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Essays on the Role of Investment-Specific Technology in Business Cycles

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Essays on the Role of Investment-Specific Technology in Business Cycles

Thesis directed by Associate Professor Martin Boileau

The recent study of Schmitt-Grohè and Uribe (2011) show that aggregate neutral productivity and investment-specific technology are cointegrated. How do the two different sources of technological progress share a common stochastic trend? I review the linkage between the cointegration of sectoral productivities and that of neutral productivity and investment-specific technology. In this paper, the linkage is investigated in two economic frameworks: a closed economy and a small open economy.

The first chapter studies U.S. business cycles by considering cointegrated sectoral productivities and investment-specific technology. Applying Johansen cointegration test to U.S. annual data constructed from the EU KLEMS database, this chapter documents that the productivities of consumption-goods and equipment sectors are cointegrated. It confirms further, using the non-linear cointegration test developed by Kapetanios et al. (2006), that the cointegration is non-linear. Also, I derive a theoretical proposition that sectoral productivities for consumption-goods and equipment are cointegrated if and only if the aggregate neutral productivity and investment-specific technology are cointegrated. Plus, I consider the non-linear cointegration of sectoral productivities to examine the role of the common stochastic trend of sectoral productivities in explaining the movements of investment-specific technology as well as those of interesting macroeconomic aggregates. For this end, I develop a two-sector dynamic stochastic general equilibrium (DSGE) model, where the non-linear cointegration of sectoral productivities is incorporated as a vector error correction model (VECM) with exponential smooth transition(ESTR) error correction term. Most of structural parameters are estimated via maximum likelihood with all significant external innovations. Simulation results show that the innovations of common stochastic trend of sectoral productivities account for half of consumption's, 79 percent of investment's, and 6 percent of hours' long-run variation.

The second chapter investigates the role of technology-embodied imports and investment-specific technology in the business cycles of a small open economy. This chapter documents that, using the EU KLEMS database, the investment-specific technology of Canada and Korea are substantially affected by foreign innovations. Considering the factors consisting investment-specific technology, I construct a dynamic stochastic general equilibrium (DSGE) model for a small open economy and do maximum likelihood estimation for the structural parameters with the Korean data. The simulation results indicates that the terms of trade shock for consumption goods and the shocks of embodied technology in imports explain a sizeable fraction of macroeconomic variations in the Korean economy along with countercyclical trade balance. It also identifies the significant role of the common trend shocks of sectoral productivities on the Korean business cycles, which is consistent to the findings of Aguiar and Gopinath (2007) arguing that shocks to trend growth are the main sources of fluctuation in emerging economies.

Dedication

This dissertation is dedicated to my parents.

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Chapter 1

Cointegrated Sectoral Productivities and Investment-Specific Technology in U.S. Business Cycles

1.1 Introduction

Since the seminal work of Greenwood et al. (1997, 2000), investment-specific technology (IST) has become a leading candidate as a main source of economic growth and fluctuation rather than aggregate neutral productivity (or total factor productivity: TFP). They also suggest that IST can be expressed by the ratio of the productivity in equipment (or capital-goods) sector to that in consumption-goods sector. There is a hardship, however, in interpreting the progress of IST as a technological progress of equipment sector.[1] Oulton (2007) comments that IST may alter without a relative change in sectoral productivities between consumption-goods and equipment sectors.[2] Furthermore, Whelan (2003) insists that a two-sector approach incorporating relatively high technological progress of durable goods better explain the long-run behavior of the U.S. economy. As another modification to the literatures for investment-specific technology, Schmitt-Grohé and Uribe

[1]Recent empirical studies show that the relative price of capital-goods does not correctly measure the relative productivity changes. Basu et al. (2010) estimate technological changes at a disaggregated industry level and aggregate them by using the U.S. input-output tables. Their finding suggests that relative price does not properly measure the relative technological change. Adopting the two-sector model calibrated on the U.S. input-output tables, Guerrieri et al. (2010) conclude that the effect of productivity shock in machinery sector is qualitatively different from that of IST shock. They argue that the productivity shock in machinery boosts consumption at all succeeding periods while IST shock reduce consumption on the impact.

[2]In the case of different factor intensity in the two sectors, Oulton (2007) points out that the relative price may change without a change in sectoral productivities.

(2011) introduce the cointegrated relation between TFP and IST, which implies the existence of common stochastic trends in TFP and IST. They further insist that the innovations in the common stochastic trends explain a sizeable fraction of volatilities of output, consumption, investment, and hours.

To investigate the features of business cycles in the U.S. economy, this paper considers the two ways of modification for investment-specific technology exhibited above. Firstly, investment-specific technology is concerned in a two-sector framework as in Whelan (2003). Because the investment-specific technology is defined as the ratio of sectoral marginal products, the investment-specific technology can be decomposed into two sectoral productivities. Ireland and Schuh (2008) establish a two-sector economy model incorporating the shocks of sectoral productivities, inspired by Whelan (2003), to study the U.S. business cycles. Their study does not reflect the fact, however, that the sectoral productivities are cointegrated; it is implied by the empirical findings of Schmitt-Grohé and Uribe (2011). To fill the gap between the empirical findings and the existing literatures with two-sector framework, this paper investigates the cointegration of sectoral productivities in the U.S. economy and incorporates it into a typical two-sector business cycles model.

To shed light on the cointegration of sectoral productivities, two independent analyses are performed. First, I conduct the Johansen cointegration test on two sectoral productivities of consumption-goods and equipment sectors, which are constructed from the EU KLEMS database[3]. The test statistics confirm the cointegration between sectoral productivities[4]. As the second way to illuminate sectoral cointegration in productivity, I establish theoretical propositions based on the findings of Schmitt-Grohé and Uribe (2011) that the aggregate neutral productivity and investment-specific technology are cointegrated. The propositions imply that the sectoral productivities are

[3]For more details about the EU KLEMS database, refer to O'Mahony and Timmer (2009). The data is available at www.euklems.net.

[4]Marquis and Trehan (2008) capture the idea that the productivities of consumption-goods and equipment shares common shocks. They fail to estimate, however, the cointegration between sectoral productivities, and just incorporate the correlation between the growth rate of the equipment productivity and that of consumption-goods productivity.

cointegrated if and only if TFP and IST are cointegrated. Thereby the sectoral cointegration is supported by the empirical findings of Schmitt-Grohé and Uribe (2011).

Applying the cointegration of sectoral productivities into a dynamic stochastic general equilibrium (DSGE) model, this paper examines the effects and roles of each external shock, such as the shocks of preference and productivities, in the U.S. business cycles. For the external shocks, as in Ireland and Schuh (2008), I consider a transitory- and a permanent-shock of preference and sectoral productivities. Especially, to incorporate the cointegration of sectoral productivities into the DSGE model, I employ the vector error correction model (VECM) framework for the cointegration of sectoral productivities. Plus, to ensure a stationary error correction dynamics, I introduce a smooth transition non-linear error correction (STR NEC) featured by exponential function into the vector error correction (VEC) system of sectoral productivities. Using the established stationary model, I perform the maximum likelihood estimation to estimate the deep parameters including sectoral capital shares without symmetric assumption. The estimated sectoral capital shares confirm the conventional wisdom that consumption-goods sector is relatively labor-intensive, whereas equipment sector is capital-intensive. More importantly, different to Ireland and Schuh (2008) which estimate an insignificant permanent (or growth-rate) shock of equipment sector, all estimated external shocks of this paper are statistically significant.

As results, I find sizeable effects of common stochastic trends in sectoral productivities to business cycles with persistence. Innovations in the common stochastic trends of consumption-goods and equipment sectors increase consumption and investment almost permanently, and explains the long-run variabilities of about 48 percent and 79 percent in consumption and investment, respectively, as well as 6 percent of hours-worked variability. Similarly to Ireland and Schuh (2008), the innovation of preference gives highly persistent and sizeable effects on hours worked. Moreover, the preference shocks account for half of consumption variability and most of hours-worked variability.

The remainder of the paper is organized as follows. Section 1.2 illuminates the cointegration in the U.S. sectoral productivities both in empirical and theoretical ways. Section 1.3 establishes a model economy incorporating the cointegrated sectoral productivities. Section 1.4 estimates the model with the maximum likelihood method and discusses the estimates. Section 1.5 examines the impulse responses and the contributions of structural shocks to forecast error variance. Lastly, Section 1.6 concludes this paper.

1.2 Cointegrated productivities

In this section I examine whether the sectoral productivities are cointegrated in two ways. First, the cointegration is tested empirically. For this, I construct annual sectoral productivities, aggregate TFP, and relative price of equipment from the EU KLEMS database and use them for unit-root tests and Johansen cointegration tests. On the other hand, Schmitt-Grohé and Uribe (2011) have found that the aggregate TFP and IST are cointegrated by using the U.S. quarterly data. By considering a neoclassical two-sector framework, I derive a proposition which shows that the empirical finding of Schmitt-Grohé and Uribe (2011) implies the cointegration of sectoral productivities in the U.S. economy.

1.2.1 Empirical evidence

To conduct the empirical cointegration test, we need to construct sectoral productivities, aggregate TFP and relative price of equipment. For this purpose I use the annual U.S. data of EU KLEMS Growth and Productivity database ranging 1970-2005. This data selection is different to that of Schmitt-Grohé and Uribe (2011), which examine the cointegration of TFP and relative price of equipment rather than sectoral cointegration. However, due to EU KLEMS, a highly disaggregated industrial productivity database, we can directly test the cointegration of sectoral productivities.

1.2.1.1 Data

EU KLEMS includes the 72 sectoral definitions. To be used in the empirical tests, 72 industrial levels have to be aggregated into two sectors; consumption-goods and equipment sectors. For aggregation, I define the equipment sector as the aggregation of Electrical and optical equipment (30t33), Machinery (29) and Transport equipment (34t35)[5], and the rest are aggregated for consumption goods sector. The Törnqvist index (or Divisia index) is applied for the aggregation. For example, log-differenced capital service input of a higher sector i is the weighted average of the log-differenced capital service of its sub-sectors; the applied formula is

$$\Delta \ln K_t^i = \sum_j \bar{\omega}_{K,j,t}^i \Delta \ln K_{j,t}^i,$$

where K_t^i is the capital service of sector i, $K_j^{i,t}$ exhibits the capital demand in sub-sector j of sector i, $j \in i$, and $\bar{\omega}_{K,j,t}^i$ is the two-period moving average of the capital input share demanded by sub-sector j out of the total demand of sector i, which satisfies $\sum_j \bar{\omega}_{K,j,t}^i = 1$, $\forall t$. The aggregations for sectoral output, intermediate input, and labor services adopt the same method as capital service. Under the growth accounting framework suggested by Jorgenson and Griliches (1967), I construct the log-differenced productivity measures by using the aggregated input and output series through the following formula:

$$\Delta \ln A_t^i = \Delta \ln Y_t^i - \bar{v}_{X,t}^i \Delta \ln X_t^i - \bar{v}_{K,t}^i \Delta \ln K_t^i - \bar{v}_{L,t}^i \Delta \ln L_t^i,$$

where A_t^i represents the Solow residual (or TFP) of sector i for $i \in \{tot, cons, equip\}$.[6] Y_t^i, X_t^i, K_t^i and L_t^i respectively denote the output, intermediate input, capital service and labor service of sector i. $\bar{v}_{l,t}^i$ indicates the two-period moving average of the share of input factor l, which satisfies $\sum_l \bar{v}_{l,t}^i = 1$, $\forall i, t$.

Price movements can be captured by the implicit GDP deflators. Log-differenced GDP deflator of sector i is formulated as

$$\Delta \ln P_t^i = \Delta \ln N.VA_t^i - \Delta \ln R.VA_t^i,$$

where $N.VA_t^i$ and $R.VA_t^i$ represent the nominal value added and real value added in sector i, $i \in \{cons, equip\}$, respectively. Then, I can construct the log-differenced relative price of equipment

[5]The number inside of parentheses indicates the industry code in the EU KLEMS database (the version of additional industry aggregation).

[6]*tot*, *cons* and *equip* stand for aggregate economy, consumption goods sector and equipment sector, respectively.

in terms of consumption ($\Delta \ln RP$) from the following:

$$\Delta \ln RP = \Delta \ln P_t^{equip} - \Delta \ln P_t^{cons}.$$

Using the log-differenced variables constructed above, I derive an index series of those variables with base year 1995. First I set the year before the starting year of each series to 100, and then apply the following formula forwardly:

$$x_{t+1} = x_t \times \exp\left(\Delta \ln x_{t+1}\right),$$

where x_t is a time-series variable, which starts with 100 and has a known $\Delta \ln x_{t+1}$, $\forall t$. Finally, I normalize indices with the value of base year 1995.

1.2.1.2 Empirical findings

Unit-root and cointegration tests are conducted for the logarithms of aggregated TFP, sectoral productivities, and relative price of equipment by using the data constructed above. As first, augmented Dickey-Fuller (ADF) and Dickey-Fuller GLS (DF-GLS) tests are performed to test the unit root. TABLE 1.1 presents the results. The ADF test fails to reject the unit-root hypothesis except for the relative price of equipment without trend. Because DF-GLS known as the increased power of test cannot reject the null hypothesis of unit-root in all tested variables (with and without trend), however, we can say that the variables are non-stationary. Furthermore, to check the order of integration of the non-stationary variables, I conduct the unit-root tests for the first-differenced logged variables, which are not reported here, and all test statistics reject the null hypothesis of unit-root. Based on the results so far, I can therefore conclude that all logged variables of aggregate TFP, TFP in consumption-goods, TFP in equipment and relative price of equipment are integrated by order one.

Schmitt-Grohé and Uribe (2011) find the cointegration of TFP and relative price of equipment with the U.S. quarterly data. To confirm the consistency of their result, I conduct Johansen cointegration tests with various sets of variables including the dataset of TFP and the relative

Table 1.1: Unit-root tests for the logarithms of productivities and relative price of equipment

Data	Test	Trend	Lags (AIC)	Test-stats.	Critical values (5%)	Null hypothesis
TFP.cons	ADF	No	1	1.15	-1.95	Accept
	ADF	Yes	1	-2.12	-3.5	Accept
	DF-GLS	No	1	-0.319	-1.95	Accept
	DF-GLS	Yes	1	-2.38	-3.19	Accept
TFP.equip	ADF	No	1	2.72	-1.95	Accept
	ADF	Yes	1	-0.46	-3.5	Accept
	DF-GLS	No	1	1.48	-1.95	Accept
	DF-GLS	Yes	1	-0.976	-3.19	Accept
TFP.tot	ADF	No	1	1.83	-1.95	Accept
	ADF	Yes	1	-1.44	-3.5	Accept
	DF-GLS	No	1	0.901	-1.95	Accept
	DF-GLS	Yes	1	-1.93	-3.19	Accept
RP	ADF	No	1	-3.07	-1.95	Reject
	ADF	Yes	1	-0.357	-3.5	Accept
	DF-GLS	No	1	1.48	-1.95	Accept
	DF-GLS	Yes	1	-0.772	-3.19	Accept

Notes: All unit-root tests fail to reject the null hypothesis of unit-root except the ADF test for RP without trend. Tests are conducted using the R program with the "urca" package. ADF stands for Augmented Dickey-Fuller, and DF-GLS stands for Dickey-Fuller Generalized Least Squares. TFP.cons, TFP.equip, TFP.tot, and RP denote the productivity of consumption-goods sector, the productivity of equipment sector, the productivity of aggregate economy, and the relative price of equipment, respectively.

price of equipment. The test results of the Johansen trace and maximum eigenvalue tests are exhibited in Table 1.2 and 1.3, respectively.

Both Johansen tests, trace and maximum eigenvalue, confirm that the system of logged aggregate TFP and sectoral productivities (db1) have one cointegrating vector, which implies logged TFP can be expressed as a linear combination of two logged sectoral productivities and one anonymous stationary series. The conventional wisdom on growth accounting and recent aggregation method of sectoral outputs also supports this result. The system of the logged relative price of equipment and sectoral productivities (db2) have cointegrated with one cointegrating vector.[7] The cointegration of logged TFP and relative price of equipment (db5) is tested and we can successfully

[7] According to Greenwood et al. (1997), the logged relative price of equipment equals the difference of logged productivity of equipment and that of consumption-goods; the implied cointegrating vector is $(1, 1, -1)$ for the system of $(\ln \text{RP}, \ln \text{TFP.equip}, \ln \text{TFP.cons})$. The estimated cointegrating vector from the Johansen test, however, fails to reproduce the implied sign of the cointegrating vector.

Table 1.2: The Johansen trace test for cointegration

Database	Cointegration rank	Lags (AIC)	Test-stats.	Critical values (5%)	Null hypothesis
db1	r <= 2	3	0.103	8.18	-
	r <= 1		13.524	17.95	Accept
	r = 0		40.328	31.52	Reject
db2	r <= 2	3	0.35	8.18	-
	r <= 1		7.31	17.95	Accept
	r = 0		37.00	31.52	Reject
db3	r <= 2	3	0.0765	8.18	-
	r <= 1		7.4565	17.95	Accept
	r = 0		37.0785	31.52	Reject
db4	r <= 2	3	0.433	8.18	-
	r <= 1		7.375	17.95	Accept
	r = 0		36.863	31.52	Reject
db5	r <= 1	3	1.62	8.18	Accept
	r = 0		21.13	17.95	Reject
db6	r <= 1	3	0.324	8.18	Accept
	r = 0		20.898	17.95	Reject

Notes: The Johansen trace tests confirm the cointegration of all specified datasets with one cointegrating vector. Tests are conducted using the R program with the "urca" package. Test models don't include both constant and trend. The dataset used for the Johansen cointegration test are defined as follows:
db1: TFP.tot, TFP.cons, TFP.equip
db2: RP, TFP.cons, TFP.equip
db3: TFP.tot, RP, TFP.cons
db4: TFP.tot, RP, TFP.equip
db5: TFP.tot, RP
db6: TFP.cons, TFP.equip

confirm the result of Schmitt-Grohé and Uribe (2011). Adding each sectoral productivity on "db5", two three-variable systems (db3 and db4) are also examined for cointegration. Interestingly, both systems accept cointegration with one cointegrating vector. The simultaneous cointegrations of the two systems of variables (the system of TFP and IST (db5), and that of TFP, IST, and an augmented sectoral productivity (db3 or db4)) let us infer the fact that sectoral productivities are cointegrated.[8] The cointegration test for sectoral productivities (db6) confirms that the inference is right.

[8] Suppose sectoral productivities are not cointegrated. Then, to make the variable system of db3 and db4 stationary, sectoral productivities should follow a stationary stochastic process. It contradicts, however, the non-stationary assumption of sectoral productivities.

Table 1.3: The Johansen maximum eigenvalue test for cointegration

Database	Cointegation rank	Lags (AIC)	Test-stats.	Critical values (5%)	Null hypothesis
db1	r = 2	3	0.103	8.18	-
	r = 1		13.421	14.9	Accept
	r = 0		26.804	21.07	Reject
db2	r = 2	3	0.35	8.18	-
	r = 1		6.96	14.9	Accept
	r = 0		29.68	21.07	Reject
db3	r = 2	3	0.0765	8.18	-
	r = 1		7.3799	14.9	Accept
	r = 0		29.6221	21.07	Reject
db4	r = 2	3	0.433	8.18	-
	r = 1		6.941	14.9	Accept
	r = 0		29.489	21.07	Reject
db5	r = 1	3	1.62	8.18	Accept
	r = 0		19.50	14.9	Reject
db6	r = 1	3	0.324	8.18	Accept
	r = 0		20.574	14.9	Reject

Notes: The Johansen maximum eigenvalue tests confirm the cointegration of all specified datasets with one conintegrating vector. Tests are conducted using the R program with the “urca” package. Test models don’t include both constant and trend. The dataset used for the Johansen cointegration test are defined as follows:
db1: TFP.tot, TFP.cons, TFP.equip
db2: RP, TFP.cons, TFP.equip
db3: TFP.tot, RP, TFP.cons
db4: TFP.tot, RP, TFP.equip
db5: TFP.tot, RP
db6: TFP.cons, TFP.equip

The cointegration among sectoral productivities indicates the possibility that the comovements of aggregate variables and sectoral comovements may arise not only from sectoral linkages but also from a common stochastic trend shared by sectors. Most of the literature in multi-sector business cycles has investigated the sectoral comovements with sectoral structural linkages: Hornstein and Praschnik (1997) and Horvath (2000) incorporate intermediate inputs into their model economy to foster sectoral linkages and find positive sectoral comovement in output and employment. However, the empirical findings in Tables 1.2 and 1.3, which exhibit the existence of a common stochastic trend in sectoral productivities, suggest that the common stochastic trend of sectoral productivities is another key to solve the sectoral comovement puzzle.

1.2.2 Theoretical approach

Schmitt-Grohé and Uribe (2011) exhibit that TFP and IST share common stochastic trends by using the U.S. quarterly data. Then, where do the common trends come from? To address this question, I first ignore the empirical results introduced in the previous subsection except for the findings of Schmitt-Grohé and Uribe (2011). There are two reasons. First, the cointegration test with annual data is sensitive to lag selection due to the small sample property. Hence, the findings of quarterly data ranging 1948-2006 are much more reliable compared to the annual data. Secondly, I show that the existence of a common trend in sectoral productivities can be proven without using the sophisticatedly disaggregated high-quality database.

Since Greenwood et al. (1997), many studies with two-sector models identify IST as the ratio of the productivity of equipment to that of consumption-goods. As such, the behavior of IST reflects the relative change of sectoral productivities. To examine the relation formally, let us consider a simplified neoclassical two-sector model as in Oulton (2007); one sector is for producing consumption-goods and the other produces equipment. A benevolent social planner would maximize aggregate social utility, $U(C_t, N_t)$, in an infinite time horizon with the given resource constraint,

$$C_t + \tilde{I}_t = \tilde{Y}_t, \tag{1.1}$$

where C_t is an aggregate consumption, $\tilde{I}_t$ is a forgone consumption or savings for investment spending, and $\tilde{Y}_t$ is households' income in terms of consumption goods. The investment spending is used for purchasing equipment and eventually contributes to capital accumulation as follows:

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{1.2}$$

where K_t is a capital stock at the beginning of period t, δ denotes depreciation rate of capital stocks, and I_t is the amount of newly produced equipment used for gross investment during period t. Note that the gross investment, I_t, is measured in the unit of equipment, whereas the investment spending, $\tilde{I}_t$, takes the unit of consumption. In capital accumulation the investment spending must be therefore transformed into the unit of equipment. Suppose that Q_t governs the linear transformation of the forgone consumption, then we can rewrite Eq.(1.2) as [9]

$$K_{t+1} = (1-\delta)K_t + \tilde{I}_t Q_t. \tag{1.3}$$

Since the nominal investment spending, $P_{c,t}\tilde{I}_t$, should equal the market value of investment, $P_{e,t}I_t$, Eq.(1.2) and Eq.(1.3) imply

$$Q_t \equiv \frac{P_{c,t}}{P_{e,t}}, \tag{1.4}$$

where $P_{c,t}$ is the market price of consumption goods, $P_{e,t}$ is the price for newly produced equipment and Q_t is known as IST by Greenwood et al. (1997).

Each representative producer of both sectors uses capital and labor in its constant return to scale production function with its own neutral technological progress as follows:

$$Y_{c,t} = Z_{c,t}F^c\left(K_{c,t}, N_{c,t}\right), \tag{1.5}$$

$$Y_{e,t} = Z_{e,t}F^e\left(K_{e,t}, N_{e,t}\right), \tag{1.6}$$

where $Y_{c,t}$ and $Y_{e,t}$ are the outputs of consumption goods and equipment sector, respectively. $K_{j,t}$ and $N_{j,t}$ stand for capital and labor inputs, respectively, of sector $j \in \{c, e\}$. The sum of each input across sectors satisfies the feasibility conditions: $N_t \geq N_{c,t} + N_{e,t}$ and $K_t \geq K_{c,t} + K_{e,t}$. Suppose that $Z_{j,t}$ represents the neutral productivity of sector j and has a random walk process as follows:

$$\ln Z_{c,t} = \ln Z_{c,t-1} + \epsilon_{c,t}, \tag{1.7}$$

$$\ln Z_{e,t} = \ln Z_{e,t-1} + \epsilon_{e,t}, \tag{1.8}$$

where both $\epsilon_{c,t}$ and $\epsilon_{e,t}$ are independent white noises. Note that both sectoral productivities follow uncorrelated random walk processes due to the independently distributed disturbances, $\epsilon_{c,t}$ and $\epsilon_{e,t}$.

Suppose both sectors are in perfect competition, then the representative firms would set their prices at marginal costs, which imply

$$\frac{P_{c,t}}{P_{e,t}} = \frac{Z_{e,t}F_1^e\left(K_{e,t}, N_{e,t}\right)}{Z_{c,t}F_1^c\left(K_{c,t}, N_{c,t}\right)}, \tag{1.9}$$

[9] Schmitt-Grohé and Uribe (2011) estimate the power of transformation as unity, which implies a linear transformation from consumption to investment.

where $F^j(\cdot,\cdot)$ is a constant-returns production function of sector j and $F_1^j(\cdot,\cdot)$ is the partial derivative with respect to the first argument. By considering the equivalence of IST and inverse relative price of equipment given by Eq.(1.4) with the properties of constant returns of production function, we can rewrite Eq.(1.9) as

$$Q_t = \frac{Z_{e,t} f^{e'}(k_{e,t})}{Z_{c,t} f^{c'}(k_{c,t})}, \tag{1.10}$$

where $k_{j,t}$ exhibits a capital per worker in sector j and $f^j(k_{j,t}) = F^j(K_{j,t}/N_{j,t}, 1)$. Suppose further that the production function is Cobb-Douglas as $f^j(k_{j,t}) = k_{j,t}^{\alpha_j}$, then Eq.(1.10) is extended by logged variables as follows:

$$\ln Q_t = \ln Z_{e,t} - \ln Z_{c,t} + S_{q,t}, \tag{1.11}$$

where $S_{q,t} = \ln\alpha_e - \ln\alpha_c - (1-\alpha_e)\ln k_{e,t} + (1-\alpha_c)\ln k_{c,t}$, and α_j indicates the capital share of sector j. Without loss of generality, we can assume that the capital-worker ratio of both sectors follows a stationary process, at most, with a deterministic trend; that is, the capital per worker has a trend-stationary stochastic process. Thus, $S_{q,t}$ is stationary. Since logged Q_t is composed of two uncorrelated random walk processes and a stationary process, the investment-specific productivity, Q_t, also has a random walk process.

On the other hand, the composite output Y_t consists of $Y_{c,t}$ and $Y_{e,t}$ with an aggregator $\Phi(\cdot)$. To make things more precise, suppose that the aggregator is Cobb-Douglas as

$$Y_t = \Phi(Y_{c,t}, Y_{e,t}) = {Y_{c,t}}^{\phi}{Y_{e,t}}^{1-\phi}, \tag{1.12}$$

where $\phi \in [0,1]$ indicates the share of output for consumption goods to the total output. Using the production functions given in Eq.(1.5) and Eq.(1.6), the composite output can be extended by logged variables as

$$\begin{aligned}\ln Y_t &= \phi\ln Z_{c,t} + (1-\phi)\ln Z_{e,t}\\ &\quad + \alpha_c\phi\ln K_{c,t} + \alpha_e(1-\phi)\ln K_{e,t}\\ &\quad + (1-\alpha_c)\phi\ln N_{c,t} + (1-\alpha_e)(1-\phi)\ln N_{e,t},\end{aligned}$$

which implies that the Solow residuals of the aggregate output from a typical growth accounting method is a linear combination of $\ln Z_{c,t}$ and $\ln Z_{e,t}$:

$$\ln A_t \equiv \phi \ln Z_{c,t} + (1-\phi) \ln Z_{e,t}, \tag{1.13}$$

where A_t represents Solow residuals or the aggregate TFP.

Then, logged A_t has to be a random walk because logged $Z_{c,t}$ and $Z_{e,t}$ are uncorrelated $I(1)$ processes by construction. Normalizing Eq.(1.13) with respect to $\ln Z_{e,t}$ and substituting for Eq.(1.11) yields

$$\ln Q_t - (1-\phi)^{-1} \ln A_t + (1-\phi)^{-1} \ln Z_{c,t} = S_{q,t}. \tag{1.14}$$

According to Eq.(1.14), a linear combination of three $I(1)$ processes gives a stationary process, which means the cointegration system of $\ln Q_t$, $\ln A_t$ and $\ln Z_{c,t}$ with the cointegrating vector of $(1, -(1-\phi)^{-1}, (1-\phi)^{-1})$. Another cointegrated relation is derived by substituting Eq.(1.13) for Eq.(1.11) with respect to $\ln Z_{c,t}$:

$$\ln Q_t + \phi^{-1} \ln A_t - \phi^{-1} \ln Z_{e,t} = S_{q,t}. \tag{1.15}$$

Eq.(1.15) implies that $\ln Q_t$, $\ln A_t$ and $\ln Z_{e,t}$ are cointegrated with the cointegration vector of $(1, \phi^{-1}, -\phi^{-1})$. These results can be summarized in the following Proposition 1:

Proposition 1. *Suppose that sectoral productivities,* $\ln Z_{c,t}$ *and* $\ln Z_{e,t}$*, follow uncointegrated* $I(1)$ *processes, then there exists a cointegrating vector that makes the system of three* $I(1)$ *processes* $(\ln Q_t, \ln A_t, \ln Z_{c,t})$ *(or* $(\ln Q_t, \ln A_t, \ln Z_{e,t})$*) stationary.*

Independent sectoral shocks are broadly assumed in most of literatures on multi-sector business cycles, including two-sector specification.[10] According to Proposition 2, however, Proposition 1 contradicts the empirical findings, indicating a cointegrated relation between TFP and IST, which is supported by Schmitt-Grohé and Uribe (2011).

[10]Consistent with Proposition 1, Ireland and Schuh (2008) introduce growth stationary (or log-difference stationary which implies $I(1)$) sectoral productivities in their two-sector model but they assume independent sectoral productivities.

