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Abstract

From a consideration of the properties near the edge of a flat finite-thickness strip and an elliptic cross-section strip, it is shown that the divergence can be handled by halting the loss calculation at a definite distance just short of the strip edge. This distance can be expressed in terms of the radius of curvature at the tip for a rounded edge, and in terms of the strip thickness for a flat edge.

1. Introduction

A commonly-used loss calculation method proceeds by taking the tangential magnetic field, as determined for the loss-less case, calculating the surface currents on the metal boundaries and then calculating the loss from these currents on the assumption that they are not affected substantially in form by the finite metal conductivity. The method can be applied if the metal thickness is everywhere sufficiently greater than the skin depth, and can even be used for infinitely thin strips or diaphragms when the very small amount of thickness needed would not alter the field substantially from the theoretical value in the ideal (zero-thickness) case. The one place where this cannot be done is for the axial current along a strip edge since the ideal current density near the edge varies as $r^{-\frac{1}{2}}$, where r is the distance from the edge. For this variation, the needed square of the current density varies as $1/r$ and produces a logarithmic divergence if integrated to the strip edge at $r = 0$. Since all such strips in practice have a certain thickness, and often a slightly rounded edge, an examination of the local fields in such cases may be expected to show how to deal with the divergence.

One method of avoiding this divergence is to simply assume a finite thickness and to carry out the much more involved analysis for the thick strip case. This has been done by Cockroft(1) for isolated rectangular conductors, and by Kaden(2) for a microstrip configuration. The problem with this method is that it requires a major change in the field calculations just in order to accommodate a local feature in the immediate neighborhood of the edge. Nosich and Shestopalov(3), assuming a rounded edge of diameter equal to the strip thickness, noted that the magnetic field from a cylinder of radius w varies as $1/kw$, so that the actual field near the edge singularity should not become greater than this. They therefore stop the integration short by the requisite amount, of the order of $(kw)^2$, where $k = 2\pi/\lambda$. Not only is this truncation limit somewhat indefinite, but it is also frequency dependent, and assumes that the edge is rounded like a circular cylinder.

A more precise analysis, combining the technique used by Cockroft with the truncation method of Nosich and Shestopalov, enables the calculations with the ideal fields to be used, with only a minor modification to take the edge features into account. Moreoever, since the effect is a very local one, in the immediate proximity of the edge, a quasi-static examination of a suitable canonical problem should be sufficient to show the essential features, providing the result can be expressed in terms of local geometrical parameters only.

2. Conformal Transformation

If an infinite strip of suitable cross-section is examined as a quasi-static problem, the stream function U and the potential V are related to the coordinates x and y by an equation of the form

$$x + iy = F(U+iV) \tag{1}$$

where F is an analytic function which can be determined from the shape of the cross-section. If n is a variable along the direction of the outward normal and s a variable along the cross-section surface, then, for a TEM wave, $\partial V/\partial n$ and $\partial U/\partial s$ are respectively proportional to the normal electric field and the tangential magnetic field at the cross-section. The latter, at the boundary surface, is proportional to the axial current density. Hence, the loss perturbation calculation involves the integral $\int I^2 \, ds \propto \int (\partial U/\partial s)^2 \, ds = \int (\partial U/\partial s) dU$ taken between appropriate limits. The use to be made of this formula is to:

i) Calculate the exact expression for a finite thickness strip.

ii) Calculate the limiting form of the expression for an infinitely thin strip where the integration stops just short of the edge.

iii) By comparing i) and ii) above, determine how far short of the edge one should stop the integration in order to equal the exact result for the finite thickness strip.

iv) Put this result in terms of local geometric parameters only.

If step iv) can be achieved, what results is a construction for using the infinitely thin strip analysis to obtain results corresponding to a thin, non-zero thickness strip, thus permitting the perturbation analysis to proceed even in this case.

The method will be illustrated for the cases of a flat edge (thin rectangular strip) and a rounded edge (thin elliptic strip).

3. Flat Edge

Figure 1 shows a flat edge with the coordinate x varying from 0 to some value D large enough that local conditions at $x = D$ do not affect appreciably the field near $x = 0$. The y coordinate at the edge varies from 0 to $-L$ and the conformal transformation between $z = x + iy$ and $W = U + iV$ comes from integrating $dz/dw = C(W^2-1)^{\frac{1}{2}}$ and determining the constants so that $z = 0$ corresponds to $W = 1$, $z = -iL$ to $W = -1$. Thus, the rectangular strip itself is the equipotential $V = 0$ and the integration gives

$$\pi z/L = W(W^2-1)^{\frac{1}{2}} - \log\,[W+(W^2-1)^{\frac{1}{2}}] \qquad (2)$$

