Price Competition, Innovation and Search in Durable-goods Markets

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Price Competition, Innovation and Search

in Durable-goods Markets

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This dissertation studies competition, innovation, and search in durable-goods markets. Specifically, it investigates three questions: First, how does counterfeit competition affect a branded firms profit and social welfare in a durable-goods market? Second, how does an imitators entry impact the durable-goods producers R&D decision when consumers care about social status? And third, how are firms' profits affected by their positions when consumers search for a durable good sequentially?

The second chapter explores why deceptive counterfeit competition may increase a branded firm's profit and total welfare. The reason is that counterfeits enable the branded firm to maintain a high price, which alleviates the time-inconsistency problem of a durable goods monopolist. The third chapter shows that, for conspicuous goods of durable nature, entry of imitators may increase the incumbent's R&D incentive. The result provides a theoretical justification for the weak protection of intellectual property in the fashion industry. The fourth chapter finds that when ex-ante heterogeneous consumers search for a durable good sequentially, a less prominent firm may earn a higher profit, because search cost enables the firm to cherry-pick high-value customers and commit to a high price.

Dedication

To my parents.

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Introduction

Durable goods have been widely studied in economics. Because they are consumed in multiple periods, durable goods can have intriguing dynamic effects on consumer behavior and firm conduct. This dissertation contributes to the economics literature on durable goods by providing new analyses on competition, innovation, and search in durable-goods markets. The analytical results shed new light on firms strategic interactions in these markets, with novel empirical and policy implications.

In the second chapter, I investigate how entry of deceptive counterfeits affects an incumbent branded firm's profit and social welfare. The conventional wisdom is that counterfeits harm producers of authentic products and reduce social welfare. However, recent empirical evidence (e.g., Qian, 2008) suggests that a branded firm's profit may actually increase with the entry of counterfeiters. This chapter provides a novel explanation of why a branded firm can benefit from counterfeit competition. I start by noticing that when the branded firm is a durable-goods producer, it has the incentive to lower its price after charging a high initial price—the classic time-inconsistency problem in inter-temporal pricing. This motivates consumers to delay their purchases. The lack of the ability for a durable-goods monopolist to commit to a high future price reduces its monopoly power and hurts its profit, an insight that originates from Coase (1972). I next observe that the entry of counterfeiters typically occurs with some delay after the introduction of the authentic product, and the branded firm may need to signal its quality with a high price after the entry of counterfeits. Hence, counterfeit competition potentially enables the branded firm to commit to a high future price, resolving the time-inconsistency problem. Consumers will then be incentivized to purchase early without delay, which increases not only the branded firms profit, but possibly also social welfare (because consumers would start consumption early and also for a longer time).

I explore these ideas in a stylized two-period model of a durable-goods monopoly, with the entry of a counterfeiter in the second period. Consumers are heterogeneous in their preferences for quality and cannot directly distinguish the authentic and the fake products before purchase in the second period. However, they may make correct inferences about which is the authentic product by observing the products prices. Although the counterfeiter can imitate the branded firms price, when that price is high enough, the imitation could be too costly to the counterfeiter due to the lower quantity of sales. I demonstrate that, when the quality difference between the branded firm and the counterfeiter is relatively large, there is indeed a separating equilibrium in the second period where the branded firm signals high quality with high prices, leading to higher profit for the branded firm and also higher total welfare, than the benchmark case where there is no counterfeiter. While models of incomplete information have the usual problem of multiple equilibria, my analysis shows, with the standard refinement of intuitive criterion, that the only type of equilibrium that can exist is separating equilibrium, and all possible separating equilibria have qualitatively similar properties: whereas competition from the counterfeiter reduces the branded firms sales in period 2, the branded firm can still have a higher profit due to the increased sales in period 1, and total welfare can also arise.

The third chapter examines the impact of imitators entry on the original designer's R&D strategy in conspicuous durable-goods markets. This chapter is motivated by the recent debate on the optimal Intellectual Property (IP) protection for designs. Unlike other industries, the IP protection level in the fashion industry is extremely low. Several attempts have been made to extend The Copyright Act to protect designs in U.S (e.g., the Innovative Design Protection Act initiated by Senator Chuck Schumer in 2012). One of their arguments is that the original designers R&D incentive needs to be protected by laws, which is a standard principle of IP protection. However, even with a low protection level, the fashion industry still has high innovation rates and fast product cycles. I demonstrate that low IP protection and high innovation rates are consistent in the fashion industry due to a special characteristic of the fashion industry—consumers buy fashion goods to signal their social status. When imitations of a design flood the market, the exclusivity of this design vanishes. Therefore, consumers are craving for a new fashion good as a signaling device, which increases the designers incentive to innovate.

I explore these ideas in a two-period durable-goods model similar to the one in the last chapter. The incumbent, who sells a durable good in period 1, decides whether to engage in R&D and launch a new fashion good in period 2. In the second period, a competitive fringe of imitators that copy the original good enter the market. Consumers care about the intrinsic utility of a product as well as its status utility, which depends on the expected types of consumers who purchase the product. Although a buyer can detect imitations when she purchases, others cannot tell whether she carries a genuine product or an imitation. Hence, imitations dampen the exclusivity of the original product. When consumers do not care about social status, I find that imitators' entry will not change the incumbent's R&D incentive. The reason is that when the incumbent keeps selling the original product in period 2, the price premium of the new product is not affected by the entry of the competitive fringe of imitators. However, for conspicuous goods, of which consumers care about the social status, imitations will weaken the exclusivity of the original product. This increases the advantage of the new product and the incumbent's R&D incentive. Unlike in Waldman (1996), where the durable-goods monopoly's R&D decision has the time-inconsistent problem so that the firm has a distortedly high incentive to launch a new product after selling old ones, for conspicuous goods the problem exists in the monopoly benchmark but vanishes with imitators competition.

The last chapter examines how firms' profits are affected by their positions in consumer ordered search. Theoretical search papers have concluded that a prominent firm earns a higher profit (e.g., Armstrong et al., 2009; Arbatskaya, 2007), because it attracts higher demand when all consumers visit it first. However, recent empirical evidence suggests that sometimes firms in less prominent positions may have higher profits (e.g., Agarwal et al., 2011). In this chapter, I find that the gap between theoretical predictions and empirical evidence can be bridged when consumers search for a durable good. As we discussed above, the durable-goods producer faces a time-inconsistency problem. It can benefit from the truncation of low-value consumers, which enables the firm to commit to a high price. Under certain conditions, costly search enables the less prominent firm to cherry-pick high value customers, alleviating the time-inconsistency problem and increasing its profit.

I build a sequential search model with two ex-ante homogeneous firms producing durable goods. I adopt the matching search set-up as in Chen and He (2011), where a consumer first learns whether the product matches her needs after search. Matching means the product provides a positive utility; otherwise the product is no value to her. The matching probability is given exogenously. Consumers' valuations are drawn from different distributions, so that they have different expected values ex-ante. The marginal search cost is increasing, as in decision fatigue theory (Levine et al. 2010). Given the search cost, a consumer who did not find her desired product from the prominent firm is more likely to search the less prominent firm if she has a high expect value from search. Thus, while the less prominent firm has a lower demand because consumers finding a match with the prominent firm will stop search, it attracts a better pool of consumers that consists of more high-value customers. Under a proper range of search cost and a low matching probability, the less prominent firm can have a higher profit, because its ability to maintain a high price overcomes the disadvantage of being searched later.

Chapter 2

Why Branded Firms May Benefit from Counterfeit Competition

2.1 Introduction

Counterfeits have become a fast growing multi-billion dollar business. In the 2007 OECD counterfeit report, the volume of counterfeits was around 200 billion dollar in international trade, 2% of world trade.[1] This figure does not include domestic consumption of counterfeits or digital products distributed via internet. The U.S. government estimated that counterfeit trade increased more than 17 fold in the past decade (U.S. Customs and Border Protection, 2008).

Counterfeits are generally viewed as harmful to both the authentic producers and consumers, especially when they are deceptive, such as counterfeits of pharmaceutical products, eyeglasses, luxury goods or even normal textile products of famous brands.[2] However, some recent empirical evidence suggests that (deceptive) counterfeits could actually benefit the branded firm. In particular, Qian (2008) finds that the average profit for branded shoes in China is higher after counterfeit entry. Qian (2011) provides further evidence that the impact of counterfeits on profit depends on the quality gap between the authentic good and the counterfeit good; the branded firm benefits from counterfeits when the quality gap is sufficiently large. In this paper, I provide a theoretical explanation of why a branded firm can indeed benefit from competition of a deceptive counterfeiter when the quality difference of their products is large enough.

[1] The Economic Impact of Counterfeiting and Piracy

[2] This does not mean consumers cannot distinguish products at all. It is just hard for buyers to tell whether the good is authentic without any other information. For example, a consumer may not be able to separate a genuine Chanel bag from a fake one only by appearance. However, if one is priced at $3,000 and the other is sold for $50, she will know that the expensive one is more likely to be authentic ex-post. On the other hand, non-deceptive counterfeits are those that consumers can easily recognize when purchasing, such as digital products.

I consider a model with an authentic durable-goods firm which sells in two periods. Without counterfeits, the branded durable-goods monopolist faces the classic time-inconsistency problem (Coase, 1972): after selling to high-value consumers in the first period at a high price, it cannot resist cutting its price in the second period. But then rational consumers will delay their purchase, forcing the monopolist to reduce its price in the first period and lower the monopolist's overall profit. Now suppose that a counterfeiter will enter the market in the second period. In order to separate its product from counterfeits, the branded firm needs to set a high price to signal its quality. Thus the presence of counterfeits enables the branded firm to commit to a high price in period 2, providing a solution to the time-inconsistency problem. This then motivates more consumers to purchase in period 1 instead of waiting to buy in period 2, even if the first-period price is high. When the quality gap is sufficiently large, this "front-loading" effect will dominate the profit loss from competition in the second period. In terms of total welfare, counterfeits are likely to decrease surplus in the second period; however, first-period welfare increases due to front loaded purchases. Early purchases contribute twice the surplus compared to late purchases because consumers can use the good for two periods. Therefore, if the quality gap is not too large, it is possible for counterfeits to increase welfare.

The results in this paper shed light on the policy towards counterfeits. Both branded firms and consumers respond to counterfeits strategically. In the model, the authentic firm separates itself from the counterfeiter through high price when the quality gap is large enough. Therefore, consumers will not be fooled by counterfeits with extremely low quality. Moreover, knowing the later counterfeit entry, consumers are more inclined to purchase early, which benefits both the authentic firm and total welfare in a dynamic context.

The existing literature has investigated varies strategies by the durable-goods monopolist to resolve the commitment problem (see, e.g., Waldman, 2003 for an excellent survey). They include leasing rather than selling the durable good (Coase, 1972; Bulow, 1982), special contracts between the monopoly and consumers (Butz, 1990), offering an inferior version (Karp and Perloff, 1996; Hahn, 2006), and product-line management (Huhn and Padilla, 1996). All of these involve tactics that the monopoly adopts to alleviate the problem. The present paper suggests a novel commitment mechanism through the competition from another firm.

Several other papers have discussed the counter-intuitive result of price- or profit- increasing competition (e.g., Chen and Riordan, 2008; Gaibaix et al., 2005; Perloff et al., 2005; Thomadsen, 2007, 2012). In those papers, competition changes the demand curve of the incumbent firm. When the competitor attracts some price-elastic consumers, the incumbent can concentrate on price-inelastic consumers by charging a higher price. However, in my paper, quality signaling leads to the higher price. In addition, in these static models, competition generally will not increase a firm's profit even if prices go up, because a monopoly will always earns higher profit than a duopoly if the price is the same. However, in a dynamic model, price-increasing competition helps the monopoly to overcome the time-inconsistency problem and boosts profit.

There are other papers that discuss deceptive counterfeits. Grossman and Shapiro (1988a), for example, discuss the problem in international trade; they show that counterfeits will decrease the total welfare and the authentic firm's profit. Qian (2014) focuses on brand-protection strategies against counterfeits, including increasing price or upgrading quality, etc. She uses a vertical differentiation model similar to my modeling of second-period competition. The main difference is that I investigate the counterfeit problem in a dynamic context. This new feature yields opposite results from hers: in her paper, the authentic firm's profit decreases with the threat of counterfeits. Also, total welfare drops when the ratio of uninformed consumer is high. However, in the present paper, the branded firm's profit and total surplus might increase even if all consumers are uninformed.

Finally, the modeling of second-period counterfeit competition is related to the literature of duopoly signaling games. Hertzendorf and Overgaard (2001), Fluet and Garella (2002) and Yehezkel (2008) study similar games with advertising. These papers focus on the role of dissipative advertising in expanding the separating equilibrium regime while I try to answer how counterfeits influence profit and welfare. Like Qian (2014), these papers only investigate the static game while my paper incorporates the signaling game into a durable-goods model.

The rest of Chapter 2 is organized as follows. Section 2.2 presents the model and reviews the monopoly benchmark. Section 2.3 investigates the effect of counterfeit competition on profit and welfare in a specific equilibrium. Section 2.4 shows that the main results continue to hold for other equilibria of the model under proper refinement. Section 2.5 concludes. All proofs are relegated to the Appendix A.

2.2 The Model and Monopoly Benchmark

I adapt the two-period durable-goods model in Tirole (1988). A branded firm sells a durable good that can be used in two periods. The quality of its product Q_A is normalized to 1. In the second period, a counterfeiter producing a low-quality clone $Q_C = C < 1$ will enter and compete with the branded firm.[3] Firms have no marginal cost to produce the good. Consumers know the quality of both products from the beginning of the game. However, they are not able to tell which good is produced by the branded firm from its appearance before their purchase.[4] This contrasts with the standard assumption that consumers can trace the producer of the good.

There is a unit mass of heterogeneous consumer indexed by the taste parameter $\theta_l \sim U[0,1]$. Consumer's utility has the linear function form:

$$U_l = \theta_l Q_i - p_i, i \in \{A, C\}, \text{ where } p_i \text{ is the price of firm } i$$

The discount factors of both firms and consumers are assumed to be 1.

Let $\mu_i(p_A, p_C)$ be the probability that consumers believe the good from firm i is the authentic good, given p_A and p_C. Unlike the traditional monopoly signaling model, there are two signal senders here. Consumer belief is based on price and the number of firms charging that price. Consumers are aware that two firms sell the good and one of them is the counterfeiter. Thus, $\mu_A(p_A, p_C) + \mu_C(p_A, p_C) = 1$ in equilibrium. In a pooling equilibrium, where $p_A = p_C$, consumers

[3] This implicitly assumes that the authentic product has a lead time advantage. Many firms have special designs on the new product so that imitators have to spend some time to learn and copy.

[4] They are aware of the counterfeit quality in the first period. The assumption can be relaxed such that consumers only know the distribution of the counterfeit quality, which will not change our result qualitatively. The underling assumption is that counterfeits are deceptive and all consumers are uninformed. An alternative assumption is that part of consumers are informed. As long as the proportion of uninformed consumers are large enough, our qualitative conclusion will hold.

cannot separate two products and $\mu_A = \mu_C = \frac{1}{2}$. In a separating equilibrium, where $p_A \neq p_C$, consumers believe that the expensive good is authentic and the cheap one is counterfeit.

Given consumer's belief, the firm's profit is represented by

$$\Pi_{it}^{k}(p_A, p_C, \mu_i), t \in \{1, 2\}, k \in \{\mathrm{P}, \mathrm{S}\}$$

The subscript i, t stands for firm type and time respectively. We use the superscript k to denote equilibrium values in the second period (P for Pooling Equilibrium and S for Separating Equilibrium). Also, assume that the separating equilibrium is selected when profits are the same for a separating and a pooling equilibrium.

The time-line of the game is as follows: the authentic firm sets the first-period price p_1 in $t = 1$. Consumers decide whether to buy or wait. The counterfeiter enters in $t = 2$ and both firms set prices simultaneously. Then consumers observe both prices and make a purchasing decision based on their beliefs.

Before analyzing the game with counterfeit competition, let's first review the benchmark monopoly model without entry.[5]

(i) When the monopoly lacks commitment power, it has an incentive to decrease the price to reap the residual demand in $t = 2$. There is a marginal consumer $\theta_1$who is indifferent between buying in $t = 1$ and in $t = 2$. Therefore, the inter-temporal incentive compatibility constraint for her is:

$$2\theta_1 - p_1 = \theta_1 - p_2$$

The right (left) hand side is her surplus from buying in $t = 1$ ($t = 2$), given that her expected second-period price is fulfilled in equilibrium ($E(p_2) = p_2$). In $t = 2$, the optimal price $p_2^M = \frac{1}{2}\theta_1$, combining with the inter-temporal incentive compatibility constraint, the monopoly's aggregate profit can be written as:

$$\Pi = (\theta_1 + p_2)(1 - \theta_1) + p_2(\theta_1 - p_2)$$

[5] Since there is only one firm here, the subscript represents time and the superscript stands for the equilibrium value in monopoly case.

The first (second) term is the profit from the first (second) period. Therefore, the marginal buyer and the monopoly's profit are $\theta_1^M = \frac{3}{5}$ and $\Pi^M = \frac{9}{20}$ respectively.

(ii) When the monopoly can commit to the same price, there will be no sale in $t = 2$ and $p_1 = 2\theta$. Hence, the monopoly's profit is as follows:

$$\Pi = 2\theta(1 - \theta)$$

This gives a optimal profit $\Pi = \frac{1}{2}$ and $\theta_1 = \frac{1}{2}$. The profit in no commitment case is lower because of the standard time-inconsistency problem: high valuation consumers will anticipate the price reduction in the future and some of them postpone purchases to the second period.