Proposition 2. *Under the assumption of uncointegrated sectoral productivities,* $\ln Z_{c,t}$ *and* $\ln Z_{e,t}$*, following* $I(1)$ *processes, if TFP* $(\ln A_t)$ *and IST* $(\ln Q_t)$ *are cointegrated, there is no such cointegrating vector that makes three* $I(1)$ *processes of* $(\ln A_t, \ln Q_t, \ln Z_{c,t})$ *(or* $(\ln A_t, \ln Q_t, \ln Z_{e,t})$*) stationary.*

PROOF: refer to APPENDIX A.1

To reconcile PROPOSITION 1 with the empirical findings of Schmitt-Grohé and Uribe (2011), I reconsider the underlying assumptions on PROPOSITION 1. First, I consider relaxing the random walk assumption from both sectoral productivities to either one of the two. This modification does not hurt the non-stationary property of the aggregate neutral and investment-specific productivities, while ensuring cointegration between them; at least one non-stationary process is enough to make any linear combination of productivities non-stationary. However, this has not been supported by data. According to TABLE 1.1, U.S. sectoral productivities constructed from the EU KLEMS database reveal that the sectoral productivities have $I(1)$ processes in both sectors.

Another possible modification is to introducing a cointegrated relation of both sectoral productivities, which is also supported by the empirical results for “db6” in TABLES 1.2 and 1.3. To derive a formal theoretical result, first of all, we have to check if this additional assumption grants the property of $I(1)$ process to TFP and IST. For the validity, the cointegrating vector has to satisfy a specific condition. It is helpful to refer to IST given in Eq.(1.11) and aggregate TFP in Eq.(1.13). Both logged TFP and IST are a special linear combination of logged sectoral productivities, $\ln Z_{c,t}$ and $\ln Z_{e,t}$, with different scale vectors; respectively, $(\phi, 1-\phi)$ and $(-1, 1)$. Now suppose that the uncovered cointegrating vector of $(\ln Z_{c,t}, \ln Z_{e,t})$ is $(1, \kappa)$. To ensure the non-stationary property of TFP and IST, κ should not be equal to $(1-\phi)/\phi$ or -1. Accordingly, if the cointegrating vector of sectoral productivities satisfies the conditions mentioned above, the non-stationarity of TFP and IST are preserved and PROPOSITION 3 follows:

Proposition 3. *Suppose* $\ln A_t$*,* $\ln Q_t$*,* $\ln Z_{c,t}$ *and* $\ln Z_{e,t}$ *follow* $I(1)$ *processes. Then,* $\ln A_t$ *and* $\ln Q_t$ *are cointegrated if and only if* $\ln Z_{c,t}$ *and* $\ln Z_{e,t}$ *are cointegrated.*

PROOF: refer to APPENDIX A.1

As we have already seen in TABLES 1.2 and 1.3, PROPOSITION 3 stands on the support of empirical findings. Consequently, an appropriate model for a two-sector economy is better to introduce the cointegrated relation of sectoral productivities. In the following section, the cointegrated sectoral productivities are incorporated into a two-sector DSGE model and are used to estimate deep parameters and analyze the role of the common stochastic trend of sectoral productivities.

1.3 Model

Throughout SECTION 1.2, I have explained why we consider the cointegration of sectoral productivities in a two-sector framework. Considering PROPOSITION 3, this section develops a two-sector business cycle model extended from Ireland and Schuh (2008); their model is established for a two-sector economy of consumption goods and equipment with both transitory and permanent shocks of preference and sectoral productivities. The main difference of this model is the cointegration of sectoral productivities. Additionally, to ensure fully mobile capital across sector, capital accumulation is allowed only at the aggregate level. Also, as real rigidities, capital adjustment cost and habit persistence in consumption are employed. Solving the competitive equilibrium, I introduce IST explicitly into the model; Ireland and Schuh (2008) regard IST as a shadow price.

1.3.1 The household

Consider that the infinitely lived representative household has the preference, described over the habit persistent consumption, C_t, and hours worked, H_t, which is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left(C_t - \xi C_{t-1} \right) - H_t / X_t \right\}, \tag{1.16}$$

where β and $\xi \in [0, 1)$, respectively, denote the subjective discount factor and the degree of habit persistence. X_t stands for the preference shock. The preference shock consists of two stochastic components: the one is a level-stationary cyclical component, $X_{l,t}$, which indicates a transitory shock and the other is a growth-stationary trend component, $X_{g,t}$, which indicates a permanent shock. The functional form of preference shocks are given by

$$X_t = X_{l,t} X_{g,t}, \tag{1.17}$$

$$\ln X_{l,t} = \rho_{xl} \ln X_{l,t-1} + \epsilon_{xl,t}, \tag{1.18}$$

$$\ln\left(\frac{X_{g,t}/X_{g,t-1}}{\eta^{xg}}\right) = \rho_{xg} \ln\left(\frac{X_{g,t-1}/X_{g,t-2}}{\eta^{xg}}\right) + \epsilon_{xg,t}, \tag{1.19}$$

where $\rho_j \in [0,1)$ and ϵ_j, respectively, indicate the autoregressive coefficients and disturbance of stochastic process which is *iid* normal with mean zero and variance σ_j^2 for $j \in \{xl, xg\}$. η^{xg} stands for the long-run steady-state growth rate of preference shock.

In this model economy, the household earns income by supplying labor force and renting capital to the firms, and spends the earned income for consumption and investment purposes. Hence, the household faces the budget constraint of

$$C_t + I_t/Q_t \leq \tilde{W}_t H_t + \tilde{R}_t K_t, \tag{1.20}$$

where $\tilde{W}_t$ and $\tilde{R}_t$ stand for the wage and rent rate in terms of the unit of consumption goods. As we have seen from Eqs.(1.1)-(1.4), investment expenditure, $\tilde{I}_t$, is equal to the gross investment in terms of consumption goods, I_t/Q_t. Capital, K_{t+1}, accumulates through investment, I_t, with capital adjustment cost and constantly depreciated previous capital stock, K_t, as follows:

$$K_{t+1} \leq (1-\delta) K_t + I_t \left[1 - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - \tau^I\right)^2\right], \tag{1.21}$$

where $\psi > 0$ is the parameter for capital adjustment cost, and τ^I denotes the steady state level of investment growth.

The representative household would maximizes its life-time utility, Eq.(1.16), subject to the budget constraint, Eq.(1.20), including the capital accumulation process, Eq.(1.21). The first-order conditions of solving the household's problem are derived as follows:

$$\Lambda_{1,t} = \frac{1}{C_t - \xi C_{t-1}} - \beta\xi\mathbb{E}_t \frac{1}{C_{t+1} - \xi C_t}, \tag{1.22}$$

$$\frac{1}{X_t} = \Lambda_{1,t}\tilde{W}_t, \tag{1.23}$$

$$\frac{\Lambda_{1,t}}{Q_t} = \Lambda_{2,t}\left[1 - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - \tau^I\right)^2 - \psi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - \tau^I\right)\right] + \beta\mathbb{E}_t\Lambda_{2,t+1}\psi\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - \tau^I\right), \tag{1.24}$$

$$\Lambda_{2,t} = \beta\mathbb{E}_t\left[\Lambda_{1,t+1}\tilde{R}_{t+1} + \Lambda_{2,t+1}(1-\delta)\right], \tag{1.25}$$

Eq.(1.20), and Eq.(1.21) with equality, in which $\Lambda_{1,t}$ and $\Lambda_{2,t}$ stand for the Lagrange multipliers on the budget constraint, Eq.(1.20), and capital accumulation process, Eq.(1.21), respectively.

1.3.2 Firms

Two producing firms represent this model economy; one produces consumption-goods and the other produces equipment. For the sake of clarity, I assume that all consumption-goods are non-durables and all equipment are durables. This assumption is consistent with the definition that I used to construct the data of two-sector productivity in Section 1.2.1. Equipment is usually demanded for the two purposes: durable consumption and investment. By assuming all consumption goods are non-durable, however, I justify that all products of the equipment sector are used for investment without being spent for consumption. This assumption is by no means at odds; if we consider a household production, the durable consumptions can be regarded as an investment for the household's production. This assumption is also applied to the construction of observed data for consumption and investment.

Each firm $i \in \{c, e\}$ uses physical capital, $K_{i,t}$, and hours worked, $H_{i,t}$, as inputs to produce its output, $Y_{i,t}$, through a Cobb-Douglas type production function of homogeneous-degree-one as

$$Y_{c,t} = A_{c,t}{K_{c,t}}^{\alpha_c}(Z_{c,t}H_{c,t})^{1-\alpha_c}, \tag{1.26}$$

$$Y_{e,t} = A_{e,t}{K_{e,t}}^{\alpha_e}(Z_{e,t}H_{e,t})^{1-\alpha_e}, \tag{1.27}$$

where α_i denotes the capital share of the production in sector i. The production technologies are affected by both transitory level shocks, which denote $A_{i,t}$, and permanent growth-rate shock, which denote $Z_{i,t}$, for $i \in \{c, e\}$. The transitory shocks of $A_{c,t}$ and $A_{e,t}$ are given as a Hicks-neutral form and are assumed independent each other; the transitory productivity shocks are supposed to have mutually uncorrelated $AR(1)$ processes as follows:

$$\ln A_{c,t} = \rho_{ac} \ln A_{c,t-1} + \epsilon_{ac,t}, \tag{1.28}$$

$$\ln A_{e,t} = \rho_{ae} \ln A_{e,t-1} + \epsilon_{ac,t}, \tag{1.29}$$

where $\rho_j \in [0, 1)$ and $\epsilon_{j,t}$ denotes the autoregressive coefficient and disturbance term which is *iid* normal with mean zero and variance σ_j^2, for $j \in \{ac, ae\}$, respectively.

The permanent productivity shocks of $Z_{c,t}$ and $Z_{e,t}$ are introduced as a labor-augmented type. Following Proposition 3, I assume that $Z_{c,t}$ and $Z_{e,t}$ are cointegrated and incorporated into the system through the vector error correction model (VECM) including the smooth transition non-linear error correction (STR NEC) as

$$\begin{bmatrix} \ln\left(\dfrac{Z_{c,t}/Z_{c,t-1}}{\bar{\eta}^{zc}}\right) \\ \ln\left(\dfrac{Z_{e,t}/Z_{e,t-1}}{\bar{\eta}^{ze}}\right) \end{bmatrix} = \begin{bmatrix} \rho_{cc} & \rho_{ce} \\ \rho_{ec} & \rho_{ee} \end{bmatrix} \begin{bmatrix} \ln\left(\dfrac{Z_{c,t-1}/Z_{c,t-2}}{\bar{\eta}^{zc}}\right) \\ \ln\left(\dfrac{Z_{e,t-1}/Z_{e,t-2}}{\bar{\eta}^{ze}}\right) \end{bmatrix} + \begin{bmatrix} f_c\left(ect_{t-1}\right) \\ f_e\left(ect_{t-1}\right) \end{bmatrix} + \begin{bmatrix} D_{cc} & D_{ce} \\ D_{ec} & D_{ee} \end{bmatrix} \begin{bmatrix} \epsilon_{zc,t} \\ \epsilon_{ze,t} \end{bmatrix}, \tag{1.30}$$

where $\epsilon_{zc,t}$ and $\epsilon_{ze,t}$ are *iid* normal with mean zero and variance σ_{zc}^2 and σ_{ze}^2, respectively, and *ect* indicates the error correction term defined as

$$ect_t = \ln Z_{c,t} - \kappa \ln Z_{e,t}, \tag{1.31}$$

which implies $Z_{c,t}$ and $Z_{e,t}$ are cointegrated with cointegrating vector $(1, -\kappa)$. The functional forms of $f_i(\cdot)$ include both linear and non-linear for $i \in \{c, e\}$; if linear, it is a typical VECM. Here I assume $f_i(\cdot)$ follows the exponential smooth transition (ESTR) functional form as

$$f_i(ect_{t-1}) = \gamma_i ect_{t-1}\left(1 - e^{-\theta(ect_{t-1}-\nu)^2}\right), \tag{1.32}$$

for $i \in \{c, e\}$, where $\theta \geq 0$ and ν is a transition parameter. According to Kapetanios et al. (2003), ect_t is geometrically ergodic or globally stationary as long as $\theta > 0$, $0 < \gamma_e < 2$ and $-2 < \gamma_c < 0$. In turn, ESTR error correction function has its own benefit by ensuring the transition dynamics stationary. For more details on this issue, I will discuss at the subsection 1.3.5.

Since firms would maximize profits in competitive markets subject to their production technology given in Eq.(1.26) and (1.27), their profit maximization should satisfy the following conditions:

$$\tilde{R}_t = \alpha_c Y_{c,t}/K_{c,t}, \tag{1.33}$$

$$\tilde{W} = (1-\alpha_c) Y_{c,t}/H_{c,t}, \tag{1.34}$$

$$Q_t \tilde{R}_t = \alpha_e Y_{e,t}/K_{e,t}, \tag{1.35}$$

$$Q_t \tilde{W} = (1-\alpha_e) Y_{e,t}/H_{e,t}, \tag{1.36}$$

Eq.(1.26), and Eq.(1.27). Accordingly, these firms' profit-maximizing conditions imply that IST is the ratio of the marginal product of capital in equipment to the marginal product of capital in consumption-goods sector, which is given as follows:

$$Q_t = \frac{\alpha_e Y_{e,t}/K_{e,t}}{\alpha_c Y_{c,t}/K_{c,t}}. \tag{1.37}$$

1.3.3 Market clearing conditions

On the equilibrium, the four markets of consumption goods, equipment, capital and labor have to be cleared. Hence, the following market clearing conditions should be satisfied:

$$C_t = Y_{c,t}, \tag{1.38}$$

$$I_t = Y_{e,t}, \tag{1.39}$$

$$K_t = K_{c,t} + K_{e,t}, \tag{1.40}$$

$$H_t = H_{c,t} + H_{c,t}. \tag{1.41}$$

In addition, the aggregate output measured by unit of consumption goods is defined as

$$\tilde{Y}_t = Y_{c,t} + Y_{e,t}/Q_t. \tag{1.42}$$

1.3.4 Solution

The variables of this model economy possess non-stationary properties granted by Z_c, Z_e and X_g of $I(1)$ stochastic processes. Consequently, we need to transform each non-stationary variable into a stationary one on the balanced growth path. Since each variable grows with different rates along the balanced growth path, the functional form of the transformation depends on each of them. Through the following transformation equations, each non-stationary variable, denoted in upper-case, is replaced by its stationary form, denoted in lower-case, : $\tilde{Y}_t = \tilde{y}_t \mathrm{T}^c_{t-1}$; $C_t = c_t \mathrm{T}^c_{t-1}$; $H_t = h_t \mathrm{T}^h_{t-1}$; $\Lambda_{1,t} = \lambda_{1,t}/\mathrm{T}^c_{t-1}$; $\Lambda_{2,t} = \lambda_{2,t}/\mathrm{T}^i_{t-1}$; $\tilde{R}_t = \tilde{r}_t \mathrm{T}^c_{t-1}/\mathrm{T}^i_{t-1}$; $\tilde{W}_t = \tilde{w}_t \mathrm{T}^c_{t-1}/\mathrm{T}^h_{t-1}$; $Q_t = q_t \mathrm{T}^i_{t-1}/\mathrm{T}^c_{t-1}$; $K_t = k_t \mathrm{T}^i_{t-1}$; $I_t = i_t \mathrm{T}^i_{t-1}$; $Y_{c,t} = y_{c,t} \mathrm{T}^c_{t-1}$; $Y_{e,t} = y_{e,t} \mathrm{T}^i_{t-1}$; $K_{c,t} = k_{c,t} \mathrm{T}^i_{t-1}$; $K_{e,t} = k_{e,t} \mathrm{T}^i_{t-1}$; $H_{c,t} = h_{c,t} \mathrm{T}^h_{t-1}$; $H_{e,t} = h_{e,t} \mathrm{T}^h_{t-1}$; $X_{l,t} = x_{l,t}$; $A_{c,t} = a_{c,t}$; $A_{e,t} = a_{e,t}$, where $\mathrm{T}^c_t = {Z_{c,t}}^{1-\alpha_c} {Z_{e,t}}^{\alpha_c} X_{g,t}$, $\mathrm{T}^i_t = Z_{e,t} X_{g,t}$ and $\mathrm{T}^h_t = X_{g,t}$.

Applying the above transformation to the non-stationary system of equations, Eqs.(1.17)-(1.42) except the redundant Eqs.(1.35) and (1.36), we obtain the stationary system of equations: the equations are presented in Appendix A.2.1. In the substitution process, I define the exogenous fundamental growth rates, denoted ηs, and the growth rates of endogenous variables, denoted τs, as follows: $\eta^{zc}_t = Z_{c,t}/Z_{c,t-1}$, $\eta^{ze}_t = Z_{e,t}/Z_{e,t-1}$ and $\eta^{xg}_t = X_{g,t}/X_{g,t-1}$; $\tau^c_t = \mathrm{T}^c_t/\mathrm{T}^c_{t-1}$, $\tau^i_t = \mathrm{T}^i_t/\mathrm{T}^i_{t-1}$ and $\tau^h_t = \mathrm{T}^h_t/\mathrm{T}^h_{t-1}$.

To solve the stationary non-linear system, I employ the method of Klein (2000). Since this solution method requires a linearized system, I log-linearize the stationary non-linear system on the steady-state values.[11]

1.3.5 Non-linear error correction

Before moving to the next section, we need to address one question: Why is a non-linear error correction considered in the model economy? A linear error correction is dominantly applied in cointegration models; Schmitt-Grohé and Uribe (2011) incorporate VECM into their model with a linear error correction. The estimated adjustment-speed coefficient with linear assumption, such as Johansen test statistics, however, does not guarantee the dynamic global stationary process of the cointegration system. Therefore, what we need now for the structural model is ensuring the

[11] The steady-state values are explicitly derived and presented in Appendix A.2.2. Also, the log-linearization method applied is explained in Appendix A.2.3.

dynamic stability of the system.

Table 1.4: Cointegrated relation of sectoral productivities

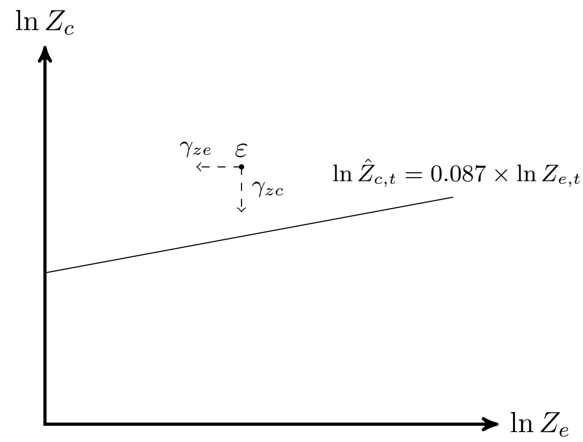
	TFP.cons	TFP.equip
Cointegration Vector	1	-0.087
Adjustment parameter	-0.653	-0.613

Notes: The estimated cointegrating vector and adjustment parameters are obtained by Johansen test for the dataset named 'db6' represented in TABLE 1.2 and 1.3. The cointegrating vector is normalized by TFP.cons. TFP.cons and TFP.equip stand for the productivity of consumption goods and equipment, respectively.

TABLE 1.4 exhibits the estimated cointegration parameters from the Johansen test for the dataset "db6" represented in TABLES 1.2 and 1.3. From TABLE 1.4, we can see that the estimated cointegrating vector, $(1, \kappa)$, is $(1, -0.087)$ and the adjustment-speed, $(\gamma_{zc}, \gamma_{ze})$, is revealed $(-0.653, -0.613)$. The adequate adjustment-speed vector, which induces stationary adjustment dynamics, is necessarily near orthogonal to the cointegrating vector. The estimated adjustment-speed vector, however, is far from the orthogonal cointegrating vector of sectoral productivities. The estimated cointegrating vector and adjustment-speed vector in TABLE 1.4 indicate that the sign of the estimated adjustment-speed vector is different to that of orthogonal vector to the long-run equilibrium represented by cointegration vector. Furthermore, FIGURE 1.1 illustrates that if the deviation point, ε, is far enough from the long-run equilibrium path, the linear adjustment from the deviation may not lead it back on the long-run equilibrium; this long travel of adjustment may arouse a dynamic instability of vector error correction system.

How can we then ensure the global stability in the cointegration system? One possible answer is to introducing a non-linear error correction dynamics into the cointegration; more specifically, the exponential smooth transition (ESTR) in error correction term. It is motivated by Kapetanios et al. (2006), who develop test statistics for cointegration under non-linear error correction assumption. To check the applicability of their model (ESTR), I test the non-linear cointegration of the annual sectoral productivities constructed from the EU KLEMS database by using the test statistics of

Figure 1.1: Linear adjustment of the cointegrated sectoral productivities



Kapetanios et al. (2006).[12] TABLE 1.5 shows the results of non-linear cointegration test for sectoral productivities. The test statistics without underlying assumption (F_{nec}) and with the assumption of zero switching point (F^*_{nec}) fail to reject the null hypothesis of no cointegration. The test statistics with the assumption of zero switching point and the unit roots process in the middle regime (t_{nec}), however, significantly reject the null hypothesis of no cointegration.

As such, the non-linear error correction dynamics between sectoral productivities is confirmed by the cointegration tests with non-linear error correction. Accordingly, if we push the assumption of linear error correction, the dynamic instability is likely to hinder the estimation of structural parameters discussed in the next section. To ensure the dynamic stationary process on the DSGE model with VECM of the sectoral productivities, I assume therefore non-linear error correction featuring exponential adjustment function.

[12] This paper considers three test statistics of Kapetanios et al. (2006); F_{nec}, F^*_{nec} and t_{nec}. The statistic of F_{nec} tests the null hypothesis of no cointegration with no underlying assumptions. The statistic of F^*_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero. The statistic of t_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero and the error correction term follows the unit roots process in the middle regime.

Table 1.5: Cointegration test under non-linear error correction assumptions

	Case	Lags(AIC)	Test statistic	Critical value(95%)	Null hypothesis
F_{nec}	Constant	3	0.908	13.73	Accept
	Trend	3	1.112	16.13	Accept
F^*_{nec}	Constant	3	1.459	12.17	Accept
	Trend	3	1.873	15.07	Accept
t_{nec}	Constant	3	-3.224	-3.22	Reject
	Trend	3	-4.477	-3.59	Reject

Notes: The statistics of F_{nec} tests the null hypothesis of no cointegration with no under-lying assumptions. The statistics of F^*_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero. The statistic of t_{nec} tests the null hypothesis of no cointegration with the assumption that the switching point is zero and the error correction term follow unit roots process in the middle regime.

1.4 Estimation

One goal of this paper is to identify the common stochastic trend of sectoral productivities, which requires estimation of the structural parameters, especially those in external stochastic processes, such as autoregressive coefficients and the standard error of disturbances. As in Ireland and Schuh (2008) and Schmitt-Grohé and Uribe (2011), I adopt the maximum likelihood estimation to estimate the deep parameters that lie on the structural model economy. The linear solution method of Klein (2000) provides the approximated solution of the non-linear system, which is defined on a state-space. Accordingly, we can employ the Kalman filter with given observable variables and construct a likelihood function.

For estimation, the growth rate of consumption, investment, and hours worked are adopted as observable variables. I construct the series of consumption and investment from the U.S. quarterly data of national income and product accounts (NIPAs) available on the BEA website.[13] To be consistent with the model economy, consumption data is constructed by aggregating non-durables and service consumption. Also, investment is constructed by aggregating "durable consumption," and "equipment and software" in NIPAs. For aggregation, as in SECTION 1.2.1, the Törnqvist index

[13]Table 1.1.4 (Price index for GDP) and Table 1.1.5 (Nominal GDP) of NIPAs are used to construct real consumption for non-durables and services, and real investment, which is redefined as the aggregate of "durable consumption," and "equipment and software" in NIPAs.

is applied. Hours worked is obtained from the Federal Reserve Bank of St. Louis' FRED website, under "hours of all persons for nonfarm business sector." All data, ranging 1948:Q2-2011:Q4, are seasonally adjusted and reconstructed in per capita terms by applying "the civilian non-institutional population age 16 and over," which is available on the BLS website.

A subset of the structural parameters is calibrated. It is quite well known that the maximum likelihood estimates of the discount factor, β, and the capital depreciation rate, δ, are extremely difficult to get. Hence, as in Ireland and Schuh (2008), I impose $\beta = 0.99$ and $\delta = 0.025$. The diagonal elements of innovation coefficients (D_{cc} and D_{ee}) of VECM, without loss of generality, are normalized to unity. The steady-state quarterly growth rates of consumption, investment, and hours worked ($\bar{\tau}^c$, $\bar{\tau}^i$, and $\bar{\tau}^h$) are calibrated as 1.0042, 1.0092, and 0.9995, respectively, from the average growth rate of the quarterly data constructed above. The cointegrating parameter, κ, and the steady-state growth rate of sectoral productivities, and preference ($\bar{\eta}^{zc}$, $\bar{\eta}^{ze}$, and $\bar{\eta}^{xg}$) are calculated from the steady-state conditions of the model economy and the steady-state growth rate of consumption, investment, and hours worked.

The rest of structural parameters are estimated via maximum likelihood. Table 1.6 presents the estimated 27 parameters with standard errors, which are computed from a parametric bootstrapping procedure as in Ireland and Schuh (2008). I generate 1000 sets of artificial data, which contain the same number of observations as the original sample, from the estimated model by assigning random disturbances for each period. The artificially generated 1000 sets of data are used to estimate 1000 sample parameters. The reported standard errors in Table 1.6 are the standard deviations of the samples. Additionally, during estimation, I allow the existence of measurement errors in the observables of the growth rates for consumption, investment, and hours worked series, which denote μ_c, μ_i, and μ_h, respectively. The estimates of these measurement errors are curbed not to exceed 25 percent of the standard error of each series.

The model estimates a significant habit-persistence parameter, ξ, of 0.2028; it is much higher than 0.08 of Ireland and Schuh (2008) but a little bit lower than 0.31 of Schmitt-Grohé and Uribe (2011). The capital adjustment-cost parameter is estimated as 0.3148, which is much lower than

Table 1.6: The maximum likelihood estimates and standard errors of the structural parameters

Parameter	Description	Estimate	Standard error
ξ	habit persistence	0.2028	0.0324
ψ	parameter for capital adjustment	0.3148	0.0392
θ	identification parameter for cointegration	0.9384	0.0267
ν	transition parameter	0.0550	0.0752
α_c	capital share of consumption-goods production	0.3307	0.0325
α_e	capital share of equipment production	0.4009	0.0747
ρ_{cc}	autoregressive parameter in VECM	0.2986	0.1479
ρ_{ce}	autoregressive parameter in VECM	0.0000	0.0525
ρ_{ec}	autoregressive parameter in VECM	0.0000	0.0757
ρ_{ee}	autoregressive parameter in VECM	0.0000	0.0402
γ_c	adjustment-speed of error correction	-0.1822	0.5258
γ_e	adjustment-speed of error correction	1.7947	0.0601
D_{ce}	correlation of innovations in VECM	0.3000	0.0762
D_{ec}	correlation of innovations in VECM	0.0225	0.1135
ρ_{xl}	autoregressive parameter of x_l	0.8910	0.1060
ρ_{xg}	autoregressive parameter of η^{xg}	0.5493	0.1200
ρ_{ac}	autoregressive parameter of a_c	0.0000	0.0829
ρ_{ae}	autoregressive parameter of a_e	0.0000	0.0476
σ_{xl}	standard error of x_l	0.0033	0.0014
σ_{xg}	standard error of η^{xg}	0.0046	0.0010
σ_{ac}	standard error of a_c	0.0029	0.0005
σ_{ae}	standard error of a_e	0.0086	0.0020
σ_{zc}	standard error of η^{zc}	0.0042	0.0010
σ_{ze}	standard error of η^{ze}	0.0200	0.0055
μ_c	measurement error of c	0.0004	0.0003
μ_i	measurement error of i	0.0078	0.0000
μ_h	measurement error of h	0.0023	0.0002

Notes: Sample period is 1948:Q2 to 2011:Q4. The observables are the growth rates of consumption, investment, and hours worked. Each of the observables is assumed to possess measurement error. During estimation $\beta = 0.99$ and $\delta = 0.025$ are imposed. The diagonal elements of VECM innovations (D_{cc} and D_{ee}) are normalized to unity.

those in existing literatures; however, the estimate is significant. Most of the two-sector models, including Ireland and Schuh (2008), assume a symmetric sectoral production technology. The symmetry assumption, however, does not reflect the reality but is done for convenience. By discarding the symmetry assumption across sectoral production, this paper estimates capital share of each sector. The maximum likelihood method estimates the capital share of consumption-goods production, α_c, as 0.3307 and that of equipment, α_e, as 0.4009; both estimated capital shares are statistically significant. The estimated sectoral capital shares are worth to compare in literatures: Ireland and Schuh (2008) estimate the capital share of 0.39 with standard error of 0.06, and Schmitt-Grohé and Uribe (2011) estimate 0.37 with standard error of 0.03. Therefore, we cannot say that the estimates of sectoral capital shares are statistically different to other literature; that is, the estimates are consistent in literatures. Moreover, the estimates correspond to the conventional wisdom that consumption-goods production is relatively labor-intensive meanwhile equipment production is capital-intensive.

The most interesting features of the estimation is the parameters for cointegration, volatility, and persistence of external innovations. The existence of cointegration can be tested by evaluating the estimate of θ.[14] If $\theta = 0$, the error-correction term of non-linear VECM will vanish; it implies a regular VAR model. Applying the standard deviation of estimated θ, we can easily test the null hypothesis of $\theta = 0$; we can reject the null because the estimated θ of 0.9349 lies far outside the two-standard deviation from the null. Accordingly, we can confirm the cointegration of sectoral productivities with the maximum likelihood estimates. The persistence parameters of the innovations in common trend (ρ_{cc}, ρ_{ce}, ρ_{ec}, and ρ_{ee}) are estimated as 0.2986 and zeros, respectively, which mean that the persistence of common trend shocks is delivered to the next period only through the consumption goods channel. The correlation parameters of innovations in common trend (D_{ce} and D_{ec}) indicate that the innovations in VECM are significantly correlated: about 30 percent of growth rate innovation of equipment, $\epsilon_{ze,t}$, is correlated to that of consumption goods, $\epsilon_{zc,t}$. Plus, the estimated adjustment-speed parameters (γ_c and γ_e), respectively -0.1825 and 1.7946, indicate that most of error-correction adjustment occurs in the equipment sector; that is, the productivity of consumption-goods sector is weakly exogenous.

