

# Ultra-low noise monolithic mode-locked solid-state laser: supplementary material

TYKO D. SHOJI,<sup>1</sup> WANYAN XIE,<sup>1</sup> KEVIN L. SILVERMAN,<sup>2</sup> ARI FELDMAN,<sup>2</sup> TODD HARVEY,<sup>2</sup> RICHARD P. MIRIN,<sup>2</sup> AND THOMAS R. SCHIBLI<sup>1,3,\*</sup>

<sup>1</sup>Department of Physics, University of Colorado, Boulder, CO 80309-0390, USA

<sup>2</sup>National Institute of Standards and Technology, 325 Broadway, Boulder, CO 80305, USA

<sup>3</sup>JILA, NIST, and the University of Colorado, Boulder, CO 80309-0440, USA

\*Corresponding author: [trs@colorado.edu](mailto:trs@colorado.edu)

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This document provides supplementary information to “Ultra-low-noise monolithic mode-locked solid-state laser,” <http://dx.doi.org/10.1364/optica.3.000995>. To motivate the design strategy of the monolithic mode-locked laser, it is helpful to understand how the system parameters influence the fundamental noise in mode-locked lasers. The summary below is applicable to any optical mode-locked oscillator. © 2016 Optical Society of America

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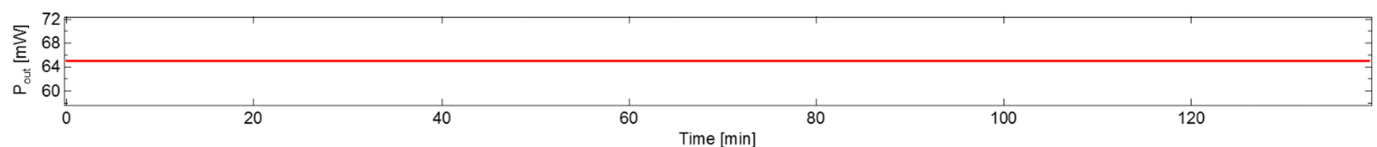


Fig. S1. Average output power of the monolithic laser over 140 minutes, showing <0.03% RMS fluctuation and an average mode-locked output power of 65 mW (24 W average intracavity).

## 1. SIMPLIFIED PROJECTED NOISE PERFORMANCE OF MODE-LOCKED LASERS

Low noise microwave generation through optical frequency division sets stringent requirements on the timing jitter and amplitude noise performance of the free-running optical frequency comb. The timing jitter of the pulse train is the timing error  $\Delta t$  of the power-weighted center of the pulse from the ideal equally-spaced temporal positions defined by the pulse repetition rate  $f_{\text{rep}}$ . In optical frequency division, the timing jitter directly translates into phase noise on the microwave carrier due to the relation  $\Delta\phi \approx 2\pi \cdot \Delta t \cdot f_{\text{osc}}$ , where  $f_{\text{osc}}$  is the carrier frequency of the extracted microwave and  $\Delta t$  is assumed to be a small fraction of an oscillator period.  $\Delta t$  can be greatly reduced by locking the optical frequency comb to a stable optical reference. However, in practice the achievable feedback bandwidth will set an upper limit of how much of this intrinsic jitter can be removed. It is therefore important that the free-running optical comb's timing jitter performance exceeds required performance at

frequencies above the achievable feedback bandwidth. Another important performance requirement is the amplitude (AM) noise of the pulsed laser, as nonlinearities in the photodetection process could convert AM noise to phase noise in the extracted microwave carrier.

We briefly summarize the dominant noise processes that typically limit the timing jitter and amplitude noise performance of a mode-locked laser. For simplicity, we ignore Kramers-Kronig-induced jitter due to a fluctuating gain as this effect does not seem to be dominant in most solid-state MLLs. As is common for characterizing the quality of microwave oscillators, we express all phase-noise terms in the form of single sided phase-noise power spectral densities, denoted as  $L(f)$ . Contributions from shot-noise are discussed in the context of photodetection. A full description of these topics can be found in [1-5].

### A. Timing jitter phase noise PSD directly driven by Amplified Spontaneous Emission (ASE)

During each round trip the pulse is subject to spontaneous emission noise from the gain medium (i.e., vacuum

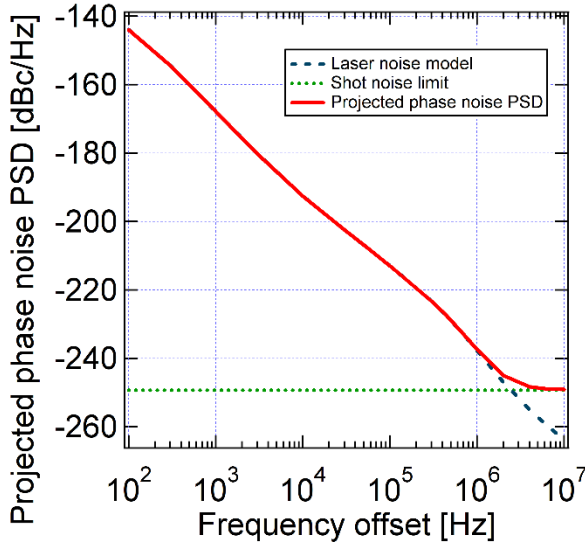


Fig. S2. Projected timing jitter phase noise spectral power density of the 1 GHz optical pulse train from the free-running monolithic laser.

