Module 4 – Personal Income Taxes

Background

- In this module, you will be calculating personal income taxes. You'll start with some simplified examples of calculations and build up to the real tax brackets. In all cases, to calculate the taxes you need to pay, you must first compute your *taxable* income. You then calculate the tax owed based on the tax rate schedules. You will not address other adjustments such as tax credits.

- The Excel workbook for this module consists of 3 tabs.
  1. The "HTC 1" tab contains information relevant to the tax code in Part 1.
  2. The "HTC 2" tab contains information relevant to the tax code in Parts 2 and 3.
  3. The "Actual Tax" tab contains information relevant to the tax code in Part 4.

- For this module, you will compute your taxes as an individual person with a "single" filing status.
  - First, the tax code provides for "deductions," meaning certain expenses that you incur can be subtracted from your taxable income. Examples of expenses that are tax deductible include state income taxes paid, interest payments on home mortgages, real estate taxes, and contributions to charities. When paying taxes, you can either deduct the total amount of these expenses (called "itemized deductions") or you can deduct a set amount determined by tax law (called the "standard deduction"). For the tax year 2020 (i.e., the taxes due in April 2021), the **standard deduction** for a single person is $12,400.
  - Second, the tax code allows a "personal exemption" for you, your spouse, and each of your dependents. The **personal exemption** is a set amount, and for the tax year 2020, the personal exemption is $0.
  - If you are a single tax filer with $47,500 in income and you take the standard deduction, you would only have to pay taxes on $47,500-$12,400-$0 = $35,100. The $12,400 and the $0 are the real numbers from the tax code for the 2020 tax year; the numbers are adjusted every year to account for inflation.

Part 0: Piecewise-defined Function

The U.S. income tax structure provides an excellent application of piecewise-defined functions.

1) To begin this Module, view the screencast *Piecewise-defined Functions*.

Part 1: Hypothetical Tax Code 1

Under Hypothetical Tax Code 1, the tax rate on all taxable income is 25%. For all of these questions, assume you take the standard deduction and one personal exemption using the values given in the background information.

1) If you earn $47,500 a year, how much tax would you pay?

2) If you earn $47,500 a year, what percentage of your total income would you pay in taxes?

3) If you earn $8,000 a year, how much would you pay in taxes? What percentage of your total income would you pay in taxes?

4) Write a function (by hand) to model the total tax paid, T(x), for a single person with a total income of x dollars.

5) Create a graph that shows taxes paid as a function of total income for total income between $0 and $80,000. In order to build a table of values to reference in the graph, you may find a Data Table useful. If you are unfamiliar with the Data Table tool, first view the screencast *Data Table Demo*.

6) Create a graph that shows the percentage of income that you would pay in taxes as a function of total income for total income between $0 and $80,000.

7) Do people who have higher total income pay more or less in taxes than people who have lower total income?

8) Do people who have higher total income pay a higher or lower percentage of their income than people who have lower total income?

Part 2: Hypothetical Tax Code 2

Under Hypothetical Tax Code 2, the tax rate on the first $25,000 of *taxable income* is 18% and the tax rate on any additional income is 27%. Another way to explain these rates is as follows:

- For taxable income from $0 to $25,000, you pay 18% of your taxable income in taxes, plus
- For taxable income above $25,000, you pay 27% of your taxable income.

Remember that there is a difference between *total income* (i.e., your salary from your employer plus other money you earn) and *taxable income* (your total income minus your deductions and your personal exemptions). Throughout these exercises, assume that you use the standard deduction. Please use the values given in the background information for those quantities.

1) The following assume a total income of $47,500 under Hypothetical Tax code 2.
   a. With a total income of $47,500 what would be your *taxable income*?

   b. Of the taxable income calculated above, how much of it would be taxed at 18%? How much of it would be taxed at 27%?

   c. Complete the following tables assuming a total income of $47,500:

Tax Brackets (for *Taxable Income*)				
Lower	Upper	Rate	Money in Bracket	Tax per Bracket
$ 0	$25,000	18%		
$25,000	No Limit	27%		

Tax Brackets (for *Total Income*)				
Lower	Upper	Rate	Money in Bracket	Tax per Bracket
$ 0				$0

		18%		
	No Limit			$2,727

      d.   What is the total tax owed under Hypothetical Tax Code 2?

