Wideband Dual-Polarized Digital Direction of Arrival Sensors

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Wideband Dual-Polarized Digital Direction of Arrival
Sensors
by
Riley Nelson Pack
B.S., University of Colorado Boulder, 2011
M.S., University of Colorado Boulder, 2011
A thesis submitted to the
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Wideband Dual-Polarized Digital Direction of Arrival Sensors
written by Riley Nelson Pack
has been approved for the Department of Electrical, Computer, and Energy Engineering

Prof. Dejan Filipović

Prof. Gregor Lasser

Date ____________________

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.
This thesis presents the design of wideband, dual-polarized direction of arrival (DOA) antennas intended for use with digital processing backends. It is shown that using a digital receiver for DOA allows for wider bandwidth and improvement of the size, weight, and complexity of the antenna system at the cost of traditional design metrics. All antennas are designed using full-wave finite element simulations and are validated with measurements.

Four-arm modulated arm width (MAW) spiral antennas are analyzed for use as digital DOA sensors for linearly-polarized signals. Counter to traditional design procedures, tightly-wound MAW spirals with a small modulation ratio perform significantly better than both a MAW spiral with a large modulation ratio and a conventional spiral. The DOA performance is analyzed using the Cramér-Rao Lower Bound (CRLB). The low modulation ratio MAW spiral exhibits less than 1° of error over a ±30° elevation and 360° azimuth field of view and over a 4.6:1 bandwidth, while the other spirals are limited to about 2:1 bandwidth or less.

A 64-element circular array of tightly-coupled dipoles over a ground plane is analyzed and measured. The effects of the major features of the array are discussed. A dielectric slab ring is placed over the elements and extends outward past them to focus the fields of the antenna outward, as well as to lower the turn-on frequency of the array. The 64 elements are combined to four sectoral outputs, which can be used separately as part of a DOA sensor or combined in-phase for omnidirectional operation over a 3.45:1 bandwidth.

The horizontally-polarized circular array is integrated with a monocone and eight vertically-polarized tapered slot antennas (TSAs) to create a wideband dual-polarized DOA sensor. The monocone is isolated from the other elements by exploiting symmetry to cancel the coupling between the monocone and the TSAs, enabling the array to be used for simultaneous transmit and receive
(STAR). The array achieves a wide bandwidth of 3.41:1 over which both return loss and isolation are high. The array is capable of providing high-accuracy DOA estimates over the entire upper hemisphere over its operating bandwidth, as characterized by the CRLB.
Dedication

To my parents Roger and Patti Pack and my brother Sean Pack.
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Chapter 1

Introduction

1.1 Motivation

Direction of arrival (DOA) sensors have been an important focus of antenna design for many years due to their wide range of applications, from aircraft navigation [1] to military applications and spectrum management. As the use of direction finding (DF) has increased, the bandwidth required of sensors has increased as well, whether to improve system accuracy or to capture a wider range of signals. Furthermore, when designing DOA sensors, attention must be given to the polarization sensitivity of the system. Otherwise, significant error can be introduced in the system by the presence of cross-polarized signals [2]. Without knowledge of the polarization of incoming signals from other sensors or via a-priori information, this error can only be compensated for sufficiently if the DOA sensor is dual-polarized. In many applications (e.g. spectrum management), all signals in an area are of interest regardless of polarization, so a dual-polarized sensor is desired anyway. Finally, many applications require sensors to detect and process signals near the noise floor, requiring the antenna to have positive gain and the backend processing to exhibit a low noise figure.

As the use of wireless spectrum grows, the ability to effectively monitor wireless frequencies has become more important, both to industry and to regulatory bodies. For example, white spaces in the spectrum can be utilized for 5G cellular, smart grid, and other next-generation wireless systems if they can be properly sensed by networks [3][4][5]. In some cases, determining the location of emitters in such a cognitive network can improve coordination and overall performance [6].
Furthermore, as the need for spectrum increases, conflicts between commercial and government entities can place a strain on innovation and impact national security [7].

In order to solve these critical challenges for future systems, networks will need to be able to measure spectrum usage in real-time, sometimes over several decades of bandwidth. In 5G alone, bands are being proposed from 400 MHz to over 100 GHz [8]. Therefore, monitoring systems will require sensors that can cover wide swaths of this bandwidth to minimize cost and complexity.

The above observations highlight the need for wideband, dual-polarized DOA sensors. However, designs by traditional methods have to trade several competing parameters, such as gain, beam ripple, element spacing, and axial ratio, which becomes challenging when the bandwidth of the sensor is increased. Many of the guidelines and design equations for DF systems are based on analog DOA algorithms such as monopulse [9] or mode comparison for spiral antennas [10]. While these algorithms are effective and reliable, they place specific requirements on the antenna design that can be a challenge when also constrained by bandwidth, size, weight, and power. Furthermore, while those algorithms offer advantages in simpler processing, they often make concessions in accuracy.

These challenges can be alleviated by taking advantage of advances in digital receivers and digital processing. Receivers with multiple decades of bandwidth are now available [11][12][13], making it possible to perform wideband DOA estimation in the digital domain. Combined with the availability of processing power, these radios make it significantly easier to implement complicated algorithms. Furthermore, digital DOA algorithms are more robust than their analog counterparts, as they are often able to approach the Cramér-Rao Lower Bound (CRLB) [14]. The CRLB quantifies the lower bound on the performance of an estimator and can give an idea of how well an antenna system can operate as a DOA sensor. As the number of averaged samples increases, algorithms such as MUSIC [15] and maximum likelihood (MLM) [16] are able to approach the theoretical limit on performance given the signal to noise ratio.

By utilizing the performance boost of digital algorithms, it is possible to relax requirements on the design of the DOA sensor for advantages in size, weight, gain, or even the ability to transmit and
receive with the same sensor. Such antennas may not measure up well to more traditional sensors intended for use with analog processors by standard metrics, but nonetheless, they can produce high quality, highly accurate DOA estimates. This thesis presents the design of two wideband DOA antennas: a modulated arm width (MAW) spiral intended for sensing linearly-polarized signals near boresight of the antenna, and a dual-polarized array on a ground plane capable of full dual-polarized DOA estimation over the entire upper hemisphere while simultaneously isolating a transmit element from the DOA receivers.

1.2 Direction of Arrival Receiver Design

The block diagram for a typical superheterodyne digital DOA receiver is shown in Figure 1.1. Note that this diagram is fundamentally similar to that of an analog DOA system, except that in an analog system there are often beam- or modeforming circuits between the antenna elements and the array, and the analog-digital converters (ADCs) and digital processing are replaced by analog processing circuitry. In the figure, each antenna element is connected to a low-noise amplifier (LNA) and then a filter designed to reject out-of-band signals. As discussed below, the order of the filter and LNA can be switched to increase input power handling at the expense of the noise figure of the system. After filtering, the signal is passed through one or more stages of mixers and filters to bring the signal either to baseband or to a low intermediate frequency (IF). Finally, the signal is quantized in the synchronized ADCs and processed in the digital DOA algorithm.

It is worth noting that recent advances in ADC bandwidths and processing power have inspired discussions about direct sampling RF front-ends, in which the outputs of the LNAs are connected directly to the ADCs. In this scheme, down-conversion and filtering are performed in the digital domain, allowing the receiver to be reconfigurable to theoretically work in any band with any signal. While this type of receiver faces several design challenges, it allows for a reduction in RF complexity and makes the receiver completely software-defined [17][18].

Designing each of the blocks in Figure 1.1 requires careful consideration of each component’s power handling and the control of error in the system. In receive-only systems, high-power blocking
Figure 1.1: Block diagram of a digital DOA receiver with optional superheterodyne RF front-end. Before being quantized at a set of synchronized ADCs, the received signals are first conditioned with one or more LNAs, filters, and mixers. In a direct conversion RF-digital system, the highlighted filters and mixer can be removed, allowing the RF to be sampled directly by wideband ADCs. The system must be designed with synchronization and calibration in mind.
signals from other nearby transmitters are of the greatest concern with respect to the power handling of the components and the selection of the filters in the front-end. However, in systems that transmit in addition to estimating the DOA of incident signals, the self-interference from the transmitter is also a major driver for the design of these components. In order to keep system size small, it is advantageous if chip-based filters can be used; however, these filters are limited in their power handling, with typical values less than 1 W of input power [19][20]. Depending on the available parts and the peak-average ratio of the signal, the actual power handling may be significantly less. Likewise, LNAs tend to be limited to approximately a 20-23 dBm 1 dB compression point at their output with 15-25 dB of gain. This means that, depending again on the peak-average ratio of the input signal and the linearity of the LNA, an LNA may start to compress with an input power in the range of -10 to 5 dBm.

In addition to power handling, one must also manage the synchronization and the magnitude and phase differences between channels. Ideally, each of the mixers in Figure 1.1 would be fed by an identical local oscillator (LO), each of the ADCs would be clocked by the same reference signal, and the components in each path would be identical. In reality, the LOs and references for the mixers and ADCs will undergo small differences in delay before reaching the mixers and each component will vary depending on the tolerances of the parts used. For the mixers, this offset will result in a slight frequency-dependent phase offset from channel to channel that can be calibrated with the rest of the components. The ADCs, however, will be sampling at slightly different times. While some compensation can be performed by resampling the signals so that they are all aligned, the synchronization of the converters is often given significant attention during design so that the offsets are minimized. Finally, the effects of the variations of each component in the processing chain can be compensated for by using a calibration routine, such as those in [21][22][23][24]. In general, the calibration procedure measures the offsets between channels and uses them to correct the measured covariance matrix of the array.

To better illustrate the trade-offs one might make when designing a front-end, consider a system in which the transmitter is outputting a signal with 30 dBm (1 W) of power. The transmit
signal is assumed to have been filtered sufficiently such that the self-interference in the receive channel is negligible. In order to connect the LNA directly to the receive antenna, the system would require somewhere on the order of 25 - 40 dB of isolation between the transmitter and each receiver, depending on the signal and the LNA. If that is not possible, then one could instead place either a filter (for frequency-division duplexed or FDD signals) or some kind of transmit cancellation circuit, such as in [25], before the LNA to reduce the power of the transmit signal in the receiver. In this case, with the same 25 - 40 dB of isolation to the receiver, the transmitter might be able to output significantly more power while still meeting the power limits of the components, depending on the amount of rejection in the filter or canceler and on the power limitations of those components. However, this gain in power handling comes at the cost of a degraded system noise figure, as the filter or canceler will incur an additional loss before the LNA.

1.3 Digital Direction of Arrival Estimation with MUSIC

Analog DOA algorithms were built to accommodate the relatively limited mathematical processing that can be done in an RF or analog circuit. Digital algorithms, on the other hand, have been designed to exploit more complicated relationships between the multiple inputs of the DOA estimator. Therefore, digital DOA algorithms tend to be less intuitive but more accurate and resistant to noise.

The algorithm described in this thesis is derived from a narrowband signal model of incident signals. For an \( N \) element array with \( D \) incident signals, the signals each have a magnitude and phase described by the \( D \times 1 \) vector \( \vec{s} \), and the \( d \)th signal has a polarization described by the Jones vector

\[
\vec{k}_d = \begin{bmatrix} \cos \alpha_d \\ \sin \alpha_d e^{j\tau_d} \end{bmatrix}
\]  

(1.1)

In (1.1), \( \alpha_d \) is the linear polarization angle and \( \tau_d \) describes how elliptical the polarization is for the \( d \)th signal. For example, \( \tau = 0 \) gives linear polarization and \( \tau = \pm 90^\circ \) with \( \alpha = 45^\circ \) gives circular polarization. For multiple incident signals, these Jones vectors are combined into a block matrix
$K$, where

$$K = \begin{bmatrix}
\vec{k}_1 & \vec{0} & \cdots & \vec{0} \\
\vec{0} & \vec{k}_2 & \cdots & \vec{0} \\
\vdots & \vdots & \ddots & \vdots \\
\vec{0} & \vec{0} & \cdots & \vec{k}_D
\end{bmatrix}$$

Finally, the voltage response of the array for the $d$th signal is

$$A'_d = \begin{bmatrix}
\vec{a}_\theta(\theta_d, \phi_d) \\
\vec{a}_\phi(\theta_d, \phi_d)
\end{bmatrix}$$

where $\vec{a}_\theta(\theta_d, \phi_d)$ and $\vec{a}_\phi(\theta_d, \phi_d)$ are the $N \times 1$ co-polarized and cross-polarized responses of the array in the direction of arrival $(\theta_d, \phi_d)$ of the $d$th signal. As with the polarization, the response of the array to all $D$ signals is captured by the matrix $A$, with

$$A = \begin{bmatrix}
A'_1 & \cdots & A'_D
\end{bmatrix}$$

Therefore, the received voltage at each antenna, $\vec{x}$, is

$$\vec{x} = AK\vec{s} + \vec{n}$$

where $\vec{n}$ is an $N \times 1$ vector containing the complex noise at each receiver.

Nearly all high resolution DOA algorithms utilize the covariance matrix of the array, which captures the relative magnitude and phase between array elements. The true covariance of the array for $D$ incident signals is

$$R_{xx} = E\left[\vec{x}\vec{x}^H\right] = AKE\left[\vec{s}\vec{s}^H\right] K^H A^H + E\left[\vec{n}\vec{n}^H\right]$$

where $(\cdot)^H$ denotes the conjugate transpose and $E[\cdot]$ represents the expected value of a random variable. Note that (1.6) assumes that the signal and noise are uncorrelated, which is reasonable in most situations. If the signal covariance matrix is defined as $R_{ss} = E\left[\vec{s}\vec{s}^H\right]$ and the noise covariance matrix is $R_{nn} = E\left[\vec{n}\vec{n}^H\right] = \lambda R_0$, then

$$R_{xx} = AKR_{ss}K^H A^H + \lambda R_0$$
The Multiple Signal Classification (MUSIC) algorithm [15] is a popular digital DOA algorithm that exploits properties of the covariance matrix in (1.7). The derivation of the algorithm assumes that $R_{ss}$ is positive definite, meaning that the incident signals must not be coherent. If the number of signals $D$ is less than the number of array elements $N$, then $AK$ only has $D < N$ columns, making $AKR_{ss}K^HA^H$ singular. In particular, this means that

$$\det(AKR_{ss}K^HA^H) = \det(R_{ss} - \lambda R_0) = 0$$ \hspace{1cm} (1.8)

Furthermore, the properties of $AK$ and $R_{ss}$ make $AKR_{ss}K^HA^H$ positive semi-definite, which means that all of the eigenvalues of $AKR_{ss}K^HA^H$ are non-negative, with $N - D$ eigenvalues being 0. Therefore, $N - D$ of the eigenvalues of $R_{xx}$ in the metric of $R_0$ are equal to a value $\lambda_{min}$, which is the smallest eigenvalue of $R_{xx}$.

Due to this relationship, each of the $\lambda_{min}$ eigenvalues is associated with an eigenvector $u_{N,d}$, with

$$AKR_{ss}K^HA^H u_{N,d} = 0 = K^HA^H u_{N,d}$$ \hspace{1cm} (1.9)

Therefore, each eigenvector $u_{N,d}$ is orthogonal to the set of effective steering vectors $AK$, and the set of $N - D$ vectors $u_{N,d}$ can be combined to form a subspace that is orthogonal to $AK$. This subspace, called the noise subspace and denoted $U_N$, can be used to find the direction of arrival of each incident signals by finding the angles at which $\|U_N^H a(\theta,\phi) \tilde{k}(\alpha,\tau)\|$ is minimized, where $a(\theta,\phi) = [\tilde{a}_\phi(\theta,\phi) \quad \tilde{a}_\varphi(\theta,\phi)]$ contains columns with the co- and cross-polarized steering vectors of the array at $(\theta,\phi)$ and $\tilde{k}(\alpha,\tau)$ is a Jones vector as defined in (1.1).

In practice, this search is implemented using the normalized MUSIC spectrum defined by [26]

$$S = \frac{\tilde{k}^H(\alpha,\tau) a^H(\theta,\phi) R_0^{-1} a(\theta,\phi) \tilde{k}(\alpha,\tau)}{\tilde{k}^H(\alpha,\tau) a^H(\theta,\phi) U_N U_N^H a(\theta,\phi) \tilde{k}(\alpha,\tau)}$$ \hspace{1cm} (1.10)

However, (1.10) requires a four-dimensional search for maxima over $(\theta,\phi,\alpha,\tau)$, which very quickly becomes computationally prohibitive. Luckily, the search can be reduced to a sweep over $\theta$ and $\phi$ by noting that the correct $\tilde{k}(\alpha,\tau) = \tilde{k}_{min}$ that maximizes (1.10) is the eigenvector associated with the minimum eigenvalue $\lambda_{min}$ of $a^H(\theta,\phi) U_N U_N^H a(\theta,\phi)$ in the metric of $a^H(\theta,\phi) R_0^{-1} a(\theta,\phi)$ [27].
In other words,

$$a^H(\theta, \phi) U_N U_N^H a(\theta, \phi) \vec{k}_{\min} = \lambda_{\min} a^H(\theta, \phi) R_0^{-1} a(\theta, \phi) \vec{k}_{\min}$$  \hspace{1cm} (1.11)

The minimum eigenvalue can be calculated by finding the minimum root of the characteristic polynomial

$$\det \left( a^H(\theta, \phi) U_N U_N^H a(\theta, \phi) - \lambda a^H(\theta, \phi) R_0^{-1} a(\theta, \phi) \right) = 0$$  \hspace{1cm} (1.12)

Therefore, $S$ in (1.10) can be computed by finding the minimum root $\lambda_{\min}$ of (1.12) at each $(\theta, \phi)$ and taking $S(\theta, \phi) = 1/\lambda_{\min}$. Then, the largest $D$ peaks in $S$ are taken as the direction of arrival of the $D$ incident signals on the array.

As an example, consider a circular array with a 1 wavelength radius of 16 omnidirectional elements in which elements alternate between $E_\theta$ polarization and $E_\phi$ polarization. Three uncorrelated (i.e. $R_{ss}$ is the identity matrix) signals are incident on the array, with directions and polarizations given in Table 1.1. The MUSIC spectrum $S$ for this scenario is shown in Figure 1.2. It can be seen in the figure that the algorithm estimates the DOA to within $0.1^\circ$ of truth and the polarization within $0.3^\circ$ of truth. Furthermore, the sharp peaks over the low background levels emphasize the high resolution of the algorithm and the absence of ambiguities in the array manifold.

### 1.4 Thesis Objectives

This thesis presents alternative design processes for designing wideband, dual-polarized DOA sensors intended to be used with digital processing backends. By evaluating these sensors in terms of the overall system DOA performance instead of traditional metrics such as gain and axial ratio,

<table>
<thead>
<tr>
<th>Signal</th>
<th>$\theta$</th>
<th>$\phi$</th>
<th>$\alpha$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10^\circ$</td>
<td>$100^\circ$</td>
<td>$60^\circ$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>$20^\circ$</td>
<td>$280^\circ$</td>
<td>$20^\circ$</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>$40^\circ$</td>
<td>$30^\circ$</td>
<td>$45^\circ$</td>
<td>$90^\circ$</td>
</tr>
</tbody>
</table>
Figure 1.2: MUSIC spectrum of the uncorrelated signals in Table 1.1 for a 16 element circular array with a radius of 1 wavelength and with element polarizations alternation between $E_\theta$ and $E_\phi$. The signal-noise ratio for each signal 20 dB and $P = 100$ snapshots are used to estimate the covariance matrix. The search is performed on a 0.1° grid. DOA estimates are accurate to within 0.1° and polarization estimates are accurate to within 0.3°.
it is shown that significant reduction in system complexity can be obtained, as well as increases in isolation between electrically-close antennas. Where possible, intuition about the relationship between antenna parameters and DOA performance is developed, resulting in new design metrics that better capture the performance of the antenna in the digital DOA system.

These design principles are illustrated through the design of two DOA sensors: a directional DOA sensor for linearly-polarized incident signals centered on boresight, and an omnidirectional dual-polarized antenna capable of simultaneous transmit and receive. In both designs, the achieved performance exceeds that of a more traditional antenna with an analog backend. For the directional antenna, implemented as a four-arm modulated arm width (MAW) spiral, conventional designs focus on polarization purity of sum and difference modes in both polarizations and require at least six arms [28]. However, it is shown that for linearly-polarized incident signals, a four-arm MAW spiral is sufficient to obtain highly accurate DOA estimates by using clean sum and difference modes in one circular polarization and a mixed polarization mode to help resolve ambiguity from the cross-polarized radiation. Despite the poor axial ratio of the third mode, the sensor produces accurate DOA estimates over a wide bandwidth, emphasizing the effectiveness of this approach.

