The Effects of Beam Stabilization on Mode Coupling in a Hollow Cylindrical Waveguide

Matthew Christopher Clark
University of Colorado Boulder

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The Effects of Beam Stabilization on Mode Coupling in a Hollow Cylindrical Waveguide

Matthew Christopher Clark
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A senior undergraduate thesis submitted to the University of Colorado at Boulder Physics Department partially fulfilling the requirements to graduate with latin honors.

Advisors:
Dr. Henry Kapteyn
Dr. Margaret Murnane

Committee Members:
Dr. Dan Adams, Dr. Henry Kapteyn
Dr. Nikolaus Correll, Dr. James Thompson
Abstract

High Harmonic Generation is a non-linear process that enables the tabletop production of coherent laser-like beams in the extreme ultraviolet and soft x-ray regions of the electromagnetic spectrum. In order to achieve significant high harmonic flux, the generated high harmonics must be phase matched. We achieve this by coupling the incident driving laser into a hollow cylindrical waveguide. The power coupling efficiency of this waveguide has an important dependence on spatial beam stability. This thesis discusses the power coupling efficiency of the incident beam into the waveguide with respect to beam position and angle at the entrance of the waveguide. It also presents an active beam stabilization technique which stabilizes both the angle and position of the beam. The performance of this beam stabilization is then be correlated to an increase in the stability of power coupling efficiency with the waveguide.
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Chapter 1

Introduction

Consistency is a quality valued by scientists in both research and industry. Physicists require consistent instruments in order to repeat important results and improve the precision of their measurements. This thesis discusses the importance and improvement of beam stability for coupling into a hollow cylindrical waveguide to perform High Harmonic Generation, a non-linear optical phenomenon which has made great contributions to the field of ultrafast x-ray laser science.

1.1 Motivation: Ultrafast X-Ray Science

In 1953, Charles Townes conceived the maser demonstrating the first amplification of electromagnetic radiation. A preface to the visible laser which was developed less than a decade later, this achievement emanated a new branch of science that has made invaluable contributions to industry and research [2]. As a result, scientists have striven to extend the boundaries of laser science by enhancing the capabilities of the devices themselves. The past 50 years have seen many breakthroughs in physics and engineering broaden the spectrum of laser light and advance the pulsed operation of lasers. Recently, these boundaries have been extended to include wavelengths in the extreme ultraviolet (EUV) and soft x-ray region, as well as pulse durations on the femtosecond and attosecond time scales.

Vast areas of research and industrial applications have been made accessible due to receding limitations on the pulse length and available wavelengths of lasers. The burgeoning field of ultrafast x-ray science renders the ability to observe the fast and the small, in addition to concentrating large amounts of energy into ultrashort pulses. These characteristics make coherent sources of ultrafast x-rays an essential tool for exploring the dynamics and structure of matter on the atomic level. [2].

Fundamental principles of diffraction physics assert that the resolution of an image is directly proportional to the wavelength of light illuminating the object [4]. Accordingly, an important application of x-ray science is as an enabling tool for observation of nanostructures which further our understanding of nanotechnology and the minute properties of cells and molecules. However, due to limited modern manufacturing methods, spatial resolution is typically restricted by the quality of x-ray optics used for microscopy rather than by the wavelength [14]. Lensless imaging techniques circumvent these hindrances by supplanting physical optics with coherent light and iterative phase retrieval algorithms [5]. If generated by a coherent source of light, the
far field diffraction pattern of an object contains the information to reconstruct an image from the modulus of its Fourier transform [5]. Researchers have also demonstrated the ability to reconstruct the 3D structure of an object from its diffraction pattern using techniques such as holography and ankylography (Fig. 1.1b) [15]. Collectively, advances in x-ray generation and diffractive imaging techniques are transforming modern perceptions of the microscope.

The turn of the 21st century also hosted great progress in the pulsed operation of lasers. Due to pulse durations on the femtosecond and attosecond time scales, ultrafast lasers are becoming increasingly common for the study of fast, dynamic processes. Pulse duration is analogous to the time it takes for a camera shutter to open and close. A camera featuring a fast shutter, or short exposure time, can capture faster events with clarity. Similarly, ultrashort pulses exhibit high temporal resolution which elucidates previously imperceptible phenomena. For instance, time resolved electron distributions during the formation of a molecule [9] (Fig. 1.2a) and magnetic switching dynamics in ferromagnetic materials [11] (Fig. 1.2b). Undoubtedly, ultrashort pulse lasers will continue to be instrumental in revealing the physics behind complex and nonlinear processes of plasma, atoms and other fascinating states of matter [2].

Experiments frequently require high peak power light. The 1990s witnessed several key developments that enable pulsed lasers to deliver such power. In 1990, the demonstration of self-modelocking Ti:sapphire lasers birthed an entirely different breed of ultrafast lasers [2]. This laser generates short pulses as a result of self-modelocking due to the intensity dependent, nonlinear Kerr effect. Furthermore, the method of chirped
pulse amplification (CPA) demonstrated a way to effectively amplify the short pulses generated by ultrafast oscillators [2]. In order to avoid damaging the amplifier, CPA temporarily reduces the peak power of a pulse by stretching it in time. This stretched pulse is then amplified and re-compressed to output a higher peak power pulse. Both advances enlivened fields of laser science which continue to reduce the cost and complexity of ultrafast technology. One particularly interesting outcome of ultrafast high energy lasers is the ability to generate coherent x-rays through the process of High Harmonic Generation (HHG) [2].

1.2 High Harmonic Generation

Preceding the breakthrough of high harmonic generation (HHG), devices capable of generating coherent x-rays were primarily limited to immense synchrotron facilities. HHG offers the capability of producing spatially and temporally coherent EUV and x-ray light on a table-top apparatus [15, 7]. Short high intensity laser pulses focused into a noble gas experience a harmonic frequency up-conversion due to nonlinear optical mechanisms of the irradiated gas [3]. McPherson et al. first observed the production of odd high harmonics as high as the 17th order of the fundamental driving frequency in 1987. At the time, the spectral intensity distribution of these high order harmonics defied the predictions of perturbative nonlinear optics [12]. This called for a novel interpretation of HHG.

1.2.1 The Semi-classical Explanation

Figure 1.3: Two depictions of HHG: The top image illustrates the generation of an EUV beam from a femtosecond laser after ionization and acceleration of an electron in classical atom-electron systems. The bottom image depicts a quantum determined electron wave-function along different stages of a high intensity pulse [7].
HHG, depicted in Fig. 1.3, is most simply understood by considering both quantum and classical perspectives when analyzing the effect of electromagnetic waves on an atom. First, the incoming high intensity pulse ionizes an electron from the atom. The electron, now free from the atomic potential, is accelerated in the electric field of the incident pulse. While in the trough of the oscillating field, the electron is accelerated back into its parent ion. To conserve energy, the atom must emit a photon equal to the sum of the electron’s ionization energy and kinetic energy delivered by the oscillating field [7].

