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Detectability of Objects in the Vicinity of Two-wire Intrusion Sensing Systems, Theoretical Studies

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DETECTABILITY OF OBJECTS IN THE VICINITY OF TWO-WIRE INTRUSION SENSING SYSTEMS, THEORETICAL STUDIES

by

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ABSTRACT

Intrusion into an area on the surface of the earth can be detected by monitoring the flow of electromagnetic energy between two parallel cables on the earth's surface. A theoretical analysis of the sensitivity of such a system has been made. The analysis is for a highly idealized system, but it is felt that the results are applicable in approximate form to a variety of practical systems.

DETECTABILITY OF OBJECTS IN THE VICINITY OF TWO-WIRE INTRUSION SENSING SYSTEMS, THEORETICAL STUDIES

1. Summary

The detection of intrusions into an area on the surface of the earth by electromagnetic sensing can be accomplished by surrounding the area with two leaky coaxial cables spaced a short distance apart and located near the surface of the earth. (It is assumed that in most applications the cables would be on the surface or slightly below.) One of the cables is then excited with a high frequency generator and the other is connected to a receiver. Energy will flow from the excited cable to the other. Any intrusion of an object into the region between the two cables will cause a change in the energy flow. Monitoring the energy flow by means of the receiver thus permits detection of such intrusions.

The manner in which the cable is excited may be either by a pulse or by a continuous signal. The use of a pulse permits the system to function somewhat in the manner of a one-dimensional radar providing information on the location of an intrusion as well as indicating its presence. The use of a continuous excitation signal does not permit the radar type of operation but is adaptable to use in a system that permits position determination in another manner. Such systems are discussed in detail in a report recently written.

In another similar type of intrusion sensing system the leaky coaxial cables are replaced by radiating cables or by non-leaky cables having discrete radiation and receiving elements periodically attached along

More generally $\Delta V_r/V_r$ can be assigned a certain minimum value for detectable intrusions. This, in turn, determines the minimum value of $\Delta Z_m/Z_m$ because

$$\Delta V_r/V_r = \Delta Z_m/Z_m$$

It follows that the relationship between an intrusion and the ΔZ_m that its causes determines the smallest intrusion that can be detected. The remainder of this analysis will be concerned with that relationship.

The electromagnetic compensation theorem provides the desired relationship. For this problem it may be stated as follows,

$$\Delta Z_m = \frac{1}{I_o I_o'} \int_s [\bar{E}' \times \bar{H} - \bar{E} \times \bar{H}'] \cdot \overline{ds} \quad .$$

The integral is a surface integral over the surface of the intrusion. $\bar{E}$ and $\bar{H}$ are the electromagnetic field vectors in the presence of the intrusion due to the transmitter exciting the coaxial cable with a current I_o. $\bar{E}'$ and $\bar{H}'$ are field vectors that would exist if the system were excited by a current I_o' into the receiver coaxial cable at the receiver terminal with no intrusion present in the system.

An exact evaluation of this integral does not seem practical for realistic intrusions but approximate evaluations for various classes of possible intrusions can supply results sufficiently accurate to serve as guidelines for designers of intrusion detection systems. Such approximate evaluations will be treated in the next section.

3. Analytical Treatment of the Problem

3.1 The Case Involving Radiating Cables

One of the simplest idealized shapes that can be used to approximate an intrusion is a horizontal cylinder located between the coaxial cables as shown in Fig. 2. Two facts should be kept in mind while attempting to evaluate the integral for the change in mutual impedance. The idealized shapes assumed for the intrusions are a great departure from actual shapes that intrusions are likely to take; and furthermore, for a single shape (except for shapes having a high degree of symmetry such as the sphere), the evaluation of the integral will be different for different orientations. Because of these facts the actual objectives of the analysis are order-of-magnitude estimates of change of impedance based upon such intrusion parameters as volume or cross-section.

Intrusions are likely to be of dense materials that reflect most of the electromagnetic energy incident upon them and absorb and dissipate the remainder in the form of heat. (Thus it is assumed that essentially none of the energy penetrates the intrusion and emerges from the opposite side.) The objective of the analysis is to determine the smallest intrusion that can be sensed so it will be assumed that the dimensions of the intrusion are much smaller than the spacing between the coaxial cables. In this initial study the presence of the earth will be ignored. Its influence will be considered later. The initial analysis will also be conducted for a system using radiating cables rather than leaky coaxial cables. The insight thus obtained will then be used in the study of the system using leaky coaxial cables.

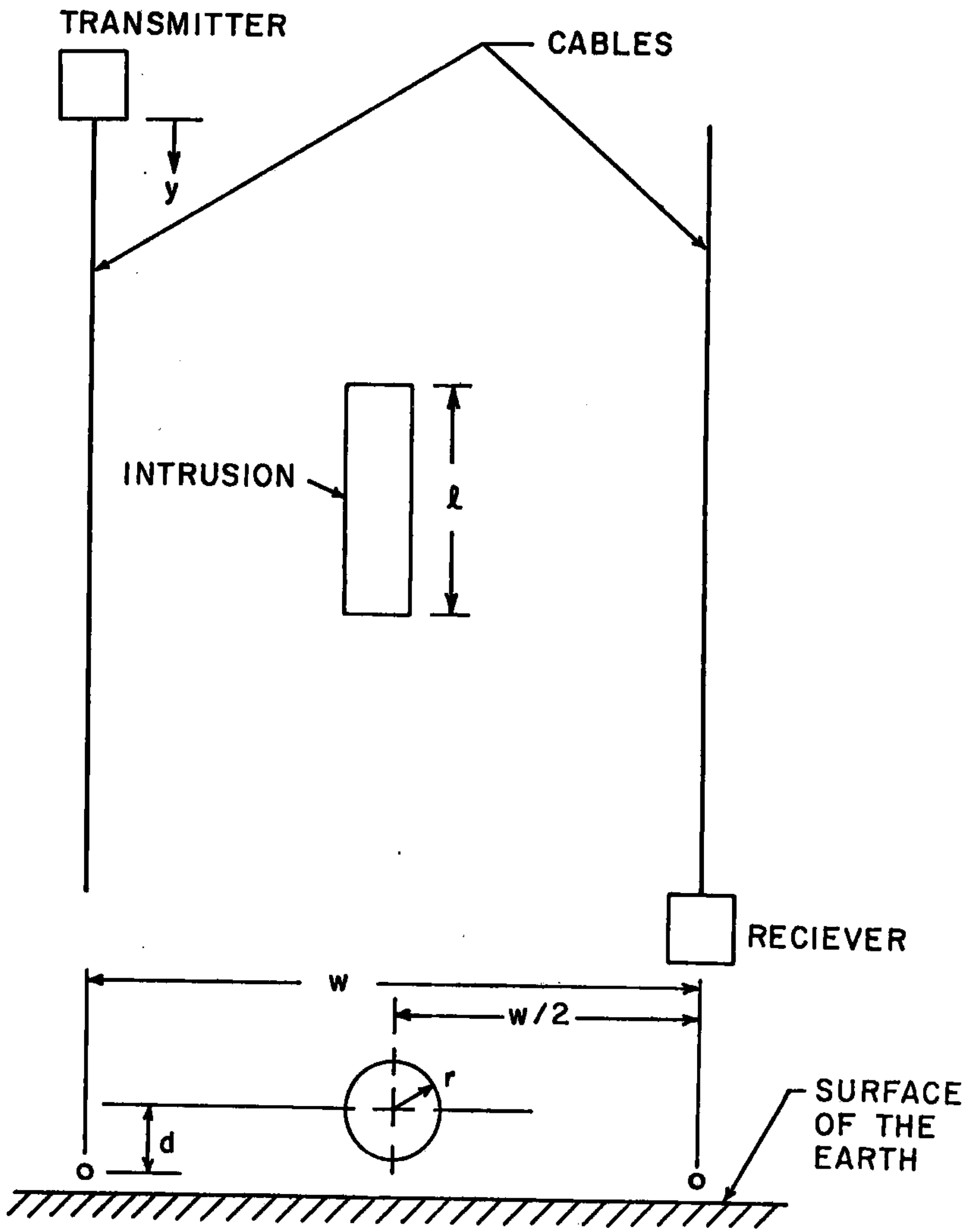

Fig. 2 Intrusion Sensing System with a Horizontal Cylindrical Intrusion

3.1.1 Solution for ΔZ_m

A cross section of the intrusion and the two sets of electromagnetic fields $\bar{E}$, $\bar{H}$, and $\bar{E}'$ $\bar{H}'$ are sketched in Fig. 3. The angle, α, is dependent upon the intrusion height, h, and the coaxial cable spacing w. α can range from π radians down to zero. Because of the assumption $w \gg r$, the primed fields on the surface of the intrusion can be expressed as

$$E_z' \cong A\eta \quad \frac{H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2} - \beta r\cos\theta)e^{ikz}}{H_o^{(2)}\ (\beta\frac{w}{2}\csc\frac{\alpha}{2})}$$

and

$$H_t' \cong A\ \frac{H_1^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2} - \beta r\cos\theta)e^{ikz}}{H_1^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})}\ \cos\theta$$

where η is an impedance, k and β are axial and radial propagation constants, subscript t denotes the tangential component of $\bar{H}'$, and A is the value of H' on the cylinder axis as z = 0. In writing these expressions, it has been assumed that the wave fronts can be considered as planar in the vicinity of the intrusion. It is now assumed that the characteristic dimensions, r and ℓ, of the intrusion are of the order of a wavelength or more so the plane-wave reflection coefficients can be used in finding approximations for $\bar{E}$ and $\bar{H}$. These approximations are

$$E_z \cong B\eta\ \frac{H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2}-\beta r\cos\phi)e^{-ikz}(1+R_\perp)}{H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})}$$

and

$$H_t \cong B\ \frac{H_1^{(2)}\ (\beta\frac{w}{2}\csc\frac{\alpha}{2} - \beta r\cos\phi)e^{-ikz}(1-R_\perp)}{H_1^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})}\cos\phi$$

where $R_\perp$ is the plane wave reflection coefficient, for perpendicular polarization, of the intrusion and B is the value of H on the cylinder axis at z = 0 and η is the same impedance as used in the expressions for

