2014

An Argument Against Commensurate Truthmakers

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Abstract

The core of the truthmaker research program is that true propositions are made true by appropriate parts of the actual world. This idea seems to give realists their best shot at capturing a robust account of the dependence of truth on the world. For a part of the world to be a truthmaker for a particular it must suffice for, or necessitate, the truth of the proposition. There are two extreme and unsatisfactory truthmaker theories. At one extreme any part of the world (up to and including the whole world) that suffices for a proposition is deemed to be a truthmaker for that proposition. At the other extreme there is only one truthmaker that can do the job, and that is the entire world, the whole show. Another possibility suggests itself, that a truthmaker be any *minimal* sufficer for a proposition. A truthmaker is minimal if it suffices but no proper part of it suffices. A minimal sufficer would be both sufficient for the truth of the proposition and, in one sense, necessary for it as well. A minimal sufficers would be *commensurate* with the proposition it makes true. Unfortunately not all propositions have minimal sufficers. But it does not follow that not every proposition has a commensurate sufficer. The problem, then, is to specify a coherent notion of commensurateness on which every truth has a commensurate sufficer. I argue that this problem may not be soluble.

An argument against commensurate truthmakers

The core of the truthmaker research program is that true propositions are made true by appropriate parts of the actual world. I have three confessions to make. First: I am (or used to be) a bit of a fan of the program. It seems to give realists their best shot at capturing a robust account of the dependence of truth on the world.   Second: I have (somewhat to my own dismay) stumbled on what appears to be a new problem for truthmaker theory. The core desiderata of the program cannot be jointly satisfied. Third: I hope I am wrong, and that someone can locate an error in the proof.

1   Truthmakers as parts of worlds

I will assume that the World is the fusion of facts not things. (If you want a term for the fusion of things, not facts, call it the Universe.) And I will assume that it is parts of the World (i.e. facts) that make propositions true, not parts of the Universe (i.e. particular things).

Let's start with an easy and favorable case for truthmaker theory. Consider some particular object x (some particular leaf, say) and the proposition (G) that *x is green*. Suppose G is true. Which part of the world makes this proposition true? This isn't one of the problem cases faced by truthmaker theorists — like a negative proposition, a universal proposition, a negative existential, a proposition about the future, and so on — and there are seemingly good candidates on offer. One such is the fact that consists in *x's being green*. The particular x alone does not make the proposition true, and neither does the property of being green. It is the having by x of the property of greenness that makes G true, and that is a fact.

So I am going to assume, without further argument, that the truthmakers of propositions are facts and that facts are obtaining states or events (from now on I will refer to them as *states*). Basic states consist in the having, by objects, of genuine properties, or the standing in genuine relations of objects. But basic states can be parts both of other basic states and of non-basic states, which are also candidates for the role of truthmakers of propositions.

I will make a few assumptions about the structure of the space of states. First, states can be *parts* of other states. (For example, all actual states are parts of the actual world). And states can *determine* other states. (The actual world as a whole determines all actual states.) But these relations are the converse of each other. The parthood relation ($\sqsubseteq$) is just the converse of the determination relation ($\sqsupseteq$). A state t *determines* a state s ($t\sqsupseteq s$) if and only if s is a part of t ($s\sqsubseteq t$). *Proper* parthood and *strict* determination are the obvious restrictions of these relations to distinct states.

For suitable classes of possible states (those which are, intuitively, *compatible*) there is the unique state which is the *fusion* of those states. The fusion of some class of compatible states is the smallest state (in the ordering by parts) of which all the states in the class are parts (i.e. it is the *least upper bound* or *supremum* of the class of states). Where S is a class of compatible states, the fusion of S will be written $\oplus S$. And the supremum of a pair of compatible states s and t will be written $s\oplus t$.[1] We can define a *world* as a maximal state.[2] A state s obtains in a world w just in case s is a part of w ($s\sqsubseteq w$).[3]

For our purposes here we can remain fairly neutral about the nature and structure of propositions. All I will assume is that proposition induce a bifurcation of the

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[1] The notion of compatibility of states can be spelt out in a number of different ways. One uses the notion of a *directed* set. A directed set is a set in which any two members of the set have an upper bound in the set (i.e. any two members of the set are parts of a third member of the set). A set's being directed is a sufficient condition for the compatibility of the states in the set. A partially ordered set is *directed complete* (it is a *dcpo)* if every directed subset has a least upper bound, or *supremum*. The supremum of a directed set of states (the fusion of the set) is that state which takes place if and only if every state in the set takes place. Alternatively one could regard any set of states that has an upper bound in the ordering to be compatible, and require that each bounded set has a least upper bound in the ordering.

[2] The existence of worlds in a dcpo is guaranteed by the fact that every chain must have a supremum. A *chain* is a set of states in which for any two distinct members of the set, s and t, either s is a proper part of t or t is a proper part of s. A chain is clearly a directed set (the larger of any two members of the set is an upper bound for the pair) and so in a dcpo, every chain has a supremum, the fusion of all the states in the chain.

[3] For simplicity I am just going to ignore the temporal dimension, which presents its own significant difficulties for truthmaker theory. This simplification might not be a problem for a four-dimensionalist who could treat the having of properties as a relation between things and times. But it might be thought problematic for a three-dimensionalist. However, in the context of my impossibility proof, the simplification is harmless. For if we cannot find parts of the world fit for the truthmaking role in this simplified atemporal framework we are not going to find them in a full blown temporal framework in which the flow of time is real.

set of worlds — the worlds in which the proposition is true and those in which it is false.[4] But this is of course compatible with propositions being structured, fine-grained entities. There can be many distinct propositions that induce the very same partition of worlds.

If every world in which a state s occurs is a world in which the proposition P is true, then s is a *sufficer* for P. Most truthmaker theorists accept that any candidate truthmaker for P must be a sufficer for P. The obtaining of s necessitates the truth of P. Call this principle:

> *Necessitation*
>
> All potential truthmakers for a proposition are sufficers for the proposition (and all the actual truthmakers for a true proposition are *actual sufficers*).

