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A NOTE ON THE EDGE ADMITTANCE OF A WIDE MICROSTRIP PATCH WITH ELECTRICALLY THIN SUBSTRATE

by

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Abstract

The reflection coefficient of a TEM wave normally incident at the truncated upper edge of a slab-loaded parallel-plate waveguide is computed from a knowledge of the tangential electric field on the top of the slab beyond the truncated edge. To approximate this "aperture" field, we use the static edge field, which results in the correct value of reflection coefficient to first order when the substrate is electrically thin. The approach of this note has implications for several problems of microstrip patch antennas and transmission lines.

1. INTRODUCTION

Analysis of microstrip patch antennas can be carried out by several techniques. A "numerically exact" approach can be used in which an integral equation for the surface electric current on the patch is formulated, and solved using, e.g., a moment method [1]. This method can be cumbersome, since the kernel of the integral equation is a Sommerfeld integral which itself requires a fair amount of numerical analysis to compute. Moreover, the unknown current must be found over the entire two-dimensional surface of the patch, while the only peculiar behavior it actually exhibits is near the edge where its singularities appear.

Other, approximate, methods are used which provide insight into the physics of the problem. The cavity model [2] is widely used, in which the fields are reckoned mainly to be under the patch and obey a two-dimensional Helmholz equation. Opinions differ, however, on what conditions should be enforced on these fields at the edges of the patch. Different ways of accounting for the fringing fields at the edge can result in resonant frequency shifts of the same order as the bandwidth, making design a difficult procedure [3]. Once accurate fringe fields at the edges are available, however, the radiation field of the patch can be accurately computed using the equivalent aperture or magnetic-current method [4] [5].

In this report we will address a simple problem--that of reflection of an incident TEM wave at the edge of a semi-infinite patch conductor.

This problem has an exact formal solution by the Wiener-Hopf method [6], [7], but extracting simple but accurate approximations for the case of an electrically thin substrate is a formidable mathematical task [3]. Also, the extension of the Wiener-Hopf method to deal with the finite patch of arbitrary shape does not seem to be possible.

We will formulate the problem as an integral equation for the electric field in the "aperture" which extends beyond the edge of the semi-infinite plane conductor along the top of the substrate to infinity. We find that an accurate expression for the reflection coefficient for the thin-substrate limit can be obtained by using known expressions for the static electric field near the edge. The idea is similar to one used some time ago by Leppington and Levine [8] to find the edge correction for the capacitance of a circular disk capacitor. This method promises to be generalizable to a patch of fairly arbitrary shape, and offers considerable efficiency insofar as it deals with an unknown (the aperture magnetic current) which is significant only over a narrow strip near the edge.

2. THE APERTURE INTEGRAL EQUATION

Consider the geometry of Fig. 1. A TEM wave in the parallel plate region $0 < z < d$ is normally incident at the edge of the upper conductor, which occupies $z = d$ from $-\infty < x < 0$. We consider the incident wave

$$E_z^i = e^{-ikx} \; ; \quad H_y^i = - \frac{e^{-ikx}}{\zeta} \qquad (0 < z < d) \tag{1}$$

where $k = k_0\sqrt{\varepsilon_r}$, $\zeta = \zeta_0/\sqrt{\varepsilon_r}$, ε_r is the relative permittivity of the substrate, and $k_o = \omega\sqrt{\mu_o\varepsilon_o}$, $\zeta_o = \sqrt{\mu_o/\varepsilon_o}$. The scattered wave will consist of only three field components, inasmuch as $\partial/\partial y = 0$:

$$\left.\begin{aligned} &H_y^s, \quad E_z^s = \frac{1}{i\omega\varepsilon}\frac{\partial H_y^s}{\partial x} \\ &E_x^s = E_x = -\frac{1}{i\omega\varepsilon}\frac{\partial H_y^s}{\partial z} \end{aligned}\right\} \tag{2}$$

We assume a time-dependence of exp($i\omega t$).