2) Write a function (by hand) to model the total tax paid, T(x), for a single person with a total income of x dollars for Hypothetical Tax Code 2.

Set up the calculations in Excel and fill in the following table based on Hypothetical Tax Code 2. The values from this question are in a data file for this exercise. Later, you will use the real tax brackets, with 7 different levels, and you will want to do those calculations in Excel, so it's good to practice with this simpler example. For the Percent of Total Income Paid in Taxes, show your answer as a percentage with two decimal places.

Hypothetical Tax Code 2		
Total Income	Taxes Owed	Percent of Total Income Paid in Taxes
$5,000	$0	
$15,000		
$25,000	$2,268	9.07%
$35,000		
$45,000	$6,552	14.56%

If this is set up properly in Excel, then it's easy to see how things change when the numbers change.

3) Create a graph that shows taxes paid as a function of total income for total income between $0 and $80,000.

4) Create a graph that shows the percentage of income that you would pay in taxes as a function of total income for total income between $0 and $80,000.

Part 3: Hypothetical Tax Code 3 (HTC 3)

HTC 3 is exactly the same as Hypothetical Tax Code 2, except that the two tax rates are 8% and 22%. The standard deduction and personal exemptions still apply, the tax rate on the first $25,000 of income is now 8%, and the tax rate on any additional income is now 22%.

1) Make a copy of your "HTC 2" tab and change the rates to create a sheet for calculating HTC 3. Use it to fill in this table:

Hypothetical Tax Code 3		
Total Income	**Taxes Owed**	**Percent of Total Income Paid in Taxes**
$5,000	$0	
$15,000		
$25,000	$1,008	
$35,000		
$45,000	$3,672	

2) How much **more** do you pay in taxes under Hypothetical Tax Code 2 compared to Hypothetical Tax Code 3,
   a. If your total income is $21,000?

b. If your total income is $32,000?

3) What component(s) of the expression that you wrote for the function that defines T(x) for HTC 2 need to be changed to define the analogous function for HTC 3?

Part 4: Real Tax Code

Now that you have your spreadsheet working, use the real tax brackets. You may need to adjust your formulas from the hypothetical tax codes so that you can drag your formulas in a way that handles all the brackets correctly.

Here are the actual tax brackets for the 2020 tax year for a single tax filer:

- For taxable income from $0 to $9,875, you pay 10% of it in taxes, plus
- For taxable income from $9,875 to $40,125, you pay 12% of it in taxes, plus
- For taxable income from $40,125 to $85,525, you pay 22% of it in taxes, plus
- For taxable income from $85,525 to $163,300, you pay 24% of it in taxes, plus
- For taxable income from $163,300 to $207,350, you pay 32% of it in taxes, plus
- For taxable income from $207,350 to $518,400, you pay 35% of it in taxes, plus
- For taxable income above $518,400, you pay 37%.

Note: The actual tax code contains additional complications, like the reduction in the personal exemptions at certain levels. Ignore these complications and consider only the tax brackets defined above, a fixed personal exemption, and fixed standard deduction.

1) Create a spreadsheet that calculates taxes for any level of total income. Using your spreadsheet, fill in the table:

Real Tax Code		
Total Income	Taxes Owed	Percent of Total Income Paid in Taxes

$5,000	$0.00	0.00%
$20,000		
$45,000	$3,714.50	
$60,000		
$75,000	$9,562.00	12.75%
$95,000		
$115,000		16.26%

2) Write a function (by hand) to model the total tax paid, T(x), for a single person with a total income of x dollars.

3) Create a graph that shows the amount of tax paid as a function of total income. Use a range of $0 to $80,000 for total income.

4) Create a graph that shows the percentage of income paid in taxes as a function of income earned. Use a range of $0 to $80,000 for income earned.

5) Looking at your second graph, estimate the total income of a person who is paying 10.00% of his or her total income in taxes.

