Femtosecond Line-by-Line Pulse Shaping with a Stabilized Frequency Comb

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Femtosecond Line-by-Line Pulse Shaping with a Stabilized Frequency Comb

by

M. S. Kirchner

B.S., Colorado School of Mines, 2005

A thesis submitted to the
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Femtosecond Line-by-Line Pulse Shaping with a Stabilized Frequency Comb
written by M. S. Kirchner
has been approved for the Department of Physics

__________________________________________
Scott Diddams

__________________________________________
Dr. Jun Ye

Date __________________

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.
Pulse shaping of femtosecond laser pulses has shown great utility across a variety of scientific fields. In the past 5 years, the ability to manipulate the individual frequency components of the light field has enabled new applications such as optical and microwave arbitrary waveform generation. This thesis details the development of this line-by-line frequency control of a phase controlled and stabilized frequency comb, which allows for absolute control of the phase and frequency of the laser pulse and enables new applications such as secure communications. Also developed is the extension of the bandwidth of line-by-line control to 640 comb modes spanning more than 13 THz with a grism (grating + prism) based pulse shaper that enables broadband shaping and short pulses. The phase and amplitude of the comb modes manipulated by the shaper are measured using spectral interferometry and frequency-resolved optical gating, allowing the generation of transform-limited pulses and arbitrary waveforms at the end of a 300 m fiber. These arbitrary waveforms are verified by background-free autocorrelation and show good agreement with the predicted waveforms. In addition, the phase noise added by the pulse shaper and fiber transmission to a remote location is measured and removed, allowing phase-stable encoding of information on the optical phase of individual frequency comb modes. Readout of this phase-encoded information is demonstrated with spectral interferometry and with dual-comb spectroscopy. The dual-comb spectroscopy shows the transfer of optical phase information to a remote location. Readout of this optical phase information can only be accomplished when both frequency combs are self-referenced and phase locked to a high-stability optical frequency reference, demonstrating a possible route to secure communications.
I dedicate this thesis first to my parents, Wilbur and Patricia, who instilled in me a thirst for knowledge in all its forms. Thank you for developing my young mind by taking an active part in my education. Thank you for always encouraging me to do my best and not allowing me to simply get by with minimal effort. I also blame my father, Wilbur, for all the money I did not make (by becoming a petroleum engineer) as a result of following his sagely advice before college: "don’t do something you do not love just because it pays well". Thanks Dad! I have found true enjoyment (and difficulty) in studying physics and optics for ten odd years and I think I have found a profession that I will love for years to come.

I also dedicate this thesis to my loving and patient wife, Jenny, who has endured many late nights of homework and thesis writing. Thank you for always being there for me and for being proud of me. Thank you for sitting through my practice talks when you had no idea what I was talking about. Thank you most of all for believing in me and being my companion.
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Chapter 1

Introduction

1.1 Introduction to pulse shaping

Pulse-shaping of ultrafast laser pulses, where a user-defined pulse shape is generated from an input pulse, is a valuable tool in many scientific endeavors [1-13]. Femtosecond pulse shapers have allowed control over photodissociation and ionization reactions by controlling the laser pulse shape in a direct feedback method [5-6], allowing selection of which branch of the photodissociation process to favor. In addition, pulse shaping has shown use in producing coherent UV and X-ray light via the focusing of very intense femtosecond laser pulses into gasses, where careful tailoring of the pulse shape has improved the amount of X-ray generation by an order of magnitude [7, 14, 15]. Pulse shaping has also shown utility in quantum control experiments, where the spectral phases of the incident beam can steer a quantum-mechanical system (such as an atom) towards a desired state [4, 10, 16]. All of these pulse shaping applications mentioned control the spectral phases of regions of a pulsed laser’s optical spectrum, not the individual frequency components of the pulsed laser. In recent years, control over these individual frequency components has been demonstrated [17], enabling better control over the properties of the shaped pulse. This line-by-line control, as it is called, has enabled the production of arbitrary optical waveforms and has enhanced the abilities of communications techniques such as OCDMA (optical code division multiple access). Applications of optical arbitrary waveform generation and line-by-line pulse shaping are still being explored, but possible applications include the production broadband optical waveforms for LIDAR (light detection and ranging) and coherent, laser-based synthetic aperture radar [18]. Furthermore,
high speed communications can benefit from the high spectral resolution and reconfigurability. Line-by-line pulse shaping allows and transfer speeds of Tb/s have been demonstrated in arbitrary modulation formats [19]. This work focuses on the development of a line-by-line pulse shaper of unprecedented bandwidth that uses a stabilized frequency comb to produce high-fidelity, low timing jitter arbitrary waveforms. Furthermore, the low noise of this shaper and comb allow the transmission of information on the optical phase of individual frequency comb modes, enabling applications such as coherent optical communication and the possibility of the secure remote transfer of information.

The fundamental technology behind pulse shaping consists of optics that apply different phase and amplitude shifts to different regions of the optical spectrum. These optics generally consist of some sort of spectrometer that separates the optical frequencies and a modulator that applies phase and/or amplitude shifts to the spread spectrum (Figure 1.1). Pulse shaping can also be accomplished solely in the time domain [20, 22], but here we focus on frequency domain pulse shaping.

Mathematically, this pulse shaping can be represented as taking the Fourier transform of an input pulse, $E_{in}(t)$, multiplying it by a complex transfer function, $H(\omega)$, and inverse Fourier transforming to obtain $E_{out}(t)$ (Equation 1.1).

$$E_{out}(t) = FT^{-1}[FT[E_{in}(t)] \cdot H(\omega)]$$  \hspace{1cm} (1.1)

The resolution with which you can apply the transfer function $H(\omega)$ depends on the resolution of your spectrometer (in most pulse shapers, around 10-100 GHz). Early pulsed sources had frequency mode spacings of tens of MHz, so pulse shapers were forced to manipulate groups of frequency components. With the development of higher repetition rate sources, pulse shaping has evolved from manipulating pulses at a limited resolution (addressing groups of frequency modes (as in [2, 23]), to manipulating the individual frequency components of the pulsed source in a line-by-line way [17, 24-27]. These line-by-line pulse shapers address single frequency components
Figure 1.1: Basic schematic representation of pulse shaping. The top schematic is an optics-centric view of pulse shaping where an input beam is spectrally spread by a dispersive element such as a grating, and the spread colors are collimated and focused with a lens to a focal plane where a modulator manipulates the phase and amplitude of the separated frequency components. The dispersed colors are then recombined with a pass through the mirror image of the spectral dispersion optics. The bottom schematic shows what is happening in the time and frequency domain. The shaper optics first perform a Fourier transform, then a linear filter is multiplied by the frequency spectrum of the input pulse, and finally additional shaper optics inverse Fourier transform the light back to the time domain for the output pulse.
of mode-locked fiber lasers, modulated cw lasers, or frequency combs. Such line-by-line shaping enables the generation of waveforms with time durations greater than the repetition period of the source and is a route to arbitrary optical waveform generation. Furthermore, the addition of carrier-envelope phase control and frequency stabilization to line-by-line pulse shaping brings full and absolute control over the phase and frequency of a light field, enabling applications such as coherent control and optical phase encoding.

1.2 This Work

1.2.1 Stabilized Line-by-line Pulse Shaping

We build on these previous results by demonstrating the ability to individually address and manipulate single frequency components selected from a 1 GHz Ti:Sapphire frequency comb that can be stabilized in both repetition rate and carrier envelope offset frequency. The absolute stabilization of the frequency modes can provide femtosecond timing jitter in the generated waveforms as well as precise control of the carrier phase within the pulse envelope. These features will expand the capabilities of traditional pulse shaping and should enable new applications in secure communication and data transfer. A unique advantage of using this octave-spanning Ti:Sapphire comb is the opportunity to perform this line-by-line manipulation over a broad range of wavelengths from 650 nm to 1050 nm while retaining the low timing jitter provided by locking the comb to a stable optical frequency reference.

1.2.2 Creating a Stabilized High Repetation Rate Source

The stabilized Ti:Sapphire frequency comb we will use for this work has a repetition rate of 1 GHz. This is too low to be resolved by a reasonably-sized grating spectrometer, so we must find a way to increase the repetition rate to above 10 GHz. For reasons outlined in chapter 2, it is difficult to construct a fully stabilized frequency comb with a repetition rate above 10 GHz. Our solution is to use a Fabry-Perot (FP) cavity to selectively transmit every $N^{th}$ comb mode,
effectively increasing the comb repetition rate by a factor of $N$. To increase the optical bandwidth transmitted through the cavity, we use a moderate finesse cavity in a double-pass configuration (described in chapter 2 and in [28]). Using this setup, we create a $\sim$20 GHz stabilized comb source with an optical spectrum centered at 965 nm and a full width at half maximum (FWHM) of 50 nm. This work can be extended to create high repetition rate combs throughout the visible and NIR spectrum.

1.2.3 Demonstrating Line-by-Line Shaping with a Grating Shaper

We constructed a line-by-line pulse shaper to fit the 20 GHz cavity-filtered source developed before. This shaper consisted of a 1200 lines/mm grating and a 1 m lens. A dual-mask liquid crystal spatial light modulator (SLM) is placed in the focal plane to apply phase and amplitude shifts to the individual frequency comb modes (chapter 3). The shaped light is retroreflected and recombined by a second pass through the lens and grating, and the output is picked off by an isolator. Because of the non-uniform angular dispersion of the grating (subsection 3.3.3), only 120 modes out of a possible 640 can be lined up one-to-one with the SLM pixels and modulated in a line-by-line way. We demonstrate simple line-by-line control of these 120 modes by attenuating every other comb mode and doubling the repetition rate to 40 GHz. In addition, we examine the effect the filter cavity and shaper setup have on the phase noise properties of the stabilized comb. We find that the cavity and shaper add phase noise above the inherent phase noise of the comb when locked to an optical standard. We further show that this phase noise can be reduced by feeding back to the path length of the shaper via a mirror mounted on a piezoelectric transducer (PZT). Using the phase-locked shaper, we demonstrate encoding and readout of discrete phase steps on the optical phase of one comb mode.

1.2.4 Extending the Bandwidth of Line-by-line Control

To take full advantage of the 640 pixels available on our SLM, it is necessary to find a way to line up every mode in the focal plane to the center of an SLM pixel. To do this we require that the
angular dispersion of the dispersive shaper element be linear across the bandwidth of the frequency comb, that is \( d^2 \theta / df^2 \approx 0 \) (where \( \theta \) is the diffraction angle and \( f \) is the optical frequency). A grating does not satisfy this requirement, but fortunately, its dispersion can be modified by a tilted dielectric interface, such as a prism. With the appropriate choice of grating, prism, and input angle, \( d^2 \theta / df^2 \) can be set to zero at the center frequency of the comb source. We demonstrate the use of a grism (grating + prism) to extend the bandwidth of line-by-line control to 640 modes separated by 21 GHz. With this grism-based shaper, each comb mode is aligned to within 10 \( \mu m \) of the SLM pixel centers. The 13.4 THz bandwidth shown is the broadest ever controlled in a line-by-line fashion.

1.2.5 Remote Transfer of Shaped Pulses

In real world applications of this line-by-line pulse shaping, it will sometimes be desirable to transmit the shaped pulses to a remote location via a fiber optic cable. For fiber lengths above a few meters, acoustic and thermal perturbations of the fiber add significant phase noise and timing jitter to the transmitted pulses [29]. This fiber noise can be reduced by actuating the round-trip group delay to a constant value. However, for line-by-line pulse shaping, monitoring the group delay, either in the microwave or optical domain, is not feasible because the group delay and pulse shape can be changed arbitrarily by the shaper. We demonstrate a new technique, based on monitoring the phase of one comb tooth and actuating on the group delay, that effectively cancels the fiber transfer phase noise while allowing arbitrary phase and amplitude masks to be applied to the modes. We demonstrate the effectiveness of this method by both measuring the phase noise across the comb and performing spectral interferometry. We then encode images onto the phase of the comb that can be read out with spectral interferometry.

1.2.6 Characterization of Shaped Pulses

Because of the high repetition rate, low peak power, and pulse complexity possible from the output of the line-by-line shaper, it is difficult to characterize the produced wave forms with
commonly-used techniques such as frequency resolved optical gating (FROG) [30, 31]. Measuring
the output pulses therefore requires some method that independently finds the amplitude and phase
of each comb mode without requiring high peak powers or non-linear optics. We investigate two
methods that accomplish this line-by-line characterization. The first is a self-referenced technique
that uses an intensity modulator to apply sidebands to each comb mode at half the repetition
rate [32]. Adjacent comb mode sidebands interfere based on the phase difference between the
comb modes. Because you integrate across the whole spectrum to extract the phase of the comb,
this method can be susceptible to noise and systematic errors. In addition, the high-frequency
modulator adds an unknown amount of dispersion from the input fiber and modulator element to
the comb phase measurement. Because of these disadvantages, we employ a second characterization
known as TADPOLE (temporal analysis by dispersing a pair of light e-fields) [33], where a strong
reference pulse is characterized by FROG [30, 31], and this reference pulse is used to characterize
the weak shaped pulse via spectral interferometry. Using this technique, we characterize the phase
and amplitude of a pulse in a line-by-line way at the end of a 300 m long fiber. The line-by-line
pulse shaper is then used to compress the pulse to its transform limit and apply several phase-only
masks to generate double and triple pulses with varying time delays. The TADPOLE results are
independently verified by background-free second harmonic (SHG) autocorrelation.

1.2.7 Remote Readout of Optical Phase-Encoded Information with a Second
Independent Frequency Comb

We demonstrate the frequency-stabilized remote transfer of a phase-encoded frequency comb
by reading out the phases of the comb modes with a second, independently frequency-stabilized
frequency comb. We combine the shaped comb at the end of a 300 m fiber with unmodulated
light from the second frequency comb. We detune the repetition rates of the two sources so that
each comb beat is at a distinct frequency. Digitizing and Fourier transforming the interferogram
directly gives the phase difference between the two combs for each comb tooth (similar to [34, 35]).
Phase information can then be encoded and read out. This phase information cannot be extracted
by a frequency comb locked to a less stable laser or even by a frequency comb microwave-locked to a high stability microwave standard like a hydrogen maser. This demonstrates a possible route to secure communication where small phase steps are encoded that can only be read out by an optically-stabilized comb with Hz-level optical linewidth.
Chapter 2

Frequency Comb Source for Line-by-Line Pulse Shaping

2.1 Source Requirements

In order to achieve line-by-line pulse shaping, it is necessary to separate the comb modes spatially so that you can address each individual frequency element. This is typically done with a high resolution device such as a grating and lens spectrometer [1], a Virtually Imaged Phased Array (VIPA) spectrometer [36], or a fiber-based Arrayed Waveguide Grating (AWG) [37]. Typical resolutions obtainable for tabletop-scale spectrometers are $\sim 5$-10 GHz for grating based spectrometers and $\sim 1$ GHz for VIPA spectrometers, while channel spacings for AWGs are typically 5-100GHz. For this work, I chose to use a grating-based spectrometer because of the relative simplicity of the setup (Figure 2.1), and because VIPA spectrometers typically operate either over a narrow spectral bandwidth or in two dimensions where modulators are more difficult obtain. Because of the limited resolution of the grating spectrometer, the frequency comb source would need to have a mode spacing (repetition rate) of greater than 10 GHz to achieve the good separation of the comb modes necessary for line-by-line pulse shaping.