To formulate the requisite integral equation for the problem, we first define Green's functions for the free space region S_1 above the substrate ($z > d$) and for the substrate region S_2($0 < z < d$) itself (Fig. 2). We denote these by G_1 and G_2 respectively, and define them by the conditions

$$\left.\begin{aligned} &(\nabla^2 + k_o^2)G_1 = -\delta(x-x')\delta(z-z') \qquad (z, z' > d) \\ &\left.\frac{\partial G_1}{\partial z'}\right|_{z'=d} = 0 \ ; \quad G_1 \to 0 \ \text{as} \ \sqrt{(x-x')^2 + (z-z')^2} \to \infty \\ &\qquad\qquad \text{if Im}(k_o) < 0 \end{aligned}\right\} \tag{3}$$

and

$$\left.\begin{aligned} &(\nabla^2 + k^2)G_2 = -\delta(x-x')\delta(z-z') \qquad (0 < z, z' < d) \\ &\left.\frac{\partial G_2}{\partial z'}\right|_{z'=0,d} = 0 \ ; \quad G_2 \to 0 \ \text{as} \ \sqrt{(x-x')^2 + (z-z')^2} \to \infty \\ &\qquad\qquad \text{if Im}(k) < 0 \end{aligned}\right\} \tag{4}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$. These functions are readily obtained by standard methods:

$$G_1(x,x';z,z') = -\frac{i}{4}\{H_0^{(2)}(k_oR) + H_0^{(2)}(k_oR_1)\} \tag{5}$$

where

$$R = \sqrt{(x-x')^2 + (z-z')^2} \quad \text{and} \quad R_1 = \sqrt{(x-x')^2 + (z+z'-2d)^2} \; ,$$

$H_o^{(2)}$ is the Hankel function of the second kind; and

$$G_2(x,x';z,z') = -\frac{i}{d}\left\{\frac{e^{-ik|x-x'|}}{2k} + \sum_{m=1}^{\infty} \frac{\cos\frac{m\pi z}{d}\cos\frac{m\pi z'}{d}}{\xi_m} e^{-i\xi_m|x-x'|}\right\} \tag{6}$$

where

$$\xi_m = [k^2 - (\frac{m\pi}{d})^2]^{\frac{1}{2}} = -i\,[\,(\frac{m\pi}{d})^2 - k^2]^{\frac{1}{2}} \tag{7}$$

Let us apply Green's theorem to the functions H_y^s and G_1 on the region S_1 and to H_y^s and G_2 on the region S_2. We find that

$$\begin{aligned} H_y^s(x,z) &= -\int_o^{\infty} G_1(x,x';z,d) \left.\frac{\partial H_y^s(x',z')}{\partial z'}\right|_{z'=d^+} dx' \\ &= i\omega\varepsilon_o \int_o^{\infty} G_1(x,x';z,d)E(x')dx' \end{aligned} \tag{8}$$

for $z > d$, and

$$\begin{aligned} H_y^s(x,z) &= \int_o^{\infty} G_2(x,x';z,d) \left.\frac{\partial H_y^s(x',z')}{\partial z'}\right|_{z'=d^-} dx' \\ &= -i\omega\varepsilon_o\varepsilon_r \int_o^{\infty} G_2(x,x';z,d)E(x')dx' \end{aligned} \tag{9}$$

We have introduced the shorthand notation

$$E(x) = E_x(x,d) \tag{10}$$

for the tangential electric field on the aperture $z = d$, $x > 0$.

The reflected TEM wave underneath the upper conductor in $x < 0$ can be identified using (9). By (2) we have

$$E_z^s = -\frac{\partial}{\partial x}\int_o^{\infty} G_2(x,x';z,d)E(x')dx' \tag{11}$$

