matching instance would then consist of the graph with edges E.

If the degree constraint function is constant over the items and constant over the slots, we could use our algorithm proposed in Section 4 to find a c-matching on $K_{p,q}$. ((Work on that algorithm; extend to non-constant u)) Unfortunately, even if we can easily find a c-matching on the bipartite graph $K_{p,q}$, our algorithms for doing so are rather deterministic. Consequently, our "random" clearance matching instances would not be very random.

Another way to generate complete clearance matching instances with at least one c-matching is to start with an empty set E. The generator proceeds very much like GEN-RANDOM1. Namely, for each item i, we add $u(i)$ random edges (i, s) to E. The constraint, however, is that not only must (i, s) not already be in E, but there must also not already be an edge $(i, s') \in E$ such that $|s - s'| < c$. Furthermore, we cannot add an edge (i, s) to E that would force the degree of s to exceed s's degree constraint.

In other words, after adding $u(i)$ edges (i, s) for each item i, the set E satisfies both the degree constraints and clearance condition of a complete clearance matching. Subsequently, we can add as many additional edges to E as we want.

We should point out several properties of the above generator. First, the generator will only generate instances with perfect c-matchings when $\sum_{i \in I} u(i) = \sum_{s \in S} u(s)$. When $\sum_{i \in I} u(i) \leq \sum_{s \in S} u(s)$, the generator will generate instances with complete c-matchings.

Secondly, there is no guarantee that our generator will halt. This is especially true when we are trying to generate instances with perfect c-matchings. Consider a stage in the execution of the algorithm in which edges have been added to E, but still more edges must be added to E before E satisfies the degree constraints of all the items. Assume $d(i, E) < u(i)$ for some item i. The algorithm must find a slot s such that $(i, s) \notin E$, $d(s, E) < u(s)$, and there does not exist an edge $(i, s') \in E$ such that $|s - s'| < c$. There is no guarantee that such a slot exists. (This can be considered analogous to a failed recursive call in the flow or matroid algorithms: i.e., the set E cannot be extended to a complete clearance matching.)

A possible approach to rectify this situation would be to remove an edges from E when we fail to be able to add new edges. The details have yet to be worked out. ((Not sure about this; need to think about it more. The generator may be better able to create an instance with a c-matching, but I doubt we can guarantee its success.))

10.3 Un-Matchable Graphs

Unfortunately, generating non-trivial clearance matching instances that are guaranteed to have no solution does not appear to be as easy as one might expect.

In other words, while it is relatively easy to generate clearance matching instances $C = (G, u, c)$, $G = (I \cup S, E)$, such that there is no set $M \subseteq E$ that satisfies C's degree constraints, it is more difficult to create instances that are guaranteed to have edge subsets $M \subseteq E$ that satisfy C's degree constraint but that do not satisfy C's clearance conditions.

One of the first questions that we wish to answer is, what makes a clearance matching instance un-c-matchable? If we could answer this question simply and completely, then the problem would be trivially solvable. Therefore, we must look for characteristics about certain instances that make them unsolvable. By forcing or exploiting these characteristics in our generators, we can generate instances that will not have solutions.

Consider a clearance matching instance $C = (G, u, c)$, $G = (I \cup S, E)$. Our first naive observation is that if, for any item $i \in I$, the induced instance with graph $G' = (\{i\} \cup S, E)$ is not c-matchable, then the entire instance C is not c-matchable. For example, consider an item $i \in I$ such that $u(i) = 2$, $d(i) = 2$, and the only edges incident on i in E are (i, s) and $(i, s + 1)$, then there is no way to satisfy i's degree constraints without violating the clearance condition. Therefore C does not have a clearance matching.

It would not be very difficult to generate instances that exploit the above property. Unfortunately, however, those instance do not fairly represent the set of all un-c-matchable instances. For example, in the clearance matching instances reduced from independent set and CNF-SAT instances (Section 3), for each item there *does* exists a set of edges that satisfies the degree constraints and clearance condition with respect to *that* item. When a CNF/IS reduced instance is un-c-matchable, it is because of the *interrelationship* between multiple items.

Our quest then is to first define what a "good" un-c-matchable instance is. We can then begin to design such a generator. Unfortunately, it would seem that, by necessity, any instance generator that exploits known properties does *not* generate completely random instances. ((This is an assertion. I would like to think about it some more))

Another possible method to generate un-c-matchable instances is to choose the number of items p, the number of slots q, and the degree constraint function u such that $C = (K_{p,q}, u, c)$ does not have a c-matching (see Section 4). Consequently, any subgraph of $K_{p,q}$ with the same degree constraints and clearance does not have a c-matching. Again, however, instances of these sorts are contrived and not purely "random."

((Re-read Iwama's paper))

10.4 Single Solution Matchable Graphs

It would be nice to be able to empirically test our clearance matching solvers on instances that are known to have one and only one solution. Our motivation is because if an instance has more than one solution, there is no guarantee that two different solvers will find the same solution.

As with generating un-c-matchable instances, generating instances with only one possible c-matching is relatively non-trivial. We have yet to propose a good solution.

((This doesn't appear to be as easy as we'd like. See the IWAMA paper or generating satisfiability problems with only one solution.))

11 Observations about Random Instances

In this section we consider the effects of various orderings of the items in random clearance matching instances. In particular, we have generated perfect clearance matching instances using the GEN-RANDOM1 generator with a range of different $|I|$, $|S|$, and $|E|$ values.

In the instances we have tested with, all the slots have been assigned a degree constraint of 1 and all the items of a particular instance have been assigned the same degree constraint. We acknowledge that this does *not* represent a significant portion of the set of all possible clearance matching instances and, as further work, we would like to explore the other random instances as well.

We have performed our tests on the random graphs themselves and on the instances with the items sorted in both non-increasing and non-decreasing order of $d(i) - u(i)$. Our first observation of these random instances is that many of the instances that did not have clearance matching also did not have any subset of edges that satisfied the instance's degree constraints. In other words, on many un-c-matchable instances, the the flow solver was unable to even find a flow f such that $|f| = \sum_{i \in I} u(i)$.

With the matroid algorithm, the results were even better. In all but three of the 54 un-c-matchable random instances that we tested, there did not exist a set $M \subseteq \mathcal{I}_1 \cap \mathcal{I}_2$ such that $|M| = \sum_{i \in I} u(i)$. In the three cases in which such a set M existed, the matroid algorithm only performed 1 recursive call before determining that the instance did not contain a c-matching. This is true regardless of whether the items were presented in the original order or sorted upon $d(i) - u(i)$.

On c-matchable instances our results were similarly pleasing. In many cases our solvers were able to find c-matchings very quickly and with a very few number of recursive calls. Our attention was thus drawn towards the pathological instances: i.e, instances that took an disproportionate amount of time to solve. A complete listing of all our instances and the amount of time and number of recursive calls required to solve them appears in Appendix C.1. Unfortunately, the number of pathological instances were very small. Thus, instead of compiling statistics as we did for the CNF reductions, we look at the pathological instances on a case-by-case basis.

Interestingly enough, what was a pathological instance when when the items were ordered one way was not necessarily a pathological instance when the items were ordered another way. In Table 1 we present the time and number of recursive calls required by the flow solver when executing on a few "pathological" c-matchable instances. We use the symbols $(d - u)+$ and $(d - u)-$ to reflect instances in which the items were sorted in non-decreasing and non-increasing order of $d(i) - u(i)$, respectively. The "nrf" field in each cell represents the number of recursive calls to FLOW-FIND performed by the solver and the "user" field represents the amount of user time spent executing the solver. In Table 2 we present a similar table for the time and number of recursive calls required by the matroid algorithm.

Some of the cells in Tables 1 and 2 were marked as killed after 90 and 600 seconds, respectively. A small point of confusion, however, could be because a cell marked as killed after 90 seconds lists the user time as being less than 90 seconds. This is because user time does not necessarily equate with "wall-clock" time: during the 90 and 600 seconds allocated to the solvers, our test computer could have performed countless other tasks. The user time reflects only the time allocated by the computer to our program.

As illustrated in Tables 1 and 2, a "pathological" instance when the items appear in one ordering is often not a pathological instance when the items appear in another order. Moreover, as mentioned before and supported by the tables, re-ordering the items of a pathological instance can make the instance *trivially* solvable (i.e., requiring only few recursive call). The last row of Table 1 demonstrates, however, that changing the order of the items in a clearance matching instance does not always make for a more efficient solution.

Unfortunately, the limited data we collected doesn't clearly indicate that one method of ordering produces much more efficiently solvable instances than the other orderings. This is in contrast to our observation that CNF instances in the $(d-u)+$ ordering are the most difficult to solve.

Let us now consider the differences between the flow and matroid solvers when operating on these random perfect clearance matching instances. Again, because both the flow and matroid solvers seemed to solve most instances relatively quickly, we are primarily interesting in pathological cases. Table 3 lists the number of recursive calls required by the two solvers on a few "pathological" instances.

When looking at the table, one should remember that the flow solver was killed if it could not find a solution in 90 seconds and that the matroid solver was similarly killed after 600 seconds. For example, consider the cell in Table 3 which lists the number of recursive calls for the matroid algorithm as 5085493 and the number of recursive calls for the flow algorithm as 1599472. Since both solvers were killed when working on that instance, the two numbers do not reflect that one solver was more "efficient" than the other. Rather, the numbers show that neither algorithm was very efficient at finding a solution for that instance.

In general, it appears that the matroid solver can find a clearance matching on a particular instance much more efficiently than the flow solver. Unfortunately, however, we cannot claim that in all circumstances the matroid solver is "better" than the network flow solver. For example, on the $(d-u)-$ ordering of the instance in the fifth column of Table 3, the matroid solver took much more time (11.75 seconds and 273484 recursive calls) than the flow solver (0.03 seconds and 1154 recursive calls).

In many circumstances it would be useful to know how many clearance matchings a particular instance has; i.e., it could be that the matroid solver and the flow solver found different c-matchings on the same instance. If we knew that an instance has one and only one c-matching, then our we may be able to make more conclusive arguments about our observations.

Instance Description	Original Instance	$(d-u)+$	$(d-u)-$
$\|I\|=30$, $\|S\|=60$ $\|E\|=408$	nrf = 3 user = 0.01s	nrf = 1442571 ♣ user = 44.82s	nrf = 2 user = 0.00s
$\|I\|=20$, $\|S\|=80$ $\|E\|=384$	nrf = 1116376 user = 38.71s	nrf = 5 user = 0.00s	nrf = 7 user = 0.00s
$\|I\|=10$, $\|S\|=90$ $\|E\|=252$	nrf = 24246 user = 0.89s	nrf = 8 user = 0.00s	nrf = 1338689 ♣ user = 44.71s
$\|I\|=10$, $\|S\|=80$ $\|E\|=512$	nrf = 700910 ♣ user = 44.80s	nrf = 21 user = 0.01s	nrf = 6 user = 0.00s
$\|I\|=10$, $\|S\|=80$ $\|E\|=368$	nrf = 7 user = 0.01s	nrf = 19 user = 0.00s	nrf = 852069 ♣ user = 45.00s
$\|I\|=10$, $\|S\|=70$ $\|E\|=196$	nrf = 1219 user = 0.04s	nrf = 1599472 ♣ user = 44.78s	nrf = 485 user = 0.03s
$\|I\|=10$, $\|S\|=100$ $\|E\|=280$	nrf = 1247 user = 0.04s	nrf = 1038060 ♣ user = 45.99s	nrf = 82 user = 0.01s
$\|I\|=10$, $\|S\|=100$ $\|E\|=280$	nrf = 818854 ♣ user = 44.57s	nrf = 1023061 ♣ user = 45.60s	nrf = 861401 ♣ user = 44.30s

Table 1: Sampling of the number of recursive calls and the time required by the flow solver when executing on random clearance matching instances. Entries marked with "♣" did not terminate after 90 seconds and were not allowed to run to completion. The number of recursive calls and time listed for the flow solver on killed instances represent only the number of recursive calls and the user time allocated to the program *before* it was killed.

Instance Description	Original Instance	$(d-u)+$	$(d-u)-$
$\|I\|=10$, $\|S\|=90$ $\|E\|=252$	nrf = 67619 user = 4.89s	nrf = 3 user = 0.01s	nrf = 10265600 ♠ user = 595.09s
$\|I\|=10$, $\|S\|=90$ $\|E\|=252$	nrf = 2931056 user = 219.45s	nrf = 35 user = 0.01s	nrf = 2369605 ♠ user = 544.89s
$\|I\|=10$, $\|S\|=80$ $\|E\|=368$	nrf = 1 user = 0.01s	nrf = 3444807 ♠ user = 383.78s	nrf = 27 user = 0.03s
$\|I\|=10$, $\|S\|=70$ $\|E\|=196$	nrf = 15 user = 0.01s	nrf = 5085493 user = 250.34s	nrf = 429 user = 0.07s

Table 2: Sampling of the number of recursive calls and the time required by the matroid solver when executing on random clearance matching instances. Entries marked with "♠" did not terminate after 600 seconds and were not allowed to run to completion.

Instance Description	Original Instance	$(d-u)+$	$(d-u)-$
$\|I\| = 10$, $\|S\| = 100$, $\|E\| = 280$	matroid = 17 flow = 818854 ♣	matroid = 5 flow = 1023061 ♣	matroid = 17 flow = 861401 ♣
$\|I\| = 10$, $\|S\| = 100$ $\|E\| = 640$	matroid = 23 flow = 19	matroid = 34407 flow = 21253	matroid = 6 flow = 27
$\|I\| = 10$, $\|S\| = 60$ $\|E\| = 168$	matroid = 1 flow = 14908	matroid = 1 flow = 1728703 ♣	matroid = 1 flow = 255
$\|I\| = 10$, $\|S\| = 70$ $\|E\| = 196$	matroid = 15 flow = 1219	matroid = 5085493 ♠ flow = 1599472 ♣	matroid = 429 flow = 485
$\|I\| = 10$, $\|S\| = 80$ $\|E\| = 224$	matroid = 18 flow = 1266784 ♣	matroid = 26 flow = 788804 ♣	matroid = 273484 flow = 1154
$\|I\| = 10$, $\|S\| = 80$ $\|E\| = 368$	matroid = 1 flow = 655150 ♣	matroid = 3444807 ♠ flow = 914898 ♣	matroid = 27 flow = 44
$\|I\| = 10$, $\|S\| = 90$ $\|E\| = 414$	matroid = 7 flow = 819309 ♣	matroid = 10841 flow = 13726	matroid = 15 flow = 81

Table 3: Sampling of the number of recursive calls required by the matroid and flow solvers solver when executing on random clearance matching instances. Entries marked with "♣" and "♠" did not terminate after 90 and 600 seconds, respectively.

12 Move-to-Front Algorithms

In this section we would like to present our "Move-to-Front" (MTF) modifications to the generic network flow and matroid algorithms presented in Sections 5 and 6, respectively.

The motivation for our MTF modifications stem from Sleator and Tarjan's paper *Amortized Efficiency of List Update and Paging Rules* [10]. ((Explain their paper a bit.))

Consider a recursive call to FLOW-FIND (Algorithm 1) on a clearance matching instance C, a fixed set M_x, a flow f, and an item *root*. As discussed in Section 5, the generic network flow algorithm tries to find a set of edges that would extend M_x into a clearance matching on C. It does this by augmenting the flow from *root* and then iteratively adding edges with flow to M_x until adding such an edge would force M_x to violate C's clearance condition. Let (i_b, l_b) be the bad edge that would force such a violation.

Now assume, as could be the case, that there is no c-matching that extends M_x. This means that the recursive call from i_b (with (i_b, l_b) voided) will fail and the algorithm will "backup" the bad edge and recurse again. The algorithm will do this until it cannot backup the bad edge any further. If $i_b \neq root$, then by reordering the list of items such that i_b immediately follows *root*, our algorithm will have less edges to "backup" through. Consequently, our algorithm could more-quickly determine that the set M_x cannot be extended to a clearance matching on C.

As can be seen, our algorithm takes a very pessimistic approach to solving the clearance matching problem. In other words, our algorithm builds upon the assumption that the original M_x cannot be extended and that (i_b, l_b) is a conflict because of the way the items are matched in the fixed set M_x. Moving i_b closer to the *root* means that we will have to perform less backups in order to rid ourselves of the erroneous fixed set M_x.

Unfortunately, it may very well be the case that M_x *can* be extended and that the edges infront of (i_b, l_b) before i_b is moved are in a clearance matching on C that extends M_x. In this circumstance, by moving i_b up in the list and augmenting flow from i_b, we could potentially "mess-up" a valid extension to M_x.

In our practical implementation (Appendices A and **??**) we have chosen to swap the positions of i_b and $root + 1$ if i_b is neither *root* nor $root + 1$. Other methods could be used to re-order the list of items. For example, we could have moved i_b to the position directly after *root* without modifying the the relative order of any of the other items in the list.

The exact same swapping procedure we described above for the flow algorithm can be used with the matroid algorithm. We shall call the flow and matroid algorithms which use the "move-to-front" technique the flowmtf and the matroidmtf algorithms, respectively.

It should be pointed out that with flowmtf and matroidmtf algorithms, the first item can never be replaced. In other words, since $root \geq 1$ and since i_b can only swap with $root + 1$, the earliest position an i_b could be placed in is the second. In order to allow the first item to be swapped out, we can introduce a dummy edge (i_0, s_0) into the clearance matching instance C such that $u(i_0) = u(s_0) = 1$ and i_0 is the first item in C's list of items. Clearly the edge (i_0, s_0) will be in every possible clearance matching on C. The first "real" item in the list of items can now be replaced. In our figures, the matroid solver that uses the move to front technique and the dummy edge (i_0, s_0) is called matroidmtf$i_0 s_0$.

13 Move-To-Front Observations

Before we begin our observations about the move-to-front solvers, let us suggest another ordering for the items in CNF-reduced clearance matching instances. Namely, let us consider instances in which the items are sorted in non-decreasing and non-increasing order of $d(i) - u(i)$. These two orderings are the same as the $(d-u)+$ and $(d-u)-$ orderings suggested in Section 11.

One might suspect that sorting the items by non-decreasing order of $d(i) - u(i)$ would make the instances easily solvable — i.e., put the forced and near forced matchings at the head of the list. Unfortunately, that did not appear to be the case with CNF-reduced instances. Namely, all the x_as were placed at the front of the list and all the x_rs near or at the end of the list. Consequently, instances in the $(d-u)+$ ordering were even less efficiently solved than instances in the $x_a x_a x_r x_r$ ordering. See Section 7 for a discussion of our observations with respect to the $x_a x_a x_r x_r$ ordering.

Appendix B.7 shows statistics of the ratio of the number of recursive calls and user time required by the matroidmtf solver over the respective parameter for the matroid solver. Unfortunately, the statistics do not suggest that the matroid MTF solver is statistically more efficient than the regular solver on all CNF-reduction orderings.

What we do see, however, is that for pathological CNF orderings such as $x_a x_a x_r x_r$ and $(d-u)+$, the matroid MTF solver appears to be much more efficient at finding solutions. Our explanation for this is that, because the only nodes with the potential of violating the clearance condition are the x_rs, if the matroid MTF solver encounters any edge that violates the clearance condition, it will move the corresponding item up in the list of items. Consequently, the problem with having all the x_a at the head of the list (see Section 7) is reduced.

Such observations are supported in part by the statistics presented in Appendix ??. The statistics in Appendix ?? show the means and confidence intervals of the ratio of the time and number of recursive calls required by the matroidmtf solver on the $x_a x_a x_r x_r$ instances over the time and number of recursive calls required by the matroidmtf solver on other instances. Our first observation is that the means and confidence intervals suggest that on unsatisfiable instances the matroidmtf solver performs equally well on the $x_a x_a x_r x_r$, $x_a x_r$, $x_r x_r x_a x_a$, and $x_r x_a$ instances. In other words, the matroidmtf appears to be able to successfully reorder the $x_a x_a x_r x_r$ instances such that the algorithm can solve them as rapidly as it solves $x_a x_r$, $x_r x_r x_a x_a$, and $x_r x_a$ instances.

Additionally, note that the sample means and confidence intervals presented in Appendix ?? are much more tightly grouped than the means and confidence intervals in Appendix B.1. This further suggests that, in general, the matroidmtf algorithm is not as effected by the original ordering of a CNF-reduced instance as the generic matroid algorithm is. This is a definite advantage because the order of the items does have a significant impact on the non-MTF solvers.

We also tested our flowmtf and matroidmtf solvers on the random instances discussed in Section 11. As suggested in our introduction to the MTF solvers, the matroidmtf and flowmtf solvers generally found solutions more quickly and with less recursive calls than the generic matroid and flow algorithms. A brief summary of the "interesting" instances is presented in Table 4. A complete listing of all the instances appears in Appendix C.1. As pointed out by the last two columns of Table 4, the flowmtf solvers may use less recursive calls that the generic flow solver on an instance in which the matroidmtf solver uses more recursive calls than the generic matroid solver, and visa-versa. In other words, although the MTF technique may help the flow solver on one clearance matching instance, the technique doesn't necessarily help the matroid solver on the same instance.

As an example of the possible extreme benifits of the MTF solvers, we cite the operation of the matroidmtf and matroid solvers on a c-matchable instance with $|I| = 10$, $|S| = 80$, and $|E| = 224$. This instance was not listed in Appendix C.1. Nevertheless, a matroid solver did not find a solution within 1091.07 seconds (more than 18 minutes) and 19390570 recursive calls. Our matroidmtf algorithm found a solution in 0.01 seconds and 22 recursive calls.

From the figures and tables in Appendices B.7 and C.1 it appears that the matroidmtf solver seldom performs more recursive calls than the matroid solver on un-c-matchable instances. When the matroidmtf solver does perform more recursive calls than the matroid solver on un-c-matchable instances, it is not by a significant amount.

As pointed out before, we can consider the MTF solvers as re-ordering the items in our clearance matching instances. Namely, consider an instance C. Let C' be the instance with re-ordered items resulting from the execution of a MTF algorithm (flowmtf or matroidmtf) on C. One question one might ask is whether post-MTF-modified instances C' yield themselves to more efficient solutions than the original pre-MTF-modified instances C.

Appendix C.2 presents information about the performance of the various solvers on post and pre-MTF-modified instances. That is, for any given instance, Appendix C.2 presents the number of recursive calls required by the flow solver on C, the flow solver on C', the flowmtf solver on C, and the flowmtf solver on C'. Appendix C.2 also contains a table which considers the performance of the matroid and matroidmtf solvers on C and C'.

Table ?? highlights a few interesting results from the above experiment. Note that the results were mixed: on several instances the regular (flow or matroid) solver performed more recursive calls on an MTF-modified instance C' than on the original instance C, and sometime the regular solver performed more recursive calls on C than on C'. Interestingly, the MTF solvers also exhibited mixed behavior: it was not always the case that the MTF solver performed less recursive calls on the MTF-modified instance C' than on C.

Instance	Original Instance	$(d-u)+$	$(d-u)-$
$\|I\| = 10$, $\|S\| = 100$ $\|E\| = 280$	flowmtf = 60 flow = 818854 ♣	flowmtf = 86893 flow = 1023061 ♣	flowmtf = 22 flow = 861404 ♣
$\|I\| = 10$, $\|S\| = 100$ $\|E\| = 280$	flowmtf = 18 flow = 783044 ♣	flowmtf = 44 flow = 54	flowmtf = 25 flow = 196
$\|I\| = 10$, $\|S\| = 100$ $\|E\| = 280$	flowmtf = 23714 flow = 1247	flowmtf = 19 flow = 1038060 ♣	flowmtf = 53 flow = 82
$\|I\| = 10$, $\|S\| = 60$ $\|E\| = 168$	flowmtf = 19 flow = 19	flowmtf = 9 flow = 44798	flowmtf = 159 flow = 1167029
$\|I\| = 10$, $\|S\| = 70$ $\|E\| = 196$	flowmtf = 29 flow = 1219 matroidmtf = 19 matroid = 15	flowmtf = 5231 flow = 1599472 ♣ matroidmtf = 84 matroid = 5085493 ♠	flowmtf = 34 flow = 485 matroidmtf = 2062 matroid = 429
$\|I\| = 10$, $\|S\| = 80$ $\|E\| = 224$	flowmtf = 344995 flow = 1266784 ♣ matroidmtf = 6 matroid = 18	flowmtf = 39576 flow = 788804 ♣ matroidmtf = 26 matroid = 26	flowmtf = 32022 flow = 1154 matroidmtf = 3 matroid = 273484

Table 4: Sampling of the number of recursive calls required by the flow and flowmtf solvers solver when executing on random clearance matching instances. Entries marked with "♣" and "♠" were killed after 90 and 600 seconds, respectively.

Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 280$	o flow = 818854 ♣ m flow = 52859 o flowmtf = 60 m flowmtf = 38	o flow = 1023061 ♣ m flow = 1393832 ♣ o flowmtf = 86893 m flowmtf = 544259	o flow = 861401 ♣ m flow = 1358 o flowmtf = 22 m flowmtf = 70
$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 280$	o flow = 783044 ♣ m flow = 91124 o flowmtf = 18 m flowmtf = 18	o flow = 54 m flow = 1035863 ♣ o flowmtf = 44 m flowmtf = 2078	o flow = 196 m flow = 1847947 ♣ o flowmtf = 25 m flowmtf = 24604
$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	o flow = 17 m flow = 81 o flowmtf = 15 m flowmtf = 273	o flow = 4 m flow = 73850 o flowmtf = 5 m flowmtf = 9	o flow = 1584021 ♣ m flow = 1 o flowmtf = 1934 m flowmtf = 1
$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	o flow = 14908 m flow = 207 o flowmtf = 24 m flowmtf = 8	o flow = 1728703 ♣ m flow = 46 o flowmtf = 12 m flowmtf = 5	o flow = 255 m flow = 28 o flowmtf = 743 m flowmtf = 8
$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	o flow = 19 m flow = 1049 o flowmtf = 19 m flowmtf = 22	o flow = 44798 m flow = 14 o flowmtf = 9 m flowmtf = 44	o flow = 1167029 ♣ m flow = 93 o flowmtf = 159 m flowmtf = 6
$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	o flow = 5 m flow = 438 o flowmtf = 4 m flowmtf = 69	o flow = 29 m flow = 171 o flowmtf = 34 m flowmtf = 14	o flow = 18 m flow = 2582622 o flowmtf = 19 m flowmtf = 331951
$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 224$	o flow = 1266784 ♣ m flow = 11 o flowmtf = 344995 m flowmtf = 12	o flow = 788804 ♣ m flow = 89 o flowmtf = 39576 m flowmtf = 20	o flow = 1154 m flow = 54 o flowmtf = 32022 m flowmtf = 47
$\|S\| = 80$ $\|E\| = 368$	m flow = 15 o flowmtf = 28 m flowmtf = 8	m flow = 1199294 ♣ o flowmtf = 26 m flowmtf = 5	m flow = 176 o flowmtf = 32 m flowmtf = 15

Table 5: Sampling of the number of recursive calls required by the flow and flowmtf solvers when operating on c-matchable random instances (o) and instances modified by the flowmtf solvers (m). Entries marked with "♣" were killed after 90 seconds.

14 Conclusions and Future Work

In this paper we have presented several observations and conclusions about the clearance matching problem. Foremost among our conclusions is that the clearance matching problem in NP-complete. We provide reductions from the X3C, clique, independent set, and CNF-satisfiability NP-complete problems to the clearance matching problem. Consequently, we were able to use the CNF-SAT generators developed for the Second DIMACS Implementation Challenge [8] to generate a suite of clearance matching test instances.

We initially proposed two algorithms for solving the clearance matching problem. The first algorithm built upon network flows and the principle of recursing on induced instances in order to extend conflict-free matchings to clearance matchings. Our second algorithm built upon the flow algorithm but used matroids to reduce the search space for a clearance matching solution.

We then performed extensive testing on clearance matching instances reduced from CNF-SAT problems and observed the efficiencies of our algorithms on isomorphic instances in which the items were presented in different orders. Our primary conclusion was that the order of the items *does* play a *significant* factor in the execution of our algorithms. We further isolated the $x_a x_a x_r x_r$ ordering as being exceedingly problematic.

We then proposed an alteration to the flow and matroids algorithm using the Move-to-Front (MTF) principle discussed in [10]. With respect to isomorphic CNF-reduced clearance matching instances, empirical tests imply that the MTF algorithms are more resilient to the ordering of the items in clearance matching instances than the generic algorithms. Unfortunately, the MTF algorithms are not always more efficient than the generic algorithms. This is true even when operating on un-c-matchable instances.

In the process of our evaluation we have drafted a large list of possible avenues for future research. Our list is broken down into three distinct but co-dependent categories: further analysis of existing data, additional experimental test cases, and open/theoretical questions.

14.1 Continued Analysis of Existing Data

Throughout the course of this paper we have considered many sources of data including differently ordered CNF-reduced instances and randomly generated instances. In comparing the execution of our solvers on isomorphic instances with different orderings of the items, we have observed interesting traits. For example, the ratio of the time required by the flow solver on unsatisfiable instances with b over instances without b seems to be dependent upon whether the x_as precede the x_rs or not (Appendix B.5).

Our primary concern with the analysis of the data discussed in the previous sections is whether we are correctly interpreting the data. That is, although we have made many observations, the observations themselves do not implicitly prove any conclusion. One of our first steps should then be to find mathematical explanations for our observations.

To initiate this step, let us list a few observations worthy of additional investigation. Many of these observations have been discussed in the preceding sections. Additionally, many of these observations can be explained intuitively. What we desire is a mathematical explanation.

With respect to the efficiency of the algorithms on CNF-reduced instances with b over instances without b, we have the following list:

- With a few exceptions, the algorithms appear to take more time on instances with b than without b. Why?

- There are a few instances in which the algorithms do take less time on instances with b than without b. Are there any properties that make certain instances perform better with b than without b? As motivation for our question, notice that the ratios of time required by the $x_a x_r$, $x_a x_r d-$, and $x_a x_r d+$ solvers over their respective instances without b all appear to be less than 1.

- On satisfiable CNF-reduced instances the sample means of the ratios of the number of recursive calls required to solve instances with b over instances without b appear to be greater than (or equal to in a few cases) 1. On unsatisfiable CNF-reduced instances, however, the ratios appear to be strictly less than 1. What is different between satisfiable (c-matchable) and unsatisfiable (un-c-matchable) instances that forces the above behavior?

- On unsatisfiable CNF-reduced instances the sample means of the ratios for the $x_a x_r$, $x_a x_r d-$, $x_a x_r d+$, and $x_a x_a x_r x_r$ instances appear to be very tightly grouped in a range (with confidence intervals) strictly less than the range of the sample means of $x_r x_a$, $x_r x_a d-$, $x_r x_a d+$, and $x_r x_r x_a x_a$. What is the difference between the two sets of orderings (those with x_a as the first element of their label and those with x_r as the first element of their label) that makes the ratios of the two groups different?

The list of observations that require further exploration extends to the ratios of the time and number of recursive calls required by the various algorithms on one ordering of the CNF-reduced instances over the other orderings of CNF-reduced instances.

- The statistics presented in Appendix B provide a lot of information. One motivational question is whether there is a better way to compare two different orderings?

- In our statistical tests, we defined only two sample spaces: one for satisfiable CNF instances and one for unsatisfiable CNF instances. Perhaps by enforcing additional constraints on our sample spaces (such as restricting sample spaces only to instances with "similar" CNF clause to variable ratios) we will learn more about how to classify CNF-reduced clearance matching instances. In other words, by continuously refining our sample spaces, we may learn how to group CNF-reduced instances as being "similar" or "comparable." Additionally, by refining our samples spaces we can expect the confidence intervals for figures similar to those in Appendix B to become smaller.

- A few of the statistics presented in Appendix B.1 show the confidence intervals for the time and number of recursive calls of the $x_a x_a x_r x_r$ ordering over a few orderings as being much larger than the confidence intervals of the $x_a x_a x_r x_r$ ordering over the other orderings. For example, with the flow solver on unsatisfiable instances, the confidence intervals for the means of the ratios with $x_a x_r d-$, $x_r x_a d-$, and $x_r x_r x_a x_a d-$ as the denominators are huge compared to the confidence intervals for the ratios with other orderings in the denominators. Can we isolate the properties that give some means large confidence intervals? We must also consider the fact that some of the properties we are observing could be the result of our CNF generators.

We also made interesting observations with respect to the statistics comparing the flow and matroid algorithms (Appendix B.6). For example, for unsatisfiable CNF-reduced instances the user time ratio of the matroid solver over the flow solver for instances with b were greater than for instances without b. Interestingly, the number of recursive calls ratio for instances with b is very similar to the ratio for instances without b. We would like to investigate the different effects of the discard node b on the flow and matroid solvers.

Additionally, while the matroid solver is clearly more efficient than the flow solver on unsatisfiable CNF-reduced instances, the matroid solver appears comparable to the flow solver on certain satisfiable instances. What is the difference between satisfiable and unsatisfiable instances that promotes this behavior? Is there something about the matroid solver that makes it "better" on unsatisfiable instances?

In addition to the user time and number of recursive calls required of our algorithms, we also recorded the maximum recursive depth of the algorithm, the number of conflicts found, the total number of edges the algorithm had to backup through, the maximum number of edges and items a single stage of the algorithm had to backup though, the number of successful and unsuccessful augmentation attempts, and the number of times the MTF technique was employed (for the flowmtf and matroidmtf algorithms). Additionally, for the matroid and matroidmtf algorithms, we recorded the number of times an augmenting path went through a $b_{i,j}$ node without going through the corresponding i node (i.e., the number of times i's adjacency was "swapped" from one edge in a matroid sub-block $S_{i,j}$ to another edge within the same sub-block; see Section 6 and Figure 5.)

We would like to incorporate some or all of the above data into out analysis.

14.2 Additional Experiments

There are also a lot of additional experiments we wish to perform. For example, it has been empirically shown that the most difficult CNF-SAT problems are when the clause to variable ratio is around 4.30. It is thought that the ratio approaches 4.25 for large v [7].[8] We would like to consider the performance of our clearance matching algorithms on CNF-reduced instances with respect to the CNF instances' clause to variable ratios.

Our figures in Appendix B contains statistics for the means of ratios with $x_ax_ax_rx_r$, x_ax_r, $x_rx_rx_ax_a$, and x_rx_a in the numerators. To be complete, we would like to analyze similar statistics with the orderings $x_ax_ax_rx_rd-$, $x_ax_ax_rx_rd+$, x_ax_rd-, x_ax_rd+, $x_rx_rx_ax_ad-$, $x_rx_rx_ax_ad+$, x_rx_ad-, and x_rx_ad+ in the numerators. We anticipate several results of the above statistics. For example, as with the x_ax_r, x_rx_a, and $x_rx_rx_ax_a$ orderings, we anticipate that on unsatisfiable CNF-reduced instances, the means of the ratios of any one of x_ax_rd-, x_rx_ad-, and $x_rx_rx_ax_ad-$ over the other two will be approximately 1. Similarly for the x_ax_rd+, x_rx_ad+, and $x_rx_rx_ax_ad+$ orderings. In other words, we anticipate additional empirical evidence to support the hypothesis that as long as the x_rs maintain the same relative order, the flow and matroid algorithms are equally efficient on the x_ax_r*, x_rx_a*, and $x_rx_rx_ax_a*$ orderings of unsatisfiable CNF-reduced instances (where $*$ implies any sorting operation on the xs; for example, $*$ could be either $d-$ or $d+$).

We would also like to create additional CNF-SAT instances using other CNF-SAT generators. This is because different generators can potentially generate CNF instances with different properties. In addition to considering all CNF instances together, we would like to separate the sample space of CNF instances by the generators that produced them.

With regards to the performance of the flow and matroid solvers on the random instances discussed in Section 11, we noticed that there were not many pathological cases in which $|I|$ was very large. We would like to generate and experiment with additional random instances.

We would also like to compare the efficiencies of our solvers with respect to the $|I|$ and $|S|$ or $|I|$ and $\sum_{i \in I} u(i)$ values. Moreover, we wish to graph the sample means of many random instances on a three-dimensional graph. For example, let the x and y axis correspond to the cardinality of I and S, respectively. For any given $|I|$ and $|S|$ pair, we could then plot the sample means $\overline{x_{|I|,|S|}}$ of the number of recursive calls (or any value we are interested in) required by a given solver as $(|I|, |S|, \overline{x_{|I|,|S|}})$. We might also include vertical lines emanating from $(|I|, |S|, \overline{x_{|I|,|S|}})$ to represent our confidence in $\overline{x_{|I|,|S|}}$. (We would need to consider enough random instances of each $|I|$ and $|S|$ combination we are interested in such that the samples for each $|I|$ and $|S|$ pair approximate a normal distribution.)