2.3 Equilibrium Analysis With Counterfeit Competition

In this section, I will first characterize the set of Perfect Bayesian Equilibrium (PBE) under counterfeit competition. I then show that there exists an equilibrium in which the counterfeit can increase the authentic firm's profit and social welfare.[6]

Standard backward induction is applied to analyze the counterfeit game. As in the benchmark, there is a marginal consumer θ_1, such that all consumers with taste parameter above θ_1 will purchase in the first period. The remaining consumers may purchase in the second period. θ_1 can be interpreted as the market size of the second period.

2.3.1 Signaling Game in Second Period

In t=2, there is a signaling game played between a pair of vertically differentiated firms and consumers. Consumers use market prices to update their beliefs. If both firms have the same price, counterfeits are indistinguishable ex-post and a pooling equilibrium is sustained. If the counterfeiter sets a lower price than the branded firm and reveals itself, there will be a separating equilibrium where consumers know for sure which goods are counterfeits.[7]

[6] In next section, I show all equilibria survive from the refinement have the desired result

[7] Because of the asymmetric information, consumers only infer the quality of the firm from its price. Hence, there is another symmetric separating equilibrium where the counterfeiter charges a higher price than the branded firm. However, I will ignore that one since in this equilibrium all consumers are paying a higher price for the fake product, which is unrealistic in real life.

In a pooling equilibrium, consumers are equally likely to pick a genuine product, leading the expected quality of the product to be $\frac{1+C}{2}$. The profit function is given by the following equation.

$$\Pi_{A2}(p_2, p_2, \frac{1}{2}) = \Pi_{C2}(p_2, p_2, \frac{1}{2}) = \frac{1}{2}(\theta_1 - \frac{2p_2}{1+C})p_2$$

In a separating equilibrium, profit functions of both firms are the same as under vertical price competition with complete information.

$$\Pi_{A2}(p_{A2}, p_{C2}, 1) = (\theta_1 - \frac{p_{A2} - p_{C2}}{1-C})p_{A2}$$

$$\Pi_{C2}(p_{A2}, p_{C2}, 0) = (\frac{p_{A2} - p_{C2}}{1-C} - \frac{p_{C2}}{C})p_{C2}$$

The counterfeiter's best response function is always $p_{C2} = \frac{C}{2}p_{A2}$ in a separating equilibrium.

The key question is when a separating equilibrium can be sustained. In the standard monopoly signaling game, the separation is attained if the single-crossing condition is satisfied: the firm with high marginal cost is willing to distort price further than the low-cost firm because the profit depends only on its own price and consumer belief. However, in a duopoly case, a firm's profit is also affected by the other firm's price. When one sets a high price, the other one faces a trade-off between favorable consumer belief and demand: if the counterfeiter decides to pool with the authentic firm, which tries to signal by pricing high, its product has 50% chance to be treated as authentic. However, the demand is low because of the uniform high price in the market. Alternatively, the counterfeiter can reveal itself with a lower price, which may be better because the upward distorted price of the branded firm mitigates competition and leaves a large market for the counterfeiter. Two incentive compatibility constraints must be satisfied to support a separating equilibrium. The first equation assures that the counterfeiter does not deviate to the authentic price and the second one implies the branded firm wants to maintain the high price.

$$\Pi_{C2}(p_{A2}, p_{C2}, 0) \geq \Pi_{C2}(p_{A2}, p_{A2}, \frac{1}{2}) \tag{2.1}$$

$$\Pi_{A2}(p_{A2}, p_{C2}, 1) \geq \Pi_{A2}(p_{C2}, p_{C2}, \frac{1}{2}) \tag{2.2}$$

Lemma 1.

(i) When the quality of the counterfeit is low ($C \leq C_1 \approx 0.604$), a set of separating equilibria can be sustained: $p_{A2}^S \in [\underline{p_2}(\theta_1, C), \overline{p_2}(\theta_1, C)]$; $p_{C2}^S = \frac{C}{2} p_{A2}^S$, where $\underline{p_2}(\theta_1, C) = \frac{2(1-C^2)}{C^2-3C+4}\theta_1$ and $\overline{p_2}(\theta_1, C) = \frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}\theta_1$.

(ii) For any quality C, there exists a set of pooling equilibria where both firms price at $p_2^P \in [0, \underline{p_2}(\theta_1, C))$.

All equilibria listed in Lemma 1 can be supported by a system of beliefs off the equilibrium path, such as the most pessimistic belief. For any separating equilibrium with $\widetilde{p}_{A2} \in [\underline{p_2}, \overline{p_2}]$ and $\widetilde{p}_{C2} = \frac{C}{2} p_{A2}$, if the out of equilibrium belief is that any deviating price $p' \neq \{\widetilde{p}_{A2}, \widetilde{p}_{C2}\}$ is conceived as a sign of counterfeits, then no firm would deviate and that particular separating equilibrium is stable. Similarly, the belief that $\mu(p', \widetilde{p}_2) = 0$, $\forall p' \neq \widetilde{p}_2$ can support all pooling equilibria.

The result is very intuitive: when the quality gap is large, the profit in a pooling equilibrium is low because of the low expected quality. The authentic firm just needs to slightly distort the price upward, which will reduce price competition and leave the counterfeiter enough profit under separating regime. For the branded firm, since price distortion is moderate, the cost of signaling is not too high. However, if two products are close substitutes, the cost of signaling for the branded firm is so high that it would rather pool with the counterfeiter.

As in other signaling games, this model also has multiple equilibria. In some pooling equilibria with low price, counterfeit competition is detrimental to the branded firm's profit. In this section, I will show that there exists an equilibrium in which both the authentic firm and the society benefit from counterfeit entry under certain conditions. In the next section, it is proved that all equilibria surviving from the Competitive Intuitive Criterion refinement have similar properties.

The equilibrium I will focus on here is the one with the highest second-period profit for the authentic firm, which is defined as the *profit-maximizing equilibrium.* It seems reasonable that consumers will believe that the authentic firm will choose the price that maximizes its second-period profit. Therefore, consumers believe the firm charging that price is the authentic firm. If both firms set that price, the good has 50% probability to be genuine. Any other price indicates a fake product. This is the pessimistic belief that supports the profit-maximizing price in t=2. Formally, consumer belief is defined as follow.

$$\mu_i(p^*_{A2}, p^*_{A2}) = \frac{1}{2}; \quad \mu_A(p^*_{A2}, p_2) = 1, \forall p_2 \neq p^*_{A2};$$
$$\mu_A(p_2, \cdot) = \mu_C(\cdot, p_2) = 0, \forall p_2 \neq p^*_{A2}$$

In this section, an extra asterisk is used in superscript to denote variables in the profit-maximizing equilibrium. Let p^{S*}_{A2} and p^{P*}_2 be the authentic price in the optimal separating and pooling equilibrium respectively. $p^*_{A2} = \arg\max[\Pi^{S*}_{A2}, \Pi^{P*}_2] \in \{p^{S*}_{A2}, p^{P*}_2\}$ is the price that maximizes the branded firm's second-period profit, which is illustrated in the following lemma.

Lemma 2. In the profit-maximizing equilibrium:

(i) if the counterfeit's quality is low enough ($C \leq C_3 \approx 0.512$), the separating equilibrium is supported as the PBE of signaling game in t=2. $p^*_{A2} = p^{S*}_{A2} = \underline{p_2}(\theta_1, C)$, $\Pi^*_{A2} = \Pi^{S*}_{A2} = \frac{4(1-C)^2(1-C^2)}{C^2-3C+4}\theta_1^2$.

(ii) If the counterfeit's quality is high ($C > C_3$), the pooling equilibrium will be selected. (a) For $C_3 < C \leq C_2 \approx 0.702$, $p^*_2 = p^{P*}_2 = \frac{1+C}{4}\theta_1$, $\Pi^*_2 = \Pi^{P*}_{A2} = \frac{1+C}{16}\theta_1^2$; (b) For $C > C_2$, $p^*_2 = p^{P*}_2 = \underline{p_2}(\theta_1, C)$, $\Pi^*_2 = \Pi^{P*}_{A2} = \frac{C(1+C)(1-C^2)}{2(C^2-3C+4)^2}\theta_1^2$.

Figure B.1 illustrates the second-period price scheme in the profit-maximizing equilibrium. For $C \in [0, C_3]$, the price $\underline{p_2}(\theta_1, C)$, which is the minimum price that prevents the counterfeiter from mimicking the branded firm, has an inverted-U shape with respect to C and is higher than the monopoly price in benchmark. The counterfeiter's profit in the pooling equilibrium increases faster with C than its profit in the separating equilibrium when C is close to 0.[8] Therefore, the authentic firm is forced to increase the price in order to reduce competition and increase the competitor's profit in the separating equilibrium. As C gets larger, the condition will be reversed and the authentic firm has no need to incur a large distortion to support the separating equilibrium. Combining these

[8] When C is close to 0, $\frac{d\Pi^P_{C2}}{dC} = \frac{1}{(1+C)^2}p^2_{A2} \geq \frac{d\Pi^S_{C2}}{dC} = \frac{1}{4(1-C)^2}p^2_{A2}$.

two segments give us an inverted-U shape price in the separating equilibrium. When $C \in (C_3, C_2]$, the price increases with C because of higher expected quality. When C is close to 1, the game converges to Bertrand Competition of homogeneous good, and the price goes down to 0.

2.3.2 The Dynamic Game

Now, I will analyze the dynamic game and illustrate why the entry of counterfeiter may generate higher profit for the incumbent. Given the second-period consumer surplus and the first-period price, the marginal buyer in the first period will be determined. The authentic firm's decision is to choose this marginal consumer to maximize total profit.

Pooling Equilibrium

In the first segment of the pooling equilibrium ($C_3 < C \leq C_2$), consumer surplus in period 2 decreases because the market is flooded with counterfeits. This pushes more consumers to buy in the first period since the authentic good can be guaranteed. However, the market price is lower than the benchmark, which makes late purchase more attractive ($p_2^{P*} = \frac{1+C}{4}\theta_1 \leq \frac{1}{2}\theta_1$). Overall, consumer surplus falls below the benchmark case and the time-inconsistency problem is mitigated. We call this effect of making consumers buy early as the *Front-Loading Effect*. On the other hand, counterfeit competition will decrease the branded firm's revenue in the second period, which is the *Competition Effect*. The change of the authentic firm's profit is determined by the magnitude of these two effects.

The marginal consumer who purchases at t=1 in the pooling equilibrium is determined by the binding incentive compatibility constraint:

$$2\theta_1 - p_1 = \frac{1+C}{2}\theta_1 - p_2^{P*}$$

The authentic firm's maximization problem is:

$$\max_{\theta_1} \ \Pi_A^{P*}(\theta_1) = (1-\theta_1)(2\theta_1 - \frac{1+C}{2}\theta_1 + p_2^{P*}) + \frac{1}{2}(\theta_1 - \frac{2p_2^{P*}}{1+C})p_2^{P*}$$

The marginal buyer θ_1^{P*} and equilibrium profit Π_A^{P*} are:

$$\theta_1^{P*} = \begin{cases} \frac{1+\frac{3-C}{4}}{2(1+\frac{11-5C}{16})} & C \in (C_3, C_2] \\ \frac{[\frac{3-C}{2}+\frac{2(1-C^2)}{C^2-3C+4}]}{2[\frac{3-C}{2}+\frac{(1-C^2)}{C^2-3C+4}+\frac{4(1-C^2)(1-C)}{(C^2-3C+4)^2}]} & C \in (C_2, 1) \end{cases}$$

$$\Pi_A^{P*} = \begin{cases} \frac{(1+\frac{3-C}{4})^2}{4(1+\frac{11-5C}{16})} & C \in (C_3, C_2] \\ \frac{[\frac{3-C}{2}+\frac{2(1-C^2)}{C^2-3C+4}]^2}{4[\frac{3-C}{2}+\frac{(1-C^2)}{C^2-3C+4}+\frac{4(1-C^2)(1-C)}{(C^2-3C+4)^2}]} & C \in (C_2, 1) \end{cases}$$

As Figure B.2 shows, when $C \in (C_3, C_2]$, θ_1^{P*} increases with C for two reasons. Individual surplus in the second period increases as the counterfeit quality rises and more customers tend to wait, which decreases the wedge between p_1 and θ_1. On the other hand, the branded firm balances the profit in each period to maximize total profit by properly choosing θ_1. It is optimal to leave more customers in the second period (increase second-period market size) because second-period profit increases with C.

When $C \in (C_2, 1)$, θ_1^{P*} first increases and then decreases in this range. When C gets close to 1, the front-loading effect disappears because p_2 is close to 0. The branded firm decreases the market size in period 2 due to fierce competition. It can be inferred that the incumbent does not benefit from counterfeit competition in this range.

Separating Equilibrium

In the separating equilibrium, the competition effect is not as strong as in the pooling equilibrium since the counterfeit quality is low. Also, as the high-quality producer, the branded firm takes a larger share of the total profit compared to the head-to-head competition in the pooling equilibrium. The mechanism of the front-loading effect is slightly different. Consumers will not be fooled ex-post but face a super monopoly price in the second period as Lemma 2 indicated. Now, the marginal buyer θ_1^P faces two options in the second period—buy the authentic good or the counterfeit.

$$2\theta_1 - p_1 = max\{\theta_1 - \underline{p_2}(C,\theta_1), C\theta_1 - \frac{C}{2}\underline{p_2}(C,\theta_1)\}$$

However, the buyer who is indifferent between a genuine product and a counterfeit in the second period must below θ_1. Therefore, the outside option is purchasing the authentic good in t=2. The incumbent's profit maximization is as follow.

$$\max_{\theta_1} \ \Pi_A(\theta_1) = (1-\theta_1)(\theta_1 + \underline{p_2}(C,\theta_1)) + \Pi_{A2}^{S*}(\theta_1)$$

In equilibrium,

$$\theta_1^{S*} = \frac{1 + \frac{2(1-C^2)}{C^2-3C+4}}{2[1 + \frac{2(1-C^2)(-C^2+C+2)}{(C^2-3C+4)^2}]}$$

$$\Pi_A^{S*} = \frac{[1 + \frac{2(1-C^2)}{C^2-3C+4}]^2}{4[1 + \frac{2(1-C^2)(-C^2+C+2)}{(C^2-3C+4)^2}]}$$

The left segment of lower curve in Figure B.2 informs that θ_1^{S*} monotonically decreases with C. As the quality gap closes, the branded firm's profit in the second period decreases. It would be better to assign less weight in the second period by decreasing θ_1^{S*}.

Profit Comparison

Proposition 1. In the profit-maximizing equilibrium, the authentic firm's profit will be higher than the monopoly benchmark if and only if the counterfeit quality is sufficiently low ($C < C_4 \approx 0.188$).

Figure B.3 illustrates Proposition 1: when the pooling equilibrium emerges in the second period, the competition effect is too strong and always dominates the front-loading effect. The authentic firm suffers from the counterfeit entry. In the first segment of the pooling equilibrium, the front-loading effect gets weaker when the quality increases (θ_1^{P*} increases with C) and the time-inconsistency problem is reinforced. However, relatively high-quality counterfeits also weaken the competition effect and raise the second-period profit. In the second segment, the competition effects gets too strong and the front-loading effect disappears.

However, if the separating equilibrium is sustained, the branded firm's profit has an inverted-U shape and can be higher than the monopoly benchmark. When the counterfeit quality is 0, the result with counterfeit competition is the same as the monopoly benchmark. In the first period, since the high second-period price makes consumers less likely to wait, the front-loading effect will be stronger when $\underline{p_2}(\theta_1, C)$ is high. Recall that $\underline{p_2}(\theta_1, C)$ has an inverted-U shape, which implies the branded firm's profit has the same curvature. On the other hand, the magnitude of the negative competition effect monotonically increases with C. Therefore, when the quality of counterfeits is low, the combination of a strong front-loading effect and a weak competition effect raises the branded firm's profit above the benchmark. As C increases, this condition will be reversed and the incumbent's profit falls below the monopoly case.

2.3.3 Welfare and Policy Implication

In terms of welfare, the conventional wisdom is that deceptive counterfeits are harmful, because they fool consumers into buying low-quality products at a relatively high price; this rationale led to trademark policy aiming to prevent consumer confusion. Grossman and Shapiro (1988) shows that free entry of deceptive counterfeits decrease welfare in international trade. However, this paper demonstrates that the impact on welfare can be quite different in a dynamic context.

In the monopoly benchmark, total surplus is given by the following equation.

$$TS^M = \int_{\theta_1^M}^{1} 2\theta d\theta + \int_{\theta_2^M}^{\theta_1^M} \theta d\theta$$

The first (second) term represents the surplus created by first (second) period transaction.[9] The total surplus decreases with θ_1, because early buyers enjoy double surplus. Given the marginal buyer in each period, $TS^M = 0.775$.