But the portion of the right side of (11) due to the reflected TEM wave can be identified as coming from the first term of G_2 as given in (6). Thus,

$$E^r_{z,TEM} = -\frac{\partial}{\partial x}\int_0^\infty [-\frac{i}{2kd}e^{-ik|x-x'|}]E(x')dx'$$
$$= -\frac{1}{2d}e^{ikx}\int_0^\infty E(x')e^{-ikx'}dx' \qquad (12)$$

since $x < 0$. Since $E^i_z = 1$ at $x = 0$, we identify the reflection coefficient Γ as

$$\Gamma = -\frac{1}{2d}\int_0^\infty E(x)e^{-ikx}dx \qquad (13)$$

The aperture field $E(x)$ can, in principle, be solved for from an integral equation which we obtain by enforcing continuity of the tangential H-field across the aperture at $z = d$, $x \geq 0$. From (8) and (9), this integral equation is

$$\int_0^\infty [K_f(x,x') + \varepsilon_r K_d(x,x') - \frac{i\varepsilon_r}{2kd}e^{-ik|x-x'|}]E(x')dx'$$
$$= \frac{i\varepsilon_r}{k}e^{-ikx} \qquad (x \geq 0) \qquad (14)$$

where the kernels are defined by

$$K_f(x,x') = G_1(x,x';d,d) = -\frac{1}{2}H_0^{(2)}(k_0|x-x'|) \qquad (15)$$

$$K_d(x,x') = G_2(x,x';d,d) + \frac{i}{2kd}e^{-ik|x-x'|} = -\frac{i}{d}\sum_{m=1}^\infty \frac{e^{-i\xi_m|x-x'|}}{\xi_m} \qquad (16)$$

3. APPROXIMATE EVALUATION OF Γ

Our strategy here will be to make a reasonable approximation to $E(x)$ in order to evaluate Γ. If the substrate is electrically thin, $kd \ll 1$, it is reasonable to suppose that, near the edge ($k|x| \ll 1$), $E(x)$ should be proportional to the static electric field distribution for the same geometry. Let us denote the static field distribution in the aperture $0 < x < \infty$ for a voltage difference V between upper plate and ground plane by $E_0(x)$:

$$\int_0^\infty E_0(x)dx = V \tag{17}$$

At a point $x < 0$ a few d behind the edge underneath the upper plate, we should still be in the quasistatic region near the edge, and yet all the higher-order cutoff modes ($m \geq 1$) have attenuated to negligible levels, leaving a TEM field of

$$E_z(x,z) \simeq 1 + \Gamma \qquad (x < 0;\ |x| \gg d;\ \ k|x| \ll 1) \tag{18}$$

and a voltage from the upper to lower plate of $-(1 + \Gamma)d$. Therefore, the quasistatic approximation to $E(x)$ should be

$$E(x) \simeq -\frac{(1+\Gamma)d}{V} E_0(x) \tag{19}$$

Now, the solution $E_0(x)$ to the static problem is also known [9]-[11], being obtained by modified Wiener-Hopf methods (though much simpler in form than the exact solution for arbitrary kd [6], [7]). Substitutiong (19) into (13), we can evaluate the resulting integral using eqns. (2.19) and (2.16) of [10]. The result, unfortunately, does not quite agree with the asymptotic development of the exact result given in [3] for $kd \ll 1$.

An alternative method for finding Γ accurately to $O(kd)$ makes use of eqn. (14) with $x = 0$. Combining it with (13) gives the expression

$$1 - \Gamma = -\frac{ik}{\varepsilon_r}\int_0^\infty [K_f(0,x') + \varepsilon_r K_d(0,x')]E(x')dx' \tag{20}$$

Now, for $kd \ll 1$, we can write approximately

$$\left.\begin{aligned} K_d(0,x') &\simeq \frac{1}{\pi}\sum_{m=1}^{\infty}\frac{e^{-m\pi x'/d}}{m} = -\frac{1}{\pi}\ln(1 - e^{-\pi x'/d}) &(21)\\ K_f(0,x') &\simeq -\frac{i}{2} - \frac{1}{\pi}\left(\ln\frac{k_0 x'}{2} + \gamma\right) &(22)\end{aligned}\right\}(x' > 0)$$

where $\gamma = 0.5772...$ is Euler's constant.

