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Abstract

Because of its proximity to the city of Naples and with a population of nearly 1 million people within its caldera, Campi Flegrei is one of the highest risk volcanic areas in the world. Since the last major eruption in 1538, the caldera has undergone frequent episodes of ground subsidence and uplift accompanied by seismic activity that has been interpreted as the result of a stationary, deeper source below the caldera that feeds shallower eruptions. However, the location and depth of the deeper source is not well-characterized and its relationship to current activity is poorly understood. Recently, a significant increase in the uplift rate has occurred, resulting in almost 13 meters of uplift by 2013 (De Martino et al., 2014; Samsonov et al., 2014b; Di Vito et al., 2016).
Here we apply a principal component decomposition to high resolution time series from the region produced by the advanced Multidimensional SBAS DInSAR technique in order to better delineate both the deeper source and the recent shallow activity. We analyzed both a period of substantial subsidence (1993-1999) and a second of significant uplift (2007-2013) and inverted the associated vertical surface displacement for the most likely source models. Results suggest that the underlying dynamics of the caldera changed in the late 1990s, from one in which the primary signal arises from a shallow deflating source above a deeper, expanding source to one dominated by a shallow inflating source. In general, the shallow source lies between 2700 and 3400 m below the caldera while the deeper source lies at 7600 m or more in depth. The combination of principal component analysis with high resolution MSBAS time series data allows for these new insights and confirms the applicability of both to areas at risk from dynamic natural hazards.

1. Introduction

A significant portion of the city of Naples lies within the Campi Flegrei caldera, along with the town of Pozzuoli and a number of densely inhabited villages, making it one of the most dangerous volcanic areas on Earth (Orsi et al., 2004; De Natale et al., 2006; Isaia et al., 2009). The last major eruption occurred at Monte Nuovo in 1538, following a period of ground uplift which interrupted a period of secular subsidence that has persisted for centuries. During that time, Campi Flegrei has undergone frequent episodes of ground subsidence and uplift accompanied by seismic activity (Troise et al., 2007). Most recently, in June of 2010 moderate uplift rates were observed that substantially increased in 2011 and further accelerated in 2012. Between 2010 and 2013, maximum uplift reached approximately 13 cm, as identified by differential Interferometric Synthetic Aperture Radar (DInSAR) (Samsonov et al., 2014).
DInSAR is used extensively today for mapping ground deformation with high spatial resolution and sub-centimeter precision over large areas, and is a suitable tool for deformation monitoring of active volcanic areas (Massonnet and Feigl, 1998; Rosen et al., 2000; Wadge, 2003; Fernández et al., 2009). A radar interferogram is calculated from two SAR images with identical characteristics acquired by space- and/or air-borne sensors at two different times and captures the intervening deformation. Spatial resolution of modern SAR sensors ranges from 1 to 20 m over areas from 10x10 km to 200x200 km. For modern satellite constellations the repeat cycle ranges from a few days to a few weeks, with the typical repeat cycle for a single satellite mission at 24 to 41 days. Repeatedly acquired SAR data from a single sensor can be used to obtain line-of-sight (LOS) time series analysis of surface displacement through the application of either Small Baseline Subset (SBAS) (Berardino et al., 2002; Usai, 2003; Samsonov et al., 2011), Persistent Scatterers (PS) (Ferretti et al., 2001) methods or their combination (Hooper, 2008). The results are limited to the time period of the individual data set and do not automatically distinguish between horizontal and vertical motion.

The Multidimensional SBAS (MSBAS) technique (Samsonov and d'Oreye, 2012) combines multiple DInSAR data sets into a single solution. Improved characteristics include lower noise and improved temporal resolution with almost uninterrupted temporal coverage. The MSBAS methodology is an extension of the original SBAS method. MSBAS addresses the data redundancy and multidimensionality of the problem by decomposing LOS DInSAR measurements into the vertical and horizontal (east-west) time series of surface deformation using ascending and descending DInSAR data. MSBAS recently has been applied to the mapping of anthropogenic (Samsonov et al., 2013, 2014a) and natural (Samsonov and d'Oreye, 2012) deformation.
2012; Samsonov et al., 2014b) ground deformation, successfully producing two-dimensional
time series with dense temporal resolution and high precision.

In this study we apply a principal component decomposition technique to an MSBAS DInSAR
time series of more than twenty years, produced from ERS-1/2, ENVISAT and RADARSAT-2
data at Campi Flegrei, Italy (Figure 1) (Samsonov et al., 2014b). Various versions of principal
component analysis (PCA) filtering techniques have been developed and applied over the past 28
years with the goal of reducing or removing the various noise sources in the position time series.