In the case of G, for example, there are tons of actual sufficers. There is the state, g, that consists in *x's having the property of being green* (assuming, for the moment, that *being green* is a genuine property[5]). But any actual state of which g is a proper part is also an actual sufficer. For example, the complete distribution of hue, saturation and brightness over the entire surface of the leaf; the fusion of all color states (i.e. the complete actual distribution of hue, saturation and brightness over all objects); the complete actual distribution of all fully determinate properties and relations over over all the objects in the actual world (i.e. the whole show, the big enchilada, all that's the case, the fusion of all actual states, the World). All these are actual sufficers for the proposition G.

In light of this surfeit of actual sufficers there are three basic choices for a truthmaker theorist. First, she could go *all inclusive* by deeming all sufficers of a proposition to be potential truthmakers.

> *All-Inclusive*
>
> All actual sufficers of a proposition are truthmakers for the proposition.

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[4] Again for reasons of simplicity, I will ignore the interesting plight of truthvalueless propositions in a theory of truthmakers.

[5] I take genuine properties to be the convex regions of a quality space. See Peter Gärdenfors, *Conceptual Spaces* (Cambridge, Mass.: MIT Press, 2000), and Graham Oddie *Value, Reality and Desire* (Oxford: OUP, 2005) chapter 5.

Necessitation and All-Inclusive jointly entail a complete theory of truthmakers:

A State is a truthmaker for *P* true if and only if it is an actual sufficer for *P*.

All Inclusive embraces a vast and varied array of states as the truthmakers for G, from least (x's being green), to largest (the actual world).

But one could also go *selective*: narrowing down the set of truthmakers to some small and privileged set of actual sufficers.

Consider the actual world — the fusion of all actual states. One and only one world is actual. The actual world not only suffices for the truth of G, but for every other truth as well.[6] As Armstrong puts it (rather moralistically), the actual world is "the most promiscuous" and "least discerning truthmaker of them all" — at least given the assumption, to which Armstrong is sympathetic, that the actual world not only suffices for every truth, but makes true every truth. Other potential truthmakers are less promiscuous, or more discerning, about the range of propositions they make true.

The world is a candidate truthmaker for a theorist who also wants to go selective. One could deem the actual world the *sole* truthmaker for all truths.[7]   Indeed it seem that this is the paradigm for a *selective* theory that embraces *undiscerning* truthmakers. Deeming the actual world the sole truthmaker of all truths is *selective*, but by its very universality, the actual world is the *least discerning* actual sufficer on offer.

Selective and Undiscerning

There is one and only one truthmaker for all truths: the actual world.

Return to the little proposition G. Of all the actual sufficers for G, the *most* discerning in our hierarchy is the state g: *x's being green*. It suffices for the truth of the proposition G (any sufficer for G must contain g as a proper part) but g does so without any extra, unnecessary, otiose, or irrelevant bits of the world. g is a *discerning* because

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[6] There are of course, problems lurking here, most obviously with the negative existentials. I will ignore these here because I think there are various ways a truthmaker theorist can make the World sufficiently rich to solve them. I have assumed that the set of states is directed complete, which entails the existence of maximal elements (i.e. worlds) which are not part of any larger states. There are no upward chains which go on forever, never culminating in a world, and every state is part of at least one world.

[7] See Jonathan Schaffer, "The most promiscuous and least discerning truthmaker", *The Philosophical Quarterly* Vol. 60, No. 239.

it doesn't make any propositions that are logically stronger than G true, and —
depending on how one goes on to develop the theory — there may also be propositions
strictly weaker than G that g does not make true either. To go discerning, then, is to
narrow the set of candidate truthmakers to actual sufficers that accomplish making-true
as *economically as possible*. They are large enough (they suffice) but they are not too
large (they don't go over the top). Because of this, a discerning truthmaker will make
true a relatively small set of propositions, those for which it is (as I will say)
commensurate. This intuitive notion of commensurateness — which it is the purpose of
this paper to develop and explore — gives us the basic framework for any *Selective and
Discerning* account:

> *Selective and Discerning*
> A state *s* is a truthmaker for a proposition *P* if and only if *s* is a *commensurate*
> actual sufficer for *P*.

2   Commensurate sufficers

In first introducing us to his conception of truthmakers in his 2004 book *Truth and
Truthmakers* David Armstrong says this:

> The idea of a truthmaker for a particular truth, then, is just some existent, some
> portion of reality, in virtue of which that truth is true. ...There is something that
> exists, in reality, ... which makes the truth true. .. This 'making' is of course, not
> the causal sense of 'making'. The best formulation of what this making is seems
> to be given by the phrase 'in virtue of'.[8]

Although Armstrong here rightly distances truthmaking from causal making, the two
cases share a striking feature. The requirement that a proposition be true *in virtue of* its
truthmaker sounds similar to what Yablo has dubbed the *commensurateness* of cause
to effect, which in turn draws on the traditional idea that the cause of a state must not
only be sufficient for its effect but also (in some sense) necessary.[9] Recall that,

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[8] Armstrong, D. M. *Truth and and Truthmakers*, (Cambridge: CUP, 2004), p. 5.

[9] Stephen Yablo, "Mental Causation", *Philosophical Review*, (1992), 101: 145-80

according to Yablo, not all the states that are causally sufficient for some state are equally good candidates for *the cause* of the state. The cause of a state must be *commensurate* with its effect, in the sense that while it must be determinate enough to causally suffice, it should not be more determinate than it is necessary for it to be. In what follows I argue that the idea behind discerning truthmakers is simply that they must be *commensurate* with the propositions they make true. Neither too large nor too small.

If the proposition G is true in virtue of any obtaining state it is true in virtue of x's instantiating greenness: the state g. But consider the proposition R that some object y, distinct from x, is red. The proposition R is not true in virtue of g, because g fails to be an actual sufficer for it (it isn't enough). But even the weaker proposition [R∨¬R] doesn't seem to be true in virtue of g, because g just carries too much excess baggage for that (it is too large). Even though it is an actual sufficer for [R∨¬R], g isn't commensurate with [R∨¬R]. It carries baggage that is strictly unnecessary for the truth of [R∨¬R].