A three-dimensional plot such as the one described above could yield interesting information about which instances are difficult and which instances are easy to solve. To aid in our visualization, we could use some interpolation

[8] Other sources cite 4.2 as the problematic ratio of clauses to variables for CNF instances.

method to fit a curve to our means and thereby possibly expose some inherent nature about our random clearance matching instances (such as a relationship between $|I|$ and $|S|$ that proves to be associated with unusually pathological instances). We could also plot the means for both un-c-matchable and c-matchable instances on the same three-dimensional graph.

Rather than simply plot the sample means and confidence intervals for a specific $|I|$ and $|S|$, we could instead adopt a box-plot approach and label the center, spread, and outliers of our samples. Identifying and analyzing the outliers may help us understand what constitutes a pathological instance.

We might also graph the percentage of instances that are c-matchable as a function of $|I|$ and $|S|$ (or $|I|$ and $\sum_{i \in I} u(i)$ or $\sum_{i \in I} u(i)$ and $\sum_{i \in I} d(i)$ or any other property). Doing so may provide us with an intuitive feel for which instances are most likely to be c-matchable and which instances are not (much like the clause to variable ratio of 4.25 is for CNF instances).

As we commented before, whenever we perform an analysis like the ones described above we must be careful about what sample instances we choose. For example, other attributes besides $|I|$ and $|S|$ are likely to effect the performance of our algorithms on clearance matching instances: we cannot expect an algorithm to perform equally well on two instances with the same number of items and slots but with completely different degree constraints. We must therefore be careful what samples we use when making conclusions from our statistics.

14.3 Open Questions

Our paper dealt primarily with clearance matching instances in which $c = 2$. How do our algorithms perform on instances in which $c > 2$. How do our algorithms compare? Does the matroid algorithm "boost" the performance of the generic flow algorithm even more than it "boosted" the performance of the generic flow algorithm on instances with $c = 2$?

Similarly, our paper considered only instances of the clearance matching problem with a slot degree constraint 1 ($u(s) = 1$ for all $s \in S$). We would like to study clearance matching instances with slot degree constraints greater than 1.

We would also like to consider the differences between perfect and complete clearance matching instances. We touched on this slightly by considering CNF-reduced instances with and without b. Our initial observations might extend the fact that because a perfect clearance matching must contain $u(v)$ edges incident to each vertex in the graph (both items and slots), a recursive call with a *root* close to the end of the items list is unlikely to succeed (and a recursive call with *root* $= |I|$ is destined to fail). This is already supported in part by the statistics showing the ratios of the number of recursive calls required by unsatisfiable CNF-reduced instances with b over instances without b (Appendix B.5).

In addition to the above, we would like to provide a more mathematical explanation for the advantages the matroid solver has over the generic network flow solver and for the advantages the MTF solvers have over the non-MTF solvers. Can we formulate a set of conditions that force the matroid algorithm to perform worse than the flow algorithm? Can we prove any relationship between the MTF and non-MTF solvers on un-c-matchable instances? From our tests with the CNF-reduced instances, the matroidmtf solver never made more than a few more recursive calls than the generic matroid solver on unsatisfiable reduced instances. Will that always be the case?

Additionally, we would like to consider other implementations of the move-to-front technique. For example, is there an advantage to keeping the relative order of all the items except the "moved-to-front" item the same? Recall that in our current implementation the "moved-to-front" item is merely swapped into its new position.

In order to aid in our experimental tests, we would also like to create better clearance matching instance generators; i.e., generators that are guaranteed to generate un-c-matchable instances, c-matchable instances with certain properties, and instances with one and only one c-matching. We would also like to be able to identify an generate "pathological" instances.

A Source Code Discussion

In this section we would like to discuss some of the source code developed throughout this project. Our actual source code appears in Appendix ??.

A.1 Tools

In order to aid in the development and testing of clearance matching solvers and clearance matching instance generators, we have developed several libraries and tools. Among these are:

libgengraph This library provides a uniform framework for manipulating, loading, and storing clearance matching instances.

All applications that manipulate clearance matching instances link with this library.

Individual solvers often convert between gengraph format clearance matching instances and their own internal representation.

The gengraph header is listed in Listing ?? and the source code is listed in Listing ??.

libdimacs The libdimacs library provides functions for loading and saving graphs as specified for the Second DIMACS Implementation Challenge [8].

The dimacs header file is listed in Listing ?? and the source code is in Listing ??.

libdcnf The libdcnf library provides functions for loading and storing DIMACS CNF instances [8].

The header file is provided in Listing ?? and the source code in Listing ??.

libdmptime Library to dump information about execution time. See Listing ?? and Listing ??.

libgenlatex Library to store clearance matching gengraph graphs in LaTeX format. See Listing ?? and Listing ??.

batch `batch` was a program developed to execute a script of commands. `batch` takes a timeout argument and kills each command in the script if the command does not terminate within the specified timeout.

This program was developed so that clearance matching solvers could be run back-to-back while stopping execution on pathological instances.

The source code is provided in Listing ??.

A.2 libgengraph Format

We would like to take a moment to briefly discuss te `libgengraph` generic clearance matching graph format. The format was heavily influenced by the DIMACS graph format used in the Second DIMACS Implementation Challenge [8].

In our discussion let us consider the a clearance matching instance C stored in a file named `instance.cm`. Each line of `instance.cm` contains a command to `libgengraph`. These lines contain the description of our clearance matching instance C. Lines cannot be more than 80 characters.

The type of command contained on a line is specified by the first character in that line. The following list breaks down the acceptible formats of a line based upon the line's first character:

`c` All lines beginning with a "`c`" are considered to be comments and are ignored by `libgengraph`.

`t <type>` Lines of the format `t <type>` specify the type of clearance matching instance C. A clearance matching instance generator can make `type` any string of alphanumeric characters. For example, our $x_a x_r$ CNF reduction algorithm (Listing ??) generates clearance matching instance files with the type `cnf_xa_xr`.

`g <nI> <nS> <nE>` Lines beginning with `g` tell the `libgengraph` library that the clearance matching instance has `nI` items, `nS` slots, and `nE` edges.

`e i<item> s<slot>` Lines beginning with `e` tell `libgengraph` that there is an edge from `item` to `slot` in the clearance matching instance.

`item` can be any number between 1 and `nI`. `slot` can be any number between 1 and `nS`. An instance file must define exactly `nE` edges.

`d [i<item>,s<slot>] <constraint>` Lines beginning with `d` specify the degree constraint of an item or slot.

`h` Lines beginning with `h` describe the "history" of the instance. For example, if a move-to-front solver changes the position of two items in a clearance matching instance, the instance's `instance.cm` file would contain a record of those swaps.

The following is a sample `instance.cm` file corresponding to a clearance matching instance with 2 items, 2 slots, and edges between the first item and the second slot and between the second item and the first slot. The degree constraint of each item and each slot is 1.

```
c this is a sample graph
t sampletype
g 2 2 2
e i1 s2
e i2 s1
d i1 1
```

```
d i2 1
d s1 1
d s2 1
h file created as a sample libgengraph clearance matching instance
h i1 i2 swapped
```

A.3 Solvers

We adopted the following conventions when dealing with clearance matching instances: Files containing clearance matching instances always have the suffix "`.cm`". For example, a valid clearance matching instance could be named "`instance.cm`". Instance files are of the format handled by `libgengraph`.

As mentioned in the text, we experimented with various algorithms to solve the clearance matching instances. The network flow solver is named `flow`, the matroid solver is named `matroid`, the flow solver that incorporated the move-to-front technique is named `flowmtf`, and the matroid solver that used move-to-front is named `matroidmtf`.

The solvers generate at least two output files. If the `flow` solver is executed on the `instance.cm` clearance matching instance with $c = 2$, then the `flow` program would generate two output files named `instance.cm.flow.2.sol` and `instance.cm.flow.2.cmsol`. In general, given any solver `solver` and any instance `instance.cm` with clearance c, the solver would generate two output files name `instance.cm.solver.c.sol` and `instance.cm.solver.c.cmsol`. The first file would contain a human-readable version of a clearance matching on `instance.cm` or, if no such matching exists, a message saying so. The later file would contain a graph in the same format used by `libgengraph` but with the c-matched edges specially marked.

Both output files would contained information about the execution of the solver on the particular instance. For example, the output files would contain the amount of time spent executing the solver, the number of recursive calls performed, the number of edges voided, etc. ((Should I give more detail about the input graph formats, the output graph formats, etc?))

When the solvers are compiled with `ANIM` defined, the solvers will store Samba animation commands in a file named `instance.cm.solver.c.anm`. The animations files could be used with the Polka/Samba system to graphically view the execution of the algorithms. See Appendix **??** for a brief discussion of Polka and Samba.

When the solvers are compiled with `DEBUG` defined, debugging output is sent to `instance.cm.solver.cm.dbg`.

In general, all our solvers incorporate different variants of the same file. In describing these files, we make heavy reference to the procedures described with respect to the flow solver in Section 5. The `solve.c` and `solve.h` files correspond roughly to the FLOW-SETUP procedure described in Algorithm 2. Namely, the code in `solve.c` is in responsible with first finding an initial "c-matching" on a clearance matching instance and then calling the `find()` function (`find.c`).

The functionality in files `find.c` and `find.h` correspond roughly to the FLOW-FIND procedure of Section 5 (Algorithm 1). The files `dfs.c` and `dfs.h` are responsible for augmenting the flow through a clearance matching instance (FLOW-AUGMENT, Algorithm 3).

The `ds.c` and `ds.h` files are responsible for maintaining the flow network corresponding to a given clearance matching instance. Among other things, they include code to convert between generic graphs as specified in `libgengraph` and clearance matching `Networks`. The `statistics.c` and `statistics.h` files were very simple and contain code to initialize and store statistics counters. The statistics themselves were updated during the execution of the solvers (such as in the `find()` function of `find.c`).

The code in `anim.c` and `anim.h` generates output understandable by the Polka/Samba animation tool. The routines in `anim.c` are only compiled into the executable solver if `ANIM` was defined.

Lastly, for any given solver (`flow`, `matroid`, ...), the `solver.c` file contains the "wrapper" code to appropriately name output files, invoke the `solve()` function in `solve.c`, and catch user interrupts. For example, the flow solver uses a file name `flow.c`.

As mentioned before, many of the files are the same between multiple instances. For example, the `flow` and `flowmtf` solvers use the same `dfs.c` and `dfs.h` files. Other files are identical with the exception of a few lines. When files are identical between instances, we include them only once in Appendix ??. When files are almost identical but contain some differences, rather than be ambiguous, we have chosen to include each of them.

flow The files corresponding to the generic network flow solver appear in Listings ??–??.

flowmtf The files corresponding to the move-to-front network flow solver appear in Listings ??–??.

matroid The files corresponding to the generic matroid solver appear in Listings ??–??.

matroidmtf The files corresponding to the move-to-front matroid solver appear in Listings ??–??.

A.4 Reducers

In Appendix ?? we include reductions from independent set and CNF instances to clearance matching instances. Our initial instances use the formats specified for the Second DIMACS Implementation Challenge (see the tools `libdimacs` and `libdcnf`). The resulting graphs are stored using the `libgengraph` format.

Because we have written a CNF to clearance matching reducer for each of the possible orderings of the items ($x_a x_r$, $x_a x_a x_r x_r$, etc.), we do not have space to list the code for each of them. Listings ?? and ?? provide reductions from CNF instances to $x_a x_r$ and $x_r x_a$_nob clearance matching instances, respectively.

A.5 Graph Generators

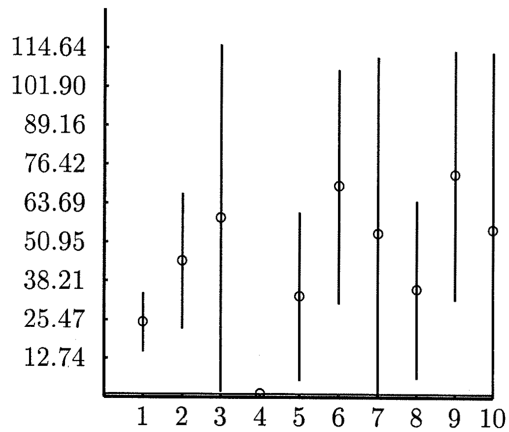
Besides some special-case generators (such as for complete biparitite graphs), our primary generator was `genrandom`. The `genrandom` program corresponds to the GEN-RANDOM1 algorithm of Section 10. See Listing ?? in Appendix ??.

`genrandom` is run with arguments corresponding to the number of items (nitems), the number of slots (nslots), the number of edges (nedges), and the degree constraints on the items (di) and slots (ds) required of the resulting graph. `random.ds`*`ds`*`.x.`*`nitems`*`.`*`nslots`*`.`*`nedges`*`.`*`label`*`.cm` contains the resulting instance. `label` is an index used to ensure that multiple random instances with the same parameters (nitems, etc.) are stored in different files.

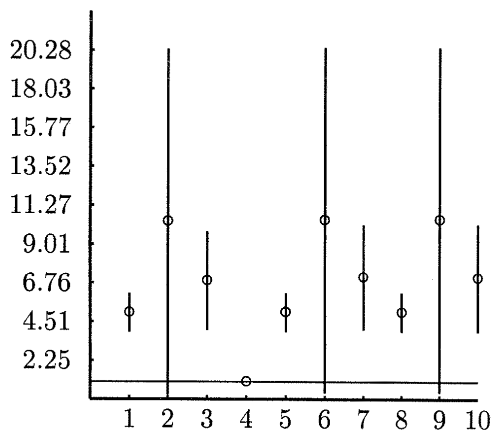
For random number generation we used to SSLeay package written by Eric Young and available at `ftp://ftp.psy.uq.oz.au/pub/Crypto/SSL/`.

B Statistics and Figures

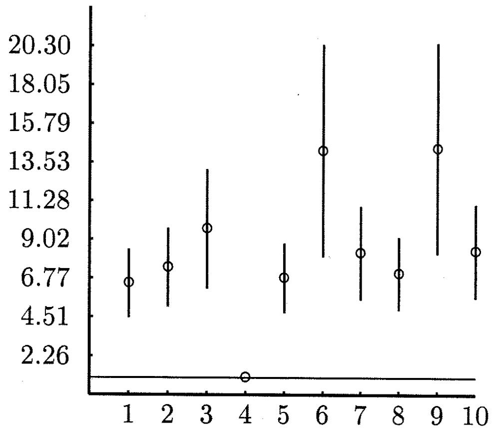
B.1 $x_a x_a x_r x_r$ CNF Reductions



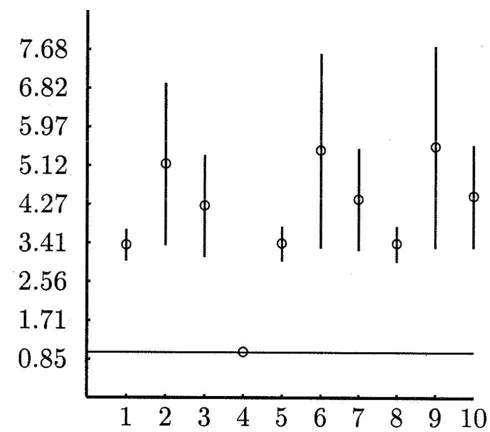
SATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 123$, $\overline{x} = 24.60$, 95% CI = $[14.88, 34.33]$. (2) $x_a x_r d-$: $n = 124$, $\overline{x} = 44.65$, 95% CI = $[22.40, 66.89]$. (3) $x_a x_r d+$: $n = 126$, $\overline{x} = 58.81$, 95% CI = $[1.83, 115.79]$. (4) $x_a x_a x_r x_r$: $n = 154$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (5) $x_r x_a$: $n = 126$, $\overline{x} = 33.09$, 95% CI = $[5.44, 60.74]$. (6) $x_r x_a d-$: $n = 114$, $\overline{x} = 69.21$, 95% CI = $[30.82, 107.59]$. (7) $x_r x_a d+$: $n = 124$, $\overline{x} = 53.65$, 95% CI = $[-4.47, 111.77]$. (8) $x_r x_r x_a x_a$: $n = 119$, $\overline{x} = 35.33$, 95% CI = $[6.09, 64.58]$. (9) $x_r x_r x_a x_a d-$: $n = 110$, $\overline{x} = 72.96$, 95% CI = $[32.02, 113.89]$. (10) $x_r x_r x_a x_a d+$: $n = 124$, $\overline{x} = 54.95$, 95% CI = $[-3.63, 113.52]$.



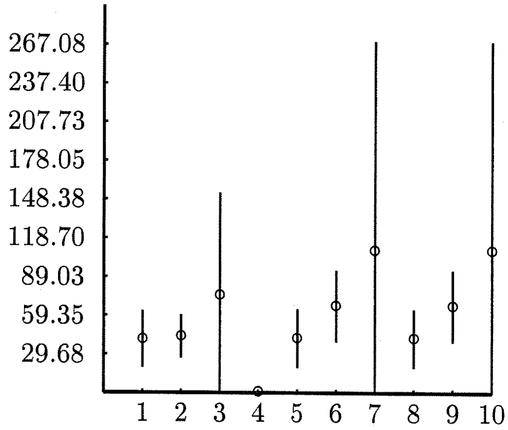
UNSATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 37$, $\overline{x} = 5.02$, 95% CI = $[3.90, 6.14]$. (2) $x_a x_r d-$: $n = 37$, $\overline{x} = 10.32$, 95% CI = $[0.27, 20.37]$. (3) $x_a x_r d+$: $n = 37$, $\overline{x} = 6.88$, 95% CI = $[4.02, 9.74]$. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (5) $x_r x_a$: $n = 37$, $\overline{x} = 5.05$, 95% CI = $[3.94, 6.16]$. (6) $x_r x_a d-$: $n = 37$, $\overline{x} = 10.41$, 95% CI = $[0.36, 20.46]$. (7) $x_r x_a d+$: $n = 37$, $\overline{x} = 7.09$, 95% CI = $[4.04, 10.14]$. (8) $x_r x_r x_a x_a$: $n = 37$, $\overline{x} = 5.05$, 95% CI = $[3.92, 6.18]$. (9) $x_r x_r x_a x_a d-$: $n = 37$, $\overline{x} = 10.43$, 95% CI = $[0.38, 20.48]$. (10) $x_r x_r x_a x_a d+$: $n = 37$, $\overline{x} = 7.04$, 95% CI = $[3.92, 10.16]$.



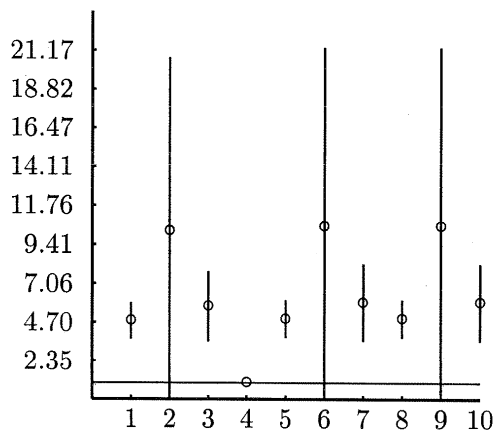
SATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 175$, $\overline{x} = 6.47$, 95% CI = $[4.48, 8.47]$. (2) $x_a x_r d-$: $n = 172$, $\overline{x} = 7.38$, 95% CI = $[5.10, 9.67]$. (3) $x_a x_r d+$: $n = 171$, $\overline{x} = 9.64$, 95% CI = $[6.16, 13.12]$. (4) $x_a x_a x_r x_r$: $n = 188$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (5) $x_r x_a$: $n = 177$, $\overline{x} = 6.77$, 95% CI = $[4.75, 8.79]$. (6) $x_r x_a d-$: $n = 159$, $\overline{x} = 14.21$, 95% CI = $[8.02, 20.40]$. (7) $x_r x_a d+$: $n = 172$, $\overline{x} = 8.24$, 95% CI = $[5.52, 10.96]$. (8) $x_r x_r x_a x_a$: $n = 171$, $\overline{x} = 7.04$, 95% CI = $[4.93, 9.15]$. (9) $x_r x_r x_a x_a d-$: $n = 167$, $\overline{x} = 14.35$, 95% CI = $[8.20, 20.51]$. (10) $x_r x_r x_a x_a d+$: $n = 167$, $\overline{x} = 8.36$, 95% CI = $[5.62, 11.10]$.



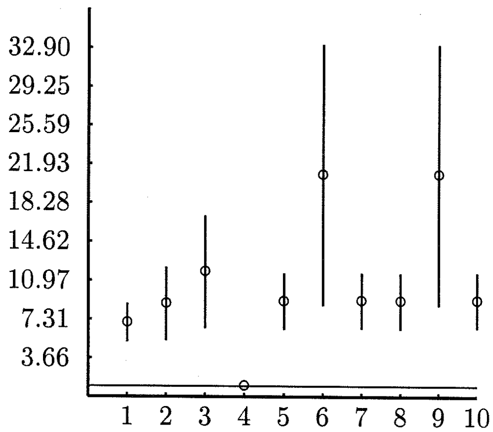
UNSATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 66$, $\overline{x} = 3.36$, 95% CI = $[3.01, 3.71]$. (2) $x_a x_r d-$: $n = 66$, $\overline{x} = 5.14$, 95% CI = $[3.36, 6.93]$. (3) $x_a x_r d+$: $n = 66$, $\overline{x} = 4.23$, 95% CI = $[3.11, 5.35]$. (4) $x_a x_a x_r x_r$: $n = 66$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (5) $x_r x_a$: $n = 66$, $\overline{x} = 3.40$, 95% CI = $[3.02, 3.78]$. (6) $x_r x_a d-$: $n = 66$, $\overline{x} = 5.45$, 95% CI = $[3.30, 7.59]$. (7) $x_r x_a d+$: $n = 66$, $\overline{x} = 4.37$, 95% CI = $[3.25, 5.49]$. (8) $x_r x_r x_a x_a$: $n = 66$, $\overline{x} = 3.39$, 95% CI = $[3.00, 3.78]$. (9) $x_r x_r x_a x_a d-$: $n = 66$, $\overline{x} = 5.53$, 95% CI = $[3.31, 7.76]$. (10) $x_r x_r x_a x_a d+$: $n = 66$, $\overline{x} = 4.45$, 95% CI = $[3.31, 5.58]$.



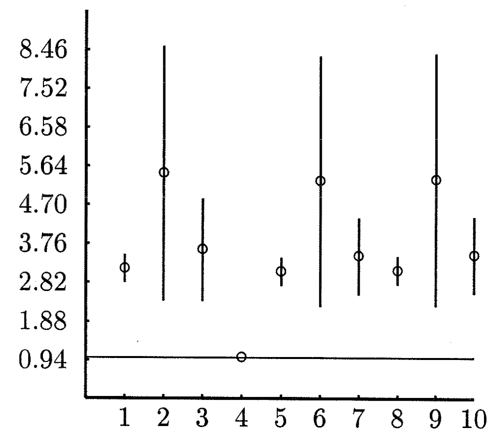
SATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 194$, $\overline{x} = 41.19$, 95% CI $= [19.56, 62.82]$. (2) $x_a x_r d-$: $n = 194$, $\overline{x} = 43.34$, 95% CI $= [26.82, 59.86]$. (3) $x_a x_r d+$: $n = 193$, $\overline{x} = 74.65$, 95% CI $= [-4.03, 153.34]$. (4) $x_a x_a x_r x_r$: $n = 194$, $\overline{x} = 1.00$, 95% CI $= [1.00, 1.00]$. (5) $x_r x_a$: $n = 194$, $\overline{x} = 41.77$, 95% CI $= [19.32, 64.21]$. (6) $x_r x_a d-$: $n = 194$, $\overline{x} = 66.66$, 95% CI $= [39.21, 94.11]$. (7) $x_r x_a d+$: $n = 194$, $\overline{x} = 109.13$, 95% CI $= [-51.48, 269.75]$. (8) $x_r x_r x_a x_a$: $n = 194$, $\overline{x} = 41.78$, 95% CI $= [19.34, 64.23]$. (9) $x_r x_r x_a x_a d-$: $n = 194$, $\overline{x} = 66.68$, 95% CI $= [39.23, 94.12]$. (10) $x_r x_r x_a x_a d+$: $n = 194$, $\overline{x} = 109.16$, 95% CI $= [-51.46, 269.77]$.



UNSATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 37$, $\overline{x} = 4.80$, 95% CI $= [3.69, 5.92]$. (2) $x_a x_r d-$: $n = 37$, $\overline{x} = 10.23$, 95% CI $= [-0.27, 20.74]$. (3) $x_a x_r d+$: $n = 37$, $\overline{x} = 5.68$, 95% CI $= [3.56, 7.80]$. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 1.00$, 95% CI $= [1.00, 1.00]$. (5) $x_r x_a$: $n = 37$, $\overline{x} = 4.92$, 95% CI $= [3.76, 6.07]$. (6) $x_r x_a d-$: $n = 37$, $\overline{x} = 10.51$, 95% CI $= [-0.33, 21.36]$. (7) $x_r x_a d+$: $n = 37$, $\overline{x} = 5.91$, 95% CI $= [3.57, 8.26]$. (8) $x_r x_r x_a x_a$: $n = 37$, $\overline{x} = 4.93$, 95% CI $= [3.77, 6.08]$. (9) $x_r x_r x_a x_a d-$: $n = 37$, $\overline{x} = 10.53$, 95% CI $= [-0.33, 21.38]$. (10) $x_r x_r x_a x_a d+$: $n = 37$, $\overline{x} = 5.92$, 95% CI $= [3.57, 8.27]$.

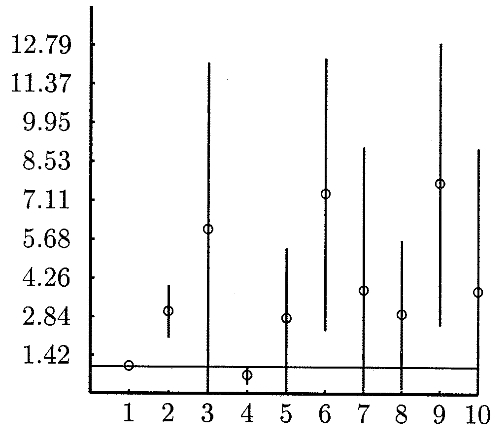


SATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: n = 241, $\bar{x}$ = 6.98, 95% CI = [5.16, 8.79]. (2) $x_a x_r d-$: n = 241, $\bar{x}$ = 8.77, 95% CI = [5.33, 12.22]. (3) $x_a x_r d+$: n = 241, $\bar{x}$ = 11.77, 95% CI = [6.48, 17.05]. (4) $x_a x_a x_r x_r$: n = 242, $\bar{x}$ = 1.00, 95% CI = [1.00, 1.00]. (5) $x_r x_a$: n = 240, $\bar{x}$ = 9.00, 95% CI = [6.34, 11.65]. (6) $x_r x_a d-$: n = 240, $\bar{x}$ = 20.92, 95% CI = [8.62, 33.23]. (7) $x_r x_a d+$: n = 239, $\bar{x}$ = 9.06, 95% CI = [6.44, 11.68]. (8) $x_r x_r x_a x_a$: n = 240, $\bar{x}$ = 9.02, 95% CI = [6.36, 11.68]. (9) $x_r x_r x_a x_a d-$: n = 240, $\bar{x}$ = 20.93, 95% CI = [8.62, 33.23]. (10) $x_r x_r x_a x_a d+$: n = 239, $\bar{x}$ = 9.09, 95% CI = [6.47, 11.71].

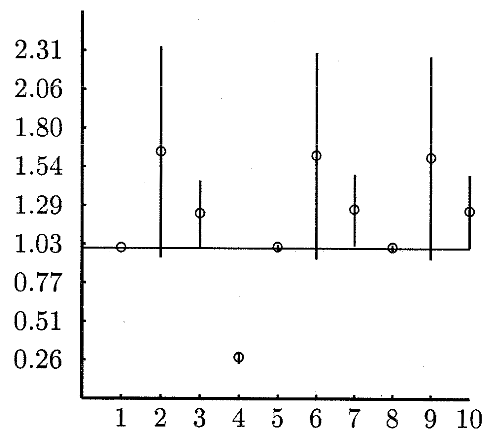


UNSATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: n = 66, $\bar{x}$ = 3.15, 95% CI = [2.82, 3.49]. (2) $x_a x_r d-$: n = 66, $\bar{x}$ = 5.46, 95% CI = [2.37, 8.55]. (3) $x_a x_r d+$: n = 66, $\bar{x}$ = 3.61, 95% CI = [2.37, 4.85]. (4) $x_a x_a x_r x_r$: n = 66, $\bar{x}$ = 1.00, 95% CI = [1.00, 1.00]. (5) $x_r x_a$: n = 66, $\bar{x}$ = 3.08, 95% CI = [2.74, 3.42]. (6) $x_r x_a d-$: n = 66, $\bar{x}$ = 5.28, 95% CI = [2.24, 8.31]. (7) $x_r x_a d+$: n = 66, $\bar{x}$ = 3.46, 95% CI = [2.53, 4.39]. (8) $x_r x_r x_a x_a$: n = 66, $\bar{x}$ = 3.11, 95% CI = [2.76, 3.45]. (9) $x_r x_r x_a x_a d-$: n = 66, $\bar{x}$ = 5.32, 95% CI = [2.25, 8.38]. (10) $x_r x_r x_a x_a d+$: n = 66, $\bar{x}$ = 3.49, 95% CI = [2.56, 4.42].

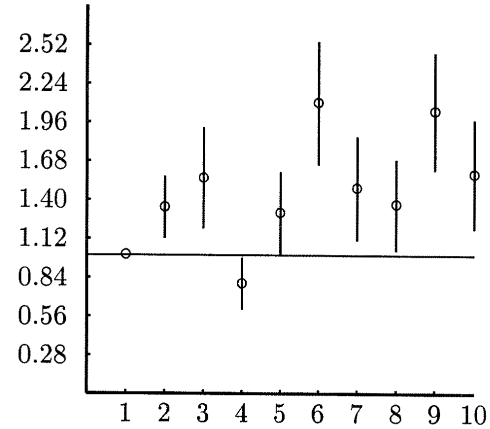
B.2 $x_a x_r$ CNF Reductions



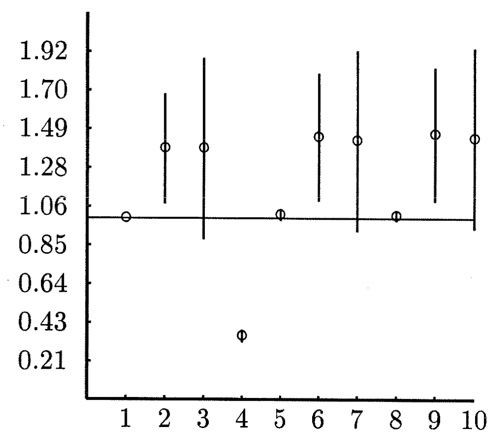
SATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 142$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (2) $x_a x_r d-$: $n = 138$, $\overline{x} = 3.03$, 95% CI = [2.07, 3.99]. (3) $x_a x_r d+$: $n = 142$, $\overline{x} = 6.03$, 95% CI = [−0.08, 12.15]. (4) $x_a x_a x_r x_r$: $n = 154$, $\overline{x} = 0.67$, 95% CI = [0.37, 0.98]. (5) $x_r x_a$: $n = 140$, $\overline{x} = 2.78$, 95% CI = [0.18, 5.38]. (6) $x_r x_a d-$: $n = 131$, $\overline{x} = 7.35$, 95% CI = [2.35, 12.34]. (7) $x_r x_a d+$: $n = 138$, $\overline{x} = 3.82$, 95% CI = [−1.43, 9.07]. (8) $x_r x_r x_a x_a$: $n = 133$, $\overline{x} = 2.94$, 95% CI = [0.21, 5.67]. (9) $x_r x_r x_a x_a d-$: $n = 127$, $\overline{x} = 7.74$, 95% CI = [2.56, 12.92]. (10) $x_r x_r x_a x_a d+$: $n = 138$, $\overline{x} = 3.78$, 95% CI = [−1.47, 9.03].



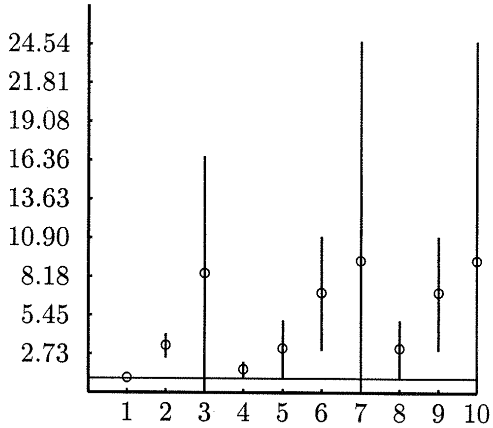
UNSATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 58$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (2) $x_a x_r d-$: $n = 55$, $\overline{x} = 1.64$, 95% CI = [0.94, 2.34]. (3) $x_a x_r d+$: $n = 57$, $\overline{x} = 1.23$, 95% CI = [1.01, 1.45]. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 0.27$, 95% CI = [0.23, 0.31]. (5) $x_r x_a$: $n = 58$, $\overline{x} = 1.01$, 95% CI = [0.99, 1.02]. (6) $x_r x_a d-$: $n = 57$, $\overline{x} = 1.61$, 95% CI = [0.93, 2.30]. (7) $x_r x_a d+$: $n = 57$, $\overline{x} = 1.26$, 95% CI = [1.02, 1.49]. (8) $x_r x_r x_a x_a$: $n = 58$, $\overline{x} = 1.00$, 95% CI = [0.99, 1.02]. (9) $x_r x_r x_a x_a d-$: $n = 58$, $\overline{x} = 1.60$, 95% CI = [0.93, 2.27]. (10) $x_r x_r x_a x_a d+$: $n = 57$, $\overline{x} = 1.25$, 95% CI = [1.00, 1.49].



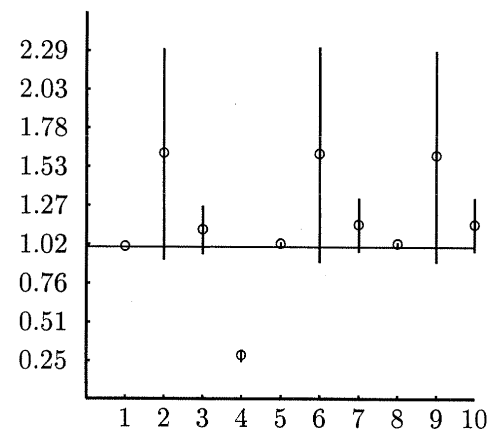
SATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 181$, $\bar{x} = 1.00$, 95% CI = [1.00, 1.00]. (2) $x_a x_r d-$: $n = 178$, $\bar{x} = 1.34$, 95% CI = [1.12, 1.57]. (3) $x_a x_r d+$: $n = 176$, $\bar{x} = 1.55$, 95% CI = [1.19, 1.91]. (4) $x_a x_a x_r x_r$: $n = 187$, $\bar{x} = 0.79$, 95% CI = [0.61, 0.98]. (5) $x_r x_a$: $n = 180$, $\bar{x} = 1.30$, 95% CI = [1.01, 1.60]. (6) $x_r x_a d-$: $n = 162$, $\bar{x} = 2.10$, 95% CI = [1.65, 2.54]. (7) $x_r x_a d+$: $n = 175$, $\bar{x} = 1.48$, 95% CI = [1.11, 1.85]. (8) $x_r x_r x_a x_a$: $n = 174$, $\bar{x} = 1.36$, 95% CI = [1.03, 1.69]. (9) $x_r x_r x_a x_a d-$: $n = 170$, $\bar{x} = 2.04$, 95% CI = [1.61, 2.46]. (10) $x_r x_r x_a x_a d+$: $n = 170$, $\bar{x} = 1.58$, 95% CI = [1.19, 1.98].