One of the most important features of the estimates is that all estimated external innovations are statistically significant. In the case of Ireland and Schuh (2008), the growth component of equipment productivity is turned out statistically insignificant. From the estimation, they con-

[14]The maximum likelihood estimates have asymptotically normal distributions. Therefore, for hypothesis tests, we can apply t-test. See Canova (2007), pp. 225-228, for details.

clude that no equipment-sector-specific technology has had permanent effects on the postwar U.S. economy. The maximum likelihood estimates of this paper suggest, however, that the insignificant external innovation of equipment productivity is due to the misspecified their structural model, which assumes independent sectoral productivities rather than sectoral cointegration or correlation. Furthermore, it is turned out that the largest disturbance among the external innovations is generated by a stochastic trend of sectoral productivities in the postwar U.S. data. The volatility of innovations in the common trend (σ_{zc} and σ_{ze}) are estimated as 0.0042 and 0.0200, respectively. The estimated volatilities of the rest of innovations (σ_{xl}, σ_{xg}, σ_{ac}, and σ_{ae}) are 0.0033, 0.0046, 0.0029, and 0.0086, respectively. The transitory level shock and the permanent growth-rate shock of preference are estimated with high persistence: the autoregressive coefficients of the transitory and permanent shocks (ρ_{xl} and ρ_{xg}) are estimated as 0.8911 and 0.5493, respectively. However, the persistence of the transitory shocks of sectoral productivities (ρ_{ac} and ρ_{ae}) are estimated as zero; that is, there are no persistence in the transitory innovations of sectoral productivities.

Table 1.7: Empirical and simulated moments

	Relative volatility			Correlation with output growth		
	Data	Model	SU2011	Data	Model	SU2011
τ^Y	1.00	1.11	0.98			
τ^C	0.58	0.64	0.56	0.76	0.69	0.41
τ^I	3.33	3.18	2.45	0.88	0.92	0.67
τ^H	0.99	0.90	-	0.59	0.65	-

Notes: τ^i, for $i \in \{Y, C, I, H\}$, denotes the growth rate of output, consumption, investment, and hours worked, respectively. Relative volatility is computed as the standard deviation of a variable divided by the standard deviation of observed output. The column of "SU2011" is the second moments reported in Schmitt-Grohé and Uribe (2011). It is noteworthy that the consumption data of this paper only includes non-durable goods and service consumption and the investment data is constructed by aggregating durable consumption and equipment. Whereas, the consumption of Schmitt-Grohé and Uribe (2011) include durable goods and the investment includes structure.

Additionally, it is worth to investigate how does the estimated model economy fit to the real data. TABLE 1.7 presents observed and simulated second moments of growth rate of output, consumption, investment, and hours worked. The second moments indicate that the model fits the data very well. The model replicates the volatility ranking of investment growth, output growth, hours worked growth and consumption growth. The model also captures procyclicality of consumption, investment, and hours worked. Because this paper consider production in a general way, which admits households' production[15], and exclude structure investment and government spending from data, a direct comparison of moments in literatures is somewhat difficult. In spite of the difference in constructing consumption and investment, the volatility ranking and procyclicality of consumption and investment are consistent to Schmitt-Grohé and Uribe (2011).

1.5 Results

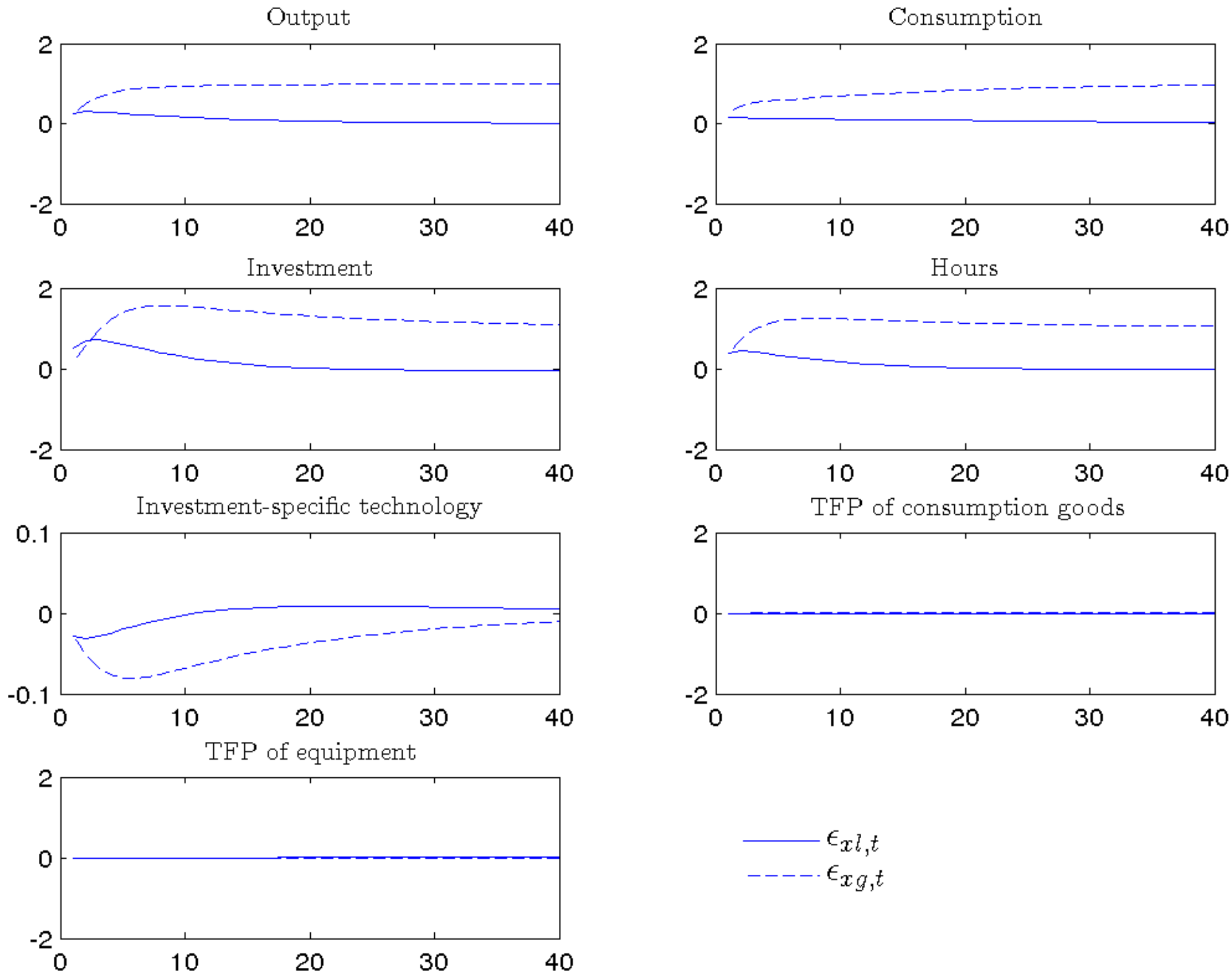
The estimated structural disturbances from Section 1.4 have different implications on the model economy. This section investigates the effects of each shock and discusses its roles in business cycles. The impulse responses in Figures 1.2-1.4 depict the responses of output, consumption, investment, hours, IST and sectoral productivities to a one-standard-deviation shock of each external innovation. Figure 1.2 displays the impulse responses to the transitory and permanent shocks in preference. Figures 1.3 and 1.4 exhibit the impulse responses to the shocks of sectoral productivities.

Figure 1.2 indicates that both transitory and permanent shocks have positive effects on all four macroeconomic aggregates: output, consumption, investment, and hours worked. In particular, the permanent shock in preference has equally sizeable effects on all macroeconomic aggregates with high persistence. This result is consistent with Ireland and Schuh (2008); they find that, among other innovations, only the permanent shock of preference has a sizeable effect on hours worked over time. Since the preference shocks are not related to the changes in productivities, they have no effect on sectoral productivities.

Another notable implication of Figure 1.2 is the decrease of IST in the short run, which recovers its original level in the long run. This fact confirms Oulton (2007)'s argument: The relative price of equipment can change without the relative change of sectoral productivities. In the model

[15] By accepting households' production, we can reclassify durable consumption into investment category as equipment.

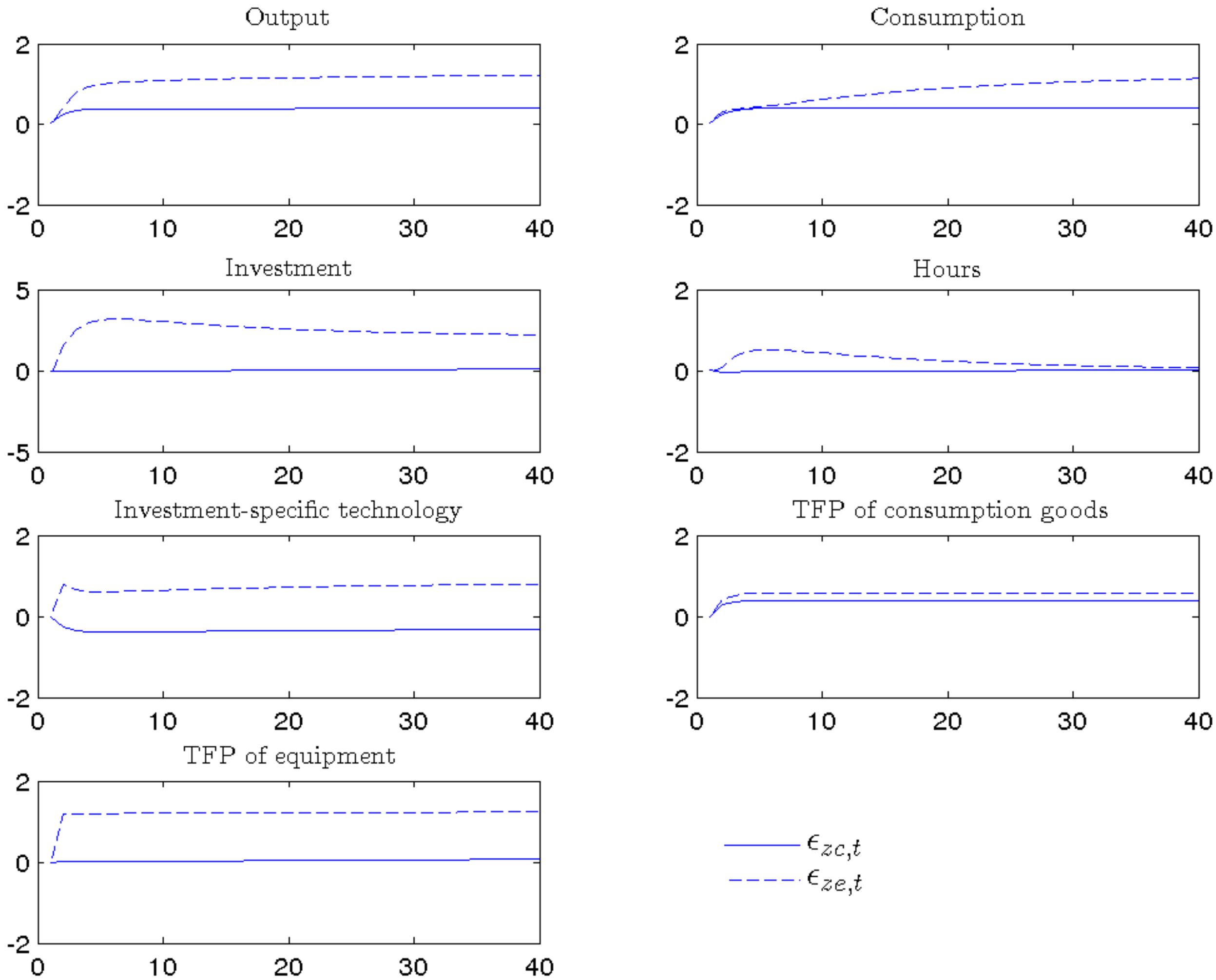
Figure 1.2: Impulse responses on preference shocks



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, productivity of consumption-goods sector, and productivity of equipment sector to a one-standard-deviation shocks of transitory and permanent innovations in preference.

economy, equipment production is capital-intensive meanwhile consumption production is labor-intensive; these are estimated rather than assumed. The positive preference shocks increase labor supply and subsequently push down equilibrium wage. Accordingly, the production of consumption-goods, which is labor-intensive, rises by accompanying a decrease in the price of consumption-goods. IST is therefore decreasing in the short run. As we can see in Figure 1.2, however, the magnitude

Figure 1.3: Impulse responses on common trend shocks of sectoral productivities

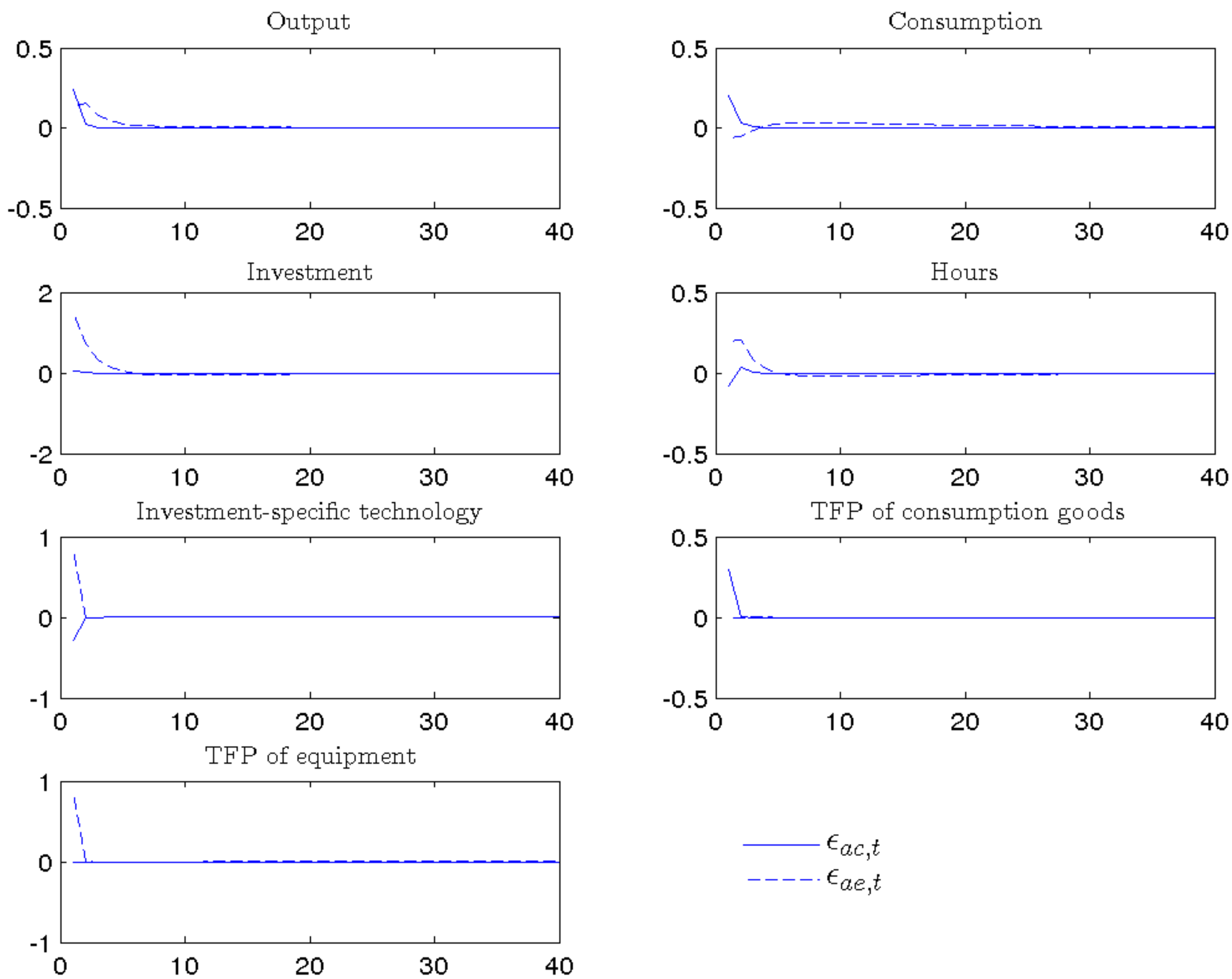


Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, productivity of consumption-goods sector, and productivity of equipment sector to a one-standard-deviation shocks of common stochastic trend in sectoral productivities.

of the effect is very limited. Consequently, we can say that Oulton's argument is right but doesn't explain significant amount of fluctuation of IST.

According to Figure 1.3, the shocks to common stochastic trend generally have persistent effects on the model but the propagation paths differ for each source of shocks. The shock arisen from $\epsilon_{ze,t}$ has a very sizeable effect on output, consumption, and investment. In particular, the

Figure 1.4: Impulse responses on transitory productivity shocks



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, productivity of consumption goods sector, and productivity of equipment sector to a one-standard-deviation shocks of transitory innovations in sectoral productivities.

effect on investment is much larger than that on consumption and remains for a long period of time. The shock on $\epsilon_{ze,t}$ also increases the hours worked in the short run and shrink rapidly to its original level. The shock arisen from $\epsilon_{zc,t}$ mostly affects the productivity of consumption-goods. The shock of $\epsilon_{zc,t}$ also increases consumption persistently. The effect of $\epsilon_{zc,t}$ on the productivity of the equipment is negligibly small; subsequently, IST decreases almost permanently. However,

Table 1.8: Forecast-error variance decomposition

Quarters ahead	ϵ_{xl}	ϵ_{xg}	ϵ_{ac}	ϵ_{ae}	ϵ_{zc}	ϵ_{ze}
			Consumption			
1	14.0	52.5	26.2	2.2	1.3	3.8
4	4.2	49.5	2.5	0.4	20.0	23.4
8	2.5	48.6	0.9	0.2	20.1	27.6
12	1.9	47.3	0.5	0.2	17.7	32.5
20	1.1	44.9	0.2	0.1	13.7	40.0
40	0.5	41.7	0.1	0.0	9.4	48.4
			Investment			
1	10.1	1.0	0.1	84.0	0.2	4.6
4	6.8	10.9	0.0	11.3	0.0	70.9
8	3.7	15.8	0.0	3.8	0.0	76.7
12	2.5	17.6	0.0	2.4	0.0	77.4
20	1.6	18.9	0.0	1.5	0.0	78.0
40	0.9	19.4	0.0	0.9	0.0	78.8
			Hours worked			
1	41.9	43.8	2.0	11.1	0.2	1.0
4	16.5	71.4	0.2	2.2	0.1	9.5
8	8.6	78.1	0.1	0.8	0.0	12.5
12	5.8	81.9	0.0	0.5	0.0	11.7
20	3.6	86.5	0.0	0.3	0.0	9.5
40	2.0	91.8	0.0	0.2	0.0	6.0

Notes: The decomposed forecast error variances in consumption, investment, and hours worked are exhibited. The decomposition consists of the contribution of all 6 shocks to each forecast error variance.

investment does not shrink from that; instead, it remains almost unchanged.

The impulse responses to the transitory shocks of sectoral productivities have effects only for short periods of time. Since these shocks are not mutually correlated, there is no cross-over effect. As we can see in Figure 1.4, the transitory productivity shock of consumption-goods sector, $\epsilon_{ac,t}$, has an effect only on consumption, while a positive transitory shock in equipment productivity, $\epsilon_{ae,t}$, leads to an increase of investment.

Table 1.8 exhibits the decomposed forecast error variances of consumption, investment, and hours worked; decomposition consists of the contribution of all 6 shocks to the forecast error variances. About half of consumption variability depends on the permanent component of preference innovations almost equally both in the short and long runs. The transitory productivity shock of consumption-goods sector takes a small portion of the variability of consumption only in the short run. Interestingly, about half of consumption variability in the long run is explained by the shocks of common trend in sectoral productivities. The variability of investment mostly explained by the shocks of common trend; especially, the shock on $\epsilon_{ze,t}$ takes about 71-79 percent of the variability after the fourth quarter predicted time horizon. The transitory productivity shock in equipment accounts for most of the one-period-ahead forecast errors for investment; however, its explanatory power shrinks radically with the increase of forecast period. The rest, around 20 percent, of investment variability is due to preference shocks. As I have pointed out before, most of the volatility of hours worked, about 90 percent, is associated with preference shocks: long-run variability of 10 percent in hours worked is explained by common stochastic trend of sectoral productivities.

1.6 Conclusion

This paper theoretically and empirically presents the existence of a cointegration in sectoral productivities, which is motivated by the findings of Schmitt-Grohé and Uribe (2011). Furthermore, I incorporate the cointegration of sectoral productivities into the two-sector model of Ireland and Schuh (2008). By introducing a non-linear vector error correction system into the model economy, I establish a dynamic stochastic general equilibrium model incorporating the cointegration of sectoral productivities. I estimate the deep parameters of the model economy with all statistically significant external innovations. The subsequent impulse-response analysis based on the estimated model finds that the innovations of common stochastic trend in sectoral productivities permanently increases both consumption and investment. There are two channels of this effect. The first one is the shock of $\epsilon_{zc,t}$, which increases the productivity of consumption-goods a lot but induce a negligible increase in equipment productivity. Consequently, the shock mostly causes consumption increase without changing investment. The second one is the shock of $\epsilon_{ze,t}$, which increases the sectoral productivities simultaneously. Since the effect of the shock on equipment productivity is twice as large compared to that on the productivity of consumption-goods sector, IST suddenly increases. Because of the positive responses of both aggregate productivities and IST, consumption and investment increase simultaneously with persistence.

The knowledge included in the paper can be applied to disentangle the sectoral comovement puzzle. The existing studies on this issue, such as Hornstein and Praschnik (1997) and Horvath (2000), address the problem by introducing intermediate inputs, which foster the sectoral linkages. The impulse responses analysis conducted in SECTION 1.5, however, indicate that a positive shock on the common trend of sectoral productivities increase sectoral outputs, consumption and investment simultaneously: that is, sectoral comovement is explained without introducing intermediate inputs. The assumptions of the paper, however, are too restrictive to be applied in the general sense: the model assumes perfect segregation between consumption goods and investment goods. Therefore, by relaxing the restrictive assumption, we may extend the idea of this study to the future study on addressing the sectoral comovement puzzle.

Chapter 2

Technology-Embodied Imports and Investment-Specific Technology in a Small Open Economy

2.1 Introduction

This paper studies the role of investment-specific technology in explaining the business cycles features for an open economy. It is closely related to the recent literatures which document the existence of investment-specific technology in the U.S. economy and argue that this kind of technological progress explains a significant fraction of economic fluctuations.[1] In order to apply the concept of investment-specific technology to an open economy, we need to consider the effect of foreign innovations to investment-specific technology. According to Eaton and Kortum (2001), the benefits from the fruits of R&D proceeded by few developed countries may spread around world through trades in capital goods that embody new technology. Lee (1995) empirically documents that international trade providing relatively cheaper foreign capital goods increases efficiency of capital accumulation and thus the growth rate of income in less developed countries.

To examine what real data tells us for the existence and sources of investment-specific technology, using the EU KLEMS database, I analyze the relation between the relative price of equipment in terms of consumption goods and the quantity of equipment both demanded and produced. The results of analysis inform us that the sources of investment-specific technology can vary by coun-

[1] Greenwood et al. (1997, 2000) suggest investment-specific technology as a main sources of economic growth and fluctuation rather than neutral productivity. Fisher (2006) also argues that technology shocks are matter a lot when investment-specific technology is introduced.

tries. In Italy and U.S., both equipment investment and production are significantly correlated to the relative price of equipment, which implies that equipment investment regardless of its sources of supply gives some effects on investment-specific technology unless a near closed economy (Case 1). The relative prices of equipment for Austria, Denmark, U.K., and Japan are largely affected by domestic production of equipment (Case 2). A large part of investment-specific technological changes in Canada and Korea might be attributed to equipment imports (Case 3). In the rest of countries, Australia, Finland, and Netherlands, the existence of investment-specific technology is not significant (Case 4). Among the four cases, it is more appropriate to investigate the countries categorized in Case 3 for the purpose of this study because the investment-specific technologies of these countries are supposed to heavily rely on the embodied technology in equipment imports.

As such, investment-specific technology of a country is affected by either domestic production or imports. Then, how do the various sources of investment-specific technology affect to the business cycles of a country? To address this question, I perform real business cycles analysis for the Korean economy categorized in Case 3. According to Boileau (2002), the terms of trade for equipment can be decomposed into terms of trade for consumption and investment-specific technological changes generated from both home and foreign. The investment-specific technology, which a country faces, consists of the same factors of terms of trade for equipment. Therefore, through this decomposition, technology-embodied imports can be incorporated to a business cycles model for an open economy. In addition to this, domestic technological processes of the Korean economy might be another important sources of investment-specific technological changes. Even though Korea have resorted to equipment imports to resume its development since the end of the Korean War, as the fruits of its endeavor, it becomes a major exporter of steel, ship, automobile, electrical equipment. Accordingly, it is more relevant to consider the domestic technological progress for equipment as a source of economic fluctuation in Korea along with equipment imports.

With the factors inducing investment-specific technological changes, I establish a dynamic stochastic general equilibrium (DSGE) model for a small open economy to analyze the business cycles in Korea. To ensure the stationarity of the small open economy model, as in Schmitt-Grohé and

Uribe (2003), I introduce external debt-elastic interest rate premium. The model economy includes eleven external innovations such as preference shocks, technology shocks, the shocks on technology-embodied imports, terms of trade shock, and the shocks of world interest rate and wage markup. In the previous chapter, I document that when we consider the permanent trend components of sectoral productivities, it is important to test if the cointegration among sectoral productivities exists and we have to reflect the empirical result to build a model economy. To investigate cointegration, Johansen tests are conducted for the Korean sectoral productivities constructed from EU KLEMS database. Test results confirm the cointegration of sectoral productivities. Different to the U.S. economy, however, cointegration tests with non-linear error correction assumption fail to reject the null hypothesis of no cointegration. In turn I assume a linear error correction function for the cointegration.

The model economy has structural parameters including the parameters for eleven external innovations. These parameters are either calibrated or estimated via maximum likelihood method. For the structural estimation I use the growth rates of output, consumption, investment, and hours worked as the observable variables. Of all the estimated structural innovations, only five shocks are statistically significant; they are the permanent shock of preference, the two transitory shocks of sectoral productivities, terms of trade shock, and the transitory shock of embodied technology in imports.

Using the estimated model economy, I conduct impulse response analysis and forecast-error variance decomposition. The main findings indicate the followings: a positive shock of terms of trade for consumption goods increase output, consumption, investment, and hours worked on its impact. The terms of trade shock also generates countercyclical movement of trade balance. The positive common trend shocks of sectoral productivities increase consumption permanently and consistently but induces J-shaped responses of output, investment, and hours worked. The J-shaped response is due to the contraction of investment caused by the sluggish investment-specific technology on impact and then growing slowly and persistently with the gradual improvement in investment-specific technology. Moreover, it is turned out that, as a single factor, the shock of terms of trade for consumption goods is the largest external force explaining the volatilities of macroeconomic aggregates in Korea. Especially, the terms of trade shock is responsible for most variability of investment. As a group of factors, the domestic technological innovations including both transitory and permanent components account for the short-run and long-run volatilities of output and consumption. Moreover, the shock of technology-embodied imports explains short-run investment volatility of about 30 percent and the common trend shocks of sectoral productivities are responsible for long-run investment volatility of about 40 percent. Also, the common trend shocks of sectoral productivities account for the overall hours worked volatility of about 30-40 percent and the permanent shock of preference accounts for long-run hours worked volatility of 27 percent. In sum, few foreign innovation such as terms of trade for consumption goods and technology-embodied imports explain a substantial portion of business cycles in Korea as well as domestic technological progresses.

This paper has its contributions on the literatures in two aspects. First, the sources of investment-specific technological changes are extended for a small open economy. Especially I incorporate technology-embodied imports to one of the main driving forces of business cycles in a small open economy. This idea basically comes from Boileau (2002), which suggests a decomposition method diversifying the source of the terms of trade for capital goods to investigates the synchronization of international business cycles with trade in capital goods. Incorporating the decomposition method to the small open economy model, we can examine the effects of foreign innovations on the investment-specific technology faced by domestic agents. This modification for investment-specific technology reflects recent criticisms on the relative price of equipment as a measurement for the investment-specific technology. Greenwood et al. (1997) suggest that investment-specific technology is measured by the relative price of equipment in terms of consumption goods. Measuring investment-specific technology as the relative price of equipment, however, may include sizeable measurement errors. Basu et al. (2010) and Guerrieri et al. (2010) point out that the relative price of equipment cannot exactly captures relative changes in sectoral productivities, which is the only sources of investment-specific technological changes in a close economy framework. Considering the possibility of measurement error in the relative price of equipment, I make the investment-specific technology be endogenously determined by the decomposed factors.

Secondly, the results of this paper attribute the domestic technological changes to the main driving forces of business cycles in the Korean economy. Furthermore, it is turned out that the common trend shocks of sectoral productivities are important sources of long-run volatility of the Korean economy. This finding is consistent to the recent literatures. Aguiar and Gopinath (2007) show that shocks to trend growth are the primary sources of fluctuation in emerging economies. According to them, since emerging markets are characterized by frequent regime switches, consequently, shocks to the growth rates is the primary sources of fluctuation rather than transitory level shocks around the trend. The model economy of this study diversifies the growth trend into four growth-rate innovations of sectoral productivities, domestic preference, and foreign technology rather than one trend component of aggregate productivity in Aguiar and Gopinath (2007). This system of diversified growth-rate innovations have its own benefit in identifying the sources of shock. Through the DSGE analysis, it is confirmed that the domestic technology is more important in transitional change in Korea.

The remainder of the paper is organized as follows. Section 2.2 investigates the sources of investment-specific technology by using the EU KLEMS database and show that the investment-specific technology of Canada and Korea are exposed to the foreign technologies. To establish structural model for the DSGE analysis of the Korean economy, I examines the cointegration of sectoral productivities with both linear (Johansen test) and non-linear error-correction assumptions. In Section 2.3, the established DSGE model for a small open economy is explained. Section 2.4 presents the values of structural parameters either calibrated and estimated. Section 2.5 discusses the results of simulation analysis of impulse responses and variance decomposition. Lastly, Section 2.6 includes concluding remarks and gives some suggestions for future studies.

2.2 Empirical motivation

In this section I investigate the existence of investment-specific technology using EU KLEMS database. Greenwood et al. (1997) confirm the existence of investment-specific technology by looking at the negative comovement between relative price of equipment and investment to GDP ratio. However, their idea fundamentally based on a closed economy. In a closed economy, equipment production and investment always have to be equal unless inventory does not change. Because a country can trade its equipment production in an open economy, however, equipment production and investment are not usually equal. Thereby I examine if the relative price of equipment shows a comovement with the equipment production or equipment investment. This attempt let us classify the examined countries into four categories.