For example, in the first successful geodetic application, Savage (1988) decomposed
displacements at Long Valley caldera into the predominant modes in order to study only the
signal that accounted for the greatest percentage of the variance, the volcanic source below the
dome. In addition, he identified the primary error sources in the data using the remaining
eigenmodes. Tiampo et al. (2004) employed a Karhunen-Loeve expansion (KLE) analysis to
study spatiotemporally correlated mass loading caused by seasonal deformation in Southern
California Integrated GPS Network (SCIGN) position series data. Dong et al. (2006) later
employed common mode error (CME) filtering using both PCA and KLE techniques in order to
identify signal and systematic error in regional GPS position time series. Zerbini et al. (2010)
applied a similar technique to GPS and gravity data in northeastern Italy and succeeded in
identifying hydrology-related correlated variations, while Chaussard et al. (2014) used a PCA
decomposition of DInSAR data to study aquifer changes in northern California. Here, because
the MSBAS data produces a time series with unprecedented duration and resolution for this
region, the PCA produces individual spatial and temporal modes at high resolution in both space
and time. Various combinations of the resulting eigenmodes are inverted using a genetic
algorithm (GA) inversion technique and a combination of simple spherical pressure models.
(Mogi, 1958). The combination of these three techniques – MSBAS DInSAR, PCA decomposition and GA inversion - results in the improved characterization of the two separate sources below Campi Flegrei and new insights into the interactions between the deeper source and the recent shallow activity. Results suggest that the underlying dynamics of the caldera changed in the late 1990s, from one in which the primary signal arises from a shallow deflating source above a deeper, expanding source to one dominated by a shallow inflating source.

In Section 2 we provide an overview of historic activity at Campi Flegrei. Section 3 provides details on the MSBAS technique and the SAR data used in this study. Section 4 describes the PCA technique and the resulting decomposition of the MSBAS time series into their eigenmodes and principal components. The GA inversion technique and its application to the significant eigenmodes are detailed in Section 5. The last section discusses the results and implications of this analysis for constraining different geophysical sources at Campi Flegrei.

2. Campi Flegrei

The Campi Flegrei caldera, located east of the city of Naples in southern Italy, was formed by two major eruptions at 35 and 15 ka (Figure 1) (Rosi et al., 1983). In modern times, a significant eruption occurred in 1538 and, since then, Campi Flegrei caldera has undergone frequent episodes of ground subsidence and uplift accompanied by seismic activity (Troise et al., 2007). Di Vito et al. (2016) has interpreted pre-eruptive magma transfer since 1538 as the result of a stationary oblate source deeper below the caldera that has been feeding shallower sources and eruptions for the last 5 ka.

The most recent uplift probably began in 1950 and included two major periods of seismic unrest in 1969–1972 and 1982–1984, before reaching a maximum value of about 3.5 m in 1985 (Gottsman et al., 2006a; Del Gaudio et al., 2010; D’Auria et al., 2011). Since 1988, slow
deflation intermittently has been interrupted by periods of seismic swarms and minor uplift, until the mid-2000s (Trasatti et al., 2015). At that time, a significant increase in the uplift rate took place, resulting in almost 13 meters of uplift by 2013 (De Martino et al., 2014; Samsonov et al., 2014b).

Between 1981 and 2001, surveys at Campi Flegrei revealed significant gravity changes. Interpretation, in conjunction with deformation data, suggested that the phenomena are the result of changes in the caldera hydrothermal systems (Bonafede & Mazzanti, 1998), activity within the subsurface magmatic reservoir (Dvorak & Berrino, 1991; Fernández et al., 2001), or some combination of the two (Gottsmann et al., 2005, 2006). Recent petrological and geochemical studies suggest that there are two magmatic sources that differ in composition, depth and size, and that the periodic episodes of uplift and unrest are the result of reinjection of CO2-rich fluids and magma (Caliro et al., 2007; Arienzo et al., 2010; Mormone et al., 2011; D’Auria et al., 2011, 2012; Moretti et al., 2013; Amoruso et al., 2014a; Trasatti et al., 2015).

Over the years, a number of different models have been ascribed to the shallow source, or sources, at Campi Flegrei. These generally include a primary inflation source between 2-4 km in depth below the caldera and some combination of shallower hydrothermal sources near the Solfatara crater (De Natale et al. 1991, De Natale et al., 2006; Gottsmann et al., 2005, 2006; Amoruso et al., 2008; Trasatti et al., 2011; Amoruso et al., 2014a; Samsonov et al., 2014b; Trasatti et al., 2015, among others), although some studies have attributed CF activity primarily to fluid injection in the hydrothermal system (Battaglia et al., 2006; Troiano et al., 2011).
Battaglia et al. (2006) inverted levelling, trilateration and gravity from the period between 1980 and 1995 and found that the inflation period during the 1980s was the result of a penny-shaped crack at a depth of approximately 3 km and the subsequent deflation was generated by a source shaped like a prolate spheroid at a depth between 1.9 and 2.2 km deep. More recently, Amoruso et al. (2015) modeled observed strain changes from March of 2010 as the result of volume changes in an offshore, slightly deeper ellipsoidal magma source at approximately 3200 m depth. Tomographic studies suggest that there is a high Vp/Vs ratio at shallow depths, indicating infiltration by hydrothermal fluids (Chiarabba and Moretti, 2006; Zollo et al., 2008). Seismic attenuation results also identify potential melt volumes at a depth of approximately 3500 and 7500 m below the caldera (De Siena et al., 2010). The models of Trasatti et al. (2011) suggest that this shallower source is fed by a deep sill, again at approximately 7500 km in depth. The recent ground deformation, 2012-2013, was modeled using DInSAR and GPS measurements as the result of a sill-like magma intrusion at approximately 3090 meters in depth (D’Auria et al., 2015), while Amoruso et al. (2014b) demonstrated that a deeper source (~3600 m), combined with the shallower Solfatara hydrothermal source, can explain the continuous GPS (cGPS) displacements since 2011. Both the subsidence period of 1993-1999 and the more recent uplift, 2007–2013, were modeled, again using DInSAR data, as the result of activity in an extended source at depths of approximately 1400 to 2000 m depth (Samsonov et al. 2014b).

