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Improving Imaging Resolution For In-Situ Measurements of Bose-Einstein Condensates

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Acknowledgements

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Without Phil’s teaching I would not have had the skills necessary to complete this thesis project. Xin joined us shortly before Phil left. She has been an exceptionally valuable resource for me. Working with Xin I’ve come to appreciate her willingness to help, her amazing work ethic, and her patience both in answering questions and dealing with the experiment. In addition to all her help with physics-related problems, Xin has taught me about the attitude an experimentalist should have. Lastly, Cathy Klauss was an essential member of the team that made this thesis possible. Like Phil, Cathy helped me learn the skills necessary to think like an experimentalist, but she was also a vital resource for me while working on my thesis project. I probably asked Cathy an average of three questions a day when we were in lab together. She always took the time to explain in detail and make sure I understood complex ideas. I am lucky that I got to work with a graduate student who was patient and caring enough to spend time assisting me with work that was not hers to do. Cathy took responsibility for helping me complete my projects, but more importantly she took responsibility for making sure that I was learning physics through my work. Cathy went above and beyond by acting as my teacher as well as my supervisor. I can’t say enough good things about the graduate students that I had the privilege of working with. They made my JILA experience special.

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Abstract

A $^{85}$Rb Bose-Einstein Condensate (BEC) is an ideal experimental tool for studying strongly interacting quantum many-body systems. In our experiment we measure the size of the BEC by analyzing images of the BEC. Unfortunately our current imaging resolution limits our ability to make accurate measurements of the BEC size in-situ. In this thesis I will describe my work to improve the imaging resolution of our experimental system, which resulted in the implementation and testing of a new and improved imaging system. I will also outline some of the novel physics we plan to investigate with the new capability to accurately image the BEC in-situ.
Introduction

Since the first observations of Bose-Einstein Condensates (BECs) with ultracold atoms in 1995 [AND] the AMO physics community has used BECs as a tool to study novel physics. The BEC is an ideal model experimental system for investigating quantum many-body physics [BLO] because it is clean and one can control the interaction strength between atoms. The ability to control the interaction strength between atoms comes from a magnetic-field Feshbach resonance. For $^{85}$Rb atom a Feshbach resonance exists at 155 G [CLA]. At a Feshbach resonance, there exists a regime called Unitarity where the two-body interactions between atoms in the gas are theoretically infinitely attractive and infinitely repulsive simultaneously [CHI]. We would like to use our experimental system to look at what happens to a BEC when the interactions are suddenly changed from weakly repulsive to this Unitarity regime where the interactions are infinitely strong. Since the ultracold gas is confined in a harmonic, or bowl-shaped trap, infinitely attractive interactions would cause the BEC to shrink, while infinitely repulsive interactions would cause the BEC, to expand. The BEC obviously cannot simultaneously shrink and expand, so we are interested in determining which actually occurs.

A challenge to measuring whether the BEC shrinks or expands after the jump to Unitarity is that the BEC size is about 15 µm in radius. Observation on this scale requires an imaging system with good imaging resolution. In our experiment, we use absorption imaging to take a picture of the cloud, and then we analyze the picture to measure the size of the cloud. The imaging resolution depends on multiple aspects of the imaging system including the sizes and focal lengths of lenses used and the alignment of those lenses. In order to make accurate measurements of cloud size, the resolution of the imaging system, as measured by the minimum spot size, should be small compared to cloud size. This thesis describes the improvements we
made to our imaging system to enable in-trap measurements of the BEC size after jumping to Unitarity. The BEC measurements enabled by the new imaging system should provide new insight into the complex physics of strongly interacting many-body systems [BLO].

I will begin by describing what a BEC is and why we are interested in studying how the size of the BEC changes after jumping to Unitarity. In the next section of this thesis, I will present a study of how different parameters of the imaging system affect resolution. Then I will illustrate the design of the imaging system we chose to implement, and explain the measured resolution for the new and old systems. Finally I will show preliminary data on the measured cloud size after jumping to Unitarity.

