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# The Transmission of Fiscal Policy Shocks: International Spillovers and Non-Linearities

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**The Transmission of Fiscal Policy Shocks: International  
Spillovers and Non-Linearities**

by

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A thesis submitted to the  
Faculty of the Graduate School of the  
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2012

This thesis entitled:  
The Transmission of Fiscal Policy Shocks: International Spillovers and Non-Linearities  
written by Stephen B. Nicar  
has been approved for the Department of Economics

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Martin Boileau

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Robert McNown

Date \_\_\_\_\_

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Nicar, Stephen B. (Ph.D., Economics)

The Transmission of Fiscal Policy Shocks: International Spillovers and Non-Linearities

Thesis directed by Associate Professor Martin Boileau

My dissertation addresses two issues that have not been adequately addressed in the empirical literature on the effects of fiscal policy: the possibility that U.S. fiscal policy has economically significant spillovers to other countries and the possibility that discretionary fiscal policy may have different effects in recessions and expansions.

In the first chapter I estimate the magnitude and direction of spillovers from U.S. fiscal policy shocks to Canada, Japan, and the U.K. I find that U.S. government spending and net tax increases can be either beneficial or harmful for our trading partners. The magnitudes are generally moderate, with the response to a government spending increase typically an order of magnitude larger than the response to a net tax increase.

In the second chapter I examine whether the spillovers to Canada vary across U.S. recessions and expansions and whether the responses vary non-linearly with the size or direction of the change in U.S. government spending. I find that there are indeed different effects in recession and expansion, but that the differences are short-lived, lasting no more than a few quarters. The use of a non-linear empirical model does not seem to matter much in estimating the direction of the responses to an increase in U.S. government spending, but the magnitudes of the effects on the real exchange rate and the trade balance are significantly larger than those estimated using a linear model.

In the third chapter I shift the focus to the domestic effects of fiscal policy in Canada. I compare the responses of several Canadian macroeconomic series to increases in government spending in recession and expansion. With the notable exception of GDP, I find that the difference in responses across regimes is generally not significant in a statistical or qualitative sense. For GDP I find that the response to government spending shocks is negative in both recessions and expansions, but the magnitude of the response in recession is significantly larger.

## Dedication

To Jenny.

## Acknowledgements

Thank you to my committee and to my friends for their patience and advice.

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## Chapter 1

### Introduction

The recent global recession has generated debate in academic and policy circles on the effects of fiscal policy. In the argument over whether the government should attempt to counteract the recession, and if so how much it should spend, there are some issues that are typically either accepted as true and significant or dismissed as insignificant, in each case without much evidence. The focus of this thesis is to address two of these issues: the first is the possibility that fiscal policy has economically significant spillovers to other countries; the second is the possibility that changes in fiscal policy may have different effects in recessions and expansions.

With respect to spillovers, their significance was often assumed by policy makers, particularly in discussion of the need for a global response to the recession. In contrast, the academic focus on closed-economy estimates of fiscal policy multipliers implicitly dismisses them, at least for the U.S. Regarding different effects over the business cycle, proponents of increased government spending or tax cuts generally assume that the effect is larger (and positive) during recession than the effect would be when the economy is producing at potential. Opponents of stimulus usually assume, if only implicitly, that there is no difference in effect across the business cycle, so that the effect one would expect during a recession is the same as it would be at full employment. None of the parties to these arguments have much empirical evidence to back up their claims.

The three papers included in this thesis provide some of the missing empirical evidence. In Chapter 2 I examine the effect of U.S. fiscal policy on GDP, the bilateral trade balance, the real bilateral exchange rate, and short-term interest rate differentials for a subset of the U.S.'s largest

trading partners - Canada, Japan, and the U.K. In Chapter 3 I look in more detail at the spillovers for U.S. government spending to Canada. Specifically, I examine whether there are different effects in recession and expansion and whether the effect varies non-linearly with the size or direction of the change in government spending. In Chapter 4 I shift the focus from the U.S., which has been the subject of most current and previous research on the effects of fiscal policy, to Canada and examine whether the domestic effects of fiscal policy there vary over the business cycle.

The rest of this introduction provides some conceptual background for the issues addressed in my thesis, as well as a discussion of the previous literature on these topics.

## 1.1 Spillovers

*The United States economy is linked to the rest of the world through goods, factor, and assets markets. The linkages are reflected in the flows of goods and services in international trade; in the relationships between goods and factor prices at home and abroad; and in the asset pricing and capital flow relationships between domestic and foreign assets markets.*

Dornbusch and Fischer (1986).

Dornbusch and Fischer's succinct description illustrates the channels through which U.S. fiscal policy can spill over to the rest of the world. In the goods market, changes in government spending can have a direct effect on the trade balance if the government purchases foreign as well as domestic goods. Fiscal policy can have indirect effects on the trade balance if changes in government spending and taxes affect the private sector's consumption decisions as well. To the extent that domestic fiscal policies affect domestic factor prices, such as wages, the relative price levels between the U.S. and other countries should change, driving changes in the real exchange rate and real interest rate differentials. These in turn may drive changes in the trade balance and capital flows between the U.S. and the rest of the world. Real interest rates differentials may also change if fiscal policies have a direct effect on the domestic interest rate.

A number of theoretical studies have developed models that can be used to investigate the

transmission of fiscal policy shocks. The predictions of the models vary, however, based on their particular structures and assumptions. In a basic undergraduate-textbook version of the Mundell-Fleming model,<sup>1</sup> with fixed prices in the short run and flexible exchange rates, a debt-financed increase in domestic government spending or a net tax decrease boosts domestic GDP. The resulting increase in real money demand causes an increase in the domestic real interest rate which leads to a real exchange rate appreciation. If the Marshall-Lerner condition holds, i.e. the sum of the elasticities of imports and exports to changes in the exchange rate is greater than one, the appreciation causes a deterioration of the trade balance. Increased imports at home imply a boost to GDP abroad. In the version of Frenkel and Razin (1996) – a two-country model – foreign output and the domestic interest rate rises, but the effect on the exchange rate and trade balance are indeterminate. The responses depend on the relative magnitudes of domestic and foreign saving and import propensities, and the domestic and foreign elasticities of money demand with respect to the interest rate and income.

Microfounded, general equilibrium models that explicitly account for spillovers include Betts and Devereux (2001), Canzoneri, Cumby, and Diba (2003), and Corsetti and Müller (2011). Assumptions related to the completeness of international asset markets, substitutability between domestic and foreign goods, and the financing of government expenditures affect the outcomes of such models. As a result, the only common prediction across these models is that an increase in domestic government spending increases domestic and foreign output.

I am aware of only a few papers that directly estimate spillover effects in a way similar to what I do in Chapter 2. Canzoneri, Cumby, and Diba (2003) estimate the effect of U.S. fiscal shocks on GDP and the real effective exchange rate for the U.K., France and Italy from 1975 to 1999. They find that U.S. government spending increases lead to significant and persistent increases in foreign GDP, while an increase in net taxes has little to no effect. Spending increases in the U.S. also cause significant exchange rate depreciations in France and the U.K., with no significant effect

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<sup>1</sup> See, e.g., Mankiw (2010), appendix to chapter 12.

for Italy, while net tax increases lead to significant appreciations in the U.K. and Italy, with no significant effect for France. Arin and Koray (2009) find that U.S. government spending shocks lead to a decrease in Canadian GDP and a real exchange rate depreciation. Corsetti and Müller (2011) find that positive government spending shocks increase output in both the U.K. and the Euro Area, while causing a real exchange rate depreciation with respect to both. The trade balance with the U.K. decreases for about four quarters, then becomes positive, while for the Euro Area a small negative effect on impact quickly becomes positive.