HHG can generate attosecond pulses, significantly shorter than that of the femtosecond driving pulse. A high harmonic burst occurs twice during a single oscillation period of the driving laser. Furthermore, a single attosecond pulse can be isolated. As detailed in (1.2) of section 1.2.3, the highest harmonic generated is directly proportional to the intensity of the driving pulse. Only the harmonic burst produced at the peak intensity of the pulse will contain the highest harmonic. The lower frequency HHG bursts can be filtered out so that only a single pulse composed of the highest harmonics remains. This enables the use of individual pulses to temporally resolve processes that take place during a single femtosecond [7].

![Figure 1.4: A depiction of high harmonic bursts generated during irradiation by a femtosecond pulse. The number of bursts decreases as the incident pulse shortens. Applying a filter to the high harmonic pulse from a 5 femtosecond driving pulse can isolate a single attosecond burst [7].](image)

### 1.2.2 Phase Matching

Due to a translucent inhomogeneous medium composed of neutral atoms and plasma, in addition to the quantum phase shifting of the laser and harmonic pulses, the driving laser pulse must traverse an inconsistent refractive index. As a result, the harmonic pulses are coherent only over a short distance before destructively interfering due to phase velocity mismatch. This distance is the coherence length, $L_c = \pi/\delta k_n$, where $\delta k_n$ is the difference in the wave number of the $n^{th}$ order harmonic and the fundamental.
In order to achieve the efficiency of HHG required for many experiments, the incident laser field must remain in phase with the generated harmonics over a distance much greater than the coherence length. This would result in a dramatic increase in flux due to the constructive interference of high harmonics. One method of achieving this is through the use of a waveguide. By adjusting the pressure inside the waveguide, the dispersion due to neutral atoms and plasma inside the waveguide can be tuned to counterbalance the dispersion of the waveguide itself. This extends the harmonic coherence length resulting in the production of a bright harmonic beam due to constructive interference [7]. Furthermore, HHG predominantly occurs at the focus of the driving beam, where the field intensity is the greatest. Thus, focusing the driving beam into a waveguide increases the high harmonic flux by guiding the region of peak intensity along the length of the waveguide.

1.2.3 Experimental Apparatus

HHG can emit photons with a maximum energy given by

\[ E_{max} = I_p + 3.2U_p \]  \hspace{1cm} (1.1)

\( I_p \) is the ionization potential of the atom. This term can be maximized by choosing noble gases such as Neon or Argon as the high harmonic medium. Noble gases have a stronger bond to their valence electrons.
and thus a much greater ionization potential\cite{7}.

$U_p$ is referred to as the ponderomotive energy. It is the average kinetic energy of the electron oscillating in an electric field.

\[
U_p = \frac{e^2 E^2}{4 m \omega^2} \propto I \lambda^2 \quad (1.2)
\]

The ponderomotive potential is directly proportional to the intensity and the square of the wavelength of the driving laser. Thus, ultrafast lasers are used to achieve high peak intensities. Longer wavelengths correspond to longer oscillation periods giving the driving pulse more time to accelerate an ionized electron. In practice, the 800 nm femtosecond pulses of a Ti:sapphire crystal effectively generate high harmonics up to 100 eV. Higher photon energies up to $>1$ keV can be generated using infrared optical parametric amplifiers (OPAs). \cite{7}.

In order to produce the coherent x-rays needed for experiments, we utilize a custom made hollow cylindrical waveguide. Fig. 1.6 shows the waveguides filled with noble gas during HHG. The efficient coupling of an incident beam into these 150 $\mu$m diameter fibers will be the focus of this thesis.

![Gas-filled waveguide](image)

**Figure 1.6:** a) Depiction of cylindrical waveguide setup \cite{7}. b) Actual waveguide implementation during HHG

### 1.3 Fiber Coupling

In order to maximize the power seen at the output of a hollow fiber, the incident laser beam must be accurately focused into the waveguide entrance. This beam fiber coupling process is most simply performed using the following setup:
The experiments performed utilized a He-Ne laser. Due to imperfections in its Gaussian form, the beam first passes through a spatial filter (SF), an aperture which removes any aberrations present in the beam. An improved gaussian distribution representing the ideal beam appears at the center of concentric rings formed due to the diffraction of the over-sized beam. These concentric rings can be blocked allowing only the ideal beam through. It is important to note that this step does not take place when using the 800 nm ultrashort Ti:Sapphire pulses which are only focused in vacuum. The spatially filtered He-Ne is next collimated by passing through a lens placed a focal length away from the laser source. A quick glance at the Lensmaker’s equation for a lens of negligible thickness shows us that the image distance $S_i$ must go to infinity if the object distance $S_o$ is equal to the focal length of the lens \[4\].

\[ \frac{1}{f} = \frac{1}{S_i} + \frac{1}{S_o} \quad (1.3) \]

This collimated beam is then reflected by two mirrors. These mirrors are used to adjust the beam position at two respective points following the second mirror. As a result the beam trajectory will trace a line between these two points allowing the experimentalist to direct the beam as required. Finally the beam is focused down to a size approximately that of fiber’s inner diameter.

Ultimately, our goal is to maximize high harmonic output by coupling the maximum amount of power from our incident beam into the waveguide. We can maximize coupling efficiency of the incident beam by focusing it down to a size equal to two thirds of the fiber’s inner diameter as the next chapter will elaborate in detail. Optimal coupling efficiency also requires that the incident beam pierce the center of the fiber entrance as well as follow a beam path parallel to the long axis of the waveguide.

1.4 The Problem: Laser Instability

Ideally, optimal coupling efficiency could be achieved with perfect accuracy and consistency. In practice, this is far from the case. Many experimentalists are constantly fighting a battle with laser stability.

Sources of laser instability can be classified in two different ways, long term and short term. Short-term instability occurs on a sub-second scale. This stability is often the result of air turbulence as well as various high frequency vibrations occurring throughout an apparatus. Long-term instabilities are noticeable
on the minute and hour scale. These instabilities are commonly a result of the expansion and contraction of optical elements, as well as support structures such as optical tables, in an experimental apparatus. This expansion occurs due to thermal fluctuations occurring in the experimental environment. Additional mechanical vibrations of varying frequency can emanate from equipment such as vacuum pumps which can prove difficult to completely isolate from the other components in a system [6].

While short term stabilities are difficult to effectively eliminate in pulsed laser systems, long term drifting due to thermal expansion and contraction can be counteracted using the beam stabilization technique presented in chapter 3 of this thesis. First, it is important to understand the detrimental effects of beam instability when coupling into a hollow fiber
Chapter 2

Modal Analysis of Fiber Coupling

In order to determine the ramifications of laser instability in our experiments, it is necessary to understand how laser light travels inside of the hollow fibers we use for HHG. Subsequently, we must understand how the laser couples into the eigenmodes of the fiber. The eigenmodes are determined by the geometry of the cylindrical waveguide, and form a basis with which we can describe the incident laser beam. Through the use of analytical and numerical methods, we can use this basis to determine how laser light travels through a cylindrical waveguide. In this case, we want to reveal the correlations between the coupling efficiency of the beam-fiber system and the position, angle, and size of the beam focus entering the fiber.