The omnidirectional antenna demonstrates another advantage to this approach. The sensor is implemented as a circular array of dual-polarized receive elements integrated with a transmit antenna, with all elements placed within approximately a wavelength at the lowest frequency of operation. Because of the close proximity of the antennas, it is difficult to obtain high isolation, which is critical for allowing simultaneous transmit and receive operation, as described above. Furthermore, some of the more effective methods used for improving isolation result in patterns that do not measure well against traditional DOA metrics. However, measurement and analysis demonstrate the effectiveness of the chosen design in terms of transmit-receive isolation and dual-polarized DOA sensing over a wide bandwidth.
1.5 Methodology

To accomplish the objectives discussed in the previous section, the following research methodology is applied. Antenna elements are first understood at the theoretical level, resulting in a set of parameters that can be analyzed for impedance and gain bandwidth as well as DOA performance. Analysis and design are supported by full-wave finite element simulations in Ansys HFSS [29]. The data from both simulation and measurement is analyzed in Matlab with a CRLB code based on [30], giving insight into the expected DOA performance of the antennas. All developed designs are fabricated and tested, with printed circuit boards (PCBs) manufactured by external vendors. The metallic and dielectric components of the array in Chapters 3 and 4 are also machined via CNC by external vendors, while the support structure in the array is 3D printed on a MakerBot Replicator with PLA filament. S-parameters for the fabricated antennas are measured with an Agilent 8719ES Vector Network Analyzer, and antenna patterns are measured using spherical near-field measurements with an NSI 700S-30 combined spherical near-field and far-field chamber at the University of Colorado Boulder.

1.6 Thesis Organization

This thesis is organized as follows:

- Chapter 2 proposes a characterization approach for four-arm modulated arm width (MAW) spiral antennas for linearly-polarized direction-of-arrival (DOA) sensing with a directly connected coherent receiver and digital backend. In this approach, performance is assessed from the system level, with the achievable field of view (FOV) used as the metric for the antenna, evaluated by means of the Cramér-Rao Lower Bound (CRLB). The resulting antenna differs from conventional MAW spiral designs, emphasizing system DOA performance over previous metrics, such as pattern quality (of importance for analog-only systems). Simulation and measurement of this MAW spiral show that the antenna can sense the DOA with 1° RMS error in azimuth and elevation over a ±30° elevation and 360° azimuth FOV across
a 4.6:1 bandwidth. This antenna is then compared with a traditional MAW spiral and a conventional spiral, which in contrast achieve less than a 2:1 bandwidth. The optimization of a MAW spiral for wider bandwidth was not carried out but is suggested as future work.

- Chapter 3 introduces the design of a novel circular array of tightly-coupled horizontally-polarized dipoles placed close to a ground plane. An approach to increase the bandwidth by cascading the dipole resonances from the ground plane below, a conducting reflector behind, a capacitive load in front, and a dielectric slab ring above is proposed. A 3.45:1 impedance bandwidth and consistent radiation patterns near the horizon are demonstrated, both as an omnidirectional radiator and as an array of four sectoral antennas. The design process is simplified by using a unit cell with periodic boundary conditions, and the effects of each resonance are characterized to provide design insight. Then, a theoretical analysis of the radiated fields of the array is performed, explaining nulls found in the radiation pattern near the horizon. A dielectric slab is added to mitigate this problem, and the effects of its dimensions on the radiation pattern and impedance match are presented. The array is fabricated and measured, and excellent agreement between simulation and measurement is observed. The array is found to provide greater than -1 dBi of horizon gain per sector (8 dBi max) with a cross-polarization ratio better than 18 dB or greater than -9 dBi of gain as an omnidirectional antenna at the horizon (8 dBi max) with a cross-polarization ratio better than 9 dB.

- Chapter 4 proposes a multi-functional wideband, dual-polarized direction of arrival (DOA) sensor integrated with a transmit monocone. The array consists of 64 tightly-coupled, horizontally-polarized dipoles combined to four sectoral outputs, eight tapered slot antennas (TSAs) combined to four outputs, and the monocone, all integrated in a circular array with a diameter of 1.15λ and a height of 0.33λ at the lowest frequency of operation. Special attention is given to the design of each type of element due to the proximity of the others. The monocone is isolated from the other elements in the array through a mix of polarization
diversity and subtraction of the coupled power due to symmetry, allowing for simultaneous transmit and receive (STAR) operation. Measured results show that the array is matched with return loss greater than 10 dB from 0.78 to 2.66 GHz, giving it a 3.41:1 bandwidth. Isolation over this bandwidth is measured to be greater than 40 dB between the monocone and the TSAs and 26.5 dB between the monocone and the tightly-coupled dipoles. Finally, the DOA performance of the array is analyzed with the Cramér-Rao Lower Bound (CRLB), which indicates that the array is capable of providing DOA estimates with less than 1.3° of error over the entire upper hemisphere with a 10 dB signal-noise ratio for the incident signal, except for a degradation of φ estimates near zenith.

• Chapter 5 provides a summary of this work, its contributions to the antenna and scientific communities, and proposes topics for future research.
Chapter 2

Performance Characterization of Four-Arm MAW Spiral Antennas for Digital Direction-of-Arrival Sensing

This chapter studies the performance of four-arm modulated arm width (MAW) spiral antennas as linearly-polarized direction-of-arrival (DOA) [31] sensors. While MAW spirals have been studied for use in dual-polarized receivers [28][32][33], previous work has focused on typical antenna properties like axial ratio and cross polarization discrimination to suggest the expected DOA performance. These approaches assume the antenna is used in conjunction with an analog mode forming network and a modal comparison DOA algorithm, as show in Figure 2.1a. This contribution handles the scenario shown in Figure 2.1b, where the antenna ports are directly connected to a set of coherent receivers. The depicted configuration, which is also popular with phased array antennas, enables for an ideal mode former to be implemented digitally, and the use of more advanced digital signal processing techniques like maximum-likelihood [16] and MUSIC [15]. The performance of these algorithms with a conventional spiral DOA sensor is discussed in [10]. As shown herein, this approach removes limitations of the mode former, allowing DOA performance over a wider bandwidth. In this work we will present a significantly different MAW spiral design when compared to a traditional configuration designed for pure modal performance and conventional antenna parameters. Combining this aperture with a fully digital coherent receiver results in a wideband sensor that can be easily realized in a practical system.

Traditional $N$-arm spiral antennas are capable of radiating $N - 1$ balanced modes with a circular polarization set by the winding sense of the arms [34]. Because its structure is defined only
Figure 2.1: Block diagram of (a) traditional MAW spiral-based DOA system with antenna, mode forming network, and comparison-based DOA algorithm, and (b) proposed system with the arms of the MAW spiral sensor directly connected to the receivers.
by the polar angle, the spiral antenna is ideally frequency independent [35]. In reality, the inner and outer radii of the spiral will limit its upper and lower frequencies of operation, respectively. However, even with these limitations, spirals with more than 10:1 bandwidth have been reported [36][37][38].

In traditional multi-mode spiral-based sensors, the voltages on the arms of the antenna are converted into received modal excitations by an analog mode former [34]. While several implementations are possible, Butler matrices [39] are often a convenient realization. However, it is difficult to build a mode former with good amplitude and phase balance over multiple octaves; even recent ~4:1 bandwidth Butler matrices exhibit greater than 1 dB of amplitude imbalance and 4.5° of phase imbalance [40][41]. While Rotman lenses [42] are effective as wideband beamformers for array antennas, they are time-based devices and would therefore not be suitable for use with a spiral antenna. On the other hand, the advent of photonics-based receivers has made it possible to receive over multi-decade bandwidths [11][12][13] with over 1 GHz of instantaneous bandwidth [43]. By combining these receivers with a wideband DOA sensor, the overall system bandwidth can be extended while reducing errors from mode former imbalances.

MAW spirals were introduced in the 1970s as a means of generating radiation patterns of both circular polarizations from spiral antennas [44]. In this geometry, the arm-width variations of the spiral create a bandstop region along the spiral that reflects currents back towards the center of the spiral. These reflected currents radiate with the opposite polarization of the winding sense, allowing for the upper half of the \( N - 1 \) modes of the antenna to radiate with the opposite polarization of the remaining conventional modes. The strength of the reflection is controlled by the modulation ratio of the arms, which is the ratio of the thick to thin section width. MAW spirals still meet the requirements for frequency independence, so they retain the wide bandwidth capability of traditional spirals.

The behavior of MAW spirals is often compared with sinuous antennas [28], as both allow for dual-polarized, multi-mode, and wideband operation. As shown in [33], the MAW spiral provides the advantage of improved axial ratio of the \( \lceil (N - 1)/2 \rceil \) mode of the antenna. For a sinuous antenna, this mode is linearly polarized, whereas for a MAW spiral, the ratio of co-polarized to
cross-polarized components can be large. In particular, for four-arm antennas, mode 2 will be of significantly higher quality for a MAW spiral than for a sinuous antenna. As we will show below, the quality of the mode 2 radiation is critical for DOA performance, making MAW spirals a better choice for dual-polarized DOA sensing than the sinuous antenna.

To obtain a lower bound for the covariance matrix of an estimator — like the one implemented in the digital backend of a DOA system — the Cramér-Rao Lower Bound (CRLB) can be used. For DOA applications, two types of CRLB are applicable: conditional and unconditional. In the conditional CRLB, the signal is assumed to be the same for all estimates, while in the unconditional CRLB, the signal is assumed to be random. Expressions for the conditional CRLB of a dual-polarized DOA estimator have been derived in [30], allowing their use in antenna DOA performance evaluation. A similar approach was followed in [45] for circular arrays, allowing for the optimization of directional elements in that array geometry.

While it is shown in [46] that the conditional lower bound is unreachable with any real DOA algorithm, for a large number of samples and high signal to noise ratio (SNR), the unconditional and conditional CRLB asymptotically converge to the same matrix. Furthermore, [14] and [47] study the asymptotic behavior of the maximum-likelihood (ML) [16] and MUSIC [15] algorithms and prove that both of these algorithms approach the CRLB under the same conditions. Therefore, the CRLB is a valuable tool for estimating the performance of a DOA system while requiring significantly less computational resources than applying a DOA algorithm over a range of incident signals [48].

In the following sections, the CRLB is used to evaluate the DOA performance of two MAW spiral designs, along with a traditional spiral antenna, in the context of a digital receiver backend. We show that for a linearly-polarized incident signal, the DOA performance of traditional MAW spiral designs can be improved by reducing the modulation ratio and using a tightly-wound spiral. This result contradicts previous design recommendations for MAW spiral antennas and highlights the importance of mode 2 in estimating DOA with a four-arm spiral. This work extends on the research showcased in [48] by presenting measurements and CRLB DOA performance of fabricated
antennas and providing an in-depth discussion of the results. Through comparison of three antennas, it is shown that lowering the modulation ratio of the antenna leads to a sensor bandwidth exceeding twice that of a more traditional MAW spiral. The chapter is organized as follows:

- Section 2.1 discusses the geometry and modes of traditional and MAW spirals.
- Section 2.2 describes the CRLB for dual-polarized DOA estimation with digital DOA backends.
- Section 2.3 presents results from parametric studies of the MAW spiral geometry on its DOA performance.
- Section 2.4 gives a description of the three fabricated antennas and presents simulation and measurement results for each, both in terms of radiation patterns and the CRLB.
- Section 2.5 provides a further discussion of the results, emphasizing intuition about the DOA performance of each antenna.

2.1 Traditional and MAW Spirals

The geometry of conventional equiangular spiral antennas is defined by three parameters: initial radius $r_0$, number of turns $n$, and expansion factor $EXP$. These values describe the shape of the center of each arm of the antenna. For MAW spirals, a fourth parameter, the modulation ratio $mod$, is required to describe the width variation of each arm. Each of these parameters has a significant effect on the radiation patterns of the antenna, which in turn determine the antenna’s DOA capabilities.

The radius of the centerline of each arm at angle $\phi$ is given by [32]

$$r = r_0 EXP^{\phi/2\pi}$$  \hspace{1cm} (2.1)

for $0 \leq \phi \leq 2\pi n$. Therefore, a low expansion factor results in a tightly wound spiral, and the outer radius of the spiral is $r_1 = r_0 EXP^n$. 
Figure 2.2: Photographs of fabricated four arm (a) conventional spiral with $\text{EXP} = 1.8$, (b) MAW spiral with $\text{EXP} = 1.8$ and $\text{mod} = 1.6$, and (c) MAW spiral with $\text{EXP} = 1.8$ and $\text{mod} = 8$. All antennas have an outer diameter of 9.54 cm and $n = 5$ turns.

The ratio of the width of the wide portion of each arm to the width of the narrow portion of each arm is defined as the modulation ratio, with $1 \leq \text{mod} < \infty$. Practically, $\text{mod}$ will be limited to $\sim 10$ due to limitations on how narrow traces can be near the feed [32]. When $\text{mod}$ is 1, the MAW spiral degenerates to a conventional spiral, and for larger $\text{mod}$, the width difference between thick and thin sections increases. The geometry (and associated parameters) of articles fabricated and tested in this paper are shown in Figure 2.2.

In general, an $N$-arm spiral can be excited by $M = N - 1$ balanced modes as well as a single unbalanced mode in which all of the arms of the spiral are excited with the same amplitude and phase [34]. This unbalanced mode, often called mode 0, is generally considered unusable for DOA applications because the radiation pattern depends heavily on the antenna’s surroundings. The $m$th balanced mode is excited by providing equal amplitude and a $2\pi m/N$ phase progression between arms, and the radiation region for mode $m$ occurs roughly at a circumference of $m\lambda$. Because spirals in this paper are fabricated on dielectric substrates, $\lambda$ is the wavelength of the wave traveling along the arm of the spiral in the active region [49]. Notably, this means that $\lambda$ will change for each mode, so the active regions will not be evenly spaced radially.
For a conventional spiral, all \( M \) balanced modes will be circularly polarized according to the winding direction of the spiral as long as the spiral is large enough to radiate them. When the spiral is not large enough to accommodate the radiation region of a given mode, the currents for that mode are reflected towards the feed and radiate at the \((N - m)\lambda\) circumference with the opposite polarization [28].

MAW spirals take advantage of this current reflection behavior by creating a bandstop region at a circumference of \( N\lambda/2 \). Therefore, the lowest \([M/2]\) modes are largely unchanged from a traditional spiral, while the remaining modes (generally referred to as the \((m - N)\)th modes) radiate with the opposite polarization. In this way, the MAW spiral can radiate modes with both left- and right-hand circular polarization. For example, a four-arm MAW spiral radiates modes 1, 2, and -1. The quality of modes 2 and -1 will be determined by the geometric parameters of the antenna. In particular, a low mod, like antenna (b) in Figure 2.2, will result in low cross-polarization contamination in mode 2 and high contamination of mode -1. On the other hand, high mod like antenna (c) in Figure 2.2 results in high contamination of mode 2 and low cross-polarization in mode -1.

2.2 Cramér-Rao Lower Bound

Traditionally, antenna engineers have designed sensors to meet requirements on the radiation patterns of an antenna or array of antennas, such as polarization discrimination, sidelobe levels, pattern slope, and other parameters that summarize its performance. However, specifications of these parameters assume a specific DOA engine implementation and often must be pessimistic in order to guarantee a specific system performance level. As desired bandwidth and field of view increase, designers need to be able to evaluate their designs on a metric tied to overall system performance.

The CRLB provides a lower bound on the statistical performance of an estimator, defined as the inverse of the Fisher information of the estimator. The CRLB for a DOA estimator with no a-priori polarization information using an arbitrary array of dual-polarized antenna elements is
derived in [30] as

\[ \Sigma_p = \frac{\sigma^2}{2P} \mathcal{R}^\dagger \left\{ (D^H \Pi_A D) \odot \left( 1_{2\times2} \bigotimes \hat{R}_{ss}^T \right) \right\} \]  

(2.2)

In (2.2), \( \Sigma_p \) is the lower bound on the covariance matrix of the system. A lower bound on the RMS error of each parameter can be obtained from the square root of the diagonal elements of \( \Sigma_p \). \( \sigma^2 \) is the variance of the noise, \( P \) is the number of snapshots used in the estimate, \( \hat{R}_{ss} \) is the covariance matrix of the signal, \( \mathcal{R}^\dagger \) is the Moore-Penrose pseudoinverse of the real part of the argument, \( 1_{2,2} \) is a \( 2 \times 2 \) matrix of ones, \( \odot \) is the Hadamard product, and \( \bigotimes \) is the Kronecker product. 

\( D = \begin{bmatrix} A^{(\theta)} K & A^{(\phi)} K \end{bmatrix} \) represents the derivatives of the radiation patterns of the antenna, with \( A \) having \( 2d \) columns containing the steering vectors of the antenna for the direction of each of the \( d \) signals present in two (ideally orthogonal) polarizations. \( K \) is a block diagonal matrix of Jones vectors for each signal, and \( A^{(\theta)} \) and \( A^{(\phi)} \) are the derivatives of \( A \) with respect to \( \theta \) and \( \phi \), respectively. Finally, \( \Pi_A = I - AA^\dagger \) is a projection operator onto the left nullspace of \( A \). When discussing DOA algorithms, the left nullspace of \( A \) is often called the noise subspace of the system and is orthogonal to the span of the columns of \( A \) [15].

Examining (2.2) more closely gives important information about the desired properties of the radiation patterns of the antenna for operation as a DOA sensor. If only a single signal is present, then (2.2) simplifies to

\[ \Sigma_p = \frac{1}{2P \text{SNR}} \frac{1}{\mathcal{R}^\dagger} \left\{ D^H \Pi_A D \right\} \]  

(2.3)

where \( \text{SNR} \) is the signal-noise ratio of the signal when received by an isotropic sensor. Effects of the gain of the antenna are captured by \( D \), which leads to an overall effective change in SNR at each elevation and azimuth angle. Furthermore, it can be shown that \( \Pi_A = \Pi_A^H \Pi_A \). This means that the argument to the pseudoinverse in (2.3) is \( (\Pi_A D)^H (\Pi_A D) \), which is a matrix whose entries contain inner products between the derivatives of \( A \) and their projection onto the noise subspace of \( A \). Therefore, each entry measures the orthogonality between \( D \) and \( A \), with larger values indicating that \( D \) and \( A \) are closer to being orthogonal. Because (2.3) takes the inverse of this matrix, the CRLB shows that the expected DOA error is reduced when the steering vector of the antenna and
its derivatives are orthogonal.

### 2.3 Parametric Study

To understand the effect of the parameters of MAW spirals on DOA performance, parametric sweeps of $EXP$ and mod are performed in Ansys HFSS [29]. The base antenna is a right-handed spiral with an inner radius of $r_0 = 5$ mm and an outer radius of 9.54 cm. Because the inner and outer radii are specified, the number of turns $n$ is set by the growth rate $EXP$. The antenna is fed with a coaxial bundle of 3.58 mm diameter 50 Ω cables and is placed on a 0.79 mm thick, 19.8 cm diameter Rogers 5870 substrate. The outer radius is chosen to support mode 2 radiation at 1 GHz. The first turn of the spiral has a fixed $mod$ of 3.8 to improve the match of the antenna, as recommended in [32], and the number of turns is selected to give the desired outer radius. Finally, the ends of the arms always terminate in a full low-impedance (wide) section, which reduces gain ripple.

The DOA performance for each design is estimated using the CRLB with a linearly-polarized plane wave incident on the antenna. The polarization angle of the signal is swept from -90° to 90° in 5° steps, and the worst-case values of the RMS elevation and azimuth errors are used to evaluate the DOA performance at each frequency. The SNR is taken to be 20 dB and $P = 1000$ snapshots are used in each estimate.

One-degree error field of view (FOV) results from a coarse sweep of $mod$ and $EXP$ at 2 GHz are shown in Figure 2.3. The FOV is defined as the smallest elevation angle at which either the elevation or azimuth estimate has an RMS error of 1° over all azimuth and polarization slant angles. Traditionally, four-arm MAW spirals are designed with low $EXP$ and high $mod$ (lower right corner of Figure 2.3), giving clean mode 1 and -1 patterns and contaminated mode 2 patterns. However, from the figure, it is clear that the best DOA performance can be obtained with low $EXP$ and low $mod$ (lower left corner of Figure 2.3).