Several recent papers have looked at the effect of U.S. fiscal shocks on the U.S. real exchange rate, terms of trade and the trade balance, though in each case they estimate the effect relative to an aggregate of other countries rather than the effect on individual countries. Enders, Müller, and Scholl (2010), Monacelli and Perotti (2010), Kim and Roubini (2008) and Ravn, Schmitt-Grohé, and Uribe (2007) all find that increases in U.S. government spending or the primary budget deficit lead to real exchange rate depreciation. Enders, Müller, and Scholl (2010) and Corsetti and Müller (2006) also find that spending shocks decrease the terms of trade. Kim and Roubini (2008) and Corsetti and Müller (2006) find that increases in the primary deficit have a small but positive effect on the current account or trade balance, while Monacelli and Perotti (2010) find a negative effect on the trade balance. Boileau and Normandin (2008), in a multi-country study including the U.S., find that U.S. tax cuts increase the external deficit.

My results from Chapter 2 are somewhat mixed. In contrast to most other studies and theoretical predictions, I find that the peak and cumulative effect of U.S. spending shocks are negative for Canadian and U.K. GDP, while positive for Japan. The effect on the real exchange rate is more in line with other studies, with peak and cumulative depreciations with respect to Canada and Japan, but an appreciation with respect to the U.K. The trade balance with Japan and the U.K. unambiguously decreases, supporting the “twin deficits” story found in other papers, while the effect for Canada starts out positive but becomes negative over time. The one result that is consistent across countries is a decrease in the short-run interest rate spread.



## 1.2 Non-Linearities

In Chapter 2 I follow the convention of most of the literature on fiscal policy and estimate the effects as if they do not vary over the business cycle. But the recent debates about the “multiplier” on federal spending have really been about the effect during *recessions*. This is another area in which empirical evidence is lacking, though several recent theoretical papers have explored non-linearities that may be triggered by the zero bound on nominal interest rates. For example, using a New Keynesian DSGE model, Woodford (2011) examines the case where a disturbance in the financial sector lowers aggregate demand and the zero bound prevents monetary policy from closing the output gap. Under these circumstances he finds that increased government spending can have multipliers substantially larger than one. Similar theoretical results have been found by Christiano, Eichenbaum, and Rebelo (2011) and Eggertsson (2011).

While these papers provide guidance for the particular circumstances in which they apply, they address non-linearity in a state that has rarely happened in the past. It is also possible that state-dependent effects of fiscal policy arise under less restrictive circumstances. In Chapter 3 I consider this possibility and, extending results from Chapter 2, estimate an empirical model of non-linear spillovers to Canada with the state of the business cycle determined by a moving average of GDP growth. Using a threshold vector autoregression model, where the regime switches discretely based on the lagged value of GDP growth, I get the somewhat counterintuitive result that government spending shocks have a negative effect on U.S. GDP in recessions but a positive one in expansions. The peak multipliers are -1.38 and 1.02, respectively. For the spillover variables – trade balance, real exchange rate, and interest rate spread – the effects in both regimes are qualitatively similar to those I estimated in Chapter 2 but larger in magnitude. In most cases the difference in response between regimes is significantly different on impact and few quarters thereafter. In addition, I find that non-linearities are mostly confined to differences across regimes.

A notable recent paper in the same vein is Auerbach and Gorodnichenko (2012a). Using a smooth transition vector autoregression, they estimate the domestic effects of U.S. government

spending in recession and expansion. They estimate peak multipliers for total government spending of 2.48 in recession and 0.57 in expansion.<sup>2</sup> Their methodology differs from mine in that the transition between regimes occurs smoothly rather than at a particular threshold value of GDP growth. Afonso, Baxa, and Slavík (2011) use a threshold VAR to determine whether the effects of fiscal shocks differ depending on financial market conditions. Specifically, they define “high stress” and “low stress” regimes using the IMF’s Financial Stress Index as the threshold variable. For the U.S. they find that the peak effect of fiscal policy shocks on GDP growth is similar in magnitude across regimes.

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<sup>2</sup> They have written a new version of this paper, Auerbach and Gorodnichenko (2012b), which calculates average multipliers across a sample of OECD countries with similar results.

## Chapter 2

### International Spillovers from U.S. Fiscal Policy Shocks

#### 2.1 Introduction

In response to the global recession that began in late 2007, policy makers called for coordinated fiscal responses, expressing a fear that spillover effects would dilute the effectiveness of policies pursued in isolation, and, implicitly, that some countries would free ride off of the difficult political decisions of others.<sup>1</sup> Were these policy makers' beliefs consistent with theoretical predictions and empirical evidence? To date there is little empirical evidence on the magnitude of fiscal policy spillovers, particularly for the United States. To the existing evidence I contribute estimates of the spillover effects of U.S. fiscal policy shocks on Canada, Japan and the U.K. from 1974 to 2007.

I find that while spillover effects of expansionary fiscal shocks are not uniform in direction or magnitude across the countries in my sample, for Canada and Japan they result in economically significant GDP increases over some portion of the response horizon. For all three countries, government spending shocks generally have larger effects than net tax shocks. Altogether, the results support the idea that some countries may benefit significantly from “free-riding” off of U.S. fiscal policy.

Several recent papers have looked at the effect of U.S. fiscal shocks on the U.S. real exchange

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<sup>1</sup> See, e.g., the speech by Dominique Struass-Kahn, former managing director of the IMF, at Oesterreichische Nationalbank, Vienna, May 15, 2009, <http://www.imf.org/external/np/speeches/2009/051509.htm>, and the “Declaration of the Summit on Financial Markets and the World Economy” (G20 Washington Summit), November 15, 2008, <http://www.g20.utoronto.ca/2008/2008declaration1115.html>.

rate, terms of trade and the trade balance, though in each case they estimate the effect relative to an aggregate of other countries rather than the effect on individual countries. Enders, Müller, and Scholl (2010), Monacelli and Perotti (2010), Kim and Roubini (2008) and Ravn, Schmitt-Grohé, and Uribe (2007) all find that increases in U.S. government spending or the primary budget deficit lead to real exchange rate depreciation. Enders, Müller, and Scholl (2010) and Corsetti and Müller (2006) also find that spending shocks decrease the terms of trade. Kim and Roubini (2008) and Corsetti and Müller (2006) find that increases in the primary deficit have a small but positive effect on the current account or trade balance, while Monacelli and Perotti (2010) find a negative effect on the trade balance. Boileau and Normandin (2008), in a multi-country study including the U.S., find that U.S. tax cuts increase the external deficit.

Canzoneri, Cumby, and Diba (2003) is the closest to what I do here. They estimate the effect of U.S. fiscal shocks on GDP and the real effective exchange rate for the U.K., France and Italy from 1975 to 1999. They find that U.S. government spending increases lead to significant and persistent increases in foreign GDP, while an increase in net taxes has little to no effect. Spending increases in the U.S. also cause significant exchange rate depreciations in France and the U.K., with no significant effect for Italy, while net tax increases lead to significant appreciations in the U.K. and Italy, with no significant effect for France.