6) Your spreadsheet can help you find that total income (where the person is paying exactly 10.00% of his or her total income in taxes) more exactly, to within $100.
   Try the guess-test-revise method by adjusting the number in the Total Income cell.   What is the total income of a person who is paying 10.00% of his or her total income in taxes?

7) Now, try using the Goal Seek tool (which is one of the What-If Analysis tools in Excel) to answer this same question. If you are unfamiliar with the Goal Seek tool, first view the screencast *Goal Seek*
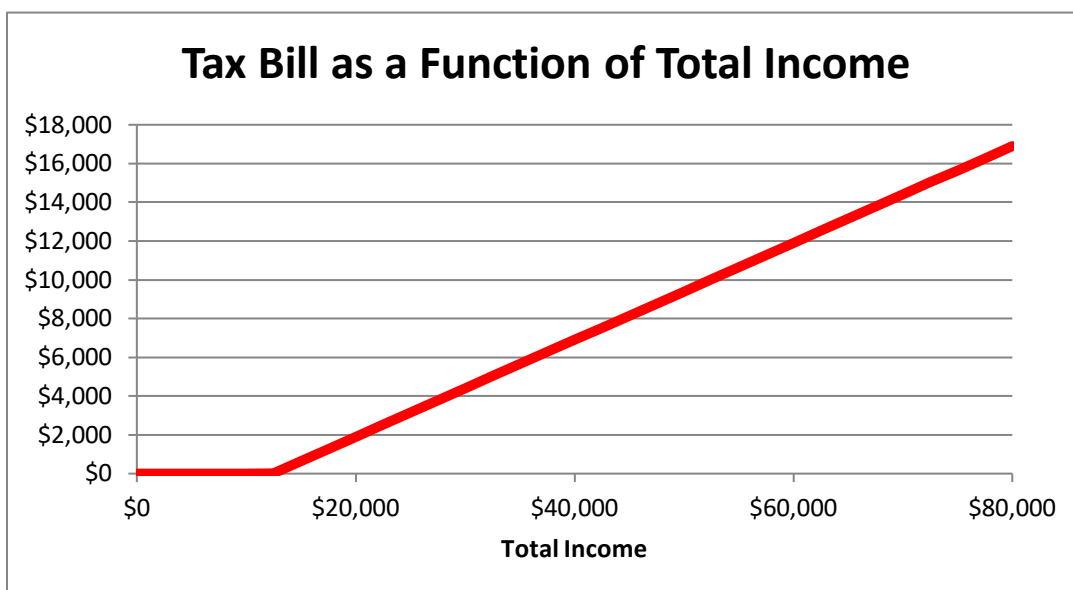
Demo.

(If you use Goal Seek, does the answer depend on the initial value of Total Income in sheet when you start? Why?)

8) Compare the Real Tax Code to Hypothetical Tax Code 3. For what ranges of total income is the tax bill higher under the Real Tax Code?
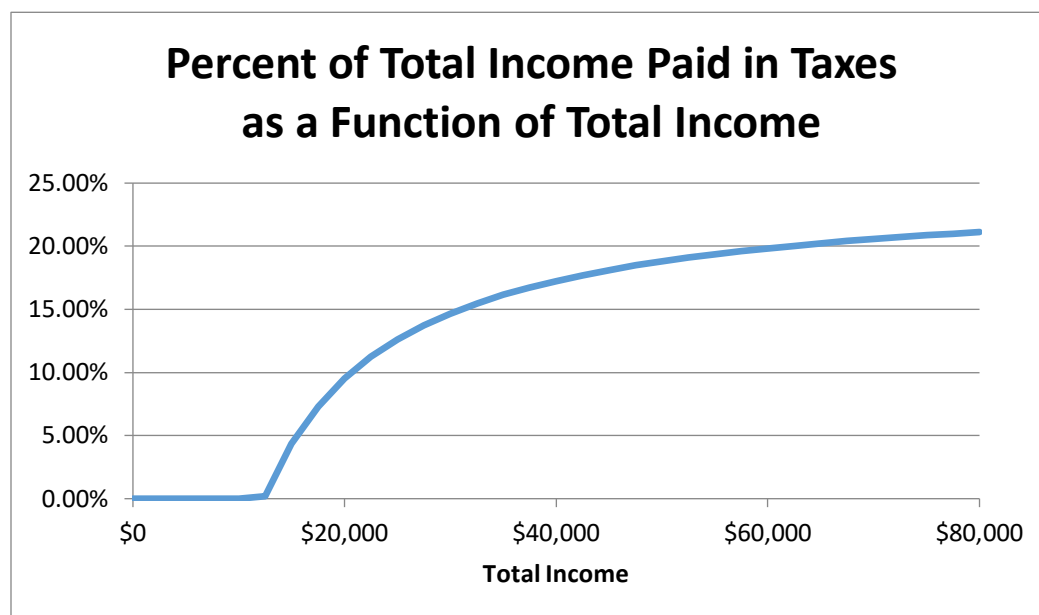
Selected Answers