Here we apply a PCA decomposition technique (Tiampo et al., 2012) and a GA inversion method to the DInSAR MSBAS data of Samsonov et al. (2014) in order to better discriminate between the potential sources at Campi Flegrei.

3. MSBAS Analysis
The theoretical derivation of the MSBAS technique is described in detail in Samsonov and d'Oreye (2012) and Samsonov et al. (2013). The technique is derived from the original SBAS method proposed in Berardino et al. (2002) and Usai (2003) but incorporates images from different satellites, coverage and look angles in order to produce two-dimensional time series of ground deformation. At least two sets of DInSAR data are needed, one from ascending and the other from descending orbits. The technique, however, efficiently handles a large number of DInSAR data sets to produce results with improved temporal resolution and precision. Basic DInSAR processing is performed outside of the MSBAS software, using either freely available (e.g. ISCEE, GMT5SAR) or commercial (GAMMA, SARscape) packages. Differential interferograms are processed, filtered, unwrapped and geocoded with the processing software and then resampled to a common grid. The final interferograms are in either angular (e.g. radian) or metric (e.g. cm or m) units, preserved during the MSBAS processing. The topographic correction is accomplished by a joint inversion that solves for the two-dimensional displacements and the residual topographic signal (Samsonov et al., 2011). The resulting deformation maps presented in this work were calculated from two decades of SAR measurements from three different SAR sensors (ERS-1/2, ENVISAT and RADARSAT-2). Individual frames are shown in Figure 1.

We processed five independent SAR data sets, described in Table 1, with uninterrupted coverage from 1992 through 2013. We applied 2x10 multilooking to four standard beams and 4x5 multilooking to one fine beam and independently processed each data set using GAMMA software (Wegmuller and Werner, 1997). All possible interferometric pairs with perpendicular baselines less than 400 m were computed and the topographic signal was removed using a 90 m resolution SRTM DEM (Farr and Kobrick, 2000). Orbital refinement to remove residual orbital
ramps was performed and minor interpolation applied to fill gaps in moderately coherent regions. The final interpolated interferograms were geocoded onto a 90x90 m grid.

TABLE 1

For the time series analysis we limited data to the Naples Bay area and resampled all interferograms to a common grid (Wessel and Smith, 1998). The final interferograms had a resolution of approximately 90x90 m. We selected only those with an average coherence above 0.5 for further processing. Over one thousand highly coherent interferograms then were used in the MSBAS processing, resulting in a time series with 385 time steps. Average error on the vertical displacement is approximately 0.09 cm and that of the east-west time series is approximately 0.07 cm (Samsonov et al., 2014b).

The results of the MSBAS processing are presented in Figure 2. Figure 2a shows the vertical change in surface height between the initial and final time steps while Figure 2b is the east-west net displacement. The associated displacement time series are shown in Figure 2c for the location designated with the pink star. They present a more complicated picture than the net subsidence of Figures 2a and 2b. The time series show more than 3 cm/yr of maximum subsidence (green dot) between 1993 and 199, centered on the caldera. Subsidence continued at a slower rate, interspersed with short periods of uplift, until 2005. Almost continuous uplift began in 2005 and accelerated to approximately 2.5 cm/yr between 2008 and 2011. Deformation is ongoing at a rate of 5 cm/yr (2011–2013). The large number of time steps and precise measurements are evident in both the vertical and east-west time series (Figure 2c). The pattern...
of deformation in Figure 2 is consistent with one or more sources of contraction and expansion located at depth below the caldera.

**FIGURE 2**

Figure 3 shows the net surface displacement for two different time periods, 1993 through 1999 and 2007 through 2013. The subsidence that occurred between 1993 and 1999 can be seen in Figure 3a, while the corresponding east-west displacement is provided in Figure 3b. Figure 3c presents the uplift period of 2007-2013 and Figure 3d shows the associated east-west displacements.

**FIGURE 3**

4. **PCA Analysis**

The Karhunen-Loeve expansion (KLE) method is a linear decomposition technique in which a dynamical system is decomposed into a complete set of orthonormal subspaces. Depending on the specific decomposition, and whether it is used to characterize the variance or correlation in the data, it also has been known as PCA or empirical orthogonal function (EOF) decomposition. The method, in one form or another, has been applied to a number of complex nonlinear systems over the last fifty years, including the ocean-atmosphere interface, turbulence, meteorology, biometrics, statistics, and geophysics (Hotelling, 1933; Fukunaga, 1970; Aubrey and Emery, 1983; Preisendorfer, 1988; Savage, 1988; Penland, 1989; Vautard and Ghil, 1989; Posadas et al., 1993; Penland and Sardeshmukh, 1995; Holmes et al., 1996; Moghaddam et al., 1998; Tiampo et al., 2002; Dong et al., 2006; Main et al., 2006; Small and Islam, 2007; Smith et al., 2007).