While the results in Figure 2.3 indicate a qualitative relationship between modulation ratio and DOA performance, there is no quantification of the performance over frequency. Figure 2.4
Figure 2.3: Elevation field of view in degrees of CRLB RMS elevation and azimuth error at 2 GHz with MAW spiral EXP varied from 1.8 to 3 in steps of 0.2 and mod varied from 1 to 8 in steps of 1. As seen, the widest field of view can be obtained with low EXP and low mod.
shows the minimum $1^\circ$ error FOV from 1.5 to 5.5 GHz in 0.5 GHz steps with finer resolution of $\text{EXP}$ and $\text{mod}$. The figure shows that a low $\text{EXP}$ and a $\text{mod}$ between 1.5 and 2 gives the best DOA performance over the entire band.

2.4 Measurements

2.4.1 Radiation Patterns

Based on the results of the parametric study, MAW spirals with the following parameters are fabricated:

- $\text{EXP}=1.8$, $\text{mod}=1.6$ (see Figure 2.2b)
- $\text{EXP}=1.8$, $\text{mod}=8$ (see Figure 2.2c)

The first antenna follows the guidelines proposed in the previous section, while the second adheres to more traditional MAW spiral parameters [28][32][33][44]. A conventional spiral with $\text{EXP}=1.8$ is also built (see Figure 2.2a). All antennas are fabricated on identical substrates as described in Section 2.3. Antennas are backed by a stack consisting of a 2.54 cm foam spacer, a single piece of 5.69 cm thick AN-77 Eccosorb, and a reflective backing. As in conventional DOA sensors, the reduced efficiency is acceptable as long as the impact on the pattern quality is minimized [34].

Spherical near-field measurements are performed for all three antennas. To reduce the impact of auxiliary mounting structures, the Mathematical Absorber Reflection Suppression (MARS) filtering technique [50] is applied. Specifically, the antenna under test is offset by 23.65 cm from the center of rotation of the measurement chamber to allow for effective filtering of chamber reflections. For each antenna, a single arm is measured and then rotated in post-processing to generate the full set of arm measurements. Because the antenna is intended for use in a system with a digital DOA backend, this synthesis is justified, as differences between arms can be calibrated out in such a system.

Because material parameters for the AN-77 absorber are not available, the simulation models for each antenna include a numerical absorber with $\epsilon_r = \mu_r = 1$ and electric and magnetic loss
Figure 2.4: Minimum elevation field of view in degrees of CRLB RMS elevation and azimuth error from 1.5 to 5.5 GHz in 0.5 GHz steps with MAW spiral $EXP$ varied from 1.8 to 2 in steps of 0.05 and $mod$ varied from 1 to 3 in steps of 0.1. As seen, the widest field of view over the bandwidth can be obtained with an $EXP$ of 1.8 and a $mod$ between 1.5 and 2.
tangents of 1. While this simple model is not expected to match the behavior of the antenna perfectly, it does appear to capture the important physics of the system. In particular, Figures 2.5 - 2.7 show the measured and simulated gain of each antenna at \( \theta = 30^\circ, \phi = 0^\circ \) from 1 to 6 GHz. These results show that the radiation patterns are stable over frequency, emphasizing the frequency-independent nature of the antenna.

Measured radiation patterns for each of the three antennas from 1.5 GHz to 6 GHz are shown in Figures 2.8 – 2.10. As expected, the \( \text{mod} = 1.6 \) MAW spiral exhibits excellent RHCP mode 1 and mode 2 radiation patterns with minimal LHCP contamination across the entire bandwidth, while mode -1 is heavily corrupted by mode 3. The \( \text{mod} = 8 \) MAW spiral also behaves as expected, with the mode 2 axial ratio now heavily degraded while the mode -1 quality is improved. This behavior is again consistent across the entire band of operation. Finally, all three modes of the traditional spiral display excellent axial ratio, providing a clear contrast to the two MAW spirals. Furthermore, Figure 2.10c shows the turn-on of mode 3 from the spiral. At 1.5 GHz, the spiral is too small to efficiently radiate mode 3, so mode -1 dominates that radiation pattern; on the other hand, at 3 GHz, the antenna can radiate mode 3, so the mode -1 contribution is significantly reduced. Simulated data are nearly identical to the measured results and are not shown for clarity of the plots.

### 2.4.2 CRLB Results

While the differences between the radiation patterns of the three measured spirals are evident, it is difficult to tell from visual inspection which antenna will perform best as a DOA sensor. As discussed above, the CRLB provides a simple method for bounding each antenna’s DOA performance. An example of the results of applying the CRLB to the low-\( \text{mod} \) MAW spiral at 2 GHz is shown in Figure 2.11. The output of the CRLB is the lower bound on the covariance matrix of the estimator, which contains the mean-square error of each variable along the diagonal of the matrix. In the figure, these variances have been calculated over linearly polarized signals with a polarization angle between -90° and 90°, with the largest error selected at each elevation and azimuth point.
Figure 2.5: Measured and simulated gain for a MAW spiral with $EXP = 1.8$ and $mod = 1.6$ at $\theta = 30^\circ, \phi = 0^\circ$. Low mod causes the MAW spiral to radiate mode 1 and mode 2 patterns with large co- to cross-polarization ratios at the expense of the mode -1 co- to cross-polarization ratio.
Figure 2.6: Measured and simulated gain for a MAW spiral with $EXP = 1.8$ and $mod = 8$ at $\theta = 30^\circ, \phi = 0^\circ$. High $mod$ causes the MAW spiral to radiate almost identical mode 1 and -1 patterns with good cross-polarization performance at the cost of poor mode 2 co- to cross-polarization ratio.
Figure 2.7: Measured and simulated gain for a traditional spiral with $EXP = 1.8$ at $\theta = 30^\circ, \phi = 0^\circ$. Once the frequency is high enough so that all three modes of the spiral radiate effectively, all three radiate RHCP patterns with good co- to cross-polarization ratios.
Figure 2.8: Measured RHCP (solid blue) and LHCP (dashed red) azimuthal gain cuts of a MAW spiral with $EXP = 1.8$ and $mod = 1.6$. $\phi$ is varied from $0^\circ$ to $360^\circ$ in $5^\circ$ steps. Insets (a) and (d) show mode 1, (b) and (e) show mode 2, and (c) and (f) show mode -1. Insets (a)-(c) show measurements at 1.5 GHz (left) and 3 GHz (right), while (d)-(f) show 4.5 GHz (left) and 6 GHz (right).
Figure 2.9: Measured RHCP (solid blue) and LHCP (dashed red) azimuthal gain cuts of a MAW spiral with $EXP = 1.8$ and $mod = 8$. $\phi$ is varied from $0^\circ$ to $360^\circ$ in $5^\circ$ steps. Insets (a) and (d) show mode 1, (b) and (e) show mode 2, and (c) and (f) show mode -1. Insets (a)-(c) show measurements at 1.5 GHz (left) and 3 GHz (right), while (d)-(f) show 4.5 GHz (left) and 6 GHz (right).
Figure 2.10: Measured RHCP (solid blue) and LHCP (dashed red) azimuthal gain cuts of a traditional spiral with $EXP = 1.8$. $\phi$ is varied from $0^\circ$ to $360^\circ$ in $5^\circ$ steps. Insets (a) and (d) show mode 1, (b) and (e) show mode 2, and (c) and (f) show mode -1. Insets (a)-(c) show measurements at 1.5 GHz (left) and 3 GHz (right), while (d)-(f) show 4.5 GHz (left) and 6 GHz (right).
Figure 2.11: Maximum RMS error in degrees, as calculated by the CRLB, over all linear polarization angles (spaced at 5° intervals) in (a) elevation and (b) azimuth of the low-mod MAW spiral at 2 GHz. The solid black lines in each figure indicate an error of 1°.
Note that in the following analysis, the large azimuth error near boresight is ignored, as the solid angle of the uncertainty due to azimuth error is small due to a \( \sin \theta \) dependence.

Figure 2.12 shows the CRLB for the \( \text{EXP} = 1.8, \text{mod} = 1.6 \) MAW spiral. Considering the agreement in the radiation patterns shown in Figures 2.5 – 2.7, it is not surprising that the CRLBs for the simulated and measured patterns agree favorably overall. There are frequencies at which the measured and simulated CRLBs differ, and these differences can be attributed to the approximated model of the AN-77 absorber described in the previous section and rippling in the measured radiation patterns. In Figure 2.8, ripple can be seen around 30° in all three modes of the antenna, especially at higher frequencies. This ripple, while small in terms of dB of gain, can nonetheless cause a large enough error in the CRLB to limit the FOV of the antenna to around 30°, especially since \( D \) in (2.2) contains the derivatives of the pattern. Furthermore, the figure shows that with this design, it is possible to obtain less than 1° of error in both elevation and azimuth over a ±30° elevation FOV from 1.2 GHz to 5.55 GHz, except for a small suck out at 1.45 GHz. Therefore, the antenna exhibits a 4.6:1 operating bandwidth for linearly polarized signals with an arbitrary (and unknown a-priori to the sensor) polarization angle. To the knowledge of the authors, this is the widest bandwidth and field of view for a single-aperture, dual-polarized DOA sensor reported in the open literature.

The performance of the low modulation ratio MAW spiral is emphasized further when compared to the traditional MAW spiral and conventional spiral. From Figure 2.13, it can be seen that the high \( \text{mod} \) MAW spiral can only meet the 1° of elevation and azimuth error over a ±30° elevation FOV from 1.25 GHz to 2.15 GHz except for a suck out at 1.55 GHz. Figure 2.14 shows that the conventional spiral can meet that performance from 1.1 GHz to 1.4 GHz and 1.65 GHz to 2.3 GHz. Therefore, both antennas can operate over less than a 2:1 bandwidth, which is significantly less than the 4.6:1 bandwidth of the low-\( \text{mod} \) sensor.
Figure 2.12: One-degree error field of view of a MAW spiral with $EXP = 1.8$ and $mod = 1.6$. The antenna allows for less than 1° of azimuth and elevation error over ±30° of elevation, 360° of azimuth, and all linear polarizations from 1.2 GHz to 5.55 GHz, except for a small suck out at 1.45 GHz.
Figure 2.13: One-degree error field of view of a MAW spiral with $EXP = 1.8$ and $mod = 8$. The antenna can sense the DOA with less than $1^\circ$ of azimuth and elevation error over $\pm 30^\circ$ of elevation and $360^\circ$ of azimuth from 1.25 GHz to 2.15 GHz, except for a suck out at 1.55 GHz.
Figure 2.14: One-degree error field of view of a traditional spiral with $EXP = 1.8$. Less than 1° of azimuth and elevation error can be obtained over a ±30° elevation and 360° azimuth field of view from 1.1 GHz to 1.4 GHz and 1.65 GHz to 2.3 GHz.
2.5 Discussion

2.5.1 Antenna Comparison

At first glance, it is surprising that the low-mod MAW spiral gives such a large performance advantage over the other two antennas. Initially, one might think that the overall clean patterns of the conventional spiral, as seen in Figure 2.10, would be superior to the poor axial ratio mode -1 of the low-mod MAW spiral, as seen in Figure 2.8. However, the performance of the traditional spiral suffers in two ways. First, because almost all the power received by the antenna is co-polarized, nearly half of the signal power is lost. In the case of the MAW spiral, mode -1 is able to capture much of the cross-polarized power, giving better sensitivity.

Second, while the conventional spiral patterns are relatively clean, even a small cross-polarized component can cause significant error in the measured voltage from each arm. If the polarization of the incident plane wave impinging from direction \((\theta_0, \phi_0)\) is described by the Jones vector 

\[
\begin{bmatrix}
k_{co} \\
k_{x}
\end{bmatrix}
\]

t, then the received voltage on a given arm, \(a_n\), is

\[
a_n = k_{co}a_{co}(\theta_0, \phi_0) + k_xa_x(\theta_0, \phi_0) \tag{2.4}
\]

or

\[
a_n = k_{co}a_{co}(\theta_0, \phi_0) \left(1 + \rho_k \frac{a_x(\theta_0, \phi_0)}{a_{co}(\theta_0, \phi_0)}\right) \tag{2.5}
\]

where \(a_{co}\) and \(a_x\) are the antenna response in the co and cross polarizations. For a linearly-polarized signal impinging on a circularly-polarized sensor, \(|k_{co}| = |k_x|\), meaning that the \(\rho_k = k_x/k_{co}\) term in (2.5) contributes only a phase shift. Therefore, for a given \(a_x\), there will be a \(\rho_k = \rho_{k,max}\) that results in the largest magnitude of \(a_n\) and another that results in the minimum, when \(\rho_k = -\rho_{k,max}\). Thus, the ratio of the maximum to minimum \(|a_n|\) is

\[
\frac{|a_{n,max}|}{|a_{n,min}|} = \frac{1 + \rho_{k,max} \frac{a_x(\theta_0, \phi_0)}{a_{co}(\theta_0, \phi_0)}}{1 - \rho_{k,max} \frac{a_x(\theta_0, \phi_0)}{a_{co}(\theta_0, \phi_0)}} \tag{2.6}
\]

For example, if the cross-polarized component is 20 dB below the co-polarized component of the radiation pattern, the polarization angle of the linearly-polarized plane wave can cause the received
power at each arm to vary by over 1.7 dB. Without a method to estimate the polarization, the conventional spiral-based estimator cannot compensate for polarization-dependent modal pattern deviations, resulting in significant error in many situations.

Likewise, it may be expected that the clean mode -1 of the mod = 8 MAW spiral, as seen in Figure 2.9, may allow for compensation of the polarization-dependent error in the DOA estimate. However, because each mode of the MAW spiral has a different phase response in \( \phi \), the polarization estimate from the antenna will depend on knowledge of the DOA of the signal. With a high modulation ratio, the cross-polarized component of mode 2 may only be 6 dB less than the co-polarized component, leading to a poor DOA solution, and therefore an overall less accurate combined DOA/polarization estimate. On the other hand, the low mod MAW spiral provides a cleaner mode 2 pattern (nearly comparable to the conventional spiral) while also measuring the cross-polarized component of the signal effectively with mode -1. This combination of pattern properties gives the low mod MAW spiral the best overall linearly-polarized DOA performance.

Finally, previous work [28] has suggested that four-arm MAW spirals cannot be used effectively for dual-polarized DOA sensing due to the contamination of mode 2 with cross-polarized radiation, instead focusing on 6- or 8-arm MAW spiral designs. In such designs, the modulation ratio is chosen to be relatively large so that modes ±1 and ±2 both exhibit polarization purity, resulting in excellent DOA performance. Therefore, using a smaller modulation ratio to obtain an improvement in DOA performance is specifically applicable to four-arm MAW spirals and has not previously been discussed in the literature. However, these designs also require more complicated mode former circuits and/or more digital receivers, resulting in higher system cost and complexity. Therefore, achieving the above DOA performance with a four-arm MAW spiral corresponds to possible improvements in size, power, and complexity over a higher arm count design.

### 2.5.2 SNR Dependence

While (2.3) indicates that for a single signal of interest the CRLB scales inversely with the SNR of the signal, it is not clear how the SNR will affect the 1° error FOV. The effect on the
FOV will depend on the slope of the CRLB at the 1° error contour. A steep slope on that contour will result in a large dependence on the SNR, while a shallow slope will result in the FOV being relatively independent of SNR.

Both types of behavior can be seen in Figure 2.15, which shows the 1° error FOV of the low-mod MAW spiral for various SNR values. For example, around 3.7 GHz, the FOV only varies by 3° when the SNR is changed from 7.5 dB to 25 dB, whereas at 2.95 GHz, the FOV changes from 45° with a 25 dB SNR to 30° with a 10 dB SNR and 0° with a 7.5 dB SNR. It is also clear from the figure that this antenna operates quite well over a wide range of frequencies down to approximately 10 dB SNR.

2.5.3 FOV Dependence on Error Threshold

The above studies characterize the DOA performance by calculating the 1° elevation and azimuth error FOV of the antenna. This error threshold is chosen arbitrarily for comparison purposes, and in many cases, a designer will have a specific accuracy requirement. As with the SNR, the effect of this threshold on the FOV will depend on the slope of the CRLB near the error threshold. A steep slope will result in a small variation in the FOV from one threshold to another. A large slope in the CRLB also likely indicates the presence of an ambiguity in the radiation patterns. While such ambiguities can appear for a variety of reasons, one common cause is rippling in the radiation pattern in both magnitude and phase. If the radiation patterns ever lose monotonicity over both \( \theta \) and \( \phi \), the DOA algorithm will not be able to distinguish between overlapping angles, resulting in an ambiguity. For example, in Figure 2.8, significant ripple can be seen in all three modes of the low-mod antenna at 6 GHz around \( \theta = 30° \). This variation is likely the cause of the \( \sim 30° \) FOV of the antenna at that frequency seen in Figure 2.12.

Figure 2.16 shows the FOV of the low-mod MAW spiral with varying error thresholds. The figure shows that the antenna can sense the DOA to within 0.3° over much of the bandwidth defined above. Furthermore, it can be seen that over a significant portion of the bandwidth, the FOV of the antenna is limited by pattern ambiguities. In the figure, these ambiguities are characterized by
Figure 2.15: $1^\circ$ error field of view of a MAW spiral with $EXP = 1.8$ and $mod = 1.6$ over various SNR values. The FOV of the antenna is relatively constant for SNR greater than 15 dB, and a significant portion of the operating band can operate with as low as 10 dB SNR.
Figure 2.16: field of view of a MAW spiral with $EXP = 1.8$ and $mod = 1.6$ over various error threshold values. Frequencies at which the FOV is relatively independent of error threshold are likely limited by pattern ambiguities.
a small variation of FOV with error threshold. This is because ambiguities will result in a large rate of change in the value of the CRLB at a given angle.

2.6 Conclusion

A four-arm MAW spiral can operate as an effective linearly-polarized DOA sensor over several octaves of bandwidth by using a low growth rate, a small modulation ratio, and a digital DOA backend. In such a design, the mode 1 and mode 2 patterns of the antenna exhibit a large ratio between co-polarized and cross-polarized components, while the cross-polarized mode -1 is highly contaminated by the co-polarized mode 3. This guideline contrasts with traditional MAW spiral designs, which recommend a large modulation ratio to provide a quality mode -1 radiation pattern. Based on measured pattern data of three designed sensors, a conventional spiral, a traditional MAW spiral, and a low modulation ratio MAW spiral, it has been shown that the new MAW spiral design exhibits significantly better DOA performance than the traditional MAW spiral and conventional spiral.
Chapter 3

Tightly-Coupled Array of Horizontal Dipoles Over a Ground Plane

An important consideration when measuring spectrum usage is the polarization coverage of the sensor. If, for example, an emitter radiates with horizontal polarization and a sensor can only measure vertically polarized signals, then the monitoring system will be unaware of the presence of the signal. In addition, even if the polarization of all emitters is known, multipath can cause a change in polarization [51]. Therefore, an effective sensor should be dual-polarized so that it can detect all signals present in its vicinity. This chapter focuses on the design of an antenna, shown in Figure 3.1, that is capable of sensing horizontally polarized signals. The integration with a vertically polarized sensor is covered in Chapter 4.

Designs for omnidirectional horizontally-polarized antennas have been known for decades. The Alford loop [52], while narrowband, remains a popular basis for more modern designs, which have been shown to cover 1.37:1 [53], 1.37:1 [54], 1.52:1 [55], and 1.67:1 [56] bandwidths while maintaining horizontally polarized radiation. Wider bandwidth can be achieved by deviating from the Alford loop concept, such as the traveling wave design in [57]. While the antenna operates from 6.7 to 16 GHz (2.39:1 bandwidth), it is only suitable for higher frequencies due to its size. Finally, the design in [58] uses an array of tightly-coupled dipoles in a circular array to achieve a 2.08:1 bandwidth with omnidirectional, horizontally polarized radiation patterns. This design highlights the promise of tightly-coupled dipole arrays for wideband circular array systems. A summary comparison of these works with the presented design is shown in Table 3.1.

Tightly-coupled dipole arrays (TCDAs) have been used previously in designs to achieve multi-
Figure 3.1: Fabricated (a) and modeled (b) 64-element tightly-coupled horizontal dipole array. The dipole elements can be seen in (b) through the transparent dielectric slab.
octave bandwidths from scanning antenna arrays [59][60][61][62][63][64]. While the specifics of each design vary, these antennas operate by balancing the capacitive reactance of the tightly-coupled elements with the one or more inductive near-field loads. By doing so, the active impedance of the dipole elements is kept relatively constant, allowing a wideband match while preserving radiation efficiency. Additional bandwidth and scanning capability can be obtained by adding one or more dielectric slab layers, which act as a combined lens and dielectric load for the dipoles.