**Scientific Goal**

**BEC**

A Bose-Einstein Condensate (BEC) is a quantum state of matter. It’s a state of matter that occurs when you get a dilute gas of atoms very cold. Unlike states of matter such as a liquid or solid, this state of matter can only be explained by the theory of quantum mechanics. The existence of BECs was theorized by Bose and Einstein in the 1920s [GRI] but because the conditions in which they can exist are so hard to achieve experimentally, it took about 70 years before BEC was realized in a lab [AND]. The realization of BEC provides the physics community with an excellent tool that can be used to investigate quantum many-body physics. BECs are useful as a model system because they are clean and highly controllable. Since we observe BECs as clouds of atoms, I will often refer to them as clouds, for this paper, the terms cloud and BEC are interchangeable. One aspect of ultracold atoms that makes them useful for exploring interesting quantum many-body phenomena is the existence of Feshbach resonances for certain atoms.
Feshbach Resonance

We make BECs of $^{85}$Rb atoms. We use $^{85}$Rb because we can take advantage of a Feshbach resonance at 155 G [CLA]. A simple way to think about a Feshbach resonance is that it is a scattering, or collision, resonance that affects the interaction strength. In quantum mechanics, there is a parameter called scattering length that is directly proportional to the interaction strength between the atoms making up the BEC. A positive scattering length means that the interactions between atoms in the BEC are repulsive while a negative scattering length means that they are attractive. For positive scattering lengths, the parameter can be thought of as an effective size of the colliding atoms. Near a Feshbach resonance, the scattering length of the atoms in a BEC is very sensitive to the external magnetic field as seen in figure 1 below. At the resonance, the scattering length, $a$, diverges. We refer to the region right around the resonance, where $a$ goes to infinity, as Unitarity.

![Feshbach Resonance](image)

**Figure 1** Dimensionless scattering length $\frac{a}{a_0}$ vs external magnetic field for $^{85}$Rb near its Feshbach resonance, according to the formula: $a = a_{bg} \left( 1 - \frac{\Delta}{B - B_{peak}} \right)$, where $a$ is the scattering length, $a_0$ is the Bohr radius, $a_{bg}$ is the background scattering length, $\Delta$ is the width of the resonance, $B$ is the magnetic field in Gauss, and $B_{peak}$ is the magnetic field where the resonance occurs [CLA].

In our experiment, we have the ability to very quickly change the external magnetic field [MAK]. With this capability, we can quickly change the interaction strength in our BEC. Going
to strong interactions is interesting because while theory can explain weakly interacting many-body systems it faces challenges when applied to a strongly interacting many-body system [GRI]. We can learn a lot about strongly interacting many-body systems by studying the strongly interacting BECs and comparing the measurements to competing theories that use different approximations. Changing the interaction strength very quickly, by “jumping” to Unitarity, is necessary because a BEC has a short lifetime when the interactions are strong.

Size of BEC at Unitarity

The BEC is held in a harmonic trapping potential. For a BEC with a positive scattering length in equilibrium, the repulsive force of interactions in the BEC balances the force from the trapping potential such that the BEC has a size given by [PET]:

\[ R_{TF} = \left( \frac{15N a}{\tilde{a}} \right)^{1/5} \tilde{a}, \]  

where \( N \) is the number of particles, \( a \) is the scattering length, and \( \tilde{a} \) is the geometric mean of the harmonic oscillator lengths given by \( \tilde{a} = \frac{\hbar}{\sqrt{m\omega}} \), where \( m \) is mass of the atoms and \( \omega \) is angular the trap frequency. Equation 1 holds when the BEC has a small positive scattering length. For negative scattering length, the interactions are attractive and the BEC collapses [DOD]. What happens to the size of a BEC after jumping to Unitarity is still not known. Determining what happens to the size of a BEC after jumping to Unitarity is the scientific goal driving my thesis. To do so we must quickly take a BEC from the weakly interacting regime to Unitarity.

The Path to Our Science Goal

In-situ Imaging

Figuring out whether the size of a BEC increases or decreases after jumping to Unitarity requires the ability to accurately measure the size of clouds in the trap. In our experiment we
make measurements by analyzing pictures that are taken through absorption imaging. This imaging has three steps. First, a beam of light that is on resonance for the $^{85}\text{Rb}$ atoms, called a probe beam, is shined on the cloud of atoms that absorbs the light, so that we have a picture of the shadow that was cast by the BEC. Next, we take another picture of the probe beam, but this time there is no BEC and therefore no shadow cast by the atoms. Then we get a signal by comparing the two pictures to see how much light was absorbed by the atoms at each pixel in the picture. Imaging works well in the limit that the object, which is the thing to be imaged, is much larger than the wavelength of light used for the imaging. For some experiments we can increase the size of the cloud by allowing the BEC to expand from the trap before imaging, but for measuring the in-trap, or in-situ, size of a BEC after jumping to Unitarity we can’t allow the BEC to expand.