The welfare in the presence of deceptive counterfeit competition is a piecewise function.

$$TS(C) = \begin{cases} TS^{S*}(C) = \int_{\theta_1^{S*}}^{1} 2\theta d\theta + \int_{\overline{\theta}_2^{S*}}^{\theta_1^{S*}} \theta d\theta + \int_{\underline{\theta}_2^{S*}}^{\overline{\theta}_2^{S*}} C\theta d\theta & \text{if } C \leq C_3 \\ TS^{P*}(C) = \int_{\theta_1^{P*}}^{1} 2\theta d\theta + \int_{\theta_2^{P*}}^{\theta_1^{P*}} \frac{1+C}{2}\theta d\theta & \text{if } C > C_3 \end{cases}$$

In the separating equilibrium, there are two marginal consumers in the second period. $\overline{\theta}_2^{S*}$ denotes the marginal consumer who is indifferent between the genuine good and the counterfeit. $\underline{\theta}_2^{S*}$ stands for the one who is indifferent between buying the counterfeit and buying nothing. Surplus is

[9] Surplus is attributed to the trading period. First-period buyer enjoys surplus in both periods but the purchase is made in the first period, therefore all surplus belongs to the first period.

discounted by C if the counterfeit is purchased. In the pooling equilibrium, expected surplus is discounted by $\frac{1+C}{2}$ for all consumers because of confusion. Comparing welfare under two cases yields the next Proposition.

Proposition 2. The entry of deceptive counterfeits increases total welfare if and only if the counterfeit quality is not too low ($C \geq C_5 \approx 0.078$).

Deceptive counterfeits have two effects on welfare. The second-period surplus decreases because of competition with incomplete information, which is the typical criticism against counterfeits. However, if the first-period welfare is taken into account, the result will be quite different. As Figure B.2 shows, there are always more sales in $t = 1$ once $C > 0$. The front-loading effect pushes consumers to buy in $t = 1$ either because of the high price or low expected quality in $t = 2$. The competition effect also forces the incumbent to reduce the market size in $t = 2$ by decreasing first-period price and expanding the market in $t = 1$. Consumers who purchase in the first period provide "double" contribution to surplus since they are guaranteed with high quality for two periods, which is the reason that total welfare could be higher under bad competition.

In Figure B.4, the middle segment demonstrates the welfare difference in the pooling equilibrium with $C \in (C_3, C_2]$. The downward pressure on welfare decreases with C because consumer confusion problem is alleviated. Since θ_1^{P*} increases with C in this range, the positive effect also decreases with C. Overall, the social welfare is higher if the pooling equilibrium is supported in $t = 2$. In the right segment of Figure B.4, the second-period price decreases with C, which implies more trade and higher welfare.

The left segment is the welfare in the separating equilibrium. In Figure B.2, as the counterfeit quality improves, the positive effect increases with C roughly at the same speed ($\frac{d^2\theta_1^{S*}}{dC^2}$ is close to 0). The second-period welfare decreases because of upward distorted prices. Since the second-period price has an inverted-U shape, the welfare in that period will be an U shape curve. Combining these two effects, it is clear why total welfare also has a U shape. When the counterfeit quality is 0, the model coincides with the benchmark. When C is small, unlike the pooling equilibrium,

θ_1 is close to the benchmark value and decreases slower compared to the second-period welfare. Therefore, when the counterfeit quality is sufficiently low, the overall welfare effect is negative.

This proposition implies that deceptive counterfeits may have a positive effect on welfare in a dynamic context, which is contrary to the traditional argument. What is more surprising is that welfare is significantly higher when counterfeits are indistinguishable ex-post. The result reminds us to think deeply about the counterfeit problem. First, branded firms actively adopt strategies against clones. Although counterfeits are deceptive ex-ante, whether they can be recognized ex-post is endogenized. If the quality of clones is low, in which case consumer confusion induced by counterfeits has a strong negative effect on welfare, the authentic firm will signal by price and rational consumer will not be fooled. If consumers cannot distinguish counterfeits from authentic goods ex-post, it must be that the quality gap is close enough. Even if consumers are diverted to counterfeits in that case, the welfare loss is relatively small. Second, consumers respond rationally to the problem. In the present paper, they are aware that surplus associated with future purchase is lowered by counterfeit competition. Thus, more people buy earlier, which is beneficial for both the branded firm and welfare. However, as I point out, when the authentic firm decides to separate itself by distorted price, the counterfeiter also charges a high price in the second period. This "price collusion" created by quality signaling might decrease welfare.

2.4 Equilibrium Refinement and Robustness

The profit-maximizing equilibrium discussed above is only one of equlibria in our model. In this section, the Intuitive Criterion (Cho and Kreps, 1987) is applied to refine equilibria. Since there are two signal senders here, I will use a competitive version as Bontems et al. (2005) and Yehezkel (2008). We will show that all pooling equilibria are eliminated with a tiny adjustment. The refinement is not applicable to separating equilibria because both firms' prices are informative.[10] However, it is proved that our general conclusion that counterfeit competition may increase the

[10] The Intuitive Criterion requires unilateral deviation. However, since the other firm charges the equilibrium price, consumers can use that information to construct the out of equilibrium belief. Therefore, I cannot simply assume a belief towards the deviating firm while the other one prices at the equilibrium path. Bester and Demuth (2011),

branded firm's profit and social welfare holds in all separating equilibria.

In previous discussion, both firms are assumed to have zero marginal cost. Now, let the authentic firm has a slightly higher marginal cost $\epsilon > 0$ which is arbitrarily close to 0. This is just a tie-breaker that helps us to eliminate all pooling equilibria. By continuity of all functions in the paper, this modification will not alter any of our results except for the existence of pooling equilibria. For convenience, I only explicitly state this adjustment in the refinement.

2.4.1 Equilibrium Refinement

Pooling Equilibrium

The basic logic of the Intuitive Criterion is equilibrium dominance: an equilibrium should be eliminated if there exists an out-of-equilibrium price such that given consumer's most favorite belief, one type of firm would be better off by deviating from the equilibrium price to that out-of-equilibrium price, while the other type of firm cannot benefit from such deviation.

In terms of pooling equilibria, the Competitive Intuitive Criterion requires that there is no p', such that

$$\Pi_{A2}(p', p_2^P, 1) \geq \Pi_{A2}(p_2^P, p_2^P, \frac{1}{2}) \tag{2.3}$$

$$\Pi_{C2}(p_2^P, p', 1) < \Pi_{C2}(p_2^P, p_2^P, \frac{1}{2}) \tag{2.4}$$

However, for every pooling equilibrium, there must exist a p' such that both equations hold, which means all pooling equilibria are eliminated. The reason is similar to the refinement in the monopoly signaling game: The authentic firm with a higher marginal cost ϵ, no matter how small it is, has a lower cost to signal its quality. Since the the profit function satisfies single-crossing property, I can always find an upward distorted price such that the authentic firm is willing to deviate to that price if consumers believe its high quality, while the counterfeiter is not willing to deviate even if people trust it.

Bontems et.al (2006) and Hertzendorf and Overgaard (2001) have discussed this issue.

Separating Equilibrium

Since the Intuitive Criterion cannot be applied to separating equilibria, Hertzendorf and Overgaard (2001) and Yehezkel (2008) use a stronger refinement named Resistance to Equilibrium Defections (REDE) to select the unique and most intuitive separating equilibrium in the duopoly signaling game, which is similar to the unprejudiced equilibrium in Bagwell and Ramey (1991).[11] Only the least distorted equilibrium survives that refinement, which is the profit-maximizing equilibrium investigated in the previous section. However, we don't need to impose that extra refinement since our main results hold in all separating equilibria, which is proved in next subsection.

2.4.2 Robustness of Results

We have shown that separating equilibria survive the refinement and pooling equilibria are eliminated. The question is whether our main conclusions regarding incumbent's profit and social welfare still hold in other separating equilibria. Let's first investigate the incumbent's profit in all separating equilibria. In the previous section, the profit-maximizing equilibrium is discussed in detail, which is the one with lowest second-period price among all separating equilibria. For any C, it can be proved that the branded firm's profit increases with the second-period price among equilibria because the front-loading effect grows faster than the competition effect. Since the branded firm can benefit from counterfeits under the equilibrium with the lowest second-period price, the result will hold under all other equilibria. Therefore, if the counterfeit quality is below C_4, the authentic firm's profit is always higher in the presence of counterfeits, no matter which separating equilibrium emerges in the second period.

In terms of the impact on welfare, there is not such a nice monotonicity property among equilibria because the welfare in $t = 2$ may be too low when the price is high in that period. However, it is verified that if C is higher than a threshold, the social welfare is higher with counterfeit

[11] Basically, REDE assumes that consumers can still make reasonable inductions from the equilibrium behavior of one sender even if they see out of equilibrium behavior from the other sender. Mathematically, if consumers observe that one good is sold at a price $\widetilde{p} \in [\underline{p_2}(C, \theta_1), \overline{p_2}(C, \theta_1)]$ and the other one is priced at $p \in [0, \underline{p_2}(C, \theta_1))$ but $p \neq \frac{C}{2}\widetilde{p}$, then they will believe the one with $\widetilde{p}$ is genuine and the other one is counterfeit. This gives the authentic firm an incentive to unilaterally deviate to the price that will maximize its profit within the separating equilibrium range. The counterfeiter never deviates because any deviation cannot fool consumers.

competition in all equilibria. The economic intuition is the same as the last section. All equilibria have higher second price (more distortion) than the one that maximizes second-period profit. Thus, the incumbent's second-period profit in other equilibria is lower than that one. When the branded firm maximizes the profit, it tends to reduce the weight on the second period (lower θ_1). Therefore, more consumers purchase in the first period and the welfare increases.

Proposition 3. All pooling equilibria are eliminated by the Competitive Intuitive Criterion. In every separating equilibrium, when $C \leq C_4$, the authentic firm's profit is higher with counterfeit competition. When $C \geq C_6 \approx 0.248$, the social welfare is higher in the presence of counterfeits.

2.4.3 Provision of a Damaged Good by the Branded Firm

This paper points out that the entry of a low-quality competitor can actually benefit the incumbent. The downside to the brand is that it takes away part of the revenue. An interesting question is whether the branded firm can overcome the competition effect by offering an inferior version itself and earn higher profit? We do observe many examples of damaged goods. Armani has a premium ready-to-wear line marketed as Giorgio Armani, relatively cheaper lines like Armani Collezioni and Emporio Armani, as well as lines distributed in shopping malls like Armani Jeans and Armani Exchange.

First, the incumbent has no incentive to provide an inferior good in the second period. Deneckere and MacAfee (1996) points out the linear utility function fails the condition that damaged goods help to raise profit. In my model, no matter what inferior quality the branded firm chooses, the optimal decision is to sell zero damaged version in t=2. The second-period price and profit are the same as monopoly benchmark. Since the price is not higher than the monopoly price, the front-loading effect does not exist. Therefore, the total profit can never be higher than the benchmark. If there is any fixed cost associated with new product introduction, the profit is always lower than the monopoly case.

Second, damaged goods introduced in the first period is not profitable as well. Hahn (2006) discusses the benefit of introducing damaged goods in durable-goods model. In his paper, part of high (low) type consumers buy a high (low) quality good in each period, which changes the ratio of consumer type. Since some low types have purchased damaged goods earlier, the firm has lower incentive to decrease price sharply later, which mitigates competition between two versions and alleviates the time-inconsistency problem. However, with continuous consumer type, it can be proved that if anyone buys a damaged good in the first period, then all consumers with higher θ must purchase (a damaged or premium version) in that period as well. Therefore, introducing damaged version in $t = 1$ only makes some higher type consumers who would purchase the premium version select damaged version, which decreases profit for sure.[12] The mechanism that helps to solve Coase Conjecture in Hahn (2006) disappears in my model and the firm would rather just offer the original version.

2.5 Conclusion

While the conventional wisdom is that deceptive counterfeits are always harmful for the authentic firm and social welfare, this paper argues that the opposite can hold in durable-goods markets. Despite the business-stealing effect, deceptive counterfeits mitigate the time-inconsistency problem for the incumbent. It is demonstrated that the effect of counterfeits crucially depends on their quality. When the quality gap is sufficiently small, pooling equilibria are sustained in the second period. The front-loading effect cannot cover the loss from the competition and the authentic firm's profit decreases with counterfeit entry. However, if the quality gap is sufficiently large, the low-quality counterfeiter only incurs a mild competition which is dominated by increased sales in the first period, and the branded firm benefits from counterfeit competition in this case. Moreover, the incumbent cannot earn higher profit by offering a damaged good because the front-loading effect then disappears. In terms of welfare, contrary to traditional arguments, it is shown that in a large quality range, the deceptive counterfeiter is actually beneficial to the society due to more earlier purchases. Surprisingly, if counterfeits remain indistinguishable ex-post, total surplus

[12] The detailed proof is available upon request

unambiguously increases.

There are several directions for future research. For instance, I only investigates the short-term effects of counterfeits. An interesting question is how the counterfeit entry affects the incumbent's incentive to innovate. Given that counterfeits may increase the branded firm's profit, there is a possibility that they promote innovation as well. Another extension is to endogenize the counterfeit entry by explicitly modeling public policies that affect its entry cost. Moreover, famous brands face many counterfeiters with different qualities in reality. It would be interesting to study how counterfeiters compete with each other and their impact on the branded firm.

Chapter 3

Imitation and New Product Launch in Conspicuous-goods Markets

3.1 Introduction

Intellectual Property (IP) protection have been widely used in many industries. For example, patents are applied to protect innovations in engineering and biotechnology. Trade secrets are used for ingredients and recipes such as Coke and Pepsi. In terms of software, copyright is the primary tool utilized by developers. However, when we talk about designs, the level of IP protection seems to be very low. In the fashion industry, there is a new business model named "fast fashion", which is led by H&M, Zara and Forever 21. When vanguard designs come out in leading fashion shows, these firms will copy and sell cheap imitations in a short time. In some countries, like Japan and India, related laws have been legislated to protect original designs.

Recently, there is a debate that whether U.S. should expand The Copyright Act to the fashion industry. The current attempt of legislation is the "Innovative Design Protection Act" (IDPA), proposed by Senator Chuck Schumer in 2012.(e.g., Grochala, 2014) The argument is that imitations steal original designers' labor fruit and branded firms' profit. One of the fundamental goals of IP protection is to provide sufficient incentive for innovators to invest in R&D. However, this argument may not hold in the fashion industry where consumers not only care about the intrinsic quality of goods, but also how to use those conspicuous goods to signal their social status (e.g., Pesendorfer, 1995). As Raustiala and Sprigman (2006) stated, the emergence of copycats weakens the exclusivity of the original design and accelerates the fashion cycle.

This paper investigates how imitators' entry affects an incumbent's product-innovation decision in a dynamic context. I apply a two-period durable-goods model to discuss this issue. The incumbent sells an old product from the first period, which will be copied by a competitive fringe of imitators that enter in the second period. The branded firm chooses whether to invest in R&D and launch another product with a new design in the second period. Compared to the monopoly benchmark, entry of imitations will dampen the exclusivity of the original product, which makes consumers craving for the new product. Despite the fact that copycats may decrease returns from the new product through intensified competition, interestingly, I find that threats of strong imitators actually incentivize the incumbent to launch the new design. The intuition is that imitations with relatiely high quality and low price expand the low-end market, which significantly reduces the status utility of the old product and creates a larger advantage for the new product.

On the other hand, unlike in Waldman (1996), where the durable-goods monopolist's R&D decision has the time-inconsistent problem so that the firm has a distortedly high incentive to launch a new product after selling old ones, for conspicuous goods the problem exists in the monopoly benchmark but vanishes with imitators competition.

This paper is related to two streams of literature. First, Pesendorfer (1995) and Rao and Schaefer (2013) both discuss firm's strategy in durable conspicuous-goods markets. Pesendorfer (1995) focuses on the fashion cycle, the length of which is determined by the R&D cost. It argues that competition will slow the cycle and raise prices. Rao and Schaefer (2013) discusses a firm's inter-temporal pricing strategy and how that strategy is affected by the intrinsic quality. Instead, my paper concentrates on how competition of copycats influences the incumbent's product-innovation decision. Moreover, contrary to Pesendorfer (1995), I demonstrate that competition can accelerate the fashion cycle and decrease prices.

Second, several papers study the product-innovation decision of a durable-goods monopolist. Nahm (2004) investigates interaction between firm's inter-temporal pricing and R&D decision. It finds that when the firm continues to sell the old product in the second period, the interaction disappears and the R&D is time-consistent. Hoppe and Lee (2003) discusses how an incumbent conducts innovation to deter entrant. These papers do not consider conspicuous goods. In the presence of status utility, the conclusion of Nahm (2004) does not hold. However, competition from imitations can restore that result.

Chapter 3 is organized as follow. Section 3.2 describes the model. Section 3.3 analyzes the benchmark monopoly case. Section 3.4 introduces competition and compares the R&D incentives. Section 3.5 discusses the R&D time-inconsistency problem. Section 3.6 concludes. All proofs are relegated to the Appendix A.

3.2 The Model

I adopt a standard two-period durable-goods model as in Tirole and Fudenberg (1998). In the first period ($t = 1$), the incumbent sells an original product (L) that can be used in two periods. There is a unit mass of heterogeneous consumer indexed by type θ_i, where $\theta_i \sim U[0, 1]$. It is assumed that all products in our model provides intrinsic utility as well as status utility to buyers. The intrinsic utility can be interpreted as the functional value of the product. For example, a Burberry trench coat can keep a buyer warm. The status utility, which will be defined later, is the one that enables consumers to signal their types through social interactions. Consumers choose to buy the original product in $t = 1$ or wait to $t = 2$.