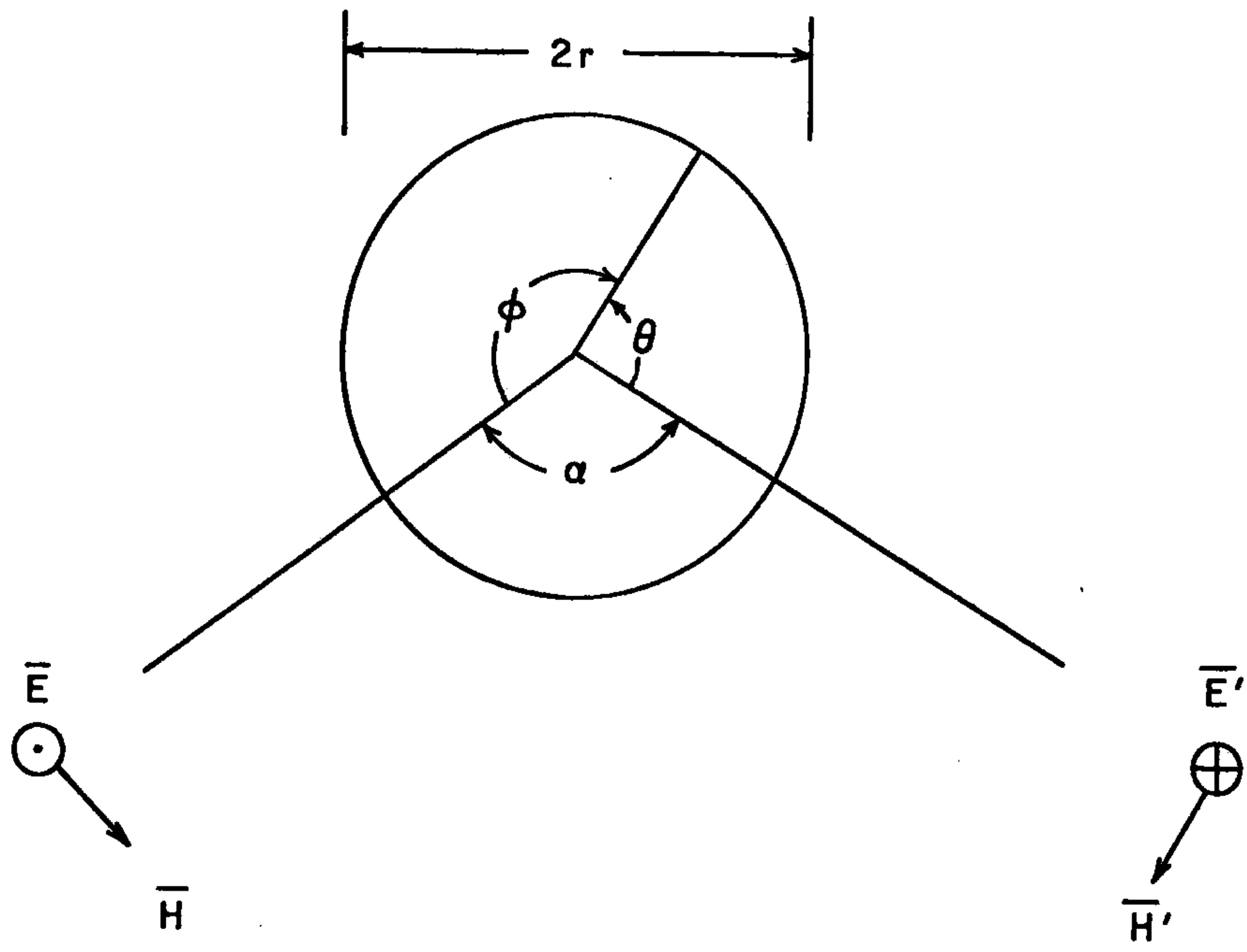

Fig. 3 Configuration of Fields Incident Upon a Horizontal Cylindrical Intrusion

E_z' and H_z'. It is assumed that βw is sufficiently large that the asymptotic approximation

$$H_n^{(2)}(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{-i(x-\frac{\pi}{4}-\frac{n\pi}{2})}$$

can be used. It is also assumed that

$$\sqrt{(\beta\frac{w}{2}\csc\frac{\alpha}{2} - \beta r\cos\theta)(\beta\frac{w}{2}\csc\frac{\alpha}{2} - \beta r\cos\phi)} \cong \beta\frac{w}{2}\csc\frac{\alpha}{2} \quad .$$

Then the normal component of the integrand in the expression for ΔZ_m can be written

$$[\bar{E}'\times\bar{H} - \bar{E}\times\bar{H}']_n \cong AB\eta\, \frac{i2e^{-i\beta w\csc\frac{\alpha}{2}}\, e^{+i\beta r(\cos\phi+\cos\theta)}}{\pi\beta\frac{w}{2}\csc\frac{\alpha}{2}\,[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2}]^2}\,[\cos\phi(1-R_\perp)$$

$$+ \cos\theta(1+R_\perp)] \ .$$

Since $\phi + \theta + \alpha = 2\pi$, $\cos\theta = \cos(2\pi - (\phi+\alpha)) = \cos(\phi+\alpha)$.

Furthermore

$$\cos\phi + \cos(\phi+\alpha) = \cos\phi + \cos\alpha\cos\phi - \sin\alpha\sin\phi$$

$$= (1+\cos\alpha)\cos\phi - \sin\alpha\sin\phi$$

$$= \sqrt{2+2\cos\alpha}\,\cos(\phi+\phi_1) = 2\cos\frac{\alpha}{2}\cos(\phi+\phi_1)$$

where

$$\phi_1 = \cos^{-1}\frac{1+\cos\alpha}{\sqrt{2+\cos\alpha}} = \cos^{-1}\sqrt{\frac{1+\cos\alpha}{2}} = \frac{\alpha}{2} \quad .$$

Thus

$$2\cos\frac{\alpha}{2}\cos(\phi+\phi_1) = 2\cos\frac{\alpha}{2}\cos(\phi+\frac{\alpha}{2}) \ .$$

Also

$$\cos\phi - \cos(\phi+\alpha) = \cos\phi - \cos\alpha\cos\phi + \sin\alpha\sin\phi$$

$$= (1-\cos\alpha)\cos\phi + \sin\alpha\sin\phi$$

$$= \sqrt{2-2\cos\alpha}\,\cos(\phi-\phi_2) = 2\sin\frac{\alpha}{2}\cos(\phi-\phi_2)$$

where

$$\phi_2 = \cos^{-1} \frac{1 - \cos\alpha}{\sqrt{2 - 2\cos\alpha}} = \cos^{-1}\sqrt{\frac{1-\cos\alpha}{2}} = \frac{\pi}{2} - \frac{\alpha}{2}$$

thus

$$2 \sin\frac{\alpha}{2} \cos(\phi - \phi_2) = 2 \sin\frac{\alpha}{2} \sin(\phi + \frac{\alpha}{2}) \quad .$$

As a consequence of the assumption that the intrusion reflects most of the incident energy and that it has dimensions of a wavelength or more, it can be assumed that $\bar{E}$ and $\bar{H}$ are negligibly small in the shadow regions so the integration for ΔZ_m can be over the range $\phi = -\frac{\pi}{2}$ to $\phi = \frac{\pi}{2}$ only. Furthermore integration over the ends* of the cylinder will be neglected and the plane wave reflection coefficient $R_\perp$, will be assumed to be independent of ϕ. (This is justified if, as is done in this case, the reflection coefficient is assumed to be nearly -1.

Then

$$\Delta Z_m = \frac{1}{I_o I_o'} \int_s [\bar{E}' \times \bar{H} - \bar{E} \times \bar{H}'] \cdot \bar{dS}$$

$$\cong \frac{r}{I_o I_o'} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{z=0}^{\ell} \frac{AB\eta \;\; 8i\, e^{-i\beta w \csc\frac{\alpha}{2}} \, e^{i\beta r 2\cos\frac{\alpha}{2}\cos(\phi + \frac{\alpha}{2})}}{\pi\beta w \csc\frac{\alpha}{2} \, [H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})]^2} [\cos\frac{\alpha}{2}\cos(\phi + \frac{\alpha}{2})$$

$$- R_\perp \sin\frac{\alpha}{2}\sin(\phi + \frac{\alpha}{2})] dz\, d\phi$$

or

$$\Delta Z_m \cong \frac{A}{I_o'} \frac{B}{I_o'} \eta \, \frac{8i\, e^{-i\beta w \csc\frac{\alpha}{2}} \ell r}{\pi\beta w \csc\frac{\alpha}{2}[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})]^2}$$

$$\int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{i\beta r 2\cos\frac{\alpha}{2}\cos(\phi + \frac{\alpha}{2})} [\cos\frac{\alpha}{2}\cos(\phi + \frac{\alpha}{2}) - R_\perp \sin\frac{\alpha}{2}\sin(\phi + \frac{\alpha}{2})] d\phi$$

*The radial component of E contributes to ΔZ_m only on the ends of the cylinder; and, since integration over the cylinder ends is neglected, it has no influence on the result.

The integrals needed to evaluate this expression are

$$\int_{\frac{\alpha}{2}-\frac{\pi}{2}}^{\frac{\alpha}{2}+\frac{\pi}{2}} e^{i\xi \cos\lambda} \cos\lambda \, d\lambda = 2J_o(\xi)\cos\frac{\alpha}{2} - 2\sum_{n=1}^{\infty} J_{2n}(\xi)\left[\frac{\cos(2n+1)\frac{\alpha}{2}}{2n+1} - \frac{\cos(2n-1)\frac{\alpha}{2}}{2n-1}\right] + i\,\frac{\pi}{2}J_1(\xi)$$

and

$$\int_{\frac{\alpha}{2}-\frac{\pi}{2}}^{\frac{\alpha}{2}+\frac{\pi}{2}} e^{i\xi \cos\lambda} \sin\lambda \, d\lambda = 2\,\frac{\sin(\xi \sin\frac{\alpha}{2})}{\xi} \quad .$$

The resulting expression for ΔZ_m is

$$\Delta Z_m \simeq \frac{A}{I_o'}\,\frac{B}{I_o}\; n\; \frac{16ie^{-i\beta w \csc\frac{\alpha}{2}}\;\ell r}{\pi\beta w \csc\frac{\alpha}{2}[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})]^2}\left\{\cos\frac{\alpha}{2} + J_o(2\beta r\cos\frac{\alpha}{2})\right.$$

$$- \sum_{n=1}^{\infty} J_{2n}(2\beta r\cos\frac{\alpha}{2})\left[\frac{\cos[2n+1)\frac{\alpha}{2}}{2n+1} - \frac{\cos(2n-1)\frac{\alpha}{2}}{2n-1} + i\,\frac{\pi}{2}J_1(2\beta r\cos\frac{\alpha}{2})\right]$$

$$\left. - R\sin^2\frac{\alpha}{2}\;\frac{\sin(\beta r\sin\alpha)}{\beta r\sin\alpha}\right\}$$

As an aid in interpretation of the above expression it may be observed that

$$\lim J_o(2\beta r\cos\frac{\alpha}{2})\cos\frac{\alpha}{2} - \sum_{n=1}^{\infty} J_{2n}(2\beta r\cos\frac{\alpha}{2})\left[\frac{\cos(2n+1)\frac{\alpha}{2}}{2n+1} - \frac{\cos(2n-1)\frac{\alpha}{2}}{2n-1}\right.$$

$$= \frac{\pi}{2}H_1(2\beta r\cos\frac{\alpha}{2}) - 1 \qquad \text{as} \quad \alpha \to 0$$

$$= 0 \qquad \text{as} \quad \alpha \to \pi$$

where $H_1(z)$ is the Struve function of order 1. Furthermore

$1.2 > H_1(z) \geq 0$ for all $z > 0$, it oscillates about $\frac{2}{\pi}$ as $z \to \infty$, $\lim H_1(z) = \frac{2}{\pi}$ and $H_1(0) = 0$. Note for the case $\alpha \cong \pi$, $\cos\frac{\alpha}{2} \cong 0$ and $\sin\frac{\alpha}{2} \cong 1$. So the last term of the above expression is the most significant for values of α near π and for moderate values of βr. For larger values of β, the term involving $J_1(2\beta r \cos\frac{\alpha}{2})$ becomes the dominant term. Then the approximation $\cos\frac{\alpha}{2} \cong \frac{2r}{w}$, and the asymptotic approximation to the Bessel function may be used to show that

$$\cos\frac{\alpha}{2} J_1(2\beta r \cos\frac{\alpha}{2}) \sim \frac{2r}{w}\sqrt{\frac{2}{\pi}}\ \frac{\cos(2\beta r\frac{2r}{w} - \frac{\pi}{4})}{\sqrt{2\beta r\frac{2r}{w}}}$$

$$= \sqrt{\frac{2}{\pi}}\ \frac{\cos(4\beta\frac{r^2}{w} - \frac{\pi}{4})}{\sqrt{\beta w}}$$