In addition, one of the goals of this work was to implement line-by-line pulse shaping on a stabilized frequency comb. Stabilizing the source comb will ensure that the output waveforms have low timing jitter (e.g. at the femtosecond level (or lower)) and are stable in time [38]. This means the source laser needs to have the ability to be stabilized in both repetition rate and offset frequency. The offset frequency can be locked by mixing doubled light from one comb mode with light from a comb mode at twice the original frequency, as shown in Equation 2.1.
Figure 2.1: Grating-based pulse shaper. A diffraction grating angularly disperses the comb modes onto a lens a focal length $f$ away, which focuses the comb modes to a common focal plane another distance $f$ away. A modulator is placed just in front of the focal plane to apply amplitude and phase shifts to the frequency comb. The mirror in the focal plane retroreflects the comb modes back through the shaper to recombine them into a single output beam. The geometry shown here is known as a $4f$ pulse shaper because the total length of the shaper is 4 times the focal length of the lens.
This method of detecting the offset frequency requires a full optical octave either directly from the laser oscillator \[39\], or from external broadening with a nonlinear element such as microstructured fiber \[40\]. The requirements for locking the repetition rate are less strict; you can either stabilize the repetition rate directly with a microwave phase lock, or you can lock one tooth of the comb to an optical reference such as the laser oscillator for an optical atomic clock \[41–46\]. Locking to an optical reference is preferable because it maps the phase stability of the optical reference onto every tooth of the frequency comb, and optical standards can have 2 or 3 orders of magnitude better phase stability than microwave references \[47\]. Therefore, the desire for a stabilized high repetition rate frequency comb requires a source that spans an optical octave, or can be broadened to do so, and has spectral overlap with one of the 5 optical standards currently available at NIST: 578 nm (Yb lattice \[44\]), 657 nm (neutral Ca \[41\]), 1064 nm (Sr lattice transfer laser \[45\]), 1068 nm (Al ion logic clock \[46\]), and 1126 nm (Hg ion clock \[42\]). After stabilizing both \(f_{\text{rep}}\) and \(f_0\), every comb mode has a well-defined frequency given by:

\[
f_n = n \cdot f_{\text{rep}} + f_0,
\]

where \(f_n\) is the frequency of the \(N^{th}\) comb mode.

There are many different technologies for producing a high repetition rate frequency comb, especially in the telecom wavelengths, but most of these combs are narrow bandwidth and do not span the optical octave necessary for offset frequency detection \[48–55\]. A promising frequency comb technology for low phase noise and good stabilization is the octave-spanning Ti:Sapphire laser \[56\]. This comb technology has shown absolute phase noise of -105 dBc/√Hz at 1 Hz offset from a 10 GHz harmonic of \(f_{\text{rep}}\) when locked to an optical standard, mirroring the stability of the optical standards \[47, 57\], and has shown residual frequency instabilities below \(10^{-19}\) \[58\]. However, it is difficult to extend the repetition rate of these systems to above 2 GHz while maintaining the
optical octave directly from the cavity \[56\]. Non-octave-spanning Ti:Sapphire frequency combs up to 10 GHz have been broadened to an optical octave via microstructured fiber \[51, 53\], but those combs have proven difficult to work with and the microstructured fiber adds some excess noise. If developed further, those high repetition rate combs will be good sources for line-by-line pulse shaping. For this work, however, a suitable compromise between high performance and ease of use is struck by using a lower repetition rate, octave-spanning oscillator and using a filter cavity to multiply the repetition rate up to the desired value.

2.2 Double-Pass Cavity Filtering to Produce a 20 GHz Frequency Comb

In addition to waveform generation \[1, 2, 17, 59\], high repetition rate mode-locked lasers and their associated frequency combs are useful for applications such as communications, frequency synthesis \[57, 60, 61\], and the calibration of astronomical spectrographs \[62, 63\]. Many of these applications require a stabilized frequency comb; however, for mode spacings above a few gigahertz, the spectral width required for frequency stabilization via self-referencing \[40\], as discussed before, is difficult to achieve because the low pulse energy does not drive non-linear processes as effectively. As a solution, we begin with an octave-spanning self-referenced 1 GHz Ti:sapphire frequency comb \[56\], then optically filter it to 20 GHz using an air-spaced Fabry-Pérot (FP) etalon in a double-pass configuration. The FP filtering approach, where a resonant cavity selects every \(N^{th}\) comb mode (increasing the repetition rate by a factor of \(N\) while decreasing the average power by a factor of \(N\) and pulse energy by \(N^2\)), has been well known for many years \[64, 65\]. More recently, the double-pass FP filtering approach has been implemented in fiber cavities over a narrow bandwidth \((\Delta \lambda/\lambda \approx 0.1\%)\) \[66\]. We demonstrate that a double-pass FP filter cavity can support both broad coupling bandwidths \((\Delta \lambda/\lambda \approx 10\%)\) and high suppression (greater than 50 dB in intensity) of off-resonant modes. The pulses from the filter cavity, centered at 960 nm, can be compressed to sub-40 fs pulses which, to our knowledge, are the shortest pulses produced at a 20 GHz rate. The residual amplitude modulation at 1 GHz on the 6 mW output is less than 2%. A semiconductor optical amplifier centered at 970 nm with a bandwidth of 45 nm can be used to compensate for
additional losses from pulse shaping (∼10dB) to maintain average power levels at the 5-10 mW level with only a small increase in pulse duration. This unique comb source with over 1500 modes is useful for low timing jitter line-by-line waveform generation and astronomical referencing, where the large mode spacing and absolute frequency stability are critical [17, 38, 62, 63]. Furthermore, the same techniques demonstrated here are generally applicable to any mode-locked laser in the visible and near infrared spectral regions.

The experiment employs an octave-spanning (550 nm to 1200 nm) Ti:Sapphire frequency comb with a repetition rate of 1 GHz [56]. A schematic of the frequency comb cavity and optical output spectrum are shown in Figure 2.2. A standard f-2f interferometer is used to detect the offset frequency for locking. The wavelength regions near 657 nm or 1068 nm are used to lock the comb to a stable CW optical reference [41, 46], or to detect the repetition rate for locking to a microwave frequency standard [67–69]. In the case of the optical reference, this approach yields comb linewidths at the ∼1 Hz level and sub-femtosecond timing jitter [47]. We then send the portion of the octave spanning spectrum from 800-1050 nm onto a double-passed filter cavity to create the high repetition rate comb. Chen et al. have shown that cavity filtering causes a minimal increase in timing jitter [65]. Figure 2.3 shows the system setup for this cavity filtering.

The portion of the spectrum in the range of 800-1050 nm (not used for stabilization) is reflected six times off planar mirrors with the same reflection band as the mirrors of the filter cavity to reduce the out-of-band light incident on the cavity. The beam is then sent through a polarizer and into the cavity, which has a mirror separation that is easily varied by a translation stage. For the purposes of this discussion, we will set the cavity length to ∼7.5 mm, yielding a repetition rate of ∼20 GHz. A standard dither lock is employed that overlaps the cavity resonances with every 20th 1 GHz comb mode, yielding a filtered mode spacing of 20 GHz. We implement the double-pass cavity by retroreflecting the single-pass cavity light back into the cavity with a curved mirror (see Figure 2.3). A broadband Faraday isolator without an input polarizer rotates the light to the orthogonal polarization for the second pass through the cavity. The use of an isolator instead of a quarter-wave plate was found to be critical to mitigate coupled cavity effects. An advantage
Figure 2.2: (top) Model of the 1 GHz octave-spanning Ti:Sapphire laser cavity. The cavity consists of four mirrors in a bow tie configuration, a laser crystal, and a glass plate for fine-tuning cavity dispersion. An output coupler provides a 1% output while three chirped mirrors compensate for the group delay dispersion of the laser crystal and glass plate to make the net cavity dispersion slightly negative. The 532 nm pump beam is coupled into the cavity through one of the chirped mirrors. The laser output power is approximately 1 W for 8 W of input pump power. (bottom) Octave-spanning optical spectrum shown on a log scale. The laser produces usable light from 550 nm to above 1200 nm.
Figure 2.3: A stabilized portion of the optical bandwidth (800-1050 nm) of a 1 GHz input frequency comb is filtered by double-passing through a 20 GHz Fabry-Pérot cavity. The forward and backward passes through the cavity are in orthogonal polarization states and the double-passed output is picked off with a polarizer. The Faraday rotator and polarizer act to reduce coupled cavity effects. The 20 GHz cavity is locked on resonance with the comb via a dither lock by actuating on a PZT behind one of the mirrors. One advantage to double-passing a single cavity instead of using two separate cavities is the need for only one locking servo.
of double-passing a single cavity over using two separate cavities is the need for only one locking servo, simplifying the control electronics.

Careful consideration is given to the choice of cavity mirrors to yield broad bandwidth and high suppression of unwanted modes. Although suppression (defined as the ratio between the intensity transmission of an on-resonant mode and its nearest off-resonant neighbor) improves with cavity Finesse, the accompanying dispersion and narrower cavity linewidth of higher Finesse cavities decrease the usable bandwidth. The narrower the cavity linewidth, the more sensitive the coupling is to cumulative phase walkoff (dispersion), so the optimal cavity mirrors require a compromise between high reflectivity and coupling bandwidth \[61, 63\].

![Figure 2.4: Dispersion shown for a typical quarter wave stack mirror. At lower cavity mirror reflectivity (lower cavity Finesse), off-resonant comb modes are not attenuated as much, leading to some transmission of the unwanted frequency comb modes. The lower Finesse cavity, however, is intrinsically less sensitive to comb mode/cavity mode walk-off due to dispersion in the cavity mirrors leading to a broader bandwidth of good coupling. The higher reflectivity mirrors (higher cavity Finesse) give greater suppression, but are more sensitive to mode walk-off caused by dispersion leading to a narrower bandwidth of good coupling.](image)

To illustrate this trade-off, we model a FP cavity consisting of two quarter-wave stack mirrors spaced by \(L=7.5\) mm (giving a repetition rate of \(\sim 20\) GHz). Quarter-wave stack dielectrics provide the simplest route to low-loss, low-dispersion mirrors of high reflectivity and can be easily analyzed. We evaluate standard equations for the characteristic matrix of a quarter-wave stack composed of
a fused silica substrate and $N$ pairs of high and low index quarter wave layers ($n_H=2.2$, $n_L=1.46$, $\lambda/4$ centered at 900 nm) \cite{70}. The reflectivity and phase shifts are calculated for $N=7$ through $N=13$ layer pairs (Finesse from 100 to 20,000).

Using these calculations, the coupling bandwidth and nearest-neighbor suppression are calculated for a 1 GHz frequency comb filtered by a 20 GHz cavity. The $\lambda_0 = 900$ nm mode is aligned with the center of the cavity mode, however, the unequally spaced cavity modes will walk off of the comb modes after a certain bandwidth because of dispersion in the mirrors.

Figure 2.5 shows that suppression is increased with Finesse at the cost of decreased coupling bandwidth; however, the suppression can be doubled (on a log scale) by double-passing the FP cavity with only a 5\% to 15\% decrease in coupling bandwidth. Therefore, a much higher bandwidth can be achieved with a double-pass cavity than with a single-pass cavity for the same suppression of off-resonant modes. For example, to obtain 60 dB of off-resonant mode suppression, a single-pass cavity will require a Finesse of around $10^4$ and have a bandwidth of $\sim 80$ nm \cite[Figure 2.5 right dotted ellipse]{}, while a double-pass cavity will require a Finesse of around 400 and have a bandwidth of $\sim 160$ nm \cite[Figure 2.5 left dotted ellipse]{}. For this reason, we have chosen to employ a moderate Finesse cavity ($\sim 300$) in a double-pass configuration to achieve high suppression and a large coupling bandwidth. The output spectrum of the octave-spanning frequency comb employed here has a broad swath of light from 800 nm to 1000 nm that is unused for stabilization purposes, so the cavity mirrors were chosen with a center wavelength of 900 nm. After the coating of the mirrors, it was realized that the majority of the comb power was in a bandwidth from 900-1020 nm with the peak at 965 nm. It would have been better to center the mirrors at 960-965 nm, but the 900 nm mirrors work well enough for these experiments.

We examine the spectral and temporal properties of the output from the filtering cavity in both single and double-pass configurations. The bandwidth of efficient coupling for the single-pass and double-pass cavities are 124 nm and 104 nm, respectively \cite[Figure 2.6a]{}. This is less than the bandwidth predicted by our simulation because the input spectrum is peaked around 960 nm and not the 900 nm center of the mirror reflectivity. Since the cavity locks to the highest power
Figure 2.5: Simulation of quarter-wave stack mirrors and associated Fabry-Pérot cavities. The coupling bandwidth (BW) decreases with increasing cavity finesse due to the increased sensitivity to dispersion. Double-pass suppression is not shown as it is exactly double the single-pass suppression, e.g., 60 dB versus 30 dB. For an off-resonant mode suppression suppression of 60 dB, a single pass cavity would require a Finesse of around $10^4$ and have a bandwidth of $\sim 80$ nm (right dotted ellipse), while a double-pass cavity will require a Finesse of around 400 and have a bandwidth of $\sim 160$ nm (left dotted ellipse).
Figure 2.6: Input and output spectra for single and double-pass cavities (power per mode). Most of the input bandwidth (50 nm FWHM) is coupled for both cases giving output bandwidths of \( \sim 40 \text{ nm (FWHM)} \). (a) Coupling ratio (defined as output power over input power) for the single and double-pass cavities. Single-pass shows 124 nm (FWHM) coupling bandwidth. Double pass shows 104 nm (FWHM) coupling bandwidth. (b) Zoomed view of high resolution (0.02 nm) optical spectrum analyzer trace showing individually resolved modes at 20 GHz.
throughput, the lock centers around this peak. The FP simulation showed a similar decrease in bandwidth for locking the cavity 60 nm away from the center of the mirror reflection bandwidth. The input power at 1 GHz is 250 mW, and the output power at 20 GHz for the single-pass cavity is 7.5 mW, while the output for the double-pass cavity is 6.0 mW. This compares to the 12.5 mW that would be expected if the cavity selected one of every 20 optical modes without additional loss. Much of the loss for the single-pass case is due to the non-ideal spatial mode of the input light (leading to inefficient coupling into the cavity optical mode) and from the rejection of modes outside the coupling bandwidth, while most of the additional loss for the second pass through the cavity results from isolator losses.

A significant advantage of the double-pass geometry is the doubling in suppression of unwanted (off-resonant) modes. One way to examine the suppression of off-resonant modes is a microwave measurement of the \( f_{\text{rep}} \) harmonics with a fast photodiode and spectrum analyzer (Figure 2.7). This microwave measurement gives a general picture of the suppression, but each harmonic beat note is the summation of all the different comb mode pair-wise interactions with that difference frequency. The microwave measurement therefore cannot reveal the mode suppression for a single on-resonant comb mode and its off-resonant neighbors, which will change across the bandwidth of the comb due to cavity dispersion. So, we turn to heterodyne detection to directly measure the powers in the comb modes near a CW diode laser. This CW diode laser can be tuned across most of the optical spectrum of the 20 GHz cavity-filtered source. Example RF spectra are shown in Figure 2.8.

The single-pass cavity shows 27 dB of off-resonant mode suppression, while the double-pass cavity provides 50 dB of off-resonant mode suppression. The impact of this off-resonant mode suppression is clearly seen in the time-domain waveforms of Figure 2.9. Here the outputs of both the single and double pass 20 GHz filter cavities were detected with a 45 GHz photodiode and displayed on a 50 GHz sampling oscilloscope.

The single-pass cavity shows more than 30% amplitude modulation at the original 1 GHz repetition rate, while the double-pass cavity shows less than 2% amplitude modulation. Figure 2.9(c)
Figure 2.7: Microwave spectra of the 20 GHz cavity-filtered comb detected with a 45GHz photodetector. The single pass cavity spectrum shows significant power in the undesired microwave harmonics. The double pass cavity spectrum shows all microwave harmonics but the 20\textsuperscript{th} suppressed by greater than 50dB.
Figure 2.8: (a) CW heterodyne beat of a 960 nm diode laser with the single-pass cavity output. In addition to the CW beat notes indicated by solid circles, harmonics of the 1 GHz repetition rate are evident indicated by dotted circles. The largest peak in the figure is the beat of the CW laser against a comb tooth tuned to the cavity resonance. The nearest neighbor off-resonant comb teeth are suppressed by 27 dB. (b) CW heterodyne beat of a 960 nm diode laser with the double-pass cavity output. Both the harmonics of the 1 GHz repetition rate and the off-resonant comb mode beat notes are attenuated greatly. The off-resonant mode suppression is increased to 50 dB.
Figure 2.9: (a) The single-pass cavity time-domain signal with 30% amplitude modulation caused by the transmission of off-resonant comb modes. (b) The double-pass cavity time-domain signal shows less than 2% modulation. (c) Intensity autocorrelation trace showing a pulse width of 36 fs after compression by SF10 prisms. The transform limit is 30 fs.
shows the autocorrelation of the 20 GHz pulses out of the double-pass filter cavity, which have been compressed with prisms to a duration of less than 40 fs. Applications such as pulse shaping will introduce inevitable losses, so we have additionally employed a broadband semiconductor optical amplifier (SOA) at 970 nm that provides up to 20 dB of gain (QPhotonics QSOA-980). With this SOA we have amplified the 20 GHz comb after a shaper from ∼100 µW up to 10 mW while still maintaining pulses as short as 70 fs with a bandwidth of 30 nm (Figure 2.10).