At the beginning of $t = 2$, the firm decides whether to invest in R&D to launch a new product (H) with a fresh design. For convenience, the R&D is deterministic: if the firm pays a fixed cost K, the new product will be produced for sure. In $t = 2$, if the new product is launched, the incumbent will make two dimensional decisions: (1) whether to keep selling L. (product-line decision). (2) prices for products that are actively sold. (pricing decision).[1] In this period, a competitive fringe of copycats enter to sell imitations (I) with low intrinsic quality. Assuming the branded firm has a lead time advantage so that it takes imitators one period to learn how to copy the product. Therefore, copycats only sell copies of the original product L. In our model, it is assumed that imitations are non-deceptive, which means that consumers know whether they are

[1] In this chapter, I only consider the second dimension and focus on the net sale case that the firm keeps selling L in $t = 2$

buying an authentic version of L or a imitation.[2] However, when they engage in social interaction ex-post, others cannot detect whether the buyer carries an imitation or an authentic product.[3] In $t = 2$, it is assumed that there is a frictionless second-hand market.[4] Therefore, consumers who purchased L in $t = 1$ can choose to keep using it or replace it with H.[5] For consumers who do not purchase in $t = 1$, they will choose to buy one of products or remain inactive in $t = 2$.

The marginal cost of the original product c^L is normalized to 0. The new high-quality product has a higher marginal cost $c^H > 0$. To generalize the model, assuming imitators' marginal cost $c^I > 0$. The marginal cost is positive because imitators may be fined by the government if they are caught. In terms of the intrinsic quality $q^l, l \in \{H, L, I\}$, assuming $q^H = H > q^L = 1 > q^I = I$. A standard assumption is that $q^H - 1 > c^H$, which implies that introducing the high quality good is efficient and beneficial. A consumer's intrinsic utility derived from a product is $\theta_i q^l$

The status utility of a good is determined by two factors: (1) the sensitivity that consumers care about social status, which is captured by the parameter λ. (2) The average type of consumers who purchase the product. For instance, if in $t = 1$, all consumers with $\theta_i \geq \theta_1$ purchase L, then the average type of L carriers is $\frac{1+\theta_1}{2}$ and the status utility of L is $\lambda\frac{1+\theta_1}{2}$ in this period. This assumption can be justified as follows: after purchase, a consumer's purchasing behavior will be observed by others and her type will be inferred by Bayesian rule. If consumers with type above θ_1 purchase L in equilibrium, then expected type of a L carrier is $\frac{1+\theta_1}{2}$.

Observers know for sure whether a consumer buys H or nothing. However, when imitators enter in $t = 2$, observers won't be able to tell whether a carrier of the product with L's design buys L or I by our assumption, which implies that customers of these two products will be pooled together by observers.

[2] Literature also discuss deceptive imitations like counterfeits. Copies in our model are distinguishable when consumers purchase because they have different prices from authentic goods or consumers buy from places that are famous for selling imitations intentionally.

[3] Many high-quality copies (A-bags) have the same appearance as authentic goods. They may also use materials that touch like genuine products.

[4] There are many examples of second hand markets for conspicuous goods. Automobiles' second hand markets have been widely studied. For textile products, such as luxury bags, clothing, etc., the second hand market is growing rapidly. Bain & Company estimates that the market is about $16 billion.

[5] It can be easily proved that they won't sell L and then buy I

3.3 The Monopoly Benchmark

This section investigates the benchmark model without imitations. Since the R&D decision is made in $t = 2$, we just need to compare the second period's profit with innovation and profit without innovation to figure out the product-innovation decision. The standard backward induction is used to analyze the Sub-game Perfect Equilibrium. As the standard durable-goods model, there is a price-skimming equilibrium: the firm charges a high price in period 1 and then lower the price to reap the residual demand in period 2. There is a cut-off marginal buyer θ_1 such that all higher type consumers purchase in $t = 1$. Currently, I focus on the equilibrium with interior solutions (market is not fully covered). A superscript $S(F)$ is used for the incumbent's profit with (without) product innovation. Equilibrium values are denoted with an upper bar.

3.3.1 No Product Innovation

Without a new product, the model is degenerated to the simple two-period model as in Tirole (1998) except that there is status utility. In $t = 2$, the marginal consumer θ_2^L is indifferent between buying L and nothing:

$$\theta_2^L + \frac{1 + \theta_2^L}{2}\lambda - p_2^L = \frac{\theta_2^L}{2}\lambda$$

In the left hand side, $(\theta_2^L + \frac{1+\theta_2^L}{2}\lambda)$ is the sum of intrinsic and status utility while p_2^L is the price of L in $t = 2$. Different from inconspicuous consumption, even if a consumer buys nothing, she still gets status utility $\frac{\theta_2^L}{2}\lambda$. Therefore, the branded firm tries to maximize its profit in $t = 2$ according to the following equation.

$$\max_{\theta_2^L} \ \pi_2^L = (\theta_1^L - \theta_2^L)(\theta_2^L + \frac{\lambda}{2})$$

The optimization yields $\overline{\theta}_2^L = \frac{1}{2}\theta_1^L - \frac{\lambda}{4}$ and $\overline{\pi}_2^F = (\frac{1}{2}\theta_1^L + \frac{\lambda}{4})^2$. Then in $t = 1$, the marginal customer θ_1^L is the one who is indifferent between buying in $t = 1$ and in $t = 2$.

$$\theta_1^L + \frac{1 + \theta_1^L}{2}\lambda + \overline{p}_2^L = p_1^L$$

We can think that buying in $t = 1$ as if consumers buy two units of non-durable goods, because a durable good can be used in two periods. The utility of the first unit is $\theta_1^L + \frac{1+\theta_1^L}{2}\lambda$ and the second unit will be priced at $\overline{p}_2^L$ in $t = 2$. Therefore, the dynamic optimization is as follows.

$$\max_{\theta_1^L} \ \pi^F = \underbrace{(1-\theta_1^L)(\theta_1^L + \frac{1+\theta_1^L}{2}\lambda + \overline{p}_2^L)}_{\pi_1^F} + \underbrace{(\theta_1^L - \theta_2^L)(\theta_2^L + \frac{\lambda}{2})}_{\pi_2^F}$$

From this equation, we get $\overline{\theta}_1^L = \frac{3}{5+2\lambda}$ and $\overline{\pi}_2^F = \frac{1}{4}(\frac{3}{5+2\lambda} + \frac{\lambda}{2})^2$.

3.3.2 With Product Innovation

When product innovation is implemented, both L and H are sold in $t = 2$. Therefore, there are two marginal buyers: the one who is indifferent between buying H and L (θ_2^H) and the one who is indifferent between buying L and nothing (θ_2^L). They are determined by next two equations.

$$\begin{aligned} q^H\theta_2^H + \frac{1+\theta_2^H}{2}\lambda - p_2^H &= \theta_2^H + \frac{\theta_2^H + \theta_2^L}{2}\lambda - p_2^L \\ \theta_2^L + \frac{\theta_2^H + \theta_2^L}{2}\lambda - p_2^L &= \frac{\theta_2^L}{2}\lambda \end{aligned}$$

Note that the status utility of L decreases compared to the non-innovation case because the high-quality product H "steals" high-type customers from L. Therefore, the branded firm maximizes its profit in $t = 2$ given by the following equation.

$$\begin{aligned} \max_{\theta_2^H, \theta_2^L} \ \pi_2^S &= [(\theta_2^H - \theta_2^L) - (1-\theta_1^L)]p_2^L + (1-\theta_2^H)p_2^H - K \\ &= \underbrace{(\theta_1^L - \theta_2^L)(\theta_2^L + \frac{\lambda}{2})}_{\pi_2^F} + \underbrace{(1-\theta_2^H)[(q^H-1)\theta_2^H + \frac{\lambda}{2}(1-\theta_1^L) - c^H]}_{pseudo\ profit\ premium\ of\ H} - K \end{aligned}$$

The second period profit can be rewritten as above. The first item is the second period profit without innovation (π_2^F). The second part can be interpreted as the pseudo profit premium of product H, where $1 - \theta_2^H$ is the demand of H and $[(q^H - 1)\theta_2^H + \frac{\lambda}{2}(1 - \theta_1^L) - c^H]$ is the pseudo price difference between H and L. Nahm (2004) finds that when the firm keeps selling L in $t = 2$, the R&D decision and inter-temporal pricing are independent, which does not hold in the presence of status utility. With a positive λ, the pseudo premium is positively affected by first period sale $(1 - \theta_1^L)$. Therefore, the pricing and R&D strategies are related. The firm's incentive to expand their first period market increases with the severity of status utility λ.

In $t = 1$, the marginal buyer θ_1^L is determined similarly as in the non-innovation case. The optimization for the total profit is as follows.

$$\max_{\theta_1^L} \pi^S = (1-\theta_1^L)(\theta_1^L + \frac{1+\theta_1^L}{2}\lambda + \overline{p}_2^L) + \overline{\pi}_2^S$$

$$\overline{\theta}_1^L = \frac{3 + \frac{c^H}{q^H-1}\lambda - \frac{3\lambda^2}{4(q^H-1)}}{5 + 2\lambda - \frac{3\lambda^2}{4(q^H-1)}}$$

$$\overline{\pi}_2^S = \frac{1}{4(q^H-1)}[q^H - 1 - c^H + \frac{\lambda}{2}\frac{2+2\lambda - \frac{c^H}{q^H-1}}{5+2\lambda - \frac{3\lambda^2}{4(q^H-1)}}]^2 - K$$

Lemma 1. With status utility, the incumbent's first period sale increases with λ. When $\frac{c^H}{q^H-1}$ is relatively small, the incumbent sells more in the first period with product innovation.

Results in Lemma 1 are straightforward. It can be easily verified that $\frac{d\bar{\theta}_1^L}{d\lambda} > 0$ for both innovation and non-innovation cases. First period demand $1-\theta_1^L$ decreases with $\frac{c^H}{q^H-1}$. Because the total quality of a product increases with λ, the firm optimally chooses to expand its sale. Since pseudo profit premium in $t = 2$ increases with first period demand, the incumbent is likely to sell more in $t = 1$.

3.4 Entry of Imitations

In this section, we do similar analysis as the last section, with entry of imitators in $t = 2$. All variables in equilibrium are denoted with an upper tilde.

3.4.1 No Product Innovation

In the second period, the incumbent's product L competes with imitations I. θ_2^I denotes the marginal consumer who is indifferent between buying I and nothing. Two marginal consumers θ_2^I and θ_2^L are determined by:

$$I\theta_2^I + \frac{1+\theta_2^I}{2}\lambda - p_2^I = \frac{\theta_2^I}{2}\lambda$$

$$\theta_2^L - p_2^L = I\theta_2^L - p_2^I$$

Since there is a competitive fringe of imitators, $p_2^I = c^I$. In the second equation, there is no status utility because L and I provide the same status by assumption. The profit maximization for the incumbent in $t = 2$ is as follows.

$$\max_{\theta_2^L} \ \pi_2^F = (\theta_1^L - \theta_2^L)[(1-I)\theta_2^L + c^I]$$

The solution is that $\tilde{\theta}_2^L = \frac{\theta_1^L}{2} - \frac{c^I}{2(1-I)}$. Taking the first period problem into account, we get $\tilde{\theta}_1^L = \frac{3-I}{5-I+2\lambda}$ and $\tilde{\pi}_2^F = \frac{1-I}{4}[\frac{3-I}{5-I+2\lambda} + \frac{c^I}{1-I}]^2$

3.4.2 With Product Innovation

In $t = 2$, the incumbent produces H and L to compete with imitators' product I. Three marginal types are given by following equations.

$$I\theta_2^I + \frac{\theta_2^H + \theta_2^I}{2}\lambda - c^I = \frac{\theta_2^I}{2}\lambda$$

$$\theta_2^L - p_2^L = I\theta_2^L - c^I$$

$$q^H\theta_2^H + \frac{1+\theta_2^H}{2}\lambda - p_2^H = \theta_2^H + \frac{\theta_2^H + \theta_2^I}{2}\lambda - p_2^L$$

The optimization in $t = 2$ is:

$$\begin{aligned}
\max_{\theta_2^H, \theta_2^L} \ \pi_2^S &= [(\theta_2^H - \theta_2^L) - (1-\theta_1^L)]p_2^L + (1-\theta_2^H)p_2^H - K \\
&= \underbrace{(\theta_1^L - \theta_2^L)[(1-I)\theta_2^L + c^I]}_{\pi_2^F} - K \\
&+ \underbrace{(1-\theta_2^H)[(q^H - 1 + \frac{\lambda^2}{4I})\theta_2^H + \frac{\lambda}{2}(1 - \frac{c^I}{I}) - c^H]}_{pseudo\ profit\ premium\ of\ H}
\end{aligned}$$

Lemma 2. When imitators enter in the second period, the incumbent's inter-temporal pricing decision and R&D strategy are independent (e.g., p_1^L, p_2^L will be the same under both cases.

The second period profit with entry can be expressed in a similar way as the monopoly case. The first term is the second period profit without innovation. However, with imitations, the pseudo profit premium of product H does not depend on the first period sale (1-θ_1^L). Therefore, competition from imitations makes the inter-temporal pricing and R&D independent, which is significantly different from the monopoly benchmark. Because of this characteristics, the R&D incentive measured by $(\pi_2^S - \pi_2^F)$ only depends on the pseudo profit premium, which can be solved by simple maximization as follows.[6]

$$\tilde{\pi}_2^S - \tilde{\pi}_2^F = (q^H - 1 + \frac{\lambda^2}{4I})[\frac{1}{2} + \frac{\frac{\lambda}{2}(1 - \frac{c^I}{I}) - c^H}{2(q^H - 1 + \frac{\lambda^2}{4I})}]^2 - K$$

3.4.3 Comparison of R&D Decision

In this part, the R&D decision will be compared between the monopoly benchmark and the competition case. For convenience, superscripts F (without innovation), S (with innovation) are also applied to marginal buyers in equilibrium.

Proposition 1. When consumers do not care about social status ($\lambda = 0$), imitators' entry does not affect the incumbent's R&D decision. When $\lambda > 0$, the incumbent is more like to introduce the new product with imitators' entry when a sufficient condition $\overline{\theta}_1^{LF} \geq \overline{\theta}_1^{LS} > \frac{c^I}{I}$ is satisfied.

The Figure B.5 is an example when $q^H = 1.5, c^H = 0.2, c^I = 0.1, I = 0.5$. When the competition effect is strong, meaning that copies have relatively high qualities but low prices, consumers can afford imitations even if their types are very low in the second period. This weakens the exclusivity of L significantly, which implies that the status utility of L shrinks a lot. Therefore, high-type consumers are eager to buy H as a signaling device.

3.5 Discussion on R&D Time-consistency

As Waldman (1996) points out, the durable-goods monopoly's R&D is also time-inconsistent: once it sells old products in the first period, the firm has a distortedly high incentive to launch a new product in the second period. This section discusses whether the firm faces the same problem in R&D for conspicuous goods. We need to compare the branded firm's R&D strategies in two

[6] The fixed cost K is the same in all cases. Therefore, we can ignore its impact when we focus on comparing the R&D incentive between monopoly and competition

cases: (1) when the firm can make commitment at the beginning of $t = 1$ (In $t = 1$, the firm announces whether it will launch a new product in $t = 2$ and the claim is binding) (2) when the firm makes decision in $t = 2$ as we discussed above.

By Lemma 1, without imitations, the R&D decision is definitely not consistent since the first period profit will be affected by the introduction of H. By Lemma 2, when imitators enter, the incumbent's R&D decision does not affect prices of L in both periods. Henceforth, the first period's profit stays the same under both cases and the R&D is time-consistent. Mathematically, $\tilde{\pi}_2^S - \tilde{\pi}_2^F = (\tilde{\pi}_1^S + \tilde{\pi}_2^S) - (\tilde{\pi}_1^F + \tilde{\pi}_2^F)$.

3.6 Conclusion

There has been a long debate on the optimal IP protection, which recently extends to the fashion industry. Given the unique feature of this industry that consumers enjoy status utility, this paper argues that we may not simply apply IP policies that work in other fields to protect designs. Despite the fact that imitations steal profits and dilute brand image, they provide a higher incentive for firms to develop new designs, thus accelerate the fashion cycle. This might be the way how the fashion industry evolves. Moreover, in a durable-goods context, the R&D incentive is time-consistent with imitations.

There are several directions to extend the model. First, the product-line choice in the second period can be considered. As Nahm (2004) points out, the interaction of R&D and inter-temporal pricing decisions depends on that choice. Second, the entry decision of imitators can be endogenized. It will be interesting to discuss whether the incumbent can use R&D to fight copycats and what the optimal level of IP protection is.

Chapter 4

Benefit of Being Less Prominent in Consumer Search

4.1 Introduction

This paper investigates a two-period duopoly ordered search problem. In contrast with traditional search literature, our model discusses how ex-ante heterogeneous consumers sequentially search for a durable good. Interestingly, we find that the less prominent firm may have a higher profit than the prominent one.

Recently, several papers have studied ordered search and all of them show that the prominent position is desirable for firms. For instance, both Armstrong et.al (2009) and Arbatskaya (2007) investigate the difference between ordered search and random search, concluding that the more prominent firm earns higher profit than other firms. However, there is some evidence shows that the firm may benefit from cherry-picking consumers by sitting in less prominent positions. Agarwal et al. (2011) runs a field experiment to test how revenue and profit of a clothing brand change with its position in Google's sponsored link. Surprisingly, both revenue and profit exhibit an inverted-U shape for longer keywords. Brands being placed at the top of the list have higher click through rate but lower conversion rate, which implies that some low-value consumers visiting their websites do not purchase. Additionally, an insurance firm named Direct Line withdrew itself from websites where low-value consumers typically look for product information. The reason is not to save fees for being listed in those websites, but to cherry-pick more valuable consumers[1] .