Therefore an approximate expression for ΔZ_m can be written in the form

$$\Delta Z_m \cong 2\,\frac{A}{I_o'}\ \frac{B}{I_o}\,\eta\quad 2\ell r D$$

where

$$D = \frac{4i\,e^{-i\beta w \csc\frac{\alpha}{2}}}{\pi\beta w[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2}]^2}\left\{+\cos\frac{\alpha}{2}\left[J_o(2\beta r\cos\frac{\alpha}{2})\cos\frac{\alpha}{2} - \sum_{n=1}^{\infty}\left[\frac{\cos(2n+1)\frac{\alpha}{2}}{2n+1} - \frac{\cos(2n-1)\frac{\alpha}{2}}{2n-1}\right] + i\,\frac{\pi}{2}J_1(2\beta r\cos\frac{\alpha}{2})\right] - R_\perp \sin^2\frac{\alpha}{2}\ \frac{\sin(\beta r\sin\alpha)}{\beta r\sin\alpha}\right\}.$$

For typical applications, α is slightly less than π, $R \approx -1$, and $\beta w >> 1$, then $D \cong 1$, and for very large values of β, D is proportional to $1/\sqrt{\beta w}$. As α decreases from π (corresponding to an increase in height of the intrusion) some of the terms in the expression for ΔZ_m increase and some decrease. The change is not great in any case. Therefore the expression

$$\Delta Z_m \cong 2 \frac{A}{I_o'} \frac{B}{I_o} \eta \quad 2\ell r D$$

is satisfactory in the "order-of-magnitude" sense which after all was the object of this analysis.

In order to further reinforce confidence in the above simplified expression for ΔZ_m, the case of a vertical cylinder rather than a horizontal cylinder will be considered. The configuration to be considered is that with a short vertical cylinder centered on the plane containing the two coaxial cables as sketched in Fig. 4. Proceeding as in the case of the horizontal cylinder, it is again assumed that the wavefronts of the electromagnetic fields incident upon the cylinder are plane, thus on the surface of the cylinder

$$E_t' \cong A\eta \frac{H_o^{(2)}(\beta\frac{w}{2} - \beta r\cos\theta)}{H_o^{(2)}(\beta\frac{w}{2})} \cos\theta\, e^{ikz}$$

$$H_t' \cong A \frac{H_1^{(2)}(\beta\frac{w}{2} - \beta r\cos\theta)}{H_1^{(2)}(\beta\frac{w}{2})} e^{ikz}$$

$$E_t \cong B\eta \frac{H_o^{(2)}(\beta\frac{w}{2} - \beta r\cos\phi)}{H_o^{(2)}(\beta\frac{w}{2})} \cos\phi\,(1 + R_{11})e^{-ikz}$$

and

$$H_t \cong B \frac{H_1^{(2)}(\beta\frac{w}{2} - \beta r\cos\phi)}{H_1^{(2)}(\beta\frac{w}{2})} (1 - R_{11})e^{-ikz}$$

where R_{11} is the plane wave reflection coefficient for parallel incidence. In this case,making assumptions similar to those made for the horizontal cylinder, the normal component of the integrand of the expression for ΔZ_m can be written (as in the case of the horizontal cylinder the radial component of E makes no contribution)

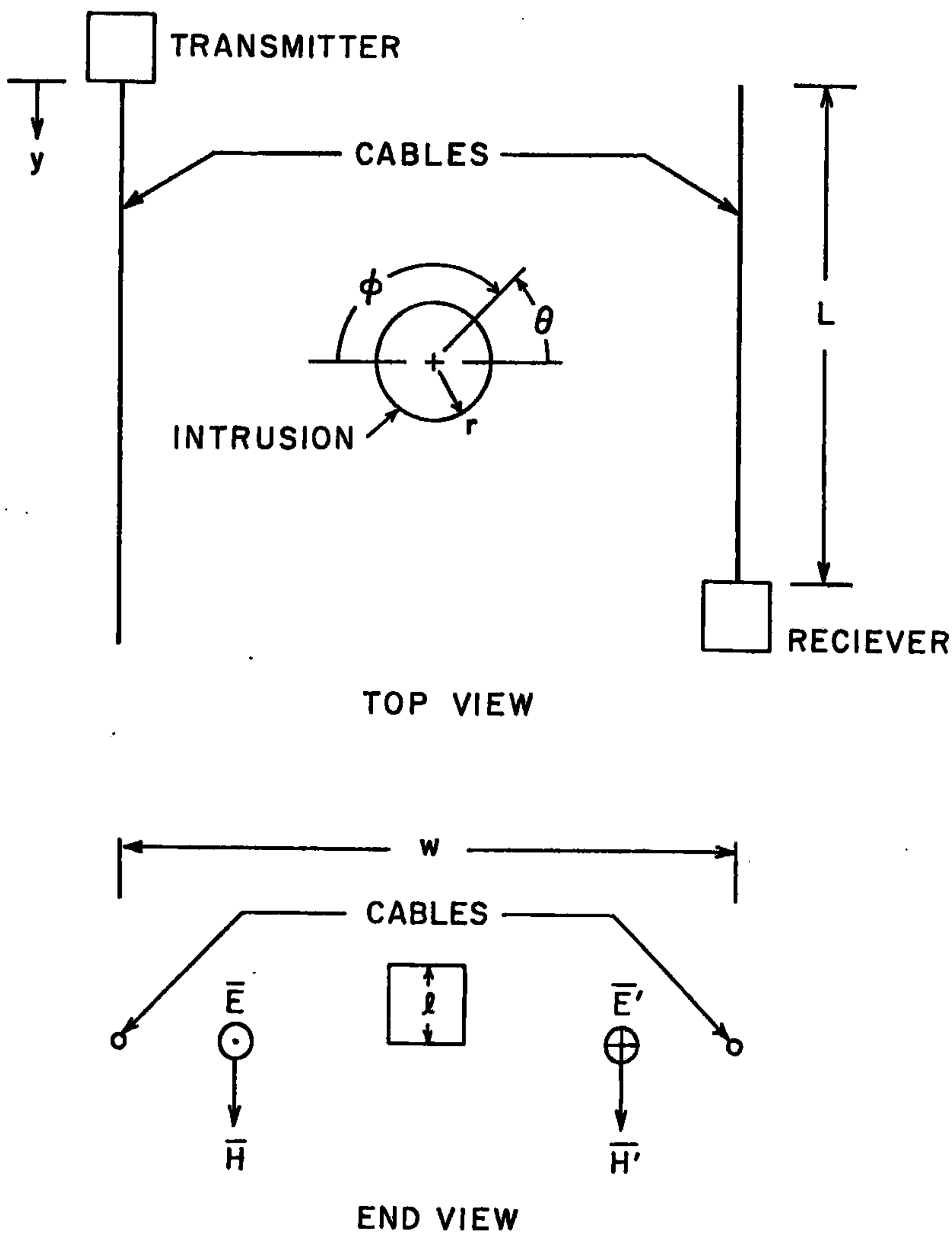

Fig. 4 Configuration of Fields Incident Upon a Vertical, Cylindrical Intrusion

$$[\bar{E}'\times \bar{H} - \bar{E}\times\bar{H}']_n = AB\eta \; \frac{i2e^{-i\beta w}e^{i\beta r}(\cos\theta + \cos\phi)}{\pi\beta\frac{w}{2}[H_o^{(2)}(\beta\frac{w}{2})]^2}[(1-R_{11})\cos\phi + (1+R_{11})\cos\theta]$$

and, since $\theta = \pi - \phi$ and $\cos\theta = -\cos\phi$,

$$\Delta Z_m \cong \frac{i}{I_o I_o'}\int_s [\bar{E}'\times\bar{H} - \bar{E}\times\bar{H}']\cdot\bar{ds} = -4\frac{A}{I_o'}\frac{B}{I_o}\eta \; \frac{ie^{-i\beta w}2R_{11}\ell r}{\pi\beta w[H_o^{(2)}(\beta\frac{w}{2})]^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\phi\, d\phi$$

or

$$\Delta Z_m = 2\frac{A}{I_o'}\frac{B}{I_o}\eta \quad 2r\ell D$$

where

$$D = \frac{-4ie^{-i\beta w}R_{11}}{\pi\beta w[H_o^{(2)}(\beta\frac{w}{2})]^2}$$

The same symbol D is used as was used in the case of the horizontal cylinder despite the fact that they are not equal, the justification being that they are nearly equal and that they are used in approximate expressions. D will also be used for other nearly equal expressions. More specifically for $R_{11} = -1$ and $\beta w >> 1$, $D \cong 1$. The intrusion in this case was centered between the coaxial cables, If it had been displaced from the center, D would have been found to be proportional to $\sqrt{\beta w}$ for large βw just as was found in the case of the horizontal cylinder. This is the same result as obtained for the horizontal cylinder. Since this result is for a cylinder with either horizontal or vertical orientation, there is reasonable justification for assuming it is approximately correct for a cylinder inclined at an angle between horizontal and vertical as well.

A more detailed examination of the expression for ΔZ_m shows it is proportional to the cross section, $2r\ell$, of the cylinder. This is gratifying since it is in agreement with the intuitive feeling that the effect of an intrusion on the receiver input voltage should be proportional to the area between the coaxial cables that has been blocked by the intrusion. Since the objective of this analysis is a result applicable to a broad variety of shapes of intrusions, it is appropriate, at this point, to inquire further into the effect of shape on the results for ΔZ_m. The observation that, for the cylinder, ΔZ_m is proportional to the cross section and is the same for two orthogonal orientations provides some justification for the assumption that ΔZ_m is given approximately by

$$\Delta Z_m \cong 2 \frac{A}{I_o'} \frac{B}{I_o} \eta \ SD$$

where $D \approx 1$ and D is proportional to $1/\sqrt{\beta w}$ for large βw and where S is the cross section of the intrusion. Here the shape of the intrusion is not involved except as it influences the cross section.

Further justification for the above result is provided by the following analysis. The integral for ΔZ_m is to be evaluated over the plane surface sketched in Fig. 5. The coaxial cables are parallel to the plane which has width g and length ℓ. The wavefronts of the fields incident upon the plane will be assumed to be plane as was done in the previous analysis of cylinders. Thus

$$E_t' \cong A\eta \frac{H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2} + \beta x \sin\theta)}{H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})} e^{ikz} \quad ,$$

$$H_t' \cong A \frac{H_1^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2} + \beta x \sin\theta)}{H_1(\beta\frac{w}{2}\csc\frac{\alpha}{2})} e^{ikz} \cos\theta \ ,$$

$$E_t \cong B\eta \frac{H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2} + \beta x \sin\phi)}{H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})} (1 + R_\perp) e^{-ikz},$$

$$H_t \cong B \frac{H_1^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2} + \beta x \sin\phi)}{H_1^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})} \cos\phi (1 - R_\perp) e^{-ikz}.$$

The radial component of E makes no contribution to ΔZ_m.

