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PROPAGATION OF RADIO WAVES IN A
COAL SEAM IN THE PRESENCE OF A CONDUCTING CABLE†

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ABSTRACT

The transmission of electromagnetic waves in an idealized coal seam
or slab is analyzed for the case where an adjacent conductor is present.
It is shown that the resultant attenuation is less than for the mode
in a conductor-free seam. The approximations made are appropriate for
low and medium frequencies.

I. INTRODUCTION

In mine communication at medium frequency range (1MHz or below), it has been observed that the attenuation of guided radio signals in a coal seam is substantially reduced in the presence of conducting rails and cables in the adjacent tunnel [Austin, 1978]. This fact is certainly consistent with the analysis by Wait and Hill [1974] for an axial conductor located anywhere inside a circular tunnel with lossy walls. A related analysis was carried out by Mahmoud and Wait [1974, 1976] for a rectangular tunnel wall. Such a phenomenon was also discussed by Lagace and Emslie [1978] in terms of the excitation efficiency of the induced current on a conducting rail or cable due to a vertical loop, but they did not actually calculate the propagation characteristics.

In this work, we consider the mathematical model of a thin wire (cable or rail) located inside a lossy dielectric slab tunnel (coal seam) of permittivity ε_1 and conductivity σ_1, and width h. The slab is then surrounded by a highly conducting medium (rock or slate bed) of permittivity ε_2 and conductivity σ_2, as depicted in Figure 1. For a signal frequency of $\omega = 2\pi f$, the wave number of the tunnel medium is given by $k_1 = [-i\omega\mu_0(\sigma_1 + i\omega\varepsilon_1)]^{\frac{1}{2}}$ and that of the surrounding medium, $k_2 \simeq (-i\omega\mu_0\sigma_2)^{\frac{1}{2}} = (1-i)d_s^{-1}$ where d_s is defined as the skin depth. The complex propagation constant of the fundamental slab mode [Wait, 1971; Wait, 1976] in the absence of cables for nonzero h, is known to have a rather simple form

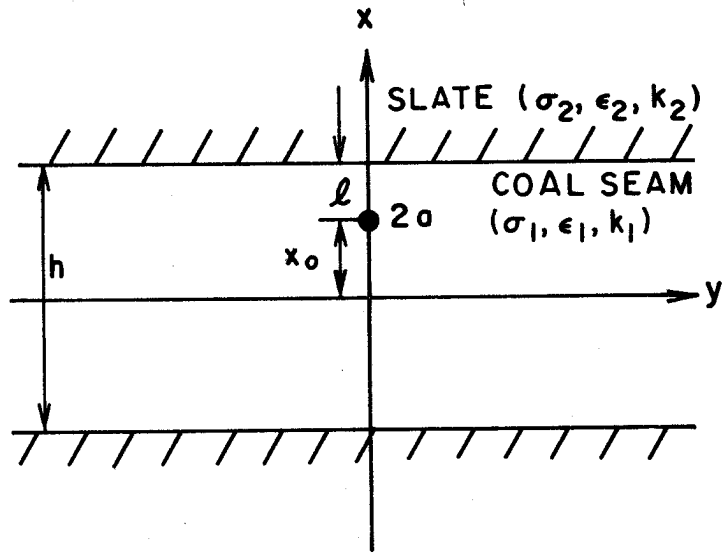$$\frac{\Gamma_s}{k_1} \simeq \left(i \frac{2}{k_2 h} - 1\right)^{\frac{1}{2}} \tag{1}$$

Figure 1

Basic slab configuration showing location of axial conductor, cable, pipe or rail in seam or tunnel.

