High-Precision Photogrammetry for Glaciology

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High-precision photogrammetry for glaciology

by

Ethan Z. Welty

B.S., University of Washington, 2007

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High-precision photogrammetry for glaciology
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has been approved for the Environmental Studies Program

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The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.
Consumer-grade digital cameras have become ubiquitous tools for documenting short-term variability in the geosciences. However, these devices were not intended for precise timekeeping and surveying, and their use as such requires management of systematic and random errors that inevitably arise.

This dissertation presents a suite of methods for registering the place and time of photographs in an absolute reference frame so that they may be analyzed and interpreted alongside other spatial and temporal data. The methods are tested on a 13-year record of 33,000 time-lapse photographs from Alaska’s Columbia Glacier. This work provides insights into the capabilities and shortcomings of consumer-grade cameras as scientific instruments, the opportunistic approaches often needed to achieve the best results, and the potential of continuous high-frequency measurements for documenting rapid geomorphic processes.

Subsecond-precision image capture times are achieved by measuring the offset to a reference clock display and accounting for the drift, precision, and reporting resolution of the camera clock. Two case studies illustrate the benefit of subsecond precision in contemporary investigations: georeferencing aerial photogrammetric surveys with camera positions time-interpolated from GPS tracklogs, and coupling videos of glacier-calving events to synchronous seismic waveforms. Retroactive dating of photographs, on the order of seconds to hours, is achieved by leveraging phenomena visible in the photographs – namely, the positions of astronomical objects in the sky or the corresponding variations in solar radiation and sea level.

Similarly, retroactive camera calibrations are achieved using surface and topographic features in the photographs – specifically, point and line features of known absolute position, the motion of static features in images due to camera rotation, and the correspondences between real images.
and images synthesized from vertical imagery. Camera motion is corrected by computing globally optimal estimates of rotation over arbitrarily-long photographic sequences.

Finally, a recently-developed tracking algorithm based on particle filtering theory is refined and applied to estimate Columbia Glacier velocities, their associated uncertainties, and the corresponding strain rate fields at 3d intervals over a 13-year period, providing an unprecedented look at the seasonal and sub-seasonal variability of tidewater glacier dynamics over long time scales.
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Chapter 1

Introduction

What can we measure about a moving landscape by photographing it? How can we capture glacier changes using photography?

Our focus on the metric power of photographs is motivated by the rapid and global proliferation of cameras and photographs in the digital age. In 1992, Tim Berners-Lee, the founder of the World Wide Web, became the first person to publish a photograph online [1]. Today, roughly 4 billion people have access to the internet [2] and 4 billion people have some form of digital camera [3]. Collectively, humankind is undertaking an unprecedented photographic survey of the physical world. The number of new photos uploaded to select photo sharing platforms rose from 300 million per day in 2012 to 1.8 billion in 2014 [4], exponential growth for what are already the largest collections of photographs ever assembled.

Although technically ‘a mosaic of millions of changeable pixels’ [5, , p. 18], digital photographs often retain stronger connections to physical reality than their analog predecessors. A growing array of electronic sensors are bundled into modern digital cameras to oversee the creation of each image and embed its ‘recipe’ into the resulting file. This can include information about the lens, camera, and camera settings, the lens focus distance, the light intensity, the capture date and time, and increasingly, the position (GPS) and orientation (compass and inclinometer) of the camera – parameters that can help reverse-engineer the geometric and radiometric characteristics of the original scene [reviewed in 6]. Key information that would have had to be written down and transmitted by the photographer now travels with the image in such a standardized way as to
become part of the image itself [7].

These annotated photographs describe on a massive scale the changing shape and appearance of the world’s built and natural environments. In dazzling demonstrations of how digital photographs can regain ‘a strong sense of the actual from deep within the virtual’ [5], the locations of 35 million geotagged images on Flickr [8] recreate the shapes of continents and the channels of human travel. At the most densely photographed sites, structure-from-motion (SfM) algorithms can leverage the stereoscopic redundancy of the many to millions of photos available to construct detailed three-dimensional (3-D) models of the scene [9, 10], then used to geometrically stabilize photographs taken from diverse perspectives into seamless time-lapse sequences [11]. As algorithms improve and the spatial and temporal coverage of photographs continue to expand, we can expect more precise and meaningful information to be recoverable from photographs, whether or not they were intentionally produced for scientific analysis.

In the research community, consumer-grade digital cameras are becoming increasingly popular and powerful tools for targeted data acquisition. These simple and cost-effective devices can collect large amounts of information in even extreme environments with relatively minimal effort or training. In glaciology for example, rephotography pairs, traditionally used to document glacier change over years to centuries [reviewed in 12], are increasingly supplanted by autonomous time-lapse camera systems continuously recording glacier motion [13], surface conditions [14], iceberg calving [15, 16, 17], and dramatic footage for documentary films [18, 19]. The number of published journal articles containing the terms ‘time-lapse camera’ and ‘glacier’ increased six-fold over the last decade (by number of Google Scholar search results), from 10 published in 2007 to 57 published in 2017. Ground-based cameras are particularly well suited for high-frequency and high-resolution observations, vertical or underwater [20] perspectives, and uninterrupted observations under cloud cover or during the polar night [21], precisely the situations in which satellite remote sensing is still inadequate. Furthermore – once calibrated – they can deliver spatiotemporal output comparable in form to that of commercial airborne and satellite imagery. However, their widespread use has not been accompanied by a corresponding availability of appropriate software tools and meth-
ods for oblique photogrammetry, and as a result, the quantitative potential of many photographic
sequences has not been fully explored.

This dissertation focuses on one prominent example of unexplored potential. Since 2004, 33
different time-lapse camera stations have been used to photograph the rapidly evolving calving front
of the Columbia Glacier, a large (1000 km)

marine-terminating glacier in Prince William Sound,
Alaska. Since glaciers are accumulated snow, their size is ultimately determined by the balance
between snowfall and melt. However, glaciers which flow into the ocean can also lose mass by
sliding rapidly into the water and calving off icebergs [22], and Columbia Glacier is a dramatic
example of such dynamic instability. Dating of previously buried trees reveals that during the
Little Ice Age (approximately 1200 – 1800), the glacier advanced gradually by 40 m/year, reaching
its maximum extended position around 1850 [23]. Destabilized by gradual thinning by the early
1980s, the glacier retreated from a shallow moraine onto a reverse-sloping bed [24], marking the
beginning of a rapid, seemingly-irreversible retreat which continues to this day [25, 26]. Overall,
the glacier accelerated from 2 m d\(^{-1}\) to up to 25 m d\(^{-1}\), while simultaneously thinning 20 m/year
and retreating 600 m/year (reversing 700 years of advance in only 15 years of retreat). At its peak,
Columbia Glacier accounted for 1% to 2% of new water added to the global oceans, with iceberg
calving accounting for 90% to 100% of the total volume loss [27].

Columbia Glacier has arguably the world’s longest and most detailed observational record
of any tidewater glacier in rapid retreat. However, most observations are glacier elevations and
velocities derived from airborne and satellite imagery with at best monthly resolution. A vigorous
repeat aerial stereophotography campaign (1 – 8 flights per year) was initiated by the U.S. Geological
Survey in 1976, before the onset of retreat, and continued through 2001 [26], then maintained
at a frequency of only 1 – 2 flights per year through 2010. Mono and stereo satellite imagery
gradually replaced the aerial photography, with roughly monthly coverage since 2011, on tempo-
ral baselines of 11 days or longer. In contrast, the time-lapse cameras have been photographing
the glacier front since 2004 at frequencies of 4 h to 45 s, resulting in a rapidly growing archive
approaching 300 000 images [28, 18]. However, the time-lapse record presents several obstacles to
precise measurement, a reminder that despite the proliferation of richly-annotated digital photographs, metadata ‘recipes’ are often lost or corrupted, and often do not meet the high levels of precision expected of scientific instruments. For example, the camera clocks were not always set correctly and human field notes were incomplete, resulting in large uncertainties in the time and date of image acquisition (‘timestamp’). Furthermore, most camera positions were never surveyed and the optical parameters of the cameras were never measured before being deployed to the field (from which most never returned), introducing large uncertainties in the correspondence between positions in the photographs and positions in the world.

In response to these challenges, we developed a suite of methods for locating photographs precisely in absolute spatial and temporal reference frames so that photographs like those from Columbia Glacier can be analyzed and interpreted alongside other high-quality spatial and temporal data. Chapter 3 tackles the problem of uncertain or unknown timestamps, recovering the date and time of photographs by leveraging astronomical phenomena, from the positions of astronomical objects in the sky to the corresponding variations in solar radiation and sea level. Chapter 2, in turn, enhances the timekeeping abilities of digital cameras, describing methods for achieving subsecond-precision timestamps to match photographs to other high-frequency instrumental records. Chapter 4 addresses the problem of retroactively recovering the position, orientation, and optical parameters of the cameras that gave rise to photographs by leveraging surface and topographic features visible in those photographs – specifically, point and line features of known position in the world, the motion of static features due to camera rotation, and the correspondences between real photographs and those synthesized from orthorectified aerial and satellite imagery. The camera orientation at the time of each photograph in a sequence is estimated using an automatic global optimization that scales to arbitrarily-long sequences, including the entire 13-year record of Columbia Glacier time-lapse photographs.

Finally, Chapter 5 highlights the potential power of calibrated photographs in the study of glaciers. A recently-developed tracking algorithm based on particle filtering theory [29] is refined and applied to the Columbia Glacier time-lapse photographs to produce glacier velocity fields, their
associated uncertainties, and corresponding strain rate fields at 3 d intervals over a 13-year period, providing an unprecedented look into the variability of glacier dynamics at sub-seasonal, seasonal, and interannual time scales simultaneously. Additional results published elsewhere include calving event volume and frequency distributions at Columbia Glacier for calving theory validation [30], terminus positions and sea-ice extent at Hansbreen, Svalbard for glacier model calibration [31], and inundation area of a braided river in Greenland as a proxy for ice sheet meltwater discharge [32].

Calibrated photographs have potential utility in the study of all processes observable in the visible spectrum, as well as in the infrared and thermal spectra as these camera technologies become more accessible [33]. Applications of the precision-timing methods of Chapter 2 have already been identified in a number of situations, for example (1) aerial topographic surveying [34], by enabling precise in-flight camera positions to be interpolated from satellite navigation tracklogs (Section 2.5); (2) ground-based topographic surveying, by synchronizing multi-camera arrays used to record sequential geometric changes of thaw slumps [35] and glacier calving fronts [36]; (3) amateur astronomy, by helping to constrain the time of observations [37]; and (4) interpreting high-frequency instrumental records – for example, by using videos of calving events synchronized with seismic waveforms to identify sources of calving seismicity [15] (Section 2.5) and using videos of penguins synchronized with logs of beak opening to quantify a variety of behaviors (ingestion of prey, breathing, vocalization, head shaking, and preening) [38]. The flexible, opportunistic calibration methods of Chapter 4 advance the use of historic or otherwise poorly-constrained oblique terrestrial and aerial photographs for mapping, which could assist, for example, in reconstructing tree line dynamics [39], coastal erosion [40, 41], and patterns of snow accumulation and melt [42, 43, 44]. Although developed for long-term glacier observation, the large-scale camera stabilization and three-dimensional surface motion tracking methods of Chapters 4 and 5 are directly applicable to the study of other surface flows, including lava flows [45], debris flows [46], landslides [47], and water currents (provided there are sufficient objects floating on the water surface) [48, 49]. Finally, reconstructing the past from photographs is an important topic in forensics. Clues found in photographs are regularly used to investigate and reconstruct past criminal activity, and used
as evidence in courts of law [reviewed in 50]. The date and time at which a photograph was taken [51], the position from which a photograph was taken, and the position of objects (and subjects) in a photograph can all prove important in solving criminal cases [52].

To forensics, photographs are physical evidence of past human actions. To the physical sciences, photographs are multi-dimensional snapshots of a dynamic physical process. Whatever their value to a particular field, the opportunity for their use has multiplied dramatically. The proliferation of digital cameras – on satellites, airplanes, and unmanned aerial vehicles (UAVs), as time-lapse cameras, web cameras [53], surveillance cameras [54], and those embedded in personal portable devices – has increased the number, frequency, and diversity of new photographs, while vast quantities of historic film terrestrial and aerial photographs are still being digitized and published online, extending the accessible photographic record deeper into the past (recent examples in glaciology include the Glaciers and Landforms Photograph Collection at content.lib.washington.edu/epicweb and the Alaska High Altitude Photography Program at pgc.umn.edu/data/aerial). Each of the following chapters represents a step forward in the interpretation and analysis of this vast photosphere.
Chapter 2

Cameras as clocks

Published as


Consumer-grade digital cameras have become ubiquitous accessories of science. Particularly in glaciology, the recognized importance of short-term variability has motivated their deployment for increasingly time-critical observations. However, such devices were never intended for precise timekeeping, and their use as such needs to be accompanied by appropriate management of systematic, rounding and random errors in reported image times. This study describes clock drift, subsecond reporting resolution and timestamp precision as the major obstacles to precise camera timekeeping, and documents the subsecond capability of camera models from 17 leading manufacturers. We present a complete and accessible methodology to calibrate cameras for absolute timing and provide a suite of supporting scripts. Two glaciological case studies serve to illustrate how the methods relate to contemporary investigations: (1) georeferencing aerial photogrammetric surveys with camera positions time-interpolated from GPS tracklogs; and (2) coupling videos of glacier-calving events to synchronous seismic waveforms.

2.1 Introduction

Digital cameras automatically record the capture date and time of every image and video file. Such time-aware imagery is leveraged throughout science and society, from forecasting weather and
monitoring global change from space to solving crimes with webcams [55], retracing city visitation patterns through internet photograph collections [8] and broadcasting plant phenophases from smartphones [56]. In glaciology, the recognized importance of short-term variability motivates ever higher-frequency observations. Rephotography pairs, traditionally used to document glacier retreat over years to centuries [57], are increasingly complemented by time-lapse cameras imaging the continual evolution of ice dynamics and surface conditions [13, 17, 14, 28]. Recent investigations of calving source mechanisms have demanded second- to subsecond-frequency time-lapse and video sequences, synchronized to GPS, seismic and other instrumental records [49, 15, 58].

Although advanced camera systems, such as those used by research satellites and astronomical observatories, are meticulously calibrated against a known time source, most digital cameras were never intended for precise temporal observation. Nevertheless, consumer-grade digital cameras are widespread and provide generally excellent image quality, and many tools have been developed for analyzing and interpreting the resulting images. Szeliski [6] provides an overview of state-of-the-art computer vision applications. These circumstances suggest that a better understanding of the timekeeping limitations of these cameras and the development of accessible calibration procedures that extend their application and reliability as scientific instruments would be useful developments, not just for glaciologists but for the broader scientific community. Whether the required accuracy is subsecond or on the order of minutes or more, confidently matching observations made by a camera to other time-aware datasets requires careful evaluation and calibration of the camera’s internal clock.

In this paper, we discuss the timekeeping limitations of consumer-grade camera and reference clocks and demonstrate an optimized and accessible approach to calibrating cameras to a reference for absolute timing. Any use of trade, firm or product names is for descriptive purposes only and does not imply endorsement by the US Government. Scripts implementing many of the steps are provided as supplementary material at igsoc.org/hyperlink/12j126. Two time-critical applications are presented: (1) using camera positions, interpolated from GPS tracklogs, as geodesic control in aerial photogrammetric surveys (Section 2.5.1); and (2) correlating high frame-rate observations of
glacier-calving events to synchronous seismic waveforms (Section 2.5.2).

2.2 Limitations of camera clocks

Any long-term consumer-grade camera deployment seeking second to minute accuracy while relying on the camera’s internal clock will need to account for the magnitude and variability of the clock’s intrinsic drift (Section 2.2.1). For subsecond-critical applications, two additional factors should be considered, whether and with what resolution a camera reports subsecond decimals (Section 2.2.2) and the true precision of the reported timestamps (Section 2.2.3).

2.2.1 Drift

Camera clocks drift and the drift can be substantial: daily subsecond to second drift can accumulate to multi-minute offsets within a few months. Clock drift varies between cameras of the same make and model, and drift, otherwise highly linear, is especially sensitive to changes in temperature, expected of any circuit-integrated oscillator [59].

Table 2.1 lists mean clock drift and temperature for a variety of cameras and treatments. The Nikon D200 (#1) and Canon 40D digital single lens reflex (DSLR) cameras were kept in a heated indoor space and compared repeatedly against Coordinated Universal Time (UTC) over 31 and 76 days (d), respectively. The weighted least-squares linear fits confirm that, at near-constant temperature, clock drift is indistinguishable from linear over month timescales (Figure 2.1). In contrast, the drift rates of the Nikon D300S and Nikon D200 (#2), deployed year-round at Columbia Glacier, Alaska, vary by a factor of four between summer and winter. Although drift rates are specific to the individual camera, the pro-level Canon 5D Mark II and Nikon D2X are the best performers by a wide margin, suggesting that some camera models may benefit from substantially superior clock hardware and design.
Table 2.1: Mean drift and air temperatures for a variety of cameras and treatments. Drift rates are for the individual camera and may not be representative of the camera model. Mean temperatures for the time-lapse deployments at Columbia Glacier were calculated from monthly averages reported for nearby Valdez, Alaska [60].

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<thead>
<tr>
<th>Camera</th>
<th>Treatment</th>
<th>Mean drift s d⁻¹</th>
<th>Mean temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nikon D200 (#1)</td>
<td>Indoors (31 days)</td>
<td>-0.187 ± 0.006</td>
<td>20</td>
</tr>
<tr>
<td>Canon 40D</td>
<td>Indoors (76 days)</td>
<td>-0.763 ± 0.006</td>
<td>20</td>
</tr>
<tr>
<td>Nikon D300S</td>
<td>Time-lapse (winter)</td>
<td>-1.101 ± 0.007</td>
<td>1</td>
</tr>
<tr>
<td>Nikon D300S</td>
<td>Time-lapse (summer)</td>
<td>-0.170 ± 0.016</td>
<td>9</td>
</tr>
<tr>
<td>Nikon D300S</td>
<td>Time-lapse (winter)</td>
<td>-1.262 ± 0.010</td>
<td>-2</td>
</tr>
<tr>
<td>Nikon D300S</td>
<td>Time-lapse (summer)</td>
<td>-0.294 ± 0.016</td>
<td>8</td>
</tr>
<tr>
<td>Nikon D200 (#2)</td>
<td>Time-lapse (winter)</td>
<td>-1.056 ± 0.007</td>
<td>1</td>
</tr>
<tr>
<td>Nikon D200 (#2)</td>
<td>Time-lapse (summer)</td>
<td>-0.295 ± 0.016</td>
<td>10</td>
</tr>
<tr>
<td>Nikon D2X</td>
<td>Mixed (297 days)</td>
<td>-0.044 ± 0.004</td>
<td>-</td>
</tr>
<tr>
<td>Canon 5D Mark II</td>
<td>Mixed (335 days)</td>
<td>+0.100 ± 0.003</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 2.1: Weighted least-squares linear fit of UTC-camera offset measurements for a Nikon D200 and a Canon 40D, evaluated using the NIST Web Clock (introduced in Section 2.3) following the methods of Section 2.4. Both cameras were kept indoors and held at near-constant temperature. The error bars are the sum of the subsecond resolution of the camera and the reported uncertainty of the NIST Web Clock at each offset measurement.
2.2.2 Resolution

Digital cameras record image-capture times following the Exchangeable image file format (Exif) standard [7]. The DateTimeOriginal tag contains the year, month, day, hour, minute and second of original data generation. Subsecond decimals, if reported, are written to the SubSecTimeOriginal tag. While no unifying standard exists for videos, capture start times can typically be found within equivalent video file tags or as Exif in accompanying image thumbnails. Whether and at what resolution a camera records subsecond decimals is of foremost concern to subsecond applications.

A survey of the camera, camcorder and camera phone models of 17 leading manufacturers, using photo-sharing and camera-review websites, found only a handful of Nikon DSLR, Canon DSLR, Kodak EasyShare compact and Nokia phone cameras implementing the SubSecTimeOriginal tag (Appendix A). Furthermore, most of the cameras that do report subsecond information do so in a manner inconsistent with expectation. Figure 2.2 compares frequency distributions of the SubSecTimeOriginal tag (a value ranging from 0 to 99 $10^{-2}$ s) for all capable Nikon and Canon DSLR camera models, compiled from thousands of user-submitted photographs on the photo-sharing website Flickr (flickr.com). All Nikon models record subsecond time with an effective resolution coarser than the expected $10^{-2}$ s, whether by clipping (Figure 2.2a), rounding (Figure 2.2b-d), or subtly favoring values at discrete intervals (Figure 2.2e). For most Canon models, the SubSecTimeOriginal tag serves only to distinguish between images taken in rapid succession and otherwise strongly favors 0 (Figure 2.2f) or 3 (Figure 2.2g). Only the Canon 5D Mark II and Canon 500D approximate a uniform distribution (Figure 2.2h) – the ideal behavior we would expect.

2.2.3 Precision

Although a camera may report subsecond decimals with high resolution, the true precision of the image times may be much less. Figure 2.3 compares the clocks of a Nikon D2X and Canon 5D Mark II against UTC over a 40 min sample period. Although the SubSecTimeOriginal tag of
Figure 2.2: Frequency distributions of the SubSecTimeOriginal tag (a value ranging from 0 to 99 $10^{-2}$ s) for all capable Nikon and Canon DSLR camera models. Each panel is representative of a subset of the 40 cameras surveyed: (a) Nikon D1, D1X and D1H – shown for 1190 photographs by 35 Nikon D1X Flickr users; (b) Nikon D5000 and D3000 – shown for 871 photographs by 118 D3000 users; (c) Nikon D70 and D70s – shown for 839 photographs by 54 D70s users; (d) Nikon D100, D40, D40x, D50, D60, D80, D7000, D5100 and D3100 – shown for 925 photographs by 71 D3100 users; (e) Nikon D2X, D2Xs, D2H, D2Hs, D3, D3S, D300, D300S, D200 and D700 – shown as an average of all ten cameras with 7790 photographs by 458 Flickr users; (f) Canon 1D Mark III, 1Ds Mark III, 1D Mark IV, 7D, 40D, 50D, 60D, 550D, 600D and 1100D – shown for 827 photographs by 128 60D users; (g) Canon 450D and 1000D – shown for 586 photographs by 82 1000D users; (h) Canon 5D Mark II and 500D – shown for 1476 photographs by 155 5D Mark II users.
the Canon 5D Mark II has a resolution approaching 0.01 s (Figure 2.2h), the finest of any model evaluated in Section 2.2.2, the timestamps reported by the camera deviated from UTC by as much as 0.8 s. The Nikon D2X timestamps, in contrast, had a precision on par with the model’s 0.08 s resolution (Figure 2.2e).

The sources of such errors are unknown. Camera manufacturers are reluctant to disclose engineering details, considering them trade secrets (personal communication from Canon Professional Services, 2010, 2011; personal communication from Nikon Support, 2010, 2011). Given the potential for substantial precision loss, the consistency of reported capture times should be evaluated for any camera considered for time-critical applications.

2.3 Limitations of reference clocks

Although relative timing may be sufficient, in most situations comparison to UTC will be desired for absolute timing. Possibly the most accessible UTC reference is the Web Clock (time.gov), provided as a free service by the US National Institute of Standards and Technology (NIST) and the US Naval Observatory (USNO). The online applet prints the current time at 1 s resolution, announcing the beginning of each new second (or ‘second rollover’), alongside the calculated accuracy, typically 100 ms on a broadband connection. Alternatively, the NIST Automated Computer Time Service driving the Web Clock may be queried directly by analog modem to stream second rollovers with 5 ms to 20 ms accuracy in a computer terminal [61]. A subsecond resolution display can be achieved by synchronizing a computer clock to UTC over the internet via the Network Time Protocol (NTP) and streaming the system time in a terminal. The accuracy, typically 5 ms to 100 ms over the public internet [62, 63], is largely a factor of the stability and reciprocity of the connection to the chosen NTP time servers, as well as the synchronization distance of those servers to a stratum-0 (reference) device (e.g. atomic clock), the sophistication of the software used, and the refresh rate of the computer monitor.

GPS satellites each maintain four onboard atomic clocks. GPS receivers, once a position lock is achieved, keep time internally to nanoseconds [64]. However, consumer-grade GPS receivers
Figure 2.3: Clocks of a Nikon D2X and a Canon 5D Mark II evaluated against UTC, calculated from rapid sequences of images taken of a 0.01 s resolution Unix computer clock synchronized to Network Time Protocol (NTP) servers (introduced in Section 2.3) following the methods of Section 2.4. The 95% confidence intervals represent the deviation of measurements in each sample set, which would include the subsecond resolution of the camera, the 0.017 s (60 Hz) refresh rate of the computer monitor, and the reported NTP accuracy (7 ms to 14 ms).
print time to their screen with as little as 1 s accuracy due to the display subroutines on some models being given lower priority by the software and single CPU (personal communication from Garmin Engineering, 2010). Figure 2.4 demonstrates this issue by comparing the time displayed by two handheld GPS (DeLorme PN-40 and Garmin eTrex Vista HCX) against an NTP-synchronized Unix system clock (using photographs as shown in Figure 2.5).

The Red Hen Blue2Can included in the study has no time display; rather it communicates by wireless Bluetooth signal with an external GPS to retrieve the time (and position) of each image capture. The GPS refreshes its broadcast of time and position only once per second and this information is passed to the Blue2Can and the camera with additional latency (personal communication from Red Hen Systems, 2011). The GPS date and time (including subsecond decimals) are written to the image Exif tags GPSTimeStamp. In practice, the time associated with an image precedes capture by about 1 s and potentially much more if the GPS signal is lost. Equivalent errors arise in cameras equipped with onboard GPS, with the added concern that the user may not be notified of signal loss. Furthermore, most models currently do not write subseconds to the GPSTimeStamp tag.

Consumer radio clocks, which synchronize to terrestrial radio time signals, are limited by the temporal accuracy of their displays, may not issue a warning when the time signal is lost and usually synchronize only periodically to the radio signal, otherwise relying on the oscillator inside the device [65]. Although insufficient for subsecond applications, the 1 s precision of consumer GPS and radio clock displays is adequate for many situations and can (and should) be used to measure the large offsets that accumulate from nonlinear drift during extended field deployments.

2.4 Calibration of camera clocks

The evaluations of camera-clock precision and drift presented in Section 2.2 relied on comparing the camera clocks with a calibrated reference. In this section, we describe a suite of methods for measuring the offset between camera time and the time displayed by a reference clock. The underlying principle of our approach is that from a picture of the reference taken with the camera
Figure 2.4: Handheld GPS (DeLorme PN-40 and Garmin eTrex Vista HCX), tethered GPS (Red Hen Blue2Can), and the NIST Web Clock evaluated against UTC, calculated from images taken of the GPS displays and a streaming 0.01 s resolution Unix computer clock synchronized (with reported 7 ms to 14 ms accuracy) to local stratum-1 NTP time servers (Figure 2.5). The GPS units were reset twice and measurements resumed only once each had acquired a three-dimensional position lock. The within-sequence variance is due to the 0.125 s (Nikon D2X photo) or 0.033 s (Canon 5D Mark II video) frame-rate, the 0.017 s refresh rate of the monitor, and the instability and latency of the GPS clock displays.
Figure 2.5: Example of a (a) Garmin eTrex Vista HCX, (b) DeLorme PN-40, (c) streaming Unix system clock, and (d) NIST Web Clock captured simultaneously by a Canon 5D Mark II (01:31:27.40) to evaluate the accuracy of handheld GPS time displays. Although photographed indoors, GPS units were propped against a window pane and successfully acquired and maintained a 3-D position and time signal.
of interest (e.g. Figure 2.5) the offset can be calculated as

\[
\text{offset} = T_c - T_r
\]

where \(T_c\) is the capture time of the image as reported by the camera and \(T_r\) is the time displayed by the reference clock in the image. If recording video, the capture time of each photograph is calculated by multiplying the frame number by the video frame-rate and adding it to the reported capture start time of the video clip.

