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# The Fundamental Properties of Tansversely Periodic Lossy Waveguides

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# THE FUNDAMENTAL PROPERTIES OF TRANSVERSELY PERIODIC LOSSY WAVEGUIDES

by

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# ABSTRACT

The basic properties of waveguide modes propagating on a transversely periodic array of lossy material are derived. Orthogonality between modes, the excitation of modes by given sources, the dependence of the propagation constants on field distributions and material parameters are investigated. Finally a set of coupled-mode equations for longitudinally tapered waveguides of this kind are obtained. These equations will provide the groundwork for the analysis of the reflection of a low-frequency plane wave by a periodic array of tapered absorbers.

# 1. INTRODUCTION

Periodic arrays of pyramid-cone absorbers made from lossy dielectric materials have been used for many years to line anechoic chambers employed for interference-free measurement of electromagnetic fields [1],[2]. These have performed well at microwave frequencies and above, but must be made uneconomically large in order to perform well below 100 MHz. Since FCC rules for the measurement of electromagnetic interference (EMI) radiated by electronic equipment specify the use of open-field test sites over a frequency range of 30-1000 MHz, it is of considerable interest to correlate such measurements (which are susceptible to bad weather conditions and interference from ambient signals) with ones made in anechoic or semi-anechoic chambers for these frequencies [3]. Indeed, at the 30 MHz end of this range, different existing measurement chambers have exhibited a wide variation in performance [4].

From a quantitative theoretical standpoint, only a few studies on the reflection of waves by pyramid-cone absorbers have been made. At high frequencies (cone dimensions large compared to a wavelength), a geometrical-optics [5] or physical optics [6] analysis can be made, both of which predict the extremely small reflection coefficients observed in practice. For arbitrary frequencies, an analysis is presented in [28] based on perturbation theory for small values of complex permittivity (not usually the case in these applications). Also for arbitrary frequencies, a numerical method is proposed in [29] based on an integral equation formulation. Though requiring large-scale computations, this approach is capable of arbitrarily accurate results in principle. However, in practice it is the low-frequency behavior which is of most interest, because precisely in this range is the reflection from the absorber at its highest. For normally incident plane waves at two-dimensional wedges, an analysis of the problem was given by

Bucci and Franceschetti [5] who modelled the problem in terms of a slowly tapered equivalent waveguide which propagates only a single mode. Essentially the same solution had been proposed a number of years earlier by Katsenelenbaum [7], though no concrete results were given there.

Although it is indicated in [7] that the formulation can be generalized to account for an obliquely incident plane wave, there are a number of subtleties involved, especially when lossy media are present. In this report, we shall develop Katsenelenbaum's work to present a complete formalism for treating the scattering of electromagnetic waves by a transversely periodic array of lossy absorbing structures. The array is to be considered as a slowly-tapered waveguide, whose fields can be described by coupled-mode equations of the usual form.

Although much work has been devoted to the study of general properties of waveguides which are periodic along their axis of propagation [8]-[13], relatively little is known even about uniform waveguides which are <u>transversely</u> periodic. The case of periodically stacked dielectric slabs (the one-dimensionally periodic case) has been studied in [14]-[17]. Two dimensional arrays of lossless dielectric rods are studied in [18] by a perturbation method and in [19] using a numerical projection method. For lossless media, results analogous to those obtained in [20] for Bloch waves in crystals can be used to derive the important properties of such periodic waveguide arrays. When the array structure is lossy, only the results of [21] (a further development of [22] and [23]) are available for the general case, and these are inadequate for many purposes.

In this report, we will develop the orthogonality and symmetry properties for the modes of lossy waveguide arrays, derive equations for the excitation coefficients of the modes by given sources, and obtain expressions for the complex propagation constants of the modes (as well as their dependences on certain parameters) in terms of integrals of their field distributions over a period cell. In the next to last section, coupled-mode equations describing propagation along longitudinally tapered transversely periodic waveguides are derived. These equations will form the basis for an analysis of plane-wave reflection from a periodic array of pyramid-cone absorbers which will be detailed in a later report.

## 2. FORMULATION OF THE PROBLEM

Consider the two-dimensionally periodic array of inhomogeneous magnetodielectric cylinders shown in FIg. 1. For simplicity, we assume that the periodic lattice is rectangular, so that the complex permeability $\mu$ and complex permittivity $\varepsilon$, which are otherwise arbitrary functions of the transverse variables x and y, satisfy the relations

$$\begin{aligned} \mu(x + pa,\ y + qb) &= \mu(x,y) \\ \varepsilon(x + pa,\ y + qb) &= \varepsilon(x,y) \end{aligned} \tag{1}$$

where p and q are arbitrary integers. The behavior of the material parameters $\varepsilon$ and $\mu$ over any <u>period</u> cell C: $\{x_0 \leqq x \leqq x_0 + a,\ \ y_0 \leqq y \leqq y_0 + b\}$ is thus infinitely replicated over the other cells of the lattice. For simplicity, we also assume that no perfect conductors are present in the lattice, although we could easily account for them by a minor modification of the subsequent analysis (e.g., by taking the limit as the conductivity approaches infinity in a certain portion of the period cell).

We seek the waveguide modes of this array. In other words, we seek source-free solutions of Maxwell's equations of the form

$$\begin{aligned} \bar{E}(x,y,z) &= \bar{\mathcal{E}}(x,y)e^{-j\beta z} \\ \bar{H}(x,y,z) &= \bar{\mathcal{H}}(x,y)e^{-j\beta z} \end{aligned} \tag{2}$$

as well as any relevant boundary conditions. When the longitudinal field components $\mathcal{E}_z$ and $\mathcal{H}_z$ are eliminated from Maxwell's equations,

$$\left.\begin{aligned} \nabla \times \bar{E} &= -j\omega\mu\bar{H} \\ \nabla \times \bar{H} &= j\omega\varepsilon\bar{E} \end{aligned}\right\} \tag{3}$$

we arrive at [24]

$$j\beta\,\bar{\mathcal{E}}_t = j\omega\mu\,\bar{\mathcal{H}}_t \times \bar{a}_z - \frac{1}{j\omega}\nabla_t\left[\frac{1}{\varepsilon}\nabla_t\cdot(\bar{\mathcal{H}}_t \times \bar{a}_z)\right]$$
$$j\beta\,\bar{\mathcal{H}}_t = j\omega\varepsilon\bar{a}_z \times \bar{\mathcal{E}}_t - \frac{1}{j\omega}\nabla_t\left[\frac{1}{\mu}\nabla_t\cdot(\bar{a}_z \times \bar{\mathcal{E}}_t)\right] \qquad (4)$$

where the subscript "t" denotes the tranverse (xy) part of a vector or operator, and $\bar{a}_z$ denotes the unit vector in the z-direction.