Likewise, actual states that are more determinate than g (of which g must be a proper part) also suffice for G, but G is not true in *virtue* of them either, since they too carry excess baggage, baggage that doesn't do any work in making G true. Truthmakers, and parts of truthmakers, have to earn their keep by doing some work! Even though G is true in virtue of g, and g is a proper part of many actual states, including the actual world w, G is not true in virtue of these larger states. Like these more encompassing bits of reality, g is large enough to do the job of making G true, but unlike them, it isn't too large. It is, as Goldilocks, might say, "just right".

All Inclusive yields the problematic *Entailment Principle*: that if *s* is a truthmaker for *P* then *s* is a truthmaker for every logical consequence of *P*. Suppose that state g is a truthmaker for proposition G. Then, since G entails both [R∨¬R] and [7+5=12], g is also a truthmaker for [R∨¬R] and [7+5=12], along with any other necessarily true proposition. g is a truthmaker for all of them, *as is every other actual state*. This truthmaker trivialization result — that a truthmaker for any proposition is also a truthmaker for every necessary proposition — clearly violates the requirement that propositions be true in *virtue* of their truthmakers. The proposition that [7+5=12] is not true in virtue of x's being green.

Some of those sympathetic to truthmakers who find this trivialization result unpalatable have suggested restricting the truthmaker requirement to contingent truths alone. Frank Jackson suggested this once.[10] And Hugh Mellor has argued, quite generally, that since necessary truths are true *come what may*, nothing is required to *make* them true.[11] In particular, x's being green doesn't make them true.

While this move is not totally unmotivated it is not an entirely happy one. Firstly it involves abandoning unrestricted *Truthmaker Maximalism* — the thesis that all truths have truthmakers. But perhaps the requirement for truthmakers can be reasonably restricted to contingent truths. Secondly, and more importantly, even if it does block a class of counterexamples to the Entailment Principle, it doesn't tackle the underlying problem. While the problem surfaces in a particularly stark form at the limit — in truthmakers for necessary truths — it is ubiquitous. At bottom it is the problem of commensurateness. Just as g is not commensurate with necessarily true propositions, the vast array of more encompassing states of which g is a part are not commensurate with the small and modest, but contingent, proposition G.

David Armstrong makes extensive use of the Entailment Principle throughout his work on truthmakers. He claims the principle helps identify truthmakers for propositions that otherwise look like they might have none — for example, truths about mere possibilities (e.g. that *G is contingent*). But, as we have seen, the Entailment Principle violates Armstrong's own *in-virtue-of* requirement, deeming states truthmakers that are clearly incommensurate with the propositions they are supposed to make true. To avoid absurd consequences like the trivialization thesis, Armstrong proposes to restrict it to *relevant* entailments: if *s* makes *P* true, and *P relevantly* entails *Q*, then *s* also makes *Q* true. So, for example, g makes G true: and since G relevantly entails the necessary truth G∨¬G, g also makes the necessary truth [G∨¬G] one true. However, G doesn't *relevantly* entail the necessary truth [R∨¬R], so a Relevant Entailment Principle would

[10] Greg Restall reports this in his "Truthmakers, entailment and necessity", *Australasian Journal of Philosophy* 74, (1996) p. 334, and calls it "Jackson's thesis".

[11] See Hugh Mellor "Contingent Facts" *Analysis* 71:1 (2011), pp. 62-68. Mellor further restricts to the thesis to what he calls "primary truths" which are presumably true "atomic" propositions, or something like that.

block at least some of the counterintuitive consequences of a pure Entailment Principle. Armstrong outsources to Relevance logicians the task of articulating a suitable theory of relevant entailment that will mesh with his other desiderata on truthmaking.

Gonzalo Rodriguez-Pereyra, also a truthmaker enthusiast, makes a similar objection the Conjunction Principle (*viz.* that a truthmaker for a conjunction is a truthmaker for each of its conjuncts).[12] He disposes of this Principle with the following example:

> Take an arbitrary conjunction, say <Peter is a man and Saturn is a planet>. Suppose that conjunctions are made true by conjunctive entities and that this conjunction is made true by the conjunctive fact that Peter is a man and Saturn is a planet. Even so it is not the case that <Peter is a man> is true in virtue of the fact that Peter is a man and Saturn is a planet. What <Peter is a man> is true in virtue of is simply the fact that Peter is man.

Again, the underlying idea is that the fusion of *Peter's being a man* and *Saturn's being a planet* is not a suitable truthmaker for the proposition that Peter is a man, because it contains excess baggage. The proposition is true in virtue of the less determinate state, and any more determinate state of which it is a part fails to be commensurate.   The Entailment Principle, whether unrestricted or restricted to relevant entailments, also yields the result that the World is a truthmaker for every truth. Suppose there is a maximally strong proposition, Truth^w, that is true in just the one world, w. Suppose further, that Truth^w is true in fact, that w is the actual world. Then w, the World, suffices for Truth^w, and no part of w suffices. So w is perfectly commensurate with Truth^w. Let P be any true proposition. Since Truth^w entails P, by the unrestricted Entailment Principle, w is a truthmaker for P. But w is hardly *commensurate* with the content of a proposition like G (that X is green). Suppose that r is the *rest* of the World — w minus just that one tiny little state g. The World is thus the fusion of g and r ($w = g \oplus r$). While the

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[12] The Entailment Principle clearly entails the Conjunction Principle. The Conjunction Principle together with the Equivalence Principle (that logical equivalents have the same truthmakers) entails the Entailment Principle. Assume Conjunction and Equivalence. Suppose *P* entails Q and *s* is a truthmaker for P. Then since *P* is equivalent to P&Q, by Equivalence *s* is a truthmaker for P&Q, and by the Conjunction Principle, *s* is a truthmaker for Q. This demonstration, of course, ignores the kinds of strictures hat could be imposed by some criteria of relevance.

proposition G is clearly true in virtue of the obtaining of g, it doesn't seem true in virtue of f r. So r is not only irrelevant to the truth of the proposition G, it is vastly so. But then, *almost all* of the World is completely irrelevant to the truth of G (Remember: g is but a tiny part of $g \oplus r$ while r is most of it). Just as the proposition that Peter is a man is not true in virtue of *Saturn's being a planet and Peter's being a man*, G is not true in virtue of the fusion of g and (the vastly irrelevant) r.