Again, Savage (1988) decomposed the deformation at Long Valley caldera into its predominant modes in order to study only the signal that accounted for the greatest percentage of the variance,
the volcanic source below the dome. In addition, he identified the primary error sources in the
data using the remaining eigenmodes.

In an application of the KLE to historic seismicity data, Tiampo et al. (2002) constructed a
correlation operator, $C(x_i,x_j)$, for seismic events over time. Subsequently, $C(x_i,x_j)$ was
decomposed into its orthonormal spatial eigenmodes and associated time series, $a_j(t)$. These
spatial and temporal pattern states were used to reconstruct the primary modes of the system,
with or without noise, and to characterize the underlying dynamics and the physical parameters
that control the observable patterns of events. The decomposition implicitly assumes that one is
dealing with a process that is both Markov and stationary in time. Anghel et al. (2004) applied a
similar methodology to modeled deformation data with the goal of identifying coherent
structures and interactions. Tiampo et al. (2004) applied the KLE technique to SCIGN data in
order to determine the principal modes of deformation for the southern California fault system,
while Dong et al. (2006) applied a similar technique to SCIGN data in order to study the CME.
More recently, it has been applied to DInSAR time series studies, primarily for better
understanding of volcanic and groundwater changes (Lipovsky, 2011; Rudolph et al., 2013;
Chaussard et al., 2014; Remy et al., 2014).

As with the EOF technique developed by Preisendorfer (1988) for the atmospheric sciences, the
KLE for displacement applications uses those $p$ time series that record the deformation history at
particular locations in space. The primary difference is that while an EOF decomposition is
based on the covariance matrix, a KLE decomposition is performed on a correlation operator
(Fukunaga, 1970). For the study at Campi Flegrei, we employ an EOF operator.
Each time series, \( y(x_i,t_i) = y^s_i, s = 1, \ldots, p \), consists of \( n \) time steps, \( i = 1, \ldots, n \). The goal is to construct a time series for each of a large number of locations for a given short period of time. If, for example, the time interval was decimated into units of days, the result could be a time series of 365 time steps for every year of data, with values of position for that location at each time step. These time series are incorporated into a matrix, \( T \), consisting of time series of the same measurement for \( p \) different locations, i.e.

\[
T = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_p] = \begin{bmatrix}
y^1_1 & y^1_2 & \cdots & y^p_1 \\
y^2_1 & y^2_2 & \cdots & y^p_2 \\
\vdots & \vdots & \ddots & \vdots \\
y^n_1 & y^n_2 & \cdots & y^n_p
\end{bmatrix}
\]

(1)

For analysis of DInSAR data, the values in the matrix \( T \) consist of horizontal or vertical position measurements. The covariance matrix, \( S(x_i, x_j) \), for these events is formed by multiplying \( T \) by \( T^T \), where \( S \) is a \( p \times p \) real, symmetric matrix.

This equal-time covariance operator, \( S(x_i, x_j) \), is decomposed into its eigenvalues and eigenvectors in two parts. The first employs the trireduction technique to reduce the matrix \( S \) to a symmetric tridiagonal matrix, using a Householder reduction. The second part employs a QL algorithm to find the eigenvalues, \( \lambda_j \), and eigenvectors, \( e_j \), of the tridiagonal matrix (Press, et al., 1992). These eigenvectors, or eigenstates, are orthonormal basis vectors arranged in order of decreasing variance that reflect the spatial relationship of events in time. If one divides the corresponding eigenvalues, \( \lambda_j \), by the sum of the eigenvalues, the result is that percent of the correlation accounted for by that particular mode. We then reconstruct the time series associated with each location for each eigenstate by projecting the initial data back onto these basis vectors in what is called a PC analysis (Preisendorfer, 1988). These time dependent expansion
coefficients, $a_j(t)$, which represent the temporal eigenvectors, are reconstructed by multiplying the original data matrix by the eigenvectors, i.e.

$$a_j(t) = \xi^T \cdot T = \sum_{s=1}^{p} \xi_j y_{i_s},$$  \hspace{1cm} (2)

where $j,s = 1, \ldots p$ and $i = 1, \ldots n$. This eigenstate decomposition technique produces the orthonormal spatial eigenmodes for this nonlinear threshold system, $e_j$, and the associated principal component time series, $a_j(t)$. These principal component time series represent the signal associated with each particular eigenmode over time. For purposes of clarity, the spatial eigenvectors are designated EOF modes and the associated time series are the principal component analysis (PCA) vectors.

PCA often is used to filter data through the identification of those modes associated with large percentages of unwanted covariance or those lower modes accounting for random noise (Preisendorfer, 1988; Penland, 1989; Dong et al., 2006). As discussed above, others have applied the technique to investigate spatiotemporally correlated geophysical signals in the position time series. The first few PCs often represent the biggest contributors to the variance of the network residual time series and the higher-order PCs are related to local site effects (Tiampo et al., 2012). Here we decompose the MSBAS time series for Campi Flegrei into the dominant eigenmodes that describe the local source physics.

Here we performed two separate analyses. The first is a covariance PCA decomposition of the east-west MSBAS displacement time series and the second is the same analysis on the vertical MSBAS displacement time series. The matrix $T$ consists of the MSBAS time series for each
pixel in the region shown in Figure 1, or 5308 locations. Each time series consists of 385 time steps, and the resulting covariance matrix, \( S(x_i, x_j) \), has dimension 5308 by 5308.

