

Evaluation of a Three-Dimensional Model for Slurry Storage Facilities

by

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Evaluation of a Three-Dimensional Model for Slurry Storage Facilities

Thesis directed by Professor Dobroslav Znidarcic

Abstract

In many mining operations large quantities of tailings in a slurry form are produced; disposing of these tailings frequently requires construction of tailings impoundment facilities (ponds). A pseudo three-dimensional model has been developed to simulate the deposition and consolidation of tailings for a given site geometry. This three dimensional model or tailings management tool (TMT) gives the tailings surface height vs. time, void ratio distributions vs. time and average dry unit weight vs. time – which aids in the design of the facilities. This thesis evaluates the model and serves as a user manual for the model. Additional features that have been added to the model are discussed. Also several potential problems with the model are discussed and possible solutions presented; areas of future model improvement are also discussed. Additionally, a parametric study has been conducted and the results will be presented.

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1.0 Introduction

In many mining operations large quantities of waste tailings in a slurry form are produced. These tailings need to be collected and stored in a storage or impoundment facility, see figure 1.0.1. These storage facilities contain the tailings material which enables the material to consolidate and then eventually the land can be reclaimed, see Marcus (1997) for a more detailed discussion.



Figure 1.0.1 – shows a containment facility for tailings from a gold mine (Orosur Mining 2011)

Designers of these tailing storage facilities need to simulate the deposition and consolidation of tailings for a given site geometry, each facility will have a unique geometry, to ensure that the facility is of adequate size. The rate of consolidation of the slurry will depend on several factors.

The rate of consolidation will depend on the site geometry and the rate of deposition – the rate of deposition will most likely change with time as the mine production changes. Additionally the rate of consolidation will also depend on the boundary conditions at the bottom of the facility – is the water free to drain or is there an impermeable boundary. The boundary condition at the top will also affect the rate of consolidation; there could be evaporation or else a surcharge load applied to the top after deposition is completed. Finally the consolidation characteristics of the material will also affect the rate of consolidation; certain materials will consolidate faster than others, and the characteristics for each material will be unique.

For any consolidation model to accurately simulate the real world conditions, all the previous factors need to be considered in the analysis. Therefore a three-dimensional model was developed that takes into account all of the previously listed factors that affect the rate of consolidation. The model gives the height of the slurry vs. time, void ratio distributions vs. time and average dry unit weight vs. time – which aids in the design of the facilities. The development of the model will be discussed in sections 2.0 and 2.1.

Additionally in many cases the facilities themselves – specifically the embankment dam – are constructed of tailings material, as this is the most cost effective option. Therefore designers also need to know the engineering properties of the slurry such as the relationship between hydraulic conductivity and void ratio and the relationship between void ratio and effective stress. The model itself doesn't provide the hydraulic conductivity vs. void ratio and void ratio vs. effective stress, but the laboratory testing used to determine the consolidation characteristics does. This testing will be discussed in section 2.2.

2.0 Development of the One-Dimensional Model

The basis for the three-dimensional consolidation model is the one-dimensional consolidation and desiccation model developed in Yao et al. (2002); the one-dimensional model only considers flow and deformations in the vertical direction. The governing equation for the one-dimensional model is the one-dimensional consolidation equation first proposed in Gibson et al. (1967); see McVay et al. (1986) for a detailed derivation and discussion. This one-dimensional consolidation equation (Gibson et al. 1967) assumes that both the hydraulic conductivity and the void ratio change when the effective stress changes. The void ratio will change due to changes in effective stress and the hydraulic conductivity will change as the void ratio changes. The one-dimensional consolidation and desiccation model also assumes the following (Yao et al. 2002):

- The soil remains fully saturated and soil shrinkage is terminated when the void ratio reaches the void ratio at the shrinkage limit
- Only one-dimensional vertical flow is considered, which results in only vertical deformations
- The soil is assumed to be horizontally homogenous
- Creep is not considered

The relationship assumed in the one-dimensional model between the void ratio and effective stress is shown in equation 2.0.1 – e is void ratio, σ' is effective stress,

and A_1 , B_1 , and Z_1 are empirical parameters. The methodology for determining the empirical parameters A_1 , B_1 , and Z_1 will be explained in section 2.2.

$$e = A_1(\sigma' + Z_1)^{B_1} \quad (2.0.1)$$

The relationship between hydraulic conductivity and effective stress assumed in the one-dimensional model is shown in equation 2.0.2 – e is the void ratio, k is the hydraulic conductivity, and C and D are empirical parameters. The methodology for determining the empirical parameters C and D will be explained in section 2.2.

$$k = Ce^D \quad (2.0.2)$$

Using the previously mentioned relationships and assumptions, with Gibson's one-dimensional consolidation equation (Gibson et al. 1967) used as the governing equation, a one-dimensional consolidation model CONDES0 or CONDES was created (Yao et al. 2002). CONDES can also serve as a desiccation model, but this feature will not be discussed. CONDES considers the filling or deposition rate, height of the column, top and bottom boundary conditions, and possible surcharge loading when calculating one-dimensional consolidation. The one-dimensional model does not allow the user to specify units as an input, only numerical values. Thus the user is required to ensure that

the units used are consistent. The model has been shown to be accurate when compared to actual consolidation results (Bartholomeeusen et al. 2002).

Since CONDES is a one-dimensional consolidation model, area is not an input. Deformations and fluid flow are one-dimensional therefore a column being analyzed with CONDES will have no area.

Initially CONDES was written in FORTRAN and was run as an executable in MS-DOS; since its release changes have been made to the original CONDES. Currently CONDES is run as an Excel (Microsoft, 2010) macro which calls the original program's algorithm written in FORTRAN to perform the consolidation calculations. Graphs are then created in Excel of the void ratio distributions vs. time, and surface height vs. time.

2.1 Development of the Three-Dimensional Model

Similar to CONDES, the three dimensional model is also run as a macro in Excel. The program, or macro in Excel, is called TMT – tailing management tool (Coffin 2010). Similar to CONDES, TMT produces the graphs of surface height vs. time, void ratio distributions vs. time, and average dry unit weight vs. time. For additional details please see section 4, this section will serve as the user manual for TMT.

The three-dimensional consolidation model simulates consolidation in a three-dimensional storage facility by repeatedly using the one-dimensional consolidation model, CONDES, discussed in section 2.0. First, the methodology behind the model's three-dimensional representation of the actual storage facility will be discussed. Then

the methodology behind simulating three-dimensional consolidation in this three-dimensional representation will be discussed.

Each storage facility will have a unique site layout or geometry. The model uses user inputted height/area pairs, see figure 2.1.1, to generate the geometry of the facility. Initially the user would input the initial area, a_0 . Next the user would specify the height, h_1 , and the area at that new height, a_1 . This process would continue until the user has reached their desired height. The user can specify up to 50 height/area pairs. In all figures the user inputted heights will always be labeled with h 's, and referred to as h-heights; similarly the user inputted areas will always be labeled with a 's.

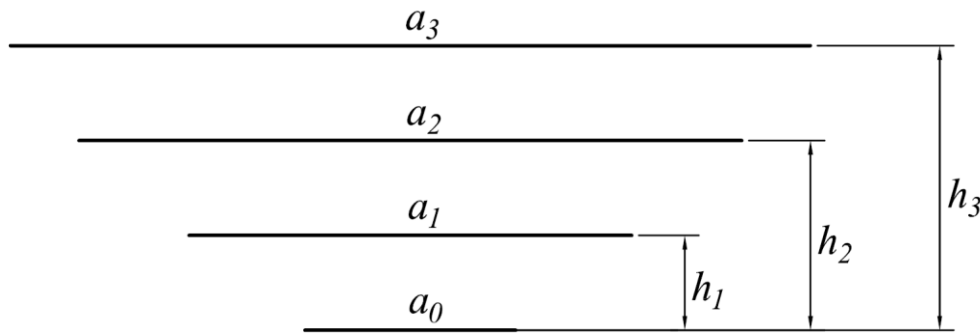


Figure 2.1.1 – the user inputted height/area pairs in profile view

Then these user inputted height/area pairs are represented as circular areas, see figure 2.1.2. The distribution or shape of the area at each height is not important because this will have no effect on the consolidation. It is important to note that due to Cavalieri's principle, also known as the principle of indivisibles, no volume is lost. Cavalieri's principle states if multiple parallel planes intersect two solids and at every corresponding

height the areas are equal then the volumes will also be equal (Weisstein 2012). This is the case with the model as well; the area at each height hasn't changed so the volumes are still equal.

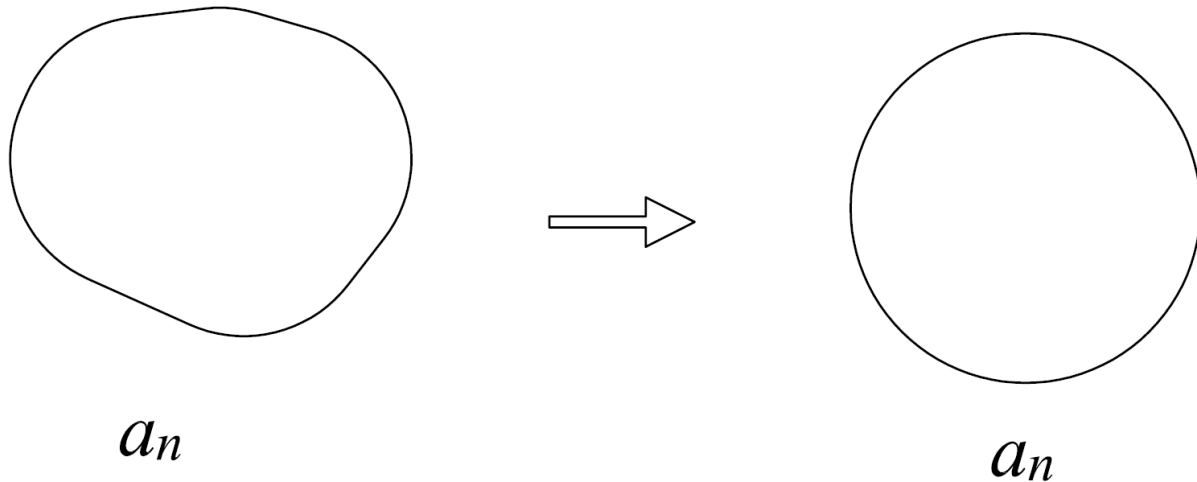


Figure 2.1.2 – user inputted area represented as a circular area shown in plan view

As mentioned previously, initially the user inputs the height/area pairs for the storage facility, see figure 2.1.1; these would most likely be taken from survey data of the storage facility or else known from the design plans. Figure 2.1.3 shows a graph of the height/area pairs taken from an actual storage facility. The user inputted ten height/area pairs and linear interpolation is used to approximate the slope between two adjacent height/area pairs. The actual volume – which is actually a very close approximation of the real world volume of the storage facility – can be found by integrating the area above this curve to the maximum height. The accuracy of this actual volume to the real-world volume of the storage facility will increase if more height/area pairs are inputted.

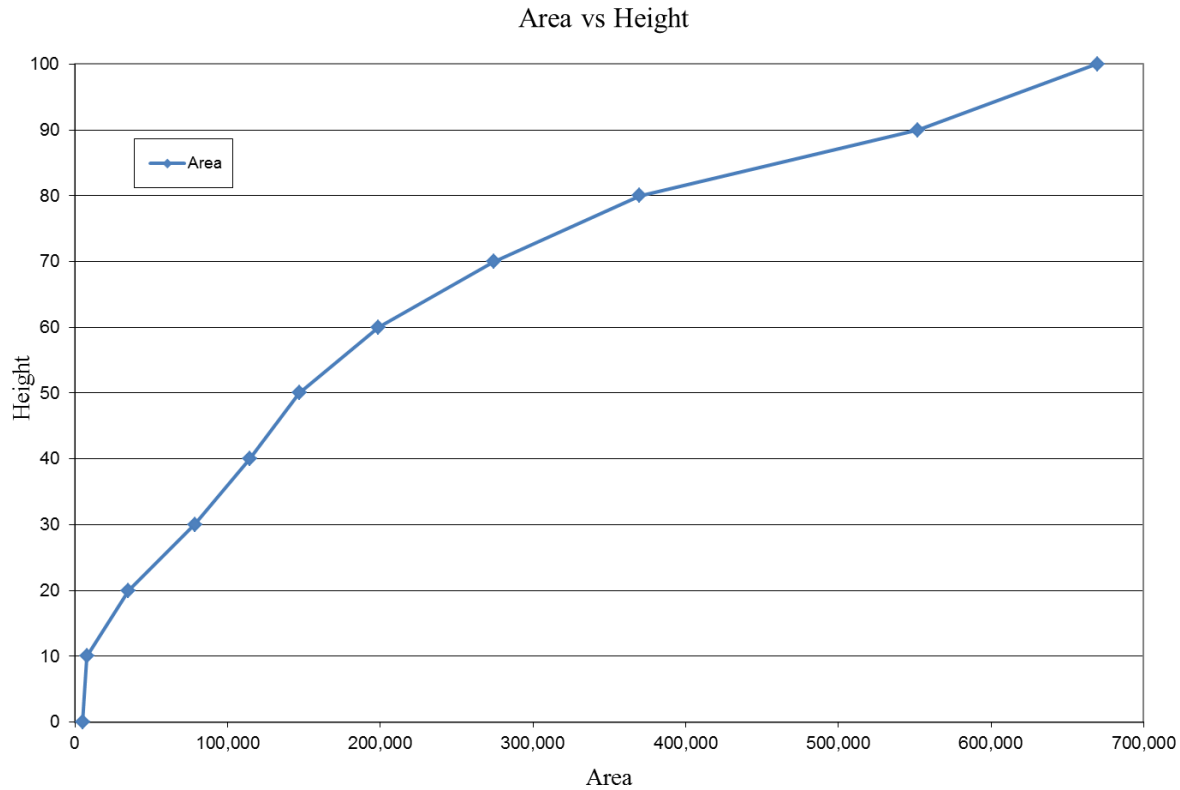


Figure 2.1.3 – graph of user inputted height/area pairs

The user can select how many columns they want to use in order to approximate the volume. Then depending on how many discretizing columns are selected, n , the height vs. area graph will be divided into $n-1$ segments. For example if the user selected four columns then the height vs. area curve would be divided into three straight line segments, see figure 2.1.4. The more columns that are chosen the closer the volume used in the model, or model volume, will be to the actual volume; however more columns are not always better, see section 5.4.

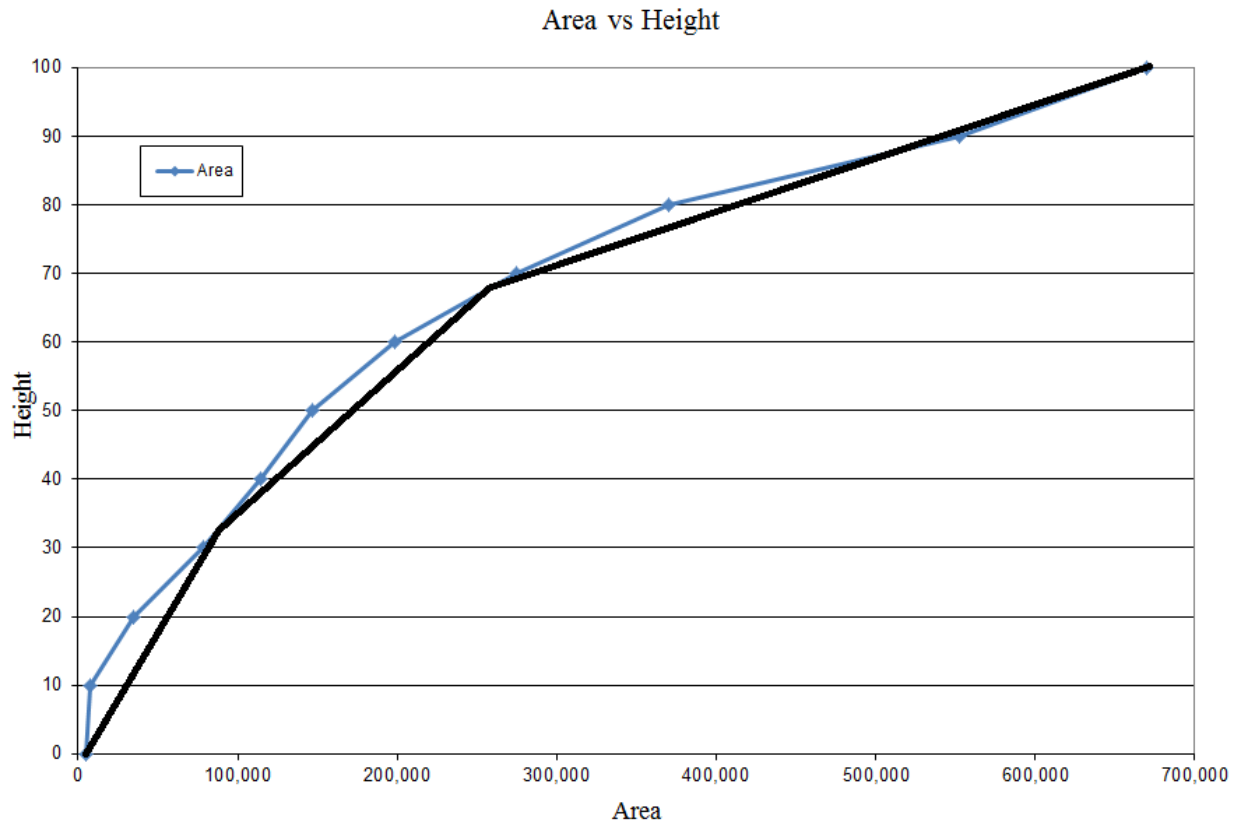


Figure 2.1.4 – height vs. area curve divided into three segments – four discretizing columns were chosen

Next the model uses an inverted cone frustum to approximate the volume defined by each line segment. For example, if four columns are chosen then the model will use three inverted cone frustums to approximate the volume, see figure 2.1.5. The first cone frustum would have a lower area equal to the initial area, a_0 , a height of 33.3 m and an upper area of 80,000 m² – this area corresponds to the area at the end of the first line segment. Similarly the second cone frustum would have a height of 33.3 m a lower area of 80,000 m² and an upper area of 260,000 m² – these areas correspond to the beginning

and end points of the middle segment. Lastly the third cone frustum would have a height of 33.3 m a lower area of approx. 80,000 m² and an upper area of 260,000 m². The third frustum would be similar to the first two. The initial area will always correspond to the initial area of the first inverted cone frustum and the final area of the final area of the last inverted cone frustum. The model considers this new discretized geometry of the storage facility to be the new geometry of the storage facility; the model volume is calculated from this discretized geometry. In all figures the heights of the cone frustums used in the discretization will be labeled with H 's and the discretized areas with A 's.