This high-fidelity 20 GHz frequency comb produced with the double-pass cavity is appropriate for waveform generation, communications, frequency synthesis, and the calibration of astronomical spectrographs. In particular, the precise frequency control and broad bandwidths should enable line-by-line femtosecond optical waveform generation with controlled carrier-envelope phase and low timing jitter.
Figure 2.10: a) Optical spectrum of 20 GHz comb after amplification by a semiconductor optical amplifier (SOA) to $\sim$10 mW. The frequency comb retains most of the input comb bandwidth because of the broad gain bandwidth of the SOA (25-35 nm). b) Compressed pulse after amplification measured by background-free autocorrelation. The pulse width is approximately 70 fs.
Chapter 3

Construction of Line-by-Line Pulse Shaper

3.1 Choice of Modulator

Following the development of the high-repetition-rate, stabilized frequency comb source described in chapter 2, it is now necessary to choose a suitable modulator to achieve line-by-line pulse shaping. There are many different reconfigurable modulator technologies to choose from, each with their own advantages and disadvantages. The first option to decide upon is the format for coupling the modes into the modulator. There are 1-dimensional [1], 2-dimensional [36, 71], and waveguide modulators available [72]. The waveguide modulators have the advantage of high-speed phase and amplitude modulation, but it is difficult to scale to a large number of modulator elements and these modulators work best at 1550 nm. Since my comb source is around 1 micron and we desire to control as many modes as possible, a waveguide modulator is not a good fit for this work. This leaves 1-dimensional and 2-dimensional modulators. In general, 2-dimensional modulators (such as liquid crystal on silicon devices) have many more pixels than 1-dimensional modulators, but generally only offer phase control. Amplitude control can be achieved through writing variable strength phase gratings onto pixel regions, with the depth of the phase grating determining the percent of power diffracted to the output of the shaper [36, 71]. This method of amplitude control effectively decreases the number of pixels available for independent modulation, but recent work has shown that this technique can control 450 modes at 10 GHz with a high-definition LCOS array (1920x1080 pixels) and a VIPA-grating shaper and should be scalable to several times that many modes [36]. At the time of starting this project, 2-dimensional modulators with a large pixel count were not as
prevalent, but there were 1-dimensional modulators available with 640 individually addressed pixels. In addition, 1-dimensional shaper configurations have been well studied and characterized [1]. For this reason, we chose to employ a 1-dimensional 640 pixel liquid crystal spatial light modulator (SLM).

Spatial light modulators are based on individually addressed cells or regions of liquid crystals. The liquid crystals are oriented so that phase and polarization shifts can be applied to light passing through the cell proportional to the drive voltage applied to the liquid crystals. The polarization shifts can be converted into amplitude shifts with a suitably oriented polarizer. One of the more common configurations of a SLM is shown in Figure 3.1.

The modulator we chose contains two separately addressable liquid crystal masks, giving independent control over the transmission and phase shift of each pixel. This modulator is divided into 640 individually addressable pixels spaced by 100 µm with 2 µm of dead space between each pixel. The transmission and phase shift of each pixel is related to the phase shifts applied to each mask ($\Phi_A$ and $\Phi_B$) as shown by Equation 3.1 and Equation 3.2.

$$T = \cos^2\left(\frac{\Phi_A - \Phi_B}{2}\right)$$  \hspace{1cm} (3.1)

$$\Phi = \frac{\Phi_A + \Phi_B}{2}$$  \hspace{1cm} (3.2)

These equations can be solved for the appropriate $\Phi_A$ and $\Phi_B$ that give a desired transmission, $T$, and phase shift, $\Phi$, as shown in Equation 3.3 and Equation 3.4.

$$\Phi_A = \Phi + \cos^{-1}\left(\sqrt{T}\right)$$  \hspace{1cm} (3.3)

$$\Phi_B = \Phi - \cos^{-1}\left(\sqrt{T}\right)$$  \hspace{1cm} (3.4)

By aligning the pulse shaper so that each mode is resolved and separable and passes through a single pixel (or group of pixels) of the SLM, arbitrary phase and amplitude masks can be applied
Figure 3.1: Typical SLM configuration. Each transparent ITO (indium tin oxide) electrode is capable of applying a voltage across the liquid crystal cell, which induces orientation changes in the liquid crystal, leading to polarization and phase changes in light passing through the cell.
to the frequency comb and line-by-line pulse shaping can be realized. Figure 2.1 shows the general design for the pulse shaper to be constructed in this work.

3.2 Calibration of Spatial Light Modulator

In order to use the SLM for pulse shaping, it is necessary to calibrate the voltage response of the liquid crystal masks. The manufacturer provides a rough calibration, however, the phase response of each liquid crystal element can change based on temperature, mechanical stress, and wavelength of operation. Therefore, it is critical to measure $\Phi_A(V)$ and $\Phi_B(V)$ directly in the shaper setup.

It is possible to directly measure the phase response of each liquid crystal pixel by performing the following procedure.

1. Focus a CW laser at the operational wavelength onto a single pixel through a polarizing beamsplitter (retroreflection mode described here, see Figure 3.2).

2. Monitor both the power incident on the pixel (with a beam pickoff or PBS) and the power from the rejection port of the polarizing beamsplitter in the retroreflected direction. Alternatively, monitor the back-reflected power via a beam pickoff or isolator.

3. While monitoring the ratio of light rejected (or transmitted) from the analysis polarizer to the light incident on the pixel, scan the voltage of SLM Mask A while keeping Mask B at a constant voltage ($V_{Bo} = 0$ was used for the scans shown in Figure 3.3).

4. The ratio of output over input power will trace a sinusoidal curve that will characterize the phase shift imparted by Mask A (for the constant Mask B voltage ($V_{Bo}$)). It is important to distinguish between the case of monitoring the SLM backreflected light that is reflected from the analysis polarizer and monitoring the SLM backreflected light that is transmitted through the analysis polarizer via a pickoff or an isolator. The former case is shown in Figure 3.2 and requires that you subtract the measured ratio from 1 ($T = 1 - R$) to give
transmission data as required by Equation 3.1. The latter case requires no processing as transmission is being monitored directly.

(5) Repeat the measurement with Mask A held at a constant phase while the voltage to Mask B is scanned \((V_{Ao} = 0\) was used for the scans shown in Figure 3.3).

Figure 3.2: SLM calibration setup. The ratio of power reflected from the analysis polarizer to the input power is monitored as the voltages to Mask A and Mask B are scanned. The 960 nm CW laser is focused onto one pixel of the modulator.

Since only a subset of the drive voltage range is needed to calculate the calibration, it is prudent to take a higher resolution scan (drive voltage step of 1 or 2) over a more limited drive voltage range. Figure 3.4a shows such a series of scans with a drive voltage step of 2 (total drive range is 4096 voltage steps). Based on the raw data, it is possible to extract the phase-voltage relationships for each mask \((\Phi_A(V)\) and \(\Phi_B(V)\)). For line-by-line pulse shaping, it is only necessary to apply phase shifts in the range of \(0 - 2\pi\), so it is possible to choose any \(2\pi\) range of phase shift for either modulator. The pixels of an SLM are driven with a digital voltage signal with a certain resolution (8-bit, 12-bit,...). The modulator we use for this work has a 12-bit resolution and so there are 4096 distinct drive voltages. Therefore, it is desirable to choose a \(2\pi\) phase shift range
Figure 3.3: Calibration curves ($T = 1 - R$) for Mask A and Mask B measured with a 960 nm CW diode laser focused onto a single pixel of the SLM (drive voltage step = 5). Both Mask A and Mask B trace out a phase shift of over $6\pi$ radians.
to maximize the number of discrete drive values, effectively maximizing the phase and amplitude shaping resolution. For this modulator, we choose a $2\pi$ phase shift range from the upper end of the drive voltage range.

There are many ways to extract the phase-voltage relationship for each mask from the raw data. The first step to extracting $\Phi_A(V)$ and $\Phi_B(V)$ is to solve Equation 3.1 for $\Phi_A(V)$ (and $\Phi_B(V)$) with the constant mask phase $\Phi_B(V_{Bo})$ (and $\Phi_A(V_{Ao})$) for the transmission scan $T_A$ (and $T_B$).

$$\Phi_A(V) - \Phi_B(V_{Bo}) = \pm 2 \cos^{-1}\left(\sqrt{T_A}\right) \quad (3.5)$$

$$\Phi_A(V_{Ao}) - \Phi_B(V) = \pm 2 \cos^{-1}\left(\sqrt{T_B}\right) \quad (3.6)$$

The phase-voltage relationships, $\Phi_A(V) - \Phi_B(V_{Bo})$ and $\Phi_A(V_{Ao}) - \Phi_B(V)$, can be extracted as shown in Figure 3.4.

The unwrapped $\Phi_A(V) - \Phi_B(V_{Bo})$ and $\Phi_A(V_{Ao}) - \Phi_B(V)$ curves shown in Figure 3.4 can be fit to smooth out the phase functions and remove measurement noise. $\Phi_B(V_{Bo})$ can then be set to any arbitrary value (usually 0), and from there you can solve directly for $\Phi_A(V)$, then you plug the calculated $\Phi_A(V_{Ao})$ into the second measurement and extract $\Phi_B(V)$. From here it is useful to invert the equations to obtain a formula for the voltage required to obtain a desired phase shift ($V_A(\Phi_A)$ and $V_B(\Phi_B)$). Again, only a voltage range that gives a total phase shift range from $0 - 2\pi$ is necessary.

### 3.3 Grating-Based Pulse Shaper Design

Once a suitable modulator is chosen, the rest of the shaper can be designed around it. The SLM chosen for this setup has pixels 100 microns wide, so the shaper setup needs to provide focused comb modes in a single plane dispersed by some integer multiple of 100 microns. Typical values for comb/SLM alignment in one-dimensional shapers are 1-3 pixel/comb mode. We chose to attempt
Figure 3.4: a) Raw data from SLM calibration scan with $V_{Bo} = 4095$ and $V_{Ao} = 3600$ with a drive voltage scan step of 2. b) Phase relationships extracted as indicated in Equation 3.5 and Equation 3.6. c) Same as b) but with the sign ambiguity removed by switching the sign every peak or valley of the drive voltage scan while keeping the trend of the phase shifts to be increasing with increasing drive voltage (either convention would work). d) Unwrapped phase showing around $6\pi$ of total phase range. e) $\Phi_B(V_{Bo})$ was chosen to be 0.05 radians to give the largest drive voltage range for a total phase shift of $0 - 2\pi$. $\Phi_A(V_{Ao})$ was determined from the calculated $\Phi_A(V)$ curve. f) Curve showing the drive voltage necessary for a given desired phase shift. This function is the main function used to calculate what drive voltages to apply to Mask A and Mask B for the desired transmission and phase.
the densest possible arrangement of 1 pixel/comb mode. In this way, we could take advantage of all 640 modulator elements provided by the SLM. For this shaper, a grating serves as a spectral angular disperser, while a lens collimates and focuses the comb modes as shown in Figure 2.1.

This shaper was designed to address one frequency comb mode per modulator element, and as a result, there is little buffer between adjacent comb modes (the two microns of pixel dead space). For this reason, it is desirable to have the spot size at the SLM masks as small as possible. However, there is some physical distance (about 2 mm) between the retroreflection mirror that is in the focal plane and the SLM masks located just in front of the mirror, so diffraction limits the achievable spot size at the masks. A diagram of the SLM optics with an example of diffraction effects for a 20 micron waist Gaussian beam at 965 nm is shown in Figure 3.5. The graph portion to the right of Figure 3.5 plots the beam waist at the second SLM liquid crystal mask as a function of the beam waist at the focal plane. In addition, the graph also plots the percentage of the integrated mode power contained within the 98 micron aperture for a single SLM pixel at 1.7mm from the focal plane (the minimum possible distance from the last SLM mask to the mirror). As shown on the graph, a beam waist of about 18 microns at the mirror minimizes the beam waist at Mask 2. The percentage of total power contained within the pixel at this beam waist is 99.98%.

The aperture and focal length of the lens and the aperture of the diffractive element as well as any aberrations in these optics will limit the minimum achievable spot size. Further consideration should be given to the sensitivity to focal plane and x-position misalignment, as the smaller spot sizes will be more sensitive to mirror-focal plane offsets but less sensitive to x-offsets from true pixel center. Figure 3.6 shows the percentage of the total beam power contained within the aperture of the second LC layer (furthest from the mirror) for different waist sizes under x and z misalignment.

Because of the increased focal plane misalignment sensitivity, we will not choose the smallest spot size allowed by diffraction, but a compromise between crosstalk and misalignment sensitivities. Each pulse shaper configuration will have a different optimum beam waist, but a good compromise for the configuration of the pulse shapers described here is a beam waist of 30-35 microns ($1/e^2$ diameter of approx. 60-70 microns). To obtain the 30-35 micron spot size at the focus, the beam
Figure 3.5: Single pixel of the Spatial Light Modulator (SLM) with a Gaussian beam propagation shown. Pixel width is 100 microns and the structure consists of two 16 micron thick liquid crystal (LC) layers each sandwiched between two glass slides, which are then glued together as shown above. The liquid crystal layer furthest from the focal plane mirror is approximately 1.7 mm from the focus. A beam with a waist of about 20 microns at the focus (1/e waist) expands to a waist of about 26.5 microns by the second LC layer (graph, blue curve, right axis). At this beam waist, 99.98% of the integrated power is contained within the SLM pixel at the second LC layer (graph, red curve, left axis).
Figure 3.6: a) Percent of mode power within SLM pixel vs. focal plane offset for various beam waists. Smaller beam waists contain more power in the pixel for perfect alignment, but are much more sensitive to focal plane offsets. b) Percent of mode power contained within SLM pixel vs. x-offset for various beam waists. Smaller beam waists are less sensitive to offsets from the pixel center.
input to the pulse shaper will need to be around 2 cm in diameter \((1/e^2)\) for a 1 m focusing lens. Large optics in some parts of the shaper (greater than 1 inch) will be required to avoid clipping the large beam, especially when the modes are dispersed spatially. In this work, a 60 mm grating and a 76.2 mm (3 inch) achromatic lens are used.

The analysis presented above was for a single wavelength focused on a single pixel, but in practice, we need to realize this beam waist and focal plane alignment across the entire bandwidth of the input frequency comb. The diffraction-limited spot size will not change much across the bandwidth of the comb for a constant input spot size, but the focal position can be changed by several millimeters across the 40 nm of possible shaped spectrum by a focusing element such as a plano-convex lens. To reduce chromatic mode size and focal position differences, it is desirable to use an achromatic focusing element such as an off-axis parabolic mirror or an achromatic doublet lens. Achromatic lenses are much cheaper and easier to obtain in a variety of focal lengths than off-axis parabolic mirrors, so I will use these in the construction of my pulse shaper. In addition, achromatic lenses have the benefit of decreasing spherical and coma aberrations compared to a singlet lens, which will improve the performance of the pulse shaper in the wings of the spectrum.