The significant eigenmodes normally are selected by examination of the eigenvalue distribution, shown in Figure 4 (Preisendorfer, 1988; Tiampo et al., 2010). The first three eigenvalue, \( \lambda_j \), account for approximately 98% of the variance in the vertical time series and 95% of the east-west time series. As a result, the first three EOFs are selected for further analysis.

Figure 5 shows the first three EOFs and the associated PCA time series for the MSBAS vertical displacement time series. Note that the spatial eigenmodes, EOF1, EOF2 and EOF3, represent the amplitude of the signal that is accounted for at each point. The value at each location is then multiplied by the associated PCA time series in order to derive the actual time history attributed to each eigenmode at each location (Equation 2, above). In general, blue pixels are correlated with each other and anticorrelated with red pixels.

EOF1 (Figure 5a) appears to be directly related to the central source expected to lie below the caldera and the associated PCA time series (Figure 5b) is similar to that for the original MSBAS time series of Figure 2. PCA2 (Figure 5d) shows a predominant linear trend that appears to represent the relationship between the Solfatara hydrothermal activity and a longer wavelength signal encompassing the larger caldera footprint (Figure 5c). EOF3 (Figure 5e) also presents a longer wavelength signal, potentially related to tropospheric error in the images. The associated PCA time series (Figure 5f) is noisy and supports the conclusion that the two earlier modes
account for most of the signal in the data, despite the fact that the third mode might be
considered significant from Figure 4.

Figure 6 shows the first three EOFs and the associated PCA time series for the MSBAS east-west
displacement time series. Note that these three eigenmodes do not necessarily represent the
same activity seen in those recovered for the vertical displacements (Figure 5). However, the
time series in PCA1 (Figure 6b) corresponds closely to that of the first time series in Figure 5,
once the opposite signs are taken into account, suggesting a similar source process. Again, the
deformation pattern in EOF1 (Figure 6a) is similar to that expected from a volcanic source
located directly below the caldera. EOF2 and EOF3 (Figure 6c and Figure 6e) are less
conclusive. Figure 6f is likely a correction to Figure 6d. However, the relatively sudden onset of
new signal in 1997 suggests that some portion of the signal is related to the volcanic activity
itself.

EOF3 (Figure 6e) has a similar, but not identical pattern to that seen in EOF2. The strong signal
seen on the western peninsula is likely a result of tropospheric noise. In addition, we again
observe a strong signal in 1997 in PCA3 (Figure 6f), a pulse of activity that tapers off but
remains observable through 2013. It should be noted that an EOF analysis is a linear
decomposition of what are inherently nonlinear processes (Preisendorfer, 1988). The result is
often a mixture of signals, particularly in the lower, shorter wavelength signals.

Figures 7 and 8 show the time series at the three locations shown by green triangles in Figure 3,
obtained by summing EOF1, EOF2 and EOF3 in consecutive order (Equation 2, above). Figure
7 shows the vertical displacement. As expected, the vertical displacement at location c is very
similar to that seen in Figure 2c. However, location b, closer to the anticipated location of the central source is dominated by the linear subsidence that initiated in 2007. That signal dies away at location a and it appears to be dominated by the secondary uplift signal again associated with the primary source below the volcano.

FIGURE 7

Given that we observe what appears to be only two separately resolvable signals in the PCA results, we inverted for two separate sources below the caldera, using a GA inversion technique and simple pressure sources.

5. GA Inversion

In order to invert for the various combinations of the three PCA modes shown above, we employed a GA inversion technique as outlined in (Tiampo et al., 2004). Briefly, geophysical inverse problems generally involve employing large quantities of measured data, in conjunction with an efficient computational algorithm that explores the model space to find the global minimum associated with the optimal model parameters. In a GA, the parameters to be inverted for are coded as genes, and a large population of potential solutions for these genes is searched for the optimal solution. The basic structure of the GA code used here is modified from Michalewicz (1992). The process begins by representing the model to be optimized as a real-value string. Starting with an initial range of models, these algorithms progressively modify the solution by incorporating the evolutionary behavior of biological systems. The fitness of each solution is measured by a quantitative, objective function, the fitness function, FV. Next, the fittest members of each population are combined using probabilistic transition rules to form a new offspring population. Copying strings according to their fitness values means that strings
with a better value of fitness have a higher probability of contributing one or more offspring in the next generation. This procedure is repeated through a large number of generations until the best solution is obtained, based on the fitness measure (Michalewicz, 1992). Those members of the population with a fitness value greater than the average fitness of the population will increase in number exponentially, accelerating the convergence of the inversion process (Holland, 1975; Goldberg, 1989).

In this study we employ the GA to invert only for the vertical displacements for two respective time periods, 1993-1997 (subsidence) and 2007-2013 (uplift), using combinations of EOF1, EOF2 and EOF3 from Figures 5 and 6. A GA inversion can be very time consuming, particularly given the large number of points available in the DInSAR analysis, the vertical deformation alone was selected for the inversion. Given the high quality of the vertical deformation from the DInSAR analysis, it was sufficient for the inversion process alone. In addition, there was no independent data set such as local continuous GPS available to use as an independent check on the model, we did not include the east-west data in the inversion. That allowed us to use the east-west deformation as independent confirmation of the model quality.