Now according to Bloch's theorem (see, e.g., [20] or [21]) such modes will be such that

$$\left.\begin{aligned}\bar{\mathcal{E}}(x,y) &= e^{-j\bar{k}_t\cdot\bar{\rho}}\,\bar{F}_E(x,y)\\ \bar{\mathcal{H}}(x,y) &= e^{-j\bar{k}_t\cdot\bar{\rho}}\,\bar{F}_H(x,y)\end{aligned}\right\} \qquad (5)$$

where the functions $\bar{F}_E$ and $\bar{F}_H$ are periodic with the same periods as the material constants (eqn. (1)) while $\bar{k}_t = \bar{a}_x k_x + \bar{a}_y k_y$ is a given vector with real components and $\bar{\rho} = \bar{a}_x x + \bar{a}_y y$. Indeed, if such an array which is semi-infinite in the z-direction ($z > 0$) is excited by an arbitrary wave in empty space ($z < 0$), the incident wave can be broken down into a spectrum of incident plane waves, each of which has a transverse variation of $\exp(-j\bar{k}_t \cdot \bar{\rho})$. Each Bloch wave with a given value of $\bar{k}_t$ is thus a part of the response to an individual plane wave excitation. We shall sometimes emphasize the dependence of the fields and propagation constant on $\bar{k}_t$ by writing $\bar{\mathcal{E}}(x,y;\,\bar{k}_t)$, $\bar{\mathcal{H}}(x,y;\,\bar{k}_t)$ and $\beta(\bar{k}_t)$.

The set of modes for this structure, for reasonably physical behaviors of $\mu$ and $\varepsilon$, is countably infinite and discrete [21] for any given $\bar{k}_t$. We denote the fields and propagation constants of these modes by the index m: $\bar{\mathcal{E}}_m$, $\bar{\mathcal{H}}_m$, $\beta_m$. As can be seen from eqn. (4) and the boundary conditions, for any forward propagating or attenuating mode (with $\mathrm{Im}(\beta_m) < 0$, or $\mathrm{Im}(\beta_m) = 0$ and $\mathrm{Re}(\beta_m) > 0$) there is a corresponding backward-going mode, with

$$\beta_{-m}(\bar{k}_t) = -\beta_m(\bar{k}_t);\quad \bar{\mathcal{E}}_{-mt}(\bar{k}_t) = \bar{\mathcal{E}}_{mt}(\bar{k}_t);\quad \bar{\mathcal{H}}_{-mt}(\bar{k}_t) = -\bar{\mathcal{H}}_{mt}(\bar{k}_t) \qquad (6)$$

We will denote forward-going modes by $m > 0$.

We must at this point make an important assumption which is quite reasonable from a physical standpoint, but which in any degree of generality is very difficult to prove with mathematical rigor. This is that the modes form a complete set.* More precisely for this case, suppose that a set of externally applied electric and magnetic current sources $\bar{J}$ and $\bar{M}$ are present which have the Bloch wavenumber $\bar{k}_t$:

$$\left.\begin{aligned} \bar{J}(x,y,z) &= e^{-j\bar{k}_t\cdot\bar{\rho}}\,\bar{F}_J(x,y,z) \\ \bar{M}(x,y,z) &= e^{-j\bar{k}_t\cdot\bar{\rho}}\,\bar{F}_M(x,y,z) \end{aligned}\right\} \qquad (7)$$

where

$$\bar{F}_{J,M}(x + pa,\ y + qb,\ z) = \bar{F}_{J,M}(x,y,z) \qquad (8)$$

Then the completeness assumption means that the resulting field has transverse components which are representable as a sum of the waveguide modes:

$$\left.\begin{aligned} \bar{E}_t(x,y,z) &= \sum_{m\gtrless 0} C_m\, e^{-j\beta_m z}\, \bar{\mathcal{E}}_{mt}(x,y) \\ \bar{H}_t(x,y,z) &= \sum_{m\gtrless 0} C_m\, e^{-j\beta_m z}\, \bar{\mathcal{H}}_{mt}(x,y) \end{aligned}\right\} \qquad (9)$$

At cross-sections z where $\bar{J}$ or $\bar{M}$ is not zero, the $C_m$ are generally functions of z. Using (6), eqn. (9) can be written in the form of a generalized transmission line representation

*In exceptional situations, so-called "adjoined" modes can arise in lossy waveguides when two ordinary modes become degenerate [25] in such a way that $N_m$ defined in eqn. (21) below becomes zero. The theory can be generalized to take account of this, but we will assume that these situations do not arise here.

$$\left.\begin{aligned} \bar{E}_t(x,y,z) &= \sum_{m>0} V_m(z)\, \bar{\mathcal{E}}_{mt}(x,y) \\ \bar{H}_t(x,y,z) &= \sum_{m>0} Z_m\, I_m(z)\, \bar{\mathcal{H}}_{mt}(x,y) \end{aligned}\right\} \tag{10}$$

where

$$\left.\begin{aligned} V_m(z) &= C_m\, e^{-j\beta_m z} + C_{-m}\, e^{j\beta_m z} \\ Z_m I_m(z) &= C_m\, e^{-j\beta_m z} - C_{-m}\, e^{j\beta_m z} \end{aligned}\right\} \tag{11}$$

and $Z_m$ is a "characteristic impedance" which is a constant for each mode, but is otherwise arbitrary.