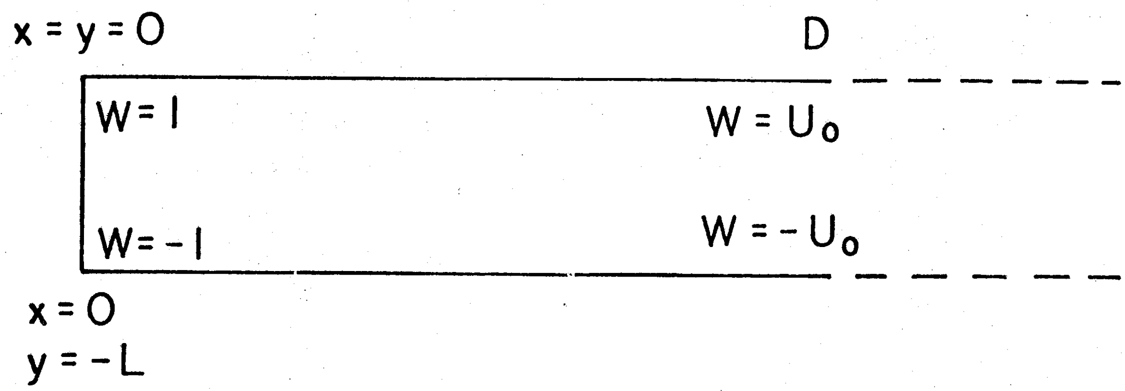

Figure 1. Rectangular Strip Cross-Section

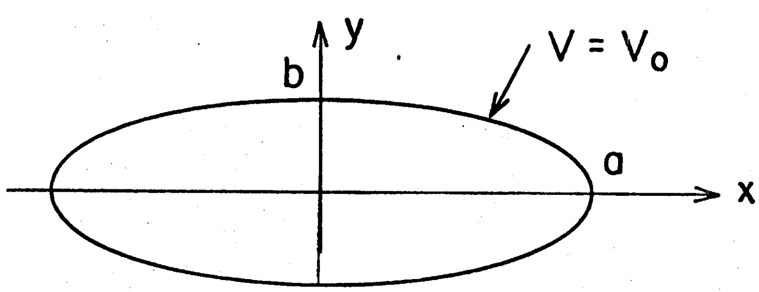

Figure 2. Elliptic Cross-Section Strip

At $V = 0$ on the strip surface, this gives

a) Upper surface, $y = 0$, $0 < x < D$,

$$\pi x/L = U(U^2-1)^{\frac{1}{2}} - \log\,[U+(U^2-1)^{\frac{1}{2}}] \qquad (3)$$

$$1 < U < U_o ,$$

where $U_o = (\pi D/L)^{\frac{1}{2}} + 0(D^{-\frac{1}{2}})$ for large D.

b) Vertical surface, $x = 0$, $-L < y < 0$,

$$\pi y/L = U(1-U^2)^{\frac{1}{2}} - \cos^{-1} U \qquad (4)$$

$$-1 < U < 1 .$$

c) Lower surface, $y = -L$, $0 < x < D$,

$$\pi x/L = -U(U^2-1)^{\frac{1}{2}} - \log\,[-U+(U^2-1)^{\frac{1}{2}}] \qquad (5)$$

$$-U_o < U < -1 .$$

The required integrations give the losses P_1 proportional to

$$P_1 = \int_1^{U_o} \frac{dU}{dx}\,dU + \int_{-1}^{1} \frac{dU}{dy}\,dU + \int_{-1}^{-U_o} \frac{dU}{dx}\,dU$$

$$= \frac{\pi}{2L}\left\{2\int_1^{U_o} \frac{dU}{(U^2-1)^{\frac{1}{2}}} + \int_{-1}^{1} \frac{dU}{(1-U^2)^{\frac{1}{2}}}\right\}$$

$$= \frac{\pi}{2L} \log\left(\frac{4\pi e^{\pi} D}{L}\right) + 0(D^{-\frac{1}{2}}) \qquad (6)$$

The corresponding equation for an infinitely thin strip to give $x = D$ at $U = U_o$ is

$$\pi x/L = U^2 \tag{7}$$

If we take the lower integration limit to $x = d$, the corresponding loss calculation gives

$$P_2 = 2\int_{U_1}^{U_o} \frac{dU}{dx}\, dU = \frac{\pi}{L}\int_{U_1}^{U_o} \frac{dU}{U} = \frac{\pi}{L}\log\frac{U_o}{U_1} \tag{8}$$

where $U_o = (\pi D/L)^{\frac{1}{2}}$, $U_1 = (\pi d/L)^{\frac{1}{2}}$.