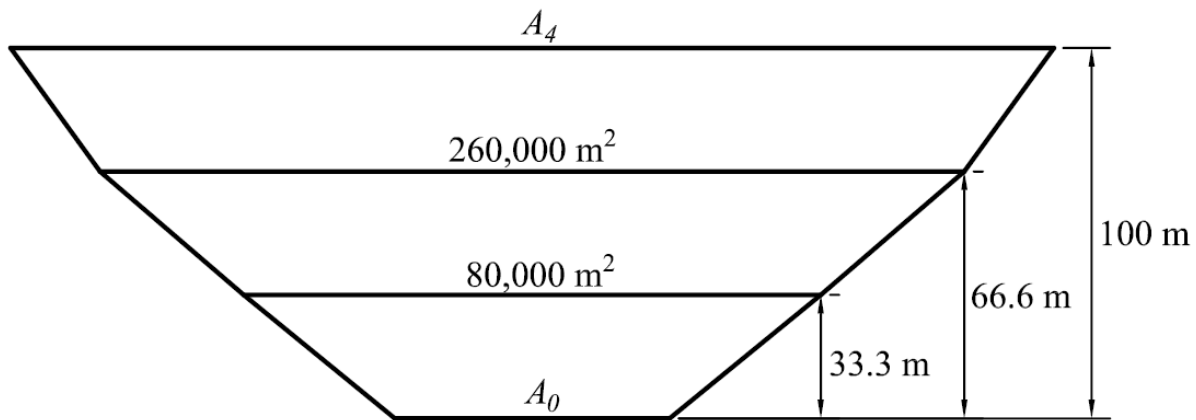
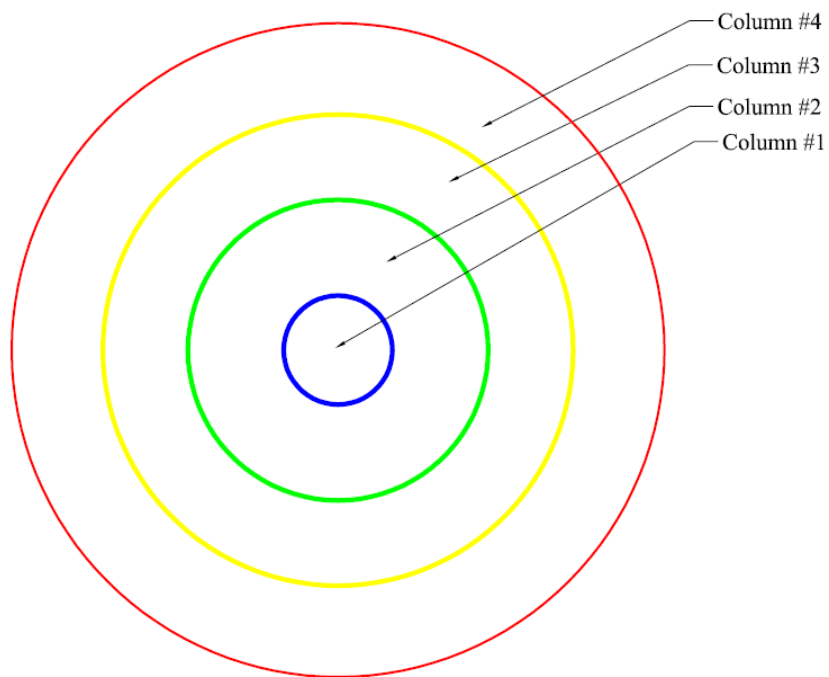
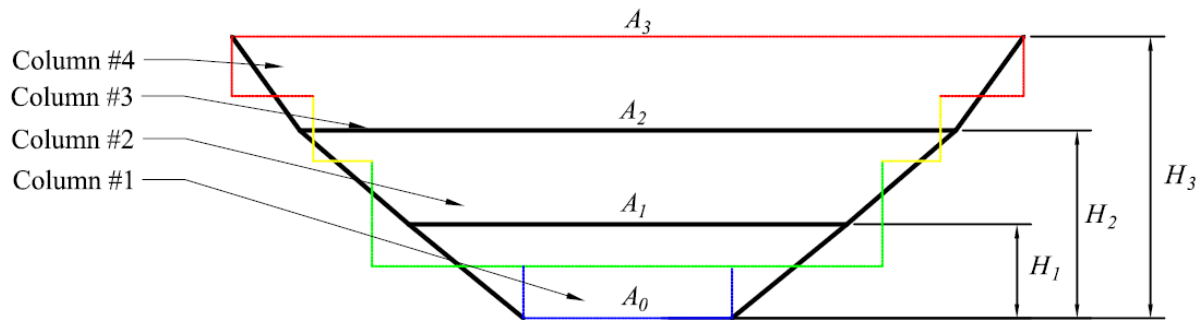


Figure 2.1.5 – Three inverted cone frustums are used to approximate the volume of the containment facility

The discretization of the storage facility is shown in figure 2.1.6. The storage facility, with the new simplified geometry, has its volume replaced by using a central cylindrical column and a number of hollow cylinder columns, or annuluses, concentric with the first column. Then the consolidation in each column can be determined using

CONDES. Each column will have a height and area – the height and area for the columns will not necessarily correspond to the user inputted height/area pairs nor the height area of the inverted cone frustums used in the discretization, see figure 2.1.6.



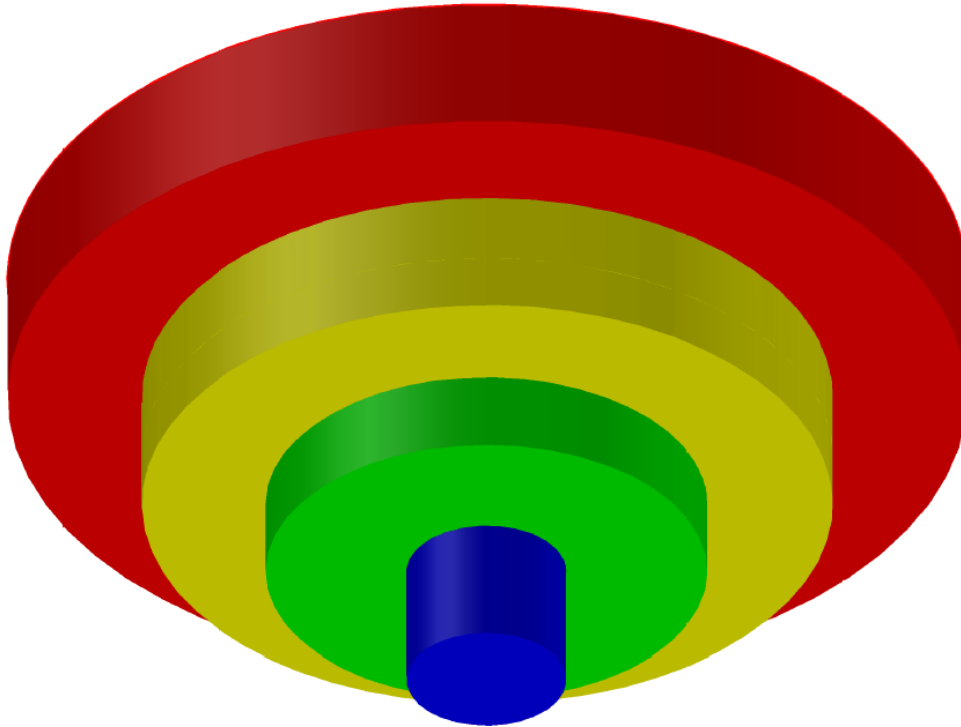


Figure 2.1.6 – the simplified geometry discretized into columns; top is a profile view, the middle is a plan view, and the third is an isometric 3-D view

The volume of each inverted cone frustum will be represented by two columns, one having the area of the lower base of the cone frustum and the other with an equal to the upper base of the cone frustum. The determination of the column heights will be discussed later in this section. In all figures the column heights will be labeled with F 's. Additionally the central or initial column will always be referred to as column #1; the adjacent column will be column #2, etc.

The geometry of the columns is what the model uses to perform the analyses; the F 's and A 's stand for fill-heights and fill-areas. The heights and areas of the columns

used in the discretization will change depending on how many columns are selected by the user due to the fact that the heights and areas of the inverted cone frustums used in the discretization will also change.

This discretization of the facility would still result in a two-dimensional consolidation problem because each column would be two-dimensional. In order to use CONDES the two-dimensional columns must be represented or transformed into one-dimensional columns. The two dimensional columns are represented as one dimensional columns by varying the filling rate that CONDES uses in the analysis. Column #1 is assigned an initial filling rate. Then for other columns, which will have larger areas, the filling rate is reduced accordingly; thus columns with a larger area will have a slower filling rate. Therefore as the area increase the filling rate will decrease, see figure 2.1.7. With a lower filling rate the one-dimensional column will take longer to fill and thus simulate a larger area.

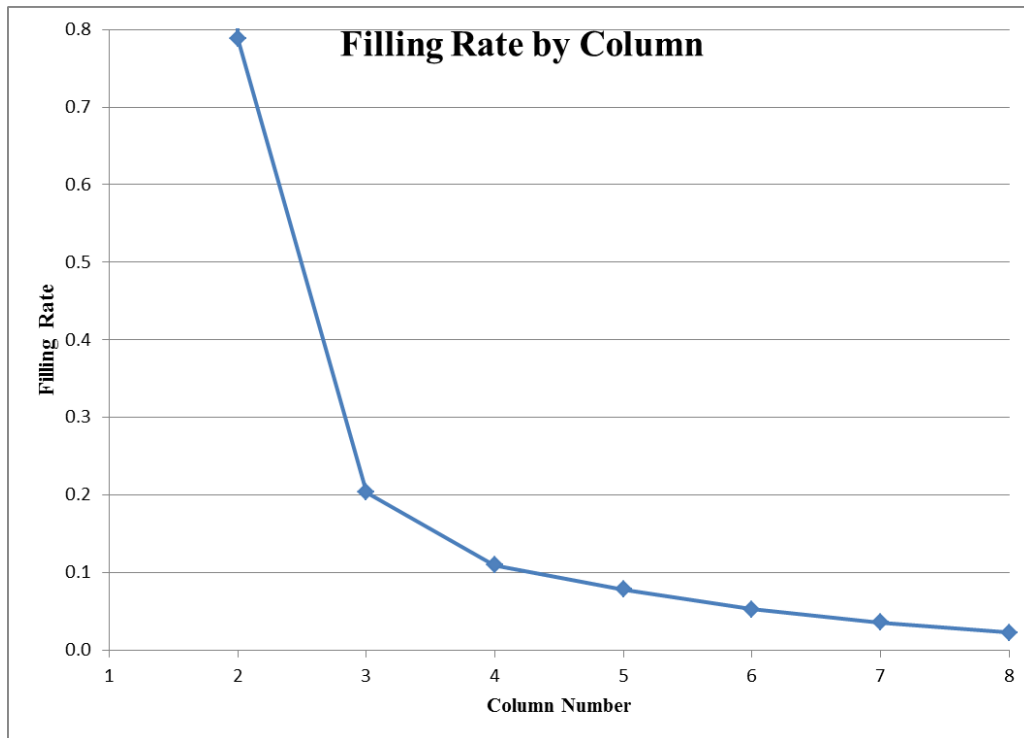


Figure 2.1.7 – graph of filling rates vs. column number taken from a sample problem; note the filling rate for column #1 is very high and off the chart, this problem will be discussed in section 5.1

The filling rate used in the analysis is the total production rate (dry mass/time) divided by the solids density & area (area of the specified column) and then multiplied by the initial void ratio plus one (initial specific volume); see equation 2.1.1 (Gjerapic et al. 2008). The initial void ratio is used because when the slurry is initially deposited no consolidation has occurred yet. The filling rate is dependent on the height of the surface level – which will determine the deposition area. This will give the rise in height with time for the slurry, see equation 2.1.1 (Gjerapic et al. 2008) and equation 3.1.1 for a

detailed derivation. Therefore as the surface level of the tailings rises – and the area increases – this filling rate will decrease, see figure 2.1.7.

$$q_i(h) = \frac{Q_s}{G_s \rho_w A_i} (1 + e_0) \quad (2.1.1)$$

Therefore between each height/area pair of the cone frustums used in the discretization there is a point where the filling rate must change, say from q_i to q_{i+1} – notation taken from Gjerapic et al. (2008). This height needs to be located such that the volume calculated from the filling rates needs to correspond to the volume of the impoundment at stage interfaces (Gjerapic et al. 2008). Figure 2.1.8 shows the segments heights or H-heights and F-heights.

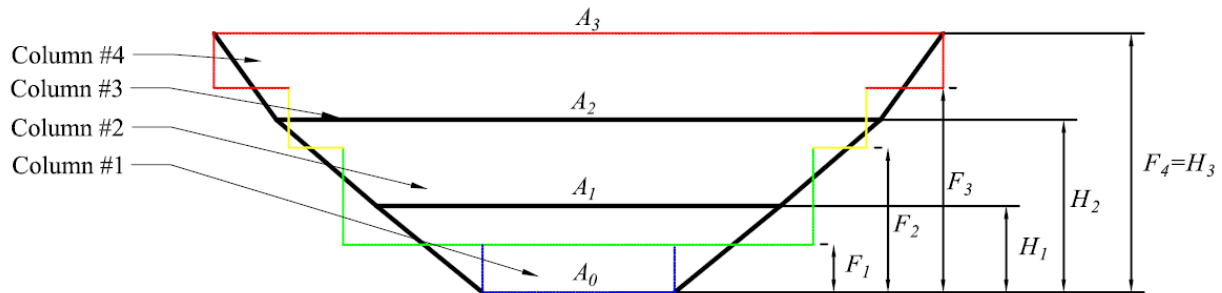


Figure 2.1.8 – H-heights and F-heights

Thus at the F-heights the filling rate is changed so that at the next H-height the volume calculated from the filling rates and the volume of the tailings facility will be

equal. For example, in figure 2.1.9 the filling rate is initially given by equation 2.1.1 where $a_i = a_0$ = initial area. At F_I the filling rate is changed, it is decreased see equation 2.1.1, so that at H_2 the volume calculated from the filling rates and the impoundment volume are equal. In order to calculate the impoundment volume the storage facility is viewed as a series of inverted cone frustums per the discretization method discussed earlier, see figure 2.1.8. The volume formula for an inverted cone frustum is used; h is the height of the cone frustum and B_1 and B_2 are the areas, see equation 2.1.2.

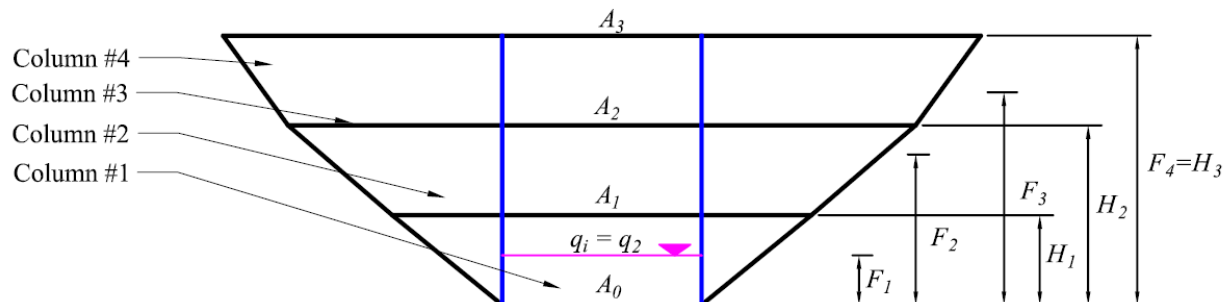
$$V = \frac{h}{3}(B_1 + B_2 + \sqrt{B_1 B_2}) \quad (2.1.2)$$

The formula used to calculate the F-heights is shown in equation 2.1.3 (Gjerapic et al. 2008), note that $\alpha_i = \sqrt{A_{I+1}/A_i}$ = geometric mean stage area. A detailed discussion about this formulation can be found in Gjerapic et al. (2008). It is important to emphasize that at each F-height the height is located so that the volumes calculated from the filling rates and the impoundment geometry are equal.

$$F_i = \begin{cases} 0 & \text{for } i = 0 \\ H_i + (H_{i+1} - H_i) \frac{2\alpha_i + 1}{3\alpha_i + 3} & \text{for } i = 1, 2, \dots, n - 1 \\ H_n & \text{for } i = n \end{cases} \quad (2.1.3)$$

The columns heights and area are chosen so that the discretized volume – that is the volume of the storage facility composed of a series of inverted cone frustums – is represented exactly by the columns. However this doesn't mean the actual volume which would be determined by the user inputted height/area pairs, is represented precisely.