In order to make use of the completeness property, we require the orthogonality property for modes on this structure. The derivation follows fairly standard lines (e.g. [24]) although we will find that, in order to have boundary terms in the transverse direction vanish, we must consider two fields given by

$$\begin{aligned} \bar{E}^a &= \bar{\mathcal{E}}_m(\bar{k}_t)\, e^{-j\beta_m(\bar{k}_t)z} \\ \bar{H}^a &= \bar{\mathcal{H}}_m(\bar{k}_t)\, e^{-j\beta_m(\bar{k}_t)z} \end{aligned} \tag{12}$$

and

$$\begin{aligned} \bar{E}^b &= \bar{\mathcal{E}}_n(-k_t)\, e^{-j\beta_n(-k_t)z} \\ \bar{H}^b &= \bar{\mathcal{H}}_n(-\bar{k}_t)\, e^{-j\beta_n(-\bar{k}_t)z} \end{aligned} \tag{13}$$

so that one of the fields corresponds to a Bloch wavenumber exactly opposite to that of the other. Since (12) and (13) represent source-free solutions to Maxwell's equations, we apply the Lorentz reciprocity theorem to them, on a volume whose transverse cross-section is any period cell C, and which extends from $z_1 \leq z \leq z_2$, for two different values of $z_1$ and $z_2$. Because of the quasi-periodicity conditions (5), and the fact that we have chosen the Bloch wavenumbers in (12) and (13) opposite to each other, the surface integrals on the transverse sidewalls (x = const and y = const) cancel out, and we are left only with integrals over the z = const walls:

$$0 = \left[ e^{-j(\beta_m(\bar{k}_t) + \beta_n(-\bar{k}_t))z_2} - e^{-j(\beta_m(\bar{k}_t) + \beta_n(-\bar{k}_t))z_1} \right] \times$$

$$\times \int_C \left[ \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_n(-\bar{k}_t) - \bar{\mathcal{E}}_n(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t) \right] \cdot \bar{a}_z dS$$

Since $z_1$ and $z_2$ are arbitrary, we have

$$\int_C \left[ \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_n(-\bar{k}_t) - \bar{\mathcal{E}}_n(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t) \right] \cdot \bar{a}_z dS = 0$$

$$\text{if } \beta_m(\bar{k}_t) + \beta_n(-\bar{k}_t) \neq 0 \tag{14}$$

Replacing n by -n and making use of (6), we can also express (14) in the form:

$$\int_C \left[ \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_n(-\bar{k}_t) + \bar{\mathcal{E}}_n(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t) \right] \cdot \bar{a}_z dS = 0$$

$$\text{if } \beta_m(\bar{k}_t) - \beta_n(-\bar{k}_t) \neq 0 \tag{15}$$

Adding (or subtracting) (14) and (15) gives the simpler relation

$$\int_C \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_n(-\bar{k}_t) \cdot \bar{a}_z dS = 0$$

$$\text{if } \beta_m^2(\bar{k}_t) \neq \beta_n^2(-\bar{k}_t) \tag{16}$$

This is the mode orthogonality relation we desire. As usual, if there is an $m \neq n$ for which $\beta_m = \beta_n$, we will assume that the corresponding mode fields have been orthogonalized using the Gram-Schmidt procedure.

If the mode set is complete, we can explicitly evaluate the expansion coefficients in (9) due to the sources of eqn. (7). When the field components

$E_z$ and $H_z$ are eliminated from the most general form of Maxwell's equations <u>with sources</u>, we obtain [26]:

$$-\frac{\partial \bar{E}_t}{\partial z} = j\omega\mu\, \bar{H}_t \times \bar{a}_z - \frac{1}{j\omega}\nabla_t[\frac{1}{\varepsilon}\nabla_t \cdot (\bar{H}_t \times \bar{a}_z)] + \bar{M} \times \bar{a}_z + \frac{1}{j\omega}\nabla_t(\frac{J_z}{\varepsilon})$$

$$-\frac{\partial \bar{H}_t}{\partial z} = j\omega\varepsilon\, \bar{a}_z \times \bar{E}_t - \frac{1}{j\omega}\nabla_t[\frac{1}{\mu}\nabla_t \cdot (\bar{a}_z \times \bar{E}_t)] + \bar{a}_z \times \bar{J} + \frac{1}{j\omega}\nabla_t(\frac{M_z}{\mu}) \tag{17}$$

(compare eqn. (4)). Note that (17) is valid even when $\varepsilon$ and $\mu$ are functions of x, y, <u>and</u> z. Taking the dot product of the first equation of (17) with $\bar{\mathcal{H}}_n(-\bar{k}_t) \times \bar{a}_z$ and integrating over C, doing a similar thing with $\bar{a}_z \times \bar{\mathcal{E}}_n(-\bar{k}_t)$ and the second equation and performing some integrations by parts with the help of (4) gives

$$-\frac{d}{dz}\int_C \bar{E}_t \times \bar{\mathcal{H}}_n(-\bar{k}_t) \cdot \bar{a}_z dS = j\beta_n(-\bar{k}_t)\int_C \bar{\mathcal{E}}_n(-\bar{k}_t) \times \bar{H}_t \cdot \bar{a}_z dS$$
$$+ \int_C [\bar{M}_t \cdot \bar{\mathcal{H}}_n(-\bar{k}_t) - J_z\, \mathcal{E}_{nz}(-\bar{k}_t)]dS \tag{18}$$

and

$$-\frac{d}{dz}\int_C \bar{\mathcal{E}}_n(-\bar{k}_t) \times \bar{H}_t \cdot \bar{a}_z dS = j\beta_n(-\bar{k}_t)\int_C \bar{E}_t \times \bar{\mathcal{H}}_n(-\bar{k}_t) \cdot \bar{a}_z dS$$
$$+ \int_C [\bar{J}_t \cdot \bar{\mathcal{E}}_n(-\bar{k}_t) - M_z\, \mathcal{H}_{nz}(-\bar{k}_t)]dS \tag{19}$$

Subtracting (18) from (19), and using (9) and (14) gives, after some further manipulation,

$$\int_C \bar{J} \cdot \bar{\mathcal{E}}_n(-\bar{k}_t) - M \cdot \bar{\mathcal{H}}_n(-\bar{k}_t)]dS = \tag{20}$$

$$= [\frac{d}{dz} + j\beta_m(\bar{k}_t)][C_m e^{-j\beta_m(\bar{k}_t)z}]\int_C [\bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_n(-\bar{k}_t) - \bar{\mathcal{E}}_n(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t)] \cdot \bar{a}_z dS$$

(if there is a $\beta_m(\bar{k}_t) = -\beta_n(-\bar{k}_t)$; for the corresponding $C_m$, $\bar{\mathcal{E}}_m$ and $\bar{\mathcal{H}}_m$); or

$= 0$ (if there is no $\beta_m(\bar{k}_t) = -\beta_n(-\bar{k}_t)$).