A common element in most TCDAs is the presence of a ground plane underneath the radiating elements [60]. In many cases, the ground plane is a requirement of the overall system due to mounting and other system-level constraints. The structure of the TCDA allows the antenna elements to resonate with the ground plane instead of being degraded as might normally happen. Even in [58] where there is no explicit ground, the elements have a virtual ground at the center of the array due to the rotational symmetry of the in-phase excitation of the elements. In the case of horizontally polarized dipoles, the traditional assumption is that a dipole placed near a ground plane will be shorted out and, therefore, ineffective. However, if the dipoles are designed to be capacitive, then the ground plane helps present a real impedance at the feed.

In this chapter, a wideband array of 64 tightly-coupled horizontal dipoles is designed and measured. Unlike the antenna in [58], which uses capacitive loads and the circular geometry of the

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<td>1.52:1</td>
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</tr>
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<td>[56]</td>
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<td>3.7</td>
<td>1.68:1</td>
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</tr>
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<td>[57]</td>
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<td>16</td>
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12-element array to widen the bandwidth, this design takes advantage of a ground plane underneath the array, a reflector towards the array center, a capacitive load, and a dielectric slab ring to achieve a 3.45:1 impedance bandwidth and horizontally-polarized radiation patterns. The contribution of each of these elements to the impedance and radiation performance of the antenna is analyzed, giving the designer intuition on how to balance the resonances to achieve a desired bandwidth. Challenges specific to a circular configuration of TCDAs are identified, and a solution in the form of the dielectric slab ring is presented. The 64 elements of the array are combined with in-phase excitations to four inputs, which can be used independently for sectoral coverage or can be combined in-phase to provide an omnidirectional radiation pattern.

The chapter is organized as follows. Section 3.1 discusses the antenna design with a periodic unit cell by examining the performance of the array in free space, over a ground plane, and with a dielectric slab so that the impact of each piece of the geometry is understood. Section 3.2 presents the design, simulation, fabrication, and measurement of the fabricated prototype.

3.1 Unit Cell Design

The design of periodic arrays with large numbers of elements is typically approached with a unit cell with periodic boundary conditions [65]. The unit cell represents the performance of an array element if it were placed in an infinite array, which is still a useful approximation for all but a few edge elements. For circular arrays, a wedge-shaped unit cell can be used to perfectly represent the operation of a single element in the array, simplifying analysis. This unit cell captures element-element coupling effects, but only describes the active performance of the element. If knowledge of the individual element performance or element-element coupling is required, then the full array must be simulated.

Because this antenna is intended for use as an omnidirectional spectrum sensor, all unit cell simulations are performed in Ansys HFSS [29] with master-slave boundaries and a $0^\circ$ phase progression on the side walls to simulate a phase mode 0 excitation of the array. Full array mode 0 radiation patterns can be obtained by rotating the unit cell pattern to represent each element’s
position in space and summing the result.

3.1.1 Unit Cell Impedance in Free Space

The basic unit cell investigated in this work is shown in Figure 3.2. The array is made up of horizontally-polarized dipoles of width $w_d$ that overlap with adjacent elements. The amount of overlap can be described by a factor $k_o$, where $k_o = 0$ represents no overlap and $k_o = 1$ represents the limiting case where the end of one dipole touches the feed of the other. To achieve this overlap, the two arms of the dipole are printed on opposite sides of the supporting substrate. The outer radius of the elements is fixed at 15.2 cm, which allows for sufficient space both to combine the elements as described in Section 3.2 and to resonate the elements with the circular ground plane described below in Section 3.1.2.

The number of elements $N$ is determined with a simplified unit cell where the overlapped dipoles are placed in free-space to understand their impedance. For this study, the outer dipole radius is fixed at 15.2 cm, $w_d = 5.1$ mm, and $k_o = 0.5$. The input resistance for various numbers of elements is shown in Figure 3.3, with the inset showing the simplified geometry. From the figure, it can be seen that as the number of elements is doubled (and therefore the unit cell angle is halved), the minimum frequency of operation is roughly halved, while the ratio of the maximum to minimum frequency remains relatively constant. Furthermore, the peak resistance of each dipole in mode 0 is reduced as the number of elements is increased. Because the final design will use a dielectric slab to load the dipoles, choosing $N=64$ elements gives an approximate operating range of 1.5 to 3 GHz based on the impedance of the dipoles. The addition of the dielectric slab will then lower this into the desired frequency range.

3.1.2 Unit Cell Impedance without Dielectric Slab

To obtain the desired operating bandwidth, the dipoles are resonated with three features within the array, as seen in Figure 3.2: the underlying ground plane at height $h_g$ from the vertical center of the dipoles, a circular ground plane on the bottom of the substrate with radius $r_r$ that
Figure 3.2: Unit cell (a) and zoomed-in top view (b) of the periodic antenna structure utilizing overlapped dipoles printed on opposite sides of a substrate. The dipoles are resonated with a reflector, capacitive load, and the ground plane underneath. Master/slave boundaries are used on the side walls to create the periodic structure and radiation boundaries are used to approximate free space above and radially outward.
Figure 3.3: Mode 0 active input resistance of the free-space unit cell overlapped dipole, shown in the inlay, for various numbers of array elements ($N$). Increasing the number of elements increases the operating frequency, reduces the peak input impedance, and increases the bandwidth over which the impedance is near 50 $\Omega$. 
Table 3.2: Parameters for Tightly-Coupled UCA (No Dielectric Slab)

<table>
<thead>
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<th>Parameter</th>
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</tr>
<tr>
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<td>$r_r$</td>
<td>58.4 mm</td>
<td>$h_g$</td>
<td>63.5 mm</td>
</tr>
</tbody>
</table>

acts as a reflector, and capacitive patch loading elements, also on the bottom of the substrate. The capacitive loads are placed a distance $g_c$ from the outside edge of the dipoles, have a radial width of $w_c$, and an angular span described by $k_c$. When $k_c = 0$, the patch width is 0, and when $k_c = 1$, the patches connect and form a continuous ring.

Each of these features contributes a separate resonance in the input impedance of the unit cell in mode 0. Figure 3.4 shows the effect of changing $h_g$ while keeping all other parameters constant, and it is clear that the lowest-frequency resonance is most strongly affected by the height above ground. Likewise, Figures 3.5 and 3.6 show the input impedance of the unit cell when $r_r$ and $w_c$ are varied, respectively. In those cases, $r_r$ most strongly controls the middle resonance and $w_c$ affects the high-frequency resonance.

By placing the three resonances so that they are spaced out relatively evenly in frequency, a roughly 3:1 impedance bandwidth can be obtained, as shown in Figure 3.7. In the figure, the reflection coefficient is shown relative to a 90 $\Omega$ transmission line, as conversion to 50 $\Omega$ will be handled by a microstrip balun and a dielectric slab, which is not included in this unit cell. The parameters used to obtain this performance are contained in Table 3.2. It is interesting that even though the dipole impedance is most consistent between 1.5 and 3 GHz, the array is able to operate below 1 GHz due to the resonances with the ground and reflector. Also, at this point the resonance from the capacitive load is not helpful in widening the match; however, this feature will allow for additional bandwidth when the dielectric slab is added in Section 3.1.4.
Figure 3.4: Variation of the mode 0 active input impedance, referenced to 90 Ω, of the unit cell in Figure 3.2 with changes in ground plane height for $r_r = 58.4$ mm and $w_c = 5.1$ mm. The frequency is swept from 0.5 GHz to 5 GHz. The first resonance in the input impedance is most affected by changes in ground plane height.
Figure 3.5: Variation of the mode 0 active input impedance, referenced to 90 Ω, of the unit cell in Figure 3.2 with changes in reflector radius for $h_g = 63.5$ mm and $w_c = 5.1$ mm. The frequency is swept from 0.5 GHz to 5 GHz. The middle resonance in the input impedance is most affected by changes in reflector radius.
Figure 3.6: Variation of the mode 0 active input impedance, referenced to 90 Ω, of the unit cell in Figure 3.2 with changes in capacitive load width for $h_g = 63.5$ mm and $r_c = 58.4$ mm. The frequency is swept from 0.5 GHz to 5 GHz. The last major resonance in the input impedance is most affected by changes in capacitive load width.
Figure 3.7: Mode 0 active reflection coefficient of the unit cell without a dielectric slab and with parameters given in Table 3.2 (blue, relative to 90 Ω) and the unit cell with a dielectric slab and parameters given in Table 3.3 (red, relative to 50 Ω). Without the slab, the array operates over a 2.7:1 bandwidth and can radiate effectively down to 1 GHz due to interactions with the ground plane and reflector. With the addition of the slab, operation is extended down to 0.73 GHz with a 3.7:1 bandwidth coverage.
3.1.3 Unit Cell Radiation Patterns without Dielectric Slab

Figure 3.8 shows the $\phi$ component of the radiation patterns for the unit cell of the array with parameters list in Table 3.2. In particular, the figure shows the $\phi = 0$ cut of the gain of the mode 0 radiation pattern for the array based on rotated and summed copies of the unit cell radiation patterns. Because of the large number of elements, the pattern has little variation as $\phi$ changes, so only a single cut is shown. While at most frequencies the array has gain at the horizon, there are frequencies at which there are large reductions in gain.

To understand the operation of the array at these frequencies, consider the current distribution along the dipoles in a unit cell in free space at 3 GHz, as shown in Figure 3.9. From the figure, it can be seen that the current along the dipole is nearly constant. Therefore, the array can be approximated as a uniform ring of azimuthal current in the $xy$-plane. The sheet has uniform current density $\vec{J}_s = J_\phi \hat{u}_\phi$ for $a \leq \rho \leq b$, where the sheet has an inner radius of $a$ and an outer radius of $b$. From [66], the electric field of such a current distribution is $\vec{E} = j\omega \vec{A}$, where $\vec{A}$ is the magnetic vector potential and is given by

$$\vec{A} = \frac{\mu}{4\pi} \int \int_S \vec{J}_s \frac{e^{-jkR}}{R} ds'$$

In (3.1), $\mu$ is the permeability of the surrounding medium, $S$ is the surface on which the current is flowing, primed coordinates are the coordinates of each current filament on $S$, $k$ is the wave number of the surrounding medium, and $R$ is the distance from the observation point to each current element. Using the far-field approximations from [66], (3.1) can be approximated in cylindrical coordinates $(\rho, \phi, z)$ as

$$\vec{A} \approx \frac{\mu}{4\pi} \frac{e^{-jk\sqrt{\rho^2 + z^2}}}{\sqrt{\rho^2 + z^2}} \int \int_S \vec{J}_s e^{-jk\rho'\sin\theta\cos(\phi - \phi')} \rho' d\rho' d\phi'$$

By evaluating the integral in (3.2) and representing $\vec{A} = A_\phi \hat{u}_\phi + A_\theta \hat{u}_\theta$, it is found that $A_\theta = 0$ and

$$A_\phi = \frac{j\mu \pi J_\phi}{4k \sin \theta} \frac{e^{-jkr}}{r} [hF(kb \sin \theta) - aF(ka \sin \theta)]$$

where

$$F(x) = J_1(x) H_0(x) - J_0(x) H_1(x)$$

(3.4)
Figure 3.8: $E_\phi$ gain at $\phi = 0^\circ$ over various $\theta$ cuts of the array synthesized from the unit cell in Figure 3.2 using the parameters in Table 3.2. The array exhibits several dips in gain at the horizon due to the azimuthal currents that govern its operation.
Figure 3.9: Current distribution of the unit cell of a ring of tightly-coupled dipoles in free space driven in mode 0 at (a) 1 GHz, (b) 1.75 GHz, and (c) 2.5 GHz. The current is uniform in the azimuthal direction (horizontal in the figure) except at the feed at the center of the element, justifying an approximation of the array as a uniform sheet of current in the azimuthal direction.
\( J_n(x) \) are Bessel functions of the first kind of order \( n \) and \( H_n(x) \) are Struve functions \([67, \text{ Ch. 11}]\) of order \( n \).

Figure 3.10 shows the \( \phi \) component of the far-field radiation patterns for the tightly-coupled dipole array described by the unit cell in Figure 3.9 and an ideal azimuthal current sheet with \( a = 14.7 \) cm and \( b = 15.2 \) cm. The array is driven in mode 0 to create a uniform current around the entire ring of dipoles. It can be seen that the free-space array behaves nearly identically to the current sheet. Furthermore, the array has several pattern nulls at the horizon caused by the oscillation of the Bessel functions in (3.3) and (3.4). These nulls also correspond well to the minima in Figure 3.8, although the addition of the ground plane, reflector, and capacitive loads has shifted the frequencies slightly and has caused the asymmetry in the pattern around the horizon.

### 3.1.4 Unit Cell with Dielectric Slab

In Figures 3.8 and 3.10, it is interesting to note that at frequencies where the gain of the array dips, radiation is focused more upward. Therefore, one approach to redirect the radiation of the array is to focus the fields outward from the center. By doing so, the array’s radiation will be biased toward the horizon at all frequencies, resulting in improved and more consistent gain.

In [58], this problem is addressed by using a ring of director elements to focus the fields away from the array. While this method is indeed effective in improving horizon gain, the patches have very little effect on the operating bandwidth of the design. Instead, in this design, a dielectric slab ring is placed on top of the array and is extended outward from the dipoles. The use of the ring is motivated by the previous use of dielectric slab loading in tightly-coupled array designs [59]. The slab has the additional benefit of reducing the turn-on frequency of the array, allowing for an amount of miniaturization. As shown in Figure 3.11, The slab has height \( h_d \), inner radius \( r_{d0} \), outer radius \( r_{d1} \), and relative permittivity \( \epsilon_{rd} \). In addition, to make the design more practical, the dipoles are now printed on either side of a Rogers RO4350B substrate (\( \epsilon_r = 3.66 \)) with thickness \( h_s = 0.508 \) mm. Finally, an aluminum cylinder of radius \( r_f = 7.62 \) cm and height \( h_f = 5.08 \) cm is placed at the center of the array to house supporting circuitry for the application. The geometry
Figure 3.10: $\phi$ gain at $\phi = 0^\circ$ over various $\theta$ values of a uniform ring of azimuth current with inner radius $a = 14.7$ cm and $b = 15.2$ cm (ideal) and the array synthesized from a uniform array of 64 dipoles in free-space when driven in mode 0 (array). The response of the array matches that of the ideal current sheet nearly identically, providing insight into the cause of the horizon directivity dips in Figure 3.8.
Figure 3.11: Final unit cell (a) and zoomed-in top view (b) for the array of tightly-coupled dipoles. The dielectric slab placed over the overlapped dipoles focuses the fields outward from the array while also reducing the turn-on frequency of the array.
Table 3.3: Parameters for Tightly-Coupled UCA (Dielectric Slab)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>64</td>
<td>$g_c$</td>
<td>2.0 mm</td>
</tr>
<tr>
<td>$w_d$</td>
<td>4.4 mm</td>
<td>$w_c$</td>
<td>3.8 mm</td>
</tr>
<tr>
<td>$k_o$</td>
<td>0.5</td>
<td>$k_c$</td>
<td>0.9</td>
</tr>
<tr>
<td>$r_r$</td>
<td>58.4 mm</td>
<td>$h_g$</td>
<td>63.5 mm</td>
</tr>
<tr>
<td>$h_d$</td>
<td>12.7 mm</td>
<td>$r_{d0}$</td>
<td>141.0 mm</td>
</tr>
<tr>
<td>$r_{d1}$</td>
<td>196.3 mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

of the final unit cell is shown in Figure 3.11. In addition to the features discussed above, the unit cell also contains a printed tapered balun to transition from an unbalanced 50 Ω microstrip line to a balanced 64 Ω twin line.

In order to provide a reasonable reduction in size while still exhibiting low loss, the slab dielectric material is selected as Eccostock HIK with $\epsilon_{rd} = 7$ and $\tan \delta < 0.002$. Because the slab is directly interfaced with the dipole array, all three dimensions have a large effect on the overall operation of the array. Figures 3.12 – 3.14 show the performance of the array when driven in mode 0 over sweeps of $h_d$, $r_{d0}$, and $r_{d1}$, respectively. All other parameters take the values from Table 3.3. From these figures, it can be seen that $r_{d1}$ has the largest effect on the dips in gain at the horizon, and that increasing $r_{d1}$ decreases the gain ripple. Likewise, increasing $r_{d0}$ up to approximately 140 mm helps reduce gain variation at the second dip, with limited effect for larger values. However, increasing both $r_{d0}$ and $r_{d1}$ too much results in a reduction in the bandwidth of the array. $h_d$ is found to have little effect on the horizon gain; instead, the slab height controls the effect of the slab on the resonance from the capacitive load. As $h_d$ increases, the resonance is brought lower in frequency. From Figure 3.12, it can be seen that a height of 12.7 mm results in the widest operating bandwidth.

Using the results of the studies in Figures 3.4 – 3.6 and 3.12 – 3.14, the final unit cell parameters are selected, as given in Table 3.3. The input reflection coefficient is shown in Figure 3.7, and the radiation patterns of the array with this unit cell are shown in Figure 3.15. The simulation
Figure 3.12: (a) Mode 0 active reflection coefficient and (b) horizon φ gain of the unit cell in Figure 3.11 over slab height variations with \( r_{d0} = 141 \) mm, \( r_{d1} = 196.3 \) mm, and all other parameters as given in Table 3.3. The slab height greatly affects the input impedance of the array but has minimal effect on the radiation patterns. Selecting \( h_d = 12.7 \) mm gives the widest operating bandwidth.
Figure 3.13: (a) Mode 0 active reflection coefficient and (b) horizon φ gain of the unit cell in Figure 3.11 over slab inner radius variations with $h_d = 12.7$ mm, $r_{d1} = 196.3$ mm, and all other parameters as given in Table 3.3. Up to approximately 140 mm, increasing the inner radius decreases the second dip in directivity, and above 140 mm, the input match starts to degrade.
Figure 3.14: (a) Mode 0 active reflection coefficient and (b) horizon φ gain of the unit cell in Figure 3.11 over slab outer radius variations with $h_d = 12.7$ mm, $r_{d0} = 141$ mm, and all other parameters as given in Table 3.3. Increasing the outer radius reduces the variation in horizon directivity by focusing the fields outward more strongly. However, increasing the outer radius also starts to restrict the operating bandwidth, making $r_{d1} = 196.3$ mm a good compromise between bandwidth and horizon directivity.
Figure 3.15: Simulated mode 0 gain patterns of the unit cell in Figure 3.11 for $\phi = 0^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. The horizon gain of the array is consistent over frequency and highly symmetrical over all values of $\phi$. 
predicts a 10 dB return loss bandwidth of 0.73 GHz to 2.71 GHz, or 3.7:1. Furthermore, the simulated mode 0 horizon gain is consistent except for a small drop around 2.2 GHz. This drop does not reduce the gain below the low-frequency gain, so it is acceptable for the design.

3.2 Full Array Design

3.2.1 Feed Network

In order to realize the unit cell in Figure 3.11 as a practical array, the 64 dipole elements must be excited correctly with a mode 0 excitation. The array is intended to be used either as a single omnidirectional element or as a set of four sectoral antennas. Therefore, the feed network is designed to provide four inputs to the array, which can then be excited in phase to create a single omnidirectional pattern.

The feed for the array is designed as a four-level corporate feed with each level consisting of meandered 2:1 combiners that spread radially. A corporate feed is chosen over a direct 16:1 splitter due to the 63.5 mm maximum radial size constraint of the feed. Because most $N : 1$ combiners rely on structures that are roughly one quarter wavelength at the lowest frequency of operation and one quarter wavelength at 700 MHz is 107 mm, the corporate feed is easier to implement in this design.

The 16:1 combiner network is shown in Figure 3.16, and the design parameters are given in Table 3.4. In the figure, line widths are denoted as $w_i$, lengths as $l_i$, and resistances as $R_i$. Because the structure is symmetric, values for each level are only called out once. Lengths between chamfered bends are measured between the largest extent of each bend. After the final bend of each level is a 0.5 mm long 50 Ω segment used to prevent overlap between levels. The design is simulated in a unit cell similar to Figures 3.2 and 3.11 above, except that the cell now covers 90° in azimuth. To simplify fabrication, the combiner is built on the same substrate (0.5 mm thick RO4350B) as the antenna elements. The splitters in each level are based off two-stage Wilkinson combiners, with the widths, lengths, and resistances tuned to provide a good match and isolation over the operating bandwidth of the unit cell and to account for the imperfect input impedance of
Figure 3.16: 16:1 combiner with 50 Ω coaxial input and 50 Ω microstrip outputs. The combiner is designed as a four-layer corporate feed with each layer derived from a two-stage Wilkinson.
the unit cell. The input of the combiner is implemented as a 50 Ω, 2.2 mm outer diameter coax to 50 Ω microstrip transition, and the final outputs are all 50 Ω microstrip to interface with the baluns of the array elements.

