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**LINE PARAMETERS OF
COPLANAR STRIPS USED
AS ELECTRO-OPTIC
MODULATOR ELECTRODES**

by

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Abstract

Approximate expressions for the transmission line parameters of coplanar strip (CPS) lines on a two-layer substrate are derived. These expressions are valid either when the strip dimensions are large compared with the thickness of the finite layer of the substrate. The formulas can serve as design tools for electro-optic traveling-wave modulators using CPS electrodes. Numerical examples are given and comparison is made with other results for CPS available in the literature.

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1. Introduction

Electro-optic modulators have been the subject of intense research for over 20 years now [1]-[3]. Lumped-element electrode designs possess a number of inherent limitations to the modulation bandwidth which can be overcome using traveling-wave electrodes with a matched load impedance termination. In the absence of attenuation effects, the fundamental limitation on the bandwidth of a traveling-wave modulator is the velocity mismatch (or equivalently, the refractive index mismatch) between the modulating (microwave) signal on the electrodes and the modulated (optical) signal in a waveguide channel in the electro-optic material [1],[2],[4],[5]. This mismatch, known as the "walk-off" between the optical and microwave signals, results in a modulation index (e.g., for a phase modulator) of

$$m = \left| \delta \frac{\omega_o N_o}{\omega_m N_m} \frac{\sin(\omega_m N_m \ell \Delta / 2c)}{\Delta} \right| \quad (1)$$

where

$\omega_o = 2\pi f_o = 2\pi c / \lambda_o =$ angular frequency of optical wave

$\omega_m = 2\pi f_m =$ angular modulating frequency

$N_o =$ effective refractive (group) index of the optical wave

$N_m =$ effective refractive (phase) index of the microwave signal

$\ell =$ modulator electrode length

$\Delta = 1 - N_o / N_m$

$\delta =$ modulation strength (proportional to the modulating voltage V_o , the relevant electro-optic coefficient r , and the overlap integral Γ (see [2]))

and c is the speed of light in vacuum. If $N_o = N_m$ ($\Delta = 0$), the modulation index becomes

$$m = \delta \omega_o N_o \ell / 2c \quad (N_o = N_m) \quad (2)$$

and is independent of the modulation frequency. Eqn. (2) is also approximately true if $f_m \ell (N_m - N_o) / c \ll 1$.

If $N_o \neq N_m$, it is clear from (1) that there is a frequency,

$$f_m^{(c)} = \frac{c}{N_m \ell \Delta} = \frac{c}{\ell (N_m - N_o)} \quad (3)$$

which results in no modulation, and serves as a measure of the bandwidth of the modulator. In this case, eqns. (2) and (3) represent the tradeoff which must be made in the design of a modulator. The electrode length must be large in order to achieve significant modulation m with minimum drive voltage V_o , but on the other hand must be small to guarantee a large bandwidth.

For given electrode structures based on given materials, N_m and N_o are fixed, and the bandwidth (3) achievable with reasonably small drive voltage is determined ahead of time. As an example, the coplanar strip (CPS) electrode structure shown in Fig. 1 has, if the substrate material is LiNbO_3 , $N_o \cong 2.2$ and $N_m \cong 4.2$. Since both (2) and (3), as well as the inherent losses of the substrate limit the performance of such a modulator, a maximum bandwidth of 17 GHz has been obtained to date with such a structure [6],[7].

In order to achieve higher bandwidths, ways of reducing $N_m - N_o$ must be found. Some progress has been made using periodic structures [8],[9] to achieve velocity match, but in general this results in a pass-band frequency response, wherein modulation in one or more stop-bands is sacrificed in order to achieve a higher upper limit on the modulation frequency. The material properties of electro-optic materials like LiNbO_3 and GaAs are fixed,

and while GaAs exhibits a closer match of N_m and N_o than LiNbO_3 , it also has a smaller electro-optic coefficient r by an order of magnitude or more [2]. The drive voltage/bandwidth product is thus not markedly improved by choosing a different material for the substrate.