Using these together with (17) and (19) in (20) gives

$$\frac{1-\Gamma}{1+\Gamma} \approx \frac{k_o d}{\sqrt{\varepsilon_r}} \left\{ \frac{1}{2} - \frac{i}{\pi} \left(\ell n \frac{k_o d}{2} + \gamma \right) - \frac{i}{\pi} \int_0^\infty \frac{E_o(x)}{V} \left[\ell n \frac{x}{d} + \varepsilon_r \, \ell n (1 - e^{-\pi x/d}) \right] dx \right\} \tag{23}$$

The remaining integral is evaluated in the Appendix. From eqn. (A.17), we get

$$\frac{1-\Gamma}{1+\Gamma} \simeq \frac{k_o d}{\sqrt{\varepsilon_r}} \left\{ \frac{1}{2} - \frac{i}{\pi} \left[\ell n (k_o d) + \gamma - 1 + 2 \, \varepsilon_r Q_o(-\delta_\varepsilon) - \varepsilon_r \, \ell n \, 2\pi \right] \right\} \tag{24}$$

where $Q_o(-\delta_\varepsilon)$ is defined in the Appendix. Equation (24) agrees precisely with the result of [3] to order kd.

The success of using (20) instead of (13) appears to be due to the fact that only the difference $1 - \Gamma$ rather than Γ itself is being computed. This point should be kept in mind when extending the method to other situations.

4. CONCLUSION

The purpose of this brief note has been to show how a rigorous result for an edge reflection problem can be duplicated up to terms of O(kd) using only the static field near the edge. The method provides a close link with the equivalent aperture or magnetic current method used for computing radiation fields of microstrip patch antennas.

This approach has been used in a separate report to evaluate the edge correction for the static capacitance of a microstrip patch of arbitrary shape, generalizing the classical Kirchhoff formula for a circular disk with air substrate. Currently under study is an application to reflection of a non-normally incident wave from the edge of a semi-infinite upper conductor. In future studies, we hope to be able to formulate an edge-aperture theory for an arbitrarily-shaped patch on an electrically thin substrate.

APPENDIX

In this Appendix, we will evaluate the integral

$$\int_0^\infty E_o(x)[\ell n \frac{x}{d} + \varepsilon_r \ell n(1 - e^{-\pi x/d})]dx$$

where $E_o(x)$ is the static electric field on the aperture $z = d$, $0 < x < \infty$. To do this we first construct static, scalar Green's functions G_{01} and G_{02} for the regions S_1 and S_2 shown in Fig. 2. In S_1 $(z > d)$, we want

$$\nabla^2 G_{01} = -\delta(x-x')\delta(z-z'); \quad \left.\frac{\partial G_{01}}{\partial z'}\right|_{z'=d} = 0 \tag{A.1}$$

(We understand as before that $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$). We choose

$$G_{01} = -\frac{1}{2\pi}[\ell n \frac{\sqrt{(x-x')^2 + (z-z')^2}}{d} + \ell n \frac{\sqrt{(x-x')^2 + (z+z'-2d)^2}}{d}] \tag{A.2}$$

and for this we have as $\rho' = (x'^2 + z'^2)^{\frac{1}{2}} \to \infty$ in $(z, z') > d$:

$$G_{01} \sim -\frac{1}{\pi} \ell n \frac{\rho'}{d} + O(\frac{1}{\rho'}) \tag{A.3}$$

In S_2 $(0 < z < d)$, we want

$$\nabla^2 G_{02} = -\delta(x-x')\delta(z-z') ; \quad \left.\frac{\partial G_{02}}{\partial z'}\right|_{z'= 0,d} = 0 \tag{A.4}$$

for which we take the solution

$$G_{02} = -\frac{|x-x'|}{2d} + G_{02}^c \tag{A.5}$$

where

$$G_{02}^c = \frac{1}{\pi} \sum_{m=1}^{\infty} \frac{\cos\frac{m\pi z}{d} \cos\frac{m\pi z'}{d}}{m} e^{-m\pi|x-x'|/d} \tag{A.6}$$