Difficulties arise with in-situ imaging because the size of the BEC, as defined by its RMS width, $\sigma \equiv 0.35R_{TF}$, is about 6 $\mu$m while the wavelength of the light we are using for imaging is $\lambda = 0.78$ $\mu$m. This means that we have to worry about the resolution of the imaging system in our experiment. The resolution is a measure of how large an infinitely small object will appear in the image. When I started this project, our experimental imaging system had a resolution of 6 $\mu$m, but to make accurate in-situ measurements of BEC size we set a design goal of 2 $\mu$m resolution for a new imaging system. Here, the resolution is given as the RMS width of the image we would expect for imaging an infinitely small object, or point object.

**Diffraction-Limited Resolution**

When one images a point source through a lens, diffraction causes the image to not be a point but a pattern referred to as an Airy pattern, as shown in figure 2 below. The width of this pattern can be estimated from the quantum mechanical uncertainty principle, which states that
the uncertainty in position multiplied by the uncertainty in momentum must always be greater than or equal to one half of Plank’s constant, \( \Delta x \Delta p \geq \frac{\hbar}{2} \).

Using a small angle approximation and the fact that the photon momentum is \( p \equiv \frac{\hbar}{\lambda} \), the maximum \( \Delta p \) for light that contributes to the image is given by \( \Delta p = \theta \frac{\hbar}{\lambda} \), where \( \theta \) is the maximum angle as indicated in figure 2. Solving for the minimum image size, \( \Delta x \), gives \( \Delta x \geq \frac{\lambda}{2\theta} \). For comparison to this estimate, the theoretical diffraction-limited resolution is often given in terms of the distance \( R \), between the maximum of the Airy disk function and the first minimum. This is called the Rayleigh criterion and is given by: \( R = \frac{0.61 + \lambda}{NA} \approx \frac{0.61 + \lambda}{r/d} \), where \( \lambda \) is the wavelength of light, \( NA = r/d \) is the numerical aperture of the lens, \( r \) is the radius of the lens, and \( d \) is the distance between the lens and object being imaged. The second equation uses a small angle approximation with \( NA = \sin \theta \approx \frac{r}{d} \). Note that our simple estimate from the quantum uncertainty principle gave a very similar result: \( \Delta x \geq \frac{0.5 \lambda}{d} \). See figure 3 below:

**Figure 2** Diffraction pattern of light through a lens and the Airy pattern it produces

**Figure 3** shows how parameters of the imaging system affect the diffraction limit of the lens according to the formula \( R = \frac{0.61 + \lambda}{\sin(\theta_{max})} \approx \frac{0.61 + \lambda}{r/d} \).
Diffraction sets the best possible resolution that can be attained. In practice, other effects such as imperfect lenses can make the achieved resolution larger than the diffraction-limited resolution. These effects are referred to as imaging aberrations.

**The Effect of Limited Resolution**

When taking pictures with an imaging system that has limited resolution, the effect of limited resolution is to artificially enlarge the image of the BEC. A BEC’s density profile is an inverted parabola [BAY], but for this thesis I will approximate it as a Gaussian. This makes it easier to understand the effect of limited resolution as shown in figure 4 below.

![Figure 4](image)

*Figure 4* The effect of limited resolution for a picture and for a BEC. The plot shows a calculated cross-section through an image of a BEC with (purple line) and without (blue line) limited resolution. Here, the BEC (blue) is approximated by a Gaussian with an RMS width of 5.8 μm, and the resolution is 5.8 μm (purple). These numbers were chosen because they correspond to average cloud size in-situ and the resolution of the old imaging system.

Mathematically this effect can be thought of as a convolution of the perfect image of the BEC with the Airy pattern created when imaging a point source through the imaging system. If we approximate the Airy pattern and the BEC density profile as two dimensional Gaussians, it is possible to get a relationship between the actual size of a BEC, the measured size of a BEC, and the resolution of the imaging system as: $\sigma_{TF}^2 + \sigma_{Res}^2 = \sigma_{Meas}^2$, where $\sigma_{TF}$ is the actual size of the BEC as calculated from the Thomas-Fermi radius of the cloud, $\sigma_{Res}$ is the Gaussian RMS.
resolution. for example, for diffraction-limited resolution, approximating the Airy disk with a Gaussian gives \( \sigma_{\text{Res}} = 0.35R \), and \( \sigma_{\text{meas}} \) is the measured RMS size of the image that is produced by the imaging system.

