Observation of Tkachenko Oscillations in Rapidly Rotating Bose-Einstein Condensates

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(February 2, 2008)

We directly image Tkachenko waves in a vortex lattice in a dilute-gas Bose-Einstein condensate. The low (sub-Hz) resonant frequencies are a consequence of the small but nonvanishing elastic shear modulus of the vortex-filled superfluid. The frequencies are measured for rotation rates as high as 98% of the centrifugal limit for the harmonically confined gas. Agreement with a hydrodynamic theory worsens with increasing rotation rate, perhaps due to the increasing fraction of the volume displaced by the vortex cores. We also observe two low-lying m=0 longitudinal modes at about 20 times higher frequency.

03.75.Lm,67.90.+z,67.40.Vs,32.80.Pj

We have all seen a cylindrically confined fluid support azimuthal flow whether we are watching water flow down a drain or a recently stirred cup of coffee. What is somewhat harder to imagine is a fluid sustaining oscillatory azimuthal flow. Instinctively one does not expect a fluid to support shear forces, and this would seem especially true in the case of zero-viscosity superfluids, but such intuition is incomplete.

The key issue is vortices. In 1955, Feynman [1] predicted that a superfluid can rotate when pierced by an array of quantized singularities or vortices. In 1957, Abrikosov [2] demonstrated that such vortices in a type II superconductor will organize into a triangular crystalline lattice due to their mutual repulsion. Not surprisingly, the Abrikosov lattice has an associated rigidity. In 1966, Tkachenko proposed that a vortex lattice in a superfluid would support transverse elastic modes [3]. First observed by Andereck et al. [4], Tkachenko oscillations have been the object of considerable experimental and theoretical effort in superfluid helium, much of which was summarized by Sonin in 1987 [5].

In the last two years it has become possible to achieve a vortex lattice state in dilute gas BEC [6–9] and recent theoretical work [10] has suggested that Tkachenko oscillations are also attainable. In this Letter we report the observation of Tkachenko oscillations in BEC. The particular strengths of BEC are that in the clean environment of a magnetically trapped gas there is no vortex pinning, and spatiotemporal evolution of the oscillation may be directly observed. Since the original submission of this paper Gordon Baym and Baksmaty et al. have independantly published theoretical works [11,12] that precisely describe our data.

We begin the experiment with a rotating condensate held in an axially symmetric trap with trap frequencies $\{\omega_\rho, \omega_z\} = 2\pi\{8.3, 5.2\}$ Hz. The condensed cloud contains 1.5-2.9 million ^{87}Rb atoms in the $|F = 1, m_F = -1\rangle$ state. The cloud rotates about the vertical, z axis. Rotation rates for the experiments described in this paper range from Ω=0.84 to Ω=0.975 (Ω defined as condensate rotation rate over ω_ρ). We have no observable normal cloud implying a T/Tc < 0.6. The means by which we prepare this condensate is identical to our previous work [9,13]. As before, rotation can be accurately measured by comparing the condensate aspect ratio to the trap aspect ratio. Vortices, which are too small to observe in trap, can be seen by turning off the trap and allowing the cloud to expand to five times its original size, or typically 380 μm FWHM, and imaging along the direction of rotation [14]. At our high rotation rates the condensate is oblate and the vortex cores are essentially vertical lines except right at the surface.

We excite lattice oscillations by two mechanisms. The first mechanism presented is based on the selective removal of atoms that has also been discussed in previous work [14]. With this method we remove atoms at the center of the condensate with a resonant, focused laser beam sent through the condensate along the axis of rotation. The width of the "blasting" laser beam is 16 μm FWHM (small compared to an in-trap condensate FWHM of 75 μm), with a Gaussian intensity profile. The frequency of the laser is tuned to the $F'' = 1 \rightarrow F' = 0$ transition of the D2 line, and the recoil from a spontaneously scattered photon blasts atoms out of the condensate. The laser power is about 10 fW and is left on for approximately one lattice rotation period (125 ms).

