Hillslope Evolution in Block-Controlled Landscapes

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Hillslope Evolution in Block-Controlled Landscapes

by

Rachel C. Glade

B.A., University of Pennsylvania, 2014

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Hillslope Evolution in Block-Controlled Landscapes

Thesis directed by Professor Robert S. Anderson

Rocky hillslopes dotted with boulder-sized blocks and covered by a thin, non-uniform soil are common in both steep landscapes and arid environments on Earth, as well as on other planets. We have long known how to read geologic structure from aerial imagery; for example, folds in layered rock generate trains of hogbacks. Yet, while the evolution of soil-mantled, convex-upward hillslopes in uniform lithology is reasonably well understood, the influence of heterogeneous lithology and geologic structure on hillslope form and evolution has yet to be properly addressed at a process level. Landscapes developed in layered sedimentary rocks feature sharp-edged landforms such as mesas and hogbacks that exhibit steep, linear to concave-upward ramps with scattered blocks calved from resistant rock layers overlying softer strata. Here I pose the question: What roles do these blocks play in landscape evolution? Using a combination of numerical modeling, fieldwork, and mathematical analysis, I demonstrate that blocks profoundly alter the style and pace of hillslope evolution in rocky landscapes. First, in a numerical model of hillslope evolution I show that the presence of discrete blocks and their interactions with the production and transport of soil can explain the characteristic concave-up hillslope profiles observed in landscapes developed in layered rock on Earth. The presence of blocks increases both the relief and the persistence of topography in these settings. I use these numerical results to develop an analytical framework that characterizes the steady-state form of layered hillslopes in horizontal, tilted, and vertical rock. I find that hillslope weathering and transport processes in the presence of blocks lead to self-organization that allows hillslopes to maintain a steady relief and form through time. Next, I present the first process-based 2D
numerical model of river canyon evolution that incorporates the roles of blocks in both hillslope and channel processes. The model reveals that channel-hillslope feedbacks driven by the delivery of large blocks from hillslopes to the channel are necessary and sufficient to develop the cross-sectional and planview morphologies of river canyons observed on Earth. Block feedbacks lead to persistent unsteadiness in the landscape, strongly modifying erosion rates over long periods of time, even under steady forcing conditions. Finally, I explore the common triangular and scalloped mapview patterns developed in tilted rocks when incised by dip-parallel streams. The work presented here demonstrates the importance of large blocks of rock in governing hillslope processes and rates, and advances our understanding of landscape evolution in layered rocks.
DEDICATION

I dedicate this manuscript to my parents, John and Debbie Glade, who have supported me throughout all my meandering paths; Will McDermid, who has listened with endless patience to all of my yammerings on about blocks and diffusion; Lentil and Bean, and the late Frog and Toad, who have all cheered me on while they hid inside my sleeves.
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CHAPTER I
INTRODUCTION

Planetary surfaces exhibit topographic signatures that are readily visible in aerial images. For example, features such as mountain ranges, exposed anticlines and synclines, mesas, and cuestas indicate underlying geologic structure and are formed by interactions among tectonics, climate, surface processes, and material properties of rock and sediment. While much research has addressed first three of these processes, the role of material properties has often been overlooked in geomorphic studies. This is particularly true of hillslopes, which are vital for models of landscape evolution because they cover the majority of Earth’s land surface. Classic geomorphic theory describes well the evolution of soil-mantled hillslopes developed in homogeneous rock. However, large-scale topographic features on Earth and other planets are commonly developed in rock that is not homogeneous. Therefore, in order to properly understand the surface of the planet and to predict its evolution over time, one must understand how hillslopes “work” in the presence of heterogeneous lithology and geologic structure.

Sediment grain size plays a key role in geomorphic systems. Grain size sets the mobility and mode of transport for sediment and controls the efficacy of various physical processes. In wind-borne and water-borne transport, a threshold must be crossed before transport occurs, and this threshold strongly depends upon grain diameter. In rivers, fine grains suspended in the fluid travel rapidly and contribute little to erosion of the riverbed. In contrast, larger mobile grains that maintain close contact with the bed can be effective at eroding the underlying bedrock. Grains that are too large, however, are generally immobile and therefore inhibit erosion by covering the bed (Sklar and Dietrich, 2004). While the effects of grain size are well known in fluvial and aeolian settings, hillslope studies often focus on the continuum motion of soil-sized particles. However, hillslopes developed in heterogeneous lithology contain a variety of sediment grain
sizes, ranging from fine-grained particles common in soil to “blocks” several meters in diameter that calve off from resistant rock strata. In this dissertation I take the first steps to incorporate grain size into process-based formulations of hillslope evolution, focusing specifically on the role of large blocks of rock, which I hereafter simply refer to as “blocks.”

Blocks are common on hillslopes in which bedrock outcrops at the surface. For example, hillslopes in granitic canyons consist of a mixture of soil-mantled patches and bare bedrock outcrops that produce discrete blocks that mantle the remainder of the hillslope. Blocks are also ubiquitous in landscapes developed in sequences of layered rock with contrasting erodibility, especially in arid environments where there are easily visible in aerial imagery. This dissertation focuses on the development of these hillslopes developed in layered rock. The reasons for this focus are threefold. First, layered landscapes are common across Earth’s surface as well as on other planets. Second, layered rocks are organized into simple geometries, allowing a spatial constraint on the sources of sediment from different rock types. Third, hillslopes developed in layered rock exhibit key morphologic features that directly contradict traditional hillslope theory, providing clear target observations for conceptual and numerical models.

The second chapter of this dissertation, entitled “Block-Controlled Hillslope Form and Persistence of Topography in Rocky Landscapes,” examines a common feature in layered landscapes known as a hogback. Hogbacks consist of a tilted resistant layer of rock overlying softer, more erodible rock. Blocks calved from the resistant layer occupy the surface of the underlying soil-mantled hillslope, which is commonly concave-up (that is, steeper at the top than the bottom). This is in direct contrast to predictions of hillslope form from classic theory, which strive to explain the common convexities in soil-mantled hilltops. Using numerical modeling and field work, I show that weathering and transport feedbacks between the blocks and soil can explain the surprising morphology of hogbacks. Further, the presence of blocks significantly
increases hillslope relief and the persistence of topography for geologically relevant periods of time.

In the third chapter, entitled “Quasi-Steady Evolution Of Hillslopes In Layered Landscapes: An Analytic Approach,” I find that in the presence of blocks, hillslopes can self-organize to maintain steady forms and erosion rates through time. I then develop analytical theory that describes the steady state evolution of hillslopes developed in layered rocks of horizontal (e.g., mesas), tilted (e.g., hogbacks), and vertical (e.g., dikes) orientations. This chapter outlines the key processes that require further study, highlighting the importance of block transport and weathering dynamics, and provides field-testable predictions.

The fourth chapter, entitled “Canyon Shape And Erosion Dynamics Governed By Channel-Hillslope Feedbacks,” explores the influence of blocks in river canyon evolution. A coupled hillslope and channel evolution model illuminates feedbacks between block delivery from the hillslopes to the channel, and subsequent changes in boundary conditions at the base of the hillslope due to altered channel incision rates. The model reproduces characteristic planform shapes of river canyons observed on Earth and demonstrates that block-related channel-hillslope feedbacks create augotenic signals that can strongly modify the signals of the hydologic and tectonic controls on baselevel. Large fluctuations in erosion rates even under steady tectonic forcing, complicating the inference of the baselevel forcing of the system from erosion rates that are documented locally in time and space.

The dissertation concludes with Chapter 5, in which I discuss ongoing work related to planform hillslope patterns in layered rock. I lay out the major findings of the dissertation, outlining the key steps in moving forward on this important problem in Geomorphology.
CHAPTER II

BLOCK-CONTROLLED HILLSLOPE FORM AND PERSISTENCE OF TOPOGRAPHY IN ROCKY LANDSCAPES

PUBLICATION NOTES


ABSTRACT

Rocky hillslopes dotted with boulder-sized blocks and covered by a thin, non-uniform soil are common in both steep landscapes and arid environments, as well as on other planets. While the evolution of soil-mantled, convex hillslopes in uniform lithology is reasonably well understood, the influence of heterogeneous lithology and geologic structure on hillslope form and evolution has yet to be properly addressed. Landscapes developed in layered sedimentary rocks feature sharp-edged landforms such as mesas and hogbacks that exhibit steep, linear-to-concave ramps with scattered blocks calved from resistant rock layers overlying softer strata. Here we show that blocks can control the persistence of topography and the form and evolution of hillslopes in these landscapes. We present a numerical model demonstrating that incorporation of feedbacks between block release, interruption of soil creep by blocks, and sporadic downslope movement of blocks are necessary and sufficient to capture the morphology and evolution of these landscapes. Numerical results are reproduced by a simple analytical solution that predicts steady-state concave hillslope form and average slope angle from block size and spacing. Our results illuminate previously unrecognized hillslope feedbacks, advancing our understanding of the geomorphology of rocky hillslopes. On a landscape scale, our findings establish a
quantitative method to address the migration of sharp edges and the persistence of topography in layered landscapes.

1. INTRODUCTION

Hillslopes cover the majority of Earth’s land surface and exert a first-order control on landscape scale (Sweeney et al., 2015). Current theory posits that a collection of hillslope processes (e.g., bioturbation and freeze-thaw) results in bulk diffusion of soil (Kirkby, 1971) and predicts the evolution of convex-up hilltops in homogeneous, soil-mantled, steady-state landscapes (e.g., Dietrich et al., 2013). However, many rocky, weathering-limited hillslopes are strongly influenced by geologic structure and heterogeneous lithology and do not exhibit this classic convex-upward form (Howard 1994; Selby, 1987; Moon, 1984; Koons, 1955; Cooke and Warren, 1973). While meaningful progress has been made over the past few decades, most notably a nonlinear flux theory that accounts for increased transport at steep gradients (Andrews and Bucknam 1987; Roering et al., 1999; 2001), models of non-local transport (Tucker and Bradley 2010), a method for modeling rock with variable susceptibility to weathering (Johnstone and Hilley, 2014), and a model of the evolution of scarps with well-developed rill networks (Ward et al., 2011), current theoretical and numerical models fail to fully capture the very elements of a landscape that reflect its geology. Many examples of rocky hillslopes are found in landscapes dominated by layered rocks across the world (King, 1957). Iconic features such as hogbacks, flatirons, mesas, and dikes are easily observed in aerial and satellite imagery and generate striking topography in otherwise flat landscapes. These features reflect one or more layers of resistant, typically coarsely jointed rock embedded within softer strata (e.g., sandstone in shale) in a horizontal, tilted, or vertical configuration. Hillslopes or “ramps” adjoining the edge of the coarsely jointed resistant layer are typically linear-to-concave-upward (Fig. 1a) and
are mantled by large resistant blocks (Howard, 1994; Ward et al., 2011) (Fig. 2). Blocks are not found beyond the base of the ramp, where the slope transitions to a nearly flat plain, suggesting that the development and persistence of local relief may be tied to the presence of the blocks (Fig. 2a).

2. FIELD OBSERVATIONS AND CONCEPTUAL MODEL

We first develop a conceptual model of the evolution of a hogback, a tilted feature that exemplifies this class of landforms (Fig. 1C). Models of hillslope evolution require treatment of both the conversion of bedrock to soil and the subsequent transport of soil downslope. Hillslopes developed in layered rock require additional acknowledgment of block release from the resistant layer, the subsequent fate of those blocks as they move downslope, and weathering and transport interactions between blocks and underlying soft rock. Topographic profiles of hogbacks across the world show that slopes adjoining hogbacks are straight to concave (Fig. 1B, 2A). Field observations of hogbacks suggest that due to the back-tilt of the sandstone with respect to the adjoining ramp developed on shale, resistant layers are undermined block-by-block (Ahnert, 1960; Oberlander, 1977) as shale is removed from the base, releasing one joint-bounded segment at a time. Blocks appear to be released by rotational sliding and deposited only a short distance downslope from the scarp.

However, blocks are found scattered along the full length of the ramp, much further from the scarp than they are initially deposited (Fig. 2), suggesting that they must move downslope at some time after release. This has most recently been shown in the Stolowe Mountains, Poland, where comparison between field data and rock-fall models demonstrates that blocks are indeed found much further from the source than would be expected from rock fall alone (Duszyński and
Figure 2.1. A) Photo of hogback in Morrison, CO, demonstrating concave up slope profile. B) Google Earth-derived topographic profiles of hogbacks across the world, normalized by distance from crest and total relief (See GSA Data Repository for locations). C) Conceptual model of ingredients required to explain hogback evolution.
We hypothesize that the presence of blocks on the slope enacts two related feedbacks. First, large blocks obstruct the downslope motion of soil, essentially serving as dams (Fig. 2B). This causes soil accumulation upslope of the block, while the ramp immediately downslope is starved of soil. Over time this generates a depression downhill of the block, into which the block may topple or slide once the relief and the local slope are sufficient. Second, the presence of the block and the perturbation of soil thickness around it alters the rate at which the underlying shale (or other easily weathered rock) can be converted to soil. Finally, the blocks themselves weather and therefore decline in size through time. We hypothesize that these weathering and transport interactions are sufficient to explain both the shape of the ramp and the distribution of blocks observed in the field.

We hypothesize that blocks should decrease in size and become more weathered with distance from the crest. To test this, we collected field data from Heil Valley Ranch along the Front Range in Colorado, a segment of the Dakota Ridge that serves as a notable example of a hogback. Our study site, a 300 m long Morrison shale slope perpendicular to the strike of the resistant Dakota sandstone layer, displays the expected concave-up topographic profile characteristic of hogbacks (Fig. 2A). The slope is well-vegetated by trees, and displays no evidence of overland flow, rills, or accumulation of soil at the base of the slope. Our data show that both block size and areal coverage decrease with distance from the crest (2A, C, D). Schmidt hammer measurements demonstrate that the compressive strength of blocks, which serves as a proxy for degree of weathering (Goudie, 2006 and references therein), also decreases with distance from the crest (Fig. 2A) (see GSA Data Repository for detailed methods). Similar data collected in the Stolowe Mountains, Poland, reveals a similar decrease in rock strength away
Figure 2.2. A) 1-m resolution LiDAR profile of a hogback at Heil Valley Ranch, CO (courtesy of BeCZO). Schmidt Hammer measurements of blocks show decreasing mean Schmidt Hammer rebound values downslope; each point represents 75-150 individual measurements; SEM error bars are smaller than data points. Mean block size and number measurements on the slope are also shown. Each block size data point is the mean value of both long and short axes of 10 blocks measured at each site. SEM error bars are shown; some are smaller than data points. B) Photo of a block dam. Red line shows average slope; white lines show altered “effective” slope due to soil accumulation behind blocks. C) Photo of block coverage near the bottom of the slope. D) Photo of block coverage further upslope.
from the crest (Duszyński and Migoń, 2015). Soil buildup can clearly be seen behind numerous blocks scattered across the ramp (Fig. 2B). While it is difficult to determine specific hillslope processes at play in any landscape, frost creep, wetting-drying cycles and tree throw are most likely the dominant sediment transport processes at this field site. See GSA Data Repository for further discussion of potential weathering and transport processes.

3. NUMERICAL MODEL

We develop a numerical model to test whether our proposed feedbacks between weathering and transport of blocks and soil can explain the shape of these landforms. Our model marries discrete and continuum approaches in order to account for the effects of blocks while using a simple formulation of hillslope soil flux. The model setup consists of a tilted resistant bedrock layer of specified thickness, dip, and fracture spacing, inter-bedded within more easily weathered bedrock. The two lithologies are differentiated by the maximum rate at which bare bedrock may be converted to soil, \( w_0 \). Soil conversion rate for both lithologies depends on soil thickness, with an exponential decrease (Ahnert, 1977; Heimsath et al., 1997) (See Fig. DR5 in GSA Data Repository). We employ a linear hillslope transport law in which soil flux depends on both the local slope and thickness of soil (e.g., Anderson, 2002; Johnstone and Hilley, 2014; Mudd and Furbish, 2007). We initiate the model with flat topography and lower the boundaries, assumed to be channels, with a prescribed constant rate of fluvial incision. For simplicity, we treat blocks as cubes of uniform size that are released when the shale hillslope ramp erodes deeply enough to initiate motion on the next joint set. A certain number of blocks, determined by layer thickness and block side length, is then removed from the scarp and deposited immediately downslope, one block in each model cell. See GSA Data Repository for algorithms capturing the processes involved.
We treat these blocks as bedrock with a low soil conversion rate and track the height of each block as it weathers. Shale underlying each block continues to produce soil at a rate determined by the sum of the thickness of soil and block height. Blocks become increasingly susceptible to downslope movement as the elevation drop between the block and the next cell increases (Tucker and Bradley, 2010).