We have in $0 < (z,z') < d$, as $|x - x'| \to \infty$:

$$G_{02}^{c} \sim O(e^{-\pi|x-x'|/d}) \tag{A.7}$$

We will apply these Green's functions in a variant of a formula due to Harrington ([12], eqn. (3-50)). Let $\bar{c}$ be any constant vector and let G be any scalar Green's function satisfying $\nabla^2 G = -\delta(x-x')\delta(z-z')$ as in (A.1) or (A.4). Let $\bar{E}$ be an electrostatic field so that $\nabla \times \bar{E} = 0$. Then using the divergence theorem and some vector identities, we can verify

$$\bar{c}\cdot\bar{E}(\bar{\rho}) = -\oint_C \bar{a}_n' \cdot \left\{[\bar{c}\times\nabla' G(\bar{\rho};\bar{\rho}')]\times\bar{E}(\bar{\rho}') + \bar{E}(\bar{\rho}')[\bar{c}\cdot\nabla' G]\right\} d\ell' \tag{A.8}$$

where $\bar{\rho} = (x,z)$ and $\bar{\rho}' = (x',z')$, and C is any closed contour in the xz-plane enclosing the point $\bar{\rho}$. We shall see that by choosing C to be C_1 or C_2 and G to be G_{01} or G_{02} that the boundary conditions on G allow for considerable simplification of the line integral in (A.8).

First put $G = G_{01}$, $\bar{c} = \bar{a}_z$, and take C to be C_1. Let $\bar{\rho} = \bar{\rho}_1 \in S_1$. Then by the boundary condition on G_{01} at $z' = d$ and the vanishing of the integral over the semicircular part of C_1 as $R \to \infty$, (A.8) reduces to

$$E_z(x_1,z_1) = \frac{\partial}{\partial x_1}\int_0^{\infty} E_0(x')G_{01}(x_1,z_1;x',d)dx' \tag{A.9}$$

where $E_0(x) \equiv E_x(x,d)$. We proceed analogously with the case when C is taken to be C_2 and $G = G_{02}$; however the first term in (A.5) must be treated carefully when taking the limit as $R \to \infty$. If $\bar{\rho}_2 \in S_2$, we obtain

$$E_z(x_2,z_2) = -\frac{\partial}{\partial x_2}\int_0^{\infty} E_0(x')G_{02}^{c}(x_2,z_2;x',d)dx' - \frac{1}{d}\int_{x_2}^{\infty} E_0(x')dx' \tag{A.10}$$

Now let $z_1 \to d^+$ and $z_2 \to d^-$, and invoke the boundary conditions on

E_z at z = d to get the following representation for the charge density at z = d:

$$\begin{aligned}\rho_s(x) &= \varepsilon_0[E_z(x,d+) - \varepsilon_r E_z(x,d-)] \\ &= \varepsilon_0 \frac{d}{dx}\int_0^\infty E_0(x')\,[K_1(x,x') + \varepsilon_r K_2(x,x')]dx' + \frac{\varepsilon_0\varepsilon_r}{d}\int_x^\infty E_0(x')dx'\end{aligned} \tag{A.11}$$

where

$$\left.\begin{aligned}K_1(x,x') &= G_{01}(x,d,x',d) = -\frac{1}{\pi}\,\ell n\,\frac{|x-x'|}{d} \\ K_2(x,x') &= G^c_{02}(x,d,x',d) \\ &= \frac{1}{\pi}\sum_{m=1}^{\infty}\frac{e^{-m\pi|x-x'|/d}}{m} = -\frac{1}{\pi}\,\ell n(1-e^{-\pi|x-x'|/d})\end{aligned}\right\} \tag{A.12}$$

We also have the conditions $E_0(x) = 0$ for $x < 0$ and

$$\int_0^\infty E_0(x)dx = V \tag{A.13}$$

where V is the voltage betwen the upper plate and the ground plane.

