

Supplement to
Scientific Report No. 20
September 1977

SIMPLE EXPRESSIONS FOR CURRENT ON A
THIN CYLINDRICAL RECEIVING ANTENNA

by

D.C. Chang, S.W. Lee[†] and L. Rispin

Prepared for

U.S. Office of Naval Research
Arlington, Virginia 22217

[†]The second author is with the Department of Electrical Engineering,
University of Illinois, Urbana, Illinois.

This project is monitored by Dr. H. Mullaney of the Office of Naval
Research under contract no. N0014-76-0318 with the University of
Colorado and in part, under contract no. N0014-76-C-0293 with the
University of Illinois.

Supplement to Scientific Report No. 20

In Scientific Report No. 20 [1] simple expressions for the currents on a finite length thin cylinder illuminated by a uniform plane wave were presented. Recently, however, a small error was discovered in the analysis. This supplement is intended to provide the necessary corrections to the affected equations and figures and to give the correct results.

The current on a semi-infinite thin cylinder was given in Eq. (27) and repeated here,

$$I^{\text{ext}}(\theta, z) = E_{\theta}^i \{ I_{S\infty}(\theta, z) - I_{S\infty}(\theta, 0) R_S(\theta, z) I_{\infty}(z) \} \quad (\text{S1})$$

which is a valid expression. Using this equation, an expression for the current on a finite length thin cylinder (34) was developed by summing the multiple reflections from both ends. It was at this point that the terms $R_S(\pi, h-z)$ and $R_S(\pi, h+z)$, ($R_S(\theta, z)$ being defined in Eq. 25) were incorrectly approximated by the term, R , (defined in (36)). As a result, discontinuity of current occurs when the incident angle is $\pi/2$. The error is usually negligibly small except near the end of the antenna. If however we do keep the exact form of R_S at $\theta = \pi$, the correct expression is given by

$$I^{\text{ext}}(\theta, z) = E_{\theta}^i \left\{ I_{S\infty}(\theta, z) + [\delta_S(\pi-\theta, h-z) I_{S\infty}(\theta, h) + \frac{R_S(\pi, h-z)}{R} C_S(\pi-\theta)] I_{\infty}(h-z) \right. \\ \left. + [\delta_S(\theta, h+z) I_{S\infty}(\pi-\theta, h) + \frac{R_S(\pi, h+z)}{R} C_S(\theta)] I_{\infty}(h+z) \right\} \quad (\text{S2})$$

where the terms $\delta_S(\theta, z)$ and R are redefined as

$$\delta_s(\theta, z) = \left[\frac{R_s(\pi, z)}{R} R_s(\theta, 2h) - R_s(\theta, z) \right] \quad (S3)$$

and

$$R = R_s(\pi, 2h) \quad (S4)$$

The other terms in (S2) retain their previous definitions. Our previous expression for the total external current (34) is easily seen to result from (S2) and (S3) if $R_s(\pi, h-z)$ and $R_s(\pi, h+z)$ are replaced by the inapplicable approximation R .

The further modification of the total current expression as was done in (38) is now unnecessary. Equation (S2) expresses the total external current on the cylinder for all values of $-h \leq z \leq +h$ and $0^\circ \leq \theta \leq 180^\circ$ within the accuracy of all of the previous approximations, specifically $ka \ll 1$ and z not close to either $+h$ or $-h$. Since there is no step-function involved in the expression, the external current is now continuous everywhere and for every incident angle. In addition, because the current at the end of a semi-infinite cylinder (S1) and (27) goes to zero as $z \rightarrow 0$, then it should be expected that the current at the end of the finite length cylinder as given by (S2) should also go to zero. To examine this situation, the terms of (S2) take the following form as $z \Rightarrow -h$,

$$I_{s\infty}(\theta, z) = I_{s\infty}(\theta, -h) = I_{s\infty}(\pi - \theta, h) \quad (S5)$$

$$R_s(\theta, h-z) = R_s(\theta, 2h) \quad (S6)$$

$$R_s(\theta, h+z) \sim \frac{1}{I_{\infty}(h+z)} \quad (S7)$$

$$\lim_{z \rightarrow -h} \delta_s(\theta, h+z) \sim \left[\frac{R_s(\theta, 2h)}{R} - 1 \right] \frac{1}{I_{\infty}(h+z)} \quad (S8)$$

$$\lim_{z \rightarrow -h} \delta_S(\pi-\theta, h-z) \sim 0 \quad (S9)$$

$$\lim_{z \rightarrow -h} I_\infty(h-z) = I_\infty(2h) \quad (S10)$$

and,

$$\lim_{z \rightarrow -h} I_\infty(h+z) \sim \frac{2\pi}{\eta} \frac{1}{\ln[2k(h+z)]} \quad (S11)$$