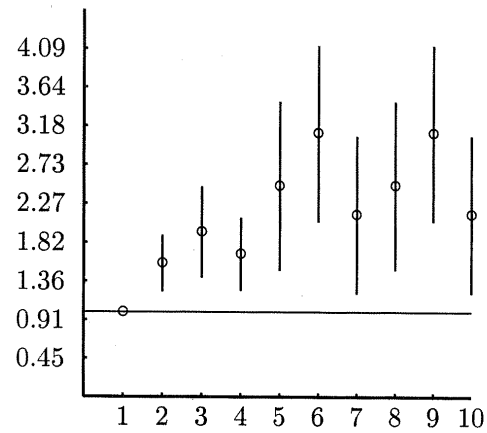
UNSATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 85$, $\bar{x} = 1.00$, 95% CI = [1.00, 1.00]. (2) $x_a x_r d-$: $n = 84$, $\bar{x} = 1.38$, 95% CI = [1.08, 1.68]. (3) $x_a x_r d+$: $n = 85$, $\bar{x} = 1.38$, 95% CI = [0.88, 1.88]. (4) $x_a x_a x_r x_r$: $n = 66$, $\bar{x} = 0.35$, 95% CI = [0.32, 0.39]. (5) $x_r x_a$: $n = 85$, $\bar{x} = 1.02$, 95% CI = [0.99, 1.05]. (6) $x_r x_a d-$: $n = 84$, $\bar{x} = 1.45$, 95% CI = [1.09, 1.80]. (7) $x_r x_a d+$: $n = 85$, $\bar{x} = 1.42$, 95% CI = [0.93, 1.92]. (8) $x_r x_r x_a x_a$: $n = 85$, $\bar{x} = 1.01$, 95% CI = [0.99, 1.04]. (9) $x_r x_r x_a x_a d-$: $n = 84$, $\bar{x} = 1.46$, 95% CI = [1.09, 1.83]. (10) $x_r x_r x_a x_a d+$: $n = 85$, $\bar{x} = 1.44$, 95% CI = [0.94, 1.94].



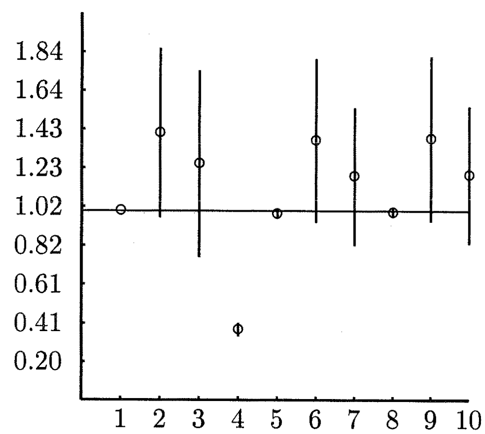
SATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 213$, $\overline{x} = 1.00$, 95% CI $= [1.00, 1.00]$. (2) $x_a x_r d-$: $n = 208$, $\overline{x} = 3.29$, 95% CI $= [2.44, 4.13]$. (3) $x_a x_r d+$: $n = 209$, $\overline{x} = 8.38$, 95% CI $= [0.12, 16.64]$. (4) $x_a x_a x_r x_r$: $n = 194$, $\overline{x} = 1.60$, 95% CI $= [1.02, 2.19]$. (5) $x_r x_a$: $n = 208$, $\overline{x} = 3.08$, 95% CI $= [1.08, 5.09]$. (6) $x_r x_a d-$: $n = 211$, $\overline{x} = 7.02$, 95% CI $= [2.99, 11.04]$. (7) $x_r x_a d+$: $n = 208$, $\overline{x} = 9.28$, 95% CI $= [-6.23, 24.78]$. (8) $x_r x_r x_a x_a$: $n = 208$, $\overline{x} = 3.08$, 95% CI $= [1.08, 5.09]$. (9) $x_r x_r x_a x_a d-$: $n = 211$, $\overline{x} = 7.02$, 95% CI $= [2.99, 11.05]$. (10) $x_r x_r x_a x_a d+$: $n = 208$, $\overline{x} = 9.28$, 95% CI $= [-6.23, 24.78]$.



UNSATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 58$, $\overline{x} = 1.00$, 95% CI $= [1.00, 1.00]$. (2) $x_a x_r d-$: $n = 55$, $\overline{x} = 1.61$, 95% CI $= [0.92, 2.30]$. (3) $x_a x_r d+$: $n = 57$, $\overline{x} = 1.11$, 95% CI $= [0.95, 1.27]$. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 0.29$, 95% CI $= [0.25, 0.33]$. (5) $x_r x_a$: $n = 58$, $\overline{x} = 1.02$, 95% CI $= [1.00, 1.03]$. (6) $x_r x_a d-$: $n = 57$, $\overline{x} = 1.60$, 95% CI $= [0.90, 2.31]$. (7) $x_r x_a d+$: $n = 57$, $\overline{x} = 1.14$, 95% CI $= [0.97, 1.32]$. (8) $x_r x_r x_a x_a$: $n = 58$, $\overline{x} = 1.02$, 95% CI $= [1.00, 1.03]$. (9) $x_r x_r x_a x_a d-$: $n = 58$, $\overline{x} = 1.59$, 95% CI $= [0.90, 2.29]$. (10) $x_r x_r x_a x_a d+$: $n = 57$, $\overline{x} = 1.14$, 95% CI $= [0.97, 1.32]$.

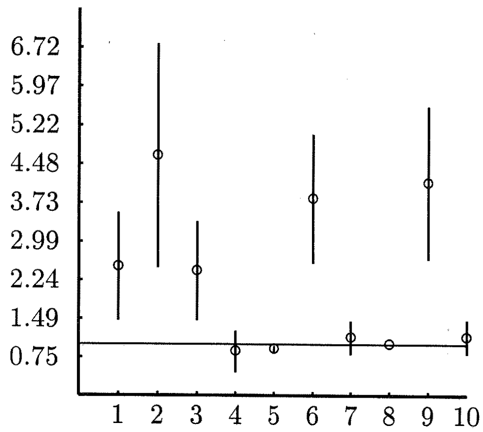


SATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: n = 247, $\overline{x}$ = 1.00, 95% CI = [1.00, 1.00]. (2) $x_a x_r d-$: n = 247, $\overline{x}$ = 1.57, 95% CI = [1.24, 1.91]. (3) $x_a x_r d+$: n = 246, $\overline{x}$ = 1.94, 95% CI = [1.41, 2.47]. (4) $x_a x_a x_r x_r$: n = 241, $\overline{x}$ = 1.68, 95% CI = [1.25, 2.11]. (5) $x_r x_a$: n = 243, $\overline{x}$ = 2.48, 95% CI = [1.49, 3.46]. (6) $x_r x_a d-$: n = 243, $\overline{x}$ = 3.10, 95% CI = [2.06, 4.13]. (7) $x_r x_a d+$: n = 242, $\overline{x}$ = 2.14, 95% CI = [1.22, 3.06]. (8) $x_r x_r x_a x_a$: n = 243, $\overline{x}$ = 2.48, 95% CI = [1.49, 3.47]. (9) $x_r x_r x_a x_a d-$: n = 243, $\overline{x}$ = 3.10, 95% CI = [2.06, 4.13]. (10) $x_r x_r x_a x_a d+$: n = 242, $\overline{x}$ = 2.14, 95% CI = [1.22, 3.07].

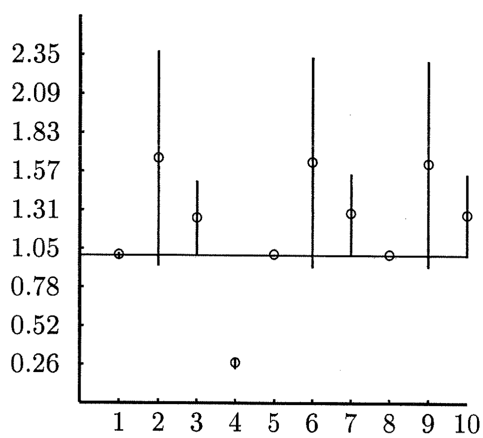


UNSATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_a x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: n = 85, $\overline{x}$ = 1.00, 95% CI = [1.00, 1.00]. (2) $x_a x_r d-$: n = 84, $\overline{x}$ = 1.41, 95% CI = [0.96, 1.86]. (3) $x_a x_r d+$: n = 85, $\overline{x}$ = 1.25, 95% CI = [0.76, 1.74]. (4) $x_a x_a x_r x_r$: n = 66, $\overline{x}$ = 0.37, 95% CI = [0.34, 0.41]. (5) $x_r x_a$: n = 85, $\overline{x}$ = 0.98, 95% CI = [0.96, 1.01]. (6) $x_r x_a d-$: n = 84, $\overline{x}$ = 1.37, 95% CI = [0.94, 1.80]. (7) $x_r x_a d+$: n = 85, $\overline{x}$ = 1.18, 95% CI = [0.82, 1.55]. (8) $x_r x_r x_a x_a$: n = 85, $\overline{x}$ = 0.99, 95% CI = [0.97, 1.01]. (9) $x_r x_r x_a x_a d-$: n = 84, $\overline{x}$ = 1.38, 95% CI = [0.94, 1.82]. (10) $x_r x_r x_a x_a d+$: n = 85, $\overline{x}$ = 1.19, 95% CI = [0.83, 1.55].

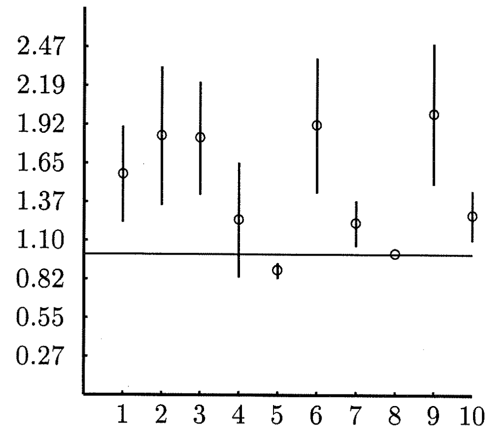
B.3 $x_rx_rx_ax_a$ CNF Reductions



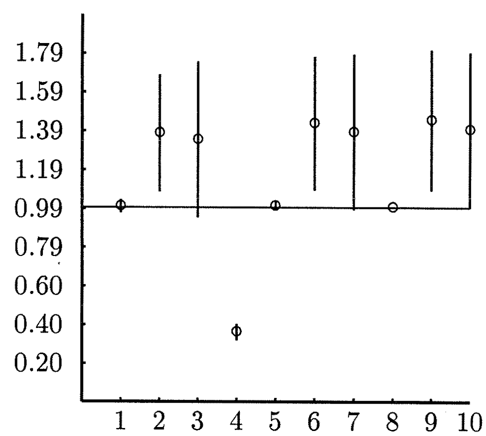
SATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_rx_rx_ax_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) x_ax_r: n = 137, $\overline{x}$ = 2.50, 95% CI = [1.46, 3.54]. (2) x_ax_rd-: n = 138, $\overline{x}$ = 4.63, 95% CI = [2.48, 6.79]. (3) x_ax_rd+: n = 142, $\overline{x}$ = 2.42, 95% CI = [1.46, 3.37]. (4) $x_ax_ax_rx_r$: n = 154, $\overline{x}$ = 0.86, 95% CI = [0.46, 1.26]. (5) x_rx_a: n = 145, $\overline{x}$ = 0.91, 95% CI = [0.85, 0.96]. (6) x_rx_ad-: n = 133, $\overline{x}$ = 3.81, 95% CI = [2.57, 5.05]. (7) x_rx_ad+: n = 138, $\overline{x}$ = 1.13, 95% CI = [0.81, 1.46]. (8) $x_rx_rx_ax_a$: n = 138, $\overline{x}$ = 1.00, 95% CI = [1.00, 1.00]. (9) $x_rx_rx_ax_ad-$: n = 129, $\overline{x}$ = 4.12, 95% CI = [2.65, 5.60]. (10) $x_rx_rx_ax_ad+$: n = 138, $\overline{x}$ = 1.14, 95% CI = [0.81, 1.47].



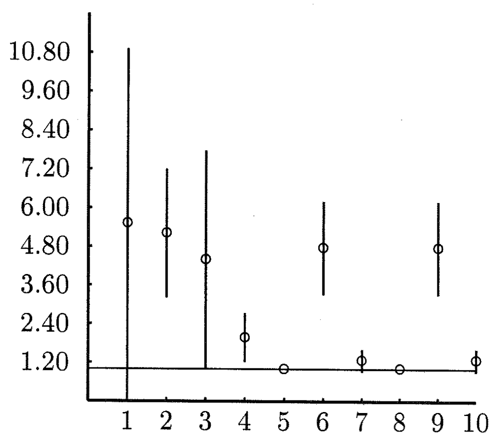
UNSATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_rx_rx_ax_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) x_ax_r: n = 58, $\overline{x}$ = 1.00, 95% CI = [0.98, 1.02]. (2) x_ax_rd-: n = 55, $\overline{x}$ = 1.65, 95% CI = [0.93, 2.38]. (3) x_ax_rd+: n = 57, $\overline{x}$ = 1.25, 95% CI = [1.00, 1.50]. (4) $x_ax_ax_rx_r$: n = 37, $\overline{x}$ = 0.27, 95% CI = [0.23, 0.31]. (5) x_rx_a: n = 58, $\overline{x}$ = 1.00, 95% CI = [1.00, 1.01]. (6) x_rx_ad-: n = 57, $\overline{x}$ = 1.63, 95% CI = [0.92, 2.34]. (7) x_rx_ad+: n = 57, $\overline{x}$ = 1.28, 95% CI = [1.01, 1.55]. (8) $x_rx_rx_ax_a$: n = 58, $\overline{x}$ = 1.00, 95% CI = [1.00, 1.00]. (9) $x_rx_rx_ax_ad-$: n = 58, $\overline{x}$ = 1.61, 95% CI = [0.92, 2.31]. (10) $x_rx_rx_ax_ad+$: n = 57, $\overline{x}$ = 1.27, 95% CI = [0.99, 1.55].



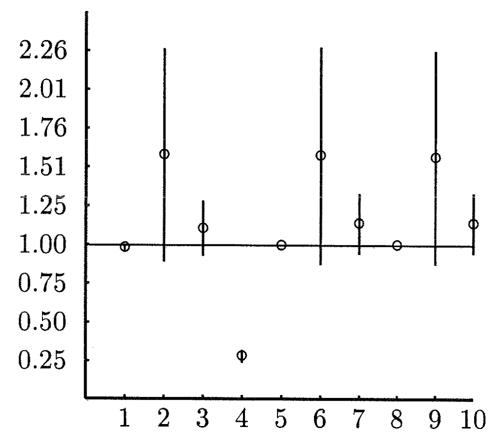
SATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the $x_r x_r x_a x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 177$, $\overline{x} = 1.57$, 95% CI = [1.23, 1.91]. (2) $x_a x_r d-$: $n = 177$, $\overline{x} = 1.84$, 95% CI = [1.35, 2.33]. (3) $x_a x_r d+$: $n = 176$, $\overline{x} = 1.82$, 95% CI = [1.42, 2.22]. (4) $x_a x_a x_r x_r$: $n = 186$, $\overline{x} = 1.24$, 95% CI = [0.84, 1.65]. (5) $x_r x_a$: $n = 184$, $\overline{x} = 0.88$, 95% CI = [0.83, 0.94]. (6) $x_r x_a d-$: $n = 162$, $\overline{x} = 1.91$, 95% CI = [1.44, 2.39]. (7) $x_r x_a d+$: $n = 178$, $\overline{x} = 1.22$, 95% CI = [1.06, 1.38]. (8) $x_r x_r x_a x_a$: $n = 178$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: $n = 170$, $\overline{x} = 2.00$, 95% CI = [1.50, 2.49]. (10) $x_r x_r x_a x_a d+$: $n = 173$, $\overline{x} = 1.27$, 95% CI = [1.10, 1.45].



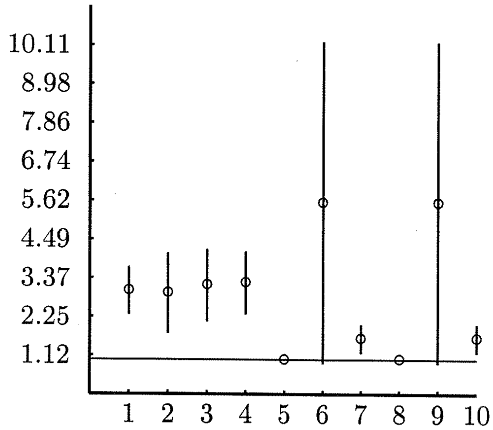
UNSATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the $x_r x_r x_a x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 85$, $\overline{x} = 1.01$, 95% CI = [0.97, 1.04]. (2) $x_a x_r d-$: $n = 84$, $\overline{x} = 1.38$, 95% CI = [1.08, 1.68]. (3) $x_a x_r d+$: $n = 85$, $\overline{x} = 1.35$, 95% CI = [0.95, 1.74]. (4) $x_a x_a x_r x_r$: $n = 66$, $\overline{x} = 0.36$, 95% CI = [0.32, 0.40]. (5) $x_r x_a$: $n = 85$, $\overline{x} = 1.01$, 95% CI = [0.99, 1.03]. (6) $x_r x_a d-$: $n = 84$, $\overline{x} = 1.43$, 95% CI = [1.09, 1.77]. (7) $x_r x_a d+$: $n = 85$, $\overline{x} = 1.38$, 95% CI = [0.99, 1.78]. (8) $x_r x_r x_a x_a$: $n = 85$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: $n = 84$, $\overline{x} = 1.45$, 95% CI = [1.09, 1.81]. (10) $x_r x_r x_a x_a d+$: $n = 85$, $\overline{x} = 1.40$, 95% CI = [1.00, 1.79].



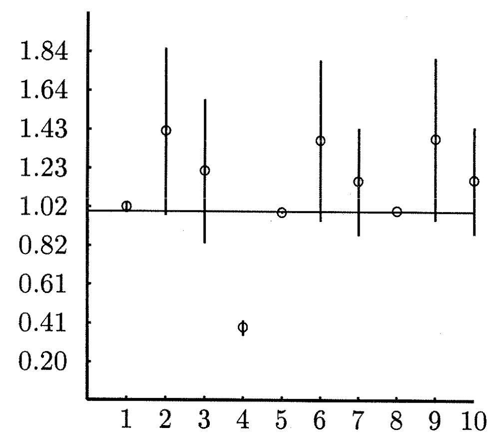
SATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_r x_r x_a x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 208$, $\overline{x} = 5.52$, 95% CI = [0.12, 10.91]. (2) $x_a x_r d-$: $n = 208$, $\overline{x} = 5.21$, 95% CI = [3.21, 7.20]. (3) $x_a x_r d+$: $n = 209$, $\overline{x} = 4.39$, 95% CI = [1.01, 7.77]. (4) $x_a x_a x_r x_r$: $n = 194$, $\overline{x} = 1.97$, 95% CI = [1.22, 2.72]. (5) $x_r x_a$: $n = 213$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (6) $x_r x_a d-$: $n = 213$, $\overline{x} = 4.76$, 95% CI = [3.31, 6.20]. (7) $x_r x_a d+$: $n = 208$, $\overline{x} = 1.27$, 95% CI = [0.92, 1.61]. (8) $x_r x_r x_a x_a$: $n = 213$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: $n = 213$, $\overline{x} = 4.76$, 95% CI = [3.31, 6.20]. (10) $x_r x_r x_a x_a d+$: $n = 208$, $\overline{x} = 1.28$, 95% CI = [0.92, 1.63].



UNSATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_r x_r x_a x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 58$, $\overline{x} = 0.98$, 95% CI = [0.97, 1.00]. (2) $x_a x_r d-$: $n = 55$, $\overline{x} = 1.58$, 95% CI = [0.89, 2.27]. (3) $x_a x_r d+$: $n = 57$, $\overline{x} = 1.11$, 95% CI = [0.93, 1.28]. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 0.28$, 95% CI = [0.24, 0.32]. (5) $x_r x_a$: $n = 58$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (6) $x_r x_a d-$: $n = 57$, $\overline{x} = 1.58$, 95% CI = [0.88, 2.28]. (7) $x_r x_a d+$: $n = 57$, $\overline{x} = 1.14$, 95% CI = [0.95, 1.34]. (8) $x_r x_r x_a x_a$: $n = 58$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: $n = 58$, $\overline{x} = 1.57$, 95% CI = [0.88, 2.26]. (10) $x_r x_r x_a x_a d+$: $n = 57$, $\overline{x} = 1.14$, 95% CI = [0.95, 1.34].

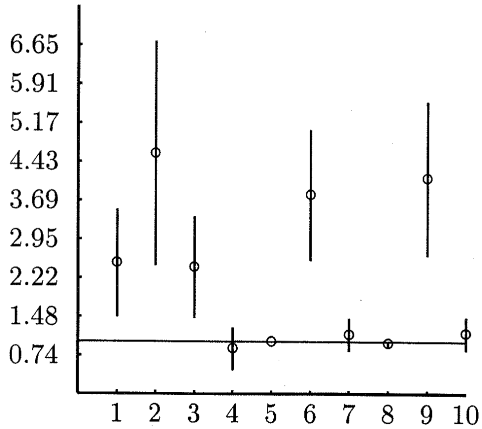


SATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_r x_r x_a x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: n = 243, $\overline{x}$ = 3.00, 95% CI = [2.30, 3.69]. (2) $x_a x_r d-$: n = 246, $\overline{x}$ = 2.93, 95% CI = [1.77, 4.10]. (3) $x_a x_r d+$: n = 246, $\overline{x}$ = 3.16, 95% CI = [2.11, 4.21]. (4) $x_a x_a x_r x_r$: n = 240, $\overline{x}$ = 3.23, 95% CI = [2.31, 4.15]. (5) $x_r x_a$: n = 247, $\overline{x}$ = 0.99, 95% CI = [0.99, 1.00]. (6) $x_r x_a d-$: n = 243, $\overline{x}$ = 5.55, 95% CI = [0.89, 10.21]. (7) $x_r x_a d+$: n = 245, $\overline{x}$ = 1.61, 95% CI = [1.19, 2.03]. (8) $x_r x_r x_a x_a$: n = 247, $\overline{x}$ = 1.00, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: n = 243, $\overline{x}$ = 5.55, 95% CI = [0.89, 10.21]. (10) $x_r x_r x_a x_a d+$: n = 245, $\overline{x}$ = 1.62, 95% CI = [1.20, 2.04].

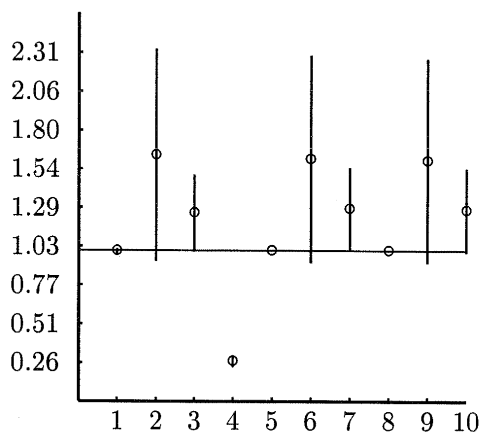


UNSATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_r x_r x_a x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: n = 85, $\overline{x}$ = 1.02, 95% CI = [1.00, 1.05]. (2) $x_a x_r d-$: n = 84, $\overline{x}$ = 1.42, 95% CI = [0.98, 1.86]. (3) $x_a x_r d+$: n = 85, $\overline{x}$ = 1.21, 95% CI = [0.83, 1.59]. (4) $x_a x_a x_r x_r$: n = 66, $\overline{x}$ = 0.38, 95% CI = [0.34, 0.42]. (5) $x_r x_a$: n = 85, $\overline{x}$ = 0.99, 95% CI = [0.99, 0.99]. (6) $x_r x_a d-$: n = 84, $\overline{x}$ = 1.37, 95% CI = [0.95, 1.80]. (7) $x_r x_a d+$: n = 85, $\overline{x}$ = 1.16, 95% CI = [0.87, 1.44]. (8) $x_r x_r x_a x_a$: n = 85, $\overline{x}$ = 1.00, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: n = 84, $\overline{x}$ = 1.38, 95% CI = [0.95, 1.81]. (10) $x_r x_r x_a x_a d+$: n = 85, $\overline{x}$ = 1.17, 95% CI = [0.88, 1.45].

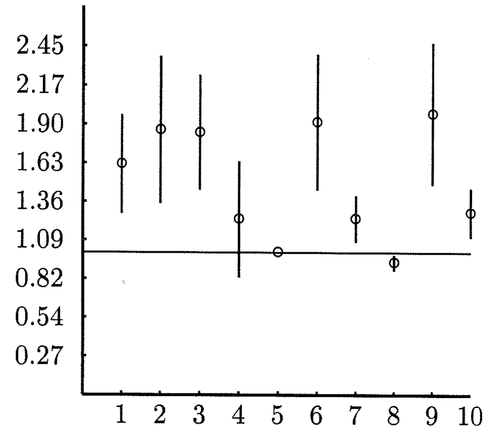
B.4 $x_r x_a$ CNF Reductions



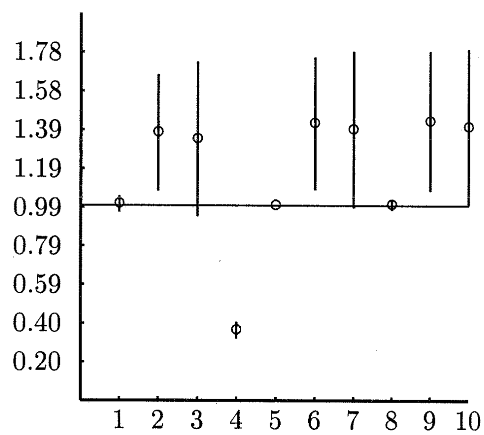
SATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_r x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 137$, $\overline{x} = 2.50$, 95% CI = $[1.47, 3.53]$. (2) $x_a x_r d-$: $n = 138$, $\overline{x} = 4.58$, 95% CI = $[2.45, 6.71]$. (3) $x_a x_r d+$: $n = 142$, $\overline{x} = 2.42$, 95% CI = $[1.45, 3.39]$. (4) $x_a x_a x_r x_r$: $n = 154$, $\overline{x} = 0.86$, 95% CI = $[0.46, 1.27]$. (5) $x_r x_a$: $n = 145$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (6) $x_r x_a d-$: $n = 133$, $\overline{x} = 3.80$, 95% CI = $[2.55, 5.05]$. (7) $x_r x_a d+$: $n = 138$, $\overline{x} = 1.14$, 95% CI = $[0.82, 1.45]$. (8) $x_r x_r x_a x_a$: $n = 138$, $\overline{x} = 0.97$, 95% CI = $[0.92, 1.01]$. (9) $x_r x_r x_a x_a d-$: $n = 129$, $\overline{x} = 4.12$, 95% CI = $[2.65, 5.59]$. (10) $x_r x_r x_a x_a d+$: $n = 138$, $\overline{x} = 1.16$, 95% CI = $[0.84, 1.48]$.



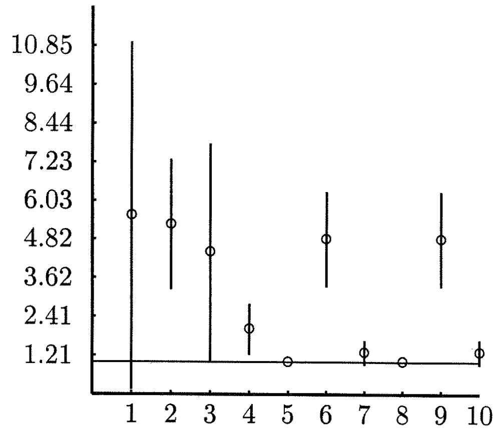
UNSATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the $x_r x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 58$, $\overline{x} = 1.00$, 95% CI = $[0.98, 1.01]$. (2) $x_a x_r d-$: $n = 55$, $\overline{x} = 1.63$, 95% CI = $[0.93, 2.34]$. (3) $x_a x_r d+$: $n = 57$, $\overline{x} = 1.25$, 95% CI = $[0.99, 1.50]$. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 0.27$, 95% CI = $[0.23, 0.30]$. (5) $x_r x_a$: $n = 58$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (6) $x_r x_a d-$: $n = 57$, $\overline{x} = 1.61$, 95% CI = $[0.92, 2.30]$. (7) $x_r x_a d+$: $n = 57$, $\overline{x} = 1.28$, 95% CI = $[1.01, 1.55]$. (8) $x_r x_r x_a x_a$: $n = 58$, $\overline{x} = 1.00$, 95% CI = $[0.99, 1.00]$. (9) $x_r x_r x_a x_a d-$: $n = 58$, $\overline{x} = 1.60$, 95% CI = $[0.92, 2.28]$. (10) $x_r x_r x_a x_a d+$: $n = 57$, $\overline{x} = 1.27$, 95% CI = $[0.99, 1.55]$.



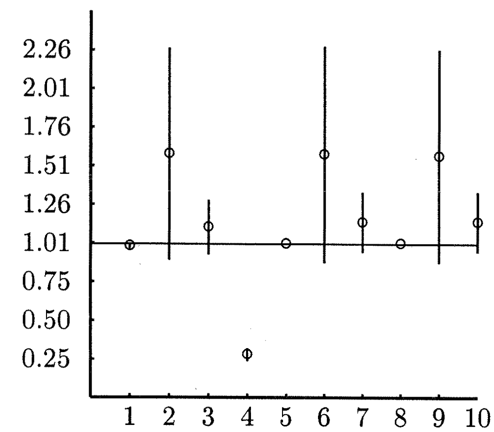
SATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the x_rx_a over the other solvers. Ratios are listed only by the denominator of the ratio: (1) x_ax_r: $n = 177$, $\overline{x} = 1.62$, 95% CI = [1.28, 1.97]. (2) x_ax_rd-: $n = 177$, $\overline{x} = 1.86$, 95% CI = [1.35, 2.37]. (3) x_ax_rd+: $n = 176$, $\overline{x} = 1.84$, 95% CI = [1.44, 2.24]. (4) $x_ax_ax_rx_r$: $n = 186$, $\overline{x} = 1.23$, 95% CI = [0.83, 1.64]. (5) x_rx_a: $n = 184$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (6) x_rx_ad-: $n = 162$, $\overline{x} = 1.92$, 95% CI = [1.44, 2.39]. (7) x_rx_ad+: $n = 178$, $\overline{x} = 1.24$, 95% CI = [1.08, 1.40]. (8) $x_rx_rx_ax_a$: $n = 178$, $\overline{x} = 0.93$, 95% CI = [0.87, 0.98]. (9) $x_rx_rx_ax_ad-$: $n = 170$, $\overline{x} = 1.98$, 95% CI = [1.48, 2.47]. (10) $x_rx_rx_ax_ad+$: $n = 173$, $\overline{x} = 1.28$, 95% CI = [1.11, 1.45].



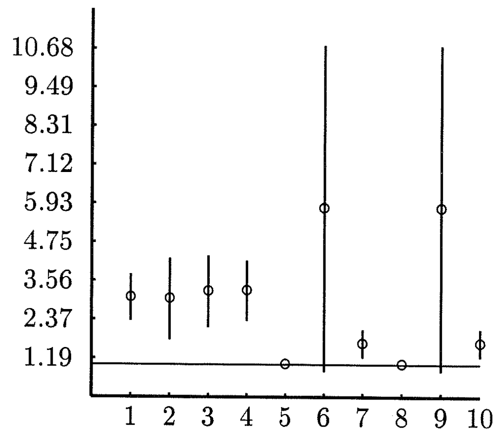
UNSATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the x_rx_a over the other solvers. Ratios are listed only by the denominator of the ratio: (1) x_ax_r: $n = 85$, $\overline{x} = 1.01$, 95% CI = [0.97, 1.05]. (2) x_ax_rd-: $n = 84$, $\overline{x} = 1.37$, 95% CI = [1.08, 1.67]. (3) x_ax_rd+: $n = 86$, $\overline{x} = 1.34$, 95% CI = [0.95, 1.73]. (4) $x_ax_ax_rx_r$: $n = 66$, $\overline{x} = 0.36$, 95% CI = [0.32, 0.40]. (5) x_rx_a: $n = 86$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (6) x_rx_ad-: $n = 84$, $\overline{x} = 1.42$, 95% CI = [1.08, 1.76]. (7) x_rx_ad+: $n = 85$, $\overline{x} = 1.39$, 95% CI = [0.99, 1.79]. (8) $x_rx_rx_ax_a$: $n = 85$, $\overline{x} = 1.00$, 95% CI = [0.98, 1.03]. (9) $x_rx_rx_ax_ad-$: $n = 84$, $\overline{x} = 1.43$, 95% CI = [1.08, 1.79]. (10) $x_rx_rx_ax_ad+$: $n = 85$, $\overline{x} = 1.40$, 95% CI = [1.00, 1.80].



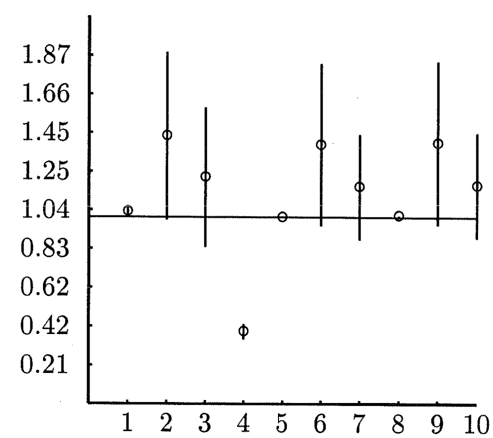
SATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_r x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 208$, $\overline{x} = 5.56$, 95% CI = [0.16, 10.95]. (2) $x_a x_r d-$: $n = 208$, $\overline{x} = 5.28$, 95% CI = [3.24, 7.31]. (3) $x_a x_r d+$: $n = 209$, $\overline{x} = 4.41$, 95% CI = [1.03, 7.80]. (4) $x_a x_a x_r x_r$: $n = 194$, $\overline{x} = 2.01$, 95% CI = [1.22, 2.80]. (5) $x_r x_a$: $n = 213$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (6) $x_r x_a d-$: $n = 213$, $\overline{x} = 4.82$, 95% CI = [3.34, 6.31]. (7) $x_r x_a d+$: $n = 208$, $\overline{x} = 1.29$, 95% CI = [0.91, 1.68]. (8) $x_r x_r x_a x_a$: $n = 213$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: $n = 213$, $\overline{x} = 4.83$, 95% CI = [3.34, 6.31]. (10) $x_r x_r x_a x_a d+$: $n = 208$, $\overline{x} = 1.30$, 95% CI = [0.91, 1.70].



UNSATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the $x_r x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 58$, $\overline{x} = 0.99$, 95% CI = [0.97, 1.00]. (2) $x_a x_r d-$: $n = 55$, $\overline{x} = 1.58$, 95% CI = [0.89, 2.27]. (3) $x_a x_r d+$: $n = 57$, $\overline{x} = 1.11$, 95% CI = [0.93, 1.29]. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 0.28$, 95% CI = [0.24, 0.32]. (5) $x_r x_a$: $n = 58$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (6) $x_r x_a d-$: $n = 57$, $\overline{x} = 1.58$, 95% CI = [0.88, 2.28]. (7) $x_r x_a d+$: $n = 57$, $\overline{x} = 1.14$, 95% CI = [0.95, 1.34]. (8) $x_r x_r x_a x_a$: $n = 58$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (9) $x_r x_r x_a x_a d-$: $n = 58$, $\overline{x} = 1.57$, 95% CI = [0.88, 2.26]. (10) $x_r x_r x_a x_a d+$: $n = 57$, $\overline{x} = 1.14$, 95% CI = [0.95, 1.34].

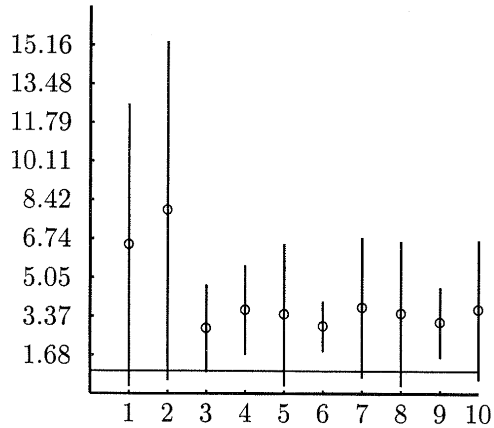


SATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_r x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 243$, $\overline{x} = 3.04$, 95% CI = $[2.33, 3.75]$. (2) $x_a x_r d-$: $n = 246$, $\overline{x} = 3.00$, 95% CI = $[1.75, 4.25]$. (3) $x_a x_r d+$: $n = 246$, $\overline{x} = 3.23$, 95% CI = $[2.13, 4.33]$. (4) $x_a x_a x_r x_r$: $n = 240$, $\overline{x} = 3.25$, 95% CI = $[2.33, 4.17]$. (5) $x_r x_a$: $n = 247$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (6) $x_r x_a d-$: $n = 243$, $\overline{x} = 5.79$, 95% CI = $[0.78, 10.79]$. (7) $x_r x_a d+$: $n = 245$, $\overline{x} = 1.64$, 95% CI = $[1.21, 2.07]$. (8) $x_r x_r x_a x_a$: $n = 247$, $\overline{x} = 1.01$, 95% CI = $[1.00, 1.01]$. (9) $x_r x_r x_a x_a d-$: $n = 243$, $\overline{x} = 5.79$, 95% CI = $[0.79, 10.79]$. (10) $x_r x_r x_a x_a d+$: $n = 245$, $\overline{x} = 1.65$, 95% CI = $[1.22, 2.08]$.