This leaves only the adjustment of N_m by modifying the electrode transmission-line structure as a possible remedy. The idea would be to concentrate relatively less of the microwave field from the electrodes in the electrically dense electro-optic material, and more of the field in surrounding media with lower ϵ . This reduces the effective dielectric constant $\epsilon_{\text{eff}} \equiv N_m^2$ accordingly. For example a ridged waveguide has been proposed for this purpose [10], and has been shown capable of achieving modulation bandwidths of 40 GHz or more. This type of waveguide, however, is not as readily incorporated into an integrated microwave or millimeter wave circuit. Yamashita et al. [11] have proposed a more promising idea of modifying the CPS electrodes as shown in Fig. 2. The electro-optic material is now reduced to a thin film directly underneath the CPS, large enough to contain the optical channel guide, but thin enough that much of the microwave field extends into a lower permittivity substrate and thus reduces the value of N_m . A single numerical example was given in [11], wherein N_m was calculated numerically from an infinite integral, similar to the method described in [12]. The numerical example used a substrate with $\epsilon_s = 4$, $d = 18.5 \mu\text{m}$, $g = 50 \mu\text{m}$ and $w = 1.34 \text{ mm}$. That is, a very thin LiNbO_3 film is needed to achieve the velocity match, and there are no published data besides this for such small d/w to use for design purposes.

There are a number of publications analyzing the quasi-TEM line parameters of CPS. The exact solution to CPS in free space can be obtained by conformal mapping [13]. Its capacitance C_o per unit length is given by

$$\frac{C_0}{\epsilon_0} = \frac{K(k')}{K(k)} \quad (4)$$

where K is the complete elliptic integral of the first kind, and the modulus k is given by

$$k = g/(g + 2w) \quad (5)$$

and $k' = \sqrt{1-k^2}$. The modification of this formula

$$C = C_0 \frac{\epsilon_f + 1}{2} \quad (6)$$

gives the capacitance for the case of a semi-infinite substrate of permittivity ϵ_f ($d \rightarrow \infty$ in Fig. 2).

Accounting for finite substrate thickness is less easy. Yamashita and his colleagues [12], [14], [15] use a variational approach to find the quasi-TEM capacitance C of strip structures like those of Fig. 2. A Fourier integral must be numerically evaluated for this purpose and so no closed-form design formulas are presented. Similarly, Knorr and Kuchler [16] have used a full-wave spectral-domain technique to analyze CPS, though again no design formulas are presented.

Other attempts have been made to treat the finite substrate case by modifying the conformal mapping solution for the case of an infinite substrate. Attempts to do this rigorously must in the end be numerical [17], and closed-form expressions do not result. If the quasi-TEM problem can be heuristically (and approximately) split into two simpler ones, each amenable to an exact solution, explicit formulas can be obtained. In an unpublished M.S. thesis, Frazita [18], as cited in [19], obtained the expression

$$\frac{C}{\epsilon_0} \cong \frac{C_0}{\epsilon_0} + \frac{\epsilon_f - 1}{2} \frac{K(k_1')}{K(k_1)} \quad (7)$$

$$k_1 = \tanh(\pi g/4d)/\tanh[\pi(g+2w)/4d] \quad (8)$$

and $k_1' = \sqrt{1-k_1^2}$. Note that $k_1 \rightarrow k$ as $d \rightarrow \infty$, so that (7) reduces to (6) in this limit as it should. We also recover eqn. (4) in the case when $\epsilon_f = 1$.

More recently, two similar expressions have been published [20], [21]. They are [20]:

$$\frac{C}{\epsilon_0} \cong \frac{C_0}{\epsilon_0} + \frac{\epsilon_f - 1}{2} \frac{K(k_2')}{K(k_2)} \quad (9)$$

and [21]:

$$\frac{C}{\epsilon_0} \cong \frac{C_0}{\epsilon_0} + \frac{\epsilon_f - 1}{2} \left(\frac{C_0}{\epsilon_0} \right)^2 \frac{K(k_2)}{K(k_2')} \quad (10)$$

where

$$k_2 = \sinh(\pi g/4d)/\sinh[\pi(g+2w)/4d] \quad (11)$$

and $k_2' = \sqrt{1-k_2^2}$. A final closed-form expression* is given in [13], and was based on a curve-fit of the data presented in [17]:

$$C \cong \left(\frac{\epsilon_f + 1}{2} \right) C_0 \left\{ 1 + \frac{kw}{d} \left[0.04 - 0.7k + 0.01 \left(1 - \frac{\epsilon_f}{10} \right) \left(k + \frac{1}{4} \right) \right] - 0.006 \left(\frac{w}{d} \right)^{1.55} \right\} \quad (12)$$

where k , again, is given by (5). Equations (9), (10) and (12) all have the proper limit of (6) as $d \rightarrow \infty$ (though they approach this limit at different rates). Equations (9) and (10) also reduce to the proper limit of (4) as $\epsilon_f \rightarrow 1$. Formula (12) does not do so, and must thus be considered inadequate for smaller ϵ_f as well as for smaller d/w as has been noted in [13].