2.1 The Eigenmodes of Hollow Cylindrical Waveguides

The eigenmodes of the electric and magnetic fields existing inside of a hollow cylindrical waveguide have been solved rigorously by Stratton [17]. However, I will briefly discuss the guided geometry of the hollow fibers our lab uses to efficiently generate high harmonics.

A hollow fiber, shown in Fig. 2.1, can be modeled as done by Marcatilli et al. [10]. The dielectric constant of the core will be that of free space ($\varepsilon_0$), while the dielectric constant of the cylindrical sheath
will be some complex constant \((\epsilon)\) such that \(\epsilon > \epsilon_0\). Note that the pressure in the waveguide can be high during HHG but the refractive index stays close to one. Deviating only slightly from that of free space, the magnetic permeability of both media will be considered constant \((\mu_0)\). In Fig. 2.1, the inner radius of the fiber is labeled “a” while the radial dimension relative to the long axis of the waveguide along \(\bar{z}\) is denoted by “\(\bar{r}\)”.

Due to internal reflections from the boundary of the waveguide, light is guided within the volume enclosed by the cylindrical walls. These cylindrically symmetric boundary conditions limit the types of electromagnetic eigenmodes allowed to propagate along the hollow fiber [17]. The cylindrical waveguide structure allows for the existence of Transverse Electric (TE), Tranverse Magnetic (TM) and Hybrid Modes [10]. This chapter will examine the Hybrid Modes (EH) which contain components of the electric and magnetic fields in all directions. EH modes are the dominant source of linearly polarized modes which will be the focus of this analysis. Additionally, only the electric field components of the EH modes will be analyzed. This is because, we are interested in finding the power of the modes our incident beam couples in to and the magnetic fields are comparatively weak. The electric field components perpendicular to the long axis of the fiber are given by the following [17]:

\[
\begin{align*}
E_{\theta,n,m} &= J_{n-1}\left( u_{n,m} \frac{r}{a} \right) \cos(n(\theta + \theta_0)) \\
E_{r,n,m} &= J_{n-1}\left( u_{n,m} \frac{r}{a} \right) \sin(n(\theta + \theta_0))
\end{align*}
\] (2.1)

In (2.1), \(J_n(u_{n,m} \xi)\) is a bessel function of the first kind where \(u_{n,m}\) is the \(m^{th}\) root of the bessel. The remainder of this analysis will only consider the case where the azimuthal phase shift \((\theta_0)\) of the modes is zero.

### 2.2 Representing an Incident Laser Beam

In order to analyze the propagation of a laser beam throughout the waveguide, it is necessary to describe the incident beam using the basis composed of the natural modes derived from the waveguide geometry. The laser light we generate in order to drive HHG, however, is linearly polarized (LP). Thus, to describe this linearly polarized radiation at the threshold between open space and the cylindrical waveguide we must first find a basis of linearly polarized modes composed of the natural modes of the waveguide.

#### 2.2.1 Finding a Linearly Polarized Basis

First, we must represent the EH modes given by (2.1) in Cartesian coordinates using the following transformation.

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
\cos(\theta) & 0 & 0 \\
\sin(\theta) & 0 & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
r \\
\theta \\
z
\end{pmatrix}
\]
Applying this transformation to (2.1) results in the following Cartesian modes:

\[ E_{x,n,m} = J_{n-1} \left( u_{n,m} \frac{r}{a} \right) \sin(\theta(n-1)) \]
\[ E_{y,n,m} = J_{n-1} \left( u_{n,m} \frac{r}{a} \right) \cos(\theta(n-1)) \]  

(2.2)

where the general form for the natural modes in Cartesian coordinates is given by:

\[ EH_{n,m} = E_{x,n,m} \hat{x} + E_{y,n,m} \hat{y} \]  

(2.3)

Furthermore, we can compose a basis formed by linearly polarized modes which occur when only one of either the x or y components remain non-zero. By inspecting the cartesian modes in (2.2), we can determine that this occurs in two situations. The first case occurs when \( n = 1 \), this is \( EH_{1,1} \), the lowest order mode. The second situation occurs when a superposition of modes results in the destructive interference of degenerate components [10]. This occurs when superimposing the \( EH_{-|n|,m} \) and \( EH_{n+2,m} \) fiber modes giving a linearly polarized basis given by

\[ LP_{n,m} = \frac{1}{2} (EH_{-|n|,m} + EH_{n+2,m}) \]  

(2.4)

Substituting Eq. (2.3) into Eq. (2.4), we get

\[ LP_{n,m} = \frac{1}{2} ((E_{x,-|n|,m} + E_{x,n+2,m})\hat{x} + (E_{y,-|n|,m} + E_{y,n+2,m})\hat{y}) \]  

(2.5)

Similarly, substituting (2.1) into Eq. (2.5):

\[ LP_{n,m} = \frac{1}{2} \left( -\left( -1 \right)^{|n|} J_{|n|} \left( \frac{r}{a} \right) - J_{|n|+1} \left( \frac{r}{a} \right) \right) \sin[\theta(|n| + 1)]\hat{x} \]
\[ + \left( -1 \right)^{|n|} J_{|n|} \left( \frac{r}{a} \right) - J_{|n|+1} \left( \frac{r}{a} \right) \right] \cos[\theta(|n| + 1)]\hat{y} \]  

(2.6)

We can simplify further by using the bessel function property \( J_{-|n|}(x) = (-1)^{|n|} J_{n}(x) \). Note this relation also implies that \( u_{-|n|} = u_{|n|+2} \). This simplification gives

\[ LP_{n,m} = \frac{1}{2} \left( (-1)^n J_{n+1} \left( \frac{r}{a} \right) - J_{n+1} \left( \frac{r}{a} \right) \right) \sin[\theta(n + 1)]\hat{x} \]
\[ + (-1)^n J_{n+1} \left( \frac{r}{a} \right) - J_{n+1} \left( \frac{r}{a} \right) \right] \cos[\theta(n + 1)]\hat{y} \]  

(2.7)

This solution is made clearer by looking at the cases for odd and even \( n \) separately. Also, note that the indices have been renumbered.
\[ LP_{n,m} = \begin{cases} 
J_{n-1} \left( u_{n,m} \frac{\pi}{2} \right) \sin(\theta(n-1)) \hat{x} & \text{if } n \text{ is odd} \\
J_{n-1} \left( u_{n,m} \frac{\pi}{2} \right) \cos(\theta(n-1)) \hat{y} & \text{if } n \text{ is even}
\end{cases} \] (2.8)

Similarly, we can consider the case where the modes of (2.4) are subtracted instead of added. Then we have

\[ LP_{n,m} = \begin{cases} 
J_{n-1} \left( u_{n,m} \frac{\pi}{2} \right) \sin(\theta(n-1)) \hat{x} & \text{if } n \text{ is even} \\
J_{n-1} \left( u_{n,m} \frac{\pi}{2} \right) \cos(\theta(n-1)) \hat{y} & \text{if } n \text{ is odd}
\end{cases} \] (2.9)

From (2.8) and (2.9) we can see that it is possible to have both x and y polarizations for n odd or even. Plotting these modes in the plane of a circular cross-section of the waveguide, we get the plots displayed in Fig. 2. for \( 1 \leq n \leq 4 \) and \( 1 \leq m \leq 3 \).