2.2.1 Sources of investment-specific technology

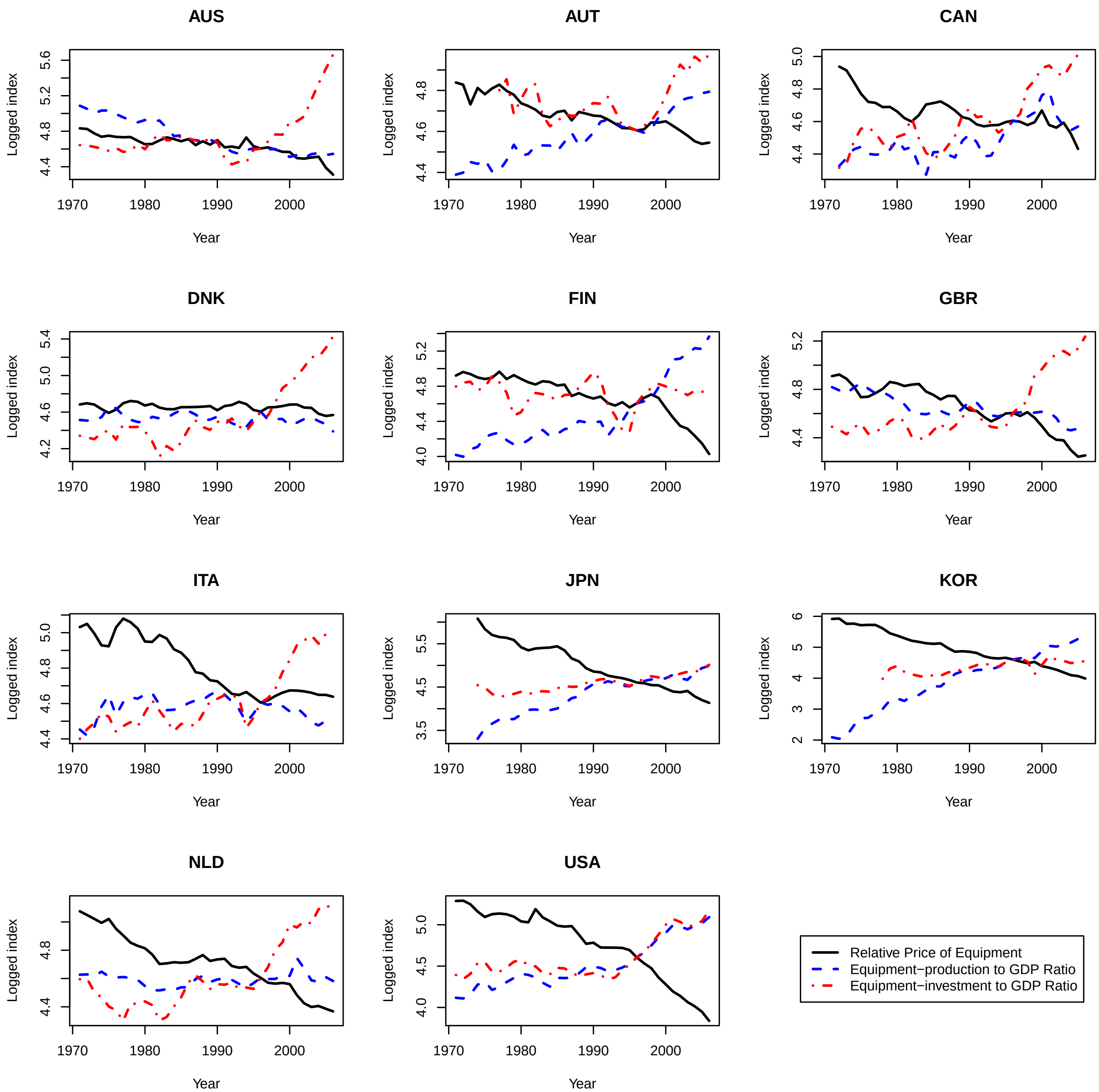
Greenwood et al. (1997) capture the concept of investment-specific technology from the negative comovement between the relative price of equipment and equipment investment to GDP ratio. They also suggest the relative price of equipment as a measurement of investment-specific technology. In a closed economy framework, furthermore, relative price of equipment represents the ratio of marginal products between consumption goods and equipment producing sectors.[2] The marginal product representation, however, may not fully identify the investment-specific technology of an open economy where imports of equipment are allowed.

To get an insight on equipment and its relative price, I examine international data constructed from EU KLEMS database.[3] FIGURE 2.1 plots the data of relative price of equipment, equipment production to GDP, and equipment investment to GDP ratio of selected countries in EU KLEMS database. Since direct measures of equipment production are not available, I approximate them by

[2]Whelan (2003) argues that a two-sector approach incorporating relatively high technological progress of durable goods better explains the long-run behavior of the U.S. economy.

[3]For more details about EU KLEMS database, see O'Mahony and Timmer (2009). The data is available at www.euklems.net.

Figure 2.1: Equipment to GDP ratio and the relative price of equipment



Notes: The indices of relative price of equipment, equipment production to GDP, and equipment investment to GDP ratio are normalized to year 1995. The plots show logged index of each data. Country codes of AUS, AUT, CAN, DNK, FIN, GBR, ITA, JPN, KOR, NLD, and USA, respectively, stand for Australia, Austria, Canada, Denmark, Finland, U.K., Italy, Japan, Korea, Netherlands, and U.S.

associating equipment with the outputs of 'electronical and optical equipment', 'machinery', and 'transport equipment'.[4] In all countries, relative price of equipment has been declined since 1970s. At the same time, equipment investment to GDP ratios exhibits increasing tendency during the data period in most of countries; that is, there exists a negative comovement in relative price and equipment investment. The negative comovement usually indicates the existence of investment-specific technology.

FIGURE 2.1 indicates, however, that in many countries the series of equipment investment is different to that of equipment production. More prominent features are observed in the data of Australia, Denmark, U.K., Italy, and Netherlands since 1990s: equipment investment increases while equipment production shows stagnant. The discrepancy between equipment investment and equipment production might heavily rely on the equipment imports. Thus investment-specific technology may be affected by domestic technological changes as well as the changes of the embodied technology in imported equipment.

Investigating the correlation of relative price and equipment investment (or equipment production) can help us to understand the origin of investment-specific technology. Suppose first that relative price of equipment is negatively correlated to both equipment production and equipment investment (CASE 1). This case is possible when technological progress in domestic equipment production coincides with balanced trades in equipment, otherwise near closed economy. If relative price of equipment has a negative correlation with equipment production but no correlation with equipment investment (CASE 2), we can conjecture that the domestic equipment-producing-technology coincide with a sizeable net exports of equipment so that excess produced equipments are purchased by foreign. An economy with low capacity of equipment production has to largely depend on imports of equipment in order to accumulate capital stock through investment. So, in this economy, the technological progress of equipment imports induces an increase of equipment

[4] Eaton and Kortum (2001) approximate capital equipment with the output of the nonelectrical equipment, electrical equipment, and instruments industries. Their equipment approximation do not include transport equipment but this paper does.

investment as well as a decrease of relative prices of equipment, whereas domestic progress in equipment producing technology hardly affect to the level of relative price (Case 3). Lastly, no negative correlations of relative price of equipment with both equipment production and equipment investment imply that investment-specific technology does not exist (Case 4).

In order to examine the sources of investment-specific technology, I analyze the correlation between the relative price of equipment and the quantity ratio of equipment (both equipment production to GDP and equipment investment to GDP). For this data analysis, I derive the percent deviation from trend for each time series of relative price of equipment, equipment production to GDP, and equipment investment to GDP; all data are constructed from EU KLEMS database. To detrend each data, HP-filter is applied with annual smoothing factor of 6.25. Table 2.1 shows that correlation coefficients of relative price of equipment with both equipment production to GDP and equipment investment to GDP for the eleven countries in EU KLEMS.

Table 2.1: Correlation coefficients between relative price and quantity ratios of equipment

	AUS	AUT	CAN	DNK	FIN	GBR	ITA	JPN	KOR	NLD	USA
ρ_{prod}	0.072	-0.383 **	-0.049	-0.647 **	-0.265	-0.349 **	-0.592 **	-0.752 **	-0.288 *	-0.086	-0.53 **
	(0.677)	(0.021)	(0.783)	(0.000)	(0.119)	(0.037)	(0.000)	(0.000)	(0.089)	(0.618)	(0.001)
ρ_{inv}	0.024	0.125	-0.577 **	0.105	0.313	-0.12	-0.374 **	0.167	-0.381 **	0.375	-0.421 **
	(0.888)	(0.511)	(0.000)	(0.541)	(0.063)	(0.485)	(0.025)	(0.354)	(0.042)	(0.024)	(0.011)

Notes: ** and * represent that the corresponding estimate is significant with level of 95% and 90%, respectively. The number inside parenthesis indicates the asymptotic **p-value** of corresponding correlation. Country codes of AUS, AUT, CAN, DNK, FIN, GBR, ITA, JPN, KOR, NLD, and USA, respectively, stand for Australia, Austria, Canada, Denmark, Finland, U.K., Italy, Japan, Korea, Netherlands, and U.S.

The first row of Table 2.1 exhibits the correlation coefficients between relative price and equipment production (ρ_{prod}) and the second row is for the relative price and equipment investment (ρ_{inv}). The results of ρ_{prod} indicate that relative price of equipment is negatively correlated to equipment production to GDP ratio in Austria, Denmark, U.K., Italy, Japan, and U.S. with significant level of 5 percent while Australia, Canada, Finland, Korea, and Netherlands don't have a significant correlation in price and production for equipment. On the other hand, it is confirmed from the results of ρ_{inv} that there are no significant negative correlations of relative price of equipment and equipment investment to GDP ratio in Australia, Austria, Denmark, Finland, U.K., Japan, and Netherlands whereas Canada, Italy, Korea, and U.S. have the negative correlation.

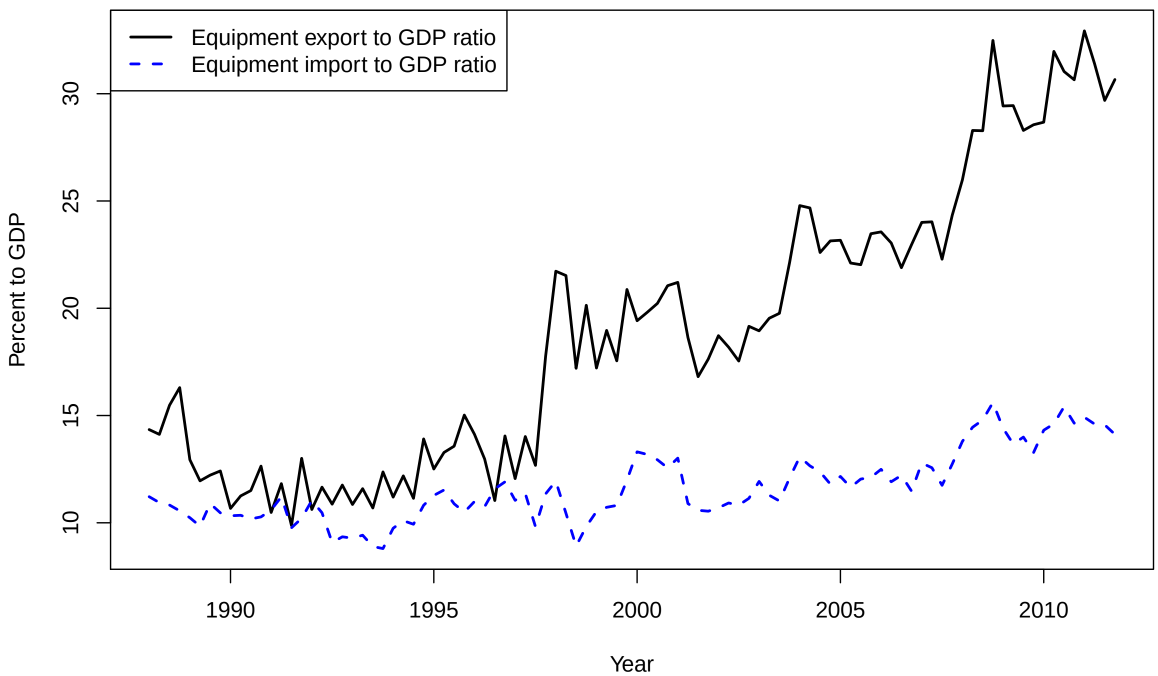
Based on the correlation coefficients, we can therefore classify the countries into the four categories defined above. Austria, Denmark, U.K., and Japan are categorized into Case 2 because only domestic production of equipment is correlated to the equipment price. Accordingly, the Case 2 countries look like to produce more equipments than domestic demands and export the excess products. Italy and U.S., in which relative prices of equipment are negatively correlated with both equipment production and equipment investment are categorized to Case 1, where domestic equipment production and equipment investment are moderately balanced. In Canada and Korea categorised to Case 3, only relative price of equipment and equipment investment have a significant negative correlation; it implies that a large fraction of decrease in relative price of equipment is attributed to the technological progress embodied in imported equipment. Lastly, the existences of investment-specific technology are not empirically supported in Australia, Finland, and Netherlands.

As we can see from the results, the sources of investment-specific technology vary across countries. In the countries of Case 2, domestic technological progresses of equipment are more important than other factors and the countries show large equipment exports. Whereas, the countries in Case 3 are significantly influenced by the technological progresses embodied in equipment imports. So, obviously, these countries have large equipment imports. Meanwhile, the sources of investment-specific technological changes in Case 1 can be both home and foreign unless the countries are in a closed economy regime.

2.2.2 Investment-specific technology in Korea

We have seen above that the investment-specific technology of a country can be generated at home, foreign, or both if it exists. Then which factor is more important in understanding the investment-specific technological changes of a country? To address this question, we need to focus on a country specific analysis. In this paper, I analyze the Korean economy and investigate the factors, which contribute to the investment-specific technology of Korea.

Figure 2.2: Exports and imports of equipment to GDP ratio in Korea



Notes: The data of export and import of equipment are collected from Korea International Trade Association (KITA: http://stat.kita.net). The GDP of Korea in term of US dollar are obtained from Bank of Korea (http://ecos.bok.or.kr).

According to the correlation analysis, the investment-specific technology of Korea have been affected by the technological change of both home and foreign with more weight on foreign factors. The historical development of Korean economy supports its technological dependence to the other countries. The production foundations of Korea had been devastated after the Korean War. Moreover, Korea has no enough natural resources to construct economical foundation for development by itself. Nonetheless, with high GDP growth rate, the Korean economy, which used to be one of the poorest countries, reached to 15th largest economy of the world (based on 2012 GDP estimates). Behind the drastic economical progress in Korea, there is an important role of equipment imports (or foreign investment). After the Korean War, it had to depend on foreign equipment to rebuild its fundamentals. Especially, in constructing the foundation of manufacturing industry, a lot of imported equipments have been used as capital investment.

Another important determinant of investment-specific technology in Korea is the domestic technological change, especially for equipment production. Through the effort to facilitate equipment industry, Korea recently can increase its amount of equipment exports. FIGURE 2.2 plots equipment exports and imports of Korea from 1988 to 2011. From FIGURE 2.2, we can see that both equipment exports and imports had remained around 10 percent to GDP during 1990s. However, the gap between equipment exports and imports have enlarged since 2000s, in which information and communication technology industry have developed. Recently the equipment exports of Korea have reached to the level of 30 percent to GDP while the ratio of equipment imports to GDP still remain on around 10 percent. In order to analyze the recent business cycles of Korea, it is therefore important to consider the technological progress of home as well as that of foreign.

2.2.3 Cointegration of sectoral productivities in Korea

To incorporate technological change of equipment sector in a business cycles model, we need to understand the systemic relations among sectoral productivities. In the previous chapter, I document that sectoral productivities of the U.S. economy are cointegrated and that if the cointegration is not considered, the corresponding implication for business cycles can be misguided. Accordingly, before proceeding a formal business cycles study, I investigate the cointegration of sectoral productivities in the Korean economy. Unit-root and cointegration tests are conducted to the logarithms of aggregated productivity, sectoral productivities, and relative price of equipment by using the data of sectoral productivities of Korea in EU KLEMS database. Since EU KLEM is a disaggregated database of 72 industries, for the analysis purpose, I need to aggregate the sectoral data into two-sector definition of consumption goods and equipment sectors. In turn, I define the equipment sector as the aggregation of 'electrical and optical equipment', 'machinery', and 'transport equipment', and the rest are used to construct consumption goods sector.

By definition, cointegation requires non-stationary variables. Accordingly, we need to examine if aggregate productivity, sectoral productivities, and relative price of equipment have a non-stationary stochastic process; that is, a unit-root. Augmented Dickey-Fuller (ADF) and Dickey-

Table 2.2: Unit-root test for the logarithms of productivities and relative price of equipment

Data	Test	Trend	Lags (AIC)	Test-stats.	Critical values (5%)	Null hypothesis
TFP.cons	ADF	No	1	3.35	-1.95	Accept
	ADF	Yes	1	-0.65	-3.5	Accept
	DF-GLS	No	1	-0.0874	-1.95	Accept
	DF-GLS	Yes	1	-1.18	-3.19	Accept
TFP.equip	ADF	No	1	4.69	-1.95	Accept
	ADF	Yes	1	-2.7	-3.5	Accept
	DF-GLS	No	1	0.838	-1.95	Accept
	DF-GLS	Yes	1	-2.8	-3.19	Accept
TFP.tot	ADF	No	1	3.96	-1.95	Accept
	ADF	Yes	1	-1.02	-3.5	Accept
	DF-GLS	No	1	0.171	-1.95	Accept
	DF-GLS	Yes	1	-1.44	-3.19	Accept
RP	ADF	No	1	-3.96	-1.95	Reject
	ADF	Yes	1	-2.82	-3.5	Accept
	DF-GLS	No	1	1.06	-1.95	Accept
	DF-GLS	Yes	1	-2.38	-3.19	Accept

Notes: All unit-root test fail to reject except the ADF test for RP without trend. Tests are conducted on R program with the 'urca' package. ADF stands for Augmented Dickey-Fuller, and DF-GLS stands for Dickey-Fuller Generalized Least Squares. TFP.cons, TFP.equip, TFP.tot, and RP denote the productivity of consumption-goods sector, the productivity of equipment sector, the productivity of aggregate economy, and the relative price of equipment, respectively.

Fuller GLS (DF-GLS) tests are performed to test the unit-root hypothesis. Table 2.2 presents the results. The ADF test fails to reject the unit-root hypothesis except for the relative price of equipment without trend. DF-GLS known as the test statistics with increased power cannot reject the null hypothesis of unit-root for all tested variables both with and without trend. I also conduct the unit-root tests for the first-differenced logged variables, which are not reported here, and all test statistics reject the null hypothesis. Based on the results so far, I can therefore conclude that all tested logged productivities and relative price of equipment are integrated by order one.

Schmitt-Grohé and Uribe (2011) find the cointegration of aggregate neutral productivity and relative price of equipment with the U.S. quarterly data. It is also suggestive to examine the cointegration of aggregate productivity and relative price of equipment with the Korean data because the cointegration of aggregate productivity and relative price of equipment implies strong systemic relation of sectoral productivities. So, I conduct Johansen cointegration tests with various

Table 2.3: Johansen trace test for cointegration

Database	Cointegration rank	Lags	Test-stats.	Critical values (5%)	Null hypothesis
db1	r <= 2	2	6.65	9.24	Accept
	r <= 1		20.68	19.96	Reject
	r = 0		58.29	34.91	Reject
db2	r <= 2	2	3.43	9.24	-
	r <= 1		12.87	19.96	Accept
	r = 0		40.88	34.91	Reject
db3	r <= 2	2	6.2	9.24	-
	r <= 1		19.1	19.96	Accept
	r = 0		44.6	34.91	Reject
db4	r <= 2	2	2.77	9.24	-
	r <= 1		12.03	19.96	Accept
	r = 0		40.00	34.91	Reject
db5	r <= 1	2	3.35	9.24	Accept
	r = 0		24.26	19.96	Reject
db6	r <= 1	2	2.81	9.24	Accept
	r = 0		26.11	19.96	Reject

Notes: Johansen trace tests confirm cointegrated relation for all specified datasets. Tests are conducted on R program with the 'urca' package. Test models don't include both constant and trend. The dataset used for Johansen cointegration test are defined as follows:
db1: TFP.tot, TFP.cons, TFP.equip
db2: RP, TFP.cons, TFP.equip
db3: TFP.tot, RP, TFP.cons
db4: TFP.tot, RP, TFP.equip
db5: TFP.tot, RP
db6: TFP.cons, TFP.equip

sets of variables including aggregate productivities, sectoral productivities, and the relative price of equipment. The test results of the Johansen trace and maximum eigenvalue tests are exhibited in TABLE 2.3 and 2.4, respectively.

Both Johansen tests, trace and maximum eigenvalue, confirm that the variable system of logged aggregate productivities and sectoral productivities (db1) have a cointegrating vector: trace test indicates two cointegrating vectors whereas maximum eigenvalue test implies one cointegrating vector. This result implies that logged aggregate productivities can be expressed as a linear combination of two sectoral productivities, which is supported by a conventional wisdom on growth accounting. The system of the logged relative price of equipment and sectoral productivities (db2)

Table 2.4: Johansen maximum eigenvalue test for cointegration

Database	Cointegration rank	Lags	Test-stats.	Critical values (5%)	Null hypothesis
db1	r = 2	2	6.65	9.24	-
	r = 1		14.03	15.67	Accept
	r = 0		37.60	22	Reject
db2	r = 2	2	3.43	9.24	-
	r = 1		9.44	15.67	Accept
	r = 0		28.01	22	Reject
db3	r = 2	2	6.2	9.24	-
	r = 1		12.9	15.67	Accept
	r = 0		25.6	22	Reject
db4	r = 2	2	2.77	9.24	-
	r = 1		9.27	15.67	Accept
	r = 0		27.96	22	Reject
db5	r = 1	2	3.35	9.24	Accept
	r = 0		20.91	15.67	Reject
db6	r = 1	2	2.81	9.24	Accept
	r = 0		23.31	15.67	Reject

Notes: Johansen maximum eigenvalue tests confirm cointegrated relation for all specified datasets with one cointegrating vectr. Tests are conducted on R program with the 'urca' package. Test models don't include both constant and trend. The dataset used for Johansen cointegration test are defined as follows:
db1: TFP.tot, TFP.cons, TFP.equip
db2: RP, TFP.cons, TFP.equip
db3: TFP.tot, RP, TFP.cons
db4: TFP.tot, RP, TFP.equip
db5: TFP.tot, RP
db6: TFP.cons, TFP.equip

have cointegrated with one cointegrating vector; it is consistent to what theory tells us.[5] The cointegration of aggregate productivities and relative price of equipment (db5) is tested and it is confirmed that the finding of Schmitt-Grohé and Uribe (2011) is also observed in the Korean data. By additionally augmenting each sectoral productivity on 'db5', two three-variable systems (db3 and db4) are constructed and examined for cointegration. Interestingly, both systems accept

[5] According to Greenwood et al. (1997), logged relative price of equipment equals the differences of logged productivity of equipment and that of consumption goods. This indicates that the implied cointegrating vector is $(1, 1, -1)$ for the system of $(\ln \text{RP}, \ln \text{TFP.equip}, \ln \text{TFP.cons})$. Johansen test estimates the cointegrating vector of $(1, 2.7872, -0.8811)$, which is consistent to the predicted sign of cointegrating vector in the literatures of investment specific technology

Table 2.5: Cointegration test under non-linear error correction assumptions

Test	Case	Lags(AIC)	Test statistic	Critical value(95%)	Null hypothesis
	Raw data	2	0.688	12.28	Accept
F_{nec}	Demeaned	3	1.003	13.73	Accept
	Detrended	1	2.622	16.13	Accept
	Raw data	2	0.712	9.06	Accept
F^*_{nec}	Demeaned	3	1.670	12.17	Accept
	Detrended	1	4.737	15.07	Accept
	Raw data	2	0.297	-2.66	Accept
t_{nec}	Demeaned	3	-3.087	-3.22	Accept
	Detrended	1	2.206	-3.59	Accept

Notes: The statistics of F_{nec} test the null hypothesis of no cointegration with no under-lying assumptions. The statistics of F^*_{nec} test the null hypothesis of no cointegration with the assumption that the switching point is zero. The statistics of t_{nec} test the null hypothesis of no cointegration with the assumption that the switching point is zero and the error correction term follow unit roots process in the middle regime.

cointegration with one cointegrating vector. The simultaneous cointegration in the three data sets of 'db3', 'db4', and 'db5' especially implies that cointegration in sectoral productivities may exist. The test result for sectoral productivities (db6) confirms that the inference is right.

In the previous chapter, I emphasize how the dynamic stability of the system of equations is important in sectoral cointegration and also suggest that non-linear error correction may increase the stability. Accordingly, it has to be checked if the application of non-linear error correction into the model system is proper and supported by data. To check the applicability, I test the non-linear cointegration of the Korean sectoral productivities constructed from the EU KLEMS database using the methods of Kapetanios et al. (2006). The statistics of F_{nec} test the null hypothesis of no cointegration with no underlying assumptions. The statistics of F^*_{nec} test the null hypothesis of no cointegration with the assumption that the switching point is zero. The statistics of t_{nec} test the null hypothesis of no cointegration with the assumption that the switching point is zero and the error correction term follows the unit roots process in the middle regime. Table 2.5 shows the test statistics. All test statistics, including both with and without underlying assumptions, fail to reject the null hypothesis of no cointegration. Therefore, incorporating non-linear error correction may not be a proper way to increase the global stability in sectoral cointegration.

2.3 Model

In this section I develop a two-sector business cycle model for a small open economy by considering technology-embodied imports. Two-sector framework is extended from Ireland and Schuh (2008) as in previous chapter. Additionally, in order to be applicable to a small open economy, I incorporate external debt-elastic interest rate premium as in Schmitt-Grohé and Uribe (2003). Most notable feature of this model is the consideration of the technology-embodied imports. Due to the embodied technology in equipment imports, firms can decrease their production costs and goods prices. Accordingly, following Boileau (2002), the technology-embodied imports is regarded as the investment-specific technology of trade counterparts. Also, the model economy embraces the cointegration of sectoral productivities via VECM.

2.3.1 Economic environments

2.3.1.1 Households

Suppose a small open economy populated by an infinite number of identical households. The representative household of this model economy would maximize its current value of life-time utility featured by habit persistent consumption as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - \xi C_{t-1}) - H_t/S_t \right], \tag{2.1}$$

where β and $\xi \in [0, 1)$, respectively, represent the subjective discount factor and the degree of habit persistence. C_t, H_t, and S_t denote the household's consumption, hours worked, and preference shock, respectively. In a small open economy, the household can access to the international financial markets in order to smooth its budget stream. Thus, the household faces a budget constraint as follows:

$$C_t + I_t/Q_t + (1 + r_t^d)\tilde{D}_{t-1} \leq \tilde{R}_t K_t + \frac{\tilde{W}_t}{\mu_t} H_t + \tilde{D}_t + \Pi_t, \tag{2.2}$$

where $\tilde{R}_t$, $\tilde{W}_t$ and $\tilde{D}_t$, respectively, denote the rental rate of capital, wage rate paid by firms, and the household's debts in terms of consumption goods. As in Schmitt-Grohé and Uribe (2011), I incorporate external wage-markup shock, $\mu_t \geq 1$. [6] Π_t represents the monopoly rent of labor union and the labor union rebates all its profit to households in a lump-sum fashion. Without loss of generality, we can assume that the debt limit of this economy is growing along the balanced growth path. Therefore, $\tilde{D}_t$ can be rewritten as $\tilde{d}_t \mathrm{T}^c_{t-1}$, where $\tilde{d}_t$ is a cyclical component and T^c_{t-1} represents the trend component of foreign debt.[7] To ensure stationarity in this small open economy model, I apply debt-elastic interest rate. So, the interest rate faced by domestic agents, r^d_t, is the sum of world interest rate, r^f_t, and a country specific interest rate premium, $\Psi_d(\cdot)$, as follows:

$$r^d_t = r^f_t + \Psi_t\left(d_t\right), \tag{2.3}$$

where d_t denotes the country's cross-sectional average level of debt, which the household takes as exogenously given. The interest rate premium is an increasing function of the debt level as follows:

$$\Psi_d(d_t) = \psi_d\left(e^{\left(d_t - \bar{d}\right)} - 1\right),$$

where ψ_d denotes risk premium parameter and $\bar{d}$ denotes steady state level of foreign debt. The investment-specific technology, Q_t, is given as the inverse relative price of equipment in terms of consumption goods as follows:

$$Q_t = \frac{P_{c,t}}{P_{e,t}}. \tag{2.4}$$

The capital held by the household accumulates through investment as the following capital accumulation process:

$$K_{t+1} \leq (1-\delta)K_t + I_t - \Psi_k\left(I_t, I_{t-1}\right), \tag{2.5}$$

where $\delta \in [0,1]$ denotes capital depreciation rate. $\Psi_k(\cdot)$ represents capital adjustment costs which is given as

$$\Psi_k\left(I_t, I_{t-1}\right) = \frac{\psi_k}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\tau}^i\right)^2 I_t, \tag{2.6}$$

[6] μ represents a wedge between the wage rate paid by firms, $\tilde{W}$, and the marginal wage rate received by households; that is, this wage wedge reflects the monopoly power of labor unions. For more detail, see Schmitt-Grohé and Uribe (2011).

[7] The implied assumption is that, along the balanced growth path, consumption spending and debt limits are growing at the same rate. Thus, the trend component of consumption and foreign debt are same to T^c.

where $\psi_k > 0$ and $\bar{\tau}^i$ indicate, respectively, the parameter of capital adjustment cost and steady state growth rate of investment.

2.3.1.2 Firms production

This economy produces consumption goods, $Y_{c,t}$, and equipment, $Y_{e,t}$. The each representative producer has its production technology as follows:

$$Y_{c,t} = a_{c,t} F\left(K_{c,t}, Z_{c,t} H_{c,t}\right), \tag{2.7}$$

$$Y_{e,t} = a_{e,t} F\left(K_{e,t}, Z_{e,t} H_{e,t}\right), \tag{2.8}$$

where $K_{j,t}$ and $H_{j,t}$ denote the capital input and the labor service for $j \in \{c, e\}$. I assume the production function $F(\cdot)$ has constant return to scale, more specifically the Cobb-Douglas function as follows:

$$F\left(K_{j,t}, Z_{j,t} H_{j,t}\right) = K_{j,t}^{\alpha_j} \left(Z_{j,t} H_{j,t}\right)^{1-\alpha_j},$$

where α_j denotes the capital share of sector $j \in \{c, e\}$. The production technology features two productivity shocks: the one represented by Hicks-neutral productivity, a_j, has a stationary stochastic process and the other represented by labor-augmented productivity, $Z_{j,t}$, has a non-stationary stochastic process, for each sector j.