Now for any given $\beta_n(-\bar{k}_t)$, only one of the choices in eqn. (20) can be true, and that choice must be the same regardless of what $\bar{J}$ and $\bar{M}$ are. So suppose that we take $\bar{J} = \bar{a}_e \delta(x - x_o)\delta(y - y_o)\delta(z - z_o)$, where $\bar{a}_e$ is some constant vector, and $\bar{M} = 0$. Then the left side of (20) is

$$\bar{\mathcal{E}}_n(x_o, y_o; -\bar{k}_t)\delta(z - z_o)$$

and unless $\bar{\mathcal{E}}_n(-\bar{k}_t)$ is identically zero (a trivial case that we exclude), then the second alternative in (20) is not allowed. This implies two things:

1) For every nontrivial mode solution $\{\bar{\mathcal{E}}_m(\bar{k}_t), \bar{\mathcal{H}}_m(\bar{k}_t), \beta_m(\bar{k}_t)\}$ of this structure, there is a corresponding backward propagating mode which has the opposite Bloch wavenumber $(-\bar{k}_t)$ and which we denote by $\{\bar{\mathcal{E}}_{-m}(-\bar{k}_t), \bar{\mathcal{H}}_{-m}(-\bar{k}_t), \beta_{-m}(-\bar{k}_t)\}$, such that $\beta_m(\bar{k}_t) = -\beta_{-m}(-\bar{k}_t)$. The further symmetries implied by (6) mean that (at least in the absence of mode degeneration) we always have $\beta_m(-\bar{k}_t) = \beta_m(\bar{k}_t)$, even though in the absence of simplifying conditions like reflection symmetry or losslessness we cannot generally find $\bar{\mathcal{E}}_m(-\bar{k}_t)$ and $\bar{\mathcal{H}}_m(-\bar{k}_t)$ in terms of $\bar{\mathcal{E}}_m(k_t)$ and $\bar{\mathcal{H}}_m(\bar{k}_t)$ in any easy way. These fields must generally be constructed specifically in each individual case.

2) The integral

$$\int_C [\bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_{-m}(-\bar{k}_t) - \bar{\mathcal{E}}_{-m}(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t)] \cdot \bar{a}_z dS$$

$$= - \int_C [\bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_m(-\bar{k}_t) + \bar{\mathcal{E}}_m(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t)] \cdot \bar{a}_z dS$$

appearing in the first alternative in (20) is not zero. Taking m = n in (14), and assuming $\beta_m \neq 0$ (a cutoff condition), we can write in complement to (16):

$$\frac{1}{2}\int_C [\bar{\mathcal{E}}_{-m}(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t) - \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_{-m}(-\bar{k}_t)] \cdot \bar{a}_z dS =$$

$$\int_C \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_m(-\bar{k}_t) \cdot \bar{a}_z dS = \int_C \bar{\mathcal{E}}_m(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t) \cdot \bar{a}_z dS \qquad (21)$$

$$\equiv N_m(\bar{k}_t)$$

which defines the norm $N_m$ of the modes corresponding to $\pm\bar{k}_t$. Evidently, $N_m$ is an even function of $\bar{k}_t$.

Hence, eqn. (20) for determining $C_m$ can be written

$$\left[\frac{d}{dz} + j\,\beta_m(\bar{k}_t)\right]\left[C_m\, e^{-j\beta_m(\bar{k}_t)z}\right] = -\frac{1}{2N_m(\bar{k}_t)} \int_C \left[\bar{J}\cdot \bar{\mathcal{E}}_{-m}(-\bar{k}_t) - \bar{M}\cdot \bar{\mathcal{H}}_{-m}(-\bar{k}_t)\right] dS \qquad (22)$$

which can be solved for $C_m$ in the usual manner of the theory of ordinary differential equations.

# 3. PARAMETRIC DEPENDENCE OF THE PROPAGATION CONSTANTS

In this section, we develop some expressions for the derivatives of $\beta$ with respect to a parameter of interest of our structure. These parameters include $\omega$, $\bar{k}_t$, and any parameter s upon which the medium parameters $\varepsilon$ and $\mu$ depend. These formulas are quite similar to analogous ones for ordinary waveguides, or for longitudinally periodic waveguides [8], [9], [12], [20]. The derivations are carried out in the Appendix; we merely quote the results here.

1) The derivative of $\beta_m(\bar{k}_t)$ with respect to $\omega$ is given by

$$\frac{\partial \beta_m}{\partial \omega} = \frac{1}{2N_m} \int_C \left[ \frac{\partial(\omega\varepsilon)}{\partial \omega} \, \bar{\mathcal{E}}_m(\bar{k}_t) \cdot \bar{\mathcal{E}}_{-m}(-\bar{k}_t) - \frac{\partial(\omega\mu)}{\partial \omega} \, \bar{\mathcal{H}}_m(\bar{k}_t) \cdot \bar{\mathcal{H}}_{-m}(-\bar{k}_t) \right] dS \qquad (23)$$

2) If $\varepsilon$ and $\mu$ depend on a parameter s which is independent of $\omega$, then

$$\frac{\partial \beta_m}{\partial s} = \frac{\omega}{2N_m} \int_C \left[ \frac{\partial \varepsilon}{\partial s} \, \bar{\mathcal{E}}_m(\bar{k}_t) \cdot \bar{\mathcal{E}}_{-m}(-\bar{k}_t) - \frac{\partial \mu}{\partial s} \, \bar{\mathcal{H}}_m(\bar{k}_t) \cdot \bar{\mathcal{H}}_{-m}(-\bar{k}_t) \right] dS \qquad (24)$$

3) The derivatives of $\beta_m$ with respect to the components of $\bar{k}_t$ are

$$\begin{aligned}
\nabla_k \beta_m &\equiv \bar{a}_x \frac{\partial \beta_m}{\partial k_x} + \bar{a}_y \frac{\partial \beta_m}{\partial k_y} \\
&= \frac{1}{2N_m} \oint_B \bar{\rho} \, \bar{a}_n \cdot \left[ \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_{-m}(-\bar{k}_t) - \bar{\mathcal{E}}_{-m}(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t) \right] d\ell \\
&= \frac{1}{2N_m} \int_C \left[ \bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_{-m}(-\bar{k}_t) - \bar{\mathcal{E}}_{-m}(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t) \right]_t dS
\end{aligned} \qquad (25)$$

where B is the boundary of C, and $\bar{a}_n$ is the unit outward normal vector to B in the xy-plane.