Part 1

1) $8,775.00
2) 18.47%
3) $0.00, which is 0.00% of total income
4) $T(x) = \begin{cases} 0 & x \leq 12{,}400 \\ 0.25(x - 12{,}400) & x > 12{,}400 \end{cases}$
5) Graph:

Tax Bill as a Function of Total Income

6) Graph:

Percent of Total Income Paid in Taxes as a Function of Total Income

7) Higher
8) Higher

Part 2

1) a. $35,100

   b. $25,000, $10,100

   d. $7,227

2) $T(x) = \begin{cases} 0 & x \leq 12{,}400 \\ 0.18(x - 12{,}400) & 12{,}400 < x \leq 37{,}400 \\ 4{,}500 + 0.27(x - 37{,}400) & x > 37{,}400 \end{cases}$

3) Graph:

Tax Bill as a Function of Total Income

4) Graph:

Percent of Total Income Paid in Taxes as a Function of Total Income
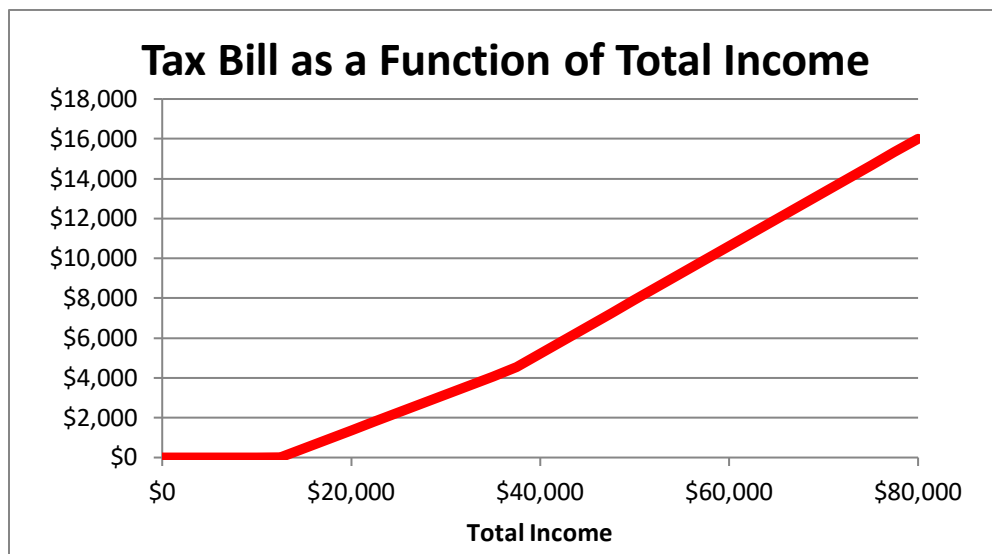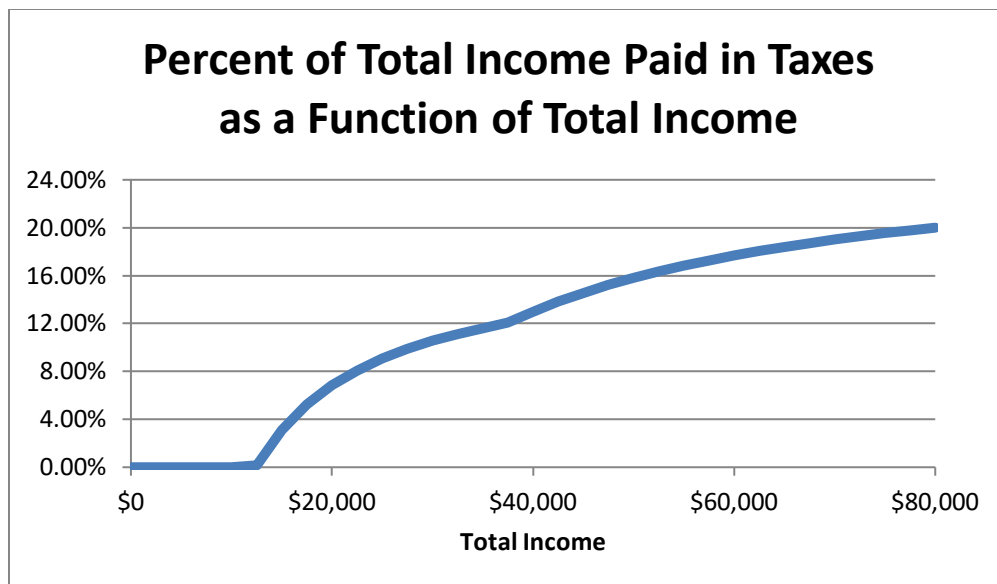
Part 3