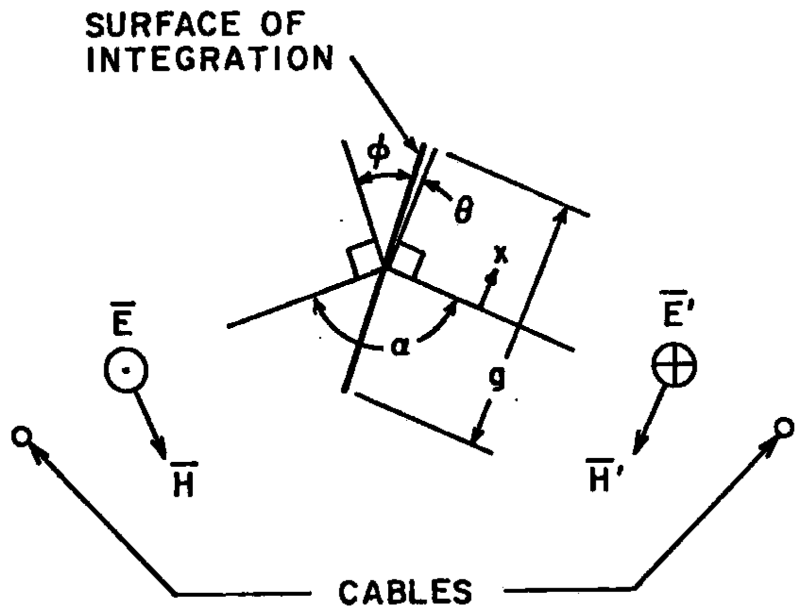

Fig. 5 Configuration of a Plane Surface in the Vicinity of Two Parallel Cables

Making the same approximations as in the previous cases, the integrand of the expression for ΔZ_m can be written

$$[\bar{E}' \times \bar{H} - \bar{E} \times \bar{H}']_n \cong AB\eta \; \frac{i2e^{-i\beta w \csc\frac{\alpha}{2}} \; e^{-i\beta x(\sin\theta + \sin\phi)}}{\pi\beta\frac{w}{2}\,[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})]^2} \, [(1-R_\perp)\cos\phi + (1+R_\perp)\cos\theta] \; ,$$

and

$$\Delta Z_m = 4\frac{A}{I_o'}\frac{B}{I_o}\eta \; \frac{ie^{-i\beta w \csc\frac{\alpha}{2}}\,\ell[(1-R_\perp)\cos\phi + (1+R_\perp)\cos\theta]}{\pi\beta w[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})]^2} \int_{-\frac{g}{2}}^{\frac{g}{2}} e^{-i\beta x(\sin\theta + \sin\phi)}dx$$

or

$$\Delta Z_m \cong 4\frac{A}{I_o'}\frac{B}{I_o}\eta \; \frac{ie^{i\beta w \csc\frac{\alpha}{2}}\,\ell[(1-R_\perp)\cos\phi + (1+R_\perp)\cos\theta]}{\pi\beta w[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})]^2} \; \frac{2\sin[\beta\frac{g}{2}(\sin\theta + \sin\phi)]}{\beta(\sin\theta + \sin\phi)} \; .$$

which can be written

$$\Delta Z_m \simeq 2\,\frac{A}{I_o'}\,\frac{B}{I_o}\,\eta \;\; \ell g \cos\phi D$$

where

$$D = \frac{i2e^{-i\beta w \csc\frac{\alpha}{2}}[(1-R)\cos\phi + (1+R_\perp)\cos\theta]}{\pi\beta w[H_o^{(2)}(\beta\frac{w}{2}\csc\frac{\alpha}{2})]^2 \cos\phi} \; \frac{\sin[\beta\frac{g}{2}(\sin\theta + \sin\phi)]}{\beta\frac{g}{2}(\sin\theta + \sin\phi)}$$

As in previous cases $D \cong 1$ for common values of the parameters and if the analysis were carried out more accurately, it could be found that D is proportional to $1/\sqrt{\beta w}$ for large βw.

In this problem the cross section of the plane, as viewed from either of the coaxial cables is $S = \ell g \cos\phi$; so ΔZ_m may be written

$$\Delta Z_m \cong 2\,\frac{A}{I_o'}\,\frac{B}{I_o}\;\eta\;\; SD$$

which is the same expression as obtained in the cases involving cylinders. This result further reinforces the assumption that the shape of the intrusion is important only to the extent that it influences the cross section. An extension of the reasoning used in this example shows that for the case of the intrusion sketched in Fig. 6 ,

$$\Delta Z_m \simeq 2 \frac{A}{I_o^{\prime}} \frac{B}{I_o} \eta \; SD$$

where α has been taken as π, $S = g\ell$ and ℓ is the length of the intrusion. ΔZ_m for this problem will change as α decreases from π (that is as the intrusion is raised above the earth) but the change will be gradual as can be reasoned using arguments similar to those used in the analysis of the cylinder. This conclusion is further reinforced by the observation that for the flat plane (Ref. Fig. 5)

$$\frac{\partial}{\partial \alpha} \Delta Z_m \Big|_{\alpha=\pi} = 0 \; .$$

One further problem will be considered in an effort to demonstrate the insensitivity of the above result for ΔZ_m to changes in shape. This involves evaluation of the integral for ΔZ_m again for a planar surface of integration but in this case for the configuration shown in Fig. 7. Again using the plane wave front approximation for the fields it is apparent that (the radial component of E will be neglected; this is

$$E_t' \cong A\eta \, \frac{H_o^{(2)}(\beta\frac{W}{2} + \beta x \sin\phi)}{H_o^{(2)}(\beta\frac{W}{2})} \cos\phi \, e^{ikz}$$

$$H_t' \cong A \, \frac{H_1^{(2)}(\beta\frac{W}{2} + \beta x \sin\phi)}{H_1^{(2)}(\beta\frac{W}{2})} \, e^{ikz}$$

$$E_t \cong -B\eta \, \frac{H_o^{(2)}(\beta\frac{W}{2} - \beta x \sin\phi)}{H_o^{(2)}(\beta\frac{W}{2})} (1 + R_{11}) \cos\phi \, e^{-ikz}$$

and

$$H_t \cong B \, \frac{H_1^{(2)}(\beta\frac{W}{2} - \beta x \sin\phi)}{H_1^{(2)}(\beta\frac{W}{2})} (1 - R_{11}) e^{-ikz}$$

roughly equivalent to the neglect of the end surfaces of the horizontal cylinder).

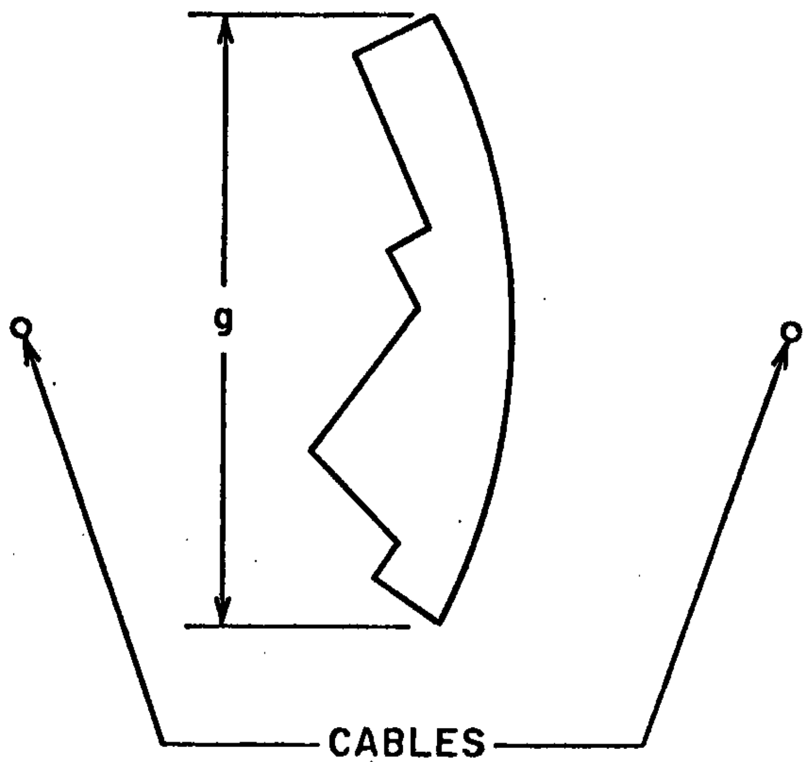

Fig. 6 Intrusion Having a More Complex Shape

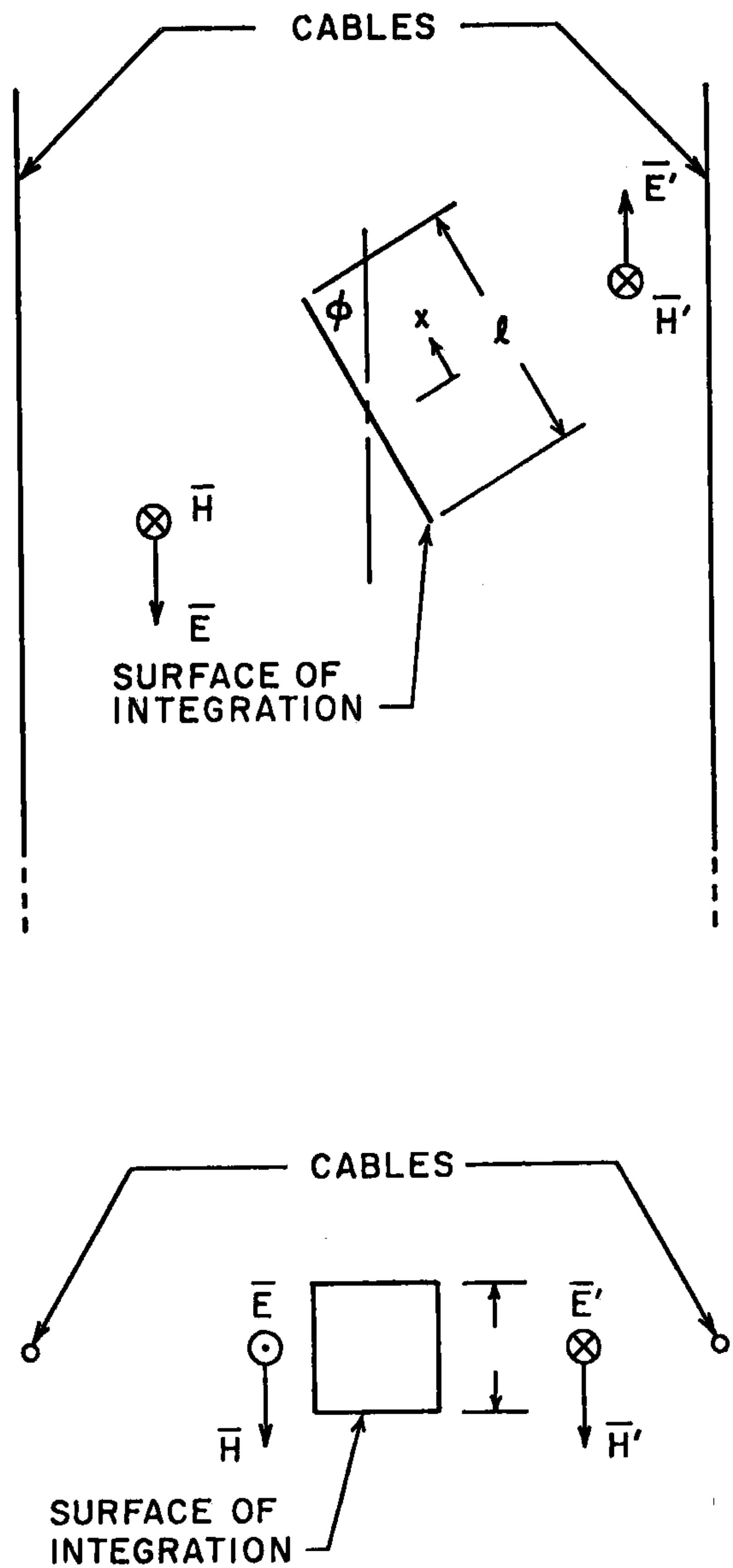

Fig. 7 Configuration of a Plane Surface in the Vicinity of Two Parallel Cables

Using approximations similar to those used before, integrand of the expression for ΔZ_m can be written