One could set up an integro-differential equation for determining E_0 by taking $x > 0$ in (A.11) and noting that $\rho_s \equiv 0$ in $x > 0$. This equation could presumably be solved by a variant of the Wiener-Hopf technique. However, we are not interested in the exact form of E_0 so much as in a certain integral of it, and for this it will be sufficient to use properties of $\rho_s(x)$ already known from previous Wiener-Hopf solutions of this static problem [9]-[11]. In particular [10], we know that

$$E_0(x) \sim \frac{Vd}{\pi\varepsilon_r x^2} \quad \text{as} \quad x/d \to \infty \tag{A.14}$$

and also that

$$Q(w) \equiv \int_{-w}^{0} \rho_s(x)dx$$
$$\sim \varepsilon_o V \left[\frac{\varepsilon_r w}{d} + \frac{1}{\pi} \ \ell n \ \frac{w}{d} + \frac{\varepsilon_r \ell n \, 2\pi + 1 - \ell n 2 - 2\varepsilon_r Q_o(-\delta_\varepsilon)}{\pi} \right]$$
$$+ \, O(\frac{d}{w} \ \ell n \ \frac{w}{d}) \quad \text{as} \quad \frac{w}{d} \to \infty \tag{A.15}$$

where

$$Q_o(-\delta_\varepsilon) = \sum_{m=1}^{\infty} (-\delta_\varepsilon)^m \ \ell n \ m; \quad \delta_\varepsilon = \frac{\varepsilon_r - 1}{\varepsilon_r + 1}$$

is a function discussed in [3] and for which simple closed-form approximations are available [13].

Upon inserting (A.11) into (A.15) and using (A.13), we obtain

$$\int_0^{\infty} E_o(x')[K_1(0,x') + \varepsilon_r K_2(0,x')]dx'$$
$$= \int_0^{\infty} E_o(x')[K_1(-w,x') + \varepsilon_r K_2(-w,x')]dx' + \frac{Q(w)}{\varepsilon_o} - \frac{\varepsilon_r V w}{d} \tag{A.16}$$

and as $w/d \to \infty$, the right side, by (A.12) and (A.15), reduces to

$$\int_0^{\infty} E_o(x')[K_1(0,x') + \varepsilon_r K_2(0,x')]dx = \frac{V}{\pi} \ [\varepsilon_r \ell n 2\pi + 1 - \ell n 2 - 2\varepsilon_r Q_o(-\delta_\varepsilon)] \tag{A.17}$$

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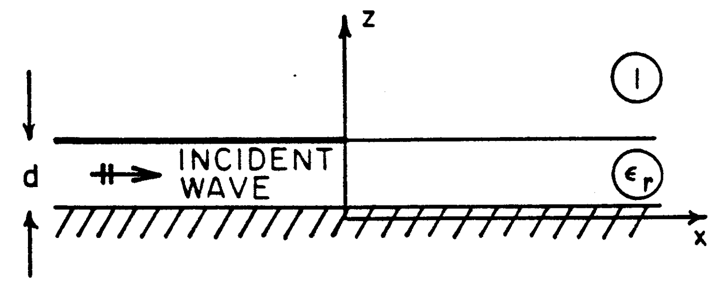

Fig. 1: TEM wave incident at the edge of a semi-infinite patch.

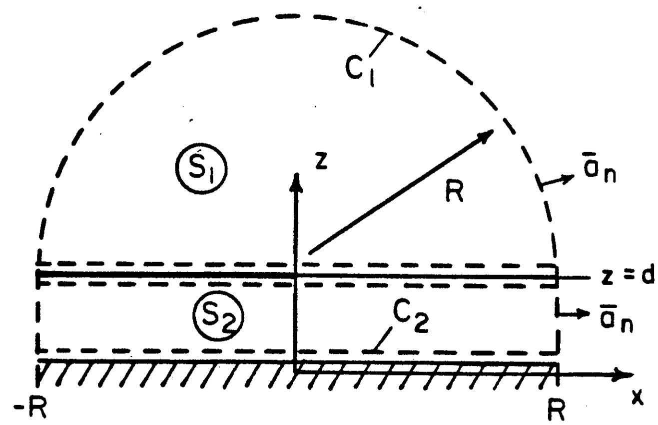

Fig. 2: Surfaces S_1 and S_2 for application of Green's theorem.