2) (a) $860.00 (b) $1,960.00

3) Here is the T(x) function for HTC3. The parts that are different from HTC2 are highlighted. Note that it is not just the marginal rates (8% and 22%) that change, but also the constant term in the last part.
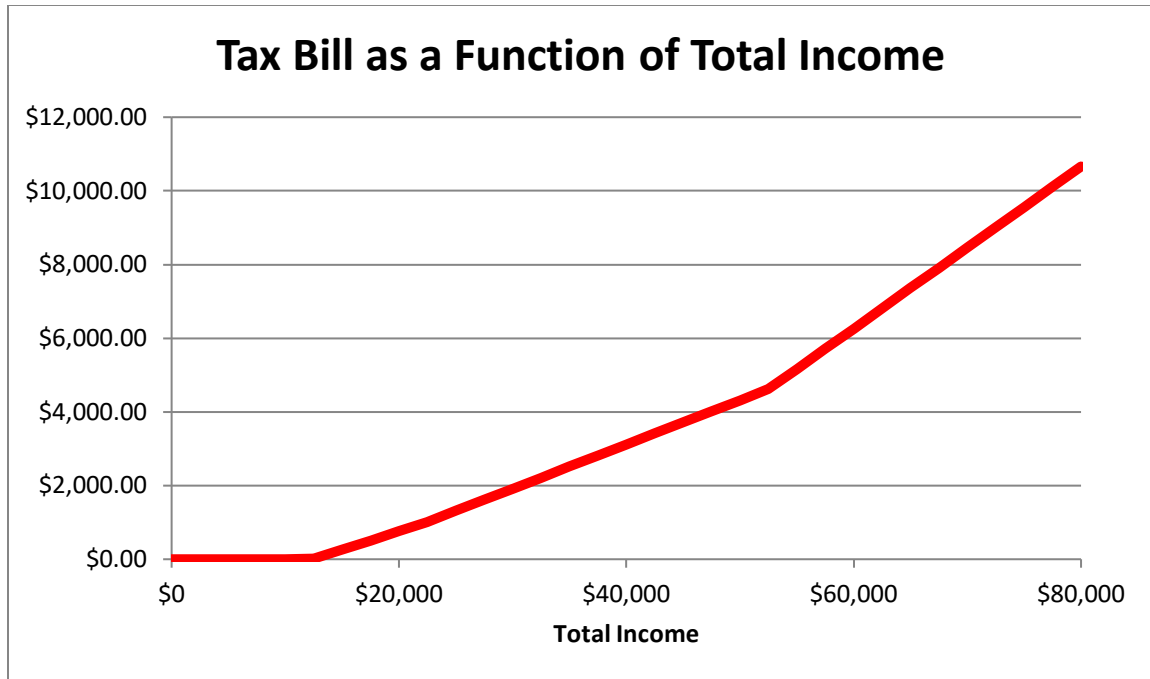
$$T(x) = \begin{cases} 0 & x \leq 12{,}400 \\ 0.08(x - 12{,}400) & 12{,}400 < x \leq 37{,}400 \\ 2{,}000 + 0.22(x - 37{,}400) & x > 37{,}400 \end{cases}$$