when the conditions that $|k_2|^2 \gg |k_1|^2$ and $k_1 h \ll 1$ are invoked. Provided that the width of the slab is small compared with the skin depth of the surrounding medium, the attenuation and propagation constants for this mode are found explicitly from (1) as $\Gamma_s = \alpha_s + i\beta_s$, where

$$\frac{\alpha_s}{k_1} \simeq \frac{\pi}{8}\left(\frac{d_s}{\sqrt{2}\,h}\right)^{\frac{1}{2}} \quad ; \quad \frac{\beta_s}{k_1} \simeq 0.85 \left(\frac{d_s}{\sqrt{2}\,h}\right)^{\frac{1}{2}} \tag{2}$$

on the assumption that k_1 is effectively real. Actually, (2) are rather crude approximations, but they illustrate very simply the functional dependence of the two parameters d_s and h. A more detailed discussion is given elsewhere [Wait, 1976], including the case where the slab width h approaches zero. The field distribution of this mode resembles that of a TEM-mode of a parallel-plate waveguide, with the exception that a significant amount of leakage into the surrounding medium may be possible when the width h is small compared to the skin-depth. Thus, the attenuation constant of this mode depends to a large extent on the penetration depth of the surrounding medium as indicated by (2).

The insertion of cables in the slab certainly will complicate the situation, since the field now has to redistribute itself in order to match the boundary conditions not only at the slab interface with the surrounding medium, but also on the cable surfaces. Depending upon the proximity of the cable to the slab, the amount of penetration into the surrounding medium can be very different, at least in principle. The purpose of this work is to investigate this specific problem.

II. AVERAGE AXIAL ELECTRIC FIELD ON A THIN-WIRE SURFACE

Our objective is to derive an expression for the attenuation and propagation constants, α and β, of a slab mode in the presence of a thin-wire which has a radius a and is at a distance x_o from the center (or a distance ℓ below the upper interface, as indicated in Figure 1). We first need to know the average axial field on the wire due to an equivalent current of the form $I_o \exp(i\omega t - \Gamma z)$ uniformly distributed on the wire surface. Later the complex propagation constant $\Gamma = \alpha + i\beta$ is to be determined.

Provided that we can invoke the thin-wire approximation, i.e. $\ell^2 \gg a^2$ and $|k_1 a|^2 \ll 1$, the formal expression for the average axial field on the wire, as shown in Appendix A , consists of a primary field ψ^P and a secondary field ψ^S .

$$\langle E_z \rangle_{\rho=a} = \frac{-iI_o}{2\pi\omega\varepsilon_1^*} (\psi^P + \psi^S) \; ; \; \varepsilon_1^* = \varepsilon_1 - i(\sigma_1/\omega), \qquad (3)$$

where

$$\psi^P \simeq (\Gamma^2+k_1^2)[\ell n(\Gamma^2+k_1^2)^{\frac{1}{2}} a + i\pi/2 + C - \ell n2]; \; C = 0.5773, \quad (4)$$

$$\psi^S = \int_0^\infty \frac{d\lambda}{u_1(\lambda^2+\Gamma^2)} \left\{ [-\Gamma^2 u_1^2 R_e(\lambda) + k_1^2 \lambda^2 R_m(\lambda)] e^{-u_1(h-2x_o)} \right.$$

$$+ \left[\frac{\Gamma^2 u_1^2 R_e(\lambda)}{1-R_e^2(\lambda) \exp(-2hu_1)} \{1-R_e(\lambda) e^{-u_1(h-2x_o)}\}^2 \right.$$

$$+ \frac{k_1^2 \lambda^2 R_m(\lambda)}{1-R_m^2(\lambda) \exp(-2hu_1)}$$

$$\cdot \left\{ \left[1 + R_m(\lambda) \, e^{-u_1(h-2x_o)} \right\}^2 \, e^{-u_1(h+2x_o)} \right\}, \qquad (5)$$

$$R_e(\lambda) = \frac{k_2^2 u_1 - k_1^2 u_2}{k_2^2 u_1 + k_1^2 u_2} \quad ; \quad R_m(\lambda) = \frac{u_1 - u_2}{u_1 + u_2} \quad , \qquad (6)$$

$$u_j = (\lambda^2 - \Gamma^2 - k_j^2)^{\frac{1}{2}} \quad \text{and} \quad \text{Re } u_j \geq 0 \text{ for } j = 1,2. \qquad (7)$$

The first term in the integrand of ψ^S , in the spectral

domain, represents exactly the contribution of the partial

image due to a line source located at a distance $\ell = h/2 - x_o$

above the upper surface. Together with the actual source, they yield

a train of images (as a result of multiple reflections between the

upper and lower interface), having a total contribution of the form

given by the second term in the integrand.