**Complications with In-Situ Imaging**

In our early attempts to take data through in-situ imaging, we saw a striking problem. We did a simple experiment where we allowed a cloud to expand. We measured the number of atoms from images taken as the cloud expanded, and the data showed the number of atoms in the BEC seemed to be increasing as the cloud size increased, as seen in Figure 5 below.

![Figure 5](image)

**Figure 5** Complications with In-situ Imaging. Here we show the results of an early experiment where we measured number from images of a BEC as it expanded. The data shows the number increasing, which is impossible; this implies that we have imaging problems.

It is impossible for the number of atoms in the BEC to increase as the cloud expands, so we concluded that the data was inaccurate due to imaging problems. We suspected that the inaccuracy was being caused by a combination of imaging problems, including saturation and resolution. In order to take in-situ data that was accurate we realized that we needed to improve our imaging system to get better resolution. Our hope was that by improving the resolution of the system we’d no longer observe this inaccuracy in data taken in-situ.
Improving the Imaging System

The “Old” Imaging System

For this thesis, I will consider the imaging system that was in place when I started my project to be the “old” imaging system. It is shown below in figure 6.

In this system, we measured $\sigma_{Res} = 5.8 \, \mu m$. However, using the formula for the diffraction-limited resolution of the system we calculate $\sigma_{Res}$ as:

$$
\sigma_{Res} = 0.35 * \frac{0.61 * \lambda}{r/d} = 0.35 * \frac{0.61 * 780 * 10^{-9} \, m}{12.7 \, mm/80 \, mm} = 1.05 \, \mu m
$$

This disagreement between the measured and calculated values for $\sigma_{Res}$ makes it evident that the resolution of the old imaging system was not diffraction limited, but aberration limited. There are a wide variety of aberrations that can affect an imaging system. To determine which types of aberrations were having the greatest effect on the resolution, I did some experimentation with a model imaging system. Through experimentation I was able to discover that the likely causes for the experimental imaging system’s resolution being worse than its diffraction limit are magnification that was too small, spherical aberrations, and imperfect tilt and alignment of the objective lens.
My Test Imaging System

In order to investigate what types of aberrations were limiting the resolution of our experimental imaging system, I constructed an analogous test imaging system. I couldn’t investigate with the imaging system in the experiment because we needed it to continue our research and we couldn’t risk upsetting that system. The test system I designed used a fiber-coupled laser source to output 785 nm light, a 3 µm pinhole for the object, and a beam profiling camera. In order to optimize the amount of light going through the pinhole, I used two beam-steering mirrors to control where the laser light was hitting the pinhole and at what angle. Additionally I put the pinhole, the camera and the objective lens on translation stages. Using translation stages helped me align the system so that I could see the object on the camera. It also helped me focus the system, including focusing the imaging beam through the pinhole, and it made it easier to investigate the effect of different parameters on the resolution of the system. The test system is shown below in figure 7.

Figure 7 The test imaging system I used to investigate the effect of aberrations on resolution. A is the beam profiling camera, B is the imaging lens, C is the objective lens, D is the pinhole, and E is the fiber-coupled laser source.

With this system I could change parameters as I wanted and I could measure the resolution of the system. I used this tool to investigate the effect of spherical aberrations, transverse offset, tilt, and magnification, on resolution.
**Spherical Aberrations**

For an ideal lens, no matter where a light ray hits the lens it will be focused to the same point. Ideal lenses are parabolic and more difficult to manufacture than spherical lenses. In practice, we use spherical lenses in our experiments because they are significantly more affordable and they come in a wider variety of focal lengths and diameters. For a spherical lens, light rays that hit at the edge of the lens will converge closer to the lens than those that hit at the center of the lens. This effect is detailed in Figure 8 below.

![Figure 8](image.png)

**Figure 8** Spherical aberrations. The diagram on the left shows an ideal lens with all light rays converging at the focal length. The diagram on the right shows a spherical lens with the light rays at the edge of the lens converging closer than those at the center. Image taken from Wikipedia.