The three-dimensional model calculates the time to reach the top of the facility in two different ways. The first method is called the upper bound method. In the upper bound method, CONDES is called only once to analyze the central column. The filling rate is changed at each F-height to simulate the change in area, see figure 2.1.8. Since only the central column is being analyzed, this method neglects the fact that the material in the middle of the facility at lower elevations will consolidate more than the natural slope material, which is usually much stiffer. This different rate of consolidation is not accounted for because only the central column is being analyzed, so only the material in the center of the facility is considered – the material at the sides of the facility is assumed to behave the same. Therefore this method will overestimate the storage volume and the amount of time required to reach the desired height, and hence it places an upper bound on the amount of time required to reach the desired height.



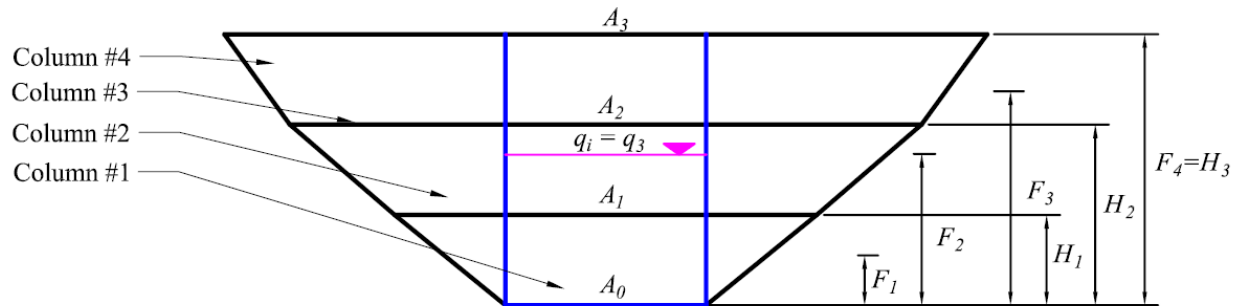


Figure 2.1.9 – the methodology behind the upper bound method

The lower bound method fixes the simplifying assumption of the upper bound method. Before the surface level reaches the first F-height, CONDES would be called once to calculate the time required for the material in column #1 to reach the first F-height. Then at the second F-height, CONDES would be called twice, see figure 2.1.9. CONDES would first be called to calculate the time required for the material in column #2 to reach the second F-height; this analysis would assume that no other material has been previously deposited in column #2. Then CONDES would be called again to calculate the time required for the material in column #1 to reach the second F-height. For this second call to CONDES, CONDES remembers the fact that column #1 already has tailings material in it from the previous step. CONDES then calculates how long it will take for column #1 to reach the second F-height. Then the time from the first step is added to this time, giving the total time required to reach the second F-height.

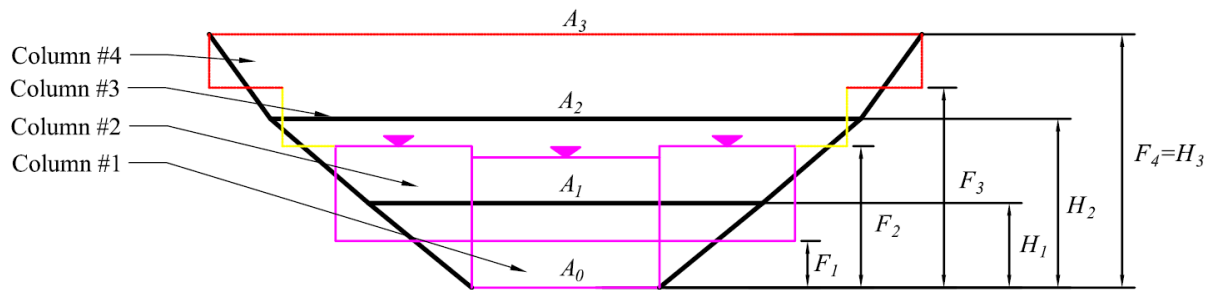


Figure 2.1.10 – the methodology behind the lower bound method

At the third F-height CONDES would have to be called three times, see figure 2.1.10. Once to calculate the time required for the material in column #3 to reach the third F-height, again this analysis would assume that no material has already been deposited in column #3. Then CONDES would have to be called twice more. Once for column #2 and then again for column #1 to calculate the time required for each of these columns to reach the third F-height; CONDES considers the previously deposited material in both of these analyses.

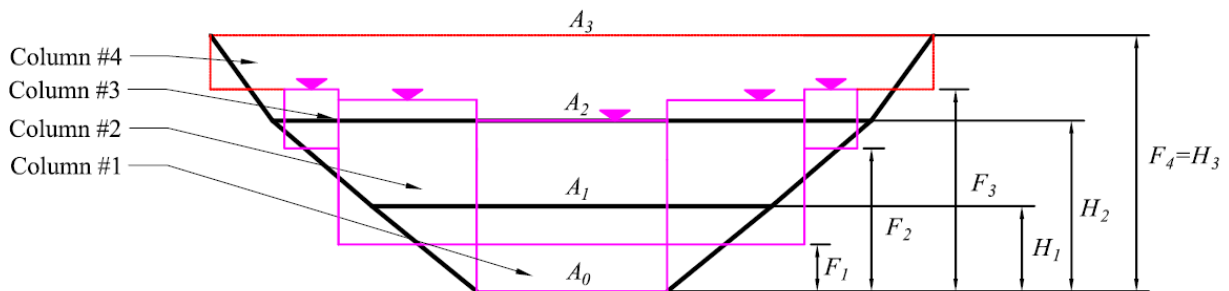


Figure 2.1.11 – the methodology behind the lower bound method

The lower bound method is much closer to an “accurate” solution than the upper bound method; it better simulates the real-world situation. The lower bound method takes into account that the material in the facility will not all consolidate at the same rate. As the analysis doesn’t include multiple iteration steps the lower bound method places a lower bound on the amount of time it would take to reach the desired height and it slightly underestimates the total storage capacity of the impoundment.

2.2 Seepage Induced Consolidation Testing

The Seepage Induced Consolidation Test, SICT, is used to determine the consolidation characteristics of soft soils and slurries (Znidarcic et al. 2011). The test will be briefly explained, for more information see Abu-Hejleh et al. (1996). The test can be divided into three general steps.

In the first step a part of the slurry, about 75 mL or 50 mL (≈ 10 cm high), is allowed to consolidate under its own weight, see figure 2.2.1. The average void ratio of this part of the slurry is considered to be the void ratio at zero effective stress. It is at this void ratio that consolidation theory applies as opposed to sedimentation theory. The amount of consolidation will depend on the consolidation characteristics of the material – some materials can have little to no consolidation.



Figure 2.2.1 – a slurry sample being allowed to consolidate under its own weight, note some consolidation has occurred as evidenced by the water on top of the sample

In the second step, the slurry is subject to a seepage force by using a flow pump to create a hydraulic gradient across the sample, see figure 2.2.2 for a typical testing setup. Using the flow pump a constant flow rate is applied across the sample and the hydraulic gradient gradually increases until the sample is fully consolidated; once the sample is fully consolidated the gradient will reach a steady state under the applied flow rate. Complete consolidation is determined by looking at increases in the gradient – a steady state gradient means that the sample has fully consolidated. Once the sample is fully consolidated the height and the head difference across the sample are both measured and recorded.

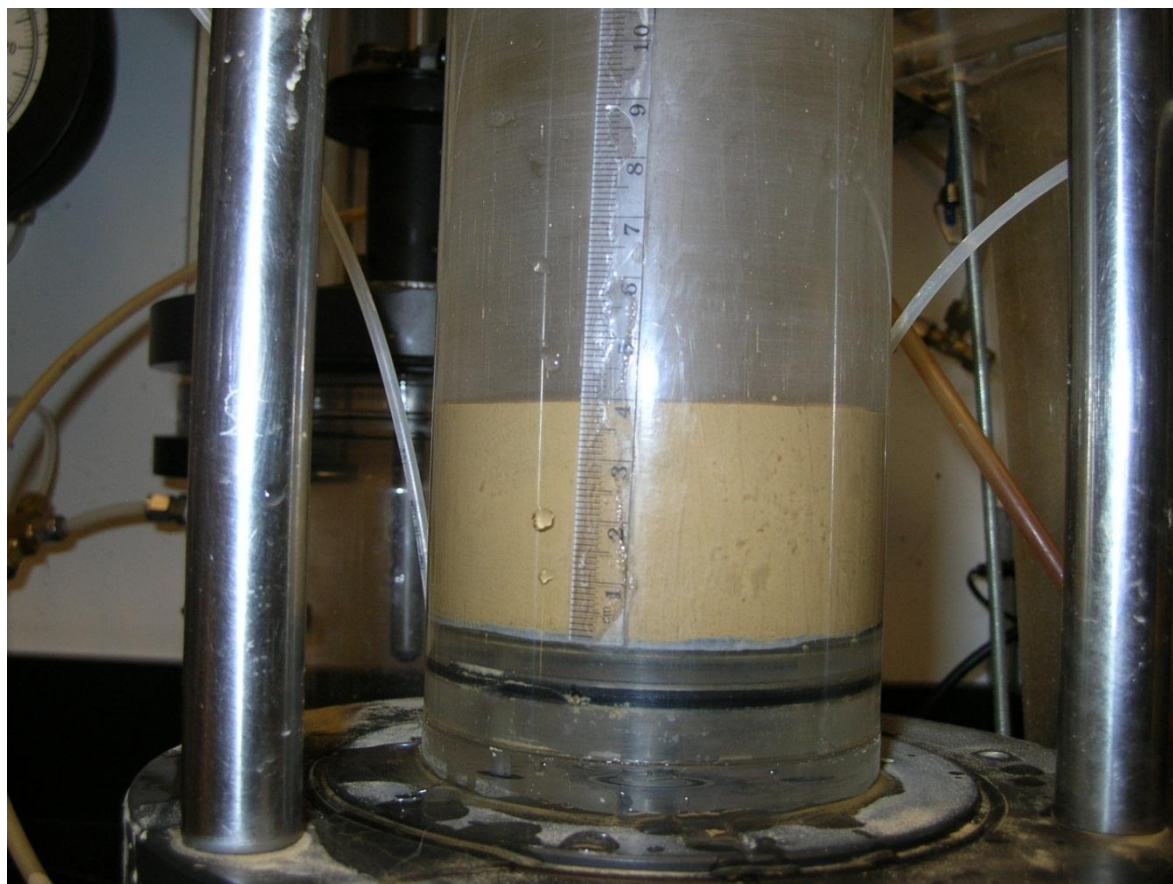




Figure 2.2.2 – shows a typical SICT setup; Top photo – the slurry is poured into a plastic cylinder; Bottom photo –a gradient created across the specimen using a flow pump. The valves at the bottom of the triaxial cell control water flow into and out of the sample, also note there is a pressure transducer at the bottom

In the third step, the sample is subjected to different stress levels by means of an air cylinder applying added stress. At each stress level hydraulic conductivity is measured by using the flow pump. Also at each stress level the change in height and change in gradient are known since the steady state measurements are known from step two. Then once the desired stress level has been reached, the sample is dried and weighed.

Once the SICT is completed the data is analyzed using the Seepage Induce Consolidation Test Analysis (SICTA) software package, see Abu-Hejleh et al. (1994) for a detailed description. The analysis procedure is based on the inverse problem solution and no simplifying and/or restrictive assumptions are made. The analysis is compatible with finite strain nonlinear consolidation theory (Znidarcic et al. 2011).

The input parameters are all known from the SICT. The void ratio at zero effective stress is known from step one. The void ratio and hydraulic conductivity at different stress levels is known from step two. Therefore the solution can determine the five parameters: A_I , B_I , C , D , and Z_I , that are used in equations 2.0.1 and 2.0.2.

3.0 New Features and Improvements to TMT

The three-dimensional model was completed in 2010 (Coffin 2010) and as mention previously TMT was released as an Excel macro. Changes have been made to the GUI – graphical user interface – to provide easier use. These changes were only cosmetic in nature and did not affect the underlying numerical methods or consolidation governing equation. Please see section 4 for a user manual on the updated version of TMT.

In addition to the cosmetic changes, changes have been made that provide information that was not in the original version of TMT. These would be the water balance calculations and the no consolidation and instantaneous consolidation curves.

It is often difficult to adequately explain the concept of consolidation. Therefore the no-consolidation curve and instantaneous consolidation curve were added to provide a further bound on the consolidation results. The no-consolidation curve represents what would happen if the slurry experienced no consolidation – i.e. filling the storage facilities with a liquid that is denser than water due to the presence of the suspended solids that never undergo sedimentation or consolidation. This would represent the fastest that the facility would be filled. The instantaneous-consolidation curve represents what would happen if the slurry consolidated instantaneously – as soon as it is placed in the facility it assumed to immediately consolidate fully. This instantaneous consolidation curve would represent the slowest that the facility could be filled.

3.1 No-Consolidation Curve

As mentioned previously the no-consolidation curve assumes no consolidation occurs. Therefore the assumption is that the storage facility is being filled with a liquid that is denser than water – i.e. a liquid with suspended particles and the particles never undergo sedimentation or consolidation. The no consolidation curve is determined by first determining the no-consolidation filling rate, which is derived in equation 3.1.1. Then once the no-consolidation filling rate is known the time required to fill the facility can be calculated by calculating the volume from the geometry. Again it is important to note that the model geometry is used – the geometry of the columns – see section 2.1.

Deposition Rate = $D_r = \frac{M_s}{T}$ = [dry unit mass of solids/time]

Density of Solids = $\rho_s = GS * \rho_w = \frac{M_s}{V_s}$ = [mass of solids/volume of solids] – as is

normally done for a phase relation of soil, the volume of solids is assumed to be 1

Initial Void Ratio = $e_0 = \frac{V_v}{V_s}$ = [volume of solids/volume of voids] – which the slurry

would have at zero effective stress is determined from SICT testing, see section 2.2

Total Initial Volume Displaced by Slurry = $1 + e_0 = \frac{V_v + V_s}{V_s}$ = [(volume of solids + volume of voids)/volume of solids]

No-Consolidation Filling Rate = $NC_r = \frac{D_r}{\rho_s} * (1 + e_0) = \frac{V_s + V_v}{T}$ = [(volume of solids + volume of voids) / time]

$$NC_r = \frac{D_r}{\rho_s} * (1 + e_0) = \frac{V_s + V_v}{T} \quad (3.1.1)$$

3.2 Instantaneous-Consolidation Curve

The instantaneous-consolidation curve assumes that the slurry material consolidates instantaneously and completely as soon as it is placed in the containment facility. The first step is to calculate the instantaneous-consolidation filling rate, see equation 3.2.1.

$$\text{Deposition Rate} = D_r = \frac{M_s}{T} = [\text{dry mass of solids/time}]$$

$$\text{Specific Gravity} = G_s$$

$$\text{Density of Water} = \rho_w$$

$$\text{Density of Solids} = \rho_s = G_s * \rho_w = \frac{M_s}{V_s} = [\text{dry mass of solids/volume of solids}] - \text{as is}$$

normally done for a phase relation of soil, the volume of solids is assumed to be 1

$$\text{Instantaneous-Consolidation Filling Rate} = IC_r = \frac{D_r}{\gamma_s} = \frac{V_s}{T} = [\text{volume of solids/time}]$$

$$IC_r = \frac{D_r}{\rho_s} = \frac{V_s}{T} \quad (3.2.1)$$

However unlike the no-consolidation case, the instantaneous-consolidation case cannot be calculated by using the volume of the storage facility. This is because that each height will have a unique corresponding height of solids. This is due to the fact that the solids will undergo self-weight compression and this effect will be different for each height – i.e. the void ratio distribution will vary for each height. Therefore the height of solids that corresponds to a given height has to be calculated. Figure 3.2.1 shows that for each height of solids there is a corresponding height of slurry. Also two different coordinate systems are used; z is used to denote the depth in the solids only coordinate system and \tilde{z} is used to denote the depth of slurry.