A relevantist might well object that even though Truth^w classically entails all truths, it may not *relevantly* entail all truths. But this is not enough to block trivialization. Where *P* is a truth not relevantly entailed by Truth^w, consider the proposition Truth$^w \wedge P$. w is the only good candidate for truthmaker of Truth$^w \wedge P$, since w is the only state that suffices for it. And Truth$^w \wedge P$ does relevantly entail *P*. So, by the Relevant Entailment Principle, w is a truthmaker for *P* too.

All Inclusive and *Selective and Undiscerning* are thus both in conflict with the desideratum that a truthmaker be commensurate with any proposition it makes true. From now on I will assume that the condition of commensurateness is a basic desideratum for an adequate theory of truthmakers, that a truthmaker theorist wants a *Selective and Discerning* account.

3 Minimal sufficers

Armstrong is sensitive to the requirement of commensurateness which his *in-virtue-of* requirement imposes. Although he still seems to endorse the World as a universal truthmaker, as we have already noted he also rightly disparages it as "the most promiscuous" and "least discerning truthmaker of them all". He goes on:

> More interesting, and of quite special importance for metaphysics, is the notion of a minimal truthmaker. If *s* is a minimal truthmaker for *P* then you cannot subtract anything from *s* and the remainder still be a truthmaker for *P*.[13]

He goes on to suggest that minimal truthmakers might be the *only* decent candidates for the status of truthmaker:

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[13] Armstrong, D. M. *Truth and and Truthmakers*, (Cambridge: CUP, 2004), p. 19.

One may object to the idea that there are any truthmakers *except* minimal truthmakers. For it, may be argued, a non-minimal truthmaker involves redundancy, and the truth in question may *not* be true in virtue of the redundant material. ... If someone wishes to say that what I call a non-minimal truthmaker for a certain proposition is really a portion of reality that has the real truthmaker as a proper art, then I have no metaphysical objection.[14]

The attraction of minimal actual sufficers is precisely that they satisfy the requirement of commensurateness. They are large enough (they suffice) but are not too large (no proper part of a minimal sufficer would be large enough). Whether or not all commensurate sufficers are minimal sufficers, it is pretty clear that *all minimal sufficers of a proposition are commensurate*.

Consider *either it is hot or dry* [H∨D]. Suppose H and D are both true. *Its being hot* (h) is a minimal sufficer for H, and *its being dry* (d) is a minimal sufficer for D. No proper part of h or of d suffices for [H∨D], so both are minimal actual sufficers for [H∨D]. The state *its being still* (s) is not a truthmaker for [H∨D] — it contains both too little (it doesn't suffice), and too much (its being still does no truthmaking work here). The fusion of *its being hot* and *its being still* (i.e. h⊕s) also suffices, but it too contains too much (the stillness factor), so it also fails to be a commensurate truthmaker for [H∨D]. Arguably h⊕d (like h⊕s) also contains too much. These facts are compatible with, and indeed suggest, Armstrong's tentative proposal that to be commensurate with *P* a state must be a minimal sufficer for *P*.

Call this notion *minimal commensurateness*, or m-commensurate for short.

A state *s* is m-commensurate with a proposition *P* if and only *s* suffices for *P* and the only part of *s* that suffices for *P* is *s* itself.

We can put the defining condition in the following form which will prove useful later on:

s suffices for *P* and for any *r⊑s* that suffices for *P*, *r=s*.

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[14] Armstrong, D. M. *Truth and and Truthmakers*, (Cambridge: CUP, 2004), p. 20-1.

Unfortunately, this is rendered problematic by an unfortunate logical fact: that there are certain contingent propositions that do not, and cannot, have any minimal sufficer.[15] So if we explicate the key notion using m-commensurateness then the *Selective and Discerning* theorist has to give up the idea that all truths have truthmakers.

4   Not all propositions have minimal sufficers

Suppose there are denumerably many electrons, and consider the proposition:

> D: *There are denumerably many electrons.*

Let e_1, e_2, e_3, ... be an enumeration of all basic electron states (i.e. e_n is the state that consists in the particular X_n being an electron).   Where $E_0 = \{e_1, e_2, e_3, ...\}$ the state $\oplus E_0$ clearly suffices for D. Is it a minimal sufficer? Consider $\oplus E_1$, where $E_1 = E_0 - \{e_1\}$. Clearly, $\oplus E_1$ also suffices for D, and $\oplus E_1$ is a proper part of E_0. So $\oplus E_0$ isn't a minimal sufficer for D. We can repeat this argument to show that there can be no minimal sufficer for D. If *d* suffices for D then *d* must have as parts infinitely many basic electron states. Let *d-* be that proper part of *d* that excludes the first electron state in *d* in the above enumeration of electron states. Then $d\text{-}\sqsubseteq d$ suffices for D, and $d\text{-}\neq d$.

Restall's argument doesn't directly refute the thesis that all truths have commensurate sufficers: what it does show is that not all commensurate sufficers need be minimal sufficers. Any state which is large enough to make D true will have to contain infinitely many electron state parts. The question is which of these states are too large to be commensurate with D. Here is an argument that the fusion of any denumerable subset of the electron states is commensurate with D.

Let $E_o = E$, and $E_{n+1} = E_n - \{e_n\}$, and consider the descending chain of states:

$$\oplus E_0 \sqsupset \oplus E_1 \sqsupset \oplus E_2 \sqsupset ... \oplus E_n \sqsupset \oplus E_{n+1} \sqsupset ...$$

All the states in this sequence are large enough for D (all suffice). In general we will make use of the following connection between parthood and commensurateness:

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[15] Armstrong attributes the following argument (or something in the ballpark) to Greg Restall, but does not cite a paper. In an email exchange Restall confirmed that he made the point in a conversation with Armstrong, not in print.