Equation (5) as it stands obviously is too complicated to yield

any physical insight regarding the behavior of ψ^S. However, simpli-

fication to the integral is possible with the assumption that the width

of the coal seam is small compared with the skin-depth of the sur-

rounding medium, i.e.,

$$|k_1|^2 h^2 < |k_2|^2 h^2 \ll 1 \qquad (8)$$

This is because the contribution to the integral in this case comes

mainly from large λ where the two reflection coefficients R_e and

R_m can be approximated by

$$R_e(\lambda) \simeq R_{eo} = \frac{k_2^2 - k_1^2}{k_2^2 + k_1^2} \quad ; \quad R_m(\lambda) \simeq 0. \tag{9}$$

The corresponding integral under this approximation is then given by

$$\psi_o^S = \Gamma^2 R_{eo} \int_0^\infty \frac{d\lambda}{u_1} \left\{ e^{-u_1(h-2x_o)} \right.$$

$$\left. + e^{-u_1(h+2x_o)} \frac{[1 - R_{eo} e^{-u_1(h-2x_o)}]^2}{1 - R_{eo}^2 e^{-2u_1 h}} \right\} \tag{10}$$

which is evaluated analytically in Appendix B. From (B.3), it is clear that ψ_o^S has only a rather weak, logarithmic dependence on h. Thus, unless the slab width is exceedingly small, one needs to retain the next higher order term in ψ^S, which is independent of h. To do that, we first subtract and add ψ_o^S from the expression of ψ^S in (5). The difference of the two is then evaluated at $h = 0$, so that $\psi^S = \psi_o^S + \delta$ and

$$\delta = \lim_{h \to 0} (\psi^S - \psi_o^S)$$

$$= 2 \int_0^\infty \left\{ \left[\Gamma^2 u_1^2 \frac{R_e}{1 + R_e} - k_1^2 \lambda^2 \frac{R_m}{1 + R_m} \right] \frac{1}{\lambda^2 - \Gamma^2} \right.$$

$$\left. - \Gamma^2 \left(\frac{R_{eo}}{1 + R_{eo}} \right) \right\} \frac{d\lambda}{u_1}$$

Interestingly, the integral actually can be carried out analytically

once the relationship that

$$2[\Gamma^2 u_1^2 \ \frac{R_e}{1 + R_e} \ + k_1^2 \lambda^2 \ \frac{R_m}{1 - R_m}] = (\lambda^2 - \Gamma^2) \ [-\Gamma^2 - k_1^2 + \frac{u_1 k_1^2}{u_2 k_2^2} \ (k_2^2 + \Gamma^2)]$$

is recognized. Substitution of this expression into the integrand

yields immediately

$$\delta \ = \ [k_1^2(\Gamma^2 + k_2^2)/k_2^2] \ \ell n \left(\frac{\Gamma^2 + k_2^2}{\Gamma^2 + k_1^2} \right)^{\frac{1}{2}} \tag{11}$$

which can then be combined with the expression for ψ_o^S in (B.3) and

ψ^P in (4) to give a delightfully simple result for the axial field on

the cable surface as

$$<E_z>_{\rho=a} \ \simeq \ \frac{-iI_o}{2\pi\omega\varepsilon_1^*} \ \left\{ (k_1^2 + \Gamma^2) \ \ell n \ \frac{2h}{a} + \Gamma^2 \Omega_o \right.$$

$$\left. - \frac{k_1^2}{k_2^2} \ (k_2^2 + \Gamma^2) \ \ell n[(k_2^2 + \Gamma^2)^{\frac{1}{2}}h] + i \ \frac{\pi}{2} + C] \right\}$$

where the parameter Ω_o is given by (B.4) as

$$\Omega_o \ = \ R_{eo} \ \ell n \ \Delta + \sum_{m=1}^{\infty} \ R_{eo}^{2m-1} \ [\ell n(m - \Delta) - 2R_{eo} \ \ell nm$$