![Figure 2.2: The normalized electric field magnitude of the linearly polarized modes of a hollow fiber. Red indicates a strong electric field while blue indicates a weak magnetic field.](image)

Notice that the profile of the lowest order mode is reminiscent of a Gaussian. This will turn out the mode we want to couple into. The structure of this mode is preferable to that of the higher order modes due to the fact that it features high intensities near it’s center and has circular symmetry in common with the waveguide and the incident beam which is well approximated as a Gaussian.
2.2.2 Decomposing an Electric Field into LP Modes

The linearly polarized basis given by (2.8) can be used to represent any linearly polarized solutions of the field inside the waveguide. The most general form of this statement can be denoted as

\[ LP = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{n,m} LP_{n,m} \]  

(2.10)

In order to find the coefficients \( A_{n,m} \) we can exploit the orthogonality of eigenstates using Fourier’s Trick. We want to obtain a linearly polarized solution. Here I will consider light polarized in the \( \hat{y} \) direction meaning we only have modes where \( n \) is even. Given some solution to the waveguide given by the function \( f(r, \theta) \), we multiply by the eigenfunction for which we want to find \( A_{n',m'} \).

\[ f(r, \theta)LP_{n',m'} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{n,m} LP_{n,m} LP_{n',m'} \]  

(2.11)

Due to orthogonality, this equation is only non-zero when \( n = n' \) and \( m = m' \). Using this property, we get the relationship (2.11) when integrating over the space of the fiber.

\[ \int_0^a \int_0^{2\pi} f(r, \theta)LP_{n',m'} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{n,m} LP_{n,m} LP_{n',m'} \delta_{n,n'} \delta_{m,m'} \]  

(2.12)

which reduces to (2.12)

\[ \int_0^a \int_0^{2\pi} f(r, \theta)LP_{n',m'}r d\theta dr = A_{n',m'} \int_0^a \int_0^{2\pi} |LP_{n',m'}|^2 r d\theta dr \]  

(2.13)

Plugging in the LP functions, the left side of (2.12) becomes

\[ \int_0^a \int_0^{2\pi} f(r, \theta)J_{n' - 1} \left( u_{n',m'} \frac{r}{a} \right) \cos(\theta(n' - 1))r d\theta dr \]  

(2.14)

While the right side of (2.12) is

\[ A_{n',m'} \int_0^a \int_0^{2\pi} J_{n' - 1} \left( u_{n',m'} \frac{r}{a} \right)^2 \cos(\theta(n' - 1))^2 r d\theta dr \]  

(2.15)

The \( \theta \) integral evaluates to \( \epsilon_{m'} \pi \) where \( \epsilon_{m'} \) is 2 for \( m' = 1 \) and 1 for \( m' > 1 \). The \( r \) integral evaluates to \( \frac{a^2 J_n(u_{n,m})^2}{2} \). The right side of (2.12) is then

\[ \frac{\epsilon_{m'} \pi a^2 J_n(u_{n,m})^2}{2} A_{n',m'} \]  

(2.16)

From here, it is simple to solve for \( A_{n',m'} \) to get
\[
A_{n',m'} = \frac{2}{\epsilon_{m'} \pi a^2 J_n(u_{n,m})^2} \int_0^a \int_0^{2\pi} f(r, \theta) J_{n'-1} \left( u_{n',m'} \frac{r}{a} \right) \cos(\theta(n'-1)) r d\theta dr
\]  

(2.17)

### 2.3 Coupling Efficiency in a Gaussian Beam Formalism

It is accurate to approximate the HHG driving laser as the TEM\(_{00}\) mode, also known as a Gaussian beam [4]. Thus, the function we want to represent using the linearly polarized eigenmodes takes the form

\[
f(\theta, r) = E_0 e^{-\frac{r^2}{w_0^2}} e^{i\vec{k} \cdot \vec{r}}
\]  

(2.18)

Where \(w_0\) is the beam waist size of the beam propagating in the \(\hat{k}\) direction and \(k = \frac{2\pi}{\lambda}\). We can find the power coupling efficiency of the \(n,m\) mode using the expression

\[
P_{n,m} = \frac{A_{n',m'} \int_0^a \int_0^{2\pi} |LP_{n',m'}|^2 r d\theta dr}{\int_0^\infty \int_0^{2\pi} |E_0 e^{-\frac{r^2}{w_0^2}} e^{i\vec{k} \cdot \vec{r}}|^2 r d\theta dr}
\]  

(2.19)

(2.18) is the ratio of the power in the \(n,m\) mode over the total power of the Gaussian beam [17]. The power of the Gaussian beam simplifies to \(\frac{\pi w_0^2}{2}\). Using this result and plugging in (2.16) and (2.8), the power can be written as.

\[
P_{n,m} = \frac{4}{w_0^2 \epsilon_{m'} \pi a^2 J_n(u_{n,m})^2} \left| \int_0^a \int_0^{2\pi} E_0 e^{-\frac{r^2}{w_0^2}} e^{i\vec{k} \cdot \vec{r}} J_{n'-1} \left( u_{n',m'} \frac{r}{a} \right) \cos(\theta(n'-1)) r d\theta dr \right|^2
\]  

(2.20)

Now we are able to vary the parameters of our incident Gaussian beam and find the theoretical power efficiency. This section will discuss the correlations between Power efficiency with respect to beam waist size, position and angle, when coupling into a hollow fiber.

#### 2.3.1 Dependencies on Beam Waist Radius

By plotting the power as a function of \(\frac{w_0}{\pi}\), we find the plot in Fig. 2.3.
It is important to notice that the lowest order mode is the most efficient. Within this mode, we can optimize coupling efficiency by making sure the waist of the beam has a radius that is 64% of the waveguide radius. Assuming that all other conditions are optimal, we can couple 98% of the incident beam’s power into the lowest order mode. If the beam is too big, power is lost because some of the power is apertured out. If the beam is too small, we begin to couple into higher order modes which do not have the same coupling efficiency as $EH_{1,1}$.

2.3.2 Dependencies on Beam Pointing and Angle

In order to analyze the effect of both beam position and angle at the entrance of the waveguide, we must make some additions to our Gaussian beam function (2.17). Consider the geometry presented in Fig. 2.4.