Hence

$$P_2 = \frac{\pi}{2L}\log\frac{D}{d} \tag{9}$$

Comparing this with eq. (6) gives

$$d = L/(4\pi e^{\pi}) \approx L/290 \tag{10}$$

Thus the usual loss calculation can be made and will yield a result equal to that for a thin strip if the integration is taken to a distance from the strip edge of approximately one three-hundredth of the strip thickness. It may be noted that eq. (10) is "local" in the sense that the extension D does not enter into the expression.

4. Rounded Edge

Figure 2 shows a rounded edge as one apex of an elliptic cross-section of semi-axes a and b. The radius of curvature at this apex is

$$R = b^2/a \tag{11}$$

The coordinates and the potentials are related by

$$x + iy = A \sin (U+iV) \tag{12}$$

where A is a scale factor. From eq. (12)

$$\begin{aligned} x &= A \sin U \cosh V \\ y &= A \cos U \sinh V \end{aligned} \tag{13}$$

and the equation of an ellipse at potential V_o is

$$\frac{x^2}{A^2 \cosh^2 V_o} + \frac{y^2}{A^2 \sinh^2 V_o} = 1 \tag{14}$$

Hence $A \cosh V_o = a$, $A \sinh V_o = b$, giving $\tanh V_o = b/a$ (15)

If the ellipse degenerates into an infinitely thin strip, then $b = 0$ and $V_o = 0$. For non-zero thin strips, b/a will be small, as will $V_o = 0$, and we shall be concerned with very small but non-zero values of V_o .

The element $ds = [(dx)^2+(dy)^2]^{\frac{1}{2}} = AdU[\cosh^2 V_o - \sin^2 U]^{\frac{1}{2}}$ from eq. (13). Hence the loss associated with the ellipse is proportional to

$$P_3 = \int_0^{2\pi} \frac{dU}{ds}\, dU = \frac{4}{A} \int_0^{\pi/2} \frac{dU}{[\cosh^2 V_o - \sin^2 U]^{\frac{1}{2}}}$$

$$= \frac{4}{a} K\,(1/\cosh V_o) \tag{16}$$

where K is the complete elliptic integral of the first kind. For V_o small the modulus is close to unity, and eq. (16) has the expansion

$$P_3 = \frac{4}{a} \log\,(4 \coth V_o) + 0(V_o^2) \approx \frac{4}{a} \log\,(4a/b) \tag{17}$$

For an infinitely thin strip, $V_0 \to 0$, $x \to a \sin U$ and the losses are given by

$$P_4 = 4 \int_0^{a-d} \left(\frac{dU}{dx}\right)^2 dx = 4 \int_0^{a-d} \frac{dx}{a^2-x^2} \approx \frac{2}{a} \log \frac{2a}{d} \qquad (18)$$

for small d .

Comparing with eq. (17) gives

$$d = b^2/8a = R/8 \qquad (19)$$

where $R = b^2/a$ is the radius of curvature at the apex. Again, this is a "local" result in that it depends only on geometrical parameters close to the edge.

5. Comparison of Flat and Rounded Edges

The total losses of a flat strip with rounded edges is proportional to log (16 a/R) from eq. (18), and of course, now depends on the strip width $2a$. For a circular rounding, the strip thickness would be $2R$. Were the edge flat, eq.(10) would apply with L replaced by $2R$, giving a loss proportional to

$$\log [2a/(2R/4\pi e^{\pi})] = \log (290a/R) = \log (16a/R) + 2.90 .$$

The extra contribution 2.90 corresponds to the excess losses associated with a flat rectangular edge versus a rounded edge, and accords with the intuitive assessment of microwave engineers that sharp (versus rounded) edges increase the loss. Just how important this term is depends on the total configuration. Thus, a 5 mm wide strip, 0.1 mm thick, with rounded edges, has a loss proportional to log (800) = 6.68.

Failure to round gives 6.68 + 2.90 = 9.58, a 43% increase. Thus, attention to the rounding of the strip edges in constructions such as microstrip can have an appreciable effect in reducing the ohmic losses of the line.

6. Conclusions

The method of halting the loss integration a determinate distance just short of an edge enables loss perturbation calculations to be made with fields calculated for structures with infinitely thin strips or diaphragms. This should simplify such calculations considerably. The method encompasses a finite strip thickness with either a flat or rounded edge. In fact, the transformation of section 3 would, if pursued to a potential surface $V_0 \neq 0$, be able to handle a variety of combinations of rounding and thickness. The effect of rounding the edge from a flat rectangular shape is shown to confirm the known appreciable effect in reducing the losses due to the high current densities associated with sharp edges.

References

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