Since we use spherical lenses in our experiment I had to determine how greatly spherical aberrations affected the resolution of the imaging system. To do so, I attached an iris to the objective lens and changed the effective radius of that lens while measuring resolution. The results of this experiment are plotted below in figure 9.
The results of my experiment pointed out two important effects of spherical aberrations. First, the experiment showed that spherical aberrations began to significantly affect resolution when the radius of the objective lens was greater than 7 mm. Second, it showed that there is a large difference in the resolution for the X and Y directions of the system when the objective lens has a large radius, but that difference becomes less substantial as the radius of the objective lens is decreased. While I have not rigorously examined this phenomenon, I generally saw that the pictures I took went from being oblong, when using the full one inch diameter objective lens, to being circular, when using a half inch diameter lens. When I did this experiment the lenses were not perfectly aligned, and I suspect this caused the image to appear oblong instead of circular. This experiment confirmed that spherical aberrations were affecting the resolution of our imaging system. Based on this experiment we realized that using a half inch diameter objective lens would get us the same resolution as using a lens with a diameter of an inch, but with the half inch lens the resolution would be diffraction-limited instead of limited by spherical aberrations.
aberrations from the lens. In addition, working with a smaller lens, the resolution-limit spot would be more likely to be circular.

Alignment

Transverse offset

One factor of the imaging system that I found has a larger effect on imaging resolution than we anticipated was the transverse offset of the objective lens with respect to the imaging axis. In an ideal imaging system the imaging beam should be centered on the objective lens. We refer to any displacement of the imaging beam from the center of the objective lens as a transverse offset. In my test system I attempted to understand the effect of transverse offset on resolution by using a micrometer attached to a translation stage to move the objective lens with respect to the probe beam. Then I would refocus the camera and measure the resolution in the direction of the translational offset for various translational offsets. The results of this experiment are detailed below, in Figure 10.

![The Effect of Transverse Offset](image)

**Figure 10** The effect of transverse offset on resolution plotted as the fractional change in resolution vs the transverse offset of the objective lens in millimeters. $R_0$ is the best achieved resolution in this experiment, $R_0 = 3.2 \mu m$. In this experiment I used the same lenses as in the spherical aberrations experiment (fig. 9), but I didn’t spend enough time aligning them exactly right to get the best possible resolution. The effect of transverse offset is actually greater than this figure may lead one to believe. The effect would be greater if the system had been achieving the best possible resolution, $R_0 = 2 \mu m$. Also here I’m only showing the effect in one direction although the resolution gets worse in both transverse directions.
Tilt

Another factor of the imaging system that I found had a profound effect on the resolution was the tilt of the object lens with respect to the imaging beam. In an ideal imaging system the imaging beam should be normal to the lens, which means that the beam should hit the lens at a right angle. We refer to any change in the angle at which the imaging beam hits the objective lens as a tilt of the lens. To investigate the effect of tilt on resolution I placed the objective lens in a mirror mount that could be tilted to an arbitrary but measurable angle. I tilted the objective lens a known amount, refocused the camera and then measured the resolution of the system for different tilts. The results of this experiment are plotted below in figure 11.

![Figure 11](image)

**Figure 11** The effect of the tilt of the objective lens on the resolution of the imaging system plotted as the fractional change in resolution vs the tilt of the objective lens in degrees. I used the same lenses as discussed above in this experiment, and again I didn’t achieve the best possible resolution. Instead I had $R_0 = 2.9 \text{ } \mu m$. The tilt of the objective lens, like its translational offset, actually has a larger effect than one may expect from this figure for the same reasons as were discussed in figure 10 above.
**Magnification**

The magnification of the imaging system is important because the camera has finite sized pixels. In order not to affect the resolution, the pixel size should be much less than the resolution times the magnification: \( \text{pixelsize} \ll R \times M \). The magnification of our system can be well approximated as \( M \approx \frac{f_i}{f_o} \) where \( f_i \) and \( f_o \) are the focal lengths of the imaging and objective lenses respectively. The old imaging system had a magnification of 2.25. The Nyquist limit gives the minimum magnification required to achieve diffraction limited resolution as:

\[ R \times M_{\text{min}} = 2 \times \text{pixelsize} \rightarrow M_{\text{min}} = 5.75, \]

where \( R \) is the diffraction limited resolution. Plugging in the diffraction limit for the old imaging system and the pixel size for the camera in the experiment (13 \( \mu \)m) we can solve for the minimum magnification as 5.75, which is much greater than 2.25.