> *If state s is a part of t and t is not too large to be commensurate with D then neither is s.*

So, if $\oplus E_n$ is commensurate with D, then $\oplus E_{n+1}$ is too, since the latter suffices for D, and is a proper part of E_n. It follows that: (i) if any element in the sequence is commensurate with D all its successors in the sequence are. Suppose for some *n* that $\oplus E_{n+1}$ is commensurate with D. Then $\oplus E_{n+1}$ is not too large for D. But the difference between $E_{n+1} = \{e_{n+1}, e_{n+2}, e_{3n+}, ...\}$ and $E_n = \{e_n, e_{n+1}, e_{n+2}, ...\}$ is only the presence of the one electron state, e_n, and it is absurd to suppose that that particular state could make the difference between being too large for D and being commensurate with D. Hence it follows that: (ii) if some state $\oplus E_{n+1}$ in the sequence is commensurate with D, every state before $\oplus E_{n+1}$ in the sequence (i.e. $\oplus E_0$ to $\oplus E_{n-1}$) is also commensurate with D. From (i) and (ii) we have that either:

    (A)   No member of the sequence is commensurate with D;

    (B)   Every member of the sequence is commensurate with D.

Now, we could repeat the argument for any enumeration of E_o demonstrating that:

    (A)   No part of $\oplus E_o$ is commensurate with D — all parts are either too large or too small;

    (B)   Non-denumerably many parts of $\oplus E_o$ are commensurate with D — i.e. the fusion of *any* denumerable subset of E_o is commensurate with D.

If we accept (A) then (assuming commensurateness) we would have to give up on the truthmaker program.

But we don't have to accept (A), at least not yet. Since *any* sufficer for D has to contain a denumerable number of basic electron states, any such state (provided it doesn't contain anything otiose) should be deemed just right for D. If this is so, rather than showing that D has *no* commensurate sufficers Restall's argument shows that there are *many* commensurate sufficers for D. Indeed, there are non-denumerably many, even though none are minimal.

But now we face an explosion problem. On pain of ending up jettisoning the commensurateness requirement and going *All Inclusive*, we have to limit the class of commensurate states from above in some principled way.

5   Structurally minimal sufficers

What would count, then, as a clear example of an incommensurate sufficer for D?   The World as a whole, of course, but what are there any less encompassing states? And where do we draw the line?

Suppose that there are denumerably infinitely many basic electron states E and denumerably infinitely many basic proton states P as well. Consider the union of these two sets: E∪P. ⊕E∪P is an actual sufficer for D. But, while ⊕E∪P is certainly large enough for D it goes way over the top. Given that all the proton states in ⊕E∪P are irrelevant to the truth of D, ⊕E∪P contains far too much excess baggage to be commensurate with D. But in what salient way does ⊕E∪P differ from ⊕E, other than the obvious fact that ⊕E is a proper part of ⊕E∪P?

There is an obvious difference between proper parts of ⊕E and proper parts of ⊕E∪P. All the proper parts of ⊕E that suffice for D have *the same structure* as ⊕E itself. States *s* and *f* have the same structure if and only if they determine the same overall distribution of properties and relations, albeit over possibly different individuals.[16] The fusion of any infinite subset of E thus has the same structure (in this sense) as ⊕E itself. For the same reason, any proper part of ⊕E that suffices for D has the same structure as ⊕E. And what seems important here is that the *structure* of ⊕E is minimal for ensuring the truth of D. Any state that suffices for D will have a part that has the structure of ⊕E, and any *proper* substructure of ⊕E will be too small to ensure the truth of D. By contrast, there are *many* parts of ⊕E∪P that suffice for D that have *a* smaller *structure* than the structure of ⊕E∪P. And some have a *much* smaller structure. But any

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[16] Note that I am employing the regular notion of isomorphism of distributions of properties and relations, not the notion of isomorphism of posets: viz. a 1-1 monotone function *f* from ↓*e* to ↓*f*. That also yields a notion of equivalence between states and an associated notion of commensurateness. Fortunately given the general argument I don't have to consider every plausible candidate.

proper substructure of ⊕E∪P that suffices for D, but is not structurally identical to ⊕E, will be too large to be commensurate with D. The structure of ⊕E is the smallest substructure of ⊕E∪P that suffices for D. This suggests that we cash out the idea of commensurateness using the concepts of *structure* and *substructure.*

Let's use *r≅s* to abbreviate: *r and s have the same structure.* We can define the notion *structural commensurateness* (or s-commensurate) thus:

> *s* is structurally commensurate with *P* iff *s* suffices for *P* and for any *r⊑s* that suffices for *P*, *r≅s*.

Note that this is formally parallel to *minimal sufficer.* Strict identity (=) and structural identity(≅) are both equivalence relations on states. Structural identity is a weaker equivalence relation than strict identity, so more states are s-commensurate with a proposition than are minimal sufficers for it. It is the structure of *s* which has to be minimal relative to *P* for *s* for to be commensurate with *P*.

The core notion of commensurateness then is this: a sufficer *s* is commensurate with *P* just in case any part of *s* that suffices for *P* is *equivalent to* (hence *no smaller than* in the salient sense) *s* itself. A proper part of a state can be as large as the whole, a whole may be no larger than some of its proper parts. The part-whole relation is not the only relevant relation of relative *size.*

Since ⊕E suffices for D, and all the proper parts of ⊕E that suffice for D are *structurally* equivalent to ⊕E, ⊕E is an s-commensurate sufficer for D. While ⊕E∪P suffices for D not all the proper parts of ⊕E∪P that suffice for D are structurally equivalent to ⊕E∪P.

6   Not all propositions have structurally minimal sufficers.

Are there cases in which a state is clearly an s-commensurate sufficer for a certain proposition *P* (it contains neither too little nor too much) but, intuitively, it isn't commensurate with P? There are, and while it takes a little work to show it, the

counterexample bears some similarity to Restall's counterexample to minimality, as I have laid it out.

Let us say that an *n-cluster* is the fusion of a set of electron states, consisting of exactly n electrons, in which each electron bears some basic equivalence relation W (*being with*, say) to every member of the set, and to nothing outside the set; and nothing outside the set bears W to any member of the set.