# 4. COUPLED-MODE EQUATIONS FOR TAPERED SECTIONS

We now permit $\varepsilon$ and $\mu$ to be functions of z, so that at each cross-section z, the mode fields $\bar{\mathcal{E}}_m$, $\bar{\mathcal{H}}_m$ and propagation constants $\beta_m$ are also (parametrically) functions of z. That is, these local normal modes are the modes which would exist on a longitudinally uniform structure whose cross-section is the same as that of the tapered structure at the given value of z. We have in mind a gentle taper of the sort shown in Fig. 2. While the coupled-mode formalism is in principle exact for any taper, it is most suitable when the taper is fairly gentle, as we shall see in future publications.

Within coupled-mode theory, we assume that a field (corresponding here to a given value of $\bar{k}_t$) can be expanded as a sum of the local normal modes:

$$\left.\begin{aligned} \bar{E}_t(x,y,z) &= \sum_{m \gtrless 0} A_m(z)\, \bar{\mathcal{E}}_{mt}(x,y,z) \\ \bar{H}_t(x,y,z) &= \sum_{m \gtrless 0} A_m(z)\, \bar{\mathcal{H}}_{mt}(x,y,z) \end{aligned}\right\} \qquad (26)$$

Note again the parametric dependence of $\bar{\mathcal{E}}$ and $\bar{\mathcal{H}}$ on z, here displayed explicity. If no impressed sources exist in the tapered region, we have from (17),

$$\left.\begin{aligned} -\frac{\partial \bar{E}_t}{\partial z} &= j\omega\mu\, \bar{H}_t \times \bar{a}_z - \frac{1}{j\omega}\, \nabla_t \left[\frac{1}{\varepsilon} \nabla_t \cdot (\bar{H}_t \times \bar{a}_z)\right] \\ -\frac{\partial \bar{H}_t}{\partial z} &= j\omega\mu\bar{a}_z \times \bar{E}_t - \frac{1}{j\omega}\, \nabla_t \left[\frac{1}{\mu} \nabla_t \cdot (\bar{a}_z \times \bar{E}_t)\right] \end{aligned}\right\} \qquad (27)$$

while for the local normal modes themselves, we again have eqn. (4):

$$\left.\begin{aligned} j\beta_m \bar{\mathcal{E}}_{mt} &= j\omega\mu\, \bar{\mathcal{H}}_{mt} \times \bar{a}_z - \frac{1}{j\omega} \nabla_t\left[\frac{1}{\varepsilon} \nabla_t \cdot (\bar{\mathcal{H}}_{mt} \times \bar{a}_z)\right] \\ j\beta_m \bar{\mathcal{H}}_{mt} &= j\omega\varepsilon\, \bar{a}_z \times \bar{\mathcal{E}}_{mt} - \frac{1}{j\omega} \nabla_t\left[\frac{1}{\mu} \nabla_t \cdot (\bar{a}_z \times \bar{\mathcal{E}}_{mt})\right] \end{aligned}\right\} \qquad (28)$$

In equation (27), dot multiplying by $\overline{\mathcal{H}}_n(-\bar{k}_t) \times \bar{a}_z$ and $\bar{a}_z \times \overline{\mathcal{E}}_n(-\bar{k}_t)$ respectively, and integrating over C as with eqns. (18) ff., we get

$$-\int_C \frac{\partial E_t}{\partial z} \times \overline{\mathcal{H}}_n(-\bar{k}_t) \cdot \bar{a}_z dS = j\beta_n(-\bar{k}_t) \int_C \overline{\mathcal{E}}_n(-\bar{k}_t) \times \bar{H}_t \cdot \bar{a}_z\, dS \qquad (29)$$

and

$$-\int_C \overline{\mathcal{E}}_n(-\bar{k}_t) \times \frac{\partial \bar{H}_t}{\partial z} \cdot \bar{a}_z\, dS = j\beta_n(-\bar{k}_t) \int_C \bar{E}_t \times \overline{\mathcal{H}}_n(-\bar{k}_t) \cdot \bar{a}_z dS \qquad (30)$$

Now we substitute (26) into (29) and (30), and use orthogonality (14) after subtracting (29) from (30):

$$A_m'(z) + j\beta_m(z)A_m(z) = \sum_{\ell \gtrless 0} A_\ell(z)C_{m\ell}(z) \qquad (31)$$

where

$$C_{m\ell}(z) = \frac{1}{2N_m} \int_C \left[\frac{\partial \overline{\mathcal{E}}_\ell(\bar{k}_t)}{\partial z} \times \overline{\mathcal{H}}_{-m}(-\bar{k}_t) - \overline{\mathcal{E}}_{-m}(-\bar{k}_t) \times \frac{\partial \overline{\mathcal{H}}_\ell(\bar{k}_t)}{\partial z}\right] \cdot \bar{a}_z dS \qquad (32)$$

Equations (31) are the analogs of the coupled-mode equations obtained in [27] and [24] for nonuniform waveguides, and are the appropriate generalizations to non-normally incident waves of the equations in [7].

As in [27] and [24], the coupling coefficients $C_{m\ell}$ for $\beta_m \neq \beta_\ell$ can be rewritten in a form explicitly dependent on $\partial\varepsilon/\partial z$ and $\partial\mu/\partial z$. The derivation is given in the Appendix, with the result that:

$$C_{m\ell}(z) = \frac{\omega}{2N_m(\beta_m - \beta_\ell)} \int_C \left[\frac{\partial \varepsilon}{\partial z}\, \overline{\mathcal{E}}_\ell(\bar{k}_t) \cdot \overline{\mathcal{E}}_{-m}(-\bar{k}_t) - \frac{\partial \mu}{\partial z}\, \overline{\mathcal{H}}_\ell(\bar{k}_t) \cdot \overline{\mathcal{H}}_{-m}(-\bar{k}_t)\right] dS$$

$$(\beta_m \neq \beta_\ell) \qquad (33)$$

Little can be said in the general case, apparently, about $C_{m\ell}(z)$ when $\beta_m = \beta_\ell$. It can only be calculated by use of (32).

# 5. CONCLUSION

In this report, we have outlined the general properties of the longitudinally propagating Bloch waveguide modes of a transversely periodic lossy array. The derivation has been done for the most general case of two-dimensional periodicity, but all of the properties apply to the case of one-dimensional periodicity when all structures and fields are independent of $y$. All integrals with respect to $y$ are eliminated in this case, while $\partial/\partial y$ is set equal to zero in any operation in which it appears.