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[1] Commentators believe the advertising expenditure that the firm spends on announcing its withdraw through TV far more exceeds the commission for those websites (see Wilson (2010)) and the primary purpose is to cherry-pick high-value consumers (see *The Economists, Jul 21st 2008, Online extra*, at `http://www.economist.com/node/11773787`).

In Armstrong et.al (2009), the prominent firm benefits from the larger fresh demand since all consumers search it first. Although a different search framework is adopted, the similar advantage for the prominent firm still exists in our model. However, the opposite result may hold in our model because we assume that consumers have heterogeneous expectations of their utility and search for a durable good. Another important assumption is that the marginal search cost is increasing, which implies consumers have to pay a higher cost to search the second firm[2] . This assumption is in line with the basic principle of marginal cost and bounded rationality. Levav et al. (2010) conducts framed and natural fields experiments to verify that consumers indeed have convex marginal cost in evaluating options and assessing utility, which are major components of the search cost. Hence, the pool of consumers that search the second firm consist of more high-value consumers because low-value ones are more likely to stop search when the search cost rises. However, similar to other papers, the demand for the less prominent firm is just a subset of the prominent one because any consumer comes to the less prominent firm must have visited the prominent one. When the good is non-durable, even if the prominent firm faces a group of low-value consumers on average, it can do at least as well as the less prominent one by setting the same price to attain a larger demand. However, in the context of durable goods, the prominent firm can not resist the temptation to decrease the price and reap the residual demand in the future, which is the standard time-inconsistency problem. This problem is alleviated for the less prominent firm because some low-value consumers are screened out by the search process, which is why the less prominent position can be more profitable.

In terms of equilibrium price, Arbatskaya (2007) argues that the price declines with the search order because consumers with a high search cost are more likely to stop search and buy from sellers at the top of the search list. Armstrong et al. (2009) assumes homogeneous search cost and obtains an opposite conclusion. The reason is that the prominent firm has a more elastic demand which contains a larger proportion of consumers who may search other firms later. In our paper,

[2] In the model, the search cost associated with the first firm is normalized to 0, which is standard in typical search literature.

the less prominent firm has a weakly higher second period price if it sells in that period, because some low-value consumers do not visit it. However, the result regarding the first period price can be ambiguous. When the search cost is relatively low, the less prominent firm's price in $t = 2$ is not too high, which makes its first period price lower than the prominent firm. If the search cost is high, the less prominent firm will have higher prices in both periods.

The paper is also related to search obfuscation. Ellison and Ellison (2009) studies the internet search engine Pricewatch and discovers that some firms try to increase consumers' search cost by using complicated product description or hiding price information. Another widely adopted strategy that can obfuscate search is obscured price. Ellison (2005), Gaibaix and Laibson (2006) explain how firms can benefit from add-on pricing. Piccione and Spiegler (2012), Zhou and Chioveanu (2013) investigate firms' incentive in using different price frames to confuse customers.

The rest of Chapter 4 is organized as follow: Section 4.2 outlines the model. Section 4.3 and 4.4 analyze the equilibrium and draw main conclusion. Section 4.5 investigates some extensions and Section 4.6 concludes. All proofs are relegated to the Appendix A.

4.2 The Model

There are two ex-ante homogeneous firms, A and B, each of which produces a durable good at zero cost that can be used in two periods. A unit mass of heterogeneous consumers search the two firms sequentially and decide the one they will patronize and when to purchase. In the benchmark model, it is assumed that all consumers search firm A first. This assumption is relaxed in section 4.5, where we show that the similar result still holds if the search order is endogenized. It is assumed that consumers pay a higher search cost to visit the second firm because they experience *Decision Fatigue* and have to spend more energy in assessing utility and evaluating options[3] . To be in line with the search literature, we assume that the first search is free and consumers have to pay homogeneous search cost $s \geq 0$ to search firm B. The search cost includes the time and efforts that

[3] This assumption is verified by psychology and economic literature. See Baumeister (2002), Baumeister (2003) and Levav et al. (2010)

consumers pay to visit the firm, understand its price scheme and learn characteristics of the product. This paper adopts a similar framework as Chen and He (2011) in modeling the search process. The major difference is that consumers are ex-ante heterogeneous in terms of their valuations in our model. There are K types in total and each type contains a measure of $\frac{1}{K}$ consumers. Consumers will never buy a product without search. Before a search, they are unaware of the firm's price and their valuation for its product[4] . After conducting a search, there is a probability β that a consumer will find a match with the firm and has positive valuation on the product. If the good does not match her preference, she derives zero utility from consumption. The matching probability β can be interpreted as the quality of the firm. If a type k consumer finds a match, her valuation for a product v is realized. $v \in [\underline{v}_k, \overline{v}_k]$ is realization of a random variable with cdf $F(v)$ on $[0, \bar{v}]$. The realization v is same for both products. Therefore, if she continues to search the other firm, her utility conditional on matching is the same v. $F(v)$ does not vary with k, which implies that the ex-post utility of different types conditional on matching is drawn from different segments of the same distribution. Following the durable-goods literature, early buyers derive $2v$ units of utility in two periods while a late purchase only provides v units. For convenience, there is no discounting for both consumers and firms.

Assumption 1. $\underline{v}_{k+1} = \overline{v}_k$, $\overline{v}_k - \underline{v}_k = \frac{\bar{v}}{K}$ $\forall k$.

Assumption 2. The hazard rate $\frac{f(v)}{1-F(v)}$ is monotonically increasing.

Assumption 1 indicates that a larger index k is associated with a higher type of consumers. It also ensures that there is no gap in ex-post utility across different types and the range of v for consumers within each type is the same. These two assumptions guarantee a well-behaved demand curve for the firm.

Time line of the model There are two periods $t = 1, 2$. In $t = 1$, firms set their first period prices p_1^i, $i \in \{A, B\}$, simultaneously. They lack commitment power on future prices. Every

[4] Assuming consumers only need to pay a unit of search cost to learn her ex-post utility and all prices of a firm. For example, if they search the firm in $t = 1$ but choose to buy in $t = 2$, then they will have free access to its price in $t = 2$ via targeted email or other advertisement.

consumer searches firm A and discovers p_1^A, whether firm A matches and her ex-post utility v if it matches. After that, she decides whether to search firm B[5] . If a consumer stops searching, she can choose to buy from A in $t = 1$, or wait until $t = 2$. If she keeps searching, she will learn p_1^B and whether firm B's product matches her demand. Then she decides whether to buy from firm B immediately or wait[6] . In $t = 2$, both firms set p_2^i together, which are automatically updated to consumers who have searched the firm in $t = 1$. Consumers then decide whether to buy at $t = 2$ or not.

4.3 Durable-goods Monopoly with General Demand

Before incorporating consumer search into the model, we first discuss the two-period durable-goods monopoly with a general demand curve. Intuitively, since all consumers search firm A freely, it faces a non-truncated demand with all consumers $v \in [0, \bar{v}]$. With a positive search cost, some low type consumers will stop searching, which renders firm B a truncated pool $v \in [\underline{v}, \bar{v}]$. Therefore, to compare profits between two firms, we have to investigate the profit under non-truncated and truncated demand respectively. In this section, three cases will be studied: (a) The monopoly faces a non-truncated demand and has commitment power. (b) The monopoly faces a non-truncated demand and has no commitment power. (c) The monopoly faces a truncated demand and has no commitment power. Superscript C, N, T are used to denote equilibrium values in three cases respectively. In the following discussion, we can think the monopoly maximizes its profit by choosing the optimal marginal buyer v_t in each period.

4.3.1 Non-truncated Demand

With commitment power

If the firm can commit, it charges the monopoly price in $t = 1$ and holds it in $t = 2$.

[5] We simply assume all searches happen in $t = 1$ because searching early is a weakly dominating strategy for consumers. If some of them search in $t = 2$, they would give up the option to buy early. Since both firms and consumers are forward looking, searching late will not affect firm's pricing strategy. Therefore, everyone prefers to search early.

[6] In equilibrium, consumers never recall. Thus we can eliminate the option that they return to firm A

Therefore, all consumers with life time utility higher than the price purchase in $t = 1$ and there is no transaction in $t = 2$, which implies that $v_1 = v_2 = v$ and $p = 2v$.

$$\max_{v}\ \pi = 2v[1 - F(v)]$$

The optimal marginal buyer v^C is an implicit solution of $v = \frac{1-F(v)}{f(v)}$, the uniqueness of which is guaranteed by Assumption 2. The firm's profit is $\pi^C = 2v^C[1 - F(v^C)]$.

No commitment power

Without commitment power on the future price, the standard time-inconsistency problem exists: The firm has an incentive to lower the price to reap the residual demand in $t = 2$, which is anticipated by forward-looking consumers. Therefore, high-value customers ask for a lower first price in $t = 1$. The problem can be solved by backward induction.

In $t = 2$, $p_2 = v_2$. The firm maximizes π_2 by choosing a proper v_2:

$$\max_{v_2}\ \pi_2 = v_2[F(v_1) - F(v_2)]$$

Therefore, v_2^N is the solution of FOC $v_2 = \frac{F(v_1)-F(v_2)}{f(v_2)}$. The uniqueness of v_2^N can be easily verified by proving the right hand side of FOC is decreasing. For convenience, we define the solution of $v_2 = \frac{F(v_1)-F(v)}{f(v)}$ as $v_2^*(v_1)$. By this definition, $v_2^N = v_2^*(v_1^N)$ and $v^C = v_2^*(\bar{v})$ in equilibrium.

In $t = 1$, the marginal buyer v_1 must be indifferent between buying in two periods. The inter-temporal incentive constraint can be written as:

$$2v_1 - p_1 = v_1 - v_2^*(v_1)$$

Therefore, $p_1 = v_1 + v_2^*(v_1)$. The firm chooses v_1 to maximize total profit.

$$\max_{v_1}\ \pi = (v_1 + v_2^*(v_1))[1 - F(v_1)] + v_2^*(v_1)[F(v_1) - F(v_2^*(v_1))]$$

In equilibrium, v_1^N implicitly solves $v_1 = \frac{1-F(v_1)}{f(v_1)}(1 + \frac{dv_2^*(v_1)}{dv_1})$. The profit can be written as $\pi^N = v_1^N[1-F(v_1^N)]+v_2^N[1-F(v_2^N)]$. Next, we prove a lemma that is technically useful for later discussion.

Lemma 1. $\frac{dv_2^*(v_1)}{dv_1} > 0$, $v_1^N > v^C > v_2^N$ and $\pi^N < \pi^C$.

The lemma shows that when the firm cannot commit, the marginal buyer $v_1^N (v_2^N)$ in the first (second) period is too high (low) compared to the optimal v^C in the commitment case, which reduces the durable-goods monopoly's profit. We can use another way to interpret the Coase Conjecture: The monopolist offers a mixed bundle of two homogeneous non-durable goods. The number of consumers who buy 2 units (the bundle), 1 unit or nothing is endogenized. When the firm commits, every buyer purchases the bundle and the market is equivalent to the pure bundling case. The firm can charge a monopoly price for every unit and earn the maximized profit. When the firm cannot commit, it has an incentive to sell to those who only buy 1 unit. However, the price of this unit has to be the same for all consumers. Since those who demand only 1 unit have lower valuations on average, the price of the second unit is too low compared to the optimal. On the other hand, the firm has to raise the first unit's to offset that effect, which makes prices for both units deviate from the monopoly price.

4.3.2 Truncated Demand

As we pointed out earlier, firm B is likely to face a pool of consumers with ex-post valuation $v \in [\underline{v}, \bar{v}]$, which is truncated from bottom. $\underline{v}$ is endogenously determined by the lowest type of consumers k who search firm B. Type k is defined as the *Search Lower Bound* in this paper. Since profit with truncated pool is determined by $\underline{v}$, the notation $\pi^T(\underline{v})$ is used for convenience. Right now, we assume $\underline{v}$ is exogenously given and focus on the firm's maximization problem. There are two differences when the firm faces a truncated pool: first, when $\underline{v}$ is close to $\bar{v}$, the firm sells to all consumers in $t = 1$ and the time-inconsistency problem disappears even if the firm lacks commitment power. Second, with some intermediate $\underline{v}$, the firm still sells in $t = 2$. However, the second period price is optimal at corner solution instead of interior solution[7] . This section investigates the range of $\underline{v}$ that supports two solutions respectively and then compares the profit between the two cases.

[7] The case that firm only sells in $t = 1$ is also categorized as corner solution since the firm prices at the corner and all consumers are served.

If $\underline{v} \geq v^C$, the firm sets $p_1 = 2\underline{v}$ and sell nothing in $t = 2$. When $\underline{v} \leq v^C$, intuitively an interior solution in $t = 2$ is sustained if $\underline{v}$ is close to 0. The game is similar to the non-truncated case. For $\underline{v}$ in a intermediate range, the firm may set $p_2 = \underline{v}$, which is the case of corner solution. However, the optimal price in $t = 2$ is determined by the difference between v_1 and $\underline{v}$ while v_1 is influenced by consumers' belief on p_2. In equilibrium, consumers' belief should be consistent (i.e $E(p_2) = p_2$) and the firm has no incentive to deviate given this belief. First, we analyze the profit maximization problem in two cases and find the range of $\underline{v}$ that can support each solution. In $t = 2$,

$$\max_{v_2 \geq \underline{v}} \; \pi_2 = v_2[F(v_1) - F(v_2)]$$

$$v_2^T = \begin{cases} \underline{v} & \text{if } v_2^*(v_1) \leq \underline{v}, \text{ corner solution} \\ v_2^*(v_1) & \text{if } v_2^*(v_1) \geq \underline{v}, \text{ interior solution} \end{cases}$$

In the corner solution equilibrium, the inter-temporal incentive constraint is $p_1 = v_1 + E(p_2) = v_1 + \underline{v}$. Then in $t = 1$,

$$\max_{v_1} \; \pi = v_1[1 - F(v_1)] + \underline{v}[1 - F(\underline{v})]$$

In this case, there is no correlation between v_1 and v_2. Henceforth, $v_1^T = v^C$ and firm's profit from the "first unit" increases. Furthermore, in the non-truncated case, consumers with valuation below v_2^N will buy nothing. If this tail is truncated and consumers believe the corner solution is attained in $t = 2$, the firm will earn higher profit ($\pi^T(v_2^N) > \pi^N$). Given the optimal v_t^T in both periods, the corner solution equilibrium can be supported only if the firm indeed prices at the corner in $t = 2$, which is satisfied if $v_2^*(v^C) \leq \underline{v}$. Therefore, $\forall \underline{v} \in [v_2^*(v^C), \bar{v}]$, the corner solution equilibrium can be sustained if consumers hold corner belief.

In the interior solution equilibrium, the model is similar to the non-truncated case and the constraint $v_2^*(v_1) \geq \underline{v}$ has to be satisfied. When $\underline{v} \leq v_2^N$, the truncated demand generates identical result as the non-truncated one because that constraint is not binding. If $\underline{v} > v_2^N$, the constraint must be binding, which implies $v_1^T = F^{-1}[F(\underline{v}) + \underline{v}f(\underline{v})] > v_1^N$ and $v_2^T = \underline{v}$. However, the firm always wants to deviate in this case. When the constraint is binding, the firm sets the same second period price $\underline{v}$ in both solutions. The corner solution yields higher profit because $v_1^T = v^C$ is the interior optimal solution of total profit maximization. Therefore, the firm has an incentive to decrease v_1^T from $F^{-1}[F(\underline{v}) + \underline{v}f(\underline{v})]$ to v^C. Moreover, consumer's belief is still consistent since v_2^T remains the same. Hence, the interior solution equilibrium can only be supported if $\underline{v} \in [0, v_2^N]$ and consumers hold interior belief.

From previous discussion, when $\underline{v} \in [v_2^*(v^C), v_2^N]$, both equilibria can be sustained depending on consumers' belief.[8] In this range, the interior solution equilibrium yields the same result as the non-truncated case. Consumers within this area buy nothing in the non-truncated case. However, if the corner solution is believed, these consumers will buy at $t = 2$. Since we do not want consumers' different beliefs to drive the result, it is assumed that when $\underline{v} \in [v_2^*(v^C), v_2^N)$, consumers believe the firm will price at interior solution in $t = 2$. It is more meaningful to compare profits between two cases under this assumption. The monopoly profit with a truncated demand is illustrated as follow and Lemma 2 concludes profit comparison.

$$\pi^T(\underline{v}) = \begin{cases} \pi^N & \text{if } \underline{v} < v_2^N, \text{ interior solution} \\ \\ v^C[1 - F(v^C)] + \underline{v}[1 - F(\underline{v})] & \text{if } v_2^N \leq \underline{v} < v^C, \text{ corner solution} \\ \\ 2\underline{v}[1 - F(\underline{v})] & \text{if } \underline{v} \geq v^C, \text{ corner solution} \end{cases}$$

Lemma 2. If $\underline{v} \in [0, v_2^N)$, the firm has the same profit with truncated and non-truncated demand. There exists a $\tilde{v} \in (v^C, \bar{v})$, such that $\forall v \in [v_2^N, \tilde{v})$, the truncated demand yields a higher profit for the firm. When $v \in (\tilde{v}, \bar{v}]$, the firm earns more with the non-truncated demand.