One might think that larger magnification is always better. However, the full size of the camera CCD divided by the magnification sets a limit on the largest object that can then be imaged. In our experiment, the same imaging system is used to image large clouds that are about 50 times larger than BECs. Imaging these large clouds is necessary for checking that the experiment is working and for diagnosing possible problems with the cooling that produces a BEC. If the new imaging system had a magnification that was too large, we couldn’t use it to see the large cloud. This concept is illustrated in figure 12 below.
The requirement that the imaging system must be able to image BECs and large clouds presented a design challenge. In order to image BECs with good resolution we needed an imaging system that had magnification greater than $M_{\text{min}}$, but to image big clouds we needed a system with smaller magnification. To meet this requirement I designed a four lens system with the objective and imaging lenses fixed in place and two more lenses, called de-mag lenses, on flipping mounts. This system is diagramed below in figure 13.

**Figure 12** A picture of a BEC (left) and a picture of the large cloud (right) taken in the old imaging system. The red boxes show how the frame of the pictures would change if we increased the magnification to be greater than $M_{\text{min}} = 5.75$.

**Figure 13** Schematic of the four lens system. The light blue circles represent the de-mag lenses. When they are flipped out of the system, an example light ray follows the red path and the system has large magnification. When they are flipped in, the light ray follows the orange path and the system has small magnification.
The New Imaging System

The New Imaging System Design

Figure 14 below shows a comparison of the new and old imaging systems.

I changed the diameter of the objective lens from one inch to half an inch to limit the effect of spherical aberrations, and I put the objective and imaging lenses in a lens tube to make alignment easier so that I could limit the effect of transverse offset and tilt on the resolution of the new system. I also added the de-mag lenses, outlined in blue in figure 14. I chose the focal lengths and spacing of the lenses in the four lens system such that when the de-mag lenses are flipped in the system has a magnification of 1, and when they are not it has a magnification of 6.6.

Implementation of the New System

Implementing the new imaging system into the experiment presented a number of challenges. The first challenge was removing the old imaging system such that if the new one didn’t work as it was expected to we could go back to the old system without much trouble. To
meet this challenge, the graduate students and I made sure to accurately diagram and document the placement of the old imaging system as we carefully removed it piece by piece. Once we had removed the old system the next challenge was aligning and focusing the new system.

We had to think carefully about how we could ensure that the alignment of the new imaging system was going to be good enough to provide good resolution. To ensure that the objective lens had no significant transverse offset we made sure that a beam of light that hits exactly at the center of a lens is not steered at all by that lens. First, we marked where the probe beam intersected the camera shutter with no lenses in place. Then we put the lens tube with the objective and imaging lenses in place. Again we looked at where the probe beam, which was now being steered by the lens, intersected the camera shutter. Then we translated the lens in both directions perpendicular to the optical axis until we saw the probe beam intersect the camera shutter at the same place that it had when there were no lenses in place. This procedure ensured that the probe beam was intersecting the lenses at their centers, which limits the effect of transverse offset on the resolution of the new system. Our next challenge was confirming that the probe beam was also normal to the lenses in the lens tube.

For a reflected beam of light, the angle of incidence is equal to angle of reflection. Thus, if we can confirm that the reflection of the probe beam off of the objective lens is overlapped with the incident beam, then they must both be normal to the reflecting surface. To get a reflection we attached a mirror to the lens tube using a lens-tube coupler that ensured that the mirror was parallel to the objective lens. Then we looked at the reflected probe beam about a meter and a half away from the lens tube. We slightly adjusted the tilt of the lens tube assembly until we saw the reflected probe beam overlap with the incident probe beam.
Focusing

Once the system was in place we had to optimize it by focusing the system. The magnification and resolution of any imaging system are very sensitive to the relative placement of lenses and the camera along the imaging axis. During the implementation of the system it is nearly impossible to place the camera and the lenses exactly in the optimum positions because the placement of the camera and the lenses needs to be correct to within a millimeter and to within fractions of a millimeter respectively. In anticipation of this challenge, we placed the camera and the lens tube assembly on translation stages that can precisely move them along the optical axis in micron increments. First we roughly focused the new imaging system by using the system to take pictures of small clouds of atoms while changing the position of the lens tube assembly. The optimum position was that in which the cloud looked the smallest. This procedure is detailed in figure 15 below.