![Power Coupling Efficiency as a Function of Beam Waist Radius](image)
The incident beam is tilted at an angle $\alpha$ from the z-axis which runs directly along the center of the fiber. $\phi$ is the angle between the positive x-axis and the input position at which the beam, represented by the propagation vector $\hat{k}$, enters the waveguide. The radial dimension $\vec{r}$ is referenced at an angle $\theta$ with respect to the polarization axis in the $\hat{y}$ direction. Using this system we can convert $\vec{r}$ and $\vec{k}$ to a Cartesian coordinate system.

\[
\vec{k} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} k_0 \sin(\alpha) \cos(\phi) \\ k_0 \sin(\alpha) \sin(\phi) \\ k_0 \cos(\alpha) \end{pmatrix},
\]

\[
\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \cos(\theta) \\ r \sin(\theta) \\ 0 \end{pmatrix}.
\] (2.21)

We can now adjust our Gaussian function to account for a change in the angle of the beam. For simplification, this analysis will only include shifts in input position and angle in the $\hat{x}$ direction, thus $\phi = 0$. This constricts the beam to only rotate about the polarization axis.

\[
f(\theta, r) = E_0 e^{-\frac{r^2}{w_0^2}} e^{ik_0 r \sin(\theta) \hat{x}}.
\] (2.22)

Furthermore, we can simulate a translational shift of the beam away from the waveguide center. Consider the relationship $r^2 = x^2 + y^2$, if shifted in the x direction by a distance $x_0$, this becomes $r^2 = (x - x_0)^2 + y^2$. Substituting (2.20) and simplifying, we get that such a shift can be represented by $r^2 + x_0^2 - 2rx_0 \cos(\theta)$. Our Gaussian now takes the form

\[
f(\theta, r) = E_0 e^{-\frac{(r^2 + x_0^2 - 2rx_0 \cos(\theta))}{w_0^2}} e^{ik_0 r \sin(\theta) \hat{x}}.
\] (2.23)

Now, by varying $x_0$ we can plot the coupling efficiency as a result of translational shifts in beam position from the center of the waveguide as done in Fig. 2.5. We can also plot the coupling efficiency as a function of a incident beam angle with respect to the long axis of the waveguide. This is done in Fig. 2.6. Note both plots show that optimal coupling efficiency can be achieved by coupling into the $EH_{1,1}$ mode. Additionally, both beam angle and input position are maximized when the beam is parallel with the long axis of the fiber and when the incident beam is focused directly into the center of the waveguide.

Figure 2.4: A diagram of the beam geometry at the waveguide
Figure 2.5

Power Coupling Efficiency as a Function of Beam Focus Position

- $LP_{1,1}$ (EH$_{1,1}$)
- $LP_{1,2}$
- $LP_{1,3}$

$Power Coupling\text{ Efficiency}$

$\Delta x/a$

- $LP_{2,1}$
- $LP_{2,2}$
- $LP_{2,3}$

$Power Coupling\text{ Efficiency}$

$\Delta x/a$

- $LP_{3,1}$
- $LP_{3,2}$
- $LP_{3,3}$

$Power Coupling\text{ Efficiency}$

$\Delta x/a$
Figure 2.6

Power Coupling Efficiency as a Function of Beam Angle

$\alpha$ [mrad]
Chapter 3

The Theory of Beam Stabilization

The effectiveness of beam stabilization is highly dependent on the geometry of the apparatus used to stabilize the beam, and the programmable logic controlling the system. By analyzing the geometry of the stabilization presented in this chapter, we can calculate the best placement of active components for optimized stability. Additionally, it is important to understand the aspects of control logic which govern the system’s response to a perturbation. This delivers insight pertaining to the effective tuning of the negative feedback loop controlling an active stabilization.

3.1 Active and Passive Stabilization

Several different beam stabilization techniques are used to attenuate fluctuations in beam pointing and angle. Each method can be categorized as either passive or active.

Less common, passive stabilization techniques function due to components which are built in to the system so that they are ever-present. For instance, padding a laser with a material which dampens vibrations that can propagate throughout a system. This aids in mechanically decoupling a laser from other oscillating components of an apparatus. Other less fundamental passive stabilizations exist such as the inventive system demonstrated by Wu et al. [19]. This ingenious design first splits a beam at some fundamental frequency. Using a mirror, the pointing of one of the resulting beams is inverted so that each beam has a relative shift in pointing which is equal in magnitude but opposite in direction. These beams are then recombined in a nonlinear crystal to produce a second harmonic beam which points in a constant direction. Such a stabilization works effectively in the limit of small pointing changes, but the large drifts resulting from thermal fluctuations can often result in a dramatic decrease in flux due to the limited angular bandwidth of second harmonic generation in a crystal. If a source of instability such as a vacuum pump is easily identifiable, simple solutions such as vibrational isolation using Sorbathane or other materials are beneficial, however, active stabilization techniques may still be necessary [19].

Active implementations of beam stabilization are the most conventional, and often most practical, solution. They use the information gathered from previous beam position measurements to predict the correction needed to counteract a current perturbation. This technique uses sensing devices such as CCD cameras or quadrant photo-diodes to obtain the position of the beam. Once found, this value can be compared to desired
The beam stabilization design presented in this thesis utilizes an active closed-loop feedback system. The beam is monitored using two CCD cameras. Using a program written in C#, the centroid of the beam is calculated and compared to the correct beam position marked by the user. A correction is applied by altering the position of the beam using two piezo-actuated mirrors which change the beam’s angle of reflection. Using this set-up we can stabilize the beam around two points which allows for not only pointing stability but also angular stability.

### 3.2 Beam Stabilization Geometry Analysis

The design displayed in Fig. 3.1 models the stabilization apparatus used in our lab. We can perform Ray Transfer Matrix Analysis in order to construct a matrix which maps the angle of the piezo-actuated mirrors (M1 & M2) to the position and angle of the beam at each camera [4].