2.4.1 Subsecond: both camera and reference

In the simplest case, both \(T_c\) and \(T_r\) include subsecond components and Equation (2.1) yields a subsecond resolution measurement of the offset between the camera and reference clocks. However, subsecond offset can be estimated even when one or both of the devices report only second rollovers. In these cases, a multi-second sequence of images is taken of the reference clock at the camera’s maximum frame-rate. First, a shutter speed faster than the target frame-rate needs to be used. To avoid buffer overrun in writing to memory, which can lead to slowing frame-rates in still image sequences and dropped frames in videos (and subsequently to a loss in the precision of the offset measurement), low-resolution output and fast memory cards are recommended for these procedures. The following subsections describe how to analyze the resulting image sequence to calculate a subsecond offset and an error estimate. Although we simply read the reference time off the photographs, a character-recognition algorithm could be developed to automate the procedure.

2.4.2 Subsecond: either camera or reference

In the case of a 1 s resolution reference clock and subsecond resolution camera clock, calculating the offset at each image yields a repeating pattern, such as the example from a Nikon D200 in Table 2.2. As a result of the stepwise nature of the reference clock sequence, the offset will reach a local maximum \(a_i\) at the frame preceding each second rollover and a local minimum \(b_i\) at the frame following each second rollover. Since the true second rollover occurs at a time between each \([a_i, b_i]\)
pair, it follows that the offset can be no more than the smallest \( b_i \) and no less than the largest \( a_i \) minus 1 s, i.e.

\[
\max(a_i) - 1 \leq \text{offset} \leq \min(b_i) \tag{2.2}
\]

For the sequence in Table 2.2, in which the true offset is constrained to the overlapping \([a_i - 1, b_i]\) intervals \([0.26 \text{s}, 0.51 \text{s}]\), \([0.34 \text{s}, 0.50 \text{s}]\), and \([0.32 \text{s}, 0.49 \text{s}]\), \( \max(a_i) - 1 = 0.34 \text{s} \) and \( \min(b_i) = 0.49 \text{s} \), yielding the interval intersection \( 0.34 \text{s} \leq \text{offset} \leq 0.49 \text{s} \), or more explicitly, \( \text{offset} = 0.415 \pm 0.075 \text{s} \). In the case of only one second-rollover in the sequence, the range of the offset is equal to the time between the single \([a - 1, b]\) pair, by definition the frame-rate of the camera between those two images. Sampling a greater number of second rollovers drives the range of the offset estimate towards zero by increasing the probability that \( \max(a_i) - 1 \approx \min(b_i) \). If we assume a uniform probability distribution function over the full range (that is, we assume the camera is being triggered randomly and we ignore any sampling bias due to the camera’s regular frame-rate), the 95% confidence interval (CI) for the example above is 0.073 s. In the reverse scenario, that of a subsecond resolution reference clock and 1 s resolution camera clock, the offset \( T_c - T_r \) reaches a local maximum \( a_i \) at the frame following each second rollover and a local minimum \( b_i \) at the frame preceding each second rollover. In this case, the true offset can be no less than the largest \( a_i \) and no more than the smallest \( b_i \) plus 1 s, i.e.

\[
\max(a_i) \leq \text{offset} \leq \min(b_i) + 1 \tag{2.3}
\]

Although ignored here for clarity, in practice the precision of both clocks is nonzero and needs to be added to the error estimate. This is made especially clear in cases when \( \max(a_i) - 1 > \min(b_i) \).

### 2.4.3 Subsecond: neither camera nor reference

In the case that both the camera and reference lack subsecond reporting, a more careful analysis is needed. Each time either the camera or reference clock rolls forward one second, the offset alternates between two consecutive values (e.g. -1 and 0, 2 and 3), yielding a periodic binary
Table 2.2: Sequence of the second component of a subsecond resolution camera clock ($T_c$), 1s resolution reference clock ($T_r$), and resulting offset ($T_c - T_r$) from photographs of the NIST Web Clock taken with a Nikon D200. Second rollovers by the reference clock are indicated by thin lines. Markers $a_i$ and $b_i$ are described in Section 2.4.2. In this case, the true camera offset is 0.415 ± 0.073 s (95% CI).

<table>
<thead>
<tr>
<th>Camera (s)</th>
<th>Reference (s)</th>
<th>Offset (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.82</td>
<td>13</td>
<td>0.82</td>
</tr>
<tr>
<td>14.08</td>
<td>13</td>
<td>1.08</td>
</tr>
<tr>
<td>14.26</td>
<td>13</td>
<td>1.26</td>
</tr>
<tr>
<td>14.51</td>
<td>14</td>
<td>0.51</td>
</tr>
<tr>
<td>14.67</td>
<td>14</td>
<td>0.67</td>
</tr>
<tr>
<td>14.92</td>
<td>14</td>
<td>0.92</td>
</tr>
<tr>
<td>15.08</td>
<td>14</td>
<td>1.08</td>
</tr>
<tr>
<td>15.34</td>
<td>14</td>
<td>1.34</td>
</tr>
<tr>
<td>15.50</td>
<td>15</td>
<td>0.50</td>
</tr>
<tr>
<td>15.74</td>
<td>15</td>
<td>0.74</td>
</tr>
<tr>
<td>15.91</td>
<td>15</td>
<td>0.91</td>
</tr>
<tr>
<td>16.08</td>
<td>15</td>
<td>1.08</td>
</tr>
<tr>
<td>16.32</td>
<td>15</td>
<td>1.32</td>
</tr>
<tr>
<td>16.49</td>
<td>16</td>
<td>0.49</td>
</tr>
</tbody>
</table>
integer sequence. By stripping subseconds from the offset sequence in Table 2.2, the following
integer sequence results – 0 1 1 | 0 0 0 1 1 | 0 0 0 1 1 | 0 – where | denotes a second rollover
by the (trailing) reference clock. In this example, the repeating five-frame pattern indicates that
the camera fired at 5 frames/s, or 1 frame every 0.2 s, a result that agrees well with the average
spacing between subsecond camera times in Table 2.2 (0.205 s) and the advertised ‘5 frames per
second’ of the Nikon D200. Since the pattern is consistent over two full cycles, or ten frames, the
apparent mean frame-rate \( f \) over that period could not differ from this estimate by \( > 10\% \), i.e.
\( f = 0.20 \pm 0.02 \) s. The use of ‘apparent’ must be stressed because besides variations in the frame-
rate of the camera, errors in the second rollover timing of both the reference and camera clocks can
alter the sequence. In practice, since errors by both clocks may mask one another, the combined
precision of the two clocks should be used instead if known to be coarser. Since the camera clock
led the reference clock by 1 s for two frames each cycle \( (n = 2) \), the offset can be no smaller than
\((n - 1)f\) (the camera clock advanced immediately before the first image in the cycle was taken,
and the reference clock advanced immediately after the second or nth image was taken) and no
larger than \((n + 1)f\) (the camera clock advanced immediately after the first image in the cycle was
taken, and the reference advanced immediately before the third or nth + 1 image was taken). This
scenario for \( n = 2 \) is depicted in Figure 2.6. The (fully bounded) offset can be expressed more
generally as

\[
\text{offset} = \min(o_i) + nf \pm (f + df),
\]

where \( \min(o_i) \) is the smallest or most negative of the offset measurements \( o_i \), \( n \) is the average
number of consecutive \( o_i \) larger than \( \min(o_i) \) within a full cycle in the sequence, \( f \) is the mean
frame-rate, and \( df \) is the uncertainty in \( f \).

Unlike in Section 2.4.2, where we assumed a uniform distribution over the range of the offset,
the probability distribution determined by this method is that of an upright isosceles triangle
(Figure 2.6c). The likelihood of a given offset decreases linearly with distance from the mean (where
the cumulative range of compatible pairs of camera and reference rollovers is maximized), until it
Figure 2.6: Schematic illustrating the bounds on the possible range of the offset for the case in which both camera and reference lack subsecond resolution (drawn for the sequence in Table 2.2, in which the camera leads the reference for two frames). Thin vertical bars represent photographs in a sequence, numbers above and below the bars are the seconds reported by the camera and reference in those images, and thick vertical bars represent possible times of second rollovers for each device. The probability (c) of a particular offset is zero at the minimum (a), which can only be achieved if the camera rollover occurs as late as possible and the reference rollover occurs as early as possible, increases linearly to the mean offset (which can be achieved by the most combinations of camera and reference rollover times), then declines linearly to zero at the maximum (b), which can only be achieved if the camera rollover occurs as early as possible and the reference rollover as late as possible.
approaches zero probability a distance $f + df$ from the mean (where the range of compatible camera and reference rollovers narrows to zero). Given this probability distribution, the 95% confidence interval is given by

$$
\text{offset} = \min(o_i) + n f \pm (1 - 1/\sqrt{20})(f + df).
$$

In our example, where $\min(o_i) = 0 \text{ s}$, $n = 2$, $f = 0.2 \text{ s}$, and $df = 0.02 \text{ s}$, this yields $\text{offset} = 0.40 \pm 0.17 \text{ s}$, which agrees with the result derived in Section 2.4.2, $0.415 \pm 0.073 \text{ s}$.

2.5 Case studies

We present two glaciological case studies where accurate knowledge of image acquisition time was critical: georeferencing aerial photogrammetric surveys at Columbia Glacier with in-flight camera positions time-interpolated from GPS tracklogs (Section 2.5.1); and synching high frame-rate observations of iceberg-calving events to the resulting seismic waveforms at Yahtse Glacier, Alaska (Section 2.5.2). The case studies are included only to highlight some of the (possibly many) applications for calibrated camera clocks; therefore, lengthy analyses of the scientific results are not conducted here.

2.5.1 Accurate geotagging for DEM creation

Leading image-based approaches to three-dimensional (3-D) modeling [reviewed by 66] are now capable of performing automated scene reconstruction on even the largest and most poorly documented image collections: for instance, rebuilding Rome from thousands [9] to millions [10] of street-level photographs. This accomplishment hinges on flexible structure-from-motion (SfM) algorithms which can triangulate from overlapping images taken from multiple perspectives (‘motion’) both the relative camera geometry and 3-D scene (‘structure’) that gave rise to the images. Therefore, although conventional ground control can be (and are best) used to scale and orient the resulting model to absolute coordinates [67, 68], surveyed camera positions are theoretically sufficient – a significant advantage for measurement programs where fixed bedrock control is unavailable.
due to topographical or logistical constraints.

On 25 May 2010, airborne vertical stereo photographs (0.40 m ground resolution) were acquired over Columbia Glacier, positioned and oriented using previously surveyed ground control, and processed to a 2 m accuracy digital elevation model (referred to as conventional DEM). Same-day oblique imagery was acquired with a Nikon D2X from the window of a second small aircraft flying the path shown in Figure 2.7 at a mean distance of 1.4 km above the glacier surface. A DeLorme PN-40 handheld GPS logged a position every second, equivalent to 32 m nominal point spacing at the average flight speed of 114 km h$^{-1}$. At such velocities, time-interpolating accurate camera positions from the tracklog requires subsecond calibration of the camera clock. The tracklog is believed to be tightly coupled to the internal rather than displayed time of the GPS device (the information required to confirm this assumption is not available from the manufacturer at this time) and therefore calibration to UTC was performed. At the onset of the 1-hour photograph acquisition period, we captured a sequence of images of the GPS which later revealed that the camera lagged behind the GPS clock display by 2.485 ± 0.054 s (95% CI). Following our return from the field, we compared the GPS display to a NTP-synchronized computer clock with millisecond accuracy and found that in 151 second-rollovers over 2 months, the GPS display lagged behind UTC by 0.173 ± 0.095 s (95% CI). A correction of 2.658 ± 0.109 s (95% CI) thus should provide the best agreement with the GPS tracklog. Camera clock drift was ignored due to the short photograph acquisition period and the small nominal drift of the Nikon D2X used (Table 2.1; −0.044 ± 0.004 s d$^{-1}$).

The photographs from our aerial survey were first processed with the open-source SfM package Bundler [70] to simultaneously calculate a sparse, relatively oriented point cloud (964 075 points) and corresponding camera positions, orientations, and lens calibration parameters. To test the time calibration estimate, absolute camera positions for all 383 images were linearly time-interpolated from the GPS tracklog for a range of camera time corrections. The SfM-computed and GPS-interpolated camera positions were then used to calculate a least-squares seven-parameter (Helmert) transformation [71] for orienting and scaling the SfM model. The root-mean-square (RMS) 3-
Figure 2.7: Map of the GPS tracklog and time-interpolated image-capture positions from the aerial photographic survey of Columbia Glacier, Alaska, conducted on 25 May 2010. The final DEM generated from the SfM model (dark hillshade) is overlaid on a regional 2007 Satellite Pour l’Observation de la Terre (SPOT) satellite DEM [69]. UTM coordinates (zone 6) reference the World Geodetic System 1984 (WGS84) ellipsoidal elevation.
D distance between the transformed model camera positions and time-interpolated GPS camera positions reaches a minimum (13.09 m) at 2.69 s, within the expected range of the camera time correction (Figure 2.8, bold line). In this case, precise knowledge of the camera-UTC offset markedly improves the agreement of the GPS positions with the relative camera geometry computed by SfM.

The RMS elevation error between the conventional DEM and transformed SfM point cloud exhibits an equivalent but lower amplitude relationship to the camera time correction (Figure 2.8, thin line). Since elevation differences depend on local slope, they offer a less sensitive confidence measure (e.g. an XY error in the transformation would result in no vertical error wherever the ground is flat, in error contours wherever the ground is sloped, and in randomly distributed error spikes over glacier crevasses). To better reflect the accuracy over the glacier surface, poorly constrained SfM points in the glacier forebay (occupied by ice mélange) and on the peaks above 600 m (13% of total points) were discarded. Furthermore, the SfM elevations were corrected for a systematic −8.02 m bias (determined from bedrock regions of known elevation) likely associated with the single-frequency handheld GPS. The RMS elevation error between the conventional DEM and transformed SfM point cloud reaches its minimum of 6.10 m at 2.69 s, providing a second independent assessment of the camera time correction. The errors are nearly equally distributed (mean −0.77 m) and could be due largely to differences in crevasse depth penetration resulting from the varying viewing angles of the oblique images or the smoothing algorithms used by Inpho Match-T in computing the conventional DEM.

Automated software packages, both commercial and open-source, are becoming available — e.g. Agisoft Photoscan [72] and VisualSFM [73] — increasing the accessibility of the SfM method. Adoption of SfM technology and application of camera time calibration methods has enabled us to produce DEMs of comparable accuracy to conventional vertical photogrammetry using only oblique photographs and a consumer-grade tracklog of camera position.
Figure 2.8: The RMS 3-D distance between the transformed SfM camera positions and time-interpolated GPS camera positions (bold line), RMS elevation error (m) between the conventional DEM and transformed SfM point cloud (thin line), and 95% CI of the predicted time correction (two vertical lines).
2.5.2 Icequake source mechanisms

Since iceberg calving was first identified as a source of seismic energy [74], glaciologists have been attempting to identify the specific sources of that energy. These studies have largely been motivated by attempts to learn about and predict calving (i.e. develop ‘calving laws’) or to remotely monitor calving fluxes for mass-balance and dynamical studies of tidewater glaciers. Various portions of the calving process have been proposed as the sources of calving seismicity, including hydrofracture and resonating water-filled cracks [75], basal slip [76], and the rotation and terminus push-off of large icebergs [77, 78].

In the present case study, we use the camera time calibrations developed in Section 2.4.3 to synchronize video of iceberg-calving events with seismograms, allowing us to draw correspondences between the directly observable calving process and the seismic record. Our efforts to constrain the seismic source of calving took place at Yahtse Glacier, an advancing tidewater glacier on the Gulf of Alaska (60.158 N, 141.388 W). Video was recorded with a Canon EOS 7D at 29.97 frames/s. We tested two commercially available standards for absolute timing by placing them together within the video: a wall clock synchronized to radio signal WWVB (Radio Shack, model 63-247) and a handheld GPS (Garmin eTrex Vista HCx). In 13 comparisons of 3 s to 5 s each, we found that the radio clock lagged the GPS display by 0.56 ± 0.57 s. Following our return from the field, we compared the GPS to a NTP-synchronized computer clock with millisecond accuracy. In 14 measurements over 2 months, we found that the GPS lagged behind UTC by 0.01 ± 0.44 s. We therefore selected the GPS unit as our time standard and used it to synchronize each of our video clips to UTC. We conservatively assume that our video is accurate to within 1 s.

Figure 2.9 presents a sequence of video frames from a typical calving event with contemporaneous seismic data (the full video is available as supplementary material at igsoc.org/hyperlink/12j126/12j126Fig9.mov). At a distance of 1.8 km from the calving event, the digitizer of the broadband seismometer is connected to a GPS antenna and is synchronized to UTC at the time of recording. On the same amplitude scale, we present two passbands of vertical channel data, 1 Hz to
5 Hz and 5 Hz to 50 Hz. Waveforms were filtered using a four-pole Butterworth filter that does not artificially offset the waveform timing, and the timing of the seismogram was adjusted to account for the travel time from the source to the receiver. We assumed a velocity of 1.9 km s\(^{-1}\), the velocity measured for the peak amplitude as it moves through a local network of seismometers.

These methods reveal that, in this case, the largest-amplitude seismic signals are at relatively low frequencies (< 5 Hz) and best associated not with ice fracture but with the splash of water seen erupting from sea level at 9 s after the calving event initiates. This result, and those from similar videos, indicates that calving seismicity is greatly influenced by calving style (i.e. submarine or subaerial calving) and how far from its neutrally buoyant position an iceberg is released [15]. Seismic methods are best suited to monitor iceberg-calving rates when the calving process is energetic, as is the case for subaerial events and submarine events released at great depths. Shallowly released submarine calving events, including some of the largest events at Yahtse Glacier, generate only gentle splashes and thus only weak seismic waves.

Owing to the 15 s to 20 s duration of this and many other calving events at Yahtse Glacier, we have been able to correlate the visual record of calving to the seismic record with an accuracy of 1 s. In the absence of clock synchronization, there would be no rigorous method for relating the mechanical calving sequence to seismic signals, hampering further analysis of calving icequakes. If care were taken to better calibrate the timing of the video sequences, more might be learned about the connections between high-frequency seismicity and ice fracturing, particularly at the initiation of the calving event (t = 0 s to 6 s in this example). However, the 0.5 s dominant period of this seismogram and uncertainties pertaining to the wave travel time would still hinder some attempts at higher-accuracy analyses.

### 2.6 Sample code

The suite of perl and bash scripts available as supplementary material at igsoc.org/hyperlink/12j126/12j126sect6 provide the basic tools needed for evaluating camera-clock offset, drift, subsecond resolution, and precision, and for subsequently correcting measured offset and drift from image-
Figure 2.9: Example calving event with synchronized video stills and filtered seismograms. In the first two frames, the top of the major detached block is outlined with a dashed line. Ice associated with the calving event is observed to begin falling at UTC 2010:09:07 22:13:50.3 (t = 0). Two passbands of seismic waveform from the vertical channel of a nearby seismometer are scaled relative to each other; the maximum unfiltered seismic amplitude is $18.3 \text{ mm s}^{-1}$ at 10.0 s. Seismograms have been corrected by 0.95 s for seismic wave travel time. Video of this calving event, synchronized with the seismogram, is available as supplementary material at ig soc.org/hyperlink/12j126/12j126Fig9.mov.
capture times. The code package requires the ExifTool command-line application and perl library [79] for reading and writing Exif, and the ImageMagick command-line application [80] for stamping capture date and time onto images. The functions are introduced below; complete documentation can be found as comment blocks inside the code.

`local_time.pl` prints out the system time at the specified interval and resolution. When run on a computer calibrated to a stable time source, this script provides a high-accuracy, subsecond resolution reference clock for measuring camera-clock precision, offset and drift.

`extract_time.pl` reads the DateTimeOriginal and SubSecTimeOriginal tags from all image files in the specified directory, writing the results alongside image filenames to a tab delimited text file in the formats ‘YYYY:MM:DD hh:mm:ss’ and ‘00’, respectively.

`timestamp.sh` stamps the DateTimeOriginal and SubSecTimeOriginal values, formatted as ‘YY YY/MM/DD hh:mm:ss.ss’, onto new versions of all jpeg images in the specified source directory. Timestamped images can help to evaluate a camera’s clock against reference clocks photographed with the camera.

`embed_time.pl` copies the values of the DateTimeOriginal and SubSecTimeOriginal tags to the XMP Description tag as ‘<DateTimeOriginal = YYYY:MM:DD hh:mm:ss(.ss)>’ for all jpeg images in the specified directory. This backup of the original capture date and time to another metadata field is recommended before any adjustments are applied so that the information can be restored with the reverse function `restore_time.pl` if an error is later realized.

`adjust_time.pl` adjusts the capture date and time of all jpeg images in the specified directory according to the camera-specific drift, the camera-reference offset, and the camera date and time at which the specified offset was measured. The original DateTimeOriginal and SubSecTimeOriginal values are overwritten with the new adjusted values.
2.7 Future work

Although low in cost, hardware requirements, and power consumption, the methods presented here are labor-intensive and in some situations only partial solutions. For instance, we provide a script to correct for mean clock drift calculated from two measurements of camera-reference offset bounding the period of interest, but accounting for undocumented deviations from this mean due to temperature modulations would require both a temperature record spanning the camera’s deployment and a physically or empirically derived model of the clock drift’s temperature dependence. Ultimately, continuous calibration to a reliable visual or electronic time signal – effectively bypassing camera timekeeping entirely – may provide the final solution for camera impartiality and immediate guaranteed precision, just as GPS has for countless other instruments. Integrated GPS already exists in some camera and camera-phone models and may soon negate the need for an external time reference in applications where second precision is sufficient and GPS signals are available. For the time being, however, the latency and slow refresh rates of these systems, as discussed in Section 2.3, currently prevent their use for subsecond observation.

For time-critical applications, we suggest two purely electronic strategies: the triggering of the camera by a time-calibrated device and alternatively the recording of the camera trigger by a time-calibrated device. A popular solution for precise and portable UTC timekeeping is a GPS receiver equipped with a pulse-per-second (PPS) dedicated time port, which announces the start of every second with millisecond or better accuracy. This electrical signal is used routinely to discipline a computer clock, which could subsequently be used to initiate video recording or still capture at predetermined times. Similarly, any GPS receiver chip could, with custom software and hardware, be used to discipline the clock’s onboard time-lapse intervalometers, an appropriately low-power solution to the challenge of filtering temperature-dependent drift from extended time-lapse deployments.

The reverse scheme – recording rather than triggering image capture -- could be made possible by the hot shoe or flash port provided on most if not all pro-level still cameras. These interfaces
are used by the camera to send an electronic signal to trigger external flashes and alternatively could be connected to a GPS event logger. By design, the timing of this signal must be precisely synchronized to the opening of the shutter, since flashes fire very short bursts of light (0.02 ms). In practice, consumer SLR cameras are capable of flash synchronization speeds of 17 ms (1/60 s) to upwards of 2 ms (1/500 s). The alignment of image capture and flash trigger can be quickly constrained by photographing the triggered flash at a range of shutter speeds.

Finally, continuous calibration to visual time signals is possible but more labor-intensive to process. Placing conventional robust reference clock displays (e.g. time-calibrated computer, research-grade GPS) within the camera field of view for the duration of the deployment is impractical in many field situations (especially if the focus is set at a great distance from the camera), but more compact solutions do exist. Precision GPS time video inserters, commonly used in the amateur astronomy community for timing occultations [81], embed millisecond accuracy timestamps directly onto intercepted analog video streams. No conventional equivalent yet exists for digital video; instead, the timestamp can be introduced optically (for video, one solution could consist of a light-emitting diode blinking to a GPS PPS signal, blinking twice every minute for easier determination of the absolute time). Any of these methods can be evaluated by photographing a reference time display with the tethered camera, as previously discussed.

2.8 Conclusions

We have reviewed obstacles to precise camera timekeeping, tested the capabilities of available consumer-grade camera models and UTC reference clocks and demonstrated calibration procedures for absolute timing, all in an effort to extend the application and reliability of digital still and video cameras for use in scientific observation. Camera clock drift is singled out as the unique source of multi-second to minute errors, while timestamp precision and resolution, onboard GPS refresh rates, and GPS and radio clock display latency pose challenges for subsecond observation. With proper calibration, subsecond imagery is well within the reach of select consumer-grade digital cameras. This represents a potentially pervasive addition to the instrumental record, both for
relative timing (e.g. measuring the rates of rapid processes) and for confidently matching visual (and acoustic) observations to the many other data already synchronized to a time standard.

At Yahtse Glacier, time-calibrating video footage of iceberg-calving events has allowed us to draw conclusions about the sources of the observed seismic signals. At Columbia Glacier, refining image-capture times has allowed us to considerably improve estimates of in-flight camera positions, which subsequently allowed us to orient our photogrammetric models without the need for additional ground control. Ultimately, careful management of camera time errors is applicable to the study of all rapid processes observable in the visible and near-infrared spectrums. Multi-second to subsecond significant processes are admittedly rare in the cryosphere: for example, supraglacial lake drainage [82], snow avalanche triggering [83], and iceberg calving as previously discussed. However, they permeate the physical Earth: volcanic eruptions [16], structural failure during earthquakes [84, 85], meteorological conditions [86, 87], nearshore wave dynamics [88, 89], and animal behavior [90, 38], to list just a few.

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2.10 Author contributions

EZW developed the calibration techniques, performed the laboratory experiments, conducted the structure-from-motion case study, wrote the included software, and drafted the manuscript. TCB conducted the icequake mechanism case study and drafted corresponding Section 2.5.2. SO’N collected many of the images used for estimating the drift of Columbia Glacier time-lapse cameras. WTP collected the images used in the structure-from-motion case study. All authors contributed to the final manuscript.
Landscapes as clocks

3.1 Introduction

Chapter 2 describes calibrating a camera clock to absolute time by placing a reference clock into the field of view of the camera. For existing photographs, precise timing may still be achievable if events of known time can be identified in the photographs.

Since August 2009, the time-lapse cameras operating at Columbia Glacier, Alaska have been calibrated to Coordinated Universal Time (UTC) by photographing Global Positioning System (GPS) and mobile phone clock displays during service visits. Image capture times are corrected for both the measured offset and the mean drift of the camera between consecutive offset measurements. The precision achieved by these corrections – on the order of seconds to a few minutes – is limited only by the temperature-variability of camera clock drift and the accuracy of the reference clock displays. Before August 2009, however, time-lapse cameras at Columbia Glacier were neither calibrated in a systematic way nor set to a consistent time zone. For these images, camera clocks were calibrated to UTC by relying on the content of the photographs: specifically, by identifying naturally-occurring events of known time. The achieved precision – on the order of seconds to hours – is limited by the framerate of the time-lapse sequences and the types of events visible in the images.