The performance of the combiner is presented in Figure 3.17, which shows the match and isolation with 50 Ω ports at the outputs as well as the match with each port terminated with the unit cell antenna of Figure 3.11. The change in performance with the unit cells loading the combiner underscores the importance of designing the combiner as part of the system and not as
Figure 3.17: Simulated performance of 16:1 combiner feed with antenna array elements attached (blue) and loaded with 50 Ω at all ports. The combiner exhibits better than 10 dB return loss and isolation and less than 1.3 dB of insertion loss (beyond the ideal splitter loss) across the operating bandwidth of the array.
a 50 Ω component. That being said, the combiner is able to operate over the bandwidth of the antenna elements, so the design successfully meets the needs of the array. Furthermore, the design is symmetric, so any amplitude and phase imbalances will result from manufacturing tolerances, which are expected to have a minimal effect on the operation of the combiner.

3.2.2 Fabrication and Performance

Using the feed network and unit cell designs from the previous sections, the 64-element array is fabricated, as can be seen in Figure 3.1. The dipoles are printed on opposite sides of a 0.5 mm thick Rogers RO4350B with an outer diameter of 39.3 cm (1.02λ at 0.78 GHz). The dielectric slab ring is implemented as 8 machined pieces of Eccostock HIK dielectric stock with a dielectric constant of 7 and loss tangent less than 0.002. Each section is secured with Nylon screws. The ring is fabricated in sections to allow full utilization of the 30 cm square sheets of the material. Finally, the array is supported by C-STOCK RH-5 foam, which has a dielectric constant of 1.09 and a loss tangent of 0.0004, and by the aluminum cylinder in the center, which also serves as a guide for the four 2.2 mm diameter coaxial cables that feed the array sectors. Finally, the array is placed over a circular ground plane with a 44.3 cm (1.15λ at 0.78 GHz) diameter.

The array is simulated in Ansys HFSS and measured in a spherical near field chamber. The simulated and measured performance of the array are shown in Figures 3.18 – 3.23. From Figure 3.18, it can be seen that the simulated and measured $S_{11}$ of the array match quite well, both for a single sector and for a mode 0 excitation. Furthermore, the coupling between adjacent sectors is insignificant, so variations in the impedance due to excitation variations are minimal. The antenna is well-matched with measured $|S_{11}| < -10$ dB from 0.78 GHz to 2.69 GHz except for a few spikes around 1 GHz, giving an operational bandwidth of 3.45:1.

The measured and simulated horizon and maximum gains of the array are shown in Figure 3.19. The array exhibits excellent agreement between simulation and measurement. Furthermore, the gain of each sector is above 0 dBi for frequencies above 0.9 GHz and the cross-polarized (θ) gain at the horizon is below -15 dBi over the entire bandwidth. The maximum gain of the
Figure 3.18: Measured and simulated sector and active mode 0 $|S_{11}|$ for the full 64-element tightly-coupled array. Low sector coupling results in little variation between the sector and mode 0 reflections. The array operates with $|S_{11}| < -10$ dB between 0.78 GHz and 2.69 GHz, resulting in a 3.45:1 bandwidth.
Figure 3.19: Measured and simulated horizon and maximum gains of a single sector and mode 0 excitations of the array shown in Figure 3.1. Simulation and measurement match very well, and the array exhibits stable gain over frequency.
sector is also within 5 dB of the horizon gain despite the presence of the ground plane. In mode 0, the horizon gain is relatively stable at approximately -5 dBi, with the maximum gain around 5 dBi. Again, the cross-polarization gain is less than -15 dBi over the entire bandwidth of the antenna.

Figures 3.20 and 3.21 show the measured and simulated normalized far-field gain patterns of a single sector of the array. The sectors have consistent gain at the horizon. Furthermore, the radiation patterns maintain a consistent shape over the operating band of the antenna. Similarly, the array’s mode 0 radiation patterns are shown in Figures 3.22 and 3.23. Due to the large number of elements, the patterns are symmetric around $\phi$ and exhibit very low wobble on wave. Again, the radiation patterns are consistent over the operating band except at the highest frequency. The scalloping in the pattern occurs in both simulation and measurement and is likely due to the Bessel-like elevation response of the uniform current sheet established by the antenna in mode 0. Regardless, the scalloping does not affect operation at the horizon, as shown in Figure 3.19.

### 3.3 Conclusion

A 64-element circular array of tightly-coupled horizontal dipoles is demonstrated. The array exhibits a good input match and consistent sectoral and mode 0 radiation patterns over a 3.45:1 bandwidth despite the radiating elements being only 0.15\(\lambda\) away from a ground plane. The electrically-close ground plane does not degrade performance; rather, the array takes advantage of the extra inductance from the ground plane, as well as a central reflector and a capacitive load, to resonate with the impedance of the coupled radiating elements, resulting in almost two octaves of operating bandwidth. The feed network of the array consists of four 16:1 combiners, which allows the antenna to be used as a set of four sectoral antennas or as a single omnidirectional element. In addition to providing radiation in different directions, the sectoral patterns could also be used to sense the direction of arrival of incoming signals, which could be useful in some sensing applications.
Figure 3.20: Measured (solid) and simulated (dashed) sector radiation patterns over $\theta$ with $\phi = 0^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The sector patterns are consistent over frequency, and the cross-polarization ratio is better than 15 dB over almost the entire bandwidth of the array.
Figure 3.21: Measured (solid) and simulated (dashed) sector radiation patterns over $\phi$ with $\theta = 90^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The sector patterns are consistent over frequency, and the cross-polarization ratio is better than 15 dB over almost the entire bandwidth of the array.
Figure 3.22: Measured (solid) and simulated (dashed) mode 0 radiation patterns over $\theta$ with $\phi = 0^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The measurements match the simulated radiation patterns in Figure 3.15 very closely. As with the sector patterns, the cross-polarization ratio is greater than 15 dB over most of the upper hemisphere, except for the nulls in the co-polarized pattern.
Figure 3.23: Measured (solid) and simulated (dashed) sector radiation patterns over $\phi$ with $\theta = 90^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The array maintains a high degree of symmetry over all azimuth angles. As with the sector patterns, the cross-polarization ratio is greater than 15 dB over most of the upper hemisphere, except for the nulls in the co-polarized pattern.
Chapter 4

Wideband Dual-Polarized Array for STAR and DOA Estimation

As discussed in Chapter 3, spectrum monitoring, among many other applications, requires the use of dual-polarized sensors in order to ensure that the sensor is not blind to any signals [68]. It is also important that the antenna has a full 360° field of view in azimuth, sufficient field of view in elevation, and wide bandwidth so that it can monitor relevant signals in an area. Furthermore, in some situations, merely detecting signals in an area is not sufficient; often, it is necessary to determine the direction of arrival (DOA) of those signals [69]. Sensors also require high sensitivity in order to detect interferers in the desired area of operation that may not be easy to detect at the monitoring station [70]. Finally, it is common for sensors to be mounted in a variety of locations, many of which may act as a ground plane for the antenna. Therefore, it is useful to be able to mount the sensors to a ground plane to allow for operators to mount them in more locations.

There are several examples in the literature of antennas that can meet many, but not all, of these criteria. For example, [71] presents a dual-polarized omnidirectional antenna in which both polarizations operate from 1.53 to 2.95 GHz, giving a 1.93:1 bandwidth. Likewise, [72] describes a multiband, dual-polarized antenna covering 0.69 to 1.03 GHz and 1.69 to 3.21 GHz. However, both of these antenna utilize only a single feed for each polarization, making it impossible to estimate the DOA of incident signals. On the other hand, systems such as in [73], [74], and [75] utilize circular arrays of wideband elements to achieve wide bandwidths and multiple antenna outputs for direction finding. Unfortunately, all of these elements are designed to be mounted on a mast elevated above a ground, making them difficult to use in some situations.
The above requirements make the design of a spectrum management sensor challenging. However, another challenge for system designers is that such a system will likely operate in conjunction with other wireless systems, some of which may need to transmit while monitoring is occurring. Because that system likely needs to utilize the same spectrum that is being monitored, one cannot use traditional frequency-based duplexing methods such as filters or diplexers. Likewise, duplexing in time would limit the effectiveness of the sensor, as it would create dead times during which no monitoring occurs. Therefore, an ideal sensor would provide sufficient isolation to allow for Simultaneous Transmit and Receive (STAR) operation.

STAR, or in-band, full-duplex, systems have recently become a focus area of research. In a STAR system, the transmitter and receiver can be active at the same time on the same frequency channel, and the self interference generated by the transmitter is mitigated so that it does not affect the sensitivity of the receiver [76]. Generally, designs utilize multiple layers of cancellation [25][77], as the power handling and cancellation floor varies for each type of canceler. In particular, systems utilize an antenna or propagation layer, an RF/analog layer, and a digital cancellation layer to achieve the necessary isolation, which can easily exceed 90-100 dB [25].

Antenna designs for STAR generally fall into two categories: monostatic configuration and bistatic configurations. In monostatic STAR, isolation is achieved between two or more antenna elements that share a common aperture [78]. Traditionally, this isolation was obtained using circulators. However, more recent designs have used either antenna properties [79] or polarization [80] to provide transmit-receive isolation of 40 dB and 50 dB, respectively. Other designs use a modeforming circuit and symmetry to provide wideband isolation [81]. On the other hand, bistatic STAR systems take advantage of the spacing between antenna elements to isolate the transmitter and receiver. In many situations, the separation does not provide sufficient isolation, so additional techniques, often involving high-impedance surfaces, are utilized [82].

The antenna array discussed in this chapter uses a combination of 64 tightly-coupled dipoles combined to four sectoral outputs, eight tapered slot antennas combined to four outputs, and a monocone antenna, as seen in Figure 4.1, to implement a wideband, dual-polarized antenna system
Figure 4.1: Fabricated (a) and modeled (b) dual-polarized array for direction of arrival sensing. The array consists of a tightly-coupled array of 64 horizontal dipoles combined to four ports, eight tapered slot antennas, and a monocone. Pairs of adjacent tapered slots are combined to provide high isolation from the cone.
that is capable of supporting STAR operation and providing high accuracy DOA estimates. The antenna uses the inherent symmetry of the design to cancel the self-interference generated by the transmit monocone in the receive co-polarized TSAs, with measured isolation greater than 40 dB over a 3.41:1 bandwidth. Likewise, the tightly-coupled dipole sectors are isolated by over 26.5 dB from the monocone due to polarization multiplexing. Finally, the eight receive elements provide the spatial information and radiation patterns to provide precise DOA estimates, as characterized through the Cramér-Rao Lower Bound (CRLB) [30]. To the knowledge of the author, this is the first implementation of a system that can perform STAR and DOA estimation in the open literature.

The chapter is organized as follows. Section 4.1 discusses the design of the array elements. Because the TCDA is described in detail in Chapter 3, attention is mainly given to the monocone and TSA elements. Section 4.2 examines the isolation of the monocone from the other array elements and describes the scheme used to decrease coupling. Finally, Section 4.3 presents the measured performance of the array, including S-parameters, radiation patterns, and DOA estimation error.

4.1 Element Design

The dual-polarized array discussed in this chapter is shown in Figure 4.1. The array consists of three types of elements: tightly-coupled dipoles designed to receive horizontally-polarized radiation, tapered slots designed to receive vertically-polarized radiation, and a monocone to transmit vertically-polarized waves, all placed over a circular ground plane. The tightly-coupled dipole array (TCDA) consists of 64 overlapped dipole elements that are combined to four outputs, as described in Chapter 3. The tapered slot antennas (TSAs) transition the incoming wave from the free space impedance to the impedance of the receiver, which is taken to be 50 Ω in this design. The TSAs take advantage of image theory over the ground plane to allow for use of only half of the slot, allowing them to fit under the TCDA. Finally, the monocone is designed to use the TCDA as its primary ground plane. As described further in Sections 4.1.2 and 4.2, the feed spokes of the TCDA act as a sufficient ground for the monocone, allowing it to approximate a bicone sufficiently and to
transmit without coupling significantly to the TSAs.

4.1.1 Tapered Slots

The TSAs in the array use an exponentially tapered slot to transition the impedance of the received wave from free space to the antenna feed impedance [83]. The taper is defined in the $xz$-plane, as shown in Figure 4.2, as

$$z = w_f e^{R_t x}$$

where $0 \leq x \leq L_t$, $L_t$ is the length of the taper, $R_t = (1/L_t) \ln(w_t/w_f)$ is the growth rate of the taper, $w_t$ is the end width of the taper, and $w_f$ is the width of the taper at the feed. Table 4.1 shows the values of these parameters as well as the others shown in Figure 4.2. The antenna is printed on a single side of a 1.52 mm thick Rogers RO4350B substrate with $\epsilon_r = 3.66$.

The slot is terminated with a wideband open circuit, implemented as a semi-circular slot with a center at $x = -x_s$ and a radius of $R_s$. The open circuit improves the match at higher frequencies and also makes the antenna physically larger, which helps extend the lower frequency of operation. The effect of the termination on the input reflection coefficient of the TSA over an infinite ground plane, as simulated in Ansys HFSS [29], can be seen in Figure 4.3. The bandwidth of the TSA covers 1.10 to 2.55 GHz with no open termination and extends to 0.86 GHz to >10 GHz with the termination. Note that the match and radiation patterns of the TSA are greatly affected by the other elements in the full array of Figure 4.1, so further performance data on the TSAs is not presented independent of the other elements.

The TSA is fed with a 2.18 mm outer diameter 50 Ω coaxial probe. The TSA element feeds alternate between $0^\circ$ and $180^\circ$ configurations, as shown in Figure 4.4. In half of the elements, the probe lies below the antenna with the center conductor of the probe soldered to the top of the slot. In the other half, the coaxial cable wraps around the wideband short and has its outer conductor soldered to the top of the slot and its inner conductor soldered to ground. More details on the rationale for and effects of this feed configuration are discussed in Section 4.2.
Figure 4.2: Model of the TSA with all dimensional parameters labeled. The design takes advantage of image theory over a ground plane to reduce the height of the antenna element and uses a wideband open circuit to extend its bandwidth.
Figure 4.3: Input reflection coefficient of the TSA with parameters in Table 4.1 with and without a wideband open circuit matching slot placed behind the feed. The slot generally improves the match of the TSA and extends both the lower and upper frequency range of the antenna.
Figure 4.4: Feed configurations for the TSAs in the array of Figure 4.1. The elements alternate between two types of feeds using a coaxial cable. In the first configuration (a), the inner conductor of the coax is attached to the TSA, while the outer conductor is attached to the ground plane. In the second configuration (b), the outer conductor of the coax is bent around the wideband open termination of the TSA and the center conductor is shorted to the ground plane. By using these two types of feeds, adjacent elements are fed 180° out of phase, which is used to improve the isolation between the monocone and the TSAs.
Table 4.1: Parameters for TSA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$L_a$</td>
<td>63.5 mm</td>
<td>$L_b$</td>
<td>1.3 mm</td>
</tr>
<tr>
<td>$L_s$</td>
<td>89.7 mm</td>
<td>$L_t$</td>
<td>53.3 mm</td>
</tr>
<tr>
<td>$R_s$</td>
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<td>$R_t$</td>
<td>76.8 m$^{-1}$</td>
</tr>
<tr>
<td>$x_s$</td>
<td>17.8 mm</td>
<td>$w_b$</td>
<td>38.1 mm</td>
</tr>
<tr>
<td>$w_f$</td>
<td>0.6 mm</td>
<td>$w_s$</td>
<td>61.0 mm</td>
</tr>
<tr>
<td>$w_t$</td>
<td>38.1 mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.1.2 Central Cone

The central element in the array is intended to act as an omni-directional, vertically polarized transmitter. In order to meet the bandwidth and symmetry requirements of the system, conic structures have been investigated for this element. In particular, top-loaded monocones and bicones are studied. The geometry of the two conic structures is shown in Figures 4.5 and 4.6. The two cones are closely related, as the monocone is simply the upper half of the bicone.

The major trade-off between the use of a monocone or a bicone is size versus peak gain. As shown in Figure 4.7b, both cones exhibit similar gains; however, the bicone does give approximately 4 dB higher gain than the monocone at the horizon but 2 dB less maximum gain at the cost of being twice as tall. The cones compared in the figure both use the parameters in Table 4.2 and are simulated in Ansys HFSS in a simplified model of the array in which the feed network for the TCDA is removed, as shown in Figures 4.5 and 4.6. Furthermore, the extra height of the bicone does not result in a reduction of the turn-on frequency of the antenna, as can be seen in Figure 4.7a. While the bicone has not been optimized for maximum impedance bandwidth, it is clear that the extra height is only contributing to the gain of the antenna and not to the match. Finally, the coupling between each cone to the TSAs beneath them is comparable. Because one of the largest challenges in this array is isolation between the cone and the other elements, the comparable coupling between the each cone and the TSAs suggests that the extra gain of the bicone does not justify doubling
Figure 4.5: Simplified monocone model used for performance comparison with a bicone. The monocone uses the TCDA without the combiner network as its ground and is placed over 8 TSAs, a metal cylinder, and a ground plane.
Figure 4.6: Simplified bicone model used for performance comparison with a monocone. The bicone is placed over 8 TSAs, a metal cylinder, a ground plane, and the TCDA without its combining network.
the height of the element. Therefore, the design uses the top-loaded monopole, as can be seen in Figure 4.1.

While the design of miniaturized monocones [84][85] and bicones [86] has been studied thoroughly in the literature, the integration of a cone into the array of Figure 4.1 requires careful consideration. In the array, the cone operates over effectively two ground planes. The first is created by the spokes and dipoles of the TCDA, and the second by the supporting ground plane under the TCDA and TSAs. To understand how the TCDA operates as a ground for the cone, consider a vertically-polarized (i.e. \( \vec{E} = E_z \hat{z} \)) wave traveling radially outward in the \( \hat{\rho} \) direction. For such a wave, the magnetic field is in the \( \hat{\rho} \times \hat{z} = -\hat{\theta} \) direction. From Maxwell’s equations, the supporting surface currents along the ground plane are determined by the boundary condition \( \vec{J} = \hat{n} \times \vec{H} \), where \( \hat{n} \) is the surface normal. Therefore, the surface currents on the ground plane are in the \( \hat{z} \times -\hat{\theta} = \hat{\rho} \) direction. Since the spokes of the TCDA run in the same direction and allow this current to flow freely, they effectively act as a ground plane for the cone, except that the spacing between spokes causes the surface to be slightly inductive.

The effect of the two grounds on the operation of the cone can be seen in Figure 4.8. The figure shows the input impedance of a monocone with a single PEC ground with a radius of 158.2 mm, two PEC grounds 63.75 mm apart, and the simplified model in Figure 4.5 without the cylinder or TSAs. The dimensions of the monocone are given in Table 4.2. In all cases, the bottom ground has a radius of 216.7 mm, and the top ground has a radius of 158.2 mm, which matches the radius of the TCDA. With a single ground, the cone is well-matched for frequencies above 0.71 GHz. However, the inclusion of either the second ground or the TCDA introduces a resonance at 0.86 GHz.

<table>
<thead>
<tr>
<th>Table 4.2: Parameters for Monocone</th>
</tr>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>( h_c )</td>
</tr>
<tr>
<td>( h_t )</td>
</tr>
<tr>
<td>( R_u )</td>
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</tbody>
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Figure 4.7: (a) S-parameters and (b) gain of the monocone and bicone in Figures 4.5 and 4.6, respectively. Both cones couple equally to the TSAs, and the bicone exhibits $\sim 4$ dB higher gain at the horizon but $\sim 2$ dB less maximum gain at low frequencies.
GHz that degrades the input match of the cone. However, introduction of the cylinder from the TCDA design prevents this resonance from forming, resulting in a turn-on frequency of 0.8 GHz. Finally, inclusion of the TSAs underneath the TCDA further reduces the turn-on frequency to 0.73 GHz. It appears that the scattering off of the TSAs makes the monocone see an electrically larger ground plane, which improves the low-frequency performance.

The final trace in Figure 4.8 shows the performance of the final monocone design with mounting features in the simplified array. The cone has 16 equally-spaced holes that start 37.6 mm from the base of the cone, each with a radius of 3 mm and a depth of 6.75 mm. These holes allow the cone to be supported by Nylon standoffs, allowing for simple integration of the cone to the TCDA. Furthermore, as seen from the figure, the holes and standoffs improve the performance of the cone above 2 GHz by reducing the input reflection coefficient of the antenna.