Model results demonstrate that our treatment of block release, weathering, and movement downslope captures the essence of hogback evolution and hits a number of targets expected from field observations (Fig. 3). Our model reproduces the characteristic concave-upward ramps. Blocks tend to cluster together, with larger blocks near the top of the ramp and smaller, more weathered blocks toward the bottom. Large blocks are not found beyond the slope break at the base of the ramp. Blocks also act as soil dams, which forces accumulation of soil upslope of each block and allows a depression to develop downslope (Fig. 3). On the dip slope (left side of Fig. 3), soil develops on the easily weathered rock above the resistant layer, and a convex hillslope evolves, ultimately leaving the hard bedrock bare at the angle of dip. Similarly, the low-angle slope beyond the block-covered ramp (right side of Fig. 3) displays a slight convexity characteristic of a typical steady-state Gilbert hillslope (Gilbert, 1909). With a constant incision rate, the ramp eventually reaches a quasi-steady-state form, in which block release rate remains constant and the ramp retreats parallel to itself and maintains a constant concave form, length, and relief (Fig. 3). Control runs using the same parameters but without blocks develop purely convex hillslopes and exhibit ~60 m less relief than runs with blocks. This demonstrates that hillslope form and the persistence of relief in our model is controlled by the presence of blocks, and not by boundary conditions (see Fig. DR6 in GSA Data Repository). Model runs using different values for thickness, block size, dip, weathering rates and incision rates preserve the concave form and general behavior illustrated by the example of Figure 2.3.
4. DISCUSSION AND CONCLUSIONS

Model results show that blocks can dictate the shape of the hillslope and profoundly influence the creation of relief and persistence of topography. The block-free, low-relief slopes on the right and left sides of Figure 2.3 can be viewed as a control case, and show the slightly convex form expected for a hillslope developed in homogeneous easily weathered lithology. We next seek to understand the quasi-steady-state form and slope angle of the ramp produced in the model. In the face of uniform rate of conversion of rock to soil, \( w \), and the simplest soil flux law \( Q = -kS \), a steady-state hillslope requires that slope, \( S \), increase linearly downhill such that \( S = \frac{wx}{k} \), where \( x \) is the distance from the crest, and \( k \) is a hillslope efficiency constant. In the absence of block dams, this results in the classic parabolic hilltop described by Gilbert (1909). In the presence of block dams, however, the local slope is altered by blocks. The effective gradient relevant to the transport of soil, \( S_{\text{eff}} \), is lower than \( S \), the gradient averaged over many such block dams (Fig. 4):

\[
S_{\text{eff}} = S - \frac{D}{Xs} \quad (1)
\]

where \( D \) is the thickness of a block, and \( Xs \) is the spacing between blocks. In order to reach steady state, the average slope must match the required steady-state slope distribution. This yields

\[
S = \frac{wx}{k} + \frac{D}{Xs} \quad (2)
\]

In the absence of blocks (\( D = 0 \)), Equation 2 recovers the classic steady-state solution. When blocks are present, the second term on the right-hand side influences, and can even dominate, the shape of the slope. For blocks of uniform size and spacing, when \( D/Xs \gg wx/k \), slopes are linear and steeper than the non-block case. For the case in which blocks decay downslope, as in our model, slope angle decreases with distance from the crest because block size decreases, leading to a concave form. Beyond the slope break, where \( D = 0 \), only the 1st
Figure 2.3. Modeled hogback evolution plotted every 400,000 years. Red squares represent locations and relative sizes of blocks, broken into four size classes. By 2 Myr, the adjoining slope reaches a quasi-steady-state of parallel retreat in which hillslope form and block release rate remain constant. Slopes are concave-upward. Blocks decrease in size as they weather and move downslope, and do not persist beyond the base of the ramp. Here sandstone thickness = 10 m, dip = 30°; k = 0.5 m/yr. Bare shale soil production rate = 10 \( \times 3 \) m/yr, bare sandstone soil production rate = 10 \( \times 5 \) m/yr, and the characteristic length scale for decline of soil production rate \( H_w = 0.2 \) m. Fluvial incision is steady at \( 5 \times 10^{-5} \) m/yr. Inset shows example of a series of soil dams in the model. (See GSA Data Repository for an animation of the model.) Inset shows an example of a block dam; black line represents soil surface, grey line represents bedrock surface.

term on the right-hand side of Equation 2 remains, and the slope should subtly increase toward the bounding stream. In Figure 2.4 we compare the average slope observed in the numerical model with the expected analytical average slope for the ramp (see Methods). The analytical solution agrees well with the steady linear decrease in slope observed in our model, as well as LiDAR-derived slope trends (see Fig. DR1 in GSA Repository). Equation 2 successfully
captures the full range of forms (convex, linear, concave) observed in the field for both homogeneous and blocky hillslopes.

Figure 2.4. A) Conceptual diagram of rationale behind analytical solution. Effective slope relevant for local sediment transport, $S_{eff}$, is lower than the average slope, $S$. B) Comparison of model with analytic solution. Slopes roughly linearly decline with distance downslope. Inset compares the numerical and analytic slopes (line is 1:1 fit).
Our results demonstrate that the feedbacks we have proposed and implemented in this 1-D model can explain the basic form and evolution of hillslopes in landscapes dominated by layered rocks. Blocks play a vital role in allowing the migration and persistence of sharp-edged features in layered landscapes; an abundance of blocks can greatly slow the lateral migration of these features (see Figs. DR6, DR11 in GSA Data Repository). Our model can easily be modified to explore the evolution of landscapes developed in layered rocks with different orientations and multiple resistant layers, as well as the effects of more complex fracture orientations and initial distribution of block locations (see Figs. DR7, DR8, & DR9 in GSA Data Repository). In future studies we will explore the 2-D effect of soil flow around blocks and the evolution of landscapes developed in folded layered rocks as they are exhumed (e.g., exposed anticlines); future field work is needed to document the specific mechanisms of block movement, which may include toppling or sliding. Further work should explore the effects of climate in the development of rocky hillslopes; for example, one could compare the evolution of landforms in this study (developed without significant overland flow or large rock-fall events) with evolution of heavily-rilled, steep, blocky slopes in very arid landscapes (as described well by Ward et al., 2011). In addition, vegetation may play an important role in the development of these landforms, as vegetation has been shown to dam soil in a manner similar to the block dams observed in this study (DiBiase and Lamb, 2013).

The importance of blocks is not limited to landscapes developed in layered rocks. Blocks have been shown to armor granitic slopes in mountainous terrain developed on crystalline rocks and to contribute to persistence of local relief (Granger et al., 2001). Recent work has shown that blocks are also important in bedrock channels (Dubinski and Wohl, 2012; Lamb and Dietrich, 2009) and glacial landscapes (Anderson, 2014; Dühnforth et al., 2010). Further, feedbacks between blocky hillslopes and stream incision in mountainous terrain can substantially alter
landscape evolution patterns and the upstream propagation of climate and tectonic signals (Shobe et al., 2016). We therefore argue that models of landscape evolution must ultimately incorporate the effects of blocks and of heterogeneous lithology in order to capture the essence of both small-scale hillslope form and large-scale persistence of topography.
CHAPTER III
QUASI-STeady EVOLUTION OF HILLSLOPES IN LAYERED LANDSCAPES: AN ANALYTIC APPROACH

PUBLICATION NOTES

ABSTRACT
Landscapes developed in layered sedimentary or igneous rocks are common on Earth, as well as on other planets. Features such as hogbacks, exposed dikes, escarpments, and mesas exhibit resistant rock layers adjoining more erodible rock in tilted, vertical, or horizontal orientations. Hillslopes developed in the erodible rock are typically characterized by steep, linear-to-concave slopes or “ramps” mantled with material derived from the resistant layers, often in the form of large blocks. Previous work on hogbacks has shown that feedbacks between weathering and transport of the blocks and underlying soft rock can create relief over time and lead to the development of concave-up slope profiles in the absence of rilling processes. Here we employ an analytic approach, informed by numerical modeling and field data, to describe the quasi-steady-state behavior of such rocky hillslopes for the full spectrum of resistant layer dip angles. We begin with a simple geometric analysis that relates structural dip to erosion rates. We then explore the mechanisms by which our numerical model of hogback evolution self-organizes to meet these geometric expectations, including adjustment of soil depth, erosion rates, and block velocities along the ramp. Analytical solutions relate easily measurable field quantities such as ramp length, slope, block size and resistant layer dip angle to local incision rate, block velocity,
and block weathering rate. These equations provide a framework for exploring the evolution of layered landscapes, and pinpoint the processes for which we require a more thorough understanding to predict their evolution over time.

1. INTRODUCTION

Current theory suggests that soil-mantled hillslopes in topographic steady state exhibit a convex-up form as a result of diffusion-like processes [e.g., Culling, 1960, Kirkby, 1971]. “Rocky” hillslopes, influenced by heterogeneous lithology or geologic structure, are more complex and often deviate from the classic steady-state form associated with soil-mantled slopes [e.g., Johnstone and Hilley, 2014; Ward et al., 2011; Howard and Selby, 1994; Howard and Kochel, 1988; Selby, 1987; Moon, 1984; Cooke and Warren, 1973; Koons, 1955]. Examples of rocky hillslopes can be found in landscapes developed in layered rock on Earth, as well as on Mars [Treiman, 2008]. In these settings, exposure of resistant layers often sets the relief of the landscape [Ward et al., 2011]. Features such as hogbacks (Figure 3.1A), exposed dikes (Figure 3.1B), and horizontal scarps (Figure 3.1C) typically exhibit steep, linear-to-concave hillslopes developed in soft rock adjoining resistant layers of varying dip angles. These “ramps” are mantled with material derived from the resistant layers, often in the form of discrete blocks [e.g., Glade et al., 2017, Ward et al., 2011, Howard and Selby, 1994]. Numerous studies have observed a positive correlation between block size and topographic slope [Poesen et al., 1998; Abrahams et al., 1985], with a decrease in block size [Glade et al., 2017] and an increase in block spacing [Duszynski et al., 2017] with distance down the ramp. The presence of resistant material typically ends at a slope break at the base of the ramp, where topographic slope shallows to become nearly horizontal [Glade et al., 2017].
The influence of lithology and geologic structure on hillslope evolution is complex; however, to improve our quantitative understanding of layered landscapes, it is useful to first consider the quasi-steady state case. Hogbacks, cuestas and gently dipping scarps migrate both laterally and vertically across a landscape. While such migrating features are unlikely to achieve steady form or erosion rates in the strictly vertical sense, a feature may maintain a constant length, relief, and curvature in the reference frame of the retreating resistant layer such that dip-parallel erosion rates remain constant (Figure 3.2B). Some early studies of landscape evolution proposed qualitative models of such parallel slope retreat [King, 1957; Koons, 1955; King, 1953; Penck and Penck, 1924]. More recently, Howard and Selby [1994] explored the erosional conditions necessary for a feature to maintain quasi-steady state through a geometric analysis supported by field observations. Ward et al., [2011] observed temporarily steady forms in a numerical model of horizontal scarp evolution in the presence of well-developed rill networks. Henceforth we use the term “steady-state” to refer to this quasi-steady state in the reference frame of the feature.

Glade et al. [2017] demonstrated that a numerical model of hogback evolution can self-organize to reach steady state. They showed that by incorporating the role of blocks, the model can reproduce first-order field observations of topographic slope and block size distribution. The presence of blocks causes soil to dam up behind them, decreasing the local slope and therefore soil flux. Average topographic slope gradient must therefore steepen to overcome this damming effect, according to the size of the block. Because blocks decrease in size away from the crest as they weather and move downslope, topographic slope also decreases, producing the characteristic concave-up profile observed in the field. Over time, the modeled feature reaches the type of steady state described above, in which curvature, ramp length and relief are constant in the moving reference frame of the feature (Figure 3.2B).
In this paper we analytically explore the mechanisms that lead to this self-organized behavior. First, we summarize the numerical model of hogback evolution presented in Glade et al. [2017]. Building upon Howard and Selby [1994], we perform a simple geometric analysis to describe the steady behavior of tilted landforms. We then develop analytic solutions that describe the behavior of the numerical model of tilted landforms. Next, we explore the steady-state behavior of the vertical end member landform using an exposed dike at Shiprock, New Mexico as a field example. We briefly outline the geometric conditions that describe steady behavior of horizontal end member landforms. We then discuss the limitations of our approach and the largest unknowns that warrant further research, outlining specific testable hypotheses derived from this study.

1.1 BACKGROUND

Glade et al. (2017) outlined the factors relevant for evolution of hillslopes developed in layered rock: 1) block release from resistant layers, 2) feedbacks between transport of blocks and soil, and 3) feedbacks between weathering of soft rock and resistant blocks. Mechanisms within each of these categories depend on local base-level control, climate, lithology, and location-specific processes [Sklar et al., 2017]. Styles of block release observed in the field include block-by-block undermining [e.g., Howard and Selby, 1994], slumping [e.g., Watson and Wright, 1963], and rockfall [e.g., Dorren, 2003; Brunsden and Prior, 1984]. Depending on the style of release and height of the scarp, blocks may travel relatively short or long initial distances. Undermined blocks can be deposited very close to the resistant layer source, with an initial size that is dependent on joint spacing [e.g., Glade et al., 2017]; alternatively, blocks released in rockfall can travel long distances and can break into smaller pieces upon impact [e.g., Wieczorek et al., 2000; Schumm and Chorley, 1966].
After initial release, resistant material may experience subsequent transport through landslides [e.g., Duszynski et al., 2017], dry ravel [DiBiase et al., 2017; Gabet 2003], earthflows [Mackey et al., 2009], ploughing due to periglacial processes [Hall et al., 2001], slow rolling, sliding, or toppling [Glade et al., 2017], or by runoff processes [Michaelides et al., 2012; Abrahams et al., 1985]. While few studies have considered the mechanics of slow, single large-grain transport on hillslopes, several have observed that it does occur. Long-term field studies tracking movement of coarse grains have found an inverse relationship between transport rates and sediment size, and a positive relationship with slope angle [Abrahams et al., 1984; Schumm, 1966]. Abrahams et al. [1984] inferred transport by overland flow, while Schumm [1966] inferred creep. However, while the grain sizes observed in these studies were larger than typical soil grains, they were much smaller than the meter-scale blocks on which we focus in this paper. While it is difficult to directly observe transport of meter-scale blocks, some studies have inferred slow transport of such large grains. Putkonen et al. [2014] studied trails of boulder fragments up to 0.4 meters in diameter in Antarctica, finding that they could be transported up to 60 meters downslope from the parent boulders. The total distance traveled by coarse material was found to correlate with slope angle; however, transport rates and mechanisms were unknown. Duszynski et al. [2017] examined large boulder transport on ramps by slow-moving landslides, and noted possible transport down the ramp by other slow processes.

Once blocks are released from a resistant layer, they may also experience a change in shape and/or size through physical and/or chemical weathering [see review in Sklar et al., 2017]. One may split block weathering into two main categories: grain-by-grain weathering, in which block size decreases slowly and steadily over time; or fragmentation, in which blocks calve off coarse fragments or split into multiple pieces. The first of these categories may be accomplished by the formation and spalling of weathering rinds [e.g., Yoo and Mudd, 2008] or grain-by-grain
weathering such as grusification [Eppes et al., 2010]. The second category includes fragmentation of blocks by splitting due to thermal fluctuations [e.g., Eppes and Keanini., 2017; Eppes et al., 2016; Eppes et al., 2010], frost cracking and other processes. Eppes et al. [2016] showed that a block must be a certain size to experience a thermal gradient sufficient to crack the block. Putkonen et al. [2014] showed that large boulders in Antarctica are more likely to break than smaller fragments. Regardless of the style of release and initial transport, the resistant material on the ramp will influence soil transport and must be either transported or weathered away to a certain degree to allow subsequent block release events [Ward et al., 2011; Howard and Selby, 1994]. While the mechanisms outlined above are paramount to understanding and predicting hillslope evolution in layered landscapes, no comprehensive model for the interplay between processes currently exists.

Figure 3.1: Photos of landscapes dominated by layered rocks. A) Tilted hogbacks along the Dakota Ridge in Fort Collins, Colorado (Photo: Robert S. Anderson) B) Vertical dikes surrounding central intrusion at Shiprock, New Mexico (Photo: Michael Collier) C) Horizontal scarps at Book Cliffs near Grand Junction, Colorado (Photo: Robert S. Anderson)
2. MODEL DESCRIPTION

Here we describe the numerical model of hogback evolution presented in Glade et al. [2017]. Our 1-D numerical model combines discrete and continuum approaches to account for the role of resistant blocks in hillslope evolution. The model setup consists of one resistant rock layer (e.g., sandstone) of prescribed thickness, dip, and joint spacing embedded within softer, more erodible rock (e.g., shale). The lithologies are distinguished by their respective maximum soil production rates, \( w_0 \). We use a simple rule in which soil production decreases exponentially with soil thickness [Ahnert, 1977; Heimsath et al., 1997]:

\[
w = w_0 e^{-H/h_w}
\]  

where \( w \) is the vertical soil production rate, \( w_0 \) is the maximum soil production rate of bare bedrock, \( H \) is the soil thickness, and \( h_w \) is a characteristic soil production depth scale. We use the term “soil production” to mean transformation of bedrock or saprolite into mobile material. We use the term “soil” loosely, in reference to mobile material. For soil transport, we use a depth-dependent linear hillslope transport rule in which soil flux depends on both local slope and soil thickness, assuming an exponential decay in soil velocity with depth [Johnstone and Hilley, 2014]:

\[
q = -kS h_\epsilon (1 - e^{-H/h_\epsilon})
\]  

where \( q \) is the volumetric soil discharge (per unit contour length) in units of \([L^3/LT]\), \( k \) is the hillslope efficiency in units of \([L/T]\), \( S \) is the local slope gradient, and \( h_\epsilon \) is a characteristic soil thickness representing the depth scale of transport. We evaluate flux at the edges of nodes, and soil thickness at the middles. Therefore, flux is calculated using the closest uphill value of soil
depth. This formulation of soil flux is valuable in that it allows flux to smoothly transition to zero when no soil is present. Therefore, the model does not make any assumptions of transport-limited or weathering-limited conditions. Flux is applied only to “soil” in the model, which includes both soil production products from the soft rock and the discrete blocks. Given the discrete component of our model, we employ a linear formulation of soil flux for both numerical stability and simplicity. While the model achieves steep average topographic slopes due to the presence of blocks, the local slope gradients relevant for soil transport remain shallow (see section 2.3.1); therefore, we expect that use of a non-linear flux rule [e.g., Pelletier and Rasmussen, 2009; Roering et al., 1999] would not substantively change the behavior of the 1-D model, but would undoubtedly be important in a 2-D model.

We initialize the model with flat topography and lower the boundaries with a constant incision rate, $\varepsilon_1$. We also force the boundaries to move laterally with the hogback, so that the model effectively operates in the frame of reference of the resistant layer. This approach ensures that the hogback feels a perfectly steady incision rate at its base without moving further away from the channel that serves as its boundary condition over time. A stationary boundary would change the imposed erosion rate at the base of the ramp over time [e.g., Mudd et al., 2007], though this only makes a small difference in the behavior of the model [Glade et al., 2017].