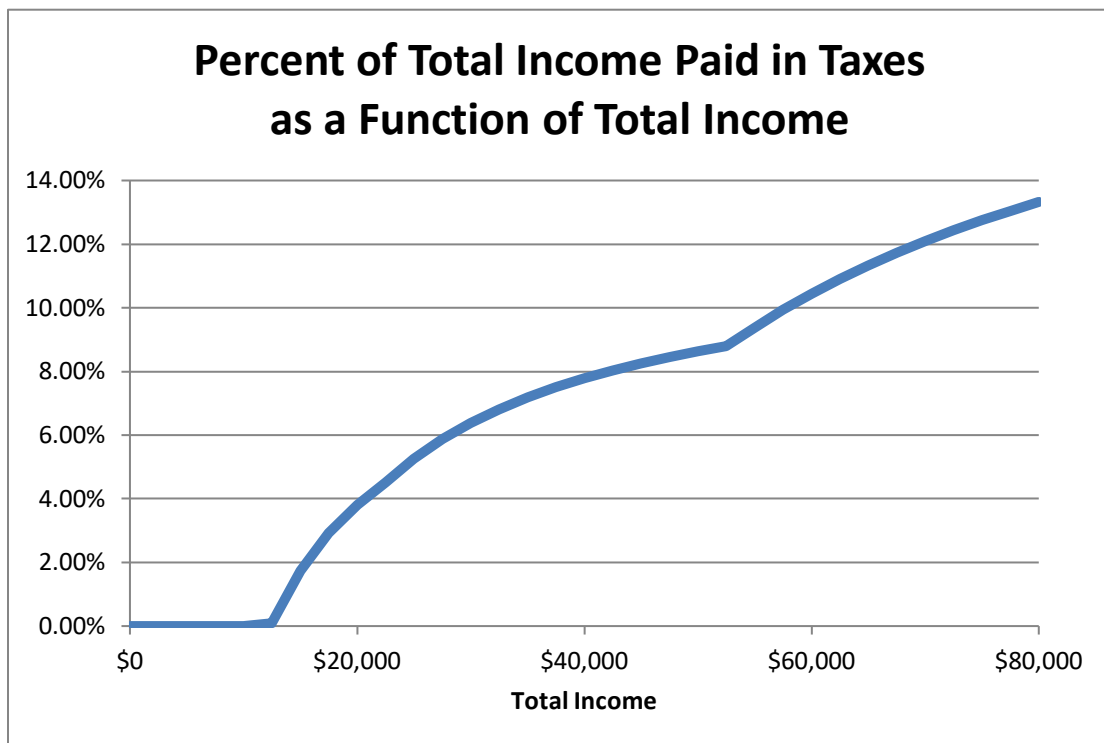
Part 4

2) Here are the first few parts of the function:

$$T(x) = \begin{cases} 0 & x \leq 12{,}400 \\ 0.10(x - 12{,}400) & 12{,}400 < x \leq 22{,}275 \\ 987.50 + 0.12(x - 22{,}275) & 22{,}275 < x \leq 52{,}525 \\ 4{,}617.50 + 0.22(x - 52{,}525) & 52{,}525 < x \leq 97{,}925 \\ etc. \end{cases}$$