$$[\bar{E}' \times \bar{H} - \bar{E} \times \bar{H}]_n \simeq AB\eta \frac{i2e^{-i\beta w}}{\pi\beta\frac{w}{2}[H_o^{(2)}(\beta\frac{w}{2})]^2}[(1-R_{11})\cos\phi + (1+R_{11})\cos\phi]$$

or

$$[\bar{E}' \times \bar{H} - \bar{E} \times \bar{H}']_n \simeq 2AB\eta \frac{i2e^{-i\beta w}}{\pi\beta\frac{w}{2}[H_o^{(2)}(\beta\frac{w}{2})]^2} \cos\phi,$$

and ΔZ_m is given by

$$\Delta Z_m \simeq 2\frac{A}{I_o'}\frac{B}{I_o}\eta \frac{i2e^{-i\beta w}}{\pi\beta\frac{w}{2}[H_o^{(2)}(\beta\frac{w}{2})]^2} \cos\phi\, g \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} dx$$

or

$$\Delta Z_m \simeq 2\,\ell g \cos\phi \frac{A}{I_o'}\frac{B}{I_o}\eta\, D$$

where

$$D = \frac{i4e^{-i\beta w}}{\pi\beta w[H_o^{(2)}(\beta\frac{w}{2})]^2}$$

and D has the same characteristics as observed previously. This is the same result as obtained for the case of Fig. 5 in which the plane was tilted in the other direction and again in this case the cross section, as seen from the direction from either one of the coaxial cables is $S = \ell g \cos\phi$; so the result may be expressed as

$$\Delta Z_m \cong 2\frac{A}{I_o'}\frac{B}{I_o}\eta\, SD$$

The case of an irregular shape such as that sketched in Fig. 8 is a direct extension of this result and it gives the same result where in this case $S = gm$.

Several examples have been given and several arguments have been advanced to support the contention that ΔZ_m as given by the expression

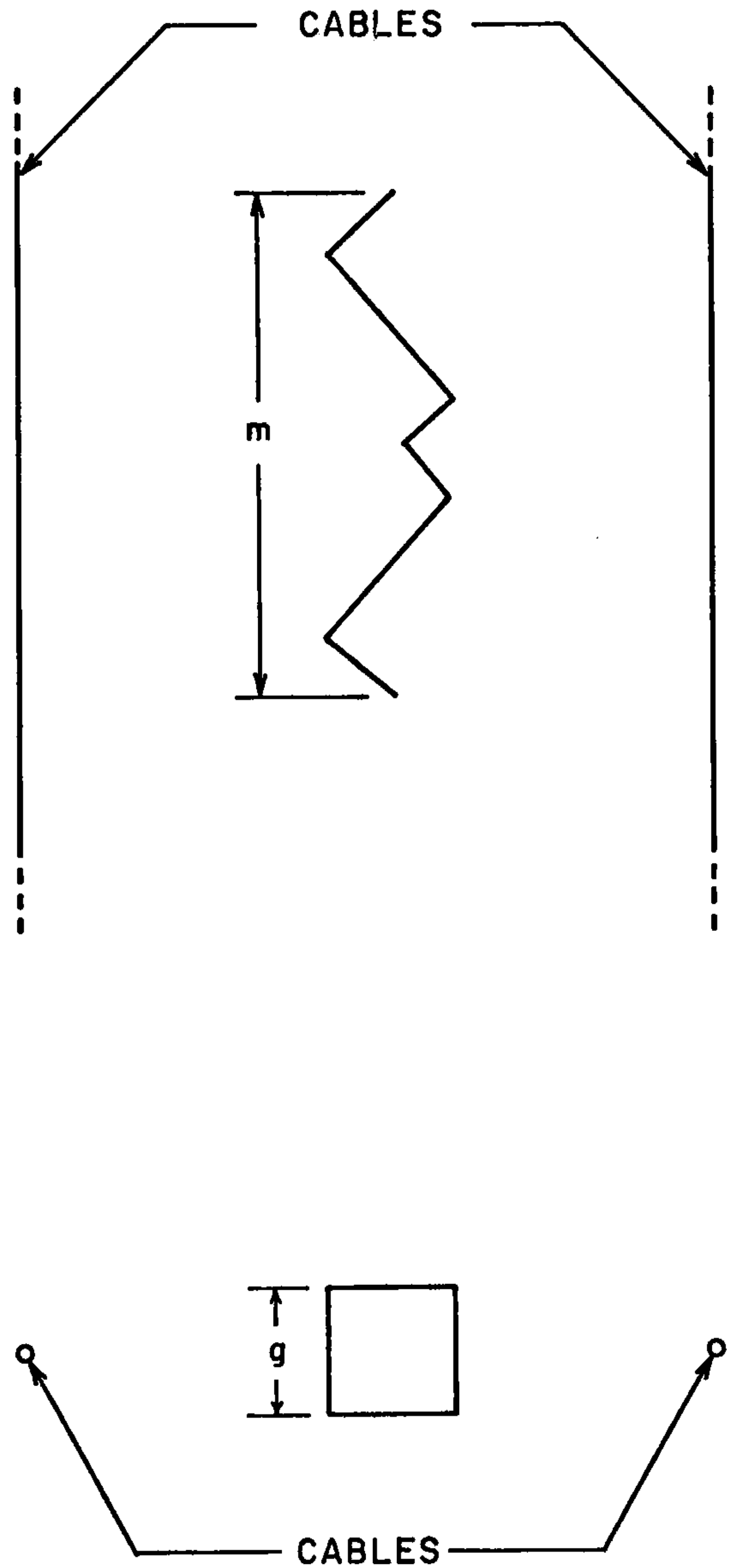

Fig. 8 Intrusion Having a More Complex Shape

$$\Delta Z_m \cong 2 \frac{A}{I_o^\tau} \frac{B}{I_o} \eta \quad SD$$

is a reasonably accurate estimate of ΔZ_m for a wide variety of shapes of intrusion near the plane containing the coaxial cables. In this case the phrase reasonably accurate estimate should be taken to mean that the true value of ΔZ_m may range from perhaps $\frac{1}{3}$ of the calculated value up to 3 times the calculated value. For many types of analyses such accuracy would be inadequate, but for the intended purpose of these results, use as guidelines in intrusion sensing system design, it is adequate and probably as accurate as can reasonably be expected because of the wide variety of shapes and orientations that intrusions may take.

In examining the approximations used to obtain the above result it appears that the assumption of plane wave fronts rather than cylindrical wave fronts for the incident fields is the most difficult to justify. However some thought about the behavior of the integrand over the surface of integration suggests that using cylindrical wave expressions would give a result different, but not greatly, from the ones obtained. It seems doubtful that the two results could differ by more than two to one. Furthermore it seems that shape variations could cause results to differ by an amount comparable or perhaps even more than the variations due to the plane wave front assumption. For these reasons, the expression for ΔZ_m will be taken as

$$\Delta Z_m = 2 \frac{A}{I_o^\tau} \frac{B}{I_o} \eta \ SD$$

where $D \simeq 1$.

The next problem to be considered is that of finding an expression for Z_m.

3.1.2 Solution for Z_m

The electromagnetic compensation theorem just used to find an approximate expression for ΔZ_m can also be used to find an approximate expression for Z_m. Consider the configuration shown in Fig. 9. A perfectly conducting vertical, thin plane midway between the radiating cables shields the cables from each other. The shielding will cause the mutual inductance between the transmitter and receiver to be zero. Then the compensation theorem can be used to find the change (from zero to a non-zero value) in mutual impedance resulting from the removal of the conducting half-plane. The change, of course, is just the mutual impedance, Z_m, that is needed. Because of the infinite extent of the plane, the fields incident upon it cannot be assumed to have plane wave fronts in this case. The fields may be taken to be

$$E_t = B\eta \frac{H_o^{(2)}(\beta r)}{H_o^{(2)}(\beta \frac{w}{2})} e^{-ikz} ,$$

$$H_t = B \frac{H_1^{(2)}(\beta r)}{H_1^{(2)}(\beta \frac{w}{2})} e^{-ikz} \cos \phi$$

$$E_t' = 0$$

and

$$H_t' = 2A \frac{H_1^{(2)}(\beta r)}{H_1^{(2)}(\beta \frac{w}{2})} \cos \phi \, e^{ikz}$$

The above expressions have been written so that A and B have the same meanings as in the previous analysis, that is they are the values of the magnetic field midway between the two coaxial cables at $\phi = 0$ and $z = 0$.

The integrand in the expression for ΔZ can be written

$$[\bar{E}' \times \bar{H} - \bar{E} \times \bar{H}']_n = 2AB\eta \frac{[H_o^{(2)}(\beta r)]^2}{[H_o^{(2)}(\beta \frac{w}{2})]^2} \cos \phi$$

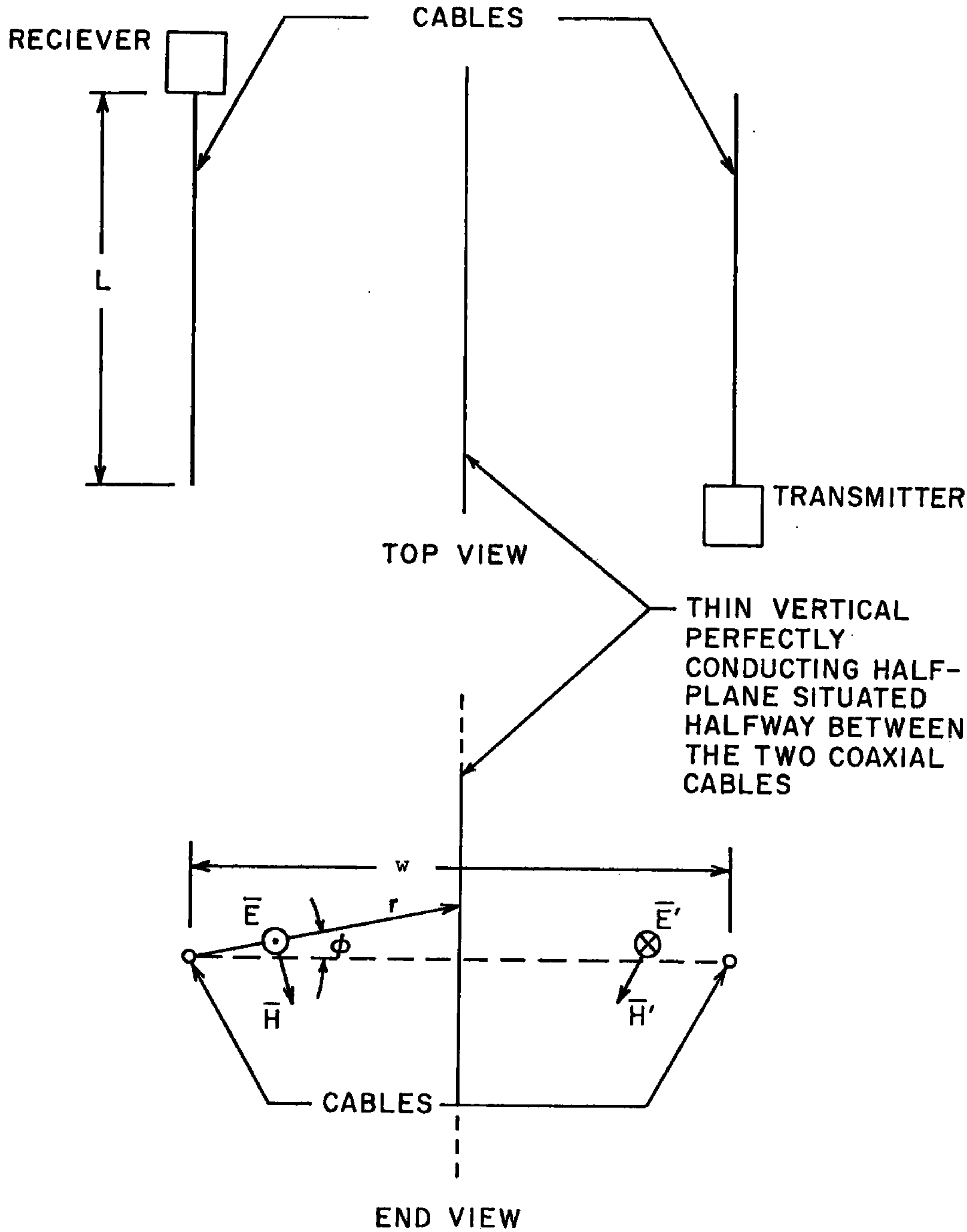

Fig. 9 Two-Cable Intrusion Sensing System with the Two Cables Shielded from Each Other by a Thin Perfectly Conducting Plane