Using (S5) thru (S11) along with the following relation derivable from the equation for $C_S(\theta)$, (37),

$$C_S(\pi-\theta) = -\frac{1}{R I_\infty(2h)} \{ C_S(\theta) + I_{S\infty}(\pi-\theta, h) R_S(\theta, 2h) \} \quad (S12)$$

The following expression for $z \Rightarrow -h$ will result,

$$\lim_{z \rightarrow -h} I^{\text{ext}}(\theta, z) \sim 0 \quad (S13)$$

A similar result also occurs for $z \Rightarrow +h$, as should be expected. Even though the analysis required that the quantity z be away from $+h$ and $-h$, a zero current at the ends is a quite acceptable result.

A complete set of graphs to replace those affected by the discussed revisions in the total external current formula are included in this supplement. The basic features remain unchanged with the exception that now the currents do reach a zero value at the ends of the cylinder. This is most readily observed in Figures S2, S3, S5 and S6. The only other difference between the original and revised graph occur at some of the phase reversals at near-nulls of the current.

The revised current expression (S2) also contains the first sought after goal of zero current for grazing incidence. To demonstrate this we

repeat the small θ forms of the terms of (S2),

$$I_{S\infty}(\theta, z) \sim \frac{2\pi}{ik\eta} \frac{e^{ikz}}{\theta \ln \theta} + 0 \left(\frac{\theta}{\ln \theta} \right) \quad (S14)$$

$$R_S(\theta, z) \sim \frac{\eta}{2\pi} \left[2C\omega + \gamma + i\frac{\pi}{2} + \ln(2kz) + 0(\theta^2) \right] = \frac{e^{ikz}}{I_\infty(z)} + 0(\theta^2) \quad (S15)$$

$$R_S(\pi - \theta, z) \sim \frac{\eta}{2\pi} \left[2C\omega + i\pi - \frac{i}{2kz} + 0(\theta^2) \right] = R_S(\pi, z) + 0(\theta^2) \quad (S16)$$

$$C_S(\pi - \theta) \sim \frac{i2\pi}{k\eta} \frac{1}{\theta \ln \theta} 0(\theta^2) = I_{S\infty}(\theta, 0) 0(\theta^2) \quad (S17)$$

$$C_S(\theta) \sim \frac{i2\pi}{k\eta} \frac{1}{\theta \ln \theta} \left[\frac{e^{ikh}}{I_\infty(2h)} + 0(\theta^2) \right] = - \frac{I_{S\infty}(\theta, h)}{I_\infty(2h)} \left[1 + 0(\theta^2) \right] \quad (S18)$$

$$\delta_S(\theta, h+z) \sim \frac{R_S(\pi, h+z)}{R} \frac{e^{i2kh}}{I_\infty(2h)} - \frac{e^{ik(h+z)}}{I_\infty(h+z)} + 0(\theta^2) \quad (S19)$$

and

$$\delta_S(\pi - \theta, h-z) \sim 0(\theta^2) \quad (S20)$$

Using the small argument forms (S14) thru (S20) in the total current expression (S2) yields,

$$I^{\text{ext}}(\theta, z) \sim 0 \left(\frac{\theta}{\ln \theta} \right) \Rightarrow 0 \quad \text{as } \theta \Rightarrow 0. \quad (S21)$$

Similarly for $\theta \Rightarrow \pi$, it can be shown that

$$I^{\text{ext}}(\theta, z) \sim 0 \left(\frac{\pi - \theta}{\ln(\pi - \theta)} \right) = 0 \quad \text{as } \theta \Rightarrow \pi \quad (S22)$$

Revised graphs for Figures 5 and 8 are included in this supplement and show very little change from the original report drawings. This is primarily due to the magnitude of the error discussed at the beginning of this supplement being very small for z near the center of the cylinder, in

these cases $z = -0.039$ m. Had a value of z been chosen away from the center, the original graphs using the original total current expression would exhibit a discontinuity in the current at $\theta = 90^\circ$. Figure S7 illustrates this jump in the magnitude of the current calculated from (38) for a position very near the end of the cylinder. The abrupt change is due to the particular construction employing step functions in the original total current expression, (38). The discontinuity, however, is now completely eliminated.

In the case of the current penetration into the end of a finite length cylinder, the use of either the original current equation (38) or the revised equation (S2) to determine the penetration current fortunately yields identical results. Thus the penetration current expression (47) does state the correct solution. The figures 12 and 13 based upon this expression are therefore also correct.

Acknowledgment

We wish to thank Ron Prehoda and Dick Swink of Naval Surface Weapons Center, Dahlgren Laboratory, for pointing out the discrepancy in our earlier result.