A *fusion of m-clusters*, exactly one for every $n > m$, is said to be *uniformly clustered above m*.

Suppose there are denumerably infinitely many electrons, each electron comes in an *m*-cluster, and that there is just exactly one *m*-cluster for every *m*. The fusion of these clusters is *uniformly clustered above 0.* Let C_0 be the smallest set of actual basic states such that $\oplus C_0$ is uniformly clustered above 0. Let $C_{n+1} \subset C_n$ be the largest subset of C_n that excludes just the basic states in the smallest cluster in C_n. $\oplus C_{n+1}$ arises from $\oplus C_n$ by lopping off the *n*-cluster from $\oplus C_n$. So for all n, $\oplus C_n$ is clustered above n.

We now have a descending chain of less and less determinate states:

$$\oplus C_0 \sqsupset \oplus C_1 \sqsupset \oplus C_2 \sqsupset \ ... \ \ \oplus C_n \sqsupset \oplus C_{n+1} \sqsupset \ ...$$

where the structure of $\oplus C_{n+1}$ is a proper substructure of $\oplus C_n$.

Consider the proposition:

B^n: *Some state uniformly clustered above n is actual*

$\oplus C_n$ is a minimal sufficer for B^n and seems (intuitively) to be commensurate with B^n.

Let B be the existential generalization of B^n: $(\exists n)B^n$. That is:

B   *There is an n such that some state uniformly clustered above n is actual.*

For every *n,* B^n entails B, so for every *n,* $\oplus C_n$ suffices for all the consequences of $B^n,$ like B. So, for each *n,* $\oplus C_n$ suffices for B. Given this, $\oplus C_n$ is commensurate with B if and only if it is not too large for B. Is $\oplus C_n$ too large for B?

If, for some n, $\oplus C_n$ is too large for B, then everything of which $\oplus C_n$ is a proper part is also too large for B. i.e. everything larger than $\oplus C_n$. (In particular, every state from $\oplus C_0$ to $\oplus C_{n-1}$ is too large for C.) If for some $n,$ $\oplus C_n$ isn't too large for B then every proper part of $\oplus C_n$ that also suffices for B isn't too large for B. (In particular, every state $\oplus C_{n+m}$ is commensurate with B.)

Suppose some members of the chain are too large and some are not. Then there must be an n such that $\oplus C_n$ is too large for B, while $\oplus C_{n+1}$ is not too large. If so, there is some finite initial segment of the chain — $\oplus C_0 \sqsupset \oplus C_1 \sqsupset \oplus C_2 \sqsupset ... \ \oplus C_n$ —all members of which are too large for B, and this is followed by an infinite descending chain $\oplus C_{n+1} \sqsupset \oplus C_{n+2} \sqsupset ...$, all members of which are *exactly the right size* for B.

This is implausible. What could make the difference between $\oplus C_n$ (too large), and $\oplus C_{n+1}$ (just right). If $\oplus C_{n+1}$ isn't too large for B then adding the one small n-cluster to its infinite stock of much larger n-clusters isn't either. So if any state in the chain $\oplus C_0 \sqsupset \oplus C_1 \sqsupset \oplus C_2 \sqsupset .....$ is commensurate with B, then every state in the chain is commensurate with B. So either every state in the sequence $\oplus C_1, \oplus C_2, \oplus C_3 ...$ is a commensurate sufficer for B or B has no commensurate sufficers. So if B is to have any commensurate sufficers at all, all of the members of the sequence $\oplus C_1, \oplus C_2, \oplus C_{3,} ...$ must be commensurate sufficers.

No state in the sequence $\oplus C_1, \oplus C_2, \oplus C_3 ...$ is *structurally* commensurate with B. For suppose that for some $n,$ $\oplus C_n$ is an s-commensurate sufficer for B. $\oplus C_{n+1}$ also suffices for B but is not structurally identical to $\oplus C_n$. $\oplus C_{n+1}$ is a proper substructure of the the structure of $\oplus C_n$. Since $\oplus C_n$ is not an s-commensurate sufficer for B, no member of the sequence $\oplus C_1, \oplus C_2, \oplus C_3 ...$ is s-commensurate with B. So if we are going to have commensurate sufficers for all propositions we need a different notion.

Just as it is implausible that no member of the chain $\oplus E_0, \oplus E_1, \oplus E_2, \oplus E_3 ...$ is a commensurate sufficer for D (rather, they all are) so too it is implausible that no member of the $\oplus C_1, \oplus C_2, \oplus C_3 ...$ is a commensurate sufficer for B. It is much more plausible that they all are. So what we need is a concept of equivalence which deems all members of the sequence $\oplus C_1, \oplus C_2, \oplus C_3 ...$ commensurate sufficers for B. Since none of these is

structurally identical to any of the others, we need some other equivalence relation (or of relative size) on states.

This suggests moving to some notion of equivalence logically weaker than structural identity. Consider *equinumerosity.* States r and s are *equinumerous* ($r \approx s$) if they have the same sheer *number* of parts i.e. there is a one-to-one mapping from the set of parts of r to the set of parts of s. And s is an *numerically commensurate* sufficer for P only if no part of s that suffices for is *smaller* than s. That is, any part of s that suffices for P is equinumerous with e.

> s is *n-commensurate* with P iff s suffices for P, and for any $r \sqsubseteq s$ that suffices
> for P, $r \approx s$.

Equinumerosity is a weaker equivalence relation than either identity or structural identity, this numerical commensurateness allows more states to be commensurate with a proposition than the previous two accounts. All the $\oplus E_n$ are n-commensurate with D, and all the $\oplus C_n$ are n-commensurate with B.

But while the first two accounts are too strong, ruling out clearly commensurate sufficers, this third account is too weak, ruling in sufficers that are way too large. For example, since $\oplus E \cup P$ and $\oplus E$ are equinumerous all the proper parts of $\oplus (E \cup P)$ that suffice for D are numerically commensurate with $\oplus (E \cup P)$. But $\oplus (E \cup P)$ is clearly too large to be commensurate with the proposition D.