* Eqn. (12) is a slightly simplified version of the formula given in [13], but is essentially identical numerically in their ranges of validity ($d/w \geq 1$).

The applicability of eqns. (7), (9) and (10) for the case of thin substrates (the case which interests us here) is less clear. Taking the limit of these formulas as $d \rightarrow 0$ gives

$$\frac{C}{\epsilon_0} \cong \frac{C_0}{\epsilon_0} + \frac{\epsilon_f^{-1}}{2} \frac{2d}{g} \quad (d \rightarrow 0) \quad (7a)$$

$$\frac{C}{\epsilon_0} \cong \frac{C_0}{\epsilon_0} + \frac{\epsilon_f^{-1}}{2} \frac{w}{d} \quad (d \rightarrow 0) \quad (9a)$$

$$\frac{C}{\epsilon_0} \cong \frac{C_0}{\epsilon_0} + \frac{\epsilon_f^{-1}}{2} \left(\frac{C_0}{\epsilon_0} \right)^2 \frac{d}{w} \quad (d \rightarrow 0) \quad (10a)$$

As pointed out in [21], formula (9a) clearly has the wrong behavior as $d \rightarrow 0$. Formulas (7a) and (10a) reduce to (4) in this limit as they should, but reach this limit in very different ways. It is thus difficult to choose between eqn. (7) and eqn. (10) for use at small values of d without more rigorous justification.

In this report, we will systematically derive an expression for C of Fig. 2 when d is small. This expression can then be used in the design of velocity-matched electro-optic modulators as discussed in the final section. The results are presented here for convenience.

$$C \cong \frac{C_0(\epsilon_s+1)/2}{1 - \frac{C_0 d}{\epsilon_0 w} \frac{\epsilon_f^2 - \epsilon_s^2}{\epsilon_f(\epsilon_s+1)}} \quad (13)$$

Capacitance per unit length of CPS when $d/w \ll 1$.

We might note that formula (13) is in agreement with (10a) for $\epsilon_s = 1$ and $\epsilon_f \gg 1$, while it is completely different from (7a).

2. Derivation of the Capacitance Formula

We will carry out a quasi-static, quasi-TEM analysis of the CPS shown in Fig. 2. This means we can solve the much simpler problem of the static capacitance per unit length of the structure. Numerical data for the case of $w/d \cong 1$ suggests that this approximation will be valid so long as $d/\lambda_0 \leq .1$ or so [16]. When d is much smaller than w , we should expect the condition to be $w/\lambda_0 \leq .1$, where λ_0 is the free space wavelength of the microwave. For example, if the highest frequency to be used is 30 GHz, we can expect the quasistatic assumption to be valid for $w \leq 1$ mm.

Another assumption we make is that the film layer (ϵ_f) is isotropic. This is to simplify the analysis, but is not a serious difficulty because it has been shown [15] (see also [22]) that the corresponding anisotropic structure shown in Fig. 3 has the same capacitance per unit length as that of Fig. 2 if

$$\epsilon_f = \sqrt{\epsilon_{xf}\epsilon_{yf}} \quad ; \quad \epsilon_s = \sqrt{\epsilon_{xs}\epsilon_{ys}} \quad ; \quad d = h \sqrt{\frac{\epsilon_{xf}}{\epsilon_{yf}}} \quad (14)$$

where h is the layer thickness in the anisotropic CPS, and $\epsilon_{xf}, \epsilon_{yf}$ (or $\epsilon_{xs}, \epsilon_{ys}$) are the elements of the diagonal permittivity tensors in the two material regions. Based on the results derived below, then, we can always obtain results for the more general anisotropic structure of Fig. 3.