References

- [1] Chang, D.C., S.W. Lee, and L.W. Rispin, "Simple Expressions for Current on a Thin Cylindrical Receiving Antenna," Technical Report No. 20, (N0014-76-C-0318 with Univ. of Colo. and N0014-76-C-0293 with Univ. of Ill.), Dept. of Elect. Eng., University of Colorado, Boulder, Colorado, 1976.

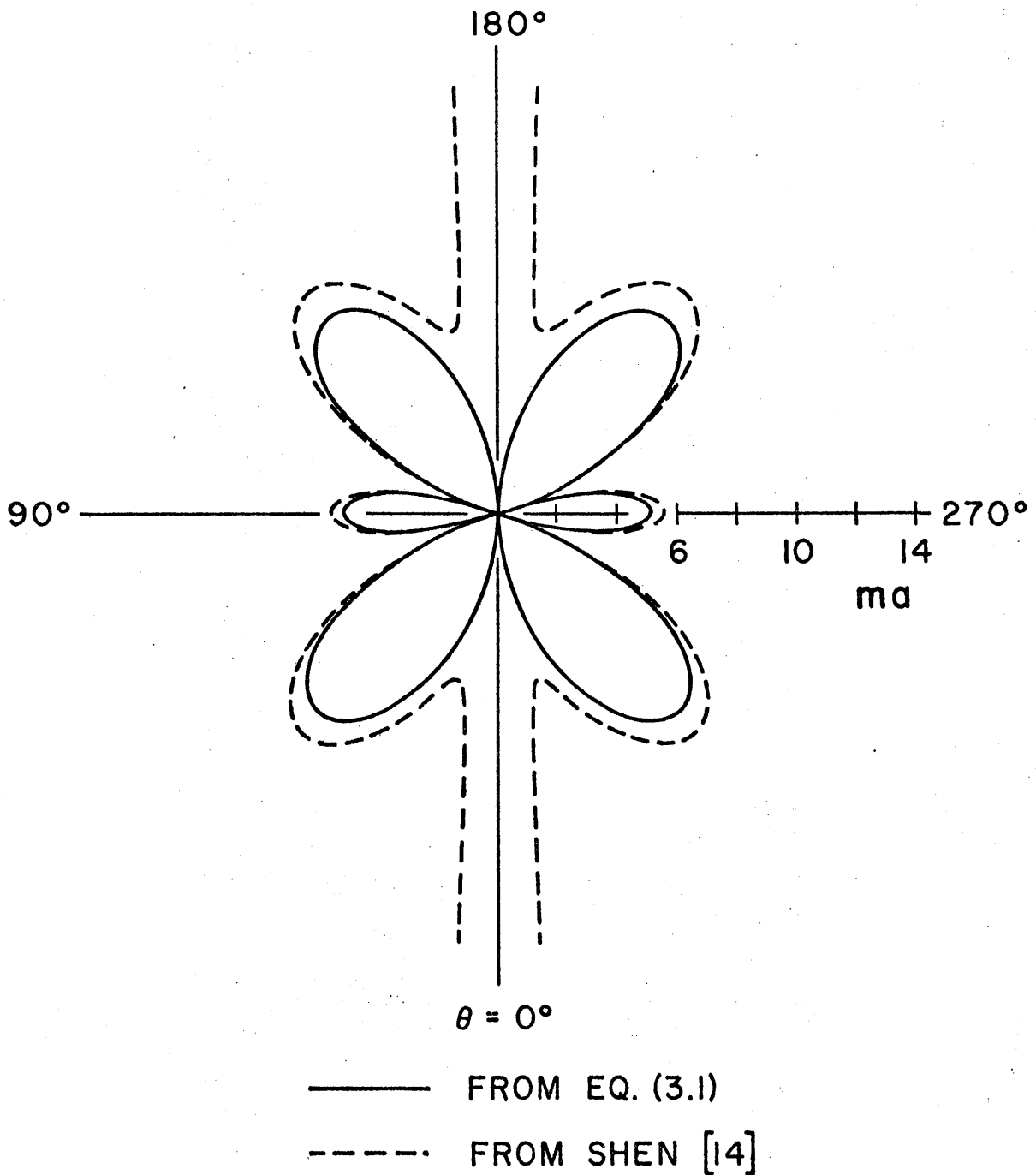


Figure S1. Magnitude of current $I^{\text{ext}}(\theta, z_0)$ on a finite length cylinder calculated from (S2.), with the same parameters denoted in Figure 5.