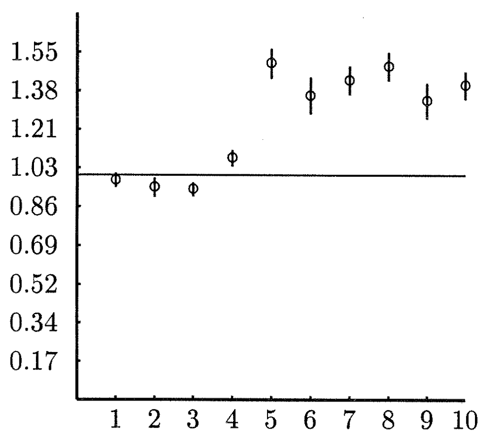


UNSATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the $x_r x_a$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 85$, $\overline{x} = 1.03$, 95% CI = $[1.00, 1.06]$. (2) $x_a x_r d-$: $n = 84$, $\overline{x} = 1.44$, 95% CI = $[0.99, 1.89]$. (3) $x_a x_r d+$: $n = 86$, $\overline{x} = 1.22$, 95% CI = $[0.84, 1.59]$. (4) $x_a x_a x_r x_r$: $n = 66$, $\overline{x} = 0.39$, 95% CI = $[0.35, 0.43]$. (5) $x_r x_a$: $n = 86$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (6) $x_r x_a d-$: $n = 84$, $\overline{x} = 1.39$, 95% CI = $[0.95, 1.83]$. (7) $x_r x_a d+$: $n = 85$, $\overline{x} = 1.16$, 95% CI = $[0.88, 1.45]$. (8) $x_r x_r x_a x_a$: $n = 85$, $\overline{x} = 1.01$, 95% CI = $[1.01, 1.01]$. (9) $x_r x_r x_a x_a d-$: $n = 84$, $\overline{x} = 1.40$, 95% CI = $[0.96, 1.84]$. (10) $x_r x_r x_a x_a d+$: $n = 85$, $\overline{x} = 1.17$, 95% CI = $[0.89, 1.46]$.

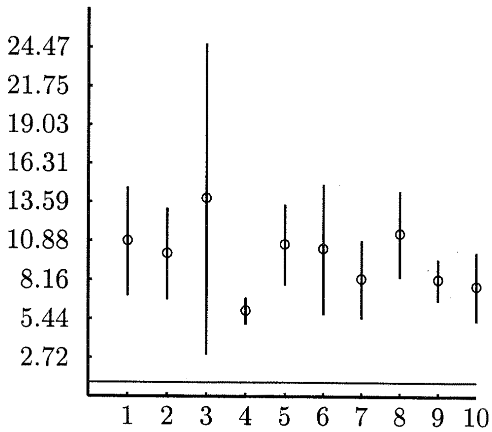
B.5 CNF Reductions With and Without the Discard Node



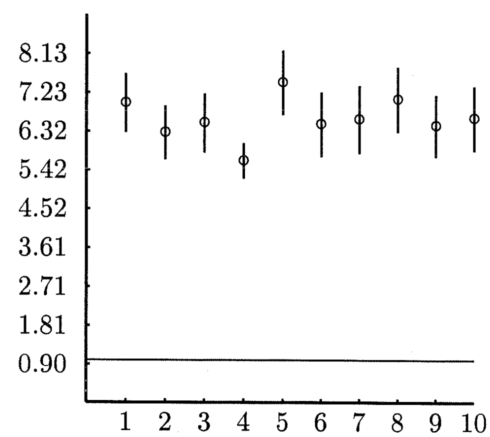
SATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the user time used by the flow solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: n = 118, $\overline{x}$ = 6.46, 95% CI = [0.33, 12.59]. (2) $x_a x_r d-$: n = 113, $\overline{x}$ = 7.95, 95% CI = [0.58, 15.31]. (3) $x_a x_r d+$: n = 124, $\overline{x}$ = 2.83, 95% CI = [0.94, 4.73]. (4) $x_a x_a x_r x_r$: n = 138, $\overline{x}$ = 3.63, 95% CI = [1.70, 5.56]. (5) $x_r x_a$: n = 121, $\overline{x}$ = 3.44, 95% CI = [0.36, 6.51]. (6) $x_r x_a d-$: n = 111, $\overline{x}$ = 2.93, 95% CI = [1.84, 4.03]. (7) $x_r x_a d+$: n = 123, $\overline{x}$ = 3.75, 95% CI = [0.70, 6.80]. (8) $x_r x_r x_a x_a$: n = 119, $\overline{x}$ = 3.49, 95% CI = [0.34, 6.63]. (9) $x_r x_r x_a x_a d-$: n = 111, $\overline{x}$ = 3.11, 95% CI = [1.57, 4.64]. (10) $x_r x_r x_a x_a d+$: n = 128, $\overline{x}$ = 3.65, 95% CI = [0.62, 6.69].



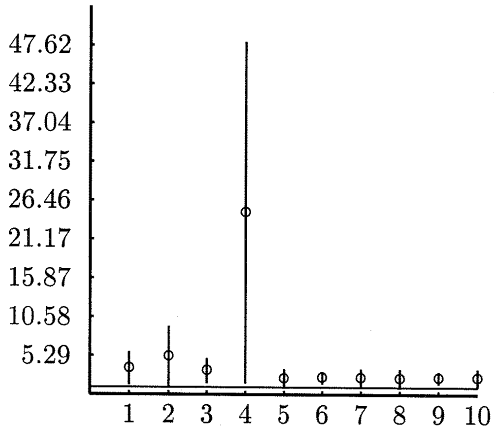
UNSATISFIABLE CNF instances; flow solver; user time. Graph of the ratio of the user time used by the flow solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: n = 57, $\overline{x}$ = 0.98, 95% CI = [0.95, 1.01]. (2) $x_a x_r d-$: n = 54, $\overline{x}$ = 0.95, 95% CI = [0.90, 0.99]. (3) $x_a x_r d+$: n = 58, $\overline{x}$ = 0.94, 95% CI = [0.91, 0.97]. (4) $x_a x_a x_r x_r$: n = 36, $\overline{x}$ = 1.08, 95% CI = [1.04, 1.11]. (5) $x_r x_a$: n = 58, $\overline{x}$ = 1.50, 95% CI = [1.43, 1.57]. (6) $x_r x_a d-$: n = 58, $\overline{x}$ = 1.35, 95% CI = [1.27, 1.44]. (7) $x_r x_a d+$: n = 60, $\overline{x}$ = 1.42, 95% CI = [1.36, 1.49]. (8) $x_r x_r x_a x_a$: n = 58, $\overline{x}$ = 1.48, 95% CI = [1.42, 1.55]. (9) $x_r x_r x_a x_a d-$: n = 59, $\overline{x}$ = 1.33, 95% CI = [1.25, 1.41]. (10) $x_r x_r x_a x_a d+$: n = 60, $\overline{x}$ = 1.40, 95% CI = [1.34, 1.46].



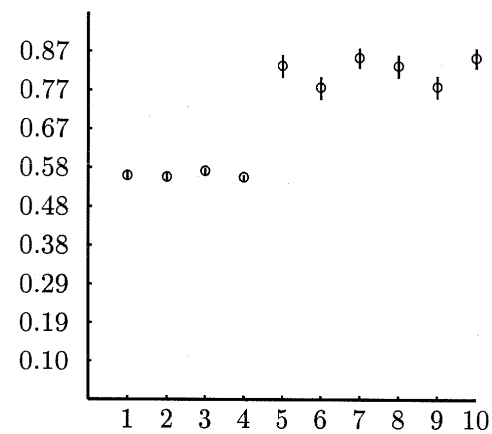
SATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the user time used by the matroid solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: $n = 126$, $\overline{x} = 10.83$, 95% CI = [7.04, 14.62]. (2) $x_a x_r d-$: $n = 124$, $\overline{x} = 9.97$, 95% CI = [6.78, 13.15]. (3) $x_a x_r d+$: $n = 130$, $\overline{x} = 13.83$, 95% CI = [2.93, 24.72]. (4) $x_a x_a x_r x_r$: $n = 141$, $\overline{x} = 5.97$, 95% CI = [5.04, 6.91]. (5) $x_r x_a$: $n = 115$, $\overline{x} = 10.61$, 95% CI = [7.81, 13.42]. (6) $x_r x_a d-$: $n = 103$, $\overline{x} = 10.29$, 95% CI = [5.75, 14.84]. (7) $x_r x_a d+$: $n = 112$, $\overline{x} = 8.21$, 95% CI = [5.49, 10.93]. (8) $x_r x_r x_a x_a$: $n = 114$, $\overline{x} = 11.36$, 95% CI = [8.34, 14.38]. (9) $x_r x_r x_a x_a d-$: $n = 96$, $\overline{x} = 8.15$, 95% CI = [6.70, 9.61]. (10) $x_r x_r x_a x_a d+$: $n = 122$, $\overline{x} = 7.69$, 95% CI = [5.28, 10.10].



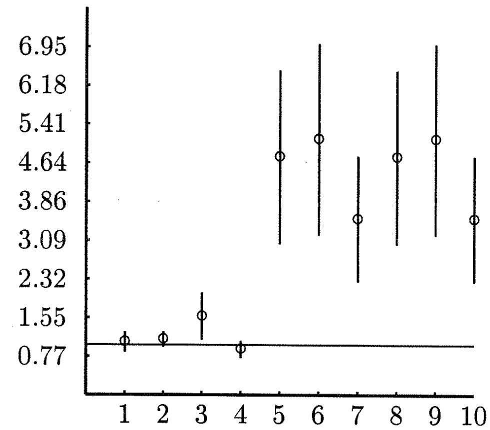
UNSATISFIABLE CNF instances; matroid solver; user time. Graph of the ratio of the user time used by the matroid solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: $n = 84$, $\overline{x} = 6.98$, 95% CI = [6.29, 7.67]. (2) $x_a x_r d-$: $n = 82$, $\overline{x} = 6.29$, 95% CI = [5.67, 6.92]. (3) $x_a x_r d+$: $n = 84$, $\overline{x} = 6.52$, 95% CI = [5.83, 7.20]. (4) $x_a x_a x_r x_r$: $n = 66$, $\overline{x} = 5.64$, 95% CI = [5.23, 6.04]. (5) $x_r x_a$: $n = 85$, $\overline{x} = 7.46$, 95% CI = [6.71, 8.21]. (6) $x_r x_a d-$: $n = 83$, $\overline{x} = 6.49$, 95% CI = [5.74, 7.24]. (7) $x_r x_a d+$: $n = 84$, $\overline{x} = 6.60$, 95% CI = [5.81, 7.39]. (8) $x_r x_r x_a x_a$: $n = 84$, $\overline{x} = 7.07$, 95% CI = [6.31, 7.83]. (9) $x_r x_r x_a x_a d-$: $n = 82$, $\overline{x} = 6.46$, 95% CI = [5.74, 7.18]. (10) $x_r x_r x_a x_a d+$: $n = 84$, $\overline{x} = 6.63$, 95% CI = [5.88, 7.39].



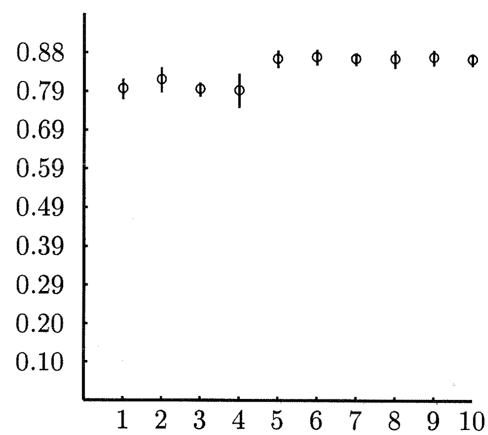
SATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the number of recursive calls used by the flow solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: $n = 211$, $\overline{x} = 3.55$, 95% CI = $[1.30, 5.79]$. (2) $x_a x_r d-$: $n = 213$, $\overline{x} = 5.19$, 95% CI = $[1.09, 9.29]$. (3) $x_a x_r d+$: $n = 211$, $\overline{x} = 3.25$, 95% CI = $[1.55, 4.95]$. (4) $x_a x_a x_r x_r$: $n = 192$, $\overline{x} = 24.81$, 95% CI = $[1.53, 48.10]$. (5) $x_r x_a$: $n = 213$, $\overline{x} = 2.19$, 95% CI = $[0.87, 3.50]$. (6) $x_r x_a d-$: $n = 217$, $\overline{x} = 2.30$, 95% CI = $[1.45, 3.15]$. (7) $x_r x_a d+$: $n = 212$, $\overline{x} = 2.27$, 95% CI = $[1.06, 3.48]$. (8) $x_r x_r x_a x_a$: $n = 213$, $\overline{x} = 2.19$, 95% CI = $[0.87, 3.50]$. (9) $x_r x_r x_a x_a d-$: $n = 218$, $\overline{x} = 2.29$, 95% CI = $[1.44, 3.14]$. (10) $x_r x_r x_a x_a d+$: $n = 212$, $\overline{x} = 2.27$, 95% CI = $[1.06, 3.48]$.



UNSATISFIABLE CNF instances; flow solver; number of recursive calls. Graph of the ratio of the number of recursive calls used by the flow solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: $n = 57$, $\overline{x} = 0.56$, 95% CI = $[0.55, 0.57]$. (2) $x_a x_r d-$: $n = 54$, $\overline{x} = 0.55$, 95% CI = $[0.54, 0.56]$. (3) $x_a x_r d+$: $n = 58$, $\overline{x} = 0.57$, 95% CI = $[0.56, 0.58]$. (4) $x_a x_a x_r x_r$: $n = 36$, $\overline{x} = 0.55$, 95% CI = $[0.55, 0.56]$. (5) $x_r x_a$: $n = 58$, $\overline{x} = 0.83$, 95% CI = $[0.80, 0.86]$. (6) $x_r x_a d-$: $n = 58$, $\overline{x} = 0.78$, 95% CI = $[0.75, 0.80]$. (7) $x_r x_a d+$: $n = 60$, $\overline{x} = 0.85$, 95% CI = $[0.82, 0.87]$. (8) $x_r x_r x_a x_a$: $n = 58$, $\overline{x} = 0.83$, 95% CI = $[0.80, 0.86]$. (9) $x_r x_r x_a x_a d-$: $n = 59$, $\overline{x} = 0.78$, 95% CI = $[0.75, 0.81]$. (10) $x_r x_r x_a x_a d+$: $n = 60$, $\overline{x} = 0.85$, 95% CI = $[0.82, 0.87]$.

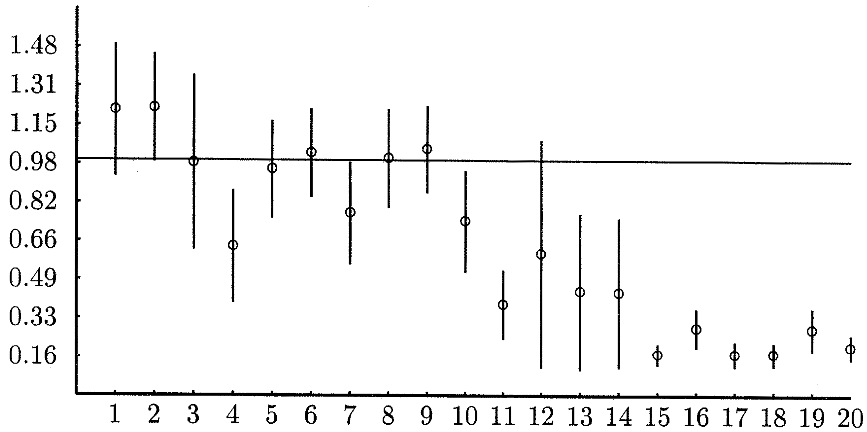


SATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the number of recursive calls used by the matroid solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: $n = 247$, $\overline{x} = 1.06$, 95% CI = $[0.86, 1.26]$. (2) $x_a x_r d-$: $n = 253$, $\overline{x} = 1.11$, 95% CI = $[0.96, 1.27]$. (3) $x_a x_r d+$: $n = 251$, $\overline{x} = 1.58$, 95% CI = $[1.10, 2.05]$. (4) $x_a x_a x_r x_r$: $n = 242$, $\overline{x} = 0.92$, 95% CI = $[0.75, 1.09]$. (5) $x_r x_a$: $n = 247$, $\overline{x} = 4.76$, 95% CI = $[3.02, 6.49]$. (6) $x_r x_a d-$: $n = 244$, $\overline{x} = 5.11$, 95% CI = $[3.20, 7.02]$. (7) $x_r x_a d+$: $n = 246$, $\overline{x} = 3.52$, 95% CI = $[2.27, 4.77]$. (8) $x_r x_r x_a x_a$: $n = 247$, $\overline{x} = 4.75$, 95% CI = $[3.02, 6.49]$. (9) $x_r x_r x_a x_a d-$: $n = 244$, $\overline{x} = 5.11$, 95% CI = $[3.20, 7.02]$. (10) $x_r x_r x_a x_a d+$: $n = 246$, $\overline{x} = 3.52$, 95% CI = $[2.27, 4.77]$.

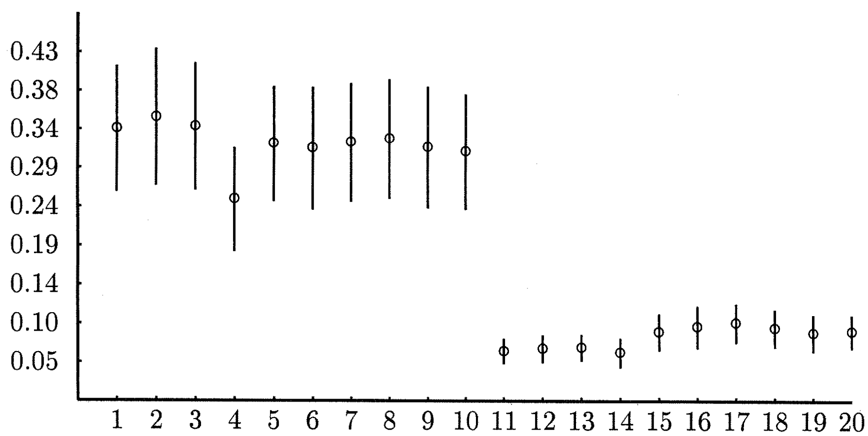


UNSATISFIABLE CNF instances; matroid solver; number of recursive calls. Graph of the ratio of the number of recursive calls used by the matroid solver concerning reductions with b over reductions without b. Ratios are listed only by the numerator of the ratio: (1) $x_a x_r$: $n = 85$, $\overline{x} = 0.79$, 95% CI = $[0.77, 0.82]$. (2) $x_a x_r d-$: $n = 84$, $\overline{x} = 0.82$, 95% CI = $[0.78, 0.85]$. (3) $x_a x_r d+$: $n = 86$, $\overline{x} = 0.79$, 95% CI = $[0.77, 0.81]$. (4) $x_a x_a x_r x_r$: $n = 66$, $\overline{x} = 0.79$, 95% CI = $[0.75, 0.83]$. (5) $x_r x_a$: $n = 86$, $\overline{x} = 0.87$, 95% CI = $[0.85, 0.89]$. (6) $x_r x_a d-$: $n = 84$, $\overline{x} = 0.87$, 95% CI = $[0.85, 0.89]$. (7) $x_r x_a d+$: $n = 85$, $\overline{x} = 0.87$, 95% CI = $[0.85, 0.89]$. (8) $x_r x_r x_a x_a$: $n = 85$, $\overline{x} = 0.87$, 95% CI = $[0.85, 0.89]$. (9) $x_r x_r x_a x_a d-$: $n = 84$, $\overline{x} = 0.87$, 95% CI = $[0.85, 0.89]$. (10) $x_r x_r x_a x_a d+$: $n = 85$, $\overline{x} = 0.87$, 95% CI = $[0.85, 0.88]$.

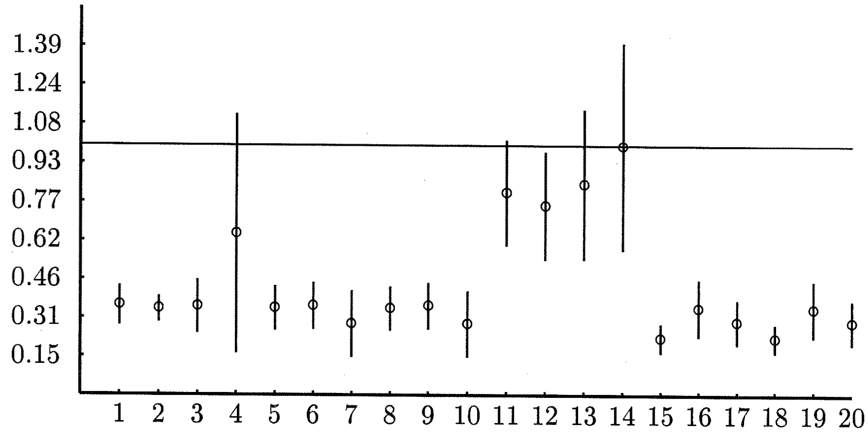
B.6 Network Flow and Matroid Solvers on CNF-Reduced Instances



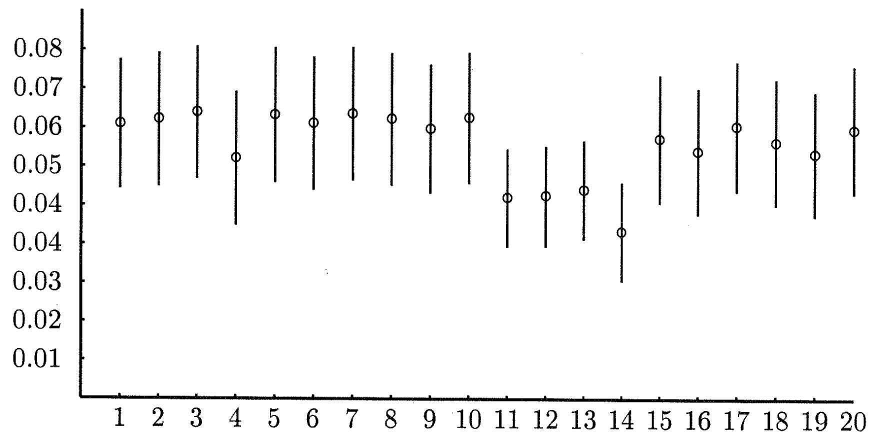
SATISFIABLE CNF instances; user time. Graph of the ratio of the matroid solver over the flow solver. Horizontal axis lists reductions: (1) $x_a x_r$: $n = 142$, $\overline{x} = 1.21$, 95% CI = [0.93, 1.49]. (2) $x_a x_r d-$: $n = 144$, $\overline{x} = 1.22$, 95% CI = [0.99, 1.45]. (3) $x_a x_r d+$: $n = 144$, $\overline{x} = 0.99$, 95% CI = [0.62, 1.36]. (4) $x_a x_a x_r x_r$: $n = 154$, $\overline{x} = 0.63$, 95% CI = [0.39, 0.87]. (5) $x_r x_a$: $n = 145$, $\overline{x} = 0.96$, 95% CI = [0.75, 1.17]. (6) $x_r x_a d-$: $n = 138$, $\overline{x} = 1.03$, 95% CI = [0.84, 1.22]. (7) $x_r x_a d+$: $n = 142$, $\overline{x} = 0.77$, 95% CI = [0.56, 0.99]. (8) $x_r x_r x_a x_a$: $n = 138$, $\overline{x} = 1.01$, 95% CI = [0.80, 1.22]. (9) $x_r x_r x_a x_a d-$: $n = 134$, $\overline{x} = 1.05$, 95% CI = [0.86, 1.23]. (10) $x_r x_r x_a x_a d+$: $n = 142$, $\overline{x} = 0.74$, 95% CI = [0.52, 0.96]. (11) $x_a x_r$_nob: $n = 120$, $\overline{x} = 0.39$, 95% CI = [0.24, 0.53]. (12) $x_a x_r d-$_nob: $n = 113$, $\overline{x} = 0.60$, 95% CI = [0.12, 1.08]. (13) $x_a x_r d+$_nob: $n = 124$, $\overline{x} = 0.44$, 95% CI = [0.11, 0.77]. (14) $x_a x_a x_r x_r$_nob: $n = 146$, $\overline{x} = 0.44$, 95% CI = [0.12, 0.75]. (15) $x_r x_a$_nob: $n = 128$, $\overline{x} = 0.17$, 95% CI = [0.13, 0.22]. (16) $x_r x_a d-$_nob: $n = 115$, $\overline{x} = 0.29$, 95% CI = [0.20, 0.37]. (17) $x_r x_a d+$_nob: $n = 126$, $\overline{x} = 0.18$, 95% CI = [0.12, 0.23]. (18) $x_r x_r x_a x_a$_nob: $n = 126$, $\overline{x} = 0.18$, 95% CI = [0.12, 0.23]. (19) $x_r x_r x_a x_a d-$_nob: $n = 116$, $\overline{x} = 0.28$, 95% CI = [0.19, 0.37]. (20) $x_r x_r x_a x_a d+$_nob: $n = 131$, $\overline{x} = 0.21$, 95% CI = [0.15, 0.26].



UNSATISFIABLE CNF instances; user time. Graph of the ratio of the matroid solver over the flow solver. Horizontal axis lists reductions: (1) $x_a x_r$: $n = 58$, $\overline{x} = 0.34$, 95% CI = [0.26, 0.42]. (2) $x_a x_r d-$: $n = 56$, $\overline{x} = 0.35$, 95% CI = [0.27, 0.44]. (3) $x_a x_r d+$: $n = 58$, $\overline{x} = 0.34$, 95% CI = [0.26, 0.42]. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 0.25$, 95% CI = [0.18, 0.31]. (5) $x_r x_a$: n = 58, $\overline{x}$ = 0.32, 95% CI = [0.25, 0.39]. (6) $x_r x_a d-$: n = 58, $\overline{x}$ = 0.31, 95% CI = [0.24, 0.39]. (7) $x_r x_a d+$: n = 60, $\overline{x}$ = 0.32, 95% CI = [0.25, 0.39]. (8) $x_r x_r x_a x_a$: n = 58, $\overline{x}$ = 0.32, 95% CI = [0.25, 0.40]. (9) $x_r x_r x_a x_a d-$: n = 59, $\overline{x}$ = 0.31, 95% CI = [0.24, 0.39]. (10) $x_r x_r x_a x_a d+$: n = 60, $\overline{x}$ = 0.31, 95% CI = [0.24, 0.38]. (11) $x_a x_r$_nob: n = 57, $\overline{x}$ = 0.06, 95% CI = [0.05, 0.08]. (12) $x_a x_r d-$_nob: n = 54, $\overline{x}$ = 0.06, 95% CI = [0.05, 0.08]. (13) $x_a x_r d+$_nob: n = 58, $\overline{x}$ = 0.07, 95% CI = [0.05, 0.08]. (14) $x_a x_a x_r x_r$_nob: n = 38, $\overline{x}$ = 0.06, 95% CI = [0.04, 0.08]. (15) $x_r x_a$_nob: n = 63, $\overline{x}$ = 0.09, 95% CI = [0.06, 0.11]. (16) $x_r x_a d-$_nob: n = 62, $\overline{x}$ = 0.09, 95% CI = [0.07, 0.12]. (17) $x_r x_a d+$_nob: n = 64, $\overline{x}$ = 0.10, 95% CI = [0.07, 0.12]. (18) $x_r x_r x_a x_a$_nob: n = 63, $\overline{x}$ = 0.09, 95% CI = [0.07, 0.11]. (19) $x_r x_r x_a x_a d-$_nob: n = 62, $\overline{x}$ = 0.08, 95% CI = [0.06, 0.11]. (20) $x_r x_r x_a x_a d+$_nob: n = 64, $\overline{x}$ = 0.09, 95% CI = [0.07, 0.11].

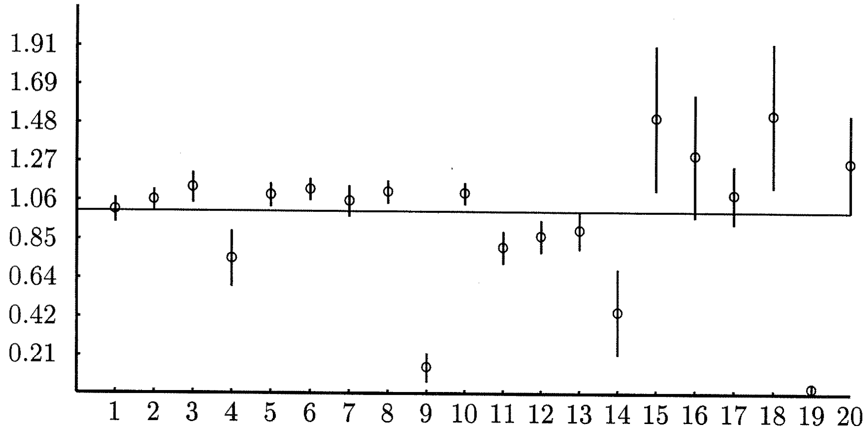


SATISFIABLE CNF instances; number of recursive calls. Graph of the ratio of the matroid solver over the flow solver. Horizontal axis lists reductions: (1) $x_a x_r$: n = 213, $\overline{x}$ = 0.36, 95% CI = [0.28, 0.44]. (2) $x_a x_r d-$: n = 214, $\overline{x}$ = 0.35, 95% CI = [0.29, 0.40]. (3) $x_a x_r d+$: n = 211, $\overline{x}$ = 0.35, 95% CI = [0.25, 0.46]. (4) $x_a x_a x_r x_r$: n = 194, $\overline{x}$ = 0.65, 95% CI = [0.17, 1.12]. (5) $x_r x_a$: n = 213, $\overline{x}$ = 0.35, 95% CI = [0.26, 0.44]. (6) $x_r x_a d-$: n = 218, $\overline{x}$ = 0.36, 95% CI = [0.26, 0.45]. (7) $x_r x_a d+$: n = 212, $\overline{x}$ = 0.29, 95% CI = [0.15, 0.42]. (8) $x_r x_r x_a x_a$: n = 213, $\overline{x}$ = 0.35, 95% CI = [0.26, 0.43]. (9) $x_r x_r x_a x_a d-$: n = 218, $\overline{x}$ = 0.36, 95% CI = [0.26, 0.45]. (10) $x_r x_r x_a x_a d+$: n = 212, $\overline{x}$ = 0.28, 95% CI = [0.15, 0.42]. (11) $x_a x_r$_nob: n = 213, $\overline{x}$ = 0.81, 95% CI = [0.60, 1.02]. (12) $x_a x_r d-$_nob: n = 213, $\overline{x}$ = 0.76, 95% CI = [0.54, 0.97]. (13) $x_a x_r d+$_nob: n = 211, $\overline{x}$ = 0.84, 95% CI = [0.54, 1.14]. (14) $x_a x_a x_r x_r$_nob: n = 200, $\overline{x}$ = 1.00, 95% CI = [0.58, 1.41]. (15) $x_r x_a$_nob: n = 220, $\overline{x}$ = 0.23, 95% CI = [0.17, 0.29]. (16) $x_r x_a d-$_nob: n = 221, $\overline{x}$ = 0.35, 95% CI = [0.23, 0.46]. (17) $x_r x_a d+$_nob: n = 215, $\overline{x}$ = 0.29, 95% CI = [0.20, 0.38]. (18) $x_r x_r x_a x_a$_nob: n = 220, $\overline{x}$ = 0.23, 95% CI = [0.17, 0.29]. (19) $x_r x_r x_a x_a d-$_nob: n = 223, $\overline{x}$ = 0.35, 95% CI = [0.23, 0.46]. (20) $x_r x_r x_a x_a d+$_nob: n = 215, $\overline{x}$ = 0.29, 95% CI = [0.20, 0.38].

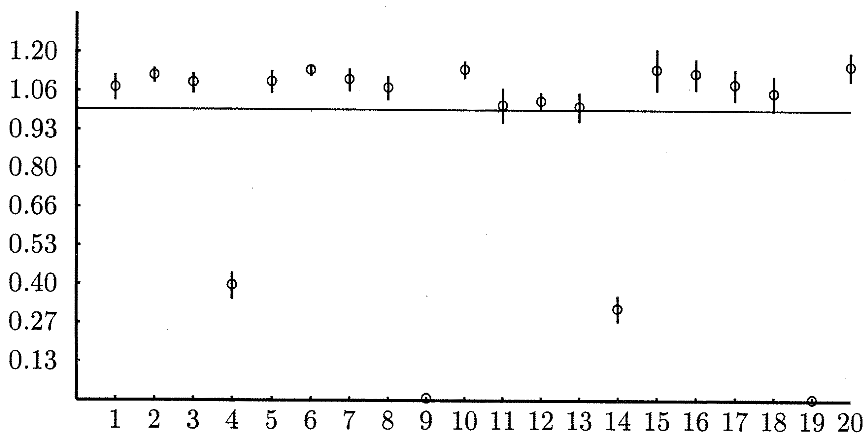


UNSATISFIABLE CNF instances; number of recursive calls. Graph of the ratio of the matroid solver over the flow solver. Horizontal axis lists reductions: (1) $x_a x_r$: $n = 58$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (2) $x_a x_r d-$: $n = 56$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (3) $x_a x_r d+$: $n = 58$, $\overline{x} = 0.07$, 95% CI = [0.05, 0.08]. (4) $x_a x_a x_r x_r$: $n = 37$, $\overline{x} = 0.05$, 95% CI = [0.04, 0.07]. (5) $x_r x_a$: $n = 58$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (6) $x_r x_a d-$: $n = 58$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (7) $x_r x_a d+$: $n = 60$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (8) $x_r x_r x_a x_a$: $n = 58$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (9) $x_r x_r x_a x_a d-$: $n = 59$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (10) $x_r x_r x_a x_a d+$: $n = 60$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (11) $x_a x_r$_nob: $n = 57$, $\overline{x} = 0.05$, 95% CI = [0.03, 0.06]. (12) $x_a x_r d-$_nob: $n = 54$, $\overline{x} = 0.05$, 95% CI = [0.03, 0.06]. (13) $x_a x_r d+$_nob: $n = 58$, $\overline{x} = 0.05$, 95% CI = [0.04, 0.06]. (14) $x_a x_a x_r x_r$_nob: $n = 38$, $\overline{x} = 0.04$, 95% CI = [0.03, 0.05]. (15) $x_r x_a$_nob: $n = 63$, $\overline{x} = 0.06$, 95% CI = [0.04, 0.07]. (16) $x_r x_a d-$_nob: $n = 62$, $\overline{x} = 0.06$, 95% CI = [0.04, 0.07]. (17) $x_r x_a d+$_nob: $n = 64$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08]. (18) $x_r x_r x_a x_a$_nob: $n = 63$, $\overline{x} = 0.06$, 95% CI = [0.04, 0.07]. (19) $x_r x_r x_a x_a d-$_nob: $n = 62$, $\overline{x} = 0.06$, 95% CI = [0.04, 0.07]. (20) $x_r x_r x_a x_a d+$_nob: $n = 64$, $\overline{x} = 0.06$, 95% CI = [0.05, 0.08].