Lemma 2 states that for a proper truncation, the firm's profit is higher than the non-truncated case. First, when a small tail is dropped ($\underline{v} \in [0, v_2^N)$), the profit is the same under both cases because those consumers will not buy in either case and they will not change the firm's pricing decisions. Second, when $\underline{v}$ is within an intermediate range, the truncated demand yields a higher profit. If $\underline{v} \in [v_2^N, v^C]$, since $\frac{d\pi^T(\underline{v})}{d\underline{v}} \geq 0$ and $\pi^T(v_2^N) > \pi^N$, it must be that $\pi^T(\underline{v}) > \pi^N$. In addition, $\pi^T(v^C) = \pi^C > \pi^N$, $\pi^T(\bar{v}) = 0 < \pi^N$ and $\pi^T(\underline{v})$ is continuous and decreasing with $\underline{v}$ in this range.

[8] By Lemma 1, $v_2^*(v^C) < v_2^*(v_1^N) = v_2^N$

There must be a unique $\tilde{v} \in (v^C, \bar{v})$ such that $\pi^T(\tilde{v}) = \pi^N$. So if $\underline{v} \in (v^C, \tilde{v}]$, $\pi^T(\underline{v}) \geq \pi^N$ as well. Finally, when too many consumers are screened out, the demand is too low and the profit is lower than the non-truncated case, which happens if $v \in (\tilde{v}, \bar{v}]$.

4.4 Full Model with Consumer Search

This section analyzes the entire model by incorporating consumers' search decision. First, optimal stopping rules for heterogeneous consumers are determined. Under some assumptions, it is true that low types are more likely to stop searching, which leaves firm B a truncated demand with smaller volume comparing to firm A's large non-truncated demand in equilibrium. With a proper combination of search cost and matching probability, firm B earns higher profit than firm A.

After searching firm A, consumers can be divided into two cohorts: matched and unmatched consumers. In our model, firm B's prices are different from firm A's. Even though a consumer's ex-post valuation v is fixed after realization, she might still search B for a lower price. In general, search decisions of two cohorts have strategic interactions and influence each other. The change of optimal stopping rule of one group will change firm's prices in equilibrium, thus alter the decision of the other cohort. Therefore, we first assume the optimal rule for one cohort while analyzing the behavior of the other one and of the firms. Then it is proved that they indeed follow that optimal rule under certain conditions.

Another major difference between this paper and existing search literature is that consumers are ex-ante heterogeneous in terms of their utility. This will change the consumers' search behavior significantly and requires further assumptions for the analysis. For example, considering a type k consumer's decision if only $k+1$ to K consumers continue to search. The firm's lowest possible price is $\underline{v}_{k+1}$, which is higher than a consumer k's ex-post valuation by Assumption 1. Since the measure of any individual is 0, even if the firm knows this type k consumer searches, it will not lower the price to capture her. Therefore, this consumer k will never search when she cannot coordinate with others in the same type. To avoid this problem, we make the following assumption.

Assumption 3. All consumers who are ex-ante identical can coordinate and choose the optimal search decision together. However, heterogeneous consumers cannot cooperate, they only consider marginal benefit and marginal cost of *unilateral* deviation.

For instance, all type k consumers who find firm A's product does not match are ex-ante homogeneous before searching B and can coordinate. However, those who find a match but have different v are heterogeneous even though they are the same type.

Assumption 3 guarantees a possibility that the marginal type will search. If firm B believes that they will search in equilibrium, it might lower the price to increase the search margin. However, the information asymmetry associates with the search model might still be a problem. Since consumers do not know the price of firm B, the search decision depends on the belief of that. In the last example, type k consumers will search if they hold optimistic belief that firm B's price is low enough so that their expected benefit outweighs search cost. We call this pro-search equilibrium where marginal consumers are inclined to search. However, type k consumers will not search if they believe that firm B holds a high price and has no intention to serve them. This is also an equilibrium since given type k consumers do not search, that high price is optimal for serving higher type consumers. And given the high price, type k consumers do not search. This is the anti-search equilibrium where marginal consumers hold pessimistic belief. In this paper, it is assumed that marginal consumers will hold optimistic belief and pro-search equilibrium is investigated.

Assumption 4. The marginal type consumers will search if their expected surplus, which is calculated given they are the search lower bound, exceeds the search cost.

There are several reasons that we concentrate on the pro-search equilibrium in later analysis: First, this is the type of equilibrium discussed in typical search papers where consumers are homogeneous in expected valuation. In all search models, there is an uninteresting equilibrium where firms' price is high enough such that no consumer search, which is similar to the anti-search equilibrium here. However, the equilibrium studied is the one that consumers will search if possible, which is the pro-search equilibrium in our model. Second, there are multiple anti-search equilibria under pessimistic belief. If the equilibrium where the search lower bound is k can be sustained, then all equilibria with the search lower bound higher than k can be as well. In some of them, our main result still hold.[9] Third, we do not want consumers' belief to be the reason that stops them searching, which truncates the demand. Under pessimistic belief, even if the search cost $s \to 0^+$, the anti-search equilibrium in which very high type consumers stop searching will exist. In that case, it is the asymmetric information of the search model that drives the result. Finally, both consumer surplus and total welfare are higher in pro-search equilibrium.

4.4.1 The Optimal Stopping Rule for Matched Consumers

Right now, it is assumed that there is a search lower bound for unmatched consumers and firm B faces a truncated ex-post demand $v \in [\underline{v}, \bar{v}]$, which is proved later. First, as we discussed above, $\underline{v} \geq v_2^N$. Since $p_2^A = v_2^N$ and $p_2^B = \underline{v}$, $p_2^B \geq p_2^A$. This implies that for all consumers with $v \leq v_1^B = v^C$ never search firm B, because they purchase in $t = 2$ while firm B has a higher price in that period. For $v \in (v^C, v_1^N]$, given $v_1^A = v_1^N$, if they buy from A, they will purchase in $t = 2$. However, if they buy from B, it will be in $t = 1$. In this case, as long as $\beta \leq \frac{v_1^N - v_2^N}{2v_1^N - v^C - \underline{v}}$, matched consumers never search. Similarly, if $v \in (v_1^N, \bar{v}]$, matched ones stop searching when $\beta \leq \frac{v_1^N - v_2^N}{2v_1^N - v^C - \underline{v}}$ is satisfied as well. Since $\underline{v} \geq v_2^N$, a sufficient condition that guarantees matched consumers to stick with firm A is given by the following lemma.

Lemma 3. If the matching probability is sufficiently small, i.e. $\beta \leq \frac{v_1^N - v_2^N}{2v_1^N - v^C - v_2^N}$, consumers who find a match with firm A's product will stop search.

It can be easily verified that if $v_1^N + v_2^N \leq v^C + \underline{v}$ (equivalent to $p_1^A \leq p_1^B$), then the condition in Lemma 3 is satisfied.

[9] Those equilibria where the search lower bound is not too high so that the firm B still has a relatively large incoming demand

4.4.2 The Optimal Stopping Rule for Unmatched Consumers

Given the optimal stopping rule for matched consumers, the next Lemma shows that there is no gap in unmatched consumers who continue to search.

Lemma 4. For unmatched consumers, if type k consumers decide to search firm B, then all consumers with higher type $k+i$, $i \in (0, K-k]$ will search as well.

Intuitively, this Lemma holds because higher type consumers have higher ex-post valuation. Type k consumers search if and only if $\beta E(U_k^B) \geq s$, where U_k^B is the net surplus of type k consumers provided by firm B's product. This equation is affected by search cost, firms' prices and consumers' ex-post valuations. First, all consumers have the same search cost. Second, if any higher type $k+i$ is going to purchase at the same time with type k, they will face the same price. Since type $k+i$ has strictly higher utility, they will search given that type k consumers search. Finally, if type $k+i$ is going to purchase at a different time, it must be that they buy at $t=1$ and type k buy at $t=2$. If buying at $t=1$ is optimal for type $k+i$, they shall get more surplus than buying at $t=2$, which dominates type k consumers' surplus at $t=2$. Therefore, $E(U_{k+i}^B) \geq E(U_k^B)$.

With the help of Lemma 4, the only thing left for the optimal stopping rule is to pin down the search lower bound k^* for a given search cost s. Obviously, when $s=0$, by assumption, everybody will search, in which case firm B faces a non-truncated demand with smaller volume because matched consumers do not search. The less prominent position can never be more profitable in that case. $\forall s > 0$, low type consumers with $\bar{v}_k < v_2^N$ do not search since they can not afford the product even if they find a match.

In our model, given the same s, it is possible that $k^* = k$ can be supported as a equilibrium but $k^* = k+i$ can not. That's because firm B's price will vary with the search lower bound. When $k+i$ is the search lower bound, the firm charges a higher price in correspondence. In general, there may be multiple equilibria under the same s. To simplify the analysis, we make the following assumption to guarantee that the search lower bound monotonically increases with s.

Assumption 5. $f'(v) \geq 0$

This assumption is stronger than the increasing hazard rate. Even if it is violated, it is still possible that firm B benefits from being searched late but the result is not as clear as the one under this assumption. Now, k^* for unmatched consumers can be determined. If $k^* = k$, two conditions need to be satisfied in pro-search equilibrium: (1) type k consumers do want to search if they are the lower bound. (2) type $k-1$ consumers do *not* want to search if they are the lower bound. Mathematically, it requires $s \in (\beta E(U_{k-1}^B)|_{k^*=k-1}, \beta E(U_k^B)|_{k^*=k}]$. Notice that the expected surplus of a certain type is conditional on k^* since the equilibrium price varies with it. It is clear that ex-post a consumer may have three options:(1) buy in t=1 (2) buy in t=2 (3) buy nothing. Therefore, $E(U_k^B)|_{k^*=k}$ depends on when she purchases. $E(U_k^B)|_{k^*=k}$ is illustrated as follow:

$$E(U_k^B)|_{k^*=k} = \begin{cases} \int_{\underline{v}_k}^{\bar{v}_k}(2v - 2\underline{v}_k)dF(v) & \text{if } \underline{v}_k \geq v^C \\ \int_{v^C}^{\bar{v}_k}(2v - v^C - \underline{v}_k)dF(v) + \int_{\underline{v}_k}^{v^C}(v - \underline{v}_k)dF(v) & \text{if } \bar{v}_k \geq v^C > \underline{v}_k \geq v_2^N \\ \int_{v^C}^{\bar{v}_k}(2v - v^C - v_2^N)dF(v) + \int_{\underline{v}_k}^{v^C}(v - v_2^N)dF(v) & \text{if } \bar{v}_k \geq v^C > v_2^N > \underline{v}_k \\ \int_{\underline{v}_k}^{\bar{v}_k}(v - \underline{v}_k)dF(v) & \text{if } v^C > \bar{v}_k > \underline{v}_k \geq v_2^N \\ \int_{v_2^N}^{\bar{v}_k}(v - v_2^N)dF(v) & \text{if } v^C > \bar{v}_k > v_2^N > \underline{v}_k \end{cases} \quad (4.1)$$

In the first case, type k is high such that if consumers within k are the search lower bound, firm B only sells in $t = 1$ and $p_1^B = 2\underline{v}_k$. In the second one, a consumer will pay $p_1^B = v^C + \underline{v}_k$ in $t = 1$ if her value is above v^C but will pay $p_2^B = \underline{v}_k$ in $t = 2$ with lower value. The third case is when the total types K is small and a type k may have all three options ex-post. The fourth case is similar to the first one except that consumers buy in the second period. In the fifth one, ex-post v is around buying in $t = 2$ or nothing, in which case $p_2^B = v_2^N$. Finally, if v is too low, consumers buy nothing , which is not listed here.

We will concentrate on the equilibria with k^* that might provide firm B higher profit. Lemma 2 states that when $\bar{v} \in [v_2^N, \tilde{v})$, $\pi^T > \pi^N$. It is implied that if the ex-post valuations of the search lower bound are within this range, there is a possibility that firm B's profit is higher than firm A's. When s is high, many consumers stop searching, the lower bound of ex-post valuation is high. There is a k_1, such that $\underline{v}_{k_1} \leq \tilde{v}$ and $\bar{v}_{k_1} > \tilde{v}$. If $k^* = k_1$, $\underline{v} = \underline{v}_{k_1}$. By this definition, k_1 is the highest k^* that may make firm B earn more. On the other hand, there is a lowest $k^* = k_2$ defined by $\underline{v}_{k_2} \geq v_2^N$ and $\underline{v}_{k_2-1} < v_2^N$. If $k^* = k_2$, $\underline{v} = \underline{v}_{k_2} \geq v_2^N$. Therefore, when $k^* \in [k_2, k_1]$, the truncated demand generates a higher profit than non-truncated one and firm B's profit may be higher than firm A's. Define k_3 as the type that is closest to v^C, such that $\underline{v}_{k_3} \leq v^C$ and $\overline{v}_{k_3} > v^C$. Next, we will show that for any β, there is a proper range of s such that every search lower bound $k^* \in [k_2, k_1]$ can be supported as an equilibrium. More importantly, in each of these equilibria, there exist a specific $\beta^*(k^*)$ such that firm B's profit is weakly higher than firm A's when $\beta \leq \beta^*(k^*)$.

Proposition 1.

1. If $f'(v) > 0$, for every $k \in [k_2, k_1]$, if $s \in (\beta E(U_{k-1}^B)|_{k^*=k-1}, \beta E(U_k^B)|_{k^*=k}]$, then type k consumers are the search lower bound ($k^* = k$). For each k^* in that range, there is a threshold matching probability $\beta^*(k^*) = \frac{\pi^T(\underline{v}_{k^*}) - \pi^N}{\pi^T(\underline{v}_{k^*})}$ such that $\forall \beta \leq \beta^*(k^*)$, $\pi^B \geq \pi^A$.
2. If $f'(v) = 0$, a similar result holds when when $k^* \in \{k_2, k_3\}$.

The intuition behind this proposition is straightforward: Assumption 1 and 5 guarantee the existence of every equilibrium $k \in [k_2, k_1]$ because the search lower bound's expected benefit from search weakly increases with k. For every equilibrium in that range, the truncated demand generates a higher profit than non-truncated one. Hence, when the disadvantage of being less prominent is sufficiently small (β is small), firm B's profit is higher than firm A. On the other hand, the condition that matched consumers stop searching firm B is also satisfied when β is low.

This proposition states that when search cost is within a proper range, profit of the less prominent firm can dominate the prominent firm's profit when the matching probability is small enough. The main reason is that costly search enables the less prominent firm to cherry-pick high-value consumers, which alleviates the time-inconsistency problem of a durable-goods producer.

A simple example: Let $v \sim U[0,1]$ and $K = 2$. In this case, $f'(v) = 0$, $v^C = 0.5$ and $v_2^N = 0.3$, all assumptions are satisfied. $\forall \beta \leq \beta^*(2) = (\frac{1}{2} - \frac{9}{20})/(\frac{1}{2}) = 0.1$, if $s \in (\beta^*(2) \int_{0.3}^{0.5} (v - 0.3) dv, \beta^*(2) \int_{0.5}^{1} 2(v - 0.5) dv] = (0.002, 0.025]$, $\frac{\pi^B}{\pi^A} = \frac{\frac{1}{2}(1-\beta)}{\frac{9}{20}} \geq 1$.

Although in this example the range of β that gives firm B higher profit is small, it is primarily due to the technical assumption that the durable good is only sold for two periods. When sale seasons are extended, the time-inconsistency problem is more severe and the range of β would be enlarged as well.

4.5 Extensions: Endogenized Search Order and Optimal Search Cost

First, we discuss the model with endogenized search order. Intuitively, there is a symmetric equilibrium where half of consumers in each type start with firm A and the other half search firm B first. In this equilibrium, firms have the same position and it is meaningless to discuss whether a firm benefits from being less prominent. It is also possible that in equilibrium, high types search A first while low types search B first. However, this is also hard to interpret because it can not be concluded which firm is searched early and which one is searched late. Therefore, we will concentrate on asymmetric equilibrium with all consumers start with the same firm in this section.

Now, the research question with endogenized search order is reduced to whether the equilibrium analyzed in previous sections can be sustained when consumers can choose their search sequences. Since firms are ex-ante homogeneous, the only thing that can alter consumers' search order is the equilibrium prices. In previous analysis, the firm being searched late always has a weakly higher second period price ($p_2^B \geq p_2^A$). Then if it also has a higher price in first period, all consumers' expected surplus are higher when they search firm A first. Therefore, it requires that $v_1^B + v_2^B \geq v_1^A + v_2^A$, which is equal to $\underline{v} \geq v_1^N + v_2^N - v^C$. Given a proper β that is small enough, when s is relatively high, the equilibrium with exogenous search order can be sustained even if consumers are strategic on their search sequence. When this condition is violated, high types wants to deviate and we will not have that equilibrium.

Second, if firm B can choose the search cost freely, what is the optimal s given a proper β?[10] This question is related to search obfuscation literature. There is evidence that the firm strategically raises consumers' search cost, including making price framework or product description difficult

[10] Assume firm A is a well-known brand and everybody can easily search it at no cost

to understand, hiding relevant information and deliberately choosing to stay at non-prominent position. The unique global maximum of $\pi^T(\underline{v})$ is attained when $\underline{v} = v^C$. When there exists a k_C with $\underline{v}_{k_C} = v^C$, the optimal search cost s^* is any $s \in (\beta E(U^B_{k_C-1})|_{k^*=k_C-1}, \beta E(U^B_{k_C})|_{k^*=k_C}]$. If that's not true, there must be a $k_C \in [k_2, k_1]$ such that $\bar{v}_{k_C} > v^C > \underline{v}_{k_C}$. Now if $\pi^T(\underline{v}_{k_C}) > \pi^T(\underline{v}_{k_C+1})$, $s^* \in (\beta E(U^B_{k_C-1})|_{k^*=k_C-1}, \beta E(U^B_{k_C})|_{k^*=k_C}]$ and k_C is the search lower bound. If the reverse condition holds, s^* should be in the range that supports $k^* = k_C + 1$.