Most existing techniques for determining the capture times of photographs are concerned with dating historical photographs of human subjects on timescales of months to years. Temporal clues – either uncovered manually by experts or automatically by trained algorithms – include the
size, paper, mounting, and chemical agents of physical prints [91, 92], the color rendition imparted by the film process [93, 94, 95, 96], and the appearance of humans or human artifacts in the photographs, ranging from clothing and hairstyles [97, 98] to the structure of cities [99]. The few examples applicable to dating natural landscapes on timescales of seconds to hours are ‘forensic astronomy’ case studies in which the positions of astronomical objects (directly visible or inferred from shadows) are used to date photographs of historic, artistic [100] or legal [101, 51] importance.

In this chapter, we describe the timekeeping opportunities and limitations of several phenomena visible in photographs, and demonstrate their use for dating both single photographs and sequences of photographs. Section 3.2 discusses the positions of astronomical objects in the sky, while Sections 3.3 and 3.4 discusses the resulting variations in solar radiation and sea level. Finally, Section 3.5 addresses the temporal ambiguity that can result from the cyclical nature of these phenomena. Time-lapse photographs from Columbia Glacier, Alaska are used to illustrate the various techniques.

3.2 Astronomical objects

3.2.1 The Sun

As the Earth rotates about its tilted axis and around the Sun, the Sun sweeps across the sky in nested arcs which precess smoothly northward (towards the June solstice) or southward (towards the December solstice) over the course of a year (Figure 3.1). As a result, the vertical angle of the Sun above the horizon (the ‘altitude’) is representative of the time of year, while the horizontal angle of the Sun east of north (the ‘azimuth’) is representative of the time of day. The capture time of an image can be inferred directly from the Sun’s position in the image – very precisely if the camera’s position, orientation, and optical parameters are known.

The Sun’s speed across the sky is about 360° per 24 h, the speed of the Earth’s rotation about its own axis. Since the angle of view of a photographic image is [103]

\[ d_{\text{angle}} = 2 \arctan \frac{d}{2f}, \]  

(3.1)
Figure 3.1: Paths of the Sun and the Moon across the sky for every day between the December and June solstices, as seen looking south from Columbia Glacier, Alaska (−147.08° W, 61.14° N). Altitude and azimuth were computed with R package oce [102].
where $d$ is the film or sensor size and $f$ is the lens focal length, the pixels of a 3872 pixel image taken with a 35 mm camera fitted with a 24 mm lens each span $0.019^\circ$ ($d_{\text{angle}}/3872$), an angle which the Sun traverses in only 4.6 s. Even when lens flare and overexposure make the Sun’s center difficult to locate precisely, the time of day can be determined to within a few minutes since the Sun moves by its angular diameter ($0.53^\circ$) in only 127.2 s. However, since the Sun’s path rises or falls by only $0.26 \pm 0.11^\circ$ per day (Figure 3.2) – a fraction of the Sun’s diameter – offsets on the order of days are difficult to resolve from the Sun alone.

### 3.2.2 The Moon

Unlike the Sun, the Moon orbits the Earth once every 27.3 d, in the same direction as the Earth’s rotation. This slows its apparent motion across the sky by only 3.5% relative to the Sun, but results in large changes to its path between consecutive days (Figure 3.1). At the same time on consecutive days, the Moon’s position changes by about $360^\circ/27.3 \approx 13.2^\circ$ (Figures 3.2 and 3.3), a distance it travels in 51 min. Thus, the Moon’s position can be used to distinguish between adjacent days if the time of day can be determined by other means to within an hour. Furthermore, since the Moon is at most $1/400\,000$ the brightness of the Sun [104], its position is not obscured by lens flare or overexposure in daytime photographs. However, since the Moon revisits a region of the sky as often as 13 times per year ($\sim 365.24 \, \text{d}/27.3$), its position — unlike the Sun’s — does not map to a unique time of year.

### 3.2.3 Stars and planets

The Earth orbits the Sun once every 365.24 d, all the while completing one rotation per day around its own axis. As seen from the rotating surface of the Earth, the Sun moves across the sky $365.24 + 1$ times in 365.24 d. For the ‘fixed’ stars, around which the Earth does not orbit, it thus takes $365.24/(365.24 + 1)$ of a solar day to complete one rotation around the Earth. This period, known as a ‘stellar day’, is approximately 235.9 s shorter than a solar day, and results in the fixed stars changing position by $0.99^\circ$ at the same time between consecutive days. Thus, like the Moon,
Figure 3.2: Changes in the position of the Sun and the Moon at solar noon between consecutive days. Altitude and azimuth were computed with R package oce [102]. Results are independent of position or time of day.
Figure 3.3: Path of the Moon (grey line) projected onto a photograph from sequence CG06-20060918 nominally taken at 2006-09-13 21:59:44 UTC. The path extends ±30 d from the time of the photograph and is marked at daily intervals (white dots): ‘0’ marks the position of the Moon in the photograph while the other dots mark the position of the Moon whenever it is again visible in the photograph at the same time on a different day. Moon altitude and azimuth were computed with Python package astropy [105].
the positions of fixed stars can be used to distinguish between adjacent (solar) days. Furthermore, because of their very small apparent size and brightness, the position of stars in an image can be more precisely determined than either the Sun or the Moon.

However, unlike the Sun, the position of the fixed stars cannot be used to resolve the time of year. The Earth’s orbit around the Sun ($300 \times 10^9$ m diameter) is dwarfed by the distance to the nearest extrasolar star ($40 \times 10^{15}$ m to Proxima Centauri), resulting in a seasonal parallax of at most 0.0004° – only 1/50 pixel for the typical camera described earlier. On longer timescales, the 25 700 year precession of the Earth’s rotational axis results in a gradual shift in star positions of up to $360°/25 700$ year $\approx 0.014°$/year (1 pixel/year) – too slow for most timing applications but useful if the capture time of a photograph is unknown on the order of decades or longer. Figure 3.4 illustrates the movement of stars over these different timescales.

The planets in our solar system, including the Earth, orbit the Sun in elliptical orbits of varying sizes and inclinations. The combined effect of the Earth’s spin and orbit and the other planets’ unique orbits results in each tracing different and complex paths across the sky [106]. Along with the Moon, their moons, and the other objects in the solar system, their configuration in the sky are temporal signatures – unique over potentially very long timescales. For example, the configuration of the Moon and Venus in Figure 3.5 is uniquely occurring for at least ±10 years.

### 3.2.4 Astronomical refraction

Light rays from astronomical objects pass through the increasingly dense gases of the atmosphere to reach an observer on the Earth’s surface. This increase in density translates to an increase in refractive index, which bends the light rays towards the normal and results in astronomical objects appearing higher in the sky than they actually are.

Refraction is zero directly overhead (90° altitude) but gradually increases towards the horizon as the light rays traverse the curved atmosphere at increasingly more oblique angles. Near the horizon, however, refraction is highly variable, since the light rays can experience strong local temperature (and hence, density) gradients as they pass near the Earth’s surface. Down to 5°
Figure 3.4: Paths of stars Altaïr (left) and Tarazed projected onto a photograph from sequence CG05-20050916 nominally taken at 2005-08-22 13:00:00 UTC. The paths are shown for camera clock offsets on the order of seconds (solid lines, dominated by the Earth’s rotation) and years (dotted lines, dominated by the precession of the Earth’s rotational axis). Star azimuth and altitude were computed with Python package astropy [105].
Figure 3.5: Positions of the Moon and Venus on a photograph from sequence AK04-20070817 nominally taken at 2007-05-20 08:59:58 UTC. The lower Moon is projected from a photograph from the following night, nominally taken at 2007-05-21 09:59:59 UTC. The boxes represent the 99.7% confidence bounds of the observed positions due to uncertainty in the camera geometry, and translate to a camera clock offset of $-13\text{s}$ to $+56\text{s}$. All paths intersecting the boxes for offsets $\pm10\text{years}$ are shown – during this period, never are two or more of the boxes again simultaneously occupied. Moon and Venus azimuth and altitude were computed with Python package astropy [105].
altitude, it is sufficient to assume the density gradient of the standard atmosphere, adjusted to
the measured temperature, pressure, and humidity at the observer. Below 5°, however, demanding
measurements of the local temperature gradients are required for precise refraction correction [107].

Figure 3.6 summarizes the relationships between refraction and its associated uncertainties
with altitude. In the absence of gross atmospheric anomalies, refraction at 5° can be determined
to within 0.01° (1 pixel) if local meteorological conditions are known, and to within 0.1° (10 pixel)
if not – still smaller than the uncertainties due to camera geometry in the scenario of Figure 3.5.

3.3 Solar radiation

Whether or not the Sun itself is visible in an image, its daily path across the sky is manifested
in the length of the shadows cast by objects in the image and the color and intensity of its light. By
inverting the equation for photographic exposure, the brightness of a scene (a proxy for available
solar radiation) can be inferred from the relative brightness of a photographic image and the camera
settings that gave rise to that image:

\[
\text{scene brightness} \sim \frac{\text{image brightness}}{\text{exposure time} \times \text{aperture area} \times \text{film speed}}
\] (3.2)

Digital cameras record exposure settings for each image following the Exchangeable image file
format (Exif) standard [7] – the ExposureTime tag records the exposure time (‘shutter speed’) in
seconds (e.g. 1/100 s, 30 s), the ApertureValue tag records the aperture f-number (e.g. f/2.8, f/8.0),
and the ISOSpeed tag records the International Organization for Standardization (ISO) linear film
speed (e.g. 100, 400). Note that the aperture area is inversely proportional to the square of the
f-number (aperture area \( \sim f\)-number\(^{-2} \)). Relative image brightness can be computed by taking the
mean of the pixel intensity values. The camera clock offset can then be estimated by comparing
the inferred scene brightness to a reference timeseries of modeled solar radiation, as illustrated in
Figure 3.7.

This method relies on seasonal changes in day length to resolve the time of year, and daily
variations in solar radiation to resolve the time of day. As shown in Figure 3.8, these measures vary
Figure 3.6: Refraction of visible light as a function of altitude angle for the full range of air pressure, temperature, and relative humidity observed at the National Oceanographic and Atmospheric Administration (NOAA) weather station at the Valdez Municipal Airport, Alaska (WBAN:26479) from 2005 through 2017. Refraction was computed by the method of, and uncertainties reported by, the C library erfa [108]. Nominal refraction values calculated by the method of Sæmundsson [109] for the standard pressure of 101 \times 10^3\ Pa and temperature of 10^\circ C are shown as a dashed line.
Figure 3.7: Scene brightness, calculated with Equation (3.2) for both the original (gray line) and corrected (black line) image times for the CGZM-20080620 sequence (top), plotted against potential direct and diffuse solar radiation (dotted line) calculated by the method of Kumar et al. [110] and Hock [111]. Although the local time zone was Alaska Daylight Time (AKDT: UTC−8 h), the camera was erroneously set to UTC−10 h.
greatly with latitude. At the equator (0° latitude), day length remains constant (placing no constraint on time of year), while the range of solar radiation remains high (placing strong constraints on time of day). As latitude increases, seasonal changes in day length increase (placing constraints on time of year), saturating at 0 h (the ‘polar night’) and 24 h (the ‘polar day’) for increasingly long periods beyond the polar circles (±66.563° latitude). The daily range of solar radiation slowly decreases (gradually weakening constraints on time of day), finally becoming constant at the poles (90° latitude).

At high latitudes, during a transition between 0 h and 24 h day length, the rate of change of day length is large enough to uniquely identify days. For other latitudes and times of year, additional information – such as sea level (Figure 3.10) – must be used to select between candidate days.

This method can also be applied to sequences of photographs taken at night or during the polar night. As the Sun descends below the horizon, solar twilight radiation drops from 10 W m⁻² at 0° altitude to 0.1 W m⁻² at −10° [112], fading past −18° to a ~0.000 01 W m⁻² background of airglow, zodiacal light, and starlight. The Moon, which contributes up to 0.003 W m⁻² of reflected solar radiation when full [104, 113], is two to three orders of magnitude brighter than the moonless night sky, a variation easily detected in photographs.

3.4 Sea level

Along the world’s coastlines, the tidal rise and fall of sea level can serve as an alternative or complementary reference clock. In Columbia Glacier time-lapse photographs, the tide is visible as a change in the waterline relative to the grounded terminus, or, when floating, as vertical motion of the glacier itself. The timing of low and high tide can be inferred to the nearest image capture times and compared against reference tide tables to estimate the camera clock offset (Figure 3.9).

The Moon contributes to (and often dominates) the combined lunar and solar tides. Since it takes 24 h 50.5 min on average for a position on Earth to return to the same position below the Moon, the times of high and low tide shift slightly each day – by up to 50.5 min on average for a
Figure 3.8: Day length (the time between sunrise and sunset), change in day length between consecutive days, and daily range of solar radiation by latitude and day of year.
Figure 3.9: Water level – inferred from vertical pixel displacements of the waterline (solid line) in images from the AK04-20070817 sequence (top) – plotted against the water level observed at the National Oceanographic and Atmospheric Administration (NOAA) tide gauge in nearby Valdez, Alaska (COOPS:9454240, dotted line). The best agreement (shown) is achieved by shifting the image times by +8 h, confirming that the camera was set to the local time zone (AKDT: UTC−8 h).
pure lunar tide. In the same way as the Moon’s position, tides can complement the Sun in helping to distinguish between adjacent days, as illustrated in Figure 3.10.

### 3.5 Singular events

The phenomena discussed above are periodic, driven by the cyclical motions of the Sun, the Moon, and other astronomical objects relative to a position on the Earth. As a result, they cannot uniquely identify the capture time of a photograph without constraints on the range of possible times. In the scenario of Figure 3.10 where both solar radiation and sea level are used to identify a best-fitting day of year and time of day, the year remains ambiguous: equivalent solutions extend at roughly annual intervals indefinitely into the past and the future.

An event that occurs only once within the bounds of possible times can resolve this ambiguity. For example, large calving events visible in Columbia Glacier time-lapse photographs have been matched to large, isolated waveforms recorded by nearby seismometers whose onboard clocks were synched to GPS satellites, yielding an unambiguous (and very precise) measurement of camera clock offset. The spatial and temporal complexity of the scene and our knowledge of the scene’s history set the limits of what can be accomplished by this method. At which times within the bounds of possible pasts has the scene looked as it did in this photograph?

Given what is known of Columbia Glacier’s previous advance [23] and its current retreat, the photograph in Figure 3.11 dates from either 2008 or from before 1200, long before the advent of photography. The combined shape and position of the terminus, medial moraines, crevasses, shadows, and icebergs in the forebay form a unique temporal fingerprint which can be matched with high certainty to an aerial photograph of known time. In contrast to proxies like solar radiation or water level which leverage only a fraction of the target image content, comparisons to reference images – whether from aerial, satellite, or terrestrial platforms – can leverage the target images in their entirety. In cases when multiple time-lapse cameras were operating simultaneously at Columbia Glacier, temporal agreement between the sequences were established by aligning them according to the general appearance of the scene, then refining the alignment based on transient
Figure 3.10: Root-mean-square (RMS) error of reference solar radiation (black line) and water level (gray line) relative to proxies inferred from images of the AK04-20070817 sequence (Figure 3.9) for a range of camera clock offsets. For any given year (the choice of which requires additional information), the solar signal identifies the time of year and the time of day while the phase shift in the lunar signal allows the date to be identified from among several candidates indistinguishable in the solar signal. For clarity, only the minimum daily values are displayed in the annual timeseries.
and singular details – for example, by identifying, for each camera, the two images bracketing a large calving event and ensuring that the time interval for all cameras intersected.

### 3.6 Discussion

The utility of photographs often hinges on knowing when they were taken. Unfortunately, although modern digital cameras embed the date and time of image capture into the resulting files, camera clocks are often set improperly, rendering the recorded timestamps unusable or at least imprecise. Furthermore, since film cameras do not record metadata (it must be written down and transmitted by the photographer), the capture time of analog photographs are rarely known with certainty. Previous studies have focused on dating photographs using human subjects, human artifacts, or the shadows cast by vertical structures – none of which are common elements in natural landscapes. Motivated by the need to fix the time of the Columbia Glacier time-lapse photographs, we have identified a range of common natural phenomena, evaluated their timekeeping potential and limitations, and used them, alone and in combination, to calibrate both single and sequences of photographs to absolute time. The achieved precision (on the order of seconds to hours) and uniqueness (on the order of a day to decades) depend mostly on the phenomena visible in the photographs.

In cases when astronomical objects are visible, precision on the order of seconds is achievable from even a single photograph under ideal conditions. If only one astronomical body is visible, the position, orientation, and optical parameters of the camera must all be known precisely by independent means, since it is the absolute position of the object in the sky that tells the time. If multiple objects are visible simultaneously, their relative positions in the sky may be sufficient (provided they are not all fixed stars, which ‘move’ together), loosening the requirements on camera geometry. Uniqueness can be increased from just a day (for a single photograph of the Sun) or up to a month (for a single photograph of the Moon) to decades or more by using the positions of additional objects or the position of the same object in additional photographs. Although astronomic refraction is negligible directly overhead, it limits the achievable precision near the
Figure 3.11: A photograph from sequence AK01b-20080921 nominally taken at 2008-08-11 19:34:52 UTC matched to aerial photographs taken at 2008-08-11 20:00:00 ± 01:00:00 UTC. To facilitate the ground-to-aerial comparison, an oblique image was synthesized from the orthophoto and digital elevation model (DEM) produced from the aerial photographs. Synthetic images are further described in Section 4.3.4; in short, the 3-D world coordinates of the DEM are projected into a mathematical model of the oblique camera and an image is formed from the intensity values of the orthophoto.
horizon, the portion of the sky most likely to be visible in photographs.

In cases where the sky is not visible, sequences of photographs may still be calibrated to absolute time with the use of geophysical rhythms visible at the surface. Modeled fluctuations in solar radiation are matched to scene brightness inferred from the photographs, and observed fluctuations in sea level are matched to the displacements of the water line visible in the photographs. The achieved precision, on the order of an hour, is mostly limited by the framerate and length of the sequence. Note that these methods rely on knowing the relative time between the images in the sequence, and cannot be used if only the order of the photographs is known. However, they do not require any knowledge of camera geometry other than a very coarse camera position for the purpose of modeling or selecting observations of sea level and solar radiation.

These examples do not consider all the natural cues that could feasibly be used to determine the date and time of a photograph. The increasing temporal coverage of satellite imagery suggests that these could be used in the future to help date terrestrial photographs. Although more difficult to predict than geophysical clocks, living organisms may serve as living clocks, displaying persistent rhythms on tidal, diurnal, and seasonal time scales [114, 51] – for example, increased feeding activity on a beach during low tide, sleep movements of plant leaves, and stages of plant flowering. It is also sometimes possible to infer the position of the Sun (or the Moon). Lalonde et al. [115] and Lalonde et al. [116] estimate the Sun’s position by fitting a physically-based model of sky appearance to a single and a sequence of photographs, respectively. Sunkavalli et al. [117] estimate the Sun’s position by fitting a photometric model of scene reflectance to a sequence of photographs, a method which does not require the sky to be visible. However, these are very complex methods which must also simultaneously infer geometric properties of the camera and the scene. If scene and camera geometry are sufficiently well constrained (for example, by the methods of Chapter 4), a much more direct approach is to calculate the illumination angle directly from the positions of the apex of an object and the apex of its shadow in a single photograph [101, 100], or for greater temporal uniqueness, from the trajectory of the shadow apex in a sequence of photographs. If detailed geometry is not available, the use of shadows may still be possible if the scene contains
planar surfaces. For example, the method of Junejo and Foroosh [118] can recover the day of year of a photographic sequence from the position of two objects and their shadow apex trajectories, provided that both shadows are cast onto a shared plane.

However successful, the methods presented and proposed are labor-intensive and intended as last resorts for poorly-documented past photographs. Future photographs should be acquired following the best practices outlined in Chapter 2. All cameras at a study site should be set to the correct date, the same and documented timezone (or UTC), and optionally kept calibrated to UTC by repeatedly photographing a reference clock display. The digital image files should be transferred and stored in such a way that the embedded camera metadata is kept intact.

3.7 Conclusions

Motivated by the need to fix the time of the Columbia Glacier time-lapse photographs, we have identified a range of phenomena visible in the photographs, evaluated their timekeeping potential and limitations, and used them to calibrate both single and sequences of photographs to absolute time. In cases when astronomical objects are visible, precision on the order of seconds and uniqueness on the order of decades is achievable from even a single photograph, limited mostly by knowledge of the camera geometry. In cases where the sky is not visible, sequences of photographs can nevertheless be calibrated to the variations in solar radiation and sea level visible at the surface, with precision on the order of an hour limited mostly by the framerate of the sequences and the sharpness of the geophysical processes being used as clocks. Uniqueness of the solution is greatly improved by combining results from phenomena operating on different temporal periods. For example, the position of the Sun indicates the time of day and time of year, while the position of the Moon helps identify the day of year. Similarly, for a sequence, solar radiation resolves the time of year to a range of possible dates while sea level helps to identify the day of year.
Chapter 4

Landscapes as calibration cues

4.1 Introduction

Digital cameras are becoming ubiquitous tools of data acquisition in the geosciences, and many have been deployed as time-lapse cameras to observe geomorphic processes over time. Time-lapse cameras are simple, cost-effective, and highly autonomous instruments, collecting large amounts of information in even extreme environments with relatively minimal effort. They are particularly well suited for high-frequency and high-resolution observations, vertical or underwater [20] perspectives, uninterrupted visibility under cloud cover, and long exposures (by moonlight and twilight) through the polar night [21, 119], precisely the situations in which conventional remote sensing is inadequate. Furthermore, once calibrated, ground-based cameras can deliver photogrammetric output comparable in form and quality (if not in spatial coverage) to that of commercial aerial and satellite imagery. However, these systems are nearly exclusively consumer-grade cameras fitted with wide-angle lenses – systems that suffer from pronounced distortion – and few have been calibrated, limiting the value of their images for detailed spatial analysis.

Ideally, a camera should be calibrated under controlled conditions before it is sent to the field – for example, by taking pictures of a calibration pattern, per Sections 4.2.3 and 4.4.2 – regardless of whether the photographs are intended for measurement, in case the camera is lost or broken and the need for analysis later arises. Of the 33 cameras used for time-lapse photography at Columbia Glacier, Alaska since 2004, only 5 were still in working order and available for retroactive lab calibration. The remaining cameras were calibrated, as described below, from only the time-lapse
photographs they acquired in the field.

Previous work on calibrating static cameras from outdoor image content – much of it motivated by the thousands of outdoor webcams available over the Internet [53] – has focused on atmospheric and astronomic features such as rainbows [120], cloud motion and shadows [121, 122, 118], solar motion and clear sky appearance [116], and relative star positions [123]. Despite the global availability of high-resolution topographic and radiometric data, photograph-to-terrain alignment has been limited to estimating camera position, orientation, and at most focal length from point features [124], land cover types [125, 126, 127], and mountain silhouettes [128, 129, 130].

In this paper, we present a general approach to calibrating a static camera from one or more landscape photographs and independent knowledge of the landscape’s topography and color. In this framework, the camera is viewed as one of a constellation of instruments – including Global Positioning System (GPS) receivers, laser altimeters, optical and radar satellite and airborne platforms – contributing to knowledge of a study area. Calibrating the camera is analogous to georeferencing the camera’s photographs so that they may be analyzed alongside other types of geospatial data. Section 4.2 reviews the fundamentals of camera geometry and camera calibration. Section 4.3 describes a variety of constraints that may be used to calibrate a static camera from landscape photographs, including image-world point and line correspondences (from conventional ground control points to horizons and coastlines), correspondences between images separated by rotation of the camera, and correspondences between real images and images synthesized from vertical imagery. Section 4.4 compares the resulting field calibrations to lab calibrations and evaluates them qualitatively with images synthesized from contemporaneous vertical imagery. Finally, Section 4.5 addresses the inevitable rotations of mounted cameras by presenting a technique for computing globally optimal estimates of camera orientations for arbitrarily-long photographic sequences and testing it on 33,000 time-lapse photographs from Columbia Glacier, Alaska.
4.2 The mathematical camera

4.2.1 Projective geometry

In mathematical terms, a camera performs a series of coordinate transformations that map an infinite 3-dimensional (3-D) world coordinate system \((x, y, z)\) onto a finite 2-D image (Figure 4.1). The camera’s local 3-D coordinate system \((x’, y’, z’)\) is transformed relative to the world by a 3-D rotation \(R\) and translation (position) \(P\) such that \(+x’\) and \(+y’\) point right and down, respectively, in the image (as viewed from the back of the camera), and \(+z’\) extends through the focal point of the lens and out the front of the camera. The process of transforming a 3-D world point \(X\) to image coordinates thus begins by transforming \(X\) to its equivalent \(X’\) in the camera’s coordinate system:

\[
X’ = R(X - P).
\]  

(4.1)

The transformed 3-D point \(X’\) is then projected onto the 2-D image plane by dividing by (and thus discarding) depth to produce so-called ‘normalized camera coordinates’ \(X_c\),

\[
X_c = \frac{1}{z'} \begin{pmatrix} x' \\ y' \end{pmatrix},
\]  

(4.2)

where \(z’\), the z-component of \(X’\), is the point’s distance from the lens focal point normal to the image plane. In the unrealistic case of a distortion-free lens, the image pixel coordinates \(X_i\) are a linear function of the normalized camera coordinates,

\[
X_i = f X_c + c,
\]  

(4.3)

where \(f\) is the focal length in pixels (either a scalar or a vector of \(x\)- and \(y\)-components) and \(c = (c_x, c_y)\) is the principal point (the intersection of the camera optical axis \(z’\) with the image plane) in pixel coordinates. To convert between pixel and physical length units, multiply (or divide) by the ratio of the camera film or sensor size to the image size in pixels. For example, to convert focal length from millimeters to pixels,

\[
f \text{ [pixel]} = f' \text{ [mm]} \frac{\text{width [pixel]}}{\text{width [mm]}}.
\]  

(4.4)
Figure 4.1: Diagram of a world point transformed to camera coordinates \((x', y', z')\) projected onto image point \((x_i, y_i)\) for a camera with position \(P\), focal length \(f\), and principal point \((c_x, c_y)\).
4.2.2 Distortion

Ideally, the principal point is at the center of the image, the focal length of the lens is as reported by the manufacturer, and the lens has no distortion, but this is not the case for real cameras. For typical rectilinear lenses, lens distortion is most often parameterized following the Brown distortion model [131] as a change to the normalized camera coordinates,

$$\Delta X_c = (k_1 r^2 + k_2 r^2 + k_3 r^6) X_c + 2 x_c y_c \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + (r^2 + 2 \begin{bmatrix} x_c^2 \\ y_c^2 \end{bmatrix}) \odot \begin{bmatrix} p_2 \\ p_1 \end{bmatrix},$$

(4.5)

where $r = \sqrt{x_c^2 + y_c^2}$, $k_{1-3}$ are radial distortion coefficients, $p_{1-2}$ are tangential distortion coefficients, and $\odot$ denotes element-wise multiplication. Radial distortion is radially-symmetric about the principal point and caused by imperfections in lens curvature, while tangential distortion is non-symmetric and due to misalignment of the lens elements. These and lesser sources of distortion are reviewed in Weng et al. [132].