![Figure 3.1: The geometry of the active beam stabilization implemented in chapter 4](image)

I will begin with the dependences on beam position and orientation at the cameras due to the movement of M1. The first camera (C1) images the beam position at the first mirror (M1). Between M1 and C1, the beam propagates over a distance equal to \( d_1 + d_2 \) in free space and then passes through the focusing lens of focal length \( f \). After the lens, the beam propagates in free space to the image plane of the M1. Using the Lensmaker’s equation (1.3), the distance to the image plane \( (S_i) \) is found to be

\[
S_i = \frac{f(d_1 + d_2)}{d_1 + d_2 - f}
\]

Using Ray Transfer Matrices, the beam position and angle along the x and y axis at C1 is given by
\[
\begin{pmatrix}
  C_{1x,y} \\
  C_{1\theta x,y}
\end{pmatrix} = A_{1,1}
\begin{pmatrix}
  M_{1x,y} \\
  M_{1\theta x,y}
\end{pmatrix}
\] (3.1)

Where \(C_{1x,y}\) is the x or y position of the beam in a plane perpendicular to the ideal beam trajectory parallel to the z-axis. \(C_{1\theta x,y}\) is the corresponding angle between the x or y axis and the z-axis. The same symbols corresponding to M1 are the respective angles and position of the beam at the mirror. From the geometry previously explained, \(A_{1,1}\) is given by

\[
A_{1,1} = \begin{pmatrix}
  1 & \frac{f(d_1+d_2)}{d_1+d_2-f} \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  1 & 0 \\
  -\frac{1}{f} & 1
\end{pmatrix}
\begin{pmatrix}
  1 & d_1+d_2 \\
  0 & 1
\end{pmatrix}
\] (3.2)

We can simplify (3.1) due to the assumption that the mirror does not translate the beam, it only alters its orientation by changing the angle of reflection. Thus, by setting \(M_{1x,y}\) equal to zero and plugging in (3.2), we can evaluate (3.1) to (3.3). Note, that there is no change in position on the camera due to a change in angle from the mirror. This is consistent with the fact that the first camera is imaging the first mirror.

\[
\begin{pmatrix}
  C_{1x,y} \\
  C_{1\theta x,y}
\end{pmatrix} = \begin{pmatrix}
  0 \\
  1 - \frac{(d_1+d_2)}{f}
\end{pmatrix} M_{1\theta x,y}
\] (3.3)

The effect of M1 on C2 is given by \(A_{1,2}\). The only difference is that C2 is at the focus of the beam and thus propagates a distance \(f\) after the lens.

\[
A_{1,2} = \begin{pmatrix}
  1 & f \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  1 & 0 \\
  -\frac{1}{f} & 1
\end{pmatrix}
\begin{pmatrix}
  1 & d_1+d_2 \\
  0 & 1
\end{pmatrix}
\] (3.4)

\[
\begin{pmatrix}
  C_{2x,y} \\
  C_{2\theta x,y}
\end{pmatrix} = \begin{pmatrix}
  0 \\
  1 - \frac{f}{(d_1+d_2)}
\end{pmatrix} M_{1\theta x,y}
\] (3.5)

The change in position and angle at the cameras due to the second mirror is given by matrices very similar to (1.2) and (1.3). However, the beam only travels a distance equal to \(d_2\) before hitting the lens. This results in the following relationship:

\[
A_{2,1} = \begin{pmatrix}
  1 & \frac{f(d_1+d_2)}{d_1+d_2-f} \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  1 & 0 \\
  -\frac{1}{f} & 1
\end{pmatrix}
\begin{pmatrix}
  1 & d_2 \\
  0 & 1
\end{pmatrix}
\] (3.6)

We can set \(M_{2x,y}\) equal to zero for the same reasons as \(M_{1x,y}\).

\[
\begin{pmatrix}
  C_{1x,y} \\
  C_{1\theta x,y}
\end{pmatrix} = \begin{pmatrix}
  d_2 + (1-d_2) \left( \frac{d_1+d_2}{d_1+d_2-f} \right) \\
  1 - \frac{d_2^2}{f}
\end{pmatrix} M_{2\theta x,y}
\] (3.7)

Similarly, the effect of change in angle of M2 results in the following effect at the second camera.

\[
A_{2,2} = \begin{pmatrix}
  1 & f \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  1 & 0 \\
  -\frac{1}{f} & 1
\end{pmatrix}
\begin{pmatrix}
  1 & d_2 \\
  0 & 1
\end{pmatrix}
\] (3.8)
\[
\begin{pmatrix}
C^2_{x,y} \\
C^2 \theta_{x,y}
\end{pmatrix} = \begin{pmatrix} f \\ 1 - \frac{d^2}{f} \end{pmatrix} M 2 \theta_{x,y} 
\] (3.9)

3.2.1 Decoupling the Mirrors

Having found the correlation between mirror orientation and the beam position on the camera, it is now possible to combine these relationships to construct the matrix presented in (3.10) that relates all mirrors and cameras.

\[
\begin{pmatrix}
C^1_{x,y} \\
C^2_{x,y}
\end{pmatrix} = \bar{A} \begin{pmatrix} M^1 \theta_{x,y} \\
M^2 \theta_{x,y} \end{pmatrix} = \begin{pmatrix} f \\ 0 \\
d_2 + (1 - d_2) \left( \frac{d_1 + d_2}{d_1 + d_2 - f} \right)
\end{pmatrix} \begin{pmatrix} M^1 \theta_{x,y} \\
M^2 \theta_{x,y} \end{pmatrix} 
\] (3.10)

The system will only be able to stabilize a beam if the vectors which form this matrix are linearly independent. This means that any row of the matrix can not be formed from a linear combination of the other rows. This is the reason we chose to image the first mirror using the first camera. The imaging condition results in a zero entry in the second row and first column is zero of the matrix in (3.10). However, this matrix still depends on \( f, d_1, \) and \( d_2 \). Further analysis of this system requires a look at the matrix determinant. The rows of a matrix are considered linearly independent if it’s determinant is nonzero. The matrix of (3.10) has the determinant

\[
\text{Det}[A] = -\frac{f^2 d_1}{f - d_1 - d_2} 
\] (3.11)

Looking at Fig. 3.2 we can see that the determinant of the matrix is decoupled by the greatest degree, that is the furthest from zero, when \( d_2 \) is relatively small when compared to \( d_1 \). This spot, the lower right corner of the plot, is where the stabilization performs best. The black line featured on the contour plot occurs when the determinant blows up to infinity because \( d_1 + d_2 \) is equal to the focal length. This represents the case when the first mirror is placed a focal length away from the lens. Using the Lensmaker’s equation (1.3) we can see that the image is infinitely far away when the object distance is equal to the focal length. In this case, it would not be physically possible to image the first mirror. It is also interesting to consider the case where \( d_1 \) is equal to zero. The value of the determinant when \( d_1 \) is zero is always equal to zero. One can interpret this case as a stabilization with only one mirror. If \( d_1 \) is zero, then the two mirrors are directly on top of each other. Thus, the mirrors are perfectly coupled because they are the same mirror.
3.3 PID Control Theory

At first glance, it appears that the most effective way of correcting for beam instability would be to alter the piezo-actuated mirrors so that the change in position equals the discrepancy in position. In practice, however, this is not the case. A more robust method involves the implementation of a Proportional Integral Derivative (PID) controller. The formula used by a PID controller to calculate the output, $u(t)$, in response to a change in beam position is [1].

$$u(t) = K \left( e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt} \right) \quad (3.12)$$

3.3.1 Proportional Feedback

The first term in (3.1) is called the proportional action.

$$u_p(t) = e(t) \quad (3.13)$$

This is the contribution to the controller output resulting from the current discrepancy in beam position. It can be adjusted changing the value of $K$, the total gain of the controller. The digital implementation of this term is equal to the difference of the most current beam centroid received from a camera frame [1].
3.3.2 Integral Feedback

The second term is called the integral action.

\[ u_i(t) = \frac{1}{T_i} \int_0^t e(t) \, dt \]  

(3.14)

This is the contribution to the controller output which comes from the sum of past proportional terms. This effectively sums previous discrepancies in beam position in order to reveal a tendency of the beam to move in any one direction over time. Responding to this information results in the removal of any offset that may occur in the position of the beam. This term can be very effective when attempting to stabilize a beam which primarily suffers from long-term instabilities such as drifting due to changing thermal conditions. This is due to weak influence from short-term instabilities. Instead of attempting to correct for an erratic error signal, the integral term merely ensures that the short-term instabilities that do occur, are centered around the desired beam position. The influence of this term on the total controller output can be altered by changing the value of \( T_i \) which has units of inverse seconds. This term has a simple digital implementation which involves maintaining a variable that adds the error in beam position for the current iteration to a running total of the error signal for past iterations \([1]\).