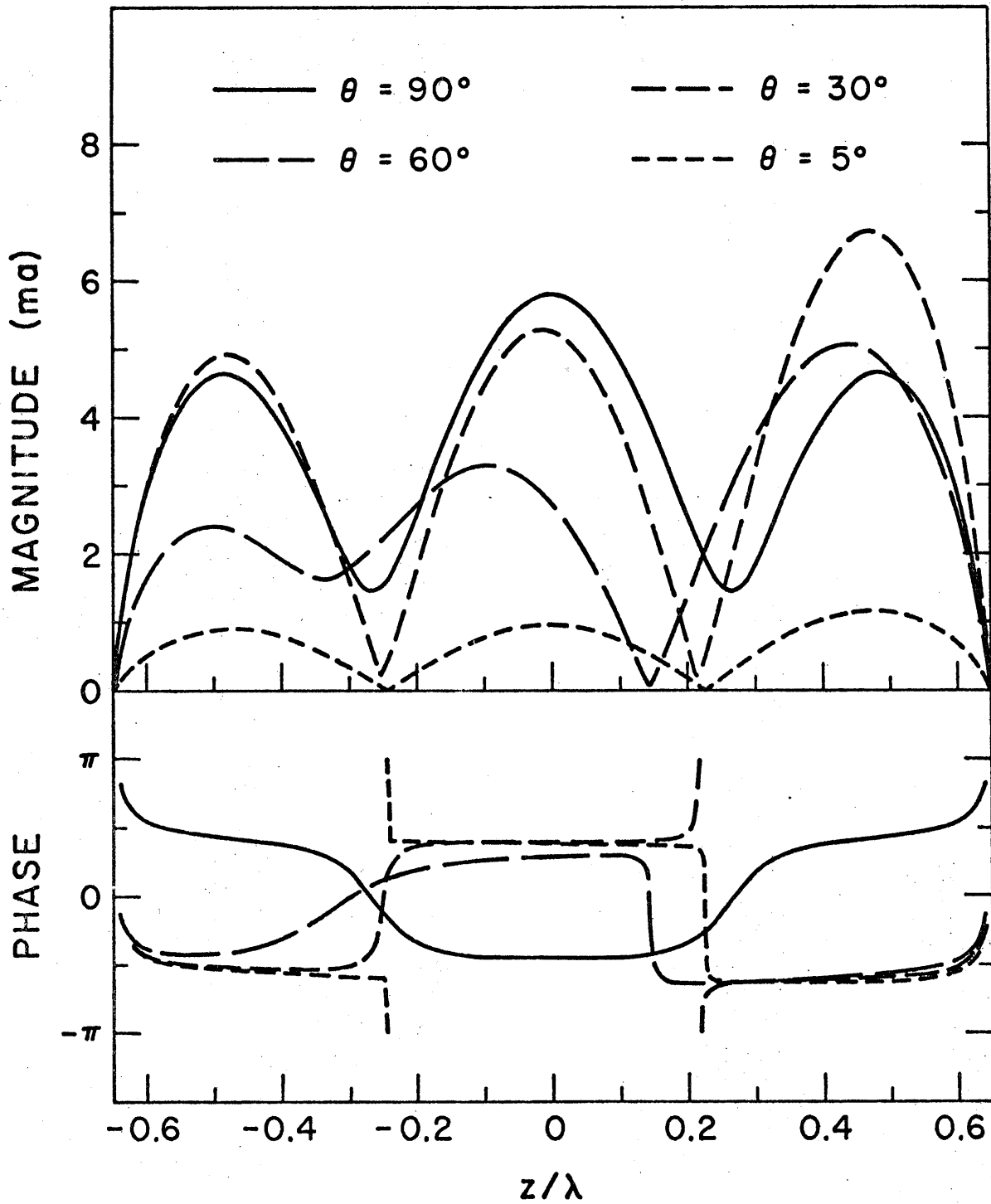


Figure S2. Magnitude and phase of the current, $I^{\text{ext}}(\theta, z)$, as a function of z calculated from (S2.), with the same parameters denoted in Figure 6.