Figure 4.2 illustrates the distribution and relative magnitude of the errors contributed by each distortion component. Overall, errors tend to be smallest at the image center and scale with distance from the image center. The largest errors are typically due to deviation of the focal length, followed by radial distortion, deviation of the principal point from the image center, and finally tangential distortion. A partial calibration should thus seek to calibrate camera parameters in that order. Figure 4.3 shows the range of errors across the image frame (in normalized camera coordinates) that result from assuming nominal values for each of the calibrated Columbia Glacier time-lapse cameras; in other words, these are the errors that were avoided by calibrating the cameras. For a given normalized error $e$, the error in pixels is $ef$, the error in physical length units at a given distance $d$ is $ed$ parallel to the image plane or $ed/\sin \theta$ on a plane inclined by $\theta$ in the error direction. For $f = 3872$ pixel, $d = 4000$ m, and $\theta = 5^\circ$ – conditions typical at Columbia Glacier – a conservative normalized error of $e = 0.005$ scales to 19.4 pixel on the image plane and 20 m to 230 m on the glacier surface depending on whether the pixel error is parallel to the surface or in the direction of the incline. The errors at the image corners are up to six times larger (up to $e = 0.028$). Calibration avoids (or rather, minimizes) potentially large errors in the mapping
between positions on an image and positions in the world.

4.2.3 Calibration

The projection of 3-D world coordinates to 2-D image coordinates is defined by the parameters of the camera model. The focal length $f$, principal point $c$, and distortion coefficients $k$ and $p$ describe the camera’s internal geometry (or ‘internal orientation’), while rotation $R$ and position $P$ describe the camera’s external geometry (or ‘pose’). The aim of a camera calibration is to find the set of parameter values $\beta$ that minimize the distance between the image coordinates predicted by the model $f(X, \beta)$ from world points $X$, and the image coordinates $X_i$ actually observed in images taken with the camera,

$$f(X, \beta) - X_i.$$  (4.6)

The world points $X$ used in the calibration may either be known explicitly or estimated from their positions in two or more images as part of a joint optimization problem.

In the field, it is often difficult or impossible to identify and survey sufficiently many precise and well-distributed world points to correctly estimate all camera parameters simultaneously. Therefore, in practice, world points are traditionally used to estimate the orientation of a camera whose position has been surveyed directly and whose internal parameters have been calibrated under controlled conditions. Lab calibrations of internal camera parameters are typically performed by taking several images of a planar calibration pattern (Figure 4.4) whose distinctive features can be detected automatically in the images [133]. The known relative positions of the points in the calibration pattern, together with the identification of their projections in images taken from different camera positions, provides the constraints necessary to simultaneously estimate the internal camera parameters and the relative position and orientation of the camera for each image. For best results, the focal length (if using a zoom lens) and focus distance (if using a variable-focus lens) must be the same for both the calibration and field images, since these settings alter the internal geometry of the lens [134].
Figure 4.2: Image position residuals due to typical deviations of the focal length from the manufacturer-reported value, the principal point from the image center, and the radial and tangential distortion coefficients from zero. The vectors are drawn at 4:1 scale and typical of a wide-angle non-metric lens.
Figure 4.3: Image position errors (in normalized camera coordinates) that result from assuming nominal parameters for Columbia Glacier time-lapse cameras, relative to the positions expected from the field calibrations. The cameras are listed in order of increasing median error. The boxes extend from the lower to upper quartile while the whiskers span the full range. The overall error mean and maximum error are 0.004 and 0.028. Errors were computed after independent optimization of camera rotation to account for the rotational ambiguity of the field calibrations performed from images taken from a single position.
Figure 4.4: Example photographs of a planar checkerboard calibration pattern.
Alternatively, the many available structure-from-motion software [listed at 135] – specifically, their underlying bundle adjustment routines [reviewed in 136] – relax the requirement of a separate calibration step by jointly estimating camera and scene geometry from the correspondences of arbitrary points between images. However, this method requires many images of a static, feature-rich scene taken from a variety of complementary camera positions – conditions which cannot be met for remote webcams or the time-lapse cameras from Columbia Glacier which are no longer in working order.

4.3 Landscape controls

4.3.1 World points

The correspondence between world points and points in an image provides a simple constraint against which to fit the parameters of the camera model. As few as two world points are sufficient to estimate the external orientation $\mathbf{R}$ of a camera whose other parameters are known. However, to parameterize the internal camera geometry, which varies non-linearly across an image, many points well-distributed across the image are needed. This presents a challenge for example at the terminus of tidewater glaciers, where the ocean, sky, and glacier surface (much of the imaged landscape) are in motion. The static world points used as control at Columbia Glacier (Figure 4.5) include foreground features surveyed with GPS, background slope features identified in high-resolution orthophotos and digital elevation models (DEM), and surveyed mountain summit elevations published on topographic maps and geodetic surveys. Nevertheless, the control points are sparse and restricted to the few narrow bands of solid ground.

Provided that the capture times of the images are themselves calibrated – for example, by the methods of Chapters 2 and 3 – and contemporaneous spatial data is available, transient and moving features such as snow fields and glacier crevasses can also be used as control, providing point coverage not otherwise possible. At Samarinbreen, a tidewater glacier in Svalbard, a GPS recorded the position of a boat carrying out a hydrographic survey in view of a time-lapse camera.
The boat’s world coordinates corresponding to each image were interpolated from the calibrated image capture times, GPS tracklog, and tide tables, resulting in abundant and spatially distributed world points in the glacier forebay (unpublished research conducted in collaboration with Jacek Jania and Waldemar Walczowski). Reversing the strategies described in Chapter 3, the position of astronomical objects can also be used as control, provided that both astronomical refraction and image capture times are well constrained.

Regardless of the precision of their world coordinates, for world points to be effective camera control, they must map to distinctive and geometrically well-defined points on the images, such as the centroids of small high-contrast ‘blobs’ [137], sharp corners [138], or the junctions of two or more thin lines [139].

4.3.2 World lines

Although the norm in built environments, sharp corners are the exception in natural landscapes. Mountains, for example, rarely culminate in a single point, making it difficult to know where precisely along their silhouette to mark the surveyed summit. Rather than rely on identifying a specific point on a line (e.g. the sharp summit), the line itself (e.g. the silhouette) can be used as control. Most existing studies focus on straight lines in built environments [140, 141, 142, 143, 144], although some studies in natural environments use arbitrary lines to orient cameras relative to mountain silhouettes [128, 129].

In our method, a world polyline (an array of connected world points) is transformed to normalized camera coordinates, interpolated to a desired density (typically $1/f$, about 1 pixel once scaled to pixels), and then projected onto an image to form a set of predicted image points. Interpolation is applied before non-linear lens distortion is applied so that straight line segments in the world are appropriately curved in the distorted image. The vertices of corresponding image polylines (interpolated to a desired density in a pre-processing step) collectively form a set of observed image points. Each observed point is assigned to the nearest predicted point and the residuals of the model are computed as the distance between the point pairs.
Figure 4.5: Panorama of world points used as control for west and northwest-facing Columbia Glacier time-lapse cameras.
Using the same error metric for both lines and points allows both to be used together in the same optimization routine. Furthermore, assigning observed points to the nearest predicted point (rather than the reverse, as in Behringer [128]) allows the image lines to be any arbitrary subset of the world lines – for example, the portions of the mountain silhouette within view of the camera and not shrouded in clouds. In the unusual case that the image lines extend past the endpoints of the reference world line, image points with residuals larger than a given tolerance can be discarded in the later stages of optimization. However, since the method assumes that all image lines match a present and known world line, if many image lines have no corresponding world lines (such as any spurious results of automatic edge detection), a more noise-tolerant line-fitting metric, such as the one developed by Baboud et al. [129], may be needed. However, in the presence of high noise, a precise tuning of the camera parameters may not be achievable.

The lines used as control at Columbia Glacier include both static and time-varying linear features of the landscape: mountain silhouettes, coastlines, glacier termini, and glacier medial moraines (Figure 4.6). A canonical 360° mountain silhouette was computed for each camera position from the best-available elevation data, while the other features were extracted from map data contemporaneous with an image. Planimetric data was sufficient to locate coastlines and glacier termini in 3-D since the elevation of the water edge is that of the tide at the time of image acquisition, while glacier medial moraines also required contemporaneous elevations to account for the rapidly lowering glacier surface. Figure 4.7 illustrates the strong constraints that the peaks and troughs of a jagged mountain silhouette places on camera parameters: radial distortion is plainly visible towards the edge of the image even after optimization of camera rotation $\mathbf{R}$ and focal length $f$.

### 4.3.3 Image points

Fixed installations such as webcams or time-lapse cameras are subject to changes in orientation, whether deliberate or as a result of wind, snowdrift, curious fauna, temperature fluctuations, or human handling during service visits. Although camera motion complicates analysis, large cam-
Figure 4.6: Sample photograph from camera station AK01b with image lines (yellow) and predicted world lines (black) of the mountain horizon, glacier terminus, and glacier medial moraines. The images lines have gaps where the features are either hidden (behind clouds or undulations in the glacier surface) or poorly defined (where the glacier terminus resembles ice mélange). The world coordinates of the time-varying glacier features were extracted from an orthophoto and DEM produced from aerial stereophotographs taken within an hour of the image capture time.
Figure 4.7: Relative errors of the predicted (solid lines) versus observed (dotted line) mountain silhouette (for a photograph from station CG06) with the cumulative addition of model parameters.
era motions move static features to different parts of the image and the relative changes of these feature positions provides constraints on the internal camera geometry. These displacements are especially valuable for parameterizing lens distortion, which varies non-linearly across the image frame.

When the translational (as opposed to rotational) motion of a mounted camera is very small relative to the object distance, the motion is well-approximated as a pure rotation and parallax can be ignored. Wang et al. [145] describe the errors associated with this assumption. In this case, no knowledge of object distance is needed; instead, control points are constructed by matching features directly between the images. Given points \( \mathbf{X}_i \) and \( \mathbf{X}_i' \) representing the same point in the world in different images with corresponding camera parameter values \( \beta \) and \( \beta' \) (but equal camera position \( \mathbf{P} \)), the observed point \( \mathbf{X}_i' \) can be predicted from \( \mathbf{X}_i \) as

\[
\mathbf{X}_i = f(f^{-1}(\mathbf{X}_i, \beta), \beta'),
\]

where \( f^{-1} \) is an inverse camera projection that transforms the image point to a synthetic world point at an arbitrary distance from the camera along the outgoing ray. As before, the objective is to find the camera parameters that minimize the residuals between the predicted and observed image points. However, in this case, the predicted image points are a function of two sets of camera parameters, \( \beta \) and \( \beta' \), which can represent any two cameras at the same position.

This method is equivalent to the image alignment step for panorama stitching [reviewed in 146]. The image-image point correspondences can be generated automatically with keypoint detection and matching techniques (e.g. SIFT: Scale-Invariant Feature Transform [147], SURF: Speeded-Up Robust Features [148]) which restrict the search to distinctive image features expected to be the most invariant to change in scale, orientation, and illumination. Excellent match accuracy can be achieved by keeping only the points whose best match distance is much shorter than the distance to their second-best match (the ‘distance ratio test’ [147]). Undesirable but valid matches corresponding to moving or very close objects (subject to more parallax) can be filtered out further using iterative model-fitting techniques (e.g. RANSAC: Random Sample Consensus [149]), provided
their apparent motion is greater than that due to errors in the camera model (Figure 4.8).

Figure 4.9 illustrates the poor image alignment obtained with only a pure rotation of an uncalibrated camera, and the improvement achieved by simultaneously adjusting internal parameters. In practice, one image pair is rarely sufficient to calibrate an uncalibrated camera: images separated by large rotations provide strong constraints on internal camera parameters, but only in the small area where the images overlap, while images separated by small rotations or rotations about a single camera axis suffer from ambiguities between possible parameter values [150, 151]. However, a robust calibration can be achieved by relying on the constraints imposed by many image pairs, as illustrated in Figure 4.10.

Image-image point correspondences can be extended to polylines using the method described in Section 4.3.2, but line detection and matching is much more difficult to automate.

4.3.4 Synthetic images

Even when high-resolution map data is available, it can be difficult and tedious to match point and line features in oblique images to their correspondences in plan view. To more easily collect control or to visually evaluate a camera calibration, synthetic images can be produced by projecting map data into imaginary cameras. This requires surface elevations and surface colors, such as an optical orthophoto or a hillshade generated from the elevations with a light source at the contemporaneous position of the Sun. By closely mimicking real images, synthetic images greatly simplify the task of identifying image-world correspondences. Abrams et al. [124] calibrated remote webcams by having members of the public manually identify building edges both in webcam images and in 3-D city models. In natural landscapes, where vertical surfaces are rare and scenes extend great distances from the camera, even modest resolution map data can produce photorealistic results that can be exploited directly by feature matching algorithms for fully automatic image-terrain alignment (Figure 4.11). Our method is essentially the reverse of Shan et al. [153], in which terrestrial photographs of building facades were warped for automated matching to oblique aerial photographs.
Figure 4.8: The residuals of matched image points (drawn at 10:1 scale) selected as inliers (yellow) or discarded as outliers (red) by RANSAC for both a camera rotation ($\mathbf{R}$) and a rotation and simultaneous estimate of radial distortion ($\mathbf{R}, k_1$). The pair of photographs, taken during the service visit between sequences AK01b-20120825 and AK01b-20130514, are shown with the second image projected and overlaid onto the first.
Figure 4.9: For a pair of photographs from sequence AK10b-20140501, the result of projecting the second image into the first is shown for two different calibrations performed from matched image points (not shown): rotation ($R$), and rotation plus various internal camera parameters ($R, f, k_1, k_2, p_1, p_2, c$).
Figure 4.10: The normalized standard deviations of camera parameters with the incremental addition of points from additional image pairs shown for a total of 40,095 points (filtered with RANSAC and the distance ratio test) from a sequence of 25 image pairs taken over 8 years by a camera at station AK01b. The vertical grey lines highlight image pairs whose addition to the model result in large reductions in parameter standard deviations. Parameter confidence intervals were computed with Python package `lmfit` [152].
When the synthetic and real camera share a common position, the image-image correspondences can be used as described in Section 4.3.3. In this case, the uncertainty is dominated by the local horizontal and vertical uncertainties of the reference spatial data. Since in image space these decrease linearly with distance from the camera, distant features are preferred. When the camera position is not known (and thus different from the one assumed for the synthetic camera), the image coordinates marked on the synthetic image can be converted back to world coordinates for use as control with cameras at other positions. In this case, however, uncertainty in the distance of the matched feature along the line of sight introduces an additional error (parallax) as the camera positions shift farther apart. Given two cameras – the first with its optical axis passing through a target point at a distance \( z + \delta z \) and the second with identical orientation but shifted a distance \( x \) perpendicular to the optical axis of the first – the angular error \( \delta \theta \) at the second camera is given by

\[
\delta \theta = \tan^{-1} \frac{x}{z} - \tan^{-1} \frac{x}{z + \delta z},
\]

(4.8)

which by the small-angle approximation, \( \tan^{-1} \theta \approx \theta \), reduces to

\[
\delta \theta \approx \frac{x \delta z}{z(z + \delta z)}.
\]

(4.9)

The error decreases with distance from the cameras (\( z \)), but also increases with the error in distance (\( \delta z \)), which increases where the surface slopes away from the camera, reaching local maximums at topographic edges (Figure 4.12). Care should thus be taken to ensure that the potential parallax at a point on the synthetic image is within the desired tolerance given the expected distance (\( x \)) between the cameras.

### 4.3.5 Measurement precision

Image object size (projected onto the image plane) decreases linearly with increasing object distance. Therefore, the precision of a measurement in the world (parallel to the image plane), \( \Delta x' \), that still achieves a target pixel precision \( \Delta x_i \) grows with increasing distance (normal to the
Figure 4.11: Automatic RootSIFT [154] keypoint matches between a photographic image from station AKJNC, taken at nominally 2012-08-13 20:53:25 UTC, and a synthetic image rendered from a DEM and orthophoto computed from Worldview satellite images taken at 2012-08-13 21:02:24 and 21:03:21 UTC. The yellow lines trace the correspondences for a subset of the matched keypoints.
Figure 4.12: Potential parallax error, expressed in normalized camera coordinates per unit camera separation (1 m), for an image synthesized from a DEM computed from aerial photographs captured on 2005-08-26.
image plane) \( z' \) as

\[
\Delta x' = \frac{\Delta x_i}{f} z'.
\]

(4.10)

For a focal length \( f = 3872 \) pixel (for example, a 3872 pixel image from a camera with a 35 mm sensor fitted with a 35 mm lens), 1 pixel corresponds to \( 2.5 \times 10^{-4} \) m at 1 m distance, \( 2.5 \times 10^{-2} \) m at 100 m, and 2.5 m at 10000 m. This is an important consideration for oblique scenes with a wide range of object distances. Foreground control points may only be placed as close to the camera as the precision of the surveying allows, while distant objects may provide strong control despite lower precision. Regardless of distance, the pixel precision with which the point can be identified in the image need also be considered. For example, a high-contrast circular target is made more recognizable by occupying a larger image region (a minimum size of 3 x 3 pixels is recommended), but the larger size decreases the precision with which its surveyed center can be identified.

### 4.3.6 Earth curvature and refraction

The curvature of the Earth causes objects to appear to be lower than would be expected from their elevations in a local rectilinear coordinate system (which represents a segment of the Earth’s curved surface as flat). Atmospheric refraction, in turn, causes objects to appear either higher or lower than they are in reality by curving the path of incoming light rays. The combined elevation correction \( \Delta z \) can be expressed as

\[
\Delta z = -\frac{d^2}{2R_e} + k \frac{d^2}{2R_e}.
\]

(4.11)

where \( d \) is the planimetric distance to the object, \( R_e \) is the radius of the Earth (which ranges from \( 6356 \times 10^3 \) m at the poles to \( 6378 \times 10^3 \) m at the equator), and \( k \), the coefficient of refraction, can be estimated from the vertical temperature gradient as [155]:

\[
k = 5.03 \frac{p}{T^2} (0.0343 + \frac{\partial T}{\partial z}),
\]

(4.12)

where \( T \) is temperature (K), \( p \) is pressure (Pa), and \( z \) is elevation (m). Although negligible at small distances, curvature and refraction scale with the square of distance, becoming significant
at large distances from the camera. For a camera level to the horizontal, curvature scales linearly with distance at a rate of $f/(2R_e)$ pixel/m. For a focal length $f = 3872$ pixel, this is equal to a rate of 0.0003 pixel/m, resulting in a 15 pixel offset at $50 \times 10^3$ m, the maximum distance to the visible horizon in Columbia Glacier time-lapse photographs.

Under standard atmospheric conditions, curvature is the larger effect by a factor of about 7. At 40 m and higher above the surface, the vertical temperature gradient is weakly negative and fairly independent of the temperature at the surface. In this stable region, the refraction coefficient $k$ varies from +0.10 to +0.20 [155], bracketing the widely-used average +0.13 [156].

The lower atmosphere, in contrast, is strongly influenced by the varying temperature at the surface, which often heats and cools faster than the surrounding air, leading to strongly negative and positive $\partial T/\partial z$, respectively. At 1 m to 5 m above the surface, $k$ can vary from −4 to +16 over vegetated ground and −14 to +18 over ice and water [155]. Although not of primary concern for the more elevated time-lapse cameras at Columbia Glacier, for which the line of sight to an object on the glacier surface travels a negligible distance through the surface boundary layer, refraction effects up to 2 pixels are evident in the most oblique views where incoming rays pass low above the glacier surface for long distances.

### 4.4 Columbia Glacier: Camera calibrations

#### 4.4.1 Field calibrations

Of the 33 digital cameras used for time-lapse photography at Columbia Glacier, 14 were calibrated with only the time-lapse photographs they acquired in the field. We used the Python package lmfit [152], based on the Levenberg-Marquardt method for non-linear least squares [reviewed in 157], to estimate the camera parameters that minimize the sum of squares of the residuals between the observed and predicted image coordinates for the variety of controls described in the previous sections. Several strategies helped ensure model convergence to the global (or at least, a strong local) minimum.
**Parameter initial values** The algorithm is more likely to converge to the global minimum if the initial parameter values are close to the optimal values. Focal lengths were estimated from the Exchangeable image file format (Exif) [7] FocalLength tags and the published sensor sizes of the camera models [158], reported by the Make and Model tags. Camera positions and orientations were surveyed in the field, estimated from field notes, or visually determined from the relative positions of overlapping mountain silhouettes subject to strong parallax.

**Parameter scaling** To ensure a reasonable initial iteration step, the step size for each parameter was scaled by each parameter’s relative contribution to a change in predicted image coordinates.

**Parameter bounds** Broad parameter bounds were enforced to prevent the algorithm from traversing unrealistic and unstable parts of the parameter space, like negative focal lengths and extreme lens distortion that violates the unique mapping between camera and image coordinates.

**Parameter growth** Rather than fitting all parameters simultaneously, which sometimes failed to converge, the full model was approached gradually by iteratively adding more parameters to the model in order of decreasing significance.

**Control coverage** The model can have more local minima if it is poorly constrained; for example, if the control cover only a small region of the image frame. Convergence was improved by including as many controls distributed across the image as were available for a given camera.

Although computationally tractable for the relatively small problems tackled here, for much larger problems, efficient implementations of Levenberg-Marquardt or other non-linear optimization methods (e.g. SBA [159], Ceres Solver [160]) should be used instead for considerable computational gains.
4.4.2 Lab calibrations

Of the 14 field-calibrated time-lapse cameras, 5 were retired from the field but still operational, allowing us to compare the field calibration to calibrations performed under controlled conditions. Since internal camera parameters vary with focus distance [134], calibrations were performed with the lenses focused near infinity to approximate the focus setting used in the field. Filling the field of view of the cameras (45° to 60°) at 3 m to 4 m distance required a 3 m calibration pattern, larger than the maximum paper size available. Therefore, rather than use a grid of regularly spaced points as described in Section 4.2.3, 12 sheets of paper with 5 targets each were distributed across a flat floor, with a few raised off the floor on stools to provide 3-D constraints (Figure 4.13). With each of the cameras, 18 images to 24 images were acquired from 6 stations to 8 stations distributed around the pattern at heights of 2 m and 4 m above the floor. At each station, the camera was rotated by −90°, 0° and 90° about its optical axis. The centers of the 60 coded targets were located in each image using the subpixel centroid detection routine in PhotoModeler and manually checked and corrected as needed. Since the relative positions of the targets was not known, they could not be used directly as control points. Instead, their corresponding positions in each of the images was used to estimate both point and camera geometries simultaneously using PhotoModeler’s bundle adjustment routine. All calibrations converged tightly, with overall root-mean-square reprojection errors of 0.113 pixel to 0.282 pixel, no point reprojection error exceeding 1.0 pixel, and image coverage of 86% to 95%.

4.4.3 Results

Results of the lab calibrations are shown in Table 4.1 alongside the results of the field calibrations. While the field calibrations adequately estimate focal length (f) and 1st and 2nd order radial distortion (k₁, k₂), the parameters with the largest contributions to the camera model, estimates of the principal point (c) and tangential distortion (p₁, p₂) are no better than chance. Some discrepancies could be due to mechanical settling of the cameras and lens in the 6 years to 14 years
Figure 4.13: The array of targets used for the lab calibrations.
between when the images used for the field calibrations were taken and the lab calibrations were performed. More likely, however, the field calibrations, which were limited to images taken from a fixed position, suffer from the inherent ambiguity between shifting the principal point and rotating the camera [150]. Numeric tests confirm that $\sim 94\%$ of the reprojection errors due to the shifts in principal point positions can be compensated for by camera rotation. Rather than evaluate each parameter in isolation, Figure 4.14 compares the nominal values and field calibrations to the lab calibrations for freely-rotating cameras. In this case, the field calibrations achieve a $65\%$ to $89\%$ reduction in mean reprojection error over using the nominal values, with most residuals relegated to the edges of the frame beyond the area of interest and the coverage of field control. Tangential distortion is a subpixel effect according to the lab calibrations, so it is not surprising that the field calibrations fail to resolve these parameters amidst the rotational ambiguity and noisy inputs.

The field calibrations of all the cameras were also evaluated qualitatively by overlaying their photographs with images synthesized from high-resolution DEMs and orthophotos produced from vertical imagery – the same data used as sources of control for the field calibrations. For 11 of the 14 cameras, contemporaneous vertical imagery was available, making it possible to evaluate the image alignment both over land and over the glacier surface (Figure 4.15). All the synthetic images agreed very closely with the real photographs, confirming that the field calibrations successfully registered the camera models to the reference spatial data.

4.5 Columbia Glacier: Camera motion

4.5.1 Method

Camera motion can be very significant in outdoor time-lapse sequences. Once the camera internal parameters are known, the rotation of the camera must be estimated for each image of interest. Doing so by manually marking world control for each image would be labor-intensive and error-prone. A more robust and automatable approach is to estimate the absolute orientation of a small subset of the images by manual or automatic methods, then compute the orientation of
Table 4.1: Nominal, field, and lab calibration parameters for several retired Columbia Glacier time-lapse cameras. The 99.7\% parameter confidence intervals were computed with Python package `lmfit` [152] for the field calibrations and by PhotoModeler for the lab calibrations.