fluctuations that couple through the gain to the laser light). This leads to a timing jitter that is typically referred to as the quantum-limited timing jitter. This noise dominates most of the noise spectrum in low-loss bulk lasers. For a  $\text{sech}^2$ -shaped pulse envelope (based on the soliton model) one can calculate the resulting single-sided phase noise spectral density for 1 Hz bandwidth at a frequency  $f$  away from the extracted microwave carrier  $f_{\text{osc}}$ :

$$L_{\text{ASE}}(f) \approx 0.26\theta g \frac{h\nu}{P} \left( \frac{f_{\text{rep}} f_{\text{osc}} \tau_p}{f} \right)^2, \quad (\text{S1})$$

where  $f_{\text{rep}}$  denotes the repetition rate,  $P$  the average intracavity power, and  $\theta$  the excess noise factor of the laser gain ( $\theta \geq 2$  in Er-doped fiber due to the quasi three-level nature of that laser material).  $g$  stands for the round-trip intensity gain and is equal to the round-trip loss,  $h\nu$  is the photon energy of the laser light, and  $\tau_p$  is the FWHM pulse duration.

### B. Phase noise PSD due to Gordon-Haus jitter

Besides the ASE-induced timing jitter discussed above, ASE also leads to fluctuations in the center frequency of the optical spectrum. Such optical frequency fluctuations couple to timing error as a second-order effect. This noise term usually dominates the timing jitter spectral density in soliton fiber lasers with large values of round-trip dispersion. For  $\text{sech}^2$ -shaped pulses (again assuming a soliton model) the resulting phase noise spectral density can be summarized as:

$$L_{\text{GH}}(f) \approx 0.25\theta g \frac{h\nu}{P} \left( \frac{f_{\text{rep}} f_{\text{osc}} \tau_p}{f} \right)^2 \frac{D^2 \Gamma_g^4}{g^2 + 9.26(f/f_{\text{rep}})^2 \Gamma_g^4 \tau_p^4}, \quad (\text{S2})$$

where  $D$  denotes the intracavity group delay dispersion and  $\Gamma_g$  is the HWHM gain bandwidth or the cavity spectral bandwidth, whichever is smaller, in units of rad/s. The other parameters are the same as above. Gigahertz-jitter is often the dominant noise term in mode-locked fiber lasers.

### C. Phase noise PSD due to self-steepening

Amplitude-to-phase coupling inside the oscillator occurs predominantly through the self-steepening effect [6]. This noise term usually dominates the noise spectrum at low Fourier frequencies:

$$L_{\text{ss}}(f) = \frac{1}{2} \left( \frac{f_{\text{rep}} f_{\text{osc}} \varphi_{\text{NL}}}{\pi f \nu} \right)^2 S_{\text{RIN}}(f), \quad (\text{S3})$$

where  $\varphi_{\text{NL}}$  is the total nonlinear phase shift per round-trip, which is a sum of  $\varphi_{\text{NL}}$  due to Kerr-nonlinearities and slow saturable absorber response, if present.  $S_{\text{RIN}}(f)$  is the intensity noise power spectral density of the laser.

It should be noted that all these noise spectra scale with the repetition rate squared. Therefore, a fair comparison of laser performance should consider this fact. The choice of repetition rate will likely be a compromise between noise performance and desired repetition rate. As demonstrated in this letter, one can achieve ultra-low noise performance even at 1 GHz repetition rate if the cavity round-trip loss is low (here  $< 1\%$  including SESAM and output coupler), the average power inside the cavity is high (here, 24 W) and the pulses are short (currently  $< 110$  fs). Large modes are beneficial to reduce the coupling between the amplitude and phase quadratures of the noise spectrum, which is usually important in OFD applications.

Based on these noise processes, the total estimated free-running phase noise PSD of the 1 GHz optical pulse train is shown in Fig. S2. This estimated noise floor is far below the detection limit of even the best commercial microwave phase noise analyzers. However, the theoretical model used in Fig. S2 is supported by the sub-attosecond timing-jitter measurements published by Hou et al. [7].

## 2. FREE-RUNNING $f_{\text{ceo}}$ LINEWIDTH ESTIMATE

A worst case estimate can be calculated by looking at the -91 dBc/Hz servo-bump at 98 kHz offset in Fig. 5, red trace. A FWHM Lorentzian line shape with 48 Hz is required to get single-sided noise spectral power at that level, and the free-running noise PSD is certainly significantly below the peak of that servo bump.

## References

1. H. A. Haus, and A. Mecozzi, "Noise of mode-locked lasers," IEEE J. Quantum Electron. **29**, 983–996 (1993).
2. R. Paschotta, "Noise of mode-locked lasers (Part I): numerical model," Appl. Phys. B **79**, 153–162 (2004).
3. R. Paschotta, "Noise of mode-locked lasers (Part II): timing jitter and other fluctuations," Appl. Phys. B **79**, 163–173 (2004).
4. F. X. Kärtner, I. D. Jung, and U. Keller, "Soliton mode-locking with saturable absorbers," IEEE J. Sel. Top. Quantum Electron. **2**, 540–556 (1996).
5. J. P. Gordon and H. A. Haus, "Random walk of coherently amplified solitons in optical fiber transmission," Opt. Lett. **11**, 665–667 (1986).
6. H. A. Haus, and E. P. Ippen, "Group velocity of solitons," Opt. Lett. **26**, 1654 (2001).
7. D. Hou, C.-C. Lee, Z. Yang, and T. R. Schibli, "Timing jitter characterization of mode-locked lasers with  $< 1$  zs/VHz resolution using a simple optical heterodyne technique," Opt. Lett. **40**, 2985–2988 (2015).