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Dielectric Image Line Coupling

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Abstract

The field distributions and the propagation constants of the TE modes of single and coupled dielectric image slab lines are derived. The coupling of modes in two parallel dielectric waveguides is studied and found to confirm results of previous work. The material is used to derive the coupling length for a 3 dB coupler.

I. INTRODUCTION

Coupling between adjacent dielectric image lines is important for assessing a) parasitic crosstalk, b) the coupling length of coupler elements.

An initial configuration of rectangular cross-section dielectric between parallel plates is investigated. By examining the propagation in single lines, and in a line pair fed both symmetrically and anti-symmetrically, the coupling in infinite parallel lines can be deduced. From this the coupling in a finite length can be found, apart from end effects which will be the subject of a further study.

An analysis of this character is not new but is redeveloped here as the first stage in a more refined calculation of the performance of actual couplers. The formulas agree with earlier results of Wilson and Reinhart,(1) Knox and Toulios,(2) and Kuester and Chang.(3) They are used to correct an erroneous formula appearing in a January 1979 contract proposal document.

II. SINGLE DIELECTRIC IMAGE LINE

Figure 1 shows a dielectric image guide which consists of a rectangular core of dielectric constant ε_r inserted between two parallel perfectly conducting planes.

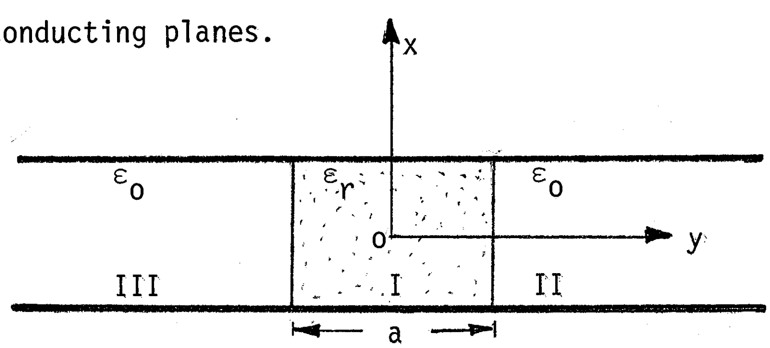

Fig. 1

It is assumed that within the guiding rod each field component varies sinusoidally with respect to y. Electric field constancy along x is assumed; $\frac{\partial}{\partial x} = 0$. The t and z dependence is assumed to be $\exp[j(\omega t - \beta z)]$ and is suppressed when not required.

For the calculation of the EM fields the cross-section is subdivided into three partial regions (I-III) and a complete set of fields is derived for each subarea. This type of structure can support two kinds of modes, namely symmetrical (even in y) and antisymmetrical (odd in y) modes. In the present analysis, only the TE even modes will be evaluated.

Building an expression for the magnetic Hertzian potential $\bar{\Pi}_m$, so that it satisfies the Vector Helmholtz equation, and using the two relations

$$\bar{H} = \nabla \times \nabla \times \bar{\Pi}_m$$

and

$$E = -j\omega\mu\nabla \times \bar{\Pi}_m$$

the field distribution can be calculated. The following field potential functions and fields are found:

Region I (Inside the rod)

* Magnetic Hertzian Vector potential:

$$\bar{\Pi}_m = \bar{a}_z \frac{j}{\omega\mu_o p_o} A \sin(p_o y) e^{-j\beta_o z}$$

* Field distributions:

$$E_x = A \cos(p_o y)\, e^{-j\beta_o z}$$

$$H_y = \frac{\beta_o}{\omega\mu_o} A \cos(p_o y) e^{-j\beta_o z} \qquad (1a)$$

$$H_z = \frac{jp_o}{\omega\mu_o} A \sin(p_o y) e^{-j\beta_o z}$$

where

$$p_o^2 = k^2 - \beta_o^2 = k_o^2\varepsilon_r - \beta_o^2 \tag{1b}$$

Regions II and III (Outside the rod)

* Magnetic Hertzian vector potential:

$$\bar{\Pi}_m = \bar{a}_z \frac{A}{j\omega\mu_o h_o} \cos(p_o \frac{a}{2}) e^{-h_o[|y|-a/2]} e^{-j\beta_o z}$$