The rest of the shaper can be designed with some flexibility, and several different lens-grating-repetition rate combinations will give the correct mode spacing in the focal plane. For the 4\(f\) pulse shaper configuration described in Figure 2.1 the mode spacing in the focal plane \((\Delta x)\) is given by:

\[
\Delta x = f \sin(\Delta \theta) \approx f \Delta \theta, \tag{3.7}
\]

where \(\Delta \theta\) is the angular separation between consecutive frequency comb modes dispersed by the grating. \(\Delta \theta\) can be found from the grating equation for a given repetition rate \((f_{rep})\), wavelength \((\lambda_0)\), grating ruling \((1/d\) lines/mm), and incidence angle \((\theta_i)\) as shown in Equation 3.8:

\[
\Delta \theta = \sin^{-1} \left( \frac{\lambda_0}{d} - \sin(\theta_i) \right) - \sin^{-1} \left( \frac{c \lambda_0}{c d + d f_{rep} \lambda_0} - \sin(\theta_i) \right), \tag{3.8}
\]

where \(c\) is the speed of light.
By using the equation above, we are able to find a grating-based shaper with a focal plane spacing between frequency comb modes of 100 $\mu$m at the center wavelength of the cavity filtered source (965 nm). This shaper consists of a 1200 lines/mm grating coupled with a 1 m focal length achromatic lens and a 20 GHz input comb. The input angle, $\theta_i$, to the grating to achieve this mode spacing is $29.4^\circ$. This input angle is within $5^\circ$ of the Littrow angle, meaning the diffraction efficiency of the grating will be near its maximum value. The theoretical mode spacing for this configuration is shown in Figure 3.7.

3.3.1 Control of 120 Comb Modes

The double-pass FP cavity (chapter 2) is configured to filter the stabilized 1 GHz frequency comb to 20 GHz. The output power of the filter cavity, as mentioned before, is around 6 mW. This output is spatially expanded and sent to the pulse shaper, consisting of a 1200 grooves/mm gold grating (60 mm width), a 1 m focal length lens (76.2 mm diameter), and a 640 pixel liquid crystal spatial light modulator (SLM) in reflection mode that is capable of both amplitude and phase control (Figure 3.8).

The grating and lens are arranged so that the spacing of the 20 GHz modes in the focal plane matches the 100 $\mu$m per pixel pitch of the SLM at the center frequency of the comb as described in section 3.3. The modulated light is retroreflected and is picked off by an optical isolator. The optical spectra at various points along the shaper are shown in Figure 3.9. The reflected light shows a bandwidth of 40 nm centered around 965 nm, which allows for pulses shorter than 50 fs. The total shaper output power is 0.9 mW. We can amplify the output to 5-10 mW using a semiconductor optical amplifier with minimal loss in bandwidth.

In principle, we can manipulate this entire bandwidth; however, in the present configuration the physical separation between modes varies across the aperture of the SLM as shown in Figure 3.7 (due to the non-constant angular dispersion of the grating). We achieve good overlap between optical modes and SLM pixels over a subset of the full aperture before the modes walk off of a SLM pixel. In this shaper configuration, we achieve a bandwidth of about 7.5 nm (120 modes)
Figure 3.7: Focal plane mode spacing ($\Delta x$) as a function of wavelength for a pulse shaper consisting of a 1200 lines/mm grating, a 1 meter lens, and a comb repetition rate of 20 GHz at an incidence angle of 29.4°. The mode spacing is near 100 $\mu$m/mode (the spacing between SLM pixels) at the center frequency of the cavity filtered source (965 nm).
Figure 3.8: System schematic of the grating-based line-by-line pulse shaper. The 1200 lines/mm grating coupled with 1 m lens gives a mode spacing near 100 µm in the focal plane at a grating incidence angle of 29.4°. This mode spacing matches the pixel pitch of the SLM over a bandwidth of approximately 120 modes, enabling line-by-line control in this region. The output of the pulse shaper is picked off by the polarizing beamsplitter (PBS) at the input of a Faraday isolator.
Figure 3.9: a) Octave-spanning laser spectrum shown on a log scale. Light at 550 and 1100 nm is used to detect and stabilize the carrier envelope offset frequency. Any one of the CW optical frequency references listed can be used to stabilize the absolute frequency of all the comb modes, leaving a broad swath of spectrum from 650-1050 nm available for pulse shaping. In this case we send light from 700-1050 nm to a 20 GHz filter cavity. b) 20 GHz output of the filter cavity showing good coupling over 100 nm. c) Output of pulse shaper showing a hard edge at each end of the aperture of the SLM. Over 600 modes are captured in the aperture of the SLM.
around 961 nm. Approaches to reduce the pixel walk-off limitations include using a spacing of 2 SLM pixels per comb mode or using a grating plus a prism (grism) to minimize dispersion in comb mode separation and provide 1 pixel per comb mode across the entire aperture of the SLM (see chapter 4).

To demonstrate the individual addressing of many comb modes, we perform basic amplitude masking on 120 comb modes from 958 to 965 nm. In this preliminary demonstration, we turned off every other mode to double the repetition rate as shown in Figure 3.10. The extinction is greater than 10 dB across this entire span, but would be improved by adjusting the mode size at the SLM, which was not yet optimized. We amplified the light from the shaper with a semiconductor optical amplifier and examined the time domain signal with a fast photodiode and oscilloscope as shown in Figure 3.10b.

![Figure 3.10: a) Zoomed in OSA spectra of all the pixels on (red trace) and every other pixel turned off (black dashed trace). b) Fast oscilloscope trace of the 120 amplified comb teeth with all teeth on showing a 20 GHz repetition rate (red trace) and every other tooth turned off showing a 40 GHz repetition rate (black trace). The photodetector bandwidth is 45 GHz.](image)

With this setup, we have shown individual control of approximately 120 comb modes around 960 nm from a filtered Ti:Sapphire frequency comb. The optical frequency of all comb teeth can be stabilized to better than one part in $10^{15}$ by locking the comb to one of the optical references, ultimately enabling novel waveform generation with femtosecond timing jitter.
3.3.2 Analysis of the Phase Noise Properties of the Pulse Shaper

When the timing jitter of the input comb source is at the femtosecond level, the excess phase noise added by the pulse shaper (and all other optical components external to the laser) will have a negative impact on the fidelity and timing stability of the generated waveform. In this work, we examine the effect of a line-by-line pulse shaper setup on the phase noise properties of a well-stabilized Ti:Sapphire frequency comb [56]. We measure the phase noise added by the 20 GHz double-passed filter cavity when locked to a stabilized, self-referenced 1 GHz frequency comb [26], as well as the phase noise added by the line-by-line pulse shaper setup. We also show how the additional phase noise can be eliminated by controlling the phase of one tooth of the comb that has passed through the shaper. This reduction in phase noise allows us to encode information on the optical phase with sub-radian precision on a timescale of many seconds.

To examine the phase noise added by the filter cavity and pulse shaper setup, we phase lock a 960 nm external-cavity diode laser (ECDL) to the stabilized 1 GHz comb, giving us a highly stable CW reference at the wavelengths that are used for pulse shaping. This locked 960 nm laser is then used to examine the phase noise of the 20 GHz frequency comb directly after the cavity and after passing through the pulse shaper [Figure 3.11].

We first measure the in-loop phase noise of the lock of the 960 nm diode laser to the 1 GHz comb using a maser-referenced network analyzer. The resulting curve is shown in [Figure 3.12] along with the integrated phase noise. The phase noise after the filter cavity and after the pulse shaper setup was measured via a heterodyne beat with the stabilized 960 nm diode laser [Figure 3.11]. As seen from the phase noise spectrum shown in [Figure 3.12], the additional path length associated with the filter cavity and shaper as well as the many bounces off turning and alignment mirrors add significant phase noise in the acoustic region and near DC. Also significant are peaks at 64 kHz and 128 kHz added by the filter cavity dither-lock servo. This added phase noise is at a level comparable to the intrinsic absolute phase noise stability of the frequency comb locked to an optical standard. The cavity and shaper will add a non-negligible amount of phase noise to a well-stabilized
Figure 3.11: A stabilized 1 GHz frequency comb is filtered to 20 GHz and then sent to a line-by-line pulse shaper. The phase noise added by the filter cavity and pulse shaper setup are measured by heterodyning with a 960 nm CW diode laser that is locked to the stabilized input comb. The beat notes both directly after the cavity and after the pulse shaper setup are characterized by a network analyzer. Mixing the 832 MHz beat note from after the pulse shaper with the 832 MHz beat note that locks the CW diode laser to the input comb results in an error signal giving the phase difference between one input comb mode and that same comb mode after propagation through the cavity and pulse shaper. This error signal can be fed back to a PZT-mounted mirror to actuate the phase difference to zero and remove shaper path length fluctuations.
frequency comb.

By mixing the 832 MHz ($f_2$) signal from after the filter cavity and shaper with the 832 MHz ($f_{\text{beat}}$) signal from the diode laser lock (see Figure 3.11), we can monitor the optical phase of one comb mode after propagation through the pulse shaper setup compared to the 1 GHz comb mode before propagation through the system. This mixing also has the advantage of removing the residual phase noise of the 960 nm diode laser. We then bandpass filter and feed this signal via a loop filter to a PZT mounted behind a mirror in the shaper setup (Figure 3.11), locking the relative phase of the comb mode that has passed through the shaper to the phase of the stable 1 GHz comb mode, effectively locking the path length of the filter cavity plus shaper setup. Because the majority of the phase noise added to the shaped comb is due to common-mode path length (group delay) fluctuations (due to mirror motion or air currents), to first order, locking the phase of the one comb tooth via group-delay actuation locks the phase of all the comb teeth. This point will be discussed in greater detail in chapter 5. Important for the validity of this postulate is the reduction of air currents in the region of the pulse shaper where the comb modes are dispersed (which could lead to non-common-mode path length fluctuations). This reduction is realized by building an airtight box around the pulse shaper.

Figure 3.13 shows the result of locking the path length. The phase noise below 300 Hz is brought down to the in-loop phase noise level and there is a reduction in phase noise out to ~1 kHz, giving a significant reduction in integrated phase noise from 0.61 radians to 0.41 radians. Further reduction of the phase noise could be accomplished by using a higher bandwidth modulator.

With one mode of the comb through the shaper phase locked, we can modulate other comb modes and read out the optical phase. The line-by-line pulse shaper allows us to individually address single comb modes (in a 120 mode region around 960 nm) and apply arbitrary phases and amplitudes to each mode [26]. With one mode phase locked at an 832 MHz offset from the 960 nm diode, we apply phase steps to a neighboring comb mode. These phase steps will show up on the beat of that comb mode with the 960 nm diode laser. Because the mode spacing is nominally at 20 GHz, the nearest neighbor comb mode beat is at approximately 19.2 GHz (~20 GHz - 832 MHz)
Figure 3.12: Relative phase noise on a frequency comb mode (approximate frequency of 311 THz) measured by a beat note with a stabilized external cavity diode laser (ECDL). The phase noise of the frequency comb is measured in various locations throughout the pulse shaping setup. The in-loop phase noise is a measure of how well the ECDL is locked to the stabilized 1 GHz input comb. After the filter cavity, there is a significant amount of phase noise added in the acoustic region and near DC as well as noise spurs at $\sim 60$ kHz and $\sim 120$ kHz added by the dither lock. The pulse shaper adds additional noise in the acoustic region and near DC that brings the phase noise to the intrinsic phase noise of the stabilized frequency comb. Dashed line shows the approximate absolute phase noise of the frequency comb locked to a high-Finesse optical cavity.
Figure 3.13: Phase noise after the filter cavity and pulse shaper compared to the phase noise in the same place with noise cancellation enabled (phase locked). Phase noise is much reduced in the lower acoustic region by the phase lock. Noise reduction is visible to slightly above 1 kHz. A faster modulator would increase the bandwidth of noise removal.
Figure 3.14: Schematic representation of beat notes of 960 nm diode laser with the 1 GHz comb (top) and the 20 GHz comb (bottom). The beat note $f_{\text{beat}}$ is used to lock the 960 nm diode laser to the 1 GHz comb. The stabilized laser is then used to measure the 20 GHz comb after the filter cavity and pulse shaper ($f_2$). Phase information can be encoded on the nearest 20 GHz mode and read out with the beat note $f_3$ (shown in Figure 3.15).
MHz). To detect the optical phase, we mix the 19.2 GHz beat down to near DC via heterodyne with a microwave synthesizer. The results are shown in Figure 3.15, where phase steps as small as \( \pi/8 \) are clearly resolved. Smaller phase steps can be resolved with additional low pass filtering.

This method of phase-encoding information at frequencies near DC could provide an interesting route to secure communications that takes advantage of the absolute phase stability of the frequency comb. One could encode information onto the optical phase of a comb at an appropriate rate and with small phase steps that would be unreadable without a similar high-stability source (i.e. another stable frequency comb). It is worth noting that common diode lasers, and even narrow linewidth solid-state or fiber lasers, have phase noise many orders of magnitude greater than is achieved with the stabilized comb presented here. In analogy to optical code division multiple access (OCDM), an additional level of technical complexity could be implemented by hiding the data on one or several of the hundreds of available comb modes and placing false information on the rest. This idea will be developed further in chapter 7.

This work has shown that grating-based pulse shapers add significant phase noise to a stabilized source. Such phase noise would have to be considered when trying to make high fidelity optical waveforms with low timing jitter. This phase noise can be reduced with a feedback loop, enabling the encoding and readout of small discrete phase steps on individual comb modes. This opens a route to new coherent optical communication schemes with a highly phase stable source.

3.3.3 Limitations of Grating Shaper: Non-uniform Angular Dispersion

As mentioned in subsection 3.3.1, the non-uniform angular dispersion of the grating causes the comb modes to be unevenly spaced in the focal plane of the shaper, making it difficult to match each pixel of the modulator with only one frequency comb mode. This shortcoming will need to be addressed if the bandwidth of line-by-line control is to be extended.

For the grating-based shaper design employed for this work, each mode in the focal plane is separated by \( \Delta x = f \Delta \theta \), as shown in Figure 2.1. It is straightforward to solve for \( \Delta x \) or \( \Delta \theta \) using the grating equation for a given frequency comb spacing, grating, lens, and input angle.
Figure 3.15: Phase steps encoded on one comb tooth and read out via a heterodyne beat which was demodulated with a synthesizer. The signal was low pass filtered with a 30 kHz cutoff. The low frequency drift in some of these traces arises from the small mis-match in frequency of the optical heterodyne ($f_3$ of Figure 3.14) and the microwave local oscillator.
(Equation 3.7 and Equation 3.8). Additionally, one can calculate how close each comb mode is to the center of its intended SLM pixel by integrating the mode spacing minus the pixel pitch (in our case, 100 µm). Figure 3.16a shows the mode spacing Δx as a function of mode number for a 21 GHz comb centered at 965 nm for a line-by-line shaper constructed with an 800 mm lens and a 1200 lines/mm grating, with the center mode spacing matched to the 100 µm pixel pitch of the SLM used in this work.

Because the mode spacing changes across the spectrum, the comb modes ’walk off’ the centers of the pixels (Figure 3.16b). This ‘walk off’ is cumulative, so the pixel centers and mode centers quickly become misaligned. Outside the center region of good alignment, the pixels and modes alias rapidly and line-by-line control is not possible in the one pixel per comb mode setup. Using two pixels or more per comb mode can mitigate this issue; however, this decreases the maximum number of modes available for control. The following chapter describes the solution to this walk-off problem using a grism (grating + prism).
Figure 3.16: (a) Mode spacing for the grating-based shaper (Δx) as a function of mode number with mode 0 at 965 nm. The mode spacing is close to 100 µm/mode only in the center. (b) Integrated residual mode walk-off shows that fewer than 200 modes stay aligned with the correct SLM pixel. Only about 100 modes are close enough to the center of the pixel to avoid spatial overlap with neighboring pixels.
Chapter 4

Extending the Bandwidth of Line-by-Line Frequency Comb Control

4.1 Grism-Based Pulse Shaper Design

As discussed in section 3.3, typical grating-based shapers have shown line-by-line control over ~100 individual comb modes [26, 73], limited by the number of modulator pixels or comb modes available and the non-uniform angular dispersion of the grating in the pulse shaper [26]. To increase the number of modes we can control in the one pixel per mode geometry, we have to alter the properties of the dispersive element so that the dispersion is flat across the region of interest, satisfying the condition $\frac{\delta^2 \theta_{out}}{\delta \lambda^2} = 0$. This is accomplished by inserting a prism on top of the grating as shown in Figure 4.1. The tilted dielectric interface at the prism-air surface maps the dispersion of the grating onto an inverse sine function from Snell’s Law, and for the right design parameters, the nonlinear angular dispersion becomes linear. Past work has combined gratings and prisms to linearize the dispersion of an astronomical spectrometer [74], but in the ultrafast optics field, grisms have been used most commonly for pulse compression [75–78].