and the expression for ΔZ is

$$\Delta Z \cong \frac{1}{I_o' I_o} \; 2AB\eta \; L\frac{w}{2} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{[H_o^{(2)}(\beta r)]^2}{H_o^{(2)}(\beta\frac{w}{2})]^2} \cos\phi \sec^2\phi \, d\phi$$

or in terms of x

$$\Delta Z \cong 2\eta \; \frac{A}{I_o'} \; \frac{B}{I_o} \, L\frac{w}{2} \, 2 \int_0^\infty \frac{[H_o^{(2)}(\beta\sqrt{x^2+(\frac{w}{2})^2})]^2}{[H_o^{(2)}(\beta\frac{w}{2})]^2} \frac{dx}{\sqrt{x^2+(\frac{w}{2})^2}}$$

The Hankel function may be replaced by its asymptotic approximations.

$$H_o^{(2)}(z) \sim e^{-iz} \, e^{i\frac{\pi}{4}} \sqrt{\frac{2}{\pi z}} = \frac{e^{-iz}}{\sqrt{\pi z}} (1+i) \; .$$

Then

$$\Delta Z \cong 2\eta \; \frac{A}{I_o'} \frac{B}{I_o} L \frac{\omega}{2} [H_o^{(2)}(\beta\frac{w}{2})]^{-2} \; \frac{2i}{\pi\beta} \; 2\int_0^\infty \frac{e^{-i\beta\sqrt{x^2+(\frac{w}{2})^2}}}{x^2+(\frac{w}{2})^2} \, dx \; .$$

The integral in the expression for Z can be transformed into

$$I = \int_{\frac{w}{2}}^\infty \frac{e^{-i2\beta z}}{z\sqrt{z^2-(\frac{w}{2})^2}} \, dz$$

which leads to the expression

$$\frac{\partial I}{\partial \beta} = -i2 \int_{\frac{w}{2}}^\infty \frac{e^{-i2\beta z}}{\sqrt{z^2-(\frac{w}{2})^2}} \, dz \; .$$

This integral can be evaluated to give the result

$$\frac{\partial I}{\partial \beta} = -\pi H_o^{(2)}(\beta w) \; .$$

Thus I can be obtained by integrating with respect to β. Using the asymptotic approximation for the Hankel function this is

$$I \cong -\pi\sqrt{\frac{2}{w}}\,(1+i)\int \frac{e^{-i\beta w}}{\sqrt{\beta w}}\,d\beta$$

$$= -\pi(1+i)\left[C\left(\sqrt{\tfrac{2}{\pi}\beta w}\right) - iS\left(\sqrt{\tfrac{2}{\pi}\beta w}\right) + K\right]$$

where $C\left(\sqrt{\frac{2}{\pi}\beta w}\right)$ and $S\left(\sqrt{\frac{2}{\pi}\beta w}\right)$ are Fresnel integrals of argument $\sqrt{\frac{2}{\pi}\beta w}$ and K is a constant of integration. Since $I \to 0$ as $\beta \to \infty$, $K = \frac{1}{2} - i\frac{1}{2}$ and

$$I \cong +\pi(1+i)\,\frac{\sqrt{2}}{w}\left[\left(\tfrac{1}{2} - C\left(\sqrt{\tfrac{2}{\pi}\,\beta w}\right)\right) - i\left(\tfrac{1}{2} - S\left(\sqrt{\tfrac{2}{\pi}\,\beta w}\right)\right)\right] .$$

Now an approximate expression for ΔZ can be written.

$$\Delta Z \cong -2\frac{A}{I_o'}\frac{B}{I_o}\,\eta\,\frac{Lw(1-i)\sqrt{2}\;2}{\beta w\left[H_o^{(2)}(\beta\frac{w}{2})\right]^2}\left[\tfrac{1}{2} - C\left(\sqrt{\tfrac{2}{\pi}\,\beta w}\right)\right) - i\left(\tfrac{1}{2} - S\left(\sqrt{\tfrac{2}{\pi}\,\beta w}\right)\right)\right]$$

but

$$\left(\tfrac{1}{2} - C\left(\sqrt{\tfrac{2}{\pi}\,\beta w}\right)\right) - i\left(\tfrac{1}{2} - S\left(\sqrt{\tfrac{2}{\pi}\,\beta w}\right)\right) = \frac{1-i}{2}\,\mathrm{erfc}\left[(1+i)\sqrt{\frac{\beta w}{2}}\right]$$

where $\mathrm{erfc}\left[(1+i)\sqrt{\frac{\beta w}{2}}\right]$ is the complementary error function of argument $\left[(1+i)\sqrt{\frac{\beta w}{2}}\right]$. Using this relation ΔZ can be written

$$\Delta Z \cong 2\,\frac{A}{I_o'}\frac{B}{I_o}\,\eta\,\frac{\sqrt{\pi}\,L\frac{w}{2}}{\sqrt{\beta\frac{w}{2}}}\,e^{i\frac{\pi}{4}}F$$

where

$$F = \frac{2}{\sqrt{\pi}}\,e^{i\frac{\pi}{4}}\,\frac{\mathrm{erfc}\left[(1+i)\sqrt{\beta\frac{w}{2}}\right]}{\sqrt{\beta\frac{w}{2}}\,\left[H_o^{(2)}(\beta\frac{w}{2})\right]^2}$$

Using the asymptotic expansion for the complementary error function

$$\operatorname{erfc}\left[(1+i)\sqrt{\beta\frac{w}{2}}\right] \sim \frac{e^{-i\beta w}}{\sqrt{\pi}(1+i)\sqrt{\beta\frac{w}{2}}}$$

it is seen that $F \simeq 1$ for $\beta w \gg 1$. It is recalled that this ΔZ is the mutual impedance Z_m so

$$Z_m \cong 2\frac{A}{I_o'}\frac{B}{I_o}\eta\left[\frac{\sqrt{\pi}\, e^{i\frac{\pi}{4}}}{\sqrt{\beta\frac{w}{2}}}\; L\frac{w}{2}\right]F$$

where

$$F \simeq 1.$$

The limitations on the accuracy of this expression must be clearly recognized. The most significant approximation used in the derivation is probably the assumption that the length, L, of the system of coupled transmission lines is short compared with the coupling length, that is length over which a complete cycle occurs; which in turn is inversely proportional to the coupling coefficient between the two cables. If the system length were not short compared with the coupling length, it would be necessary to use a value for the propagation constant, k, in the expressions for E and H which is corrected to account for the coupling.

3.1.3 Solution for $\Delta V/V$

Now the quantity $\Delta V_r/V_r$ discussed as a meaningful measure relating to the sensitivity of the system can be approximately evaluated.

$$\frac{\Delta V_r}{V_r} = \frac{\Delta Z_m}{Z_m} \cong \frac{2\frac{A}{I_o'}\frac{B}{I_o}\eta\; SD}{2\frac{A}{I_o'}\frac{B}{I_o}\eta\;\frac{\sqrt{\pi}\, e^{i\frac{\pi}{4}}}{\sqrt{\beta\frac{w}{2}}}\; L\frac{w}{2}F}$$

Since $D \simeq 1$ and $F \simeq 1$

$$\frac{\Delta V_r}{V_r} \cong \frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} \; \frac{S}{L\frac{w}{2}} \sqrt{\beta \frac{w}{2}}$$

If dimensions are normalized to the radial wavelength λ, where $\lambda = \frac{2\pi}{\beta}$, then

$$\frac{\Delta V_r}{V_r} \simeq \frac{s_\lambda}{L/\lambda \sqrt{w/4\lambda}} e^{-i\frac{\pi}{4}}$$

where s_λ is the cross section of the intrusion in square wavelengths.

3.2 The Case Involving Non-Radiating Cables

3.2.1 Solution for ΔZ_m

The previous analysis was for radiating cables; now the case of non-radiating cables will be considered. It will be assumed that the fields in the vicinity of the intrusion in this case, are given by the same sorts of expressions as were used in the case of the radiating cable except for replacement of the Hankel functions by modified Bessel functions of the second kind. This seems a reasonable approach on the basis of the observation that the fields in the vicinity of an isolated radiating cable are described in terms of Hankel functions and those in the vicinity of an isolated non-radiating cable by modified Bessel functions of the second kind. The use of the plane wave reflection coefficient in the manner that was done for the case of the radiation cable and the restriction of the range of integration so as not to include the "shadows" region are more difficult to justify in the case of a non-radiating cable. However, since order-of-magnitude guidelines are the objective of this analysis, the same sorts of approximations as used for the radiating cable will be used in this case. Considering again the geometry sketched in Fig. 3, it will be assumed that the fields can be expressed as

$$E_z' \cong A\eta \,\frac{K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2}\;\beta r\cos\theta)}{K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2})}\quad e^{ikz}$$

$$H_t' \cong A\,\frac{K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2}-\beta r\cos\theta)}{K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2})}\cos\theta\, e^{ikz},$$

$$E_z \cong B\eta\,\frac{K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2}-\beta r\cos\theta)}{K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2})}(1+R_\perp)e^{-ikz},$$

$$H_t \cong B\,\frac{K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2}-\beta r\cos\phi)}{K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2})}(1-R_\perp)\cos\phi\, e^{-ikz}$$

Assuming the asymptotic approximations $K_n(z) \sim \sqrt{\frac{\pi}{2z}}\; e^{-z}$, can be used for the modified Bessel function, the integrand of the expression for ΔZ_m may be written

$$[\bar{E}'\times\bar{H} - \bar{E}\times\bar{H}']_n \cong AB\eta\;\frac{\pi e^{-\beta w\csc\frac{\alpha}{2}}\; e^{\beta r(\cos\phi+\cos\phi)}}{2\beta\frac{w}{2}\csc\frac{\alpha}{2}K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2})K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2})}[\cos\phi\;(1-R_\perp)+\cos\theta\;(1+R_\perp)]$$

where it has also been assumed that

$$(\beta\frac{w}{2}\csc\frac{\alpha}{2}-\beta r\cos\theta)(\beta\frac{w}{2}\csc\frac{\alpha}{2}-\beta r\cos\phi)]^{\frac{1}{2}} \simeq \beta\frac{w}{2}\csc\frac{\alpha}{2}\,.$$