4.2 Isolation

The array in Figure 4.1 contains 4 TCDA sectors, 8 TSAs, and a monocone. Considering that all of these elements fit within a 19.6 cm (0.46λ at 700MHz) radius cylinder with a total height of 12.7 cm (0.3λ at 700MHz), significant coupling between elements is expected. Because the monocone will act as the transmit antenna, the relevant coupling is between the monocone and all other elements. Coupling between the receive TSAs and TCDA sectors is less important, as that coupling can be compensated for via calibration.

4.2.1 Monocone-TSA Isolation

As seen in Figure 4.7a, the monocone is only isolated from the receive TSAs by approximately 20 to 25 dB below 1.5 GHz. Efforts to reduce this coupling are complicated by the desire for wide bandwidth (~4:1), high transmit efficiency, and positive maximum gain in the TSA elements to provide high receive sensitivity. Many methods considered in traditional bistatic STAR systems, such as corrugations [87], high-impedance surfaces [82] and traps [88] are limited by the bandwidth or are less effective when antennas are placed closely together, and use of absorber [89] requires
Figure 4.8: $S_{11}$ of a monocone with parameters given in Table 4.2 over various ground configurations. In the "Single Ground" case, the cone is placed over a circular ground of radius 158.2 mm. In all other configurations, an additional ground of radius 216.7 mm is placed 63.5 mm below the cone and top ground. In the full array configuration, the interaction between the cone and the other antenna elements improves the match of the cone so that it operates well between 0.73 GHz and above 3 GHz.
reducing the efficiency too low for the cone to be an effective transmitter (e.g. much less than 70%).

Therefore, the design of this antenna is approached more like a monostatic STAR system, even though the phase centers of each element are separated in space. Because the system is circularly symmetric, the coupling between the cone and each TSA is the same (assuming that each TSA is fed the same). This symmetry means that if adjacent TSA received voltages are subtracted from each other, the coupling between that combined output and the cone can be theoretically eliminated. In practice, fabrication differences, differences between the two types of feeds, and amplitude and phase imbalance between the TSAs will limit the isolation; however, as shown in Section 4.3, significant measured improvement in the isolation can be obtained using this method.

The simulated isolation between the monocone and the TSAs in the array is shown in Figure 4.9. Simulations are again performed in Ansys HFSS. Coupling to both types of TSA feeds is captured, as well as the coupling when adjacent TSAs are combined in-phase (i.e. added together). Elements are combined in-phase due to the inherent 180° phase shift between adjacent TSA feeds. Because the system is symmetric, the isolation should ideally be infinite; however, the simulated value is limited to approximately 50 dB due to asymmetries in the mesh in HFSS. Regardless, combining adjacent elements results in an improvement of up to 30 dB in the isolation between the monocone and the TSAs.

Combining adjacent elements does not come without a cost. For two elements with free-space radiation patterns of \( \vec{g}(\theta, \phi) \) placed at a radius \( R \) from the center of the array and placed with an azimuth offset of \( \phi = \phi_0 \), the electric field resulting from their subtraction is

\[
\vec{E}(\theta, \phi) = \vec{g}(\theta, \phi)e^{-jkR \sin \theta \cos \phi} - \vec{g}(\theta, \phi - \phi_0)e^{-jkR \sin \theta \cos(\phi - \phi_0)}
\] (4.2)

where \( k \) is the free-space wave number. Therefore, if \( \vec{g}(\theta, \phi) \) has even symmetry with \( \phi \), then at \( \phi = \phi_0/2 \),

\[
\vec{E}(\theta, \phi_0/2) = \vec{g}(\theta, \phi_0/2)e^{-jkR \sin \theta \cos(\phi_0/2)} - \vec{g}(\theta, -\phi_0/2)e^{-jkR \sin \theta \cos(-\phi_0/2)} = 0
\] (4.3)

This null along \( \phi = \phi_0/2 \) can be seen in Figure 4.10, which shows the Ansys HFSS simulated radiation patterns of the TSAs in the array of Figure 4.1b. However, the patterns are wide enough
Figure 4.9: Simulated coupling between the transmit monocone and the TCDA sector and TSA elements in the array. Coupling is shown from the monocone to the following elements: a TCDA sector, a TCDA sector when the TCDA is flipped over, a TSA element fed like in Figure 4.4a, a TSA element fed like in Figure 4.4b, and a set of combined TSA elements. The monocone couples strongly to the TSA elements due to their proximity and shared polarization. Coupling to adjacent TSAs is equal in magnitude and out of phase, allowing for significant cancellation of the coupled power from the monocone. Coupling to the TCDA sector is mostly because the monocone is using the TCDA combiner as its ground. Flipping the TCDA over so that the ground of the combiner primarily acts as the ground of the monocone significantly improves isolation.
that an incoming signal can be received sufficiently well by the surrounding TSA elements for DOA estimation, as will be shown in Section 4.3.2.

Due to the symmetry in the system, theoretically any of the TSA elements could be subtracted from any other in the array to achieve high isolation from the cone. However, as shown in Figure 4.11, the larger spacing between elements causes additional rippling in the radiation patterns. As discussed further in Section 4.3.2, this rippling does not necessarily mean that the array would perform worse from a DOA estimation perspective, as long as the rippling does not cause ambiguities in the antenna manifold, which is not explored here.

### 4.2.2 Monocone-TCDA Sector Isolation

At first glance, the use of the TCDA as the ground for the monocone might appear to be a poor choice for a system that requires high transmit to receive isolation. As discussed above, currents for the cone will travel radially outward towards the spokes in the TCDA. However, because the TCDA is cross-polarized to the monocone, very little coupling occurs, as seen in Figure 4.9.

One method of reducing the coupling further would be to flip the TCDA over so that the ground for the combiner network acts as the ground for the monocone. Figure 4.9 shows that doing so results in an improvement of 10 dB or more in the isolation between the TCDA and the monocone. Furthermore, looking closer at the isolation in both cases, it can be seen that the coupling appears to fall into two distinct regions. At low frequencies, the standard and flipped TCDA configurations both exhibit the same coupling. Only at frequencies above approximately 1.5 GHz does the flipped configuration begin to give an advantage. This observation indicates that at lower frequencies, coupling primarily occurs through the coupled dipoles and not through the combiner network, as the top and bottom of the TCDA are only symmetric after the baluns in the spokes complete their transition to the balanced feed of the dipoles. This also means that a majority of the ground currents for the monopole are contained within a circle of radius $0.8\lambda$, as the wavelength at 1.5 GHz is 20 cm and the array has a radius of 15.8 cm.
Figure 4.10: Simulated gain patterns of combined adjacent TSAs over $\phi$ at $\theta = 90^\circ$ at (a) 0.7 GHz, (b) 1.1 GHz, (c) 1.5 GHz, (d) 1.9 GHz, (e) 2.3 GHz, and (f) 2.7 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The elements are oriented so that one lies along $\phi = 315^\circ$ and the other lies along $\phi = 0^\circ$. Combining adjacent elements, which are fed $180^\circ$ out of phase, reduces coupling with the monocone but creates a null between the two elements at $\phi = 337.5^\circ$. 
Figure 4.11: Simulated gain patterns of combined TSAs over $\phi$ at $\theta = 90^\circ$ at 1.9 GHz when the TSAs are (a) adjacent and separated by (b) one, (c) two, and (d) three other TSAs. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. One element lies along $\phi = 315^\circ$, and the other lies at $\phi = 0^\circ$, $\phi = 45^\circ$, $\phi = 90^\circ$, and $\phi = 135^\circ$ for (a), (b), (c), and (d), respectively. Due to the feed structure, (a) and (c) are generated by adding the TSA signals, whereas (b) and (d) are generated by subtracting the two TSAs. Combining TSAs that are further apart from each other results in more rippling in the radiation patterns.
4.3 Fabrication and Measurement

The full dual-polarized array is fabricated using the design information in Chapter 3 and the above sections. The completed antenna can be seen in Figure 4.1a. The monocone and TSA elements are added to the existing TCDA from Chapter 3, which is not modified. In particular, while flipping the TCDA shows promising results in simulation, it is not possible to implement with the existing manufactured parts. The monocone is machined out of 6061-T6 aluminum and is supported by 8 Nylon standoffs. The top of the cone is manufactured separately so that the standoffs can be attached more easily to the cone. The feed for the monocone consists of a 2.2 mm diameter, 50 Ω coaxial cable that is fed through a hole in the center of the TCDA, with the inner conductor attached to the cone with conductive epoxy and the outer conductor attached to the ground of the TCDA combiner.

The TSAs are fabricated on 1.5 mm Rogers RO4350B and are secured with a 3D printed support structure. The support structure is printed on a MakerBot Replicator with PLA filament. As discussed above, the TSAs are fed with alternating phase between elements. Each feed is implemented with a 2.2 mm diameter, 50 Ω coaxial cable that is fed through a hole in the ground plane. For half of the TSAs, the inner conductor is soldered to the TSA and the outer conductor is attached to the ground with conductive epoxy; for the other half, the outer conductor is soldered along the wideband open termination and the inner conductor is epoxied to the ground plane. In both cases, it is important to epoxy the end of the TSA to the ground plane to ensure connectivity of the termination. Any gap between the TSA and the ground plane will detune the element.

Figures 4.12 and 4.13 compare the measured performance of the antenna with simulations in Ansys HFSS. Only a single TCDA sector and TSA pair are shown because all elements have comparable measurements. Both figures show excellent agreement between simulation and measurement. Furthermore, the TCDA sectors have a return loss better than 10 dB from 0.78 to 2.66 GHz except for a few small spikes around 1 GHz. The monocone is well-matched with $|S_{11}| < -10$ dB from 0.7 to 2.93 GHz, and the TSAs operate for frequencies greater than 0.73 GHz. Therefore,
Figure 4.12: Measured and simulated S-parameters of a TCDA sector and the monocone in the fabricated array of Figure 4.1a. The TCDA sector has $|S_{11} < -10|$ dB between 0.78 and 2.66 GHz, while the cone is well-matched from 0.7 to 2.93 GHz. Over the operating bandwidth of the array, the cone to TCDA sector coupling is less than -26.5 dB.
Figure 4.13: Measured and simulated S-parameters of a set of TSAs and the monocone in the fabricated array of Figure 4.1a. The (F) traces represent the S-parameters of the TSA with the flipped feed in 4.4b, and the (C) traces represent those of combined adjacent TSAs. The TSAs have $|S_{11}| < -10$ dB for frequencies above 0.73 GHz. Over the operating bandwidth of the array, the cone to combined TSA coupling is less than -40 dB.
the array can operate over the entire bandwidth of the TCDA, giving a 3.41:1 bandwidth. Over this bandwidth, the isolation between the monocone and the TCDA is greater than 26.5 dB, and the isolation between the cone and combined TSAs is greater than 40 dB.

4.3.1 Radiation Patterns

Measured radiation patterns for the array are obtained via spherical near field measurements in an anechoic chamber. As with S-parameter measurements, only the monocone, a single TCDA sector, and a single TSA are shown due to the similarity between elements. The gain of the monocone is shown in Figure 4.14. Overall, the measured and simulated gain match well, and the cone exhibits positive maximum gain and horizon gain better than -5 dBi. Normalized E-plane and H-plane cuts of the monocone are shown in Figures 4.15 and 4.16, respectively. The patterns show that the monocone acts very similarly to a standard monopole over a finite ground, with peak gain squinted up from the horizon and a null at zenith.

Figure 4.17 shows the measured and simulated gain of a TCDA sector, and the radiation patterns for the sector are displayed in Figures 4.18 and 4.19. Again, measurement and simulation show excellent agreement. The sector has at least a 10 dB front-back ratio, radiates with positive gain at the horizon over almost the entire operating bandwidth, and has cross-polarized gain more than 15 dB less than the co-polarized gain. In general, note that the sector performance matches very well with its standalone performance in Figures 3.19 – 3.21.

Finally, the gain of both a single TSA and a combined set of adjacent TSAs is shown in Figure 4.20. The single element has stable gain around 0 dBi at the horizon and approximately 5 dBi maximum gain over the bandwidth of the array. The combined element exhibits more gain variation than a single element but still radiates well at the horizon. The normalized radiation patterns of the single and combined TSA elements are shown in Figures 4.21 – 4.24. As expected, the combined elements have a null at -22.5°, which falls at the angular midpoint between the two elements. However, the radiation patterns are generally broad in azimuth and are focused at elevations near the horizon, making them effective as DOA sensors, as shown in the next section.
Figure 4.14: Measured and simulated gain of the monocone in the fabricated array of Figure 4.1a. The cone has gain around -4 dBi at the horizon but positive maximum gain, as the ground plane squints the beams upward slightly. Measured gain shows excellent agreement with simulation results.
Figure 4.15: Measured (solid) and simulated (dashed) monocone radiation patterns over $\theta$ with $\phi = 0^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The antenna radiates similarly to a classical monopole over a finite-sized ground plane.
Figure 4.16: Measured (solid) and simulated (dashed) monocone radiation patterns over $\phi$ with $\theta = 90^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The antenna radiates with a high degree of symmetry with $\phi$. 
Figure 4.17: Measured and simulated gain of a TCDA sector in the fabricated array of Figure 4.1a. The sector has positive gain at the horizon and over 5 dBi of maximum gain. The cross-polarized gain at the horizon is at least 15 dB less than the co-polarized gain.
Figure 4.18: Measured (solid) and simulated (dashed) TCDA sector radiation patterns over $\theta$ with $\phi = 0^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The radiation patterns match the standalone patterns of Figure 3.20 very closely.
Figure 4.19: Measured (solid) and simulated (dashed) TCDA sector radiation patterns over $\phi$ with $\theta = 90^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The radiation patterns match the standalone patterns of Figure 3.21 very closely.
Figure 4.20: Measured and simulated gain of a single TSA element (a) and of a combined set of TSAs (b) in the fabricated array of Figure 4.1a. The single TSA has flat gain of approximately 0 dBi of gain at the horizon and maximum gain of approximately 5 dBi. The combined TSA set has more variation at the horizon but exhibits similar maximum gain to the single TSA.
Figure 4.21: Measured (solid) and simulated (dashed) radiation patterns of a single TSA element over $\theta$ with $\phi = 0^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. At lower frequencies, the element focuses its radiation near the horizon, while at higher frequencies the patterns squint upwards slightly due to the influence of the ground plane.
Figure 4.22: Measured (solid) and simulated (dashed) radiation patterns of a single TSA element over $\phi$ with $\theta = 90^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. The broad H-plane patterns of the single element allow the combined TSA elements to cover all $\phi$ angles despite the null in the radiation pattern.
Figure 4.23: Measured (solid) and simulated (dashed) radiation patterns of a set of combined TSA elements over $\theta$ with $\phi = 0^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. Combining adjacent elements causes the patterns to be shaped strangely and for the cross-polarized radiation of the elements to be much stronger than for a single TSA. However, because the TCDA elements have clean patterns, the overall DOA error is not greatly affected.
Figure 4.24: Measured (solid) and simulated (dashed) radiation patterns of a set of combined TSA elements over $\phi$ with $\theta = 90^\circ$ at (a) 0.75 GHz, (b) 1.15 GHz, (c) 1.55 GHz, (d) 1.95 GHz, (e) 2.35 GHz, and (f) 2.75 GHz. $\phi$-polarized radiation is shown in blue while $\theta$-polarized radiation is shown in red. Combining adjacent elements in-phase causes a null at the angular midpoint between them due to the 180° phase difference between their feeds.
4.3.2 CRLB Results

While it is important to understand the electromagnetic performance of an antenna system, the ultimate goal for this antenna array is to be used to estimate the DOA of an incident signal. As in Chapter 2, the DOA performance of the array will be characterized using the CRLB given in (2.3). Because this array receives both $\theta$- and $\phi$-polarized waves equally well, the CRLB is evaluated for all polarization angles ($-90^\circ \leq \alpha \leq 90^\circ$ and $-90^\circ \leq \tau \leq 90^\circ$).

The CRLB results for the standard array configuration with adjacent TSA elements combined together to achieve isolation from the monocone are shown in Figure 4.25. The figure shows that the array is capable of estimating the DOA of an incident signal at the horizon with less than 1.3$^\circ$ of error in $\theta$ and 0.6$^\circ$ of error in $\phi$. Furthermore, the $\theta$ estimates improve as the signal moves towards zenith, and $\phi$ estimates have less than 1$^\circ$ of error until $\theta < 14^\circ$. At such low elevation angles, the azimuth is less critical to the overall knowledge of the signal’s direction, so this error does not significantly degrade the performance of the array. Therefore, the array is capable of operating as a dual-polarized DOA sensor over the entire upper hemisphere over a 3.41:1 bandwidth.

As discussed above, it is also possible to combine TSA elements that are not adjacent. While the radiation patterns from such configurations appear to be less effective for DOA estimation due to increased rippling in the patterns, the CRLBs for all possible configurations are calculated and presented in Figures 4.26 – 4.28. When TSAs are combined with one or two elements in between, the overall DOA performance is indeed degraded, especially in the case of two element separation. However, when the combined TSA elements are directly across from each other in the array, the DOA estimation error is comparable to when the combined elements are adjacent. Looking more closely at Figure 4.11, it can be seen that while the adjacent TSA patterns are certainly cleaner than when the elements are across from each other, the latter patterns are nearly omnidirectional at the horizon. This means that of the four sets of combined TSAs, the overall received signal level will be higher when the elements are across from each other than when they are adjacent.
Figure 4.25: Maximum CRLB for (a) $\theta$ and (b) $\phi$ estimates of the DOA of a single incident signal over all $\phi$ and all polarization angles $\alpha$ and $\tau$ when the combined TSA outputs are taken from adjacent TSAs. $\phi$ is swept from $0^\circ$ to $360^\circ$ in $5^\circ$ steps, $\alpha$ is swept from $-90^\circ$ to $90^\circ$ in $5^\circ$ steps, and $\tau$ is swept from $-90^\circ$ to $90^\circ$ in $5^\circ$ steps. The SNR of the signal is 10 dB and $P = 1000$ samples are used to estimate the covariance matrix. The array is capable of estimating $\theta$ and $\phi$ with less than $1.3^\circ$ of error over the entire upper hemisphere except near zenith, where the $\phi$ estimate is poor.
Figure 4.26: Maximum CRLB for (a) $\theta$ and (b) $\phi$ estimates of the DOA of a single incident signal over all $\phi$ and all polarization angles $\alpha$ and $\tau$ when the combined TSA outputs are taken from elements separated by one additional TSA. $\phi$ is swept from 0° to 360° in 5° steps, $\alpha$ is swept from -90° to 90° in 5° steps, and $\tau$ is swept from -90° to 90° in 5° steps. The SNR of the signal is 10 dB and $P = 1000$ samples are used to estimate the covariance matrix. Estimation accuracy is degraded in this configuration compared to when adjacent TSA elements are combined.
Figure 4.27: Maximum CRLB for (a) $\theta$ and (b) $\phi$ estimates of the DOA of a single incident signal over all $\phi$ and all polarization angles $\alpha$ and $\tau$ when the combined TSA outputs are taken from elements separated by two additional TSAs. $\phi$ is swept from $0^\circ$ to $360^\circ$ in $5^\circ$ steps, $\alpha$ is swept from $-90^\circ$ to $90^\circ$ in $5^\circ$ steps, and $\tau$ is swept from $-90^\circ$ to $90^\circ$ in $5^\circ$ steps. The SNR of the signal is 10 dB and $P = 1000$ samples are used to estimate the covariance matrix. This configuration gives the worst overall DOA estimates for the array compared to other TSA combination schemes.
Figure 4.28: Maximum CRLB for (a) $\theta$ and (b) $\phi$ estimates of the DOA of a single incident signal over all $\phi$ and all polarization angles $\alpha$ and $\tau$ when the combined TSA outputs are taken from elements across from each other in the array. $\phi$ is swept from $0^\circ$ to $360^\circ$ in $5^\circ$ steps, $\alpha$ is swept from $-90^\circ$ to $90^\circ$ in $5^\circ$ steps, and $\tau$ is swept from $-90^\circ$ to $90^\circ$ in $5^\circ$ steps. The SNR of the signal is 10 dB and $P = 1000$ samples are used to estimate the covariance matrix. Despite the extra rippling in the radiation patterns, this configuration meets or exceeds the performance of the array with adjacent TSA elements being combined.
4.4 Conclusion

An array that is capable of simultaneously transmitting while estimating the DOA of incident signals over a 3.41:1 bandwidth is presented. Isolation of $> 40$ dB is achieved between the transmit monocone and sets of combined TSA elements, and $26.5$ dB of isolation is attained between the cone and TCDA sectors. If the sectors are flipped over, simulation shows that this isolation can be improved by up to 10 dB, especially at higher frequencies. The estimation error of the array for an incident signal of arbitrary elliptical polarization is less than $1.3^\circ$ in $\theta$ and $1^\circ$ in $\phi$ over the entire upper hemisphere, except for an increase in $\phi$ error near zenith.
Conclusions and Future Work

5.1 Summary

In this thesis a new approach to the design of wideband, dual-polarized direction of arrival sensors for digital backends is developed. Instead of relying on traditional design metrics, the CRLB is used to design antennas for digital DOA sensors. Using this approach, two DOA sensors are designed: a four-arm MAW spiral capable of DOA estimation near boresight and a dual-polarized circular array with omnidirectional DOA coverage and an isolated transmit monocone. While neither antenna exhibits radiation patterns that would be expected to perform well for DOA estimation, both are found to produce accurate direction of arrival estimates over 4.6:1 and 3.41:1 bandwidths, respectively. Furthermore, the MAW spiral requires only four synchronized receivers to achieve this performance, as compared to six or eight for conventional MAW spiral designs. Reducing the number of receivers results in a significant reduction in overall system complexity, as synchronization and calibration become much more complicated as more receivers are added. Likewise, the omnidirectional array is able to perform DOA estimates while simultaneously transmitting from the monocone. To the knowledge of the author, this is the first antenna design that demonstrates combining these two capabilities over wide bandwidth and arbitrary signal polarization.