We assume that the source models are a combination of either one or two simple Mogi pressure sources with either positive or negative pressures for both time periods. Here the vertical and radial components of displacement in a half-space are defined as:

\[ U_z = \frac{3\Delta Vd}{4\pi R^{3/2}} \]  \hspace{1cm} (3)

and

\[ U_r = \frac{3\Delta Vr}{4\pi R^{3/2}}. \]  \hspace{1cm} (4)
Here $U_z$ and $U_r$ are the vertical and radial displacement, respectively, $d$ is the depth to the source, $R$ is the radial distance to a point on the surface, and $\Delta V$ is the change in volume of the source, here converted to the change in radius, $r$ (Mogi, 1958).

The GA inversion solved for the x and y location of each source, in UTM coordinates, the radius of the spherical pressure source, $r$, and the depth to each sphere, $d$. The initial search range of parameters for the GA was the spatial extent of the original InSAR images (Figure 2), $r$ values between 20 and 200 m, and depths, $d$, of 1000 to 14000 m below the surface.

The inversion results for six different cases are shown in Table 2. The first case inverts EOF1 alone for the time period 2007-2013 for one positive source, two positive sources and two sources, one of which is negative and another which is positive. The second case inverts for EOF1 alone for the time period 1993-1999 for one negative source, two negative sources and two sources, one of which is negative and another which is positive. The third case inverts the sum of EOF1 and EOF2, 2007-2013, for the same three different source options as in Case 1. The fourth case inverts the sum of EOF1 and EOF2, 1993-1999, for the same three different source options as in Case 2. The fifth case inverts the sum of EOF1, EOF2 and EOF3, 2007-2013, for the same three different source options as in Case 1. The sixth case inverts the sum of EOF1, EOF2 and EOF3, 1993-1999, for the same three different source options as in Case 2. It should be noted that a number of other configurations, both for individual and summed EOF modes and for different source types, were tested as well, but none provided better solutions than those presented in Table 2.

6. Results

Table 2 shows the time periods chosen for inversion analysis and the resulting parameters for the associated inflation or deflation (x and y location, $d$ and $\Delta V$). Also provided are the root-mean-
square (RMS) between the forward model produced by the best solution for each case and the actual data seen in Figure 3, and the associated reduced chi-square value. Here the RMS value is estimated using the error values for each of the 5308 locations provided in the Supplementary Material of Samsonov et al., (2008).

The results shown in Table 2 demonstrate that the addition of a second source of the same polarity to the inversion does not improve the RMS. In each of those cases, the GA attempts to minimize the size of that second source while either moving it to the same location as the first source or as deep as possible in the medium. On the other hand, the addition of a second source of opposite polarity always significantly improves the RMS of the solution.

Table 2 also shows that the RMS significantly improves with the addition of both EOF2 and EOF3 to EOF1. The best solution for the 2007-2003 time period uses the sum of modes EOF1, EOF2 and EOF3 and results in a shallower, positive source at approximately 3400 m in depth and deeper, negative source at approximately 7624 m in depth. The best solution for the 1993-1999 time period uses the sum of modes EOF1, EOF2 and EOF3 and results in a shallower, negative source at approximately 2750 m in depth and deeper, negative source at approximately 8014 m in depth.

Figures 8 through 13 present the modelled results and residuals for the six different cases. We omitted results for two sources with the same polarity because of the lack of increase in fitness associated with those solutions. Figure 8 shows the forward model and residuals for the 1993-1999 inversion of EOF1 alone, with both one negative source (Figures 8a and 8b) and for two sources of opposite polarity (Figures 8c and 8d). The residuals are calculated as the difference between the forward model from the best fit inversion for that decomposition and the original MSBAS results for that time period. Figure 9 is for the same time period, 1993-1999, and the
same two cases, one source (Figures 9a and 9b) and two sources (Figures 9c and 9d). However, here the forward model is the result of the inversion of the summation of PCA modes EOF1 and EOF2. Figure 10 also represents the 1993-1999 time period and the same two cases, but the forward model is the result of the inversion of the summation of modes EOF1, EOF2 and EOF3. Note that the addition of EOF3 relocates the second source from the north of the caldera to south, similar to the results of Amoruso et al. (2015).

Figure 11 presents the forward model results residuals for the 2007-2013 inversion of EOF1 alone, with both one positive source (Figures 11a and 11b) and for two sources of opposite polarity (Figures 11c and 11d). Figure 12 also shows the results for the 2007-2013 time period and the same two cases, but here the forward model is the result of the inversion of the summation of modes EOF1 and EOF2. Figure 13 also represents the period 2007-2013 and the same two cases, but here the forward model is the result of the inversion of the summation of modes EOF1, EOF2 and EOF3. Here all three inversions place the second source in the south of the caldera. The final model suggests that both sources are further south than expected and minimize the residuals from the Solfatara region.