B.7 Matroid and Matroid MTF Solvers on CNF-Reduced Instances

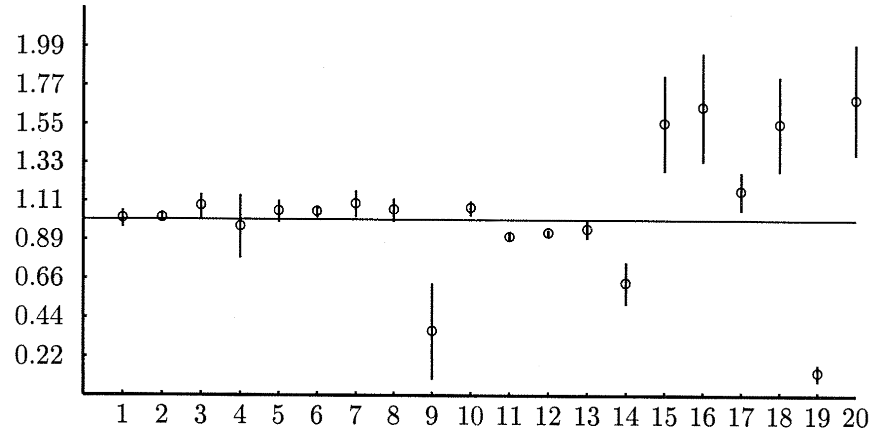


SATISFIABLE CNF instances; user time. Graph of the ratio of the matroidmtf solver over the matroid solver. Horizontal axis lists reductions: (1) $x_a x_r$: $n = 181$, $\overline{x} = 1.01$, 95% CI = [0.94, 1.07]. (2) $x_a x_r d-$: $n = 184$, $\overline{x} = 1.06$, 95% CI = [1.00, 1.12]. (3) $x_a x_r d+$: $n = 181$, $\overline{x} = 1.13$, 95% CI = [1.04, 1.21]. (4) $x_a x_a x_r x_r$: $n = 187$, $\overline{x} = 0.74$, 95% CI = [0.59, 0.89]. (5) $x_r x_a$: $n = 184$, $\overline{x} = 1.09$, 95% CI = [1.02, 1.15]. (6) $x_r x_a d-$: $n = 162$, $\overline{x} = 1.12$, 95% CI = [1.06, 1.18]. (7) $x_r x_a d+$: $n = 179$, $\overline{x} = 1.05$, 95% CI = [0.97, 1.14]. (8) $x_r x_r x_a x_a$: $n = 178$, $\overline{x} = 1.10$, 95% CI = [1.04, 1.17]. (9) $(d-u)+$: $n = 106$, $\overline{x} = 0.14$, 95% CI = [0.06, 0.22]. (10) $(d-u)-$: $n = 154$, $\overline{x} = 1.10$, 95% CI = [1.04, 1.16]. (11) $x_a x_r$_nob: $n = 137$, $\overline{x} = 0.80$, 95% CI = [0.71, 0.89]. (12) $x_a x_r d-$_nob: $n = 129$, $\overline{x} = 0.86$, 95% CI = [0.77, 0.95]. (13) $x_a x_r d+$_nob: $n = 137$, $\overline{x} = 0.90$, 95% CI = [0.79, 1.00]. (14) $x_a x_a x_r x_r$_nob: $n = 148$, $\overline{x} = 0.45$, 95% CI = [0.21, 0.68]. (15) $x_r x_a$_nob: $n = 125$, $\overline{x} = 1.51$, 95% CI = [1.11, 1.91]. (16) $x_r x_a d-$_nob: $n = 115$, $\overline{x} = 1.30$, 95% CI = [0.97, 1.64]. (17) $x_r x_a d+$_nob: $n = 124$, $\overline{x} = 1.09$, 95% CI = [0.93, 1.25]. (18) $x_r x_r x_a x_a$_nob: $n = 124$, $\overline{x} = 1.53$, 95% CI = [1.13, 1.92]. (19) $(d-u)+$_nob: $n = 102$, $\overline{x} = 0.03$, 95% CI = [0.00, 0.06]. (20) $(d-u)-$_nob: $n = 101$, $\overline{x} = 1.26$, 95% CI = [1.00, 1.53].

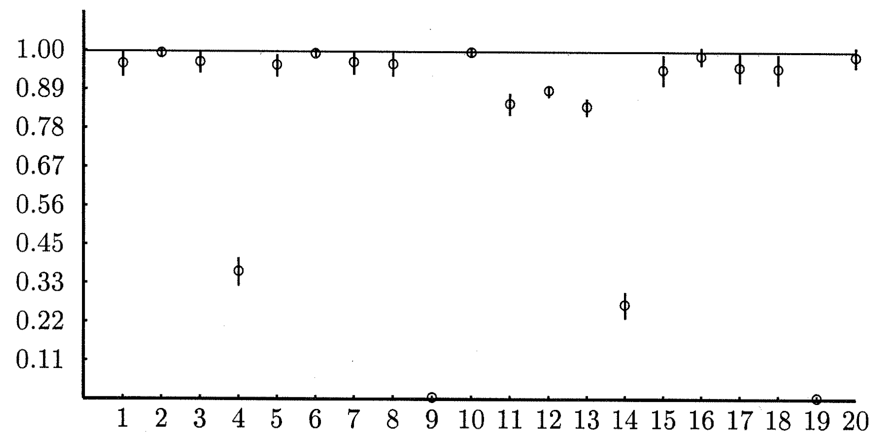


UNSATISFIABLE CNF instances; user time. Graph of the ratio of the matroidmtf solver over the matroid solver. Horizontal axis lists reductions: (1) $x_a x_r$: $n = 85$, $\overline{x} = 1.07$, 95% CI = [1.03, 1.12]. (2) $x_a x_r d-$: $n = 83$, $\overline{x} = 1.12$, 95% CI = [1.09, 1.14]. (3) $x_a x_r d+$: $n = 85$, $\overline{x} = 1.09$, 95% CI = [1.06, 1.12]. (4) $x_a x_a x_r x_r$: $n = 66$,

$\overline{x}$ = 0.39, 95% CI = [0.35, 0.44]. (5) $x_r x_a$: n = 85, $\overline{x}$ = 1.09, 95% CI = [1.06, 1.13]. (6) $x_r x_a d-$: n = 83, $\overline{x}$ = 1.13, 95% CI = [1.11, 1.15]. (7) $x_r x_a d+$: n = 84, $\overline{x}$ = 1.10, 95% CI = [1.06, 1.14]. (8) $x_r x_r x_a x_a$: n = 82, $\overline{x}$ = 1.07, 95% CI = [1.03, 1.11]. (9) $(d-u)+$: n = 9, $\overline{x}$ = 0.01, 95% CI = [0.00, 0.01]. (10) $(d-u)-$: n = 70, $\overline{x}$ = 1.14, 95% CI = [1.11, 1.17]. (11) $x_a x_r$_nob: n = 90, $\overline{x}$ = 1.01, 95% CI = [0.95, 1.07]. (12) $x_a x_r d-$_nob: n = 89, $\overline{x}$ = 1.03, 95% CI = [1.00, 1.06]. (13) $x_a x_r d+$_nob: n = 89, $\overline{x}$ = 1.01, 95% CI = [0.96, 1.06]. (14) $x_a x_a x_r x_r$_nob: n = 79, $\overline{x}$ = 0.31, 95% CI = [0.27, 0.36]. (15) $x_r x_a$_nob: n = 90, $\overline{x}$ = 1.14, 95% CI = [1.07, 1.21]. (16) $x_r x_a d-$_nob: n = 90, $\overline{x}$ = 1.12, 95% CI = [1.07, 1.18]. (17) $x_r x_a d+$_nob: n = 90, $\overline{x}$ = 1.09, 95% CI = [1.03, 1.14]. (18) $x_r x_r x_a x_a$_nob: n = 90, $\overline{x}$ = 1.06, 95% CI = [1.00, 1.12]. (19) $(d-u)+$_nob: n = 29, $\overline{x}$ = 0.00, 95% CI = [0.00, 0.00]. (20) $(d-u)-$_nob: n = 75, $\overline{x}$ = 1.15, 95% CI = [1.10, 1.20].

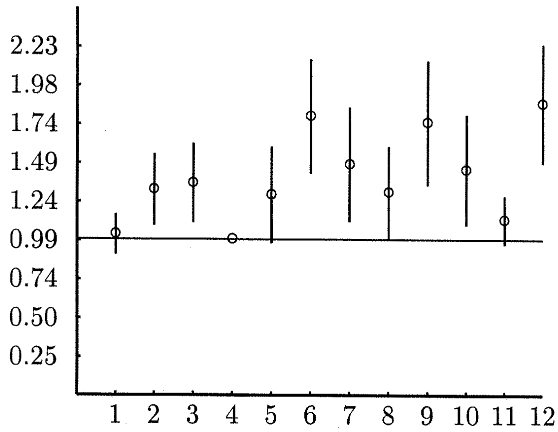


SATISFIABLE CNF instances; number of recursive calls. Graph of the ratio of the matroidmtf solver over the matroid solver. Horizontal axis lists reductions: (1) $x_a x_r$: n = 247, $\overline{x}$ = 1.00, 95% CI = [0.96, 1.05]. (2) $x_a x_r d-$: n = 253, $\overline{x}$ = 1.01, 95% CI = [0.98, 1.03]. (3) $x_a x_r d+$: n = 251, $\overline{x}$ = 1.08, 95% CI = [1.01, 1.15]. (4) $x_a x_a x_r x_r$: n = 241, $\overline{x}$ = 0.96, 95% CI = [0.78, 1.14]. (5) $x_r x_a$: n = 247, $\overline{x}$ = 1.05, 95% CI = [0.98, 1.11]. (6) $x_r x_a d-$: n = 243, $\overline{x}$ = 1.04, 95% CI = [1.01, 1.08]. (7) $x_r x_a d+$: n = 246, $\overline{x}$ = 1.09, 95% CI = [1.01, 1.16]. (8) $x_r x_r x_a x_a$: n = 247, $\overline{x}$ = 1.05, 95% CI = [0.99, 1.12]. (9) $(d-u)+$: n = 129, $\overline{x}$ = 0.36, 95% CI = [0.09, 0.64]. (10) $(d-u)-$: n = 212, $\overline{x}$ = 1.07, 95% CI = [1.03, 1.11]. (11) $x_a x_r$_nob: n = 258, $\overline{x}$ = 0.90, 95% CI = [0.88, 0.93]. (12) $x_a x_r d-$_nob: n = 258, $\overline{x}$ = 0.92, 95% CI = [0.91, 0.94]. (13) $x_a x_r d+$_nob: n = 258, $\overline{x}$ = 0.94, 95% CI = [0.89, 1.00]. (14) $x_a x_a x_r x_r$_nob: n = 249, $\overline{x}$ = 0.64, 95% CI = [0.52, 0.76]. (15) $x_r x_a$_nob: n = 257, $\overline{x}$ = 1.56, 95% CI = [1.28, 1.83]. (16) $x_r x_a d-$_nob: n = 256, $\overline{x}$ = 1.65, 95% CI = [1.33, 1.96]. (17) $x_r x_a d+$_nob: n = 258, $\overline{x}$ = 1.16, 95% CI = [1.05, 1.28]. (18) $x_r x_r x_a x_a$_nob: n = 257, $\overline{x}$ = 1.55, 95% CI = [1.28, 1.83]. (19) $(d-u)+$_nob: n = 135, $\overline{x}$ = 0.13, 95% CI = [0.08, 0.18]. (20) $(d-u)-$_nob: n = 226, $\overline{x}$ = 1.69, 95% CI = [1.37, 2.01].

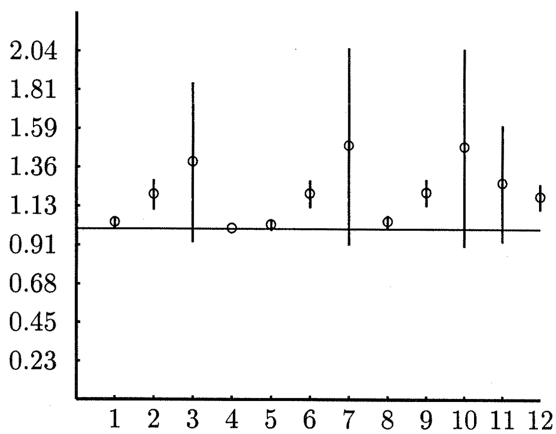


UNSATISFIABLE CNF instances; number of recursive calls. Graph of the ratio of the matroidmtf solver over the matroid solver. Horizontal axis lists reductions: (1) $x_a x_r$: $n = 85$, $\overline{x} = 0.96$, 95% CI = [0.93, 1.00]. (2) $x_a x_r d-$: $n = 83$, $\overline{x} = 0.99$, 95% CI = [0.98, 1.01]. (3) $x_a x_r d+$: $n = 85$, $\overline{x} = 0.97$, 95% CI = [0.94, 1.00]. (4) $x_a x_a x_r x_r$: $n = 66$, $\overline{x} = 0.37$, 95% CI = [0.33, 0.41]. (5) $x_r x_a$: $n = 85$, $\overline{x} = 0.96$, 95% CI = [0.93, 0.99]. (6) $x_r x_a d-$: $n = 83$, $\overline{x} = 0.99$, 95% CI = [0.98, 1.01]. (7) $x_r x_a d+$: $n = 84$, $\overline{x} = 0.97$, 95% CI = [0.93, 1.00]. (8) $x_r x_r x_a x_a$: $n = 82$, $\overline{x} = 0.96$, 95% CI = [0.93, 1.00]. (9) $(d-u)+$: $n = 9$, $\overline{x} = 0.00$, 95% CI = [0.00, 0.01]. (10) $(d-u)-$: $n = 70$, $\overline{x} = 1.00$, 95% CI = [0.98, 1.01]. (11) $x_a x_r$_nob: $n = 91$, $\overline{x} = 0.85$, 95% CI = [0.82, 0.88]. (12) $x_a x_r d-$_nob: $n = 91$, $\overline{x} = 0.89$, 95% CI = [0.87, 0.90]. (13) $x_a x_r d+$_nob: $n = 91$, $\overline{x} = 0.84$, 95% CI = [0.82, 0.87]. (14) $x_a x_a x_r x_r$_nob: $n = 79$, $\overline{x} = 0.27$, 95% CI = [0.23, 0.31]. (15) $x_r x_a$_nob: $n = 91$, $\overline{x} = 0.95$, 95% CI = [0.90, 0.99]. (16) $x_r x_a d-$_nob: $n = 91$, $\overline{x} = 0.99$, 95% CI = [0.96, 1.01]. (17) $x_r x_a d+$_nob: $n = 91$, $\overline{x} = 0.95$, 95% CI = [0.91, 1.00]. (18) $x_r x_r x_a x_a$_nob: $n = 91$, $\overline{x} = 0.95$, 95% CI = [0.91, 0.99]. (19) $(d-u)+$_nob: $n = 29$, $\overline{x} = 0.00$, 95% CI = [0.00, 0.00]. (20) $(d-u)-$_nob: $n = 75$, $\overline{x} = 0.98$, 95% CI = [0.95, 1.01].

B.8 $x_a x_a x_r x_r$ CNF Reductions and the MTF Solvers

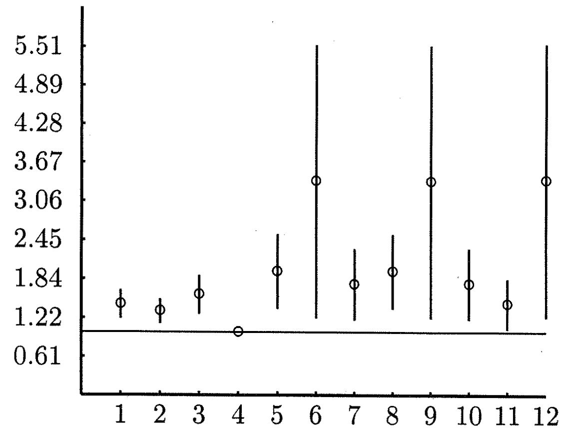


SATISFIABLE CNF instances; matroidmtf solver; user time. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 185$, $\overline{x} = 1.03$, 95% CI = [0.90, 1.16]. (2) $x_a x_r d-$: $n = 179$, $\overline{x} = 1.32$, 95% CI = [1.09, 1.55]. (3) $x_a x_r d+$: $n = 188$, $\overline{x} = 1.36$, 95% CI = [1.11, 1.61]. (4) $x_a x_a x_r x_r$: $n = 195$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (5) $x_r x_a$: $n = 186$, $\overline{x} = 1.28$, 95% CI = [0.98, 1.59]. (6) $x_r x_a d-$: $n = 184$, $\overline{x} = 1.79$, 95% CI = [1.42, 2.16]. (7) $x_r x_a d+$: $n = 177$, $\overline{x} = 1.48$, 95% CI = [1.11, 1.85]. (8) $x_r x_r x_a x_a$: $n = 192$, $\overline{x} = 1.30$, 95% CI = [1.01, 1.60]. (9) $x_r x_r x_a x_a d-$: $n = 173$, $\overline{x} = 1.75$, 95% CI = [1.35, 2.15]. (10) $x_r x_r x_a x_a d+$: $n = 182$, $\overline{x} = 1.45$, 95% CI = [1.09, 1.80]. (11) $(d-u)+$: $n = 181$, $\overline{x} = 1.12$, 95% CI = [0.97, 1.28]. (12) $(d-u)-$: $n = 175$, $\overline{x} = 1.87$, 95% CI = [1.49, 2.26].

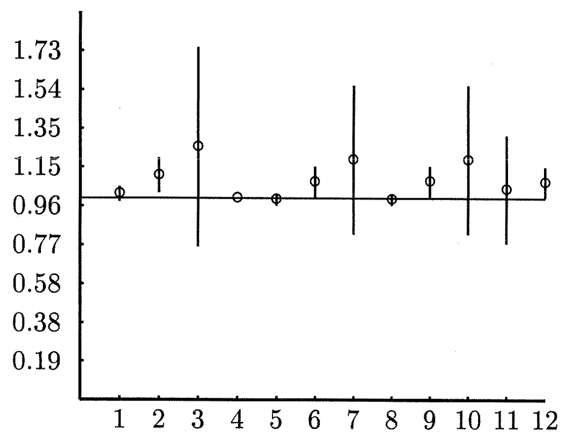


DS

UNSATISFIABLE CNF instances; matroidmtf solver; user time. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 85$, $\overline{x} = 1.03$, 95% CI = [1.00, 1.07]. (2) $x_a x_r d-$: $n = 83$, $\overline{x} = 1.20$, 95% CI = [1.11, 1.29]. (3) $x_a x_r d+$: $n = 85$, $\overline{x} = 1.39$, 95% CI = [0.92, 1.85]. (4) $x_a x_a x_r x_r$: $n = 85$, $\overline{x} = 1.00$, 95% CI = [1.00, 1.00]. (5) $x_r x_a$: $n = 85$, $\overline{x} = 1.02$, 95% CI = [0.99, 1.05]. (6) $x_r x_a d-$: $n = 83$, $\overline{x} = 1.20$, 95% CI = [1.12, 1.28]. (7) $x_r x_a d+$: $n = 84$, $\overline{x} = 1.48$, 95% CI = [0.91, 2.06]. (8) $x_r x_r x_a x_a$: $n = 82$, $\overline{x} = 1.04$, 95% CI = [1.00, 1.08]. (9) $x_r x_r x_a x_a d-$: $n = 82$, $\overline{x} = 1.21$, 95% CI = [1.13, 1.29]. (10) $x_r x_r x_a x_a d+$: $n = 84$, $\overline{x} = 1.48$, 95% CI = [0.90, 2.05]. (11) $(d-u)+$: $n = 82$, $\overline{x} = 1.27$, 95% CI = [0.92, 1.61]. (12) $(d-u)-$: $n = 83$, $\overline{x} = 1.19$, 95% CI = [1.11, 1.26].



SATISFIABLE CNF instances; matroidmtf solver; number of recursive calls. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 247$, $\overline{x} = 1.44$, 95% CI = $[1.21, 1.66]$. (2) $x_a x_r d-$: $n = 248$, $\overline{x} = 1.33$, 95% CI = $[1.14, 1.52]$. (3) $x_a x_r d+$: $n = 247$, $\overline{x} = 1.59$, 95% CI = $[1.29, 1.89]$. (4) $x_a x_a x_r x_r$: $n = 248$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (5) $x_r x_a$: $n = 244$, $\overline{x} = 1.95$, 95% CI = $[1.37, 2.54]$. (6) $x_r x_a d-$: $n = 243$, $\overline{x} = 3.38$, 95% CI = $[1.22, 5.53]$. (7) $x_r x_a d+$: $n = 242$, $\overline{x} = 1.75$, 95% CI = $[1.19, 2.31]$. (8) $x_r x_r x_a x_a$: $n = 244$, $\overline{x} = 1.95$, 95% CI = $[1.37, 2.54]$. (9) $x_r x_r x_a x_a d-$: $n = 243$, $\overline{x} = 3.37$, 95% CI = $[1.22, 5.53]$. (10) $x_r x_r x_a x_a d+$: $n = 242$, $\overline{x} = 1.75$, 95% CI = $[1.20, 2.31]$. (11) $(d-u)+$: $n = 246$, $\overline{x} = 1.44$, 95% CI = $[1.05, 1.84]$. (12) $(d-u)-$: $n = 242$, $\overline{x} = 3.40$, 95% CI = $[1.23, 5.56]$.



UNSATISFIABLE CNF instances; matroidmtf solver; number of recursive calls. Graph of the ratio of the $x_a x_a x_r x_r$ over the other solvers. Ratios are listed only by the denominator of the ratio: (1) $x_a x_r$: $n = 85$, $\overline{x} = 1.02$, 95% CI = $[0.98, 1.06]$. (2) $x_a x_r d-$: $n = 83$, $\overline{x} = 1.11$, 95% CI = $[1.03, 1.20]$. (3) $x_a x_r d+$: $n = 85$, $\overline{x} = 1.25$, 95% CI = $[0.76, 1.75]$. (4) $x_a x_a x_r x_r$: $n = 85$, $\overline{x} = 1.00$, 95% CI = $[1.00, 1.00]$. (5) $x_r x_a$: $n = 85$, $\overline{x} = 0.99$, 95% CI = $[0.96, 1.02]$. (6) $x_r x_a d-$: $n = 83$, $\overline{x} = 1.08$, 95% CI = $[1.00, 1.16]$. (7) $x_r x_a d+$: $n = 84$, $\overline{x} = 1.19$, 95% CI = $[0.82, 1.56]$. (8) $x_r x_r x_a x_a$: $n = 82$, $\overline{x} = 0.99$, 95% CI = $[0.97, 1.02]$. (9) $x_r x_r x_a x_a d-$: $n = 82$, $\overline{x} = 1.08$, 95% CI = $[1.01, 1.16]$. (10) $x_r x_r x_a x_a d+$: $n = 84$, $\overline{x} = 1.19$, 95% CI = $[0.82, 1.56]$. (11) $(d-u)+$: $n = 82$, $\overline{x} = 1.04$, 95% CI = $[0.78, 1.31]$. (12) $(d-u)-$: $n = 83$, $\overline{x} = 1.08$, 95% CI = $[1.00, 1.15]$.

C Tables

C.1 Random Instances

Table 6: Number of recursive calls required by the flow, matriod, flowmtf, matroidmtf, and matroidmtf$i_0 s_0$ solvers when operating on *c-matchable* random instances. Entries marked with "♣" and "♠" did not terminate after 90 and 600 seconds, respectively.

	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
1	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 1000$	flow = 29 matroid = 4 flowmtf = 13 matroidmtf = 3 matroidmtf$i_0 s_0$ = 5	flow = 29 matroid = 4 flowmtf = 13 matroidmtf = 3 matroidmtf$i_0 s_0$ = 5	flow = 29 matroid = 4 flowmtf = 13 matroidmtf = 3 matroidmtf$i_0 s_0$ = 5
2	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 1000$	flow = 57 matroid = 42 flowmtf = 9 matroidmtf = 8 matroidmtf$i_0 s_0$ = 7	flow = 57 matroid = 42 flowmtf = 9 matroidmtf = 8 matroidmtf$i_0 s_0$ = 7	flow = 57 matroid = 42 flowmtf = 9 matroidmtf = 8 matroidmtf$i_0 s_0$ = 7
3	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 1000$	flow = 84 matroid = 4 flowmtf = 14 matroidmtf = 8 matroidmtf$i_0 s_0$ = 8	flow = 84 matroid = 4 flowmtf = 14 matroidmtf = 8 matroidmtf$i_0 s_0$ = 8	flow = 84 matroid = 4 flowmtf = 14 matroidmtf = 8 matroidmtf$i_0 s_0$ = 8
4	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 280$	flow = 818854 ♣ matroid = 17 flowmtf = 60 matroidmtf = 7 matroidmtf$i_0 s_0$ = 3	flow = 1023061 ♣ matroid = 5 flowmtf = 86893 matroidmtf = 5 matroidmtf$i_0 s_0$ = 2	flow = 861401 ♣ matroid = 17 flowmtf = 22 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3
5	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 280$	flow = 783044 ♣ matroid = 1 flowmtf = 18 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 54 matroid = 3 flowmtf = 44 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 196 matroid = 1 flowmtf = 25 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
6	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 280$	flow = 1247 matroid = 2 flowmtf = 23714 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 1038060 ♣ matroid = 3 flowmtf = 19 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 82 matroid = 2 flowmtf = 53 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
7	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 460$	flow = 20 matroid = 3 flowmtf = 22 matroidmtf = 2 matroidmtf$i_0 s_0$ = 3	flow = 30 matroid = 19 flowmtf = 7 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 13 matroid = 4 flowmtf = 15 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4
8	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 460$	flow = 23 matroid = 8 flowmtf = 22 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 72 matroid = 12 flowmtf = 24 matroidmtf = 4 matroidmtf$i_0 s_0$ = 3	flow = 107 matroid = 3 flowmtf = 11 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3
9	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 460$	flow = 30 matroid = 4 flowmtf = 15 matroidmtf = 5 matroidmtf$i_0 s_0$ = 6	flow = 26 matroid = 80 flowmtf = 35 matroidmtf = 5 matroidmtf$i_0 s_0$ = 5	flow = 306 matroid = 3 flowmtf = 2905 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4
10	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 640$	flow = 6 matroid = 4 flowmtf = 13 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4	flow = 10 matroid = 36 flowmtf = 11 matroidmtf = 7 matroidmtf$i_0 s_0$ = 7	flow = 472 matroid = 4 flowmtf = 12 matroidmtf = 5 matroidmtf$i_0 s_0$ = 6
11	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 640$	flow = 338 matroid = 3725 flowmtf = 11 matroidmtf = 6 matroidmtf$i_0 s_0$ = 6	flow = 33 matroid = 18 flowmtf = 12 matroidmtf = 6 matroidmtf$i_0 s_0$ = 7	flow = 8 matroid = 20 flowmtf = 14 matroidmtf = 3 matroidmtf$i_0 s_0$ = 4

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
12	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 640$	flow = 19 matroid = 23 flowmtf = 21 matroidmtf = 7 matroidmtf$i_0 s_0$ = 8	flow = 21253 matroid = 34407 flowmtf = 32 matroidmtf = 7 matroidmtf$i_0 s_0$ = 9	flow = 27 matroid = 6 flowmtf = 33 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4
13	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 820$	flow = 13 matroid = 4 flowmtf = 12 matroidmtf = 6 matroidmtf$i_0 s_0$ = 7	flow = 229 matroid = 4 flowmtf = 18 matroidmtf = 4 matroidmtf$i_0 s_0$ = 6	flow = 15 matroid = 34 flowmtf = 16 matroidmtf = 7 matroidmtf$i_0 s_0$ = 8
14	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 820$	flow = 60097 matroid = 4 flowmtf = 60097 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4	flow = 60202 matroid = 3 flowmtf = 13 matroidmtf = 4 matroidmtf$i_0 s_0$ = 2	flow = 9 matroid = 21 flowmtf = 12 matroidmtf = 6 matroidmtf$i_0 s_0$ = 4
15	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 820$	flow = 28 matroid = 12 flowmtf = 10 matroidmtf = 5 matroidmtf$i_0 s_0$ = 5	flow = 40 matroid = 6 flowmtf = 14 matroidmtf = 6 matroidmtf$i_0 s_0$ = 7	flow = 29 matroid = 13 flowmtf = 9 matroidmtf = 3 matroidmtf$i_0 s_0$ = 2
16	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
17	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	flow = 3 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
18	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	flow = 21 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 4 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 3	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
19	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	flow = 3 matroid = 3 flowmtf = 3 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 4 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
20	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 4 flowmtf = 4 matroidmtf = 2 matroidmtf$i_0 s_0$ = 3	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
21	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	flow = 3 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 4 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
22	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3
23	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
24	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	flow = 1 matroid = 4 flowmtf = 1 matroidmtf = 4 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 4 flowmtf = 1 matroidmtf = 4 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 4 flowmtf = 1 matroidmtf = 4 matroidmtf$i_0 s_0$ = 2

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
25	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 56$	flow = 3 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1331 matroid = 1 flowmtf = 51 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
26	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
27	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
28	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
29	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 6 matroid = 3 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 3
30	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	flow = 4 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 10 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
31	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	flow = 9 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 29 matroid = 2 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 2	flow = 2 matroid = 2 flowmtf = 15 matroidmtf = 2 matroidmtfi_0s_0 = 2
32	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 8 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 16 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2
33	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	flow = 4 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2
34	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 34 matroid = 8 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
35	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	flow = 4 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
36	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 3 flowmtf = 2 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 3 matroid = 4 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3
37	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 4	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
38	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
39	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	flow = 6 matroid = 2 flowmtf = 3 matroidmtf = 6 matroidmtfi_0s_0 = 5	flow = 6 matroid = 2 flowmtf = 3 matroidmtf = 6 matroidmtfi_0s_0 = 5	flow = 6 matroid = 2 flowmtf = 3 matroidmtf = 6 matroidmtfi_0s_0 = 5
40	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2
41	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	flow = 88 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 13 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtfi_0s_0 = 1
42	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	flow = 9 matroid = 12642 flowmtf = 9 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 3516 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2
43	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	flow = 8 matroid = 583 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 14 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
44	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	flow = 10 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
45	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	flow = 32 matroid = 1 flowmtf = 21 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2328 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
46	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	flow = 67 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 5 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 5	flow = 370 matroid = 894 flowmtf = 4 matroidmtf = 5 matroidmtfi_0s_0 = 5
47	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	flow = 29 matroid = 62 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 28 matroid = 2 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 4	flow = 44 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2
48	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	flow = 11 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 1 flowmtf = 14 matroidmtf = 1 matroidmtfi_0s_0 = 1
49	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	flow = 23 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 1 flowmtf = 15 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 24 matroid = 20 flowmtf = 9 matroidmtf = 2 matroidmtfi_0s_0 = 2
50	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	flow = 15 matroid = 1 flowmtf = 15 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7 matroid = 3 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 3	flow = 97 matroid = 12 flowmtf = 10 matroidmtf = 3 matroidmtfi_0s_0 = 2

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
51	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	flow = 9 matroid = 3 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 5 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 25 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1
52	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	flow = 135 matroid = 18 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 3 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 94 matroid = 14 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 2
53	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	flow = 5 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 4 flowmtf = 5 matroidmtf = 4 matroidmtfi_0s_0 = 4
54	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	flow = 2 matroid = 46 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 2	flow = 58 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2
55	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 16 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 6	flow = 7 matroid = 2 flowmtf = 8 matroidmtf = 4 matroidmtfi_0s_0 = 5
56	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	flow = 5 matroid = 3 flowmtf = 5 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 5 matroid = 3 flowmtf = 5 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 5 matroid = 3 flowmtf = 5 matroidmtf = 3 matroidmtfi_0s_0 = 3
57	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	flow = 3 matroid = 14 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 2	flow = 3 matroid = 14 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 2	flow = 3 matroid = 14 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 2
58	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	flow = 3 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 3	flow = 3 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 3	flow = 3 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 3
59	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	flow = 8 matroid = 1 flowmtf = 12 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 16 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 189943 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2
60	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	flow = 1336 matroid = 1 flowmtf = 11 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 23 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 256 matroid = 2 flowmtf = 15 matroidmtf = 2 matroidmtfi_0s_0 = 2
61	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 15 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4725 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
62	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	flow = 9 matroid = 2 flowmtf = 11 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 7 matroid = 3 flowmtf = 11 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 7 matroid = 2 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 3
63	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	flow = 28 matroid = 2 flowmtf = 17 matroidmtf = 2 matroidmtfi_0s_0 = 5	flow = 7 matroid = 2 flowmtf = 13 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 5 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
64	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	flow = 6 matroid = 3 flowmtf = 7 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 3 matroid = 2 flowmtf = 7 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 24 matroid = 4 flowmtf = 3 matroidmtf = 2 matroidmtf$i_0 s_0$ = 4
65	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	flow = 2 matroid = 5 flowmtf = 2 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 421 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 4 flowmtf = 2 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4
66	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	flow = 7 matroid = 4 flowmtf = 4 matroidmtf = 5 matroidmtf$i_0 s_0$ = 3	flow = 24 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtf$i_0 s_0$ = 4	flow = 6 matroid = 3 flowmtf = 6 matroidmtf = 6 matroidmtf$i_0 s_0$ = 5
67	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	flow = 480 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 8 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 11 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
68	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	flow = 5 matroid = 3 flowmtf = 4 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4	flow = 4 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 6 matroid = 2 flowmtf = 7 matroidmtf = 4 matroidmtf$i_0 s_0$ = 2
69	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	flow = 9 matroid = 5 flowmtf = 4 matroidmtf = 4 matroidmtf$i_0 s_0$ = 2	flow = 10 matroid = 12 flowmtf = 2 matroidmtf = 3 matroidmtf$i_0 s_0$ = 5	flow = 4 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
70	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	flow = 7 matroid = 9 flowmtf = 8 matroidmtf = 7 matroidmtf$i_0 s_0$ = 6	flow = 5 matroid = 2 flowmtf = 6 matroidmtf = 6 matroidmtf$i_0 s_0$ = 6	flow = 12 matroid = 3 flowmtf = 5 matroidmtf = 3 matroidmtf$i_0 s_0$ = 5
71	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 500$	flow = 9 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 9 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 9 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
72	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 500$	flow = 1 matroid = 6 flowmtf = 1 matroidmtf = 6 matroidmtf$i_0 s_0$ = 6	flow = 1 matroid = 6 flowmtf = 1 matroidmtf = 6 matroidmtf$i_0 s_0$ = 6	flow = 1 matroid = 6 flowmtf = 1 matroidmtf = 6 matroidmtf$i_0 s_0$ = 6
73	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 500$	flow = 8 matroid = 13 flowmtf = 9 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4	flow = 8 matroid = 13 flowmtf = 9 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4	flow = 8 matroid = 13 flowmtf = 9 matroidmtf = 4 matroidmtf$i_0 s_0$ = 4
74	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	flow = 17 matroid = 1 flowmtf = 15 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 4 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1584021 ♣ matroid = 1 flowmtf = 1934 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
75	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	flow = 14908 matroid = 1 flowmtf = 24 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1728703 ♣ matroid = 1 flowmtf = 12 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 255 matroid = 1 flowmtf = 743 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
76	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	flow = 19 matroid = 1 flowmtf = 19 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 44798 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1167029 ♣ matroid = 1 flowmtf = 159 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
77	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 276$	flow = 160 matroid = 1 flowmtf = 14 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 360792 matroid = 41 flowmtf = 12 matroidmtf = 2 matroidmtfi_0s_0 = 2
78	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 276$	flow = 6465 matroid = 239 flowmtf = 35 matroidmtf = 5 matroidmtfi_0s_0 = 5	flow = 16 matroid = 2 flowmtf = 8 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 681 matroid = 5279 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 3
79	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 276$	flow = 39 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 15 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 39 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
80	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 384$	flow = 795 matroid = 2 flowmtf = 11 matroidmtf = 2 matroidmtfi_0s_0 = 3	flow = 39 matroid = 13 flowmtf = 10 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 5 matroid = 2 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 3
81	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 384$	flow = 29 matroid = 36 flowmtf = 11 matroidmtf = 6 matroidmtfi_0s_0 = 3	flow = 8 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 43 matroid = 5 flowmtf = 7 matroidmtf = 5 matroidmtfi_0s_0 = 8
82	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 384$	flow = 6 matroid = 3 flowmtf = 5 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 16 matroid = 3 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 10 matroid = 3 flowmtf = 8 matroidmtf = 2 matroidmtfi_0s_0 = 4
83	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 492$	flow = 4 matroid = 3 flowmtf = 5 matroidmtf = 3 matroidmtfi_0s_0 = 6	flow = 5 matroid = 3 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 9 matroid = 5 flowmtf = 9 matroidmtf = 5 matroidmtfi_0s_0 = 5
84	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 492$	flow = 25 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 16 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 3	flow = 5 matroid = 10 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 3
85	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 492$	flow = 2 matroid = 4 flowmtf = 2 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 14 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 4	flow = 10 matroid = 838 flowmtf = 2 matroidmtf = 4 matroidmtfi_0s_0 = 4
86	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 600$	flow = 3 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
87	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 600$	flow = 4 matroid = 4 flowmtf = 4 matroidmtf = 7 matroidmtfi_0s_0 = 7	flow = 4 matroid = 4 flowmtf = 4 matroidmtf = 7 matroidmtfi_0s_0 = 7	flow = 4 matroid = 4 flowmtf = 4 matroidmtf = 7 matroidmtfi_0s_0 = 7
88	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 600$	flow = 38 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 38 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 38 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1
89	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	flow = 24 matroid = 1 flowmtf = 434 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 21 matroid = 2 flowmtf = 13 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 5 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
90	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	flow = 5 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 29 matroid = 1 flowmtf = 34 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 18 matroid = 1 flowmtf = 19 matroidmtf = 1 matroidmtfi_0s_0 = 1
91	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	flow = 1219 matroid = 15 flowmtf = 29 matroidmtf = 19 matroidmtfi_0s_0 = 21	flow = 1599472 ♣ matroid = 5085493 ♠ flowmtf = 5231 matroidmtf = 84 matroidmtfi_0s_0 = 29	flow = 485 matroid = 429 flowmtf = 34 matroidmtf = 2062 matroidmtfi_0s_0 = 21
92	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	flow = 6 matroid = 2 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 9 matroid = 2 flowmtf = 8 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 29388 matroid = 4802 flowmtf = 4 matroidmtf = 5 matroidmtfi_0s_0 = 6
93	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	flow = 41 matroid = 2 flowmtf = 10 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 92 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1
94	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	flow = 14 matroid = 2 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 145 matroid = 1 flowmtf = 17 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 230 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1
95	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	flow = 64 matroid = 2 flowmtf = 10 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 10 matroid = 4 flowmtf = 8 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 8 matroid = 152 flowmtf = 8 matroidmtf = 4 matroidmtfi_0s_0 = 4
96	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	flow = 10 matroid = 4 flowmtf = 7 matroidmtf = 5 matroidmtfi_0s_0 = 5	flow = 5 matroid = 5 flowmtf = 6 matroidmtf = 5 matroidmtfi_0s_0 = 5	flow = 4 matroid = 3 flowmtf = 6 matroidmtf = 4 matroidmtfi_0s_0 = 4
97	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	flow = 5 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 5 matroid = 2 flowmtf = 7 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 2 matroid = 49 flowmtf = 2 matroidmtf = 4 matroidmtfi_0s_0 = 4
98	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	flow = 17 matroid = 4 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 274 matroid = 49 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 2	flow = 49 matroid = 3 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 4
99	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	flow = 21 matroid = 2 flowmtf = 8 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 22 matroid = 8 flowmtf = 6 matroidmtf = 8 matroidmtfi_0s_0 = 8	flow = 19 matroid = 3 flowmtf = 6 matroidmtf = 4 matroidmtfi_0s_0 = 6
100	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	flow = 15 matroid = 4 flowmtf = 9 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 7 matroid = 4 flowmtf = 9 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 39 matroid = 15 flowmtf = 11 matroidmtf = 4 matroidmtfi_0s_0 = 7
101	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	flow = 14 matroid = 2 flowmtf = 11 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 14 matroid = 2 flowmtf = 11 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 14 matroid = 2 flowmtf = 11 matroidmtf = 2 matroidmtfi_0s_0 = 2
102	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	flow = 17 matroid = 20 flowmtf = 7 matroidmtf = 5 matroidmtfi_0s_0 = 8	flow = 17 matroid = 20 flowmtf = 7 matroidmtf = 5 matroidmtfi_0s_0 = 8	flow = 17 matroid = 20 flowmtf = 7 matroidmtf = 5 matroidmtfi_0s_0 = 8