<table>
<thead>
<tr>
<th>Camera</th>
<th>Parameter</th>
<th>Nominal</th>
<th>Field</th>
<th>Lab</th>
<th>Match (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>canon-40d-01</td>
<td>$f_x$ (mm)</td>
<td>24.0</td>
<td>24.561 ± 0.008</td>
<td>24.616 ± 0.002</td>
<td>99.78 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>$f_y$ (mm)</td>
<td>24.0</td>
<td>24.692 ± 0.022</td>
<td>24.620 ± 0.002</td>
<td>99.71 ± 0.10</td>
</tr>
<tr>
<td></td>
<td>$c_x$ (mm)</td>
<td></td>
<td>0.148 ± 0.008</td>
<td>−0.089 ± 0.003</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>$c_y$ (mm)</td>
<td></td>
<td>0.149 ± 0.048</td>
<td>0.300 ± 0.003</td>
<td>50.00 ± 16.36</td>
</tr>
<tr>
<td></td>
<td>$k_1$</td>
<td></td>
<td>−0.131 ± 0.002</td>
<td>−0.107 ± 0.001</td>
<td>81.67 ± 2.11</td>
</tr>
<tr>
<td></td>
<td>$k_2$</td>
<td></td>
<td>0.179 ± 0.008</td>
<td>0.107 ± 0.003</td>
<td>60.03 ± 4.21</td>
</tr>
<tr>
<td></td>
<td>$p_1$</td>
<td></td>
<td>−0.00101 ± 0.00017</td>
<td>0.00047 ± 0.00003</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>$p_2$</td>
<td></td>
<td>0.00387 ± 0.00004</td>
<td>0.00211 ± 0.00004</td>
<td>5.34 ± 1.07</td>
</tr>
<tr>
<td>nikon-d200-11-28</td>
<td>$f_x$ (mm)</td>
<td>28.0</td>
<td>28.701 ± 0.006</td>
<td>28.800 ± 0.002</td>
<td>99.66 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>$f_y$ (mm)</td>
<td>28.0</td>
<td>28.798 ± 0.007</td>
<td>28.802 ± 0.002</td>
<td>99.97 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>$c_x$ (mm)</td>
<td></td>
<td>−0.107 ± 0.005</td>
<td>−0.036 ± 0.003</td>
<td>33.50 ± 4.32</td>
</tr>
<tr>
<td></td>
<td>$c_y$ (mm)</td>
<td></td>
<td>0.038 ± 0.013</td>
<td>0.012 ± 0.003</td>
<td>37.47 ± 20.32</td>
</tr>
<tr>
<td></td>
<td>$k_1$</td>
<td></td>
<td>−0.117 ± 0.002</td>
<td>−0.122 ± 0.001</td>
<td>95.89 ± 2.01</td>
</tr>
<tr>
<td></td>
<td>$k_2$</td>
<td></td>
<td>0.121 ± 0.006</td>
<td>0.134 ± 0.004</td>
<td>90.85 ± 7.30</td>
</tr>
<tr>
<td></td>
<td>$p_1$</td>
<td></td>
<td>−0.00007 ± 0.00009</td>
<td>−0.00019 ± 0.00004</td>
<td>49.87 ± 49.87</td>
</tr>
<tr>
<td></td>
<td>$p_2$</td>
<td></td>
<td>−0.00071 ± 0.00003</td>
<td>−0.00024 ± 0.00004</td>
<td>33.44 ± 6.63</td>
</tr>
<tr>
<td>nikon-d200-12-28</td>
<td>$f_x$ (mm)</td>
<td>28.0</td>
<td>28.746 ± 0.003</td>
<td>28.815 ± 0.002</td>
<td>99.76 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>$f_y$ (mm)</td>
<td>28.0</td>
<td>28.843 ± 0.006</td>
<td>28.816 ± 0.002</td>
<td>99.90 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>$c_x$ (mm)</td>
<td></td>
<td>0.175 ± 0.003</td>
<td>−0.041 ± 0.003</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>$c_y$ (mm)</td>
<td></td>
<td>0.083 ± 0.008</td>
<td>−0.090 ± 0.003</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>$k_1$</td>
<td></td>
<td>−0.122 ± 0.001</td>
<td>−0.121 ± 0.001</td>
<td>98.86 ± 0.69</td>
</tr>
<tr>
<td></td>
<td>$k_2$</td>
<td></td>
<td>0.141 ± 0.002</td>
<td>0.132 ± 0.004</td>
<td>93.82 ± 4.19</td>
</tr>
<tr>
<td></td>
<td>$p_1$</td>
<td></td>
<td>0.00053 ± 0.00006</td>
<td>−0.00042 ± 0.00004</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>$p_2$</td>
<td></td>
<td>0.00140 ± 0.00002</td>
<td>−0.00035 ± 0.00004</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td>nikon-d2x</td>
<td>$f_x$ (mm)</td>
<td>28.0</td>
<td>28.655 ± 0.008</td>
<td>28.778 ± 0.002</td>
<td>99.57 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>$f_y$ (mm)</td>
<td>28.0</td>
<td>28.748 ± 0.011</td>
<td>28.706 ± 0.002</td>
<td>99.85 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>$c_x$ (mm)</td>
<td></td>
<td>0.026 ± 0.016</td>
<td>0.065 ± 0.003</td>
<td>40.71 ± 26.44</td>
</tr>
<tr>
<td></td>
<td>$c_y$ (mm)</td>
<td></td>
<td>−0.108 ± 0.043</td>
<td>−0.121 ± 0.003</td>
<td>65.40 ± 12.74</td>
</tr>
<tr>
<td></td>
<td>$k_1$</td>
<td></td>
<td>−0.112 ± 0.003</td>
<td>−0.121 ± 0.001</td>
<td>92.15 ± 2.69</td>
</tr>
<tr>
<td></td>
<td>$k_2$</td>
<td></td>
<td>0.126 ± 0.008</td>
<td>0.135 ± 0.003</td>
<td>92.22 ± 7.11</td>
</tr>
<tr>
<td></td>
<td>$p_1$</td>
<td></td>
<td>0.00251 ± 0.00021</td>
<td>−0.00027 ± 0.00004</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>$p_2$</td>
<td></td>
<td>−0.00038 ± 0.00004</td>
<td>−0.00025 ± 0.00004</td>
<td>67.33 ± 17.42</td>
</tr>
<tr>
<td>nikon-e8700</td>
<td>$f_x$ (mm)</td>
<td>8.9</td>
<td>9.052 ± 0.004</td>
<td>9.032 ± 0.002</td>
<td>99.79 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>$f_y$ (mm)</td>
<td>8.9</td>
<td>9.079 ± 0.004</td>
<td>9.034 ± 0.002</td>
<td>99.51 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>$c_x$ (mm)</td>
<td></td>
<td>0.008 ± 0.010</td>
<td>0.022 ± 0.003</td>
<td>48.64 ± 48.64</td>
</tr>
<tr>
<td></td>
<td>$c_y$ (mm)</td>
<td></td>
<td>−0.038 ± 0.009</td>
<td>0.018 ± 0.003</td>
<td>0.00 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>$k_1$</td>
<td></td>
<td>−0.201 ± 0.003</td>
<td>−0.210 ± 0.002</td>
<td>95.30 ± 2.01</td>
</tr>
<tr>
<td></td>
<td>$k_2$</td>
<td></td>
<td>0.209 ± 0.005</td>
<td>0.207 ± 0.004</td>
<td>95.45 ± 0.66</td>
</tr>
<tr>
<td></td>
<td>$p_1$</td>
<td></td>
<td>0.00219 ± 0.00025</td>
<td>0.00059 ± 0.00009</td>
<td>27.99 ± 7.27</td>
</tr>
<tr>
<td></td>
<td>$p_2$</td>
<td></td>
<td>0.00076 ± 0.00007</td>
<td>−0.00036 ± 0.00009</td>
<td>0.00 ± 0.00</td>
</tr>
</tbody>
</table>
Figure 4.14: Image position residuals for the nominal and field calibration parameters relative to the positions expected from the lab calibrations after optimization of camera rotation. The vectors are drawn at 5:1 scale. Each panel is labeled with the mean and maximum error expressed in normalized camera coordinates.
Figure 4.15: Synthetic images, produced for both nominal (top) and field calibrated camera parameters from an orthophoto and DEM, overlaid on the original photographic image. The poor alignment achieved with the nominal camera parameters is evidenced by a blurry appearance. The photograph, from station AK01, was taken at 2007-09-22 23:02:10 ± 01:00:00 UTC, while the aerial photographs used to produce the orthophoto and DEM were taken within 30 min of the ground-based photograph (as determined from the evolution of shadows in the time-lapse sequence).
intervening images relative to the absolutely-oriented images. This approach can make use of any stationary features visible in both images, making it possible to automate with the keypoint matching routines described in Section 4.3.3. Although Messerli and Grinsted [161] propose projecting matched image points out of the reference image onto a digital elevation model (DEM) and back into the target image, the projection of image to world coordinates is unnecessary if both images were taken from the same position.

The relative rotation between two images taken from the same position can be estimated from two or more ray correspondences. To propagate absolute orientation from reference images through the image sequence, one could compute the relative rotation between each adjacent image pair. However, this would inevitably cause errors to steadily accumulate outwards from the reference images; for example, \( R_{2 \rightarrow 3} R_{1 \rightarrow 2} \neq R_{1 \rightarrow 3} \). Instead, one can optimize camera orientations simultaneously to achieve globally optimal estimates. Antone and Teller [162], working with spatial clusters of images in urban environments, optimized all \( N \) cameras in a cluster simultaneously using the correspondences between all camera pairs. In natural landscapes, however, surface appearance can change gradually over time (e.g. snow field recession, gully formation), leading to visually coherent but spatially erroneous matches between images more distant in time, while the motion of shadows over the course of a day can similarly bias matches between images taken near in time.

To strike a balance between these opposing effects while minimizing computational costs, we matched each image to all other images within a day in each direction (or to a minimum number of nearest neighbors if gaps exist). To buttress the solution against long-term drift, each image is also matched to a few images more distant in time – on the scale of months, to avoid biased matches from gradual but coherent appearance changes, to years, to target similar seasonal conditions. Ideally, two or more reference images are present to anchor the solution at different positions in the match chain. To ensure high match accuracy, points whose best match is only marginally better than the second-best match (a ‘distance ratio’ approaching unity) are discarded. Images with very few filtered matches are also discarded; they typically contain a low amount of detail (e.g. under- or overexposure, low atmospheric visibility) and fail to register properly.
The optimization problem can quickly become very large. For a sequence with a 2 h framerate and a symmetric 1 d matching window, every additional day of images (12 images) adds 36 model parameters and the matches from 156 image pairs. Several additional strategies to Section 4.4.1 had to be adopted for computational gains:

**Outlier-robust loss function** Filtering outlier image points for each image pair using iterative model-fitting techniques such as RANSAC adds considerable computational cost. Instead, we reduce the influence of any outliers by performing robust least squares by minimizing the sum of ‘soft’ L1 norms (an approximation of absolute residuals) rather than L2 norms (squares of residuals).

**Normalized residuals** For two images separated by a pure rotation, the normalized camera coordinates \(X_c\) and \(X'_c\) of the matching points in each image are related by a relative rotation \(R'\); that is \(X'_c = R'X_c\). If the internal camera parameters are held fixed in the optimization, the camera coordinates can be precomputed and the model residuals computed in camera space, avoiding the computational costs of additional coordinate transformations.

**Sparse Jacobian matrix** Each camera’s orientation parameters only interact with the point projections associated with that camera, resulting in a sparse block structure for the Jacobian (the matrix of all first-order partial derivatives). Precomputing this sparse structure and computing only its non-zero elements greatly reduces computational costs.

**Block processing** Very long image sequences can be processed in overlapping blocks of images, starting with a block that includes a reference image, and fixing the orientation of previously oriented images for use as reference images in subsequent blocks.

In the case of Columbia Glacier, where most of the imaged landscape is in motion, precautions had to be taken to restrict matches to static landscape features. Very large camera rotations (‘motion breaks’) were identified visually and reference images were chosen such that one or more occupied each period between motion breaks. Land masks can be generated for each reference image.
by either manually tracing polygons on the image, projecting world polygons onto the image, or projecting polygons traced on a reference image onto the image. Intervening images are assigned to the nearest reference image occupying the same period between motion breaks. Keypoint detection is restricted to the land masks, reducing computational time and storage and ensuring that all keypoint matches correspond to static features. Furthermore, each image is initialized with the solved orientation of their assigned reference image, ensuring that initial orientations for all images are within a certain tolerance. These improved initial guesses speed up model convergence and allow erroneous feature matches with initial residuals beyond this tolerance to be filtered out. Images cluttered by snow, rain, and ice moving across the image (forming drifts, stalagtites, or affixed to the window of the camera housing) were manually removed, out of concern that these would lead to spurious matches that would bias the global orientation estimates.

### 4.5.2 Results

Over 33 000 candidate images, spanning nearly 13 years (from 2004-06-24 to 2017-05-12), were selected at a nominal 2 h spacing from 15 different camera positions (‘stations’). A total of 29 motion breaks were identified (all but 12 of which were due to servicing), slicing the record into 44 periods. These were each populated by at least one reference image, with more for very long periods, for a total of 78 reference images and land masks. For each station, RootSIFT (a square root L1-normalized SIFT descriptor with superior match performance [154]) keypoints for each image were matched to those of the 12 nearest images in either direction (at least one day in either direction), as well as to images roughly 10 days, 1 month, 3 months, and 1 year in either direction. Only the best 50 keypoint matches of each image pair were retained, and the 393 images (1.2%) with fewer than 50 matches total were removed. Remarkably, despite gaps in coverage up to 8 months and drastic changes in appearance over winter, the matches successfully linked together all the images at each station, including the 9 year sequence from station AK01b. Stations with fewer than 2000 images were processed as a single block, while stations AK01b and AK10b were processed incrementally in 1500 image blocks.
By definition, the globally-optimal estimates of camera rotation ensure that the relative rotation between two images is the same regardless of the path taken; for example, that $R_{2\to3}R_{1\to2} = R_{1\to3}$. This however does not mean that the solutions are optimal, only that they are self-consistent. Although there are no correct answers against which to compare the results, they can be assessed visually by checking whether world points projected into the rotated cameras consistently project onto the same features in the images. Animations confirm excellent alignment throughout the record, with residual motion either unperceivable or on the order of 1 to 3 pixels (a normalized error of up to 0.001). Errors appear to be largest at the image edges, where camera calibration errors are largest, suggesting that it may be possible, at additional computational cost, to refine field calibrations further while simultaneously solving for the camera rotations.

The solved camera rotations reveal distinctive temporal patterns in camera instability. The cameras experienced much larger and more erratic motion in the winter than in the summer, as summarized in Table 4.2 and illustrated in Figure 4.16. A review of the winter images suggests that the gradual large magnitude motions are driven by pressure on the camera assemblies from snow drift and accumulation, while the sudden large motions are caused by wind gusts, which are on average more common and achieve higher speeds in winter. In the extreme case of sequence AK12-20110516 (Table 4.2), the camera pivoted a total of 85° between September 2010 and February 2011 in a series of large rotations each corresponding to violent wind gusts (as evidenced by spindrift snow and cresting waves in the forebay). The camera was mounted on a corner bracket itself mounted with U-bolts to a standard aluminum instrument tower. The U-bolts could not withstand the torque inflicted by the strongest winds and the camera assembly rotated around the tower post.

Under even the calmest conditions, the cameras continue to experience instability, albeit of a much smaller magnitude. The diurnal variability in the temperature of the ground or camera assembly appears to drive a small but persistent diurnal change in camera orientation (Figure 4.17). Overall, the mean errors from not correcting for camera motion are on the same order as those from not calibrating the cameras, but the maximum errors due to camera motion are several orders of magnitude larger.
Figure 4.16: Camera motion relative to the first image in a sequence – expressed as the minimum, maximum, and mean normalized errors that would result from not correcting for camera motion – for a variety of camera stations during winter (top) and summer months. Images within a day of service visits were ignored to avoid human-caused camera motion. The summer timeseries start after the service visit, while the winter timeseries are shifted to start at the beginning of winter conditions. Line colors are only used to distinguish between sequences in each panel, so matching colors between the panels are not meaningful.
Table 4.2: Mean (maximum) camera motion statistics relative to the first image in a sequence, expressed as both the relative rotation angles and the normalized errors that would result from not correcting for camera motion. Images within a day of service visits were ignored to avoid human-caused camera motion. Azimuth is the rotation about the $+z$ axis, altitude the rotation relative to the $xy$ plane, and roll the rotation of the camera about its optical axis $z’$.

<table>
<thead>
<tr>
<th>Sequences</th>
<th>Normalized error</th>
<th>Azimuth [°]</th>
<th>Altitude [°]</th>
<th>Roll [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer (May – September)</td>
<td>0.002 (0.024)</td>
<td>0.06 (1.00)</td>
<td>0.08 (0.99)</td>
<td>0.08 (1.23)</td>
</tr>
<tr>
<td>Winter (November – March)</td>
<td>0.004 (0.090)</td>
<td>0.11 (3.80)</td>
<td>0.19 (3.60)</td>
<td>0.16 (3.04)</td>
</tr>
<tr>
<td>AK12-20110516</td>
<td>0.250 (22.800)</td>
<td>27.42 (85.32)</td>
<td>0.59 (2.61)</td>
<td>0.50 (3.16)</td>
</tr>
</tbody>
</table>

Figure 4.17: The estimated pan-tilt camera rotation (relative to the first image) for a subset of sequence AK01b-20100820 versus the air temperature observed at the National Oceanographic and Atmospheric Administration (NOAA) weather station at the Valdez Municipal Airport, Alaska (WBAN:26479). The dashed lines mark local solar noon on each day, beginning with 2010-05-16 13:41:00 AKDT. The 0.05 deg amplitude of the diurnal motion corresponds to a 0.001 normalized error (~3 pixel for this camera).
4.6 Discussion

Ideally, camera internal parameters are measured in the lab and position is surveyed in the field so that only the orientation of the camera needs to be estimated using landscape control. However, for photographs such as those from the Columbia Glacier time-lapse program, well-distributed control is needed to retroactively calibrate and position the camera. This chapter identifies a variety of conventional and novel controls that can be used, alone or in combination, to precisely reconstruct the camera geometry that gave rise to a single or sequence of landscape photographs:

(1) Static and transient world points.

- Foreground features surveyed directly with GPS.
- Background features identified in orthophotographs (‘orthos’) and digital elevation models (DEMs).
- Surveyed mountain summit elevations published on topographic maps and geodetic surveys.

(2) Moving world points, provided that image capture times are sufficiently well constrained (for example, by the methods of Chapter 2).

- Features whose positions are tracked by GPS, such as GPS rovers on the glacier surface or boats surveying the glacier forebay.
- Astronomical objects, if they are high enough above the horizon that astronomic refraction is well constrained.

(3) World polylines, including horizons, coastlines, glacier termini, and medial moraines.

(4) World points and polylines extracted from images synthesized from contemporaneous orthos and DEMs.
Image points and polylines matched between photographs separated by large camera rotations. These may only be used to constrain internal camera parameters, not external camera parameters.

Regardless of the landscape controls employed, some general guidelines should be considered when planning a camera calibration campaign. First, the precision required in the world to achieve a desired precision in an image grows linearly with increasing distance from the camera. Therefore, foreground control may only be as close to the camera as the precision of the surveying allows, while distant features may provide precise control despite lower positional precision. Regardless of distance, the precision with which the feature can be identified in the image need also be considered. For example, a circular target is made more recognizable by occupying a larger image region (a minimum size of 3 x 3 pixels is recommended), but the larger size decreases the precision with which its surveyed center can be identified. For best results, features should map to distinctive and geometrically well-defined features on the image – for example, the centroids of small high-contrast ‘blobs’ [137], sharp corners [138], or the junctions of two or more thin lines [139].

The novel controls developed in this chapter open up new opportunities for content-based camera calibration. World polylines efficiently constrain large areas of the image, a major advantage over traditional point correspondences. Horizon lines in particular transect the entire image, and in rugged mountain terrain, have jagged peaks which impose strong constraints on internal camera parameters. The horizon is also the only landscape feature that remains distinct when the landscape is blanketed in snow. Whether a seasonal shift of the visible horizon is a concern depends on the distance to the horizon and the expected snow depth on the ridge line. Importantly, the horizon is a promising candidate for automation. The world coordinates of the horizon are automatically extracted from topographic data, and although the image coordinates of the horizon were traced manually, the horizon is very distinctive in most images, suggesting that its detection can be readily automated by naive or supervised edge-filtering methods [128, 163].

As discussed, points automatically matched between photographs can be used to estimate
internal camera parameters and the relative rotations of the camera. However, they cannot be used to recover the absolute orientation of the camera. This limitation was addressed by automatically matching points between real photographs and synthetic images produced by projecting map data into imaginary cameras. Since the synthetic images map to known world coordinates, the matched points are an automatable equivalent of the world points collected manually by traditional methods. Paired with global high-resolution satellite data products [164], this method could be used to automate photo-to-terrain registration anywhere on Earth.

It is instructive for future field campaigns and analysis to rank the relative sources of error. They are listed below in decreasing order of importance, based on the results from the Columbia Glacier time-lapse cameras. The errors are expressed in normalized camera coordinates. For a given normalized error $e$, the equivalent error in pixels is $ef$ (where $f$ is the focal length in pixels) and the error in physical length units (parallel to the image plane) is $ed$ (where $d$ is the distance from the camera along the optical axis).

**Camera motion** Updating the camera orientation for each photograph prevented mean (and maximum) errors of $e = 0.002$ (0.024) in summer sequences and $e = 0.004$ (0.090) in winter sequences. Although the mean errors are equivalent to those resulting from uncalibrated cameras (see below), the maximum errors are an order of magnitude larger. The diurnal (presumably thermal) fluctuations in camera motion are estimated to be roughly $e = 0.001$, while the much larger rotations are caused by wind gusts and pressure from snowdrifts and snow accumulation. Flotron [165] built an insulated box around their time-lapse camera to prevent thermal expansion and contraction of the camera assembly. Such a construction would also reduce or negate the other sources of camera motion. However, the residual motion after correction was estimated visually to be up to $e = 0.001$, so other sources of error should be addressed before building protective structures.

**Camera calibration** Field calibrating the cameras prevented mean (maximum) errors of $e = 0.004$ (0.028). The majority of the cameras, including all of those fitted with prosumer-
grade fixed focal length lenses, had maximum errors less than $e = 0.010$. Two of the three outlier cameras were fitted with consumer-grade zoom lenses set to the widest focal length of all lenses, 18 mm (35 mm equivalent), supporting the general assertion that both cheaper and wider lenses exhibit more distortion. As a starting point for planning, the average distortion of different lens models can be compared using lens databases like lensfun (lensfun.sourceforge.net), Adobe (helpx.adobe.com/camera-raw/kb/supported-lenses.html), and DXOMARK (dxomark.com/lenses), but these do not replace the need for calibrating the specific camera body and lens combination actually used in the field.

**Earth curvature** Use of a local rectilinear coordinate system (which represents a segment of the Earth’s curved surface as flat) introduces a vertical error of $e = 8 \times 10^{-8} \text{m}^{-1}$. Correcting for this effect prevented errors of up to $e = 0.004$ at $50 \times 10^3 \text{m}$, the maximum distance to the visible horizon in Columbia Glacier time-lapse photographs. This correction was critical when estimating the absolute orientation of photographs using distant horizons.

**Atmospheric refraction** At 40 m and higher above the surface, the vertical temperature gradient is fairly independent of the temperature at the surface. In this zone, the vertical errors due to refraction are an order of magnitude smaller than those caused by Earth curvature, roughly $e = 8 \times 10^{-9} \text{m}^{-1}$. Cameras should be placed at elevated positions overlooking the surface of interest to avoid lines of sight traveling long distances through near-surface air layers where refraction effects can be very large. Daily fluctuations of $e = 0.0005$ (at distances on the order of 1 km) are evident in extremely oblique views at Columbia Glacier where incoming rays skim the glacier surface.

Since camera motion contributes the largest errors, and must be corrected on a per-image basis, correcting for camera motion is the most critical and sensitive step in preparing images for analysis. The novel moving-window global optimization presented here is remarkably successful at stabilizing the entire 13-year record of time-lapse photographs at Columbia Glacier. However, residual motions of up to $e = 0.001$ persist, suggesting that further refinements are possible. Al-
though the SIFT keypoint descriptor (and as an extension RootSIFT, the improved variant used in this study), ranks highly among established methods for robustness to rotation and illumination changes [166], some modern descriptors significantly outperform SIFT in these situations, including soft Ordinal Spatial Intensity Distribution (soft OSID) [167], Multisupport Region Order-Based Gradient Histogram (MROGH) and Multisupport Region Rotation and Intensity Monotonic Invariant Descriptor (MRRID) [168], and Mixed Intensity Order Pattern (MIOP) [169]. Alternatively, keypoint matching could be followed by a second refinement step acting at a higher pixel density, such as dense optical flow [170] or robust variants of classical image cross-correlation such as orientation correlation [171]. Finally, recent developments on matching large stable regions [144] and edges [129], rather than point features, are promising directions for resolving camera motion in the most challenging cases when snow cover and poor visibility reduce the availability of distinctive point features. Finally, recall that images cluttered by snow, rain, and ice moving across the image (forming drifts, stalagtites, or affixed to the window of the camera housing) were manually removed, out of concern that these would lead to spurious matches that would bias the global orientation estimates. Testing should be performed to determine what, if any, impact these images actually have on the results. Since most of these images suffer from low detail, their automatic removal may already be assured by the threshold placed on images with too few matches to other images.

4.7 Conclusions

we have reviewed the fundamentals of camera geometry and camera calibration, developed a set of strategies to retroactively calibrate static cameras from one or more outdoor photographs, and demonstrated their use on a large collection of time-lapse photographs from Columbia Glacier, Alaska. Traditional ground control points are complimented by the novel use of polyline features like mountain horizons and coastlines to achieve much greater levels of image coverage. Static features matched between photographs, traditionally used to calibrate rotating cameras, are enhanced by the novel use of synthetic images produced by projecting map data into imaginary cameras. Since the synthetic images map to known world coordinates, the matched points are an automatable
equivalent of the world points collected manually by traditional methods. Paired with global high-resolution satellite data products [164], this technique can be used to automate photo-to-terrain registration anywhere on Earth. Camera motion is identified as the most significant obstacle to time-lapse image analysis; a novel moving-window global optimization of camera orientations successfully stabilizes 33,000 time-lapse photographs spanning 15 camera positions over 13 years. The results from Columbia Glacier inform a series of recommendations and priorities for future camera deployments and calibration efforts.

4.8 Acknowledgements

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5.1 Introduction

The previous chapters developed methods to calibrate photographs to absolute spatial and temporal reference frames so that they may be analyzed together and alongside other spatial and temporal data. In this chapter, we demonstrate the value of calibrated photographs in the study of glaciers. We deploy a novel feature-tracking algorithm to 32,000 calibrated Columbia Glacier time-lapse photographs to produce glacier velocity fields, their associated uncertainties, and corresponding strain rate fields at 3 d intervals over 13 years, producing an unprecedented high-frequency, long-term record of glacier dynamics.

5.2 Methods

The tracking algorithm used in this work (and introduced in the next section) was originally developed by Brinkerhoff [29], who applied the method to Columbia Glacier time-lapse photographs from June 2013 – May 2014, when a large number of reference elevation and velocity datasets were available and the glacier geometry was relatively stable. The sections that follow describe extensions and adaptations of the method made to address the rapid changes in glacier geometry and dynamics, and sparse reference elevation and velocity observations, encountered elsewhere in the period of record. The result is a flexible and scalable pipeline for deploying the tracking algorithm across a wide range of glacier-camera configurations.
5.2.1 The particle filtering approach

Provided with oblique images of a glacier surface, we seek to recover the glacier’s motion over the time interval bracketed by the images. The method we use should meet the particular challenges presented by glacier landscapes and the limitations of typical glacier time-lapse camera configurations. First, the method should gracefully handle the cluttered and repetitive appearance of a typical glacier surface. The method needs to also be robust to intermittent loss of visibility or changes in appearance due to poor weather, lighting, or obstruction by foreground objects. Since camera positions are typically limited to just a few areas of flat, stable ground at low elevations around the glacier perimeter, the method should be able to handle highly-oblique perspectives and arbitrary camera configurations. Finally, despite the many possible sources of error, the method should ideally produce a robust estimate of the overall measurement uncertainty.

Given two sequential images of the glacier surface, the conventional approach begins by matching (or ‘tracking’) features between the images. Whether matching is done manually [26] or with an automated method – for example by matching local feature descriptors (per Section 4.3.3) or by using template-based image cross-correlation [172] (which identifies the optimal pixel offset between a reference and test sub-image) – only the best match is typically retained for further analysis. Over the self-similar texture of a glacier surface, however, other alternative matches may be nearly as likely, in which case discarding these alternative hypotheses may lead to errors in tracking and underestimates of the uncertainties.

In a second step, the retained image matches are converted to motion in world coordinates. For a static feature, any image displacement is the result of camera motion, while for a moving feature, it is the result of both camera and surface motion. Provided the camera geometry is known for both images (for example, by the techniques of Chapter 4), the trajectory of the feature can be recovered by transforming its coordinates in each image into world coordinates. However, the transformation from 2-D to 3-D is problematic: while the direction to the feature is known (defining an outgoing ray), the distance to the feature is unknown. In the case of a single camera,
the transformation to world coordinates is performed by intersecting the outgoing ray with a surface on which the feature is assumed to lie. Although their intersection is well-defined when the ray and the surface are close to perpendicular, as the angle between them decreases, the intersection becomes sensitive to errors in ray direction and surface topography, and prone to discontinuities due to spurious occlusions by the foreground surface. If two or more cameras are present, an alternative is to match the feature in each camera to find the near-intersection of the outgoing rays. However, the additional step of matching features between images from each camera is only reliable if the camera positions are close together (especially in the present case of oblique views of a rough surface), in which case the camera rays are close to parallel and place only weak constraints on each other.