Solving for the output angle of a grism as a function of wavelength or frequency allows one to find $\Delta \theta$ between comb modes spaced by a given repetition frequency. For the geometry shown in Figure 4.1 the output angle of a grism is:

$$\theta_{out} = \sin^{-1} \left[ n(\lambda) \sin \left( \sin^{-1} \left[ \frac{m \lambda}{n(\lambda)} \sin \left( \sin^{-1} \left[ \frac{1}{n(\lambda)} \sin \left( \theta_{in} - \alpha \right) \right] + \alpha \right) \right] - \alpha \right) \right] + \alpha, \quad (4.1)$$

where $\theta_{in}$ and $\theta_{out}$ are the input and output angles as defined in Figure 4.1 $\alpha$ is the apex angle of the prism, $n(\lambda)$ is the index of the prism as a function of wavelength, $m = 1$ is the diffraction
Figure 4.1: Reflection grism geometry. The angled dielectric interface of the prism serves to alter the dispersion of the grating. Input and output angles are measured clockwise with respect to grating normal.

Our grism:
\( \alpha = 60^\circ \)

1/d = 1200

prism material: UVFS

\( \theta_{in} = -2.1^\circ \)

\( \theta_{out} = 3.0^\circ \)
order, \( d \) is the groove spacing, and \( \lambda \) is the wavelength of the comb mode.

To design our grism, we attempt to set \( \delta^2 \theta_{\text{out}}/\delta \lambda^2 = 0 \) or \( \delta^2 \theta_{\text{out}}/\delta f_{\text{optical}}^2 = 0 \) while keeping \( \Delta x = f \Delta \theta \approx 100 \, \mu m \). We found that this can be accomplished for a 21 GHz comb centered at 965 nm using an equilateral UV fused silica (UVFS) prism, a 1200 lines/mm grating, a 750 mm focal length achromatic lens, and \( \theta_{\text{in}} = -2.1^\circ \). This solution also operates the grating near Littrow so the grating efficiency is maximized. We had some flexibility in choosing our comb repetition rate because we are using a cavity filtered 1 GHz comb and can select repetition rates in integer multiples of 1 GHz [28]. However, this solution is not unique, and in general, one can find a solution with off-the-shelf parts for many comb repetition rates and center wavelengths. The theoretical solution showed a mode spacing near 100 \( \mu m \)/mode and much less integrated walk-off than the grating-based shaper, with more than 600 modes within 10 \( \mu m \) of pixel center (Figure 4.2).

To measure the mode spacing in our grism-based pulse shaper, we replaced the grating element in the shaper with a fabricated grism designed as described above. The fabricated grism was constructed from a 50 mm grating and an equilateral UVFS prism with 50 mm side lengths. The fabricated grism showed a single-pass efficiency of 80\% when operated at the incidence angle used in the pulse shaper (very near the Littrow condition). The beam waist into the grism shaper was around 6.6 mm (\( \sim 10 \, mm \, 1/e^2 \) width) and 2 inch and larger optics were used throughout the pulse shaper. A CCD camera was placed in the focal plane of the pulse shaper to image the comb modes and measure the mode spacing (Figure 4.3). A line-out from each image was fit with an array of Gaussians to extract the mode spacing. The measured mode spacing was compared to the theoretical model to find the current input angle. Minor adjustments to the input angle fine-tuned the mode spacing to achieve the correct mode spacing in the focal plane.

A series of \( \sim 10 \) images was taken by translating the CCD camera to cover the entire dispersed frequency comb (spanning over 6 cm), and the mode spacings were extracted from each image. The ten mode spacing measurements were then stitched together and compared with our theoretical model (Figure 4.4). The integrated walk-off from the measured mode spacings shows that there are 640 comb modes within \( \pm 10 \, \mu m \) of pixel center (Figure 4.4b), as predicted by theory.
Figure 4.2: (a) Mode spacing for the grism-based shaper ($\Delta x$) as a function of mode number with mode 0 at 965 nm. The mode spacing is close to 100 $\mu$m/mode across the entire bandwidth of the pulse shaper (640 modes). (b) Integrated mode walk-off shows that more than 600 modes are within 10 $\mu$m of pixel center.

Figure 4.3: Example CCD image showing 70 comb modes spaced by 100 $\mu$m. Ten such images were taken to span the whole aperture of the SLM. The mode spacings were extracted from each of these images to measure the dispersed comb mode spacings to compare with theory.
Figure 4.4: a) Mode spacing ($\Delta x$) as a function of mode number with the theory (dashed) and measurement (solid) showing good agreement. b) Measured integrated mode walk-off (solid) also shows good agreement with theory (dashed).
These CCD measurements also allowed us to extract the beam waist of comb modes in different areas of the spectrum. Beam waist is important because it determines how much crosstalk there will be between neighboring pixels, as discussed in section 3.3. As was also discussed in section 3.3, the optimal electric field beam waist for this SLM in reflection mode is about 30-35 µm (21-25 µm intensity waist). We measured an intensity waist of 25 µm at the focus across most of the comb (Figure 4.5). At this measured waist, 99.3% of the power is contained within the correct pixel for perfect alignment, decreasing to 98.4% for 10 µm of misalignment. The measured beam waist expands in the wings of the spectra due to a small amount of chromatic aberration in the shaper lens, but even at the largest waist, 95% and 94% of the power is contained in the correct pixel for perfect alignment and 10 µm of misalignment, respectively.

It would be better to have a smaller beam waist at the focus to allow for the expansion of mode size in the wings of the spectrum, but wavefront distortions on the large beam limit the achievable spot size in the focus to approximately 25 µm. To find the minimum achievable spot size, a CW diode laser was coupled into the filter cavity and sent through the shaper setup. A CCD camera on a translation stage was placed in the focal plane to image the beam at a range of focal positions. The input beam size was then varied by changing the lenses in the expansion telescope. Figure 4.6 and Figure 4.7 show the results of focal spot measurements with the CW diode laser for two diffraction-limited spot sizes. These measurements indicate that wavefront aberrations in grism are limiting the achievable beam intensity waist to above 25 µm. For this reason, we configure the input spot size into the pulse shaper to give a diffraction-limited spot size of around 25 µm.

After characterizing the mode spacing and beam waist, we inserted a 640 pixel amplitude and phase SLM with 100 µm pixel pitch into the focal plane of the grism-based line-by-line pulse shaper. To achieve good pixel alignment over the entire aperture of the SLM, we first attenuate every other pixel on the SLM and look at the comb on an optical spectrum analyzer (OSA). If there is good alignment between the pixels and comb modes, the trace on the OSA will look like a frequency comb with double the comb mode spacing. Coarse alignment can be obtained by translating the SLM with a translation stage or translating the comb modes with a mirror, and fine adjustments can
Figure 4.5: Intensity beam waist measured across the aperture of the SLM shows beam waists near 25 µm for most of the comb. The beam waist expands in the wings of the spectrum due to chromatic variation in the shaper lens.
Figure 4.6: Beam waist measurements for a diffraction limited beam waist (intensity) of approximately 25 µm (input beam waist (intensity) of 4.7 mm). Without the grism in the shaper, the beam reaches the diffraction limit in the focus and the beam is circular. With the grism in the shaper, the beam becomes elliptical and the horizontal and vertical foci occur at different focal planes. This indicates that the grism is adding wavefront aberration to the beam. The horizontal beam waist almost reaches the diffraction limit, but at a different focal position than before.
Figure 4.7: Beam waist measurements for a diffraction limited beam waist (intensity) of approximately 21 µm (input beam waist (intensity) of 5.6 mm). Without the grism in the shaper, the beam again reaches the diffraction limit in the focus and the beam is circular. With the grism in the shaper, the beam becomes even more elliptical and aberrated than the weaker focus case shown in Figure 4.6. The horizontal beam waist does not reach the diffraction limit, showing that this input beam diameter is too large for the amount of aberration in the shaper.
be made by tuning the comb repetition rate slightly. The well-aligned case is shown in Figure 4.8.

Good alignment of the comb to the SLM pixels is established across the entire 13.4 THz bandwidth ($\Delta \lambda = 41$ nm). This is verified in the measurement of the ’off’ state modes which show attenuation of 10-20 dB. This could be improved through better calibration of the SLM on a pixel-by-pixel basis. There is some attenuation of the ’on’ state modes, especially in the lower end of the spectrum, possibly due to an increase in beam waist in this region, or residual misalignment.

To our knowledge, this 640 mode and 13.4 THz is the largest number of modes and bandwidth of optical frequency that has been addressed in a line-by-line manner. The expansion in both the number of modes and the bandwidth controlled makes more complex arbitrary optical and microwave waveforms possible. Furthermore, this grism technique should be similarly effective in pulse shapers at 1550 nm or other dispersive systems where constant angular dispersion is desirable.
Figure 4.8: (top) OSA traces showing the shaped frequency comb on a linear scale with all pixels in the 'on' state (solid red) and pixels alternating between 'on' and 'off' (dashed black). (below) Zoomed sections of above traces showing the alignment of the comb modes with the SLM pixels is good across the whole bandwidth of the frequency comb. The additional attenuation of the 'on' modes in some parts of the spectrum are either due to residual misalignment or an increase in mode waist in the wings of the spectrum.
Chapter 5

Remote Transfer of a Phase-Modulated Frequency Comb

5.1 Motivation

The transfer of stable optical signals from one location to another location over optical fiber has shown great utility across a broad range of experiments. The detection and removal of the transmission noise of these fiber links has been thoroughly studied (for a good review, see [29]). The stable transmission of arbitrary waveforms over optical fiber, however, has not been demonstrated before. For the transmission of frequency combs and other pulsed sources, fiber noise is generally detected by monitoring the group delay of pulses travelling through the fiber. The detection of fiber noise for arbitrary waveforms is complicated, however, by the fact that the group delay and pulse shape of pulses being transmitted through the fiber can be changed arbitrarily. Such group delay and pulse shape changes would change the lock point of the servo with each waveform update, introducing noise onto the fiber transfer. In this chapter, we demonstrate a new method for the detection of fiber noise that enables the stable transfer of arbitrarily shaped pulses over an optical fiber regardless of waveform updates. Such stable transfer will enable the direct transmission of optical phase information to a remote location that can be read out with a second frequency comb (chapter 7).

5.2 Method for Detecting Path-Length Fluctuations

Fiber fluctuations are generally measured by monitoring a back-reflection from the remote end of the fiber, commonly from a flat-polished fiber tip. This back-reflected light samples the
round trip phase delay of the fiber. In general, it is assumed that the round trip phase delay is twice the one-way phase delay such that stabilizing the round trip phase stabilizes the one way phase. This assumption holds for noise frequencies that are less than $1/\tau_{rt}$ ($\sim$100 kHz for a 1 km fiber). There are a variety of phase detection and fiber noise cancellation schemes well summarized by Foreman, et. al. [29]. For transfer of pulsed sources and frequency combs, removal of fiber noise is accomplished through actuation of the group delay, and phase detection is usually accomplished through detection of the round trip optical or microwave group delay. Since line-by-line pulse shaping can arbitrarily change the group delay and pulse shape of the remote pulse, the detected fiber group delay would change with each new mask applied and the noise cancellation lock point would change. In addition, optical detection schemes such as described by Chen, et. al. would be similarly challenged by mask changes [65]. To overcome these difficulties, we employ a new phase detection scheme that allows line-by-line phase control of the transmitted comb. Phase detection in this scheme is accomplished by monitoring the phase of one comb tooth and locking it to a fixed value by actuating on the group delay (in this case the length of the fiber). The phase monitoring is accomplished by using a CW diode laser as a transfer oscillator to compare the phase of the back-reflected comb tooth to the phase of the same comb tooth before propagation through the fiber. The detected phase of comb mode m is:

$$\phi_m = \text{MOD} \left[ \frac{\omega n(\omega) L_c}{c}, 2\pi \right],$$  \hspace{1cm} (5.1)

where $\omega$ is the optical angular frequency, $n(\omega)$ is the frequency-dependent index of refraction, $L$ is the length of the fiber, and $\text{MOD}[a, b]$ denotes $a$ modulo $b$. This detected phase is related to the phase delay, $T_\phi$, by Equation 5.2

$$T_\phi = \frac{\phi}{\omega} = \frac{n(\omega) L_c}{c}$$ \hspace{1cm} (5.2)

However, the stabilization of a pulse train at the end of a fiber requires that the group delay be stabilized. The group delay is related to the phase delay by Equation 5.3

$$T_g = T_\phi + \frac{L}{c}, \frac{\partial n(\omega)}{\partial \omega}$$ \hspace{1cm} (5.3)
Because the frequency comb is stabilized, $\omega$ is assumed to be constant in time, while both the length of the fiber and the index of refraction can vary with time. When either the length of the fiber or the index of the fiber changes with time (as a result of acoustic and thermal perturbations), a significant amount of noise can be added to the group delay of the comb. To first order, however, locking the phase delay of one frequency comb tooth by actuating on the length of the fiber effectively locks the group delay to a constant value. There will be a small amount of noise on the group delay caused by $n(\omega)$ and $\partial n(\omega)/\partial \omega$ changing with time, but this noise is shown later to be sufficiently small that it will not affect the phase stability of the frequency comb.

Using only one comb tooth to remove noise on the group delay of a frequency comb transmitted over an optical fiber enables arbitrary line-by-line masks to be applied to the rest of the frequency comb. To allow arbitrary line-by-line pulse shaping, one simply chooses not to apply phase variations to this one comb tooth with the pulse shaper, so that the detected fiber phase does not change with the application of pulse shaping masks. Since an offset in absolute optical phase does not affect the pulse shape, any arbitrary line-by-line mask can be applied by first subtracting a constant phase from all the comb modes such that the mask applies zero phase shift to the mode used for phase locking.

### 5.3 Measurement of Comb Stability at Remote End of Fiber

To verify that the stabilization of the phase of one comb tooth by group delay actuation sufficiently stabilizes the phase of all the comb teeth, it is necessary to measure the phase noise of individual comb modes across the whole bandwidth of the comb. This can be accomplished by heterodyning the light from the remote end of the fiber with stable light from the original frequency comb. In this case, the remote light is shifted in frequency by 80 MHz via an acousto-optic modulator (AOM) to facilitate the detection of the microwave beat notes. The shaped frequency comb in this work consists of 640 individually controllable 21 GHz comb modes (chapter 4 and [27]). Because each of these comb modes will have its own 80 MHz beat note, it is necessary to spectrally disperse the combined light to detect the beat notes of isolated comb modes. By using a
Figure 5.1: System overview of fiber noise cancellation scheme. Light from a stabilized 1 GHz Ti:Sapphire laser is cavity filtered up to 21 GHz and sent to a line-by-line pulse shaper whose output is coupled into a single mode fiber. A small amount of the stabilized 1 GHz light is also picked off to provide a reference for the fiber noise cancellation. A 965 nm external cavity diode laser (ECDL) is heterodyned with and locked to one tooth of the stable frequency comb (\(f_{\text{beat1}}\)). The ECDL is also heterodyned with the light that is retro-reflected from the remote end of the long fiber (\(f_{\text{beat3}}\)). The two microwave heterodyne beat notes are mixed to find the optical phase difference between the stable comb tooth and the comb tooth that has propagated through the shaper and fiber system. This mixing has the added benefit of removing the noise of the ECDL. The phase difference error signal is then fed back via a loop filter to a fiber stretcher to stabilize the fiber length. A similar process is followed to stabilize the path length of the line-by-line pulse shaper using a mirror mounted on a PZT (using \(f_{\text{beat2}}\)).
A high resolution spectrometer, a single 21 GHz comb tooth was allowed to pass through an iris and onto an avalanche photodiode along with the 1 GHz reference light. The 80 MHz beat note contains the phase noise of the comb mode at the end of the fiber with respect to the stable reference comb mode. Rotating the spectrometer grating allows the selection of single comb teeth from across the entire 13 THz bandwidth of the comb. The detected 80 MHz beat note is analyzed with a network analyzer in demodulation mode.