4.6 Conclusion

This paper explores the ordered search problem when ex-ante heterogeneous consumers search a durable good. Increased marginal search cost screens out low-value consumers and prevents them from searching the second firm. When search cost is within certain range, the second firm faces a truncated demand with a proper search lower bound and maintains a high price, which mitigates the standard time-inconsistency problem of the durable-goods seller. When the matching probability is sufficiently low, the less prominent firm do not lose too much matched consumers and has higher profit than the prominent one.

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Appendix A

Derivations and Proofs

A.1 Proofs of Chapter 2

A.1.1 All Equilibria in Second Period

To sustain a separating equilibrium, the incentive compatibility constraint for the counterfeiter requires that:

$$(\frac{p_{A2} - \frac{C}{2}p_{A2}}{1-C} - \frac{\frac{C}{2}p_{A2}}{C})\frac{C}{2}p_{A2} \geq \frac{1}{2}(\theta_1 - \frac{2p_{A2}}{1+C})p_{A2}$$
$$p_{A2} \geq \frac{2(1-C^2)}{C^2-3C+4}\theta_1 = \underline{p_2}(\theta_1, C)$$

This equation is derived from Eq(2.1) by plugging the best response function of the counterfeiter. Similarly, the incentive compatibility constraint for the authentic firm requires that:

$$(\theta_1 - \frac{p_{A2} - \frac{C}{2}p_{A2}}{1-C})p_{A2} \geq \frac{1}{2}(\theta_1 - \frac{C}{1+C}p_{A2})\frac{C}{2}p_{A2}$$
$$p_{A2} \leq \frac{(4-C)(1-C^2)}{2(2-C)(1+C) - C^2(1-C)}\theta_1 = \overline{p_2}(\theta_1, C)$$

Therefore, when $\overline{p_2}(\theta_1, C) \geq \underline{p_2}(\theta_1, C)$, a separating equilibrium exists. Otherwise, only pooling equilibria can be supported.

$$\frac{(4-C)(1-C^2)}{2(2-C)(1+C) - C^2(1-C)}\theta_1 \geq \frac{2(1-C^2)}{C^2-3C+4}\theta_1$$

This implies that when $C \leq C_1 \approx 0.604$, a separating equilibrium can be supported.

For pooling equilibria, as long as Eq(2.1) is violated, the counterfeiter is willing to pool with the authentic firm. Therefore, $\forall C$, if $p^P_{A2} = p^P_{C2} = p^P_2 \in [0, \underline{p_2}(C, \theta_1))$, a pooling equilibrium can be sustained by certain out of equilibrium beliefs. *Q.E.D.*

A.1.2 Selection of Optimal Equilibrium

In separating equilibria, it can be easily shown that all authentic prices are higher than the unconstrained optimal price. Since the profit function is a concave parabola, $p_{A2}^{S*} = \underline{p_2}(\theta_1, C)$. In any separating equilibrium, the branded firm's profit decreases with the counterfeit quality in the second period because of intensified competition.

In pooling equilibria, when $C < C_2 \approx 0.702$, the unconstrained optimal price is always less than $\underline{p_2}(\theta_1, C)$. Therefore, the optimal price is the unconstrained optimal, $p_2^{P*} = \frac{1+C}{4}\theta_1$. Fixing the market size, the authentic firm's profit increases with C within this range. That is because consumer confusion is alleviated, which enables the firm to raise the price. However, when $C > C_2$, the quality gap is small and the competition is intense. The unconstrained optimal is higher than $\underline{p_2}(\theta_1, C)$. Since the profit function is a concave parabola as well, $p_2^{P*} = \underline{p_2}(\theta_1, C)$.

As Lemma 1 indicates, when $C \leq C_1$, both types of equilibria exist and $\Pi_{A2}^* = Max\ \{\Pi_{A2}^{S*}, \Pi_{A2}^{P*}\}$. Given $C_1 < C_2$, the price of the optimal pooling equilibrium is $p_2^{P*} = \frac{1+C}{4}\theta_1$. Since $\frac{d\Pi_{A2}^{S*}}{dC} < 0$ and $\frac{d\Pi_{A2}^{P*}}{dC} > 0$, there is a cut-off quality $C_3 \approx 0.512$ such that the optimal separating equilibrium is chosen if $C \leq C_3$ and the pooling equilibrium would be selected for $C_3 < C \leq C_1$. When the counterfeit quality is low, the profit in a separating equilibrium is high because of moderate distortion while the profit in a pooling equilibrium is low due to low expected quality. As the quality of the fake good increases, the profit associates with the pooling equilibrium increases. For $C > C_1$, separating equilibria cannot exist and the only candidate is pooling equilibria. When $C_1 < C \leq C_2$, the equilibrium price is the unconstrained optimal price. If $C > C_2$, the price is the binding price $\underline{p_2}(C, \theta_1)$. *Q.E.D*

A.1.3 Profit Comparison

(1) When $C > C_3$, the pooling equilibrium is sustained. There are two second-period prices given different C, both of which can be written as $p_2^{P*} = F(C)\theta_1^{P*}$. Firstly, I will prove $\forall C \in (C_3, 1)$, $\frac{\partial \Pi_A^{P*}}{\partial F(C)} > 0$.

$$\frac{\partial \Pi_A^{P*}}{\partial F(C)} = \frac{(\frac{3-C}{2} + F(C))[(3-C)(\frac{1}{2} - \frac{F(C)}{1+C}) + \frac{F(C)}{2}]}{4(\frac{3-C}{2} + \frac{F(C)}{2} + \frac{F(C)^2}{1+C})^2}$$

Since $0 < F(C) \leq \frac{1+C}{4}$, $\frac{\partial \Pi_A^{P*}}{\partial F(C)} > 0$.

Given this property, it can be shown that even with the larger second-period price $p_2^{P*} = \frac{1+C}{4}\theta_1, \forall C \in (C_3, 1)$, counterfeit competition in the pooling equilibrium still cannot increase the branded firm's profit.

If $p_2^{P*} = \frac{1+C}{4}$,

$$\frac{d\Pi_A^{P*}}{dC} = \frac{(1 + \frac{3-C}{4})(-2 + \frac{35C-53}{64})}{(1 + \frac{11-5C}{16})^2} < 0, \forall C \in (C_3, 1)$$

Therefore, $\forall C \geq C_3, \Pi_A^{P*}(C) \leq \Pi_A^{P*}(C_3)$. Since $\Pi_A^{P*}(C_3) < \Pi^M$, we have $\Pi_A^{P*}(C) < \Pi^M$, $\forall C \geq C_3$.

(2) When $C \leq C_3$, the separating equilibrium is supported in the second period. If $C = 0$, the model is degenerated to the monopoly benchmark. $\Pi^M = \Pi_A^{S*}$.

Let $\Delta\Pi(C) = \Pi_A^{S*} - \Pi^M$, then $\frac{d\Delta\Pi(C)}{dC}|_{C=0} = 0.045 > 0$. So there must exist some C that is low enough such that the authentic firm's profit would increase under the competition.

On the other hand, the only root $C_4 \in (0, 1]$ of $\Delta\Pi(C) = 0$ is $C_4 \approx 0.188$. Henceforth, $\Delta\Pi(C) \geq 0$ if $C \leq C_4$ and vice versa. *Q.E.D.*

A.1.4 Welfare Comparison

(1) In the pooling equilibrium, similar to proof of proposition 1, I can write $p_2^{P*} = F(C)\theta_1^{P*}$. Firstly, I will show that $\frac{\partial TS^{P*}(C)}{\partial F(C)} < 0$ for any $C > C_2$.

$$TS^{P*}(C) = (1 - (\theta_1^P)^2) - \frac{1+C}{4}[1 - \frac{4}{(1+C)^2}F(C)^2]$$

$$\frac{\partial TS^{P*}(C)}{\partial F(C)} = 2(\theta_1^{P*})^2[\frac{\partial \theta_1^{P*}(C)}{\partial F(C)}(\frac{1+C}{4} - \frac{F(C)^2}{1+C} - 1) - \frac{F(C)}{1+C}\theta_1^{P*}]$$

Plugging in θ_1^{P*} and $\frac{\partial \theta_1^{P*}(C)}{\partial F(C)}$, it is easy to verify that $\frac{\partial TS^{P*}(C)}{\partial F(C)} < 0$.

Given this property, it is proved that with the larger second-period price $p_2^{P*} = \frac{1+C}{4}\theta_1, \forall C \in (C_3, 1)$, counterfeit competition in pooling equilibrium still increases the total welfare. When $p_2^{P*} =$

$\frac{1+C}{4}\theta_1$, $\theta_2^{P*} = \frac{1}{2}\theta_1^{P*}$.

$$
\begin{aligned}
TS^{P*}(C) &= \int_{\theta_1^{P*}}^{1} 2\theta d\theta + \int_{\theta_2^{P*}}^{\theta_1^{P*}} \frac{1+C}{2}\theta d\theta \\
&= 1 - (\theta_1^{P*})^2(1 - \frac{3(1+C)}{16}) \\
\Delta TS^{P*}(C) &= TS^{P*}(C) - TS^{M}(C) = \frac{5}{8}(\theta_1^{M})^2 - (\theta_1^{P*})^2(1 - \frac{3(1+C)}{16}) \\
\frac{d\Delta TS^{P*}(C)}{dC} &= \frac{8(1+C)}{(27-5C)^3} > 0
\end{aligned}
$$

Therefore, $\Delta TS^{P*}(C) \geq \Delta TS^{P*}(C_3)$, $\forall C > C_3$. Since, $\Delta TS^{P*}(C_3) > 0$, deceptive counterfeits always yield a higher welfare under the pooling equilibrium.

(2) In the separating equilibrium,

$$
\overline{\theta}_2^{S*} = \frac{2-C}{2(1-C)}\underline{p_2}(C, \theta_1), \underline{\theta}_2^{S*} = \frac{1}{2}\underline{p_2}(C, \theta_1)
$$

$$
\begin{aligned}
TS^{S*}(C) &= \int_{\theta_1^{S*}}^{1} 2\theta d\theta + \int_{\frac{(2-C)(1+C)}{C^2-3C+4}\theta_1^{S*}}^{\theta_1^{S*}} \theta d\theta + \int_{\frac{1-C^2}{C^2-3C+4}\theta_1^{S*}}^{\frac{(2-C)(1+C)}{C^2-3C+4}\theta_1^{S*}} C\theta d\theta \\
&= 1 - \frac{1}{2}(\theta_1^{S*})^2[1 + \frac{(1+C)^2(4-3C)(1-C)}{(C^2-3C+4)^2}]
\end{aligned}
$$

Since $\frac{d\Delta TS^{S*}(C)}{dC}|_{C=0} < 0$, if C is sufficiently low, $TS^{S*}(C) < TS^{M}(C)$. Moreover, there is only one root $C_5 \in (0, 1]$ such that $\Delta TS^{S*}(C) = 0$. Therefore, $\forall C \leq C_5$, $TS^{S*}(C) \leq TS^{M}(C)$ and vice verse. Henceforth, the counterfeit entry increases total welfare if its quality $C \geq C_5 \approx 0.078$. *Q.E.D.*

A.1.5 Equilibria Refinement and General Results

(i) The elimination of all pooling equilibria.

Firstly, $\forall p \in [0, \underline{p_2}(C, \theta_1))$, there exists a $p < p' < p + (1-C)\theta_1$, such that (3) is binding. Choosing a δ that is arbitrarily close to 0. Then $\Pi_{A2}(p+\delta, p, 1) > \Pi_{A2}(p, p, \frac{1}{2})$ and $\Pi_{A2}(p + (1 - C)\theta_1, p, 1) = 0 < \Pi_{A2}(p, p, \frac{1}{2})$. Therefore, by the continuity of profit function, there must exist a $p < p' < p + (1-C)\theta_1$ that makes $\Pi_{A2}(p', p, 1) = \Pi_{A2}(p, p, \frac{1}{2})$.

Plug p' and Eq(2.3) into Eq(2.4),

$$
\begin{aligned}
&\Pi_{C2}(p,p',1)-\Pi_{C2}(p,p,\frac{1}{2})\\
=&(\theta_1-\frac{p'-p}{1-C})p'-\frac{1}{2}(\theta_1-\frac{2p}{1+C})p\\
=&\epsilon(\theta_1-\frac{p'-p}{1-C})(\frac{p-p'}{p-\epsilon})<0
\end{aligned}
$$

Hence, for every pooling equilibrium, there is a price p' that the authentic firm wants to deviate and the counterfeiter does not given consumer's best belief.

Now let's make some preliminary definition for separating equilibria

$$\underline{p_2}(C,\theta_1)=\frac{2(1-C^2)}{C^2-3C+4}\theta_1,\overline{p_2}(C,\theta_1)=\frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}\theta_1$$

For convenience, let

$$\underline{K}(C)=\frac{2(1-C^2)}{C^2-3C+4},\overline{K}(C)=\frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}$$

In any separating equilibrium, the authentic firm's price is between $\underline{K}(C)\theta_1$ and $\overline{K}(C)\theta_1$.

(ii) For incumbent's profit:

$$
\begin{aligned}
\Pi_A^S&=\frac{1}{4}\frac{[1+K(C)]^2}{[1+\frac{2-C}{2(1-C)}K(C)^2]}\\
\frac{\partial\Pi_A^S}{\partial K(C)}&=\frac{[1+K(C)][1-\frac{2-C}{2(1-C)}K(C)]}{2(1+\frac{2-C}{2(1-C)}K(C)^2)^2}
\end{aligned}
$$

Since $1-\frac{2-C}{2(1-C)}K(C)\geq 1-\frac{2-C}{2(1-C)}\overline{K}(C)>0$, $\frac{\partial\Pi_A^S}{\partial K(C)}>0$. The profit-maximizing equilibrium is the one that yields lowest total profit for the incumbent. In that equilibrium, when $C\leq C_4$, the profit with counterfeit entry is higher. Therefor no matter which separating equilibrium is sustained in the second period, $\Delta\Pi_A^S(C)\geq 0$ if $C\leq C_4$.

(iii) For total welfare:

$$\Delta TS^S(C,K(C))=0.225-\frac{1}{8}\frac{[1+K(C)]^2[1+\frac{4-3C}{4-4C}K(C)^2]}{[1+\frac{2-C}{2-2C}K(C)^2]^2}$$

When $C=0$,

$$\Delta TS^S(0,K(0))=0.225-\frac{1}{8}\frac{[1+K(0)]^2}{1+K(0)^2}$$

Since $\frac{[1+K(0)]^2}{1+K(0)^2}$ increases with $K(0)$,

$$\Delta TS^S(0, K(0)) \leq 0.225 - \frac{1}{8}\frac{[1+\underline{K}(0)]^2}{1+\underline{K}(0)^2} = 0$$

When $C = C_1$, $\forall K(C_1)$,

$$\begin{aligned}\Delta TS^S(C_1, K(C_1)) &= 0.225 - \frac{1}{8}\frac{[1+K(C_1)]^2[1+\frac{4-3C_1}{4-4C_1}K(C_1)^2]}{[1+\frac{2-C_1}{2-2C_1}K(C_1)^2]^2} \\ &> 0.225 - \frac{1}{8}\frac{[1+K(C_1)]^2}{[1+\frac{2-C_1}{2-2C_1}K(C_1)^2]} \\ &\geq 0.225 - \frac{1}{8}\frac{[1+\overline{K}(C_1)]^2}{[1+\frac{2-C_1}{2-2C_1}\overline{K}(C_1)^2]} > 0\end{aligned}$$

Therefore, when $C = 0$, the welfare differences under all separating equilibria are negative. However, when $C = C_1$, the welfare differences under all separating equilibria are strictly positive. By continuity of the welfare difference function, there must exist a threshold C_6 such that as long as $C \geq C_6$, the welfare is higher in presence of deceptive counterfeits for any separating equilibrium. Numerically, I find that $C_6 \approx 0.248$. *Q.E.D.*

A.2 Proof of Chapter 3

A.2.1 Comparison of R&D Incentive

$$\begin{aligned}\tilde{\pi}_2^S - \tilde{\pi}_2^F =& (1-\tilde{\theta}_2^{HS})[(q^H - 1 + \frac{\lambda^2}{4I})\tilde{\theta}_2^{HS} + \frac{\lambda}{2}(1-\frac{c^I}{I}) - c^H] - K \\ \overline{\pi}_2^S - \overline{\pi}_2^F =& \frac{1}{4}(\overline{\theta}_1^{LS} + \frac{\lambda}{2})^2 - \frac{1}{4}(\overline{\theta}_1^{LF} + \frac{\lambda}{2})^2 \\ &+ (1-\overline{\theta}_2^{HS})[(q^H-1)\overline{\theta}_2^{HS} + \frac{\lambda}{2}(1-\overline{\theta}_1^{LS}) - c^H] - K\end{aligned}$$

When $\lambda = 0$, it can be easily verified that $\tilde{\pi}_2^S - \tilde{\pi}_2^F = \overline{\pi}_2^S - \overline{\pi}_2^F$. Therefore, the incumbent has the same innovation incentive.