2.3.1.3 Composite final goods and Relative prices

The household uses two final goods: it purchases consumption goods to maximize its preference and invest the rest of its income (or forgone consumption) for the capital accumulation, which will be rented out to the producing firms. To satisfy its own demand of consumption goods, C_t, and equipment, I_t, the household purchases domestic products and imports. Accordingly, the final goods are composite bundles as follows:

$$C_t = \left\{ (1-\omega_c)^{\frac{1}{\zeta_c}} Y_{c,t}^{h}{}^{\frac{\zeta_c - 1}{\zeta_c}} + \omega_c^{\frac{1}{\zeta_c}} C_t^{f}{}^{\frac{\zeta_c - 1}{\zeta_t}} \right\}^{\frac{\zeta_c}{\zeta_c - 1}}, \tag{2.9}$$

$$I_t = \left\{ (1-\omega_e)^{\frac{1}{\zeta_e}} Y_{e,t}^{h}{}^{\frac{\zeta_e - 1}{\zeta_e}} + \omega_e^{\frac{1}{\zeta_e}} I_t^{f}{}^{\frac{\zeta_e - 1}{\zeta_t}} \right\}^{\frac{\zeta_e}{\zeta_e - 1}}, \tag{2.10}$$

where ω_j and ζ_j denote trade openness and elasticity of substitution for goods $j \in \{c, e\}$, respectively. C_t^f and I_t^f denote the imported consumption goods and equipment, respectively. The implied price index of each composite goods, therefore, can be written as follows:

$$P_{c,t} = \left\{(1-\omega_c){P_{c,t}^h}^{1-\zeta_c} + \omega_c {P_{c,t}^f}^{1-\zeta_c}\right\}^{\frac{1}{1-\zeta_c}}, \tag{2.11}$$

$$P_{e,t} = \left\{(1-\omega_e){P_{e,t}^h}^{1-\zeta_e} + \omega_e {P_{e,t}^f}^{1-\zeta_e}\right\}^{\frac{1}{1-\zeta_e}}, \tag{2.12}$$

where $P_{j,t}^h$ and $P_{j,t}^f$ denote the price of home-produced and foreign-produced goods $j \in \{c, e\}$, respectively.

From the two composite price indices, we can derive the relative prices. The concept of relative price is very useful to transform the nominal terms into real terms or in terms of consumption goods. Also, it facilitates the decomposition of investment-specific technology into meaningful units. As in Boileau (2002), the relative prices can be easily decomposed into the domestically generated investment-specific technology, Q_t^h, the embodied technology in equipment imports, Q_t^f, and the terms of trade for consumption goods, $tot_{c,t}$. The technologies and the terms of trade are defined by relative prices as follows: $Q^h = P_c^h/P_e^h$, $Q^f = P_c^{f*}/P_e^{f*}$ and $tot_c = P_c^h/P_c^f$, where the superscript of '$f*$' means the price of foreign goods in a foreign country.

Using the composite price indices given in Eq.(2.11) and Eq.(2.12), we can derive the following relative prices:

$$rp_{c,t}^h \equiv \frac{P_{c,t}}{P_{c,t}^h} = \Phi_c\left(1, \frac{P_{c,t}^f}{P_{c,t}^h}\right) = \Phi_c\left(1, \frac{1}{tot_{c,t}}\right), \tag{2.13}$$

$$rp_{c,t}^f \equiv \frac{P_{c,t}}{P_{c,t}^f} = \Phi_c\left(\frac{P_{c,t}^h}{P_{c,t}^f}, 1\right) = \Phi_c\left(tot_{c,t}, 1\right), \tag{2.14}$$

$$rp_{e,t}^h \equiv \frac{P_{e,t}}{P_{e,t}^h} = \Phi_e\left(1, \frac{P_{e,t}^f}{P_{e,t}^h}\right) = \Phi_e\left(1, \frac{Q_t^h}{tot_{c,t}Q_t^f}\right), \tag{2.15}$$

$$rp_{e,t}^f \equiv \frac{P_{e,t}}{P_{e,t}^f} = \Phi_e\left(\frac{P_{e,t}^h}{P_{e,t}^f}, 1\right) = \Phi_e\left(\frac{tot_{c,t}Q_t^f}{Q_t^h}, 1\right), \tag{2.16}$$

where $\Phi_j(n_1, n_2) = \left((1-\omega_j){n_1}^{1-\zeta_j} + \omega_j {n_2}^{1-\zeta_j}\right)^{1/(1-\zeta_j)}$ for $j \in \{x, e\}$.[8] These relative prices also

[8] The terms of trade for equipment, $\frac{P_e^h}{P_e^f}$, is decomposed as follows: $\frac{P_e^h}{P_e^f} = \frac{P_e^h}{P_c^h}\frac{P_c^h}{P_c^f}\frac{P_c^f}{P_e^f} = \frac{P_e^h}{P_c^h}\frac{P_c^h}{P_c^f}\frac{P_c^{f*}}{P_e^{f*}} = \frac{tot_c Q^f}{Q^h}$.

establish the relationship between investment-specific technology in Eq.(2.4) and domestically generated investment-specific technology, Q_t^h, as follows:

$$Q_t \equiv \frac{P_{c,t}}{P_{e,t}} = \frac{P_{c,t}}{P_{c,t}^h}\frac{P_{c,t}^h}{P_{e,t}^h}\frac{P_{e,t}^h}{P_{e,t}} = \frac{rp_{c,t}^h Q_t^h}{rp_{e,t}^h}. \tag{2.17}$$

As such investment-specific technology is decomposed into domestic investment-specific technology, terms of trade for consumption goods, and embodied technology in equipment imports.

2.3.1.4 External shocks

In this model economy, I introduce eleven external shocks, which follow either a transitory level-stationary process (denote lower-case) or a permanent growth-stationary process(denote upper-case). The preference shock, S_t, consists of transitory s_l and permanent S_g given as follows:

$$S_t = s_{l,t} S_{g,t}, \tag{2.18}$$

$$\ln s_{l,t} = \rho_{sl} \ln s_{l,t-1} + \epsilon_{sl,t}, \tag{2.19}$$

$$\ln\left(\frac{S_{g,t}/S_{g,t-1}}{\eta^{sg}}\right) = \rho_{sg} \ln\left(\frac{S_{g,t-1}/S_{g,t-2}}{\eta^{sg}}\right) + \epsilon_{sg,t}, \tag{2.20}$$

where η^{sg} stands for the long-run steady state growth rate of preference shock. The shocks on the embodied technology in equipment imports, Q^f, consists of transitory q_l^f, and permanent Q_g^f as follows:

$$Q_t^f = q_{l,t}^f Q_{g,t}^f, \tag{2.21}$$

$$\ln q_{l,t}^f = (1-\rho_{ql}) \ln \bar{q}_l^f + \rho_{ql} \ln q_{l,t-1}^f + \epsilon_{ql,t}, \tag{2.22}$$

$$\ln\left(\frac{Q_{g,t}^f/Q_{g,t-1}^f}{\eta^{qg}}\right) = \rho_{qg} \ln\left(\frac{Q_{g,t-1}^f/Q_{g,t-2}^f}{\eta^{qg}}\right) + \epsilon_{qg,t}, \tag{2.23}$$

where $\bar{q}_l^f$ and η^{qg}, respectively, denote the steady state level and growth rate of the embodied technology in imports. There are five transitory stochastic variables known as world interest rate, wage markups, terms of trade for consumption goods, and two Hicks-neutral productivity shocks, which are respectively given as follows:

$$\ln r_t^f = (1-\rho_r)\ln \bar{r}^f + \rho_r \ln r_{t-1}^f + \epsilon_{r,t}, \tag{2.24}$$

$$\ln \mu_t = (1-\rho_\mu)\ln \bar{\mu} + \rho_\mu \ln \mu_{t-1} + \epsilon_{\mu,t}, \tag{2.25}$$

$$\ln tot_{c,t} = \rho_{tot} \ln tot_{c,t-1} + \epsilon_{tot,t}, \tag{2.26}$$

$$\ln a_{c,t} = \rho_{ac} \ln a_{c,t-1} + \epsilon_{ac,t}, \tag{2.27}$$

$$\ln a_{e,t} = \rho_{ae} \ln a_{e,t-1} + \epsilon_{ae,t}, \tag{2.28}$$

where $\bar{r}^f$ and $\bar{\mu}$ denote the steady state values of the world interest rate and wage markups, respectively. $\rho_j \in [0,1)$ and ϵ_j, respectively, denote the autoregressive coefficients and disturbances of stochastic process j which is *iid* normal with zero mean and variance of σ_j^2 for $j \in \{sl, sg, ql, qg, r, \mu, tot, ac, ae\}$.

From the empirical analysis performed in Section 2.2, we already have found that the sectoral productivities are cointegrated in the Korean economy. Accordingly, I incorporate the cointegrated sectoral productivities through the vector error correction model (VECM) as follows:

$$\begin{bmatrix} \ln\left(\dfrac{Z_{c,t}/Z_{c,t-1}}{\eta^{zc}}\right) \\ \ln\left(\dfrac{Z_{e,t}/Z_{e,t-1}}{\eta^{ze}}\right) \end{bmatrix} = \begin{bmatrix} \rho_{cc} & \rho_{ce} \\ \rho_{ec} & \rho_{ee} \end{bmatrix} \begin{bmatrix} \ln\left(\dfrac{Z_{c,t-1}/Z_{c,t-2}}{\eta^{zc}}\right) \\ \ln\left(\dfrac{Z_{e,t-1}/Z_{e,t-2}}{\eta^{ze}}\right) \end{bmatrix} + \begin{bmatrix} f_{zc}\left(ect_{t-1}\right) \\ f_{ze}\left(ect_{t-1}\right) \end{bmatrix} + \begin{bmatrix} D_{cc} & D_{ce} \\ D_{ec} & D_{ee} \end{bmatrix} \begin{bmatrix} \epsilon_{zc,t} \\ \epsilon_{ze,t} \end{bmatrix} \tag{2.29}$$

where ρ_j for $j \in \{cc, ce, ec, ee\}$ denote the vector autoregressive coefficients and ϵ_i implies the disturbance of stochastic process, which is *iid* normal with zero mean and variance of σ_i^2, for $i \in \{zc, ze\}$. η^{zc} and η^{ze} denote the steady state growth rate of the productivities of consumption goods and equipment producing sectors, respectively. The system has an error-correction term given as

$$ect_t = \ln Z_{c,t} - \kappa \ln Z_{e,t}. \tag{2.30}$$

The error-correction term implies that $Z_{c,t}$ and $Z_{e,t}$ are cointegrated with the cointgrating vector $(1, -\kappa)$. The function $f_i(\cdot)$ can be either a linear or a non-linear function for $i \in \{zc, ze\}$. In here I assume that $f_i(\cdot)$ is a typical linear function as follows:

$$f_i(ect_t) = \theta_i ect_t, \tag{2.31}$$

where θ_i denotes speed-of-adjustment parameter for $i \in \{zc, ze\}$. The assumption of linear error-correction term is consistent with the empirical result of SECTION 2.2, in which sectoral cointegration is confirmed while the null hypothesis of no cointegration with non-linear error correction cannot be rejected.

2.3.2 Competitive equilibrium

To solve the model economy, we need the optimization conditions of households and firms as well as market clearing conditions. In what follows, I would explain for the conditions. Under the economy described above, the representative household would maximize its life-time utility given in Eq.(2.1) subject to its budget constraint, Eq.(2.2), and a capital accumulation process, Eq.(2.5). Therefore, the following necessary conditions have to be satisfied for the household's optimization:

$$\Lambda_{1,t} = \frac{1}{C_t - \xi C_{t-1}} - \beta \mathbb{E}_t \frac{\xi}{C_{t+1} - \xi C_t}, \tag{2.32}$$

$$\frac{1}{S_t} = \Lambda_{1,t} \frac{\tilde{W}_t}{\mu_t}, \tag{2.33}$$

$$\Lambda_{1,t} = \beta \mathbb{E}_t \Lambda_{1,t+1} \left(1 + r_t^d\right), \tag{2.34}$$

$$\Lambda_{2,t} = \beta \mathbb{E}_t \left[\Lambda_{2,t+1}(1-\delta) + \Lambda_{1,t+1} \tilde{R}_{t+1}\right], \tag{2.35}$$

$$\frac{\Lambda_{1,t}}{Q_t} = \Lambda_{2,t} \left(1 - \frac{\partial \Psi_k \left(I_t, I_{t-1}\right)}{\partial I_t}\right) - \beta \mathbb{E}_t \Lambda_{2,t+1} \frac{\partial \Psi_k \left(I_{t+1}, I_t\right)}{\partial I_t} \tag{2.36}$$

$$C_t + I_t/Q_t + (1 + r_{t-1}^d)\tilde{d}_{t-1} \mathrm{T}_{t-2}^c = \tilde{R}_t K_t + \frac{\tilde{W}_t}{\mu_t} H_t + \tilde{d}_t \mathrm{T}_{t-1}^c + \Pi_t, \tag{2.37}$$

$$K_{t+1} = (1-\delta) K_t + I_t - \Psi_k \left(I_t, I_{t-1}\right), \tag{2.38}$$

$$r_t^d = r_t^f + \Psi_d(d_t). \tag{2.39}$$

Because the households of this economy are identical, the average level of debt, d_t, has to be equal to the household's debt, $\tilde{d}_t$. Accordingly, the interest rate for foreign debt of Eq.(2.39) can be rewritten with respect to $\tilde{d}_t$ as follows:

$$r_t^d = r_t^f + \Psi_t(\tilde{d}_t). \tag{2.40}$$

Two representative firms producing consumption goods and equipment have production technology as follows:

$$Y_{c,t} = a_{c,t} K_{c,t}{}^{\alpha_c} \left(Z_{c,t} H_{c,t} \right)^{(1-\alpha_c)}, \tag{2.41}$$

$$Y_{e,t} = a_{e,t} K_{e,t}{}^{\alpha_e} \left(Z_{e,t} H_{e,t} \right)^{(1-\alpha_e)}. \tag{2.42}$$

Since the two producers face competitive goods and factor markets, their profit maximization behaviors have to satisfy the following conditions:

$$\tilde{R}_t = \alpha_c \frac{1}{rp^h_{c,t}} \frac{Y_{c,t}}{K_{c,t}}, \tag{2.43}$$

$$\tilde{W}_t = (1-\alpha_c) \frac{1}{rp^h_{c,t}} \frac{Y_{c,t}}{H_{c,t}}, \tag{2.44}$$

$$Q^h_t = \frac{\alpha_e Y_{e,t}/K_{e,t}}{\alpha_c Y_{c,t}/K_{c,t}}. \tag{2.45}$$

The aggregate output is expressed in terms of consumption goods as follows:

$$\tilde{Y}_t = \frac{Y_{c,t}}{rp^h_{c,t}} + \frac{Y_{e,t}}{rp^h_{e,t} Q_t}. \tag{2.46}$$

Additionally, the value of aggregate output in terms of consumption goods, $\tilde{Y}_t$, must equal the value of firms' production cost. Therefore, the following equation should hold:

$$\tilde{Y}_t = \tilde{W}_t H_t + \tilde{R}_t K_t. \tag{2.47}$$

The final consumption and investment are the composite of domestic products and imports as in Eq.(2.9) and (2.10). We can therefore derive the domestic demands for the goods as follows:

$$Y^h_{c,t} = (1-\omega_c) \left(rp^h_{c,t} \right)^{\zeta_c} C_t \tag{2.48}$$

$$C^f_t = \omega_c \left(rp^f_{c,t} \right)^{\zeta_c} C_t \tag{2.49}$$

$$Y^h_{e,t} = (1-\omega_e) \left(rp^h_{e,t} \right)^{\zeta_e} I_t \tag{2.50}$$

$$I^f_t = \omega_e \left(rp^f_{e,t} \right)^{\zeta_e} I_t. \tag{2.51}$$

Because this model economy is described in real terms or in terms of consumption goods, we have to include the relative price system of Eqs.(2.13)-(2.16) into the system of equations. Also, we need the relation between the investment-specific technology, Q_t, and the domestically generated investment-specific technology, Q_t^h, which is described in Eq.(2.17). Lastly, to be a complete system, the model economy should satisfy the following market clearing conditions:

$$K_t = K_{c,t} + K_{e,t}, \tag{2.52}$$

$$H_t = H_{c,t} + H_{e,t}, \tag{2.53}$$

$$C_t + I_t/Q_t + \tilde{TB}_t = \tilde{Y}_t, \tag{2.54}$$

$$\tilde{TB}_t = (1 + r_{t-1}^d)\tilde{D}_{t-1} - \tilde{D}_t. \tag{2.55}$$

2.3.3 Solution

Due to the non-stationary stochastic processes of $S_{g,t}$, $Z_{c,t}$, and $Z_{e,t}$, the variables of the model economy possesses non-stationary properties either. Therefore, we need to transform the non-stationary variables into stationary ones onto the balanced growth path. Since each variable grows with different rate along the balanced growth path, the functional forms of transformation for the variables are different. Through the following transformation equations, each non-stationary variable, denoted by upper-case, is replaced by its stationary form, denoted by lower-case, : $\tilde{Y}_t = \tilde{y}_t \mathrm{T}_{t-1}^c$ $C_t = c_t \mathrm{T}_{t-1}^c$; $I_t = i_t \mathrm{T}_{t-1}^i$; $H_t = h_t \mathrm{T}_{t-1}^h$; $H_{c,t} = h_{c,t} \mathrm{T}_{t-1}^h$; $H_{e,t} = h_{e,t} \mathrm{T}_{t-1}^h$; $\Lambda_{1,t} = \lambda_{1,t}/\mathrm{T}_{t-1}^c$; $\Lambda_{2,t} = \lambda_{2,t}/\mathrm{T}_{t-1}^i$; $\tilde{R}_t = \tilde{r}_t/\mathrm{T}_{t-1}^q$; $\tilde{W}_t = \tilde{w}_t \mathrm{T}_{t-1}^c/\mathrm{T}_{t-1}^h$; $Q_t = q_t \mathrm{T}_{t-1}^q$; $Q_t^h = q_t^h \mathrm{T}_{t-1}^q$; $K_t = k_t \mathrm{T}_{t-1}^i$; $K_{c,t} = k_{c,t} \mathrm{T}_{t-1}^i$; $K_{e,t} = k_{e,t} \mathrm{T}_{t-1}^i$; $Y_{c,t} = y_{c,t} \mathrm{T}_{t-1}^c$; $Y_{c,t}^h = y_{c,t}^h \mathrm{T}_{t-1}^c$; $C_t^f = c_t^f \mathrm{T}_{t-1}^c$; $Y_{e,t} = y_{e,t} \mathrm{T}_{t-1}^i$; $Y_{e,t}^h = y_{e,t}^h \mathrm{T}_{t-1}^i$; $I_t^f = i_t^f \mathrm{T}_{t-1}^i$; $\tilde{TB}_t = \tilde{tb}_t \mathrm{T}_{t-1}^c$; $\tilde{D}_t = \tilde{d}_t \mathrm{T}_{t-1}^c$, where $\mathrm{T}_t^c = {Z_{c,t}}^{1-\alpha_c} {Z_{e,t}}^{\alpha_c} S_{g,t}$, $\mathrm{T}_t^i = Z_{e,t} S_{g,t}$, $\mathrm{T}_t^h = S_{g,t}$, and $\mathrm{T}_t^q = Q_{g,t}^f = \mathrm{T}_t^i/\mathrm{T}_t^c$.

Applying the above transformation to the non-stationary system of equations, Eqs.(2.13)-(2.17) and Eqs.(2.32)-(2.55), we can obtain the stationary system of equations: the equations are presented in Appendix B.1.1. In the substitution process, I define the exogenous growth rates, denote ηs, and the growth rates of endogenous variables, denote τs, as follows: $\eta_t^{zc} = Z_{c,t}/Z_{c,t-1}$, $\eta_t^{ze} = Z_{e,t}/Z_{e,t-1}$ and $\eta_t^{sg} = S_{g,t}/S_{g,t-1}$; $\tau_t^c = \mathrm{T}_t^c/\mathrm{T}_{t-1}^c$, $\tau_t^i = \mathrm{T}_t^i/\mathrm{T}_{t-1}^i$ and $\tau_t^h = \mathrm{T}_t^h/\mathrm{T}_{t-1}^h$.

To solve the stationary non-linear system, I employ the method of Klein (2000). Since this solution method requires a linearized system, I log-linearize the stationary non-linear system on the steady-state values.[9]

2.4 Estimation

The main goal of this paper is to examine the contribution of foreign innovations to the economic fluctuation of Korea. For the business cycle analysis, the structural parameters of the above model economy have to be determined. I calibrate a subset of the structural parameters and estimates the remaining parameters using the model economy as the data generating process.

Table 2.6: Calibrated parameter values

Parameter	Description	Value
β	subjective discount factor	0.9954
$\bar{r}^f$	steady state value of world interest rate	0.0123
δ	depreciation rate	0.025
$\bar{\mu}$	steady state value of wage markup	1.1
D_{cc}	a diagonal element of innovation coefficient in VECM	1
D_{ee}	a diagonal element of innovation coefficient in VECM	1
ζ_c	elasticity of substitution of consumption goods	0.70
ζ_e	elasticity of substitution of equipment	0.25
γ	the share of consumption goods to aggregate output	0.7242
ω_c	openness parameter for consumption goods	0.0672
ω_e	openness parameter for equipment	0.1986
$\bar{\tau}^c$	steady state value of consumption growth rate	1.0077
$\bar{\tau}^e$	steady state value of investment growth rate	1.0125
$\bar{\tau}^h$	steady state value of hours worked growth rate	0.9947

Table 2.6 exhibits the descriptions and values of calibrated parameters. The steady state growth rates of consumption, investment and hours worked use their period averages of corresponding observable variables; the assigned values are 1.0077, 1.0125, and 0.9947, respectively. I assign the value of 10 percent to the steady state value of wage markup and 0.025 to the quarterly depreciation rate. Following Kim (2012), the elasticity of substitution of consumption goods, ζ_c, is set to

[9] More details for the dynamic solution method are explained in the appendices of previous chapter.

0.7 and that of equipment, ζ_e, to 0.25. The diagonal elements of innovation coefficients of VECM are normalized to unity; that is, D_{cc} and D_{ee} are set to unity. The trade openness parameters and the share of consumption goods sector are determined by referring the input-output table of year 2000, which is available on the website of Bank of Korea. I use the value of 6.72 percent for the trade openness of consumption goods, ω_c, and the value of 19.9 percent for that of equipment, ω_e. The share of consumption goods to the aggregate output, γ, is set to 72.42 percent.

The annual interest rate of 5 percent is imposed to the steady state value of world interest rate, $\bar{r}^f$. The steady state conditions of the model economy, exhibited in Appendix B.1.2, imply that the subjective discount factor, β, is determined by other calibrated parameters as follows:

$$\beta = \frac{\bar{\tau}^c}{1 + \bar{r}^d}.$$

Accordingly, the value of 0.9954 is assigned to β. Since this model economy incorporate external debt-elastic interest rate premium, the steady state conditions requires the value of steady state debt level. From the observable data used in the maximum likelihood estimation, we can get the average trade-balance-to-output ratio of 2.87 percent. By matching to the average trade balance ratio, I let the steady state debt level, $\bar{\bar{d}}$, be computed with other related parameters.

With the calibrated parameters, I estimate the remaining parameters using maximum likelihood as in Schmitt-Grohé and Uribe (2011) and Ireland and Schuh (2008). Before proceeding estimation, it is necessary to define the real aggregate output of the model economy because the structural parameters are estimated by observing the growth rates of macroeconomic aggregates, which will be explained shortly after. In nominal terms or terms of consumption goods, defining aggregate output is straightforward as in Eq.(2.46). The method of aggregation for real sectoral outputs, however, are not simple as nominal aggregation because each real sectoral output has different unit-measures. In order to aggregate real sectoral outputs, Horvath (2000) and Whelan (2003) use Divisia index, which is internally consistent with the method of national accounting.[10] For parameter estimation, instead of Divisia index, I approximate the aggregate output using

[10]Divisia weights the growth rate of each sector by its current share in the corresponding nominal aggregate outputs. Accordingly, the weights may change over time. Divisia index is a theoretical construct to create index

Cobb-Douglas aggregator as follows:

$$Y_t = {Y_{c,t}}^{\gamma} {Y_{e,t}}^{1-\gamma}, \tag{2.56}$$

where γ represents the share of nominal consumption goods out of nominal aggregate output. With simple manipulation, we can derive the following equality from Eq.(2.56):

$$g_t^Y = \gamma g_t^C + (1-\gamma) g_t^I, \tag{2.57}$$

where g_t^Y, g_t^C, and g_t^I represent the production growth-rates of aggregate output, consumption-goods, and equipment, respectively.[11] Eq.(2.57) implies that the growth rate of aggregate output is the weighted sum of the production growth rates of consumption goods and equipment. As such, we can see that the Cobb-Douglas aggregator resembles Divisia index except for restricting the weights to constant. Additionally, the implied price index, P_t, corresponding to the aggregate output, Y_t, is given by

$$P_t = \left(\frac{P_{c,t}^h}{\gamma} \right)^{\gamma} \left(\frac{P_{e,t}^h}{1-\gamma} \right)^{1-\gamma}. \tag{2.58}$$

To estimate structural parameters, I adopt the growth rate of output, consumption, investment, and hours worked to the observable variables. These series of observables, except for the growth rate of hours worked, are constructed from the Korean data of national income statistics available on the website of Bank of Korea (BOK). I reconstruct the consumption and investment for the purpose of this study because households' durables have to be regarded as equipments. To do this, applying Törnqvist index, I derive the price deflator of each item in GDP expenditure and aggregate to the newly defined deflators for consumption and investment. These aggregated deflators are applied to calculate real consumption and investment from their nominal aggregates. Hours worked is constructed from the labor statistics of Ministry of Employment and Labor. Since hours worked on the database are only available since 1993, I extrapolate the series of hours worked

series for continuous data. In practice, however, we usually use a discrete approximation to a continuous Divisia index, which is called Törnqvist index, because most of economic data are discrete.

[11] On the balanced growth path of the model economy, the growth rate of consumption-goods production is equal to that of consumption expenditure. The growth rate of equipment production is equal to that of investment expenditure.

Table 2.7: The maximum likelihood estimates and standard errors of the structural parameters

Parameter	Description	Estimate	Standard error
ξ	habit persistence	0.1469	0.0744
ψ_k	parameter for capital adjustment	2.0927	0.3670
ψ_d	parameter for debt-elastic premium	0.0109	0.0121
α_c and α_e	capital elasticity of production	0.4453	0.0555
ρ_{cc}	autoregressive parameter in VECM	0.5834	0.2454
ρ_{ce}	autoregressive parameter in VECM	0.4647	0.3249
ρ_{ec}	autoregressive parameter in VECM	0.0243	0.1656
ρ_{ee}	autoregressive parameter in VECM	0.5242	0.2617
θ_c	adjustment speed of error correction	-0.6798	0.5770
θ_e	adjustment speed of error correction	0.8858	0.6070
D_{ce}	correlation of innovations in VECM	0.7011	0.2000
D_{ec}	correlation of innovations in VECM	0.1461	0.1853
ρ_{sl}	autoregressive parameter of s_l	0.5795	0.1208
ρ_{sg}	autoregressive parameter of s_g	0.1862	0.3228
ρ_{ac}	autoregressive parameter of a_c	0.8764	0.0777
ρ_{ae}	autoregressive parameter of a_e	0.1353	0.1180
ρ_{tot}	autoregressive parameter of tot_c	0.7411	0.0634
ρ_{ql}	autoregressive parameter of q_l^f	0.3117	0.2032
ρ_{qg}	autoregressive parameter of q_g^f	0.8250	0.1137
ρ_{rf}	autoregressive parameter of r^f	0.1241	0.1364
ρ_{μ}	autoregressive parameter of μ	0.1037	0.1085
σ_{sl}	standard error of s_l	0.0000	0.0031
σ_{sg}	standard error of s_g	0.0043	0.0021
σ_{ac}	standard error of a_c	0.0110	0.0019
σ_{ae}	standard error of a_e	0.0496	0.0047
σ_{tot}	standard error of tot_c	0.2700	0.0308
σ_{ql}	standard error of q_l^f	0.3250	0.0766
σ_{qg}	standard error of q_g^f	0.0531	0.0686
σ_{rf}	standard error of r^f	0.0016	0.0718
σ_{μ}	standard error of μ	0.0000	0.0015
σ_{zc}	standard error of z_c	0.0029	0.0020
σ_{ze}	standard error of z_e	0.0007	0.0024
ϕ_y	measurement error of y	0.0030	0.0018
ϕ_c	measurement error of c	0.0003	0.0010
ϕ_i	measurement error of i	0.0001	0.0023
ϕ_h	measurement error of h	0.0001	0.0007

Notes: The sample period is 1988:Q1 to 2011:Q3. The observables are the growth rate of output, consumption, investment, and hours worked. Each of the observables is assumed to possess measurement error.

before 1993 by using the growth rate of labor-input index served by Korea Productivity Center. All observed data, ranging 1988:Q1-2011:Q3, are seasonally adjusted and reconstructed in per capita terms by applying "economically active population 15 years or over," which is available on the website of Statistics Korea.

Using the constructed observable variables, I obtain the maximum likelihood estimates in Table 2.7. Along with that, the standard errors of estimates are computed via parametric bootstrapping procedure as in Ireland and Schuh (2008). I generate 1,000 sets of artificial data from the estimated model by assigning random disturbances to each period, which have the same length of actual data. Artificial data of 1,000 sets are used to estimate 1,000 sample parameters. The standard errors of Table 2.7 are the standard deviations of the samples. Additionally, the estimation allow the existence of measurement errors in observable variables. In the estimation, I curb the estimates of these measurement errors not to exceed 25 percent of the standard error of each series.

The model estimates the habit persistence of $\xi = 0.1469$; it is quite lower than Rhee (2004)[12], which estimates habit formation of 0.6 by using Korean Household Panel Studies. The parameters for capital adjustment costs and debt-elastic premium are estimated as $\psi_k = 2.0927$ and $\psi_d = 0.0109$, respectively; it implies that the real rigidity of capital accumulation in Korea is significantly important. In estimation, I assume that each production sectors, consumption goods and equipment, have symmetric production technology. The capital share of both consumption goods and equipment production, α_c and α_e, is estimated to 0.4453 with significance.[13] The estimated capital share is higher than 0.36, which is used in many literatures of the Korean business cycles, but lower than 0.5395 of Kim (2004)[14].