Figure 5.2: Relative phase noise of individual comb modes (∼310 THz optical carrier) at the remote end of a 300 m fiber. Phase noise on all comb modes is reduced within a ∼30 kHz bandwidth by a phase-locked loop. Modes further from the CW lock point show slightly more noise near DC, leading to increased integrated phase noise (right). This increase in phase noise is most likely due to dispersion noise in the fiber as well as non-common mode noise in the shaper at positions where the comb modes are dispersed. Integrated phase noise with the noise canceller off is above 14 radians.

As shown in Figure 5.2, with no fiber noise cancellation, the relative phase noise at the end of a 300 m fiber is quite high, giving an integrated phase noise above 14 radians (7 fs timing jitter). This shows that the length fluctuations in the fiber ruin the coherence of the stable input frequency comb. In contrast, with the fiber noise cancellation enabled, the phase noise is much reduced within the ∼30 kHz bandwidth of the servo. The integrated phase noise increases away from the stabilization comb mode; however, all modes show integrated phase noises below 40 mrad (20 as) from 60 kHz to 0.125 Hz. The increase in phase noise arises from uncompensated index and index dispersion noise.
in the fiber and from non-common mode path length fluctuations in the regions of the shaper where 
the comb modes are spatially dispersed. The V-shaped integrated phase noise curve indicates that 
the noise increases linearly away from the lock point, indicating dispersion noise as the probable 
source of the excess noise. The low integrated phase noise, however, demonstrates that the fiber 
noise cancellation is working well for the entire frequency comb (upholding the assumption from the 
section 5.2) and that the frequency comb at the remote end of the fiber will replicate the stability of 
the input comb. Also important for this work, this stabilization method allows arbitrary phase and 
amplitude shifts to be applied to every comb mode but the one being stabilized, allowing stabilized 
arbitrary optical waveform generation at the end of a fiber.

5.4 Demonstration of pulse shaping with stabilized remote transfer

To demonstrate the power of this fiber noise cancellation technique, we perform spectral 
interferometry at the remote end of the fiber (similar to [79]). Comb light from the shaper delivered 
to the remote end of the fiber is combined on a beamsplitter with the same 1 GHz reference comb 
light as used for the noise cancellation, this time without the 80 MHz frequency shift so that 
interference occurs at DC. The combined light is then spectrally dispersed in two dimensions with 
a high resolution VIPA and grating spectrometer that can resolve and image all 640 21-GHz comb 
modes simultaneously. Figure 5.3. The VIPA-grating spectrometer was necessary because we do 
not have a CCD large enough to resolve and image all 640 comb modes using a one-dimensional 
grating spectrometer, but it is possible to image all 640 comb modes in two dimensions.

Since we can resolve all of the 21 GHz comb lines with the VIPA spectrometer, it is possible 
to do line-by-line phase characterization of the shaped comb. The spectral interferogram seen by 
the camera is given by:

\[ I_n = |E_1|^2 + |E_2|^2 + 2|E_1||E_2|\cos(\Delta \phi_n), \]  

(5.4)

where \(E_1\) and \(E_2\) are the electric field amplitudes of the 1 GHz reference comb modes and 21
Figure 5.3: Light that has been manipulated by the line-by-line pulse shaper (shaped light) is coupled into a 300 m long optical fiber. At the remote end of the 300 m long fiber, the shaped light is combined with light picked off of the input 1 GHz comb on a non-polarizing beam splitter. The combined light is then coupled into a high-resolution VIPA spectrometer that disperses the comb modes in two dimensions. A CCD camera records the spectral interference between the shaped frequency comb and the reference pulse. A computer processes the images and extracts the phase difference between the shaped pulse and the reference pulse.
GHz comb modes, and $\Delta \phi_n$ is the relative phase shift between the $n^{th}$ 21 GHz comb mode and its corresponding 1 GHz reference comb mode. By blocking each of the combs in succession, we can measure $|E_1|^2$ and $|E_2|^2$ for background subtraction. In addition, a reference comb mode is turned off and an additional image is taken so that software can label each of the frequency comb modes according to the pixel of the spatial light modulator that controls them. One final image is taken where the fiber phase is dithered rapidly, averaging the $2|E_1||E_2|\cos(\Delta \phi_n)$ term to zero for all modes, such that the image records $|E_1|^2 + |E_2|^2$. Using these reference images, the software assigns each comb mode to the correct pixel of the spatial light modulator and extracts the $\Delta \phi$ for each comb mode. This spectral interference pattern is very sensitive to optical phase fluctuations, so with the fiber noise canceller disabled, the pattern changes rapidly and the spectral phase cannot be retrieved. With the noise canceller enabled, the spectral interference pattern is stationary and can be characterized effectively. Figure 5.4 shows a schematic picture of the phase extraction method, while Figure 5.5 show the result of this method.

Figure 5.6 and Figure 5.7 show the result of scanning one pixel’s phase with the pulse shaper and monitoring the spectral interferogram measured intensity. Ten separate interferograms were recorded for each phase step to show the repeatability of the measurement. Phase steps of $\pi/32$ are resolvable using this method.

Once the relative phases are measured for each comb mode, the pulse shaper is used to apply the inverse phase shift to bring all the $\Delta \phi$s to zero (with the exception of the comb mode used for fiber noise cancellation). It is simple from there to encode phase information or an image onto the phases of the comb modes that can be read out by this spectral interferometry (Figure 5.8).

The fiber stabilization technique presented here allows the stable delivery of arbitrary optical pulses to a remote location. The fiber can remain stabilized even with arbitrary mask changes that vary the group delay and pulse shape dramatically. The transmission fiber length could be increased to several kilometers with similar stability as demonstrated here, and longer transmission fibers could be used at the expense of short term instability. In addition, the pulses at the output of the comb replicate the phase stability and timing jitter of the input frequency comb, so this
Figure 5.4: Schematic view of phase extraction method. First, the $I_{tot}$ image has the background removed by subtraction of the $|E_1|^2 + |E_2|^2$ image. This background-subtracted image is then divided by $\sqrt{|E_1|^2 \cdot |E_2|^2}$ to arrive at an image that is proportional to $\cos(\Delta \phi_n)$. The resulting image is shown in Figure 5.5.
Figure 5.5: Phase difference image (proportional to $\cos(\Delta \phi_n)$) extracted by a computer. The image has been masked to one VIPA free spectral range. The red and yellow comb modes are positive in value ($\cos(\Delta \phi_n) \geq 0$), while the blue modes are negative in value ($\cos(\Delta \phi_n) \leq 0$). To find the phases of each comb mode, the power in each comb mode area in the image is integrated and saved. These integrated powers are normalized to $\pm 1$ and the inverse cosine is taken.
Figure 5.6: Integrated power in the extracted spectral interferogram of one comb mode as the phase of that comb mode is scanned in phase steps of $\pi/16$ by the line-by-line shaper. Ten spectral interferograms were recorded for each phase step to show the scatter in the measurement. These phase steps are well resolved away from the maximum and minimum of the cosine function (where the measurement is most sensitive to phase steps).
Figure 5.7: Integrated power in the extracted spectral interferogram of one comb mode as the phase of that comb mode is scanned in phase steps of $\pi/32$ by the line-by-line shaper. Ten spectral interferograms were recorded for each phase step to show the scatter in the measurement. These phase steps are resolved away from the maximum and minimum of the cosine function (where the measurement is most sensitive to phase steps).
Figure 5.8: a) Spectral interferometry image from CCD camera. b) Background-subtracted and normalized spectral interferogram. Image has been masked to one VIPA free spectral range. c) Interference pattern of one comb mode as phase is scanned from 0 to $4\pi$ radians. d) Each mode phase corrected to have zero phase difference between reference and shaped light (except stabilization mode). e) Image encoded in the phase of the comb modes. f) Same phase applied as e), but with fiber noise canceller turned off. Image is no longer discernable and the interference pattern changes rapidly with time.
technique also has application for the remote distribution of frequency combs for ultra-stable optical and microwave frequency dissemination.
Chapter 6

Characterization of Shaped Pulses

6.1 Intensity Modulator Phase Measurement

The characterization of very complex pulses has been an area of intense study in recent years \cite{25, 32, 80, 85}, so there are a multitude of characterization schemes that could be employed to measure the pulses generated by a line-by-line pulse shaper. One difficulty in measuring the pulses from the line-by-line pulse shaper in this work is the relatively low peak power of the pulses. Any measurement approach relying on non-linear effects would suffer from low signal-to-noise. As a result, we decided to first try a linear pulse characterization technique based on using an intensity modulator to apply sidebands to each comb mode at $f_{\text{rep}}/2$ \cite{32}. Sidebands from neighboring frequency comb modes will interfere based on the phase difference between the two modes and the phase of the microwave drive signal. Two interference patterns are needed to resolve a phase ambiguity; one with the microwave phase set to 0° and one with the microwave phase set to 45°. These patterns can be measured with a high-resolution optical spectrum analyzer (OSA) with sufficient resolution to resolve both the comb modes and the generated sidebands. The measured sideband intensity for the sideband between the $n^{th}$ and $(n+1)^{th}$ frequency comb mode is:

$$I_{0^\circ} = c \left[ |a_n|^2 + |a_{n+1}|^2 + 2|a_n||a_{n+1}|\cos(\phi_n - \phi_{n+1}) \right], \quad (6.1)$$

$$I_{45^\circ} = c \left[ |a_n|^2 + |a_{n+1}|^2 + 2|a_n||a_{n+1}|\sin(\phi_n - \phi_{n+1}) \right], \quad (6.2)$$
where $a_n$ and $\phi_n$ are the amplitude and phase of the $n^{th}$ comb mode and $c$ is the modulation strength (defined as the ratio of the sideband peak to the carrier peak). The modulation parameter $c$ is generally around 0.01-0.1 and can be extracted from the analysis of the optical spectra.

Figure 6.1: Schematic of the intensity modulator phase measurement. An intensity modulator is driven at a frequency of $f_{rep}/2$ producing sidebands at this frequency on each comb mode. The sidebands from neighboring comb modes will interfere based on the phase difference between the two comb modes. The microwave phase of the modulator drive frequency is set to either 0° or 45° to produce two phase measurements in accordance with Equation 6.1 and Equation 6.2.

Using Equation 6.1 and Equation 6.2 along with the OSA traces for both microwave phase drives allows extraction of the phase difference between each neighboring pair of comb modes such that a simple integration will yield the phase of all the comb modes.

This technique is self-referenced, meaning it does not require an independently characterized reference pulse, and has shown sensitivities down to average powers of 100 nW \[32\]. There are several disadvantages of this technique, however, that will be discussed in the following section.

6.2 960 nm Intensity Modulator Phase Measurements

We obtained an intensity modulator capable of modulating light around 960 nm at speeds above 10 GHz to perform this measurement. A schematic of the pulse measurement system is shown in Figure 6.2.
Figure 6.2: Characterization of the line-by-line pulse shaper output with an intensity modulator and optical spectrum analyzer (OSA). Twenty percent of the 21 GHz comb is split off, amplified by a semiconductor optical amplifier (SOA), and detected by a 25 GHz photodetector. The 21 GHz signal is then divided by 2 and sent through a phase shifter and onto the intensity modulator (microwave bandpass filters and amplifiers not shown). The intensity modulators applies sidebands to each comb mode that are resolved and measured with a high-resolution OSA.
We use a fiber splitter to pick off 20% of the light for repetition rate detection. This light is amplified by an SOA and sent onto a 25 GHz detector. The resulting 21 GHz microwave tone is divided by 2, sent through a phase shifter, variable attenuator, and 10 GHz power amplifier and onto the intensity modulator. To perform a phase measurement, we take three optical spectra: one with the RF modulation disabled, one with the RF modulation enabled, and one with the RF modulation enabled and shifted by 45°. Because the resolution of the OSA was not quite enough to resolve both the comb modes and the sidebands, the intensity modulator bias is tuned to attenuate the carrier completely so the measurement operates in the carrier-suppressed mode. Example OSA spectra are shown in [Figure 6.3].

A computer program (MATLAB) is used to analyze the spectra and extract the phase difference, Δφ_n, for each pair of comb modes. Those phase differences were then integrated to find the phase of each comb mode (with an undetermined but irrelevant offset phase and group delay). These extracted phases are shown in [Figure 6.3c] and [Figure 6.3d].

After measuring the phases of all the comb modes, an inverse mask was applied with the pulse shaper to attempt to bring the shaped comb to the transform limit (Δφ_n = 0 for all n). After applying the inverse phase mask, however, the measured Δφ_n's showed significant scatter around zero as shown in [Figure 6.4a]. The integrated phase of the comb therefore showed a significant amount of phase walkoff and the resulting pulse is not measured to be transform-limited.

The residual phase scatter shown by the second measurement could be due to noise on the original measurement of Δφ_n that accumulates with the integration that obtains φ_n, or it could be due to an imperfect phase correction applied by the pulse shaper. Additional iterations bring the average measured Δφ closer to zero, but the scatter around 0 remains much the same (±0.5 radians). Independent measurements of the comb mode phases with the TADPOLE technique [section 6.3] indicate that the SLM can compensate the phases φ_n to zero with lower scatter than shown by the intensity modulator measurement. It is more likely then that this technique as implemented here is too noisy to characterize this comb source. In addition, there is an unknown amount of dispersion added by the input fiber and modulator crystal that would be difficult to
Figure 6.3: a) Example OSA data showing the optical spectrum when the intensity modulator is off (used for determining the mode amplitudes), and when the intensity modulator is on at both 0° and 45°. The measurements with the modulator on had the modulator bias set to minimum transmission so that the mode carriers are suppressed and the sidebands are easier to resolve. b) Zoomed version of a) showing the resolved comb modes and their sidebands. c) Extracted phase difference between neighboring comb modes ($\Delta \phi_n$). d) Integrated $\Delta \phi_n$ giving the phase of every comb mode.
Figure 6.4: a) Measurement of $\Delta \phi_n$ with a phase correction mask applied that should have driven all the $\Delta \phi$'s to zero. There is much scatter and some offset around $\Delta \phi = 0$ that is most likely due to a combination of measurement noise and measurement systematic error along with some SLM calibration error. Additional iterations of correction bring the average offset closer to zero, but the scatter on $\Delta \phi$ remains approximately the same. b) Integrated phase after correcting the measured phase from Figure 6.3d. The phase measured here has significant walkoff from zero, showing that either the phase correction is not driving the phase to zero correctly or the measurement has a large amount of noise.
measure without taking apart and destroying the modulator. For these reasons, we decided to examine other characterization techniques as described in the following section.

6.3 Pulse Measurement with TADPOLE and Verification with Autocorrelation

Because of the previously mentioned drawbacks of the intensity modulator phase measurement, it is instructive to obtain an independent characterization of the phase and amplitude properties of the shaped 21 GHz comb. The spectral interferometry measurement described in section 5.4 does a good job of characterizing the relative phase difference between the shaped comb and a reference comb; however, it is desirable to obtain an absolute characterization of the shaped comb output. The shaped comb light would be difficult to characterize directly using methods such as FROG or SPIDER because of its low peak power and complex nature, but the 1 GHz reference pulse used for the spectral interferometry of section 5.4 does not suffer from these limitations. So, to characterize the absolute pulse shape of the shaped comb, we combine a FROG measurement of the reference pulse with the spectral interferometry discussed in the previous section (this type of measurement has been referred to as TADPOLE (temporal analysis by dispersing a pair of light e-fields) and is discussed in detail in [33]).