We can formulate the problem as an integral equation for the unknown surface charge density $\sigma(x)$ on one of the strips, $a \leq x \leq b$, where

$$\left. \begin{aligned} a &= g/2 \\ b &= w + g/2 \end{aligned} \right\} \quad (15)$$

The charge density on the other strip, $-b < x < -a$, will be antisymmetric:

$$\sigma(-x) = -\sigma(x)$$

The integral equation can be obtained by the standard technique of representing the potential ϕ at any point (x,y) as a Fourier transform with respect to x [25], [26]. The result is that in the plane containing the strips ($y = 0$), the potential is given by

$$\phi(x,0) = \frac{1}{\pi\epsilon_0(\epsilon_f+1)} \int_a^b \sigma(x')G(x,x')dx' \quad (16)$$

where

$$G(x,x') = 2 \int_0^\infty \frac{\sin \alpha x \sin \alpha x'}{\alpha} \left(\frac{1 + \delta_{fs} e^{-2\alpha d}}{1 - \delta_f \delta_{fs} e^{-2\alpha d}} \right) d\alpha \quad (17)$$

and

$$\delta_f = \frac{\epsilon_f - 1}{\epsilon_f + 1} ; \quad \delta_{fs} = \frac{\epsilon_f - \epsilon_s}{\epsilon_f + \epsilon_s} \quad (18)$$

Once eqn. (16) is solved for $\sigma(x)$, the capacitance per unit length can be found from

$$C = \frac{1}{V} \int_a^b \sigma(x) dx \quad (19)$$

where V is the potential difference between the strips (their potentials are $\pm V/2$). The effective index of a signal on this coplanar line is then

$$N_m = \sqrt{\frac{C}{C_0}} \quad (20)$$

where C_0 is the capacitance for the case $\epsilon_f = \epsilon_s = 1$.

It is of some interest to see how the expression (4) for C_0 arises from the solution of an integral equation. Setting $\epsilon_f = \epsilon_s = 1$, the Green's function in (17) reduces to

$$G_0(x,x') = 2 \int_0^\infty \frac{\sin \alpha x \sin \alpha x'}{\alpha} d\alpha = \ln \left| \frac{x+x'}{x-x'} \right| \quad (21)$$

the integral being found in ([27], eqn. 3.741.1). For the air-filled CPS, then, (16) reduces to

$$\Phi(x,0) = \frac{1}{2\pi\epsilon_0} \int_a^b \sigma_0(x') \ln \left| \frac{x+x'}{x-x'} \right| dx' \quad (22)$$

and an integral equation for σ_0 is obtained by forcing $\Phi(x,0)$ to be equal to $V/2$ on the strip $a \leq x \leq b$:

$$\int_a^b \sigma_0(x') \ln \left| \frac{x+x'}{x-x'} \right| dx' = \pi\epsilon_0 V \quad (23)$$

The solution to (23) is found by known methods in the Appendix. From (A.6) and (A.9), we have

$$\sigma_0(x) = \frac{b\epsilon_0 V}{K(k)} \frac{1}{\sqrt{(b^2-x^2)(x^2-a^2)}} \quad (24)$$

where K is the complete elliptic integral of the first kind and modulus $k = a/b$. Finally, from (A.11) and (19), we have that

$$C_0 = \epsilon_0 \frac{K(k')}{K(k)} \quad (25)$$

i.e., precisely the result given in eqn. (4).

If $\epsilon_f = \epsilon_s \neq 1$, the eqn. for $\sigma(x)$ is still easy to solve, since $G(x,x')$ is still equal to $G_0(x,x')$ because of $\delta_{fs} = 0$. Only the factor of $(\epsilon_f+1)/2$ needs to be added to the right side of (23), (24) and (25) to describe this case. This is in agreement with the result in eqn. (6).