We employ a discrete rule for erosion of the resistant layer, in which blocks of uniform size are released when the adjoining shale hillslope erodes enough to “undermine” the resistant layer at a specified relief threshold between the edge of the resistant layer and the next downslope cell [Ward et al., 2011]. For simplicity, we assume that negligible weathering occurs on the backslope (i.e., the intact resistant layer) meaning that the blocks are released in an unweathered state and no soil production occurs on the resistant layer; this generally agrees with field observations of features with resistant layers (Figure 3.2A; Howard and Selby, 1994). Once
blocks are removed, they are deposited immediately downslope, with one block in each model cell. Note that the width of a block is therefore constrained to be equivalent to node spacing (dx) in the model; while this may be viewed as a limitation of the model, it allows rapid computation of discrete block movement. Blocks may stack on top of one another if they happen to land in an occupied cell. To conserve mass, when a block lands in a cell, the current soil thickness is memorized and restored once the block moves to another cell. After blocks are deposited, we track the thickness of each block as it weathers vertically through time with a constant weathering rate, $w_{0h}$, that is lower than the maximum weathering rate of the soft rock (allowing soil on top of blocks to influence weathering rate of the blocks does not substantively change the behavior of the model [Glade et al., 2017]). We refer to the breakdown of blocks over time as “weathering”, which therefore encompasses all processes that lead to their reduction in size.

Blocks decrease in height only—block width remains the length of a model cell. Weathering products from the blocks turn into mobile soil. When weathering blocks reach a cutoff size of 0.2 meters, we cease to treat them as blocks and treat the mass as soil instead.

Field observations suggest that, at least in some settings, blocks must move downslope after initial deposition [Duszinsky et al., 2017; Glade et al., 2017; Duzsinski and Migon, 2015; Abrahams et al., 1984]. We use a simple discrete rule for motion that relates block velocity to the current block diameter. In our model, a block can only move when the relief between the bottom of the block and the next model cell is equal to the current height of the block; therefore, the waiting time (t) for a block to move depends on its current thickness (D) and the vertical erosion rate ($\varepsilon_2$) just downhill of the block:

$$t = \frac{D}{\varepsilon_2}$$ (3)
We assume that the block travels a characteristic distance $dx$ each time it moves (in the model, this is equivalent to the node spacing). Therefore, in our simplified model the block velocity, $v_b$, for any given block diameter is simply:

$$v_b = \frac{dx}{\varepsilon_2}$$  \hspace{1cm} (4)

If downhill vertical erosion rate $\varepsilon$ is constant, then we expect blocks to move slowly when they are large and more quickly when they are small. Block velocity increases linearly with downhill erosion rate and is inversely related to block size. This behavior mimics the behavior inferred from the field studies outlined in Section 1.1. Our parameterization does not directly relate block transport to slope gradient; rather, the correlation between block speed and gradient arises naturally from the fact that a steeper gradient yields a shorter waiting time between block undermining events, all else equal. This approach avoids the complication of deciding over which length to measure slope angle, which could be problematic given the inherent spatial variability in slope in our model due to the presence of blocks.

### 2.1 ANALYTIC SOLUTION FOR SPATIAL BLOCK SIZE DISTRIBUTION

The simple model rules for block motion described in Eqns. 3 and 4 can be solved analytically to show that in the face of constant vertical erosion rate on the ramp, block size decays exponentially with distance from the crest. This exponential spatial block size distribution is scaled by the ratio between the weathering rate of the blocks and the erosion rate at the base of the ramp.

Eqn. 4 gives the instantaneous velocity of a block for a given downhill vertical erosion rate and block diameter. If vertical erosion rate $\varepsilon_2$ is constant, then blocks move slowly when
they are large and more quickly when they are small. Using Eqn. 4 and the mean value theorem, we can estimate the mean block velocity over its lifetime as:

\[ \bar{v}_b = \frac{1}{(D_0 - D)} \int_D^{D_0} \frac{\varepsilon_2 dx}{D} dD \]  

(5)

where \(D_0\) is the original block diameter. Evaluating this integral yields the mean velocity as a function of current block size, \(D\):

\[ \bar{v}_b = \frac{1}{(D_0 - D)} \varepsilon_2 dx [\ln(D_0) - \ln(D)] \]  

(6)

We can use this expression for block velocity to derive the expected block size decline with distance from the crest in the face of a uniform erosion rate and a constant weathering rate of the blocks. Here for simplicity we assume that blocks are initially deposited relatively close to the top of the ramp. First, we relate block size to the amount of time the block has been exposed to weathering:

\[ D = D_0 - w_b T_w \]  

(7)

where \(D\) is the current block diameter, \(D_0\) is the initial block diameter, \(w_b\) is the weathering rate of the block, and \(T_w\) is the total time a block has been exposed to weathering. If we assume the block begins to weather only once it is released from the resistant layer, then \(T_w\) is equivalent to the amount of time the block has spent on the ramp. Therefore:
\begin{equation}
T = \frac{x}{v_b} \tag{8}
\end{equation}

where \( x \) is the distance from the crest. Substituting our expression for mean velocity into this timescale, and then substituting the timescale into the equation for \( D \), we obtain:

\begin{equation}
D = D_0 - \frac{w_b x (D_0 - D)}{\varepsilon_2 x [\ln(D_0) - \ln(D)]} \tag{9}
\end{equation}

Isolating \( D \), we obtain an expression for block size decline with distance from the crest:

\begin{equation}
D = D_0 e^{-\frac{w_b x}{\varepsilon_2 dx}} \tag{10}
\end{equation}

Eqn. 10 predicts that with a uniform erosion rate, constant block weathering rate, and a linear relationship between block velocity, diameter and erosion rate, block size should decrease exponentially with distance from the crest. The exponential decay is scaled by the product of the step length, \( dx \), and the ratio between erosion rate and block weathering rate, \( w_b / \varepsilon \). The step length and the vertical erosion rate influence the rate of block motion; therefore, higher \( dx \) and \( \varepsilon_2 \) serve to lengthen the decay length scale, leading to larger block sizes at greater distances downslope. In contrast, a higher block weathering rate \( w_b \) leads to rapid decay of block size through time, shortening the decay length scale to produce smaller blocks at shorter downslope distances. Under the assumptions of the model, Eqn. 10 suggests that the ratio of block weathering rate to vertical erosion rate of the landform may be estimated from spatial block size distribution alone. We explore this idea further for the case of a vertical dike in Section 4.
2.2 MODEL PARAMETERS, DEFINITIONS, AND ASSUMPTIONS

Here we outline definitions and parameter values we use in the numerical model in this study. We strictly define “ramp” in our model as the entire portion of the hillslope mantled with blocks. This generally corresponds to a clear break in topographic slope. The ramp extends from the “crest” (the edge of the resistant layer, which also is the highest elevation point in the model) to the block furthest from the crest. We use the term “steady-state” to refer to a quasi-steady state of constant resistant layer dip-parallel erosion rate. In order to easily compare our model with an analytic solution, we use a resistant layer with a thickness of 1 meter such that only 1 block is produced in each release event. This results in shorter ramps than one might expect in the field; however, model behavior is not qualitatively different from cases with thicker resistant layers (as in Glade et al., [2017]). We avoid non-dimensionalizing the model to allow for a more direct interpretation of model results with respect to field settings. See Table 1.1 for a full list of parameter values used in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Resistant layer thickness</td>
<td>1 m</td>
<td>Characteristic weathering depth (h&lt;sub&gt;W&lt;/sub&gt;)</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Resistant layer dip angle (θ)</td>
<td>30 degrees</td>
<td>Hillslope efficiency (k)</td>
<td>0.5 m/yr</td>
</tr>
<tr>
<td>Initial block height (D&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>1 meter</td>
<td>Characteristic soil thickness for transport (h&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Maximum weathering rate of soft rock</td>
<td>1x10&lt;sup&gt;-3&lt;/sup&gt; m/yr</td>
<td>Incision rate (ε)</td>
<td>(1x10&lt;sup&gt;-5&lt;/sup&gt; m/yr – 1x10&lt;sup&gt;-4&lt;/sup&gt; m/yr)</td>
</tr>
</tbody>
</table>
Maximum weathering rate of hard rock \( (w_0) \) & 1x10^{-5} m/yr & Relief threshold & 3 m \\

Table 1.1: Numerical model parameter values used in this study

The applicability of our numerical model of hogback evolution depends on a number of assumptions. In order to observe the qualitative predictions of our model—block mantled, concave-up ramps—there must be a contrast in weathering rate between the blocks and the underlying rock. Our model does not predict a threshold ratio at which the qualitative behavior is observed; however, lower weathering contrasts between lithologies modulate the relief and length of ramps developed. While the model is not limited to steady state behavior, in order to reach steady state the maximum weathering rate of soft rock must be greater than the base level incision rate; otherwise, weathering cannot produce enough soil to allow erosion rate across the feature to keep up with incision. In its current form, our model is suitable for settings in which long transport distances (i.e., rockfall) are not present. We assume that the processes that transport soil downslope are different from the processes that move blocks; i.e., blocks move more slowly and sporadically than the surface of the soil layer. This assumption is most fitting for cases in which soil thickness is smaller than block thickness, though it remains to be seen to what extent it is unrealistic for cases with thicker soil. Because our model is 1-D, in order for soil to move past a block it must flow over the top of the block. This is a simplification that may not apply in field settings, where soil can flow both over and around blocks; however, the soil-damming effect of blocks has been observed in the field [Glade et al., 2017]. Blocks in our model may stack or inhabit cells next to each other, creating “trains” of blocks and subtle steps in
the topography (similar to step-like features described in Duszynski et al., 2017); however, our model is not suited for thick talus slopes where a majority of blocks are not embedded in soil.

3. TILTED LANDFORMS

Here we explore, in detail, the behavior of our numerical model of tilted landform evolution. Examples of hillslopes developed in tilted, layered rock include hogbacks, cuestas, and homoclinal ridges. Figure 3.2A shows a photo of a hogback in Morrison, Colorado. The resistant sandstone layer of the Dakota Formation overlies the more erodible shale and siltstone of the Morrison Formation. Blocks of resistant rock are released from the edge of the resistant layer and mantle the concave-up ramp, which is developed in the shale layer. The Dakota Ridge (Figure 3.1A), which stretches from Wyoming to New Mexico, is a classic example of a hogback. We outline the conditions necessary for steady evolution of tilted landforms, developing analytic solutions that explain the behavior of a 1-D numerical model of hogback evolution. We demonstrate that measurements of ramp length, block size, and topographic slope may be ultimately used to estimate incision rates and weathering rates. We also discuss which individual processes we must better understand to fully utilize this approach.

3.1 GEOMETRIC ANALYSIS

For features retreating both laterally and vertically, the retreat rate of the resistant layer in the down-dip direction, \( d \), may be decomposed into a purely horizontal component, \( c \), and a purely vertical component, \( \epsilon \). At steady state, the feature must maintain constant relief with respect to the base of the ramp; therefore, \( \epsilon \) must equal the vertical erosion rate defined in the reference frame of the feature, at the base of the ramp, \( \epsilon_1 \) (Figure 3.2B). The dip-parallel erosion rate of the resistant layer, \( d \), is related to \( \epsilon_1 \) via the dip angle:
\[ d = \frac{\varepsilon_1}{\sin \theta} \]  

For a given resistant layer thickness and erosion rate at the base of the ramp, features with shallow dips must have a higher dip-parallel erosion rate. With each block release event, layers with shallow dips experience less rapid vertical lowering than more steeply dipping layers; therefore, they must retreat more rapidly to keep pace with base level lowering. In Figure 3.3 we compare our numerical model results to the dip-parallel erosion rates calculated from Eqn. 11. Small errors in measurement and time averaging contribute to the slight deviation from perfect correlation, especially for shallow dips where a small difference in relief reduction translates into a large difference in down-dip erosion because the horizontal component of scarp retreat is larger. In the field, the dip-parallel erosion rate from Eqn. 11 may be converted to a volumetric erosion rate [Howard et al., 1994]. This would allow the observable volume of resistant blocks on the ramp to be used to compare resistant layer erosion rates.
Figure 3.2: A) Photo of a hogback in Morrison, CO. Ramp (right) demonstrates typical concave-up profile. Backslope (left) mirrors bedding plane. Reprinted from Glade et al. (2017), Figure 3.5A. B) Schematic of quasi-steady state conditions for a hogback. Vertical arrows show higher vertical erosion rate required for steeper slopes. Diagonal arrows show steady down-dip erosion rate, d, in the reference frame of the moving feature. T1-T3 indicate profiles through time. Red squares represent typical spatial distribution of blocks.

Now we examine the pattern of vertical erosion rates required for steady behavior on the ramp. From a purely geometric sense, for the ramp profile to remain steady in the moving frame of reference, vertical erosion rates must adjust according to topographic slope (see arrows at the top of the ramp in Figure 3.2B). To maintain steady state, the diagonal arrows of erosion rate d in Figure 3.2B must maintain a constant length. This requires that the vertical arrows, labeled $\varepsilon_2$ at
the top of the ramp and $\epsilon_1$ at the bottom and representing the vertical component of erosion, maintain a length that depends on the topographic slope. For concave-up ramps, $\epsilon_2$ must be larger than $\epsilon_1$. The vertical erosion rate $\epsilon_2$ at the top of the ramp is:

$$\epsilon_2 = \epsilon_1 \left(1 + \frac{\tan \alpha}{\tan \theta}\right)$$

(12)

where $\epsilon_1$ is the vertical erosion rate of the resistant layer in the reference frame of the layer (equal to the vertical erosion rate at the base of the ramp), $\alpha$ is the local topographic slope angle at the top of the ramp, and $\theta$ is the dip angle of the resistant layer (Figure 3.2B). Eqn. 12 illustrates that at steady state, vertical erosion rates on the ramp must be higher for steeper topographic slopes. This relationship allows us to predict the spatial trend in erosion rates along the ramp; for a concave-up ramp, in which slope angle decreases away from the crest, vertical erosion rates must also decrease, illustrated by a decreasing length in vertical arrows downslope in Figure 3.2B. Note from Eqn. 12 that the relationship between $\epsilon_2$ and slope angle $\alpha$ is approximately linear, except for very steep topographic slopes where the $\tan \alpha$ term increases nonlinearly.

Whether block release occurs due to undermining, slumping, or catastrophic rockfall, the release of resistant material depends on erosion of softer rock on the ramp. Therefore, erosion rates on the ramp must adjust to allow the resistant layer to achieve the necessary erosion rate for a given dip angle. Because features with shallow dip angles require a higher volumetric erosion rate, they may adjust to have higher vertical erosion rates to allow efficient undermining of the resistant layer. Using field data from the Henry Mountains, Howard and Selby [1994] suggested that this may be accomplished by higher relief and steeper slopes between the crest of the feature and the base of the ramp; we demonstrate in Section 3.2 that our numerical model adjusts in a similar way. More research is needed to identify a mechanistic link between ramp slope angle and block release rate (See Section 5).
Figure 3.3: Comparison of dip-parallel erosion rate at steady state observed in numerical model (open black circles) and calculated from Eqn. 5 (red dots) for a hogback developed in a hard layer of 1 m thickness, with an incision rate of $3 \times 10^{-5}$ m/yr. All parameters are held constant except dip angle.

3.2 SELF-ORGANIZATION OF A NUMERICAL HOGBACK

Here we illustrate the self-organized behavior of the numerical model and qualitatively discuss the feedbacks that allow it to reach a steady state consistent with the geometric constraints outlined in section 3.1. This exploration serves two purposes. First, a rigorous understanding of the behavior of a numerical model is essential for validation and accurate interpretation of results. Second, careful analysis of different components of the model can provide insight into feedbacks that may be useful for guiding fieldwork and interpreting real landscapes.
Figure 3.4 shows the spatial distribution of four parameters measured from the numerical model after it has reached steady state: topographic slope gradient, block diameter, weathering rate, and sediment flux. We will explore each of these components individually, then as a whole to diagnose the self-organized behavior that leads to the observed trends.

Figure 3.4: Four metrics computed by the numerical hogback model, averaged spatially over 4 cells and averaged temporally over 250,000 years after reaching steady state, using parameter values in Table 1.1 and incision rate = 3x10^{-5} m/yr. Measured in the frame of reference of the hogback (x=0 follows the crest). Weathering rates shown exclude blocks.

3.2.1 TOPOGRAPHIC SLOPE

The steady-state topographic slope gradient in the model adjusts to allow the ramp to transport all the sediment that is produced upslope, in accord with classic steady-state hillslope theory [Glade et al., 2017]. At steady state, the dip-parallel rate of lowering of the bedrock interface (or soil production rate) must be uniform, and the soil flux must therefore increase with
distance from the divide [Gilbert 1909]. In the presence of blocks, which act as a barrier for soil discharge, the slope must adjust to allow efficient soil transport (Figure 3.5). Therefore, the average topographic slope must be steeper where blocks are large, and need not be as steep where they are small. A general 1-D analytic equation for the topographic slope in the presence of blocks, assuming the simplest form of the soil flux equation, is:

\[ S = \frac{wx}{k} + \frac{D}{x_s} \]  \hspace{1cm} (13)

where \( S \) is the spatially averaged slope over a distance larger than the block spacing, \( w \) is the vertical soil production rate, \( x \) is the distance from the crest, \( k \) is the hillslope efficiency in units of \([L^2/T]\), \( D \) is the local block height, and \( x_s \) is the local spacing between blocks [Glade et al., 2017]. The term \( wx/k \) is equivalent to the local soil slope \( S_{eff} \), which is relevant for soil flux (Figure 3.5A).