The introduction of the euro in 1999 and the resulting common monetary policy has motivated work on fiscal policy spillovers among euro-area countries. Beetsma, Giuliadori, and Klaassen (2006) and Giuliadori and Beetsma (2005) focus on international trade spillovers. Beetsma, Giuliadori, and Klaassen (2006), using a sample of 14 countries, find that domestic government spending increases and net tax decreases significantly increase imports from other euro area countries, with spending having the larger impact. Giuliadori and Beetsma (2005), likewise find that expansionary fiscal shocks in France, Germany and Italy lead to significant increases in imports from other euro area countries.

## 2.2 VAR Specification and Identification

I estimate the effects of fiscal shocks using a vector autoregression (VAR) on quarterly data from 1974:1 - 2007:4. The baseline specification for the VAR is of the form

$$x_t = \sum_{j=1}^p A_j x_{t-j} + u_t, \quad (2.1)$$

where  $x_t$  is the vector of endogenous variables,  $A_j$  is the coefficient matrix on lag  $j$ ,  $u_t$  is the vector of reduced-form residuals and  $p$  is the lag length of the VAR. I start with a baseline specification that includes U.S. real government consumption and investment ( $g_t$ ), U.S. real net taxes ( $t_t$ ), U.S. real GDP ( $y_t$ ), foreign real GDP ( $y_t^*$ ), and the real bilateral trade balance ( $tb_t$ ). Alternately, I include as a fifth variable the real bilateral exchange rate ( $q_t$ ) and the ex-post real short term interest rate differential ( $r_t - r_t^*$ ). U.S. net taxes are current tax and transfer receipts net of transfer, subsidy and interest payments. All variables except the interest rate differential are in natural logs and GDP, spending and net tax variables are per capita. The VAR is estimated in levels, with four lags, a constant and a linear time trend.

### 2.2.1 Identification

The residuals from an unrestricted VAR like (2.1) will, in general, be correlated across equations. As a result, the residuals from the equations for  $g$  and  $t$  cannot be interpreted as exogenous fiscal shocks – some method must be used to recover the uncorrelated structural shocks from the residuals. The relationship between the VAR residuals ( $u_t$ ) and the desired structural shocks ( $\varepsilon_t$ ) can be written as

$$u_t = B \varepsilon_t, \quad (2.2)$$

where  $E[\varepsilon_t \varepsilon_t'] = I$ . Two popular methods for recovering structural shocks require imposing specific restrictions on  $B$ .<sup>2</sup>

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<sup>2</sup> Another approach used in the fiscal policy literature does not identify fiscal policy shocks from measures of total government spending. Instead, measures of federal defense spending are used as instruments for government spending and included in a VAR. See Ramey (2011).

The first method, originally suggested by Sims (1980),<sup>3</sup> uses the Cholesky factorization of the estimated residual covariance matrix ( $\widehat{\Sigma}_u$ ) for  $B$ .<sup>4</sup> This imposes a recursive ordering in which a shock to one variable has a contemporaneous effect on variables following it in the ordering, but no contemporaneous effect on those preceding it. One problem with this approach is technical: the impulse responses it generates may not be robust to alternate orderings of the variables. So the effect of changes in government spending or net taxes on GDP, for example, may change with the ordering of the variables. The larger the covariance between the residuals of the model the more sensitive the results will be to reordering. Another problem is conceptual: the contemporaneous effects it identifies may conflate discretionary policy responses with automatic changes in government spending or net taxes over the business cycle. For example, government transfers vary systematically over the business cycle by design and do not reflect discretionary policy changes. Since changes in the net taxes variable indirectly capture changes in transfers, the impulse responses to net tax shocks using a recursive identification reflect more than just responses to discretionary policy changes.

The second method, first used by Blanchard and Perotti (2002) in the fiscal policy context, seeks to deal with the conceptual problem of Cholesky identification by using external information to identify the contemporaneous response of government spending and net taxes to changes in output.<sup>5</sup>

They use this information to derive restrictions on  $B$  that have the effect of isolating discretionary policy responses. For government spending they find no systematic response to changes in output at a quarterly frequency. In addition, they assume that policy makers take at least a quarter to make any discretionary changes in spending in response to changes in output. As a result, unanticipated government spending shocks are just the residuals from the spending equation in a

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<sup>3</sup> Among the studies cited above, Corsetti and Müller (2006) and Kim and Roubini (2008) use this method.

<sup>4</sup> The Cholesky factorization results in a lower triangular matrix  $P$  such that  $PP' = \widehat{\Sigma}_u$ . In this case, then,  $B = P^{-1}$ .

<sup>5</sup> Among the studies cited above, Canzoneri, Cumby, and Diba (2003) and Monacelli and Perotti (2010) use this identification method.

VAR. This is equivalent to a recursive ordering with spending ordered before GDP. For net taxes, they use OECD estimates of the elasticity of taxes and transfers to changes in output to control for non-discretionary changes in net taxes. These estimates indicate a positive automatic response of net taxes to changes in GDP. Once the automatic response is controlled for, Blanchard and Perotti find a negative relationship between net taxes and GDP. A problem with this approach is that the results may be sensitive to the particular estimates used to control for the automatic responses. These estimates are obtained from regressions and therefore subject to estimation uncertainty which is not reflected in the resulting impulse responses.

### **2.2.2 Sign Restriction Methodology**

In this paper I use the sign restriction approach of Mountford and Uhlig (2009) which I believe avoids the problems associated the identification methods above. The basic idea is to specify a minimal number of assumptions on what the impulse responses should look like, find a large number of candidate  $B$  matrices that produce impulse responses that satisfy the restrictions, and calculate point estimates and percentile bands from the resulting distribution of impulse responses. The advantage of this methodology over the recursive approach is that, like Blanchard and Perotti, I can specify the restrictions in a way that isolates discretionary policy shocks. In addition, the results are not dependent on the ordering of the variables. The advantage of this method over the Blanchard and Perotti approach is that it allows me to deal with the same problems they were addressing but in a more general way. Rather than specify a particular quantitative structural relationship *a priori* based on uncertain estimates of contemporaneous correlations, I generate many candidate structural relationships that share qualitative implications. This better accounts for the inherent uncertainty of the estimates.

#### **Specifying the Restrictions**

As mentioned above, the key to identifying discretionary policy shocks is controlling for the automatic response of spending and net taxes to changes in GDP. I accomplish this by following

Mountford and Uhlig (2009) and first identifying a “business cycle shock” which captures these automatic responses and requiring government spending and net tax shocks to be orthogonal to the business cycle shock. To identify a business cycle shock I make one critical assumption: increases in net taxes do not cause increases in U.S. GDP; if output and tax revenue are both increasing it must be the result of an improvement in the business cycle. Specifically, I define a business cycle shock as a positive co-movement in the impulse responses of net taxes and U.S. GDP for quarters zero through four. In principal, the improvement in the business cycle could be the result of an increase in government spending. Accordingly, for a business cycle shock I also require that the increase in net taxes be greater than any change in government spending for quarters zero through four.<sup>6</sup>

A “government spending shock” is defined as increase in the impulse response of government spending for quarters zero through four that is also orthogonal to a business cycle shock. In certain draws that meet this restriction, it may also be the case that net taxes are decreasing over the same horizon. This is a fiscal change that is usually considered to have similar qualitative results as an increase in government spending. As a result, for government spending shocks I also require that the increase in government spending be greater in magnitude than any change in net taxes over the same horizon. I define “net tax shocks” analogously: an increase in the impulse response of net taxes for quarters zero through four that is orthogonal to a business cycle shock and which is greater than any change in government spending over the same horizon. Government spending and net tax shocks must also be mutually orthogonal. No restrictions are placed on the impulse responses of any of the other variables in the system. A summary of the identifying sign restrictions is provided in Table 2.1.