* Field distributions:

$$E_x = A \cos(\frac{p_o a}{2}) e^{-h_o[|y|-a/2]} e^{-j\beta_o z}$$

$$H_y = \frac{\beta_o}{\omega\mu_o} \cos(p_o \frac{a}{2})\, e^{-h_o[|y|-a/2]} e^{-j\beta_o z} \tag{2a}$$

$$H_z = \mathrm{sgn}(y) \frac{jh_o}{\omega\mu_o} A \cos(p_o \frac{a}{2}) e^{-h_o[|y|-a/2]} e^{-j\beta_o z}$$

where

$$h_o^2 = \beta_o^2 - k_o^2 \tag{2b}$$

The transverse propagation constants h_o and p_o are related by

$$h_o^2 + p_o^2 = k_o^2\,(\varepsilon_r - 1) \tag{3a}$$

and

$$\cot(p_o \frac{a}{2}) = \frac{p_o}{h_o} \tag{3b}$$

Equation (3b) comes upon matching the tangential components at the boundaries of Region I.

III. TWO PARALLEL DIELECTRIC IMAGE LINES

Figure 2 shows two coupled dielectric image lines inserted between two parallel perfectly conducting ground planes. All assumptions used in the previous section, relating t,x,y, and z dependence, the cross section, and the linearity of the medium hold here.

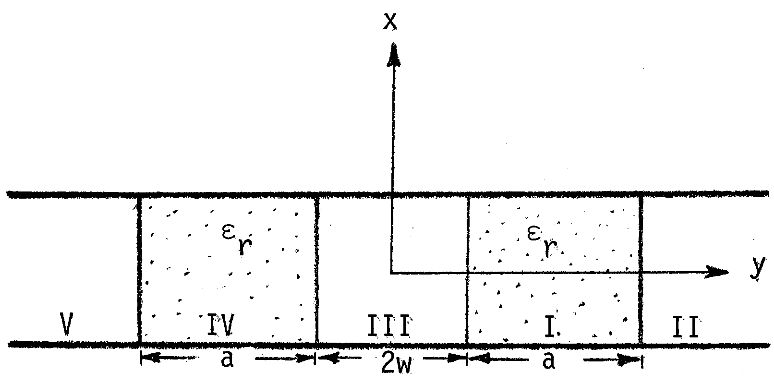

Fig. 2

For the calculation of the field distributions, the field region is subdivided into five partial regions (I-V).

Figure 3 shows the field distribution of the symmetrical and anti-symmetrical modes.

We shall examine in detail the symmetrical case.

Region I (Inside rod no. 1)

* Magnetic Hertzian vector potential:

$$\bar{\Pi}_m = \bar{a}_z \, [A \cos(py) + B \sin(py)] \, e^{-j\beta z}$$

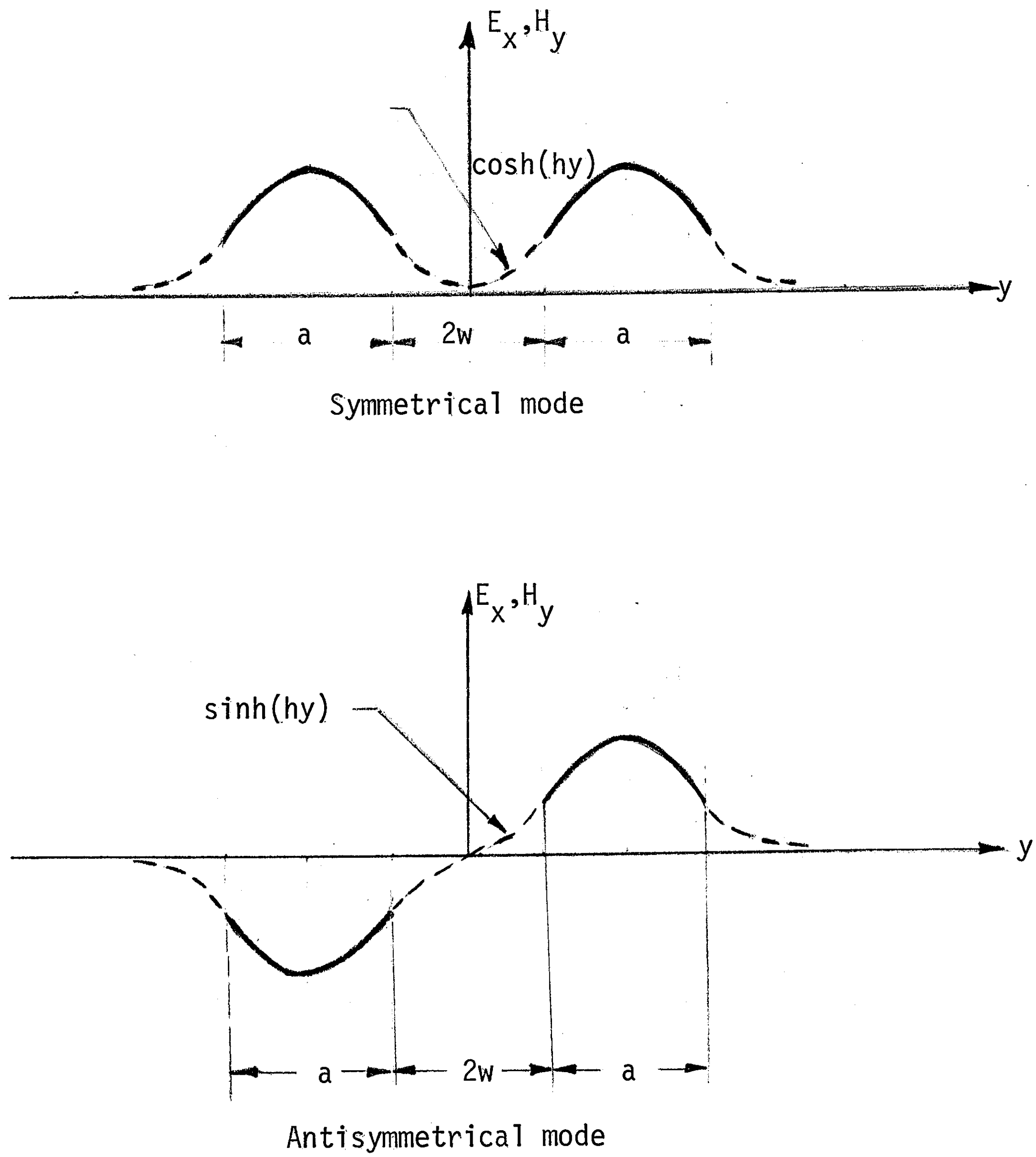
E_x, H_y
cosh(hy)
y
a
2w
a
Symmetrical mode
E_x, H_y
sinh(hy)
y
a
2w
a
Antisymmetrical mode

Fig. 3

* Field distribution

$$E_x = j\omega\mu p[A \sin(py) - B \cos(py)]e^{-j\beta z}$$

$$H_y = j\beta p[A \sin(py) - B \cos(py)]e^{-j\beta z} \qquad (4a)$$

$$H_z = p^2[A \cos(py) + B \sin(py)]e^{-j\beta z}$$

where

$$p^2 = k_o^2\varepsilon_r - \beta^2 \qquad (4b)$$

<u>Region II</u>. $[y > w + a]$

* Magnetic Hertzian vector potential

$$\bar{\Pi}_m = \bar{a}_z \; C \; e^{-hy} \; e^{-j\beta z}$$

* Field distribution

$$E_x = j\omega\mu_o h \; C \; e^{-hy} \; e^{-j\beta z}$$

$$H_y = j\beta h \; C \; e^{-hy} \; e^{-j\beta z} \qquad (5a)$$

$$H_z = - \; h^2 C \; e^{-hy} \; e^{-j\beta z}$$

where

$$h^2 = \beta^2 - k_o^2 \qquad (5b)$$

From (4b) and (5b), and by addition, we have

$$p^2 + h^2 = k_o^2 \; (\varepsilon_r - 1) \qquad (5c)$$

Region III

* Magnetic Hertzian potential

$$\bar{\Pi}_m = \bar{a}_z E \sinh(hy)\, e^{-j\beta z}$$

* Field distribution

$$E_x = -j\omega\mu_o hE \cosh(hy)\, e^{-j\beta z}$$

$$H_y = -j\beta hE \cosh(hy)\, e^{-j\beta z} \tag{6}$$

$$H_z = -h^2 E \sinh(hy)\, e^{-j\beta z}$$

where as before

$$h^2 = \beta^2 - k_o^2 .$$

Field distribution for regions IV and V will not be demonstrated here because it is exactly identical to that of Region I and II respectively except of the arbitrary constant coefficients and changing y to -y in Region V.