The effect of this blasting laser is to remove a small (barely observable) fraction of atoms from the center of the condensate. This has two consequences. First, the average angular momentum per particle is increased by the selective removal of low angular momentum atoms from the condensate center. This increase then requires a corresponding increase in the equilibrium condensate radius [14]. Secondly, the atom removal creates a density dip in the center of the cloud. Thus, after the blasting pulse, the condensate has fluid flowing inward to fill the density dip and fluid flowing outward to expand the radius. The Coriolis force acting on these flows causes the inward motion to be diverted in the lattice rotation direction and the outward flow to be diverted in the opposite direction. This sheared fluid flow drags the vortices from their equilibrium configuration and sets the initial conditions for the lattice oscillation as can be seen from the expanded images in Fig. 1.

The second method of exciting the Tkachenko oscillation is essentially the inverse of the previous method. Instead of removing atoms from the cloud we use a red-detuned optical dipole potential to draw atoms into the middle of the condensate. To do this we focus a 850 nm laser beam onto the condensate. The beam has 3 μW of power and a 40 μm FWHM. It propagates along the direction of condensate rotation and its effect is to create a 0.4 nK deep Gaussian dip in the radial trapping potential. This beam is left on for 125 ms to create an inward fluid flow similar to before. The resulting Tkachenko oscillation was studied for $\Omega = 0.95$, and found to be completely consistent with the atom removal method. It is not surprising that these two methods are equivalent since one works by creating a dip in the interaction potential and the other creates a similar dip in the trapping potential.

For these experiments, data is extracted by destructively imaging the vortex lattice in expansion and fitting the lattice oscillation. To perform this fit we find a curvilinear row of vortices going through the center of the cloud and fit a sinewave to the locations of the vortex centers, recording the sine amplitude. This is done for all three directions of lattice symmetry [see Fig. 1], with the amplitudes averaged to yield the net fit amplitude of the distortion.

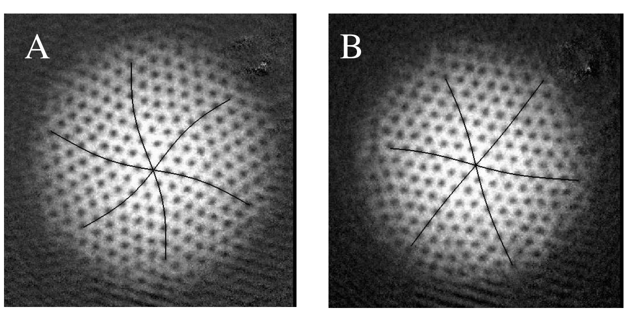

FIG. 1. (1,0) Tkachenko mode excited by atom removal (a) taken 500 ms after the end of the blasting pulse (b) taken 1650 ms after the end of the blasting pulse. BEC rotation is counterclockwise. Lines are sine fits to the vortex lattice.

The resulting oscillation [see Fig. 2] is heavily damped and has a Q value of 3-5 for the data presented. Here Q is given by $Q=2\pi f \tau_{damping}$, where $\tau_{damping}$ is the exponential-damping time constant for the oscillation. We are able to increase this to a Q of 10 by exciting lower amplitude oscillations (40% of the previous amplitude) and by better mode matching of the blasting beam to the shape and period of the oscillation (40 μm FWHM beam width and 500 ms blasting time). Measured frequencies for the high-amplitude oscillations are the same as for the low-amplitude, high-Q case so we do not believe that we are seeing anharmonic shifts [15].

Because of the characteristic s-bend shape and the low resonant frequency of these oscillations [see Fig. 3(a)] we interpret them to be the (n=1,m=0) Tkachenko oscillations predicted by Anglin [10]. Here (n,m) refer to the radial and angular nodes, respectively, in the presumed quasi-2-D geometry. The calculations of Ref. [10] predict that these lattice oscillations should have a frequency of $\nu_{10} = 1.43\epsilon\Omega(\frac{\omega_\rho}{2\pi})$ for the (1,0) mode and $\nu_{20} = 2.32\epsilon\Omega(\frac{\omega_\rho}{2\pi})$ for the (2,0) mode. Here $\epsilon = b/R_\rho$ denotes the nearest-neighbor vortex spacing, b, over the radial Thomas-Fermi radius, R_ρ. For our system these predicted frequencies are around 1-2 Hz and are therefore far slower than any of the density-changing coherent oscillations of the condensate except for the m=-2 surface wave [9,16–18]. In addition the shape of the observed oscillation agrees well with theory. Specifically, the prediction [10] that the spatial period of a sinewave fit to a row of vortices in a (1,0) oscillation should be 1.33 R_ρ is in perfect agreement with our data.