3.3.3 Derivative Feedback

The third term of (3.1) is referred to as the derivative action.

\[ u_d(t) = T_d \frac{de(t)}{dt} \]  

(3.15)

This is the contribution to the controller output resulting from the rate of change in the beam position. It is normally used to improve the stability of the PID controller. In an active stabilization, a short period of time rests in between the output of the controller correcting for a perturbation, and the cameras noticing that correction. As a result, the feedback system always has a delayed correction. The derivative action attempts to correct this by predicting future beam position discrepancies as a function of the rate of change in past beam positions \([1]\). Unfortunately, short-term instabilities which occur on time scale much shorter than the frame rate of our cameras can cause this term to blow-up almost instantaneously. This often results in a huge over correction in beam position which leads to an unstable system. At first thought, it seems simple to find the difference between the current and previous beam positions and divide it by the period of time required for one iteration of the feedback loop. Unfortunately, our stabilization does not have the temporal resolution to accurately predict the motion of these short term instabilities, the derivative term is not representative of the future behavior of the laser beam. However, this ailment can be alleviated by using a higher order backward difference formula. Higher order backward difference formula’s of the nth order take into account n previous data points in order to improve the accuracy in the rate of change of discrete systems \([8]\). The stabilization implemented in this thesis used a 6th order backward formula of the form

\[ e'_n = \frac{49}{20} e_{n-6} - 6e_{n-5} + \frac{15}{2} e_{n-4} - \frac{20}{3} e_{n-3} + \frac{15}{4} e_{n-2} - \frac{6}{5} e_{n-1} + \frac{1}{6} e_n }{t_n - t_{n-6}} \]  

(3.16)

Even with this addition, the PID controller used to stabilize a laser beam usually benefits from a very
small gain factor $T_d$ compared to the gain factors of other action terms. This limits the influence it has on the controller output. Thus, preventing the possibility of an unstable system resulting from a brief but large change in beam position.

### 3.3.4 Tuning the PID

### 3.4 Digital Low-Pass Filtering

The integration of a low-pass filter was found to be crucial when designing a PID controlled beam stabilization. Low-pass filters attenuate high frequency instabilities while letting low frequency instabilities pass unimpeded [16]. This aids in filtering out abrupt changes in beam position which may result from temporary conditions such as someone touching the table supporting the experiment. These instabilities, although brief, can cause the PID controlled stabilization to become unstable and oscillate uncontrollably due to overcorrection. Digital low-pass filters can be implemented using the code presented in (3.17).

$$e'_n = \alpha e_n - (1 - \alpha)e'_{n-1} \quad (3.17)$$

The factor $\alpha$ is a constant that is set by the user to tune the cut-off frequency of the filter. The time constant of this digital filter is found using (3.18).

$$RC = \delta_T \left( \frac{1 - \alpha}{\alpha} \right) \quad (3.18)$$

$\delta_T$ is the sampling period of the program which in this case corresponds to the inverse frame rate of the camera. This enables the user to set alpha to filter out instabilities which exhibit periods below the value of (3.18). In general, you want to set $\alpha$ so that $RC$ is equal to the Nyquist period. This is twice the period of time it takes for the stabilization to run one iteration. Instabilities that occur on a time scale faster than this cannot be sufficiently sampled, let alone stabilized.
Chapter 4

Beam Stabilization in Practice

This chapter discusses the methods utilized when implementing the beam stabilization discussed previously. Furthermore it demonstrates the ability of this system to stabilize the angle and position of the beam.

4.1 Apparatus

![Figure 4.1: Experimental apparatus during data collection](image)

Fig. 4.1 displays the apparatus used for data collection when stabilizing the beam. It is a fusion of Fig. 1.7 and Fig. 3.1 including a few additions. First, a piezo-actuated mirror was added before the first and
second stabilizing mirrors. This is used to simulate instabilities of various frequencies. Also, a mirror was
added in between M1 and M2 in order to decouple the two actuated mirrors by lengthening the distance
between them. Finally, two cameras (C3 & C4) were added in order to take additional data using cameras
which did not operate inside the closed-loop feedback system stabilizing the beam. C3 was used to monitor
the beam 5cm after the focal point. Because the waveguide is 5cm long, this point helped to determine the
angle of the beam entering the waveguide. Finally, C4 was used to image the output of the waveguide. This
aided in the alignment of the beam into the waveguide by allowing us to monitor the beam profile at the
fiber output. This enabled us to adjust the beam until the mode visible at the output resembled the \( EH_{1,1} \)
mode and the power emanating from the output was at a maximum.

4.2 Finding the Centroid of the Beam

In order to determine the position of the beam, an algorithm was implemented in C\# to find the centroid
of the frames captured by the cameras. The formula to find the centroid of an image is analogous to finding
the center of mass of an object. However, here we will find the center of the light distribution incident on
the CCD camera. Let’s first find the x component of the centroid. We need the first moment of the image
which is given by (4.1) [18].

\[
\mu_1 = \sum_{i=1}^{\# \text{ of pixels}} x_i P_i \tag{4.1}
\]

The sum occurs over all pixels forming an image. The term inside the sum is the pixel value of the \( i^{th} \)

pixel weighted by it’s x axis position. The ccd cameras used in this experiment read out an 8 bit gray-scale
image so \( P_i \) will be a value between 0 and 255. In order to determine the center of the distribution we must
divide the first moment by the sum of all pixel values in the image [18].

\[
P = \sum_{i=1}^{\# \text{ of pixels}} P_i \tag{4.2}
\]

Dividing (4.1) by (4.2) we find the centroid of the beam to be

\[
C_x = \frac{\mu_1}{P} = \frac{\sum_{i=1}^{\# \text{ of pixels}} x_i P_i}{\sum_{i=1}^{\# \text{ of pixels}} P_i} \tag{4.3}
\]

An identical process can be used to find the y centroid.