There are of course infinitely many possible equivalence relations and associated notions of commensurateness, so a potted critique of candidates doesn't constitute an impossibility proof.

7   The argument pattern generalized

Whether or not we require unrestricted truthmaker maximalism, two basic desiderata for any adequate theory of truthmakers are:

> *Restricted Truthmaker Maximalism*:
> Every true contingent proposition has at least one truthmaker,

Commensurateness:

Any truthmaker for a proposition must be commensurate with that proposition. And, for there to be an adequate notion of commensurateness there must be an associated relation of equivalence on states (label it ≈) satisfying the equivalence schema:

Equivalence Schema
s is *commensurate* with P iff s suffices for P, and for any $r \sqsubseteq s$ that suffices for P, $r \approx s$.

This simply spells out the idea that, for s to be commensurate with P, s must be both large enough for P (i.e. suffice for it) but *not too large* (any parts of s that suffice for P must be *no larger* than s itself). Since a state can never be larger than a whole of which it is a part, it follows that all parts of s that suffice for P must be *equivalent* to s.

If all propositions had *minimal* sufficers then these principles could be jointly satisfied, since minimal sufficers are clearly commensurate and we can capture it with the equivalence relation of strict state identity. If, for example, the set of possible states were finite then all propositions would, *ipso facto*, have minimal sufficers. But as soon as the class of states is sufficiently interesting and the class of propositions sufficiently rich, then as Restall's argument shows, there will be lots of propositions that do not have minimal sufficers. In sufficiently rich frameworks such propositions are by no means rare or unusual. So on the minimalist criterion for commensurateness, some propositions lack truthmakers. But it we switch to a conception of commensurateness to allow in the states that are intuitively commensurate with the problematic propositions at hand, we end up counting as commensurate states that are not.   So is there any notion of commensurateness that gets everything right?

Firstly, I distill the common feature of the problematic states, and the propositions associated with them, that seems to generate the counterexamples to each notion of commensurateness. I will show that if the classes of states and propositions are rich and interesting enough to contain what I will call *large* states and *large* propositions the requirement of commensurateness collapses into all-inclusiveness.

First we need the ancillary notion of the *largeness* of a state which plays an important role the counterexamples above. First a state *s* is *infinite* if it generates the now familiar infinitely descending chain: $s \sqsupset s_0 \sqsupset s_1 \ldots \sqsupset s_n \sqsupset \ldots$. Call the infinite chains that emanate from a state *s* the *downward chains* of *s* (for short). Obviously a state that spouts one downward chain spouts infinitely many downward chains, and every member of such a downward chain is itself an infinite state. Being infinite is a necessary but not quite a sufficient condition for a state to count as large. Let I^s be the set of infinite parts of *s*. A state *f* is *strictly below* state *s* if *f* is below every state in I^s. (That is, *f* is a lower bound on I^s, or *f* is a part of every member of I^s.) The extra element to *s*'s being large is that no state strictly below *s* guarantees that some or other infinite part of *s* occurs. Intuitively, the state *s* is large provided it and all its infinite parts are "beyond the reach" of all the states that are strictly below it.

First we define what it is for *s* to be *within reach* of a state below it. For this, we need a familiar concept from the theory of partial orders. $\uparrow S$, or the *upset* of *S,* is the set of all states that are above, or determine, some member of *S*. (And $\uparrow s$ is short for $\uparrow\{s\}$, the upset of the singleton of *s*, the set of all states that determine *s,* or of which *s* is a part.) $\uparrow f \subseteq \uparrow S$ if and only if the obtaining of *f* ensures that some member or other of *S* obtains.

> Proof: Suppose $\uparrow f \subseteq \uparrow S$. Since $f \in \uparrow f$, $f \in \uparrow S$. So *f* determines some particular member of *S* and hence ensures that some member or other of *S* obtains. Suppose $\uparrow f \not\subseteq \uparrow S$. Let *h* be an element of $\uparrow f$ not in $\uparrow S$. Since $h \in \uparrow f$, $f \sqsubseteq h$. Let *s* be some arbitrary member of *S*. If $s \sqsubseteq h$ then $h \in \uparrow S$: contradiction. Hence no member *s* of *S* is a part of *h*. If *h* obtains then *f* obtains but no member of *S* obtains. So the obtaining of *f* doesn't guarantee the obtaining of some member or other of *S.*

We say that *s* is within reach of *f* if and only if *f* guarantees the occurrence of some or other infinite part of *s*: i.e. $\uparrow f \subseteq \uparrow I^s$. And *s* is *beyond the reach* of a state *f* below *s* if and only if *s* is not within reach of *f. T*hat is, *f* is below *s* and $\uparrow f \not\subseteq \uparrow I^s$.

Large states

A state s is large if and only if s is infinite and s is beyond the reach of every state f that is strictly below s (i.e. $\uparrow f \not\subseteq \uparrow I^s$).

Of course, this notion of largeness is very coarse-grained. But we do not need it to be any finer-grained for our purposes. In fact its coarseness is a bonus. With the notion of a large state we can now characterize *large propositions*.

Large propositions

A proposition is *large* if and only if all its sufficers are large.

The proposition D (that *there are denumerably many electrons*) and the proposition B (that *for some n, an n-cluster is actual*) are both clearly large. I will show this for D.

Proof Any state that suffices for D will be the fusion of some infinite set of electron states, E say.[17] $\oplus E$ is infinite, and $I^{\oplus E}$ is the set of fusions of infinite subsets of E. For any two subsets S and T of E, f is below both $\oplus S$ and $\oplus T$ iff f is below $\oplus(S \cap T)$. Let E_1 and E_2 be two infinite subsets of E such that $E_1 \cup E_2 = E$ and $E_1 \cap E_2 = \varnothing$. $\oplus E_1$ and $\oplus E_2$ are both in $I^{\oplus E}$. So any state strictly below s must be below $\oplus \varnothing$ i.e. below the null state $\mathbf{0}$. But there is only one such state, the null state itself. Since $\uparrow \mathbf{0}$ is the set of all states and $\uparrow I^s$ isn't, $\uparrow \mathbf{0} \not\subseteq \uparrow I^s$.