### **Generating Candidate Responses**

First, it’s useful to note that the Cholesky factorization mentioned above is only one of an

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<sup>6</sup> The magnitude restrictions included for each of the shocks are intended to deal with what Fry and Pagan (2009) call the “multiple shocks problem.” See below for additional detail.



Table 2.1: Identifying Sign Restrictions

	Business Cycle Shock	Gov't Spending Shock	Net Tax Shock
U.S. GDP ( $y_t$ )	+		
Gov't Spending ( $g_t$ )	$g_t < t_t$	+	$g_t < t_t$
Net Taxes ( $t_t$ )	+	$t_t < g_t$	+

**Notes:** This table shows the restrictions on the sign and relative magnitude of impulse responses for each identified shock. The restrictions are imposed for impact and the following four quarters.

arbitrarily large number of matrices for which  $BB' = \widehat{\Sigma}_u$ . As Fry and Pagan (2007) emphasize, each of these matrices represents a separate structural model, all of which are observationally equivalent in the sense that they produce residuals with the same covariance structure. To generate each set of impulse responses, I start with the Cholesky factorization ( $P$ ) and multiply by an orthonormal matrix  $Q$  which has the property that  $Q'Q = QQ' = I$ . Accordingly,  $PQQ'P' = \widehat{\Sigma}_u$ . The sign restrictions I impose are restrictions on the responses of the first three variables in the VAR (government spending,  $g_t$ , net taxes,  $t_t$ , and U.S. GDP,  $y_t$ , respectively). So the matrix  $Q$  that I construct makes use of a Givens rotation in three dimensions. Specifically, for each set of candidate responses I draw  $(\theta_1, \theta_2, \theta_3)$  from  $U[0, \pi]$  and calculate

$$Q_3 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_2) & 0 & -\sin(\theta_2) \\ 0 & 1 & 0 \\ \sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_3) & -\sin(\theta_3) \\ 0 & \sin(\theta_3) & \cos(\theta_3) \end{bmatrix},$$

where  $Q_3$  is the upper left  $3 \times 3$  section of  $Q$ . The rest of  $Q$  consists of ones on the diagonal and zeros everywhere else.

### Potential Problems with the Methodology

Fry and Pagan (2007) and Fry and Pagan (2009) identify and discuss two potential problems when using the sign restriction methodology: the “multiple shocks problem” and the “multiple models problem.” The multiple shocks problem arises when a set of responses could have been

generated by more than one potential shock; not enough information is specified to discriminate between the potential shocks. An example of this in my case is a shock which produces an increase on impact and for the following four quarters in both net taxes and government spending. With only restrictions on the sign of the impulse responses this could be identified as either a government spending or net tax shock. As mentioned above, I impose additional restrictions on the magnitude of the responses to deal with this problem.

The multiple models problem arises from the fact that each set of impulse responses represents a different model of the relationship between the variables in the VAR. One way to present my results would be, for example, to provide the median response of foreign GDP to an identified net tax shock, calculating the median with reference to the range of responses of foreign GDP only. Then I could give the median response of the interest rate differential, calculating the median with reference to the range of responses of the interest rate differential only, and so on for the other variables. The problem with this approach is that if I don't consider the responses of all variables to a particular shock, there's no reason to expect that all of the median responses are coming from the same set of impulse responses. The result for each variable will likely be coming from a different model. To deal with this, Fry and Pagan (2007) suggest presenting the set of median responses that are "closest" to the median calculated across all candidate responses that meet the restrictions.

For this paper I generated candidate matrices  $Q$  until I had 1000 that each contained a business cycle, government spending and net tax shock identified using the restrictions discussed in the paper. After I generated the corresponding impulse responses, I calculated the median and standard deviation of the responses to both the government spending and net tax shocks at each time horizon for each variable across all 1000 responses. I then calculate standardized deviations from the median by subtracting the median and dividing by the standard deviation. I calculate squared deviations for each variable in each candidate set of responses and then sum across the variables in that set of responses. This results in a single number that represents how close each set of responses is to the median across all variables and sets of responses. The set of responses with the lowest number is chosen as the median in the results I present.

Table 2.2: Government Spending shock

	impact	4 qrts	8qrts	12 qrts	20 qrts	peak	
<b>Real GDP (%)</b>							
Canada	0.07*	0.17*	0.08*	-0.08*	-0.22*	-0.22*	(20)
Japan	-0.04*	-0.05*	0.16*	0.34*	0.55*	0.55*	(20)
U.K.	-0.04*	-0.09	-0.06*	0.01	0.06*	-0.15*	(1)
<b>Trade Balance (%)</b>							
Canada	0.27*	0.38*	0.59*	0.63*	0.54*	0.63*	(13)
Japan	-0.64*	-2.25*	-1.25*	-0.58*	0.19*	-2.41*	(3)
U.K.	-2.08*	-1.98*	-1.69*	-0.96*	0.69*	-2.55*	(3)
<b>Real Exchange Rate (%)</b>							
Canada	0.18*	-0.40	-0.89*	-1.30*	-1.75*	-1.75*	(20)
Japan	-0.72*	-0.98*	-0.98*	-0.92*	-1.14*	-1.14*	(20)
U.K.	-0.11	1.06*	0.25*	-0.13	-0.68*	1.19*	(2)
<b>Interest Rate Differential (basis points)</b>							
Canada	8.22*	-22.43*	-18.82*	-9.82*	-10.58*	-27.52*	(5)
Japan	6.83	-30.79*	-30.25*	-26.67*	-1.54	-31.21*	(5)
U.K.	4.10	-23.70*	-10.62*	-9.99*	-9.35*	-23.70*	(4)

**Notes:**

Response to a one-period, 1% increase in the level of real U.S. government spending per capita. An asterisk indicates that zero falls outside two-standard-error bands. The number in parentheses is the quarter in which the peak effect occurs.

## 2.3 Results

Graphs of the impulse responses to a positive government spending shock and a positive net tax shock are given in Figures 2.1 - 2.4. Tables 2.2 and 2.3 provides more detailed numerical results. Using the sign restrictions explained above, I generated 1000 sets of impulse responses. The point estimates are the median values across the identified responses. Two-standard-error bands were calculated from the standard error for each variable's response across all of the identified responses.

Table 2.3: Net Tax shock

	impact	4 qrts	8qrts	12 qrts	20 qrts	peak	
<b>Real GDP (%)</b>							
Canada	0.000	0.044	0.058*	0.036*	0.033*	0.063*	(7)
Japan	0.008*	-0.027*	-0.069*	-0.098*	-0.118*	-0.118*	(20)
U.K.	0.013*	0.031	0.041*	0.052*	0.033*	0.052*	(12)
<b>Trade Balance (%)</b>							
Canada	0.026*	0.118*	0.032	-0.048	-0.117*	0.128*	(5)
Japan	-0.311*	0.069	-0.002	0.053	0.123*	-0.311*	(0)
U.K.	-0.006	-0.095	0.009	0.097*	0.233*	-0.248*	(1)
<b>Real Exchange Rate (%)</b>							
Canada	-0.077*	-0.235*	-0.267*	-0.300*	-0.354*	-0.354*	(20)
Japan	0.134*	0.046	0.268*	0.360*	0.379*	0.379*	(20)
U.K.	-0.007	-0.150*	-0.057	-0.036	-0.095*	-0.163*	(3)
<b>Interest Rate Differential (basis points)</b>							
Canada	5.011*	6.053*	4.936*	5.430*	4.150*	8.937*	(3)
Japan	-1.286	5.354*	4.754*	3.258*	-0.613*	6.703*	(3)
U.K.	2.640*	10.706*	9.396*	5.232*	3.945*	10.984*	(6)

**Notes:**

Response to a one-period, 1% increase in the level of real U.S. net taxes per capita. An asterisk indicates that zero falls outside two-standard-error bands. The number in parentheses is the quarter in which the peak effect occurs.