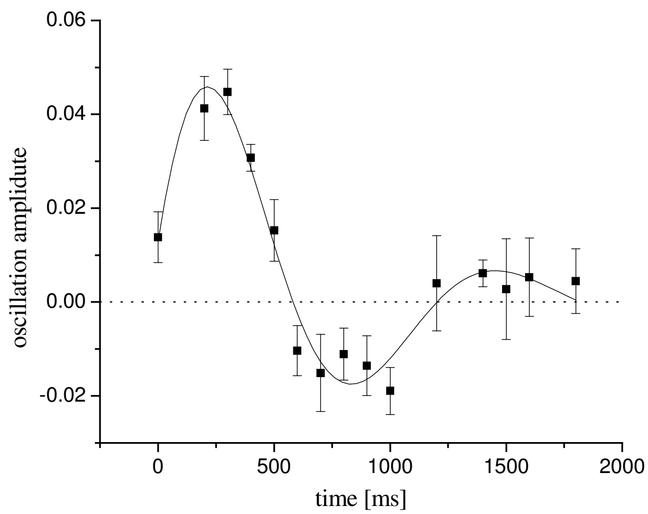

FIG. 2. Measured oscillation amplitude for a typical excitation Ω=0.92 and 2.2×10^6 atoms. Fit is to a sinewave times an exponential decay and yields a frequency of 0.85 Hz and a Q of 3. The oscillation amplitude is expressed as the average amplitude of the sinewave fits to the vortex oscillation in units of the radial Thomas-Fermi radius (roughly the azimuthal displacement of a vortex a distance 0.33 R_ρ from the condensate center). Both values are in expansion.