4.3 Calibrating the Piezo-Actuated Mirrors

To implement the beam stabilization, the Matrix A of (3.10) must be found. This matrix enables the
stabilization system to relate a change in angle of a mirror to a change in position of the beam. Although
the analysis of section 3.2.1 offers insight into the geometry of the stabilization, the elements of this matrix
can be found in practice using a much more direct method.

For small angles, the voltage applied to the piezo mirrors has a linear relationship with the change in
beam position detected by the CCD’s. This allows us to calibrate the matrix by applying a voltage to one axis of the mirror and measuring the change in position. The element of the matrix $A$ relating the movement of the first mirror to the movement of the beam on the first camera is then the ratio of the change in position to the change in voltage. In practice, this process was automated in the beam stabilization code in order to take several position readings at several voltages between 0 and 150 volts. This data was then averaged to create a more robust and accurate calibration. This process results in the following matrix:

$$\begin{bmatrix}
C_{1x} \\
C_{1y} \\
C_{2x} \\
C_{2y}
\end{bmatrix} \rightarrow \vec{C} = \vec{A} \vec{V} = \begin{bmatrix}
A_{1,1} & A_{1,2} & A_{1,3} & A_{1,4} \\
A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} \\
A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} \\
A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4}
\end{bmatrix} \begin{bmatrix}
V_{1\theta_x} \\
V_{1\theta_y} \\
V_{2\theta_x} \\
V_{2\theta_y}
\end{bmatrix}$$ \hspace{1cm} (4.4)

Consider a change in position of the beam due to some source of instability. We can fill the vector $\vec{C}$ so that it is comprised of the detected change in positions on each camera. Inverting the Matrix $\vec{A}$ and multiplying by $\vec{C}$ gives the vector $\vec{M}$ which represents the voltages that must be applied to the mirrors in order to counteract this instability.

### 4.4 Tuning the PID and Low-pass Filter

Choosing the correct values for the gain coefficients $K$, $T_i$, and $T_d$ is a crucial step in optimizing the stability of a laser beam. Unfortunately, the frame rate of our camera’s ($\sim 13$Hz) is not fast enough to capture the information required to stabilize many of the short term instabilities seen on a time scale below a couple hundred milliseconds. As a result, many traditional PID tuning methods, such as the Ziegler-Nichols Method [1], can not be used.

The method I used when stabilizing the beam is simple. Start with the proportional term. With the gain of the derivative and integral terms set to zero, increase the value of $K$ slowly until oscillations in the beam begin to grow. Half of this number is usually a reasonable value for $K$. With the proportional term set, repeat the same method with the integral and derivative terms.

Once the PID is tuned, you can find the value of $\alpha$ which optimizes the effect of the low-pass filter. This value is more intuitive to select because you can calculate the corresponding time constant using (3.18). This allows us to choose an $\alpha$ which filters out the short term instabilities which are noticeably effecting the PID controller [16].

From this point, fine tuning the system is done by trial and error. Furthermore, values for the gain coefficients used in this experiment are arbitrary. This is because each experiment experiences instabilities with dissimilar qualities and each stability apparatus features a slightly different geometry. This results in the need to tune each individual beam stabilization independently [1].
4.5 Results

4.5.1 Position Stability

The data collected from the CCD cameras gives an x and y centroid of the beam as a function of time. The distance of the beam from the point which we stabilize to was found using the distance formula. Assuming that the beam is stabilized around the origin in the x-y plane, the discrepancy in beam position is simply \( \sqrt{C_x^2 + C_y^2} \). A 10\(^{-2}\) Hz sinewave was applied to the modulating mirror resulting in a 30 \( \mu m \) wide instability. The position data at the focus of the beam, also the fiber entrance, is presented in Fig. 4.2.

![Position Stability](image)

Figure 4.2

The stability of the beam is measured by the standard deviation which is listed in Fig. 4.2. The stabilization was able to reduce the standard deviation of the beam position by a factor of 5.75. Assuming all other beam parameters are optimized, we can find the stability of the power coupling efficiency (2.19) corresponding to the measured position stability. The standard deviation of the beam without stabilization results in a power coupling efficiency within 96.89%. The beam with stabilization was able to stabilize the power coupling efficiency within 98.04%.

4.5.2 Angular Stability

By collecting position data at the focus of the beam and a position 5cm away from the focus, we are able to find the angle of the beam entering the waveguide. Again, we find the distance in the x-
y plane between the beam fiber entrance centroid and the centroid at the end of the fiber using 
\[ d = \sqrt{(C_{x2} - C_{x1})^2 + (C_{y2} - C_{y1})^2}. \] Then the angle is found using 
\[ \alpha = \arctan \frac{d}{5.0 \text{ cm}}. \]

![Angular Stability](image)

Figure 4.3

It is immediately noticeable that the angle was not stabilized by nearly as much. The standard deviation was only reduced by a factor of 1.86. This is most likely due to the limited resolution of the CCD imaging the first mirror. The source of instability from the modulating mirror was placed in close proximity to the first actuated mirror. Thus, the oscillations in position at the first mirror are smaller than the oscillations observed at the focus of the beam. The image of the beam at the first mirror was also de-magnified, further diminishing the oscillations observed on the camera. If the de-magnified oscillations were too small for the CCD to detect any significant change, the stabilization would only be correcting the position at the focus and not the first mirror. As a result, only one point would be stabilized leading to effective position stabilization but minuscule angular stabilization. When looking at the power coupling efficiency corresponding to these angular stabilities we find that the beam without stabilization results in an efficiency within 97.78% while the stabilized beam results in an efficiency within 97.99%. Comparing these values to the power coupling efficiency with respect to position stability, we can see that the system is more tolerant to fluctuations in beam angle.
Chapter 5

Conclusions

5.1 Summary

The process used by our lab to generate high harmonics is highly dependent on the hollow fiber used as a waveguide to phase match the harmonic beam. It was found that the coupling efficiency of an incident Gaussian beam is strongly correlated to beam stability parameters such as position and angle at the entrance of the fiber. We found that it is advantageous to couple the incident beam into the $EH_{1,1}$ mode for optimal power transfer into the fiber.

The active stabilization presented in chapter 4 is capable of stabilizing the beam’s position by a factor of 5.75 and beam’s angle by a factor of only 1.86 for instabilities on the scale of a few minutes. However, sub-milliradian instabilities in beam angle have very little effect on the fiber coupling efficiency of the incident beam. Thus, the output power of the waveguide is not significantly diminished by small changes in beam angle. It follows that the beam stabilization presented by this thesis can stabilize the coupling efficiency of the incident beam into the fiber and thus stabilize the high harmonic output. This gives HHG experiments the ability to produce enduring bright harmonics.

5.2 Future Work

Moving forward it would be beneficial to test the harmonic output generated with a stabilized beam against a beam without stabilization. This would either confirm the predictions of this thesis or justify further investigation into the subject. Furthermore, it would give us information which may narrow down the source of power loss leading to a reduction in high harmonic output.
Bibliography