Now let us consider the case $\epsilon_f \neq \epsilon_s$, but the film layer thickness d is small compared with the other characteristic dimensions of the problem (primarily $d \ll w$). We express the Green's function G as the sum of its value at $d = 0$ plus a correction term which becomes small as $d \rightarrow 0$:

$$\begin{aligned}
G(x, x') &= \frac{2(\epsilon_f + 1)}{\epsilon_s + 1} \int_0^\infty \frac{\sin \alpha x \sin \alpha x'}{\alpha} d\alpha + G_1(x, x') \\
&= \frac{\epsilon_f + 1}{\epsilon_s + 1} G_0(x, x') + G_1(x, x')
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
G_1(x, x') &= - \frac{2(\epsilon_f - \epsilon_s)}{\epsilon_s + 1} \int_0^\infty \frac{\sin \alpha x \sin \alpha x'}{\alpha} \left(\frac{1 - e^{-2\alpha d}}{1 - \delta_f \delta_{fs} e^{-2\alpha d}} \right) d\alpha \\
&= \frac{\epsilon_f}{\epsilon_f - 1} \sum_{m=1}^{\infty} (\delta_f \delta_{fs})^m \ln \left[\frac{1 + \left(\frac{2md}{x+x'} \right)^2}{1 + \left(\frac{2md}{x-x'} \right)^2} \right]
\end{aligned} \tag{27}$$

The second way of expressing G_1 in (27) is obtained by expanding $(1 - \delta_f \delta_{fs} e^{-2\alpha d})^{-1}$ as a geometrical series and integrating term-by-term with the help of ([27]), eqn. 3.947.1):

$$\int_0^\infty \frac{\sin \alpha x \sin \alpha x'}{\alpha} e^{-\alpha y} d\alpha = \frac{1}{2} \ln \frac{y^2 + (x+x')^2}{y^2 + (x-x')^2} \quad (y \geq 0) \tag{28}$$

Now G_1 will be small unless $|x \pm x'|$ is comparable to or smaller than d . If we focus our attention to the case when $w = b - a \gg d$, then the interval over which G_1 is significant is rather narrow compared to w . We thus approximate G_1 by evaluating its integrand for small d and integrating:

$$\begin{aligned}
G_1(x, x') &\cong - \frac{2(\epsilon_f - \epsilon_s)}{\epsilon_s + 1} \int_0^\infty \frac{\sin \alpha x \sin \alpha x'}{\alpha} \frac{2\alpha d}{1 - \delta_f \delta_{fs}} d\alpha \\
&= - \frac{2d(\epsilon_f^2 - \epsilon_s^2)(\epsilon_f + 1)}{\epsilon_f(\epsilon_s + 1)^2} \int_0^\infty \sin \alpha x \sin \alpha x' d\alpha \\
&= - \frac{\pi d(\epsilon_f^2 - \epsilon_s^2)(\epsilon_f + 1)}{\epsilon_f(\epsilon_s + 1)^2} \delta(x - x')
\end{aligned} \tag{29}$$

if $x + x'$ is always positive.

Enforcing the condition $\phi = V/2$ on $a \leq x \leq b$, we use (26) and (29) in (16) to obtain an approximate integral equation for $\sigma(x)$:

$$\int_a^b \sigma(x') G_0(x, x') dx' = \pi \epsilon_0 V \left(\frac{\epsilon_S + 1}{2} \right) + F_1 \sigma(x) \quad (a < x < b) \quad (30)$$

where

$$F_1 = \frac{\pi d (\epsilon_f^2 - \epsilon_S^2)}{\epsilon_f (\epsilon_S + 1)} \quad (31)$$

Because of the approximations we have made, eqn. (30) will not hold near the edges of the strip ($x - a$ or $b - x = 0(d)$). However, the effect of this error on the capacitance C from (19) is expected to be small.

One further approximation to eqn. (30) will render it explicitly solvable. We replace the small term $F_1 \sigma(x)$ on the right side of (30) by its average value over the interval (a, b) . By (19), this average value will be $(\epsilon_S + 1) F_1 CV/w$, where C is the yet unknown capacitance per unit length of the CPS. We thus have

$$\int_a^b \sigma(x') G_0(x, x') dx' = V(\epsilon_S + 1) \left[\frac{\pi \epsilon_0}{2} - \frac{F_1 C}{\epsilon_f + 1} \frac{C}{w} \right] \quad (32)$$