![Figure 15](image)

*Figure 15 shows the focusing procedure. These are images of small clouds labeled with the reading from the micrometer on the translation stage. When focusing, we look for where the imaged cloud looks the smallest, these pictures all look alike, but there is small variation in the cloud size. In this case we found that the micrometer should read 5.28 mm if we want optimum resolution.*

We repeated this procedure moving the camera. After the rough focusing described here the graduate students performed additional fine-tuning of both the focus and the lens tilt.
Results

Improved Resolution

With the new imaging system installed and optimized, we imaged a BEC and compared that image to the images we got in the old system. See figure 16 below.

![Figure 16](image)

Figure 16 A comparison of BECs imaged in the old (left) and new (right) imaging systems. The plots show traces through the images. The BEC imaged in the old system has a $\sigma_{TF} = 7.5 \ \mu m$ and was measured as, $\sigma_{Meas} = 9.6 \ \mu m$. The BEC imaged in the new system has an $\sigma_{TF} = 3.5 \ \mu m$ and was measured as, $\sigma_{Meas} = 4.1 \ \mu m$. The pixel size at the object = \( \frac{\text{pixelsize}}{\text{Magnification}} \) is 5.7 $\mu m$ px in the old system and 2.0 $\mu m$ px.

The most meaningful comparison we can make between the two systems is comparing the resolution in each. We use the formula, $\sigma_{Res}^2 = \sigma_{Meas}^2 - \sigma_{TF}^2$, to solve for the resolution. We calculated a resolution of 5.8 $\mu m$ in the old system and 2.2±0.1 $\mu m$ in the new system. The resolution of the new system is close to our goal of 2.0 $\mu m$.

Demagnification System

Figure 17 below shows the effectiveness of the demagnification system.

![Figure 17](image)

Figure 17 images take with demagnification system. On the left is an image of a large cloud taken with the old imaging system. The red box represents the field of view of the new system without demagnification. On the right is an image of a large cloud with the demagnification system.
**In-situ Imaging Test**

With the old system, we found that if we measured the number of atoms from images of a cloud as it expanded, that number increased. This implied that we had imaging problems. Once we implemented the new imaging system and found that we had improved the imaging resolution, we repeated this experiment with the new imaging system. The results are plotted below in figure 18.

The results of this experiment are informative. Figure 18 shows two clouds with about 40,000 atoms expanding in trap as we change the scattering length from 150 $a_0$ to 500 $a_0$. The data in blue comes from the new imaging system and the data in red comes from the old imaging system. The first piece of information that we can get from these results is further proof that we’ve improved the resolution. The cloud at 150 $a_0$ is measured as 8 $\mu$m and 11 $\mu$m in the new and old imaging systems respectively. The actual size of the cloud, as calculated, is $\sigma_{TF} = 7.6 \mu$m, so the fact that we measure a size much closer to the actual size with the new imaging system confirms that we have improved the resolution. Additionally, the data from the new imaging system shows no artificial increase in number as the cloud expands.
BEC Size after Jumping to Unitarity

We used the new imaging system to investigate the size of a BEC following a sudden jump to Unitarity. The results are plotted in figure 19 below.

We start by creating a BEC in an external magnetic field of 163 G where the scattering length is 150 a₀, then we quickly change (jump) the magnetic field to 155 G where the BEC’s scattering length is infinite. We repeat this process taking images of the BEC at different times after the jump, with the results shown in figure 19 above. We find that the BEC size increases after jumping to Unitarity. This suggests that the interactions in the many-body system are effectively repulsive at Unitarity.

Conclusion

In my thesis project I designed, tested and implemented an imaging system that achieved a resolution of 2.1, which is about a factor of three better than the resolution of the old system. This imaging system was used to observe that BEC size increases after jumping to Unitarity. In
the future, it will be interesting to examine more carefully the evolution of the shape of the BEC following the jump to Unitarity. The improved imaging system will also allow us to better resolve changes in the shape of expanded clouds, which reveals the momentum distribution of the BEC after jumping to Unitarity.
Works Cited