$$+ \ R_{eo}^2 \ \ell n(m + \Delta)] \quad ; \quad \Delta = \ell/h \tag{13}$$

and, therefore, involves only the geometric ratio of ℓ and h, and R_{eo}. It is of interest to note that the derivation leading to (12) so far only involves the assumption of $|k_2|^2 h^2 \ll 1$ and $\ell^2 \gg a^2$. In actual application, the additional condition that $|k_2|^2 \gg |k_1|^2$ also applies so that (12) and (13) can be further simplified to

$$\langle E_z \rangle_{\rho=a} \simeq \frac{-i\,I_o}{2\pi\omega\varepsilon_1^*} \left\{ (k_1^2 + \Gamma^2)\,\ell n\,\frac{2\ell_e}{a} - k_1^2\,[\ell n(ik_2\ell_e) + C] \right\} \qquad (14)$$

$$\simeq \frac{+iI_o}{2\pi\omega\varepsilon_1^*} \left\{ -\Gamma^2 \ell n\,\frac{2\ell_e}{a} + k_1^2\,[\ell n(ik_2 a/2) + C] \right\} \qquad (15)$$

with the effective height ℓ_e defined as

$$\ell_e = h\,e^{\Omega_o} \simeq \ell\,\exp\left(\sum_{m=1}^{\infty} \ell n\,\frac{m^2 - \Delta^2}{m^2} \right)$$

$$\simeq \ell\,\exp\left(-\frac{\pi^2\Delta^2}{6} \right); \qquad \Delta \doteq \ell/h\,. \qquad (16)$$

The result, as given by (14), is identical in form with the case of a thin-wire located at a distance ℓ_e above a conducting half-space [Chang and Wait, 1974]. Since the actual distance to the upper interface is ℓ and $\ell_e/\ell = \exp(-\pi^2\Delta^2/6) < 1$ according to (16), one might conclude that the net effect of the interaction between upper and lower interfaces is to bring the cable a little closer to the upper interface. Obviously, the periodic nature of the partial "images" in the spectral domain is insignificant in the spatial domain when the tunnel width is small compared with the skin-depth.

III. ATTENUATION AND PROPAGATION IN A CABLE-TUNNEL SYSTEM

For the case of a cable which is located inside a coal seam, having a known series impedance Z_i, the complex propagation constant Γ of the fundamental mode can be obtained by enforcing the boundary condition on the cable surface. Because we have assumed the cable radius is small, we can replace equivalent current I_o by $2\pi a$ times the average angular magnetic field, $\langle H_\phi \rangle$. The use of the impedance condition, $\langle E_z \rangle = 2\pi a Z_i \langle H_\phi \rangle$ and (15) then provides an appropriate expression for Γ as follows:

$$\Gamma^2 \ln \frac{2\ell_e}{a} + i\omega\varepsilon_1^* [i\omega\mu_o(\ell n i k_2 a/2 + C) + 2\pi Z_i] = 0 \tag{16}$$

or

$$\Gamma^2 = YZ \tag{17}$$

where

$$Y = i2\pi\omega\varepsilon_1^* [\ell n 2\ell_e/a]^{-1} \tag{18}$$

and

$$Z = -i\omega\mu_o(2\pi)^{-1}[\ell n(i k_2 a/2) + C] + Z_i \tag{19}$$

Equation (17) resembles that for a lossy transmission-line system, with Z, Y being the distributed series impedance and shunt admittance, respectively. Since we have assumed that the slab (coal seam) width is very small compared with the skin-depth, it is not surprising to see that the series impedance Z consists mainly of the self-inductance of the cable immersed in the ambient medium (slate), and the surface impedance Z_i. On the other hand, because of the high conductivity contrast between the slate and the coal seam, the shunt admittance is essentially that of a wire located in a perfectly-conducting parallel-plate region. As we mentioned before, the series of images produced by the multiple reflection between the two plates results in an apparent image at a distance $2\ell_e$ away from the cable itself. For a conducting rail, $Z_i = 0$ so that

$$\frac{\Gamma}{k_1} = \left[\frac{\ln(ik_2a/2) + C}{\ln \frac{2\ell_e}{a}} \right]^{\frac{1}{2}} ; \quad \ell_e = \ell e^{-(\pi\ell/h)^2/6)} \quad (20)$$