Since the right side of (32) is constant, this equation has the same form as that of (A.1) in the Appendix. Applying (A.11) and (19), we have that

$$\int_a^b \sigma(x) dx = CV = V(\epsilon_S + 1) \frac{C_0}{\epsilon_0} \left[\frac{\epsilon_0}{2} - \frac{F_1 C}{\pi w (\epsilon_f + 1)} \right] \quad (33)$$

or from (31),

$$C = \frac{C_0 (\epsilon_S + 1) / 2}{1 - \frac{C_0 d}{\epsilon_0 w} \frac{\epsilon_f^2 - \epsilon_S^2}{\epsilon_f (\epsilon_S + 1)}} \quad (34)$$

as presented in eqn. (13). From (20), then, it easily follows that the effective permittivity for the quasi-TEM mode of this CPS is

$$\epsilon_{\text{eff},m} = N_m^2 = \frac{(\epsilon_s + 1)/2}{1 - \frac{C_o}{\epsilon_o} \frac{d}{w} \frac{\epsilon_f^2 - \epsilon_s^2}{\epsilon_f(\epsilon_s + 1)}} \quad (35)$$

Finally the characteristic impedance of the mode is:

$$Z_c = \frac{\zeta_o}{N_m} \frac{\epsilon_o}{C_o} \quad (36)$$

where $\zeta_o = (\mu_o/\epsilon_o)^{\frac{1}{2}} \cong 377 \Omega$ is the wave impedance of free space.

3. PROPOSED DESIGN OF ELECTRO-OPTIC MODULATOR

In this section we will apply formula (35) to the design of an electro-optic modulator in the form proposed by Yamashita et al. [11] shown in Fig. 3. The index-matched condition of $N_o = N_m$ is what we want to achieve. If this is met, then according to eqn. (36), the characteristic impedance of the CPS will be fixed at

$$Z_c = \frac{Z_{co}}{N_o} \quad (37)$$

where

$$Z_{co} = \frac{\zeta_o \epsilon_o}{C_o} = \zeta_o \frac{K(k)}{K(k')} \cong \frac{\pi \zeta_o}{2 \ln(8w/g)} \quad (38)$$

is the characteristic impedance of the CPS in air. The approximation in the last term is accurate for $w \geq g$.

According to (35), therefore, our design equation will be

$$N_o^2 \left[1 - \frac{K(k')}{K(k)} \frac{d}{w} \frac{\epsilon_f^2 - \epsilon_s^2}{\epsilon_f(\epsilon_s + 1)} \right] = \frac{\epsilon_s + 1}{2} \quad (39)$$

or, in terms of the actual parameters of the anisotropic substrate (cf. (14)):

$$N_o^2 \left[1 - \frac{K(k')}{K(k)} \frac{h}{w} \frac{\epsilon_{xf} \epsilon_{yf} - \epsilon_{xs} \epsilon_{ys}}{\epsilon_{yf} (\sqrt{\epsilon_{xs} \epsilon_{ys}} + 1)} \right] = \frac{\sqrt{\epsilon_{xs} \epsilon_{ys}} + 1}{2} \quad (40)$$

A design procedure would then go as follows:

1. Choose a value of d (equivalently, h) large enough to accommodate an optical channel waveguide comfortably.
2. Choose a ratio of w/g which will achieve a desired value of $Z_c = Z_c^*$.

From (37) and (38), this gives

$$\frac{w}{g} = \frac{1}{8} \exp \left(\frac{\pi \zeta_o}{2 N_o Z_c^*} \right) \quad (41)$$

3. Choose a ratio of d/w so that $N_o = N_m$. From (39), (37) and (38), we have

$$\frac{d}{w} = \frac{\epsilon_f(\epsilon_s+1)}{\epsilon_f^2 - \epsilon_s^2} \left[N_o^2 - \frac{\epsilon_s+1}{2} \right] \frac{N_o Z_c^*}{\zeta_o} \quad (42)$$

The value of ϵ_s is somewhat arbitrary (it must be less than $2N_o^2 - 1$ to assure that (42) gives a positive value for d). For given N_o , ϵ_f and Z_c^* , eqn. (42) can be satisfied with the smallest possible w (for $\epsilon_f \gg \epsilon_s$) if $\epsilon_s = N_o^2 - 1$. However, the eventual choices of material will be dictated by considerations of compatibility between the two substrate layers and ease of fabrication.

As an illustration, let us choose as our electro-optic material LiNbO_3 , with $\epsilon_{xf} = 28$ and $\epsilon_{yf} = 43$, while $N_o^2 = 5$.