To avoid these errors and limitations, Brinkerhoff [29] approach the problem in reverse: Rather than project sequentially-matched image coordinates outward to estimate the original motion, they model the motion, projecting sequences of world coordinates into the images for validation. Their approach (depicted graphically in Figure 5.1), based on particle filtering theory [173], updates the probability distribution of the modeled motion (specifically, position and velocity on an assumed glacier surface) based on the information contained in the images. In practice, this is achieved by representing the motion of the glacier for a particular starting point in time and space with a large number of ‘particles’, each with horizontal velocities sampled from an initial probability distribution. The particle positions and velocities are evolved forward (or backward) in time based on their current position and velocity (and small random accelerations to mimic high-frequency glacier velocity variations). Whenever an image is available, the particles are projected onto the image and each deemed more or less likely based on the similarity of the image (in a neighborhood centered around the particle’s current position) to that of an earlier image (in a neighborhood centered around the particle’s earlier position) computed with image cross-correlation. Unlikely particles are pruned while likely particles are replicated for the next time step, with the effect of updating the probability distribution of the motion as additional images are considered in turn.

Although this method requires an initial probability distribution of glacier velocity, the re-
Initialize $x'$ (cyan dot)

Extract reference sub-image

Initialize particles (red dots) from prior

For all images $d_k$

Evolve particles with dynamical model

Resample proportional to likelihood

Extract test sub-image around prediction mean (yellow dot)

Compute likelihood from reference and test images

Evaluate likelihood for each particle

Figure 5.1: Schematic of the feature tracking algorithm used in this paper. Reprinted with permission from Brinkerhoff [29].
sults are not overly sensitive to the quality of this initial guess if some simple precautions are taken (Section 5.2.7). Furthermore, the method comes with a number of distinct advantages. It produces a probability distribution (and thus any desired uncertainty statistic) as a natural output. Furthermore, by using a large population of particles, the tracking algorithm can explore all of the likely matches identified at each time step (in proportion to the likelihood of each match). Over time, transient hypotheses are culled in favor of the more persistent ones, providing protection from detection errors and occlusions. During stretches of low quality images (typically due to bad weather), when no likely matches are found, the particles evolve according to the motion model until visibility is once again restored. Critically, since the motion model operates in world coordinates, the projection from image to world coordinates is avoided entirely. This allows each camera present to contribute independently to the computation of particle likelihood. Since there is no need to match features between images from each camera, cameras can be placed at arbitrarily large angles from each other, providing strong orthogonal constraints on the particle trajectory, as illustrated in Figure 5.2.

5.2.2 Three-dimensional motion model

The original motion model used by Brinkerhoff [29] limits the model state variables to map-plane coordinates by requiring particles to move tangent to an assumed glacier surface. We generalize the motion model to three dimensions (3-D) in order to account for periods of flotation and the large elevation uncertainties that arise at Columbia Glacier as a result of rapid glacier thinning and infrequent elevation observations.

For a given particle, the model state variables are the 3-D coordinates $\mathbf{x}$ and 3-D velocities $\mathbf{v}$, which for time step $k$ of length $\Delta t$ yields the discrete difference equations

$$x_k = x_{k-1} + \Delta t v_{k-1} + \frac{\Delta t^2}{2} a$$

(5.1)

$$v_k = v_{k-1} + \Delta t a.$$  

(5.2)
Figure 5.2: Schematic illustrating the benefit of two (or more) oblique cameras for resolving horizontal motion. Although neither camera can constrain both the orientation and magnitude of motion, together the two cameras can resolve the motion uniquely. Reprinted with permission from Brinkerhoff [29].
\( \mathbf{a} \) are random accelerations, described as the multivariate normal distribution

\[
\mathbf{a} \sim \mathcal{N}_3(0, \Sigma_a),
\]

where \( \Sigma_a = \text{diag}(\sigma_{ax}^2, \sigma_{ay}^2, \sigma_{az}^2) \) is a diagonal covariance matrix specifying the assumed characteristic variance in glacier accelerations in dimensions \( x, y, \) and \( z \). Normal distributions \( \mathcal{N} \) of one variable are written as \( \mathcal{N}(\mu, \sigma^2) \), where \( \mu \) is the mean, \( \sigma \) is the standard deviation, and \( \sigma^2 \) is the variance. We write the multivariate equivalent as \( \mathcal{N}_n(\mu, \Sigma) \), where \( n \) is the number of variables, \( \mu \) is a vector of \( n \) means, and \( \Sigma \) is an \( n \times n \) covariance matrix.

The initial state of the particle is specified by the normal distributions

\[
\mathbf{x}_0 \sim \mathcal{N}_3(\bar{\mathbf{x}}_0, \Sigma_{x_0})
\]
\[
\mathbf{v}_0 \sim \mathcal{N}_3(\bar{\mathbf{v}}_0, \Sigma_{v_0})
\]

where \( \bar{\mathbf{x}}_0 \) and \( \bar{\mathbf{v}}_0 \) are the means, and \( \Sigma_{x_0} \) and \( \Sigma_{v_0} \) the diagonal covariance matrices, of the initial particle position and velocity. The known (or, at least, assumed) surface elevations are specified as the spatially-varying normal distribution

\[
Z(x,y) \sim \mathcal{N}(\mu_z(x,y), \sigma_z^2(x,y)).
\]

where \( \mu_z(x,y) \) and \( \sigma_z^2(x,y) \) are the mean and variance of elevation at position \( (x,y) \). The likelihood \( \mathcal{L} \) of a particle – how well the particle matches observations – is computed from both image(s) and elevations as

\[
\mathcal{L} \sim \exp \left( -\sum_i \frac{\ell_i(u_i, v_i)}{\sigma_i^2} - \frac{(\mu_z(x,y) - z)^2}{\sigma_z^2(x,y)} \right),
\]

where \( \ell_i(u, v) \) is the area-averaged sum-of-squares error between a template sub-image and image \( i \) centered over the position of the particle \( (u_i, v_i) \) in image \( i \), \( \sigma_i^2 \) is the assumed variance in intensity values between the template sub-image and image \( i \) (due to illumination change, surface deformation, and residual camera motion), and \( (x,y,z) \) is the particle position.

As in the original method of Brinkerhoff [29], we use normal distributions to model the initial particle velocities, the random accelerations, and now also the uncertainties of the surface
elevations. Although specific situations may justify more catered choices, the normal distribution is computationally and mathematically convenient and a reasonable average approximation of uncertainties when the distribution of the uncertainties is not known explicitly.

5.2.3 Reference elevations

Computing particle likelihood $L$ from Equation (5.7) requires a normal distribution of elevation at each horizontal position. However, elevation data is typically deterministic – only one value (a reasonable estimate of the mean) is provided. In the absence of published uncertainties, a reasonable global estimate of the measurement variance $\sigma_m^2$ can be calculated directly from the spread of repeat elevation measurements of static landscape features (e.g. bare bedrock).

Additional error is driven by the movement of the glacier surface, which transports local surface features (e.g. crevasses, seracs) downglacier and thus invalidates small-scale spatial variability in elevation measurements. To spread out error evenly over the glacier, we fill crevasses and smooth out small-scale surface roughness [per 161] by applying a maximum filter, followed by a gaussian filter, to gridded elevations. The variance in elevation due to surface roughness, $\sigma_r^2$, can be estimated from the spread of elevation differences between the smoothed and original surface. The elevation at time $t$ (at a given horizontal position) is described as the normal distribution

$$Z(t) \sim \mathcal{N}(\mu_t, \sigma_t^2),$$  \hspace{1cm} (5.8)

where the overall variance $\sigma_t^2 = \sigma_m^2 + \sigma_r^2$ per the sum of two uncorrelated variables. Observed elevations are linearly interpolated to estimate the distribution of elevations at times for which no contemporaneous observations are available. Given observed elevations $Z(t = a)$ and $Z(t = b)$, the interpolated elevation $Z(t = i)$ is nominally

$$Z(i) \sim \mathcal{N}(\mu_a + r(\mu_b - \mu_a), \sigma_a^2 + r^2(\sigma_a^2 + \sigma_b^2)), $$  \hspace{1cm} (5.9)

where $r = (i - a)/(b - a)$. Linear interpolation supposes that the rate of change of glacier elevation is constant between $t = a$ and $t = b$. However, as Figure 5.3 reveals, surface elevations at Columbia
Glacier exhibit strong seasonal variability. Out of an abundance of caution, we add to the variance in Equation (5.9) an additional term $\sigma_s^2$ scaled such that $3\sigma_s$ (the 99.7% confidence interval) equals the difference to the nearest observed $Z$; that is,

$$3\sigma_s = (\mu_b - \mu_a) \frac{\min| i - a |, | b - i |}{b - a}.$$

(5.10)

While this approach is sensible over the glacier interior, the assumption of linearity is dramatically violated at the glacier margin as the glacier advances or retreats. Within the regions between the glacier termini (calving fronts) at $t = a$ and $t = b$, one of the elevation measurements will be of the water rather than of the glacier surface. We chose not to track points over these areas.

Appropriate propagation of elevation uncertainties are of particular concern given the highly oblique viewing angles typical of ground-based time-lapse cameras. The influence of elevation errors on the positional accuracy of optical and radar instruments is reviewed in Palubinskas et al. [174]. For a camera viewing a flat horizontal surface from an altitude angle $\theta$, an elevation error $\Delta z$ results in a horizontal position error $\Delta x = \Delta z / \tan \theta$ and a camera-object distance error $\Delta d = \Delta z / \sin \theta$.

Since time-lapse cameras are fixed, the position error is constant in time and thus has a negligible influence on velocity measurements unless velocity varies considerably over that distance. However, as a result of the linear scaling between image and world coordinates, the distance error results in a corresponding velocity percent error $\Delta v / v = \Delta d / d$ (for the component of velocity parallel to the image plane). For example, an altitude angle $\theta = 10^\circ$ and elevation error $\Delta z = 20$ m (an undesirable configuration experienced at Columbia Glacier during periods of rapid thinning and infrequent elevation observations) result in a distance error $\Delta d = 115$ m, or a 10% velocity error at a distance $d$ of 1150 m.

5.2.4 Reference velocities

Whereas glacier surface elevations are expected to vary smoothly on subseasonal time scales, velocities are subject to both strong seasonal [26, 175, 176, 177] and shorter-term variability [178,
Figure 5.3: Surface elevations at fixed positions on the centerline of Columbia Glacier for two
periods with a high frequency of observations. Elevations in the top panel are from Krimmel [26],
while elevations in the bottom panel are from the datasets listed in Table 5.1. The glacier surface
tends to be higher in spring and lower in fall, perhaps due to snow accumulation and advection
of thicker ice and snow from above outpacing dynamic thinning and melt through the winter and
early spring.
22, 179, 180, 181, 182], driven by changes in water availability (e.g. precipitation, melt), subglacial drainage, and backstress (e.g. tides, large calving events, ice mélange). Furthermore, velocity maps are typically produced by tracking features in repeat optical [183] or radar [184] satellite imagery with wide temporal baselines. Such maps do not reach the terminus, since features close to the terminus that are visible in the first image will have either calved off or been subjected to large deformations (and thus made unrecognizable) by the time of the second image. As a result, most available velocity maps have gaps in coverage and represent long-term averages not representative of the full range of possible velocities. Therefore, rather than interpolate velocities in time as for elevations, we reduce repeat velocity observations to a temporally-weighted spatially-varying normal distribution of horizontal velocity. This can achieve both complete coverage and, if these observations span the full range of conditions experienced by the glacier during the period of interest (e.g. multiple years, different times of year, different distances from the terminus as the glacier advances or retreats), a reasonable (lower) estimate of the variance at each location.

Given a sorted timeseries of velocities \( \{v_1, v_2, \ldots, v_n\} \), in either the \( x \) or \( y \) dimension, observed at times \( \{t_1, t_2, \ldots, t_n\} \) starting from a particular horizontal position, we define the weight of each observation \( w_i \) as
\[
 w_i = \min((t_i - t_{i-1}) + (t_{i+1} - t_i), \Delta t_{\text{max}}),
\] (5.11)
where the maximum weight, \( \Delta t_{\text{max}} \), is chosen such that, for example, isolated observations cannot count for more than 1/2 year given the strong seasonal variability overlaid on the interannual trend. The weighted mean \( \bar{v} \) is given by
\[
 \bar{v} = \frac{\sum_i v_i w_i}{\sum_i w_i},
\] (5.12)
and the resulting normal distribution is given by
\[
v \sim \mathcal{N} \left( \bar{v}, \frac{\sum_i (v_i - \bar{v})^2 w_i}{\sum_i w_i} \right).
\] (5.13)

The procedure is repeated for each horizontal velocity component \( v_x \) and \( v_y \). To estimate the vertical component \( v_z \), we integrate over the spatial derivatives of the time-interpolated elevations.
$Z(x, y)$ along the horizontal velocity components computed above:

$$v_z \sim \mathcal{N} \left( \frac{\partial Z}{\partial x} \bar{v}_x + \frac{\partial Z}{\partial y} \bar{v}_y, \sigma_{\bar{v}_x}^2 + \sigma_{\bar{v}_y}^2 \right), \quad (5.14)$$

where $\bar{v}_x$ and $\bar{v}_y$ are the mean of $v_x$ and $v_y$, and the spatial derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ are computed numerically as the central differences $(Z(x + \Delta x, y) - Z(x - \Delta x, y)) / 2\Delta x$ (and similarly for $y$). This result considers only the uncertainty in the magnitude and orientation of the horizontal particle velocity; it assumes that neighboring values of $Z(x, y)$ are perfectly correlated and thus that their difference have zero variance. Although this assumption is reasonable for the elevation measurement variance $\sigma_m^2$, it may be desirable to propagate the elevation roughness variance $\sigma_r^2$.

### 5.2.5 Probability of flotation

The vertical trajectory of a point on the glacier is well-approximated by the local slope of the glacier surface, as was assumed in Equation (5.14). However, when a glacier is floating, ocean tides drive an additional, periodic vertical motion of the surface. In order to initialize particles in these regions with suitably large variances of vertical velocity and acceleration, we extend the stochastic surface elevations to the calculation of a flotation probability $P_f$. To the elevation of the glacier ice $Z_i$, we introduce additional normal distributions $Z_w$ and $Z_b$ for the elevation of the water level and the glacier bed,

$$Z_i \sim \mathcal{N}(\mu_i, \sigma_i^2) \quad (5.15)$$
$$Z_w \sim \mathcal{N}(\mu_w, \sigma_w^2) \quad (5.16)$$
$$Z_b \sim \mathcal{N}(\mu_b, \sigma_b^2). \quad (5.17)$$

$Z_w$ can be estimated from modeled or observed tidal heights expressed relative to a mean tidal datum, such as Mean Sea Level (MSL), which closely matches 0 elevation relative to the local geoid. The maximum possible ice thickness $h_{\text{max}}$ is given by

$$h_{\text{max}} \sim \mathcal{N}(\mu_i - \mu_b, \sigma_i^2 + \sigma_b^2), \quad (5.18)$$
while the maximum ice thickness required for flotation, $h_f$, is given by

$$h_f \sim \mathcal{N}(\rho(\mu_w - \mu_b), \rho(\sigma_w^2 + \sigma_b^2)), \quad (5.19)$$

where $\rho = \rho_w/\rho_i$ is the ratio of the densities of water $\rho_w$ and ice $\rho_i$. Although presented here as predetermined, the densities could also be treated as stochastic, for example to account for reductions in water salinity from fresh water inputs [185, 186] or reduced effective glacier density due to snow, firn, and air content [187, 188, 189, 190]. The flotation probability $P_f$ is given by

$$P_f = P(h_f > h_{\text{max}}) = P(h_{\text{max}} - h_f \leq 0) = \Phi(\mu_\delta), \quad (5.20)$$

where $\Phi$ is the cumulative distribution function, and $\mu_\delta$ the mean, of $\delta = h_{\text{max}} - h_f$, the normal distribution

$$\delta \sim \mathcal{N}(\mu_i + \rho\mu_b - \rho\mu_w, \sigma_i^2 + (\rho - 1)^2\sigma_b^2 + \rho^2\sigma_w^2). \quad (5.21)$$

The flotation probability is considered in the computation of the initial state of the particles, described in Section 5.2.6.

5.2.6 Per-point model parameters

For each nominal starting point $(x, y)$ at time $t$, we parameterize the initial particle positions and velocities in terms of the reference elevations, velocities, and flotation probability previously described. The initial mean and variance $\bar{x}_0$ and $\Sigma_{x_0}$ of the particle positions are

$$\bar{x}_0 = \left( x, y, \mu_z(t, x, y) \right), \quad (5.22)$$

$$\Sigma_{x_0} = \text{diag}(\sigma_x^2, \sigma_y^2, \sigma_z^2(t, x, y) + P_f^2(t, x, y)\sigma_w^2),$$

where $(\sigma_x^2, \sigma_y^2)$ are the chosen variances about the mean position $(x, y)$, $\mu_z(t, x, y)$ and $\sigma_z^2(t, x, y)$ are the mean and variance of the reference glacier surface elevations, $P_f(t, x, y)$ is the probability of flotation, and $\sigma_w^2$ is the variance of water surface elevation (e.g. due to tides). The initial mean
and variance \( \bar{v}_0 \) and \( \Sigma_{v_0} \) of the particle velocities are

\[
\bar{v}_0 = \begin{pmatrix}
  v_x(x, y) , & v_y(x, y) , & v_z(t, x, y)
\end{pmatrix}
\]

\[
\Sigma_{v_0} = \text{diag}(\begin{pmatrix}
  s^2\sigma_{vx}^2 , & s^2\sigma_{vy}^2 , & \sigma_{vz}^2 + P_f^2(t, x, y)\sigma_{vz,f}^2
\end{pmatrix})
\]

(5.23)

where \((v_x(x, y), v_y(x, y), v_z(t, x, y))\) and \((\sigma_{vx}^2, \sigma_{vy}^2, \sigma_{vz}^2)\) are the mean and variance of the reference velocity, \(s\) is a scaling factor to account for short-term variance in velocity being more (or less) than the long-term variance, and \(\sigma_{vz,f}^2\) is the variance in vertical velocity due to flotation.

The variances \(\Sigma_a\) of the random accelerations \(a\) are defined as

\[
\Sigma_a = \text{diag}(\begin{pmatrix}
  c^2\sigma_{vx}^2 , & c^2\sigma_{vy}^2 , & \sigma_{min_{az}}^2 + P_f^2(t, x, y)\sigma_{az,f}^2
\end{pmatrix})
\]

(5.24)

where \((\sigma_{vx}, \sigma_{vy})\) are the \(x\) and \(y\) components of \(\Sigma_{v_0}\) resulting from Equation (5.23), \(c\) is a scaling factor to account for the larger accelerations experienced by regions with higher velocities, and \(\sigma_{min_{az}}^2\) is the minimum variance, and \(\sigma_{az,f}^2\) the variance due to flotation, of vertical acceleration.

5.2.7 Forward, backward, and repeat tracks

The tracking algorithm is relatively robust to the initial particle state. The algorithm is expected to converge as long as the distribution of the particles has a sufficient probability density at the true motion being measured (relative to the number of particles being used to numerically approximate the distribution). The previous sections described our methods for estimating reasonable initial particle states so that tracking converges quickly. To further ensure rapid convergence, we track each point twice, the second time starting with particle velocities sampled from the final particle velocity distribution of the first run. Of these two runs, we select the run with the smallest mean standard error in horizontal velocity (the mean of \(\sqrt{\sigma_{vx}^2 + \sigma_{vy}^2}\) over all time steps) and discard the alternate run.

An additional weakness of the tracking algorithm is its sensitivity to the quality of the sub-image sampled from the initial image as a template for matching to subsequent images. Outdoor images can suffer from high noise, low detail, and occlusion from under or overexposure, extreme
lighting conditions, and poor visibility due to lens flare or inclement weather. Since the algorithm works equally well forwards and backwards in time, as in Brinkerhoff [29], we track each point both forward starting from the first image and backwards starting from the last image (each run is then reinitialized and filtered as described above). We take the mean of the two runs at each time step by weighing them by the inverse of the variance of each velocity component. The forward run tracks a feature downglacier from an initial position \((x, y)\) at the time of the first image, while the backwards run tracks a feature upglacier from the same initial position \((x, y)\) at the time of the last image. The result of averaging these two tracks is an Eulerian estimate of the glacier flow field passing through \((x, y)\).

The benefit of these strategies is illustrated by the example results of Figure 5.4. The first image (a) was taken during a storm and suffers from low detail; as expected, the forward run (b), which uses the first image as a template, fails to converge, even after being run a second time reinitialized with the final particle state of the initial run (c). The backward run (d), which uses the superior last image as a template, converges successfully (e) and is further improved by a repeat run reinitialized at the final converged state of the initial run (f).

5.3 Application to Columbia Glacier

5.3.1 Images

Before the advent of digital imaging, film time-lapse cameras were used at the Columbia Glacier to observe short-term glacier dynamics [191, 192, 193]. Since 2004, digital time-lapse cameras have been used to photograph the glacier front with framerates of 4 h to 45 s, resulting in a growing archive approaching 300,000 images [28, 18]. 31,777 of these images, sampled at a nominal spacing of 2 h from the 15 camera positions (‘stations’) with clear views of the glacier surface, were selected for analysis. Figure 5.5 lists the temporal coverage of each station, while Figure 5.6 displays a sample time-lapse photograph from each station.

Although only one of these stations was operating at any given time before 2008, from
Figure 5.4: Mean and standard deviation of horizontal speed for a point tracked over a 3 d interval from photographs with nominal 2 h spacing. Results are shown for the initial forward run (first to last image), initial backward run (last to first image), and the corresponding repeat runs reinitialized with the final particle velocities of the initial runs.
summer 2008 onwards, multiple stations (up to four) were often operating simultaneously. The camera model parameters corresponding to each image were estimated following the methods of Chapter 4 and the image capture times were calibrated to Coordinated Universal Time (UTC) to within 1 min following the methods of Chapter 2 or to within 1 h following the methods of Chapter 3. For sets of cameras operating simultaneously, temporal agreement was verified by identifying, for each camera, the two images bracketing a large calving event (or other near-instantaneous visible event), and ensuring that the time interval of all cameras intersected.

5.3.2 Image bins

The images were grouped into optimal 3 d bins. First, the maximal temporal coverage interval for each station was cut at any instantaneous change violating the assumptions of the tracking algorithm – namely, a change in camera, image pixel size, large instantaneous camera rotations (> 1° roll or > 2° pan-tilt), or large temporal gaps (> 2 d) – to ensure they would not occur within a bin. Intervals of excessive camera motion (> 1° roll or > 2° pan-tilt within a sliding 3 d window) were cut out entirely. The resulting intervals of continuity for each station were then cut at the interval endpoints of all stations to produce the intersected intervals of continuous coverage for all stations available at each time. To ensure maximum temporal coverage despite staggered station coverage, short station intervals (< 1 d) resulting only from other stations’ cuts were appended to adjacent multi-camera coverage intervals, and any remaining short intervals (< 1 d) isolated from neighboring intervals by a temporal gap or station cut were dropped. Finally, the resulting intervals of coverage were divided into bins of equal length that evenly divided the interval and least deviated from the nominal length (3 d).

5.3.3 Glacier elevations

To estimate reasonable initial parameter distributions (‘priors’) for the motion model at each starting position and time, we made use of elevation and velocity data from a variety of vertical airborne and satellite optical and radar imagery.
Figure 5.5: Temporal coverage for each camera station selected for analysis.
Figure 5.6: Sample time-lapse photograph for each camera station selected for analysis.
The elevation datasets (‘digital elevation models’, or DEMs) selected for analysis are listed in Table 5.1. They were obtained from four different sources: (1) AeroMetric (now Quantum Spatial, www.quantumspatial.com) aerial stereophotographs [194] (referred to here as aerometric); (2) ArcticDEM produced from Worldview satellite optical stereo imagery [195] (arcticdem); (3) airborne interferometric synthetic aperture radar (IfSAR or InSAR) from the Alaska Mapping Initiative (ita.cr.usgs.gov/IFSAR_Alaska) (ifsar-alaska); and (4) InSAR from the TanDEM-X twin satellite [196] (tandem).

The aerometric elevations, expressed in World Geodetic System 1984 (WGS84) ellipsoid heights, were used as the reference to which to register the other elevation datasets. This choice was motivated by their high resolution (2 m), low measurement standard error ($\sigma_m = 1.5 \text{ m}$), excellent agreement with GPS control point surveys, and because they were relied upon for calibrating many of the time-lapse cameras (per the methods of Chapter 4). A vertical adjustment $\Delta z$ was applied to each of the other elevation datasets as needed to correct for a mean offset relative to the aerometric elevations over select snow-free bedrock areas, while their measurement standard errors $\sigma_m$ were estimated from the residuals following the adjustment (Table 5.2). Additional corrections, such as affine transformations or elevation-dependent debiasing [197, 164], were not performed as any gains from these methods would be small (over the low-angled glacier surface visible to the time-lapse cameras) relative to the uncertainties introduced by time interpolation and surface roughness (see Section 5.2.3). The remaining elevation sources are further discussed below.

arcticdem Elevations were selected from ArcticDEM strips with the best spatial coverage over Columbia Glacier. They are expressed in the same vertical datum as aerometric (WGS84 ellipsoid heights) and no vertical adjustment was applied.

ifsar-alaska Elevations were converted from North American Vertical Datum of 1988 (NAVD88) geoid heights to WGS84 ellipsoid heights using the Earth Gravitational Model 2008 (EGM-2008, Pavlis et al. [198]); no additional vertical adjustments were applied. Since these elevations were only available for a single date range (as a blend of observations spanning
2010-07-20 ± 10 d), the measurement error $\sigma_m$ was assumed to be the same as reported in the published quality report available for an adjacent cell: 1.5 m, increased to 2 m to account for the additional error introduced by the temporal blending.

**tandem** Vijay and Braun [196] do not explicitly state the vertical datum of their results. Because of the poor data quality over land, we used contemporaneous arctidem as a reference over glacier ice to estimate a consistent mean vertical error of $-10$ m for the subset of strips used in this study. Across all strips, we identified four distinct error types, with mean vertical errors ranging from $-10$ m to $+8$ m.

All elevation datasets were linearly interpolated to a common 20 m grid and smoothed (per Section 5.2.3) in a sliding 5 x 5 cell (100 m) neighborhood to fill crevasses and smooth surface details. By comparing the smoothed and original elevations, we estimated an overall glacier surface roughness $\sigma_r$ of 3 m.

### 5.3.4 Glacier flotation

For the flotation calculation, we used the observed and modeled bed elevations from Enderlin et al. [177] and assigned a conservative standard error $\sigma_b$ of 20 m based on the uncertainties reported by Enderlin et al. [199] and Enderlin et al. [177] and the rapid rates of sediment deposition in the glacier forebay following retreat [200]. For the water surface, we assumed a standard error $\sigma_w$ of 2 m based on the amplitude of tidal fluctuations in sea level observed at the permanent National Oceanographic and Atmospheric Administration (NOAA) tide gauge in nearby Valdez, Alaska (COOPS:9454240). A detailed comparison of the tide at Valdez to those measured in 2005 in the glacier forebay [201] indicates that the identity, phase, and amplitude of the major tidal constituents at both locations were equal to within their uncertainties. The tides at Valdez are therefore a suitable proxy for those at the Columbia Glacier terminus.
Table 5.1: Date and source of the elevation datasets selected for analysis.