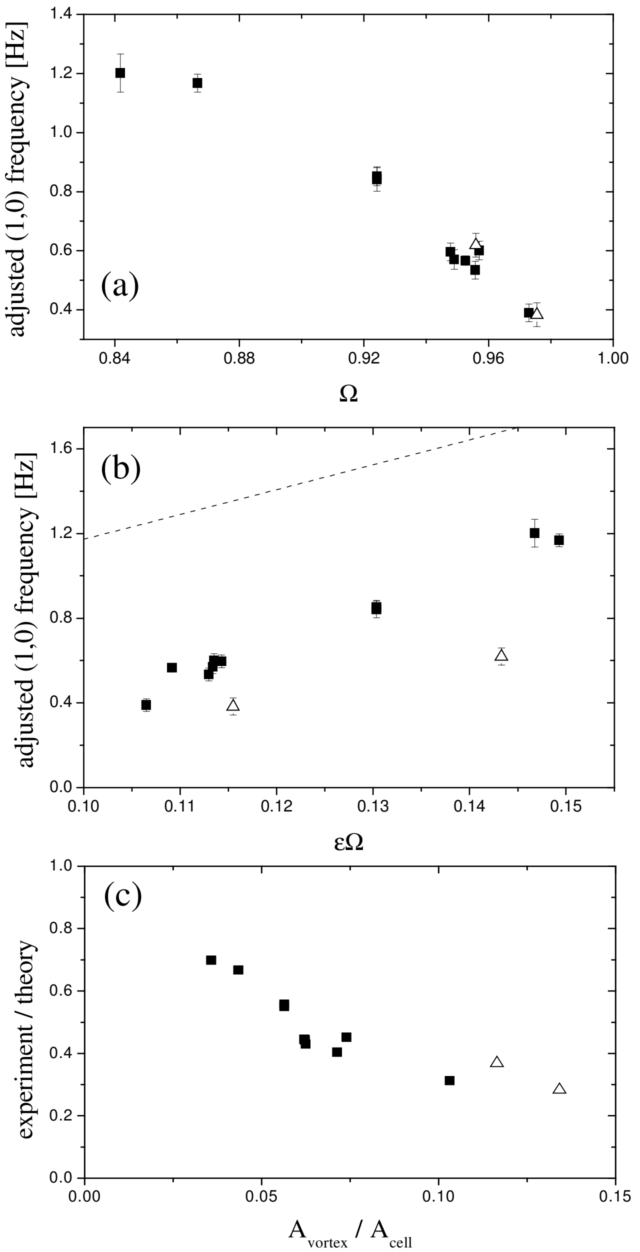

FIG. 3. Plot (a) shows the adjusted[15] (n=1,m=0) Tkachenko oscillation frequencies as a function of scaled rotation rate Ω. Plot (b) shows the (1,0) frequency as a function of the theory parameter $\epsilon\Omega$. The dotted line is the theory line $\nu_{10} = 1.43\epsilon\Omega(\frac{\omega_\rho}{2\pi})$ from Ref.[10]. Note that the low number data shows much worse agreement with theory. Plot (c) demonstrates the divergence from theory as the ratio of vortex core area to unit cell area increases. A_{vortex} is $\pi\xi^2$ where the healing length $\xi = (8\pi na)^{-1/2}$ (here n is density-weighted average density and a is the s-wave scattering length). Lattice cell area A_{cell} is $\sqrt{3}b^2/2$ (here b is the nearest-neighbor vortex spacing). For all plots black squares and triangles refer to high and low atom number experiments, respectively.

The predicted frequencies are however problematic. To make the comparison to the theory presented in Ref. [10] we excite lattice oscillations in the condensate for $\epsilon\Omega$ ranging from 0.10 to 0.15. This is achieved by varying number and rotation rate. Over this range of $\epsilon\Omega$ the oscillation frequencies measured are consistently lower than those predicted by theory as can be seen in Fig. 3(b). For the slowest rotations, Ω=0.84 ($\epsilon\Omega = 0.15$,N=2.5×10^6), we observe frequencies that are as close as 0.70 of the predicted value. However, at larger rotation rates, Ω=0.975 ($\epsilon\Omega = 0.10$,N=1.7×10^6), the agreement is considerably worse (the measured value is 0.31 of the of the predicted value). One possible explanation for this general discrepancy is that the calculations are done in 2-D and ignore the issues of vortex bending at the boundary and finite condensate thickness [19]. In those cases, however, one would expect better agreement at high rotation rates where the condensate aspect ratio is more 2-D. A more likely explanation is that the continuum theory, used in the Anglin calculation, is breaking down as the vortex core size to vortex spacing becomes finite [19]. This suggests that at high rotation and lower atom number we are entering a new regime. To further explore this possibility we reduced the atom number to N=$7 - 9 \times 10^5$, while keeping $\epsilon\Omega$ roughly the same. This should increase the core size and exacerbate the problem. As can be seen in Fig. 3(b) and Fig. 3(c) the agreement with theory is significantly worse under these conditions.

We are also able to excite the (2,0) mode. We note that atom removal creates an s-bend in the lattice that is centered on the atom removal spot. To write two s-bends onto the lattice one could imagine removing atoms from an annular ring instead of a spot. To make this ring we offset the blasting beam half a condensate radius and leave it on for 375 ms (three full condensate rotation periods). As one can see this does lead to an excitation of the (2,0) oscillation (see Fig. 4). We measure the frequency of this mode as before. For 2.3 million atoms and $\Omega = 0.95$ we measure a lattice oscillation frequency of 1.1 ± 0.1 Hz, distinctly lower than the theoretical prediction [10] of 2.2 Hz for our parameters. It is interesting that the predicted *ratio* of frequencies, ν_{20}/ν_{10}, is 1.63, in agreement with the experimental value, 1.8 ± 0.2, measured at $\epsilon\Omega = 0.12$.