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
103	$\lvert I\rvert = 10$ $\lvert S\rvert = 70$ $\lvert E\rvert = 700$	flow = 23 matroid = 5 flowmtf = 9 matroidmtf = 5 matroidmtfi_0s_0 = 5	flow = 23 matroid = 5 flowmtf = 9 matroidmtf = 5 matroidmtfi_0s_0 = 5	flow = 23 matroid = 5 flowmtf = 9 matroidmtf = 5 matroidmtfi_0s_0 = 5
104	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 224$	flow = 1266784 ♣ matroid = 18 flowmtf = 344995 matroidmtf = 6 matroidmtfi_0s_0 = 6	flow = 788804 ♣ matroid = 26 flowmtf = 39576 matroidmtf = 26 matroidmtfi_0s_0 = 26	flow = 1154 matroid = 273484 flowmtf = 32022 matroidmtf = 3 matroidmtfi_0s_0 = 3
105	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 368$	flow = 7 matroid = 10577 flowmtf = 8 matroidmtf = 19 matroidmtfi_0s_0 = 6	flow = 19 matroid = 2 flowmtf = 8 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 852069 ♣ matroid = 14 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2
106	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 368$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 24 matroid = 32 flowmtf = 5 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 37 matroid = 72 flowmtf = 2 matroidmtf = 4 matroidmtfi_0s_0 = 6
107	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 368$	flow = 655150 ♣ matroid = 1 flowmtf = 28 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 914898 ♣ matroid = 3444807 ♠ flowmtf = 26 matroidmtf = 21 matroidmtfi_0s_0 = 24	flow = 44 matroid = 27 flowmtf = 32 matroidmtf = 28 matroidmtfi_0s_0 = 28
108	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 512$	flow = 115 matroid = 3 flowmtf = 27 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 8 matroid = 12 flowmtf = 11 matroidmtf = 6 matroidmtfi_0s_0 = 5	flow = 12 matroid = 4 flowmtf = 17 matroidmtf = 3 matroidmtfi_0s_0 = 3
109	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 512$	flow = 700910 ♣ matroid = 2 flowmtf = 505064 ♣ matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 21 matroid = 33 flowmtf = 14 matroidmtf = 5 matroidmtfi_0s_0 = 9	flow = 6 matroid = 8 flowmtf = 9 matroidmtf = 3 matroidmtfi_0s_0 = 4
110	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 512$	flow = 183 matroid = 7 flowmtf = 9 matroidmtf = 11 matroidmtfi_0s_0 = 11	flow = 5 matroid = 6 flowmtf = 7 matroidmtf = 6 matroidmtfi_0s_0 = 6	flow = 26 matroid = 25 flowmtf = 12 matroidmtf = 5 matroidmtfi_0s_0 = 9
111	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 656$	flow = 30 matroid = 15 flowmtf = 8 matroidmtf = 8 matroidmtfi_0s_0 = 8	flow = 6 matroid = 3 flowmtf = 11 matroidmtf = 2 matroidmtfi_0s_0 = 3	flow = 26874 matroid = 7 flowmtf = 41 matroidmtf = 4 matroidmtfi_0s_0 = 2
112	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 656$	flow = 57 matroid = 2 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 10 matroid = 4 flowmtf = 8 matroidmtf = 5 matroidmtfi_0s_0 = 5	flow = 8 matroid = 5 flowmtf = 9 matroidmtf = 7 matroidmtfi_0s_0 = 7
113	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 656$	flow = 7 matroid = 34 flowmtf = 8 matroidmtf = 7 matroidmtfi_0s_0 = 9	flow = 28 matroid = 7 flowmtf = 14 matroidmtf = 10 matroidmtfi_0s_0 = 7	flow = 11 matroid = 34 flowmtf = 15 matroidmtf = 8 matroidmtfi_0s_0 = 8
114	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 800$	flow = 10 matroid = 6 flowmtf = 10 matroidmtf = 7 matroidmtfi_0s_0 = 7	flow = 10 matroid = 6 flowmtf = 10 matroidmtf = 7 matroidmtfi_0s_0 = 7	flow = 10 matroid = 6 flowmtf = 10 matroidmtf = 7 matroidmtfi_0s_0 = 7
115	$\lvert I\rvert = 10$ $\lvert S\rvert = 80$ $\lvert E\rvert = 800$	flow = 12 matroid = 12 flowmtf = 11 matroidmtf = 9 matroidmtfi_0s_0 = 6	flow = 12 matroid = 12 flowmtf = 11 matroidmtf = 9 matroidmtfi_0s_0 = 6	flow = 12 matroid = 12 flowmtf = 11 matroidmtf = 9 matroidmtfi_0s_0 = 6

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
116	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 800$	flow = 76 matroid = 4 flowmtf = 11 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 76 matroid = 4 flowmtf = 11 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 76 matroid = 4 flowmtf = 11 matroidmtf = 3 matroidmtfi_0s_0 = 3
117	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	flow = 36794 matroid = 2 flowmtf = 3671 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 26271 matroid = 36 flowmtf = 104 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 132 matroid = 21 flowmtf = 94 matroidmtf = 5 matroidmtfi_0s_0 = 3
118	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	flow = 912849 ♣ matroid = 2931056 flowmtf = 43841 matroidmtf = 25 matroidmtfi_0s_0 = 25	flow = 142716 matroid = 35 flowmtf = 76 matroidmtf = 17 matroidmtfi_0s_0 = 14	flow = 709407 ♣ matroid = 2369605 ♠ flowmtf = 553166 ♣ matroidmtf = 93 matroidmtfi_0s_0 = 93
119	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	flow = 24246 matroid = 67619 flowmtf = 81 matroidmtf = 28 matroidmtfi_0s_0 = 30	flow = 8 matroid = 3 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 1338689 ♣ matroid = 10265600 ♠ flowmtf = 106396 matroidmtf = 40 matroidmtfi_0s_0 = 23
120	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	flow = 819309 ♣ matroid = 7 flowmtf = 52 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 13726 matroid = 10841 flowmtf = 428346 matroidmtf = 3 matroidmtfi_0s_0 = 7	flow = 81 matroid = 15 flowmtf = 89 matroidmtf = 4 matroidmtfi_0s_0 = 3
121	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	flow = 9 matroid = 3 flowmtf = 1583 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 508 matroid = 3 flowmtf = 921 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 24 matroid = 2 flowmtf = 23 matroidmtf = 2 matroidmtfi_0s_0 = 2
122	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	flow = 32 matroid = 1 flowmtf = 35 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 144 matroid = 4 flowmtf = 9 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 486074 ♣ matroid = 2 flowmtf = 95 matroidmtf = 2 matroidmtfi_0s_0 = 2
123	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	flow = 4 matroid = 3 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 4	flow = 12 matroid = 1 flowmtf = 25 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 8 matroid = 6 flowmtf = 7 matroidmtf = 5 matroidmtfi_0s_0 = 4
124	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	flow = 28 matroid = 4 flowmtf = 13 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 8 matroid = 2 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 59 matroid = 3 flowmtf = 11 matroidmtf = 6 matroidmtfi_0s_0 = 5
125	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	flow = 32 matroid = 3 flowmtf = 11 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 92 matroid = 3 flowmtf = 12 matroidmtf = 5 matroidmtfi_0s_0 = 5	flow = 34 matroid = 12 flowmtf = 17 matroidmtf = 7 matroidmtfi_0s_0 = 8
126	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	flow = 111 matroid = 5 flowmtf = 9 matroidmtf = 5 matroidmtfi_0s_0 = 7	flow = 6 matroid = 4 flowmtf = 7 matroidmtf = 8 matroidmtfi_0s_0 = 7	flow = 36 matroid = 4 flowmtf = 10 matroidmtf = 2 matroidmtfi_0s_0 = 3
127	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	flow = 19 matroid = 6 flowmtf = 14 matroidmtf = 8 matroidmtfi_0s_0 = 9	flow = 10 matroid = 3 flowmtf = 9 matroidmtf = 3 matroidmtfi_0s_0 = 5	flow = 2484 matroid = 27 flowmtf = 7 matroidmtf = 4 matroidmtfi_0s_0 = 4
128	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	flow = 285 matroid = 7 flowmtf = 14 matroidmtf = 7 matroidmtfi_0s_0 = 7	flow = 138 matroid = 21 flowmtf = 11 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 751 matroid = 34 flowmtf = 10 matroidmtf = 9 matroidmtfi_0s_0 = 9

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
129	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	flow = 9 matroid = 3 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 9 matroid = 3 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 9 matroid = 3 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 4
130	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	flow = 26 matroid = 39 flowmtf = 11 matroidmtf = 10 matroidmtfi_0s_0 = 8	flow = 26 matroid = 39 flowmtf = 11 matroidmtf = 10 matroidmtfi_0s_0 = 8	flow = 26 matroid = 39 flowmtf = 11 matroidmtf = 10 matroidmtfi_0s_0 = 8
131	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	flow = 95 matroid = 96 flowmtf = 11 matroidmtf = 10 matroidmtfi_0s_0 = 10	flow = 95 matroid = 96 flowmtf = 11 matroidmtf = 10 matroidmtfi_0s_0 = 10	flow = 95 matroid = 96 flowmtf = 11 matroidmtf = 10 matroidmtfi_0s_0 = 10
132	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	flow = 6 matroid = 12 flowmtf = 5 matroidmtf = 4 matroidmtfi_0s_0 = 5	flow = 7 matroid = 129 flowmtf = 3 matroidmtf = 6 matroidmtfi_0s_0 = 5	flow = 5 matroid = 3 flowmtf = 9 matroidmtf = 5 matroidmtfi_0s_0 = 4
133	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	flow = 5 matroid = 3 flowmtf = 5 matroidmtf = 4 matroidmtfi_0s_0 = 5	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 6 matroid = 3 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 3
134	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	flow = 4 matroid = 2 flowmtf = 8 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
135	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	flow = 4 matroid = 5 flowmtf = 7 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 4 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2
136	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	flow = 35 matroid = 3 flowmtf = 10 matroidmtf = 4 matroidmtfi_0s_0 = 2	flow = 15 matroid = 7 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 5	flow = 4 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 4
137	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	flow = 5 matroid = 5 flowmtf = 6 matroidmtf = 5 matroidmtfi_0s_0 = 3	flow = 7 matroid = 3 flowmtf = 9 matroidmtf = 3 matroidmtfi_0s_0 = 4	flow = 17 matroid = 6 flowmtf = 7 matroidmtf = 12 matroidmtfi_0s_0 = 12
138	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	flow = 1 matroid = 3 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 3 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 3 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2
139	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	flow = 9 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 9 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 9 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 3
140	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	flow = 5 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 5	flow = 5 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 5	flow = 5 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 5
141	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	flow = 18 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 54 matroid = 1 flowmtf = 15 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7 matroid = 3 flowmtf = 12 matroidmtf = 3 matroidmtfi_0s_0 = 3

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
142	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	flow = 1272046 ♣ matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7714 matroid = 2 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 21148 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1
143	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	flow = 67 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 953 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtfi_0s_0 = 1
144	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1
145	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 20 matroid = 20 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1
146	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	flow = 5 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 64 matroid = 3 flowmtf = 4 matroidmtf = 4 matroidmtfi_0s_0 = 5	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2
147	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 192$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
148	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 192$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
149	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 192$	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
150	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 344$	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
151	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 344$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 4
152	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 344$	flow = 3 matroid = 2 flowmtf = 2 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 5 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
153	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 496$	flow = 3 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
154	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 496$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
155	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 496$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
156	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 648$	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
157	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 648$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 3
158	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 648$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
159	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 800$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
160	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 800$	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
161	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 800$	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
162	$\lvert I\rvert = 20$ $\lvert S\rvert = 60$ $\lvert E\rvert = 1200$	flow = 3 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1
163	$\lvert I\rvert = 20$ $\lvert S\rvert = 60$ $\lvert E\rvert = 1200$	flow = 4 matroid = 3 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 3 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 3 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2
164	$\lvert I\rvert = 20$ $\lvert S\rvert = 60$ $\lvert E\rvert = 1200$	flow = 6 matroid = 3 flowmtf = 10 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 6 matroid = 3 flowmtf = 10 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 6 matroid = 3 flowmtf = 10 matroidmtf = 3 matroidmtfi_0s_0 = 3
165	$\lvert I\rvert = 20$ $\lvert S\rvert = 60$ $\lvert E\rvert = 288$	flow = 14 matroid = 332 flowmtf = 8 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 21 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 367 matroid = 2 flowmtf = 11 matroidmtf = 2 matroidmtfi_0s_0 = 2
166	$\lvert I\rvert = 20$ $\lvert S\rvert = 60$ $\lvert E\rvert = 288$	flow = 4 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 55 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 73337 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1
167	$\lvert I\rvert = 20$ $\lvert S\rvert = 60$ $\lvert E\rvert = 288$	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
168	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	flow = 3 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 6 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 4 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2
169	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2
170	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	flow = 3 matroid = 2 flowmtf = 4 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 7 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
171	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	flow = 2 matroid = 6 flowmtf = 2 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
172	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	flow = 22 matroid = 2 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 4	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 10 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2
173	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	flow = 19 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 11 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
174	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 3 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 2 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 3
175	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
176	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 9 flowmtf = 6 matroidmtf = 4 matroidmtfi_0s_0 = 3
177	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	flow = 3 matroid = 4 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 3 flowmtf = 7 matroidmtf = 4 matroidmtfi_0s_0 = 5	flow = 16 matroid = 3 flowmtf = 4 matroidmtf = 3 matroidmtfi_0s_0 = 4
178	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	flow = 14 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2
179	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 8 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 6 matroid = 2 flowmtf = 7 matroidmtf = 2 matroidmtfi_0s_0 = 2
180	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	flow = 5 matroid = 13 flowmtf = 5 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 5 matroid = 13 flowmtf = 5 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 5 matroid = 13 flowmtf = 5 matroidmtf = 4 matroidmtfi_0s_0 = 4

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
181	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	flow = 9 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 9 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 9 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
182	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
183	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 51 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
184	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	flow = 1116376 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 3	flow = 5 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7 matroid = 3 flowmtf = 9 matroidmtf = 4 matroidmtfi_0s_0 = 6
185	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	flow = 4 matroid = 7 flowmtf = 9 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 12 matroid = 3 flowmtf = 7 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 12 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
186	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	flow = 25 matroid = 2 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 4	flow = 114 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 8 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1
187	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
188	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	flow = 3 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1
189	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	flow = 6 matroid = 2 flowmtf = 7 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 3 matroid = 2 flowmtf = 4 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 6 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 5
190	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	flow = 2 matroid = 3 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 22 matroid = 7 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 19 matroid = 18 flowmtf = 10 matroidmtf = 4 matroidmtfi_0s_0 = 3
191	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	flow = 1 matroid = 6 flowmtf = 1 matroidmtf = 3 matroidmtfi_0s_0 = 2	flow = 7 matroid = 1 flowmtf = 7 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 4 flowmtf = 9 matroidmtf = 6 matroidmtfi_0s_0 = 6
192	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
193	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	flow = 4 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
194	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3
195	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
196	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 8 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2
197	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 3 matroidmtfi_0s_0 = 6	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2
198	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	flow = 7 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
199	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2
200	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
201	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	flow = 3 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1442571 ♣ matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
202	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	flow = 2 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
203	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
204	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 26 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
205	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 3 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2
206	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	flow = 2 matroid = 5 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 267 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
207	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	flow = 7 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1
208	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 6 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 2 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 2
209	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
210	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	flow = 3 matroid = 3 flowmtf = 2 matroidmtf = 4 matroidmtfi_0s_0 = 3	flow = 5 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 3 flowmtf = 8 matroidmtf = 3 matroidmtfi_0s_0 = 3
211	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 7 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
212	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	flow = 3 matroid = 2 flowmtf = 6 matroidmtf = 6 matroidmtfi_0s_0 = 4	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
213	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	flow = 4 matroid = 2 flowmtf = 6 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 5 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 2 flowmtf = 5 matroidmtf = 3 matroidmtfi_0s_0 = 3
214	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	flow = 6 matroid = 2 flowmtf = 2 matroidmtf = 3 matroidmtfi_0s_0 = 2	flow = 7 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 3 matroidmtfi_0s_0 = 3
215	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	flow = 3 matroid = 3 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 5 matroid = 2 flowmtf = 3 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 4 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 2
216	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	flow = 3 matroid = 4 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 3 matroid = 4 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 4	flow = 3 matroid = 4 flowmtf = 3 matroidmtf = 4 matroidmtfi_0s_0 = 4
217	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
218	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
219	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 5 matroid = 651 flowmtf = 9 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 10 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
220	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	flow = 97885 matroid = 2 flowmtf = 9 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 148 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
221	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	flow = 26 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
222	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
223	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
224	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 4 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
225	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
226	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	flow = 3 matroid = 3 flowmtf = 2 matroidmtf = 3 matroidmtf$i_0 s_0$ = 3	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
227	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 3 matroid = 1 flowmtf = 8 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
228	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	flow = 1 matroid = 3 flowmtf = 1 matroidmtf = 3 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
229	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 4 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
230	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
231	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
232	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	flow = 5 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 5 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 5 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
233	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
234	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
235	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1
236	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 21 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtfi_0s_0 = 1
237	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 6 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 2 flowmtf = 6 matroidmtf = 2 matroidmtfi_0s_0 = 2
238	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1
239	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
240	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 3 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
241	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	flow = 2 matroid = 1 flowmtf = 9 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 4 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtfi_0s_0 = 3
242	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	flow = 3 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 8 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
243	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	flow = 1 matroid = 2 flowmtf = 1 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1
244	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	flow = 2 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 2 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtfi_0s_0 = 2	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtfi_0s_0 = 1
245	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	flow = 3 matroid = 2 flowmtf = 3 matroidmtf = 3 matroidmtfi_0s_0 = 3	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtfi_0s_0 = 1	flow = 8 matroid = 1 flowmtf = 4 matroidmtf = 1 matroidmtfi_0s_0 = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
246	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	flow = 2 matroid = 2 flowmtf = 5 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 2 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
247	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	flow = 4 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 3 flowmtf = 3 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2	flow = 2 matroid = 2 flowmtf = 2 matroidmtf = 2 matroidmtf$i_0 s_0$ = 2
248	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	flow = 3 matroid = 1 flowmtf = 5 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
249	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 6 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
250	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 1 matroid = 1 flowmtf = 1 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1
251	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1	flow = 2 matroid = 1 flowmtf = 3 matroidmtf = 1 matroidmtf$i_0 s_0$ = 1

Table 7: Number of recursive calls required by the flow, matriod, flowmtf, and matroidmtf solvers when operating on *un-c-matchable* random instances. Entries marked with "♣" and "♠" did not terminate after 90 and 600 seconds, respectively.

	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
1	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 131$	flow = 9 matroid = 0 flowmtf = 9 matroidmtf = 0	flow = 11 matroid = 0 flowmtf = 11 matroidmtf = 0	flow = 4 matroid = 0 flowmtf = 4 matroidmtf = 0
2	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 131$	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
3	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 137$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
4	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 26$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
5	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 27$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
6	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 31$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
7	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 56$	flow = 20 matroid = 1 flowmtf = 20 matroidmtf = 1	flow = 836 matroid = 1 flowmtf = 20 matroidmtf = 1	flow = 8037 matroid = 1 flowmtf = 25 matroidmtf = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
8	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 56$	flow = 2168 matroid = 1 flowmtf = 47 matroidmtf = 1	flow = 14 matroid = 1 flowmtf = 5 matroidmtf = 1	flow = 7688 matroid = 1 flowmtf = 85 matroidmtf = 1
9	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 38$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
10	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 39$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
11	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 41$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
12	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 52$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
13	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 55$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
14	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 56$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
15	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 62$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
16	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 67$	flow = 17 matroid = 0 flowmtf = 12 matroidmtf = 0	flow = 9 matroid = 0 flowmtf = 9 matroidmtf = 0	flow = 4 matroid = 0 flowmtf = 4 matroidmtf = 0
17	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 69$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
18	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 80$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
19	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 81$	flow = 59 matroid = 0 flowmtf = 22 matroidmtf = 0	flow = 38 matroid = 0 flowmtf = 38 matroidmtf = 0	flow = 25 matroid = 0 flowmtf = 25 matroidmtf = 0
20	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 84$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
21	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 96$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
22	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 96$	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
23	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 97$	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
24	$\|I\| = 10$	flow = 0	flow = 0	flow = 0

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 80$ $\|E\| = 102$	matroid = 0 flowmtf = 0 matroidmtf = 0	matroid = 0 flowmtf = 0 matroidmtf = 0	matroid = 0 flowmtf = 0 matroidmtf = 0
25	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 106$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
26	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 109$	flow = 3 matroid = 0 flowmtf = 3 matroidmtf = 0	flow = 30 matroid = 0 flowmtf = 14 matroidmtf = 0	flow = 5 matroid = 0 flowmtf = 5 matroidmtf = 0
27	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 224$	flow = 452191 ♣ matroid = 1 flowmtf = 584535 ♣ matroidmtf = 1	flow = 580494 ♣ matroid = 1 flowmtf = 365047 matroidmtf = 1	flow = 479539 ♣ matroid = 1 flowmtf = 606320 ♣ matroidmtf = 1
28	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 121$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
29	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 123$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
30	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 126$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 1 matroid = 0 flowmtf = 1 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
31	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 134$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
32	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 135$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
33	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 138$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
34	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 55$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
35	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 57$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
36	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 57$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
37	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 79$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
38	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 81$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
39	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 84$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
40	$\|I\| = 20$ $\|S\| = 80$	flow = 0 matroid = 0	flow = 0 matroid = 0	flow = 0 matroid = 0

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|E\| = 110$	flowmtf = 0 matroidmtf = 0	flowmtf = 0 matroidmtf = 0	flowmtf = 0 matroidmtf = 0
41	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 110$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
42	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 111$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
43	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 80$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
44	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 84$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
45	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 84$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
46	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 116$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
47	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 122$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
48	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 123$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
49	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 111$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
50	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 113$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
51	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 114$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
52	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 133$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
53	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 139$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0
54	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 141$	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0	flow = 0 matroid = 0 flowmtf = 0 matroidmtf = 0

C.2 MTF-Ordered Random Instances

Table 8: Number of recursive calls required by the flow and flowmtf solvers when operating on c-matchable random instances (o) and instances modified by the flowmtf solvers (m). Entries marked with "$\clubsuit$" did not terminate after 90 seconds.

	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
1	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 1000$	o flow = 29 m flow = 97 o flowmtf = 13 m flowmtf = 13	o flow = 29 m flow = 97 o flowmtf = 13 m flowmtf = 13	o flow = 29 m flow = 97 o flowmtf = 13 m flowmtf = 13
2	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 1000$	o flow = 57 m flow = 723 o flowmtf = 9 m flowmtf = 12	o flow = 57 m flow = 723 o flowmtf = 9 m flowmtf = 12	o flow = 57 m flow = 723 o flowmtf = 9 m flowmtf = 12
3	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 1000$	o flow = 84 m flow = 31 o flowmtf = 14 m flowmtf = 13	o flow = 84 m flow = 31 o flowmtf = 14 m flowmtf = 13	o flow = 84 m flow = 31 o flowmtf = 14 m flowmtf = 13
4	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 280$	o flow = 818854 $\clubsuit$ m flow = 52859 o flowmtf = 60 m flowmtf = 38	o flow = 1023061 $\clubsuit$ m flow = 1393832 $\clubsuit$ o flowmtf = 86893 m flowmtf = 544259	o flow = 861401 $\clubsuit$ m flow = 1358 o flowmtf = 22 m flowmtf = 70
5	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 280$	o flow = 783044 $\clubsuit$ m flow = 91124 o flowmtf = 18 m flowmtf = 18	o flow = 54 m flow = 1035863 $\clubsuit$ o flowmtf = 44 m flowmtf = 2078	o flow = 196 m flow = 1847947 $\clubsuit$ o flowmtf = 25 m flowmtf = 24604
6	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 280$	o flow = 1247 m flow = 840528 $\clubsuit$ o flowmtf = 23714 m flowmtf = 382064 $\clubsuit$	o flow = 1038060 $\clubsuit$ m flow = 537982 $\clubsuit$ o flowmtf = 19 m flowmtf = 343488 $\clubsuit$	o flow = 82 m flow = 103 o flowmtf = 53 m flowmtf = 10
7	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 460$	o flow = 20 m flow = 643326 $\clubsuit$ o flowmtf = 22 m flowmtf = 421718 $\clubsuit$	o flow = 30 m flow = 126 o flowmtf = 7 m flowmtf = 22	o flow = 13 m flow = 73 o flowmtf = 15 m flowmtf = 67
8	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 460$	o flow = 23 m flow = 7 o flowmtf = 22 m flowmtf = 7	o flow = 72 m flow = 8 o flowmtf = 24 m flowmtf = 4	o flow = 107 m flow = 5 o flowmtf = 11 m flowmtf = 5
9	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 460$	o flow = 30 m flow = 2116 o flowmtf = 15 m flowmtf = 978	o flow = 26 m flow = 77 o flowmtf = 35 m flowmtf = 83	o flow = 306 m flow = 959170 $\clubsuit$ o flowmtf = 2905 m flowmtf = 51
10	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 640$	o flow = 6 m flow = 6 o flowmtf = 13 m flowmtf = 9	o flow = 10 m flow = 7 o flowmtf = 11 m flowmtf = 7	o flow = 472 m flow = 38 o flowmtf = 12 m flowmtf = 8
11	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 640$	o flow = 338 m flow = 25 o flowmtf = 11 m flowmtf = 25	o flow = 33 m flow = 20 o flowmtf = 12 m flowmtf = 11	o flow = 8 m flow = 98 o flowmtf = 14 m flowmtf = 12
12	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 640$	o flow = 19 m flow = 10 o flowmtf = 21 m flowmtf = 12	o flow = 21253 m flow = 29 o flowmtf = 32 m flowmtf = 29	o flow = 27 m flow = 30 o flowmtf = 33 m flowmtf = 34
13	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 820$	o flow = 13 m flow = 17 o flowmtf = 12 m flowmtf = 18	o flow = 229 m flow = 31 o flowmtf = 18 m flowmtf = 19	o flow = 15 m flow = 9 o flowmtf = 16 m flowmtf = 10
14	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 820$	o flow = 60097 m flow = 84 o flowmtf = 60097 m flowmtf = 17	o flow = 60202 m flow = 19 o flowmtf = 13 m flowmtf = 15	o flow = 9 m flow = 10 o flowmtf = 12 m flowmtf = 10
15	$\lvert I \rvert = 10$ $\lvert S \rvert = 100$ $\lvert E \rvert = 820$	o flow = 28 m flow = 19 o flowmtf = 10	o flow = 40 m flow = 30 o flowmtf = 14	o flow = 29 m flow = 19 o flowmtf = 9

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m flowmtf = 12	m flowmtf = 16	m flowmtf = 19
16	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2
17	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	o flow = 3 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 2 m flow = 3 o flowmtf = 3 m flowmtf = 3	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2
18	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	o flow = 21 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 4 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 4 m flow = 1 o flowmtf = 3 m flowmtf = 1
19	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1
20	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 4	o flow = 2 m flow = 5 o flowmtf = 4 m flowmtf = 4	o flow = 2 m flow = 12 o flowmtf = 2 m flowmtf = 4
21	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	o flow = 3 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 4 m flow = 2 o flowmtf = 4 m flowmtf = 6	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2
22	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	o flow = 3 m flow = 4 o flowmtf = 3 m flowmtf = 2	o flow = 3 m flow = 4 o flowmtf = 3 m flowmtf = 2	o flow = 3 m flow = 4 o flowmtf = 3 m flowmtf = 2
23	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 4	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 4	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 4
24	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1
25	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 56$	o flow = 3 m flow = 10 o flowmtf = 5 m flowmtf = 3	o flow = 1331 m flow = 4 o flowmtf = 51 m flowmtf = 9	o flow = 5 m flow = 16 o flowmtf = 3 m flowmtf = 6
26	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1
27	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 4	o flow = 2 m flow = 14 o flowmtf = 2 m flowmtf = 3
28	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	o flow = 1 m flow = 18 o flowmtf = 1 m flowmtf = 2	o flow = 3 m flow = 4 o flowmtf = 2 m flowmtf = 4	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2
29	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	o flow = 2 m flow = 5 o flowmtf = 2 m flowmtf = 5	o flow = 2 m flow = 12 o flowmtf = 2 m flowmtf = 4	o flow = 6 m flow = 1 o flowmtf = 6 m flowmtf = 1
30	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	o flow = 4 m flow = 2 o flowmtf = 5 m flowmtf = 2	o flow = 10 m flow = 3 o flowmtf = 3 m flowmtf = 3	o flow = 2 m flow = 5 o flowmtf = 2 m flowmtf = 5
31	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	o flow = 9 m flow = 35 o flowmtf = 5 m flowmtf = 8	o flow = 29 m flow = 141 o flowmtf = 4 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 15 m flowmtf = 5

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
32	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	o flow = 3 m flow = 3 o flowmtf = 3 m flowmtf = 6	o flow = 8 m flow = 4 o flowmtf = 6 m flowmtf = 5	o flow = 16 m flow = 1 o flowmtf = 5 m flowmtf = 1
33	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	o flow = 4 m flow = 2 o flowmtf = 5 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 6
34	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 34 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 1 m flow = 4 o flowmtf = 1 m flowmtf = 3
35	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	o flow = 4 m flow = 6 o flowmtf = 2 m flowmtf = 4	o flow = 1 m flow = 4 o flowmtf = 1 m flowmtf = 4	o flow = 1 m flow = 30 o flowmtf = 1 m flowmtf = 2
36	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	o flow = 3 m flow = 9 o flowmtf = 3 m flowmtf = 3	o flow = 5 m flow = 3 o flowmtf = 2 m flowmtf = 3	o flow = 3 m flow = 2 o flowmtf = 3 m flowmtf = 2
37	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 2	o flow = 1 m flow = 8 o flowmtf = 1 m flowmtf = 3
38	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2
39	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	o flow = 6 m flow = 3 o flowmtf = 3 m flowmtf = 6	o flow = 6 m flow = 3 o flowmtf = 3 m flowmtf = 6	o flow = 6 m flow = 3 o flowmtf = 3 m flowmtf = 6
40	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	o flow = 1 m flow = 7 o flowmtf = 1 m flowmtf = 8	o flow = 1 m flow = 7 o flowmtf = 1 m flowmtf = 8	o flow = 1 m flow = 7 o flowmtf = 1 m flowmtf = 8
41	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	o flow = 88 m flow = 2 o flowmtf = 8 m flowmtf = 2	o flow = 7 m flow = 5 o flowmtf = 7 m flowmtf = 2	o flow = 13 m flow = 55833 o flowmtf = 7 m flowmtf = 11
42	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	o flow = 9 m flow = 5 o flowmtf = 9 m flowmtf = 4	o flow = 4 m flow = 7 o flowmtf = 4 m flowmtf = 5	o flow = 3 m flow = 25 o flowmtf = 4 m flowmtf = 6
43	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	o flow = 8 m flow = 3 o flowmtf = 3 m flowmtf = 3	o flow = 14 m flow = 118 o flowmtf = 5 m flowmtf = 2	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 3
44	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	o flow = 10 m flow = 4 o flowmtf = 5 m flowmtf = 4	o flow = 5 m flow = 3 o flowmtf = 6 m flowmtf = 5	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1
45	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	o flow = 32 m flow = 11 o flowmtf = 21 m flowmtf = 2	o flow = 3 m flow = 6076 o flowmtf = 7 m flowmtf = 3	o flow = 2328 m flow = 7 o flowmtf = 3 m flowmtf = 3
46	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	o flow = 67 m flow = 15223 o flowmtf = 9 m flowmtf = 6	o flow = 3 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 370 m flow = 289 o flowmtf = 4 m flowmtf = 5
47	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	o flow = 29 m flow = 2010443 ♣ o flowmtf = 7 m flowmtf = 2	o flow = 28 m flow = 49 o flowmtf = 7 m flowmtf = 6	o flow = 44 m flow = 20 o flowmtf = 5 m flowmtf = 4
48	$\|I\| = 10$	o flow = 11	o flow = 4	o flow = 5