Comparison of (20) with (1) now enables us to assess the importance of the conducting rails and cables in mine communication. It is clear that, in the absence of these conductors, waves are guided by the upper and lower surfaces of the coal seam with the surrounding medium. Thus, the width of the slab compared with the skin-depth determines the extent of penetration into the surrounding medium, and hence the attenuation as well as the phase constants. With the presence of a parallel conductor, however, the field concentration is largely controlled by the size of the conductor because a substantial amount of longitudinal current can now flow on the conductor surface and the surrounding medium acts more like a return path. As an extreme case, the propagation constant approaches to the value corresponding to the wave number of the medium in the immediate vicinity of the conductor in the limit of a vanishingly small radius. As is evident by the dependence of h in (20) the width of the slab also becomes a relatively less important parameter in determining the attenuation and phase constants. Instead it is the ratio of the radius and height of the conductors. Dependence on height compared with the skin depth of the surrounding medium is also rather weak because of the logarithmic nature of this dependence. In fact, the attenuation and propagation constants can be explicitly derived from (20) as $\Gamma = \alpha_c + i\beta_c$, with

$$\frac{\alpha_c}{k_1} = \frac{\pi - 2\phi_2}{4(L_aL_b)^{\frac{1}{2}}} ; \quad \frac{\beta_c}{k_1} = \left(\frac{L_a}{L_b}\right)^{\frac{1}{2}} \quad (21)$$

where $L_a = -(\ln|k_2|a/2 + C)$, $L_b = \ln(2\ell_e/a)$, and $k_2 = |k_2| \exp(-i\phi_2)$ where ϕ_2 is a phase angle. Again k_1 is assumed real.

To illustrate the nature of the results, we show in Fig. 2 a comparixon for the case with and without the conductor. The attenuation constant is plotted as a function of frequency for a special case where the seam width h = 3m, and the conductor is distance ℓ = 1m from the seam surface, the radius of the conductor a = 0.02m. The conductivity σ_2 of the bounding material (i.e., slate) is 2×10^{-2}mhos/m. The dashed line is for the case when the conductor is absent as computed from (2), and the solid line is when the conductor is present as computed from (20). For the latter case, the values based upon a numerical evaluation of the exact expression as given in (3)-(5) are also included. From the figure, it is clear that for frequencies of practical interest, attenuation of the guided electromagnetic waves is indeed substantially reduced as a result of the conducting rail's presence.