Thus

$$\Delta Z_m \cong \frac{A}{I_o}\frac{B}{I_o}\,\eta\;\frac{\pi e^{-\beta w\csc\frac{\alpha}{2}}\;2rL}{2\beta\frac{w}{2}\csc\frac{\alpha}{2}K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2})K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2})}\int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\beta r(\cos\theta+\cos\phi)}[\cos\phi(1-R_\perp) + \cos\theta(1+R_\perp)]d\phi$$

The integral, I, in this expression can be written

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\beta r 2\cos\frac{\alpha}{2}\cos(\phi+\frac{\alpha}{2})}[\cos\frac{\alpha}{2}\cos(\phi+\frac{\alpha}{2}) - R_\perp \sin\frac{\alpha}{2}\sin(\phi+\frac{\alpha}{2})]d\phi$$

or

$$I = \cos\frac{\alpha}{2}\int_{\frac{\alpha}{2}-\frac{\pi}{2}}^{\frac{\alpha}{2}+\frac{\pi}{2}} e^{2\beta r\cos\frac{\alpha}{2}\cos t}\cos t\,dt - R_\perp \sin\frac{\alpha}{2}\int_{\frac{\alpha}{2}-\frac{\pi}{2}}^{\frac{\alpha}{2}+\frac{\pi}{2}} e^{2\beta r\cos\frac{\alpha}{2}\cos t}\sin t\,dt$$

The integrals needed to evaluate this expression are

$$\int_{\frac{\alpha}{2}-\frac{\pi}{2}}^{\frac{\alpha}{2}+\frac{\pi}{2}} e^{\zeta\cos\lambda}\cos\lambda\,d\lambda = 2\Big[I_o(\zeta)\cos\frac{\alpha}{2} + \frac{\pi}{2}I_1(\zeta) + \sum_{k=1}^{\infty}(-1)^k I_{2k}(\zeta) \cdot \Big[\frac{\cos(2k+1)\frac{\alpha}{2}}{2k+1} - \frac{\cos(2k-1)\frac{\alpha}{2}}{2k-1}\Big]\Big]$$

and

$$\int_{\frac{\alpha}{2}-\frac{\pi}{2}}^{\frac{\alpha}{2}+\frac{\pi}{2}} e^{\zeta\cos\lambda}\sin\lambda\,d\lambda = \frac{e^{\zeta\sin\frac{\alpha}{2}} - e^{-\zeta\sin\frac{\alpha}{2}}}{\zeta} = 2\sin\frac{\alpha}{2}\,\frac{\sinh(\zeta\sin\frac{\alpha}{2})}{\zeta\sin\frac{\alpha}{2}}$$

The expression for ΔZ_m may now be written

$$\Delta Z_m \cong 2\frac{A}{I_o}\frac{B}{I_o}\eta\,\frac{\pi e^{-\beta w\csc\frac{\alpha}{2}}\,2rL}{2\beta\frac{w}{2}\csc\frac{\alpha}{2}K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2})K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2})}\Big\{\cos\frac{\alpha}{2}\Big[I_o(2\beta r\cos\frac{\alpha}{2})\cos\frac{\alpha}{2}$$

$$+ \frac{\pi}{2}I_1(2\beta r\cos\frac{\alpha}{2}) + \sum_{k=1}^{\infty}(-1)^k I_{2k}(2\beta r\cos\frac{\alpha}{2})\Big[\frac{\cos(2k+1)\frac{\alpha}{2}}{2k+1} - \frac{\cos(2k-1)\frac{\alpha}{2}}{2k-1}\Big]\Big]$$

$$- R_\perp \sin^2\frac{\alpha}{2}\,\frac{\sinh(\beta r\sin\alpha)}{\beta r\sin\alpha}\Big\}$$

For $\alpha \cong \pi$, $R_\perp \cong -1$ and for moderate values of βr, the last term in the above expression is dominant and is equal to unity, but for larger values of β the term involving $I_1(2\beta r\cos\frac{\alpha}{2})$ becomes dominant. Then the approximation $\cos\frac{\alpha}{2} \simeq \frac{2r}{w}$ and the asymptotic approximation

$$I_1(z) \sim \frac{e^z}{\sqrt{2\pi z}} ,$$

to the modified Bessel function can be used to show that

$$\cos\frac{\alpha}{2}\, I_1(2\beta r \cos\frac{\alpha}{2}) \sim \frac{2re^{2\beta r\frac{2r}{w}}}{w\sqrt{2\pi 2\beta r\frac{2r}{w}}} = \frac{e^{4\beta\frac{r^2}{w}}}{\sqrt{2\pi\beta w}}$$

Therefore an approximate expression for ΔZ_m is

$$\Delta Z_m \cong 2\, \frac{A}{I_o'}\, \frac{B}{I_o}\, \eta \quad 2rLD$$

where

$$D = \frac{\pi e^{-\beta w \csc\frac{\alpha}{2}}}{\beta w \csc\frac{\alpha}{2}\, K_o(\beta\frac{w}{2}\csc\frac{\alpha}{2})\, K_1(\beta\frac{w}{2}\csc\frac{\alpha}{2})} \cos\frac{\alpha}{2}\Bigg[I_o(2\beta r \cos\frac{\alpha}{2}) \cos\frac{\alpha}{2}$$

$$+ \frac{\pi}{2}\, I_1(2\beta r\cos\frac{\alpha}{2}) + \sum_{k=1}^{\infty} (-1)^k I_{2k}(2\beta r \cos\frac{\alpha}{2}) \left[\frac{\cos(2k+1)\frac{\alpha}{2}}{2k+1} - \frac{\cos(2k-1)\frac{\alpha}{2}}{2k-1}\right]$$

$$- R_\perp \sin^2\frac{\alpha}{2}\, \frac{\sinh(\beta r \sin\alpha)}{\beta r \sin\alpha} \Bigg\}$$

The function $D \simeq 1$ for common values of the parameters and it is proportional to $e^{4\beta\frac{r^2}{w}}/\sqrt{\beta w}$ for large β. As before the symbol, S will be used for $2rL$, the cross section of the intrusion; then

$$\Delta Z_m \cong 2\, \frac{A}{I_o'}\, \frac{B}{I_o}\, \eta \quad SD .$$

3.2.2 Solution for Z_m

The analysis for the mutual impedance of the two coaxial cables in the case of non-radiating cables proceeds as follows: The expressions for the fields are taken to be

$$E_z \cong B\eta \frac{K_o(\beta r)}{K_o(\beta \frac{w}{2})} e^{-ikz} ,$$

$$H_t \cong B \frac{K_1(\beta r)}{K_1(\beta \frac{w}{2})} \cos\phi \, e^{-ikz} ,$$

$$E_t' \cong 0 ,$$

and

$$H_t' \cong 2A \frac{K_1(\beta r)}{K_1(\beta \frac{w}{2})} \cos\phi \, e^{ikz} .$$

The integrand of the expression for the mutual impedance is

$$[\bar{E}' \times \bar{H} - \bar{E} \times \bar{H}']_n = 2AB\eta \frac{[K_o(\beta r)]^2}{K_o(\beta \frac{w}{2}) K_1(\beta \frac{w}{2})} \cos\phi$$

and in terms of an integral over the coordinate, x,

$$\Delta Z = 2\eta \frac{A}{I_o'} \frac{B}{I_o} L \frac{w}{2[K_o(\beta \frac{w}{2})]^2} \int_{-\infty}^{\infty} \frac{[K_o(\beta\sqrt{x^2 + (\frac{w}{2})^2})]^2}{\sqrt{x^2 + (\frac{w}{2})^2}} dx$$

Replacing the modified Bessel function by its asymptotic approximation permits the above expression to be written

$$\Delta Z \cong 2\eta \frac{A}{I_o'} \frac{B}{I_o} L \frac{w\pi 2}{2K_o(\beta \frac{w}{2}) K_1(\beta \frac{w}{2}) 2\beta} \int_0^{\infty} \frac{e^{-2\beta\sqrt{x^2 + (\frac{w}{2})^2}}}{x^2 + (\frac{w}{2})^2} dx .$$

The integral in the above expression can be transformed as follows:

$$I = \int_0^{\infty} \frac{e^{-2\beta\sqrt{x^2 + (\frac{w}{2})^2}}}{x^2 + (\frac{w}{2})^2} dx = \int_{\frac{w}{2}}^{\infty} \frac{e^{-2\beta z}}{z\sqrt{z^2 - (\frac{w}{2})^2}} dz$$

where z is simply a variable of integration not related to the coordinate, z. This leads to the expression

$$\frac{\partial I}{\partial \beta} = -2 \int_{\frac{w}{2}}^{\infty} \frac{e^{-2\beta z}}{\sqrt{z^2 - (\frac{w}{2})^2}} dz \ .$$

The integral can be evaluated to give the result

$$\frac{\partial I}{\partial \beta} = -2 \, K_o \, (\beta w)$$

If the asymptotic approximation is used for the modified Bessel function, the above expression can be integrated to give

$$I \cong -2\sqrt{\frac{\pi}{2}} \int \frac{e^{-\beta w}}{\sqrt{\beta w}} d\beta = -\frac{\sqrt{2\pi}}{w} \operatorname{erf}\sqrt{\beta w} + C$$

where $\operatorname{erf}\sqrt{\beta w}$ is the error function of argument $\sqrt{\beta w}$ and C is a constant of integration which can be evaluated using the requirement that $\lim I = 0$ as $\beta \to \infty$. This dictates that $C = 1$; so I can be written in terms of the complementary error function.

$$I \simeq \frac{\sqrt{2\pi}}{w} \operatorname{erfc} \sqrt{\beta w}$$

It follows now that ΔZ can be written

$$\Delta Z \simeq 2\eta \ \frac{A}{I_o'} \frac{B}{I_o} \sqrt{\frac{\pi}{2}} \ \frac{Lw}{\sqrt{\beta w}} \ F$$

where

$$F = \frac{(\pi)^{3/2} \operatorname{erfc} \sqrt{\beta w}}{\sqrt{\beta w}\, K_o (\beta\frac{w}{2}) K_1 (\beta\frac{w}{2})}$$

It may be noted that for common values of the parameters $F \simeq 1$ and $\lim F \simeq 1$ as $\beta \to \infty$.

3.2.3 Solution for $\Delta V/V$

Since ΔZ is an approximation to Z_m, an expression for $\Delta V_r/V_r$ can now be written.

$$\frac{\Delta V_r}{V_r} = \frac{\Delta Z_m}{Z_m} \simeq \frac{2\eta \, \frac{A}{I_o'} \, \frac{B}{I_o} \, SD}{2\eta \, \frac{A}{I_i'} \, \frac{B}{I_o} \sqrt{\frac{\pi}{2}} \, \frac{Lw}{\sqrt{\beta w}} \, F}$$

Since $D \cong 1$ and $F \cong 1$

$$\frac{\Delta V_r}{V_r} \cong \frac{S\sqrt{\beta w}}{\sqrt{\frac{\pi}{2}} \, Lw} .$$

If the dimensions are expressed in terms of a quantity λ defined by $\lambda = \frac{2\pi}{\beta}$ and s is the cross section in units of λ^2, the above expression becomes*

$$\frac{\Delta V_r}{V_r} \cong \frac{\sqrt{2}\, s_\lambda}{(L/\lambda)\sqrt{w/2\lambda}} = \frac{s_\lambda}{(L/\lambda)\sqrt{w/4\lambda}}$$

It is of interest to note that the result for $\Delta V_r/V_r$ for non-radiating cables is the same in magnitude as for radiating cables. The only difference in the expression is a factor, $e^{-i\frac{\pi}{4}}$ which has a magnitude of unity and it is present in the expression for the radiating cables.