Because the model economy incorporates the cointegration of sectoral productivities, we can

[12]Rhee (2004) finds that the estimate of habit formation for non-durables and services consumption is marginally significant at 10 percent level.

[13]It is well known that the maximum likelihood estimates have asymptotically a normal distribution. Accordingly, we can apply t-test for an hypothesis test. For more details, see Canova (2007).

[14]Kim (2004) calibrates the capital share using the national account of Korea.

investigate the features of sectoral cointegration in the Korean economy. The model economy estimates autoregressive coefficients in VECM of $\rho_{cc} = 0.5834$ and $\rho_{ee} = 0.5242$ with significance but the cross-sectoral autoregressive coefficients, $\rho_{ce} = 0.4647$ and $\rho_{ec} = 0.0243$, are not significant; this implies that the most of permanent shocks of the Korean sectoral productivities are delivered to the next period with low cross-sector effects. Also, the estimates of $D_{ce} = 0.7011$ and $D_{ec} = 0.1461$ indicate that the permanent innovations of sectoral productivities are correlated. The estimates of $\sigma_{zc} = 0.0029$ and $\sigma_{ze} = 0.0007$ imply that the permanent shocks in sectoral productivities do not have large effects on short-run fluctuation of the Korean economy.

Among other shocks, the permanent shock of preference ($\sigma_{sg} = 0.0043$), the transitory shocks of sectoral productivities ($\sigma_{ac} = 0.0110$ and $\sigma_{ae} = 0.0496$), the terms of trade shock ($\sigma_{tot} = 0.2700$), and the transitory shock of embodied technology in imports ($\sigma_{ql} = 0.3250$) are revealed as significant external forces affecting to the economic fluctuation of Korea. Moreover, the transitory preference shock ($\rho_{sl} = 0.5795$), the transitory shock of consumption goods production ($\rho_{ac} = 0.8764$), the terms of trade shock ($\rho_{tot} = 0.7411$), and the permanent shock of technology embodied imports ($\rho_{qg} = 0.8250$) show significantly high persistence. Additionally, it turns out that there are no significant measurement error of observables in this estimation. In summary, these estimates of structural parameters imply that the largest sources of economic volatility in Korea are foreign shocks such as the terms of trade shock and the transitory shock of technology-embodied imports. Furthermore, the terms of trade shock gives highly persistent effect on the Korean economy.

Table 2.8 presents observed and simulated second moments of growth rate of output, consumption, investment, and hours worked. The second moments indicate that the model fits the data quite well. The model replicates the similar volatility pattern of data: the highest volatility of investment growth rate, the lowest volatility of consumption growth rate, and the medium volatility of output and hours worked with almost same level. The model also captures procyclicality of consumption, investment, and hours worked.

Table 2.8: Empirical and simulated moments

	Relative volatility		Correlation with output growth	
	Data	Model	Data	Model
τ^Y	1.00	1.23		
τ^C	0.79	0.80	0.18	0.36
τ^I	3.26	3.68	0.36	0.30
τ^H	1.05	1.20	0.24	0.51

Notes: τ^i, for $i \in \{Y, C, I, H\}$, denotes the growth rate of output, consumption, investment, and hours worked, respectively. Relative volatility is computed as the standard deviation of a variable divided by the standard deviation of observed output growth. It is noteworthy that the consumption data of this paper only includes non-durables and service consumption and the investment data is constructed by aggregating durable consumption and equipment.

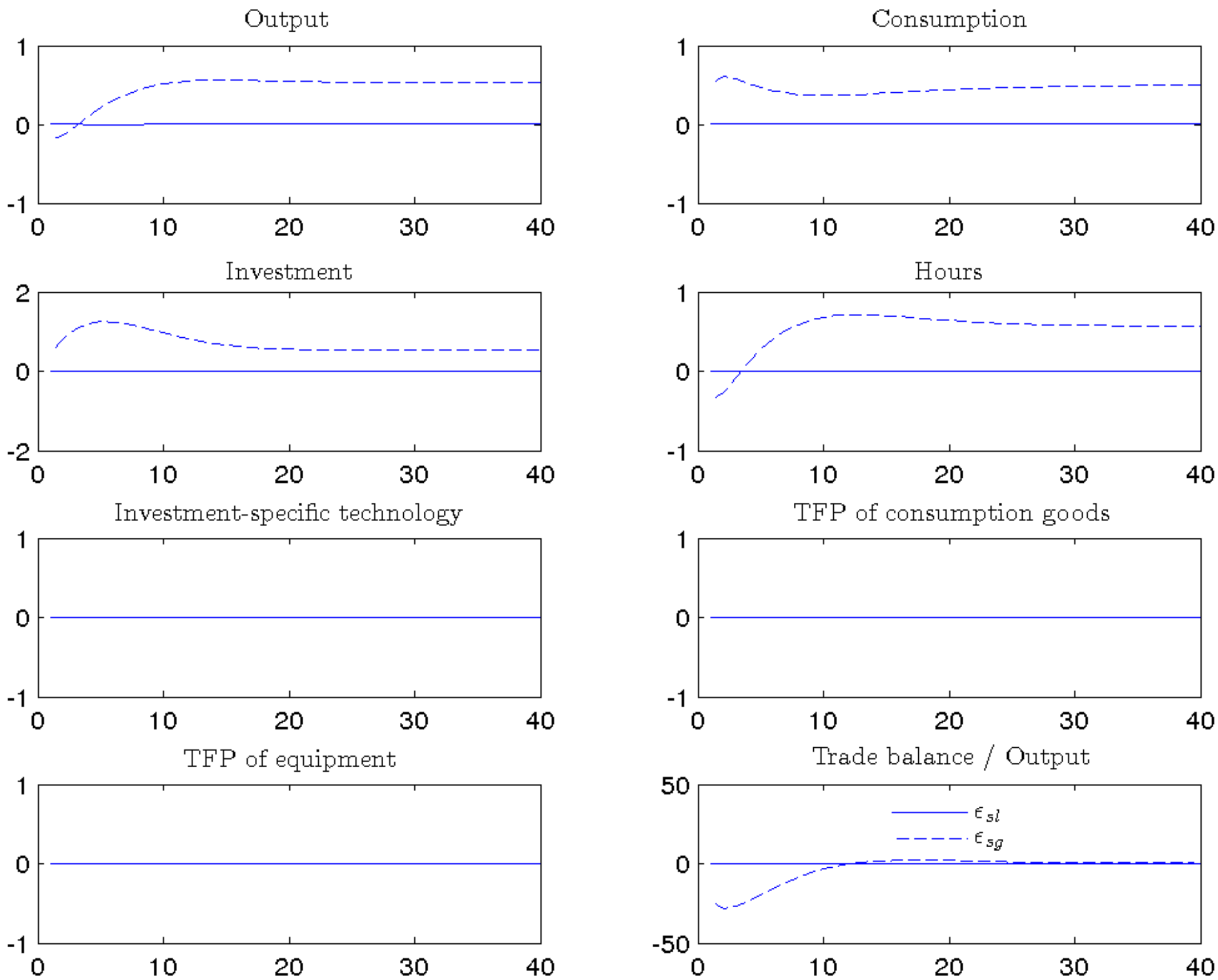
2.5 Results

In this section, using the estimated structural model from the previous sections, I investigate the implication of the considered external innovations to the Korean economy. Firstly, to examine the effects of each external shock, I conduct impulse response analysis for the eleven external shocks. The plots of FIGURE 2.3-2.8 represent the impulse responses of output, consumption, investment, hours worked, investment-specific technology, sectoral productivities and the ratio of trade balance to GDP corresponding to a one-standard-deviation of each shock.

FIGURE 2.3 depicts the impulse responses of macroeconomic aggregates to the transitory and permanent shocks of preference. In a closed economy, with a positive preference shock, the household would consume more by working more. In an open economy, however, the household doesn't have to increase its hours worked to increase consumption because it can import more consumption goods from a foreign country. On the initial impact, consumption therefore increases persistently with trade deficit rather than increase of output. However, the country has to rebalance the trade deficit in the lasting periods by increasing its production. Meanwhile, there are no impulse responses for a transitory shock of preference because the corresponding standard deviation turns out zero.

The impulse responses on the transitory shocks of sectoral productivities are presented in FIGURE 2.4. When the productivity level of consumption goods increase temporarily, the household

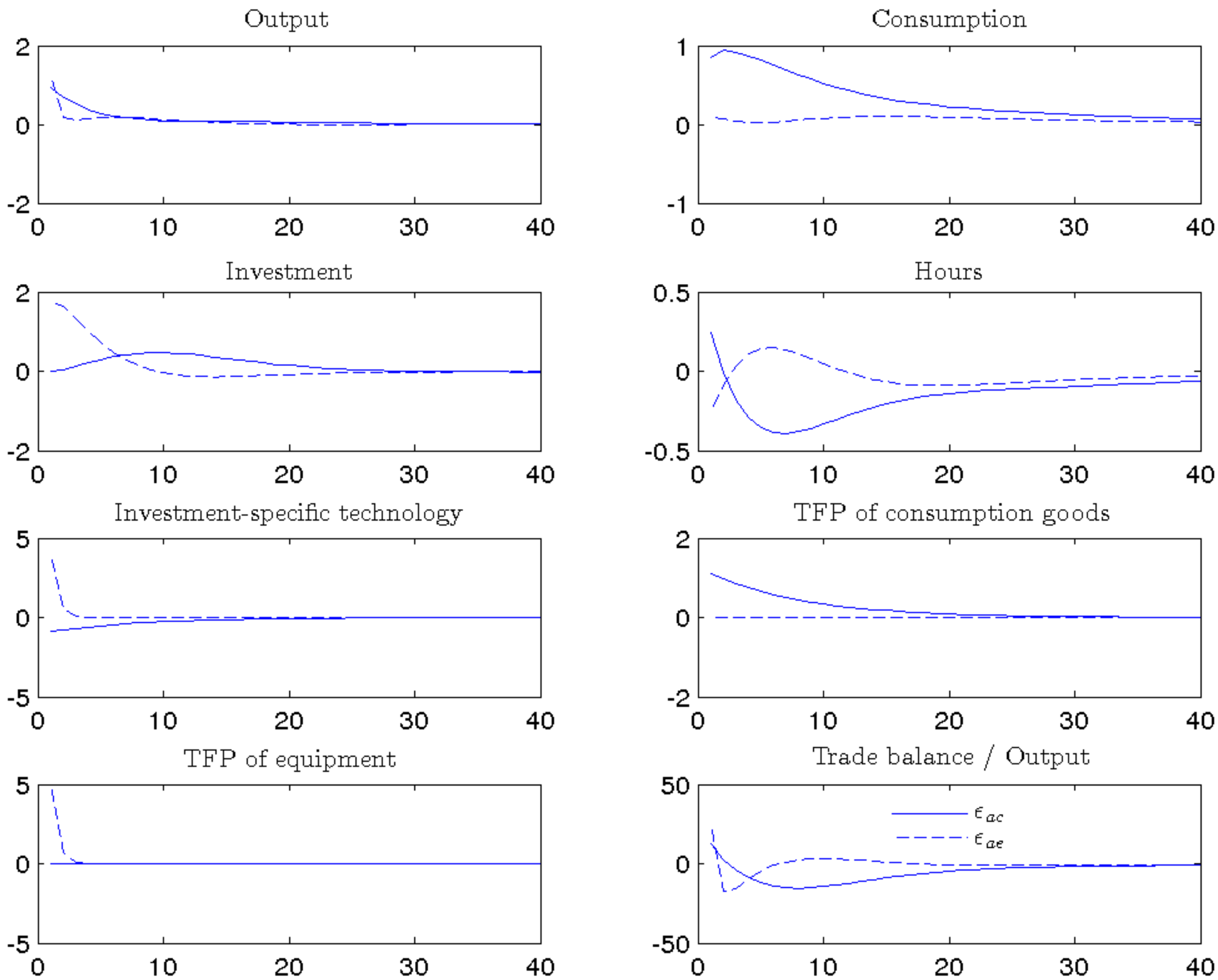
Figure 2.3: Impulse responses on the transitory and permanent shocks of preference



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, TFP of consumption goods sector, and TFP of equipment sector to a one-standard-deviation shocks to the level and growth rate of preference.

knows that it is transitory and doesn't remain longer. Accordingly, the household would work and produce more output in present and smooth its consumption over time. Due to the temporary increase in the productivity of consumption goods, investment-specific technology decreases on the impact and then gradually recover its original level by inducing a hump shaped response of investment expenditure. On the other hand, the transitory shock on the equipment productivity increase investment expenditure by enhancing the efficiency of investment without a significant effect on consumption. Hours worked, however, contract on the impact due to the substitution

Figure 2.4: Impulse responses on the transitory shocks of sectoral productivities

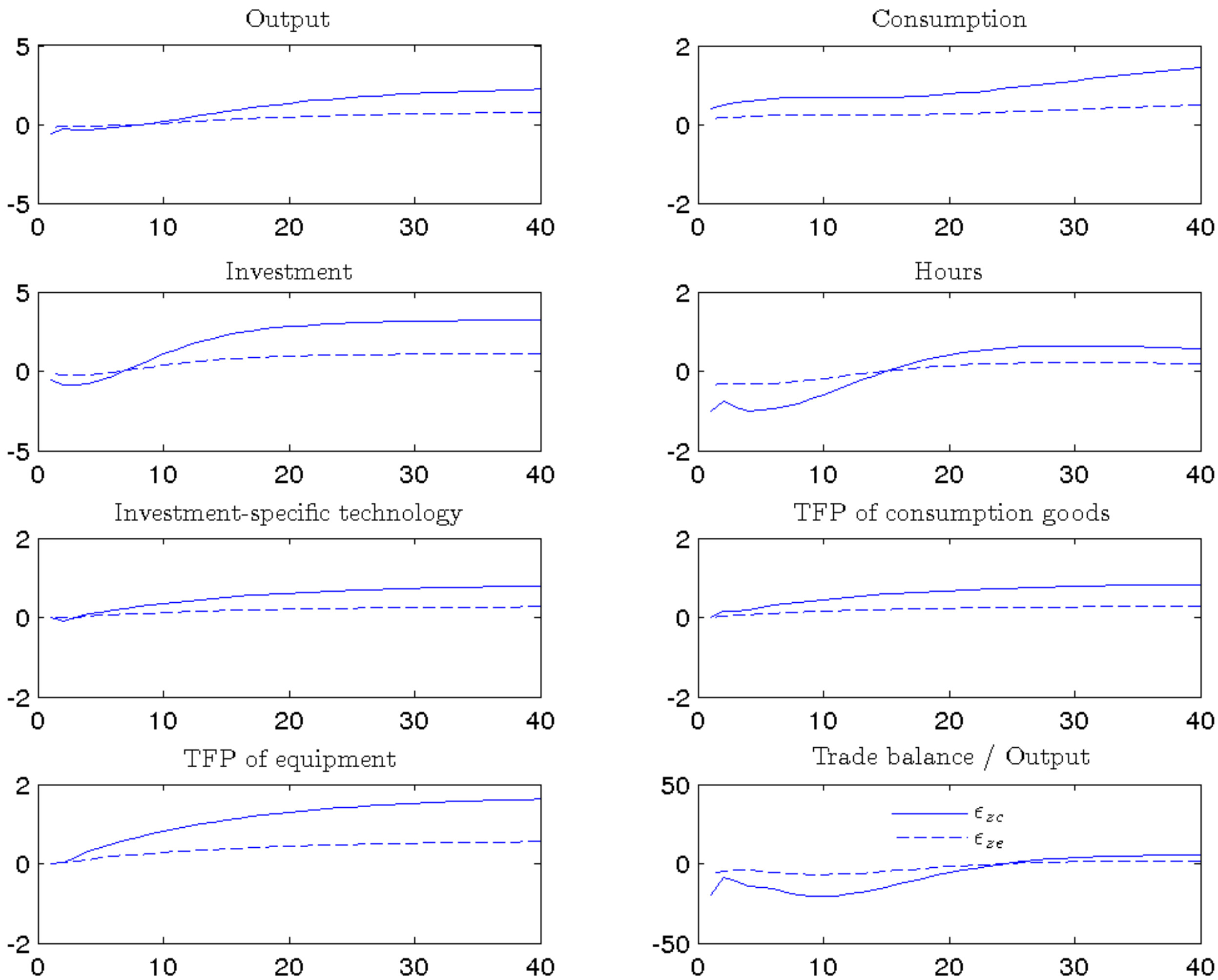


Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, TFP of consumption goods sector, and TFP of equipment sector to a one-standard-deviation shocks to the levels of sectoral productivities.

effect of investment. After the initial contract, hours worked exhibit a hump shaped response.

Figure 2.5 exhibits the impulse responses on the permanent shocks of the sectoral productivities. Because the sectoral productivities are cointegrated, the productivities share a common stochastic trend affected by two external shocks ϵ_c and ϵ_e in this model economy. In this case, both shocks cause similar patterns of impulse responses. The common trend shocks induce the permanent increases of sectoral productivities. Since the productivity enhancement of equipment sector is overall larger than that of consumption goods, however, investment-specific technology

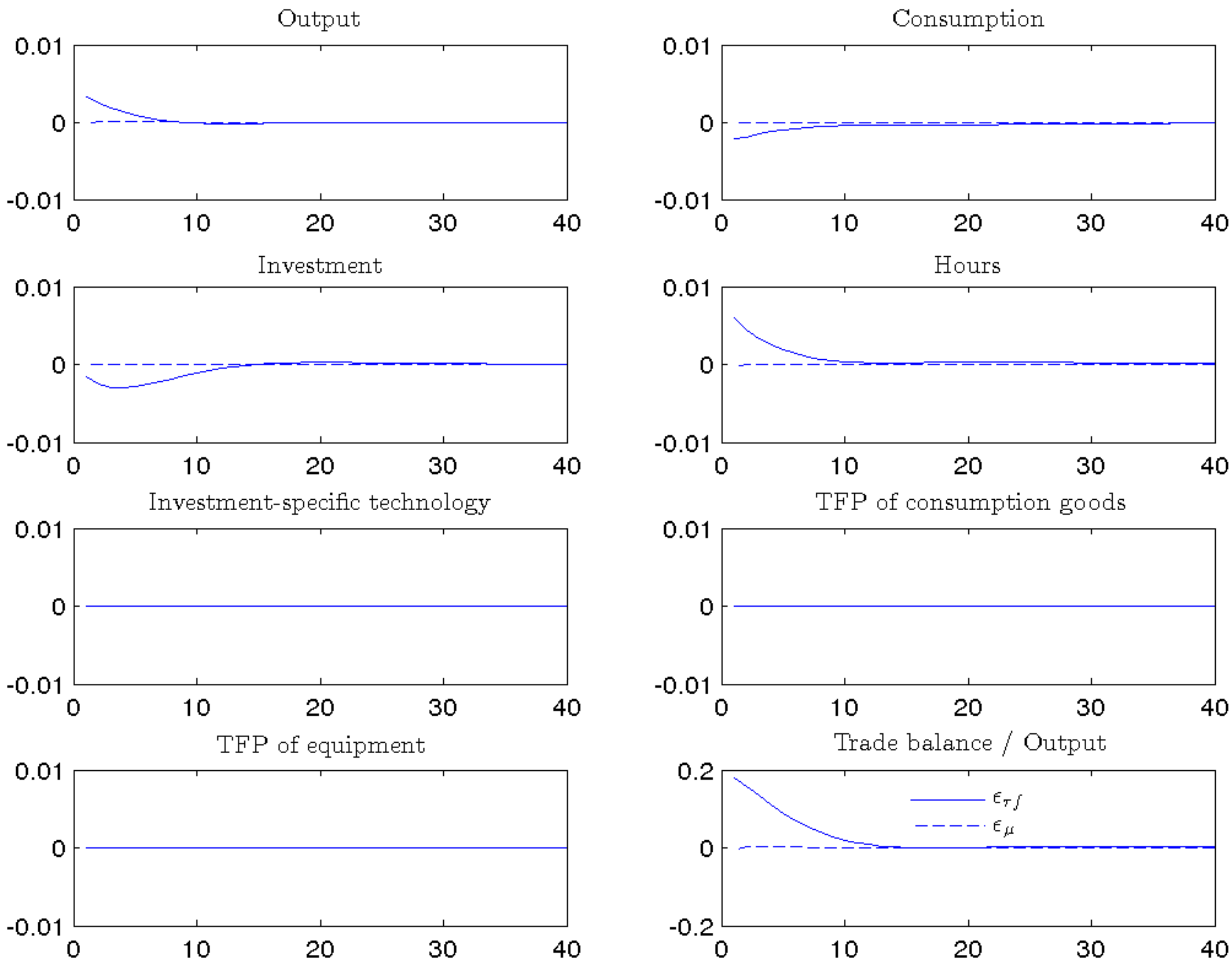
Figure 2.5: Impulse responses on the permanent shocks of sectoral productivities



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, TFP of consumption goods sector, and TFP of equipment sector to a one-standard-deviation shocks to the growth rates of sectoral productivities.

also displays permanent increase except for the brief decrease on the impact. These patterns of productivities induce a J-shaped response of investment spending, contracting investment spending with sluggish investment-specific technology at first, and then growing slowly and persistently with the gradual improvement in investment-specific technology. Because the permanent technology shock on common trend increase the income stream of the household, it would smooth its consumption by increasing current consumption. Excessive increase of consumption, however, induces trade deficit on impact and remained about twenty quarters. It is worth to note that investment

Figure 2.6: Impulse responses on world interest rate and wage-markup shock

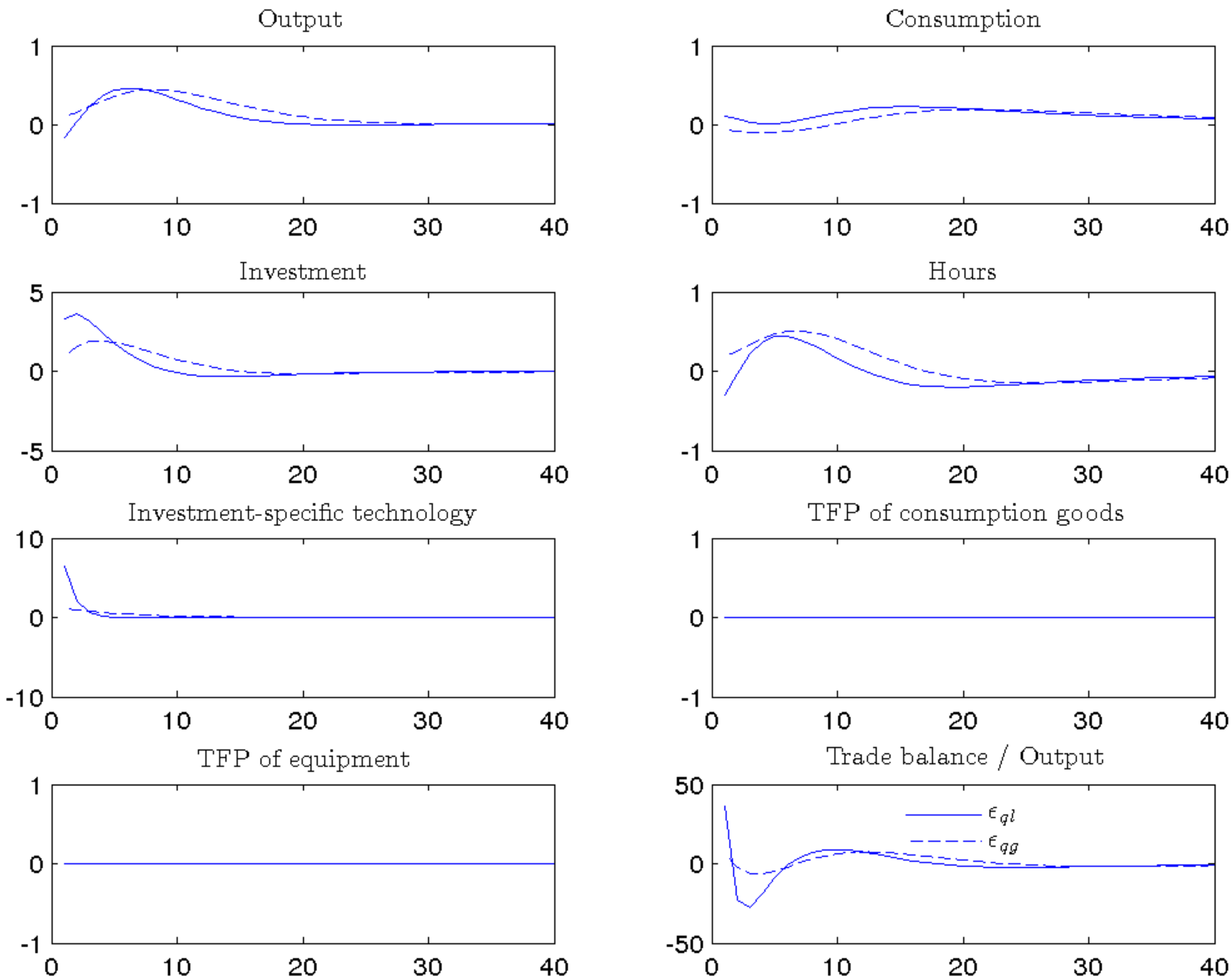


Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, TFP of consumption goods sector, and TFP of equipment sector to a one-standard-deviation shock to the world interest rate and the wage-markup.

and hours show similar responses on common trend shocks. The J-shaped response of investment and hours worked indicate that the substitution effect of investment-specific technology is offset by the income effect of aggregate productivity.[15]

[15] Aggregate productivity represents the overall productivity level of an economy whereas investment-specific technology indicates the relative improvement of equipment producing technology in terms of consumption goods. Accordingly, the simultaneous improvements of sectoral productivities with relative significance of equipment productivity enhance aggregate productivity as well as investment-specific technology.

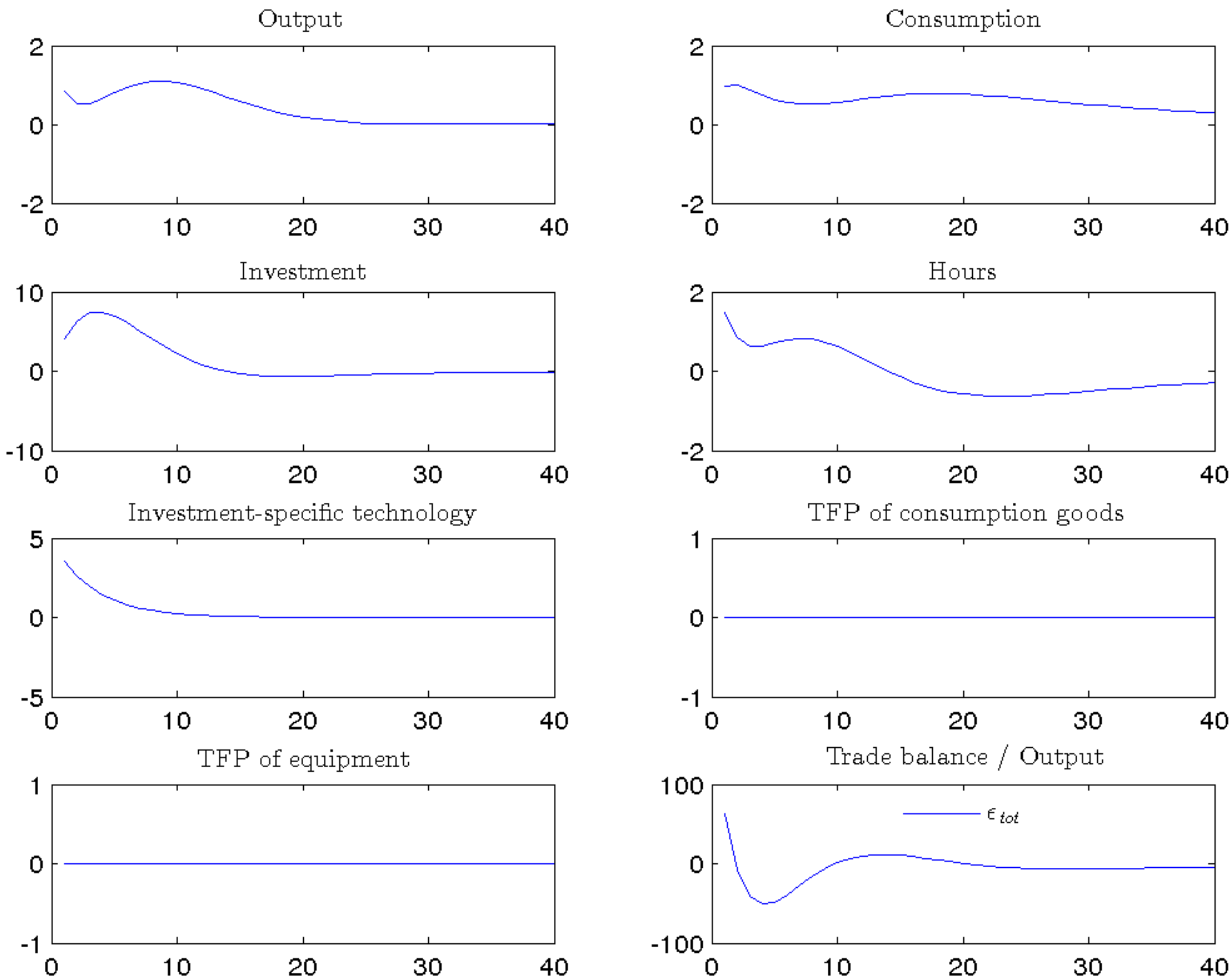
Figure 2.7: Impulse responses on the shocks of embodied technology in imports



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, TFP of consumption goods sector, and TFP of equipment sector to a one-standard-deviation shocks to the level and growth rate of embodied technology in equipment imports.

The model economy includes the shocks of wage-markup and world interest rate. A positive shock of wage markup may contract the production and employment of an economy with trade deficits. The wage-markup shock in Figure 2.6, however, has no significant effects on most of macroeconomic aggregates in this model. The shock of world interest rate in Figure 2.6 causes consumption and investment to contract on impact because of the intertemporal opportunity cost of current consumption and the cost of investment. Whereas, the surge of world interest rate induces the inflow of foreign assets, which requires surplus in trade balance. Thereby, the corresponding

Figure 2.8: Impulse responses on the terms of trade for consumption goods



Notes: Each panel shows the percentage deviation of output, consumption, investment, hours worked, IST, TFP of consumption goods sector, and TFP of equipment sector to a one-standard-deviation shock to the terms of trade for consumption goods.

output and hours worked display positive responses.[16] These impulse responses, however, are negligibly small.