Thus, the framework has been set up for the study of oblique plane-wave reflection from a periodic array of tapered (pyramid-cone or wedge) absorbers, generalizing the approach of [7] and [5] from the case of normal incidence. To address a specific geometry of absorber, we are still faced with the problem of finding $\overline{\mathcal{E}}_m$, $\overline{\mathcal{H}}_m$ and $\beta_m$ -- at least for all the above-cutoff modes--and evaluating the $C_{m\ell}$, in order to be able to solve eqns. (31). This task can be simplified if we are content with a low-frequency approximation to the fundamental modes only. All these questions will be addressed at length in future reports.

# APPENDIX

## Derivation of the Formulas of Section 3

### A.1

Consider the fields

$$\bar{E}^a = \bar{\mathcal{E}}_m(\bar{k}_t)\, e^{-j\beta_m z}$$
$$\bar{H}^a = \bar{\mathcal{H}}_m(\bar{k}_t)\, e^{-j\beta_m z} \qquad \text{(A.1)}$$

which are source-free. Consider also the fields

$$\bar{E}^b = \frac{\partial}{\partial\omega}\left(\bar{\mathcal{E}}_{-m}(-\bar{k}_t)\, e^{j\beta_m z}\right) = \left[\frac{\partial \bar{\mathcal{E}}_{-m}(-\bar{k}_t)}{\partial\omega} + jz\,\frac{\partial\beta_m}{\partial\omega}\,\bar{\mathcal{E}}_{-m}(-\bar{k}_t)\right]e^{j\beta_m z}$$
$$\text{(A.2)}$$
$$\bar{H}^b = \frac{\partial}{\partial\omega}\left(\bar{\mathcal{H}}_{-m}(-\bar{k}_t)e^{j\beta_m z}\right) = \left[\frac{\partial \bar{\mathcal{H}}_{-m}(-\bar{k}_t)}{\partial\omega} + jz\,\frac{\partial\beta_m}{\partial\omega}\,\bar{\mathcal{H}}_{-m}(-\bar{k}_t)\right]e^{j\beta_m z}$$

which are <u>not</u> source-free, but by differentiating Maxwell's equations by $\omega$ must be supported by the sources

$$\bar{J}^b = j\,\frac{\partial(\omega\varepsilon)}{\partial\omega}\,\bar{\mathcal{E}}_{-m}(-\bar{k}_t)e^{j\beta_m z}$$
$$\text{(A.3)}$$
$$\bar{M}^b = j\,\frac{\partial(\omega\mu)}{\partial\omega}\,\bar{\mathcal{H}}_{-m}(-\bar{k}_t)e^{j\beta_m z}$$

Let us now apply Lorentz reciprocity [24] integrated over the volume whose cross-section is a period cell C and which extends from $z_1$ to $z_2$ (as in the derivation of (14)).

From the quasiperiodicity of (A.1) and (A.2), the transverse boundary terms add to zero, while the remaining surface and volume integrals give, after some cancellations and rearrangement,

$$\frac{\partial \beta_m}{\partial \omega} = \frac{1}{2N_m} \int_C \left[ \frac{\partial(\omega\varepsilon)}{\partial \omega} \quad \bar{\mathcal{E}}_m (\bar{k}_t) \cdot \bar{\mathcal{E}}_{-m}(-\bar{k}_t) - \frac{\partial(\omega\mu)}{\partial \mu} \bar{\mathcal{H}}_m(\bar{k}_t) \cdot \bar{\mathcal{H}}_{-m}(-\bar{k}_t)\right] dS \tag{A.4}$$

where $N_m$ is defined by eqn. (21). We have left open the possibility that $\varepsilon$ and $\mu$ may be functions of $\omega$.

## A.2

In an almost identical fashion, suppose that $\varepsilon$ and $\mu$ depend upon a parameter s, which is independent of $\omega$. Then instead of (A.4), we have

$$\frac{\partial \beta_m}{\partial s} = \frac{\omega}{2N_m} \int_C \left[\frac{\partial \varepsilon}{\partial s} \quad \bar{\mathcal{E}}_m(\bar{k}_t) \cdot \bar{\mathcal{E}}_{-m}(-\bar{k}_t) - \frac{\partial \mu}{\partial s} \quad \bar{\mathcal{H}}_m(\bar{k}_t) \cdot \quad \bar{\mathcal{H}}_{-m}(-\bar{k}_t)\right] dS \tag{A.5}$$

## A.3

Next, let us differentiate by a component (say $k_x$) of $\bar{k}_t$, rather than by $\omega$ or s. We have

$$\bar{E}^b = \left[\frac{\partial \bar{\mathcal{E}}_{-m}(-\bar{k}_t)}{\partial k_x} + jz \frac{\partial \beta_m}{\partial k_x} \quad \bar{\mathcal{E}}_{-m}(-\bar{k}_t)\right] e^{j\beta_m z}$$

$$\bar{H}^b = \left[ \frac{\partial \bar{\mathcal{H}}_{-m}(-\bar{k}_t)}{\partial k_x} + jz \frac{\partial \beta_m}{\partial k_x} \quad \bar{\mathcal{H}}_{-m}(-\bar{k}_t)\right] e^{j\beta_m z} \tag{A.6}$$

and $\bar{E}^b$, $\bar{H}^b$ is source-free ($k_x$ does not explicitly affect the terms appearing in Maxwell's equations). The quasiperiodicity of $\bar{E}^b$, $\bar{H}^b$, has, however, been affected. Indeed, we now have that (cf. (5))

$$\frac{\partial \bar{\mathcal{E}}_{-m}(-\bar{k}_t)}{\partial k_x} = jx \, \bar{\mathcal{E}}_{-m}(-\bar{k}_t) + e^{j\bar{k}_t \cdot \bar{\rho}} \text{ (periodic function)}$$

and similarly for $\partial \bar{\mathcal{H}}_{-m}(-\bar{k}_t)/\partial k_x$ . The Lorentz reciprocity theorem now gives

$$\frac{\partial\beta_m}{\partial k_x} = \frac{1}{2N_m}\oint_B x\,\bar{a}_n \cdot [\,\bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_{-m}(-\bar{k}_t) - \bar{\mathcal{E}}_{-m}(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t)]d\ell \qquad (A.7)$$

where B is the boundary contour of C and $\bar{a}_n$ is the transverse outward unit normal to B. A similar formula can be derived for the derivative with respect to $k_y$, and we have:

$$\nabla_k\beta_m \equiv \bar{a}_x\frac{\partial\beta_m}{\partial k_x} + \bar{a}_y\frac{\partial\beta_m}{\partial k_y}$$

$$= \frac{1}{2N_m}\oint_B \bar{\rho}\,\bar{a}_n \cdot [\,\bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_{-m}(-\bar{k}_t) - \bar{\mathcal{E}}_{-m}(-\bar{k}_t) \times \bar{\mathcal{H}}_m(\bar{k}_t)]d\ell \qquad (A.8)$$

Now, as can be readily proven, the transverse divergence of the bracketed vector in the integrand of (A.8) is zero. Use of the two-dimensional divergence theorem thus results in

$$\nabla_k\beta_m = \frac{1}{2N_m}\int_C [\,\bar{\mathcal{E}}_m(\bar{k}_t) \times \bar{\mathcal{H}}_{-m}(-\bar{k}_t) - \bar{\mathcal{E}}_{-m}(-k_t) \times \bar{\mathcal{H}}_m(\bar{k}_t)]_t dS \qquad (A.9)$$

which is our desired result.