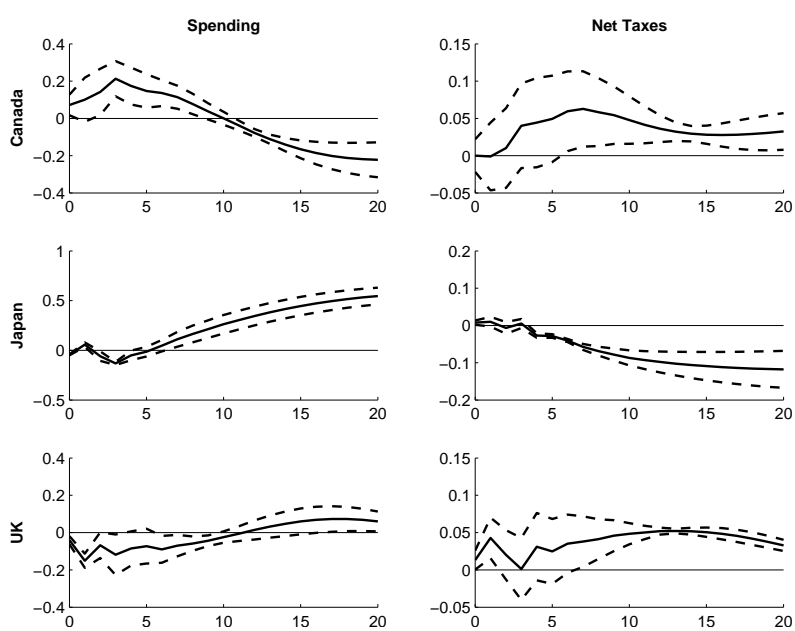


Figure 2.1: Spillover Effect on Foreign GDP

Each graph displays the response to a positive 1% shock to the indicated variable.

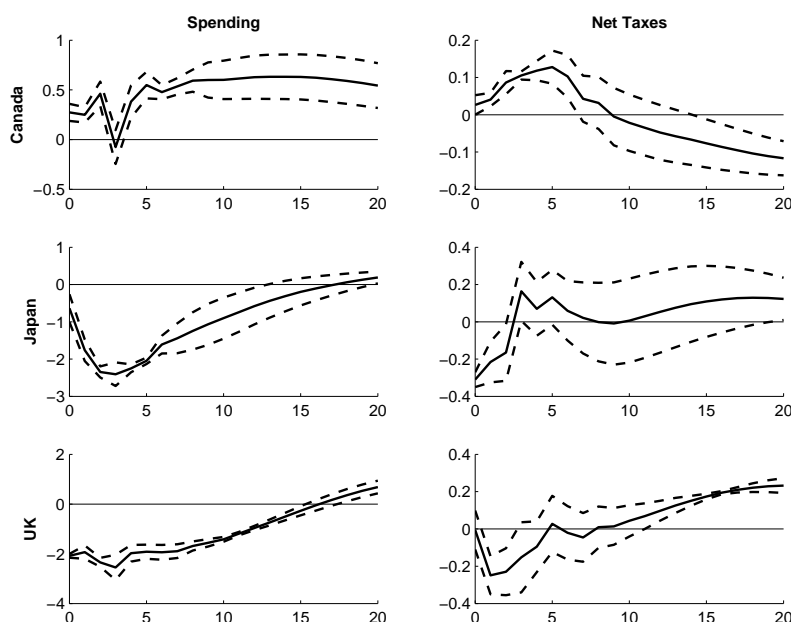


Figure 2.2: Spillover Effect on Trade Balance

Each graph displays the response to a positive 1% shock to the indicated variable.

### 2.3.1 Foreign GDP

An increase in U.S. government spending initially has a positive effect on Canadian GDP, peaking at 0.21%, but the effect becomes negative after about 10 quarters. For Japan the effect alternates between negative and positive for the first 4 quarters, but is consistently positive thereafter, peaking at 0.55%. The effect on U.K. GDP is negative until around the tenth quarter, then becomes positive. The peak effect is a decrease of 0.15%. For all three countries, zero falls outside the two-standard-error bands over most of the response horizon. A temporary increase in U.S. net taxes has a positive effect on Canadian and U.K. GDP, peaking at 0.063% and 0.052% respectively. There is initially a positive impact on Japan's GDP, but the effect becomes negative after 4 quarters, peaking at -0.118%.

### 2.3.2 Bilateral Trade Balance

With respect to Canada, the effect of a government spending shock improves the trade balance and the effect is persistent. An increase in spending leads to a decrease in the trade balance with

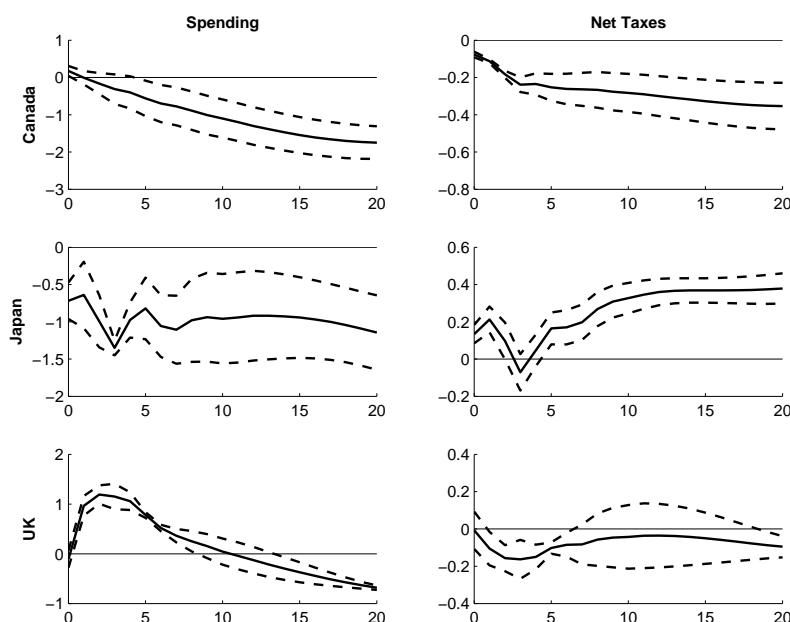


Figure 2.3: Spillover Effect on Real Exchange Rate

Each graph displays the response to a positive 1% shock to the indicated variable.

Japan and the U.K., though the effect becomes positive near the end of the response horizon. The peak effects are improvements of 0.63% with respect to Canada and a deterioration of 2.41% and 2.55% with respect to Japan and the U.K. In all cases zero falls outside the two-standard-error bands over most of the response horizon.