The coefficients C and E of Regions II and III are given in terms of A and B.

$$C = \frac{p}{h}\left\{A \sin[p(w+a)] - B \cos[p(w+a)]\right\} e^{h(w+a)}$$

and

$$E = -\frac{p}{h} \cdot \frac{1}{\cosh(hw)} \cdot \left\{A \sin(pw) - B \cos(pw)\right\}$$

By matching the tangential field components E_x and H_z at y = w and = w + a, we obtain

$$\tanh(hw) = \frac{p}{h} \cdot \frac{A \cos(pw) + B \sin(pw)}{A \sin(pw) - B \cos(pw)} \tag{7}$$

and

$$-\frac{h}{p} = \frac{A\cos[p(w+a)] + B\sin[p(w+a)]}{A\sin[p(w+a)] - B\cos[p(w+a)]} \tag{8}$$

Using (8) to substitute for $\frac{B}{A}$ in (7) we have

$$\frac{h}{p}\tanh(hw) = \frac{p\sin(pa) - h\cos(pa)}{p\cos(pa) + h\sin(pa)}$$

which can be written in the form

$$\frac{h}{p}\tanh(hw) = \tan[pa - \tan^{-1}(\frac{h}{p})] \tag{9}$$

If each of the two dielectric waveguides (in isolation) has a mode with the same propagation constant β_o, then when the two waveguides are placed parallel, as in Fig. 2, the double waveguide system supports two new modes whose propagation constants are $\beta_+ = \beta_o + \Delta\beta$ and $\beta_- = \beta_o - \Delta\beta$. These two modes are symmetric and anti-symmetric combinations of approximately the original modes in the isolated waveguides. The shift in propagation constant, $\Delta\beta$, is related to the beat length L and will now be calculated.

From equation (9) we have

$$\begin{aligned} pa &= \tan^{-1}(\frac{h}{p}) + \tan^{-1}[\frac{h}{p}\tanh(hw)] \\ &= 2\tan^{-1}(\frac{h}{p}) + \tan^{-1}\left\{\frac{\frac{h}{p}[\tanh(hw)-1]}{1 + \frac{h^2}{p^2}\tanh(hw)}\right\} \end{aligned}$$

For large values of hw, which is a restriction on the separation between the two waveguides, we have

$$pa \simeq 2\tan^{-1}\left(\frac{h}{p}\right) + \tan^{-1}\left\{\frac{-2\,\frac{h}{p}\,e^{-2hw}}{1 + \frac{h^2}{p^2}}\right\}$$

$$\simeq 2\tan^{-1}\left(\frac{h}{p}\right) - \frac{h}{p}\,\frac{2e^{-2hw}}{1 + \frac{h^2}{p^2}} \qquad (10)$$

Because of coupling, the propagation constants in the transverse plane will be affected too, as follows:

$$\begin{aligned} p &= p_o(1 + \delta) \\ h &= h_o(1 + \Delta) \end{aligned} \qquad (11)$$

By squaring and adding we have

$$p^2 + h^2 = p_o^2(1 + \delta)^2 + h_o^2(1 + \Delta)^2$$

$$\simeq p_o^2 + h_o^2 + 2(p_o^2\delta + h_o^2\,\Delta)$$

According to eqns. (3-a) and (5-c) we should have

$$p_o^2\delta + h_o^2\Delta = 0$$

that is

$$\Delta = -\left(\frac{p_o}{h_o}\right)^2\delta \qquad (12)$$

By means of (11), (10) becomes

$$p_o a(1+\delta) \simeq 2\tan^{-1}\frac{h_o}{p_o}(1+\Delta-\delta) - \frac{h_o}{p_o}\,\frac{2\,e^{-2h_o w}}{1+h_o^2/p_o^2} \tag{13}$$

By using Taylor's expansion of the inverse tangent term, eq. (13) becomes:

$$p_o a(1+\delta) \simeq p_o a + 2\,\frac{h_o}{p_o}\,\frac{(\Delta-\delta)}{1+h_o^2/p_o^2} - \frac{h_o}{p_o}\cdot\frac{2\,e^{-2h_o w}}{1+h_o^2/p_o^2} \tag{14}$$

Thus by eqs. (12) and (14)

$$\delta = \delta_s = -\frac{h_o\,e^{-2h_o w}}{k_o^2(\varepsilon_r-1)(\frac{a}{2}+1/h_o)} \tag{15}$$