One of the important external shocks originated foreign is the embodied technology in equip-

[16] According to Neumeyer and Perri (2005), in emerging economics real interest rates are countercyclical and lead the business cycles. Also, other literatures such as Jung et al. (2011) document countercyclical interest rate shock for the Korean economy. The impulse response of output and hours worked does not reflect actual features of the Korean business cycles.

ment imports. In the previous chapter, I document that investment-specific technology may alter without the changes of sectoral productivities in a closed economy. As we can see from Figure 2.7, in an open economy framework, positive shocks on embodied technology in equipment imports increase the investment-specific technology without a relative change in sectoral productivities. This finding indicates that the investment-specific technology in an open economy can alter by more various factors than a closed economy. The investment-specific technological change induced by both transitory and permanent shocks of technology-embodied imports increases investment, output, and hours worked in the initial periods with limited effects on consumption. Additionally, it is also confirmed that the shocks on embodied technology in imports induces a countercyclical movement of trade balance, which is broadly documented in literatures. Output boom caused by an improvement in technology-embodied imports generate a large trade deficit with more demand for investment.

Figure 2.8 shows the impulse responses on the shock of terms of trade for consumption goods. The terms of trade shock gives positive effect on output, consumption, investment, and hours worked. A positive shock of terms of trade for consumption goods implies the increase in domestic consumption goods price relative to foreign. Accordingly, domestic consumption-goods-producing firms would export more to achieve more marginal profits. In this model economy, a positive shock of terms of trade for consumption goods also generates a positive responses of investment-specific technology by making the relative price of equipment cheaper. In turn, investment increases by responding to this shock. Moreover, similarly to the shock on the growth rate of embodied technology in imports, the terms of trade shock for consumption goods generates countercyclical trade balance.

Table 2.9 presents the decomposed forecast error variances of output, consumption, investment, and hours worked. Of all eleven external shocks, the transitory shock of preference, wage markup shock, and the shock of world interest rate have no contribution to the variabilities of the macroeconomic aggregates. In short-run, the transitory shocks of sectoral productivities explain more than half of volatility for output but in mid-run the explanatory weights shift to the terms

Table 2.9: Forecast error variance decomposition

Quarters ahead	ϵ_{sl}	ϵ_{sg}	ϵ_{mu}	ϵ_{ac}	ϵ_{ae}	ϵ_{c}	ϵ_{e}	ϵ_{ql}	ϵ_{qg}	ϵ_{rf}	ϵ_{tot}
Output											
1	0.0	1.2	0.0	24.9	43.2	9.0	1.0	0.8	0.3	0.0	19.6
4	0.0	1.2	0.0	28.4	25.2	11.0	1.2	3.5	2.8	0.0	26.7
8	0.0	4.4	0.0	15.6	13.6	6.8	0.7	8.0	6.9	0.0	44.0
12	0.0	8.6	0.0	10.3	9.2	6.2	0.7	7.0	7.9	0.0	50.1
20	0.0	12.4	0.0	6.0	5.3	28.4	3.4	4.2	5.6	0.0	34.7
40	0.0	8.0	0.0	1.6	1.4	68.8	8.1	1.1	1.5	0.0	9.4
Consumption											
	0.0	12.5	0.0	33.6	0.4	7.2	0.8	0.5	0.3	0.0	44.7
4	0.0	13.9	0.0	35.7	0.2	11.6	1.3	0.2	0.4	0.0	36.8
8	0.0	13.0	0.0	35.3	0.1	18.6	2.1	0.2	0.4	0.0	30.3
12	0.0	12.4	0.0	31.3	0.2	23.0	2.7	0.6	0.4	0.0	29.3
20	0.0	12.1	0.0	22.1	0.4	27.1	3.2	1.6	0.8	0.0	32.7
40	0.0	11.7	0.0	10.2	0.3	48.0	5.7	1.1	1.0	0.0	22.1
Investment											
1	0.0	0.7	0.0	0.0	9.5	0.9	0.1	34.0	3.0	0.0	51.9
4	0.0	1.5	0.0	0.0	3.7	1.0	0.1	17.3	4.6	0.0	71.8
8	0.0	2.4	0.0	0.2	2.4	0.8	0.1	11.8	5.3	0.0	76.9
12	0.0	3.1	0.0	0.4	2.3	2.2	0.3	11.0	5.4	0.0	75.4
20	0.0	3.4	0.0	0.4	2.0	12.0	1.4	9.8	4.7	0.0	66.3
40	0.0	3.2	0.0	0.3	1.4	36.0	4.2	6.6	3.2	0.0	45.2
Hours worked											
1	0.0	3.7	0.0	1.6	1.7	27.4	3.0	2.3	0.9	0.0	59.4
4	0.0	2.6	0.0	2.0	1.0	39.6	4.5	3.1	4.3	0.0	42.9
8	0.0	6.1	0.0	4.2	0.9	37.4	4.2	5.1	7.5	0.0	34.7
12	0.0	12.6	0.0	4.9	0.7	33.7	3.7	4.3	8.2	0.0	31.9
20	0.0	22.6	0.0	4.9	0.7	28.5	3.1	4.1	6.9	0.0	29.1
40	0.0	27.0	0.0	3.3	0.6	31.1	3.5	3.1	4.7	0.0	26.9

Notes: The decomposed forecast error variance in output, consumption, investment, and hours worked are exhibited. The decomposition consists of the contributions of all 11 shocks to the forecast error variances.

of trade shock for consumption goods. In long-run, the shocks on common trend for sectoral productivities account for about 77 percent of output fluctuations. For consumption volatility, the growth rate shock of preference is consistently responsible around 12 percent. In short-run and mid-run, most of variability for consumption counts on the transitory shock of consumption goods productivity and the terms of trade shock for consumption goods. In long-run, the explanatory portion of common trend shocks for consumption variability increase to about 40 percent level. The terms of trade shock is responsible for overall investment volatility of 45-77 percent. The short-run investment volatility is explained by the transitory shock of technology embodied imports and the common trend shocks explains long-run variability of 36 percent. Lastly, the hours worked volatility of about 26-59 percent is explained by terms of trade shock and about 30-44 percent by common trend shocks of sectoral productivities. The permanent shock of preference is responsible for the long-run variability of hours worked.

In summary, from the impulse response and variance decomposition analysis, we can see that the terms of trade shock, as a single factor, induces the typical movements of business cycles and it accounts for most of volatilities of macroeconomic aggregates in the Korean economy. Besides the common trend shocks of sectoral productivities explains large volatilities of macroeconomic aggregates especially in the long-run. As such, the external shocks related to a trade counterpart are highly important to disentangle the business cycles features of Korea. Meanwhile, domestic technological changes also contribute to explain significant amount of macroeconomic volatilities.

2.6 Conclusion

This paper documents that few foreign innovations such as terms of trade for consumption goods and technology-embodied imports are responsible for substantial macroeconomic variations in the Korean economy. Using EU KLEMS database, the paper presents that there exists a sizeable expenditure for equipment imports in Canada and Korea; this fact implies that Canada and Korea are exposed to foreign innovations which affect to investment-specific technology. Considering the factors for investment-specific technology, I construct a dynamic stochastic general equilibrium model in a small open economy framework. The structural parameters of the model economy are either calibrated and estimated via maximum likelihood. The impulse response and variance decomposition analysis imply that terms of trade shock for consumption goods and the shock of embodied technology in imports give significant effects to macroeconomic aggregates with countercyclical trade balance.

Even though the shock of technology-embodied imports explains overall variation of the Korean economy, its magnitude is relatively smaller than terms of trade shock for consumption goods. The relatively small role of technology-embodied import in Korea is, however, attributed to the time period of this model analysis. The Korean economy have shown rapid growth since 1988 with its heavy manufacturing industries; that is, during the time period of this analysis, Korea have produced lots of equipment by itself. Therefore, to study the role of technology-embodied imports in the Korean economic fluctuation, we need to analyze the Korean data including 1970s when Korea had heavily relied on foreign equipment to build its capital stock. Also, we may investigate the role of foreign innovations in Canadian economic fluctuation because the Canadian economy have imported lots of equipment from foreign countries such as U.S.

Another plausible modification of this study is to introducing working capital into the model. The model economy of this paper fails to replicate a well-known empirical finding that real interest rate is countercyclical. Therefore, a future study need to extend the model economy to generate a countercyclical movement of real interest rate. One possible modification is to incorporate working capital as an labor market friction as in Neumeyer and Perri (2005).

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Appendix A

Technical notes for Chapter 1

A.1 Proofs

A.1.1 Proof for Proposition 2

Suppose $\ln A_t$ and $\ln Q_t$ are cointegrated, then there exists $(1, \psi)$ such that $\ln A_t + \psi \ln Q_t = S_t^1$, where S_t^1 is a stationary stochastic process. Suppose a negation that there exist a cointegrating vector $(1, \mu_1, \mu_2)$ in the system of $(\ln A_t, \ln Q_t, \ln Z_{c,t})$ and assume that S_t^2 is another stationary process which is independent of S_t^1, then

$$\begin{aligned} & \ln A_t + \mu_1 \ln Q_t + \mu_2 \ln Z_{c,t} = S_t^2 \\ \rightarrow \quad & S_t^1 - \psi \ln Q_t + \mu_1 \ln Q_t + \mu_2 \ln Z_{c,t} = S_t^2 \\ \rightarrow \quad & (\mu_1 - \psi) \ln Q_t + \mu_2 \ln Z_{c,t} = S_t^2 - S_t^1 \\ \rightarrow \quad & (\mu_1 - \psi)(\ln Z_{e,t} - \ln Z_{c,t}) + \mu_2 \ln Z_{c,t} = S_t^2 - S_t^1 \\ \rightarrow \quad & (\mu_1 - \psi) \ln Z_{e,t} + (\mu_2 - \mu_1 + \psi) \ln Z_{c,t} = S_t^2 - S_t^1 \end{aligned}$$

Since *RHS* of the above equation is stationary, *LHS* has to be stationary either. Since $\ln Z_e$ and $\ln Z_c$ are not cointegrated, to make *LHS* stationary, the following condition has to be satisfied:

$$\mu_1 - \psi = 0, \ \ and$$

$$\mu_2 - \mu_1 + \psi = 0,$$

which implies $\mu_2 = 0$. However, $\mu_2 = 0$ contradicts the assumption that $(\ln A_t, \ln Q_t, Z_{c,t})$ is a cointegrated system. Therefore, $(\ln A_t, \ln Q_t, \ln Z_{c,t})$ is not cointegrated.

The proof for $(\ln A_t, \ln Q_t, \ln Z_{e,t})$ is omitted because of the similarity to the above $\square$

A.1.2 Proof for Proposition 3

Case1: $\ln A_t$ and $\ln Q_t$ are cointegrated $\Longrightarrow$ $\ln Z_{c,t}$ and $\ln Z_{e,t}$ are cointegrated.

Suppose $\ln Z_{c,t}$ and $\ln Z_{e,t}$ consist of random walk components ($\mu_{c,t}$ and $\mu_{e,t}$) and stationary components ($e_{c,t}$ and $e_{e,t}$) as follows:

$$\ln Z_{c,t} = \mu_{c,t} + e_{c,t}$$

$$\ln Z_{e,t} = \mu_{e,t} + e_{e,t},$$

then $\ln A_t$ and $\ln Q_t$ are represented as follows:

$$\begin{aligned}
\ln A_t &= \phi \ln Z_{c,t} + (1-\phi)\ln Z_{e,t} \\
&= \phi\mu_{c,t} + (1-\phi)\mu_{e,t} + \phi e_{c,t} + (1-\phi)e_{e,t} \\
\ln Q_t &= \ln Z_{e,t} - \ln Z_{c,t} \\
&= \mu_{e,t} - \mu_{c,t} + e_{e,t} - e_{c,t}.
\end{aligned}$$

Since $\ln A_t$ and $\ln Q_t$ are cointegrated, there exists $(1, \psi)$ such that $\ln A_t + \psi \ln Q_t = S_t$ where S_t is a stationary process. $\ln A_t + \psi \ln Q_t$ can be rewritten as

$$\begin{aligned}
\ln A_t + \psi \ln Q_t &= \phi\mu_{c,t} + (1-\phi)\mu_{e,t} + \psi\mu_{e,t} - \psi\mu_{c,t} + D \\
&= (\phi-\psi)\mu_{c,t} + (1-\phi+\psi)\mu_{e,t} + D,
\end{aligned}$$

where D is a stationary process, defined as $\phi e_{c,t} + (1-\phi)e_{e,t} + \psi e_{e,t} - \psi e_{c,t}$. Suppose further that $\mu_{c,t}$ and $\mu_{e,t}$ are not cointegrated, then the cointegrated $\ln A_t$ and $\ln Q_t$ requires the following conditions:

$$\phi - \psi = 0, \ \ \textit{and}$$

$$1 - \phi + \psi = 0.$$

The two equations, however, cannot be solved simultaneously. Therefore, $\mu_{c,t}$ and $\mu_{e,t}$ have to be cointegrated, which further implies the cointegration of $\ln Z_{c,t}$ and $\ln Z_{e,t}$.

Case2: $\ln Z_{c,t}$ and $\ln Z_{e,t}$ are cointegrated $\Longrightarrow \ln A_t$ and $\ln Q_t$ are cointegrated.

Since $\ln Z_{c,t}$ and $\ln Z_{e,t}$ are cointegrated, there exists a cointegrating vector $(1, \kappa)$ such that $\ln Z_{c,t} + \kappa \ln Z_{e,t} = S_t$ with a stationary S_t. $\ln A_t$ and $\ln Q_t$ can be rewritten as

$$\ln A_t = (1 - \phi - \kappa\phi) \ln Z_{e,t} + \phi S_t$$
$$\ln Q_t = (1 + \kappa) \ln Z_{e,t} - S_t.$$

Then there exists a linear combination for $\ln A_t$ and $\ln Q_t$ such that

$$\begin{aligned} \ln A_t - \frac{1 - \phi - \phi\kappa}{1 + \kappa} \ln Q_t &= (1 - \phi - \kappa\phi) \ln Z_{e,t} + \phi S_t - (1 - \phi - \kappa\phi) \ln Z_{e,t} + \frac{1 - \phi - \kappa\phi}{1 + \kappa} S_t \\ &= \frac{1}{1 + \kappa} S_t. \end{aligned}$$

Therefore, $\ln A_t$ and $\ln Q_t$ are cointegrated with the cointegrating vector $\left(1, -\frac{1-\phi-\phi\kappa}{1+\kappa}\right)$ □

A.2 Model solution

A.2.1 Stationary system

A.2.1.1 The household's conditions

$$\lambda_{1,t} = \frac{1}{c_t - \xi c_{t-1}/\tau^c_{t-1}} - \beta\xi \mathbb{E}_t \frac{1}{c_{t+1}\tau^c_t - \xi c_t} \tag{A.1}$$

$$\frac{1}{x_{l,t}\eta^{xg}_t} = \lambda_{1,t}\tilde{w}_t \tag{A.2}$$

$$\lambda_{1,t}/q_t = \lambda_{2,t}\left[1 - \frac{\psi}{2}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \tau^I\right)^2 - \psi\frac{i_t}{i_{t-1}}\tau^i_{t-1}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \tau^I\right)\right]$$
$$+\beta\psi\mathbb{E}_t\left(\frac{i_{t+1}}{i_t}\right)^2 \tau^i_t \left(\frac{i_{t+1}}{i_t}\tau^i_t - \tau^I\right) \tag{A.3}$$

$$\lambda_{2,t}\tau^i_t = \beta\mathbb{E}_t\left\{\lambda_{1,t+1}\tilde{r}_{t+1} + \lambda_{2,t+1}(1-\delta)\right\} \tag{A.4}$$

$$c_t + i_t/q_t = \tilde{w}_t h_t + \tilde{r}_t k_t \tag{A.5}$$

$$k_{t+1}\tau^i_t = (1-\delta)k_t + i_t\left[1 - \frac{\psi}{2}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \tau^I\right)^2\right] \tag{A.6}$$

A.2.1.2 The firms' conditions

$$y_{c,t} = a_{c,t}(k_{c,t})^{\alpha_c}(\eta_t^{zc}h_{c,t})^{1-\alpha_c} \tag{A.7}$$

$$y_{e,t} = a_{e,t}(k_{e,t})^{\alpha_e}(\eta_t^{ze}h_{e,t})^{1-\alpha_e} \tag{A.8}$$

$$\tilde{r}_t = \alpha_c y_{c,t}/k_{c,t} \tag{A.9}$$

$$\tilde{w}_t = (1-\alpha_c)y_{c,t}/h_{c,t} \tag{A.10}$$

$$q_t = \frac{\alpha_e y_{e,t}/k_{e,t}}{\alpha_c y_{c,t}/k_{c,t}} \tag{A.11}$$

A.2.1.3 Market clearing conditions

$$k_t = k_{c,t} + k_{e,t} \tag{A.12}$$

$$h_t = h_{c,t} + h_{e,t} \tag{A.13}$$

$$c_t = y_{c,t} \tag{A.14}$$

$$i_t = y_{e,t} \tag{A.15}$$

$$\tilde{y}_t = y_{c,t} + y_{e,t}/q_t \tag{A.16}$$

A.2.1.4 Growth rates

$$\tau_t^c = (\eta_t^{zc})^{1-\alpha_c}(\eta_t^{ze})^{\alpha_c}\eta_t^{xg} \tag{A.17}$$

$$\tau_t^i = \eta_t^{ze}\eta_t^{xg} \tag{A.18}$$

$$\tau_t^h = \eta_t^{xg} \tag{A.19}$$

A.2.1.5 Observable variables

$$\tau_t^C = \tau_{t-1}^c \frac{c_t}{c_{t-1}} \tag{A.20}$$

$$\tau_t^I = \tau_{t-1}^i \frac{i_t}{i_{t-1}} \tag{A.21}$$

$$\tau_t^H = \tau_{t-1}^h \frac{h_t}{h_{t-1}} \tag{A.22}$$

A.2.1.6 Exogenous stochastic processes

$$ect_t - ect_{t-1} = \ln \eta_t^{zc} - \kappa \ln \eta_t^{ze} \tag{A.23}$$

$$\begin{bmatrix} \ln\left(\eta_t^{zc}/\eta^{zc}\right) \\ \ln\left(\eta_t^{ze}/\eta^{ze}\right) \end{bmatrix} = \begin{bmatrix} \rho_{cc} & \rho_{ce} \\ \rho_{ec} & \rho_{ee} \end{bmatrix} \begin{bmatrix} \ln\left(\eta_{t-1}^{zc}/\eta^{zc}\right) \\ \ln\left(\eta_{t-1}^{ze}/\eta^{ze}\right) \end{bmatrix} + \begin{bmatrix} f_c(ect_{t-1}) \\ f_e(ect_{t-1}) \end{bmatrix} + \begin{bmatrix} D_{cc} & D_{ce} \\ D_{ec} & D_{ee} \end{bmatrix} \begin{bmatrix} \epsilon_{zc,t} \\ \epsilon_{ze,t} \end{bmatrix} \tag{A.24}$$

$$\ln x_{l,t} = \rho_{x,l} \ln x_{l,t-1} + \epsilon_{xl,t} \tag{A.25}$$

$$\ln(\eta_t^{xg}/\eta^{xg}) = \rho_{xg} \ln(\eta_{t-1}^{xg}/\eta^{xg}) + \epsilon_{xg,t} \tag{A.26}$$

$$\ln a_{c,t} = \rho_{ac} \ln a_{c,t-1} + \epsilon_{ac,t} \tag{A.27}$$

$$\ln a_{e,t} = \rho_{ae} \ln a_{e,t-1} + \epsilon_{ae,t} \tag{A.28}$$

A.2.2 Steady states

The steady-state values of the variables in the model economy are determined by exogenously given parameter set, Θ, and the long-run average of the deterministic growth rates: η^{zc}, η^{ze} and η^{xg}. Substituting these parameters and growth rates into the Eqs.(A.17)-(A.19), we can get the steady-state of endogenous growth rates:

$$\tau^c = (\eta^{zc})^{1-\alpha_c}(\eta^{ze})^{\alpha_c}\eta^{xg} \tag{A.29}$$

$$\tau^i = \eta^{ze}\eta^{xg} \tag{A.30}$$

$$\tau^h = \eta^{xg}. \tag{A.31}$$

Using Eqs.(A.20)-(A.22), additionally, the long-run growth rate of the non-stationary variables are obtained as follows: $\tau^C = \tau^c$, $\tau^I = \tau^i$, and $\tau^H = \tau^h$.

The household's optimization conditions exhibited in Eqs.(A.1)-(A.6), respectively, implies the following conditions on steady states:

$$\lambda_1 c = \Phi_1, \tag{A.32}$$

$$1/\eta^{xg} = \lambda_1 \tilde{w}, \tag{A.33}$$

$$\lambda_1/q = \lambda_2, \tag{A.34}$$

$$\lambda_2 \tau^i = \beta \left\{ \lambda_1 \tilde{r} + \lambda_2(1-\delta) \right\}, \tag{A.35}$$

$$c + i/q = \tilde{w}h + \tilde{r}k, \tag{A.36}$$

$$i = \Phi_i k, \tag{A.37}$$

where $\Phi_1 = \frac{\tau^c - \beta\xi}{\tau^c - \xi}$ and $\Phi_i = \tau^i - 1 + \delta$. Also, Eqs.(A.34) and (A.35) indicates

$$\tilde{r}q = \bar{rq}, \tag{A.38}$$

where $\bar{rq} = \tau^i/\beta - 1 + \delta$.

Market clearing conditions, Eqs.(A.12)-(A.16), give the important steady-state equalities, respectively, as follows:

$$k = k_c + k_e \tag{A.39}$$

$$h = h_c + h_e \tag{A.40}$$

$$c = y_c \tag{A.41}$$

$$i = y_e \tag{A.42}$$

$$\tilde{y} = y_c + y_e/q \tag{A.43}$$

By considering Eq.(1.35) with stationary transformation, Eqs.(A.37), (A.38), (A.39) and (A.42), we can write the steady-state capital of each sector in terms of aggregate capital stock:

$$k_e = \Pi k, \tag{A.44}$$

$$k_c = (1-\Pi)k, \tag{A.45}$$

where $\Pi = \alpha_e \Phi_i / \bar{r}q$. Eq.(A.8), with Eqs.(A.37), (A.42) and (A.44), implies the steady-state of h_e;

$$h_e = \Phi_{he} k, \tag{A.46}$$

where $\Phi_{he} = (\Phi_i / \Pi^{\alpha_e})^{1/(1-\alpha_e)} / \eta^{ze}$.

From (A.14), we can see that in steady-state $c = y_c$. Eqs.(A.32) and (A.33) gives

$$\frac{\tilde{w}}{c} = \frac{1}{\eta^{xg} \Phi_1}. \tag{A.47}$$

Applying Eq.(A.47) to Eq.(A.10), the steady-state level of h_c is obtained as the following:

$$h_c = \Phi_{hc}, \tag{A.48}$$

where $\Phi_{hc} = (1 - \alpha_c)\eta^{xg}\Phi_1$. With the implicit steady-state condition for q, $q = \frac{(1-\alpha_e)y_e/h_e}{(1-\alpha_c)y_c/h_c}$, Eqs.(A.11), (A.44), (A.45), (A.46) and (A.48), we can get the steady-state level of capital stock as follows:

$$\bar{k} = \frac{\alpha_c \Pi (1 - \alpha_e) \bar{h}_c}{\alpha_e (1 - \Pi)(1 - \alpha_c) \Phi_{he}}. \tag{A.49}$$

A.2.3 Log-linearization

To calculate a numerical solution for the decision rules of this model economy, I linearize the system of equations given in Section A.2.1 on its steady-state value of Section A.2.2. Instead of solving for the log-linearized system by hand, I have derived the linearized system on `Matlab` by applying the standard-method of log-linearization with the `Symbolic` toolbox in `Matlab`.

From here, I briefly describe the standard-method of log-linearization with a simple example. Suppose that we have an equation given as follows:

$$f(X_t) + g(Y_t) = 0, \tag{A.50}$$

where X and Y are strictly positive variables. Using the identity $X = e^{\ln X}$, we can rewrite Eq.(A.50) as

$$f\left(e^{\ln X_t}\right) + g\left(e^{\ln Y_t}\right) = 0. \tag{A.51}$$

Taking the first-order Taylor expansion for Eq.(A.51) with respect to $\ln X$ and $\ln Y$ around the steady-state values, $\ln \bar{X}$ and $\ln \bar{Y}$, we can have

$$f(\bar{X}) + f'(\bar{X})(\ln X_t - \ln \bar{X}) + g(\bar{Y}) + g'(\bar{Y})(\ln Y_t - \ln \bar{Y}) = 0. \tag{A.52}$$

Using the identity of $f(\bar{X}) + g(\bar{Y}) = 0$ and letting $\hat{x} = \ln X - \ln \bar{X}$ and $\hat{y} = \ln Y - \ln \bar{Y}$, Eq.(A.52) is simplified as

$$f'(\bar{X})\hat{x} + g'(\bar{Y})\hat{y} = 0. \tag{A.53}$$

This standard-method of log-linearization can be coded on `Matlab` as follows:

```
ff_lv = subs(ff, {xx}, {exp(xx)});
grad = jacobian(ff_lv,xx);,
```

where `ff` stands for the system of equation before log-linearized and `xx` indicates a set of variables in the system. In the first line, `Matlab`, using the identity of $X = e^{\ln X}$, substitutes `xx` to logged `xx`. And then, take derivatives with respect to logged `xx` on the second line. With the simple two-line code, we can linearize more complicated system of equations easily.

Through the above method, I linearize the non-linear system of equations, Eqs.(A.1)-(A.28) around their steady state values.

A.2.4 Solving the model

This section explains the solution method of the model economy. I adopt the generalized Schur method (QZ Decomposition) of Klein (2000), and Gomme and Klein (2011); they develop solution algorithm in both first- and second-order approximation with tensor-free mechanism. In what follows, I describe the solution procedure of the model by using their way of explanation. Also, I would announce that for practical reason I employ the code, '`gx_hx.m`', written by Schmitt-Grohé and Uribe (2004), which is available on their web site. The algorithm is actually same to Klein (2000) for the first-order approximation.

The system of log-linearized equations from the previous section consists of a state vector, x, and a non-state vector, y, for period t and $t+1$ as follows:

$$\mathbb{E}_t\left[f(x_{t+1}, y_{t+1}, x_t, y_t)\right] = 0 \tag{A.54}$$

where f maps $\mathbb{R}^{2n_x+2n_y}$ into $\mathbb{R}^{n_x+n_y}$. The state vector and the non-state vector for time t of the model are defined as

$$x_t = [\hat{c}_{t-1}, \hat{i}_{t-1}, \hat{h}_{t-1}, \hat{k}_{t-1}, \hat{ect}_t, \hat{\eta}^{xg}_{t-1}, \hat{\eta}^{zc}_{t-1}, \hat{\eta}^{ze}_{t-1}, \hat{\eta}^{zc}_t, \hat{\eta}^{ze}_t, \hat{x}_{l,t}, \hat{\eta}^{xg}_t, \hat{a}_{c,t}, \hat{a}_{e,t}, \hat{\epsilon}_{zc,t}, \hat{\epsilon}_{ze,t}],$$

$$y_t = [\hat{\tau}^C_t, \hat{\tau}^I_t, \hat{\tau}^H_t, \hat{\tilde{y}}_t, \hat{c}_t, \hat{i}_t, \hat{h}_t, \hat{q}_t, \hat{x}_t, \hat{y}_{c,t}, \hat{y}_{e,t}, \hat{k}_{c,t}, \hat{k}_{e,t}, \hat{h}_{c,t}, \hat{h}_{e,t}, \hat{\tilde{w}}_t, \hat{\tilde{r}}_t, \hat{\tau}^c_t, \hat{\tau}^i_t, \hat{\tau}^h_t, \hat{\lambda}_{1,t}, \hat{\lambda}_{2,t}],$$

where $\hat{}$ indicates the percent deviation from steady-state value. The linearized system of equations, Eq.(A.54), can be written as

$$A\begin{bmatrix} x_{t+1} \\ \mathbb{E}_t y_{t+1} \end{bmatrix} = B\begin{bmatrix} x_t \\ y_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}, \tag{A.55}$$

where A denotes the coefficient matrix of the time $t+1$ variables including both state and non-state, and B is the coefficient matrix of the time t variables. Note that both A and B are $(n_x+n_y)\times(n_x+n_y)$ matrices. The theorem of generalized Schur form presented in Golub and Van Loan (1996) is required here.

Theorem [Generalized Schur Form] 1. *Let A and B be $n \times n$ matrices. If there is a $z \in \mathbb{C}$ such that $|B - zA| \neq 0$, then there exist matrices Q, Z, S and T such that*

*(1) Q **and** Z **are Hermitian**, i.e. $Q^H Q = QQ^H = I_n$ and similarly for Z, where H denotes the Hermitian transpose.*

(2) T and S upper triangular.

(3) $QA = SZ^H$ and $QB = TZ^H$.

(4) There is no i such that $s_{ii} = t_{ii} = 0$.

Moreover, the matrices Q, Z, S and T can be chosen in such a way as to make the diagonal entries s_{ii} and t_{ii} appear in any desired order.

For ordering of i, the ones satisfying $|s_{ii}| > |t_{ii}|$ will be chosen to appear first; these s_{ii} and t_{ii} pairs are called stable generalized eigenvalues.