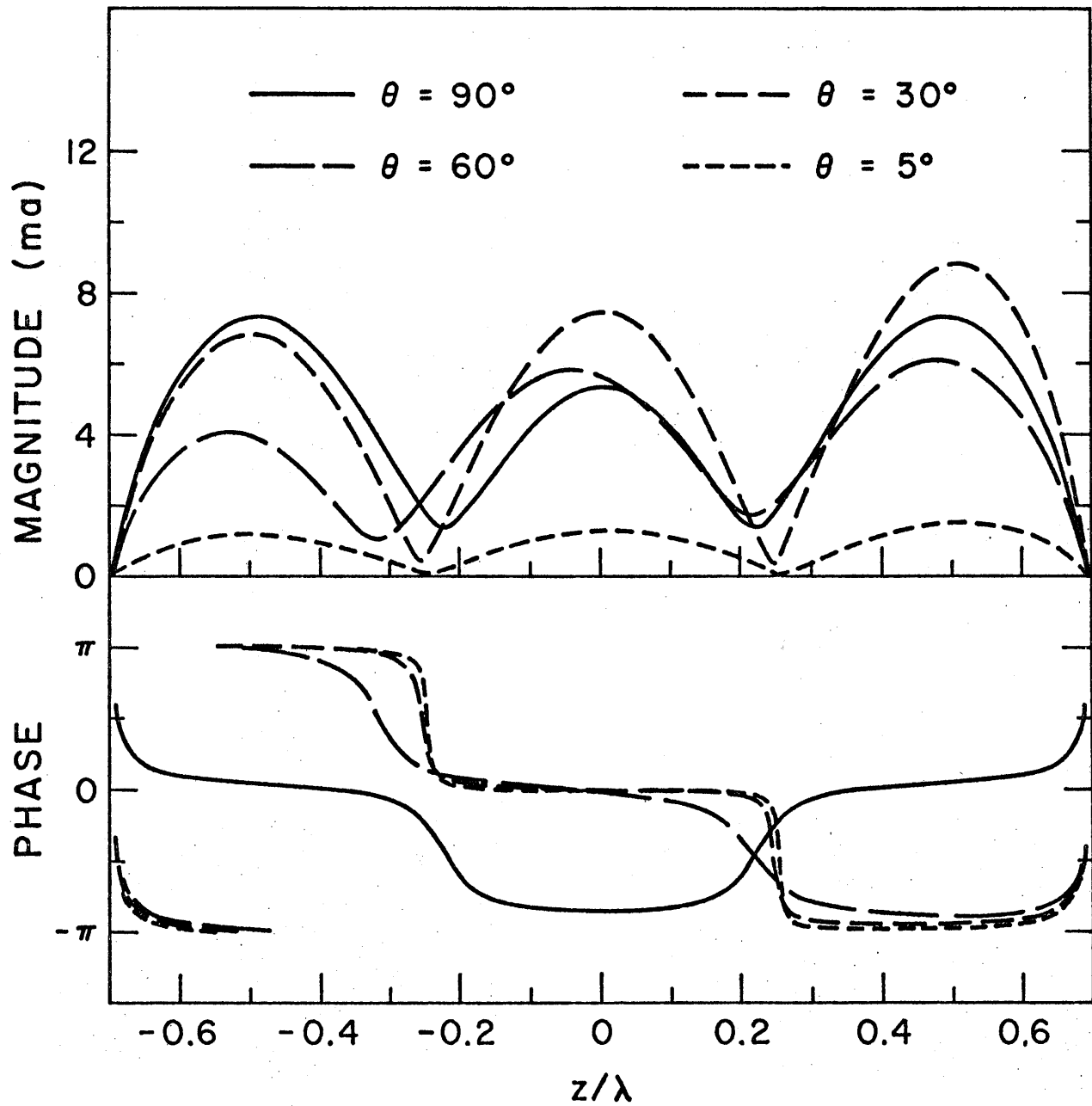


Figure S3. Magnitude and phase of the current, $I^{\text{ext}}(\theta, z)$, as a function of z calculated from (S2.), with the same parameters denoted in Figure 7.