Equation (15) is restricted to the symmetrical case. By exactly the same procedure, for the antisymmetrical case, we find

$$\delta_a = \frac{h_o\,e^{-2h_o w}}{k_o^2(\varepsilon_r-1)(\frac{a}{2}+1/h_o)} = -\delta_s \tag{16}$$

Now, for the symmetrical case we had

$$\beta_+ = \beta_s = [k^2 - p_s^2]^{\frac{1}{2}}$$

By means of (11), it becomes

$$\beta_s \simeq \sqrt{k^2 - p_o^2}\cdot\left[1 - \frac{\delta_s p_o^2}{k^2 - p_o^2}\right]$$

or

$$\beta_s \simeq \beta_o\left[1 - \frac{\delta_s p_o^2}{\beta_o^2}\right] \tag{17}$$

The antisymmetrical case has the same form, thus

$$\beta_- = \beta_a = \beta_o \left[1 - \frac{\delta_a p_o^2}{\beta_o^2} \right] \tag{18}$$

By means of (17) and (18)

$$\Delta\beta = \frac{\beta_+ - \beta_-}{2} = \frac{\beta_s - \beta_a}{2}$$

and thus

$$\Delta\beta = \frac{p_o^2 h_o \, e^{-2h_o w}}{\beta_o k_o^2 (\varepsilon_r - 1)(\frac{a}{2} + \frac{1}{h_o})} \tag{19}$$

III. COUPLING

If at a pont $z = 0$ the field is represented by a value E_o in one line and 0 in the other, this can be decomposed into

$$E_1 = \frac{1}{2} E_o + \frac{1}{2} E_o = E_o$$
$$E_2 = \frac{1}{2} E_o - \frac{1}{2} E_o = 0 \tag{20}$$

that is, a symmetrical field of amplitude $\frac{1}{2} E_o$ and an antisymmetrical field of the same amplitude. After traversing a distance z along the line, the amplitudes become

$$E_1 = \frac{1}{2} E_o \, e^{-j\beta_+ z} + \frac{1}{2} E_o e^{-j\beta_- z}$$
$$E_2 = \frac{1}{2} E_o \, e^{-j\beta_+ z} - \frac{1}{2} E_o \, e^{-j\beta_- z} \tag{21}$$

The <u>relative</u> phase of the two components of E_1 or E_2 is

$$e^{j(\beta_+ - \beta_-)z} = e^{j2z\Delta\beta} \tag{22}$$

Hence the beat wavelength L, in which all the power is transferred out and back into line one is given by

$$L = \pi/\Delta\beta \tag{23}$$

In a distance L/2 the power transfers completely from one line to the other, and in a distance L/4 the amplitudes in the two lines become proportional to cos(π/4) and sin(π/4), i.e., half the power is in each line. The (uncorrected) length for a 3dB coupler is accordingly

$$L_{3dB} = \pi/4\Delta\beta \tag{24}$$

The transmission coefficient T, which is the fraction of power coupled in a distance ℓ, is therefore given by

$$T = \sin^2(\ell\Delta\beta) = \sin^2(\pi\ell/4L_{3dB}) \tag{25}$$

In the contract proposal RS-2141 of Jan. 1979, an expression (their eq. 4) is given which, in the present notation would be written

$$|T| = \ell\Delta\beta(1 + 2/h_o a)/h_o \tag{26}$$

This result appears to be wrong on the following points.

1. It refers to field amplitude, not power.
2. It is valid for ℓ small only.
3. It is dimensionally incorrect and is missing a factor $h_o/(1 + 2/h_o a)$.

The formula of eq. (25) should be used for the design of 3dB or other couplers.

IV. FUTURE WORK

With the development of the formulas in Section III, it will now be necessary to develop transmission line equations for situations in which a finite length coupler is joined to terminal lengths via curved guide sections. From this analysis the back coupling and reflection, as well as corrections to the coupling length, should ensue, with application to a determination of coupler bandwidth, and possible modifications thereto.

References

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(2) R.M. Knox and P.P. Toulios, "Integrated circuits for the millimeter through optical frequency range," in Proc. Symp. Submillimeter Waves (New York), March 31-April 2, 1970.

(3) E.F. Kuester and D.C. Chang, "Non-degenerate surface-wave mode coupling of a system of dielectric waveguides," Rept. 9, EM Lab., Univ. of Colo. at Boulder, October, 1979.