<table>
<thead>
<tr>
<th>Date</th>
<th>Source</th>
<th>Date</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004-06-18</td>
<td>aerometric</td>
<td>2012-05-07</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2004-07-07</td>
<td>aerometric</td>
<td>2012-06-17</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2005-08-11</td>
<td>aerometric</td>
<td>2012-07-17</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2005-08-27</td>
<td>aerometric</td>
<td>2012-08-13</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2006-07-12</td>
<td>aerometric</td>
<td>2012-10-12</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2006-07-27</td>
<td>aerometric</td>
<td>2012-11-23</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2007-09-22</td>
<td>aerometric</td>
<td>2013-03-26</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2008-08-11</td>
<td>aerometric</td>
<td>2013-06-10</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2009-08-03</td>
<td>aerometric</td>
<td>2013-07-12</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2009-08-27</td>
<td>aerometric</td>
<td>2013-11-19</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2010-05-25</td>
<td>aerometric</td>
<td>2014-04-17</td>
<td>tandem</td>
</tr>
<tr>
<td>2010-06-02</td>
<td>aerometric</td>
<td>2014-05-31</td>
<td>tandem</td>
</tr>
<tr>
<td>2010-07-20</td>
<td>ifsar-alaska</td>
<td>2014-06-22</td>
<td>tandem</td>
</tr>
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<td>2010-09-06</td>
<td>arcticdem</td>
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<td>tandem</td>
</tr>
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<td>2011-06-18</td>
<td>tandem</td>
<td>2015-01-18</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2011-07-21</td>
<td>tandem</td>
<td>2015-02-27</td>
<td>arcticdem</td>
</tr>
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<td>2011-08-12</td>
<td>tandem</td>
<td>2015-04-23</td>
<td>arcticdem</td>
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<td>2011-09-03</td>
<td>tandem</td>
<td>2015-05-27</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2011-12-11</td>
<td>tandem</td>
<td>2015-08-01</td>
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</tr>
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<td>2012-01-02</td>
<td>tandem</td>
<td>2015-08-24</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2012-02-04</td>
<td>tandem</td>
<td>2015-09-30</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2012-03-08</td>
<td>tandem</td>
<td>2016-06-14</td>
<td>arcticdem</td>
</tr>
<tr>
<td>2012-03-29</td>
<td>arcticdem</td>
<td>2016-08-20</td>
<td>arcticdem</td>
</tr>
</tbody>
</table>

Table 5.2: Original vertical datum, applied vertical adjustment $\Delta z$, and residual measurement standard error $\sigma_m$ by elevation source.

<table>
<thead>
<tr>
<th>Source</th>
<th>Vertical datum</th>
<th>$\Delta z$ m</th>
<th>$\sigma_m$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>aerometric</td>
<td>WGS84 ellipsoid</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>arcticdem</td>
<td>WGS84 ellipsoid</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>ifsar-alaska</td>
<td>NAVD88 geoid</td>
<td>EGM2008</td>
<td>2</td>
</tr>
<tr>
<td>tandem</td>
<td>-</td>
<td>$+10$</td>
<td>3</td>
</tr>
</tbody>
</table>
5.3.5 Glacier velocities

The velocities used to estimate the canonical velocity distributions were compiled from five sources (Table 5.3): (1) U.S. Geological Survey aerial stereophotographs [26] (referred to here as krimmel); (2) AeroMetric aerial stereophotographs (Yushin Ahn, unpublished, per the method of Ahn and Howat [202]) (aerometric); (3) Landsat 4, 5 and 7 satellites [203] (landsat); (4) TerraSAR-X synthetic aperture radar (SAR) satellite [177] (terrasar); (5) and Global Land Ice Velocity Extraction from Landsat 8 (GoLIVE) [183, 204] (golive). Due to the setback of velocities from the terminus, it was necessary to go as far back as 1996 to achieve coverage to the position of the terminus at the start of the time-lapse record in June 2004. Velocities occurring outside the interval of interest were assigned weights diminishing with distance from the interval endpoints to reduce their influence on the results.

Unlike the other datasets, krimmel are not computationally-derived gridded velocities but rather sparse features tracked manually between adjacent flight pairs with an analytic stereoplotter. These data were conservatively interpolated to a regular 100 m grid by identifying the clusters of contiguous coverage in each flight pair using a shared nearest neighbor method [205] and applying cubic interpolation at grid cell centers within the boundaries of a tight concave hull computed for each region [206].

The landsat and golive velocities both suffer from high noise levels due to a loss of coherence driven by changes in surface appearance (e.g. deformation, snowfall, cloud cover) and the relatively coarse spatial resolution of the Landsat sensors (15 m to 30 m). These were filtered by computing median values in a 3 x 3 neighborhood (900 m for golive and 90 m to 180 m for landsat), retaining only the cells with values deviating less than 1 m d−1 from the median for both the x and y components, and discarding isolated clusters of fewer than 3 cells. To combat residual noise, landsat were further filtered by computing the median orientation of velocity in a sliding temporal window of 5 for each grid cell – i.e. the median of the orientations at times $t = \{t_{i-2}, t_{i-1}, t_i, t_{i+1}, t_{i+2}\}$ – and keeping only the cells within 18° of the median.
Figure 5.7 plots the computed reference velocities in terms of magnitude and orientation. As expected, both the largest magnitudes and variability in speed are centered over the deep centers of the glacier channels and concentrated in the lower region occupied by the terminus. The variability in orientation is concentrated along the channel margins. Although partly a result of the higher relative measurement errors in these slow-moving regions, this pattern may also reflect flow rotations along the shear margins driven by the large fluctuations in speed in the channel centers and the changes in glacier geometry (and flow) with the fluctuation of the terminus position.

5.3.6 Terminus positions

Terminus positions were traced for all reference elevations datasets in order to identify the regions between the terminus positions of intervening elevations, a region in which we chose not to interpolate elevations and thus not to track points. To capture when and where the terminus retreated upglacier of this region and avoid tracking points in open water, we complemented these with terminus positions traced from TerraSAR-X (Alexander Zinck, unpublished), Landsat 7 [207, 208] and Landsat 8 orthorectified imagery. All terminus traces were clipped at their first intersection with the west and east fjord coastlines to remove spurious segments extending down the coast.

5.3.7 Glacier outline

We traced the maximal extent of the glacier, over the area visible to the cameras, from an AeroMetric orthophoto from 2004-06-18 (6 days before the first time-lapse photograph), and used this outline as the initial guess of glacier extent over the full period.

5.3.8 Tracking point selection

Initial positions for tracking were defined as the cell centers of a regular 100 m square grid. The seemingly straightforward task of identifying which points to track with which stations at the time of each tracking interval was complicated by the rapidly changing glacier geometry. The following procedure is illustrated in Figure 5.8. For each time $t$ (the earliest of the image capture
Table 5.3: Number, temporal coverage, and temporal resolution of the velocity datasets selected for analysis, summarized by source.

<table>
<thead>
<tr>
<th>Source</th>
<th>Count</th>
<th>Temporal coverage</th>
<th>Temporal resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>krimmel</td>
<td>22</td>
<td>1996-02-17 – 2001-04-19</td>
<td>14 – 362 days</td>
</tr>
<tr>
<td>landsat</td>
<td>82</td>
<td>1996-04-11 – 2010-09-16</td>
<td>16 – 32 days</td>
</tr>
<tr>
<td>aerometric</td>
<td>5</td>
<td>2004-06-18 – 2010-05-25</td>
<td>8 – 24 days</td>
</tr>
<tr>
<td>terrasar</td>
<td>46</td>
<td>2011-02-11 – 2013-10-28</td>
<td>11 – 22 days</td>
</tr>
<tr>
<td>golive</td>
<td>22</td>
<td>2013-11-18 – 2016-10-25</td>
<td>16 – 32 days</td>
</tr>
</tbody>
</table>

Figure 5.7: Reference maps of the mean and standard error of the horizontal components of velocity for Columbia Glacier for years 2004 to 2016 expressed in polar coordinates (speed and orientation).
times in each bin), we interpolated elevations from the two nearest elevation datasets on either side of \( t \). The terminus positions corresponding to these elevations, as well as the nearest intervening terminus position if one was available, were snapped to the glacier outline and each used to crop the glacier outline; the glacier outline at time \( t \) was taken to be the intersection (i.e. the minimal extent) of the resulting crops. Candidate points were rejected if they fell outside the glacier outline or outside the coverage of the interpolated elevations. Viewsheds were computed for each station from the mean of the interpolated elevations and points were assigned to a station for tracking if they were both visible from the station’s position and their camera projection fell within image bounds. Since even modest elevation errors close to a station can radically impact the computed viewshed (in the extreme case, the station lies beneath the interpolated surface, resulting in nowhere being visible), we ignored the elevations in a 400 m radius around each station and instead masked out points occluded by foreground features (and any points not originating from the glacier surface) by testing whether their camera projection fell within the land masks produced in Section 4.5 for resolving camera motion.

### 5.3.9 Tracking point parameters

We computed the initial particle states as described in Section 5.2.6. Although most parameters are derived directly from the reference glacier velocities and elevations, others are chosen based on heuristics. Initial horizontal positions were assigned a small but non-zero standard error of \( (\sigma_x, \sigma_y) = (2 \text{ m}, 2 \text{ m}) \) (Equation (5.22)) to account for a potential shift between the mean initial position \((x, y)\) of the particles and the true position of the feature depicted in the center of the sub-image used as the template. The standard errors of the long-term horizontal velocities (Figure 5.7, upwards of 5 m d\(^{-1}\) in the center channel) were scaled by \( s = 1.25 \) (Equation (5.23)) to account for the possibility of even larger short-term variations. Similarly, the horizontal acceleration standard errors were computed by scaling the horizontal velocity standard errors by \( c = 0.75 \text{ d}^{-1} \) (Equation (5.24)) to account for rapid accelerations, observed by ground-based optical and radar surveying, of upwards of 8 m d\(^{-1}\) [209, 210]. We chose standard errors for vertical velocity and
Figure 5.8: Map illustrating the point selection procedure, showing all camera stations (yellow dots and labels), the coast used for clipping terminus traces (solid grey line), the maximal glacier extent (dotted black line), the glacier extent once cropped by terminus traces (grey shading), selected camera fields of view (yellow lines), and the resulting tracking point selection (black dots, small: visible to one camera, large: visible to both cameras). The holes in tracking point coverage are caused by terrain occlusion. The basemap is a hillshade of the aerometric elevations from 2009-08-03, while the points are set back to the 2009-08-27 terminus corresponding to the next available elevations.
acceleration due to flotation of $\sigma_{vzf} = 3.5 \text{ m d}^{-1}$ and $\sigma_{azf} = 12 \text{ m d}^{-1}$ based on the trajectory of an optical survey marker near the floating terminus in 2009 (W. Tad Pfeffer and Shad O’Neel, personal communication).

5.3.10 Tracking and data reduction

We used template sub-images with a size of 15 x 15 pixels, which Brinkerhoff [29] found to be a good compromise between feature stability (low deformation) at smaller sizes and feature uniqueness at larger sizes. To account for changes in feature appearance between images (due to illumination changes, surface deformation, and residual camera motion), we used a standard error of $\sigma_i = 0.3$ globally in the particle likelihood calculation (Equation (5.7)). For normalized sub-images (pixel values with mean 0 and variance 1), this assumes that the temporal variance in pixel values is an order of magnitude less than their spatial variance. We tracked each point with 10 000 particles; more particles better approximate the velocity probability distribution, although at the expense of increased computational cost.

For each initial point position, the tracking algorithm produces a population of particle velocities (and positions) at the time of each image in the tracking interval. We expressed the particle velocities at each time by a multivariate normal distribution with a diagonal covariance matrix (as for the initial particle state, per Equation (5.5)), then reduced the distributions to a single time-averaged normal distribution by taking their inverse-variance weighted mean and propagating their (correlated) variances. To eliminate outliers in the resulting time-averaged velocity fields, we replaced the distribution at each point by the distribution of the point within 100 m (the 5-cell neighborhood) with the median mean value, repeating for each velocity component. From the (median-filtered) means of the horizontal velocity components, we also computed strain rate fields for each tracking interval, expressed as the magnitude and orientation of the extensional and compressive principal strain rates.
5.4 Results

The result is 1067 velocity (and corresponding velocity uncertainty and strain rate) fields on a fixed 100 m square grid, time-averaged over intervals of 2.96 ± 0.54 d arranged end-to-end from 2004-06-24 to 2016-08-20 (the date of the last available elevation observations, see Table 5.1). Of these, 553 fields between 2008-06-18 and 2016-01-02 are constructed from photographs from two or more simultaneously operating camera stations. The remaining 514 fields are derived from photographs from only one camera.

5.4.1 Data quality

Before presenting and interpreting our results, we must confirm that they are consistent with other contemporaneous observations. To do this, we temporally average the time-lapse velocities to match the time intervals of airborne and satellite-derived velocities from Table 5.3 and compare the velocities at each grid point. Figure 5.9 shows representative comparisons of aerometric, terrasar, landsat, and golive velocities with equivalent time-lapse velocities derived exclusively from multiple cameras. With the exception of aerometric velocities (of which only two fields overlap multiple-camera results), agreement is excellent, with robust (L1-norm) linear regression slopes of 1 ± 0.05, \( R^2 \) coefficients of 0.98 to 1.0, and biases of ±0.75 m d\(^{-1}\). The ±σ uncertainty of the time-lapse velocities encompass the reference velocities in nearly all cases.

A single camera can only distinguish the components of motion parallel to its image plane. At increasingly oblique angles to the glacier surface, the magnitude and direction of horizontal motion become difficult to resolve simultaneously – much like an underdetermined system with fewer equations than unknowns. Therefore, in oblique configurations, two or more cameras separated by large (ideally, right) angles are needed to resolve the horizontal flow field uniquely in two dimensions. The difference in quality between one and multiple cameras is plainly visible in our results. Figure 5.10 shows representative comparisons of terrasar velocities with time-lapse velocities derived from only one camera. In clear conditions, multiple cameras produce velocity
Figure 5.9a: Comparisons of select airborne and satellite-derived velocities from Table 5.3 to those produced by temporally averaging time-lapse velocities (derived from multiple cameras) over the same time interval. The scatter plots compare time-lapse mean speeds and their ±σ uncertainties (grey bars) on the y-axis to reference speeds on the x-axis. The dotted grey line is 1:1 and the red line is the result of a robust (L1-norm) linear regression. The corresponding maps show the reference velocities (black) besides the time-lapse velocities (red) shifted right a half grid cell (50 m) to facilitate comparison. The solid grey line is the glacier terminus.
Figure 5.9b: continued
distributions with smaller uncertainties and significantly shifted means relative to the initial guess. In contrast, a single camera is often unable to improve upon the initial guess, instead producing as output the input velocity distribution (Figure 5.11). Henceforth, unless otherwise specified, results are limited to those derived from two or more cameras.

Uncertainties also increase as the number of photographs in a tracking interval decline. This is mostly manifested in higher uncertainties over the winter, when there are fewer photographs available due to longer nights (the cameras were not configured to take long exposures) and the increased likelihood of poor visibility due to clouds, snow, or ice in front of the cameras. Figure 5.12 shows the strong seasonality in the number of photographs available per tracking interval and the correlation between velocity uncertainty and number of photographs.

### 5.4.2 Velocities

Figure 5.13 plots all multiple-camera glacier speeds (2008 through 2015) by distance from the terminus, averaged over 1 km wide longitudinal bins along the glacier centerline (shown on a map in Figure 5.14). The corresponding speeds from the velocity datasets listed in Table 5.3 are overlaid on our results. Although this comparison is biased by the differences in spatial coverage, the agreement between the two is generally excellent, but with our 3 d averages rendering far more detail than can be captured with 11 d to 32 d satellite revisit times. The two complement each other: The reference velocities fill gaps in winter when the ground-based cameras were prone to burial or failure, helping to complete seasonal trends, while our results capture short-term glacier dynamics over long periods and extend the observational record to several years preceding the proliferation of high-resolution, high-frequency satellite imagery. Strong seasonal and sub-seasonal variability is evident, with near-terminus centerline speeds ranging from 2 m d$^{-1}$ in the fall (a minimum on 2012-09-30) to upwards of 20 m d$^{-1}$ to 25 m d$^{-1}$ in the spring, and rapid accelerations and decelerations of up to 10 m d$^{-1}$ over just a few days.

Alongside glacier speeds, Figure 5.13 includes three additional meteorological timeseries. The plotted air temperature and precipitation were measured at the National Water and Climate
Figure 5.10: Comparisons of select reference airborne and satellite-derived velocities from Table 5.3 to those produced by temporally-averaging time-lapse velocities (derived from a single camera) over the same time interval. The scatter plots compare time-lapse mean speeds and their $\pm \sigma$ uncertainties (grey bars) on the $y$-axis to reference speeds on the $x$-axis. The grey line is 1:1.
Figure 5.11: Output particle speed distributions (red) versus input particle speed distributions (grey) for a two-camera (left) and a single-camera (right) velocity field. The grey line is 1:1. Error bars are $\pm \sigma$ uncertainties.
Figure 5.12: Maximum number of photographs per camera station used for tracking (top), for single-camera intervals in grey and multi-camera intervals in black. The uncertainty in multi-camera glacier speeds (bottom) – specifically, the median speed $\sigma$ in a 1 km wide longitudinal bin stretching from 0.5 km to 2.5 km from the terminus. The boxes extend from the lower to upper quartile while the whiskers span the full range.
Center weather station in Tatitlek, Alaska (wcc.sc.egov.usda.gov/nwcc/site?sitenum=1076), 35 km southeast of the 2008 terminus. Temperatures were adjusted from the station elevation (15 m) to an average basin elevation (500 m) using the standard atmosphere lapse rate (6.49 K km\(^{-1}\)). The plotted discharge was measured at the U.S. Geological Survey gauging station on the Knik River (nwis.waterdata.usgs.gov/nwis/dv?site_no=15281000), which drains a glaciated catchment 120 km west of the 2008 terminus.

**Columbia Glacier, Alaska; Recent Ice Loss and Its Relationship to Seasonal Terminal Embayments, Thinning, and Glacial Flow** [211] found that calving rate at Columbia Glacier (in 1976 – 1978) was very strongly correlated with Knik River discharge, suggesting that although draining a different catchment, the Knik River is a reasonable proxy for the freshwater discharge at the glacier.

Figure 5.15 adds additional geometric context to the observed glacier speeds. The plotted height above buoyancy, ice thickness, and water depth were computed from the glacier bed elevations described in Section 5.3.4 and the glacier surface elevations listed in Table 5.1, with ice thickness computed explicitly as a function of flotation. In 2000 – 2004, Columbia Glacier retreated past a major constriction, grounding the terminus and slowing retreat. As the glacier continued to thin, flotation was first achieved upglacier of the terminus, above an overdeepening in the fjord. By 2007, the terminus itself retreated over the overdeepening and the glacier entered a phase of rapid retreat that lasted until the regrounding of the terminus at some point in 2010. The precise timings of these transitions are not precisely known, largely due to uncertainties in the glacier bed elevations. In the time-lapse photographs, widespread buoyant flexure is visible across the terminus as early as August 2007 and as late as June 2010, and bobbing of the terminus is plainly visible in 2008 – 2009, which supports the overall timing of flotation and grounding suggested by Figure 5.15. During the transition from floating to grounded, the glacier retreated to shallower water but thinned steadily, such that the terminus remained near flotation throughout the grounded period.

The transition from floating to grounded in 2010 corresponds to a shift in the overall flow dynamics of the glacier. The floating period (2008 – 2009) is distinguished by sustained high speeds throughout the year and erratic short-term variations seemingly uncorrelated with either
Figure 5.13a: Glacier speeds (m d$^{-1}$, left y-axis), averaged over 1 km wide longitudinal bins, plotted by distance from the terminus – 0 km to 2 km (red), 2 km to 4 km (brown), and 4 km to 6 km (grey) – and as the maximum from 0 km to 6 km (black). The lines are bin means and the shadings are the $\pm 3\sigma$ uncertainty of the means (mostly a function of spatial coverage in each bin). The colored bars are corresponding speeds from the velocity datasets listed in Table 5.3. Top orange lines are air temperature (°C, dark > 0°C, light < 0°C), bottom blue lines are precipitation (mm d$^{-1}$), and the tall cyan lines are discharge (from 0 m$^3$ s$^{-1}$ to 40 m$^3$ s$^{-1}$).
Figure 5.13b: continued
Figure 5.13c: continued
Figure 5.14: The 1 km wide centerline strip used for sampling (white band) overlaid on an aerometric orthophoto from 2008-08-11. Terminus positions (dashed white lines) are drawn for 2008-08-11 and 2015-11-17, bracketing the multiple-camera velocity record. The sampling bins – 0 km to 2 km (red), 2 km to 4 km (brown), and 4 km to 6 km (grey) from the terminus – are drawn from their downglacier start on 2008-08-11 to their upglacier end on 2015-11-17 to represent their full spread over 2008 – 2015.
Figure 5.15: Glacier height above buoyancy (m, top), ice thickness (m, middle), and water depth (m, bottom), averaged over 1 km wide longitudinal bins, plotted by distance from the terminus – 0 km to 2 km (red), 2 km to 4 km (brown), and 4 km to 6 km (grey). The lines are bin medians and the shadings span the full range of bin values.
warmer temperatures or rainfall. Although the large gaps in winter and spring make it difficult to reconstruct the seasonal variability during these years, the observations suggest that speeds peak in May – July, dip only slightly over the summer, then recover over the fall and winter. The drop in speeds over summer is very modest in 2008 (13.5 m d\(^{-1}\) to 10.5 m d\(^{-1}\), or 22\%, at 0 km to 2 km from the terminus) and 2009 (27\%), when the terminus is floating, and somewhat more distinct in 2010 (43\%) as the terminus retreats to shallower water. By 2011, the seasonal variability is much more pronounced, with summer slowdowns of 60\% or more, peaking in 2012 at 88\% (16.5 m d\(^{-1}\) to 2 m d\(^{-1}\)). During this latter period, speeds reach a minimum in October – November, then gradually recover over the winter, making up in roughly 8 months the speed lost in 4 months over the summer. Speedups earlier in the summer (May – July) tend to be correlated with periods of warmer temperatures, but this association weakens through the summer. The decoupling of speed with temperature is also evident on seasonal timescales: although average air temperatures peak in July – August, speeds begin to wane by May – June while temperatures are still increasing. Speedups later in the summer and fall tend instead to be correlated with intense rainfall, although these variations are more uncertain since visibility is poor during bad weather.

By winter, speeds are insensitive to both variations in temperature and precipitation, presumably because temperatures are too low to produce abundant melt or rainfall. However, any conclusions about short-term behavior during the winter are tenuous. Only one winter (2013 – 2014) is captured in full by multiple cameras and, as discussed in the previous section, uncertainties are higher in winter.

The computed correlations between changes in speed and changes in air temperature and precipitation, shown in Figure 5.16, support the seasonality suggested by the timing of the large speedups in Figure 5.13. Both temperature and precipitation are poorly correlated with glacier speed in winter and early spring, when freezing temperatures dominate. The correlation with temperature is strong at the start of the melt season, then gradually fades over summer and fall. In contrast, the correlation with precipitation rises gradually over summer, peaking in fall.
Figure 5.16: Correlation between glacier speed and (orange) air temperature and (blue) precipitation by 2-month period for all observations 2011 – 2015. The curves are the frequency distribution of correlation coefficients computed for each point in a 1 km wide longitudinal bin stretching from 0 km to 2 km from the terminus.
5.4.3 Strain rates

The strain rate fields are computed from the spatial derivatives of velocity [212] and are thus sensitive to noise, whether noise in the velocity measurements or noise in the glacier flow field. Therefore, we limit ourselves to only presenting strain rate fields with coherent features, good spatial coverage, and high certainty due to favorable camera geometry, clear images, and fast glacier flow.

The series of strain rate fields in Figure 5.17, from June – October 2012, illustrates the major patterns in behavior observed throughout the record. During this period, the glacier slowed from 15 m d$^{-1}$ to 2 m d$^{-1}$ while the overall position of the terminus remained fixed, resulting in continuous coverage near the terminus. The absolute orientations and relative magnitudes of the principal strains remain stable over the summer as the glacier slows, even on the 3 d resolution of our results. Near the terminus, the extensional principal strains are generally oriented perpendicular to the long-term average (in this case, concave) shape of the terminus. Short-term reorganization of the flow field is limited to a small region immediately upglacier of the terminus, presumably in response to the constant changes to the terminus geometry by calving (Figure 5.18). Furthermore, the magnitude and extent of this disturbance is largest earlier in the summer when speeds and calving rates are highest (Figure 5.19). By October, following a dramatic slowdown to 2 m d$^{-1}$, noise dominates the strain rate fields (Figure 5.17, bottom right). Strain rate fields for other very slow periods are similarly incoherent, but it is difficult to discern whether this is predominantly measurement noise or low organization of the flow field at velocity minimums.

During this period, the terminus was just downglacier of a bend formed below a glacier confluence (top right), made sharper by an accumulation of slow-moving ice against the valley wall to the northwest. The zones of compression below the confluence and along the outside margin of the bend are clearly visible in Figure 5.17. Early in the summer when speeds are at a maximum, the crevasses advecting from upglacier are not aligned with the dominant direction of flow and extension as they approach the terminus, resulting in these crevasses intersecting with the terminus
Figure 5.17: Principal strain rates (extension in yellow, compression in red, scale 10 000:1) overlaid on contemporaneous orthophotos derived from Worldview satellite images.
Figure 5.18: Mean glacier speed (m d\(^{-1}\), top left), mean principal strain rate (d\(^{-1}\), top right), and the 2-d variance in strain rate (d\(^{-2}\)) computed from the original 3 d and downsampled 18 d velocities, for 2012-06-05 (start of multi-camera observations) to 2012-09-14 (start of large rainstorms affecting data quality). The strain rate variance was estimated as \(\sqrt{\text{det} \Sigma}\), where \(\Sigma\) is the covariance matrix of the \(x\) and \(y\) components of the principal strain rate vectors [213].
Strain rate variance: 3-day ($d^{-2}$) 
2012-06-05 to 2012-07-25 

Strain rate variance: 3-day ($d^{-2}$) 
2012-07-25 to 2012-09-14 

Figure 5.19: The 2-d variance in strain ($d^{-2}$), per Figure 5.18, computed separately for 2012-06-05 to 2012-07-25 and 2012-07-25 to 2012-09-14.
at a sharp angle – whereas calving presumably occurred along crevasses formed parallel to the terminus (e.g. those visible in 2012-06-05). By 2012-10-12, the background crevasses make a larger turn towards facing the terminus, presumably because the compression into the bend decreased as the glacier slowed.

Figure 5.20 shows strain rate fields from other years with similar features. The flow field above the terminus tends to mirror the complex shape of the terminus but is highly disorganized in comparison to elsewhere on the glacier. As in 2012, the glacier makes a sharp bend above the terminus (2010-07-20), resulting in a misalignment between the background crevasse field and the calving margin.

5.5 Glaciological implications

Our multi-year high-frequency record of glacier velocities supports and extends the conclusions of Brinkerhoff [29] that were based on results for a single year (June 2013 – May 2014). Specifically, we find the strong seasonal and sub-seasonal velocity variations evident in Brinkerhoff [29] reproduced across all years from 2008 through 2015. Our results strongly support the conclusions, advanced by previous studies, that (1) fast glacier flow is dominated by basal sliding, (2) primarily driven by high basal water pressure countering ice overburden pressure, and thus (3) responsive to changes in water inputs and the relative efficiency of the subglacial drainage system [214, 215, 216]. In the following sections, we explain the overall observed seasonality in sliding speed as the evolution of the drainage system with respect to seasonal water availability, interpret the synoptic fluctuations relative to the evolving state of the drainage system, and discuss the interannual variability in terms of large changes to the terminus geometry.