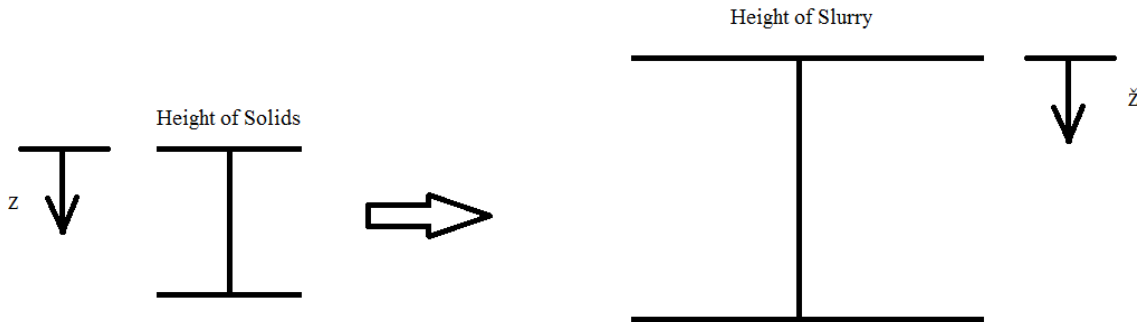


Figure 3.2.1 – each height of solids corresponds to a unique height of slurry

The first step is to derive an equation for the height of slurry that corresponds to a given height of solids, see equation 3.2.2. The height of slurry will be equal to the summation of the height of the voids and the height of the solids. As mentioned previously the void ratio will change with depth due self-weight compression. The equation for a partial height of solids, h_s , that corresponds to a partial height of slurry, h , is very similar.

$$\text{Total Height of Solids} = H_s$$

$$\text{Weight of Solids} = W_s$$

$$\text{Total Height of Slurry} = H$$

$$\text{Area} = A$$

Specific Gravity = G_s

Unit Weight Water = γ_w

Effective Stress = σ'

Consolidation Parameters (determine from SICT) – A_1, B_1, Z_1

$$H_s = \frac{W_s}{G_s * \gamma_w * A}$$

$\sigma' = (G_s - 1) * \gamma_w * z$ – this is the effective stress at depth z in the solids only coordinate system

$$H = \int_0^{H_s} (1 + e) dz$$

$$H = \int_0^{H_s} (1 + A_1(\sigma' + Z_1)^{B_1}) dz \text{ – from equation 2.0.1}$$

$$H = \int_0^{H_s} \left(1 + A_1((G_s - 1) * \gamma_w * z + Z_1)^{B_1} \right) dz$$

Solve the integral via substitution: $x = (G_s - 1) * \gamma_w * z + Z_1$ then $dx = \frac{dz}{(G_s - 1) * \gamma_w}$

$$H = \int_0^{H_s} dx + \int_0^{H_s} \frac{A_1 x^{B_1}}{(G_s - 1) * \gamma_w} dx$$

$$H = x|_0^{H_s} + \frac{A_1}{(G_s - 1) * \gamma_w} * \frac{x^{B_1+1}}{B_1 + 1} \Big|_0^{H_s}$$

Then substitute $x = (G_s - 1) * \gamma_w * z + Z_1$ back in and you have the final form

$$H = H_s + \frac{A_1}{(G_s-1)*\gamma_w} * \left(\frac{((G_s-1)*\gamma_w*H_s + Z_1)^{B_1+1} - Z_1^{B_1+1}}{B_1+1} \right) \quad (3.2.2)$$

Once equation 3.2.2 is known, the height of slurry that corresponds to a height of solids can be calculated. In the three-dimensional model an iterative procedure is used to calculate the height of slurry. An initial value for H_s (1/5 of H) is assumed, which implies that the average void ratio for the column is approximately four. This value is chosen because experience has shown that this is a reasonable assumption for the final average void ratio of many slurry materials. The final height of solids is not dependent on this initial value. This method converges rapidly, typically within ten iterations.

After the initial value for H_s is guessed, equation 3.2.2 is rearranged so that an H_s term is on both sides of the equal sign, see equation 3.2.3. Then H_{s1} is used to calculate H_{s2} and this process is repeated until H_{sn} and H_{sn+1} are within the desired tolerance level; which is 10^{-5} for the model.

$$H - \frac{A_1}{(G_s-1)*\gamma_w} * \left(\frac{((G_s-1)*\gamma_w*H_{sn} + Z_1)^{B_1+1} - Z_1^{B_1+1}}{B_1+1} \right) = H_{sn+1} \quad (3.2.3)$$

Since the height of slurry that corresponds to a given height of solids is unique – i.e. the void ratio distribution profile is unique – the height of solids for each height of slurry must be calculated individually. Thus the height of solids at each F-height, see

figure 2.1.8, must be calculated individually. For example if $H = 1$ m corresponds to $H_s = 0.5$ m it is not true that $H = 2$ m corresponds to $H_s = 1$ m.

3.3 Water Balance

In many mining operations access to water is limited. The reasons for this could be varied, but they include: political, environmental, site location, and monetary considerations. Thus the water expelled during the consolidation process is often needed to continue mining operations. Due to this fact it was decided to add a water balance calculation – which calculates the amount of water expelled during consolidation – to the three-dimensional model.

The water expelled during consolidation is calculated at each F-height, see figure 2.1.3. The first step is to determine the time required using the lower bound method, see section 2.1, to reach the F-height. The lower bound method is used because it better simulates the real world situation. The amount of slurry that would be present, assuming no consolidation occurs, can be calculated by using equation 3.1.1. The difference between the volume calculated via the no-consolidation method and the volume calculated via the lower bound method will give the volume of water expelled during consolidation when the slurry reaches that F-height. By using this method a curve can be generated – linear interpolation was used – that will give an approximation of the amount of water expelled during consolidation for each day.

The water balance calculation does not consider either the top or bottom boundary condition. Thus evaporation is not considered nor is the type of bottom boundary – i.e.

impermeable or pervious. Additionally the water balance calculation only gives the total amount of water expelled during consolidation. The water balance doesn't give the amount of water that was expelled at either the top or bottom boundary; this would depend on the specific boundary conditions that were used in the model.

4.0 User Manual for TMT

This section will serve as the user manual for the TMT. Multiple changes have been made to the original GUI (Coffin 2010). It should be noted that all changes to the GUI were to improve usability of the model and no changes were made to the original algorithm. This section will discuss each tab and how it can affect the model.

For each tab the user needs to click on the input/action button after entering their data. Failure to click on the button will result in the data not being updated.

4.1 Start Tab

The Start tab is shown in figure 4.1.1. This tab allows the user to label the current project and to select a save destination for the project. The project title will be displayed in the Excel tab Material and Analysis. When TMT performs either a deposition or post deposition analysis it will overwrite the previous analysis/project; a new save destination should be selected for each project to prevent previous work from being overwritten.

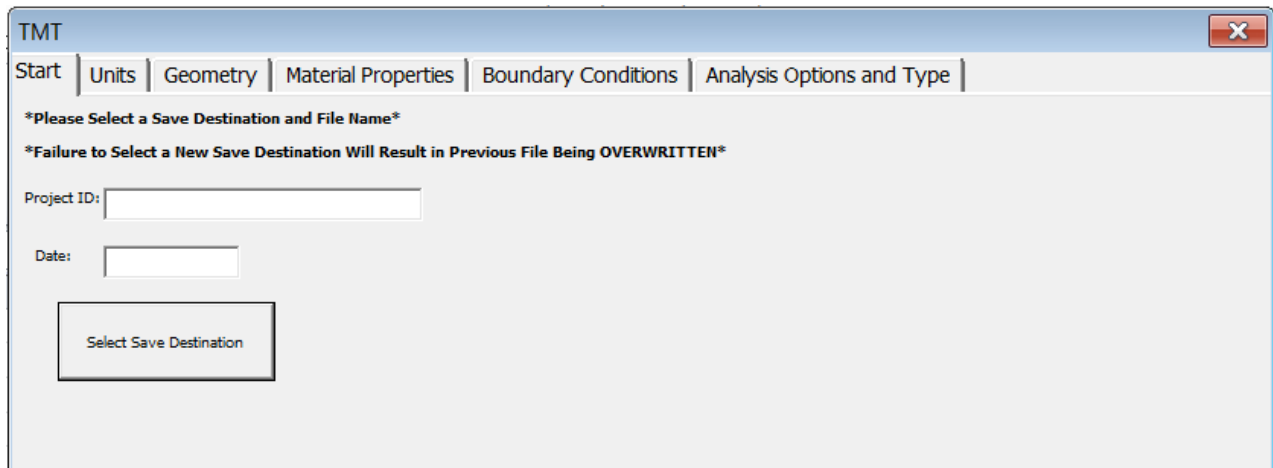


Figure 4.1.1 – the Start tab allows the user to label the current project and select a save destination

4.2 Units Tab

The Units tab is shown in figure 4.2.1. The units tab allows the user to select the units that will be used for mass and time. The time units can be either days or years and the mass units can be either tonnes (metric) or tons (U.S. standard). The selection of these units will determine the units that will be used for: length/area/volume, unit weight of water, deposition rate, hydraulic conductivity, and stress. The units used for each input parameter are shown at the bottom of the tab; also the units have been automatically selected in the other tabs. The user needs to make sure that the numerical values they are entering correspond to the units shown.

Originally the units for each of these parameters were selected separately. This had the potential for the units to be inconsistent. For example the deposition rate units [mass/time] would have to be consistent with the units for the hydraulic conductivity

[length/time] and vice-versa. It was decided that to have the user pick two units and the units for the other inputs would be determined from these – in order to minimize the potential for errors. However, as mentioned previously CONDES & TMT do not accept non-numerical inputs – the user is required to ensure that the units are consistent. CONDES & TMT have no internal check to ensure correct units are being used.

TMT

Start | **Units** | Geometry | Material Properties | Boundary Conditions | Analysis Options and Type

Select Time and Mass Units

* Selection of time units will determine the time units for the deposition rate(s) and hydraulic conductivity (C parameter). When entering these values later make sure the time units are correct. *

Days
Years

Selection of mass units will determine the units used for the stress (Z parameter), unit weight of water, and the length/area/volume units

Tonnes
Tons

Tonne = 1000 kg --> meter & kPa

Ton = 2000 lb --> feet & psf

Input Selected Units

Units Used in Analysis

The units used in the analysis are shown below

When entering values make sure units are consistent

Length/Area/Volume: Meters

Deposition Rate Units: Tonnes/Day

Unit Weight of Water: 9.81 kN/m³

Hydraulic Conductivity (C parameter): m/day

Stress Units: kPa

Figure 4.2.1 – the Units tab allows the user to select the units that will be used in the analysis

4.3 Geometry Tab

The Geometry tab is shown in figure 4.3.1. In the Geometry tab the user can enter the height/area pairs that will form the truncated cone that simulates the site geometry; these heights are the h-heights and their corresponding areas are labeled as $a_0, a_1 \dots$ etc., as shown in figure 2.1.1. The F-heights and their corresponding areas and F-heights are determined by the number of columns chosen – which will be discussed in section 4.4.

TMT

Start | Units | **Geometry** | Material Properties | Boundary Conditions | Analysis Options and Analysis Type

The units used are determined by the units selected for mass and time.

Length Units:

Units for Production Rate:

Height/Area Pairs

#	Height	Area	#	Height	Area
#1	0	70690	#16		
#2	13.33	121500	#17		
#3	26.67	186000	#18		
#4	40	264200	#19		
#5			#20		
#6			#21		
#7			#22		
#8			#23		
#9			#24		
#10			#25		
#11			#26		
#12			#27		
#13			#28		
#14			#29		
#15			#30		

Input Height/Area Pairs

Production Schedule - Tonnes/Year

#	Production Rate	Ending Time
#1	730000	15
#2		
#3		
#4		
#5		
#6		
#7		
#8		
#9		
#10		
#11		
#12		
#13		
#14		
#15		

Input Production Rates

Figure 4.3.1 – the Geometry tab allows the user to enter the height area pairs and the production schedule

In the Geometry tab the user can also input the production schedule. The deposition rate format should be [dry mass/time] – the actual units will depend on the selections made in the Units tab.

4.4 Material Properties

The Material Properties tab, see figure 4.4.1, allows the user to enter the consolidation parameters as determined from SICTA. Also the specific gravity needs to be inputted, this would be typically determined from a pycnometer test on the slurry. Again, the user needs to be very attentive to ensure that the values entered correspond to the units shown – otherwise the analysis will not make sense.

Figure 4.4.1 – the Material Properties tab allows the user to enter the consolidation parameters determined from SICTA and pycnometer test

4.5 Boundary Conditions

The Boundary Conditions tab is shown in figure 4.5.1. In the Boundary Conditions tab the user selects the top and bottom boundary conditions.

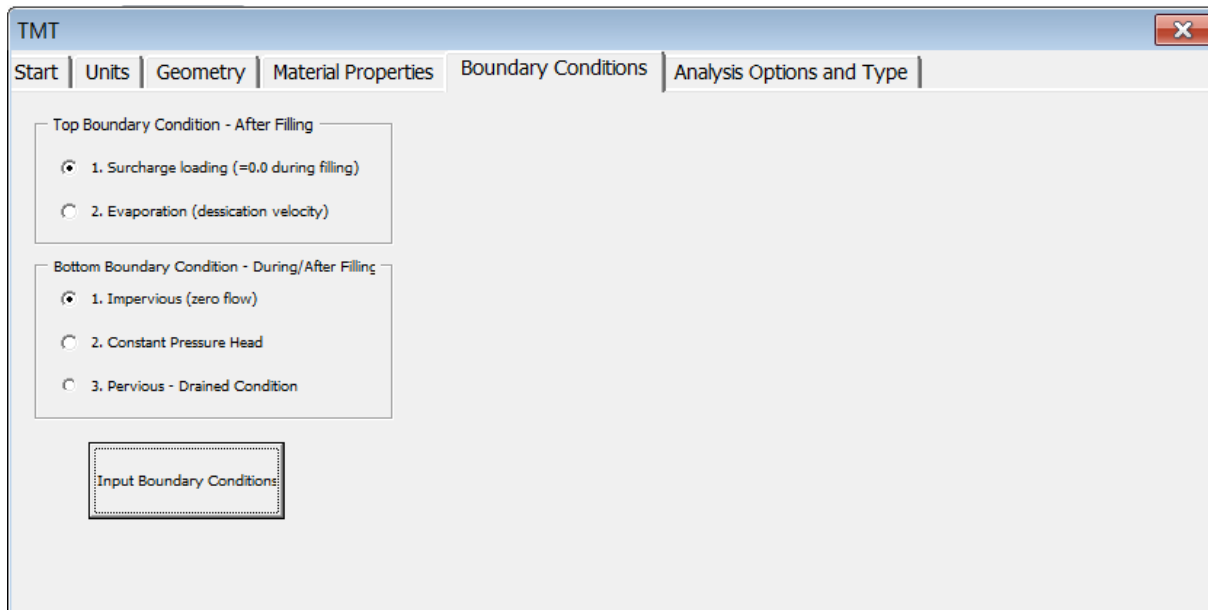


Figure 4.5.1 – the Boundary Conditions tab allows the user to select the top and bottom boundary conditions

There are two options for the top boundary condition; either surcharge loading or evaporation. The surcharge loading option assumes that during deposition no surcharge load is applied. After deposition is complete, a post deposition analysis can be performed. A post deposition analysis allows the user to simulate stress on the top boundary; this will be discussed in section 4.6. The evaporation option assumes that during deposition no water is lost due to evaporation and a post deposition analysis will account for the loss of surface water due to evaporation.

There are three options for the bottom boundary condition: impervious, constant head, and pervious. The impervious option assumes that there is no flow across the bottom boundary. For the pervious boundary condition it is assumed that the pressure head is equal to the height of the slurry above the point at all times; therefore no additional downward seepage forces are imposed on the tailings. The constant head option maintains a constant pressure head across the bottom boundary.

4.6 Analysis Options and Analysis Type

The Analysis Options and Analysis Type tab is shown in figure 4.6.1 and it allows the user to select a variety of options. This tab will allow the user to select the number of columns that are used to approximate the volume of the inverted cone frustum, see figure 2.1.4. The more columns that are selected the more accurate the approximation will be.

The user is also allowed to select the minimum void ratio. The minimum void ratio is the void ratio at which the void ratio cannot be reduced further – i.e. consolidation is complete. Note when performing a post deposition analysis when this minimum void ratio is reached CONDES will terminate; the minimum void ratio has no effect on the deposition analysis.

Additionally the user can select the maximum time interval. This is the maximum “jump” in time that CONDES will use. Thus if the results need to be very precise select a small maximum time interval. However in most cases to speed things up a maximum time interval of 5-10 days (120-240 hours) is acceptable.

TMT

Start | Units | Geometry | Material Properties | Boundary Conditions | Analysis Options and Analysis Type

Analysis Options

Initial Number of Columns
4

Minimum Void Ratio
0.5

Maximum Time Interval
0.1

Input

Deposition Analysis

Perform Deposition Analysis

Cancel Analysis

Cancel Analysis

Post Deposition Analysis - Please Enter Times For Analysis and Select Which Columns to Analyze

Ending Time for Deposition Analysis was 11.5. Please Enter Elapsed Times After Deposition Has Ended That You Want To See Results For

#1		#6	
#2		#7	
#3		#8	
#4		#9	
#5		#10	

Please List the Columns You Wish to Analyze (Central Column is One, The Adjacent Column is Two, etc.)

Column #		Column #	
Column #		Column #	
Column #		Column #	
Column #		Column #	
Column #		Column #	

Perform Post Deposition Analysis

Figure 4.6.1 – the Analysis Options and Analysis Type tab allows the user to select the type of analysis and specify the number of columns, minimum void ratio, and maximum time interval

Next the user can select either deposition analysis or post deposition analysis. A deposition analysis will begin at the beginning of deposition – i.e. before any material has been added to the facility. Deposition analysis will perform the standard analysis to

determine how long it will take the material to fill the storage facility – both upper and lower bound solutions will be obtained.

Post deposition analysis starts from the ending time of the previous run – i.e. post deposition analysis begins from the previous ending time. In post deposition analysis new material is not deposited but the consolidation of the material already present is determined. The user can select the times when the void ratio distribution will be displayed. The user can also select the columns they want to use in the post deposition analysis – i.e. not all the columns are analyzed only the columns that the user selects.

4.7 Excel Spreadsheet

After the user has entered the information in the GUI and the analysis has been performed; the information and results will be written to the Excel sheet. This section will cover the various tabs and the information on each.

The first tab, Material and Analysis, lists the: project name, date, material consolidation parameters – determined from SICTA, specific gravity, number of columns (filling stages) used in the model, boundary conditions, minimum void ratio, maximum time interval, and the type of analysis performed, see figure 4.7.1. If a post deposition analysis is performed then the number of post deposition times is given and also which columns are being analyzed. Additionally the hydraulic conductivity and void ratio for different stress levels are calculated by using equations 2.0.1 and 2.0.2.