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 40$ $\|E\| = 184$	m flow = 25 o flowmtf = 4 m flowmtf = 5	m flow = 3 o flowmtf = 6 m flowmtf = 3	m flow = 6 o flowmtf = 14 m flowmtf = 7
49	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	o flow = 23 m flow = 9 o flowmtf = 6 m flowmtf = 6	o flow = 3 m flow = 4 o flowmtf = 15 m flowmtf = 5	o flow = 24 m flow = 6 o flowmtf = 9 m flowmtf = 6
50	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	o flow = 15 m flow = 89 o flowmtf = 15 m flowmtf = 5	o flow = 7 m flow = 8 o flowmtf = 7 m flowmtf = 3	o flow = 97 m flow = 16 o flowmtf = 10 m flowmtf = 7
51	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	o flow = 9 m flow = 3 o flowmtf = 7 m flowmtf = 6	o flow = 5 m flow = 15 o flowmtf = 5 m flowmtf = 5	o flow = 25 m flow = 6 o flowmtf = 8 m flowmtf = 5
52	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	o flow = 135 m flow = 3 o flowmtf = 6 m flowmtf = 5	o flow = 3 m flow = 5 o flowmtf = 3 m flowmtf = 2	o flow = 94 m flow = 2 o flowmtf = 7 m flowmtf = 4
53	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	o flow = 5 m flow = 4 o flowmtf = 4 m flowmtf = 6	o flow = 4 m flow = 24 o flowmtf = 6 m flowmtf = 3	o flow = 4 m flow = 19 o flowmtf = 5 m flowmtf = 6
54	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	o flow = 2 m flow = 9 o flowmtf = 4 m flowmtf = 3	o flow = 58 m flow = 4 o flowmtf = 6 m flowmtf = 5	o flow = 4 m flow = 5 o flowmtf = 6 m flowmtf = 6
55	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	o flow = 3 m flow = 4 o flowmtf = 3 m flowmtf = 4	o flow = 16 m flow = 2 o flowmtf = 5 m flowmtf = 2	o flow = 7 m flow = 5 o flowmtf = 8 m flowmtf = 6
56	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	o flow = 5 m flow = 9 o flowmtf = 5 m flowmtf = 7	o flow = 5 m flow = 9 o flowmtf = 5 m flowmtf = 7	o flow = 5 m flow = 9 o flowmtf = 5 m flowmtf = 7
57	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	o flow = 3 m flow = 3 o flowmtf = 4 m flowmtf = 2	o flow = 3 m flow = 3 o flowmtf = 4 m flowmtf = 2	o flow = 3 m flow = 3 o flowmtf = 4 m flowmtf = 2
58	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	o flow = 3 m flow = 18 o flowmtf = 5 m flowmtf = 8	o flow = 3 m flow = 18 o flowmtf = 5 m flowmtf = 8	o flow = 3 m flow = 18 o flowmtf = 5 m flowmtf = 8
59	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	o flow = 8 m flow = 20 o flowmtf = 12 m flowmtf = 510	o flow = 16 m flow = 223 o flowmtf = 3 m flowmtf = 9	o flow = 189943 m flow = 2224357 o flowmtf = 5 m flowmtf = 4
60	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	o flow = 1336 m flow = 4 o flowmtf = 11 m flowmtf = 3	o flow = 2 m flow = 3 o flowmtf = 7 m flowmtf = 3	o flow = 256 m flow = 793 o flowmtf = 15 m flowmtf = 9
61	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	o flow = 3 m flow = 48 o flowmtf = 3 m flowmtf = 14	o flow = 15 m flow = 15 o flowmtf = 8 m flowmtf = 18	o flow = 4725 m flow = 1581 o flowmtf = 3 m flowmtf = 13
62	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	o flow = 9 m flow = 10 o flowmtf = 11 m flowmtf = 10	o flow = 7 m flow = 7 o flowmtf = 11 m flowmtf = 6	o flow = 7 m flow = 25 o flowmtf = 7 m flowmtf = 29
63	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	o flow = 28 m flow = 385 o flowmtf = 17 m flowmtf = 8	o flow = 7 m flow = 18825 o flowmtf = 13 m flowmtf = 9	o flow = 5 m flow = 516 o flowmtf = 4 m flowmtf = 53
64	$\|I\| = 10$ $\|S\| = 50$	o flow = 6 m flow = 11	o flow = 3 m flow = 3	o flow = 24 m flow = 2

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|E\| = 230$	o flowmtf = 7 m flowmtf = 5	o flowmtf = 7 m flowmtf = 6	o flowmtf = 3 m flowmtf = 4
65	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	o flow = 2 m flow = 50 o flowmtf = 2 m flowmtf = 6	o flow = 421 m flow = 3 o flowmtf = 5 m flowmtf = 3	o flow = 2 m flow = 9 o flowmtf = 2 m flowmtf = 2
66	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	o flow = 7 m flow = 7 o flowmtf = 4 m flowmtf = 6	o flow = 24 m flow = 3 o flowmtf = 5 m flowmtf = 3	o flow = 6 m flow = 28 o flowmtf = 6 m flowmtf = 8
67	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	o flow = 480 m flow = 5 o flowmtf = 7 m flowmtf = 7	o flow = 8 m flow = 5 o flowmtf = 5 m flowmtf = 5	o flow = 11 m flow = 8 o flowmtf = 4 m flowmtf = 5
68	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	o flow = 5 m flow = 3 o flowmtf = 4 m flowmtf = 4	o flow = 4 m flow = 13 o flowmtf = 7 m flowmtf = 9	o flow = 6 m flow = 6 o flowmtf = 7 m flowmtf = 4
69	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	o flow = 9 m flow = 8 o flowmtf = 4 m flowmtf = 8	o flow = 10 m flow = 6 o flowmtf = 2 m flowmtf = 7	o flow = 4 m flow = 5 o flowmtf = 2 m flowmtf = 5
70	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	o flow = 7 m flow = 3 o flowmtf = 8 m flowmtf = 7	o flow = 5 m flow = 9 o flowmtf = 6 m flowmtf = 8	o flow = 12 m flow = 5 o flowmtf = 5 m flowmtf = 5
71	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 500$	o flow = 9 m flow = 6 o flowmtf = 7 m flowmtf = 8	o flow = 9 m flow = 6 o flowmtf = 7 m flowmtf = 8	o flow = 9 m flow = 6 o flowmtf = 7 m flowmtf = 8
72	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 500$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2
73	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 500$	o flow = 8 m flow = 9 o flowmtf = 9 m flowmtf = 6	o flow = 8 m flow = 9 o flowmtf = 9 m flowmtf = 6	o flow = 8 m flow = 9 o flowmtf = 9 m flowmtf = 6
74	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	o flow = 17 m flow = 81 o flowmtf = 15 m flowmtf = 273	o flow = 4 m flow = 73850 o flowmtf = 5 m flowmtf = 9	o flow = 1584021 ♣ m flow = 1 o flowmtf = 1934 m flowmtf = 1
75	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	o flow = 14908 m flow = 207 o flowmtf = 24 m flowmtf = 8	o flow = 1728703 ♣ m flow = 46 o flowmtf = 12 m flowmtf = 5	o flow = 255 m flow = 28 o flowmtf = 743 m flowmtf = 8
76	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 168$	o flow = 19 m flow = 1049 o flowmtf = 19 m flowmtf = 22	o flow = 44798 m flow = 14 o flowmtf = 9 m flowmtf = 44	o flow = 1167029 ♣ m flow = 93 o flowmtf = 159 m flowmtf = 6
77	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 276$	o flow = 160 m flow = 3 o flowmtf = 14 m flowmtf = 4	o flow = 5 m flow = 14 o flowmtf = 8 m flowmtf = 3	o flow = 360792 m flow = 3 o flowmtf = 12 m flowmtf = 8
78	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 276$	o flow = 6465 m flow = 18 o flowmtf = 35 m flowmtf = 31	o flow = 16 m flow = 8 o flowmtf = 8 m flowmtf = 11	o flow = 681 m flow = 3 o flowmtf = 7 m flowmtf = 3
79	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 276$	o flow = 39 m flow = 15 o flowmtf = 4 m flowmtf = 15	o flow = 15 m flow = 7 o flowmtf = 6 m flowmtf = 9	o flow = 39 m flow = 3 o flowmtf = 4 m flowmtf = 3
80	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 384$	o flow = 795 m flow = 7 o flowmtf = 11	o flow = 39 m flow = 61 o flowmtf = 10	o flow = 5 m flow = 40 o flowmtf = 7

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m flowmtf = 10	m flowmtf = 5	m flowmtf = 12
81	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 384$	o flow = 29 m flow = 211 o flowmtf = 11 m flowmtf = 5	o flow = 8 m flow = 141 o flowmtf = 7 m flowmtf = 15	o flow = 43 m flow = 24 o flowmtf = 7 m flowmtf = 18
82	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 384$	o flow = 6 m flow = 9 o flowmtf = 5 m flowmtf = 9	o flow = 16 m flow = 6 o flowmtf = 6 m flowmtf = 8	o flow = 10 m flow = 14 o flowmtf = 8 m flowmtf = 9
83	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 492$	o flow = 4 m flow = 13 o flowmtf = 5 m flowmtf = 6	o flow = 5 m flow = 35 o flowmtf = 5 m flowmtf = 10	o flow = 9 m flow = 86 o flowmtf = 9 m flowmtf = 6
84	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 492$	o flow = 25 m flow = 4 o flowmtf = 3 m flowmtf = 4	o flow = 3 m flow = 4 o flowmtf = 5 m flowmtf = 9	o flow = 5 m flow = 5 o flowmtf = 4 m flowmtf = 8
85	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 492$	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 5	o flow = 14 m flow = 5 o flowmtf = 6 m flowmtf = 5	o flow = 10 m flow = 2 o flowmtf = 2 m flowmtf = 2
86	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 600$	o flow = 3 m flow = 5 o flowmtf = 6 m flowmtf = 6	o flow = 3 m flow = 5 o flowmtf = 6 m flowmtf = 6	o flow = 3 m flow = 5 o flowmtf = 6 m flowmtf = 6
87	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 600$	o flow = 4 m flow = 20 o flowmtf = 4 m flowmtf = 7	o flow = 4 m flow = 20 o flowmtf = 4 m flowmtf = 7	o flow = 4 m flow = 20 o flowmtf = 4 m flowmtf = 7
88	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 600$	o flow = 38 m flow = 8 o flowmtf = 9 m flowmtf = 6	o flow = 38 m flow = 8 o flowmtf = 9 m flowmtf = 6	o flow = 38 m flow = 8 o flowmtf = 9 m flowmtf = 6
89	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	o flow = 24 m flow = 1085 o flowmtf = 434 m flowmtf = 38	o flow = 21 m flow = 11 o flowmtf = 13 m flowmtf = 12	o flow = 5 m flow = 2712442 ♣ o flowmtf = 6 m flowmtf = 5205
90	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	o flow = 5 m flow = 438 o flowmtf = 4 m flowmtf = 69	o flow = 29 m flow = 171 o flowmtf = 34 m flowmtf = 14	o flow = 18 m flow = 2582622 o flowmtf = 19 m flowmtf = 331951
91	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	o flow = 1219 m flow = 119 o flowmtf = 29 m flowmtf = 24	o flow = 1599472 ♣ m flow = 9 o flowmtf = 5231 m flowmtf = 10	o flow = 485 m flow = 67 o flowmtf = 34 m flowmtf = 12
92	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	o flow = 6 m flow = 1028 o flowmtf = 7 m flowmtf = 11	o flow = 9 m flow = 35 o flowmtf = 8 m flowmtf = 9	o flow = 29388 m flow = 74 o flowmtf = 4 m flowmtf = 8
93	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	o flow = 41 m flow = 4351 o flowmtf = 10 m flowmtf = 24	o flow = 3 m flow = 1298107 ♣ o flowmtf = 4 m flowmtf = 9	o flow = 92 m flow = 76 o flowmtf = 5 m flowmtf = 8
94	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	o flow = 14 m flow = 100 o flowmtf = 6 m flowmtf = 4	o flow = 145 m flow = 395557 o flowmtf = 17 m flowmtf = 50	o flow = 230 m flow = 16 o flowmtf = 8 m flowmtf = 26
95	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	o flow = 64 m flow = 12 o flowmtf = 10 m flowmtf = 7	o flow = 10 m flow = 9 o flowmtf = 8 m flowmtf = 13	o flow = 8 m flow = 16 o flowmtf = 8 m flowmtf = 6
96	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	o flow = 10 m flow = 3 o flowmtf = 7 m flowmtf = 7	o flow = 5 m flow = 14 o flowmtf = 6 m flowmtf = 8	o flow = 4 m flow = 33 o flowmtf = 6 m flowmtf = 11

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
97	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	o flow = 5 m flow = 14616 o flowmtf = 5 m flowmtf = 10	o flow = 5 m flow = 7 o flowmtf = 7 m flowmtf = 9	o flow = 2 m flow = 54 o flowmtf = 2 m flowmtf = 5
98	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	o flow = 17 m flow = 6 o flowmtf = 7 m flowmtf = 8	o flow = 274 m flow = 4 o flowmtf = 7 m flowmtf = 5	o flow = 49 m flow = 6 o flowmtf = 7 m flowmtf = 6
99	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	o flow = 21 m flow = 10 o flowmtf = 8 m flowmtf = 9	o flow = 22 m flow = 9 o flowmtf = 6 m flowmtf = 7	o flow = 19 m flow = 8 o flowmtf = 6 m flowmtf = 10
100	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	o flow = 15 m flow = 8 o flowmtf = 9 m flowmtf = 8	o flow = 7 m flow = 16 o flowmtf = 9 m flowmtf = 8	o flow = 39 m flow = 23 o flowmtf = 11 m flowmtf = 9
101	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	o flow = 14 m flow = 3 o flowmtf = 11 m flowmtf = 7	o flow = 14 m flow = 3 o flowmtf = 11 m flowmtf = 7	o flow = 14 m flow = 3 o flowmtf = 11 m flowmtf = 7
102	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	o flow = 17 m flow = 10 o flowmtf = 7 m flowmtf = 6	o flow = 17 m flow = 10 o flowmtf = 7 m flowmtf = 6	o flow = 17 m flow = 10 o flowmtf = 7 m flowmtf = 6
103	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	o flow = 23 m flow = 7 o flowmtf = 9 m flowmtf = 6	o flow = 23 m flow = 7 o flowmtf = 9 m flowmtf = 6	o flow = 23 m flow = 7 o flowmtf = 9 m flowmtf = 6
104	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 224$	o flow = 1266784 ♣ m flow = 11 o flowmtf = 344995 m flowmtf = 12	o flow = 788804 ♣ m flow = 89 o flowmtf = 39576 m flowmtf = 20	o flow = 1154 m flow = 54 o flowmtf = 32022 m flowmtf = 47
105	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 368$	o flow = 7 m flow = 328 o flowmtf = 8 m flowmtf = 72	o flow = 19 m flow = 5 o flowmtf = 8 m flowmtf = 5	o flow = 852069 ♣ m flow = 2124351 ♣ o flowmtf = 6 m flowmtf = 1522861 ♣
106	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 368$	o flow = 3 m flow = 12 o flowmtf = 3 m flowmtf = 12	o flow = 24 m flow = 79 o flowmtf = 5 m flowmtf = 15	o flow = 37 m flow = 18 o flowmtf = 2 m flowmtf = 11
107	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 368$	o flow = 655150 ♣ m flow = 15 o flowmtf = 28 m flowmtf = 8	o flow = 914898 ♣ m flow = 1199294 ♣ o flowmtf = 26 m flowmtf = 5	o flow = 44 m flow = 176 o flowmtf = 32 m flowmtf = 15
108	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 512$	o flow = 115 m flow = 1355 o flowmtf = 27 m flowmtf = 52	o flow = 8 m flow = 48 o flowmtf = 11 m flowmtf = 14	o flow = 12 m flow = 5 o flowmtf = 17 m flowmtf = 5
109	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 512$	o flow = 700910 ♣ m NO DATA o flowmtf = 505064 ♣ m NO DATA	o flow = 21 m flow = 19634 o flowmtf = 14 m flowmtf = 30	o flow = 6 m flow = 18 o flowmtf = 9 m flowmtf = 9
110	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 512$	o flow = 183 m flow = 19 o flowmtf = 9 m flowmtf = 20	o flow = 5 m flow = 23 o flowmtf = 7 m flowmtf = 8	o flow = 26 m flow = 12 o flowmtf = 12 m flowmtf = 12
111	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 656$	o flow = 30 m flow = 283 o flowmtf = 8 m flowmtf = 10	o flow = 6 m flow = 38 o flowmtf = 11 m flowmtf = 7	o flow = 26874 m flow = 42 o flowmtf = 41 m flowmtf = 16
112	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 656$	o flow = 57 m flow = 6 o flowmtf = 7 m flowmtf = 6	o flow = 10 m flow = 30 o flowmtf = 8 m flowmtf = 10	o flow = 8 m flow = 19 o flowmtf = 9 m flowmtf = 10
113	$\|I\| = 10$	o flow = 7	o flow = 28	o flow = 11

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 80$ $\|E\| = 656$	m flow = 10 o flowmtf = 8 m flowmtf = 12	m flow = 58 o flowmtf = 14 m flowmtf = 13	m flow = 27 o flowmtf = 15 m flowmtf = 13
114	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 800$	o flow = 10 m flow = 7 o flowmtf = 10 m flowmtf = 7	o flow = 10 m flow = 7 o flowmtf = 10 m flowmtf = 7	o flow = 10 m flow = 7 o flowmtf = 10 m flowmtf = 7
115	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 800$	o flow = 12 m flow = 15 o flowmtf = 11 m flowmtf = 6	o flow = 12 m flow = 15 o flowmtf = 11 m flowmtf = 6	o flow = 12 m flow = 15 o flowmtf = 11 m flowmtf = 6
116	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 800$	o flow = 76 m flow = 12 o flowmtf = 11 m flowmtf = 9	o flow = 76 m flow = 12 o flowmtf = 11 m flowmtf = 9	o flow = 76 m flow = 12 o flowmtf = 11 m flowmtf = 9
117	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	o flow = 36794 m flow = 1076256 ♣ o flowmtf = 3671 m flowmtf = 570369 ♣	o flow = 26271 m flow = 35 o flowmtf = 104 m flowmtf = 56	o flow = 132 m flow = 91 o flowmtf = 94 m flowmtf = 27860
118	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	o flow = 912849 ♣ m flow = 840307 ♣ o flowmtf = 43841 m flowmtf = 44	o flow = 142716 m flow = 2350995 ♣ o flowmtf = 76 m flowmtf = 456326 ♣	o flow = 709407 ♣ m NO DATA o flowmtf = 553166 ♣ m NO DATA
119	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	o flow = 24246 m flow = 77381 o flowmtf = 81 m flowmtf = 10273	o flow = 8 m flow = 1570273 ♣ o flowmtf = 7 m flowmtf = 195994	o flow = 1338689 ♣ m flow = 23244 o flowmtf = 106396 m flowmtf = 11600
120	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	o flow = 819309 ♣ m flow = 18 o flowmtf = 52 m flowmtf = 34	o flow = 13726 m flow = 15 o flowmtf = 428346 m flowmtf = 6	o flow = 81 m flow = 3 o flowmtf = 89 m flowmtf = 4
121	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	o flow = 9 m flow = 87 o flowmtf = 1583 m flowmtf = 66	o flow = 508 m flow = 1422894 ♣ o flowmtf = 921 m flowmtf = 16	o flow = 24 m flow = 17 o flowmtf = 23 m flowmtf = 9
122	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	o flow = 32 m flow = 560090 o flowmtf = 35 m flowmtf = 978372 ♣	o flow = 144 m flow = 39 o flowmtf = 9 m flowmtf = 380382 ♣	o flow = 486074 ♣ m flow = 30 o flowmtf = 95 m flowmtf = 8
123	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	o flow = 4 m flow = 33 o flowmtf = 3 m flowmtf = 12	o flow = 12 m flow = 16 o flowmtf = 25 m flowmtf = 13	o flow = 8 m flow = 118 o flowmtf = 7 m flowmtf = 104
124	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	o flow = 28 m flow = 6 o flowmtf = 13 m flowmtf = 6	o flow = 8 m flow = 1202823 ♣ o flowmtf = 7 m flowmtf = 9	o flow = 59 m flow = 95 o flowmtf = 11 m flowmtf = 91
125	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	o flow = 32 m flow = 9 o flowmtf = 11 m flowmtf = 9	o flow = 92 m flow = 319 o flowmtf = 12 m flowmtf = 12	o flow = 34 m flow = 15 o flowmtf = 17 m flowmtf = 10
126	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	o flow = 111 m flow = 11 o flowmtf = 9 m flowmtf = 13	o flow = 6 m flow = 105 o flowmtf = 7 m flowmtf = 15	o flow = 36 m flow = 10 o flowmtf = 10 m flowmtf = 10
127	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	o flow = 19 m flow = 40 o flowmtf = 14 m flowmtf = 7	o flow = 10 m flow = 28 o flowmtf = 9 m flowmtf = 11	o flow = 2484 m flow = 84 o flowmtf = 7 m flowmtf = 7
128	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	o flow = 285 m flow = 21 o flowmtf = 14 m flowmtf = 11	o flow = 138 m flow = 154 o flowmtf = 11 m flowmtf = 11	o flow = 751 m flow = 47 o flowmtf = 10 m flowmtf = 12
129	$\|I\| = 10$ $\|S\| = 90$	o flow = 9 m flow = 34	o flow = 9 m flow = 34	o flow = 9 m flow = 34

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|E\| = 900$	o flowmtf = 3 m flowmtf = 13	o flowmtf = 3 m flowmtf = 13	o flowmtf = 3 m flowmtf = 13
130	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	o flow = 26 m flow = 14 o flowmtf = 11 m flowmtf = 13	o flow = 26 m flow = 14 o flowmtf = 11 m flowmtf = 13	o flow = 26 m flow = 14 o flowmtf = 11 m flowmtf = 13
131	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	o flow = 95 m flow = 8 o flowmtf = 11 m flowmtf = 9	o flow = 95 m flow = 8 o flowmtf = 11 m flowmtf = 9	o flow = 95 m flow = 8 o flowmtf = 11 m flowmtf = 9
132	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	o flow = 6 m flow = 3 o flowmtf = 5 m flowmtf = 3	o flow = 7 m flow = 6 o flowmtf = 3 m flowmtf = 7	o flow = 5 m flow = 4 o flowmtf = 9 m flowmtf = 5
133	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	o flow = 5 m flow = 5 o flowmtf = 5 m flowmtf = 7	o flow = 1 m flow = 5 o flowmtf = 1 m flowmtf = 8	o flow = 6 m flow = 3 o flowmtf = 6 m flowmtf = 4
134	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	o flow = 4 m flow = 2 o flowmtf = 8 m flowmtf = 5	o flow = 4 m flow = 2 o flowmtf = 5 m flowmtf = 3	o flow = 3 m flow = 4 o flowmtf = 4 m flowmtf = 7
135	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	o flow = 4 m flow = 6 o flowmtf = 7 m flowmtf = 8	o flow = 4 m flow = 12 o flowmtf = 5 m flowmtf = 6	o flow = 2 m flow = 5 o flowmtf = 2 m flowmtf = 6
136	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	o flow = 35 m flow = 6 o flowmtf = 10 m flowmtf = 8	o flow = 15 m flow = 6 o flowmtf = 6 m flowmtf = 6	o flow = 4 m flow = 6 o flowmtf = 3 m flowmtf = 14
137	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	o flow = 5 m flow = 33 o flowmtf = 6 m flowmtf = 5	o flow = 7 m flow = 3 o flowmtf = 9 m flowmtf = 3	o flow = 17 m flow = 3 o flowmtf = 7 m flowmtf = 7
138	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 4	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 4	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 4
139	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	o flow = 9 m flow = 2 o flowmtf = 8 m flowmtf = 2	o flow = 9 m flow = 2 o flowmtf = 8 m flowmtf = 2	o flow = 9 m flow = 2 o flowmtf = 8 m flowmtf = 2
140	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	o flow = 5 m flow = 5 o flowmtf = 8 m flowmtf = 6	o flow = 5 m flow = 5 o flowmtf = 8 m flowmtf = 6	o flow = 5 m flow = 5 o flowmtf = 8 m flowmtf = 6
141	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	o flow = 18 m flow = 3 o flowmtf = 4 m flowmtf = 2	o flow = 54 m flow = 2 o flowmtf = 15 m flowmtf = 3	o flow = 7 m flow = 53 o flowmtf = 12 m flowmtf = 5
142	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	o flow = 1272046 ♣ m flow = 5 o flowmtf = 7 m flowmtf = 7	o flow = 7714 m flow = 7 o flowmtf = 6 m flowmtf = 8	o flow = 21148 m flow = 2 o flowmtf = 8 m flowmtf = 2
143	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	o flow = 67 m flow = 13 o flowmtf = 5 m flowmtf = 17	o flow = 953 m flow = 2 o flowmtf = 9 m flowmtf = 2	o flow = 3 m flow = 203173 o flowmtf = 7 m flowmtf = 9
144	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	o flow = 2 m flow = 7 o flowmtf = 2 m flowmtf = 5	o flow = 2 m flow = 5 o flowmtf = 6 m flowmtf = 9	o flow = 5 m flow = 7 o flowmtf = 9 m flowmtf = 7
145	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	o flow = 2 m flow = 4 o flowmtf = 2	o flow = 20 m flow = 8 o flowmtf = 7	o flow = 3 m flow = 6 o flowmtf = 8

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m flowmtf = 6	m flowmtf = 4	m flowmtf = 6
146	$\lvert I\rvert = 20$ $\lvert S\rvert = 100$ $\lvert E\rvert = 860$	o flow = 5 m flow = 41 o flowmtf = 6 m flowmtf = 5	o flow = 64 m flow = 1 o flowmtf = 4 m flowmtf = 1	o flow = 3 m flow = 6 o flowmtf = 3 m flowmtf = 9
147	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 192$	o flow = 1 m flow = 4 o flowmtf = 1 m flowmtf = 4	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2
148	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 192$	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 4 m flow = 3 o flowmtf = 4 m flowmtf = 3
149	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 192$	o flow = 2 m flow = 15217 o flowmtf = 3 m flowmtf = 5	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 3	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1
150	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 344$	o flow = 2 m flow = 2 o flowmtf = 4 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 3
151	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 344$	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 4	o flow = 2 m flow = 3 o flowmtf = 3 m flowmtf = 3
152	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 344$	o flow = 3 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 5 m flow = 2 o flowmtf = 4 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2
153	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 496$	o flow = 3 m flow = 3 o flowmtf = 2 m flowmtf = 2	o flow = 3 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 2 m flow = 13 o flowmtf = 2 m flowmtf = 3
154	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 496$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 3
155	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 496$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2
156	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 648$	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2
157	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 648$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 2 m flow = 1 o flowmtf = 5 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1
158	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 648$	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 5 o flowmtf = 1 m flowmtf = 6
159	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 800$	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 2	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 2	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 2
160	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 800$	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2
161	$\lvert I\rvert = 20$ $\lvert S\rvert = 40$ $\lvert E\rvert = 800$	o flow = 2 m flow = 1 o flowmtf = 4 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 4 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 4 m flowmtf = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
162	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 1200$	o flow = 3 m flow = 3 o flowmtf = 5 m flowmtf = 4	o flow = 3 m flow = 3 o flowmtf = 5 m flowmtf = 4	o flow = 3 m flow = 3 o flowmtf = 5 m flowmtf = 4
163	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 1200$	o flow = 4 m flow = 3 o flowmtf = 6 m flowmtf = 3	o flow = 4 m flow = 3 o flowmtf = 6 m flowmtf = 3	o flow = 4 m flow = 3 o flowmtf = 6 m flowmtf = 3
164	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 1200$	o flow = 6 m flow = 1 o flowmtf = 10 m flowmtf = 1	o flow = 6 m flow = 1 o flowmtf = 10 m flowmtf = 1	o flow = 6 m flow = 1 o flowmtf = 10 m flowmtf = 1
165	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 288$	o flow = 14 m flow = 4 o flowmtf = 8 m flowmtf = 3	o flow = 21 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 367 m flow = 1 o flowmtf = 11 m flowmtf = 1
166	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 288$	o flow = 4 m flow = 3 o flowmtf = 3 m flowmtf = 6	o flow = 55 m flow = 2 o flowmtf = 6 m flowmtf = 5	o flow = 73337 m flow = 4 o flowmtf = 9 m flowmtf = 2
167	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 288$	o flow = 2 m flow = 3615036 ♣ o flowmtf = 4 m flowmtf = 4	o flow = 1 m flow = 6 o flowmtf = 1 m flowmtf = 3	o flow = 2 m flow = 3 o flowmtf = 4 m flowmtf = 6
168	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	o flow = 3 m flow = 5 o flowmtf = 5 m flowmtf = 3	o flow = 6 m flow = 10 o flowmtf = 8 m flowmtf = 2	o flow = 4 m flow = 3 o flowmtf = 5 m flowmtf = 3
169	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	o flow = 1 m flow = 8 o flowmtf = 1 m flowmtf = 2	o flow = 4 m flow = 7 o flowmtf = 3 m flowmtf = 6	o flow = 2 m flow = 3 o flowmtf = 4 m flowmtf = 5
170	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	o flow = 3 m flow = 3 o flowmtf = 4 m flowmtf = 8	o flow = 7 m flow = 3 o flowmtf = 6 m flowmtf = 6	o flow = 2 m flow = 2 o flowmtf = 4 m flowmtf = 4
171	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	o flow = 2 m flow = 4 o flowmtf = 2 m flowmtf = 13	o flow = 2 m flow = 7 o flowmtf = 3 m flowmtf = 3	o flow = 1 m flow = 15 o flowmtf = 1 m flowmtf = 5
172	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	o flow = 22 m flow = 9 o flowmtf = 6 m flowmtf = 4	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 4 m flow = 7 o flowmtf = 6 m flowmtf = 6
173	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	o flow = 19 m flow = 7 o flowmtf = 5 m flowmtf = 3	o flow = 11 m flow = 3 o flowmtf = 2 m flowmtf = 4	o flow = 3 m flow = 3 o flowmtf = 6 m flowmtf = 7
174	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 4 m flow = 3 o flowmtf = 4 m flowmtf = 3	o flow = 2 m flow = 4 o flowmtf = 4 m flowmtf = 9
175	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	o flow = 3 m flow = 5 o flowmtf = 3 m flowmtf = 3	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 2	o flow = 5 m flow = 1 o flowmtf = 6 m flowmtf = 1
176	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	o flow = 2 m flow = 3 o flowmtf = 4 m flowmtf = 8	o flow = 3 m flow = 2 o flowmtf = 3 m flowmtf = 5	o flow = 3 m flow = 4 o flowmtf = 6 m flowmtf = 6
177	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	o flow = 3 m flow = 9 o flowmtf = 7 m flowmtf = 11	o flow = 3 m flow = 4 o flowmtf = 7 m flowmtf = 4	o flow = 16 m flow = 4 o flowmtf = 4 m flowmtf = 9
178	$\|I\| = 20$	o flow = 14	o flow = 3	o flow = 4
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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 80$ $\|E\| = 1296$	m flow = 2 o flowmtf = 6 m flowmtf = 4	m flow = 4 o flowmtf = 5 m flowmtf = 7	m flow = 1 o flowmtf = 4 m flowmtf = 1
179	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 6	o flow = 8 m flow = 1 o flowmtf = 4 m flowmtf = 1	o flow = 6 m flow = 13 o flowmtf = 7 m flowmtf = 6
180	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	o flow = 5 m flow = 2 o flowmtf = 5 m flowmtf = 2	o flow = 5 m flow = 2 o flowmtf = 5 m flowmtf = 2	o flow = 5 m flow = 2 o flowmtf = 5 m flowmtf = 2
181	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	o flow = 9 m flow = 3 o flowmtf = 6 m flowmtf = 6	o flow = 9 m flow = 3 o flowmtf = 6 m flowmtf = 6	o flow = 9 m flow = 3 o flowmtf = 6 m flowmtf = 6
182	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	o flow = 2 m flow = 3 o flowmtf = 6 m flowmtf = 9	o flow = 2 m flow = 3 o flowmtf = 6 m flowmtf = 9	o flow = 2 m flow = 3 o flowmtf = 6 m flowmtf = 9
183	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	o flow = 4 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 3 m flow = 3 o flowmtf = 2 m flowmtf = 5	o flow = 51 m flow = 199109 o flowmtf = 6 m flowmtf = 5
184	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	o flow = 1116376 m flow = 2 o flowmtf = 6 m flowmtf = 4	o flow = 5 m flow = 6 o flowmtf = 5 m flowmtf = 5	o flow = 7 m flow = 5 o flowmtf = 9 m flowmtf = 5
185	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	o flow = 4 m flow = 3 o flowmtf = 9 m flowmtf = 4	o flow = 12 m flow = 6 o flowmtf = 7 m flowmtf = 7	o flow = 12 m flow = 432 o flowmtf = 6 m flowmtf = 4
186	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	o flow = 25 m flow = 1 o flowmtf = 8 m flowmtf = 1	o flow = 114 m flow = 8 o flowmtf = 4 m flowmtf = 11	o flow = 8 m flow = 5 o flowmtf = 8 m flowmtf = 6
187	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	o flow = 3 m flow = 3 o flowmtf = 3 m flowmtf = 3	o flow = 3 m flow = 3 o flowmtf = 3 m flowmtf = 3	o flow = 2 m flow = 5 o flowmtf = 2 m flowmtf = 5
188	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	o flow = 3 m flow = 4 o flowmtf = 2 m flowmtf = 4	o flow = 3 m flow = 1 o flowmtf = 5 m flowmtf = 1	o flow = 4 m flow = 4 o flowmtf = 5 m flowmtf = 4
189	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	o flow = 6 m flow = 10 o flowmtf = 7 m flowmtf = 6	o flow = 3 m flow = 3 o flowmtf = 4 m flowmtf = 3	o flow = 4 m flow = 1 o flowmtf = 3 m flowmtf = 1
190	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 22 m flow = 3 o flowmtf = 3 m flowmtf = 11	o flow = 19 m flow = 9 o flowmtf = 10 m flowmtf = 13
191	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 3	o flow = 7 m flow = 5 o flowmtf = 7 m flowmtf = 5	o flow = 3 m flow = 3 o flowmtf = 9 m flowmtf = 2
192	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 2	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2
193	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	o flow = 4 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 2 m flow = 3 o flowmtf = 3 m flowmtf = 4	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 2
194	$\|I\| = 30$ $\|S\| = 60$	o flow = 1 m flow = 1	o flow = 2 m flow = 2	o flow = 2 m flow = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|E\| = 1104$	o flowmtf = 1 m flowmtf = 1	o flowmtf = 3 m flowmtf = 2	o flowmtf = 3 m flowmtf = 1
195	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 5	o flow = 2 m flow = 3 o flowmtf = 6 m flowmtf = 2
196	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 8 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 5
197	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 3	o flow = 2 m flow = 4 o flowmtf = 3 m flowmtf = 4
198	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	o flow = 7 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 7 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 7 m flow = 1 o flowmtf = 3 m flowmtf = 1
199	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 5	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 5	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 5
200	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	o flow = 4 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 4 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 4 m flow = 1 o flowmtf = 3 m flowmtf = 1
201	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	o flow = 3 m flow = 1 o flowmtf = 5 m flowmtf = 1	o flow = 1442571 ♣ m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 3
202	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 2	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1
203	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 4 o flowmtf = 1 m flowmtf = 5
204	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	o flow = 2 m flow = 1 o flowmtf = 4 m flowmtf = 1	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 3	o flow = 26 m flow = 1 o flowmtf = 3 m flowmtf = 1
205	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 2 m flow = 4 o flowmtf = 2 m flowmtf = 2	o flow = 4 m flow = 6 o flowmtf = 5 m flowmtf = 2
206	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 267 m flow = 3 o flowmtf = 2 m flowmtf = 4
207	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	o flow = 7 m flow = 5 o flowmtf = 6 m flowmtf = 8	o flow = 4 m flow = 4 o flowmtf = 5 m flowmtf = 6	o flow = 3 m flow = 4 o flowmtf = 8 m flowmtf = 6
208	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 4	o flow = 6 m flow = 2 o flowmtf = 3 m flowmtf = 9	o flow = 2 m flow = 3 o flowmtf = 6 m flowmtf = 3
209	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 4	o flow = 2 m flow = 5 o flowmtf = 4 m flowmtf = 5
210	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	o flow = 3 m flow = 12 o flowmtf = 2	o flow = 5 m flow = 7 o flowmtf = 5	o flow = 4 m flow = 4 o flowmtf = 8