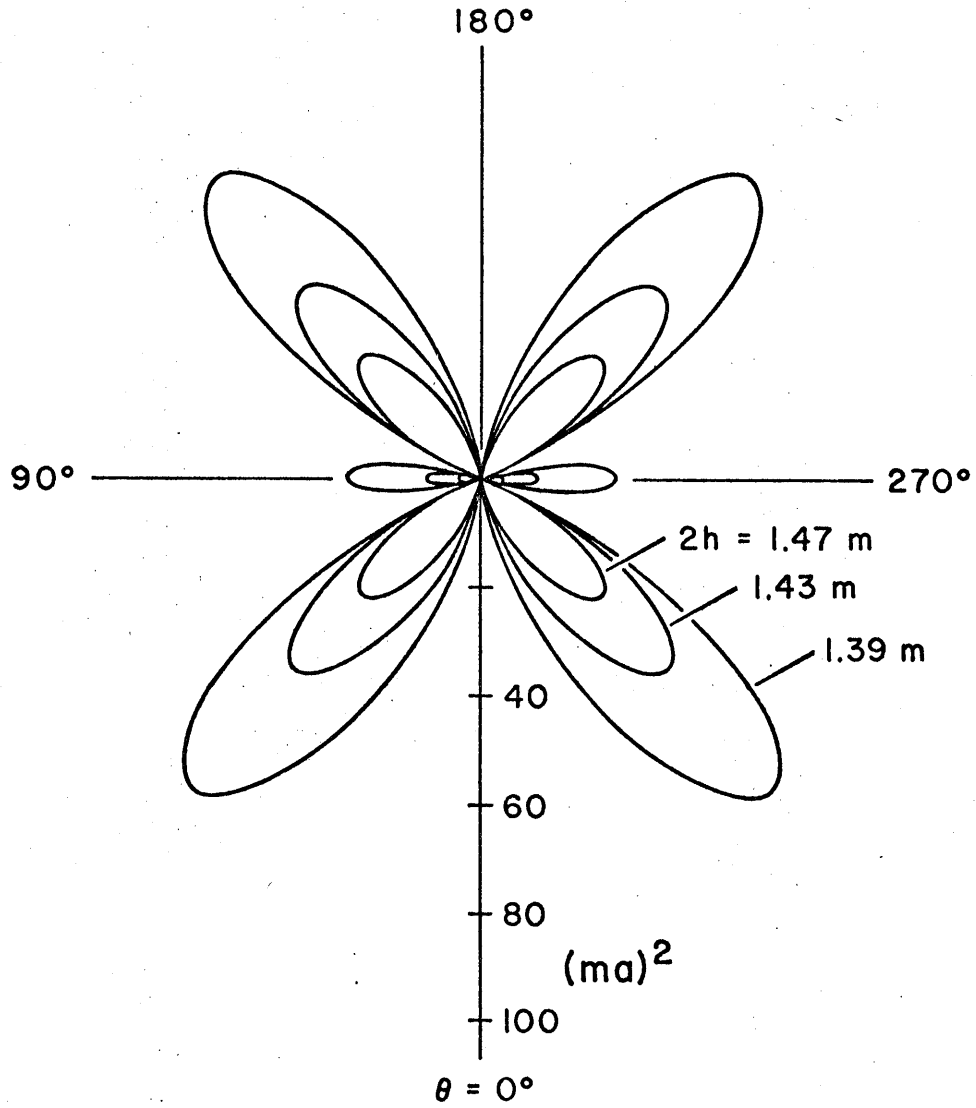


Figure S4. Squared magnitude of the exterior current, $|I^{\text{ext}}(\theta, z_0)|^2$, calculated from (S2.), with the same parameters denoted in Figure 8.