Actually, for the results shown in Fig. 2, the seam was replaced by free space but the results apply qualitatively for an effectively lossless seam of any permittivity if distances are scaled accordingly, e.g., $k_1 d$ and $k_2 d$ are fixed. Of course, the theory presented is valid for a dissipative seam where σ_1 is comparable with the displacement term $\varepsilon_1 \omega$ but we do not show such results here.
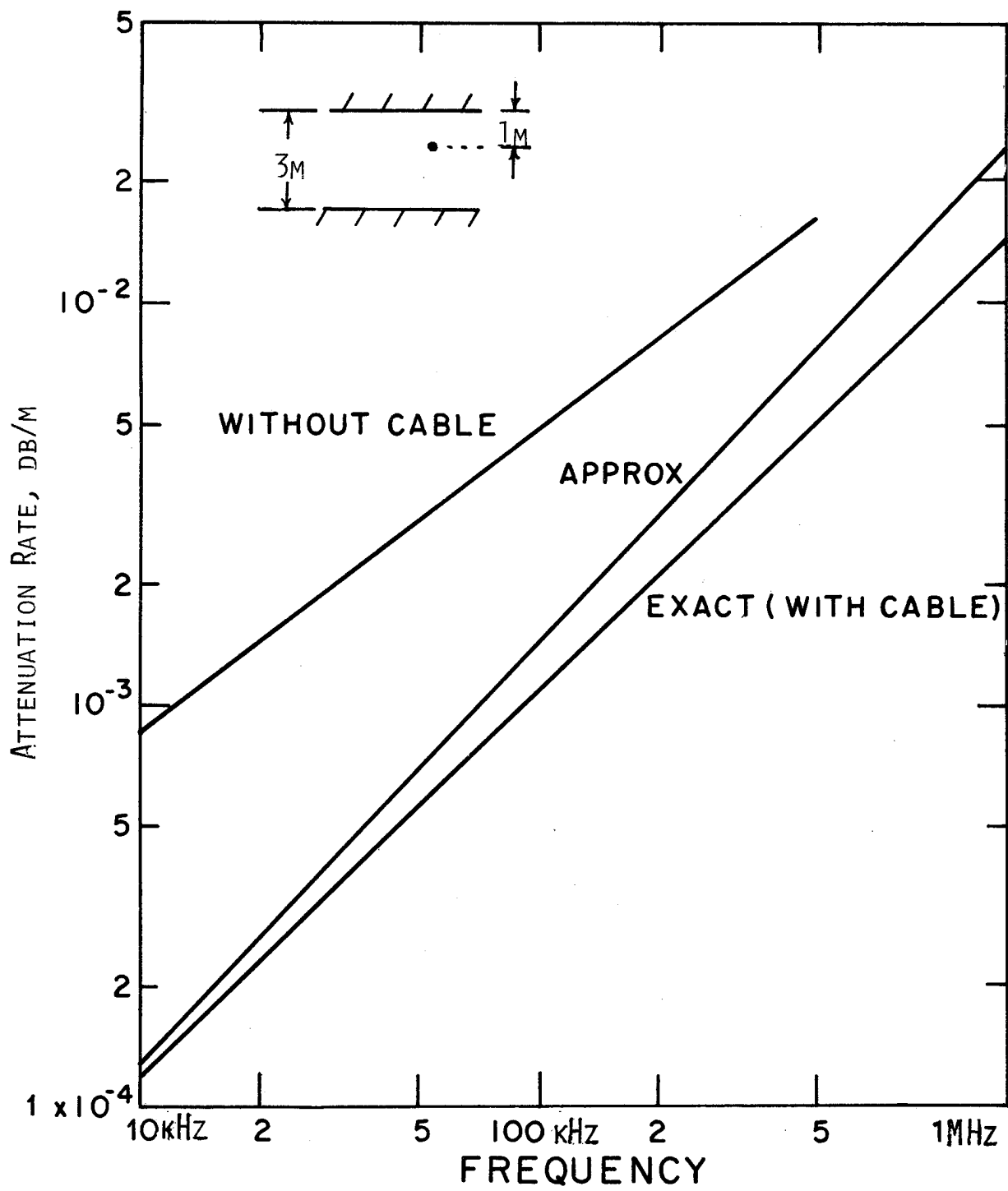
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Figure 2

APPENDIX A

Derivation of Axial Field Expressions

Here we derive an expression for the axial electric field at the surface of a thin wire carrying a filamental current $I_o \exp(-\Gamma z)$. The relevant geometry is indicated in Fig. 1. Following an earlier formulation for a similar problem [Wait, 1977], it is a fairly simple matter to derive explicit expressions for the x directed electric and magnetic Hertz vectors within the slab region. These so-called electric and magnetic Hertz potentials π_e and π_m must be of the respective forms:

$$\pi_e = \frac{-i\Gamma I_o}{4\pi\omega\varepsilon_1^*} \int_{-\infty}^{\infty} (\mp e^{\pm u_1(x-x_o)} + A_e e^{u_1 x} + B_e e^{-u_1 x}) e^{-i\lambda y} \frac{d\lambda}{\lambda^2 - \Gamma^2} \tag{A.1}$$

$$\pi_m = \frac{i I_o}{4\pi} \int_{-\infty}^{\infty} (e^{\pm u_1(x-x_o)} + A_m e^{u_1 x} + B_m e^{-u_1 x}) e^{-i\lambda y} \frac{\lambda d\lambda}{u_1(\lambda^2 - \Gamma^2)} \tag{A.2}$$

where $\varepsilon_1^* = \varepsilon_1 - i(\sigma_1/\omega)$ and where ($\mp$) designates the region $x \gtrless x_o$; A_e, B_e and A_m, B_m are as yet undetermined constants. However, since in the region $x > x_o$, B_e, B_m are part of the incoming wave that are incident onto the upper interface and A_e, A_n are the reflected wave, we have

$$A_e e^{u_1 h/2} = R_e(-e^{u_1 x_o} + B_e)e^{-u_1 h/2} \tag{A.3}$$

$$A_m e^{u_1 h/2} = R_m(e^{u_1 x_o} + B_m)e^{-u_1 h/2} \tag{A.4}$$

where R_e and R_m are as defined by (6).