The DOA capabilities of four-arm MAW spirals in the context of digital processing backends are studied in detail. Through parametric simulations, it is found that tightly-wound MAW spirals with a small modulation ratio provide the best DOA performance over wide bandwidth. This is
because they preserve the polarization quality of modes 1 and 2 while still providing sufficient cross-polarized information from mode 3/-1. On the other hand, MAW spirals with a high modulation ratio provide clean modes 1 and -1 and a mode 2 with poor axial ratio, while conventional spirals exhibit modes 1, 2, and 3, resulting in almost no cross-polarized information. Through the use of the CRLB, it is found that the quality of mode 2 and the availability of some power in both polarizations for polarization estimation is critical for wideband DOA accuracy. The low modulation ratio MAW is capable of estimating the DOA of a linearly polarized incident signal with arbitrary linear slant angle with less than 1° of error over a ±30° elevation and 360° azimuth field of view from 1.2 to 5.55 GHz, giving it a 4.6:1 bandwidth. The high modulation ratio MAW and conventional spiral are limited to operation from 1.25 to 2.15 GHz for the former and 1.1 to 1.4 GHz and 1.6 to 2.3 GHz for the latter.

Omnidirectional sensing of horizontally-polarized signals with antennas placed over a large ground plane is a big challenge. When wide bandwidth and overall low electrical height above ground are required, the problem appears nearly unsolvable. In this thesis, a design of a 64-element tightly-coupled array of horizontal dipoles over a ground plane that tackles these challenges is introduced. The effects of the spacing to a ground plane, the distance between the dipoles and a reflector at the center of the array, and capacitive loads placed next to the dipoles are characterized for use in future designs. A wideband array design is achieved by spreading the resonances from these features out in frequency. The array is found to emulate a uniform current sheet, which allows for its wide bandwidth but also causes nulls in the radiation pattern at the horizon due to the Bessel-like variation of the electric field. The depths of these nulls are reduced considerably, along with the turn-on frequency of the antenna, through the use of a high dielectric constant slab ring over the dipole elements. With the slab, the array operates from 0.78 to 2.69 GHz, giving it a 3.45:1 bandwidth. The 64 elements are combined in-phase to four sectoral outputs. Each sector has at least -1 dBi co-polarized gain at the horizon, with 8 dBi maximum gain. If the sectors are combined in-phase, mode 0 is obtained with greater than -9 dBi gain at the horizon and 8 dBi maximum gain.
On their own, high accuracy, wideband, dual-polarized DOA sensors and high isolation, dual-polarized STAR systems present numerous challenges in their designs, especially in the presence of a ground plane. Furthermore, STAR-capable antennas exhibit radiation patterns that are usually not designed for DOA estimation. Despite these challenges, a wideband, dual-polarized array that is capable of operating as a high-accuracy DOA sensor while also transmitting is designed, fabricated, and measured. The array uses a 64-element tightly-coupled array in four sectors to sense horizontally-polarized waves, eight tapered slot antennas to sense vertically-polarized radiation, and a monocone to transmit. Because the entire structure is circularly symmetric, the monocone couples evenly to all of the tapered slots. Therefore, adjacent slots are subtracted from each other, resulting in significant cancellation of the coupling from the cone. On the other hand, the tightly-coupled array is used as the ground for the monocone, and the cross-polarization between the cone and the dipoles allows for isolation. If the array is flipped over so that the combiner network is better shielded from the monocone, additional isolation can be achieved. The array operates from 0.78 to 2.66 GHz and gives 40 dB of monocone-tapered slot isolation and 26.5 dB of monocone-dipole sector isolation over more than 3.4:1 bandwidth. As a DOA sensor, the array can estimate elevation with less than 1.3° of error over the entire upper hemisphere (0° ≤ θ ≤ 90°) and azimuth with less than 1° of error for 14° ≤ θ ≤ 90°. Near zenith, azimuth errors result in a small solid angle error, making azimuth estimation less important. Furthermore, different combinations of tapered slot antennas are shown to work well for DOA estimation, as subtracting opposite TSAs gives similar DOA performance as subtracting adjacent elements.

5.2 Original Contributions

The original contributions of this thesis are as follows:

- A digital receiver driven DOA antenna design approach was proposed and successfully demonstrated.

- The effect of MAW spiral design parameters (EXP and mod) on wideband DOA perfor-
mance is characterized. An understanding of the relationship between these parameters, the modes of the antenna, and DOA performance is developed. It is found that a tightly-wound MAW spiral with a low mod allows for the design of a wideband DOA antenna capable of sensing incident signals with arbitrary linear polarization.

- A tightly-wound low modulation ratio MAW spiral is found to be more suitable for digital DOA estimation than its conventional high modulation ratio counterpart. The antenna is found to be capable of estimating the DOA of a signal from 1.2 to 5.55 GHz with less than 1° of error over ±30° elevation and 360° azimuth, as characterized by the CRLB. The design is compared with a high modulation ratio MAW spiral and a conventional spiral, both of which are limited to less than a 2:1 bandwidth.

- A 64-element array of tightly-coupled horizontal dipoles over a ground plane is designed, fabricated, and measured. This design overcomes the inherent challenges of sensing horizontal polarization at the horizon near a ground plane. Despite being placed 1/6 of a wavelength from the ground plane at the lowest frequency, each of the sectors exhibits -1 dBi gain at the horizon over its matched bandwidth of 0.78 to 2.69 GHz. Achieving this gain over a 3:45:1 bandwidth is a significant improvement over the state of the art.

- Theoretical analysis is performed on the tightly-coupled array to understand nulls in the horizon gain when all elements are combined in phase as a mode 0 excitation. This theory is used to design a dielectric slab ring for the tightly-coupled dipole array that mitigates the horizons nulls for the array, resulting in a wideband array with consistent radiation patterns over its operating bandwidth.

- A wideband, dual-polarized circular array of a monocone, eight tapered slot antennas, and a 64-element tightly-coupled dipole array is designed, fabricated, and measured. The array operates from 0.78 to 2.66 GHz and provides less than 1.3° elevation error over the upper hemisphere and less than 1° of error for 14° ≤ θ ≤ 90° for a signal with any polarization.
This DOA performance is achieved while also demonstrating for the first time integration of DOA estimation and STAR operation in a single antenna system.

- Coupling between the monocone and all other array elements is analyzed. Isolation of 40 dB to the tapered slots is achieved by subtracting adjacent slots from each other, while 26.5 dB of isolation to the tightly-coupled dipole sectors is shown despite using the dipole array as the ground of the monocone. The isolation scheme is designed to provide high isolation while also enabling DOA estimation with a digital backend.

- DOA performance of the dual-polarized array is characterized with different combination schemes between tapered slot antennas. Subtracting adjacent and opposite elements results in similar performance, despite the extra rippling when opposite antennas are combined. This study emphasizes that while traditional intuition might state that a design will not perform well as a DOA sensor, high accuracy estimates can be achieved with a variety of antenna designs when a digital backend is used.

5.3 Future Work

The work presented in this thesis can be extended in many interesting directions, including the following areas.

5.3.1 High-Efficiency Cavity-Backed MAW Spiral for DOA Sensing

The MAW spiral designs presented in Chapter 2 are all fabricated using absorber to minimize the effect of the backward wave radiated from the spiral. Using this absorber results in very low radiation efficiency on the order of 30-50%. In narrowband applications, spiral antennas are regularly backed with a reflective cavity. While this cavity can increase the efficiency significantly, it degrades the axial ratio off boresight and can exacerbate rippling in the radiation patterns due to the reflection of higher-order modes.

It would be interesting to study partially- and fully-reflective cavity designs for MAW spirals.
in the context of digital DOA receivers. Even improving the efficiency to the 60-70% range could increase gain (and therefore receive sensitivity) and could also allow MAW spirals to be used as dual-polarized transmit antennas. It’s possible that the cavity could be designed to preserve the quality of mode 2 over a reasonable bandwidth to enable DOA estimation and high-efficiency operation.

5.3.2 Optimization of Four-Arm MAW Spirals for Digital DOA Receivers

In Chapter 2, four-arm MAW spiral DOA estimation performance for linearly-polarized incident signals is characterized. However, the characterization and analysis does not result in an optimal design in any formal sense. Further gains in DOA estimation bandwidth and field of view may be obtainable by formally optimizing the antenna for DOA estimation via the CRLB. Careful selection of the objective function would need to be performed, followed by a combination of theory and simulation.

5.3.3 MAW Spiral Terminations for Extended DOA Bandwidth

Previous work on MAW spirals has shown that the impedance at the end of the arms greatly affects performance [32]. However, that study was only performed in terms of gain and only included different spiral widths at the ends of the antenna. Placing an arbitrary complex lumped or distributed load between the ends of the spirals could improve the low-frequency performance by changing both the shape and phasing of the different modes of the spiral. In particular, the low-frequency performance of the four-arm MAW spiral in this thesis is inconsistent, with large changes occurring over small bandwidths when the frequency is around 1 GHz.

Lossless terminations could also be used in conjunction with the cavity integration mentioned above. By controlling the modes caused by reflections off of the end of the spiral and the modes reflected from the cavity, a designer might be able to achieve some cancellation of rippling and an improvement in return loss, at least at lower frequencies.
5.3.4 Increased Isolation in the Dual-Polarized Array

As mentioned in Chapter 4, simulation indicates that the isolation between the monocone and the TCDA can be improved by flipping the TCDA. It would be valuable to validate the simulation results with a fabricated antenna.

If further isolation improvement is desired, one could also investigate an approach similar to the TSAs. The TCDA could be modified to instead combine the 64 elements to eight outputs, with adjacent outputs subtracted to give high isolation from the cone. The effect of the subtraction on the match, radiation patterns and DOA performance would need to be evaluated.

5.3.5 Improved Combined Tapered Slot Patterns

The theory presented in Chapter 4 indicates that the null in the radiation patterns of the combined TSAs occurs because the elements have even symmetry in the gain pattern. If they instead had odd symmetry, the result would be an increase in gain at the midpoint between the elements. Research could be done on different element designs that could lead to odd symmetry in azimuth, which could make the antenna more useful for applications other than DOA estimation.

5.3.6 Mitigation of Near-Field Effects on Dual-Polarized Array Isolation

While the dual-polarized array shows excellent promise as a STAR DOA estimation sensor, the isolation measurements were made in an ideal environment without nearby reflectors. In a real environment, it might not always be possible to remove sources of reflection or asymmetry. However, in applications where only a narrow bandwidth is being used at any given time, a tuning circuit could be used to tweak the amplitude and phase between combined TSAs to compensate for the effects of near-field interactions. This tuning would also affect DOA estimation, so a careful system design would need to be completed.
Bibliography


Appendix A

Calibration and Receiver Design

The antenna designs in this thesis show promise as DOA sensors for use with digital backends. However, the design of the digital receiver and integration of the antenna and receiver introduce several challenges and possibilities for error. As discussed in Chapter 1, Figure 1.1 shows the block diagram of a typical superheterodyne receiver. In order to support DOA estimation, the ADCs used in the receiver must be synchronized so that they are sampling the incident signal(s) at the same time. Furthermore, the mixers in the front-end before the ADCs must be driven by the same LO. Otherwise, phase noise and drift can introduce phase errors between the channels over time. Finally, all components must either be thermally stable or must remain the same temperature to prevent variations between channels due to thermal drift.

In most DOA receivers, synchronization and calibration schemes are designed along with the physical ADC and front-end hardware. Designing with the necessary precision for high-quality DOA estimation requires careful consideration of the components used, how they are clocked, and the lengths of the traces on the circuit boards, among many other factors. In this appendix, an attempt is made at implementing a digital DOA receiver with a low-cost software defined radio (SDR) that was not inherently designed for synchronized operation. With this limitation in mind, an RF front-end (RFFE) is designed to support calibration of magnitude and phase offsets between channels of the SDR. Furthermore, calibration schemes for several types of offsets and imbalances are developed. Finally, measured results are presented based on two schemes for obtaining the reference patterns used to estimate the DOA of an incident signal.
A.1 Receiver Design

A.1.1 Radio Characterization

The receiver studied in this appendix contains four Realtek R820T2 RTL2832U software defined radio (SDR) USB dongles. The dongles are sold commercially as DAB and FM tuners; however, custom drivers have been developed that give access to the raw I/Q stream from the device [90]. The internal ADC has 8-bit resolution for both I and Q and can sample as fast as 2.4 million samples per section (Msps) without dropping bits. In the experiments in this appendix, the sampling rate is set to 1.048576 Msps to make real-time capture easier and to reduce the chance for dropped bits. The radio IC supports integration with several front-end ICs, including the R820T2 chip used in this version of the dongle. The tuner allows the dongle to receive signals from 0.024 – 1.766 GHz. Because the dongles were not originally designed for coherent, synchronized operation, modified radios with a common reference are instead used [91]. A picture and block diagram of the receiver are shown in Figure A.1. While the receiver has a total of eight receivers, only four are used in this experiment.

While it is clear from the block diagram in Figure A.1b that each radio is run off of the same reference clock, the true synchronization of the radios must be determined in order to understand the performance limitations of the system. The measured magnitude and phase of the cross-covariance between channel 1 and the other channels over time relative to the first covariance sample is shown in Figure A.2. The figure shows that while the receivers maintain a rough phase lock over time, there is drift of up to 8° over 60 seconds. Note that this phase drift is present even when phase dithering is turned off in the driver, as described in [91] and [92]. With phase dithering turned on, the phase of each channel varies wildly, leading to inconsistent results. Without compensation, this phase drift would result in significant error in DOA estimates over time. However, the inset of the figure shows that the channels are relatively stable over the first 200 ms, allowing for calibration and measurement on that time scale.
Figure A.1: Picture (a) and block diagram (b) of the coherent receiver used in this experiment. The receiver uses an external clock to synchronize the radios. While eight receivers are included in the design, only four are used at this time. Both images are sourced from [91].
Figure A.2: Receiver phase drift over 60 seconds. Receiver covariance phase drifts by up to $8^\circ$ over 60 seconds, which would cause significant DOA estimation error without compensation. The inset shows that the receivers are relatively stable over the first 200 ms, indicating that calibration can correct this phase drift over short time periods.
A.1.2 RF Front-End Design

In order to compensate for the phase variation of the receiver, an RF front-end (RFFE) is placed between the antenna and the coherent receiver. A picture and block diagram of the RFFE are shown in Figure A.3. Each channel of the RFFE has four major components: a Peregrine Semiconductor PE42423 [93] single-pole dual-throw (SPDT) switch that selects between a reference signal input and the antenna input, a Mini-Circuits TSS-53LNB+ [94] LNA to improve sensitivity, an Analog Devices ADL5801 [95] mixer to extend the frequency range to 6 GHz, and a Mini-Circuits LFCN-400+ [96] 400 MHz low-pass filter to reject harmonics from the mixer. These components are used to create an identical processing chain for each receiver, with the mixer used to translate the measured frequency from a range of 1 to 6 GHz down to an IF of 0.35 GHz. The mixer can use either high-side or low-side LO injection, although high-side injection requires samples in the radio to be complex-conjugated to compensate for the spectral inversion [98]. The RFFE uses an Analog Devices ADF4356 [97] integrated synthesizer and voltage-controlled oscillator (VCO) as the local oscillator (LO) for the mixer. The VCO has two differential outputs that are instead treated as single-ended and fed to each of the four mixers. While this scheme results in a phase shift between channels and a slight degradation in the return loss of the LO output, it saves on components and allow for a very wideband split. Furthermore, calibration is performed on the RFFE to handle other mismatches, so this phase imbalance has no overall effect on system performance.

The input reflection coefficient of the RFFE antenna and reference ports is shown in Figure A.4. It can be seen that the four channels are well-matched and show very similar responses over frequency. While the match is only better than 10 dB up to approximately 5 GHz, the sensitivity is sufficient up to 6 GHz to make measurements in the anechoic chamber. Furthermore, reflections from the mismatch can be corrected, as described in Section A.3. The gain of the RFFE channels is shown in Figure A.5. The figure shows the gain of all four channels over frequency as well as the gain of the first channel over frequency and input power. All channels have similar responses, and the antenna and reference port gains track each other well for each channel. The RFFE shows ex-
Figure A.3: Picture (a) and block diagram (b) of the RF front-end used in this experiment. The RFFE contains four identical signal processing chains and a common LO used to extend the frequency range of the SDRs.
Figure A.4: Input reflection coefficient of the RFFE antenna (ANT) and reference (REF) ports with the switch connected to (a) the antenna port and (b) the reference port. Each set of ports is consistent over frequency and has a reflection coefficient better than -7 dB from 0.9 to 5.6 GHz.
Figure A.5: RFFE gain over frequency. (a) shows the gain of all channels over frequency in both switch configurations while (b) shows the gain of the first channel over input power and frequency with the switch connected to the antenna port. All channels are consistent, especially between the two switch states. The RFFE also shows excellent linearity for less than -20 dBm of input power.
cellent stability over power, especially for input powers less than -20 dBm. Therefore, nonlinearities are unlikely to cause unwanted distortion in the measurements.

A.2 Measurement Setup

The goal of this experiment is to obtain a characterization of the performance of a MAW spiral-based digital DOA system, using the CU Boulder anechoic chamber. The chamber is isolated from other signals, is designed to provide suppression of reflections from the walls, and provides precise control of the orientation of the antenna. A block diagram of the setup is shown in Figure A.6. The MAW spiral, RFFE, and synchronized radios are mounted on the positioner in the chamber so that the orientation of the antenna can be varied relative to the incident signal transmitted by the NSI-RF-RGP-10 wideband probe from the other side of the chamber. A four-way splitter is also mounted on the positioner to provide a signal for the reference inputs of the RFFE. The splitter is implemented as a set of cascaded three-section Wilkinson dividers, as shown in Figure A.7. It is built on FR-4 to simplify fabrication, so the insertion loss is higher than would be obtained with a higher performance substrate. Regardless, loss in this path is not a large concern, as the path loss across the chamber is much higher. The splitter also has acceptable phase imbalance of approximately $\pm 4^\circ$. Furthermore, imbalances in the splitter are corrected in processing, as described in Section A.3.3.

The chamber vector network analyzer (VNA) is used as the source for the experiment. In normal chamber measurements, the VNA transmits on one port through the probe and receives the signal through the antenna under test and back into its other port. In this setup, the transmit path of the VNA instead goes through a two-way splitter. The phase relationship between the two outputs of the splitter is not important due to the calibration scheme in use, so any type of splitter can be used. One output of the splitter is fed to the RGP-10 probe, while the other is attached to the four-way splitter mounted on the positioner. In this way, the receivers can be given a directly-wired copy of the transmit signal that is equal for all four receivers. In order to make integration with the chamber easier, the reference signal is routed from the two-way splitter through the rotary
Figure A.6: Experiment setup for the characterization of a MAW spiral-based digital DOA receiver. The MAW spiral, RFFE, radio, and splitter are mounted on the positioner in an anechoic chamber so that the direction of arrival of the signal from the probe can be changed via the positioner.
Figure A.7: Four-way splitter (a) picture, (b) port reflection coefficients, (c) input-output transfer coefficients, and (d) input-output phase imbalance. The splitter is well-matched over the entire measured bandwidth and has less than $\pm 4^\circ$ of phase imbalance. The insertion loss is between 1.7 and 3.5 dB, which stems primarily from the lossy FR-4 substrate.
joints of the positioner and finally into the four-way splitter. Feeding the reference signal this way removes the need for an extra RF cable hanging from the antenna and receivers in the chamber.