With an increase in net taxes the trade balance with Canada initially improves but the effect becomes negative after nine quarters. The trade balance with Japan initially falls, with a positive effect occurring around three quarters after the shock. After basically no effect on impact for the U.K, the effect becomes negative for several quarters and then becomes positive again for the rest of the response horizon. The magnitude of the effects are smaller than for spending shocks; the peak effects are improvements of 0.13% for Canada, and a deterioration of 0.25% and 0.31% for Japan and the U.K.

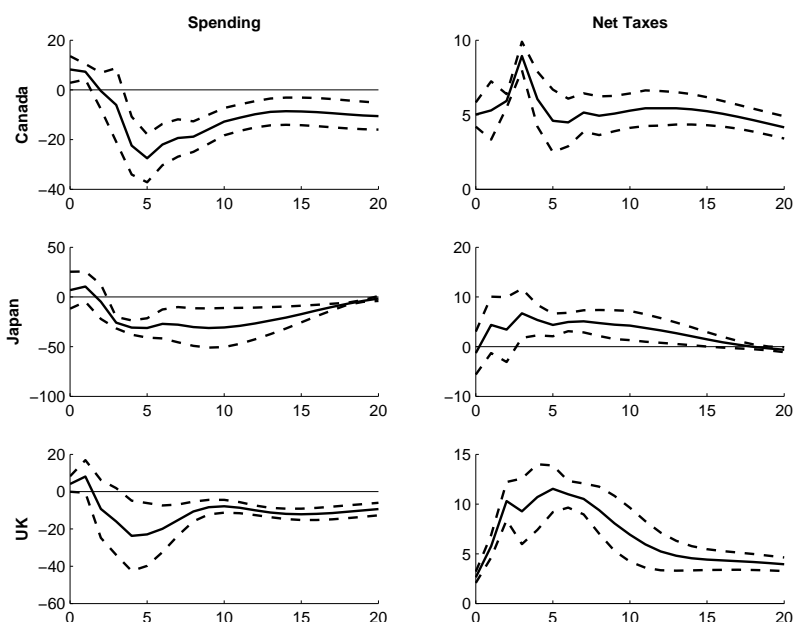


Figure 2.4: Spillover Effect on Interest Rate Differential

Each graph displays the response to a positive 1% shock to the indicated variable.

### 2.3.3 Real Exchange Rate

A government spending shock leads to a persistent real exchange rate depreciation with respect to both Canada and Japan. For the U.K., a depreciation on impact quickly becomes an appreciation and then a depreciation again after ten quarters. The peak effect ranges from a 1.19% appreciation relative to the U.K. (eventually becoming an 0.68% depreciation) to a 1.14% depreciation relative to Japan to a 1.75% depreciation relative to Canada. A net tax increase results initially in a depreciation again with respect to Canada, but an appreciation with respect to Japan and a depreciation with respect to the U.K. Once again, the peak effects are significantly smaller than for government spending shocks: depreciations of 0.35% and 0.16% with respect to Canada and the U.K., and an appreciation of 0.38% with respect to Japan.

### 2.3.4 Interest Rate Differential

For all three countries, a positive spending shock leads to a decrease in the interest rate differential. This implies a decrease in the U.S. rate, a decrease in the foreign rate or both. Net



tax shocks have the opposite result for all three countries. The peak effects for spending shocks are -28, -31 and -23 basis points for Canada, Japan and the U.K. respectively. For net tax shocks the peak effects are 9, 7 and 11 basis points for Canada, Japan and the U.K. If the effect of net tax decreases is symmetric to that of increases, then this is the only variable for which the estimated direction of the effect is consistent across both types of “expansionary” shocks.

### **2.3.5 Sign Restriction vs. Other Identification Methods**

In this section I consider the results generated from the sign restriction identification relative to those generated by recursive identification and the Blanchard and Perotti methodology. For the recursive identification the U.S. variables were ordered government spending, net taxes, then GDP. For the Blanchard and Perotti methodology I used the values reported in their paper to estimate a VAR using my data sample. Figure 2.5 shows the results for government spending shocks and Figure 2.6 shows the results for net tax shocks. Since there is no reason to think that the relative difference in identifications should vary by country, I only report results for the U.S. - Canada specifications.

From the figure it is clear that the government spending shocks identified from the sign restriction approach generate nearly identical responses to those identified by a recursive ordering with government spending ordered first. For the Blanchard and Perotti methodology, the shape of the responses is very similar, but the magnitudes on impact are smaller. For the foreign variables, this is the result of additional zero restrictions I had to impose to identify the model.<sup>7</sup> For all of the variables except the GDP variables, the Blanchard and Perotti responses track the recursive and sign restriction responses very closely after the first few quarters. For U.S. and Canadian GDP, the magnitude of the response is lower than the recursive and sign restriction responses for about eight quarters. Overall the similarity of the recursive and sign restriction results supports Blanchard and Perotti’s finding that there are no systematic changes in government spending in

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<sup>7</sup> In Blanchard and Perotti (2002) they estimate a three-variable VAR consisting only of U.S. government spending, net taxes, and GDP.

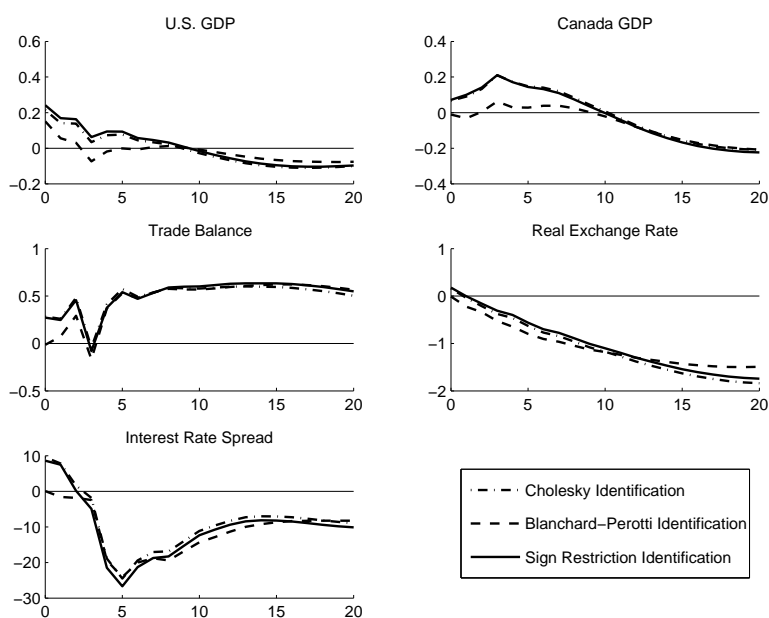


Figure 2.5: Impulse Responses by Identification Method: Government Spending

Each graph displays the response to a positive 1% shock to U.S. Government Spending.