Likewise, for the lower interface

$$B_e e^{u_1 h/2} = R_e(e^{-u_1 x_o} + A_e)e^{-u_1 h/2} \tag{A.5}$$

$$B_m e^{u_1 h/2} = R_m(e^{-u_1 x_o} + A_m)e^{-u_1 h/2} \tag{A.6}$$

Using (A.3) to (A.6), we solve for the A's and B's:

$$A_{e,m} = R_{e,m} \, e^{-2u_1 h} \left[\frac{R_{e,m} \, e^{-u_1 x_o} + e^{u_1(x_o+h)}}{1 - R_{e,m}^2 \, e^{-2u_1 h}} \right] \tag{A.7}$$

$$B_{e,m} = R_{e,m} \, e^{-2u_1 h} \left[\frac{\mp R_{e,m} \, e^{u_1 x_o} + e^{u_1(h-x_o)}}{1 - R_{e,m}^2 \, e^{-2u_1 h}} \right] \tag{A.8}$$

Substitution of (A.7) and (A.8) into (A.3) to (A.6) then yield the expression for π_e and π_m. To obtain the axial electric field, we only need to use the relationship

$$E_z = \frac{\partial^2}{\partial x \, \partial z} \, \pi_e + i\omega\mu_o \frac{\partial}{\partial y} \, \pi_m$$

to obtain from (A.3) to (A.6) the following expression

$$E_z = \frac{iI_o}{4\pi\omega\varepsilon_1} \left[(k_1^2 + \Gamma^2) \int_{-\infty}^{\infty} e^{\mp u_1(x-x_o)} \, e^{-i\lambda y} \, \frac{d\lambda}{u_1} \right.$$

$$+ 2 \int_{-\infty}^{\infty} \frac{1}{\lambda^2 - \Gamma^2} \left\{ \frac{+\Gamma^2 u_1^2 R_e}{\Delta_e} [R_e e^{-2u_1 h} \cosh u_1(x-x_o) - e^{-u_1 h} \cosh u_1(x+x_o)] \right.$$

$$\left. \left. + \frac{k_1^2 \lambda^2 R_m}{\Delta_m} [R_m e^{-2u_1 h} \cosh u_1(x-x_o) + e^{-u_1 h} \cosh u_1(x+x_o)] \right\} \right] ; \tag{A.9}$$

where

$$\Delta_{e,m} = 1 - R_{e,m}^2 \, e^{-2u_1 h} \tag{A.10}$$

In deriving (A.9), the identity $\Gamma^2 u_1^2 + \lambda^2 k_1^2 = (\lambda^2 - \Gamma^2)(k_1^2 + \Gamma^2)$ was used in obtaining the first integral which is known to be $2K_o(ir[\Gamma^2 + k_1^2]^{\frac{1}{2}})$; $r = [(x-x_o)^2 + y^2]^{\frac{1}{2}}$ and K_o is the modified Bessel function of the

second kind.   On the wire surface, the average axial field is then

obtained by setting $r = a$, $x = x_o$ and $y = 0$:

$$\langle E_z \rangle_{\rho=a} = \frac{iI_o}{2\pi\omega\epsilon_o} \left\{ (k_1^2 + \Gamma^2) K_o(ia [\Gamma^2 + k_1^2]^{\frac{1}{2}}) \right.$$

$$+ \int_0^\infty \left[\frac{+\Gamma^2 u_1^2 R_e}{\Delta_e} (R_e e^{-u_1 h} - \cosh 2u_1 x_o) + \frac{k_1^2 \lambda^2 R_m}{\Delta_m} (R_m e^{-u_1 h} \right. \quad \text{(A.11)}$$

$$\left. \left. + \cosh 2u_1 x_o) \right] e^{-u_1 h} \frac{d\lambda}{(\lambda^2 - \Gamma^2)} \right\}$$

Provided the radius is small, K_o can be approximated by $-\ln[ia(\Gamma^2 + k_1^2)^{\frac{1}{2}}]$.