This simple equation is valid for a linear flux rule with uniform soil production rate \( w \) and hillslope efficiency \( k \). In our model, however, vertical soil production rate is not uniform, but is modified by soil thickness; in addition, soil discharge is related not only to hillslope efficiency and slope, but also to soil thickness. Block size and spacing are also not necessarily spatially constant. Therefore, the full form of the slope equation is:

\[ S(x) = \int \frac{w(x)}{k(x)} dx + \frac{D(x)}{x_s(x)} \]  \hspace{1cm} (14)

where \( x \) is the distance from the hogback crest. Substituting our chosen rules for soil production and soil transport, we obtain:

\[ S(x) = \int \frac{\left( w_0 e^{-\frac{H(x)}{h_w}} \right)}{kh_s \left( 1 - e^{-\frac{H(x)}{h_t}} \right)} dx + \frac{D(x)}{x_s(x)} \]  \hspace{1cm} (15)
The slope depends on the integral of weathering rate divided by hillslope efficiency from the crest to the x-coordinate, as well as the local block size and spacing. Glade et al. [2017] showed that numerical model results are captured well by this equation. In the presence of large blocks, the term \( \frac{D(x)}{x_e(x)} \) is generally larger than the effective slope term and dominates the average slope trend; therefore, if block size decreases down the ramp (or if spacing increases), topographic slope also decreases and the ramp is concave-up. Note that Eqn. 15 may be re-derived for any chosen soil flux or weathering rule. Additionally, the 1-D nature of our model requires that soil flows over the top of blocks; in the field, 2-D flow allows soil to flow around blocks. This likely does not change the general feedbacks described in this paper; however, it could lower the absolute topographic slopes necessary to efficiently transport soil.

3.2.2 BLOCK SIZE DISTRIBUTION

Figure 3.4B shows block size as a function of distance from the crest once the model has reached steady state. Block size decreases nearly linearly away from the crest. At first glance this result is surprising, given the block transport rules outlined in the model description. According to our rules for block transport, for a constant erosion rate downhill of a block, smaller blocks should move faster than large blocks. In Section 2.1 we showed that this should result in an exponential decrease in block size with distance from the crest if vertical erosion rate is constant. However, in the hogback model we observe a linear decrease. Because the blocks experience a constant weathering rate through time, the observed linear distribution indicates that at steady state, the blocks experience an essentially constant velocity. Eqn. 4 shows that in the model, block velocity depends only on block diameter and downhill erosion rate (since we set the transport step length dx to be a constant). For the blocks to move at a constant velocity, the
vertical erosion rate downhill of each block must offset the effect of decreasing block diameter due to weathering. Model results show that the vertical erosion rate on the ramp at steady state is not constant; Figure 3.4C shows that the soil production rate, which at steady state equals the vertical erosion rate, decreases approximately linearly with distance from the crest. The observed decrease in vertical erosion rate away from the crest is necessary for the feature to maintain a steady state, as expected from our geometric analysis in the previous section for a ramp with a concave-up profile (Figure 3.2B). This explains why the block size distribution linearly decreases with distance from the crest; while smaller blocks should theoretically move more easily, the erosion rate decrease away from the crest offsets this effect and gives each block approximately the same velocity.
Figure 3.5: A) Schematic illustration of average topographic slope from Eqn 7. Average slope gradient, $S$, is steeper than local slope, $S_{eff}$, according to block diameter and spacing. White represents bedrock, yellow is soil, brown are blocks. B) Photo of a block dam from Heil Valley Ranch, Boulder, Colorado. Red line indicates average slope ($S$). White lines indicate local slope ($S_{eff}$). $Q$ = soil flux; $x$ = distance from the crest. Reprinted from Glade et al., 2017.
3.2.3 SOIL PRODUCTION RATE AND SOIL FLUX

The observed decrease in soil production rate (and therefore vertical erosion rate) away from the crest (Figure 3.4C) is accomplished through an adjustment of soil depth according to Eqn. 1; soil depth is thin at the top of the ramp, and thickens downslope. However, this does not necessarily mean that soil is accumulating at the base. Figure 3.4D demonstrates that the soil discharge increases roughly linearly with distance from the crest, meeting the steady-state soil discharge requirements (the trend should in fact not be exactly linear, because soil depth and therefore weathering rate is not spatially constant). The down-slope increase in soil discharge is made possible by the dependence of the soil flux rule on soil depth (Eqn. 2); greater soil discharge occurs at the base of the slope because the soil is thicker. Down-slope soil thickening allows the feature to efficiently transport all of the available material in order to reach steady state in the frame of reference of the hogback. This surprising result demonstrates that important feedbacks result from soil depth-dependent rules for both soil production and transport. These attributes of the transport and soil production rules are not nuances; rather, they are fundamental to the forms of these geomorphic features.

3.2.4 HOLISTIC VIEW

We discuss the interactions between each piece of the model holistically in an effort to illuminate the trends presented in Figure 3.4. The self-organized behavior can be summarized as follows:

1) Blocks steadily decrease in size through weathering as they move down the ramp.

2) The presence of blocks influences average slope according to Eqn. 9. Larger blocks necessitate a steeper average slope to allow efficient sediment transport. Because block
size decreases down the ramp, slope angle also decreases, leading to a concave-up profile.

3) A concave-up slope profile necessitates a decline in vertical erosion rate from the top of the ramp to the bottom due to geometric considerations (section 3.1).

4) Decreasing vertical erosion rates are accompanied by adjustment of soil depth, with thinner soil at the top of the ramp and thicker soil at the bottom.

5) A combination of decreasing block size and decreasing erosion rates down the ramp leads to a constant block velocity (Eqn. 4).

6) Constant block velocity results in a linear decrease in block size, and therefore a linear decrease in topographic slope gradient and erosion rate down the ramp.

7) Depth-dependent soil flux allows flux to increase away from the crest, as required for steady-state forms.

More broadly, the numerical model demonstrates that the seemingly complex form of a hogback and its concave-upward, block-strewn rampart below can be explained as the consequence of just a few basic principles of rock weathering, soil transport, and block displacement. Glade et al. [2017] discussed how small-scale processes (such as block dams) influence larger scale topography. Here we have illustrated that topography in turn influences block and sediment dynamics, leading to a self-organized quasi-steady state. Similar complex behavior has been described in other self-organized geomorphic systems, such as dunefields and sorted bedforms [Murray et al., 2014]. The details of our results depend on the imposed assumptions and rules in the model; variations on these rules would undoubtedly influence the detailed behavior of the model feature. More work is needed to constrain realistic weathering, transport, and block transport rules that determine the form of the slope (Eqn. 15). However, the
importance of the general feedbacks observed likely extends beyond the numerical model and into the field. For example, our analysis shows that depth-dependent rules for weathering and transport of soil are essential for the ability of the landform to meet the geometric constraints outlined in section 3.1. Interpretations of topography, soil depth, and flux trends on hillslopes in layered landscapes should incorporate the possibility of these feedbacks.

3.3 ANALYTIC SOLUTION FOR RAMP LENGTH

Armed with simple geometric constraints and a qualitative understanding of the self-organized behavior of the system, we develop an analytic solution for ramp length that quantitatively explains the behavior of the model. We show that with assumptions of block transport and weathering rules, estimates of relative incision and weathering rates can be obtained from field measurements of ramp length, topographic slope, and block size. With this analysis we aim to demonstrate the potential utility of using hillslopes developed in layered rock for estimating weathering and erosion rates.

3.3.1 General case

First, we derive a generic expression for the steady state length of a ramp. This expression is applicable for features of any resistant layer dip angle. In all cases, ramp length should be equal to the sum of two relevant distances: the distance traveled by a block before it weathers away, and the lateral distance the cliff retreats (in the opposite direction) during that time (Figure 3.6):

\[ L = \bar{x}_i + \frac{D_0}{\bar{w}_b} (\bar{v}_b + \bar{e}) \]  \hspace{1cm} (16)
where $L$ is the horizontal length of the ramp, $\bar{x}_t$ is the mean initial distance traveled by a block when released, $D_0$ is the initial block diameter, $\bar{w}_b$ is the mean weathering rate of the block, $\bar{v}_b$ is the mean horizontal velocity of the block after it initially lands, and $\bar{c}$ is the mean horizontal retreat rate of the resistant layer (Figure 3.2B). The quantity $(D_0/\bar{w}_b)$ is equivalent to the timescale for complete weathering of one block after it is released from the resistant layer. While this equation is cast in terms of our 1-D numerical model, it should be generically applicable. For example, the initial distance term $x_t$ can be modified to account for rockfall processes; the block velocity $v_b$ can be modified for any rule of block motion downslope; $w_b$ can be modified for any rule of block weathering. This most general expression for ramp length is also valid whether the feature is in steady state or not. If blocks do not move (horizontally) once they land ($v_b = 0$), then Eqn. 16 shows that ramp length scales as the ratio between cliff retreat rate and block weathering rate, similar to the talus weathering ratio first posited by Schumm and Chorley [1966]. However, the presence of blocks very far from the initial source and their decrease in size downslope indicates that at least for some features, blocks experience downslope transport after the initial deposition [Duszynski and Migon, 2017; Glade et al., 2017; Duszynski and Migon, 2015]. We therefore allow for block transport downslope in the following analysis.

Figure 3.6: Schematic of ramp length. Total length is composed of total block transport distance and total cliff retreat during the lifetime of 1 block ($T_2-T_1$).
3.3.2 STEADY-STATE CASE

Using a similar approach to Eqn. 11, we can relate the horizontal retreat rate of the resistant layer to the vertical erosion rate at the base of the slope:

\[ c = \frac{\varepsilon_1}{\tan \theta} \]  \hspace{1cm} (17)

where \( \theta \) is the dip angle of the resistant layer (Figure 3.2). Substituting Eqn. 17 into Eqn. 16 relates the block-release term in the equation for \( L \) to erosion rate at the base of the ramp, a quantity that is useful to know but notoriously difficult to measure:

\[ L = \bar{x}_t + \frac{D_0}{w_b} \left( \bar{v}_b + \frac{\varepsilon_1}{\tan \theta} \right) \]  \hspace{1cm} (18)

We require knowledge of the post-deposition velocity of blocks, \( v_b \), in order to close the ramp length equation. For this we must have independent knowledge about the way in which blocks move. In our numerical model, block velocity is related to block diameter and erosion rate, and is essentially constant at steady state. We also know that block weathering rate, initial distance, and erosion rate at the base of the ramp are constant. For simplicity, we develop a solution for this case. Note that Eqn. 18 does not depend on these particular assumptions; one could replace our assumed rules with alternative formulations.

Because blocks move at a roughly constant velocity, we can use the velocity of a block at the very top of the ramp to estimate the block velocity along the entire ramp. At the top of the ramp, blocks fall with an initial diameter \( D_0 \). This initial size should be readily measurable in the field, and possibly representative of the joint spacing of the resistant layer. Using Eqn. 4, the velocity of a block at the top of the ramp is:
\[ v_b = \frac{(x-x_i)}{t} = \frac{dx}{D_0/\varepsilon_2} = \frac{dx \varepsilon_2}{D_0} \]  

(19)

where \( \varepsilon_2 \) is the vertical erosion rate just downhill of the block and \( dx \) is a characteristic block transport distance. From the geometric analysis outlined in section 2.2, we know that this vertical erosion rate at the top of the ramp is a function of base level lowering rate, resistant layer dip, and local topographic slope (Eqn. 12). For convenience, we can also simply cast this erosion rate at the top of the ramp as a multiple of the vertical erosion rate at the base of the cliff, \( \varepsilon_1 \):

\[ \varepsilon_2 = \gamma \varepsilon_1 \]  

(20)

where \( \gamma = \left(1 + \frac{\tan \alpha}{\tan \theta}\right) \).

The multiplicative factor \( \gamma \) represents the erosion rate needed at the top of the ramp relative to that at the bottom to achieve steady state. \( \gamma \) can be viewed as a proxy for the difficulty of eroding the resistant layer.

Plugging our expression for \( \varepsilon_2 \) into Eqn. 19 and simplifying, the expression for the length of the ramp at steady state becomes:

\[ L = \bar{x}_i + \left(\frac{\varepsilon_1}{w_b}\right) (\gamma dx + \frac{D_0}{\tan \theta}) \]  

(21)

Eqn. 21 may be re-arranged to isolate field-measurable quantities on one side, solving for the ratio of two fundamental speeds in the problem: erosion rate at the slope base vs. weathering rate of the blocks, \( 1/w_b \). Substituting back in the expression for \( \gamma \):
To test whether Eqn. 22 explains the behavior of the numerical model, we ran the model with different values of $\varepsilon_1/w_b$. We then used measurements of ramp length and $\alpha$ from the model to solve Eqn. 22 and back-calculate the expected $\varepsilon_1/w_b$. Figure 3.7 shows our calculated results compared to the known values input into the model. The proximity of the data points to the black 1:1 line indicate that our analytical solution captures the behavior of the numerical model.

Eqn. 21 indicates that ramp length is directly related to the ratio between ramp-base erosion rate and the weathering rate of the blocks, $\varepsilon_1/w_b$. Because ramp length is controlled by the presence of resistant blocks, $\varepsilon_1/w_b$ also represents the velocity of the blocks relative to their weathering rate. A higher value of $\varepsilon_1/w_b$ indicates that blocks can move farther before weathering away completely, therefore developing a longer ramp. A smaller value of $\varepsilon_1/w_b$ indicates that blocks weather more quickly, and the ramp will be shorter. This relationship between ramp length and $\varepsilon_1/w_b$ is scaled by specific block release and transport characteristics, such as initial block size, characteristic transport distance, and efficiency of block release (subsumed into $\gamma$). We note that the Eqns. 16, 21 and 22 are largely independent of both soil flux and soil production rules. Knowledge of these rules is not needed to estimate $\varepsilon_1/w_b$. 

$$\frac{\varepsilon_1}{w_b} = \frac{(L-x_i)\tan \theta}{dx(\tan \theta + \tan \alpha) + D_0} \quad (22)$$
3.3.3 DISCUSSION OF FIELD IMPLICATIONS

Eqn. 21 predicts that the length of the ramp should increase with the ratio of ramp-base erosion rate to block weathering rate in a way that is predictable with knowledge of the geometry and a few measureable parameters. First, one must know something about the initial distance of block travel, \( x_i \). For features in which blocks are commonly deposited almost immediately next to the source, this term is essentially zero. However, where \( x_i \) is substantial, a mean distance could be used, provided the distribution of \( x_i \) can be constrained. The initial block size \( D_0 \) can be measured from joint spacing in the resistant layer. In rock with non-uniform joint spacing, a mean or distribution of initial sizes may be used [Sklar et al., 2017]. The characteristic distance a block moves, here taken to be \( dx \), is more difficult to measure; more work is needed to estimate \( dx \) based on the inferred type of block transport. The resistant layer dip, \( \theta \), and the topographic slope at the top of the ramp, \( \alpha \), are easily obtained in the field, as is the horizontal length of the
ramp. With these pieces, one may estimate the ratio between incision rate, $\varepsilon_1$, and the weathering rate of the blocks, $w_b$. If knowledge of the block-related parameters is not known, one may still compare multiple field sites with similar properties to estimate differences in erosion rate or block weathering rate, if it can be assumed that blocks behave in similar ways between the sites. This is a simple, low-cost way to obtain rough estimates of incision and weathering rates in the field and to compare rates across field sites.

At steady state, the topographic slope at the top of the ramp, $\alpha$, contains information about several parameters. First, we can interpret $\alpha$ as the slope required to allow erosion of the resistant layer to keep up with base level change. As the processes of block release are not well known, this slope may illuminate release mechanisms. $\gamma$, the ratio between $\alpha$ and the dip angle $\theta$, could be useful in comparing between field sites. If, for example, two field sites have equal resistant layer dip but different topographic slopes at the top of the slope (and therefore different $\gamma$ values), a higher $\gamma$ value at one site may indicate greater lithologic resistance, coarser joint spacing, or the presence of blocks on the ramp that are more difficult to move. Second, because the topographic slope at the top of the ramp is related to the vertical erosion rate through Eqn. 6, $\alpha$ may reveal information about soil production rates without the use of cosmogenic radionuclides. Measurements of slope and soil depth could aid in determining weathering rules at a given site. Third, soil transport efficiency is also subsumed into the slope at the top of the ramp; one can assume that in climates capable of driving more efficient hillslope transport, a lower slope would be needed at the top of the ramp. Further work is needed to untangle these details.

We have shown that the analysis of ramp length in this section correctly predicts the behavior of our numerical model and may be useful in determining erosion and weathering rates, or at least their ratio, in the field. However, we make several key assumptions. Informed by the model, we assume constant block velocity, a linear relationship between block motion and
vertical erosion rate, and a constant weathering rate of the blocks. In addition, our analytical results work best for cases in which blocks are surrounded by some amount of soil (i.e., blocks are not clustered together in talus). These assumptions will likely not hold for every field site. Our analysis illuminates some of the most important unknowns in the problem of blocky hillslope evolution in layered landscapes: block release, transport, and weathering mechanisms. We hope this analysis provides incentive for careful study of these processes.