(That B is large is an exercise for the reader.)

If there are large elements in the space of possible states Ω, then there will be many such, and any sufficiently rich propositional framework designed for such a space will itself contain a rich set of large propositions.

It follows immediately that:

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[17] Strictly speaking we would need to specify the space of events with basic events consisting in states like *X's being an electron*. I am ruling out weird spaces in which there is a basic property *D* of fusions of particulars that consists in that particular being a scattered denumerable infinity of electrons.

(i)   *s* is large iff every infinite part of *s* (i.e. every member of I^s) is large.

Proof: Suppose *s* is large and *r*∈I^s, *r*≠*s*. For every state *f* strictly smaller than *s*, ↑*f*⊄↑I^s. *f* is strictly below *s* iff *f* is below every member of I^s. Since I^r⊆I^s *f* is also strictly below *r*, and for ever y state *f* strictly smaller than *r*, ↑*f*⊄↑I^r. So *r* is large. Suppose that for every *r*, if *r*∈I^s then *r* is large. Since *s*∈I^s it follows that *s* is large.

Let Ω be the set of possible states, Ω^{Large} the subset of large states in Ω, and let *s* be any member of Ω^{Large}. Consider the following proposition:

L^s *Some large part of s occurs.*

Clearly all the members of I^s — all and only the large parts of *s* — suffice for L^s. If any of them obtains L^s is true. So L^s looks like it might be a large proposition. We show that it is.

(ii) L^s is a large proposition.

Proof: Suppose *r* suffices for L^s and *r*∉I^s. Either there is some member *i* of I^s that *r* determines or there isn't. Suppose the former. *r* determines *i* ∈ I^s. Since *i* is large, for every *f* strictly below *i*, ↑f ⊄↑I^i. Since *r*⊐*i*, every state strictly below *r* is also *t* is large. Suppose *t* does not determine any large part of *s*. If *t* is below (see **). Suppose *t* is large part of L^s then *t* is not a member of any any of *i*'s descending chains.   any sufficer for L^s has to determine the occurrence of a large part of *s,* any sufficer for L^s also has to be large (from (i)). So L^s is a large proposition. Since every sufficer for L^s contains some large part of *s* as a proper part, every *commensurate* sufficer for L^s must contain some large part of *s* as a proper part (provided there are commensurate sufficers for L^i).

From an intuitive point of view it seems that here, as in the earlier cases, either all the large parts of *s* are not too large for L^s or all of them are too large. Demanding that there be a precise point below *s* in each descending chain that *s* sprouts, at which

point we flip from *too large* to *just right* just seems gratuitous. (The following is intended to spell this out although I am not sure it makes it any plainer or more obvious.)

Suppose some large parts of s commensurate with L^s but not all are commensurate with L^s. Then there will be non-denumerably many chains that end up in the non-commensurate regions of s. Where $s = c_0$, let $c_0 \sqsupset c_1 \sqsupset c_2 \sqsupset ... \ c_n \sqsupset ...$ be one such chain. If c_n is commensurate with L^s then c_n isn't too large, so all the elements of the chain below c_n will also be commensurate with L^s. If c_n is not commensurate with L^s then c_n is too large, and all the elements of the chain above c_n will also be incommensurate with L^s. So in each such descending chain there will be a cut off — c_{n+1} will be exactly the right size while its predecessor in the chain, c_n, will be too large. So an initial finite section of the chain $c_0 \sqsupset c_1 \sqsupset c_2 \sqsupset ... \ c_n$ will consist of states that are all too large for L^s, and an infinite final section $c_{n+1} \sqsupset c_{n+2} \sqsupset ... \ c_{n+m}...$will consist of members that are *exactly the right size* for L^s. But nothing in the structure of states or their relative sizes seems to justify the change from too large to exactly right at precisely that point c_{n+1} below s. That there have to be such cutoff points in each such chain seems as implausible here as in the earlier cases.

So in each chain either all the states in the chain will be too large for L or none of the states in the chain are too large for L^s. And from that it follows that either all large parts of s are commensurate with L^s or none of the large parts of L are commensurate with L.

If none of the large parts of L^s are commensurate with L^s then (since any sufficer for L^s has to contain such a part) L^s would have no commensurate sufficers, and hence no truthmakers in any world. So on pain of jettisoning the truthmaker program, all the large parts of s have to be commensurate with L^s. Given the schema for commensurateness that entails:

If d is any large part of s then $d \approx s$.

Since there was nothing special about the state s and the associated proposition L^s, we have effectively proved:

(ii)   Every large state is equivalent to (i.e. no larger than) all its large parts.

Suppose that s and j are any two distinct but compatible large states. Then $i \oplus j$ exists, a state of which both s and j are parts. Since $i \oplus j$ has large parts, $i \oplus j$ is itself large. By (ii) $i \oplus j$ is equivalent to all its large parts, so in particular $i \oplus j \approx s$ and $i \oplus j \approx j$. By transitivity of equivalence we have $i \approx j$. That is:

(iii)   Any two large compatible states are equivalent.

Suppose that P is any large proposition, and let s be a sufficer for P. Since P is large, s is large. By (iii), s is equivalent to all of its large parts. So s suffices for P and any part r of s that suffices for P is also large. Since r and s are compatible large states, by (iii) we have that $r \approx s$. So any part r of s that suffices for P is equivalent to s. And that, of course, entails:

s is commensurate with P.

But there was nothing special about P and s. So:

(iv)   If P is any large proposition, every sufficer for P is commensurate with P.

For a large proposition every state large enough to ensure its truth is also exactly the right size. A State is thus a truthmaker for a large proposition if and only if it is an actual sufficer. Thus if the state space contains large states, then for the class of large propositions the truthmaker theorist is in a pickle: she is either forced to abandon the requirement for truthmakers for a whole class of propositions or else forced to embrace all actual sufficers of those propositions as genuine truthmakers. The assumption that all propositions have genuinely commensurate sufficers thus appears to be unsustainable.