## A.4

Finally, let $\varepsilon$ and $\mu$ depend on z, put

$$\bar{E}^a = \bar{\mathcal{E}}_{-m}(-\bar{k}_t)e^{+j\beta_m z}$$
$$\bar{H}^a = \bar{\mathcal{H}}_{-m}(-\bar{k}_t)e^{+j\beta_m z} \qquad (A.10)$$

(which are source-free fields), and

$$\bar{E}^b = \left[\frac{\partial\ \bar{\mathcal{E}}_\ell(\bar{k}_t)}{\partial z} - j\,\frac{\partial(z\beta_\ell)}{\partial z}\ \bar{\mathcal{E}}_\ell\ (\bar{k}_t)\right]e^{-j\beta_\ell z}$$
$$\bar{H}^b = \left[\frac{\partial\,\bar{\mathcal{H}}_\ell(\bar{k}_t)}{\partial z} - j\,\frac{\partial(z\beta_\ell)}{\partial z}\ \bar{\mathcal{H}}_\ell(\bar{k}_t)\right]e^{-j\beta_\ell z} \tag{A.11}$$

which must be supported by the sources

$$\bar{J}^b = j\omega\,\frac{\partial\varepsilon}{\partial z}\ \bar{\mathcal{E}}_\ell(\bar{k}_t)\ e^{-j\beta_\ell z}$$
$$\bar{M}^b = j\omega\,\frac{\partial\mu}{\partial z}\ \bar{\mathcal{H}}_\ell(\bar{k}_t)\ e^{-j\beta_\ell z} \tag{A.12}$$

We apply Lorentz reciprocity again to the same volume whose cross-section is the period cell C, and make use of orthogonality (14). The final result is

$$(\beta_m - \beta_\ell)C_{m\ell} + \frac{\partial\beta_m}{\partial z}\ \delta_{\ell m} = \frac{\omega}{2N_m}\int_C \left[\frac{\partial\varepsilon}{\partial z}\ \bar{\mathcal{E}}_\ell(\bar{k}_t)\cdot\bar{\mathcal{E}}_{-m}(-\bar{k}_t) - \frac{\partial\mu}{\partial z}\ \bar{\mathcal{H}}_\ell(\bar{k}_t)\cdot\bar{\mathcal{H}}_{-m}(-\bar{k}_t)\right]dS \tag{A.13}$$

where $C_{m\ell}$ is given by (32), and

$$\delta_{m\ell} = 1 \quad \text{if} \quad \beta_m = \beta_\ell$$
$$= 0 \quad \text{if} \quad \beta_m \neq \beta_\ell \tag{A.14}$$

When $m = \ell$, eqn. (A.13) reduces to a special case of (A.5), but gives no information about $C_{mm}$ itself. When $m \neq \ell$, however, we get

$$C_{m\ell} = \frac{\omega}{2N_m(\beta_m-\beta_\ell)}\int_C \left[\frac{\partial\varepsilon}{\partial z}\ \bar{\mathcal{E}}_\ell(\bar{k}_t)\cdot\bar{\mathcal{E}}_{-m}(-\bar{k}_t) - \frac{\partial\mu}{\partial z}\ \bar{\mathcal{H}}_\ell(\bar{k}_t)\cdot\bar{\mathcal{H}}_{-m}(-\bar{k}_t)\right]dS \tag{A.15}$$

which is an alternate form for eqn. (32).

# References


[1] H. Severin, "Nonreflecting absorbers for microwave radiaton," IRE Trans. Ant. Prop., vol. 4, pp. 385-392 (1956).

[2] W.H. Emerson, "Electromagnetic wave absorbers and anechoic chambers through the years, "IEEE Trans. Ant. Prop., vol. 21, pp. 484-490 (1973).

[3] R.F. German, personal communication, 1986.

[4] R.F. German, "Comparison of semi-anechoic chamber and open-field site attenuation measurements," 1982 IEEE Internat. Symp. Electromagnetic Compatibility, Santa Clara, CA, 1982, pp. 260-265.

[5] O.M. Bucci and G. Franceschetti, "Scattering from wedge-tapered absorbers," IEEE Trans. Ant. Prop., vol. 19, pp. 96-104 (1971).

[6] A.K. Cherepanov, "Reflection of eletromagnetic waves from an absorptive pyramidal surface" [Russian], Radiotekh. Elektron., vol. 19, pp. 1749-1753 (1974) [Engl. transl. in Radio Eng. Electron. Phys., vol. 19, no. 8, pp. 120-123 (1974)].

[7] B.Z. Katsenelenbaum, "The problem of a normally incident plane electromagnetic wave on a periodic boundary separating two dielectrics," [Russian], Radiotekh. Elektron., vol. 5, pp. 1929-1932 (1960) [Engl. transl. in Radio Eng. Electron., vol. 5, no. 12, pp. 77-82 (1960)].

[8] D.A. Watkins, Topics in Electromagnetic Theory. New York: Wiley, 1958, chapter 1.

[9] M.G. Krein and G. Ya. Lyubarskii, "The theory of transmission bands of periodic waveguides," [Russian], Prikl. Mat. Mekh., vol. 25, pp. 24-37 (1961) [Engl. transl. in PMM, vol. 25, pp. 29-48 (1961)].

[10] M.J. Gans, "A general proof of Floquet's theorem, "IEEE Trans. Micr. Theory Tech., vol. 13, pp. 384-385 (1965).