4. VERTICAL LANDFORMS

4.1 GEOMETRIC ANALYSIS

In section 3.1, we outlined a geometric analysis of conditions required for steady evolution of tilted features. We noted that Eqns. 11 and 12 break down for dip angles of 90°. Geometrically, vertical features (Figure 3.8A) should in fact be much simpler than tilted or horizontal ones, because erosion of the resistant layer has no lateral component. Therefore, it is possible that the feature may reach a steady state in which vertical erosion rates are spatially and temporally constant across the entire feature (Figure 3.8B). In this scenario, steady state is defined such that time-averaged vertical erosion rates are constant, to allow for stochastic lowering of the vertical layer that may cause short-term unsteadiness.
Figure 3.8: A) Photo of Shiprock, New Mexico. Foreground is southernmost near-vertical dike exhibiting mantle of basalt blocks on concave-up shale ramp. Background is central intrusion. B) Schematic of steady-state evolution of an exposed dike. At steady state, vertical erosion rates are spatially and temporally constant.
4.2 AN EXPOSED VERTICAL DIKE AT SHIPROCK, NEW MEXICO

Figure 3.8A shows an example of an exposed vertical dike at Shiprock, New Mexico. Six basalt dikes, which radiate from a central intrusion known as Shiprock, were originally intruded into shale approximately 1000 meters below the surface [Ehrenberg, 1978]. Since then, the surrounding shale has eroded to expose the resistant dikes, which now form several-kilometer long ridges. Steep, concave-up shale ramps adjoining both sides of the vertical dike are mantled with basalt from the dike in the form of both large blocks and centimeter-scale chips. Vegetation on the ramps is sparse, consisting of low-lying shrubs and bunch grasses. While these ramps are similar to those developed on hogbacks, there are several differences between vertical and tilted features that warrant attention before drawing direct comparisons. First, while erosion of underlying soft rock is often tied to resistant block release through undermining on tilted features, this mechanism of block release is not likely on vertical features. Figure 3.8A illustrates the height of the exposed dike, which towers in places up to 20 meters over the top of the adjoining shale ramp. The large notch in the wall in Figure 3.8A also illustrates the spatial heterogeneity of lowering of the dike. Some sections appear to be lowering uniformly, where the top of the dike is horizontal; other sections clearly experienced a large block release event, indicated by large notches in the wall. This leads to the second major difference; while rockfall is undoubtedly important in many layered landscapes, the dip of tilted features allows blocks to land and to stop moving close to the source [Glade et al., 2017; Duszynski et al., 2015]. However, rockfall may be more prevalent in vertical features, in which a block released from the top of the resistant layer may fall a substantial distance before landing, and may travel a significant distance down the ramp before stopping. In the following sections we present findings from our field site at Shiprock and discuss possible interpretations of the data, in light of our numerical modeling results.
4.3 FIELDWORK

Fieldwork was conducted at Shiprock in the summer of 2015. Qualitative observations at Shiprock, New Mexico suggest that several hillslope processes shape the landscape. First, as mentioned above, rockfall occurs in some sections of the dike; however, the initial transport distances and possible shattering of falling blocks upon landing are unclear. Second, blocks also appear to be released from the dike block-by-block, especially where there is a free edge (for example, along the small rectangular section of the dike extending above the rest in Figure 3.8A). Third, blocks also appear to slowly move downslope, with small gaps behind large blocks observed after a thunderstorm, suggesting they slid a short distance (~mm) downslope. Block dams (Figure 3.5) are observed at Shiprock, especially near the top of the ramp where blocks are largest; soil buildup behind the blocks leads to enhanced lowering immediately downhill of the blocks. Fourth, blocks weather in situ after they land on the ramp, producing both small, flat, cm-scale “chips” of basalt, but also splitting into larger blocks. These chips must move downslope as well, as they are found far from any parent blocks near the base of the ramp (Figure 3.9B). Finally, the presence of both blocks and chips appears to influence soil transport. Topographic slopes are concave-up to the base of the ramp, where the basalt cover ends. Developed rill networks are not common on the ramp, indicating that the concave-up slope is not attributable to slopewash or rilling processes [Dunne and Aubry, 1986], but is instead likely due to the decrease in basalt grain size and cover down ramp (Figure 3.9A and 9B); this echoes our findings from our numerical model of tilted landscapes. Where blocks are large, we hypothesize that they decrease soil transport by means of block dams. However, centimeter-scale chips likely do not cause soil dams; they must armor the ramp from erosion through another mechanism, perhaps by inhibiting rainsplash or other relevant processes. A positive relationship between topographic
slopes and grain size has been observed in other landscapes in arid settings with both large blocks and smaller resistant grains [Poesen et al., 1998; Abrahams et al., 1985].

4.3.1 FIELD DATA

We collected grain size measurements and topographic profiles at two transects for the southernmost dike at Shiprock (Figure 3.8A). Each transect begins near the top of the ramp and ends at the base, where topographic slope transitions to be nearly flat, coincident with loss of basalt coverage. We chose transects that displayed no evidence of rilling, landsliding, or catastrophic rockfall. We took photographs of the ground with a scale at each point along each transect and analyzed them using ImageJ [Schneider et al., 2012] to obtain grain size distributions. We also used a handheld laser rangefinder to obtain topographic profiles. In Figures 9C and 9E, we present the visible long axis grain size against distance from the dike for transects 1 and 2, respectively. Both the mean and D$_{84}$ grain size decrease down-ramp for both transects; the D$_{84}$ grain size trend is well fit by an exponential. The D$_{25}$ remains essentially constant, with a slight decrease down-ramp. Figures 9D and 9F show the topographic profiles (left y axes) and slope trends (right y axes) for each transect. Profiles are concave-up, as indicated by decreasing slope with distance from the dike.
4.3.1.1 INTERPRETATION

By plotting a range of percentiles in grain-size data, we can see how trends in both large and small grains evolve with distance from the dike. The D₈₄ represents large basalt blocks derived from the dike; we are interested in this large size class because it may contain information about the evolution of the feature (as shown in our analytic explorations of ramp length and grain size distributions in previous sections). Because this large grain size is the size that matters for our previous analytic solutions, we fit a trend to the D₈₄ data. A decaying exponential trendline fits both profiles. This matches the analytic solution for grain size distribution derived in section 2.1. Recall that the exponent in the analytical solution (Eqn. 10) is scaled by the ratio of block weathering rate and vertical erosion rate, along with a characteristic
transport distance of each block \((w_b/e^2 dx)\). Assuming that the observed grain size distributions in Figure 3.9 are well described by the analytical solution, and that Shiprock is approximately in steady state, then the best fit exponent of \(-0.0125\) may contain useful information about these parameters. While it is difficult to constrain erosion rates or weathering rates at our field site independently in order to fully test our analytic solution, a simple calculation shows that the best fit exponent may provide a reasonable estimate of \((w_b/e^2 dx)\). Base level at Shiprock is the San Juan River; estimates of incision rates on the San Juan suggest an order of magnitude of about \(10^{-4}\) m/yr [Wolkowinsky and Granger, 2004; Steelquist et al., 2017]. While it is more difficult to constrain weathering rates of basalt blocks, studies of rind weathering on basalt clasts estimate advance rates between \(10^{-9}\) and \(10^{-7}\) m/yr (Sak et al., 2004; Navarre-Stitchler and Brantley, 2007). Assuming that rind weathering products rapidly fall away from a block, these rates may be a fair estimate for rates of size reduction of basalt clasts. Assuming clasts may have a characteristic transport distance \(dx\) of anywhere from 0.01 to 1 meter, and dividing block weathering rate estimates by incision rate estimates, we obtain a range of values of \((w_b/e^2 dx)\) between \(10^{-5}\) and \(10^{-1}\) m\(^{-1}\). The best fit exponent of 0.0125 falls within this range, suggesting our approach may be useful in estimating the ratio between block weathering and incision rates in the field. We acknowledge that the observed exponential trend is non-unique; many combinations of weathering and transport rules can lead to the observed exponential decrease in grain size downslope. Weathering, incision rates and transport distances must be better constrained at a given field site to fully test our analytical solution; however, the above analysis demonstrates the potential utility of the method.

In contrast to the \(D_{84}\), which decreases away from the dike, the \(D_{25}\) remains essentially constant (with only a slight linear decrease). We interpret this to mean that the \(D_{25}\) fraction represents the weathering products of the large grains; this interpretation agrees with qualitative
field evidence, which indicates that large grains both break into blocks and cleave off small basalt chips. At the top of the ramp, grain size ranges widely as there are both large boulders and small weathering products derived from them. Further away from the dike, this range narrows as boulders weather and produce more small chips. At the base of the ramp, chips are the only basalt cover present.

Each transect has one outlier in the $D_{84}$. In transect 2, the outlier labeled in Figure 3.9E indicates rockfall. A large boulder at this data point clearly deviates from the general exponential decay of boulder size. This may be useful, because it can help to differentiate boulders that slowly weather and move downslope over time from those that arrive far downslope as intact large boulders after rockfall. The outlier in transect 1 is likely due to poor image quality near the top of the ramp, where abundant vegetation obscured any basalt.

McGrath et al. [2013] report a similar exponential decrease in grain size with distance from a source in an arid landscape with a horizontal resistant layer. Their study showed that the decrease in size could be due to weathering alone. If the position of the edge of the resistant layer is retreating laterally with time, as in the case for a tilted or horizontal layer, then blocks may weather in place and appear to be farther from the source as the source retreats through time, without moving in an absolute sense. However, exposed dikes are unique in that the resistant source remains at a constant position (x-coordinate) through time. Therefore, the decrease in block size observed in our data likely occurs due to a combination of block weathering (both spalling of small chips and splitting into large chunks) and transport. Whether transport is accomplished initially through rockfall, or slowly as blocks move over time, is not easily discernible. However, given the qualitative evidence at the site, we interpret the data to suggest that while some blocks have been deposited at initially large distances downslope, most move slowly over time in a way similar to that proposed for tilted features. A study of boulder
weathering in Antarctica [Putkonen et al., 2014] reports results consistent with our interpretation; large boulders produced both small and large fragments. The distance of these fragments from the parent boulder scaled directly with the topographic slope and inversely with fragment size.

5. DISCUSSION

Here we briefly discuss the evolution of another end member, horizontal landforms. We then outline future work, specifically outlining testable hypotheses resulting from this study.

5.1 HORIZONTAL LANDFORMS

Figures 10A and 10B show an example of a ramp beneath a scarp in horizontally layered rock. While resistant layers on vertical features such as dikes experience mostly vertical erosion, and therefore can theoretically achieve true topographic steady state, horizontal features experience mostly lateral erosion, and therefore have no way to maintain constant relief with respect to a lowering base level. However, in the reference frame of the scarp we argue that a number of conditions may allow the feature to achieve a steady form. First, if the local channel experiences only lateral migration and no vertical incision, the feature can reach a steady state in which ramp length, curvature and relief are constant, as is the case for tilted and vertical landforms (Figure 3.10C). Analysis of this case could be done incorporating the effects of a laterally translating boundary condition [e.g., Perron et al., 2012]. Second, if the channel is stationary and incising, then as long as the lateral retreat rate of the feature can keep up with the steepening of the hillslope such that the slope lengthens and steepens an equal amount, the feature can maintain constant curvature; however, ramp length will not remain constant but will increase over time as relief increases over time. This case requires that the lateral retreat rate of the resistant layer equal the vertical incision rate of the channel (Figure 3.10D). However, the
length of the ramp will likely cease to increase when the length of the ramp exceeds the distance a block can possibly travel before weathering away.

Though horizontal scarps differ from hogbacks in their inability to vertically lower, the feedbacks between blocks and soil outlined in Section 3 should hold as the feature adjusts to erode as quickly as possible in order to keep up with base level incision. If the ramp is concave-up, then vertical erosion rates must be higher at the top of the ramp than at the bottom. With depth-dependent soil production and transport, this should result in thinner soil at the top of the ramp than the bottom. If the ramp is linear, then vertical erosion rates can be relatively uniform (except where the resistant layer has retreated to lengthen the ramp). The rapid erosion rate required for features of shallow dip may be accomplished by an enhanced probability of landsliding or rilling (see Ward et al. [2011] for more exploration of the role of rilling processes in scarp evolution). More work is needed to determine whether the dominance of these processes is correlated with resistant layer dip angle, or largely controlled by vegetation and climate (e.g., contrast Figure 3.10A with 10B).
Figure 3.10: A) Photo of horizontal scarp with coarse debris and pronounced rilling at North Caineville Plateau, Utah (Photo: Robert S. Anderson). B) Google Earth image of horizontal scarp with vegetation and few rills near Castle Rock, Colorado. C) Steady behavior of horizontal scarp with lateral channel migration maintains constant curvature, ramp length, and relief. D) Steady behavior of horizontal scarp with channel incision maintains constant curvature, but non-constant ramp length and relief. Tan(θ) is the average slope of the hillslope profile.

5.2 FUTURE STUDIES

Our study highlights some of the most pressing unknowns in the problem of hillslope evolution in layered landscapes: the processes involved in block release, transport, and weathering. The analytic approach detailed in this paper results in a number of testable hypotheses. We outline the major testable predictions below:

1) Average topographic slope on ramps developed in layered rock is directly related to local block size.
2) If a tilted landform is in steady state with a concave-up ramp profile, and soil production and flux depend on soil depth, then soil should be thinner at the top of the ramp than at the bottom.

3) Ramp lengths of landforms developed in similar lithology, climate and vegetation conditions (i.e., where block weathering rates are similar) may be compared to estimate relative base level incision rates between the two landforms (features with longer ramps have experienced faster incision rates).

4) Ramp lengths of landforms developed in similar climate and vegetation conditions with similar incision rates, but where lithology differs, may be used to estimate relative block weathering rates between the two landforms.

5) Spatial grain size distributions on vertical landforms in steady state contain information about relative base level incision and block weathering rates, \( \frac{w_b}{\xi \, dx} \).

6) Dip-parallel erosion rates should increase with decreasing resistant layer dip angle.

Future work should constrain physical processes of block release and transport. Lithology, joint spacing, groundwater sapping [see review in Howard et al., 1994], and vegetation likely play a role in erosion of the resistant layer. Careful field studies and physical experiments of block release could shed light on these processes. More work is also needed to understand block transport mechanisms in the field and their dependence on slope, erosion rate, and specific factors such as overland flow, soil saturation, and vegetation. While we have shown that blocks affect soil transport by damming, additional feedbacks between the presence of large blocks and soil creep mechanisms should be explored, as well as the effect of different soil flux rules [e.g., Ferdowsi et al., 2018; Roering et al., 2008]. Exploration of the relative roles of transport and weathering in producing observed block size distributions through numerical
modeling, field studies, and physical experiments would greatly improve our understanding of these landscapes. Ultimately, this should lead to development of a quantitative framework analogous to models of coupled clast transport, abrasion, and fragmentation in fluvial systems [e.g., Le Bouteiller et al., 2011; Attal and Lave, 2006]. Linking geometric analysis of dip-parallel erosion rates required for steady hillslope evolution in layered landscapes with similar analysis done for fluvial systems (e.g., Perne et al. [2017]) would be beneficial. In addition, a better understanding of the size of blocks once they reach channels would greatly improve landscape evolution models, since the presence of blocks may influence channel evolution and therefore local base level [Sklar et al., 2017; Shobe et al., 2016].

5. CONCLUSIONS

Our work has shown that an understanding of block release, weathering and transport is essential for a full understanding of hillslope evolution in layered rock. Analysis of our numerical model illuminates the potential self-organized behavior of these landscapes and demonstrates the importance of depth-dependent weathering and soil flux in the ability of layered landscapes to reach steady state. Our analytic results could allow valuable estimation of weathering and erosion rates in the field, as well as a much deeper understanding and prediction of the large-scale evolution of landscapes developed in sedimentary rock. While our assumptions are not universally applicable, the approach could be refined to accommodate other formulations for block transport, soil transport, and weathering processes.
CHAPTER IV

CANYON SHAPE AND EROSION DYNAMICS GOVERNED BY CHANNEL-HILLSLOPE FEEDBACKS

PUBLICATION NOTES

This chapter has been revised and is currently resubmitted to Geology as: Glade, R.C.*, Shobe, C.M.*, Anderson, R.S., and Tucker, G.E. (in review) Canyon shape and erosion dynamics governed by channel-hillslope feedbacks. *Equal author contributions.

ABSTRACT

The geologic history of major river canyons is strongly debated, as is the extent to which river canyons record climatic and tectonic signals. Fluvial and hillslope processes work in concert to control canyon evolution; rivers both set the boundary conditions for adjoining hillslopes and respond to delivery of hillslope-derived sediment. But what happens when canyon walls deliver boulders that are too large for a river to carry? River canyons commonly host large blocks of rock derived from resistant hillslope strata. Such blocks have recently been shown to control the shapes of hillslopes and channels and the pace of erosion by inhibiting sediment transport and bedrock erosion. Here we develop the first process-based model for canyon evolution that incorporates the roles of blocks in both hillslope and channel processes. Our model reveals that two-way negative channel-hillslope feedbacks driven by the delivery of large blocks to the river result in characteristic planview and cross-sectional river canyon forms. Internal negative feedbacks strongly reduce the rate at which erosional signals pass through landscapes, leading to persistent local unsteadiness even under steady tectonic and climatic forcing. Surprisingly, while the presence of blocks in the channel initially slows incision rates, subsequent channel oversteepening due to block armoring substantially increases incision rates.
This interplay between channel and hillslope dynamics results in highly variable long-term erosion rates. These autogenic channel-hillslope dynamics can mask external signals, such as changes in rock uplift rate, complicating the interpretation of landscape morphology and erosion histories.

1. INTRODUCTION

River canyons, steep-walled valleys often developed in bedrock, evolve through a combination of deepening by river incision and widening by hillslope processes. Considerable effort has been expended on establishing the timing and mechanisms of canyon evolution (e.g., Cook et al., 2009; Schildgen et al., 2009; Flowers and Farley, 2012), with a focus on understanding landscape response to climatic and tectonic forcing. The traditional view of canyon erosion is that river incision, driven by tectonics, climatic perturbations, or changes in substrate erodibility, lowers the canyon bottom. Adjacent hillslopes then respond to river incision by rockfall, landsliding, and/or diffusive sediment transport (e.g., Mudd and Furbish, 2007; Gallen et al., 2011). Under the assumption that sediment delivered to the channel is mobile, patterns and timescales of river incision control hillslope form and dominate canyon evolution. The majority of prior work on canyon development has embraced this view and drawn conclusions about the timing of canyon evolution under the assumption that hillslopes respond passively to river incision. However, canyon-confined rivers do not operate in isolation from their adjacent hillslopes (Egholm et al., 2013; Attal et al., 2015; Bennet et al., 2016; DiBiase et al., 2018; Shobe et al., 2016; Shobe et al., 2018; Golly et al., 2017). Hillslopes make up the majority of canyon planview area and are often the primary source of sediment to the rivers. Steep canyon walls with substantial bare bedrock exposure and sufficient fracture density commonly release large (several meter diameter) pieces of rock into the channel (DiBiase et al.,
2018; Glade et al., 2017; Glade and Anderson, 2018; Howard and Selby, 1994). Large grain delivery to rivers can inhibit incision over large space and time scales (Shobe et al., 2016; Shobe et al., 2018), even damming rivers for short periods of time (Korup et al., 2006; Ouimet et al., 2007; Castleton et al., 2016). We propose that slowing or cessation of river incision must then influence the hillslopes by reducing the rate of hillslope steepening. Block delivery is therefore a negative feedback on both river and hillslope erosion. To constrain canyon evolution rates and process dynamics, it is critical to understand the interactions between incising, canyon-confined rivers and their adjacent hillslopes.