1. Choose $d = 15 \mu\text{m}$ ($h = 18.6 \mu\text{m}$) to comfortably accommodate the optical waveguide
2. Suppose we wish to achieve $Z_o^* = 50 \Omega$. Then by (41),

$$\frac{w}{g} = 24.96$$

3. By (42), if we suppose that $\epsilon_s = 4$, $\frac{d}{w} = 0.109$.
4. By these results, $w = 0.138 \text{ mm}$, and $g = 5.5 \mu\text{m}$, both of which are quite reasonable values.

The very large values of w/g which tend to be required in this design are actually also quite desirable for the distinct reason that lower electrode losses are achieved in this range. In principle, therefore, a very attractive possibility exists for the implementation of large bandwidth electro-optic modulators using the structure of Fig. 3.

As a second example, let us choose (for mechanical reasons) a thicker electro-optic layer as follows:

1. Choose $d = 100 \mu\text{m}$ ($h = 124 \mu\text{m}$).
2. If we want $Z_C^* = 50 \Omega$, we again have $\frac{w}{g} = 24.96$.
3. If $\epsilon_S = 4$, then once again we have $\frac{d}{w} = 0.109$.
4. With these numbers, $w = 0.917 \text{ mm}$, and $g = 36.8 \mu\text{m}$. The gap width may be somewhat large in terms of the most efficient structure, but this may be an acceptable trade-off in view of the greater ease of fabricating the electro-optic layer.

4. DISCUSSION AND CONCLUSION

There are some difficulties of a practical nature associated with the proposed design which must be overcome. A suitable substrate material (ϵ_s) might be found which could be bonded to a layer of LiNbO_3 of $\sim 20 \mu\text{m}$ thickness. Possibilities include Si or SiO_2 . However, the most common method of forming the optical channel waveguide in LiNbO_3 is diffusion. This process requires such high temperatures that thermal stresses are likely to detach the electro-optic layer from the substrate. A challenging problem in materials science is thus posed.

A similar derivation to that of section 2 could be done for a coplanar waveguide (CPW) on such a layered substrate. Formulas valid for large d/w can also be obtained, and will be reported in a future publication.

APPENDIX

Consider the integral equation

$$\int_a^b \sigma(x') \ln \left| \frac{x+x'}{x-x'} \right| dx' = C_0 = \text{constant on } a \leq x \leq b \quad (\text{A.1})$$

We may solve this equation by transforming it into a form whose solution is well-known. First, we differentiate (A.1) with respect to x and make the changes of variable $t = x^2$, $t' = x'^2$. The result is

$$\int_a^{b^2} f(t') \frac{dt'}{t-t'} = 0 \quad (a \leq x \leq b) \quad (\text{A.2})$$

where $f(t) = \sigma(\sqrt{t})$, and the bar through the integral sign denotes that the Cauchy principle value is to be taken at $t = t'$. Use of the Schwinger transformation

$$\left. \begin{aligned} t &= \frac{a^2 + b^2}{2} + \frac{b^2 - a^2}{2} y \\ t' &= \frac{a^2 + b^2}{2} + \frac{b^2 - a^2}{2} y' \end{aligned} \right\} \quad (\text{A.3})$$

casts (A.2) into standard form:

$$\int_{-1}^1 \frac{g(y')}{y-y'} dy' = 0 \quad (-1 \leq y \leq 1) \quad (\text{A.4})$$

where $g(y) = f(t)$. The solution of (A.4) is well-known [23]:

$$g(y) = \frac{C_1}{\sqrt{1-y^2}} \quad (\text{A.5})$$

where the constant C_1 must be determined by recourse to the original equation (A.1), to retrieve the constant lost by differentiation.

Retracing our changes of variable, this gives us

$$\sigma(x) = \frac{C_2}{\sqrt{(b^2-x^2)(x^2-a^2)}} \quad (\text{A.6})$$

The constant C_2 is obtained by substituting (A.6) into (A.1):

$$C_2 \int_a^b \frac{\ln \left| \frac{x+x'}{x-x'} \right|}{\sqrt{(b^2-x'^2)(x'^2-a^2)}} dx' = C_0 \quad (\text{A.7})$$