The following equation can be derived from Eq.(A.55) by taking conditional expectation.

$$A\mathbb{E}_t \begin{bmatrix} x_{t+1} \\ y_{t+1} \end{bmatrix} = B \begin{bmatrix} x_t \\ y_t \end{bmatrix} \tag{A.56}$$

According to the above theorem, there exist upper triangular matrices S and T satisfying $QA = SZ^H$ and $QB = TZ^H$. Consequently, by premultiplying Q in both sides of Eq.(A.56), we can rewrite Eq.(A.56) as

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \mathbb{E}_t \begin{bmatrix} s_{t+1} \\ u_{t+1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} s_t \\ u_t \end{bmatrix}, \tag{A.57}$$

where

$$\begin{bmatrix} s_t \\ u_t \end{bmatrix} \equiv Z^H \begin{bmatrix} x_t \\ y_t \end{bmatrix}, \tag{A.58}$$

and s_t and u_t have same length as x_t and y_t respectively. The last block of Eq.(A.57) can be written out as

$$S_{22}\mathbb{E}_t[u_{t+1}] = T_{22}u_t.$$

If S_{22} and T_{22} constitute a (weakly) unstable matrix pair, $|s_{ii}| < |t_{ii}|$ (for weakly $|s_{ii}| \leq |t_{ii}|$), then any solution to Eq.(A.55) with bounded variance must satisfy $u_t = 0$, $\forall t$ (for weakly, unless $\Sigma = 0$). Given $u_t = 0$, $\forall t$, the first block of Eq.(A.57) should hold

$$S_{11}\mathbb{E}_t[s_{t+1}] = T_{11}s_t. \tag{A.59}$$

If S_{11} and T_{11} constitute a stable matrix pair, $|s_{ii}| > |t_{ii}|$, then S_{11} is invertible. Hence we may write

$$\mathbb{E}_t[s_{t+1}] = S_{11}^{-1}T_{11}s_t. \tag{A.60}$$

Rewrite Eq.(A.58) as

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = Z \begin{bmatrix} s_t \\ u_t \end{bmatrix} \tag{A.61}$$

where $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$.

If Z_{11} is invertible, then we may found the first-order approximation of policy rules as follows:

$$y_t = \underbrace{Z_{21}Z_{11}^{-1}}_{F} x_t \tag{A.62}$$

$$x_{t+1} = \underbrace{Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}}_{P} x_t + \varepsilon_{t+1}. \tag{A.63}$$

A.3 Estimating model parameters[1]

Applying Kalman filter, I construct a log-likelihood function and find a parameter set, Θ, such that maximizes the likelihood function. The log-likelihood function that I want to construct is given as follows[2];

$$\mathcal{L}(d|\Theta) = -\frac{Tl}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln|\Sigma_{t|t-1}| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t\Sigma_{t|t-1}^{-1}\varepsilon_t, \tag{A.64}$$

where T shows the time-length of the observed-vector, d, and l is the number of element of vector d, and ε_t and $\Sigma_{t|t-1}$ indicate the one-period-ahead forecast error of the observed-vector and its mean-square error, respectively.

$$x_{t+1} = Cx_t + v_{t+1},$$

$$d_t = Dx_t + w_t,$$

where x and d respectively represent the state-vector of $k \times 1$, and the observed-vector of $l \times 1$. v and w stand for the stochastic disturbance of the state-vector and the measurement error, respectively, with $\mathbb{E}(vv') = \Sigma_v$ and $\mathbb{E}(ww') = \Sigma_w$. Due to the recursive nature of the state-space, the task will start from determining the initial conditions, mean and mean-square error, for the one-period-ahead

[1]Writing this section, I found great usefulness in Hamilton (1994), Canova (2007), and the technical appendix of Ireland and Schuh (2008).

[2]The log-likelihood function is derived by using prediction error decomposition for the computational purpose. For more detail, see Canova (2007), pp.221-225.

forecast of the state-vector:

$$x_{1|0} = \mathbb{E}(x_1),$$
$$\mathbb{E}(x_1 - x_{1|0})(x_1 - x_{1|0})' = \Omega_{1|0}$$
$$vec(\Omega_{1|0}) = [I_{k^2} - (C \otimes C)]^{-1}\, vec(\Sigma_v),$$

where $x_{1|0}$ implies the expected value of x_1 on the information available at time 0. With the predetermined state-vector, we can find the one-period-ahead forecast for observed-vector and its mean-square error:

$$d_{t|t-1} = Dx_{t|t-1}, \tag{A.65}$$
$$\Sigma_{t|t-1} = D\Omega_{t|t-1}D' + \Sigma_w, \tag{A.66}$$

and the forecast error, ε, is written as

$$\varepsilon_t = d_t - d_{t|t-1}, \tag{A.67}$$

where d_t is the observed-vector at time t. Substituting Eqs.(A.66) and (A.67) into Eq.(A.64) recursively, we can construct the log-likelihood function. To move next period's forecast, we have to update the state-vector with the information of time t.

Using the formula for updating a linear projection, we can update state equation estimates[3]:

$$\begin{aligned} x_{t|t} &= x_{t|t-1} + \mathbb{E}(x_t - x_{t|t-1})(d_t - d_{t|t-1})' \\ &\quad \times \mathbb{E}(d_t - d_{t|t-1}))(d_t - d_{t|t-1})' \times (d_t - d_{t|t-1}) \\ &= x_{t|t-1} + \Omega_{t|t-1}D'\Sigma_{t|t-1}^{-1}\varepsilon_t, \end{aligned} \tag{A.68}$$
$$\Omega_{t|t} = \Omega_{t|t-1} - \Omega_{t|t-1}D'\Sigma_{t|t-1}^{-1}D\Omega_{t|t-1}. \tag{A.69}$$

The next period's forecast of the state-vector are then given as follows:

$$\begin{aligned} x_{t+1|t} &= Cx_{t|t} \\ &= Cx_{t|t-1} + K_t\varepsilon_t, \end{aligned} \tag{A.70}$$

[3]see Hamilton (1994), pp.92-100

$$
\begin{aligned}
\Omega_{t+1|t} &= \mathbb{E}(x_{t+1} - x_{t+1|t})(x_{t+1} - x_{t+1|t})' \\
&= \mathbb{E}(C(x_t - x_{t|t}) + v_{t+1})(C(x_t - x_{t|t}) + v_{t+1})' \\
&= C(x_t - x_{t|t})(x_t - x_{t|t})'C' + \Sigma_v \\
&= C\Omega_{t|t}C' + \Sigma_v,
\end{aligned} \tag{A.71}
$$

where K_t implies the Kalman-gain given by

$$K_t = C\Omega_{t|t-1}D'\Sigma_{t|t-1}^{-1}. \tag{A.72}$$

A.4 Evaluating the model: Variance decomposition[4]

This section ascertains how to decompose the forecast error variance for the observable variables, such as consumption, investment, and hours worked into percentage due to each of the model shocks.

We can rewrite the state space equation and decision rule as follows:

$$x_{t+1} = Px_t + v_{t+1}, \tag{A.73}$$

$$y_t = Fx_t. \tag{A.74}$$

Eq.(A.73) can be rewritten as MA representation:

$$
\begin{aligned}
(1 - PL)x_t &= v_t \\
x_t &= \textstyle\sum_{j=0}^{\infty} P^j v_{t-j}
\end{aligned} \tag{A.75}
$$

The s-period-ahead forecast error of state vector on the information of time t is

$$x_{t+s} - x_{t+s|t} = \sum_{j=0}^{s-1} P^j v_{t+s-j}, \tag{A.76}$$

and MSE of the forecast is exhibited as

$$\mathbb{E}[(x_{t+s} - x_{t+s|t})(x_{t+s} - x_{t+s|t})'] \equiv \Sigma_{x,s} = \Sigma_v + P\Sigma_v P' + P^2\Sigma_v P'^2 + \ldots + P^{s-1}\Sigma_v P'^{s-1}. \tag{A.77}$$

[4]This section reconstructs the technical appendix of Ireland and Schuh (2008). I add definitions of some variables to fit to the model economy and try to increase the readability.

Next we can get the forecast error of the non-state vector of Eqs.(A.74) as

$$y_{t+s} - y_{t+s|t} = F(x_{t+s} - x_{t+s|t}). \tag{A.78}$$

Then MSE of the forecast for non-state vector is

$$\mathbb{E}[(y_{t+s} - y_{t+s|t})(y_{t+s} - y_{t+s|t})'] \equiv \Sigma_{y,s} = F\Sigma_{x,s}F'. \tag{A.79}$$

What we are interested in this analysis is mainly on the behavior of non-stationary aggregate variable such as consumption, investment, and hours worked per worker. Accordingly, we would get the variance decomposition for these non-stationary variables. In what follows, I describe the procedure for the variance decomposition of consumption as an example.

From the model solution given above we can rewrite the decision rule for consumption growth rate as follows:

$$\ln C_t - \ln C_{t-1} - \ln g^c = F_{gc}x_t, \tag{A.80}$$

where F_{gc} indicate the row for the consumption growth (g_t^c) in matrix F. Then we can derives the following s-period-ahead forecasts from Eq.(A.80):

$$\ln C_{t+s} - \ln C_t - s\ln g^c = F_{gc}\sum_{j=1}^{s} x_{t+j}, \tag{A.81}$$

$$\ln C_{t+s|t} - \ln C_t - s\ln g^c = F_{gc}\sum_{j=1}^{s} x_{t+j|t}. \tag{A.82}$$

Then the forecast error and MSE of forecast are derived as

$$\ln C_{t+s} - \ln C_{t+s|s} = F_{gc}\sum_{l=1}^{s}\left(x_{t+l} - x_{t+l|t}\right) = F_{gc}\sum_{l=1}^{s}\sum_{j=0}^{l-1} P^j v_{t+l-j} \tag{A.83}$$

$$\mathbb{E}\left[\ln C_{t+s} - \ln C_{t+s|t}\right]\left[\ln C_{t+s} - \ln C_{t+s|t}\right]' = F_{gc}\mathbb{E}\left[\sum_{l=1}^{s}\sum_{j=0}^{l-1} P^j v_{t+l-j}\right]\left[\sum_{l=1}^{s}\sum_{j=0}^{l-1} P^j v_{t+l-j}\right]' F_{gc}', \tag{A.84}$$

where $\sum_{l=1}^{s}\sum_{j=0}^{l-1} P^j v_{t+l-j}$ is extended as

$$\begin{aligned}
\sum_{l=1}^{s}\sum_{j=0}^{l-1} P^j v_{t+l-j} &= \sum_{l=1}^{s}\left\{v_{t+l} + P v_{t+l-1} + \ldots + P^{l-1} v_{t+1}\right\} \\
&= [\{v_{t+1}\} \\
&\quad + \{v_{t+2} + P v_{t+1}\} + \ldots \\
&\quad + \left\{v_{t+s} + P v_{t+s-1} + \ldots + P^{s-1} v_{t+1}\right\}] \\
&= v_{t+s} + (I+P) v_{t+s-1} + \ldots + (I + P + \ldots + P^{s-1}) v_{t+1}.
\end{aligned}$$

Then the middle term of Eq.(A.84) is represented as

$$\mathbb{E}\left[\sum_{l=1}^{s}\sum_{j=0}^{l-1} P^j v_{t+l-j}\right]\left[\sum_{l=1}^{s}\sum_{j=0}^{l-1} P^j v_{t+l-j}\right]' = \Sigma_v + (I+P)\Sigma_v(I+P)' + \ldots + (I+P+\ldots+P^{s-1})\Sigma_v(I+P+\ldots+P^{s-1})'. \tag{A.85}$$

Appendix B

Technical notes for Chapter 2

B.1 Model solutions

B.1.1 Stationary system

B.1.1.1 The household's conditions

$$\lambda_{1,t} = \frac{1}{c_t - \xi c_{t-1}/\tau^c_{t-1}} - \beta \mathbb{E}_t \frac{\xi}{c_{t+1}\tau^c_t - \xi c_t} \tag{B.1}$$

$$\frac{1}{s_t} = \lambda_{1,t} \frac{\tilde{w}_t}{\mu_t} \tag{B.2}$$

$$\lambda_{1,t}\tau^c_t = \beta \mathbb{E}_t \lambda_{1,t+1} \left(1 + r^d_t\right) \tag{B.3}$$

$$\lambda_{2,t} = \beta \mathbb{E}_t \frac{1}{\tau^i_t} \left[\lambda_{2,t+1}(1-\delta) + \lambda_{1,t+1}\tilde{r}_{t+1}\right] \tag{B.4}$$

$$\begin{aligned} \frac{\lambda_{1,t}}{q_t} &= \lambda_{2,t} \left[1 - \psi_k \frac{i_t}{i_{t-1}}\tau^i_{t-1} \left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \bar{\tau}^i\right) - \frac{\psi_k}{2}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \bar{\tau}^i\right)^2\right] \\ &+ \beta\psi_k \mathbb{E}_t \lambda_{2,t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 \tau^i_t \left(\frac{i_{t+1}}{i_t}\tau^i_t - \bar{\tau}^i\right) \end{aligned} \tag{B.5}$$

$$c_t + \frac{i_t}{q_t} + (1 + r^d_{t-1})\frac{\tilde{d}_{t-1}}{\tau^c_{t-1}} = \tilde{r}_t k_t + \frac{\tilde{w}_t}{\mu_t}h_t + \tilde{d}_t + \pi_t \tag{B.6}$$

$$k_{t+1}\tau^i_t = (1-\delta)k_t + i_t \left[1 - \frac{\psi_k}{2}\left(\frac{i_t}{i_{t-1}}\tau^i_{t-1} - \bar{\tau}^i\right)^2\right] \tag{B.7}$$

$$r^d_t = r^f_t + \psi_d \left[e^{(\tilde{d}_t - \bar{d})} - 1\right] \tag{B.8}$$

B.1.1.2 Firms' conditions

$$y_{c,t} = a_{c,t} {k_{c,t}}^{\alpha_c} \left(\eta_t^{zc} h_{c,t}\right)^{1-\alpha_c} \tag{B.9}$$

$$y_{e,t} = a_{e,t} {k_{e,t}}^{\alpha_e} \left(\eta_t^{ze} h_{e,t}\right)^{1-\alpha_e} \tag{B.10}$$

$$\tilde{r}_t = \frac{\alpha_c}{rp_{c,t}^h} \frac{y_{c,t}}{k_{c,t}} \tag{B.11}$$

$$\tilde{w}_t = \frac{1-\alpha_c}{rp_{c,t}^h} \frac{y_{c,t}}{h_{c,t}} \tag{B.12}$$

$$q_t^h = \frac{\alpha_e y_{e,t}/k_{e,t}}{\alpha_c y_{c,t}/k_{c,t}} \tag{B.13}$$

B.1.1.3 Final goods demands

$$y_{c,t}^h = (1-\omega_c) \left(rp_{c,t}^h\right)^{\zeta_c} c_t \tag{B.14}$$

$$c_t^f = \omega_c \left(rp_{c,t}^f\right)^{\zeta_c} c_t \tag{B.15}$$

$$y_{e,t}^h = (1-\omega_e) \left(rp_{e,t}^h\right)^{\zeta_e} i_t \tag{B.16}$$

$$i_t^f = \omega_e \left(rp_{e,t}^f\right)^{\zeta_e} i_t \tag{B.17}$$

$$c_t = \left\{ (1-\omega_c)^{\frac{1}{\zeta_c}} {y_{c,t}^h}^{\frac{\zeta_c-1}{\zeta_c}} + \omega_c^{\frac{1}{\zeta_c}} {c_t^f}^{\frac{\zeta_c-1}{\zeta_c}} \right\}^{\frac{\zeta_c}{\zeta_c-1}} \tag{B.18}$$

$$i_t = \left\{ (1-\omega_e)^{\frac{1}{\zeta_e}} {y_{e,t}^h}^{\frac{\zeta_e-1}{\zeta_e}} + \omega_e^{\frac{1}{\zeta_e}} {i_t^f}^{\frac{\zeta_e-1}{\zeta_e}} \right\}^{\frac{\zeta_e}{\zeta_e-1}} \tag{B.19}$$

B.1.1.4 Relative prices

$$rp_{c,t}^h = \left\{ (1-\omega_c) + \omega_c \left(\frac{1}{tot_{c,t}} \right)^{1-\zeta_c} \right\}^{\frac{1}{1-\zeta_c}} \tag{B.20}$$

$$rp_{c,t}^f = \left\{ (1-\omega_c) \left(tot_{c,t}\right)^{1-\zeta_c} + \omega_c \right\}^{\frac{1}{1-\zeta_c}} \tag{B.21}$$

$$rp_{e,t}^h = \left\{ (1-\omega_e) + \omega_e \left(\frac{q_t^h \tau_{t-1}^i / \tau_{t-1}^c}{tot_{c,t} q_t^f} \right)^{1-\zeta_e} \right\}^{\frac{1}{1-\zeta_e}} \tag{B.22}$$

$$rp^f_{e,t} = \left\{ (1-\omega_e) \left(\frac{tot_{c,t} q^f_t}{q^h_t \tau^i_{t-1} / \tau^c_{t-1}} \right)^{1-\zeta_e} + \omega_e \right\}^{\frac{1}{1-\zeta_e}} \tag{B.23}$$

$$q_t = \frac{rp^h_{c,t}}{rp^h_{e,t}} q^h_t \tag{B.24}$$

B.1.1.5 Aggregate output

$$y_t = (y_{c,t})^{\gamma} (y_{e,t})^{1-\gamma} \tag{B.25}$$

$$\tilde{y}_t = \tilde{w}_t h_t + \tilde{r}_t k_t \tag{B.26}$$

$$\tilde{y}_t = \Gamma \frac{y_t}{rp^h_{c,t} \left(q^h_t\right)^{1-\gamma}} \tag{B.27}$$

B.1.1.6 Growth rates

$$\tau^c_t = (\eta^{zc}_t)^{1-\alpha_c} (\eta^{ze}_t)^{\alpha_c} \eta^{sg}_t \tag{B.28}$$

$$\tau^i_t = \eta^{ze}_t \eta^{sg}_t \tag{B.29}$$

$$\tau^h_t = \eta^{sg}_t \tag{B.30}$$

B.1.1.7 Observable variables

$$\tau^Y_t = \left(\tau^c_{t-1}\right)^{\gamma} \left(\tau^i_{t-1}\right)^{1-\gamma} \frac{y_t}{y_{t-1}} \tag{B.31}$$

$$\tau^C_t = \tau^c_{t-1} \frac{c_t}{c_{t-1}} \tag{B.32}$$

$$\tau^I_t = \tau^i_{t-1} \frac{i_t}{i_{t-1}} \tag{B.33}$$

$$\tau^H_t = \tau^h_{t-1} \frac{h_t}{h_{t-1}} \tag{B.34}$$

B.1.1.8 Market clearing conditions and other equations

$$k_t = k_{c,t} + k_{e,t} \tag{B.35}$$

$$h_t = h_{c,t} + h_{e,t} \tag{B.36}$$

$$\tilde{tb}_t = (1 + r^d_{t-1}) \frac{\tilde{d}_{t-1}}{\tau^c_{t-1}} - \tilde{d}_t \tag{B.37}$$

$$\tilde{y}_t = c_t + i_t/q_t + \tilde{tb}_t \tag{B.38}$$

$$s_t = s_{l,t} s_{g,t} \tag{B.39}$$

$$q^f_t = q^f_{l,t} q^f_{g,t} \tag{B.40}$$

B.1.1.9 Exogenous stochastic processes

$$\ln s_{l,t} = \rho_{sl} \ln s_{l,t-1} + \epsilon_{sl,t} \tag{B.41}$$

$$\ln s_{g,t} = (1 - \rho_{sg}) \ln \eta^{sg} + \rho_{sg} \ln s_{g,t-1} + \epsilon_{sg,t} \tag{B.42}$$

$$\ln q^f_{l,t} = (1 - \rho_{ql}) \ln \bar{q}^f_l + \rho_{ql} \ln q^f_{l,t-1} + \epsilon_{ql,t} \tag{B.43}$$

$$\ln q^f_{g,t} = (1 - \rho_{qg}) \ln \eta^{qg} + \rho_{qg} \ln q^f_{g,t-1} + \epsilon_{qg,t} \tag{B.44}$$

$$\ln r^f_t = (1 - \rho_r) \ln \bar{r}^f + \rho_r \ln r^f_{t-1} + \epsilon_{r,t}, \tag{B.45}$$

$$\ln \mu_t = (1 - \rho_\mu) \ln \bar{\mu}^f + \rho_\mu \ln \mu_{t-1} + \epsilon_{\mu,t}, \tag{B.46}$$

$$\ln tot_{c,t} = \rho_{tot} \ln tot_{c,t-1} + \epsilon_{tot,t}, \tag{B.47}$$

$$\ln a_{c,t} = \rho_{ac} \ln a_{c,t-1} + \epsilon_{ac,t}, \tag{B.48}$$

$$\ln a_{e,t} = \rho_{ae} \ln a_{e,t-1} + \epsilon_{ae,t}, \tag{B.49}$$

$$\begin{bmatrix} \ln z_{c,t} \\ \ln z_{e,t} \end{bmatrix} = \begin{bmatrix} 1-\rho_{cc} & -\rho_{ce} \\ -\rho_{ec} & 1-\rho_{ee} \end{bmatrix} \begin{bmatrix} \eta^{zc} \\ \eta^{ze} \end{bmatrix} + \begin{bmatrix} \rho_{cc} & \rho_{ce} \\ \rho_{ec} & \rho_{ee} \end{bmatrix} \begin{bmatrix} \ln z_{c,t-1} \\ \ln z_{e,t-1} \end{bmatrix} + \begin{bmatrix} f_c\left(ect_{t-1}\right) \\ f_e\left(ect_{t-1}\right) \end{bmatrix} + \begin{bmatrix} D_{cc} & D_{ce} \\ D_{ec} & D_{ee} \end{bmatrix} \begin{bmatrix} \epsilon_{zc,t} \\ \epsilon_{ze,t} \end{bmatrix} \tag{B.50}$$

$$ect_t - ect_{t-1} = \ln z_{c,t} - \kappa \ln z_{e,t} \tag{B.51}$$

B.1.2 Steady states

The steady state values of the variables in the model economy are determined by exogenous parameter set, Θ, including the long-run average growth rates of sectoral productivities for consumption-goods and equipment, and hours worked: η^{zc}, η^{ze} and η^{sg}. Substituting these parameters into the Eqs.(B.28)-(B.30), we can get the steady-states of endogenous growth rates:

$$\bar{\tau}^c = (\eta^{zc})^{1-\alpha_c}(\eta^{ze})^{\alpha_c}\eta^{sg} \tag{B.52}$$

$$\bar{\tau}^i = \eta^{ze}\eta^{sg} \tag{B.53}$$

$$\bar{\tau}^h = \eta^{sg}. \tag{B.54}$$

Using Eqs.(B.31)-(B.34), additionally, the observable variables in the model economy are obtained as follows: $\bar{\tau}^C = \bar{\tau}^c$, $\bar{\tau}^I = \bar{\tau}^i$, and $\bar{\tau}^H = \bar{\tau}^h$.

The household's optimization conditions exhibited in Eqs.(B.1)-(B.8), respectively, implies the following conditions on steady-states:

$$\lambda_1 c = \Phi_1, \tag{B.55}$$

$$1/\bar{s} = \lambda_1 \frac{\tilde{w}}{\bar{\mu}}, \tag{B.56}$$

$$\bar{\tau}^c = \beta\left(1+\bar{r}^d\right), \tag{B.57}$$

$$\lambda_2\tau^i = \beta\left[(1-\delta)\lambda_2 + \tilde{r}\lambda_1\right], \tag{B.58}$$

$$\lambda_1 = \lambda_2 q^h, \tag{B.59}$$

$$c + i/q + (1+\bar{r}^d)\frac{\tilde{d}}{\bar{\tau}^c} = \frac{\tilde{w}}{\bar{\mu}}h + \tilde{r}k + \tilde{d} + \pi, \tag{B.60}$$

$$i = \Phi_i k, \tag{B.61}$$

$$\bar{r}^d = \bar{r}^f, \tag{B.62}$$

where $\Phi_1 = \frac{\bar{\tau}^c - \beta\xi}{\bar{\tau}^c - \xi}$ and $\Phi_i = \bar{\tau}^i - 1 + \delta$. Also, Eqs.(B.58) and (B.59) indicates

$$\tilde{r}q^h = \bar{r}q, \tag{B.63}$$

where $\bar{r}q = \bar{\tau}^e/\beta - 1 + \delta$. Eqs.(B.55) and (B.56) imply

$$c\bar{w} \equiv \frac{c}{\tilde{w}} = \Phi_1\frac{\bar{s}}{\bar{\mu}}. \tag{B.64}$$

From the firms' profit maximization, we can have the following steady state conditions:

$$\frac{y_c}{h_c} = (\eta^{zc})^{1-\alpha_c}\left(\frac{k_c}{h_c}\right)^{\alpha_c}, \tag{B.65}$$
$$\frac{y_e}{h_e} = (\eta^{ze})^{1-\alpha_e}\left(\frac{k_e}{h_e}\right)^{\alpha_e}, \tag{B.66}$$
$$\tilde{r} = \alpha_c\frac{y_c}{k_c}, \tag{B.67}$$
$$\tilde{w} = (1-\alpha_c)\frac{y_c}{h_c}, \tag{B.68}$$
$$\tilde{r}q^h = \alpha_e\frac{y_e}{k_e}, \tag{B.69}$$
$$\tilde{w}q^h = (1-\alpha_e)\frac{y_e}{h_e}. \tag{B.70}$$

Eqs.(B.63), (B.66) and (B.69) imply

$$\frac{k_e}{h_e} = \eta^{ze}\left(\frac{\alpha_e}{\bar{rq}}\right)^{\frac{1}{1-\alpha_e}} \equiv k\bar{h}e \tag{B.71}$$

and Eqs.(B.67)-(B.70) implies the following equality:

$$\frac{\alpha_e y_e/k_e}{\alpha_c y_c/k_c} = \frac{(1-\alpha_e)y_e/h_e}{(1-\alpha_c)y_c/h_c}$$
$$\longrightarrow \quad \frac{\alpha_e/(1-\alpha_e)}{\alpha_c/(1-\alpha_c)} = \frac{k_e/h_e}{k_c/h_c}. \tag{B.72}$$

Let $\mathrm{A} \equiv \frac{\alpha_e/(1-\alpha_e)}{\alpha_c/(1-\alpha_c)}$, then we can express k_c/h_c, from the Eq.(B.72), as follows:

$$k\bar{h}c = \frac{k\bar{h}e}{\mathrm{A}} \tag{B.73}$$

Considering Eqs.(B.67) and (B.68) with Eqs.(B.65) and (B.73), we can derive the steady state conditions for $\tilde{r}$ and $\tilde{w}$ as follows:

$$\bar{\tilde{r}} = \alpha_c\,(\eta^{zc})^{1-\alpha_c}\,k\bar{h}c^{\alpha_c-1}, \tag{B.74}$$

$$\bar{\tilde{w}} = (1-\alpha_c)\,(\eta^{zc})^{1-\alpha_c}\,k\bar{h}c^{\alpha_c}. \tag{B.75}$$

Then we can use Eqs.(B.74) and (B.75) to calculate $\bar{q}$ and $\bar{c}$:

$$\bar{q} = \bar{rq}/\bar{\tilde{r}} \tag{B.76}$$

$$\bar{c} = \bar{cw}\cdot\bar{\tilde{w}} \tag{B.77}$$

Also, using Eq.(B.37) with other steady states previously derived, we can obtain the steady state of $\tilde{tb}$ as follows:

$$\bar{\tilde{tb}} = \left(\frac{1+\bar{r}^d}{\bar{\tau}^x} - 1\right)\bar{\tilde{d}}. \tag{B.78}$$

Using the implied demand function of $y_{c,t}$ and the price index corresponding to y_t from Eq. (B.25), the steady state condition of $\tilde{y}_t$ is derived as follows:

$$\tilde{y} = y_c/\gamma. \tag{B.79}$$

From Eqs.(B.26), (B.14), and (B.79), we can derive the following condition:

$$\begin{aligned}
&\frac{(k_c)^{\alpha_c}(\eta^{zc}h_c)^{1-\alpha_c}}{\gamma} = \bar{\tilde{w}}h + \bar{\tilde{r}}k \\
\longrightarrow\ &\frac{(k_c/h_c)^{\alpha_c}(\eta^{zc})^{1-\alpha_c}h_c}{\gamma} = \bar{\tilde{w}}h + \bar{\tilde{r}}k \\
\longrightarrow\ &\frac{k\bar{h}c^{\alpha_c}(\eta^{zc})^{1-\alpha_c}h_c/h}{\gamma} = \bar{\tilde{w}} + \bar{\tilde{r}}\frac{k}{h}
\end{aligned} \tag{B.80}$$

Suppose that the share of capital and labor for consumption goods are denoted ϕ_k and ϕ_h; that is, $k_c = \phi_c k$ and $h_c = \phi_h h$. Then k/h can be expressed as follows:

$$\begin{aligned}
\frac{k}{h} &= \frac{k_c}{h} + \frac{k_e}{h} \\
&= \frac{h_c}{h}\frac{k_c}{h_c} + \frac{h_e}{h}\frac{k_e}{h_e} \\
&= \phi_h k\bar{h}c + (1-\phi_h)k\bar{h}e \\
&= k\bar{h}e + (k\bar{h}c - k\bar{h}e)\phi_h
\end{aligned} \tag{B.81}$$

Eqs.(B.80) and (B.81) imply that

$$\bar{\phi}_h = \frac{\bar{\tilde{w}} + \bar{\tilde{r}}\cdot k\bar{h}e}{\frac{k\bar{h}c^{\alpha_c}(\eta^{zc})^{1-\alpha_c}}{\gamma} - \bar{\tilde{r}}(k\bar{h}c - k\bar{h}e)}. \tag{B.82}$$

From Eq.(B.81), we can easily know that the steady state level of caplital per hours worked, k/h, is given as

$$\frac{k}{h} \equiv k\bar{h} = \bar{\phi}_h k\bar{h}c + (1-\phi_h)k\bar{h}e. \tag{B.83}$$

Since $\bar{kh c} = \frac{\phi_k}{\phi_h} \bar{kh}$, the share of capital for consumption production, ϕ_k, is expressed as

$$\bar{\phi}_k = \frac{\bar{khc}}{\bar{kh}} \bar{\phi}_h. \tag{B.84}$$

The market clearing conditions of this economy require that the following equation should hold:

$$\bar{\tilde{r}} k + \bar{\tilde{w}} h = \bar{c} + \frac{\Phi_i k}{\bar{q}} + \bar{\tilde{t}}\bar{b}. \tag{B.85}$$

Therefore, we can derive the steady state value of hours worked, $\bar{h}$, as follows:

$$\left\{ \left(\bar{\tilde{r}} - \frac{\Phi_i}{\bar{q}} \right) \frac{k}{h} + \bar{\tilde{w}} \right\} h = \bar{c} + \bar{\tilde{t}}\bar{b} \tag{B.86}$$

$$\longrightarrow \quad \bar{h} = \frac{\bar{c} + \bar{\tilde{t}}\bar{b}}{\left(\bar{\tilde{r}} - \frac{\Phi_i}{\bar{q}} \right) \bar{kh} + \bar{\tilde{w}}} \tag{B.87}$$

Accordingly, the steady state value of aggregate capital, k, is obtained as

$$\bar{k} = \bar{kh} \cdot \bar{h} \tag{B.88}$$