3) Graph

Tax Bill as a Function of Total Income

4) Graph

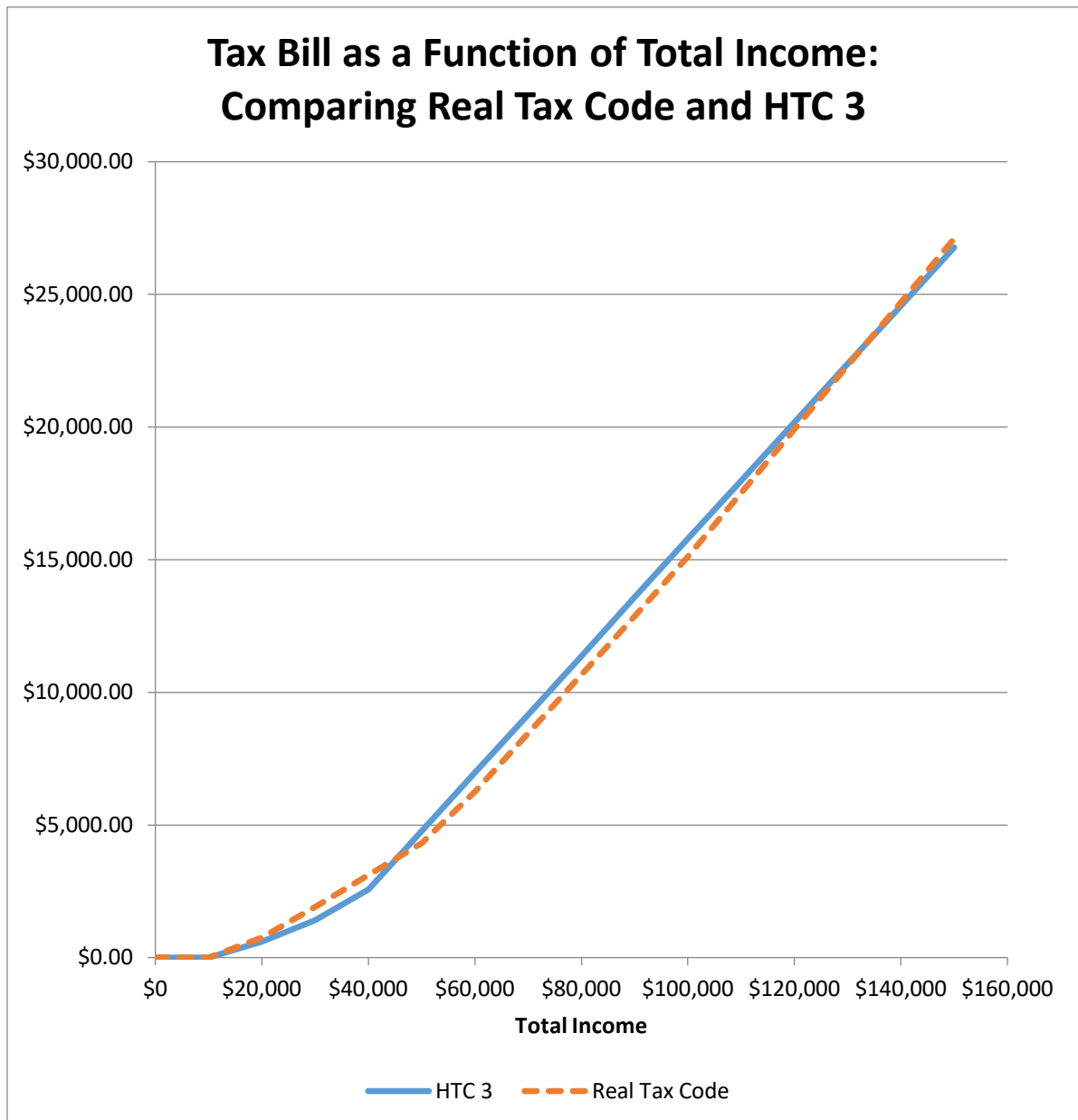## Percent of Total Income Paid in Taxes as a Function of Total Income

5-7) The exact answer is $57,816.67, but answers within about plus or minus $40 from that answer will give 10.00% to two decimal places.

8) The Real Tax Code is higher for total incomes greater than $12,400 up to $45,425.00 (when both tax bills are $3,765.50), and is also higher at total incomes greater than $133,425.00 (when both tax bills are $23,125.50). You can see the crossing points in the graph below, but to find the exact values, Goal Seek is useful.

Tax Bill as a Function of Total Income: Comparing Real Tax Code and HTC 3