A very straightforward interpretation of the above result is possible ; the ratio $\Delta V_r/V_r$ equals the ratio of the cross section of the intrusion in units of λ^2 as sketched in part (a) of Fig. 10 to the area sketched in part (b).

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*The quantity λ cannot be called a wavelength in this case as was done in the case of the radiating cable because in this case there is no radiation; it could more properly be called a distance-of-decay characterizing the fields in the radial direction.

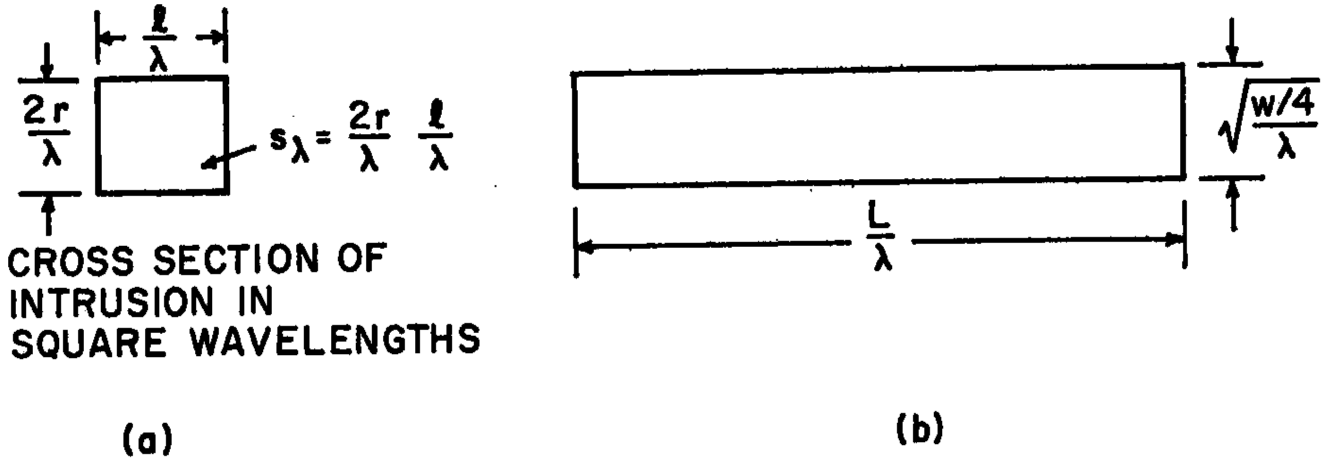

Fig. 10 Sketch of Areas Used to Determine the Sensitivity of an Intrusion Sensing System. Sensitivity is the Ratio of the Area in Part (a) to that in Part (b).

4. Extensions and conclusions

The analysis of the previous section yielded an approximate result for the sensitivity of an isolated intrusion sensing system using radiating cables. The result takes a rather simple form relating the change ΔV in receiver input voltage V resulting from an intrusion of cross section s_λ (where $s_\lambda = S/\lambda^2$; λ is the wavelength in the radial direction and S is the cross section in m^2) in the following manner

$$\frac{\Delta V}{V} \approx \frac{s_\lambda \, e^{-i\frac{\pi}{4}}}{L/\lambda \sqrt{\frac{W}{4}/\lambda}} \quad .$$

It is seen that the sensitivity of a system depends upon its length, L, and one quarter of the spacing between the cables or $\frac{W}{4}$. If these are expressed in wavelengths, λ , the result is the dimensionless quantities L/λ and $\frac{W}{4}/\lambda$. Now consider a rectangle having sides measuring L/λ and $\sqrt{\frac{W}{4}/\lambda}$. The area* of this rectangle is easily calculated (it is the product, $L/\lambda \sqrt{\frac{W}{4}/\lambda}$). The quantity $|\Delta V/V|$ can now be described as the ratio of the cross section s_λ of the intrusion (as viewed from the transmitter cable) measured in square wavelengths to the area defined above.

A diagram may make the concepts somewhat simpler. The characteristic area which serves to characterize the system sensitivity is sketched in Fig. 11. An intrusion having a cross section in square wavelengths, of say 0.1 of the characteristic area will produce a voltage change at the receiver input of ΔV such that $|\Delta V/V| = 0.1$ or $|\Delta V| = 0.1|V|$.

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*The term area as used here may be confusing since it is the product of two dimensionless quantities rather than the product of two lengths. Nevertheless it appears that the concepts involved are most easily discussed with the aid of the term area used in the sense defined here.

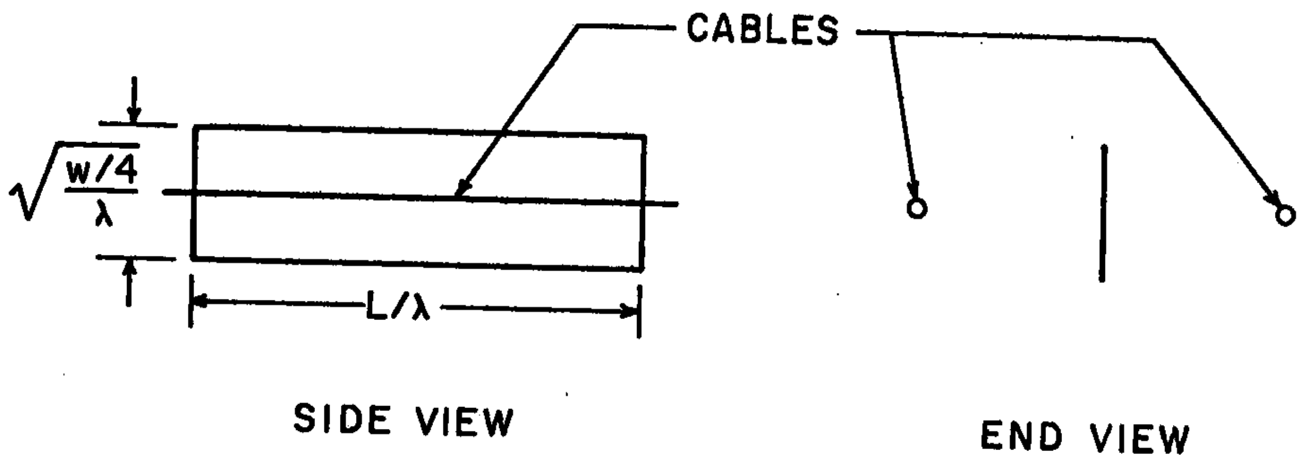

Fig. 11 Characteristic Area in Terms of Which System Sensitivity is Defined

Note that the characteristic area increases as $w^{\frac{1}{2}}$ so the minimum detectible cross section will also increase as $w^{\frac{1}{2}}$. The derivation of these results also suggests (though it was not proven) that the region within which intrusions are detectible extends approximately to distances $\sqrt{\frac{W}{4}/\lambda}\ /2$ units (that is λ-units) above and below the plane of the cables.

The case of an intrusion sensing system utilizing non-radiating cables was analyzed in a manner similar to that for radiating cables. The approximations are more difficult to justify in this case, but it is felt that the results are sufficiently accurate to use for order-of-magnitude estimates as guidelines for system designers. The result of the analysis is the same expression as for the case of radiating cables except that the phase factor $e^{-i\frac{\pi}{4}}$ is missing and the parameter is not wavelength in this case but rather it is a characteristic distance defined by $\lambda = \frac{2\pi}{\beta}$ where β is the multiplicative constant in the argument of the modified Bessel function, $K_o(\beta r)$, occurring in the description of the fields of the cables. In terms of magnitude only, the results are the same, The result in this case is

$$\frac{\Delta V}{V} \simeq \frac{S_\lambda}{L/\lambda\sqrt{\frac{W}{4}/\lambda}} \quad .$$

The analyses in both the radiating cable case and non-radiating cable case were made for isolated cable pairs rather than for cable pairs in close proximity to the earth which is the case of actual interest. The analysis were made by expressing the fields in the vicinity of the intrusion in terms of the fields that would exist at the location of the center of the intrusion if the intrusion were absent. It can be argued that the presence of the earth near the cables would drastically alter the fields at the location of the center of the intrusion but in the expression for $\Delta V/V$ these fields occur in both the numerator and in the denominator and thus cancel out. In other words the presence of the earth will greatly reduce the magnitude of ΔZ_m but it will also greatly reduce the magnitude of Z_m. It is expected that the reduction will be approximately the same in the two terms so the ratio $\Delta Z_m/Z_m$ will be approximately the same as in the absence of the earth. For these reasons it is expected that the results derived for the case in which the earth is not present are also approximately correct for the case in which the cables are in close proximity to the earth. In this regard it is well to recall that the objectives of the analysis are order-of-magnitude guidelines for use in system planning and design.

An alternative configuration for the basic two-wire intrusion sensing system of Fig. 1 is that having the transmitter and receiver on the same end of the system rather than on opposite ends. The consequence of such a modification is the inclusion of an additional factor e^{-i2kz} in the integrand of the expressions for both ΔZ_m and Z_m. This in turn modifies ΔZ_m by the factor $e^{-i2kz_o} \frac{\sin k\ell}{k\ell}$ where z_o is the z-coordinate of the center of the intrusion and ℓ is the dimension of the intrusion along the axis. Z_m is modified by the factor $e^{-ikL} \frac{\sin kL}{kL}$. It follows that the sensitivity of the system as measured by the ratio $\Delta V_r/V_r$ is modified by the factor

$$e^{-ik(2z_o-L)} \left(\frac{\sin k\ell}{\sin kL}\right)\left(\frac{\ell}{L}\right)$$

The term $e^{-ik(2z_o-L)}$ always has a magnitude of one. The factor ℓ/L is substantially less than one for all configurations for which the analysis is applicable. The factor $(\sin k\ell/\sin kL)$ could be significantly greater than one for certain combinations of values of ℓ and L, however, for such combinations, signal levels at the receiver input are likely to be so low that system noise would adversely influence the operation of the system. It is concluded therefore that the system having transmitter and receiver at the same end is significantly less sensitive than that having them at opposite ends and therefore generally should not be used. However, if such an arrangement is needed, the sensitivity can be substantially restored simply by placing a perfectly conducting termination on the end of the receiver cable opposite from the receiver. This makes the system nearly the same as if the receiver and transmitter were on opposite ends and will nearly restore the sensitivity to the level thereby attainable.

The entire analysis in this report has been based on the assumption that the field incident upon the intrusion consists of an electric field oriented axially with respect to the cables and a magnetic field oriented azimuthally. Other waveguiding systems may produce fields of different configurations but it is expected that an analysis based upon those different configurations will produce results essentially similar to those obtained above. Actually it is felt that the results of this investigation, as expressed in terms of the ratio of the cross section of the intrusion to a characteristic cross section based upon the geometry of the system (ref. pp. 33 and 38) are reasonably insensitive to the nature of the fields of the waveguiding structures, to the presence of the earth, to the height of the intrusion (up to a limit), and to the shape and composition of the intrusion.