FIGURE8
FIGURE9
FIGURE10
FIGURE11
FIGURE12
FIGURE13
Displacements in the east-west direction were modeled in order to assess how well the source models agreed with the complete displacement field. Figure 14 presents the results from the inversion of the summation of modes EOF1, EOF2 and EOF3 for both time periods. Figure 14a shows the modeled east-west displacements for the two source model derived for 1993-1999, as given in Figure 10. Figure 14b shows the residuals between the model of Figure 14a and the actual displacements. Figure 14c are the modeled east-west displacements for the two source model of Figure 13, the time period 2007-2013. Figure 14d presents the residuals between model shown in Figure 14c and actual the displacements. The results for both models are in good agreement with the actual data, although the displacements associated with the subsidence model (Figures 14a and 14b), 1993-1999, suggest that the modeled displacements are slightly underfit by the model. The wavelength of the residual signal suggests that is the difference is contained in the shallow source.

FIGURE 14

7. Conclusions

In this work we applied, for the first time, a PCA decomposition analysis to the advanced MSBAS DInSAR time series of ground deformation in the Campi Flegrei caldera. The MSBAS time series incorporate ERS-1/2, ENVISAT and RADARSAT-2 data and result in nearly twenty years of data, with uninterrupted temporal coverage for 2003-2013. The PCA analysis produces three significant eigenmodes for both the vertical and east-west time series. These time series were inverted using a GA technique for simple Mogi pressure sources and a variety of cases. The fit to the actual data increases progressively with the addition of each mode, suggesting that each contains important information related to the source mechanisms. The best fit occurs for an inversion that sums all three modes (EOF1, EOF2 and EOF3) and for two sources with opposite
polarity, for both the period of subsidence (1993-1999) and the period of uplift (2007-2013). In
the first case, a shallower source is deflating while a deeper source inflates; in the second case, a
shallower source is inflating while the deeper source deflates. The time series for EOF2 and
EOF3 suggest that a sharp pulse in activity occurred between 1997 and 2002, potentially
indicating that the dynamics of the system changed significantly. This hypothesis is supported
by a similar uplift signal seen in levelling data from Amoroso et al. (2014a), at the same time that
the CO2/H2O ratio in local fumaroles starts to increase, potentially as a result of an increased
contribution of the magmatic component (Chiodini et al., 2012). It has been suggested that this
change was driven by magma fed from a deeper magma chamber, such as that found in our
inversion for the sum of modes EOF1, EOF2 and EOF3 (Zollo et al., 2008; Amoroso et al.,
2014a; Di Vito et al., 2016). Incorporation of all three modes is necessary to significantly
improve the fit and model the two sources together.
Past work, using various combinations of geodetic data, including leveling, trilateration, GPS,
gravity and DInSAR, have found that the shallower source can be fit better using different
geometries and some combination of shallower hydrothermal sources (see, e.g. De Natale et al.
1991; Battaglia et al., 2006; De Natale et al., 2006; Gottsmann et al., 2005, 2006; Amoruso et al.,
2008; Trasatti et al., 2011; Chiodini et al., 2012; Amoruso et al., 2014a,b; Samsonov et al.,
2014b; Trasatti et al., 2015; Di Vito et al., 2016). Here we found that two simple, spherical
sources of opposite polarity, one deeper and the second shallow, provided an adequate fit to the
data without resorting to sills or spheroidal magma chambers.
The final models for both periods place the shallower source at between 2750 and 3400 m below
the caldera, at either the upper or lower edge of the gas bearing rock layer (Figure 1). The
deeper source is more stable, at 7600 to 8000 meters in depth, also as suggested by earlier work
(Trasatti et al., 2011, 2015). Expansion of the existing SAR data set using new satellite data (e.g. Sentinel-1a and 1b) will help to better characterize these sources with time.

This study provides evidence for the effectiveness of PCA in denoising large geophysical data sets, including DInSAR data. Dense time series are critical to the process and, as a result, suggests that MSBAS time series will be of increasing importance in the accurate and reliable estimation of natural and anthropogenic hazards.

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References


Figure 1: (a) Map of the Naples region, Italy, with Campi Flegrei caldera outlined in the red box (city of Naples, orange star). Black boxes identify frames for each of the ERS-ENVISAT (T129 and T036) and RADARSAT-2 (S3, both ascending and descending, and F6N) radar image frames. (b) Simplified geologic cross section of the caldera structure.
Figure 2: MSBAS results, 1992-2013, for the images outlined in Figure 1 (see Table 1 for details). a) Vertical component of deformation, 1992-2013; b) east-west component of deformation, 1992-2013; c) time series of vertical and east-west components identified in (a) and (b) by pink triangle. Modified from Samsonov et al., 2014.
Figure 3: Net surface deformation for two time periods chosen from the time series of Figure 1. 