When $\overline{\theta}_1^{LF} \geq \overline{\theta}_1^{LS}$, $\frac{1}{4}(\overline{\theta}_1^{LS} + \frac{\lambda}{2})^2 - \frac{1}{4}(\overline{\theta}_1^{LF} + \frac{\lambda}{2})^2 \leq 0$. On the other hand, when $\overline{\theta}_1^{LS} > \frac{c^I}{I}$, $(\tilde{\pi}_2^S - \tilde{\pi}_2^F)|_{\theta_2^H = \overline{\theta}_2^{HS}} > (1-\overline{\theta}_2^{HS})[(q^H-1)\overline{\theta}_2^{HS} + \frac{\lambda}{2}(1-\overline{\theta}_1^{LS}) - c^H]$. Since $\tilde{\theta}_2^{HS}$ is the one that maximizes $\tilde{\pi}_2^S - \tilde{\pi}_2^F$, we have $\tilde{\pi}_2^S - \tilde{\pi}_2^F > (\tilde{\pi}_2^S - \tilde{\pi}_2^F)|_{\theta_2^H = \overline{\theta}_2^{HS}}$. Combining both inequalities, it is obvious that $\tilde{\pi}_2^S - \tilde{\pi}_2^F > \overline{\pi}_2^S - \overline{\pi}_2^F$. Q.E.D

A.3 Proofs of Chapter 4

A.3.1 Profit with Truncated Demand

$$v_2^*(v_1) = \frac{F(v_1) - F(v_2^*(v_1))}{f(v_2^*(v_1))}$$

$$\frac{dv_2^*(v_1)}{dF(v_1)} = \frac{\frac{1}{f(v_2^*(v_1))}}{1 + \frac{f^2(v_2^*(v_1)) + f'(v_2^*(v_1))[F(v_1) - F(v_2^*(v_1))]}{f^2(v_2^*(v_1))}}$$

By Assumption 2, $\frac{f^2(v_2^*(v_1)) + f'(v_2^*(v_1))[F(v_1) - F(v_2^*(v_1))]}{f^2(v_2^*(v_1))} \geq 0$. Given that $f(v)$ is positive everywhere, $\frac{dv_2^*(v_1)}{dF(v_1)} > 0$. Therefore, $\frac{dv_2^*(v_1)}{dv_1} = \frac{dv_2^*(v_1)}{dF(v_1)} \cdot \frac{dF(v_1)}{dv_1} > 0$.

Now, let's compare the value of v_1^N, v^C, v_2^N. Since $v_2^N = v_2^*(v_1^N)$, $v^C = v_2^*(\bar{v})$ and $v_1^N < \bar{v}$, $\frac{dv_2^*(v_1)}{dv_1} > 0$, it is clear that $v^C > v_2^N$. On the other hand, v^C and v_1^N are solutions for $v = \frac{1-F(v)}{f(v)}$ and $v_1 = \frac{1-F(v_1)}{f(v_1)}(1 + \frac{dv_2^*(v_1)}{dv_1})$ respectively. By Assumption 2 and $\frac{dv_2^*(v_1)}{dv_1} > 0$, it must be $v_1^N > v^C$.

Finally, $\Pi^C = 2v^C[1 - F(v^C)]$, $\Pi^N = v_1^N[1 - F(v_1^N)] + v_2^N[1 - F(v_2^N)]$. Since v^C is the optimal solution for $v^C[1 - F(v^C)]$ and $v_1^N > v^C > v_2^N$, we have $\Pi^N < \Pi^C$. Q.E.D.

A.3.2 Profit Difference between Two Firms

(1) $f'(v) > 0$: first, I prove the existence of equilibria. To support k_2 as the search lower bound, $s \in (\beta E(U_{k_2-1}^B)|_{k^*=k_2-1}, \beta E(U_{k_2}^B)|_{k^*=k_2}]$. We just need to prove $E(U_{k_2-1}^B)|_{k^*=k_2-1} < E(U_{k_2}^B)|_{k^*=k_2}$ to show that such s exists. Here, I focus on the case that K is big so that type k_2 consumers only consider buy in $t = 2$ or nothing. When K is small and consumers have other options, the proof is quite similar. From Equation(4.1) and definition of k_2

$$E(U_{k_2-1}^B)|_{k^*=k_2-1} = \int_{v_2^N}^{\bar{v}_{k_2-1}} (v - v_2^N) dF(v)$$

$$E(U_{k_2}^B)|_{k^*=k_2} = \int_{\underline{v}_{k_2}}^{\bar{v}_{k_2}} (v - \underline{v}_{k_2}) dF(v)$$

Since $v_2^N > \underline{v}_{k_2-1}$, $E(U_{k_2-1}^B)|_{k^*=k_2-1} < \int_{\underline{v}_{k_2-1}}^{\bar{v}_{k_2-1}} (v - \underline{v}_{k_2-1}) dF(v)$. By Assumption 1. $\int_{\underline{v}_{k_2-1}}^{\bar{v}_{k_2-1}} (v - \underline{v}_{k_2-1}) dF(v) = \int_0^{\frac{\bar{v}}{K}} t dF(t + \frac{k_2-2}{K}\bar{v})$, $E(U_{k_2}^B)|_{k^*=k_2} = \int_0^{\frac{\bar{v}}{K}} t dF(t + \frac{k_2-1}{K}\bar{v})$. Assumption 5 ensures that $\int_0^{\frac{\bar{v}}{K}} t dF(t + \frac{k_2-2}{K}\bar{v}) < \int_0^{\frac{\bar{v}}{K}} t dF(t + \frac{k_2-1}{K}\bar{v})$. Therefore, $\beta E(U_{k_2-1}^B)|_{k^*=k_2-1} < \beta E(U_{k_2}^B)|_{k^*=k_2}$.

For $k \in [k_2+1, k_1]$, if type k and type $k-1$ are going to buy in the same period ex-post, the similar method can be applied. For example, both types buy at $t=2$, then $E(U_k^B)|_{k^*=k} = \int_{\underline{v}_k}^{\bar{v}_k}(v-\underline{v}_k)dF(v) = \int_0^{\frac{\bar{v}}{K}} tdF(t+\frac{k-1}{K}\bar{v})$ and $E(U_{k-1}^B)|_{k^*=k-1} = \int_{\underline{v}_{k-1}}^{\bar{v}_{k-1}}(v-\underline{v}_{k-1})dF(v) = \int_0^{\frac{\bar{v}}{K}} tdF(t+\frac{k-2}{K}\bar{v})$. By Assumption 5, $E(U_k^B)|_{k^*=k} > E(U_{k-1}^B)|_{k^*=k-1}$. If both types buy at $t=1$ ex-post, from Eq (1), the ex-post utility being integrated just doubles the previous case and the proof is the same. If ex-post type k consumers have a chance to buy earlier than type $k-1$, then they get higher utility than if they postpone the purchase and buy at the same time with type $k-1$. However, when two types buy together, the higher type always have more surplus. In all, equilibria can be sustained $\forall k \in [k_2, k_1]$.

Second, it can be verified that $\pi^B \geq \pi^A$ is possible in all these equilibria. All consumers search firm A and β of them find a match, which means $\pi^A = \beta\pi^N$. Since matched consumers stop searching, the maximum volume for firm B is $1-\beta$. If k^* is the search lower bound, firm B faces a distribution that only contains unmatched consumers with $k \geq k^*$, which implies $\pi^B = \beta(1-\beta)\pi^T(k^*)$. When $k^* \in [k_2, k_1]$, by the definition of k_2 and k_1, $\underline{v} \in [v_2^N, \underline{v}_1]$ and $\pi^T(\underline{v}_{k^*}) > \pi^N$. For any $k^* \in [k_2, k_1]$, if $\beta \leq \frac{\pi^T(\underline{v}_{k^*})-\pi^N}{\pi^T(\underline{v}_{k^*})}$, $\pi^B \geq \pi^A$.

(2)$f'(v) = 0$: when $f'(v) = 0$, similar as the previous proof, $\beta E(U_{k_2-1}^B)|_{k^*=k_2-1} = \beta E(U_{k_2}^B)|_{k^*=k_2}$. By previous discussions, when $s=0$, all consumers search. When $s \in (0, \beta E(U_{k_2}^B)|_{k^*=k_2}]$, the search lower bound is k_2 and firm B may have a higher profit. When $s \in (\beta E(U_{k_2}^B)|_{k^*=k_2}, \beta E(U_{k_3}^B)|_{k^*=k_3}]$, the search lower bound is k_3 (it can be easily proved that $E(U_{k_3}^B)|_{k^*=k_3} > E(U_{k_2}^B)|_{k^*=k_2}$ since type k_3 consumers may purchase in $t=1$, which yields higher surplus than buying in $t=2$). Therefore, if $k^* = \{k_2, k_3\}$, corresponding $\beta^*(k^*)$ defined in the $f'(v) > 0$ case guarantees that firm B has higher profit.

Recall that β also needs to be small enough such that matched consumers do not search, which is guaranteed if $\beta \leq \frac{v_1^N - v_2^N}{2v_1^N - v^C - v_2^N}$. It can be proved that $\frac{v_1^N - v_2^N}{2v_1^N - v^C - v_2^N} \geq \frac{1}{2} > \frac{\pi^C - \pi^N}{\pi^C} \geq \frac{\pi^T(\underline{v}_{k^*})-\pi^N}{\pi^T(\underline{v}_{k^*})}$. The first inequality is given by the fact that $v^C > v_2^N$ and the entire fraction decreases with v^C. The second one is because $\pi^N > \frac{1}{2}\pi^C$, since the firm can always sell nothing in $t=1$ and obtain

$\frac{1}{2}\pi^C$ by setting the monopoly price in $t = 2$. The third one is proved by $\pi^T(\underline{v}_{k^*}) \leq \pi^C$. Therefore, when $\beta \leq \beta^*(k^*)$, matched consumers will never search. Q.E.D.

Appendix B

Figures

Figure B.1: Equilibrium Price in t=2

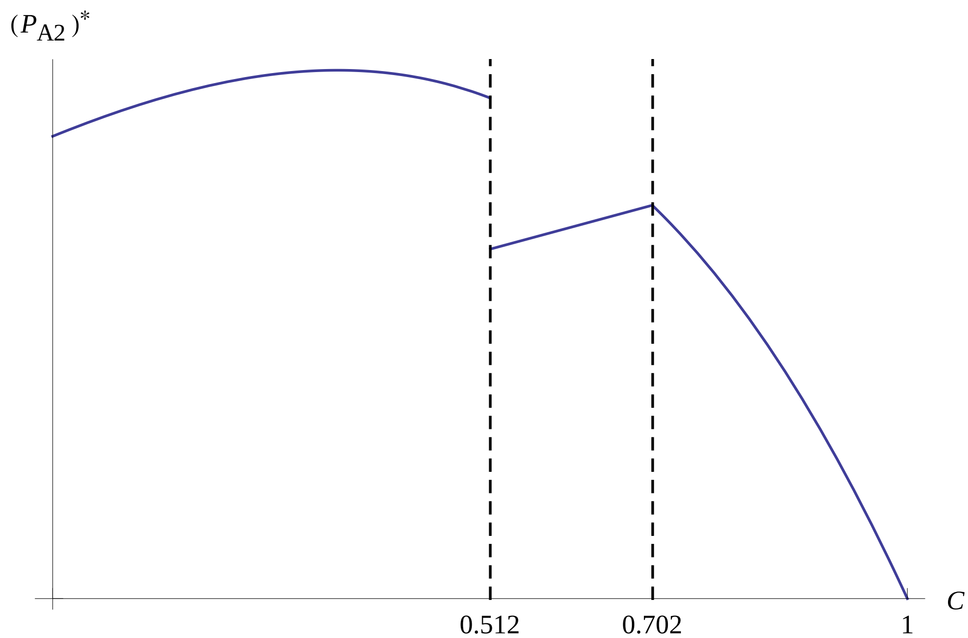

Figure B.2: Marginal Buyer in t=1

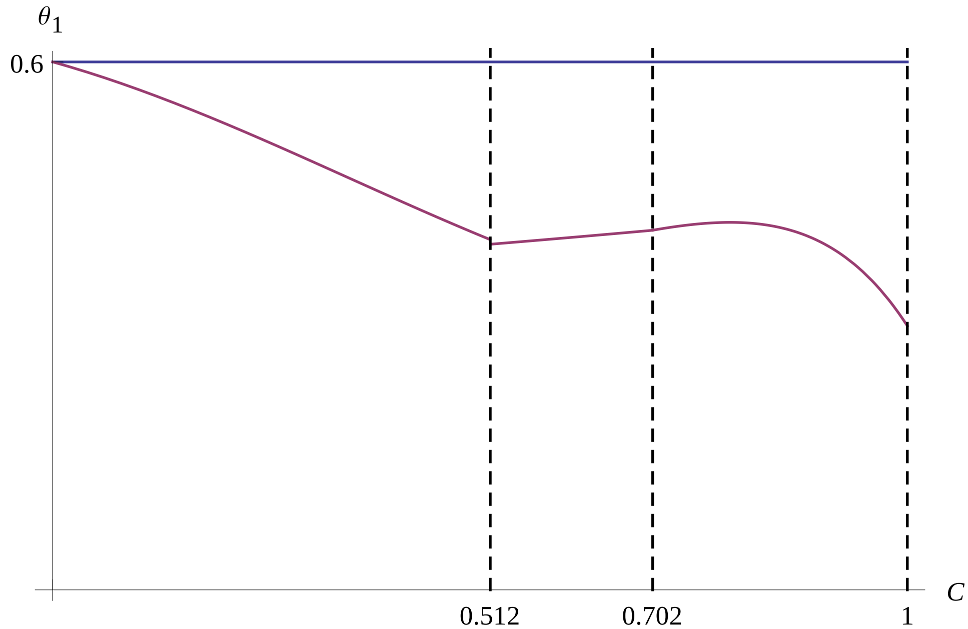

Figure B.3: Profit Difference

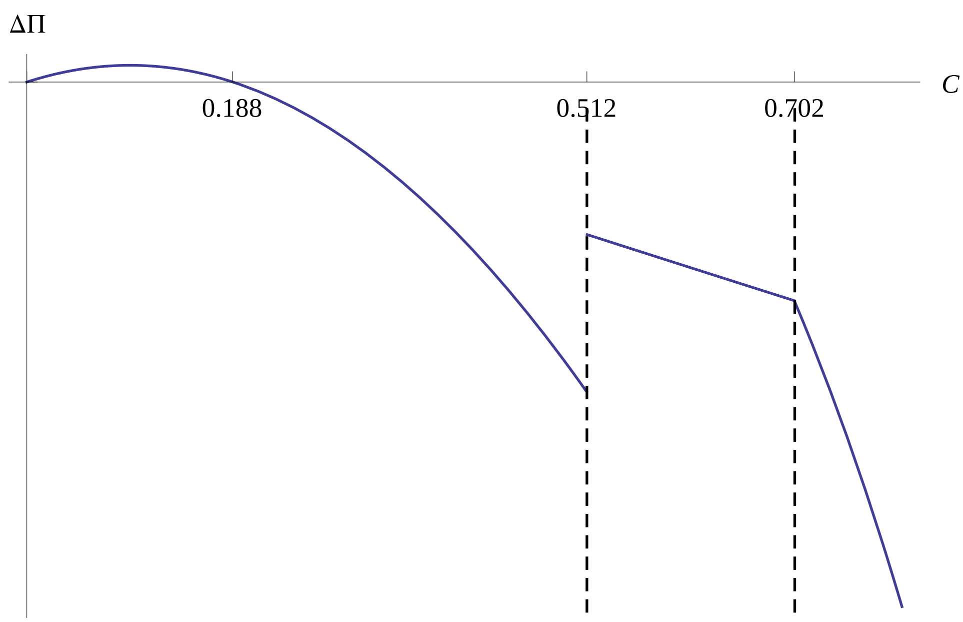

Figure B.4: Welfare Difference

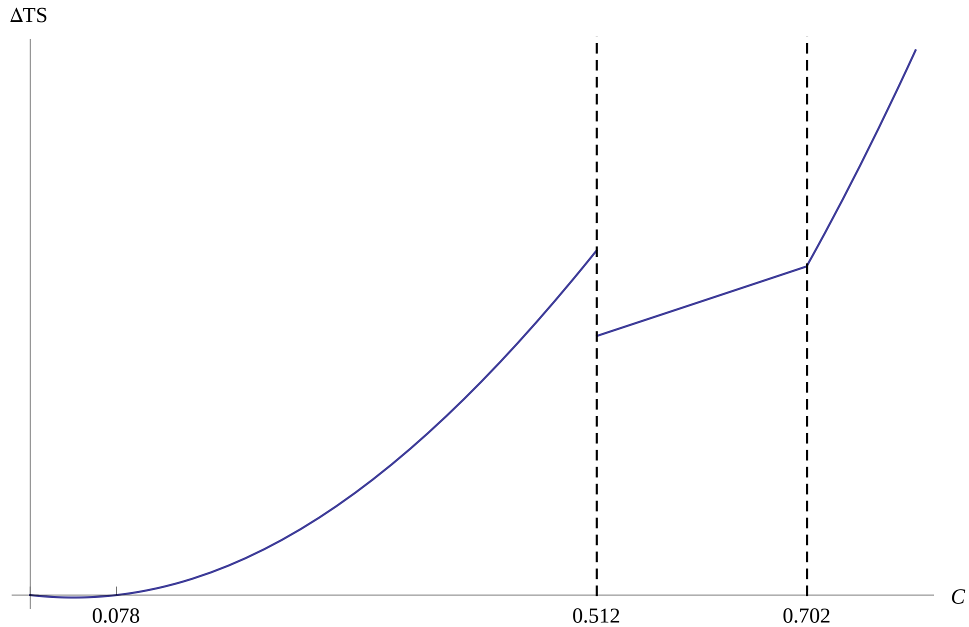

Figure B.5: Comparison of R&D Incentive