Figure 6.5 shows the experimental set up and extracted phases of the reference pulse (measured with SHG FROG) and the extracted phase of the 21 GHz comb at the end of a 300 m long fiber (measured with spectral interferometry). The FROG trace and recovered spectral amplitudes and phases are shown in Figure 6.6. The FROG setup employed a 100 μm thick BBO crystal to ensure phase matching across the bandwidth of the reference pulse. The inherent phase ambiguity in FROG is removed by choosing the solution with positive dispersion (since it is known that the pulse travels through several thick optics). The method for extracting the phase from the spectral interferometry measurement is discussed in detail in section 5.4. Again, the phase ambiguity in the spectral interferometry measurement is removed by assuming net positive dispersion (since the pulse travels through a long, positive dispersion fiber). The measured phases from FROG and
Figure 6.5: TADPOLE system schematic. 1 GHz reference light is characterized in a SHG FROG setup. That same light is then used to compare against the shaped light from the frequency comb in a spectral interferometry setup. Both retrieved phase are shown.
spectral interferometry are added together to give the absolute phase of each comb mode.

Once the absolute phase for each comb mode is determined through TADPOLE, the line-by-line pulse shaper can be used to subtract this phase out and a transform-limited pulse is obtained at the output of the long fiber. In addition, arbitrary waveform masks can be added to this zero phase mask to obtain arbitrary optical waveforms at the end of the long fiber.

### 6.3.1 Verification with 2nd Order Autocorrelation

By using the TADPOLE measurement, we are able to characterize and correct the phase of each 21 GHz mode to obtain a transform limited pulse at the end of the 300 m fiber. Furthermore, we can apply phase and amplitude masks to generate arbitrary waveforms over a noise-cancelled fiber. The TADPOLE method will directly give the amplitude and phases of the individual frequency comb components, and a time-domain waveform can be calculated from TADPOLE, however, we wanted to independently verify the arbitrary waveforms with a time domain technique. For this purpose, we used background-free second harmonic generation (SHG) autocorrelation. Although an autocorrelation measurement is not very sensitive to subtle pulse differences, it will serve in this case only as a verification of the more powerful TADPOLE technique.

A series of numerically optimized phase masks were created to test the capabilities of the system, consisting of double pulses of varying delay times, a triple pulse, and a square pulse. All of the phase masks included the correction needed to zero the phase as measured by TADPOLE. Phase-only shaping was chosen to maximize the amount of power available for pulse characterization (approx. 0.5 mW), and as a result, the double and triple pulses showed some satellite pulses and the square pulse had oscillations around the target flat-top intensity. Higher quality arbitrary waveforms could be easily generated at the expense of total pulse power via phase + amplitude shaping. These waveforms were then measured with background-free SHG autocorrelation and the measured traces were compared with the predicted autocorrelation traces.

For each phase mask applied to the pulse shaper, the measured and theoretical autocorrelation traces match up well, showing that the TADPOLE measurement effectively characterizes the phases
Figure 6.6: Measured and retrieved FROG traces for the 1 GHz reference pulse. The measured FROG trace is asymmetric because of spatial chirp and mode variations with frequency. The FROG error for a 256 grid is 0.011. The retrieved intensity and phase are also shown along with the measured optical spectrum obtained with an optical spectrum analyzer (OSA). The retrieved intensity matches the OSA measured intensity with some narrowing most likely due to the spatial chirp interacting with the acceptance angle of the BBO crystal.
Figure 6.7: Various waveforms produced by correcting the pulse phase at the end of a long fiber and applying phase masks. (First column) Predicted pulse intensity profile. Transform-limited pulse (first row) shows a pulse duration of 70 fs, consistent with an independently obtained optical spectrum (from and optical spectrum analyzer). Amplitude shaping could bring this transform limit to under 40 fs. (Second column) Measured (red) and predicted (blue) SHG autocorrelation signals. Measured and predicted autocorrelation traces match up well. (Third column) Phase masks applied to pulse shaper. Phase masks were optimized numerically with projection onto convex sets.
of the 21 GHz comb and that the pulse shaper is applying the phase masks correctly. This result demonstrates that we can measure and remove the phase offset due to transmission through a long fiber and obtain sub-100 fs pulses at the output. In addition, we have demonstrated the production and measurement of arbitrary optical waveforms and have verified those waveforms in both the frequency and time domain with TADPOLE and SHG autocorrelation.
Remote Readout of Optical Phase Information with a Second Frequency Comb

7.1 Route to Secure Communications

Stable oscillators and clocks play important roles in most modern communication links, and very stable or accurate clocks can be critical components for some types of secure communication, such as spread spectrum methods. Recently developed optical atomic frequency standards in combination with femtosecond laser frequency combs provide new capabilities and opportunities along these lines, and in principle could be used for applications such as distribution of secure keys over an optical network. The advantage of optical frequency standards comes from the exceptional stability and high accuracy that can be achieved. State of the art optical frequency references are based on highly stabilized lasers probing narrow atomic resonances of laser-cooled atoms, which can provide a very high Q reference or oscillator for encoding or detecting frequency changes or even absolute frequencies. The femtosecond laser frequency comb permits the transfer of this high frequency stability and accuracy to virtually any region of the optical spectrum, thereby opening the possibility for spread spectrum encoding on \( \sim 10^6 \) channels over hundreds of THz of spectral bandwidth.

The basic idea is very simple; by using a frequency standard with the best stability and accuracy you can transmit information from one place to another (say over an optical fiber) that others (with a lesser frequency reference) could not detect. The information to be hidden is encoded in the optical frequency and can be realized in a number of different ways: as the absolute optical frequency itself, as a frequency offset, or as a small frequency or phase modulation (PM) imposed
on the stable laser frequencies. We demonstrate such phase modulation and readout by connecting two frequency combs locked to high-performance optical frequency standards over a noise-cancelled fiber. The readout frequency comb detects the small phase shifts imparted on the transmission comb’s modes by a line-by-line pulse shaper. When the frequency comb is locked to a less stable source (such as a 10 kHz linewidth laser or a hydrogen maser-referenced synthesizer), these phase steps cannot be seen.

Encoding information as the optical frequency or phase is secure in the technological sense that an eavesdropper could not detect the signal using traditional high-stability oscillators or atomic frequency standards. Even the highest quality microwave atomic standards (cold cesium or rubidium fountains) approaching the quantum projection noise limit and translated to the visible with optical frequency combs do not have sufficient stability (or equivalently low enough phase-noise) to measure optical frequencies with the precision required to see the signals transmitted by the optical standards. Thus the eavesdropper would also require an optical frequency standard based high-Finesse, environmentally isolated optical cavities or laser-cooled atoms. Although optical frequency standards are of growing interest to the atomic-physics/metrology communities only a handful of such systems have been developed to date. Moreover, the use of a frequency comb in conjunction with the optical frequency standard adds yet another layer of security, in that data can potentially be encoded on any (or all) of the frequency elements of the octave-spanning optical comb.

7.2 System Setup

Two separate octave-spanning 1 GHz Ti:Sapphire frequency combs are employed for this work. Each comb is independently stabilized to the laser oscillators used for optical clocks. The frequency comb used for pulse shaping (called TF1) is locked to the 657 nm laser oscillator (with a current operating linewidth of 1 Hz and drift less than 10 Hz/s) for the neutral Ca clock at NIST \cite{41}. The readout frequency comb (called TF2) is locked to the 578 nm laser oscillator (with a current operating linewidth of 250 mHz and drift of less than 0.2 Hz/s) for the Yb lattice clock at NIST \cite{73}. The readout comb (TF2) can also be locked to a $\sim$10 kHz linewidth laser at 578 nm
or the repetition rate can be locked to a microwave synthesizer that is referenced to a hydrogen maser. When a frequency comb is locked to an optical reference, each tooth of the frequency comb mirrors the linewidth and stability of the optical reference \[47\]. When the frequency comb is locked to a microwave reference, small phase errors in the lock get multiplied by the index of the comb mode in the optical domain \(n \approx 3 \times 10^5\) at 960 nm, so the optical linewidth can be significantly greater than would be expected from the stability of the source (typical optical linewidths are \(\sim 100\) kHz when locking the repetition rate to a maser-referenced synthesizer). A 300 m noise-cancelled fiber link (chapter 5) carries light from a line-by-line pulse shaper (chapter 4) that modulates the first comb to the second comb’s location. Although the two combs were in the same lab, they are independent and this experiment could be done with two combs in different laboratories connected by an optical fiber. The comb modes that have been manipulated by the line-by-line pulse shaper are heterodyned with comb modes from the readout comb as shown in Figure 7.1.

The 1 GHz combs are detuned in repetition rate \(\Delta f_{\text{rep}} \approx 1\) kHz so that beat notes between the teeth of the two combs are spectrally separated in the microwave domain (Figure 7.2). Since the shaped comb is filtered up to 21 GHz, the beat notes are separated by \(21 \cdot \Delta f_{\text{rep}}\). Digitizing the interference pattern generated by the beating of the two combs allows the phases of the comb teeth to be extracted with a computer via a simple Fourier transform (FFT). For all of the data shown here, the interference pattern was sampled by an oscilloscope with 8-bit resolution at 160 MHz for 50 ms, giving 8,000,000 total data points. Figure 7.3 shows the interference pattern and the Fourier transformed power spectrum for three measurements; one with the readout comb locked to a high-stability optical reference (Yb optical clock oscillator), one with the readout comb locked to a \(\sim 10\) kHz linewidth laser, and one with the readout comb locked to a high-stability microwave reference (hydrogen maser). The comb mode beats appear as a region of microwave power with a center frequency determined by the offset frequencies of the combs and the optical frequencies of the reference lasers. Zooming in on the comb mode beats shows that for the readout comb locked to an optical reference of high stability, there are 640 discrete and Fourier-limited beat notes \((1/50\) ms = 20 Hz) separated by approximately 25 kHz, demonstrating the detection of the 640 comb
Figure 7.1: Two independent octave-spanning frequency combs are self-referenced and locked to high stability optical references. Light from the first comb is filtered to 21 GHz and sent through a line-by-line pulse shaper, where arbitrary phase and amplitude shifts are applied to the 640 individually-addressed 21 GHz comb modes. The comb light from the pulse shaper is then sent to the second comb’s location over a 300 m, noise-cancelled fiber link. At the remote location, light from the shaped comb and from the second stable frequency comb are heterodyned and the interference pattern produced by the two combs is detected via balanced photodetection. The photodetected signal is recorded by an analog to digital converter and sent to a computer for analysis. Because the two frequency combs are detuned in repetition rate, beat notes between the teeth of the two combs are spectrally separated in the microwave domain and the phase of individual frequency comb modes can be read out.
Figure 7.2: Two frequency combs with slightly detuned repetition rates, when heterodyned, give rise to a series of optical beats in the microwave domain, each beat separated by $\Delta f_{\text{rep}}$. In this work, one of the combs is filtered from 1 GHz to 21 GHz, so the microwave beat notes are separated instead by $21 \cdot \Delta f_{\text{rep}}$. The microwave beats are offset from zero frequency by some $f_{\text{offset}}$ determined by the offset frequencies of the two combs, the difference in $f_{\text{rep}}$, and the particular comb modes that are interfering. These microwave beats contain information about the amplitude of the comb modes as well as the phase difference between the two comb modes.
modes being addressed by the line-by-line shaper. The comb mode beat notes for the case where
the readout comb is locked to a 10 kHz linewidth laser are still resolvable, but show a linewidth
similar to that of the locking laser and there is no discernible carrier. For the case where the readout
laser is locked to a microwave standard, the comb mode beat notes are no longer resolvable.

Each microwave beat note’s amplitude is proportional to a multiplication of the amplitudes
of both comb teeth. The phase of the beat note is simply the optical phase difference between
the comb modes that are beating against each other (with an offset phase determined by the
choice of $t = 0$). This phase difference can be extracted from the FFT when a carrier frequency
can be resolved and determined. When the readout comb is locked to the high stability optical
reference, this carrier is easily determined and the phase can be extracted from the complex FFT
data. Figure 7.4 shows the extracted phase for each beat note when the pulse shaper is applying no
phase mask. The unwrapped phase shows a large amount of dispersion as expected for transmission
through a long fiber.

To determine the repeatability of the measurement, a second interference pattern was recorded
with the same phase mask except that phase and amplitude shifts were applied to modes 500
through 502 so that the modes could be easily labeled by the number of the SLM pixel that con-
trols them. The time between the two measurements was on the order of 5 minutes. Figure 7.5
shows the result of subtracting the measured phases from each other. With the exception of modes
500 through 502, the subtracted phases are very close to zero and show a deviation of less than a
tenth of a radian. A linear group delay and phase offset were subtracted from the measurement.

In the case of a less stable locking standard such as the 10 kHz linewidth laser, there is no
discernible carrier so the beat note center frequency must be extracted by some means such as a fit
or a weighted mean. Phase measurements at these extracted center frequencies can be performed,
but the result is indistinguishable from noise (example shown in Figure 7.6). When the readout
comb is locked to a microwave standard, no individual beat notes can be extracted, and again, the
measured phase is simply noise.
Figure 7.3: Captured interferograms for the read out comb locked to three separate sources (red: high-stability optical reference, green: 10 kHz linewidth laser, and blue: high stability microwave reference). The interferograms do not contain any peaks because the shaped comb is spread out greatly in the time domain. The power spectrum resulting from the Fourier transform of each interference pattern is shown at a variety of zooms. The full microwave spectrum for all three data sets show that there is a region of microwave power that contains the beat notes of the two combs heterodyned against each other. The oscillation on this microwave envelope is due to amplitude modulation of the read out comb, possibly due to an etalon. A zoomed view of the optical reference power spectrum shows that individual comb modes are well resolved and have a Fourier-limited bandwidth (20 Hz in this case). For the 10 kHz linewidth laser, the comb modes are still resolved in the zoomed view, but are no longer transform limited. Zooming in on one comb mode shows the linewidth to mirror that of the 10 kHz laser. No discernible carrier is seen. In the case of a microwave reference, the comb modes are no longer resolvable and the beat notes all merge together.
Figure 7.4: (top) Measured optical phase difference between the shaped comb and the read out comb. (bottom) Unwrapped optical phase difference between the two combs. The large amount of GDD is due to the 300 m transfer fiber between the two combs.
Figure 7.5: (top) Phase difference between two measurements spaced by 5 minutes. Comb modes 500, 501, and 502 were modulated to enable the labeling of the microwave beat notes by the SLM pixel controlling the comb mode, but the phase and amplitude of the rest of the pixels remained the same. The phase difference between the two measurements is less than 0.1 radians. (bottom) Zoomed view of the phase difference between two similar measurements gives a good picture of how stable the measurement of the comb mode phase differences is on a 5 minute time scale.
7.3 Information Encoding and Readout

Once the phase difference between the two combs has been measured (Figure 7.4), it can be compensated to zero using the shaper (if desired) and phase steps can be applied to individual comb modes and read out with the second frequency comb. In the example shown in Figure 7.6, a phase-only mask was applied to line-by-line pulse shaper. Each mode was set to either 0 or $\pi$ radians. The phase extracted from the phase-encoded measurement was subtracted from the phase extracted from the first measurement with no phase mask. The subtracted phase shows a phase offset and group delay offset that are related to the choice of $t = 0$ in each measurement and the slight drift of the optical standards. The phase offset and group delay offset are removed to give the result shown in Figure 7.6. The measured phase and applied phase agree to within 0.2 radians. Also shown in Figure 7.6 is the extracted phase for the same applied mask, but with the readout comb locked to a 10 kHz linewidth laser. The phase encoding is not read out correctly in this case.

As a simple example of how such a technique can be employed to send information, the mask shown in Figure 7.6 encodes an image that can be read out by lining up the comb mode phases into 16 rows of 40 modes each. The extracted images for the readout frequency comb locked to the high stability optical reference and to the 10 kHz linewidth laser are shown in Figure 7.7.