Pink triangle is as shown in Figure 2; green triangles identify location of time series in Figures 7 and 8.
Figure 4: Eigenvalue plots showing the percentage of variance accounted for by each eigenvector mode for the decomposition of MSBAS time series of surface displacement in a) the vertical direction and b) the east-west direction.
Figure 5: First three eigenmodes for vertical displacement, 1993-2013. a) First spatial eigenmode (EOF1); b) principal component time series associated with EOF1 (PCA1); c) second spatial eigenmode (EOF2); d) principal component time series associated with EOF2 (PCA2); e) third spatial eigenmode (EOF3); f) principal component time series associated with EOF3 (PCA3). Here blue is anticorrelated with red, EOF plots a, c, and e.
Figure 6: First three eigenmodes for east-west displacement, 1993-2013. a) First spatial eigenmode (EOF1); b) principal component time series associated with EOF1 (PCA1); c) second spatial eigenmode (EOF2); d) principal component time series associated with EOF2 (PCA2); e) third spatial eigenmode (EOF3); f) principal component time series associated with EOF3 (PCA3). Here blue is anticorrelated with red, EOF plots a, c, and e.
Figure 7: Time series of vertical displacement for combinations of the first three EOFs at the locations shown by green triangles in Figure 3. Vertical displacement at location a, Figure 3 is shown for a) EOF1, b) EOF1 and EOF2, summed, and c) EOF1, EOF2 and EOF3, summed. Vertical displacement at location b, Figure 3 is shown for d) EOF1, e) EOF1 and EOF2, summed, and f) EOF1, EOF2 and EOF3, summed. Vertical displacement at location c, Figure 3 is shown for g) EOF1, h) EOF1 and EOF2, summed; and i) EOF1, EOF2 and EOF3, summed.
Figure 8: Modelled displacements and residuals in the vertical direction inverted for EOF1 for the time period 1993-1999 (subsidence). a) Displacements for a single source model, location shown by green star, at a depth of 1665 meters below the surface (mbs); b) residuals between model shown in (a) and actual displacements (Figure 2); c) displacements for a two source model, locations shown by green stars, at depths of 2069 (south, negative) and 13802 (north, positive) mbs; d) residuals between model shown in (c) and actual displacements (Figure 2). All displacements and residuals in cm; source details are given in Table 2.
Figure 9: Modelled displacements and residuals in the vertical direction inverted for EOF12 for the time period 1993-1999 (subsidence). a) Displacements for a single source model, location shown by green star, at a depth of 1690 mbs; b) residuals between model shown in (a) and actual displacements (Figure 2); c) displacements for a two source model, locations shown by green stars, at depths of 2102 (south, negative) and 14834 (north, positive) mbs; d) residuals between model shown in (c) and actual displacements (Figure 2). All displacements and residuals in cm; source details are given in Table 2.
Figure 10: Modelled displacements and residuals in the vertical direction inverted for EOF123 for the time period 1993-1999 (subsidence).  a) Displacements for a single source model, location shown by green star, at a depth of 1623 mbs; b) residuals between model shown in (a) and actual displacements (Figure 2); c) displacements for a two source model, locations shown by green stars, at depths of 2749 (south, negative) and 8014 (north, positive) mbs; d) residuals between model shown in (c) and actual displacements (Figure 2). All displacements and residuals in cm; source details are given in Table 2.
Figure 11: Modelled displacements and residuals in the vertical direction inverted for EOF1 for the time period 2007-2013 (uplift). a) Displacements for a single source model, location shown by green star, at a depth of 1671 mbs; b) residuals between model shown in (a) and actual displacements (Figure 2); c) displacements for a two source model, locations shown by green stars, at depths of 2892 (south, positive) and 8454 (north, negative) mbs; d) residuals between model shown in (c) and actual displacements (Figure 2). All displacements and residuals in cm; source details are given in Table 2.
**Figure 12:** Modelled displacements and residuals in the vertical direction inverted for EOF12 for the time period 2007-2013 (uplift).  

- (a) Displacements for a single source model, location shown by green star, at a depth of 1810 mbs; 
- (b) residuals between model shown in (a) and actual displacements (Figure 2); 
- (c) displacements for a two source model, locations shown by green stars, at depths of 2818 (south, positive) and 9340 (north, negative) mbs; 
- (d) residuals between model shown in (c) and actual displacements (Figure 2). All displacements and residuals in cm; source details are given in Table 2.
Figure 13: Modelled displacements and residuals in the vertical direction inverted for EOF123 for the time period 2007-2013 (uplift). a) Displacements for a single source model, location shown by green star, at a depth of 1987 mbs; b) residuals between model shown in (a) and actual displacements (Figure 2); c) displacements for a two source model, locations shown by green stars, at depths of 3402 (south, positive) and 7624 (north, negative) mbs; d) residuals between model shown in (c) and actual displacements (Figure 2). All displacements and residuals in cm; source details are given in Table 2.
**Figure 14**: Modelled displacements and residuals in the east-west direction for the best fit two source model, using EOF123, as given in Table 2.  

a) Modelled displacements for a two source model, location shown by green star, for the 1993-1999 (subsidence); b) residuals between model shown in (a) and actual displacements (Figure 2b); c) displacements for the two source model, locations shown by green stars, for the time period 2007-2013 (uplift); d) residuals between model shown in (c) and actual displacements (Figure 2b). All displacements and residuals in cm.
Table 1: Seven DInSAR data sets providing continuous coverage from 1993 through 2013 used in this study. Included are incidence angle $\phi$ (degrees), azimuth angle $\theta$ (degrees), the number of available SLC SAR images, $N$, and the number of computed highly coherent interferograms, $M$.

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<th>$\phi$</th>
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<th>$M$</th>
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Total $M = 1271$

Combined coverage: 19930110-20130803

Total number of unique time steps = 385 (48 repeated by different sensors)
Table 2: GA inversion results for different combinations of EOF modes for each time period, 1993-1999 (9399) and 2007-2013 (0713), and multiple source types. Here ‘Opposing’ refers to two sources with opposite polarity.

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