Project:	Example				Analysis Type				
Date:	11-Jan				2 Post-Deposition Analysis				
Runs:	63								
A =	3.3				Boundary Condition				
B =	-0.21				Top: Surcharge Loading				
C =	9.10E-03	m/yr			Bottom: Impervious				
D =	5.50E+00								
Z =	0.08	kPa			Number of Filling Stages				
					4				
Gs =	3.666								
					Minimum Void Ratio				
Gamma W =	9.81	kN/m ³			0.5				
Effective Stress	Void Ratio	Hydraulic Conductivity			Maximum Time Interval				
					0.1				
0.1	4.730496	46.88439522							
0.2	4.311325	28.1449535			Post-Deposition Analysis Times		Post-Deposition Columns		
0.4	3.849934	15.10199884			1		1		
0.8	3.389788	7.49877936					2		
1.6	2.959368	3.553332444							
3.2	2.571464	1.640717356							
6.4	2.228862	0.747306793							
12.8	1.929441	0.337998863							
25.6	1.669164	0.152330131							
51.2	1.443527	0.068529585							
102.4	1.248187	0.030802068							
204.8	1.079193	0.013838406							
409.6	0.933041	0.006215762							
819.2	0.806665	0.002791603							
1638.4	0.697399	0.001253685							

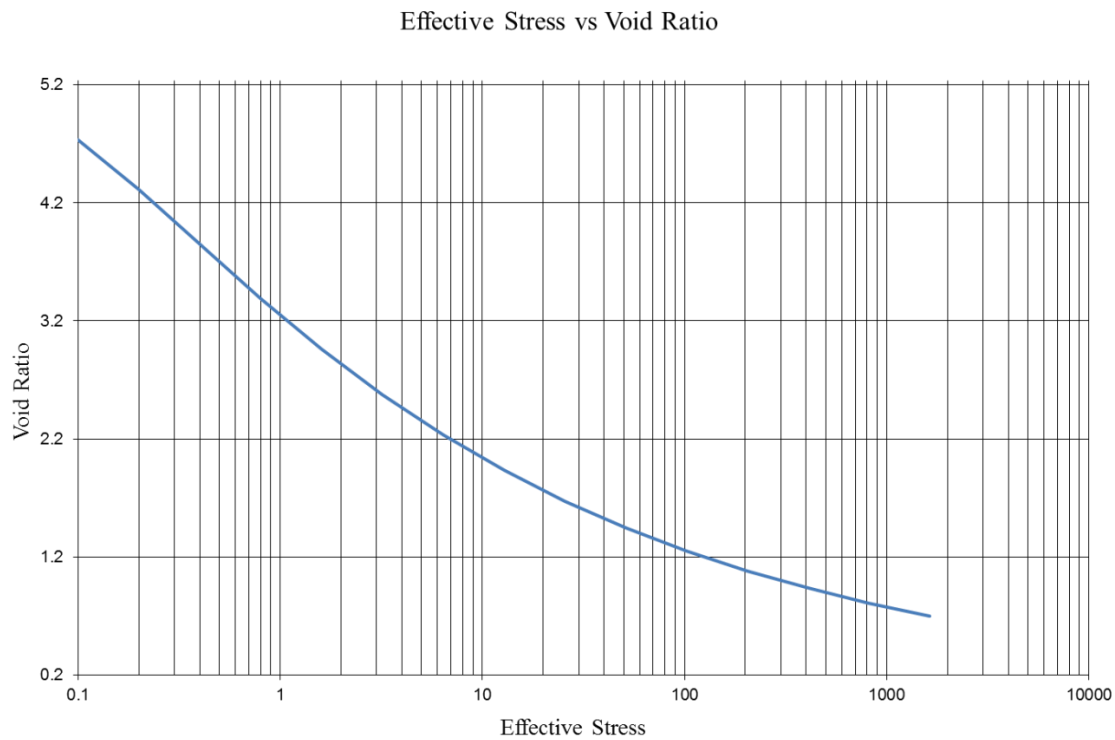
Figure 4.7.1 – shows the Materials and Analysis tab in Excel

The Geometry tab shows the heights and areas that the user entered in the Geometry tab in the GUI, see figure 4.7.2. The volume is calculated using the volume formula for an inverted cone frustum, see equation 2.1.2. The volume must be calculated for consecutive height/area pairs and then added to the previous volume to obtain the correct volume – i.e. the first and last height/area pairs cannot be used to calculate the final volume as this would assume a different cone frustum than actually exists. Additionally the deposition schedule is shown in the Geometry tab.

Number	Height	Area	Volume		Deposition Rate	Units	Starting Time	Ending Time
Pair Number	Meters	Meters^2	Meters^3		730000	Tonnes/Year	0	15
1	0	7.07E+04	0					
2	13.33	1.22E+05	1265754					
3	26.67	1.86E+05	3301570					
4	40	2.64E+05	6286949					

Figure 4.7.2 – shows the Geometry tab in Excel

The sigma' vs e and the k vs e tabs show the graphs of effective stress vs. void ratio and hydraulic conductivity vs. void ratio respectively, see figure 4.7.3. These curves are generated using the table of values in the “Materials and Analysis” tab.



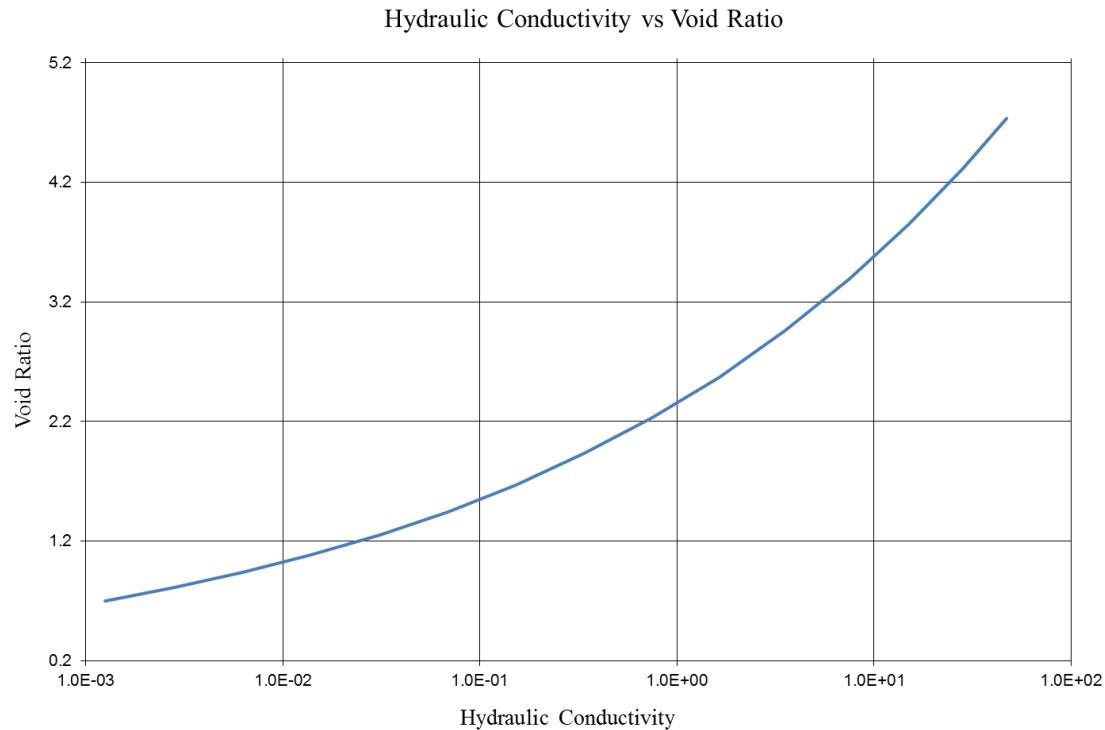


Figure 4.7.3 – sample graphs of effective stress vs. void ratio and hydraulic conductivity vs. void from the sigma' vs e & k vs e tabs in Excel

The Results tab lists information from the analysis. The time required to reach the F-heights is listed for both the upper & lower bound methods, see figure 4.7.4. Additionally for both methods, at each F-height the average dry unit weight is listed. The average dry unit weight is calculated by converting the volume of solids deposited to a mass of solids – which will be different for both the upper and lower bound methods – and dividing by the area, see Coffin (2010) for additional details. The column next to the average dry unit weights in the upper and lower bound methods is an estimation of the error in calculating the average dry unit weights (Coffin 2010). This estimate is calculated by comparing the developed solids mass – the solids mass calculated by the

model – to the estimated solids mass – which is obtained by multiplying the filling rates by the time elapsed. A more detailed discussion about the specifics of the error estimation is given in Coffin (2010).

Upper Bound									
Time	Height	Average Dry Unit Weight							
0	0	5.44180565		Time =	0.88				
0.87569	6.972652	12.5763024	-0.01249		51.00	6.97	5.61	0.40	
3.804207	20.24068	12.6591641	-0.02191		50.00	6.87	3.04	0.55	
8.381821	33.53716	12.6402219	-0.03615		49.00	6.69	2.38	0.61	
11.75198	40.00713	12.7072466	-0.05073		48.00	6.53	2.10	0.64	
					47.00	6.38	2.00	0.65	

Lower Bound									
Time	Height	Average Dry Unit Weight							
				TIME =	0.88				
0.87569025	6.972652	12.7139346			51.00	6.97265	5.60874	0.39527	
3.78461442	20.24461	12.8606523			50.00	6.86611	3.04022	0.64816	
8.28902488	33.5358	12.9560257			49.00	6.68779	2.37955	0.64605	
11.4618662	40.00318	13.0462491	-0.0025		48.00	6.52915	2.1041	0.65028	
12.4618662	38.25623				47.00	6.37921	1.99822	0.65188	

Figure 4.7.4 – the time required to reach each F-height and the average dry unit weight for both methods in the Results tab in Excel

Additionally for both methods, at the times when the slurry reached the F-heights the void ratio distribution is given as a series of 51 void ratios; these series will be used to

generate the void ratio distribution curves, see figure 4.7.5. The first column is the node number, the second column is the height, the third column is the void ratio, and the fourth column is solids content.

Time =	11.75			
	51.00	40.01	5.61	0.40
	50.00	39.09	2.17	0.63
	49.00	38.16	2.14	0.63
	48.00	37.25	2.10	0.64
	47.00	36.34	2.08	0.64
	46.00	35.44	2.05	0.64
	45.00	34.55	2.03	0.64
	44.00	33.66	2.02	0.65
	43.00	32.78	2.00	0.65
	42.00	31.90	1.98	0.65
	41.00	31.03	1.97	0.65
	40.00	30.16	1.95	0.65
	39.00	29.30	1.94	0.65
	38.00	28.44	1.92	0.66
	37.00	27.59	1.91	0.66
	36.00	26.74	1.90	0.66
	35.00	25.89	1.88	0.66
	34.00	25.05	1.87	0.66
	33.00	24.21	1.85	0.66
	32.00	23.37	1.84	0.67
	31.00	22.54	1.82	0.67
	30.00	21.72	1.81	0.67
	29.00	20.89	1.80	0.67
	28.00	20.08	1.78	0.67
	27.00	19.26	1.76	0.68
	26.00	18.45	1.75	0.68
	25.00	17.65	1.73	0.68
	24.00	16.85	1.72	0.68
	23.00	16.06	1.70	0.68
	22.00	15.27	1.68	0.69
	21.00	14.48	1.67	0.69
	20.00	13.70	1.65	0.69
	19.00	12.93	1.63	0.69
	18.00	12.16	1.61	0.69
	17.00	11.40	1.59	0.70
	16.00	10.64	1.57	0.70
	15.00	9.89	1.55	0.70
	14.00	9.14	1.53	0.71
	13.00	8.40	1.51	0.71
	12.00	7.67	1.49	0.71
	11.00	6.94	1.47	0.71
	10.00	6.22	1.45	0.72
	9.00	5.50	1.43	0.72
	8.00	4.79	1.41	0.72
	7.00	4.09	1.39	0.73
	6.00	3.39	1.37	0.73
	5.00	2.70	1.35	0.73
	4.00	2.02	1.32	0.73
	3.00	1.34	1.30	0.74
	2.00	0.67	1.28	0.74
	1.00	0.00	1.26	0.74

Figure 4.7.5 – the void ratio distribution at a given time in the Results tab in Excel; the first column is the node number, the second column is the surface height, the third column is the void ratio distribution, the fourth column is solids content

The Results tab also lists the time required to reach the F-heights when assuming no-consolidation and instantaneous-consolidation properties for the slurry, see figure 4.7.6. Also for each of the two assumptions, the volume of material deposited up to each F-height is given. Lastly the amount of water expelled during consolidation up to each F-height is given.

No Consolidation			
Time	Height	Volume	
0	0	0	
0.4	6.965808	526391.46	
1.6	20.23556	2105565.8	
3.5	33.52786	4605925.2	
4.8	40	6316697.5	

Instantaneous Consolidation			
Time	Height	Volume	
0	0	0	
1	6.965808	199127.114	
4.5	20.23556	896072.0131	
10.2	33.52786	2031096.563	
14.2	40	2827605.019	

Water Balance		
Time	Volume	
0	0	
0.9	691967.8033	
3.8	2895816.72	
8.3	6345567.68	
11.5	8846759.924	

Figure 4.7.6 – shows the information for the no-consolidation, instantaneous-consolidation, and water balance calculations in the Results tab in Excel

The LB Results tab lists the F-heights, which are listed under the column heading “F-heights”, and their corresponding areas, which are listed under the column heading “F-height Areas”. Table 4.7.1 shows a sample table, which was taken from an analysis conducted on actual mine geometry. Also the filling rates used for each column are given; they are given in the cell immediately below the cell listing the filling rate #.

The times that were calculated using the Lower bound method to reach the F-heights are given in the table. For example table 4.7.1 it took 0.3 days to fill the first column to the first F-height (7.7 m) – see row 1 of the table – refer to figure 2.1.8 as a reference. Then it took 16.41 days to fill the second column to the second F-height, and then it took an additional 0.01 days to fill the first column – which had previous material from the first step – up to the second F-height. Therefore the final number in the row, listed under the column heading “Filling Rate #1”, gives the total time required to fill to that F-height from the previous F-height. The summation of the first column will give the total time it took to fill the facility, up to the selected F-height.

	Filling Rate #1	Filling Rate #2	Filling Rate #3	Filling Rate #4	Filling Rate #5	Filling Rate #6	Filling Rate #7	Filling Rate #8	Filling Rate #9	
F-Heights	2.89E+01	7.88E-01	2.04E-01	1.10E-01	7.87E-02	5.31E-02	3.59E-02	2.28E-02	1.73E-02	F-Height Areas
7.7	0.3									400
19.4	16.42	16.41								14662
31.6	76.17	76.16	75.30							56753
43.9	155.99	155.98	155.28	152.62						105373
56.5	229.52	229.51	228.96	227.86	223.46					146869
69.0	355.87	355.86	355.25	354.00	352.16	343.25				217701
81.5	552.36	552.35	551.73	549.67	547.86	544.12	527.58			322177
93.9	901.74	901.73	901.04	898.44	894.96	890.89	883.29	849.71		506691
100.0	602.05	602.04	601.63	600.46	598.22	595.70	592.38	585.38	541.38	669642

Table 4.7.1 – times at which surface level reaches F-height calculated by using the lower bound method

There is an error that frequently occurs in this table. This problem will be discussed in section 5.6.

The LB RESULTS tab also gives the void ratio distribution, as calculated using the lower bound method, as time changes for each column. It is important to note that these void ratio distributions are different than the void ratio distributions given in the Results tab. These void ratio distributions are given for each column; as opposed to only the central column that is given in the Results tab. For example, initially column #1 is filled up to the first F-height and the void ratio distribution is given for this column at the first F-height, see figure 4.7.7. Then columns #1 and #2 both need to be filled from the first F-height to the second F-height; and the void ratio distributions for both columns are given, see figure 4.7.8.