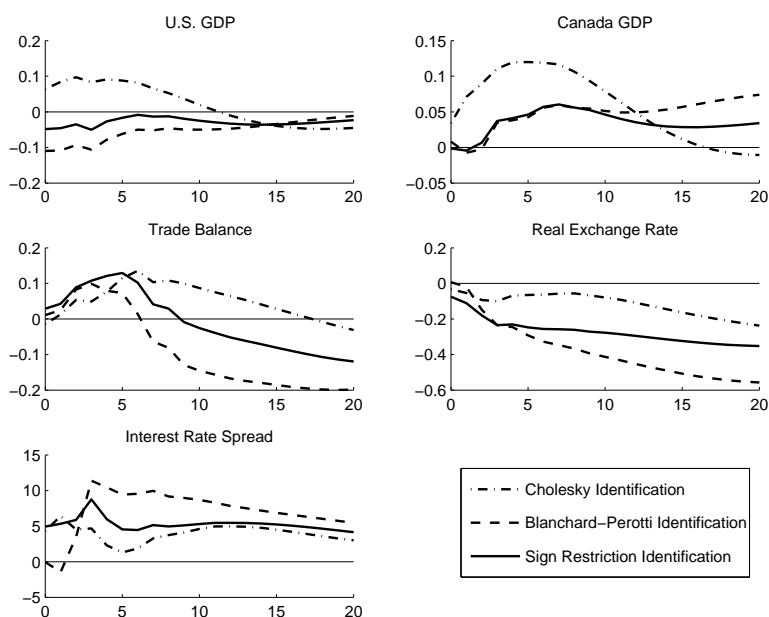


Figure 2.6: Impulse Responses by Identification Method: Net Taxes

Each graph displays the response to a positive 1% shock to U.S. Net Taxes.

response to GDP at a quarterly frequency. As a result, it does not seem to matter which of the methods are used to estimate responses to government spending shocks.

A much clearer difference is evident with the responses of U.S. and Canadian GDP to net tax shocks. In the case of U.S. GDP, the response on impact to a positive net tax shock is negative for both the sign restriction and Blanchard and Perotti approaches while in the recursive specification it is positive. This difference in responses supports the conclusion that a simple recursive specification does not adequately control for automatic co-movements in net taxes and GDP. With Canadian GDP, the three identification methods all show a positive effect but the recursive specification shows a larger effect on impact and much larger peak effect. This difference is likely driven by the difference in responses for U.S. GDP. For the trade balance, real exchange rate, and interest rate difference the qualitative results for each identification method are similar: an increase in the trade balance, depreciation of the exchange rate, and increase in the interest rate difference. The sign restriction approach generates responses whose magnitudes typically fall between the recursive and Blanchard and Perotti methods. Overall, the identification method makes a difference for identifying the effects on GDP, but not so much for the other variables.

### **2.3.6 Open Economy vs. Closed Economy Multiplier**

In this section I compare the estimated multipliers on U.S. fiscal policy shocks in a closed-economy empirical model relative to an open-economy model. Basic economic intuition suggests that the multipliers on government spending and net tax shocks should be smaller in an open economy since any change in private demand that results from changes in fiscal policy will fall in part on imports rather than domestic production. Table 2.4 shows numerical results for the two specifications, while Figure 2.7 shows the impulse responses.<sup>8</sup>

The peak effect of a one-dollar increase in government spending is a \$1.23 increase in U.S. GDP in both specifications and occurs on impact. In the open-economy specification, however,

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<sup>8</sup> The open-economy specification includes Canadian GDP and the U.S.-Canada trade balance as the foreign variables.

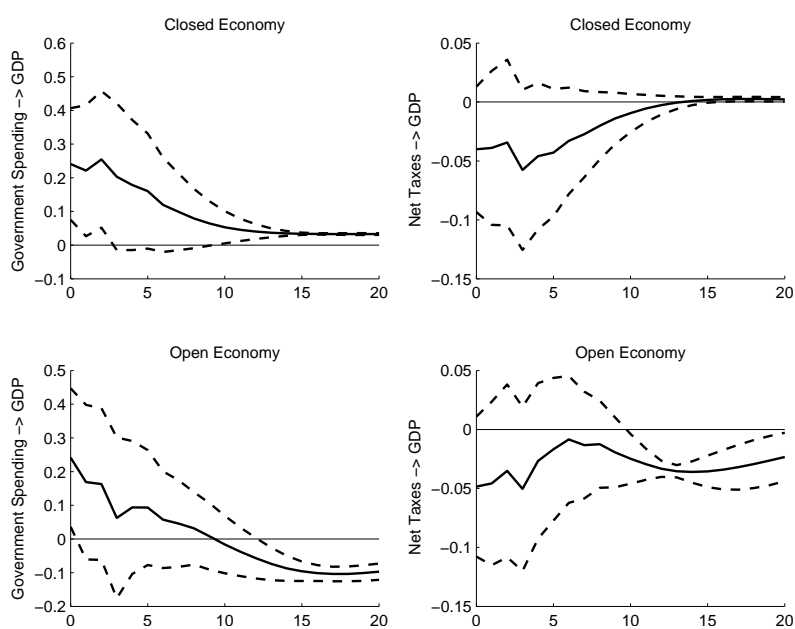


Figure 2.7: Closed Economy vs. Open Economy IRFs

Each graph displays the response of U.S. GDP to a positive 1% shock to the indicated variable.

Table 2.4: Open vs. Closed Economy Multipliers

	impact	4 qrts	8qrts	12 qrts	20 qrts	peak	
<b>Open Economy</b>							
Spending	1.23	0.48	0.16	-0.30	-0.50	1.23	(0)
Net Taxes	-0.35	-0.17	-0.08	-0.25	-0.19	-0.35	(0)
<b>Closed Economy</b>							
Spending	1.23	0.91	0.40	0.21	0.17	1.23	(0)
Net Taxes	-0.32	-0.36	-0.16	-0.02	0.02	-0.46	(3)

**Notes:**

Response, in dollars, to a one-period, \$1 increase in the level of real U.S. government spending or U.S. net taxes per capita. The number in parentheses is the quarter in which the peak effect occurs.

the effect decreases more rapidly than in the closed-economy specification and becomes negative after ten quarters. After twenty quarters the effect is a decrease of \$0.50 in the open-economy specification, while in the closed-economy specification the effect is still positive at \$0.17. For an increase in net taxes, one again the peak effects are similar, with a one-dollar increase in net taxes leading to a peak decrease in U.S. GDP of \$0.46 in the closed-economy specification and a peak decrease of \$0.35 in the open-economy specification. In the closed-economy specification the effect gradually approaches zero, while in the open-economy specification the negative effect persists for the entire impulse response horizon.

## 2.4 Conclusion

The response to U.S. fiscal shocks is not uniform across countries or type of shock. In general the response to government spending shocks is much larger – often an order of magnitude – than the response to net tax shocks. Qualitatively, the effect of spending shocks are sometimes expansionary for the foreign country. For example, U.S. spending shocks lead to significant increases in GDP in both Canada and Japan over some part of the response horizon, though in Canada they eventually lead to decreases. Net tax increases are expansionary for Canada and the U.K. over most of the horizon, though they are contractionary for Japan after about 4 quarters. Spending increases lead

to real exchange rate depreciations with respect to Canada and Japan, an appreciation with respect to the U.K., and improve the trade balance with Canada, but not with Japan and the U.K. Net tax increases cause exchange rate appreciations with respect to Japan but depreciations with respect to Canada and the U.K. For Canada and the U.K. net tax increases have the same effect on the trade balance as spending increases. But again, for both the real exchange rate and trade balance the magnitudes are much smaller than for spending shocks.

This paper relied on an identification method for fiscal shocks that has conceptual benefits over other common identification methods. As a practical matter, however, when government spending shocks are identified from total government spending and investment in a VAR each of the three identification methods discussed produces similar results. For net tax shocks the identification method matters, particularly for the short-term responses.