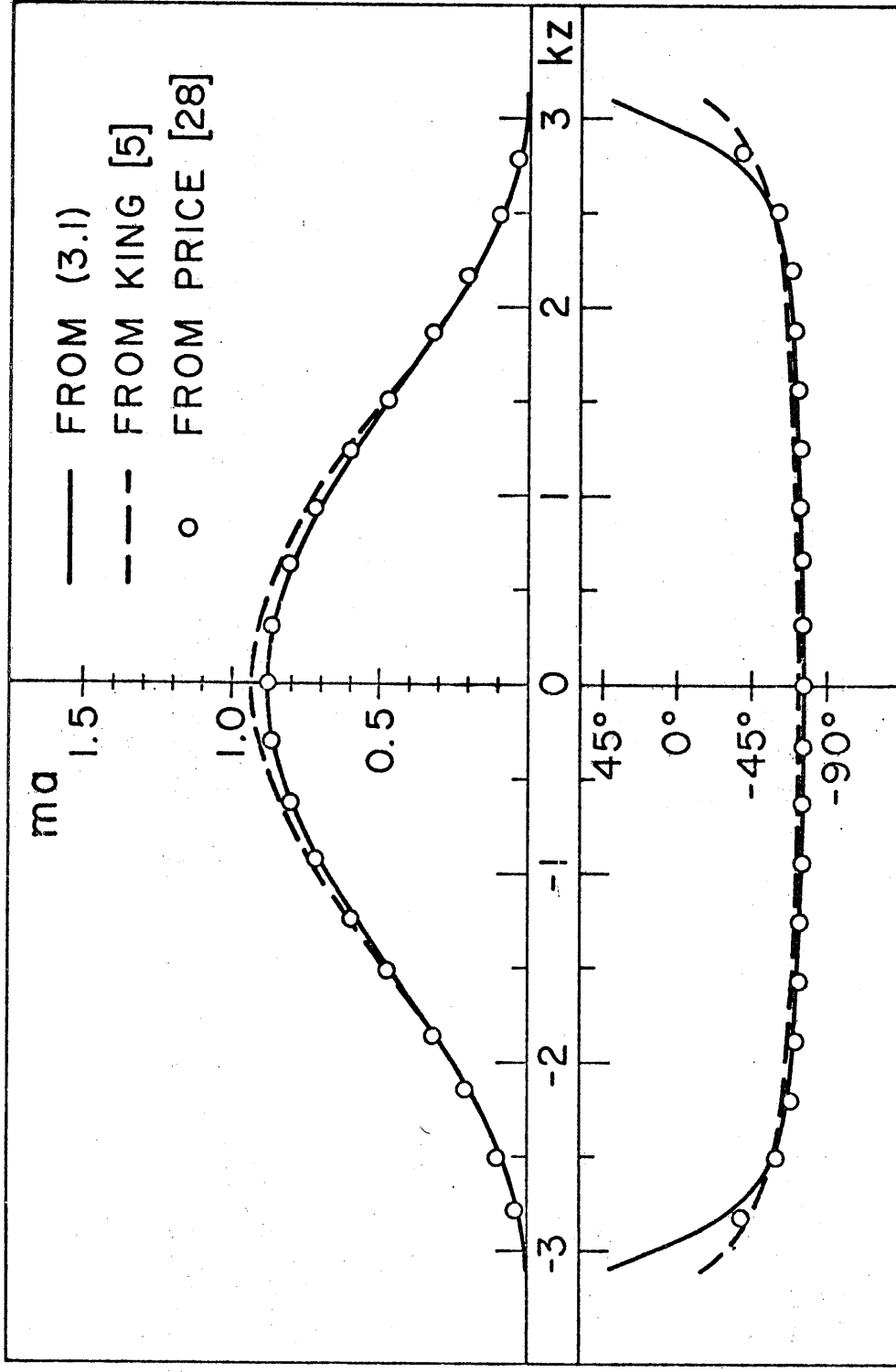


Figure S5. Current distribution on a full-wave cylinder illuminated by a unit incident plane wave calculated from (S2.), with the same parameters denoted in Figure 9.