5.5.1 Seasonal variability

The strong seasonality evident in our results is consistent with decades of observations at Columbia Glacier [217, 26, 218], as well as observations from other marine-terminating glaciers such as Hubbard in Alaska [219] and Helheim and Kangerlussuaq in Greenland [220]. The observations
Figure 5.20: Principal strain rates (extension in yellow, compression in red, scale 10 000:1) overlaid on contemporaneous orthophotos derived from aerometric and alaska-ifsar imagery.
support the following explanation, drawn from past work, based on the premise that water pressure at the bed is controlled by a delicate balance between the rate of water input to the bed (in the form of melt and rainfall) and the corresponding rate of expansion or contraction of the suglacial drainage system.

Persistent positive air temperatures in spring trigger the rapid melting of the snow accumulated over the winter. The rapid increase in water volume reaching the glacier bed overwhelms the drainage system, leading to high basal water pressures, low effective pressures, and fast sliding speeds. As the transmissivity of the drainage system increases to accommodate the additional water (through the increased size and connectivity of cavities and channels), ever more water is needed to maintain high pressures. Glacier speed begins to decline by early summer as meltwater production is outpaced by the increase in drainage efficiency, even as air temperatures continue to increase (Figure 5.13). Thus, maximum speed is strongly correlated not with the largest peak in Knik River discharge (here a proxy for the rate of water input), but with the first peak. Following the slowdown, even larger peaks in water input induce brief and relatively modest speedups, supporting the conclusion that a highly-connected drainage system has evolved to carry large amounts of water and is kept open by high water inputs through the summer.

By fall, water inputs decline as air temperatures drop, precipitation transitions from rain to snow, and a growing snowpack absorbs any rainfall. The water still reaching the bed is amply accommodated by the mature drainage system; as a consequence, water pressure and sliding speed reach a minimum. With frictional and turbulent heat flux at a minimum, creep closure [221, 222] finally overcomes melt and begins to destroy the efficient drainage system formed over the summer, promoting a gradual increase in water pressure and sliding speed over the winter, when the background basal melt is likely to dominate. The low speeds and strain rates during the fall may further help to reduce subglacial and englacial storage to a minimum [223, 224], further promoting sliding over the winter despite the low volumes of water present. Over winter and early spring, increasingly more water drives higher water pressure and sliding speed through an inefficient drainage system until a surge of meltwater in late spring once again drives the formation
of an efficient drainage system and a drop in speeds.

### 5.5.2 Synoptic variability

The relatively smooth seasonal cycle is overlaid with high-frequency, often large variations in sliding speed. As observed by previous studies at Columbia Glacier [178, 225, 210], these speedups are often correlated with surges of water inputs due to either increased temperatures (enhanced melt) or large rainstorms. Our multi-year high-frequency velocity record further reveals that the sensitivity of the glacier to each of these drivers varies seasonally. The correlation with temperature is strong at the start of the melt season, then fades over summer, while correlation with precipitation rises gradually over summer, peaking in fall (Figure 5.16).

The seasonal transition from melt to rainfall-dominated glacier speeds likely reflects the seasonal source of water reaching the bed of Columbia Glacier. At the start of the melt season, abundant snow covers the entire catchment. An increase in temperature drives enhanced melt over both glaciated and non-glaciated surfaces, while rainfall tends to be absorbed by snow and firn. As temperatures continue to rise, the correlation between temperature and water input begins to weaken, even though glacier ice melts faster than snow [226] and ice melt increases greater than linearly with increasing temperature [227]. Instead, the reduced importance of temperature is likely due to the decreasing melt area as the surface snow is removed from non-glaciated surfaces. The rainfall catchment, meanwhile, remains unchanged and, in the absence of surface snow, drains more quickly to the glacier bed. This pattern is apparent in the Knik River discharge (Figure 5.13): the declining baseline discharge from melt is punctuated by large spikes in discharge correlated with large rainstorms. The lower melt in the fall further enhances the correlation between precipitation and speed by lowering the baseline that a rainstorm must overcome to overwhelm the drainage system.

There are too few observations from which to draw conclusions about the glacier’s winter response to rainstorms or enhanced melt. Nevertheless, the series of speedups in December 2015, correlated with periods of positive temperatures, suggests that liquid water can induce faster speeds
at any time of year. Slight increases in temperature could significantly disrupt the ‘typical’ seasonal behavior of temperate glaciers, especially at the low-lying termini of tidewater glaciers like Columbia Glacier.

5.5.3 Interannual variability

While the overall seasonality in glacier speed described above is consistent across all years in the record, the magnitude and timing of the seasonal fluctuation varies considerably between years. Overall, the record spans two dynamically-distinct periods: a floating period (2008 – 2009), distinguished by sustained high speeds and low sensitivity to fluctuations in water input, and a grounded period (2011 – 2015), distinguished by large seasonal variations in speed and high sensitivity to fluctuations in water input (Figure 5.13). In the transition from floating to grounded, the glacier retreated to shallower water but thinned steadily, such that the terminus remained close to flotation throughout the grounded period (Figure 5.15). These differences in the geometry of the glacier terminus may help explain the large differences in glacier behavior.

First, sliding speed is controlled by the net pressure at the glacier bed, so speeds generally increase as glacier termini approach flotation. Near flotation, speeds are very sensitive to fluctuations in water pressure, since little additional water pressure is required to reach buoyancy. However, once the front achieves flotation, it is no longer sensitive to further water inputs. In this region, longitudinal stress gradients control ice speed [216]. The (very modest) summer slowdowns experienced by the floating front may be a result of longitudinal coupling to slowdowns experienced by the glacier above the grounding line. Second, thinner ice results in less ice overburden pressure, and thus slower rates of creep closure in the subglacial drainage system [221]. This may have favored the formation of a more efficient drainage system, and thus large summer slowdowns, during the grounded period. Third, a shallower grounding line decreases the baseline marine water pressure that must be overcome by the subglacial drainage system [27], further reducing the ability of the glacier to maintain high basal water pressures (and speeds) during the grounded period.

The interannual variability within each period is more difficult to explain. That said, it seems
plausible that the overall timing and rate of water input drives the variability in seasonal speedup and slowdown between years, at least when the glacier was grounded. The rapid onset of melt in 2011, 2012, and 2013 is correlated with pronounced spring maximum speeds, while an unusually large early-season rainstorm in 2015 coincides with the spring maximum, followed shortly by a large secondary peak as melt accelerates. Conversely, the slow onset of melt in 2014 may explain the absence of a prominent spring maximum (although it may have been missed by the May – June gap in coverage). Finally, as proposed by [29], large spikes in water input may briefly increase speeds but ultimately enhance summer slowdown. The largest surges in water input (all caused by large rainstorms) occur in 2012 – 2014, the years with the lowest fall minimum speeds.

5.5.4 Strain rate evolution

Finally, our results provide rare insight into the short term fluctuations of the two-dimensional flow field. Most high-frequency observations of glacier motion are sparse point measurements collected from optical or GPS surveying of markers, or autonomous GPS rovers, placed on the ice. Terrestrial radar interferometry [reviewed by 228] has emerged as a powerful method for measuring glacier speeds at very high frequencies. However, similar (but opposite) to optical cameras, interferometry can only resolve displacements along the line of sight (LOS). Two-dimensional velocity fields required either two radars operating simultaneously or projecting LOS displacements onto average flow orientations determined from vertical imagery or speckle tracking of the radar intensity images. Furthermore, these instruments remain large, expensive, and power-hungry, and glacier deployments have so far been limited to a few weeks [210, 229, 230]. For now, arrays of two or more time-lapse cameras are unique in providing high-frequency flow fields over extended periods, allowing us to investigate strain rates at different temporal scales.

We find that short-term reorganization of the flow field is narrowly constrained to the narrow region immediately upglacier of the terminus. The magnitude and extent of the disturbance is largest when speeds and calving rates are highest, suggesting that these fluctuations are a response to the constant changes to the terminus geometry by calving. Immediately upglacier, strain rates
tend to mirror the average shape of the terminus, but elsewhere, the flow field is remarkably stable over long time scales, suggesting that the dynamic changes at the calving front only propagate a small distance upglacier and are quickly overwhelmed by persistent features in the flow field regulated by the glacier bed and valley walls.

5.6 Methodological improvements

5.6.1 Planning future camera deployments

Although there are a number of ways the computational methods can be improved, the position and number of camera stations cannot be stressed enough. Even two cameras, if separated by a large angle, are significantly more successful at resolving a time-varying two-dimensional flow field than a single camera. Furthermore, advanced planning is important to achieve good spatial coverage from oblique cameras; many of our results suffer from holes in coverage Figures 5.8 and 5.9, areas of the glacier not visible from oblique angles due to undulations in the surface. Since the motion model and particle state are all described in spatial coordinates, the method easily lends itself to realistic simulations. We suggest establishing a testing framework in which input elevations and velocities drive the evolution of orthorectified image tiles projected into camera models to produce synthetic images with which to test the tracking algorithm. This would provide a means to carry out realistic sensitivity tests and a means to identify optimal camera configurations (e.g. camera position, resolution, and focal length) for future time-lapse deployments.

5.6.2 Improving single-camera results

As discussed in Section 5.4.1, a single oblique camera has difficulty uniquely resolving both horizontal components of motion, even when the vertical component is prescribed by elevation observations. However, as Figure 5.7 demonstrates, flow direction tends to be stable over time even though the magnitude of flow is highly variable. By expressing particle velocities and accelerations in cylindrical coordinates, flow direction could be constrained independently of flow magnitude,
and thus more tightly than is possible with cartesian coordinates [29]. This has the potential to greatly improve the results that can be extracted from the many existing single-camera time-lapse sequences from Columbia Glacier and elsewhere.

### 5.6.3 Template image selection

The algorithm’s performance is limited by the quality of the image used as a template for tracking. Forward and reverse runs provide protection from weak templates Section 5.2.7, but only if either the first or last image is of sufficient quality. Although images with poor detail over land areas are filtered out in the camera orientation step (Section 4.5), quality can vary across the images from localized effects like lens flare, fog, and snow crystals or water droplets suspended in the atmosphere or adhered to the viewing window. This problem could be addressed by evaluating local image quality automatically and building image bins on a per-point basis (although at the cost of a different tracking interval for each point). For example, the region of each image centered on the projection of the point could be described by the magnitude of local image gradients or descriptors derived from grey level co-occurrence matrices (GLCM) [231] and assigned a goodness likelihood based on a binary classifier (e.g. logit regression) trained on a small training set of ideal glacier image tiles.

The use of a constant-size, axis-aligned image template for tracking assumes that the feature, as represented on the image plane, experiences negligible rotation or scaling. As a precaution, images were binned such that this assumption was not violated by a lens change or large camera rotation during tracking. Tracking over short time periods further minimizes the deformation the surface patch can experience by its own motion – namely, moving towards or away from the camera, moving from the edge to the center of the image frame, and rotating in space. At the expense of computational complexity, these cases could be addressed by using scale- and rotational-invariant template matching methods [232, 233]. Alternatively, the extensive knowledge of the camera-surface geometry in our case could be employed to transform the template over time to better match the evolving image projection.
5.6.4 Extending tracking to the terminus

We chose not to track points between the terminus positions of consecutive elevation observations since the elevations in these regions could not be estimated by linear interpolation (Section 5.2.3). This results in gaps in coverage close to the calving front (e.g. Figure 5.9), despite the fact that high-frequency oblique images are well-suited for tracking features in this region. Short of modeling the 2-d evolution of the glacier surface, elevations and their uncertainties could be estimated by extrapolating the two nearest elevation observations (preceding the retreat or proceeding the advance) or propagating the elevation changes observed immediately upglacier of the uppermost terminus and increasing the uncertainty accordingly.

Tracking features right up to the terminus also requires high-frequency observations of the terminus position, since otherwise motion of the water surface may be mistaken for glacier motion. We used terminus positions traced manually from all available aerial and satellite imagery (Section 5.3.6), but these are insufficient for tracking the fluctuations in front position visible in the time-lapse photographs. Local texture analysis is a promising direction for inferring terminus position from the photographs themselves. This tedious task has so far been performed manually, whether by the researchers [234, 31] or by members of the public (Ethan Welty, unpublished). Edge detection and linking have been tried with some success in clear water conditions when the glacier terminus and the water surface form a distinctive edge (Timothy Bartholomaus, personal communication) but fail in the presence of reflections, ice mélange, or other visual clutter. A joint texture, color, and motion technique with memory of previous results has shown promising results in distinguishing the glacier surface even in the challenging situation when ice mélange fills the forebay (Nathan Jacobs, personal communication).

5.6.5 Higher frequency observations

Our results were computed from 3 d tracks of 2 h photographs (up to 36 photographs per camera station). We found that 3 d averages were a good compromise between high detail and
high uncertainty on the one hand, and low detail and low uncertainty on the other (thanks to a larger number of images and larger glacier displacement). However, at this scale, we miss the response to individual calving events, tidal (semidiurnal) and solar (diurnal) forcings, and smooth over the response to rainstorms and other short-lived meteorological phenomena – begging the question whether robust higher-frequency results can be extracted from the photographs. One approach could be to use more frequent images within shorter tracker intervals; if the apparent glacier motion between consecutive images (the signal) is still larger than the pixel size, residual camera motion, and atmospheric refraction (the noise), the denser sampling of images may impose additional constraints to offset the smaller glacier motion being measured. Another approach would be to include acceleration in the particle state (rather than apply random accelerations to the particle velocities) to estimate both velocity and its first derivative so that they can be used simultaneously to estimate the shape of the velocity function over time (e.g. Hermite interpolation).

5.7 Conclusions

The novel probabilistic feature-tracking algorithm of [29] is expanded to handle large uncertainties in glacier surface elevations, and complimented by automatic generation of the model priors, tracking points, and tracking intervals, forming a flexible and scalable pipeline for deploying the algorithm on a wide range of glacier-camera configurations. The method is applied to 32,000 Columbia Glacier time-lapse photographs, spanning 13 years and 15 camera positions, to produce glacier velocity fields, their associated uncertainties, and corresponding strain rate fields at 3-d intervals. When temporally downsampled, the results derived from two or more time-lapse cameras agree very closely with contemporaneous reference (airborne and satellite derived) velocity fields, while also capturing significant short-term variability not visible in the longer-baseline reference velocities. In contrast, the results derived from only one camera rarely improve on the initial guess, stressing the need for multiple cameras in future time-lapse campaigns.

The multiple-camera results (2008 – 2015) resolve the motion of Columbia Glacier on an unprecedented level of breadth and detail, providing spatially detailed insight into sub-seasonal,
seasonal, and interannual time scales simultaneously. On seasonal scales, the glacier accelerates slowly over the winter to a spring maximum, then decelerates rapidly to a fall minimum. Our results agree with previous findings, and further support the hypothesis that fast glacier flow is (1) primarily driven by high basal water pressure approaching the ice overburden pressure and (2) that the water pressure is modulated by the balance between the rate of water reaching the bed and the rate of expansion or closure (and corresponding efficiency or inefficiency) of the suglacial drainage system. The smooth seasonal cycle is overlaid by large high-frequency fluctuations in speed. As observed by previous studies, these speedups are often correlated with increased input rate of liquid water due to either increased air temperatures (enhanced melt) or precipitation. Our results reveal that the sensitivity of the glacier to these meteorological drivers varies seasonally, transitioning from melt to rainfall-dominated water inputs over the course of the summer, which we hypothesize to be driven by a transition from snowmelt to rainfall-dominated discharge in the glacier catchment. Between years, sharp accelerations in discharge at the start of the melt season are correlated with prominent spring maximuma (2011 – 2013), while a slow onset of melt may conversely lead to the absence of a prominent spring peak (2014). As suggested by [29, 235], large pulses of water may briefly increase speeds but ultimately enhance the summer slowdown. The largest surges in water input (all caused by large late-summer rainstorms) occur in 2012 – 2014, the years with the lowest fall minimuma. Taken together, the observations strongly support the hypothesis that on seasonal and sub-seasonal scales, glacier flow is largely controlled by fluctuations in the timing and rate of water reaching the bed.

On longer timescales, the record spans two dynamically distinct periods: a thin grounded glacier terminating in shallow water (2011 – 2015), distinguished by large seasonal variations in speed and high sensitivity to fluctuations in water input, and a thick floating glacier terminating in deeper water (2008 – 2010), distinguished by year-round high speeds and low sensitivity to fluctuations in water input to the bed. We hypothesize that the change in dynamics was primarily driven by changes to the glacier geometry. During the preceding fast floating period, (1) the flotation at the terminus and low height above buoyancy further upglacier increased the sensitivity of the
glacier to elevated water pressures at the bed, (2) the thicker ice increased the rate of creep closure, discouraging the formation of an efficient drainage system, and (3) the deeper water increased the baseline marine water pressure which had to be overcome by the subglacial drainage system – all of which would maintain low effective pressures and thus fast and sustained basal sliding.

Finally, our results provide rare insight into the short-term fluctuations of the two-dimensional glacier flow field. We find that short-term reorganization of the flow field is narrowly constrained to a region of a few hundred meters upglacier of the terminus, suggesting that these fluctuations are a response to the constant changes to the terminus geometry by calving. Elsewhere, the flow field is remarkably stable over long time scales, suggesting that the dynamic changes at the calving front only propagate a small distance upglacier and are quickly overwhelmed by persistent features in the flow field regulated by the glacier bed and valley walls.

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Chapter 6

Conclusions

Ground-based cameras are simple and cost-effective instruments which can collect large amounts of information in even extreme environments with relatively minimal effort or training. Furthermore, they are particularly well suited for high-frequency and high-resolution observations, oblique perspectives, and uninterrupted observations under cloud cover or near-darkness, precisely situations in which satellite remote sensing is still inadequate. Furthermore, they can deliver spatiotemporal output comparable in form to that of commercial aerial and satellite imagery, and of equal or better precision. However, ground-based systems require specialized techniques and careful management of systematic and random errors, and as a result, the quantitative potential of terrestrial photogrammetry in this context has not been fully explored. The 13-year time-lapse program at Columbia Glacier suffered from mundane but severe errors: many camera clocks were set to wrong dates or unknown timezones, most camera positions were never surveyed, none of the cameras were calibrated, and the cameras, while firmly mounted, rotated from image to image. The potential value of these photographs for glaciological research drove us to develop methods to achieve high spatial and temporal precision despite these challenging conditions.

In Chapter 3, we tackled the problem of uncertain or unknown timestamps. We recovered the year, date, and time of photographs by leveraging visible geophysical phenomena of known time, from the positions of astronomical objects in the sky to the corresponding variations in solar radiation and sea level. We found that precision on the order of seconds was achievable from even a single photograph of astronomical objects, while in their absence, sequences of photographs could
nevertheless be calibrated, on the order of an hour, to variations in solar radiation or sea level visible at the surface. The periodicity of these natural clocks results in cyclical ambiguities which can be resolved by combining processes with different periods. For example, the length of daylight constrains the month of year while the timing of lunar tides constrains the day of the month. The increasing frequency of satellite imagery suggests that these could also be used in the future to help date terrestrial photographs.

In Chapter 2, we turned to enhancing the timekeeping abilities of digital cameras. Subsecond-precision image capture times were achieved by measuring the offset to a reference clock display and accounting for the drift, precision, and reporting resolution of the camera clock. We demonstrated their utility in two contemporary geophysical investigations: georeferencing aerial photogrammetric surveys with camera positions time-interpolated from GPS tracklogs, and coupling videos of glacier-calving events to synchronous seismic waveforms. Since 2009, Columbia Glacier time-lapse cameras have been kept calibrated to within seconds of Coordinated Universal Time (UTC) by photographing Global Positioning System (GPS) and mobile phone clock displays during service visits, negating the need for the complex forensics of Chapter 3.

In Chapter 4, we addressed the challenge of estimating the position, orientation, and optical properties of static cameras retroactively from one or more of their photographs. Registering photographs to an absolute spatial reference frame was achieved by solving for the camera parameters that minimized the reprojection errors of visible surface and topographic features of known position. We extended conventional ground control points (e.g. mountain summits) to arbitrary line features (e.g. mountain horizons) to achieve higher levels of image coverage. We extended pairwise image matches, traditionally used to calibrate rotating cameras, to matches with synthetic terrestrial images produced by projecting aerial imagery (draped over elevation data) into imaginary cameras. Since the synthetic images map to known world coordinates, the matched points are an automatable equivalent of manually collected ground control. We used these various controls to calibrate 14 Columbia Glacier time-lapse cameras, and confirmed the success of our method by the excellent alignment between the original photographs and synthetic images generated from contemporane-
ous vertical imagery. Finally, to correct for camera motion, we developed a novel moving-window global optimization to estimate the orientation of the camera for each photograph in an arbitrarily long sequence. Our method successfully stabilized 33,000 selected Columbia Glacier time-lapse photographs spanning 15 camera positions over 13 years.

Finally, in Chapter 5, we deployed a probabilistic feature-tracking algorithm to the calibrated Columbia Glacier time-lapse photographs to produce glacier velocity fields, their associated uncertainties, and corresponding strain rate fields at 3 days intervals over 13 years, producing an unprecedented high-frequency, long-term record of glacier dynamics. We found that Columbia Glacier exhibits strong variations in flow speed at synoptic, seasonal, and interannual time scales, all of which are consistent with the hypothesis that fast glacier flow is (1) primarily driven by high basal water pressure approaching the ice overburden pressure and (2) that the water pressure is modulated by the balance between the rate of water reaching the bed and the rate of expansion or closure (and corresponding efficiency or inefficiency) of the suglacial drainage system. When temporally downsampled, the results derived from two or more time-lapse cameras agree very closely with contemporaneous aerial (airborne and satellite derived) velocity fields, while also capturing significant short-term variability not visible in the longer-baseline aerial velocities. In contrast, the results derived from only one camera rarely improved on the initial guess, stressing the need for multiple cameras in future time-lapse campaigns.

This dissertation offers insights into the capabilities and shortcomings of consumer-grade digital cameras as scientific instruments, the opportunistic approaches often needed to achieve the best results, and the potential of continuous high-frequency measurements for documenting rapid geomorphic change. For the Columbia Glacier time-lapse photographs, the chapters trace a methodical journey from futility to utility, a journey that other poorly-documented photographs can now follow. Even within glaciology, many other large time-lapse sequences remain unexplored, for example those by other research groups in Greenland (Andreas Vieli, personal communication) and Svalbard (Jacek Jania, personal communication) and those from other Extreme Ice Survey (extremeicesurvey.org) cameras in North America, Europe, Greenland, and Antarctica. The meth-
ods may even translate to space, where a growing constellation of miniaturized satellites equipped with consumer-grade camera modules are producing sequences of photographs with opportunities and challenges reminiscent of those from terrestrial cameras: much higher frequencies and, in some cases, also higher uncertainties [236, 237].

To maximize access to the methods of this dissertation, they have been implemented as a Python package called **glimpse** (Glacier Image Particle Sequencer), to be published on GitHub upon journal publication (and available by request until then). Although originally designed for rapid and precise analysis of glacier time-lapse photographs, it can be used with any outdoor photographic sequences. Some of the features for spatial camera calibration include:

- Model a distorted camera and perform incoming and outgoing ray projections, accounting for earth curvature, atmospheric refraction, and uncertainties in camera parameters.

- Convert between different camera model formats, including MATLAB, Agisoft PhotoScan, and EOS Systems PhotoModeler, accounting for uncertainties in camera parameters.

- Calibrate single or multiple cameras simultaneously – controlling which camera parameters are held fixed, varied, or synced across groups of cameras – using points and polylines in image and world coordinates.

- Load manual vector image annotations and spatial data as inputs for camera calibration.

- Compute the visible horizon for a position from elevation data as input for camera calibration.

- Generate photorealistic synthetic images (and depth maps) from a camera model, elevations, and orthorectified imagery for automated control point collection and camera model validation.

Highly optimized and parallelized implementations of the camera stabilization algorithm of Chapter 4 and the tracking algorithm of Chapter 5 are at the center of an efficient motion-extraction
pipeline which can be deployed on a wide range of camera-surface configurations. These features include:

- Stabilize external orientations of photographic sequences of arbitrary length using automatic feature matching and globally-optimal orientation estimates.

- Temporally interpolate between repeat elevation observations, propagating their uncertainties accordingly.

- Compute camera viewsheds to inform tracking point selection.

- Compute the velocities (and corresponding uncertainties) of points visible in sequences of photographs taken from one or more camera positions, using a predefined or custom particle motion model.

The temporal calibration methods have not yet been incorporated into \textit{glimpse}. Scripts implementing the subsecond-precision methods of Chapter 2 are however available at igsoc.org/hyperlink/12j126/12j126sect6.

However successful, the forensics of Chapters 3 and 4 are labor-intensive and should be regarded as last resorts for poorly-documented past photographs. We hope that future photographs will be acquired in a manner more conducive to scientific analysis. For example, all cameras at a study site should be set to the correct date, the same and documented timezone (or UTC), and optionally kept calibrated to absolute time by repeatedly photographing a reference clock display. Cameras should be calibrated under controlled conditions before they arrive in the field in case the camera is lost or broken and the need for analysis later arises. Similarly, the positions of mounted cameras should always be surveyed directly so that only the external orientation of the camera needs to be estimated from landscape controls. Finally, and critically, the resulting digital image file should be transferred and stored in such a way that the embedded camera metadata is kept intact.
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Appendix A

Subsecond capability of digital camera models

In a broad survey of 1693 Apple, Blackberry, Canon, Fujifilm, HTC, Kodak, Leica, Motorola, Nikon, Nokia, Olympus, Panasonic, Pentax, Ricoh, Samsung, Sigma, and Sony camera, camcorder, and cameraphone models – most of the models still in active use today (as measured by the number of daily users on Flickr) – only select Nikon DSLR (Table A.1), Canon DSLR (Table A.2), Kodak EasyShare compact (Table A.3), and Nokia phone cameras were found to report a subsecond component of capture time. Tables A.1 to A.3 list subsecond resolution, when applicable, alongside maximum still and video framerates. Table A.4 lists the DSLR and interchangeable lens cameras, from brands other than Nikon or Canon, confirmed to lack subsecond reporting.
Table A.1: Subsecond capability of Nikon DSLR camera models. Models are sorted by series (from professional to entry-level), then by release year. Subsecond resolution types reference the panels of Figure 2.2.

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Table A.2: Subsecond capability of Canon DSLR camera models. Cameras are sorted by series (from professional to entry-level), then by release year. Subsecond resolution types reference the panels of Figure 2.2.

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Table A.3: Subsecond capability of many recent (2005-present) M, V, and Z-series Kodak EasyShare compact camera models. Cameras are sorted by series, then by release year. Subsecond resolution types, when sufficient image samples were available, reference the panels of Figure 2.2. All listed cameras are capable of recording video at a minimum framerate of 24 frames/s, but some may be capable of higher speeds.

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Table A.4: DSLR and interchangeable lens camera models (excluding Canon and Nikon) confirmed to not write to the SubSecTimeOriginal tag. Only models from the listed makers were evaluated, and all models by the listed makers were found to lack subsecond reporting, with the exception of the Sony SLT-A55 ‘V’ version (with integrated GPS) which writes with subsecond resolution to the GPSTimeStamp tag.

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