On the other hand, the terms inside the square bracket can be rearranged

according to

$$R_{e,m}(R_{e,m} e^{-u_1 h} \mp \cosh 2u_1 x_o) = \mp R_{e,m} e^{-u_1(h-2x_o)}$$

$$\mp \frac{R_{e,m} e^{-u_1(h+2x_o)}}{1 - R_{e,m}^2 e^{-2hu_1}} [1 - R_{e,m} e^{-u_1(h-2x_o)}]^2 \quad \text{(A.12)}$$

where the upper sign is for the R_e and lower sign for R_m. Substitution of

(A.12) into (A.11) yields immediately the expressions given in (3) - (5).

APPENDIX B

Evaulation of E_{zo}^s

The integral given in (10) is evaluated in this appendix. Recog-

nizing that $\left| R_{eo} \exp(-2u_1 h) \right| < 1$, we can first expand the denominator

in series to obtain

$$e^{-u_1(h+2x_o)} \frac{[1-R_{eo} e^{-u_1(h-2x_o)}]^2}{1 - R_{eo}^2 e^{-2u_1 h}} = \sum_{m=0}^{\infty} R_{eo}^{2m} e^{-2mu_1 h}$$

$$\left\{ e^{-u_1(h+2x_o)} - 2R_{eo} e^{-u_1 h} + R_{eo}^2 e^{-u_1(3h-2x_o)} \right\}$$

so that each term in the series can be integrated analytically according to the identity

$$K_o(x[\Gamma^2 + k_1^2]^{\frac{1}{2}}) = \int_0^{\infty} e^{-u_1 x} \frac{d\lambda}{u_1} \quad .$$

Consequently, we obtain from (10) the alternative expression

$$\psi_o^s = \Gamma^2 R_{eo} \left[K_o(\zeta_1[h-2x_o]) + \sum_{m=0}^{\infty} R_{eo}^{2m}\{K_o([(2m+1)h+2x_o]\zeta_1) \right.$$

$$\left. - 2R_{eo} K_o(2[m+1]h\zeta_1) + R_{eo}^2 K_o([(2m+3)h-2x_o]\zeta_1)\} \right] \qquad (B.1)$$

where $\zeta_1 = i(\Gamma^2 + k_1^2)^{\frac{1}{2}}$. Now since we have assumed the seam size is small compared with the guided wavelength as well as the wavelength of the medium, $|\zeta_1|h$ generally is much smaller than 1. A small-argument expansion of K_o then yields approximately,

$$E_{zo}^s = -\Gamma^2 R_{eo} \left[\left(\frac{2}{1+R_{eo}}\right)\left[\ln 2h(\Gamma^2+k_1^2)^{\frac{1}{2}} i\pi/2 + C - \ln 2\right] + \ln(h - 2x_o)/2h \right.$$

$$+ \sum_{m=1}^{\infty} R_{eo}^{2(m-1)} \{ \ln[(2m-1)h+2x_o]/2h - 2R_{eo} \ln m$$

$$+ R_{eo}^2 \ln[(2m+1)h-2x_o]/2h \} \Bigg] \qquad (B.2)$$

Defining $\Delta = \ell/h = (\frac{1}{2} - \frac{x_o}{n})$ as the ratio of the distance to the upper

boundary and the tunnel width and using the explicit expression of

R_{eo} in (9), we have from (B.2) the simplified result

$$E_{zo}^S = -\Gamma^2 \left[\frac{k_2^2 - k_1^2}{k_2^2} \right] [\ln 2h(\Gamma^2 + k_1^2)^{\frac{1}{2}} + i\pi/2 + C - \ln 2] - \Gamma^2 \Omega_o \qquad (B.3)$$

where $\Omega_o = R_{eo} \ln\Delta + \sum_{m=1}^{\infty} R_{eo}^{2m-1} [\ln(m-\Delta) - 2R_{eo} \ln m + R_{eo}^2 \ln(m+\Delta)] \ (B.4)$

Expression (B.3) can then be used with the expression δE derived in

(11), and E_z^P in (4) to obtain the total axial electric field expres-

sion given in (12).

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