TIME =	0.87569025	
	6.972652481	5.60874
	6.866112443	3.04022
	6.687786621	2.37955
	6.529151943	2.10410
	6.379207732	1.99822
	6.232740355	1.95380
	6.087780161	1.93319
	5.943591946	1.92097
	5.79992354	1.91163
	5.656679668	1.90365
	5.513799041	1.89689
	5.371220915	1.891375949
	5.228884223	1.887063568
	5.086734199	1.883758474
	4.944727517	1.881199306
	4.802834074	1.879119621
	4.66103612	1.877288938
	4.519326007	1.875528602
	4.377703616	1.873710251
	4.236174128	1.871745083
	4.094746393	1.869570798
	3.953431917	1.867140194
	3.812244335	1.864413167
	3.671199178	1.861352278
	3.530313799	1.857920848
	3.38960734	1.854083131
	3.249100659	1.849805439
	3.108816192	1.845057502
	2.968777758	1.839813898
	2.829010281	1.834055031
	2.68953951	1.82776714
	2.550391728	1.820942246
	2.411593474	1.813577784
	2.273171295	1.805675946
	2.135151522	1.797243277
	1.997560066	1.788290093
	1.860422255	1.778829638
	1.723762702	1.768877656
	1.58760519	1.758452084
	1.451972569	1.747572797
	1.316886661	1.736261643
	1.182368169	1.724542213
	1.048436594	1.712439615
	0.915110164	1.699980407
	0.782405766	1.687192532
	0.650338876	1.674105304
	0.518923489	1.660749583
	0.388172039	1.647157806
	0.258095314	1.633364583
	0.128702316	1.619408246
	0	1.605334283

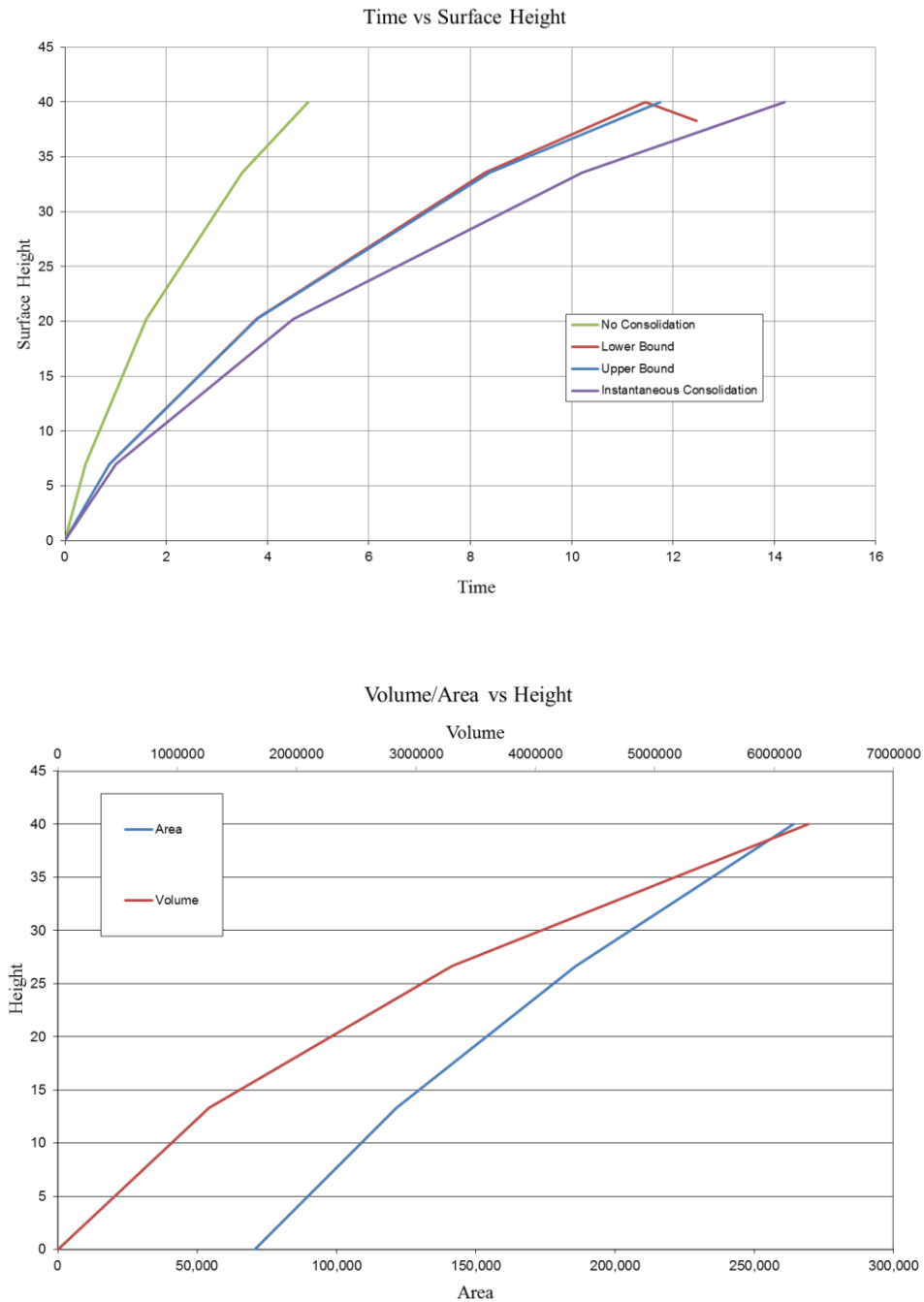
Figure 4.7.7 – a sample void ratio distribution at the first F-height, as shown in the LB

RESULTS tab in Excel

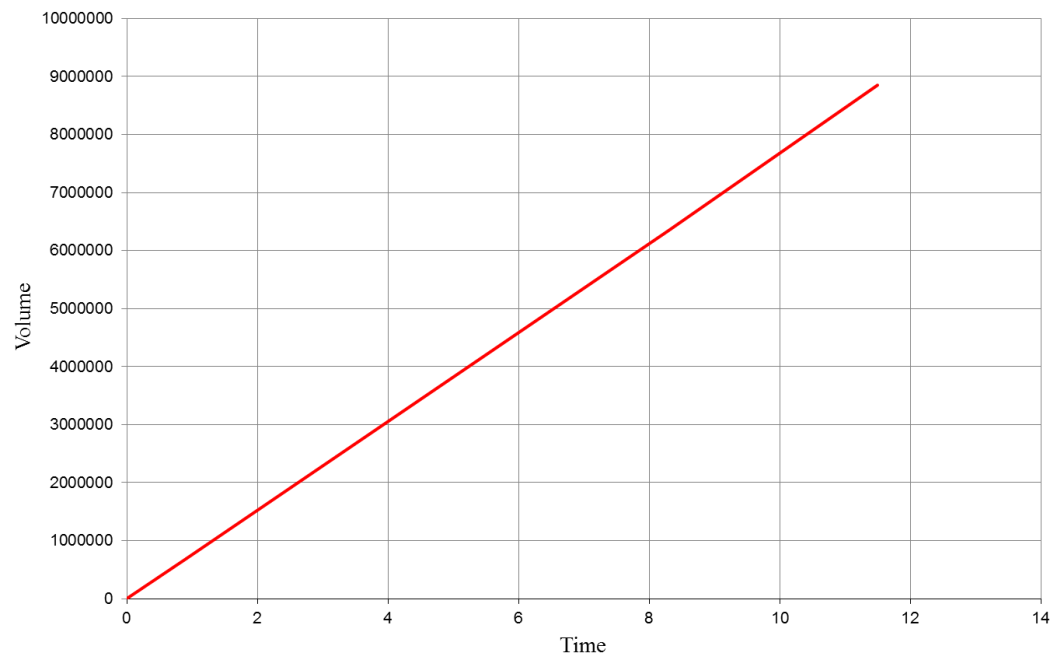
TIME =	2.908924174			TIME =	2.887807264	
	20.24461437	5.60873657			13.27590355	5.60873657
	19.9530597	1.990026046			12.99497552	2.224442591
	19.51367857	2.008499988			12.70319425	2.012019432
	19.07800675	1.971611583			12.42036415	1.981598534
	18.64603483	1.957704625			12.13876233	1.977761288
	18.21497104	1.957060496			11.85731267	1.977294419
	17.78366439	1.959952662			11.57587955	1.977260968
	17.35194838	1.962368165			11.29445602	1.977001739
	16.9199722	1.963518484			11.0130873	1.976036929
	16.48791086	1.963578808			10.73185452	1.974102805
	16.05590767	1.962739461			10.45085246	1.971163422
	15.62408866	1.96105287			10.17017244	1.9673087
	15.1925778	1.958509562			9.889895365	1.962656035
	14.76150074	1.955102108			9.610091948	1.957299795
	14.33098276	1.950843641			9.330825593	1.951300356
	13.90114597	1.945762523			9.052155317	1.944692172
	13.47210786	1.939892498			8.774137622	1.937495919
	13.04398104	1.933266103			8.496827325	1.929727686
	12.61687364	1.925911753			8.220277728	1.92140393
	12.19089001	1.917853152			7.944540486	1.912543125
	11.76613132	1.909110393			7.669665419	1.903165559
	11.34269606	1.899700056			7.395700374	1.893292432
	10.92068052	1.88963613			7.122691191	1.882944924
	10.50017905	1.87893099			6.850681738	1.872143527
	10.08128431	1.86759622			6.579713989	1.860907704
	9.664087231	1.855643304			6.309828134	1.849255835
	9.248677085	1.843084145			6.041062672	1.83720523
	8.835141336	1.829931535			5.773454516	1.824772278
	8.423565467	1.816199892			5.507039061	1.81197266
	8.014032774	1.801904795			5.241850244	1.798821647
	7.606624191	1.787063054			4.977920565	1.78533437
	7.201418108	1.771692782			4.715281098	1.771525901
	6.798490179	1.755813377			4.453961487	1.757411456
	6.397913138	1.739445504			4.193989924	1.743006573
	5.999756617	1.722611062			3.935393111	1.728327256
	5.604086965	1.705333205			3.678196212	1.713390025
	5.210967037	1.687636754			3.422422804	1.698211967
	4.820455968	1.669547852			3.168094822	1.682810777
	4.432608982	1.651094071			2.915232495	1.667204791
	4.047477167	1.632304579			2.663854289	1.651412989
	3.665107239	1.613210193			2.413976848	1.635454985
	3.285541302	1.59384336			2.165614931	1.619350996
	2.908816613	1.574238135			1.91878136	1.603121788
	2.534965348	1.554430148			1.67348697	1.586788624
	2.16401437	1.534456673			1.429740567	1.57037316
	1.795985005	1.514356295			1.187548895	1.553897349
	1.43089287	1.494168791			0.946916613	1.537383347
	1.068747699	1.473935039			0.707846278	1.520853437
	0.709553192	1.453696953			0.470338331	1.504330207
	0.353306867	1.433497337			0.23439102	1.487837927
	0	1.413378649			0	1.471409194

Figure 4.7.8 – a sample void distributions at the second F-height, as shown in the LB RESULTS tab in Excel

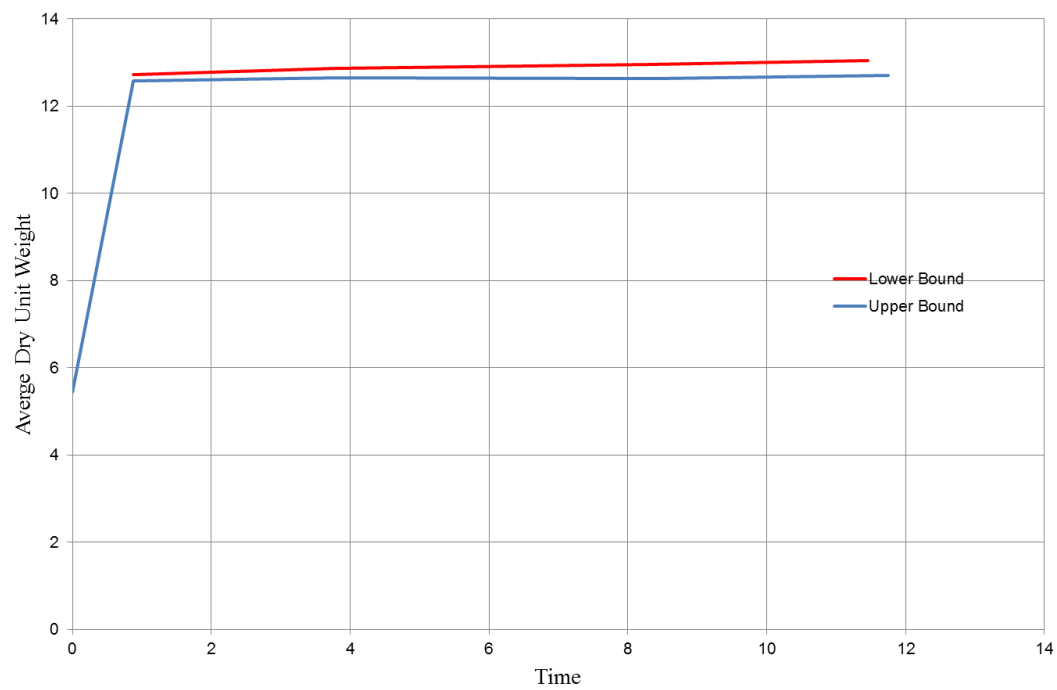
The next five tabs: Time vs Surface Height, Volume-Area vs Height, Water Balance, Time vs. Avg. Dry Unit Weight, UB – Void Ratio Dist., and LB – Void Ratio Dist. are graphs that have been generated using the data discussed previously, see figure 4.7.10.



Water Expelled Due to Consolidation
Time vs. Volume



Time vs. Average Dry Unit Weight



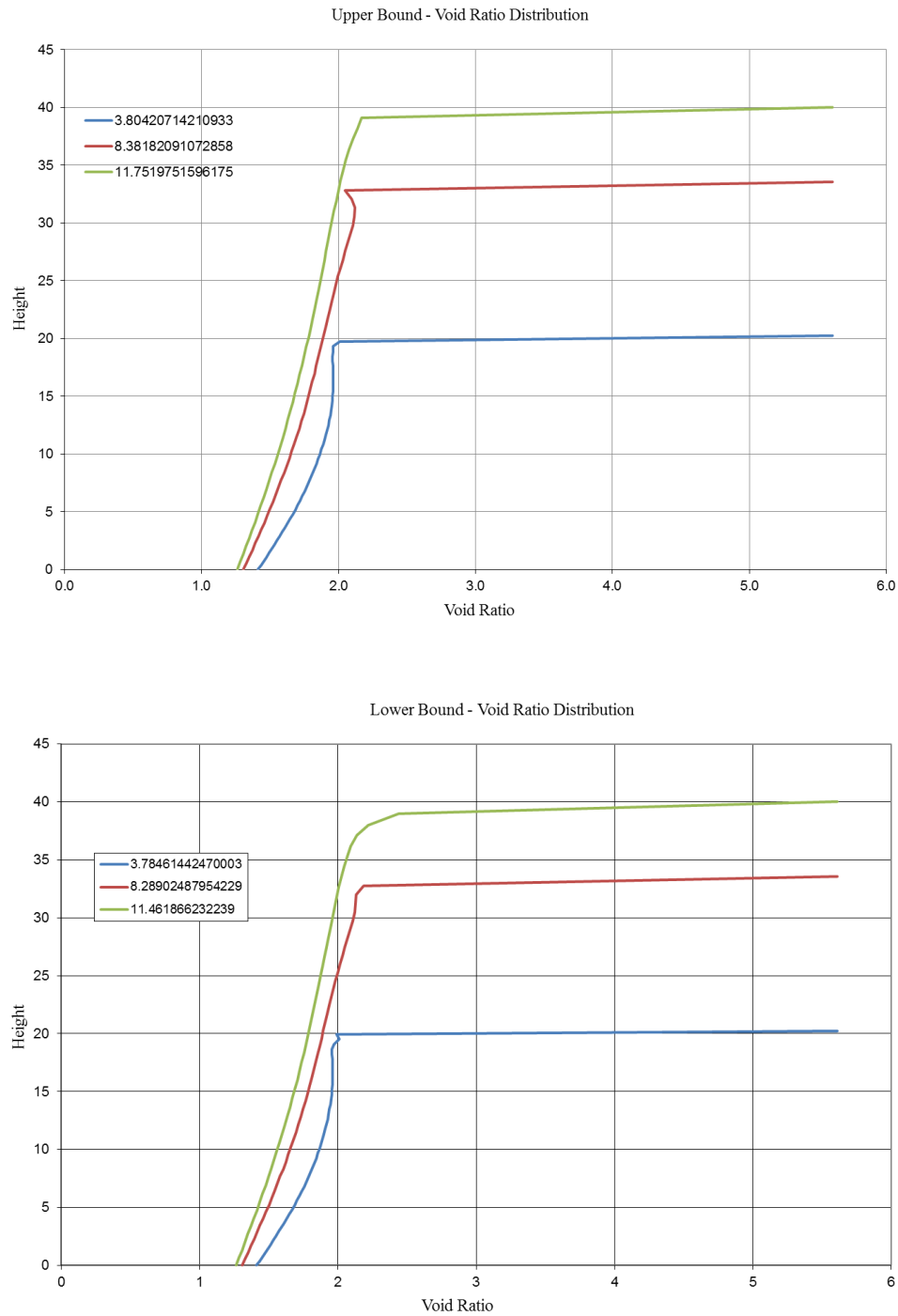


Figure 4.7.9 – sample graphs from the Time vs Surface Height, Volume-Area vs Height, Water Balance, Time vs. Avg. Dry Unit Weight, UB – Void Ratio Dist., and LB – Void Ratio Dist. tabs respectively

The final two tabs CURRENT LB and CURRENT UB are final void ratio distributions when the analysis stopped. These results are typically not used; but could be used to determine the void ratio profiles in case the programs terminates before the analysis is complete.

5.0 Potential Problems and Troubleshooting Techniques

The three-dimensional model and TMT never changes the problem to force convergence – i.e. input values are never changed for easier calculations. Thus TMT isn't forced to converge each time and sometimes it won't. After using TMT extensively several potential problems were discovered; these will be discussed and potential solutions will also be presented.

5.1 Recommended Initial Area

In TMT, the central column or column #1 in the discretization of the storage facility always has the area of the initial area, a_0 , – see figure 2.1.6 – this can create calculation issues. Table 4.7.1 shows a sample table from the LB Results tab, the filling rate for the first column is greater by a factor of 35 than the filling rate for column two. The other filling rates differ only by a factor of 1.5 to 4 for this example, which is typical. This can lead to problems that will be discussed in sections 5.5 and 5.5.

The first initial area is smaller by a factor of approximately 35 than the second area. Additionally the area of the initial column is 0.06% of the area for the final column.

This difference in area is very taxing, from a numerical stability issue, on the CONDES algorithm. Thus a small initial area creates problems that will be discussed in sections 5.4 and 5.5.

Due to these potential problems it is recommended that the initial area be approximately 0.2% - 0.5% of the final area and/or differ from the second area by a factor of 1-4. These recommendations are not exact but they have been found to increase the likelihood of convergence. TMT can be run successfully with a small initial area but a larger initial area greatly increases the probability of a successful analysis.

5.2 Runtime Error

Before CONDES is called, the average dry unit weights for both the upper and lower bound methods are calculated in the Excel macro – in Visual Basic. Occasionally, on approximately 1-2%, of the runs this can cause problems. A “Run-time error ‘6’ ” message dialog box will pop up, see figure 5.2.1. The user should just click the End button and then rerun the analysis.