We develop a numerical model that explicitly treats block dynamics on both hillslopes and in channels. We then examine the influence of these negative channel-hillslope feedbacks on river canyon evolution, with two guiding questions:

1) Are negative channel-hillslope feedbacks necessary and sufficient to explain the shapes of cross-sections and planforms of natural canyons?

2) How do these feedbacks affect long term erosion dynamics in river canyons responding to base level fall?

Block delivery is likely important in any block-producing landscape. However, as a simplified test case, we focus on river canyons developed in layered rock (Figures 15 and 16) in which a resistant caprock (e.g., sandstone) overlies softer rock (e.g., shale). In this simplified geologic setting the caprock acts as a line source of blocks, with block size dictated by caprock thickness and joint spacing. The softer, underlying layer produces soil but no blocks. Canyons developed in layered rock often exhibit key morphologic features: a characteristic bell-shaped
planform during transient response to baselevel fall (Figure 4.1A,B), block-mantled channels, and steep hillslopes (Figure 4.1C-F).

Figure 4.1: Blocky river canyons on Earth. A,B) Map view of canyons developed in layered rock. C-F) Photographs and cross sections of river canyons in layered rock with a range of canyon depths and widths. Large blocks are present in channels and on hillslopes. Insets show close-ups of blocks in channels. Photos and profiles from Google Earth.

2. CONCEPTUAL MODEL

Several processes (e.g., landsliding, debris flows) may influence channel-hillslope coupling. We focus on the delivery of large blocks from hillslopes to channels (Figure 4.2), which we hypothesize control river canyon form. During the early stages of canyon evolution, channel incision causes failure of the caprock (Ward et al., 2011), which delivers large blocks of
rock to the hillslopes and channel. The presence of blocks on the hillslopes inhibits soil erosion, stalling subsequent block release from the caprock (Glade et al., 2017; Glade and Anderson, 2018). Blocks in the channel, if they are too large to be transported, reduce the river incision rate by armoring the bed and increasing hydraulic roughness (Shobe et al., 2016; Shobe et al., 2018). Prolonged inhibition of river incision reduces the rate of hillslope steepening and hence the rate of block delivery to the channel. Thus as long as hillslopes supply blocks to the channel (Figure 4.2), erosion rates both in the channel and on the hillslopes are expected to be highly variable in time. Eventually, the hillslopes retreat far enough from the channel that blocks weather during hillslope transport to a size at which they no longer inhibit river incision (Figure 4.2). From then on, the channel lowers at a rate that is unaffected by blocks delivered from the hillslopes.
Figure 4.2: Conceptual model of canyon evolution in layered rock. A) When the caprock is close enough to the channel to deliver large, erosion-inhibiting grains, canyon evolution is governed by interactions between unsteady channel and hillslope evolution, even when the baselevel forcing is steady in time. Reduction of soil transport by blocks (inset) strongly influences hillslope form, yielding linear to concave-up hillslope profiles. B) Once the caprock has retreated far enough from the channel that blocks no longer inhibit incision upon arrival in the channel, the river incises at a steady rate (assuming a steady forcing) and the channel-adjacent portions of the hillslope become convex-upward.
3. NUMERICAL MODEL

We test the conceptual model shown in Figure 4.2 with a series of numerical experiments, coupling models for channel (Shobe et al., 2016) and hillslope (Glade et al., 2017) evolution that incorporate the effects of large blocks of rock (see Supplementary Material). The model domain, designed to represent the layered landscapes in Figures 15 and 16, consists of a horizontal resistant layer of rock overlying softer, more erodible rock (domain is 2 km wide x 1 km long with 5 m resolution). A channel of uniform initial slope, forced with a constant baselevel lowering rate at its downstream end, incises the weaker, underlying rock. The channel permanently occupies the center of the model domain and has a constant width and discharge. The rest of the model domain operates under hillslope process laws (Glade et al., 2017; Glade and Anderson, 2018). We use this model to investigate erosion dynamics and timescales in a river canyon in which blocks released from the resistant caprock cause interactions that govern both hillslope and channel evolution. In interpreting model behavior, we focus on the role of the two unique elements of our model: the geometry of horizontal layered rock, and the role of block feedbacks.

The model couples two existing one-dimensional numerical models: a hillslope evolution model (Glade et al., 2017; Glade and Anderson, 2018) and a fluvial incision model (Shobe et al., 2016; Shobe et al., 2018), both in the presence of blocks. The hillslope model uses continuum depth-dependent linear soil diffusion (Johnstone and Hilley, 2015) and the exponential soil production function (Ahnert, 1976; Heimsath, et al., 1997). Blocks, which are treated as discrete particles, experience a steady weathering rate, and are released and gradually transported down the steepest slope according to a local relief threshold. The fluvial incision model employs a modified shear-stress incision rule that accounts for the inhibition of erosion by blocks that both cover the channel bed and extract momentum from the flow. Block motion in the channel is
calculated using a force balance (Larsen and Lamb, 2016), and blocks are abraded in proportion to the shear stress exerted on them. The two models were coupled using the Landlab modeling toolkit (Hobley et al., 2017).

4. RESULTS AND DISCUSSION

Our model results in characteristic planform and cross-sectional features that agree with first-order field observations (Figure 4.1; Figure 4.3). At early times, the model reproduces the bell-shaped planform (Figure 4.1A,B; Figure 4.3) observed in the field. The planform shape results from two unique features of our model: 1) hillslope response times in horizontally layered rock, and 2) channel-hillslope block feedbacks. First, the curvature of the planform shape is dictated by both the hillslope and channel adjustment timescales (e.g., Mudd and Furbish, 2007). In layered rocks, the hillslope adjustment timescale is complicated by the geometry of the system in which the hill crest is pinned at the elevation of the resistant caprock; this leads to a hillslope that is always increasing in length and relief through time, and the scarp retreat rate therefore decreases through time. This geometry results in the gentle planform curvature observed in the control case (Figure 4.3C). In the blocky case, in addition to geometric constraints, the hillslope and channel adjustment timescales are heavily modified by the presence of blocks (Figure 4.4). This results in the sharp kink in the planform (Figure 4.3), which corresponds to the location in the channel that has roughly reached steady state (Supplementary Information). Our results indicate that the planform bell shape is a unique result of canyons developed in horizontally bedded rock, and is further modified by block dynamics. Key differences in planform shape between control and blocky cases predicted in our model may be testable at well-constrained field sites.
Incrementally increasing block size to explore the model parameter space, we find that diagnostic features grow more pronounced as the erosion-inhibiting effects of blocks increase (Figure 4.3, see also Supplementary Information). Canyon cross-sections at early times (Figure 4.3A,B) show linear-to-concave-up canyon walls mantled with blocks derived from the overlying resistant layer. Channel longitudinal profiles exhibit knickpoints associated with channel steepening in response to block delivery. At later times, when the scarp has retreated far from the channel, blocks mantle only the upper portion of the hillslopes, leaving the lower portions to become convex-up because they are no longer influenced by the blocks (Figure 4.3C-D). Only at later times do channel profiles become linear, as expected for a steady channel reach (Figure 4.3D).
Figure 4.3: Model output of a river canyon incised into horizontal layered rock. A): Close-up view of the model at 100,000 years. Blocks are illustrated by cubes with size and color corresponding to their size, plotted as scaled markers which allows size to be shown independently of model grid resolution. Blocks in the chanjonel are often larger than those on the hillslope, illustrating that they were transported from farther upstream. B) Time series of model canyon evolution at 100ky, 200ky, and 400ky. Vertical plots on left show channel profiles. Horizontal plots on bottom show cross-section profiles 150 meters upstream. C) Control run with 1 meter thick caprock, no blocks. D) Planform comparison for 4 blocky and 4 control model runs, each with 0.1 meter, 0.5 meter, 0.8 meter, and 1 meter thick caprock. Models are shown at equivalent stage of fluvial response to baselevel fall, not at the same absolute time. E) Comparison of time to steady fluvial incision for runs with different block sizes.
Tracking the erosion rate through time at three points along the channel and hillslope (Figure 4.4) reveals that negative channel-hillslope feedbacks control rates of canyon evolution, and that these feedbacks persist for much of early canyon evolution. Near baselevel, the channel quickly equilibrates to the imposed baselevel lowering rate, with steady river erosion occurring until incision triggers block delivery from the hillslopes. Block delivery to the channel, which causes increased bed armoring and hydraulic drag, results in a 100-kyr period of very unsteady 1000-year averaged river erosion rates. This in turn causes unsteadiness in the hillslope boundary condition. 1000-year averaged channel erosion rates vary between zero and four times the imposed baselevel lowering rate, indicating that block dynamics can force erosion rate variations comparable to those caused by manifold changes in rock uplift rates. Only after 100 kyr of erosion rate oscillations driven by negative channel-hillslope feedbacks do the channel and hillslope experience relatively steady erosion.

Further up the channel, the response to baselevel lowering is delayed because block delivery from the hillslopes slows the upstream propagation of the baselevel signal. The channel erosion rate is minimal until the channel has time to steepen to the point where it can transport blocks delivered from the hillslopes. After the channel has steepened enough to mobilize the blocks and initiate incision, a >200 kyr period occurs in which both the channel and hillslope erosion rates oscillate about the imposed baselevel lowering rate. The most striking example of internal dynamics produced by channel-hillslope feedbacks in the model is shown in Figure 4.4A, in which a period of persistent disequilibrium in both hillslope and channel erosion rates occurs between 300 and 450 kyr, illustrating the ability of autogenic feedbacks to result in chaotic dynamics even under constant forcing (Jerolmack and Paola, 2010). This surprising prolonged increase in erosion rate is driven by prior channel steepening resulting from frequent block
Figure 4.4: Erosion rates at three points along the channel (red) and on the adjacent hillslope (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream, and upper plot is 600 m upstream. Pink and gray lines are 10-year averaged erosion rates; red and black lines are 1000-year averaged rates. All erosion rates are normalized by the imposed baselevel fall rate $\varepsilon_0$. Dashed black lines show $\varepsilon/\varepsilon_0 = 1$. Block delivery feedbacks delay upstream propagation of the baselevel lowering signal by approximately 100 kyr over the no-blocks response time. Once erosion commences in the channel and on the adjacent hillslope, two-way block delivery feedbacks cause 1000-year erosion rates to deviate significantly (~4x) from the imposed baselevel fall rate. Feedbacks lengthen the response time of the channel and hillslopes to the baselevel perturbation. Without block delivery feedbacks, this adjustment takes place over a few tens of kyr (see Supplementary Material). Full adjustment of the channel and hillslope to
baselevel fall occurs once the caprock, the source of erosion-inhibiting blocks, has retreated far enough that blocks are transportable when they reach the channel.

delivery from the hillslopes; the oversteepened channel erodes rapidly for 150 kyr, triggering increased hillslope erosion, until the channel slope and erosion rate return to equilibrium conditions. Whereas the erosion rates in the channel and channel-adjacent hillslopes become steady at this point, the erosion rates farther up the hillslope never become steady because the hillslope is lengthening and gaining relief through time (Supplementary Material). This is a unique feature of hillslope erosion in the presence of horizontal layered rock (Glade et al., 2018), but the long-lived unsteadiness caused by negative block delivery feedbacks is applicable to all landscapes with a source of blocks. The importance of blocks in stalling river erosion will vary with the scale and hydrology of drainage basins, but our analysis suggests that it may be important in many canyon landscapes.

Exploration of model parameter space reveals that greater block delivery increases both the amplitude of erosion rate perturbations and the timescale required for the canyon to experience near-steady erosion rates (see Supplementary Information). This finding has implications for the applicability of common stream power type approximations for understanding landscape response to perturbations. When substantial mass is delivered to the channel as blocks, basic inversion techniques are more likely to yield incorrect erosion rates during transient canyon evolution. However, recent work has shown that the stream power approximation may be modified to include the effects of hillslope-derived blocks by incorporating a variable erosion threshold to account for transient block delivery to the channel (Shobe et al., 2018). More work is needed to finalize a quantitative framework for extracting forcing signals from the form of block-controlled landscapes.
5. CONCLUSIONS

This work presents the first model of canyon evolution capable of matching observations of planview form, cross-sectional shape, and the presence of large grains. While we explored the example of a simple caprock canyon, our results apply to any landscape in which blocks are delivered from hillslopes to channels. We observe complex erosion rate dynamics due to negative, two-way feedbacks between incising channels and delivery of difficult-to-transport grains from the adjacent hillslopes. Simulations that did not include block delivery feedbacks did not match observed canyon morphology (Supplementary Material). Further, we emphasize that model results illustrate a minimum effect of large grains in canyon evolution; there are many additional elements of reality that would amplify the effects described here. For example, rockfall and landsliding would lead to more rapid delivery of large sediment over short periods of time. Landscapes where blocks are sourced along the entire hillslope (for example, in well-fractured granitic rock (Granger et al., 2001)) will also experience a greater influx of blocks to the channel than the line source explored in our model.

Our results imply that channel-driven models for canyon evolution may be overly simplistic, even when canyons evolve under a steady external forcing. Changes to channel incision rates caused by hillslope sediment delivery, in addition to changes in hillslope erosion rate due to unsteady channel incision, set both the shapes of canyons and the timescales of their evolution. Autogenic channel-hillslope feedbacks substantially modify baselevel signals for hundreds of thousands of years and increase the time required for canyon landscapes to equilibrate to an imposed baselevel forcing. Bedrock canyon evolution can only be understood as the product of a coupled channel-hillslope system in which large blocks of rock play a critical role.
CHAPTER V
DISCUSSION AND CONCLUSIONS

The studies above have used numerical modeling, mathematical analysis, and field data to demonstrate the importance of blocks in hillslope evolution. Below, I outline ongoing work that aims to understand planform patterns in layered landscapes. I then summarize the major findings in the three studies presented here.

1. ONGOING WORK

Layered landscapes exhibit characteristic planview patterns that have not yet been explained; for example, exposed synclines and anticlines are often flanked by triangular or scalloped patterns that outline sedimentary layers (Figure 5.1). These patterns likely reflect hillslope response to local base level signals controlled by dip-parallel streams that cut through layered landforms (Figure 5.2). I hypothesize that the curvature of these planform patterns is controlled by stream spacing, channel incision rates, dip angle, lithologic parameters, and hillslope efficiency. Using the 2-D numerical hillslope evolution model described in Chapter IV, I am exploring the influence of these controls on the development of planview patterns, paying special attention to the transition between scalloped (parabolic) and flatiron (triangular) shapes. A quantitative understanding of these patterns may allow estimation of key parameters from readily-available aerial imagery.

A close-up view of a flatiron in Jukan, Iran indicates that block release is the major erosion mechanism that produces these patterns (Figure 5.3). Therefore, this problem is well-suited to be studied with the numerical model presented in Chapter IV. Preliminary results
Figure 5.1. Scallops and flatiron patterns flanking a syncline in Jukan, Iran

Figure 5.2. Planview patterns at The Cockscomb, at Grand Staircase National Monument, Utah
Figure 5.3. Close-up view of a flatiron in Jukan, Iran indicates that block release processes control planview patterns.

show that the model is capable of capturing both scalloped (Figure 5.4) and triangular (Figure 5.5) patterns. The curvature of the scallop shape is dictated by the hillslope response timescale to incision at the boundaries. Previous work has shown that the hillslope adjustment timescale is directly related to the hillslope length squared, and inversely related to the hillslope diffusivity (Fernandes and Dietrich, 1997; Mudd and Furbish, 2007). In the numerical model shown in Figures 4 and 5, however, there is a complicating factor due to the presence of the scarp and the 2-D nature of soil motions along the scarp. I define the width of the planform shape as the total down-dip distance from the nose of the scarp to the base—that is, the location where the scarp is exposed at the domain boundary (Figure 5.4). The maximum with, \( W_{\text{max}} \), is simply a function of the hillslope relief and the resistant layer dip angle, \( \theta \):

\[
W_{\text{max}} = \frac{\text{relief}}{\tan(\theta)}
\]
where the relief is a function of the hillslope response timescale.

The curvature of the planform shape along the scarp front, $dW/dx$, is controlled by the block release rate through time. Because block release in the model is a function of the vertical erosion rate at the cell immediately downhill of the scarp, $E$ (see model description in Chapter V), the curvature can be written as:

$$\frac{dW}{dx} = \frac{d}{dx} \int_0^T E(x, t)$$

Where $T$ is the total time since the beginning of incision of the boundaries. The curvatures therefore reflects the difference in the integrated erosion rates through time along the scarp, which again will be controlled by the hillslope response timescale. Further work is needed to determine a precise formulation of the hillslope response timescale in this type of system.

The development of the triangular shape in Figure 5.5 is more enigmatic. While the landform starts out as a scallop-shaped scarp, through time it transitions to a triangular shape. This behavior is unexpected. Looking more closely at the soil depth in Figure 5.5, the triangular nose is composed of exposed bedrock is both the resistant layer and the underlying soft rock. It appears that similar features exist in Jukan, Iran, in which a triangular nose in the resistant rock extends into exposed bedrock in the underlying soft rock (Figure 5.7). These “bedrock fins” only appear in runs with high incision rates at the boundaries; therefore, the fins may be a result of high erosion rates relative to the efficiency of hillslope erosion processes. I am currently working to understand this surprising result.
Figure 5.4. Perspective views of scallop shape produced by the 2-D numerical model

Figure 5.5. Triangular shape developed in the blocky 2-D numerical model.
Figure 5.6. Planview of a flatiron produced by the control run of the 2-D numerical model shown through time. Colors indicate soil thickness.