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m flowmtf = 6	m flowmtf = 10	m flowmtf = 6
211	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 4	o flow = 2 m flow = 5 o flowmtf = 2 m flowmtf = 9	o flow = 7 m flow = 6 o flowmtf = 6 m flowmtf = 5
212	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	o flow = 3 m flow = 3 o flowmtf = 6 m flowmtf = 3	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 5	o flow = 4 m flow = 61 o flowmtf = 4 m flowmtf = 3
213	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	o flow = 4 m flow = 3 o flowmtf = 6 m flowmtf = 3	o flow = 5 m flow = 4 o flowmtf = 5 m flowmtf = 3	o flow = 2 m flow = 3 o flowmtf = 5 m flowmtf = 3
214	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	o flow = 6 m flow = 3 o flowmtf = 2 m flowmtf = 3	o flow = 7 m flow = 4 o flowmtf = 4 m flowmtf = 3	o flow = 1 m flow = 4 o flowmtf = 1 m flowmtf = 6
215	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	o flow = 3 m flow = 5 o flowmtf = 5 m flowmtf = 2	o flow = 5 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 4 m flow = 2 o flowmtf = 2 m flowmtf = 7
216	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1
217	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 6	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 6	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 6
218	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	o flow = 4 m flow = 4 o flowmtf = 3 m flowmtf = 5	o flow = 4 m flow = 4 o flowmtf = 3 m flowmtf = 5	o flow = 4 m flow = 4 o flowmtf = 3 m flowmtf = 5
219	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	o flow = 2 m flow = 54 o flowmtf = 2 m flowmtf = 4	o flow = 5 m flow = 4 o flowmtf = 9 m flowmtf = 7	o flow = 10 m flow = 2 o flowmtf = 8 m flowmtf = 2
220	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	o flow = 97885 m flow = 18 o flowmtf = 9 m flowmtf = 7	o flow = 148 m flow = 4 o flowmtf = 4 m flowmtf = 4	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 3
221	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	o flow = 26 m flow = 3 o flowmtf = 5 m flowmtf = 4	o flow = 1 m flow = 14 o flowmtf = 1 m flowmtf = 5	o flow = 2 m flow = 5 o flowmtf = 5 m flowmtf = 5
222	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 3
223	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 3 m flow = 1 o flowmtf = 4 m flowmtf = 1
224	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 3	o flow = 4 m flow = 2 o flowmtf = 2 m flowmtf = 3
225	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 3	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2
226	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	o flow = 3 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
227	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	o flow = 2 m flow = 2 o flowmtf = 4 m flowmtf = 6	o flow = 3 m flow = 1 o flowmtf = 8 m flowmtf = 1	o flow = 2 m flow = 3 o flowmtf = 2 m flowmtf = 3
228	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 2 m flow = 3 o flowmtf = 3 m flowmtf = 3	o flow = 2 m flow = 4 o flowmtf = 2 m flowmtf = 4
229	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 5 m flowmtf = 3	o flow = 4 m flow = 1 o flowmtf = 4 m flowmtf = 1
230	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 2 m flow = 3 o flowmtf = 3 m flowmtf = 5	o flow = 1 m flow = 5 o flowmtf = 1 m flowmtf = 2
231	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1
232	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	o flow = 5 m flow = 3 o flowmtf = 6 m flowmtf = 2	o flow = 5 m flow = 3 o flowmtf = 6 m flowmtf = 2	o flow = 5 m flow = 3 o flowmtf = 6 m flowmtf = 2
233	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1
234	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 3 m flow = 3 o flowmtf = 2 m flowmtf = 3
235	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 4 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2
236	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 2 m flow = 479 o flowmtf = 2 m flowmtf = 2	o flow = 21 m flow = 2 o flowmtf = 6 m flowmtf = 2
237	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 6 m flow = 3 o flowmtf = 4 m flowmtf = 2	o flow = 3 m flow = 20 o flowmtf = 6 m flowmtf = 5
238	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 2 m flow = 7 o flowmtf = 2 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 4 m flowmtf = 3
239	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 3 m flow = 2 o flowmtf = 4 m flowmtf = 2	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1
240	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 3	o flow = 3 m flow = 1 o flowmtf = 2 m flowmtf = 1
241	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	o flow = 2 m flow = 3 o flowmtf = 9 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 3	o flow = 4 m flow = 2 o flowmtf = 2 m flowmtf = 2
242	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	o flow = 3 m flow = 8 o flowmtf = 3 m flowmtf = 3	o flow = 2 m flow = 1 o flowmtf = 2 m flowmtf = 1	o flow = 8 m flow = 1 o flowmtf = 2 m flowmtf = 1
243	$\|I\| = 50$	o flow = 1	o flow = 1	o flow = 2

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 100$ $\|E\| = 3040$	m flow = 4 o flowmtf = 1 m flowmtf = 6	m flow = 1 o flowmtf = 1 m flowmtf = 1	m flow = 2 o flowmtf = 2 m flowmtf = 2
244	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	o flow = 2 m flow = 3 o flowmtf = 4 m flowmtf = 3	o flow = 2 m flow = 2 o flowmtf = 5 m flowmtf = 2	o flow = 1 m flow = 1 o flowmtf = 1 m flowmtf = 1
245	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	o flow = 3 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2	o flow = 8 m flow = 25 o flowmtf = 4 m flowmtf = 3
246	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	o flow = 2 m flow = 1 o flowmtf = 5 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 2 m flow = 2 o flowmtf = 2 m flowmtf = 2
247	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	o flow = 4 m flow = 1 o flowmtf = 5 m flowmtf = 1	o flow = 2 m flow = 2 o flowmtf = 3 m flowmtf = 2	o flow = 2 m flow = 5 o flowmtf = 2 m flowmtf = 2
248	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	o flow = 3 m flow = 3 o flowmtf = 5 m flowmtf = 5	o flow = 1 m flow = 3 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 3
249	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	o flow = 2 m flow = 2 o flowmtf = 6 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 6 m flowmtf = 2	o flow = 2 m flow = 2 o flowmtf = 6 m flowmtf = 2
250	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2	o flow = 1 m flow = 2 o flowmtf = 1 m flowmtf = 2
251	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1	o flow = 2 m flow = 1 o flowmtf = 3 m flowmtf = 1

Table 9: Number of recursive calls required by the matroid (matr) and matroid MTF (matrmtf) solvers when operating on c-matchable random instances (o) and instances modified by the matroidmtf solvers (m). Entries marked with "♣" and "♠" did not terminate after 90 and 600 seconds, respectively

	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
1	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 1000$	o matr = 4 m matr = 8 o matrmtf = 3 m matrmtf = 9	o matr = 4 m matr = 8 o matrmtf = 3 m matrmtf = 9	o matr = 4 m matr = 8 o matrmtf = 3 m matrmtf = 9
2	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 1000$	o matr = 42 m matr = 17 o matrmtf = 8 m matrmtf = 7	o matr = 42 m matr = 17 o matrmtf = 8 m matrmtf = 7	o matr = 42 m matr = 17 o matrmtf = 8 m matrmtf = 7
3	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 1000$	o matr = 4 m matr = 4 o matrmtf = 8 m matrmtf = 8	o matr = 4 m matr = 4 o matrmtf = 8 m matrmtf = 8	o matr = 4 m matr = 4 o matrmtf = 8 m matrmtf = 8
4	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 280$	o matr = 17 m matr = 543 o matrmtf = 7 m matrmtf = 9	o matr = 5 m matr = 15 o matrmtf = 5 m matrmtf = 18	o matr = 17 m matr = 7 o matrmtf = 3 m matrmtf = 9
5	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 280$	o matr = 1 m matr = 12 o matrmtf = 1 m matrmtf = 5	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 1 m matr = 80 o matrmtf = 1 m matrmtf = 27
6	$\|I\| = 10$ $\|S\| = 100$	o matr = 2 m matr = 1	o matr = 3 m matr = 9486	o matr = 2 m matr = 3

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|E\| = 280$	o matrmtf = 3 m matrmtf = 1	o matrmtf = 3 m matrmtf = 3	o matrmtf = 2 m matrmtf = 3
7	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 460$	o matr = 3 m matr = 3 o matrmtf = 2 m matrmtf = 3	o matr = 19 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 4 m matr = 228 o matrmtf = 4 m matrmtf = 4
8	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 460$	o matr = 8 m matr = 10 o matrmtf = 2 m matrmtf = 4	o matr = 12 m matr = 3 o matrmtf = 4 m matrmtf = 3	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2
9	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 460$	o matr = 4 m matr = 5 o matrmtf = 5 m matrmtf = 5	o matr = 80 m matr = 7658 o matrmtf = 5 m matrmtf = 5	o matr = 3 m matr = 26 o matrmtf = 4 m matrmtf = 4
10	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 640$	o matr = 4 m matr = 3 o matrmtf = 4 m matrmtf = 3	o matr = 36 m matr = 52 o matrmtf = 7 m matrmtf = 3	o matr = 4 m matr = 4 o matrmtf = 5 m matrmtf = 3
11	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 640$	o matr = 3725 m matr = 20 o matrmtf = 6 m matrmtf = 7	o matr = 18 m matr = 314211 ♣ o matrmtf = 6 m matrmtf = 9	o matr = 20 m matr = 4 o matrmtf = 3 m matrmtf = 4
12	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 640$	o matr = 23 m matr = 166 o matrmtf = 7 m matrmtf = 3	o matr = 34407 m matr = 3 o matrmtf = 7 m matrmtf = 7	o matr = 6 m matr = 28 o matrmtf = 4 m matrmtf = 12
13	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 820$	o matr = 4 m matr = 28 o matrmtf = 6 m matrmtf = 8	o matr = 4 m matr = 33 o matrmtf = 4 m matrmtf = 5	o matr = 34 m matr = 6 o matrmtf = 7 m matrmtf = 8
14	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 820$	o matr = 4 m matr = 41 o matrmtf = 4 m matrmtf = 2	o matr = 3 m matr = 7 o matrmtf = 4 m matrmtf = 11	o matr = 21 m matr = 4 o matrmtf = 6 m matrmtf = 5
15	$\|I\| = 10$ $\|S\| = 100$ $\|E\| = 820$	o matr = 12 m matr = 153 o matrmtf = 5 m matrmtf = 5	o matr = 6 m matr = 7 o matrmtf = 6 m matrmtf = 6	o matr = 13 m matr = 8 o matrmtf = 3 m matrmtf = 8
16	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
17	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
18	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 128$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
19	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	o matr = 3 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 4 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
20	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 4 m matr = 2 o matrmtf = 2 m matrmtf = 5	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
21	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 164$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 3	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
22	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	o matr = 2 m matr = 4 o matrmtf = 3	o matr = 2 m matr = 4 o matrmtf = 3	o matr = 2 m matr = 4 o matrmtf = 3

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m matrmtf = 2	m matrmtf = 2	m matrmtf = 2
23	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
24	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 200$	o matr = 4 m matr = 2 o matrmtf = 4 m matrmtf = 2	o matr = 4 m matr = 2 o matrmtf = 4 m matrmtf = 2	o matr = 4 m matr = 2 o matrmtf = 4 m matrmtf = 2
25	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 56$	o matr = 1 m matr = 7 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3
26	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 11 o matrmtf = 1 m matrmtf = 2
27	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 2
28	$\|I\| = 10$ $\|S\| = 20$ $\|E\| = 92$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
29	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2
30	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
31	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 138$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 38 o matrmtf = 3 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
32	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
33	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2
34	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 192$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 2	o matr = 8 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
35	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
36	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 3 m matr = 7 o matrmtf = 3 m matrmtf = 3	o matr = 4 m matr = 2 o matrmtf = 3 m matrmtf = 2
37	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 246$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
38	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
39	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	o matr = 2 m matr = 1 o matrmtf = 6 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 6 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 6 m matrmtf = 1
40	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 300$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2
41	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
42	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	o matr = 12642 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 3516 m matr = 1 o matrmtf = 2 m matrmtf = 1
43	$\|I\| = 10$ $\|S\| = 30$ $\|E\| = 84$	o matr = 583 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
44	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
45	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
46	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 112$	o matr = 1 m matr = 85 o matrmtf = 1 m matrmtf = 7	o matr = 5 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 894 m matr = 2 o matrmtf = 5 m matrmtf = 2
47	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	o matr = 62 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 46069 o matrmtf = 2 m matrmtf = 3	o matr = 2 m matr = 7 o matrmtf = 2 m matrmtf = 2
48	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 93 o matrmtf = 1 m matrmtf = 3
49	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 184$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 20 m matr = 2 o matrmtf = 2 m matrmtf = 2
50	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 4	o matr = 3 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 12 m matr = 3 o matrmtf = 3 m matrmtf = 3
51	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	o matr = 3 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
52	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 256$	o matr = 18 m matr = 4 o matrmtf = 2 m matrmtf = 4	o matr = 3 m matr = 5 o matrmtf = 3 m matrmtf = 3	o matr = 14 m matr = 9 o matrmtf = 2 m matrmtf = 3
53	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 4	o matr = 4 m matr = 1 o matrmtf = 4 m matrmtf = 1
54	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 328$	o matr = 46 m matr = 5 o matrmtf = 3 m matrmtf = 5	o matr = 2 m matr = 4 o matrmtf = 2 m matrmtf = 6	o matr = 2 m matr = 4 o matrmtf = 2 m matrmtf = 3
55	$\|I\| = 10$	o matr = 1	o matr = 2	o matr = 2

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 40$ $\|E\| = 328$	m matr = 4 o matrmtf = 1 m matrmtf = 4	m matr = 3 o matrmtf = 2 m matrmtf = 3	m matr = 2 o matrmtf = 4 m matrmtf = 4
56	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2
57	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	o matr = 14 m matr = 3 o matrmtf = 3 m matrmtf = 2	o matr = 14 m matr = 3 o matrmtf = 3 m matrmtf = 2	o matr = 14 m matr = 3 o matrmtf = 3 m matrmtf = 2
58	$\|I\| = 10$ $\|S\| = 40$ $\|E\| = 400$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 5	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 5	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 5
59	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
60	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 23 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 10 o matrmtf = 2 m matrmtf = 11
61	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 140$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
62	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 3 m matr = 2 o matrmtf = 4 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 3
63	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	o matr = 2 m matr = 4 o matrmtf = 2 m matrmtf = 3	o matr = 2 m matr = 17 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
64	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 230$	o matr = 3 m matr = 3 o matrmtf = 3 m matrmtf = 3	o matr = 2 m matr = 28 o matrmtf = 3 m matrmtf = 4	o matr = 4 m matr = 3 o matrmtf = 2 m matrmtf = 3
65	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	o matr = 5 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 5	o matr = 4 m matr = 8 o matrmtf = 4 m matrmtf = 2
66	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	o matr = 4 m matr = 4 o matrmtf = 5 m matrmtf = 4	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 5	o matr = 3 m matr = 11 o matrmtf = 6 m matrmtf = 6
67	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 320$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 9519 o matrmtf = 2 m matrmtf = 3
68	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	o matr = 3 m matr = 2 o matrmtf = 4 m matrmtf = 6	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 3 o matrmtf = 4 m matrmtf = 3
69	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	o matr = 5 m matr = 2 o matrmtf = 4 m matrmtf = 5	o matr = 12 m matr = 3 o matrmtf = 3 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 3
70	$\|I\| = 10$ $\|S\| = 50$ $\|E\| = 410$	o matr = 9 m matr = 1 o matrmtf = 7 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 6 m matrmtf = 2	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2
71	$\|I\| = 10$ $\|S\| = 50$	o matr = 1 m matr = 4	o matr = 1 m matr = 4	o matr = 1 m matr = 4

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\lvert E\rvert = 500$	o matrmtf = 1 m matrmtf = 4	o matrmtf = 1 m matrmtf = 4	o matrmtf = 1 m matrmtf = 4
72	$\lvert I\rvert = 10$ $\lvert S\rvert = 50$ $\lvert E\rvert = 500$	o matr = 6 m matr = 3 o matrmtf = 6 m matrmtf = 3	o matr = 6 m matr = 3 o matrmtf = 6 m matrmtf = 3	o matr = 6 m matr = 3 o matrmtf = 6 m matrmtf = 3
73	$\lvert I\rvert = 10$ $\lvert S\rvert = 50$ $\lvert E\rvert = 500$	o matr = 13 m matr = 4 o matrmtf = 4 m matrmtf = 7	o matr = 13 m matr = 4 o matrmtf = 4 m matrmtf = 7	o matr = 13 m matr = 4 o matrmtf = 4 m matrmtf = 7
74	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 168$	o matr = 1 m matr = 9 o matrmtf = 1 m matrmtf = 9	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
75	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 168$	o matr = 1 m matr = 10 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 556 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
76	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 168$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
77	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 276$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 41 m matr = 1 o matrmtf = 2 m matrmtf = 1
78	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 276$	o matr = 239 m matr = 296 o matrmtf = 5 m matrmtf = 2	o matr = 2 m matr = 6 o matrmtf = 4 m matrmtf = 6	o matr = 5279 m matr = 16 o matrmtf = 3 m matrmtf = 3
79	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 276$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3
80	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 384$	o matr = 2 m matr = 6 o matrmtf = 2 m matrmtf = 6	o matr = 13 m matr = 5 o matrmtf = 4 m matrmtf = 5	o matr = 2 m matr = 10878 o matrmtf = 2 m matrmtf = 4
81	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 384$	o matr = 36 m matr = 5 o matrmtf = 6 m matrmtf = 3	o matr = 1 m matr = 33 o matrmtf = 1 m matrmtf = 3	o matr = 5 m matr = 4 o matrmtf = 5 m matrmtf = 4
82	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 384$	o matr = 3 m matr = 6 o matrmtf = 3 m matrmtf = 6	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 3 m matr = 2 o matrmtf = 2 m matrmtf = 2
83	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 492$	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 4	o matr = 3 m matr = 11 o matrmtf = 2 m matrmtf = 6	o matr = 5 m matr = 3 o matrmtf = 5 m matrmtf = 3
84	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 492$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 16 m matr = 3 o matrmtf = 2 m matrmtf = 6	o matr = 10 m matr = 4 o matrmtf = 3 m matrmtf = 5
85	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 492$	o matr = 4 m matr = 17 o matrmtf = 3 m matrmtf = 4	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 3	o matr = 838 m matr = 3 o matrmtf = 4 m matrmtf = 4
86	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 600$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 4
87	$\lvert I\rvert = 10$ $\lvert S\rvert = 60$ $\lvert E\rvert = 600$	o matr = 4 m matr = 1 o matrmtf = 7	o matr = 4 m matr = 1 o matrmtf = 7	o matr = 4 m matr = 1 o matrmtf = 7

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m matrmtf = 1	m matrmtf = 1	m matrmtf = 1
88	$\|I\| = 10$ $\|S\| = 60$ $\|E\| = 600$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3
89	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
90	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 7 o matrmtf = 1 m matrmtf = 2
91	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 196$	o matr = 15 m matr = 1 o matrmtf = 19 m matrmtf = 1	o matr = 5085493 ♠ m matr = 2 o matrmtf = 84 m matrmtf = 2	o matr = 429 m matr = 2 o matrmtf = 2062 m matrmtf = 2
92	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 4802 m matr = 21 o matrmtf = 5 m matrmtf = 2
93	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	o matr = 2 m matr = 53 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1975 o matrmtf = 1 m matrmtf = 4
94	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 322$	o matr = 2 m matr = 3 o matrmtf = 3 m matrmtf = 4	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 62 o matrmtf = 1 m matrmtf = 22
95	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	o matr = 2 m matr = 13 o matrmtf = 2 m matrmtf = 8	o matr = 4 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 152 m matr = 11 o matrmtf = 4 m matrmtf = 5
96	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	o matr = 4 m matr = 3 o matrmtf = 5 m matrmtf = 3	o matr = 5 m matr = 2 o matrmtf = 5 m matrmtf = 2	o matr = 3 m matr = 1 o matrmtf = 4 m matrmtf = 1
97	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 448$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 3 o matrmtf = 4 m matrmtf = 3	o matr = 49 m matr = 5 o matrmtf = 4 m matrmtf = 6
98	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	o matr = 4 m matr = 2 o matrmtf = 3 m matrmtf = 3	o matr = 49 m matr = 38 o matrmtf = 3 m matrmtf = 9	o matr = 3 m matr = 5 o matrmtf = 3 m matrmtf = 4
99	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 8 m matr = 3 o matrmtf = 8 m matrmtf = 5	o matr = 3 m matr = 9 o matrmtf = 4 m matrmtf = 4
100	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 574$	o matr = 4 m matr = 3 o matrmtf = 4 m matrmtf = 2	o matr = 4 m matr = 4 o matrmtf = 4 m matrmtf = 6	o matr = 15 m matr = 5 o matrmtf = 4 m matrmtf = 8
101	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	o matr = 2 m matr = 12 o matrmtf = 2 m matrmtf = 7	o matr = 2 m matr = 12 o matrmtf = 2 m matrmtf = 7	o matr = 2 m matr = 12 o matrmtf = 2 m matrmtf = 7
102	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	o matr = 20 m matr = 81 o matrmtf = 5 m matrmtf = 6	o matr = 20 m matr = 81 o matrmtf = 5 m matrmtf = 6	o matr = 20 m matr = 81 o matrmtf = 5 m matrmtf = 6
103	$\|I\| = 10$ $\|S\| = 70$ $\|E\| = 700$	o matr = 5 m matr = 4 o matrmtf = 5 m matrmtf = 4	o matr = 5 m matr = 4 o matrmtf = 5 m matrmtf = 4	o matr = 5 m matr = 4 o matrmtf = 5 m matrmtf = 4

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
104	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 224$	o matr = 18 m matr = 1 o matrmtf = 6 m matrmtf = 1	o matr = 26 m matr = 396084 ♣ o matrmtf = 26 m matrmtf = 17	o matr = 273484 m matr = 25 o matrmtf = 3 m matrmtf = 19
105	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 368$	o matr = 10577 m matr = 42 o matrmtf = 19 m matrmtf = 26	o matr = 2 m matr = 62 o matrmtf = 2 m matrmtf = 4	o matr = 14 m matr = 4 o matrmtf = 2 m matrmtf = 3
106	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 368$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 32 m matr = 3 o matrmtf = 4 m matrmtf = 2	o matr = 72 m matr = 1 o matrmtf = 4 m matrmtf = 1
107	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 368$	o matr = 1 m matr = 17 o matrmtf = 1 m matrmtf = 17	o matr = 3444807 ♠ m matr = 1 o matrmtf = 21 m matrmtf = 1	o matr = 27 m matr = 2 o matrmtf = 28 m matrmtf = 3
108	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 512$	o matr = 3 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 12 m matr = 5 o matrmtf = 6 m matrmtf = 3	o matr = 4 m matr = 4 o matrmtf = 3 m matrmtf = 4
109	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 512$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 33 m matr = 3 o matrmtf = 5 m matrmtf = 4	o matr = 8 m matr = 4 o matrmtf = 3 m matrmtf = 4
110	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 512$	o matr = 7 m matr = 17 o matrmtf = 11 m matrmtf = 5	o matr = 6 m matr = 2448 o matrmtf = 6 m matrmtf = 3	o matr = 25 m matr = 3 o matrmtf = 5 m matrmtf = 4
111	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 656$	o matr = 15 m matr = 83 o matrmtf = 8 m matrmtf = 7	o matr = 3 m matr = 35 o matrmtf = 2 m matrmtf = 6	o matr = 7 m matr = 34 o matrmtf = 4 m matrmtf = 7
112	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 656$	o matr = 2 m matr = 4 o matrmtf = 2 m matrmtf = 4	o matr = 4 m matr = 82 o matrmtf = 5 m matrmtf = 5	o matr = 5 m matr = 42 o matrmtf = 7 m matrmtf = 2
113	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 656$	o matr = 34 m matr = 4 o matrmtf = 7 m matrmtf = 4	o matr = 7 m matr = 3 o matrmtf = 10 m matrmtf = 3	o matr = 34 m matr = 6 o matrmtf = 8 m matrmtf = 6
114	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 800$	o matr = 6 m matr = 5 o matrmtf = 7 m matrmtf = 6	o matr = 6 m matr = 5 o matrmtf = 7 m matrmtf = 6	o matr = 6 m matr = 5 o matrmtf = 7 m matrmtf = 6
115	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 800$	o matr = 12 m matr = 7 o matrmtf = 9 m matrmtf = 8	o matr = 12 m matr = 7 o matrmtf = 9 m matrmtf = 8	o matr = 12 m matr = 7 o matrmtf = 9 m matrmtf = 8
116	$\|I\| = 10$ $\|S\| = 80$ $\|E\| = 800$	o matr = 4 m matr = 5 o matrmtf = 3 m matrmtf = 5	o matr = 4 m matr = 5 o matrmtf = 3 m matrmtf = 5	o matr = 4 m matr = 5 o matrmtf = 3 m matrmtf = 5
117	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 36 m matr = 4 o matrmtf = 2 m matrmtf = 5	o matr = 21 m matr = 228 o matrmtf = 5 m matrmtf = 3
118	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	o matr = 2931056 m matr = 888601 ♣ o matrmtf = 25 m matrmtf = 4	o matr = 35 m matr = 5 o matrmtf = 17 m matrmtf = 5	o matr = 2369605 ♠ m matr = 398487 ♣ o matrmtf = 93 m matrmtf = 105258
119	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 252$	o matr = 67619 m matr = 583 o matrmtf = 28 m matrmtf = 3	o matr = 3 m matr = 8 o matrmtf = 3 m matrmtf = 3	o matr = 10265600 ♠ m matr = 6 o matrmtf = 40 m matrmtf = 6
120	$\|I\| = 10$	o matr = 7	o matr = 10841	o matr = 15

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 90$ $\|E\| = 414$	m matr = 3 o matrmtf = 3 m matrmtf = 4	m matr = 2 o matrmtf = 3 m matrmtf = 2	m matr = 3 o matrmtf = 4 m matrmtf = 4
121	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	o matr = 3 m matr = 3 o matrmtf = 3 m matrmtf = 3	o matr = 3 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 10 o matrmtf = 2 m matrmtf = 2
122	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 414$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 5	o matr = 4 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 2 m matr = 25 o matrmtf = 2 m matrmtf = 3
123	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	o matr = 3 m matr = 57 o matrmtf = 3 m matrmtf = 7	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 6 m matr = 4 o matrmtf = 5 m matrmtf = 8
124	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	o matr = 4 m matr = 4 o matrmtf = 4 m matrmtf = 7	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 4	o matr = 3 m matr = 5 o matrmtf = 6 m matrmtf = 5
125	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 576$	o matr = 3 m matr = 193994 ♣ o matrmtf = 3 m matrmtf = 5	o matr = 3 m matr = 4 o matrmtf = 5 m matrmtf = 10	o matr = 12 m matr = 16 o matrmtf = 7 m matrmtf = 6
126	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	o matr = 5 m matr = 64 o matrmtf = 5 m matrmtf = 3	o matr = 4 m matr = 5 o matrmtf = 8 m matrmtf = 5	o matr = 4 m matr = 21 o matrmtf = 2 m matrmtf = 4
127	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	o matr = 6 m matr = 2 o matrmtf = 8 m matrmtf = 5	o matr = 3 m matr = 4 o matrmtf = 3 m matrmtf = 6	o matr = 27 m matr = 4 o matrmtf = 4 m matrmtf = 6
128	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 738$	o matr = 7 m matr = 111 o matrmtf = 7 m matrmtf = 7	o matr = 21 m matr = 5 o matrmtf = 4 m matrmtf = 7	o matr = 34 m matr = 8 o matrmtf = 9 m matrmtf = 7
129	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	o matr = 3 m matr = 5 o matrmtf = 4 m matrmtf = 9	o matr = 3 m matr = 5 o matrmtf = 4 m matrmtf = 9	o matr = 3 m matr = 5 o matrmtf = 4 m matrmtf = 9
130	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	o matr = 39 m matr = 127 o matrmtf = 10 m matrmtf = 7	o matr = 39 m matr = 127 o matrmtf = 10 m matrmtf = 7	o matr = 39 m matr = 127 o matrmtf = 10 m matrmtf = 7
131	$\|I\| = 10$ $\|S\| = 90$ $\|E\| = 900$	o matr = 96 m matr = 3 o matrmtf = 10 m matrmtf = 4	o matr = 96 m matr = 3 o matrmtf = 10 m matrmtf = 4	o matr = 96 m matr = 3 o matrmtf = 10 m matrmtf = 4
132	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	o matr = 12 m matr = 7 o matrmtf = 4 m matrmtf = 2	o matr = 129 m matr = 2 o matrmtf = 6 m matrmtf = 2	o matr = 3 m matr = 3 o matrmtf = 5 m matrmtf = 3
133	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	o matr = 3 m matr = 4 o matrmtf = 4 m matrmtf = 3	o matr = 2 m matr = 4 o matrmtf = 2 m matrmtf = 5	o matr = 3 m matr = 3 o matrmtf = 3 m matrmtf = 3
134	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1240$	o matr = 2 m matr = 7 o matrmtf = 2 m matrmtf = 3	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 3
135	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	o matr = 5 m matr = 2 o matrmtf = 4 m matrmtf = 2	o matr = 2 m matr = 4 o matrmtf = 2 m matrmtf = 3	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
136	$\|I\| = 20$ $\|S\| = 100$	o matr = 3 m matr = 4	o matr = 7 m matr = 5	o matr = 2 m matr = 9

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering .
	$\|E\| = 1620$	o matrmtf = 4 m matrmtf = 5	o matrmtf = 3 m matrmtf = 6	o matrmtf = 3 m matrmtf = 2
137	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 1620$	o matr = 5 m matr = 2 o matrmtf = 5 m matrmtf = 3	o matr = 3 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 6 m matr = 2 o matrmtf = 12 m matrmtf = 2
138	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	o matr = 3 m matr = 2 o matrmtf = 2 m matrmtf = 3	o matr = 3 m matr = 2 o matrmtf = 2 m matrmtf = 3	o matr = 3 m matr = 2 o matrmtf = 2 m matrmtf = 3
139	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	o matr = 3 m matr = 14 o matrmtf = 3 m matrmtf = 5	o matr = 3 m matr = 14 o matrmtf = 3 m matrmtf = 5	o matr = 3 m matr = 14 o matrmtf = 3 m matrmtf = 5
140	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 2000$	o matr = 3 m matr = 3 o matrmtf = 3 m matrmtf = 4	o matr = 3 m matr = 3 o matrmtf = 3 m matrmtf = 4	o matr = 3 m matr = 3 o matrmtf = 3 m matrmtf = 4
141	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 10 o matrmtf = 1 m matrmtf = 3	o matr = 3 m matr = 1 o matrmtf = 3 m matrmtf = 1
142	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	o matr = 1 m matr = 5 o matrmtf = 1 m matrmtf = 3	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 3
143	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 480$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
144	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
145	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 20 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 5 o matrmtf = 1 m matrmtf = 4
146	$\|I\| = 20$ $\|S\| = 100$ $\|E\| = 860$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 3	o matr = 3 m matr = 3 o matrmtf = 4 m matrmtf = 3	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 3
147	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 192$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
148	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 192$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
149	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 192$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
150	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 344$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 2
151	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 344$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
152	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 344$	o matr = 2 m matr = 1 o matrmtf = 4	o matr = 2 m matr = 1 o matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m matrmtf = 1	m matrmtf = 1	m matrmtf = 1
153	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 496$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3
154	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 496$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
155	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 496$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
156	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 648$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
157	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 648$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 4 m matrmtf = 2
158	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 648$	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 5	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
159	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 800$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 4
160	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 800$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
161	$\|I\| = 20$ $\|S\| = 40$ $\|E\| = 800$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
162	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 1200$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
163	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 1200$	o matr = 3 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 3 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 3 m matr = 1 o matrmtf = 2 m matrmtf = 1
164	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 1200$	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2
165	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 288$	o matr = 332 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2
166	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 288$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
167	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 288$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
168	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 3 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
169	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
170	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 516$	o matr = 2 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 2 m matr = 6 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
171	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	o matr = 6 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
172	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 10 m matr = 1 o matrmtf = 2 m matrmtf = 1
173	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 744$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 6 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
174	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 3 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1
175	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 4
176	$\|I\| = 20$ $\|S\| = 60$ $\|E\| = 972$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 7	o matr = 9 m matr = 2 o matrmtf = 4 m matrmtf = 2
177	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	o matr = 4 m matr = 3 o matrmtf = 2 m matrmtf = 3	o matr = 3 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 3 m matr = 4 o matrmtf = 3 m matrmtf = 6
178	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3	o matr = 2 m matr = 4 o matrmtf = 2 m matrmtf = 6	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
179	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1296$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 3	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 5 o matrmtf = 2 m matrmtf = 3
180	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	o matr = 13 m matr = 3 o matrmtf = 4 m matrmtf = 5	o matr = 13 m matr = 3 o matrmtf = 4 m matrmtf = 5	o matr = 13 m matr = 3 o matrmtf = 4 m matrmtf = 5
181	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 4	o matr = 1 m matr = 4 o matrmtf = 1 m matrmtf = 4
182	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 1600$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
183	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
184	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 384$	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 4	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 3 m matr = 1 o matrmtf = 4 m matrmtf = 1
185	$\|I\| = 20$	o matr = 7	o matr = 3	o matr = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 80$ $\|E\| = 384$	m matr = 314 o matrmtf = 3 m matrmtf = 2	m matr = 1 o matrmtf = 4 m matrmtf = 1	m matr = 6 o matrmtf = 1 m matrmtf = 3
186	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 3
187	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
188	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 688$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 3	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 11 o matrmtf = 1 m matrmtf = 2
189	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	o matr = 2 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 6 m matr = 2 o matrmtf = 3 m matrmtf = 2
190	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 3	o matr = 7 m matr = 3 o matrmtf = 4 m matrmtf = 4	o matr = 18 m matr = 2 o matrmtf = 4 m matrmtf = 2
191	$\|I\| = 20$ $\|S\| = 80$ $\|E\| = 992$	o matr = 6 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 1 m matr = 7 o matrmtf = 1 m matrmtf = 4	o matr = 4 m matr = 1 o matrmtf = 6 m matrmtf = 1
192	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
193	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
194	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1104$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 3 m matrmtf = 2
195	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
196	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
197	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1452$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
198	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
199	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
200	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 1800$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
201	$\|I\| = 30$ $\|S\| = 60$	o matr = 1 m matr = 1	o matr = 2 m matr = 1	o matr = 1 m matr = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|E\| = 408$	o matrmtf = 1 m matrmtf = 1	o matrmtf = 2 m matrmtf = 1	o matrmtf = 1 m matrmtf = 1
202	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
203	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 408$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
204	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
205	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 3 m matr = 1 o matrmtf = 2 m matrmtf = 1
206	$\|I\| = 30$ $\|S\| = 60$ $\|E\| = 756$	o matr = 5 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
207	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
208	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 6 o matrmtf = 1 m matrmtf = 3	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1
209	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1134$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
210	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	o matr = 3 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 3 m matr = 5 o matrmtf = 3 m matrmtf = 2
211	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
212	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 1656$	o matr = 2 m matr = 1 o matrmtf = 6 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 4
213	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	o matr = 2 m matr = 2 o matrmtf = 3 m matrmtf = 4	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1
214	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	o matr = 2 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 3 o matrmtf = 3 m matrmtf = 3
215	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2178$	o matr = 3 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 3	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
216	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	o matr = 4 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 4 m matr = 1 o matrmtf = 4 m matrmtf = 1	o matr = 4 m matr = 1 o matrmtf = 4 m matrmtf = 1
217	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	o matr = 1 m matr = 1 o matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
		m matrmtf = 1	m matrmtf = 1	m matrmtf = 1
218	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 2700$	o matr = 1 m matr = 6 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 6 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 6 o matrmtf = 1 m matrmtf = 2
219	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 651 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
220	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
221	$\|I\| = 30$ $\|S\| = 90$ $\|E\| = 612$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
222	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
223	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
224	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1328$	o matr = 1 m matr = 5 o matrmtf = 1 m matrmtf = 4	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
225	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 3	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
226	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	o matr = 3 m matr = 2 o matrmtf = 3 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
227	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 1952$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
228	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	o matr = 3 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
229	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 3
230	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 2576$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
231	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 3	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 3	o matr = 2 m matr = 3 o matrmtf = 2 m matrmtf = 3
232	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
233	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 3200$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
234	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
235	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
236	$\|I\| = 40$ $\|S\| = 80$ $\|E\| = 704$	o matr = 1 m matr = 129 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
237	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
238	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2
239	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 1080$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
240	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
241	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
242	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 2060$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
243	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
244	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 2 m matr = 2 o matrmtf = 2 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
245	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 3040$	o matr = 2 m matr = 1 o matrmtf = 3 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
246	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 5	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
247	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	o matr = 1 m matr = 3 o matrmtf = 1 m matrmtf = 2	o matr = 3 m matr = 1 o matrmtf = 2 m matrmtf = 1	o matr = 2 m matr = 1 o matrmtf = 2 m matrmtf = 1
248	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 4020$	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 2 o matrmtf = 1 m matrmtf = 2	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
249	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1
250	$\|I\| = 50$	o matr = 1	o matr = 1	o matr = 1

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	Instance	Original ordering	$(d-u)+$ ordering	$(d-u)-$ ordering
	$\|S\| = 100$ $\|E\| = 5000$	m matr = 2 o matrmtf = 1 m matrmtf = 2	m matr = 2 o matrmtf = 1 m matrmtf = 2	m matr = 2 o matrmtf = 1 m matrmtf = 2
251	$\|I\| = 50$ $\|S\| = 100$ $\|E\| = 5000$	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1	o matr = 1 m matr = 1 o matrmtf = 1 m matrmtf = 1

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