This specific run-time error is an overflow error. When calculating the average dry unit weights occasionally the algorithm will divide by a very small number – basically zero – and this creates too large of a number for the Visual Basic code to handle. This error occurs at random and there are no lingering effects – once the analysis has been restarted it will proceed fine.

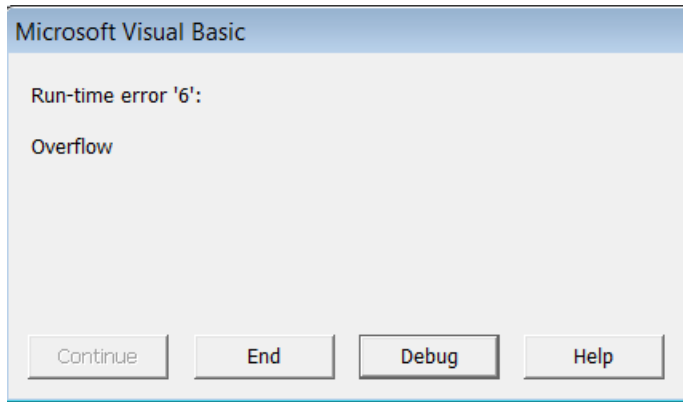


Figure 5.2.1 – Run-time error ‘6’ is an overflow error that occurs when the average dry unit weights are calculated

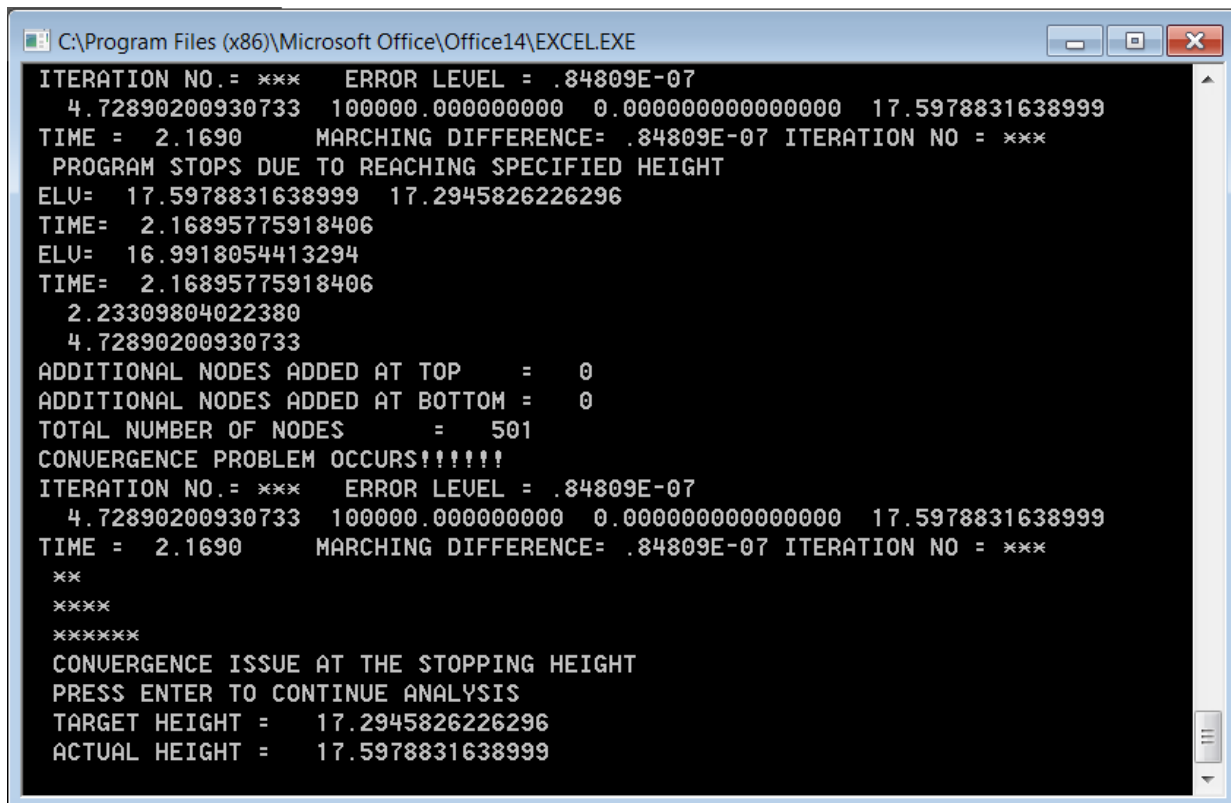
5.3 Minimum Void Ratio Reached

When performing a post deposition analysis if CONDES reaches the minimum void ratio it will terminate the analysis. This normally occurs for two reasons. The first and by far the most common is that the minimum void ratio is set too high, see section 4.6. The second is that there could be an inconsistency in the units for the values inputted – the numerical values inputted correspond to different units.

5.4 Convergence Issue

Sometimes CONDES cannot converge to the exact F-height and when this occurs CONDES stops the analysis and displays an error message, see figure 5.4.1. This type of error is fairly common and is serious. The actual difference is small, normally not greater than 0.5. However when this convergence error occurs it can produce errors in

the void ratio distribution, see figure 5.4.2. There are two void ratio profiles for time 115.5 and the second one in blue is clearly wrong. This is an extreme example normally the multiple void ratios for the same time are reasonably close together, but still differ. Therefore when there is a convergence issue at an F-height the void ratio distribution should be examined closely and it is recommended that changes are made to the input parameters and the analysis is performed again.



```

C:\Program Files (x86)\Microsoft Office\Office14\EXCELE.EXE
ITERATION NO.= *** ERROR LEVEL = .84809E-07
4.72890200930733 100000.000000000 0.000000000000000 17.5978831638999
TIME = 2.1690 MARCHING DIFFERENCE= .84809E-07 ITERATION NO = ***
PROGRAM STOPS DUE TO REACHING SPECIFIED HEIGHT
ELU= 17.5978831638999 17.2945826226296
TIME= 2.16895775918406
ELU= 16.9918054413294
TIME= 2.16895775918406
2.23309804022380
4.72890200930733
ADDITIONAL NODES ADDED AT TOP = 0
ADDITIONAL NODES ADDED AT BOTTOM = 0
TOTAL NUMBER OF NODES = 501
CONVERGENCE PROBLEM OCCURS!!!!!!
ITERATION NO.= *** ERROR LEVEL = .84809E-07
4.72890200930733 100000.000000000 0.000000000000000 17.5978831638999
TIME = 2.1690 MARCHING DIFFERENCE= .84809E-07 ITERATION NO = ***
**
***
*****
CONVERGENCE ISSUE AT THE STOPPING HEIGHT
PRESS ENTER TO CONTINUE ANALYSIS
TARGET HEIGHT = 17.2945826226296
ACTUAL HEIGHT = 17.5978831638999

```

Figure 5.4.1 – convergence issue at an F-height

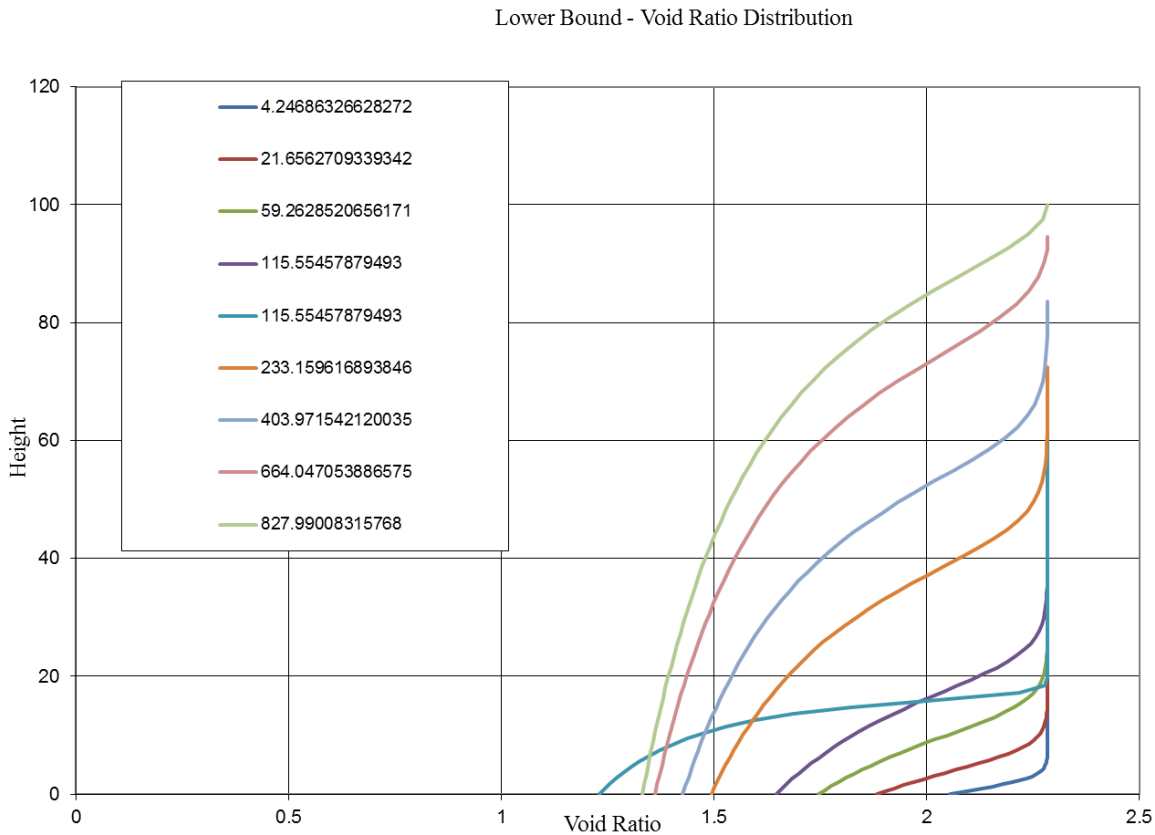


Figure 5.4.2 – problem in void ratio distribution due to convergence error

It is recommended that the number of columns be changed first, the number can be increased or decreased; please see section 2.1 for a discussion of the discretization procedure. Changing the number of columns only affects the model and not the site or material properties. It is not always better to have a high number of columns because this can create columns that have low height and large areas – which can be taxing from a numerical standpoint. Additionally have too few columns will not approximate the volume accurately. From experience generally between 4-12 columns is sufficient.

Additionally from experience increasing or decreasing the number of columns by 1-4 can alleviate some convergence issues.

When the Z_I parameter, see equation 2.0.1, is small – less than 0.1 kPa – this can trigger convergence issues as well. This low value for Z_I corresponds to a high initial void ratio and a high rate of void ratio change with low values of effective stress, and this is very numerically taxing on the algorithm. Changing the Z_I parameter to a larger value will help reduce this effect; Z_I should only be changed slightly – just enough to perturb the calculations. Changing Z_I by too much will result in the analysis not accurately simulating the correct real world situation.

It is recommended that the other parameters from SICT not be changed, as these are dependent on the properties of the material and changing them, even slightly, could result in different material behavior. The most effective solution appears to be increasing the initial area, see section 5.1. In most cases increasing the initial area eliminated or significantly reduced the instances of non-convergence.

Finally due to the uncompromising nature of TMT – i.e. parameters are not changed to force convergence – sometimes the convergence issues cannot be eliminated. However as long as the void ratio distributions are critically analyzed, in most cases the non-convergence will not cause unsolvable problems.

5.5 LB RESULTS Tab F-height Time Problem

As previously mentioned there is an error that can occur with the filling times in the LB Results tab, see Table 5.5.1. In row two column one, the time decreases. The

time should actually be the same; this is a numerical error. Typically this error occurs when there is a small initial area but it can occur with large initial areas as well. The time to fill the final column is so small that it creates an error which causes the time counting procedure to go back to an earlier time step, and this causes the problem where the time appears to decrease. This error, when it occurs, can be significant for lower times. The results need to be analyzed and if the time for the final column is found to decrease, in actuality the time should be the same.

	Filling Rate #1	Filling Rate #2	Filling Rate #3	Filling Rate #4	Filling Rate #5	Filling Rate #6	Filling Rate #7	Filling Rate #8	Filling Rate #9	Filling Rate #10		
F-Heights	35.851	2.634	0.642	0.317	0.223	0.163	0.115	0.082	0.054	0.043		Areas
6.62	0.214											400
17.29	4.033	4.601										5444
28.10	17.409	18.025	18.002									22347
39.05	37.607	37.869	37.869	37.344								45211
50.14	56.292	56.343	56.312	56.288	54.440							64400
61.27	0.000	79.857	79.857	79.747	78.817	75.100						87858
72.38	117.605	117.384	117.379	117.122	115.996	113.097	107.097					124606
83.53	170.812	170.810	170.790	169.972	168.137	165.588	161.010	151.264				174354
94.55	260.076	260.072	259.691	257.968	255.259	252.455	248.947	243.271	228.984			266008
100.00	163.943	163.938	163.598	162.618	161.541	160.356	158.808	155.968	151.291	137.503		334821

Table 5.5.1 – errors that can occur with lower bound filling times

6.0 Potential Areas of Improvements

Two prospective changes to the three-dimensional model will be discussed. Currently when calculating the water balance only the total amount of water expelled is given, see section 3.3. It would be better if the water balance calculations considered the top and bottom boundary conditions and could therefore tell how much water was

expelled at each separate boundary. In many mining operations water is scarce and a more accurate calculation of the available water would be very useful.

Additionally it would be desirable to have average dry unit weights for both the no consolidation and instantaneous consolidation conditions. This would place an additional upper and lower bound on the values, see sections 3.1 and 3.2. This would provide consistent results for both the time vs. dry unit weights graph and the time vs. surface height graphs.

7.0 Parametric Study

A short parametric study will be presented. A more detailed study could potentially develop a relationship between consolidation parameters and consolidation behavior. The purpose of this study is to illustrate potential problems that can occur when using TMT and how hydraulic conductivity can affect the consolidation behavior.

The consolidation characteristics of the material are taken from Gjerapic et al. (2008) and are listed in table 7.0.1. The deposition rate and layout geometry are also from Gjerapic et al. (2008) and are given in table 7.0.1. The volume calculations are performed by formula for the volume of an inverted cone frustum, see equation 2.1.2.

Material Properties		Impoundment Geometry				Deposition Rate
A_1	3.3	Stage	Height (m)	Area (m ²)	Volume (m ³)	730,000 tonne/year
B_1	-0.21	1	0	7.07E+04	0	
C	0.091 (m/year)	2	13.33	1.22E+05	1.27E+06	
D	5.50E+00	3	26.67	1.86E+05	3.30E+06	
Z_1	0.08 (kPa)	4	40	2.64E+05	6.29E+06	
G_s	3.66					

Table 7.0.1 – material properties, layout geometry, and deposition rate used in the parametric study

Initially the analysis was run for the original material properties as given in table 7.0.1. The void ratio distributions calculated by the lower bound method are shown in figure 7.0.1 and the surface height vs. time is shown in figure 7.0.2. Originally the material is very close to consolidating instantly, see figure 7.0.2. Additionally the void ratio distributions show this behavior, see figure 7.0.1. The void ratio distributions show a sharp decrease from the initial void ratio which is 5.6, see equation 2.0.1, and the final void ratios which will vary for each height (1.0 – 1.2).

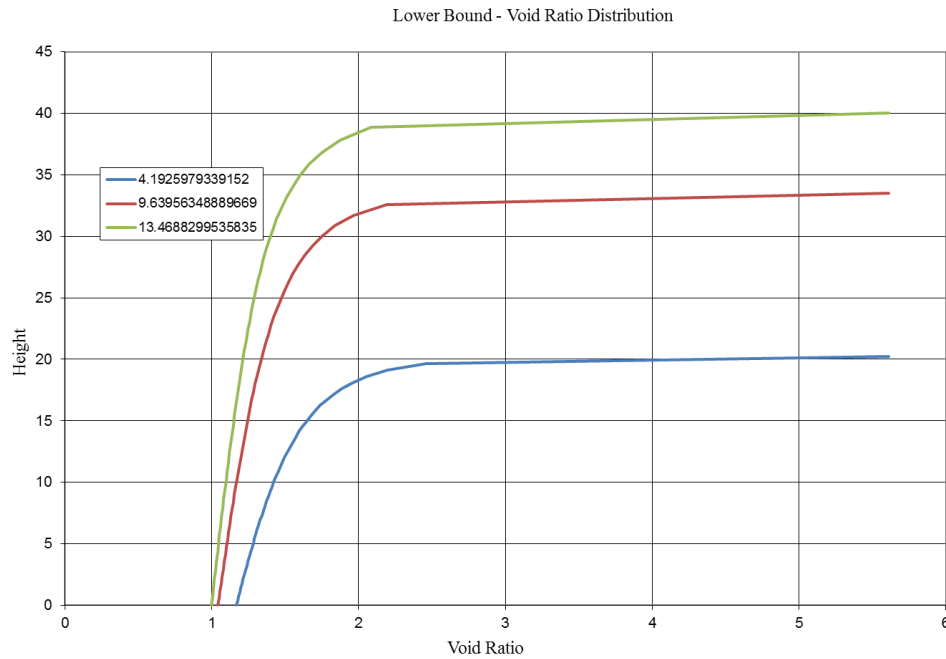


Figure 7.0.1 – void ratio distributions calculated by the lower bound method for the original parameters

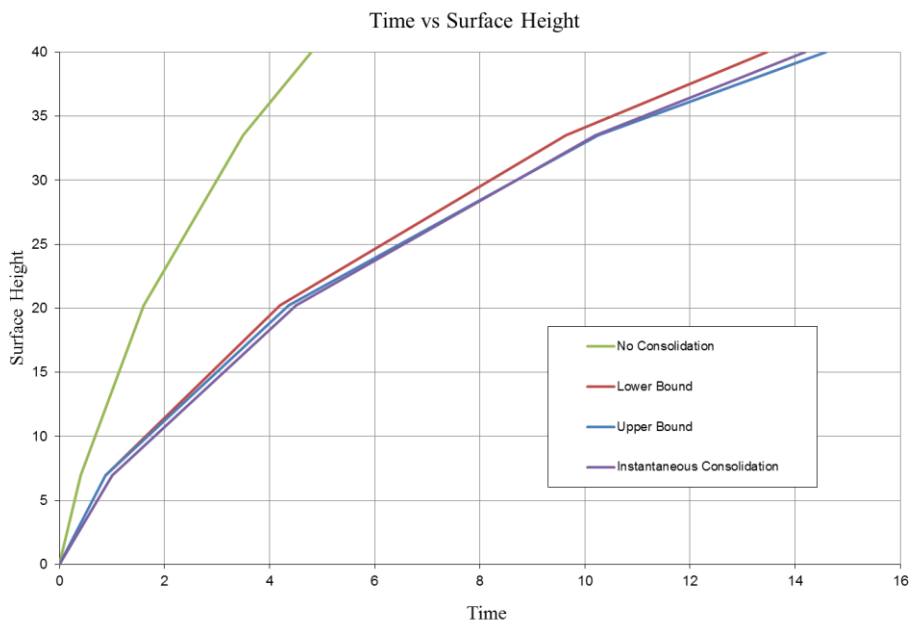


Figure 7.0.2 – surface height vs. time for the original parameters

The initial hydraulic conductivity was reduced by a factor of 10 until an error message was produced. Figure 7.0.3 shows the height vs. time when the hydraulic conductivity was reduced by a factor of 10^3 . The curve has moved to the left which is due to the fact that the material is undergoing less consolidation – due to the decrease in hydraulic conductivity.