Figure 5.7. Potential bedrock fins in Jukan, Iran.
2. CONCLUSIONS

Rocky hillslopes mantled with large blocks are common across Earth’s surface. My work has shown that these blocks profoundly alter landscape morphology, erosion rates, and persistence of topography. I have shown that the evolution of hogbacks, characterized by concave-up hillslope profiles, can be explained by accounting for weathering and transport feedbacks between blocks and soil. Additionally, blocks allow topographic features such as hogbacks and mesas to persist for longer periods of time than hillslopes without blocks. I have demonstrated that hillslopes in layered rocks of all orientations—horizontal, tilted, and vertical—can reach a quasi-steady state in which hillslope morphology and erosion rates remain constant in the reference frame of a retreating topographic feature. The analytic framework I developed for hillslope evolution in layered landscapes points to the future work needed to accurately predict landscape evolution in layered rock. Finally, I have shown that a numerical model that account for coupled hillslope-channel interactions in the presence of blocks illuminates the evolution of river canyons. Block dynamics result in autogenic feedbacks that result in unique canyon morphology, and can heavily modify baselevel forcing signals.

These studies highlight the importance of large blocks of rock in geomorphic systems, and represent the first attempts to explicitly incorporate sediment grain size into studies of hillslope evolution. They also clarify the physical processes for which we require a better understanding: namely, block weathering, block transport, and block release. I hope that future studies can lean on the ideas presented in this dissertation, testing predictions in the field and developing process-based formulations of block dynamics.


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APPENDIX A

SUPPLEMENTARY INFORMATION FOR CHAPTER I: BLOCK-CONTROLLED HILLSLOPE FORM AND PERSISTENCE OF TOPOGRAPHY IN ROCKY LANDSCAPES

CONTENTS

I) Detailed field methods and locations

II) Detailed numerical modeling methods

III) We present additional model runs and discussion that serve 1) to demonstrate that the results are insensitive to the details of the chosen soil production algorithm, 2) to amplify our argument that it is the blocks that matter in the development of these landforms, 3) to explore the influence of select parameters, including initial spatial block distributions and relative weathering rates.

IV) Animation of model (See video in GSA Data Repository)

I) FIELD METHODS

a) TOPOGRAPHIC PROFILES IN FIG. 1B

Topographic profiles were collected from Google Earth using the elevation profile tool. Profiles were chosen in locations that lack evidence of significant human influence or drainage networks and wereawn perpendicular to the strike of the resistant layer. Profiles were not preferentially selected on concave-up slopes, and start just beyond the slope break at the base of the slope. Hogbacks in the eastern United States and Europe were discounted due to extreme human influence.

<table>
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<tr>
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</tr>
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</table>

**Table A.1. Coordinates of topographic profiles derived from Google Earth**

b) **TOPOGRAPHIC PROFILE IN FIG 2A**

Topographic profile was obtained from 1-m resolution LiDAR from Boulder Creek Critical Zone Observatory August 2010 Snow-Off LiDAR Survey collected 8/21/2010-8/26/2010. Data is available online at OpenTopo.sdsc.edu/datasets. Profile coordinates are 40.148693 N, -105.295822 W. Figure A.1 shows additional profiles from Heil Valley Ranch, as well as slope values.
Fig. A.1 Elevation profiles and trends in slope derived from 1-meter resolution LiDAR data of hogbacks adjacent to the Front Range, Colorado. The magnitude of the slope declines with distance downhill from the top of the ramp.

c) FIELD WORK

i) BLOCK SIZE

We conducted field work at Heil Valley Ranch Open Space, north of Boulder, Colorado. The profile begins at 40.152270 N, -105.296935 W. Data was collected at 10 sites along a profile running perpendicular from the crest. Each site was 100 square meters, and spaced 30 meters Art. At each site we counted every block over 10 cm in diameter; we also chose at random 10 blocks and measured the surface-parallel long and short axis. The third axis was not measured due to partial burial of the blocks by soil or pine needles. Block size data are shown in Table A.2.
Table A.2. Block size data from Heil Valley Ranch, Boulder, Colorado. L (length) values represent the longer of two axes measured. W (width) values represent shorter axis measured. Values have units of meters. Site numbers begin at base of slope (Site 1) and increase in elevation.

<table>
<thead>
<tr>
<th>Site #</th>
<th>Site 1</th>
<th>Site 2</th>
<th>Site 3</th>
<th>Site 4</th>
<th>Site 5</th>
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</table>

ii) SCHMIDT HAMMER

For each of 10 blocks at each field site, we collected 15 Schmidt Hammer (Type N) measurements, totaling 150 Schmidt hammer measurements per site. This is well above the suggested number of Schmidt hammer readings for sandstone (Niedzielski et al., 2009). While blocks were derived from a constant resistant layer of Dakota Sandstone, a small number of blocks were composed of a conglomerate; these were removed from the Schmidt Hammer data, since the absence of a flat surface from which to collect measurements biases the Schmidt Hammer to give low readings. At sites where this occurred, calculation of standard error incorporated the smaller sample size; however, sample size always remained greater than the number suggested by Niedzielski et al., 2009. See Table A.3 for complete Schmidt hammer data.
Table A.3 Schmidt Hammer rebound values. Columns show mean (M) rebound value and standard deviation (S), for each block (each mean and standard deviation is composed of 15 individual measurements). Combined standard deviation and SEM for each site is reported in the bottom rows. Site numbers begin at base of slope (Site 1) and increase in elevation. Red values show blocks of conglomerate lithology, which were not included in final mean, st. dev or SEM calculations due to biased readings (see text above).
When using the Schmidt Hammer, it is possible that measurements may be biased due to size effects of the blocks; small block may give artificially low readings, especially when measurements are taken too close to cracks or edges (Sumner and Nel, 2002; Demirdag et al., 2009). To explore whether this effect biases our data, we plot Schmidt Hammer rebound value for each block against that block’s mean size (Fig. A.2) and length of the shortest axis (Fig. A.3). The lack of a positive trend in either plot assures us that block size does not skew the Schmidt Hammer results; that is, the decrease in Schmidt Hammer rebound values is not simply an artefact of decreasing block size.

**Fig. A.2.** Comparison of Schmidt Hammer rebound value with mean block size for each measured block (the mean of measured long and short axes).

**Fig. A.3.** Comparison of Schmidt Hammer rebound value with short axis length for each measured block.
The Schmidt Hammer data presented in Fig. 2 of the main text displays a pronounced decrease in rebound values beginning around 200 meters from the crest. At this point, both mean rebound values and mean block size decrease abruptly. This could be an indication of a shift from dominantly physical weathering processes to dominantly chemical weathering processes. Schmidt Hammer rebound values can be used as a proxy for chemical weathering in sandstone (Tugrul, 2004), but do not respond to physical or mechanical breakdown or rocks. Therefore, we posit that the relatively constant (with a slight decrease) in rebound values over the first 200 m represents the dominance of mechanical weathering or splitting of blocks over a smaller chemical weathering component, whereas the rapid decrease in rebound values beyond 200 m represents a shift to dominantly chemical weathering. A potential mechanism for this shift is a change in efficiency of thermally-induced cracking. Field experiments have shown that thermally induced cracking of rocks can be an efficient process of mechanical weathering of large blocks (Eppes et al., 2016). However, the efficiency of cracking due to thermal stresses has been shown to decrease with decreasing block size (AlA.ed et al., 2015). Further work is needed to test this hypothesis. At present we suggest that it is a plausible explanation for the trend in Schmidt Hammer data in Figure 2.

II) MODELING METHODS

The numerical model links algorithms for block release, mobile regolith (soil) production by weathering, and both soil and block transport. The governing equations are discretized and solved using an Euler forward finite difference scheme.

BLOCK RELEASE AND PLACEMENT

Joints are assumed to be oriented perpendicular to the dip of the resistant layer. Joint-bounded blocks are released from the resistant layer when the elevation $A_op$ between the edge of the hard layer and the shale slope is:

\[ z(r) - z(s) = \frac{D}{\sin \beta} \quad (3) \]

where $z(r)$ is the elevation of the base of the resistant layer at its edge, $z(s)$ is the elevation of the shale slope immediately beneath the resistant layer, $D$ is the thickness of a block (or the distance between joints) and $\beta$ is the dip of the resistant layer. Blocks are deposited on the shale ramp immediately adjacent to the resistant layer, one in each cell. The number of blocks to be placed
depends on the thickness of the jointed layer and the specified block size. See Fig. A.4 for an illustration of equation 3.

\[ w = w_0 e^{-H/H_w} + w_1 \frac{H}{H_w} e^{-H/H_w} \quad (4) \]

The first term represents a simple exponential decline with soil thickness, with a bare beA.ock rate, \( w_0 \), and a characteristic soil depth \( H_w \) at which the beA.ock conversion rate has declined to 1/e of that on bare beA.ock. The second term is a gamma function with a peak at \( H=H_w \) and a peak of \( w_1 \). We capture the pure exponential case by setting \( w_1=0 \).

We honor the difference between the two beA.ock types in the model by specifying a ratio of the bare beA.ock weathering rates, \( w_{0\text{shale}} \) and \( w_{0\text{sandstone}} \) (e.g., \( w_{0\text{shale}}/w_{0\text{sandstone}} = 100 \)). Blocks weather vertically according to Eq. (4), using \( w_{0\text{sandstone}} \). Therefore, blocks weather in the same manner as the shale, but more slowly. Any soil buildup on top of the blocks contributes to a lowering of block weathering rate according to Eq. (4). Soil thickness under each block is maintained until the block moves.

**Fig. A.4.** Definition sketch of parameters relevant to block release in Eq. 3.
SOIL TRANSPORT
Our algorithm for downslope discharge of soil, $q = L^3/LT$, honors the dependence on slope and on soil thickness. Here we cast the dependence as

$$q = -kS h_* \left( 1 - e^{-\frac{H}{h_*}} \right)$$

(5)

where $k$ is a soil transport efficiency or here a characteristic velocity with units of $L/T$, $S$ the local slope, $H$ the local soil thickness, and $h_*$ a characteristic soil thickness representing the depth scale of any particular transport process (Anderson, 2002; Johnstone and Hilley, 2015). The first factor captures a simple flux dependence on slope. The term in brackets assures that transport vanishes at zero soil thickness: as $H/h_*$ approaches zero, soil discharge vanishes. At the other end-member of thick soil, $H/h_*>1$, transport becomes insensitive to soil thickness and is governed instead simply by the local slope. The use of a linear flux law ensures that any non-convex hillslope form generated by the model can be attributed to the presence of blocks, and is not the result of a non-linear flux law.

BLOCK MOVEMENT
Blocks may move downslope only when sufficient relief exists. This will evolve as soil is transported away from the cell immediately downslope of a block. We define a threshold for block motion in which a block will move one cell downhill if relief is greater than or equal to twice the current height of a block. Therefore, as blocks weather and their size decreases, they move more easily. This choice of rule for block movement is simply a parameterization; more work is needed to understand the specific processes that allow blocks to move downslope, which may include toppling or sliding. However, setting a relief threshold for movement in the model essentially asserts that blocks are more likely to move when the local slope is steeper; this is likely to be a valid parameterization regardless of the specific transport method.

BOUNDARY CONDITIONS
We specify an elevation history on each lateral boundary of the calculation space, representing incision of local streams. We present results for constant river incision, which is required for the system to reach a steady state form. Here we choose channel incision rate to be $5 \times 10^{-5} \text{ m/yr}$. 


COMPARISON OF ANALYTICAL AND NUMERICAL MODELS

The slope is reported for every 10-m increment of the ramp in the numerical model, beginning at 15 meters from the peak where the slope is not affected by initially placed blocks. For comparison with the analytic model, we also report the analytical slope (eqn. 2) using the mean block size (D), spacing (X_s), and the soil production rate (w), for the same 10-m increments. We use mean weathering rate from the model, which accounts for the lower weathering rate of the blocks. As we employ the soil depth-dependent flux law (Eq. 5) in the numerical model, we also incorporate the depth-dependent term in the analytical solution for the comparison shown in Figure 4.

III) ADDITIONAL MODEL RUNS

COMPARISON OF MODELS WITH EXPONENTIAL VS. HUMPED SOIL PRODUCTION RULES

Fig. A.5. Comparison of model using exponential soil production rule employed in the main text (top) with model using humped soil production rule (bottom) (following Anderson and Humphrey,
1989, and Mudd and Furbish, 2007). Both runs employ a bare beA.ock weathering rate of $1 \times 10^{-5}$ m/yr for the resistant layer, and $1 \times 10^{-3}$ m/yr for the shale. For the humped soil production rule (see methods Eq. 4) we set $w_1 = 1 \times 10^{-4}$ m/yr. Differences between model runs are subtle, and do not substantially alter the conclusions: both rules result in concave ramps with significant relief.

**COMPARISON OF BLOCK MODEL WITH CONTROL RUN**

![Comparison of block model with control run](image)

**Fig. A.6.** Comparison of numerical model presented in the text (top) with control model in which blocks are not included (bottom). Models share the same boundary conditions, a constant lowering rate of $5 \times 10^{-5}$ m/yr. The control run employs a bare beA.ock soil production rate of $1 \times 10^{-5}$ m/yr for the resistant layer (red), and $1 \times 10^{-3}$ m/yr for the shale. An initial steep cliff develops due to the difference in soil production rates, but the step declines over time, resulting in a parabolic profile with an offset along the hard layer. Slopes in the control run are purely convex, as expected in steady state diffusive systems. The difference between the model runs illustrates the importance of blocks in creating concave slopes rather than convex, and in maintaining significant relief. Relief in the control run is approximately 60 m smaller than that in the model with blocks.

**INITIAL BLOCK PLACEMENT**
The numerical model presented in the main text assumes that blocks released from the resistant layer are deposited a short distance away from the cliff. We argue that this simplified view is a good approximation for hogbacks with a relatively thin resistant layer, where blocks are back-tilted and do not have much vertical distance to fall before landing on the slope below. However, hogbacks, mesas, or other scarps may have substantial relief between the resistant layer and the adjoining slope, allowing for further initial transport of the blocks. Here we compare the original model presented in the main text (Fig. A.7) with two modified model outputs that include further initial transport distance of blocks immediately after release. Figure A.8 shows a model run in which blocks are deposited immediately downslope of the cliff, but with 1 model cell in between each block. This does not substantially alter the first-order results. Figure A.9 shows a model run in which blocks have a probability of being initially placed far from the crest according to a truncated normal distribution. This allows most blocks to be deposited near the crest, while some blocks have the opportunity to travel much further. This also does not substantially alter the first-order effects; however, the trend in block size downslope is more scattered, due to stochasticity. Further work is needed to quantify the effects of different initial block placement distributions.

Fig. A.7. Original numerical model results, presented in main paper. Lower subset shows block size distribution.
Fig. A.8. Model output for initial block placement beginning immediately downslope of the crest with 1 model cell in between each block. All other parameters are identical to model output presented in main paper. Lower subset shows block size distribution.
Fig. A.9. Model output for initial block placement with a truncated normal distribution, using a standard deviation of 20 meters from the crest. All other parameters are identical to model output presented in main paper. Lower subset shows block size distribution.

RELATIVE WEATHERING RATES

Here we briefly explore the effect of changing the ratio between the maximum weathering rate of the blocks and that of the underlying shale. Results presented in the main paper show a hogback in which the weathering rate of the blocks is 100 times lower than that of the shale. Fig. A.10 shows that when blocks have a weathering rate that is 50 times lower than that of the shale, the resulting concave-up slope is shorter in both length and relief. Conversely, Fig. A.11 shows that when the weathering rate of the blocks is 1000 times lower than that of the shale, the resulting slope is much greater in both length and relief.
Fig. A.10. Model output for a relatively low contrast between weathering rates of blocks and underlying shale (weathering rate of blocks is 50 times lower than that of shale). Resulting block-covered slope is shorter in length and relief than results in main paper.

Fig. A.11. Model output for a relatively high contrast between weathering rates of blocks and underlying shale (weathering rate of blocks is 1000 times lower than that of shale). Resulting block-covered slope is greater in length and relief than results in main paper.


APPENDIX B

SUPPLEMENTARY INFORMATION FOR CHAPTER II: QUASI-STEADY EVOLUTION OF HILLSLOPE DEVELOPED IN LAYERED LANDSCAPES: AN ANALYTIC APPROACH

This supplementary material contains the data collected at Shiprock, New Mexico in the summer of 2015. Two topographic elevation profiles were taken along the east-facing ramp of the southern dike using a laser rangefinder. Photographs of the ground were also taken along these transects with a scale in locations without significant plant coverage. These photographs were analyzed using ImageJ to measure the visible long axis of every grain. We used this data to calculate mean, d25 and d84 grain size at each point in each transect.

Table B.1 contains two topographic elevation profile along with their respective mean, d25 and d84 grain size data. Table B.2 contains grain size measurements from these transects.

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<th>Distance from dike [m]</th>
<th>Relative elevation [m]</th>
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<th>Relative elevation [m]</th>
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Table B.1. Topographic elevation profiles and grain size data from two transects at Shiprock, New Mexico.
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<th>Mean grain size [cm]</th>
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Table B.2. Grain size measurement profiles from two transects at Shiprock, New Mexico. Visible long axis of every grain was measured using ImageJ.
APPENDIX C

First, we present supplementary methods along with coordinates for field sites and profiles presented in the main text. We provide results from a control run corresponding to the 1 meter block case presented in the main text, alongside corresponding blocky model plots (Figures C.1-C.11). Next, we provide supplementary discussion regarding the planform shape that is not essential to the main text (Figures C.12 – C.13). Finally, we provide results from 3 additional model runs with varying block size parameters and their 3 corresponding control runs (Figures C.14-C.28).