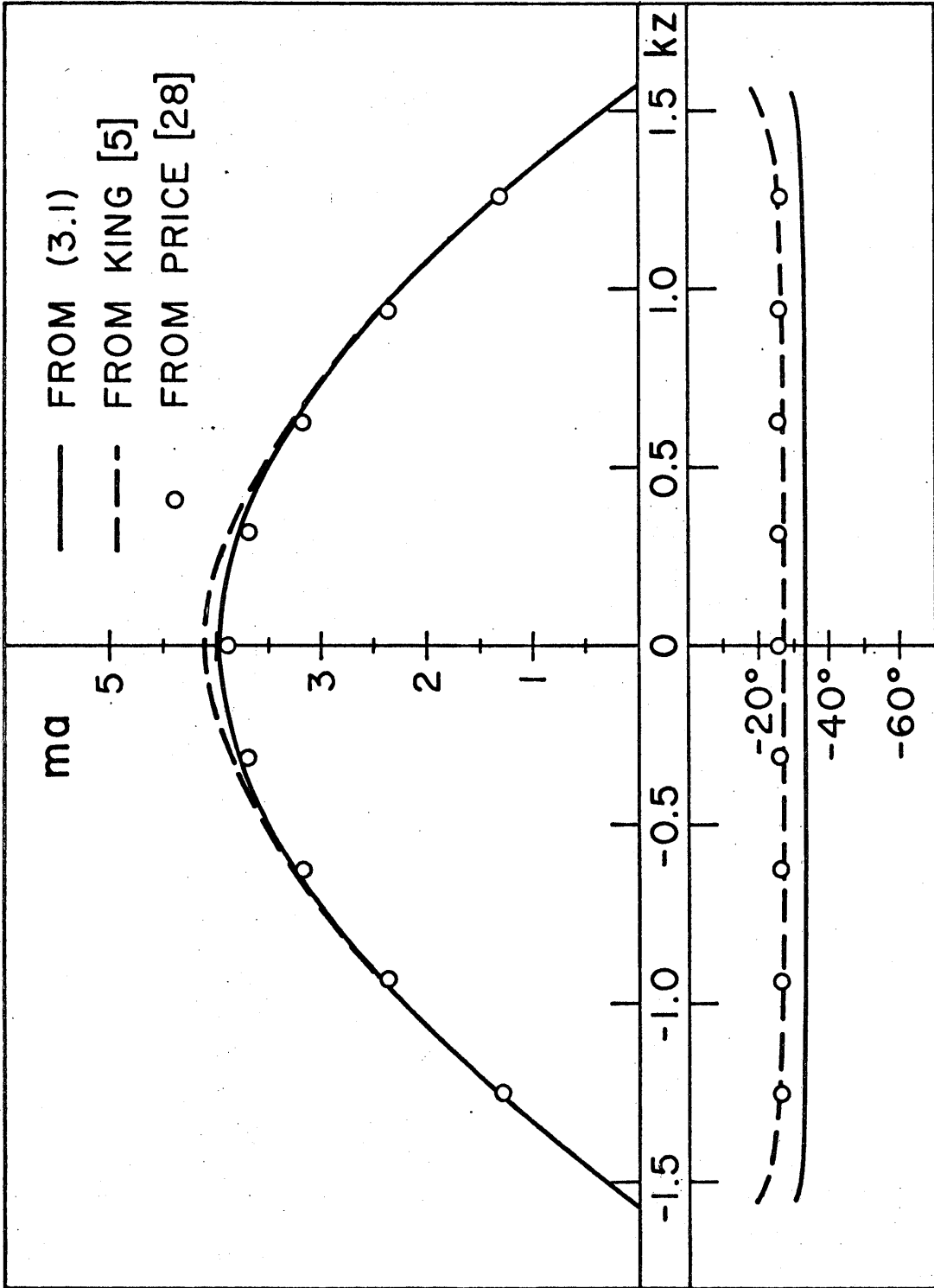


Figure S6. Current distribution on a half-wave cylinder illuminated by a unit incident plane wave calculated from (S2.), with the same parameters denoted in Figure 10.

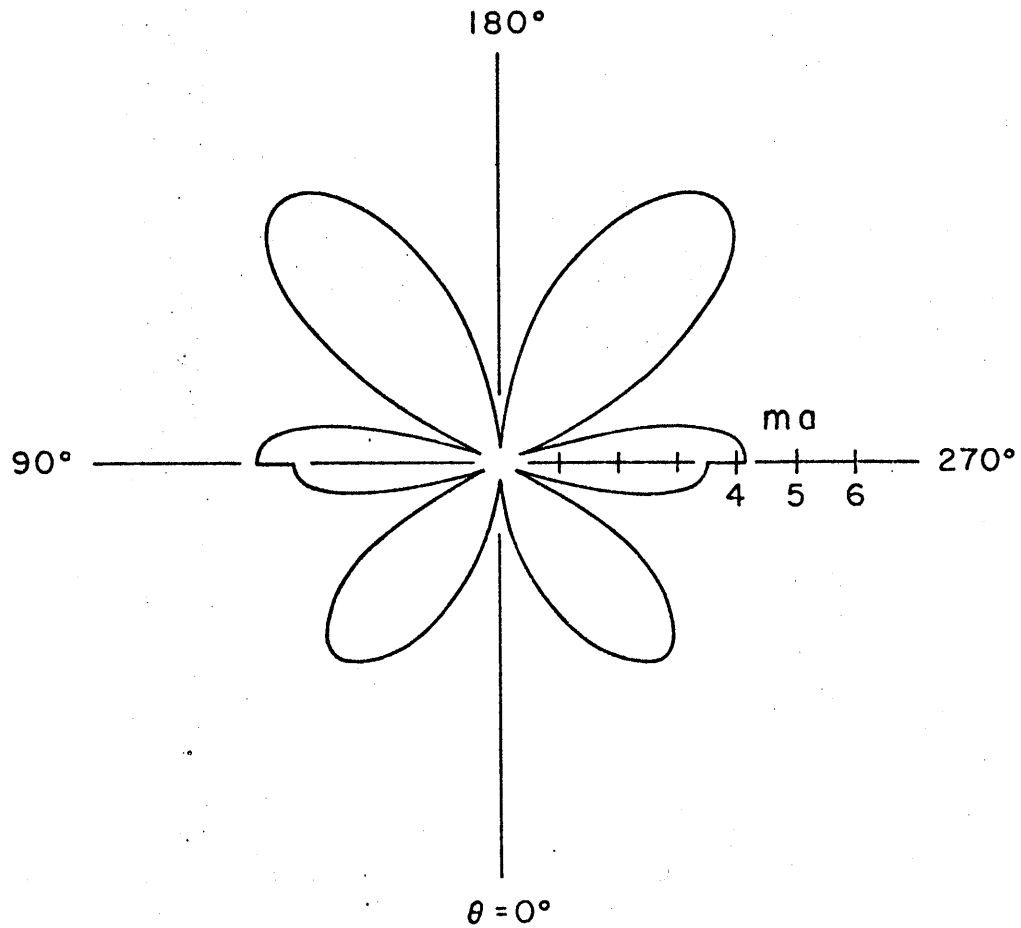


Figure S7. Magnitude of the current, $I^{\text{ext}}(\theta, z_0)$ on a finite length cylinder at $z_0 = -0.67$ m, as a function of the incident angle θ_0 , demonstrating the incorrect behavior of the original current formula (38), ref. [1], at $\theta = 90^\circ$. The other parameters are $2h = 1.39$ m, $a = 0.05$ m, $E^i = 1.0$ v/m and $\lambda = 1.0$ m.