Vortex motion and condensate fluid motion are intimately linked [5]. In Tkachenko oscillations, the moving of vortices must also entail some motion of the underlying fluid, and pressure-velocity waves in the fluid must conversely entrain the vortices. Very generally, for a substance composed of two interpenetrating materials, one of which has an elastic shear modulus and one of which does not (in our case, the vortex lattice and its surrounding superfluid, respectively), one expects to find three distinct families of sound waves in the bulk: (i) a shear, or transverse, wave, (ii) a common-mode pressure or longitudinal wave, and (iii) a differential longitudinal wave, with the lattice and its fluid moving against one another [20]. The presence of strong Coriolis forces makes the distinction between longitudinal and transverse waves problematic, but the general characteristics of the three families should extend into the rotating case. For instance, one can still readily identify the Tkachenko modes discussed thus far as the transverse wave. Our assumption is that the common-mode longitudinal waves are nothing other than the conventional hydrodynamic shape oscillations studied previously [17,18].

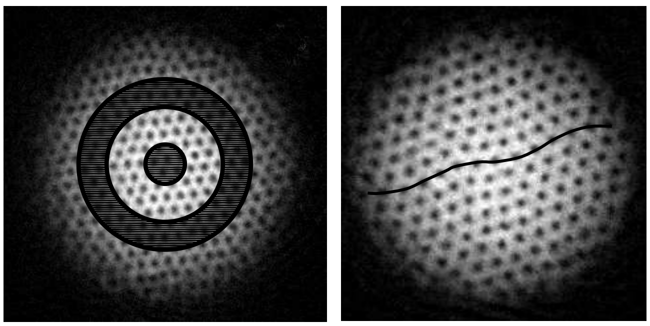

FIG. 4. On the left are the locations where atoms are removed from the cloud. For the (1,0) excitations the atoms are removed from the shaded region in the center. For the (2,0) mode atoms are removed from the shaded ring half a condensate radius out. Image on the right is the resulting (2,0) mode, where the black line has been added to guide the eye.

To excite the common-mode longitudinal wave, we use the dipole force from the 850 nm red-detuned laser described earlier. In order to excite a broad spectrum of modes we shorten the laser pulse to 5 ms, widen the excitation beam to a 75 μm FWHM Gaussian profile, and increase the laser power to 1 mW, resulting in a 30 nK deep optical potential. We find that this pulse excites three distinct m=0 modes: the first is the (1,0) Tkachenko s-bend mode at about 0.6 Hz already discussed. The second is a radial breathing mode in which the condensate radius oscillates at 16.6 ± 0.3 Hz (or $2.0 \pm 0.1\frac{\omega_\rho}{2\pi}$). This mode has been previously observed [21], and our observed frequency is consistent with hydrodynamic theory for a cloud rotating at $\Omega = 0.95$ [17]. As the radius of the fluid density oscillates, so does the mean lattice spacing of the vortex lattice, but we observe no s-type bending of the lattice at this frequency. The fact that the frequency of the lowest m=0 radial longitudinal mode is more than 20 times that of the transverse mode demonstrates how relatively weak the transverse shear modulus is.

The same laser pulse excites a third mode, at the quite distinct frequency of 18.5 ± 0.3 Hz. This mode manifests as a rapid s-bend distortion of the lattice indistinguishable in shape from the 0.6 Hz (1,0) Tkachenko oscillation. 18.5 Hz is much too fast to have anything to do with the shear modulus of the lattice, and we were very tempted to identify this mode as a member of the third family of sound-waves, the differential longitudinal waves. Simulations by Cozzini and Stringari [22], however, show that our observed frequency is consistent with a higher-order, hydrodynamic mode of the rotating fluid that can be excited by an anharmonic radial potential such as our Gaussian optical potential. Moreover, they show that the radial velocity field of their mode is distorted by Coriolis forces so as to drag the lattice sites into an azimuthally oscillating s-bend distortion that coincidentally resembles the Tkachenko mode. It is worth noting that without the presence of the lattice to serve as tracers for the fluid velocity field, it would be very difficult to observe this higher-order mode, since this mode has very little effect on the mean radius of the fluid. In any case, the mode at 18.5 Hz appears to be yet another member in the family of common-mode longitudinal waves. So far we have been unable to observe a mode we can assign to the family of differential longitudinal waves.

We would like to acknowledge James Anglin, Marco Cozzini, Sandro Stringari John Toner and Gordon Baym for their useful discussion. We are also appreciative of additional calculations done by Anglin and Cozzini. The work presented in this paper was funded by NSF and NIST.

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