Methods
The existing 1-D hillslope (Glade et al., 2017) model was converted into two dimensions, allowing two-dimensional soil and block transport. One small modification was made to the existing 1-D fluvial incision model (Shobe et al., 2016). We altered the block spacing term, 2, to account for channel width: 2 = dx wn, where dx is the longitudinal node spacing, w is constant channel width, and n is the number of blocks in a node. The hillslope and fluvial use a different treatment of blocks with respect to grid cell size. In the hillslope model, block occupation of a cell is binary. If blocks occupy a cell, that entire cell experiences the weathering rate defined for blocks, and gains topography equal to block height. The implication of this is that blocks on the hillslope have a volume dx2Hb, where dx is node spacing and Hb is the height of one block. Thus block-occupied model cells contain a volume of rock with imposed initial volume corresponding to one release event. This can be understood as a collection of a number of blocks of equal volume moving en masse on the hillslope. In the fluvial incision model, blocks must be small relative to grid cell size to uphold the assumptions inherent in calculations of fluid drag on blocks. Once a group of blocks reaches the channel, the total block volume in that cell is converted into an imposed number of individual cubic blocks of equal volume. Each of those is treated independently with regards to calculations of fluid drag, block motion, and block degradation.

Parameters
Here we list key parameters in the model. For a complete list of model inputs, see model driver (Data Availability Section).

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<tr>
<th>Parameter Name</th>
<th>Value</th>
<th>Unit</th>
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<td>Channel width</td>
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<td>Discharge</td>
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<td>Block critical stress</td>
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<tr>
<td>Block motion relief threshold</td>
<td>2*block height</td>
<td>m</td>
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Table C.1. Model parameters used in main text

**Sensitivity Analysis**

The key findings in this paper—persistent unsteadiness in erosion rates, hillslope morphology, and landscape planform evolution—hold for a wide range of values of hillslope diffusivity, block weathering rate, block motion threshold, channel bed erodibility, channel block erodibility, and river discharge. We also found that results are not sensitive to model domain size, timestep, and initial channel slope. Extensive sensitivity analysis has been conducted on both individual component models in previous work (Shobe et al., 2018; Glade et al., 2018).

To examine the importance of blocks as a control on canyon evolution, we varied the thickness of the caprock layer, and therefore the thickness of the blocks delivered to the canyon. We conducted simulations using caprock thickness values of 0.1 m, 0.5 m, 0.8 m, and 1.0 m, the last of which is presented in the main text. Because the control, or no-blocks, cases involve the caprock failing before it immediately breaks apart into soil, changing the caprock thickness changes the mass flux of material to the canyon even in the control cases. We ran a control run to accompany each of the different blocky simulations. We present figures showing the planview
canyon evolution, channel and hillslope erosion rates, and channel bed cover time series in the supplementary information for all model runs.

The sensitivity analysis of caprock thickness reveals a consistent trend: the thicker the caprock (i.e., the larger the blocks), the more the evolution of canyon topography deviates from the no-blocks cases. The propagation of the baselevel forcing through the system slows when larger blocks are released from the caprock. Bigger blocks cause channel and hillslope erosion rates to deviate more strongly from the imposed baselevel lowering rate. We also find that the channel takes longer to equilibrate to the baselevel forcing (main text Figure 3) with larger blocks. When blocks are larger, the canyon planview becomes more kinked, as opposed to the linear scarp edges in the control cases.

Aside from block size, which we have explored, the other parameters most strongly influencing the effects of blocks are the weathering rate of the blocks on the hillslope, their erodibility coefficient in the channel, and the width and discharge of the river. The most complex effects are likely to arise in situations in which channel width is allowed to freely evolve in conjunction with bed cover and slope. Future work will be needed to elucidate the complete couplings between block delivery, erodibility, river discharge, and channel width.

Field Locations
Here we record coordinates for the beginning and end points of profiles presented in the main text.

Great Escarpment, South Africa planview photo:
6519553.4S 319436.9E

Mund’s Canyon, Arizona planview photo:
3864351.1N, 435999.5S

Canyonlands National Park, Utah
37.99547 N, 109.5733 W : 37.99442 N, 109.5687 W

Gorges du Tarn, France
44.23557 N, 3.21985 E : 44.23477 N, 3.242456 E

Great Escarpment, South Africa

Columbia River Basalt, Oregon
42.97428 N, 119.033 W : 42.97542 N, 119.0314 W
**Control run with 1 meter thick caprock corresponding to main text model run**

We conducted a control run of the BlockLab model to compare with results from the model run discussed in the main manuscript with 1 meter blocks. In the control run we use the same parameters for uplift rate, block release threshold, hillslope and channel parameters as the model run presented in the main manuscript. However, once blocks are “released” from a 1 meter thick resistant layer in the control run, they turn into soil on the hillslope instead of discrete blocks with a low weathering rate.

Control run: 1 meter thick caprock with no blocks

![Figure C.1.: Model output from control run, in which a river canyon incises into horizontally layered rock but no blocks are produced. This figure corresponds to the blocky model run presented in Figure 3 in the main text. Left: Model landscape at 100k years into the model run. Middle) Model landscape 200k years into the model run. Right) Model landscape 400k years into the model run. Vertically-oriented plots show topographic channel profiles. Horizontally-oriented plots show cross-sectional topographic profile 150 meters upstream. The slight planform curvature of the scarp edge is due to the horizontal geometry of the resistant layer. Near baselevel, the inflection point in the planform results from a decrease in block release rate from the scarp over time, which causes a decreasing rate of lengthening over time (Figure C.10). As the hillslope retreats, hillslope profiles must constantly lengthen and gain relief (Figure C.5), leading to a decrease in vertical erosion rate at the top of the hillslope over time (in the reference frame of the retreating scarp) (Figure C.8). Because blocks are released according to a relief threshold, this decrease in erosion rate leads to a decrease in block release rate, and therefore an inflection point in the planform. The planform inflection point further upstream is due to the propagation of the baselevel signal; because the channel is initiated with a non-zero slope, all points upstream of baselevel experience a finite amount of incision even before the baselevel signal has reached them. Therefore, by the time the signal reaches the channel nodes upstream, the relief between the channel and the hillslope is slightly greater over time. This means that the hillslope can release blocks more slightly more quickly at the upstream nodes, leading to an inflection in the planform shape.\[112\]
Figure C.2.: Channel and hillslope erosion rate histories from control run (shown in Figure C.1), shown at the channel node adjacent to baselevel (bottom), 300 m (middle), and 600 m (top) upstream of baselevel forcing. This figure corresponds to Figure 4 in the main text. Channel incision history shown in red, left axis labels; hillslope erosion history shown in black, right axis labels. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages. Control model reaches steady state ~ 1 order of magnitude faster than model run with blocks. Hillslope erosion rates are shown for the hillslope node closest to the channel. Erosion rates further up the hillslope never become steady, due to the geometry of a horizontally-oriented resistant layer (Figures C.6, C.7, C.8).
Figure C.3: River erosion rates and bed cover fractions for the 1m caprock control simulation (blocks released from the caprock immediately turn to soil) shown in Figure C.1. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
Figure C.4: River erosion rates and bed cover fractions for the 1m caprock simulation shown in main text Figures 3 and 4. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
Figure C.5: Hillslope profiles through time at three points along the channel in the control case. Time slices are shown from 0 to 250 k years. Portions of the hillslope close to the channel reach near-steady state, as described in the main text, while the portions of the hillslope far from the channel experience progressively declining vertical erosion rates through time as the hillslope lengthens.
Figure C.6: Hillslope profiles at varying distance upstream for the blocky model presented in the main text. At early times, profiles are linear-to-concave-up. At later times, profiles are linear-to-concave-up in the upper portions of the hillslope, and convex-up in the lower portions.
Supplementary Fig. 7: Hillslope gradient as a function of distance from the channel at three different distances upstream from baselevel in the control case. As a steady-state diffusive hillslope should display a pattern of slopes that increases linearly toward the channel, these patterns of hillslope gradients suggest persistent hillslope unsteadiness. Note: Slope, erosion rate and curvature plots are not shown for the blocky case because they are difficult to interpret due to the noise of the blocks. However, the general trends are similar to those observed in control run due to the geometry of a hillslope with a horizontally retreating resistant caprock.
Figure C.8: 1,000-year averaged hillslope vertical erosion rate as a function of distance from the channel at three different distances upstream from baselevel in the control case. Erosion rates at all times decline with distance from the channel because the upper boundary of the hillslope is pinned at constant elevation by the presence of the resistant overlying layer. Large spikes are caused by the periodic delivery of large quantities of soil from the resistant layer, in lieu of blocks for the control case. Note: Slope, erosion rate and curvature plots are not shown for the blocky case because they are difficult to interpret due to the noise of the blocks. However, the general trends are similar to those observed in control run due to the geometry of a hillslope with a horizontally retreating resistant caprock.
Figure C.9: Local hillslope curvature as a function of distance from the channel at three different distances upstream from baselevel in the control case. Curvature at all times declines with distance from the channel. Curvature at points on the hillslope close to the channel is many times higher close to baselevel, where the hillslope has steepened more substantially in response to river incision. Large spikes are caused by the periodic delivery of large quantities of soil from the resistant layer, in lieu of blocks for the control case. Note: Slope, erosion rate and curvature plots are not shown for the blocky case because they are difficult to interpret due to the noise of the blocks. However, the general trends are similar to those observed in control run due to the geometry of a hillslope with a horizontally retreating resistant caprock.
Figure C.10: Plots of hillslope length over time in the control run. Length is defined by distance from the channel to the base of the resistant scarp. The rollover in length illustrates a decrease in block release rate over time due to a decrease in vertical erosion rates at the top of the hillslope over time (Figure C.8).
Figure C.11: Plots of hillslope length over time in the Blocklab model presented in the main text. Length is defined by distance from the channel to the resistant scarp. The rollover in length illustrates a decrease in block release rate over time due to a decrease in vertical erosion rates at the top of the hillslope over time (Figure C.8). Similar to the control run, this is due to the horizontal geometry of the resistant layer- as the hillslope lengthens and steepens through time, the vertical erosion rate at the top of the slope (in the reference frame of the retreating scarp) decreases through time. However, channel-hillslope feedbacks in the Blocklab model alter the planform shape compared to the control run. A sharp kink in the planform scarp edge (main text, Figure 3) corresponds to the location where the channel is roughly equilibrated with baselevel at a given time (Figure C.12, C.13). Above this kink, the channel is oversteepened and channel incision rates increase up the channel (until the lip of the knickpoint).
Supplementary explanation of planform shape

In both the control and blocky models, cliff retreat rate is a function of the vertical erosion rate at the top of the slope \( e \), the retreat length of one block \( dx \), and the relief threshold for a block to fall. Therefore, the length of the hillslope at any given location and point in time \( t_2 \) is the integral of the cliff retreat rate:

\[
L = \int_{t_1}^{t_2} dx \frac{e(t)}{\text{threshold}}
\]

Where \( t_1 \) is the time at which the cliff first experiences incision. The change in hillslope length along the channel—that is, the planform shape—is therefore dictated by two things: the erosion rate trend through time, \( e(t) \), and the time at which the clock starts, \( t_1 \).

First, a note about the geometrical setup of the model. Hillslope response timescales have been well-described by, for example, Fernandes and Dietrich 1997. In a sense, the hillslope length trends in our model reflect an increase in hillslope response time through time as the hillslope lengthens (seen in Figure C.11). However, the adjustment timescales are complicated by the presence of a horizontal resistant layer, which leads to a hillslope that is always lengthening and increasing in relief through time. This in turn leads to a non-steady hillslope profile and erosion rate trend through time at the top of the hillslope, \( e(t) \), as seen for the control run in Figure C.8. A full analytical solution for the hillslope morphology and change through time—at least for the control case—could possibly be obtained by solving the diffusion equation in the case of a lowering boundary condition on one end (the incising channel), and a constant—but moving—source on the other side (a pinned elevation at the cliff edge) (Carslaw and Jager, 1947).

In the control run, we can assume that the channel incision rate adjusts to the baselevel signal quickly and predictably once the signal has reached a given location (Figure C.2). Therefore, the hillslope erosion rate trend through time, \( e(t) \), will be the same at all points along the channel (starting at \( t_1 \) for each given location). The only difference in hillslope erosion rate is due to the baslevel signal propagation delay, which modifies \( t_1 \) in the equation above. In essence, as you move up the channel, the hillslope integrates the same erosion rate function over shorter and shorter periods of time. The value of \( t_1 \) up the channel would be well described by the knickpoint propagation timescale presented in Whipple and Tucker, 1999.

In the blocky run, however, we cannot assume that the channel quickly responds to the baselevel signal, due to the presence of the blocks. As seen in figure 4 in the main text, the channel response is quite complicated. This means two things: first, the knickpoint propagation timescale that controls \( t_1 \) will be altered by block dynamics in the channel. Second, because the channel experiences fluctuations in incision rate after the baselevel signal has arrived, the hillslope erosion rate function through time, \( e(t) \), will be different for every different point along the channel. Thus the kink in the planform profile observed in the blocky runs is due to a combination of these two effects, and corresponds to the location where the channel erosion rate has become roughly steady (Figures C.12, C.13).
Figure C.12: Channel erosion rate (difference in channel profiles over 5000 years) shown with planform view at 200k years. Note that the kink in the planform shape coincides with a change in erosion rate up the channel, above which the channel is oversteepened, and below which the channel approaches an equilibrium state.
Figure C.13: Channel erosion rate (difference in channel profiles over 5000 years) shown with planform view at 400k years. Note that the kink in the planform shape coincides with a change in erosion rate up the channel, above which the channel is oversteepened, and below which the channel approaches an equilibrium state.

Additional Parameter Space Exploration Model Runs
Here we present 6 additional model runs: blocky models with 0.1, 0.5, and 0.8 initial block heights, along with their corresponding control runs with a resistant layer thickness equal to block height in which “blocks” that are released immediately turn into mobile soil. For each caprock thickness, we present figures for the blocky model above the corresponding control model illustrating: canyon morphology; channel and hillslope erosion rates; channel erosion rates and bed cover fraction.
0.1 meter thick caprock runs

Blocky model run: 0.1 meter thick block-producing caprock

Control run: 0.1 meter thick caprock with no blocks

Figure C.14: Top: Blocky model run with 0.1 meter thick caprock. Bottom: Control run with 0.1 meter thick caprock. Note that because blocks are small, control run and blocky model are not easily distinguishable.
Figure C.15: Channel and hillslope erosion rate histories from blocky model run with 0.1 m thick caprock, shown at the channel node adjacent to baselevel (bottom), 300 m (middle), and 600 m (top) upstream of baselevel forcing. Channel incision history shown in red, left axis labels; hillslope erosion history shown in black, right axis labels. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages. Hillslope erosion rates are shown for the hillslope node closest to the channel.
Figure C.16: Channel erosion rates and bed cover fractions for blocky model run with 0.1 m thick caprock. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
Figure C.17: Channel and hillslope erosion rate histories from control model run (blocks immediately turn to soil when released from the caprock) with 0.1 m thick caprock, shown at the channel node adjacent to baselevel (bottom), 300 m (middle), and 600 m (top) upstream of baselevel forcing. Channel incision history shown in red, left axis labels; hillslope erosion history shown in black, right axis labels. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages. Hillslope erosion rates are shown for the hillslope node closest to the channel.
Figure C.18: River erosion rates and bed cover fractions for control model run (blocks immediately turn to soil when released from the caprock) with 0.1 m thick caprock. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
0.5 meter thick caprock runs

Figure C.19: Top: Blocky model run with 0.5 meter thick caprock. Bottom: Control run with 0.5 meter thick caprock. While cross-sectional profiles are similar, planform and channel profiles have different features through time.
Figure C.20: Channel and hillslope erosion rate histories from blocky model run with 0.5 m thick caprock, shown at the channel node adjacent to baselevel (bottom), 300 m (middle), and 600 m (top) upstream of baselevel forcing. Channel incision history shown in red, left axis labels; hillslope erosion history shown in black, right axis labels. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages. Hillslope erosion rates are shown for the hillslope node closest to the channel.
Figure C.21: River erosion rates and bed cover fractions for blocky model run with 0.5 m thick caprock. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
Figure C.22: Channel and hillslope erosion rate histories from control model run (blocks immediately turn to soil when released from the caprock) with 0.5 m thick caprock, shown at the channel node adjacent to baselevel (bottom), 300 m (middle), and 600 m (top) upstream of baselevel forcing. Channel incision history shown in red, left axis labels; hillslope erosion history shown in black, right axis labels. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages. Hillslope erosion rates are shown for the hillslope node closest to the channel.
Figure C.23: River erosion rates and bed cover fractions for control model run (blocks immediately turn to soil when released from the caprock) with 0.5 m thick caprock. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
0.8 meter thick caprock runs

Figure C.24: Top: Blocky model run with 0.8 meter thick caprock. Bottom: Control run with 0.8 meter thick caprock. Profiles and planform exhibit key differences discussed in the main text.
Figure C.25: Channel and hillslope erosion rate histories from blocky model run with 0.8 m thick caprock, shown at the channel node adjacent to baselevel (bottom), 300 m (middle), and 600 m (top) upstream of baselevel forcing. Channel incision history shown in red, left axis labels; hillslope erosion history shown in black, right axis labels. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages. Hillslope erosion rates are shown for the hillslope node closest to the channel.
Figure C.26: River erosion rates and bed cover fractions for blocky model run with 0.8 m thick caprock. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
Figure C.27: Channel and hillslope erosion rate histories from control model run (blocks immediately turn to soil when released from the caprock) with 0.8 m thick caprock, shown at the channel node adjacent to baselevel (bottom), 300 m (middle), and 600 m (top) upstream of baselevel forcing. Channel incision history shown in red, left axis labels; hillslope erosion history shown in black, right axis labels. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages. Hillslope erosion rates are shown for the hillslope node closest to the channel.
Figure C.28: River erosion rates and bed cover fractions for control model run (blocks immediately turn to soil when released from the caprock) with 0.8 m thick caprock. Erosion rates (normalized to baselevel lowering rate) are shown at three points along the channel (red) and proportion of the channel bed covered by blocks (black) through time. Bottom plot is adjacent to baselevel, middle plot is 300 m upstream of baselevel, and upper plot is 600 m upstream from baselevel. Pink and gray lines are 10-year averages; red and black lines are 1000-year averages.
References