Although it is possible to apply very small phase steps to the shaped comb modes (SLM phase resolution $\sim \pi/500$), the inherent noise on the frequency combs and optical references will limit the resolution of the phase readout. While measurements taken 5 minutes apart shown phase differences under 0.1 radians (Figure 7.5), measurements with longer time between them shown phase differences of about 0.25 radians. This phase error could possibly be reduced by shortening the time between measurements so that the drift of the optical cavities is not a factor, but current measurements are limited in time by the transfer of data from the oscilloscope to the computer. Furthermore, the phase noise of the optical locks on both frequency combs add some noise to the measurement. Figure 7.8 shows the results of an attempt to encode and readout phase steps of $\pi/16$ (0.2 radians) on the comb modes. The measured phase is close to the applied phase, but not close
Figure 7.6: a) Comparison of the measured and applied phase shifts for a region of the comb spectrum. Both of the frequency combs were phase locked to high-stability optical references. The shaped comb modes were modulated either to 0 or $\pi$ radians and the extracted phase shifts show agreement with the applied mask to within 0.2 radians. b) The same phase mask was applied to the shaped comb, but this time the read out comb was locked to a 10 kHz linewidth laser. The phase mask is unable to be extracted with the read out comb locked in this way.
Figure 7.7: a) Picture view of the extracted phases shown in Figure 7.6. By aligning the extracted comb mode phases into 16 rows of 40 modes each, an image is extracted. b) Same image encoding as above, but with the read out comb locked to a 10 kHz linewidth laser instead of a high-stability optical reference. The image is not extracted and looks like noise.
enough that the phase steps can be resolved into specific $\pi/16$ phase bins. The standard deviation of the phase error is 0.2 radians with a maximum phase error of $\pm 0.4$ radians. This indicates that $\pi/4$ and possibly $\pi/8$ phase encoding could be realized with this method on 640 channels. Further refinement of the data capture and phase extraction techniques should yield improvements that would allow $\pi/32$ (0.1 radian) phase steps to be resolved.

The encoding and readout of optical phase information between two combs separated by 300 m of optical fiber demonstrates the possibility of secure communications. The encoded phase steps are not visible on the amplitudes of the comb modes and so can only be read out with an optical phase detection method. Furthermore, only a frequency comb locked to a high stability optical reference (with Hz-level optical linewidth) has the sufficient phase stability to read out the encoded phase. This phase encoding and detection system, as currently implemented, is not practical for high speed communications, but the limitations on the transfer speed are not fundamental. Line-by-line pulse shaping with GHz update rates have been demonstrated [86], and data collection and analysis could be sped up considerably with dedicated hardware. Overall, it should be possible to transmit this optical phase information at MB/s rates. This is slow compared to the state-of-the-art data rates of current telecommunication systems, but the added layer of security enabled by optical phase encoding is worth developing further.
Figure 7.8: Attempted encoding and readout of a phase mask with $\pi/16$ (0.2 radian) phase steps. The extracted phase and applied phase agree within 0.4 radians, so the discretized phase steps are not extracted with good fidelity. This indicates that minimum steps of $\pi/8$ are required for encoding of phase information in this implementation. It is likely that drift in the optical cavities contributes to this phase error along with the relatively slow retrieval of data from the sampling oscilloscope (30 minutes between this measurement and the reference phase measurement).


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Appendix A

Octave-Spanning Chirped Mirrors

A.1 Chirped Mirror Overview

Before starting graduate school, I designed and had fabricated a pair of octave-spanning chirped mirrors that have found some utility in various lasers and system setups around the Diddams lab. To ensure that the design of these mirrors and their performance specs are not lost to posterity, I have included this appendix to my thesis.

A ’chirped mirror’ is a generic name for a mirror that provides negative group delay dispersion (GDD) upon reflection. This negative dispersion is achieved by chirping the thickness of the reflective layers within the thin film stack such that wavelengths closer to blue are reflected in the surface layers while colors closer to the red are reflected from layers deep in the stack. This causes the red end of the spectrum to accumulate more phase delay than the blue end of the spectrum, leading to negative GDD. Chirped mirrors are useful for pulse compression and for laser cavity construction. In laser cavities, the chirped mirrors replace bulky prisms or gratings to manage net cavity dispersion, so the whole cavity can become much more compact.

Designing chirped mirrors can be a time-consuming process. For most chirped mirrors, there is a small reflection for all colors off of the surface layers, leading to interference effects that give ripples in the GDD around the desired value. This ripple can be almost eliminated over narrow wavelength regions with an appropriately designed anti-reflection (AR) coating and numerical optimization. For broader bandwidth designs, however, this ripple cannot be eliminated. The solution to reducing the ripple in the GDD is then to design two mirrors, the second with the opposite sign
of ripple as the first, such that reflecting off of both mirrors reduces or cancels the ripple and gives the desired GDD. These ripple-compensated mirrors are used in broadband laser cavities and for the external compression of very short pulses.

The chirped mirror pair I designed was intended for use in an octave-spanning Ti:Sapphire laser cavity consisting of four mirrors in a bowtie configuration [87]. The mirrors were designed to provide negative GDD with offsetting ripple over a bandwidth of an optical octave, specifically, from 600-1200 nm. The mirrors were also designed to have bandwidth of high reflection ($R \geq 99.9\%$) over the same span. I designed the mirrors using TFCalc\textsuperscript{TM}, a thin-film design software, using SiO\textsubscript{2} as the low index layer and TiO\textsubscript{2} as the high index layer. A multitude of designs were used as the starting point for numerical optimizations; most of them derivatives of designs from the paper by Matuschek, et al. [88, 89]. The targets for GDD were set to correct a combination of Ti:Sapphire and fused silica dispersion up to 4\textsuperscript{th} order (values calculated from [90]). The net cavity dispersion used for the design of the chirped mirrors is shown in Figure A.1. The cavity I originally designed the mirrors for had no output coupler mirror; instead, it was to use a small reflection off of one surface of a tilted prism (the other surface was at Brewster’s angle) as the output coupler. As a result, the mirrors were designed such that four mirrors compensate the dispersion of the Ti:Sapphire crystal and the output prism.

Heavy numerical optimization led to the final designs henceforth called M1 and M2. M1 has a transmission window around 532 nm to allow transmission of the pump light into the cavity. The layer stacks for M1 and M2 are shown in Table A.1 and Table A.2. The mirrors were designed for 14° incidence angle.

These mirrors had reflectivity above 99.9% from 650-1200 nm and compensating GDD ripple as shown in Figure A.2 and Figure A.3.

If I had to design some chirped mirrors again, knowing more about how these octave-spanning oscillators work, I would have made a few changes. First, I would have made a pump transmission window in both mirrors, so that the laser builder would not be constrained to using M1 as the first cavity mirror and also so that retroreflection of the pump light could be implemented to increase
Figure A.1: Net cavity dispersion assuming 3 mm of Ti:Sapphire and 5 mm of fused silica. The large amount of fused silica in the cavity was employed to increase the dispersion required from each mirror to make the design easier to optimize.
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Table A.2: M2 Layer Stack
Figure A.2: Theoretical reflectivity of both M1 and M2 at 14° incidence angle. Both mirrors show greater than 99.9% reflectivity over the range from 650-1200 nm. From 600-650 nm, it was necessary to relax the reflectivity requirement to ≥ 99% so that the GDD could be better optimized. There is a pump window in M1 spanning 10 nm around 532 nm.
Figure A.3: Theoretical GDD of both M1 and M2 at 14° incidence angle. The target GDD that cancels the dispersion of the cavity is also shown. Both M1 and M2 show large oscillations around the average value, but the oscillations are out of phase with each other and cancel quite well.
efficiency. Second, I would abandon the prism output coupler idea in favor of a traditional output coupler design that takes advantage of increased output coupling in the wings of the spectrum, giving a much broader output spectrum than the prism case. This would require some changes to the design; namely, the cavity dispersion would need to be managed by 3 mirrors instead of four, and the three mirrors would need to have some way of offsetting GDD ripple. This offsetting ripple could be brought about by increasing the ripple on one of the mirrors such that the ripple from two bounces off of the M1 design could be compensated by only one bounce off of the M2 design. Additionally, experimentation by Tara Fortier has shown that ripple compensation is not necessary for the construction of an octave-spanning oscillator. Tara built an octave-spanning Ti:Sapphire laser with three M1 mirrors (giving extreme amount of GGD ripple) and an output coupler. Its output spectrum was not as broad as the laser with some ripple-compensation, but it was still a usable octave-spanning laser. The ripple appears not to broaden the pulse, but only produce satellite pulses that sap some power out of the main pulse. These satellite pulses are possibly attenuated by the Kerr lens effect. So, if coating cost were too high for two mirror designs, a single chirped mirror design could be made that could be used for all three non-output-coupling mirrors in the cavity that could still yield an octave-spanning laser.

A.2 Characterization of the Chirped Mirrors

The mirror designs mentioned above were produced by Research Electro Optics (REO) in Boulder, CO. To characterize the fabricated mirrors, we needed to measure both the phase shift and the reflectivity/loss of the mirrors. To measure the GDD, we constructed a white light interferometer (WLI). A WLI uses a source with a very short coherence length, typically some kind of light bulb or filament, and a dispersion-balanced scanning Michelson interferometer. The setup for our WLI is shown in Figure A.4.

The two arms of the interferometer have to be matched in length to within the coherence length of the source (a few microns). The scanning mirror is then scanned back and forth across the zero delay point. A reference laser is used to calibrate the distance scan of the scanning mirror.
Figure A.4: Light from a broadband, incoherent source is collimated and split into two arms by a broadband metallic beamsplitter. The metallic beamsplitter coating is a thin layer of inconel with approximately 35% percent transmission and reflection. The dispersion due to the glass backing of the beamsplitter is balanced by a matched glass plate in the reflection arm of the interferometer. The length of one arm of the interferometer is scanned across the zero-delay point by a movable mirror. Metal mirrors providing broadband reflection and zero dispersion are used throughout, and the mirror under test is placed in one arm of the interferometer. A CW reference laser is coupled through the same path as the white light and the fringes are monitored. The CW fringes provide a distance reference to calibrate the scan of the movable mirror. The white light interferogram is detected with either a Si detector (visible) or a Ge detector (NIR). The white light interferogram and CW fringes are recorded by an oscilloscope and analyzed by a computer.
With no dispersion imbalance between the two arms, the interference pattern is a measurement of the coherence length of the source (along with the response of the detector), so the interference signal looks like a short pulse [Figure A.5]. If one of the metal mirrors in one arm is substituted for a chirped mirror with dispersion, the short pulse interference pattern spreads out and has much more structure [Figure A.6]. A Fourier transform directly gives the phase response of the mirror [91]. A program was designed that captured the interference traces from the oscilloscope and extracted the phase response of the chirped mirrors. Example screenshots of this program are shown in Figure A.6 and Figure A.5.

Using this white light interferometer, we characterized the GDD of both M1 and M2 from 600-1200 nm. It was necessary to use two detectors (one Si, one Ge) to capture interference patterns across this broad spectrum. The results of these measurements are shown in Figure A.7 and Figure A.8. Also measured was the GDD with both M1 and M2 in the interferometer, leading to the results shown in Figure A.9. The ripple for both mirrors matched with the ripple predicted from theory (with a small wavelength shift) and the ripple cancellation also worked well.

The reflectivity and loss of the mirrors also needed to be characterized to verify that these mirrors would make good cavity mirrors. Early coating runs and test shots showed that absorption might be on the order of 0.1%, which would reduce the performance of the laser quite significantly. Increasing the oxygen flow during the coating runs reduced this loss below a level that could be measured easily by the coating engineers at REO, so I constructed a Fabry-Perot cavity using a M1 or M2 mirror, and a 'super mirror' with very high reflectance and low loss. This cavity was swept across resonance and the transmission was monitored. The mirrors were tested at two different wavelengths (657 nm and 1112 nm) to test the loss in two different reflectivity regions. A system diagram along with a sample transmission trace is shown in Figure A.10.

The interesting structure on the transmission trace arrives from frequency shifts of the light in the cavity due to bouncing off the moving mirror [92, 93]. This effect can be modeled, so I decided to fit the transmission data to the models from the papers and extract the cavity finesse in that way. Using this method, I found mirror reflectivities as shown in Figure A.11.
Figure A.5: Screen capture of white light interferometer control program with balanced dispersion in the interferometer arms. The short pulse (yellow) is the white light interference pattern and the sinusoidal fringes (blue) are the fringes from the HeNe reference laser. The lower graphs show the phase, group delay (GD), and group delay dispersion (GDD) extracted from the measurement. The dispersion of the two arms is balanced within a few fs².
Figure A.6: Screen capture of white light interferometer control program with a chirped mirror in one of the interferometer arms. The interference pattern (yellow) is spread out in time and shows more structure. The lower graphs show the phase, group delay (GD), and group delay dispersion (GDD) extracted from the measurement. The mirror shows positive dispersion in this case, meaning that the program captured the interferogram when the mirror was moving in the opposite direction as assumed in the phase extraction program. A simple sign reversal gives the correct phase measurement and shows that this mirror provides negative dispersion from 750-1000 nm with some ripple around the mean. This mirror was an early design and test shot that was not as broad band as the final designs.
Figure A.7: White light interferometer measurement of M1 GDD shows the GDD fringes of the M1 coating match well with the theoretical GDD fringes (with a slight offset). The slight offset in the GDD fringes is possibly due to layer thickness errors in the coating or, more likely, to a slight change between the theoretical indices of refraction of the high and low index layers and the actual indices of refraction. Two measurements for each detector are shown to demonstrate the repeatability of the measurement.
Figure A.8: White light interferometer measurement of M2 GDD shows the GDD fringes of the M2 coating match well with the theoretical GDD fringes (again with a slight offset). The slight offset in the GDD fringes is again possibly due to layer thickness errors in the coating or, more likely, to a slight change between the theoretical indices of refraction of the high and low index layers and the actual indices of refraction. Two measurements for each detector are shown to demonstrate the repeatability of the measurement.
Figure A.9: White light interferometer measurement with both M1 and M2 in one arm of the interferometer. The measured dispersion of the mirror pair shows reduced oscillations consistent with the theoretical ripple cancellation.
Figure A.10: A ringdown cavity is used to characterize the reflectivity of the fabricated M1 and M2 coatings. The cavity consists of one mirror under test and one 'super mirror' with reflectivity greater than 99.99%. The cavity is swept across resonance with a stable CW laser and the transmission is monitored as a function of time. For certain sweep speeds, the cavity transmission peak has oscillations on the ringdown side of the signal as shown by the inset data. This structure comes from the frequency shifts imparted on the intracavity light as the mirror moves across the resonance. Light that has been in the cavity longer has a greater frequency shift, so there are a range of optical frequencies in the cavity, leading to interference effects described in [92, 93].
Figure A.11: Experimental Fabry-Perot ringdown data shown with fit to that data for both M1 and M2 at 657 nm and 1112 nm. The fits indicate that the reflectivities of both M1 and M2 are very close to their expected values, showing that the mirror coatings have low loss. The 657 nm measurements show a larger decrease in reflectivity than the 1112 nm measurements, but the reflectivity of both M1 and M2 are quite sensitive to spectral shifts in the coating at 657 nm, so the decrease in reflectivity is not necessarily loss. M1 and M2 are much less sensitive to spectral shifts at 1112 nm and the 1112 nm light penetrates much farther into the mirror stack, so those measurements are a better indication of the loss.
These results, coupled with transmission measurements by REO, show that the loss for the mirrors is below 0.016%, which indicate these mirrors will be useful for laser cavities as hoped. These mirrors were incorporated into an octave-spanning Ti:Sapphire laser cavity in 2010. The octave-spanning laser with these mirrors shows a larger bandwidth than the octave-spanning laser built with commercially available chirped mirrors. A comparison of the output spectra of the two lasers is shown in Figure A.12. In conclusion, the mirrors M1 and M2 are useful for building broadband laser cavities and for general pulse compression of ultra-short pulses.
Figure A.12: Comparison of the optical spectrum of an octave-spanning Ti:Sapphire laser made with commercial chirped mirrors (TF1) with the optical spectrum of an octave-spanning Ti:Sapphire laser made with the chirped mirrors M1 and M2 (TF2). TF2 has much more light in the wings of the spectrum, showing that the chirped mirrors M1 and M2 produce a shorter pulse in the cavity, leading to enhanced spectral broadening in the crystal.