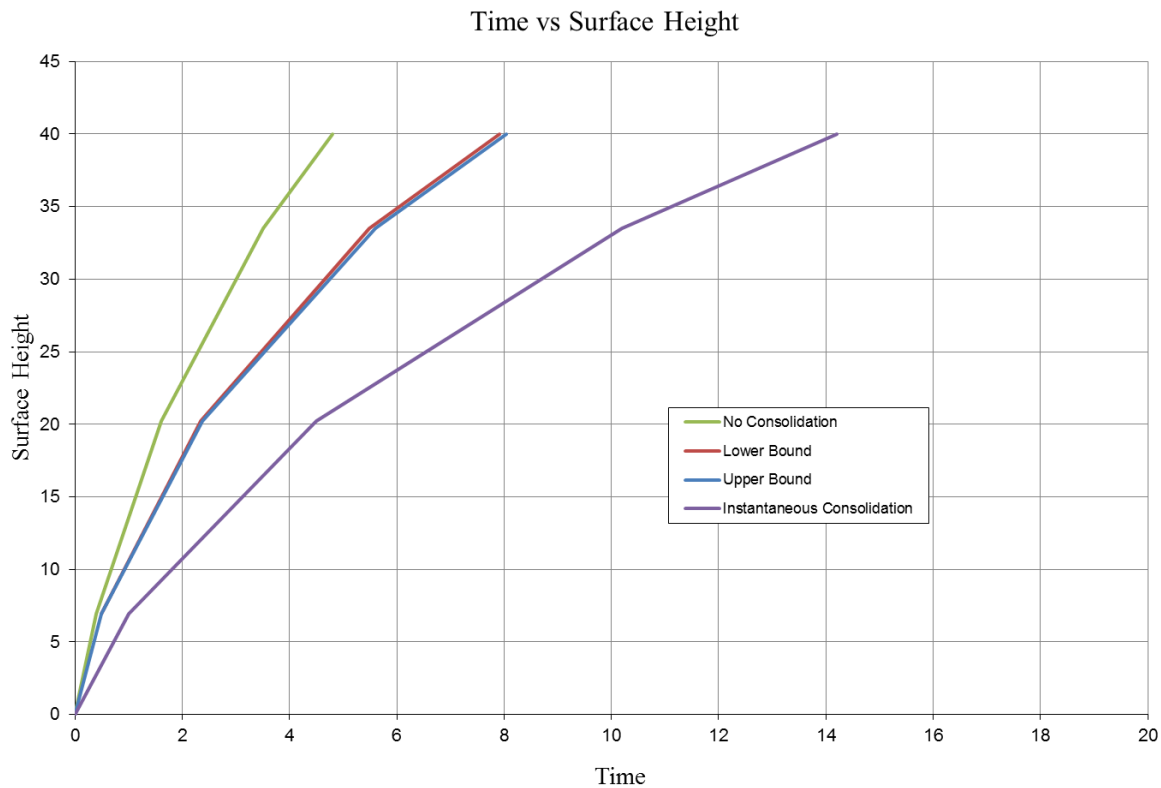


Figure 7.0.3 – height vs. time when hydraulic conductivity was reduced by a factor of 10^3

The last analysis that produced a result was when the hydraulic conductivity was reduced by a factor of 10^5 . The void ratio distributions are shown in figure 7.0.4 and the

height vs. time graph is shown in figure 7.0.5. The void ratio distributions show very little consolidation has occurred, hence the near vertical void ratio distribution. Also figure 7.0.5 shows that the upper and lower bound curve are almost identical to the no consolidation curve.

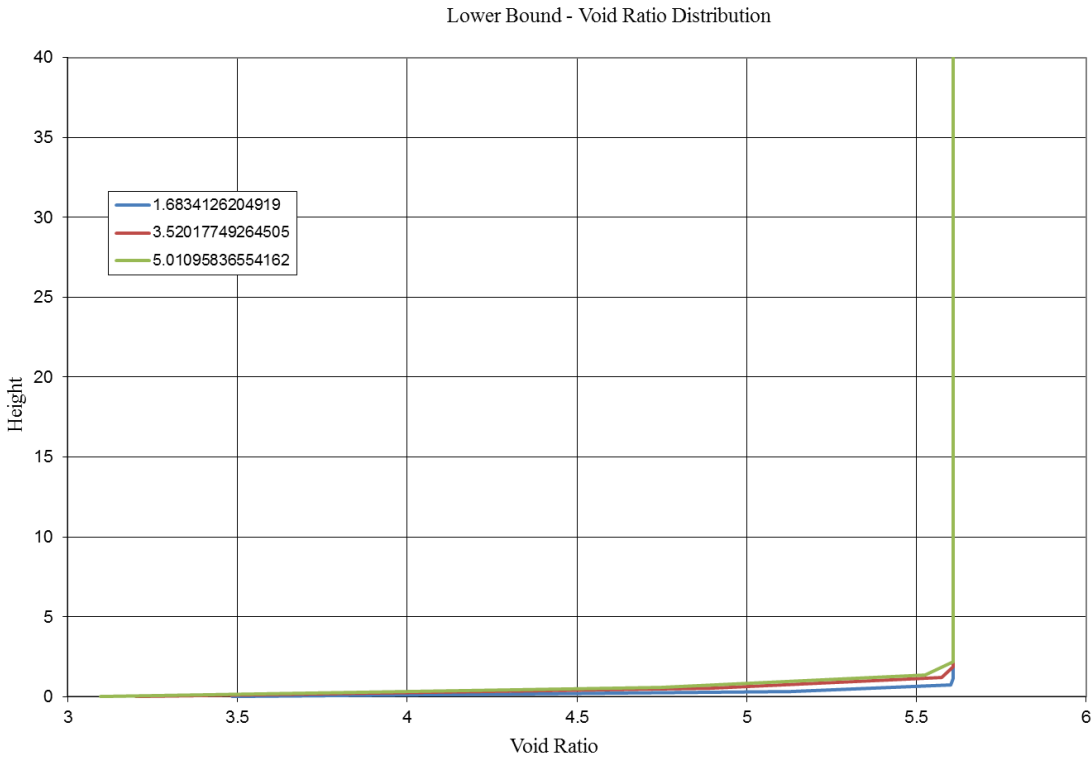


Figure 7.0.4 – void ratio distributions when the hydraulic conductivity was reduced by a factor of 10^5

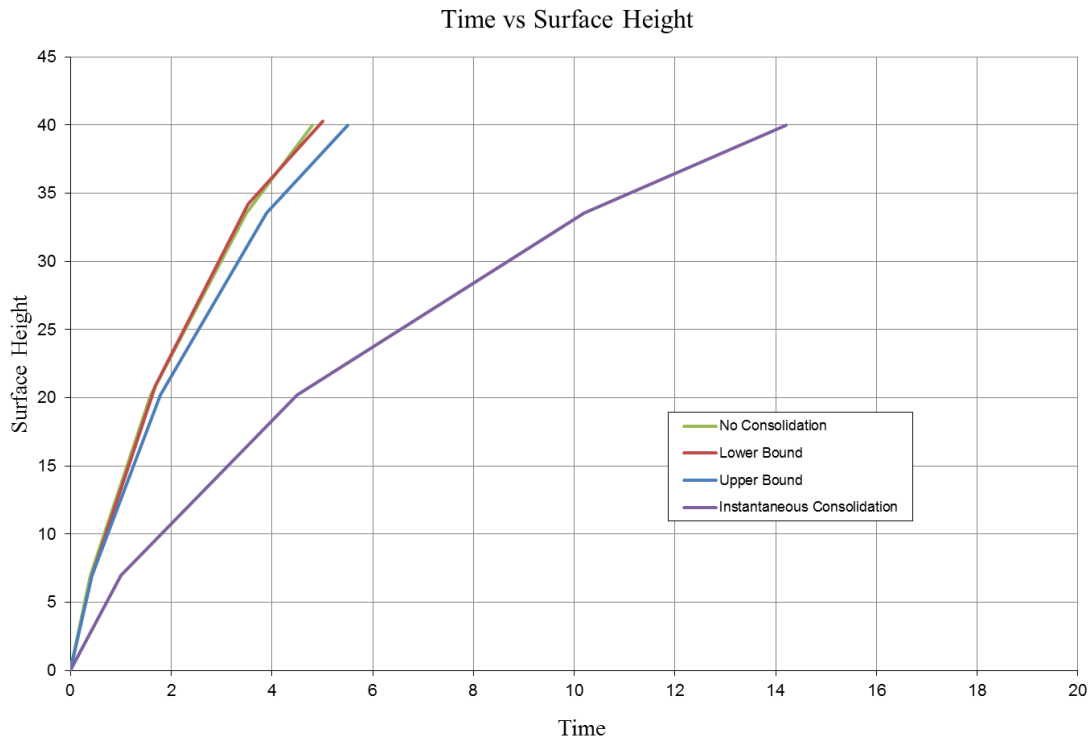


Figure 7.0.5 – surface height vs. time graph when the hydraulic conductivity was reduced by a factor of 10^5

When the hydraulic conductivity was reduced by a factor of 10^6 the analysis produced an error message, see figure 7.0.6. This was most likely due to the fact that the material was undergoing very little consolidation and as a result the hydraulic gradients were very high and produced numerical issues that the model couldn't handle.

```

C:\Program Files (x86)\Microsoft Office\Office14\EXCEL.EXE
ITERATION NO.= 24 ERROR LEVEL = .16416E-08
ITERATION NO.= 25 ERROR LEVEL = .17355E-10
ITERATION NO.= 26 ERROR LEVEL = .14566E-12
ITERATION NO.= 27 ERROR LEVEL = .23768E-14
ITERATION NO.= 28 ERROR LEVEL = .15801E-16
ITERATION NO.= 29 ERROR LEVEL = .24993E-18
ITERATION NO.= 30 ERROR LEVEL = .21597E-20
ITERATION NO.= 31 ERROR LEVEL = .21954E-22
18.6161923956122 100000.000000000 0.000000000000000 6.95900293693837
TIME = 0.42774 MARCHING DIFFERENCE= .13254E-02 ITERATION NO = 31
PROGRAM STOPS DUE TO REACHING SPECIFIED HEIGHT
ELU= 6.95900293693837 6.96580807934655
TIME= 0.427741363804928
6.96580807934655 6.96580807934655
UD 6.96580807934655 6.96580807934655
6.96580807934655 20.2355634119473
6.96580807934655 33.5278599310801
6.96580807934655 40.000000000000000
THE DIFFERENCE OF FINAL SAMPLE HEIGHT BETWEEN THE STEADY STATE AND CURR
ENT STATE IS***** %
-FINAL STEADY STATE INFORMATION IS AVAILABLE-
TIME = 0.4277
10.8296633695022
INAPPROPRIATE COMBINATION OF BOUNDARY CONDITIONS AND MATERIAL PROPE
RTIES!
PLEASE ADJUST THE BOUNDARY CONDITION OR MODIFY THE MATERIAL PROPERTIES
!

```

Figure 7.0.6 – error message when hydraulic conductivity was too low

Similarly the hydraulic conductivity was also increased by factors of 10 to try and produce another situation where the model would “crash” or not produce an answer.

Initially the material behavior was very close to instantaneous consolidation, see figure 7.0.2. When the hydraulic conductivity was increased by a factor of 10^5 , an identical error message was produced, see figure 7.0.6.

However when the hydraulic conductivity was increased by factors of 10^2 , 10^3 , and 10^4 an interesting behavior in the model was observed. The model did not crash but rather got stuck at one elevation – the particular elevation varied depending on the factor.

It is believed that this behavior was due to the fact that the material could not reach convergence at the given height, see section 5.4. When the height is very close to the convergence point the model then adds more material to try and increase the elevation. However since the material was basically consolidating instantaneously the elevation could never reach the desired height. This behavior is interesting but for most materials it would never occur naturally.

References

- Abu-Hejleh, A. N., Znidarcic, D., and Barnes, B.L., 1996, Consolidation Characteristics of Phosphatic Clays, *Journal of Geotechnical Engineering*, ASCE, New-York, Vol. 122, No. 4. pp. 295-301.
- Abu-Hejleh, A.N. & Znidarcic, D. 1994. Estimation of the Consolidation Constitutive Relations. *Computer Methods and Advances in Geomechanics*, Siriwardane & Zaman (eds) Balkema, Rotterdam, pp. 499-504.
- Bartholomeeusen, G. & Sills, G.C. & Znidarcic, D. & Van Kesteren, W. & Merckelbach, L.M. & Pyke, R. & Carrier, W.D. & Lin, H. & Penumadu, D. & Winterwerp, H. & Masala, S. & Chan, D 2002. Numerical Prediction of Large Strain Consolidation. *Geotechnique*: Volume 52, Issue 9, November 1, 2002 pp. 639-648.
Available from:
<http://www.icevirtuallibrary.com/content/article/10.1680/geot.2002.52.9.639>
- Coffin, J.G. 2010. A Three-Dimensional Model for Slurry Storage Facilities. Doctoral Thesis, University of Colorado, Boulder 2010.
- Gjerapic, G. & Johnson, J. & Coffin, J. & Znidarcic, D 2008. Determination of Tailings Impoundment Capacity via Finite-Strain Consolidation Models. International GeoCongress Conference. ASCE Proceedings Paper GeoCongress, March 2008.
Available from:
http://ascelibrary.org/proceedings/resource/2/ascecp/311/40972/99_1

Gibson, R.E. & England, G.L. & Hussey, M.J. 1967. The Theory of One-Dimensional Consolidation of Saturated Clays – Finite Non-Linear Consolidation of Thin Homogenous Layers. *Geotechnique*: Volume 17, Issue 3, 1967, pp. 261-273.
Available from:

<http://www.icevirtuallibrary.com/content/article/10.1680/geot.1967.17.3.261>

Marcus, Jerold J. (editor) 1997. Mining Environmental Handbook – Effects of Mining on the Environment and American Environmental Controls on Mining. Imperial College Press ISBN 1860940293.

McVay, M. & Townsend, F. & Bloomquist, D. 1986. Quiescent Consolidation of Phosphatic Waste Clays. American Society of Civil Engineers *Journal of Geotechnical Engineering*, 112(11):1033-1049, February 1986. Available from:

http://ascelibrary.org/gto/resource/1/jgendz/v112/i11/p1033_s1

Microsoft Corporation. Microsoft Excel 2010 [Computer Software]. Redmond, WA.

Orosur Mining 2011. Image Gallery. Available from:

http://www.orosur.ca/media/download_library/

Weisstein, Eric W 2012. "Cavalieri's Principle." From MathWorld--A Wolfram Web Resource. Available from:

<http://mathworld.wolfram.com/CavalierisPrinciple.html>

Znidarcic, D. & Yao, D. 1997. User's Manual for Computer Program CONDES0.

Prepared by the University of Colorado, Boulder under a grant from the Florida

Institute of Phosphate Research, October 1997. Available from:

[http://fipr1.state.fl.us/fipr/fipr1.nsf/0/c3d53550b82be09685256b2f00575a8f/\\$FILE/04-055-134Final.pdf](http://fipr1.state.fl.us/fipr/fipr1.nsf/0/c3d53550b82be09685256b2f00575a8f/$FILE/04-055-134Final.pdf)

Znidarcic, D & Miller, R. & van Zyl, D. & Fredlund, M. & Wells, S. 2011.

Consolidation Testing of Oil Sands Fine Tailings. Proceedings from Tailings and

Mine Waste 2011, Vancouver, British Columbia, November 6-9, 2011. Available

from: <http://www.infomine.com/publications/docs/Znidarcic2011.pdf>