This equation is identically satisfied for all $a \leq x \leq b$, so in particular if we let $x \rightarrow b$, and use the following result from [24]:

$$\int_a^b \frac{\ln \left(\frac{b+x'}{b-x'} \right)}{\sqrt{(b^2-x'^2)(x'^2-a^2)}} dx' = \frac{\pi}{b} K(k) \quad (\text{A.8})$$

where $K(k)$ is the complete elliptic integral of the first kind of modulus $k = a/b$, then

$$C_2 = \frac{bC_0}{\pi K(k)} \quad (\text{A.9})$$

It might also be noted from [24] that

$$\int_a^b \frac{dx}{\sqrt{(b^2-a^2)(x^2-a^2)}} = \frac{K(k')}{b} \quad (\text{A.10})$$

where $k' = \sqrt{1-k^2} = \sqrt{1-a^2/b^2}$. We thus have that

$$\int_a^b \sigma(x) dx = \frac{C_0}{\pi} \frac{K(k')}{K(k)} \quad (\text{A.11})$$

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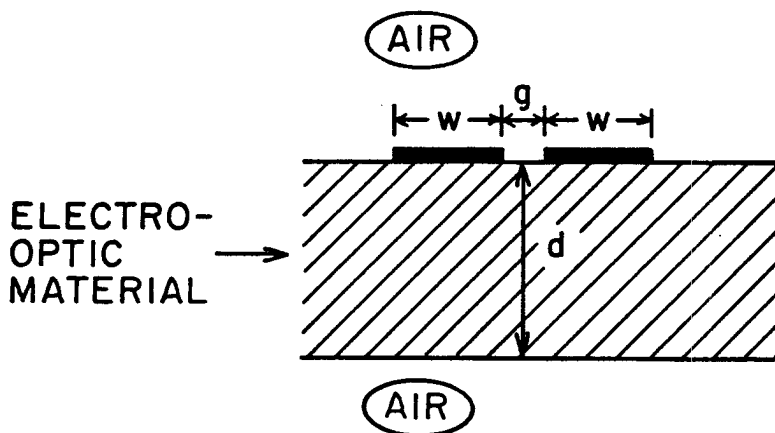


Fig. 1: Coplanar strip line as modulator electrodes (cross-section).

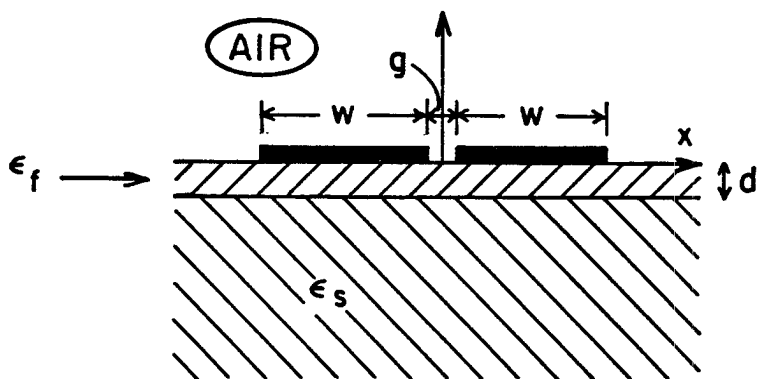


Fig. 2: Coplanar strip line on a thin electro-optic film over a lower- ϵ substrate.

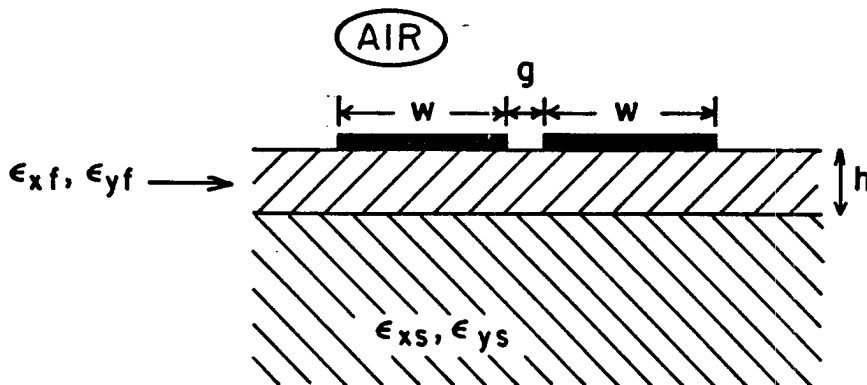


Fig. 3: Coplanar strip line on an anisotropic two-medium substrate.