[11] A.B. Manenkov, "Excitation of open periodic waveguides" [Russian], Izv.VUZ Radiofizika, vol. 19, pp. 263-270 (1976) [Engl. transl. in Radiophys. Quantum Electron., vol. 19, pp. 183-188 (1976)].

[12] M.V. Fedoryuk, "Propagation of waves in periodic waveguides" [Russian], Dokl. Akad. Nauk SSSR, vol. 242, pp. 574-577 (1978) [Engl. transl. in Sov. Phys. Dokl., vol. 23, pp. 662-664 (1978)].

[13] P.R. McIsaac, "A general reciprocity theorem," IEEE Trans. Micr. Theory Tech., vol. 27, pp. 340-342 (1979).

[14] S.M. Rytov, "Electromagnetic properties of a finely stratified medium" [Russian], Zh. Exp. Teor. Fiz. vol. 29, pp. 605-616 (1955) [Engl. transl. in Sov. Phys. JETP, vol. 2, pp. 466-475 (1956)].

[15] R.E. Collin, "Reflection and transmission at a slotted dielectric interface," Canad. J. Phys., vol. 34, pp. 398-411 (1956).

[16] D.S. Jones, The Theory of Electromagnetism. Oxford: Pergamon Press, 1964, pp. 348-351.

[17] L.R. Lewis and A. Hessel, "Propagation characteristics of periodic arrays of dielectric slabs," IEEE Trans. Micr. Theory Tech., vol. 19, pp. 276-286 (1971).

[18] V.F. Andreev, "Analysis of periodic systems of longitudinal elements" [Russian], Radiotekh. Elektron., vol. 18, pp. 40-48 (1973) [Engl. transl. in Radio Eng. Electron. Phys., vol. 18, no. 1, pp. 33-40 (1973)].

[19] V.M. Krekhtunov and V.A. Tyulin, "Diffraction of electromagnetic waves at the junction of two-dimensionally periodic arrays of dielectric rods" [Russian], Radiotekh. Elektron., vol. 26, pp. 2265-2272 (1981).

[20] F. Odeh and J.B. Keller, "Partial differential equations with periodic coefficients and Bloch waves in crystals," J. Math. Phys., vol. 5, pp. 1499-1504 (1964).

[21] V.I. Derguzov, "Incidence of a plane wave on a lossy periodic structure" [Russian], in Problemy matematicheskogo Analiza, vyp. 6 (N.N. Ural'tseva, ed.). Leningrad: Izdat. Leningrad. Univ., 1977, pp. 30-60.

[22] V.I. Derguzov, "Wave propagation in periodic structures in a prescribed direction" [Russian], Sibirsk. Mat. Zh., vol. 14, pp. 300-321 (1973) [Engl. trans. in Siberian Math. J., vol. 14, pp. 206-221 (1973)].

[23] V.I. Derguzov, "On the discreteness of the spectrum of non-self-adjoint periodic boundary-value problems" [Russian], Sibirsk. Mat. Zh., vol. 15, pp. 292-298 (1974) [Engl. transl. in Siberian Math. J., vol. 15, pp. 205-209 (1974)].

[24] F. Sporleder and H. -G. Unger, Waveguide Tapers Transitions and Couplers. Stevenage, UK: Peter Peregrinus, 1979, chapter 2.

[25] V.V. Shevchenko, "Excitation of waveguides in the presence of adjoined waves" [Russian], Radiotekh. Elektron., vol. 31, pp. 456-465 (1986) [Engl. transl. to appear].

[26] L.B. Felsen and N. Marcuvitz, Radiation and Scattering of Waves. Englewood Cliffs, NJ: Prentice-Hall, 1973, pp. 265-268.

[27] B.Z. Katsenelenbaum, "Irregular waveguides with variable dielectric filling" [Russian], Radiotekh. Elektron., vol. 3, pp. 890-895 (1958) [Engl. transl. in Radio Eng. Electron., vol. 3, no. 7, pp. 48-55 (1958)].

[28] O.S. Mergelyan, "Diffraction of a plane electromagnetic wave by a crimped dielectric surface" [Russian], Izv. VUZ Radiofizika, vol. 15, pp. 1233-1238 (1972) [Engl. transl. in Radiophys. Quantum Electron., vol. 15,pp. 941-945 (1972)].

[29] E.M. Inspektorov, "Calculation of the reflection from the surface of radio-absorbing material," [Russian], Izv. VUZ Radioelektronika, vol. 27, no. 11, pp. 103-105 (1982).

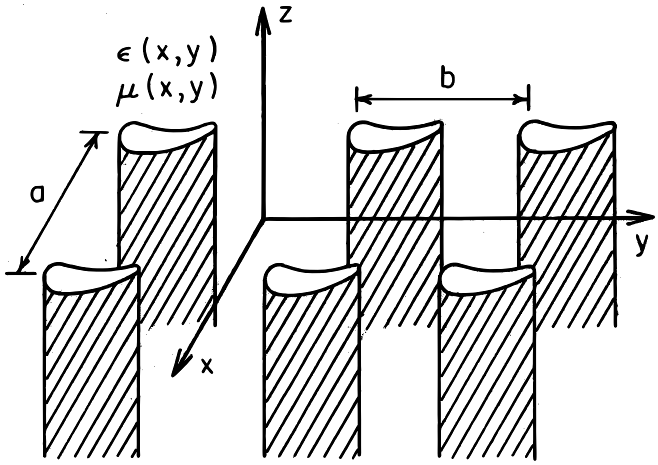


Figure 1: A transversely periodic waveguide.

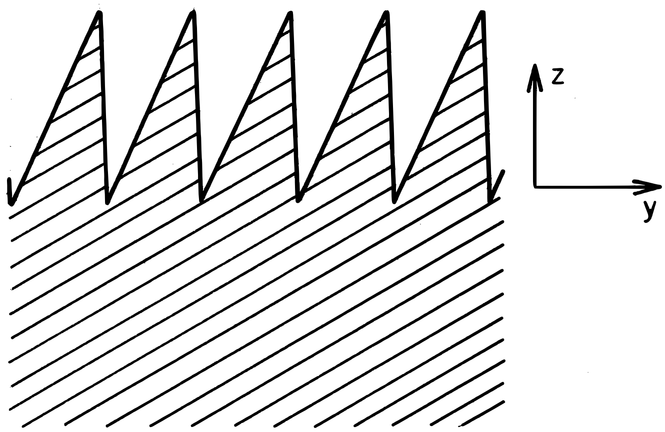


Figure 2: Tapered transversely periodic waveguide.