The receivers and the RFFE are controlled via USB from a PC running a custom Python script. The script creates a thread for each receiver that reads the I/Q samples and pushes them into queues, which are read by two processing threads. The first processing thread writes the data to a file every 10 seconds, and the second determines when to flip the switches between the antenna and reference ports. The script does not coordinate directly with the chamber control software. Instead, the VNA is setup to alternate between two frequencies: a "dummy" frequency outside of the bandwidth of the receivers and the desired measurement frequency. This alternation creates a pulse train in each of the receivers. Therefore, the processing thread looks for the beginning of pulses. When a pulse begins, the switches are set to connect the reference ports to the receivers for calibration for 50 ms, as described below in Section A.3. Then, the switches are flipped to the antenna ports, allowing for measurement of the covariance matrix for DOA estimation. DOA error can be determined because the VNA outputs a single pulse per positioner azimuth and elevation setting. Therefore, the true DOA can be inferred by counting the number of pulses and associating that with the position of the antenna at that time.

A.3 Calibration

There are several sources of error that can reduce the accuracy of DOA measurements by this digital receiver. First, due to limitations in the driver software, the receivers cannot be started simultaneously. Therefore, each channel of the receiver is offset in time from the others. This offset must be removed so that the time synchronization of the receivers is preserved. Second, the phase drift of the radios must be characterized and compensated for. Likewise, mismatches in the RFFE and SDR must also be measured so that they can be removed from measurements. Third, imbalances in the splitter used to feed the reference ports of the RFFE must be characterized. Finally, because the arms of the MAW spiral are strongly coupled, reflections between the antenna and the RFFE must be accounted for in the expected radiation patterns of the spiral. The following
sections will address each of these sources of error.

A.3.1 Timing Alignment

When the switches are in the reference position, all four receivers receive the same signal. Therefore, timing alignment can be achieved by correlating the received signals in time and finding the peak of the correlation over a range of delays. However, for this correlation to be successful and to correctly identify the calibration and measurement periods for each channel, the start and stop sample for each channel is found by finding the sample at which the moving average of the amplitude of each channel is greater than a threshold $A_{\text{start}}$. After experimentation, $A_{\text{start}}$ is set to 0.02, and the moving average is taken over 100 samples. Once the pulse start times are found, each channel is correlated with the first receiver channel with sliding delays of $\pm 10000$ samples, although the actual offset should be much smaller due to the coarse synchronization above. Finally, the channels are synchronized by delaying them by the offset associated with the peak in the correlation output spectrum.

Figure A.8 shows the measured results of synchronization before and after the pulse start indices and correlation offsets are applied. It is clear that the process succeeds in synchronizing the channels to within one sample, as the samples only appear to differ by the noise in the system. While the signal could be interpolated to refine this alignment to better than one sample, that level of precision is not required for this experiment because narrowband signals are in use. In this case, any sub-sample timing misalignment will appear as a constant phase shift, which will be calibrated out in the following section.

A.3.2 Radio Phase Drift and RFFE Imbalances

Both the phase drift of the radio and imbalances in the radio and RFFE are compensated for in the same manner. At the beginning of each pulse from the VNA, the switches on the RFFE are commanded to connect the reference input to the radios. Because all four radios receive the same signal (other than the imbalances in the splitter discussed below), the offsets in magnitude and
Figure A.8: Timing alignment of receiver channels before (a) and after (b) the timing alignment procedure is performed. The algorithm uses power thresholding as a coarse alignment indicator and then refines the alignment by correlating each channel with the data from the first receiver.
phase between each of the four channels can be characterized. After the calibration time has ended,
the switch is commanded to connect the antenna arms to the receivers. The voltages received by
the radios on each channel are divided by the average magnitude and phase during the calibration
phase; that is,

\[ \hat{v}_k = \frac{v_k}{1/N_{cal} \sum_{n=0}^{N_{cal}-1} v_{cal,k}[n]} \]  

where \( N_{cal} \) is the number of samples over which the calibration offset is averaged, \( v_k \) is the voltage
on the \( k \)th channel during the measurement phase, \( v_{cal,k}[n] \) is the \( n \)th sample on the \( k \)th channel
during the calibration phase, and \( \hat{v}_k \) is the compensated voltage on the \( k \)th channel during the
measurement phase. All measured voltages are taken after the timing alignment procedure is
applied. The calibration voltages are all referenced to the calibration voltage on channel 1 so that
variations in the phase between samples due to the non-zero frequency of the measured signal do
not cause the average value to be 0.

Figure A.9 shows the magnitude and phase of signals captured with the switch connecting
the reference ports of the RFFE to the receiver. The signals in the figure have been processed
with a moving average of 100 samples so that general trends can be seen more easily. As expected,
the calibration step removes the magnitude and phase difference between the channels so that the
average deviation from channel 1 is 0 in magnitude and phase. Note that this calibration must be
performed for each pulse, as the drift of the receiver causes the calibration coefficients to change
over time.

A.3.3 Splitter Imbalances

Splitter imbalances are handled after phase drift and RFFE imbalances by using measured
S-parameters of the 4:1 splitter connected to the reference ports. In order to take reflections off of
the RFFE into account, the S-parameters are renormalized to the input impedances of the reference
ports. From [99], the \( S \) and \( Z \) matrices of a system are related by

\[ S = F (Z - Z_R^H) (Z + Z_R)^{-1} F^{-1} \]  

(A.2)
Figure A.9: Phase drift and RFFE imbalance calibration results. (a) and (c) show the magnitude and phase of each channel of the receiver before compensation while (b) and (d) show the magnitude and phase after compensation. All signals have been processed with a moving average filter of length 100 to make it easier to see the various signals. The calibration successfully removes the average deviation between channels so that all channels have the same mean magnitude and phase.
where \( Z_R = \text{diag}(Z_{R,n}) \), \( Z_{R,n} \) is the reference impedance at port \( n \), \( F = \text{diag}\left(1/\left[2\sqrt{\Re\{Z_{R,n}\}}\right]\right) \), and \( \text{diag}(\cdot) \) represents a diagonal matrix with entries given by the argument. By rearranging (A.2), it can be found that

\[
Z = F^{-1} (I - S)^{-1} \left( Z_R^H Z_R^{-1} + S \right) Z_R F
\] (A.3)

where \( I \) is the identity matrix. Note that if \( Z_R \) is real, as is often the case, this simplifies to \( Z = F^{-1} (I - S)^{-1} (I + S) Z_R F \). Therefore, if the original S-parameter matrix \( S_0 \) is referenced to \( Z_R = Z_0 \) and has a corresponding \( F = F_0 \), the renormalized S-parameters referenced to \( Z_R \) can be found by using (A.3) with \( S_0, Z_0, \) and \( F_0 \) to calculate the impedance matrix of the system (which is invariant to reference impedance changes). Then, (A.2) can be used to convert to the new set of reference impedances.

After renormalization, the splitter imbalances are corrected by taking

\[
\tilde{v}_k = \hat{v}_k \frac{\tilde{S}_{2,1}}{\tilde{S}_{k+1,1}}
\] (A.4)

where \( \tilde{S} \) contains the renormalized S-parameters of the splitter and \( \hat{v}_k \) is the final compensated receive voltage. Port 1 of the S-parameters is the common port of the splitter, and port \( k + 1 \) for \( k \) between 1 and 4 is connected to receiver \( k \). As in (A.1), the corrections are made in reference to the first receiver port. Figure A.10 shows the magnitude and phase imbalance of the splitter over frequency after the S-parameters have been renormalized to the reference port impedances. While the difference between ports is overall relatively small, a 0.8 dB magnitude difference and 8° phase difference between ports could result in a DOA error of several degrees for the system. Furthermore, when Figures A.10b and A.7d are compared, it can be seen that renormalization increases the phase imbalance by almost 5°. This observation emphasizes the importance of renormalizing the S-parameters of the splitter to correctly account for the RFFE input impedances.

A.3.4 Antenna – RFFE Impedance Interactions

In addition to compensating for mismatches in the receive and calibration chains, errors in the expected radiation patterns of the antenna must also be corrected. In particular, because the
Figure A.10: Magnitude (a) and phase (b) imbalance of the four-way splitter after renormalization to the reference port impedances. The splitter has up to 0.8 dB of magnitude imbalance and 8° of phase imbalance. Also, when (b) is compared with Figure A.7d, it can be seen that cascading the splitter and receiver increases the phase imbalance by almost 5°.
arms of the spiral antenna are tightly-coupled, reflections off of the RFFE can cause variations in the received voltages in the other arms of the antenna. While these reflections are usually small, even a reflection 10 dB below the signal level that couples to adjacent arms by 10 dB can cause ripping in magnitude by up to $20 \log_{10}(1 \pm 10^{-20/20}) \approx \pm 0.9$ dB. Figure A.11 shows the measured S-parameters of the MAW spiral. Comparing the antenna S-parameters from the figure with the reflection coefficients of the RFFE antenna ports in A.4, these example values are reasonable estimates of the reflected and coupled values for the measured system.

Compensation for these reflections can be handled by using the S-parameters, $S_a$, of the antenna and the input reflection coefficients of each channel of the RFFE, $\rho_k$, when the switches are set to connect the antenna to the receivers. Define $\Gamma = \text{diag}(\rho_k)$. Then, without loss of generality, consider the first port of the antenna. By the definition of S-parameters,

$$\vec{b} = \begin{bmatrix} b_1 \\ \vec{b} \end{bmatrix} = \begin{bmatrix} S_{a11} & \tilde{S}_{a1n} \\ \tilde{S}_{an1} & \tilde{S} \end{bmatrix} \begin{bmatrix} a_1 \\ \tilde{a} \end{bmatrix} = S_a \tilde{a}$$

(A.5)

where $\tilde{a}$ and $\vec{b}$ are the input and output power waves of the antenna, respectively, and $\tilde{a}$ and $\vec{b}$ contain the power waves for all ports of the antenna except port 1. $S_a$ is partitioned into a block matrix to fully capture the effects of the split input power waves on the split output power waves. Note that this partition can be done for any port $n$ of the antenna by changing the definitions of $\tilde{a}$ and $\vec{b}$ to split out the $n$th port instead of the first port. Then, define

$$\tilde{\Gamma}_1 = \text{diag}(\rho_2, \ldots, \rho_N)$$

(A.6)

where $N$ is the number of ports of the antenna (and $N = 4$ for the spiral antenna used in this case). Again, this definition can be updated for the $n$th port by removing the $n$th row and column of $\Gamma$ to get $\tilde{\Gamma}_n$. Because $a_1$ is the input to the system, the other input power waves in $\tilde{a}$ must be constrained by the reflections off of the RFFE. Therefore,

$$\tilde{a} = \Gamma_1 \vec{b}$$

(A.7)
Figure A.11: S-parameters of the four-arm MAW spiral. Only parameters associated with the first arm of the antenna are shown due to its symmetry. Adjacent arms are coupled by between -10 and -15 dB. This coupling can cause variations in the received voltages at the arms of the antenna if power is reflected off of the RFFE.
By substituting (A.7) into (A.5) and solving for \( \tilde{b} \), one finds that

\[
\tilde{b} = \left( I - \tilde{S}\tilde{\Gamma}_1 \right)^{-1} \tilde{S}_{an1} a_1
\]

(A.8)

and, from (A.7),

\[
\tilde{a} = \tilde{\Gamma}_1 \left( I - \tilde{S}\tilde{\Gamma}_1 \right)^{-1} \tilde{S}_{an1} a_1
\]

(A.9)

At this point, the relative power levels at each port of the antenna are known. Therefore, if the co- and cross-polarized electric fields of the antenna are given by \( E_{co,n} \) and \( E_{x,n} \) at the \( n \)th port when all other ports are terminated in the reference impedances used to find \( S_o \), then the total co-polarized fields at port 1 for the antenna terminated in the RFFE are

\[
\tilde{E}_{co,1}(\theta, \phi) = \sum_{n=1}^{N} a_n E_{co,n}(\theta, \phi)
\]

(A.10)

where \( a_1 \) is arbitrarily set to 1 and \( a_2 \) through \( a_N \) are calculated from (A.9). \( \tilde{E}_{x,1} \) is calculated similarly but uses \( E_{x,n} \) instead of \( E_{co,n} \). The same procedure can then be used to find the compensated radiation patterns at each port of the antenna.

To appreciate the error that can result from these reflections, consider Figure A.12, which shows the bias in a MUSIC estimate using the MAW spiral connected to the RFFE at 2.5 GHz for an \( E_{\theta} \)-polarized incident signal at various angles. From the figure, it can be seen that the reflections off of the RFFE can cause up to a 1° error in \( \theta \) and 4° in \( \phi \) for \( \theta \) between 15° and 30°. From Figure 2.12, the expected error in this range when the estimator is unbiased is less than 1°, so this bias is significant. The figure also shows the MUSIC bias when patterns that are corrected for the RFFE reflections are used as the reference patterns for the MUSIC algorithm. With the new patterns, the estimator is unbiased, showing the effectiveness of the calibration.

### A.4 Results

Two experiments are performed using the setup in Figure A.6, both of which use the integrated radio system shown in Figure A.13. The figure shows the full integrated radio system, including the MAW spiral, RFFE, synchronized receivers, splitter, USB hub, and extension cord
Figure A.12: Bias in MUSIC estimates caused by reflections at the ports of the MAW spiral antenna at 2.5 GHz. (a) and (b) show the bias in MUSIC estimates of $\theta$ and $\phi$ when the antenna is terminated in the RFFE antenna port impedances but the measured radiation patterns are used as the expected patterns, whereas (c) and (d) show the bias in $\theta$ and $\phi$ when the corrected patterns are used as the reference instead. Estimates are calculated by evaluating (1.12) on grid with steps of 0.25° in $\theta$ and 0.25° in $\phi$ for a signal with no noise. Even with small reflections, the bias in the algorithm can be as large as 4°, especially in $\phi$. 
Figure A.13: Pictures of the integrated digital DOA receiver. (a) shows the receiver mounted in the anechoic chamber while (b) shows the system with a wall and the lid removed. The RFFE can be seen up front, with the radio placed behind it and the splitter mounted on the right wall.
for AC power, all contained in a 3D printed box. The box is 3D printed on a MakerBot Replicator with PLA filament. In the first, antenna pattern measurements are made on the full DOA receiver system using the CU Boulder anechoic chamber. These measurements are used as the $a$ matrix in (1.12) when estimating the DOA of the signal from the probe. In the second experiment, antenna pattern measurements are instead made using the receiver system itself. The system first measures $E_\phi$ and $E_\theta$ at the desired positions in the chamber. Then, the probe is rotated $45^\circ$ and the measurements are made again. The DOA of the probe is estimated using the initial measurements in the reference polarizations. More detailed setup descriptions and results are presented in the following sections.

A.4.1 Chamber-Based DOA Estimation Experiment

As described above, the first experiment performed is a characterization of the DOA estimation performance of the receiver when previously measured radiation patterns are used as the expected antenna response. The patterns are measured for each arm of the four-arm MAW spiral with the antenna integrated on top of the rest of the system. The antenna arms are connected one at a time to what is normally the reference input of the system, allowing for access through the box. The rest of the ports are terminated in the RFFE antenna ports, and the RFFE is configured to connect the antenna ports to the receivers so that the impedance seen by the other antenna arms is correct. Because the antenna will be used in the exact same configuration when connected to the receiver, the raw near-field measurements from the chamber are used, as opposed to fields obtained from a near-field to far-field transform as would be done when using the system outdoors.

Once the pattern measurements are completed, the chamber is reconfigured to the setup in Figure A.6. The chamber control software is instructed to sweep over azimuth and elevation angles for an $E_\theta$ polarized probe and then repeat the positions with an $E_\phi$ polarized probe. As the positioner moves from orientation to orientation, the receiver measures the received voltage on each arm of the antenna. These voltage measurements are aligned and corrected using the procedures in Section A.3. Finally, DOA estimates are obtained by using the MUSIC-based algorithm in [100].
The algorithm implements FFT-based interpolation of the measured antenna pattern in order to produce DOA estimates between the measured grid points, with the actual DOA estimate based on the dual-polarized MUSIC spectrum in (1.10).

The error of the DOA estimates made from measurements at 2.5 GHz is shown in Figure A.14. For these measurements, \( \theta \) is swept from 0\(^\circ\) to 48\(^\circ\) in 4\(^\circ\) steps and \( \phi \) is swept from 0\(^\circ\) to 360\(^\circ\) in 12\(^\circ\) steps. From the figure, it can be seen that while the system is generally able to produce coarse estimates of the DOA of the incident signal, estimates of both \( \theta \) and \( \phi \) have several degrees of error at most angles. \( \theta \) estimates tend to have lower error than those of \( \phi \) for both polarizations.

A full error analysis is not performed here; however, there are several likely sources of error. First, at the time of the measurements, the positioner had significant backlash in its elevation stage. Due to this backlash, both the chamber-measured patterns and the receiver measurements had several degrees of uncertainty in their elevation. Second, the receiver is placed in a 3D-printed box directly behind the antenna. It is likely that there is near-field coupling between the antenna and the receivers, leading to differences in the received voltages compared to the voltage at the VNA when measuring the antenna patterns. Finally, there may be other sources of coupling between channels, which could be compensated if it were to be measured.

**A.4.2 Receiver-Based DOA Estimation Experiment**

The second experiment involves two separate measurements using the receivers. First, the voltage on each each receiver is measured over all angles for \( E_\theta \) and \( E_\phi \) polarizations. As above, \( \theta \) is stepped by 4\(^\circ\) between points, \( \phi \) is incremented by 12\(^\circ\), and all measurements are made at 2.5 GHz. These measurements are processed to calculate the steering vector of the antenna for each polarization over \( \theta \) and \( \phi \). Because the phase of the receivers drifts, these measurements are all made relative to the voltages on channel 1; therefore, there is a magnitude and phase offset between the measured \( E_\theta \) and \( E_\phi \) from the true response of the antenna. However, because (1.12) estimates the DOA independent of the polarization of the signal, this offset only results in a polarization error.
Figure A.14: DOA estimation error when using the near-field chamber radiation patterns as the reference patterns for the MUSIC algorithm. (a) and (b) show the mean error in $\theta$ and $\phi$ for an $E_{\theta}$ polarized incident wave, whereas (c) and (d) show the mean error in $\theta$ and $\phi$ for an $E_{\phi}$ polarized incident wave. $\theta$ estimates are best near boresight, as expected, while $\phi$ estimates improve as the signal moves away from boresight. Positioner alignment issues are the expected cause of higher error near $\phi = 180^\circ$. 
Once the receiver response has been characterized, the probe is rotated 45° so that the polarization is now slant-45° linear, described with $\alpha = \pm 45^\circ$ and $\tau = 0$. The voltages measured in this configuration are used to estimate the DOA of the signal. Figure A.15 shows the DOA estimation error for this experiment. The results in the figure are overall more accurate than those in Figure A.14. It is important to note that in this experiment, the backlash in the positioner was resolved, so positioner error is not expected to be a major cause of error. Furthermore, coupling between the antenna and the receivers is captured by the measurements of the reference patterns.

A.5 Conclusion

An integrated DOA system composed of the low-mod MAW spiral from Chapter 2, a four-channel RF front-end (RFFE), and a four-channel synchronized receiver is designed, fabricated, and tested. The RFFE is designed to extend the frequency range of the receivers, as well as to facilitate correction of phase drift in the receiver and magnitude and phase imbalance between channels in the system. Correction methods for these sources of error are developed along with compensation techniques for pattern distortions caused by reflections from the RFFE. Finally, error in DOA estimates based on reference patterns obtained by standard chamber measurements and on measurements by the receivers are presented. The system shows the feasibility of the antenna for digital DOA estimation, even if there is higher than expected error at some angles.
Figure A.15: DOA estimation error when using the receiver-measured radiation patterns as the reference patterns for the MUSIC algorithm. (a) and (b) show the mean error in $\theta$ and $\phi$ for an $\alpha = -45^\circ$ and $\tau = 0^\circ$ polarized incident wave, whereas (c) and (d) show the mean error in $\theta$ and $\phi$ for an $\alpha = 45^\circ$ and $\tau = 0^\circ$ polarized incident wave. The estimates are generally better than those in Figure A.14, as coupling error is accounted for by using receiver measurements as the reference patterns.