## Chapter 3

### Non-Linearities in U.S. Fiscal Policy Spillovers

#### 3.1 Introduction

The recent recession generated a spirited debate on the desirability and effects of fiscal policy. While this debate has been specifically about the effect of fiscal policy during recessions, the majority of past and recent research on fiscal policy has relied on linear empirical or theoretical models. These models, by construction, do not allow for different effects for fiscal policy across the business cycle. In this paper I examine non-linear responses to fiscal policy in the context of the international transmission of shocks. Specifically, I extend my earlier work on the international transmission of fiscal policy shocks from the U.S. to Canada to allow for different responses in recessions and expansions. I also consider other forms of non-linearity in the responses: within each regime I consider whether the economy responds differently to positive shocks than negative shocks and whether the size of the response increases proportionately with the size of the fiscal shock. I find that there are different effects in recession and expansion, but that the differences are not significant beyond a few quarters after the shock. I also find that for some variables the response to positive and negative spending shocks differs in the recession regime as well as the degree to which the response changes in proportion to the size of the shock.

## 3.2 Methodology

### 3.2.1 Threshold Vector Autoregression

To allow for different responses to fiscal shocks across the business cycle, I employ a variation on a traditional vector autoregression called a threshold vector autoregression (TVAR). This is a form of regime-switching regression in which the VAR coefficients are allowed to vary discretely across an arbitrary number of regimes, defined with reference to values of a “threshold variable.” The regime that the model is in at any point in time is determined by the (usually lagged) value of the threshold variable at that time and within each regime the model is linear. In the discussion that follows, the lag at which the threshold variable causes regime shifts is called the *delay* ( $d$ ) and the value of the threshold variable at which regime shifts occur is called the *threshold value* ( $\gamma$ ).

Although the business cycle can in principle be divided into more than two regimes, to minimize the number of parameters to be estimated I consider a model with two regimes – one for expansion ( $E$ ) and one for recession ( $R$ ). A TVAR incorporating these two regimes has the following specification:

$$Y_t = A^E Y_t + B^E(L)Y_{t-1} + (A^R Y_t + B^R(L)Y_{t-1})I(thresh_{t-d} < \gamma) + U_t. \quad (3.1)$$

$Y_t$  is the vector of endogenous variables,  $B^i(L)$  is the lag polynomial matrix of coefficients for regime  $i \in \{E, R\}$ ,  $A^i$  represents the contemporaneous relationship between the endogenous variables in regime  $i$ , and  $U_t$  are the structural disturbances.  $I(\cdot)$  is an indicator function equal to one for the recession regime and zero otherwise. As discussed in further detail in Section 3.3.1, I alternately use functions of U.S. and Canadian real GDP growth as the threshold variables ( $thresh_t$ ), so a recession regime occurs when U.S. or Canadian GDP growth, lagged  $d$  periods, falls below some threshold  $\gamma$  that is estimated along with the other parameters of the model.

The specification in (3.1) illustrates a couple of features that make a TVAR model useful for my analysis. First, conditional on the regime the model is linear in its parameters and is straightforward to estimate via least squares. Second, by defining the threshold variable as a



function of one of the endogenous variables I can model regime-switching as an endogenous process. This implies that shocks to any of the variables in  $Y_t$  may – via their impact on the variable underlying  $thresh_t$  – induce a shift between the recession and expansion regimes which can be reflected in the impulse responses.

### 3.2.2 Non-Linear Impulse Responses

Koop, Pesaran, and Potter (1996) identify three properties of linear impulse response functions that make them unsuitable for analyzing non-linear models such as a TVAR. The first is a *symmetry property*: in the context of my model, this means a negative one-standard-deviation shock to government spending has exactly the opposite effect of a positive one-standard-deviation shock. The second is a *shock-linearity property*: a two-standard-deviation shock has exactly twice the effect of a one-standard-deviation shock. Third, and perhaps most important for my analysis, is a *history independence property*: the effect is the same regardless of whether the shock occurs during a recession or an expansion.

To deal with the non-linear dynamics implied by a TVAR model I use the Generalized Impulse Response Function (GIRF) proposed by Koop, Pesaran, and Potter. Defining the impulse response function as a revision in conditional expectations, the GIRF at time horizon  $t + h$  for an arbitrary shock  $u_t$  is

$$GIRF_Y(h, u_t, \Omega_{t-1}) = E[Y_{t+h}|u_t, \Omega_{t-1}] - E[Y_{t+h}|\Omega_{t-1}], \quad (3.2)$$

where  $Y_{t+h}$  is the vector of endogenous variables at  $t + h$  and  $\Omega_{t-1}$  is the information set up to the period before the shock. This definition implies that the impulse response functions depend on the initial conditions (i.e. whether the model is in a recession or expansion regime at the time of the shock) and that there is no restriction regarding the size or symmetry of the shocks.

With this formulation I can explore the full dynamic effects of a shock to government spending by considering shocks of different sizes and signs, conditional on both the recession and expansion regimes. To calculate the effect of a shock to government spending during a recession regime, for example, I use the following algorithm.

- (1) Use the estimated threshold value to identify observations that fall into the recession regime. These constitute the set of recession “histories”  $\Omega_{t-1}^R$ .
- (2) From the set of regression residuals corresponding to the observations in  $\Omega_{t-1}^R$  randomly draw, with replacement, a set of  $h + 1$  innovations, where  $h$  is the desired impulse response horizon.
- (3) Using a particular history  $\omega_{t-1}^R \in \Omega_{t-1}^R$  as the initial condition and the innovations drawn in step 2, simulate  $Y_t$  for  $t = 0 \dots h$  using the estimated TVAR coefficients. The threshold variable is simulated at the same time to allow for regime switching.
- (4) Repeat step 3 using the same initial condition and innovations, but replace the first innovation with the desired shock to government spending.
- (5) Repeat steps 2 - 4 a large number of times, using the same initial condition  $\omega_{t-1}^R$ . Averaging over all of the simulations results in estimates of the conditional expectations  $E[Y_{t+h}|u_t, \omega_{t-1}^R]$  and  $E[Y_{t+h}|\omega_{t-1}^R]$ .
- (6) Repeat steps 2 - 5 for each  $\omega_{t-1}^R \in \Omega_{t-1}^R$  to get  $E[Y_{t+h}|u_t, \Omega_{t-1}^R]$  and  $E[Y_{t+h}|\Omega_{t-1}^R]$ .

In the empirical analysis below, I compute four shocks for each regime: positive and negative one- and two-standard-deviation shocks to U.S. government spending.

### 3.3 Empirical Analysis

#### 3.3.1 Data and VAR specification

I estimate the model using quarterly data for the U.S. and Canada from 1974:1 - 2007:4, starting with a baseline specification that includes U.S. real government consumption and investment ( $g_t$ ), U.S. real net taxes ( $t_t$ ), U.S. real GDP ( $y_t$ ), Canadian real GDP ( $y_t^*$ ), and the U.S.-Canada bilateral trade balance ( $tb_t$ ). Alternately, I include as a fifth variable the real bilateral exchange rate ( $q_t$ ) and the ex-post real short term interest rate differential ( $r_t - r_t^*$ ). U.S. net taxes are defined

as current tax and transfer receipts net of transfer, subsidy and interest payments. All variables except the interest rate differential are in natural logs. I estimate the TVAR in levels, including two lags of the endogenous variables, a constant, and a linear time trend that are all allowed to vary between the two regimes.<sup>1</sup>

Because the variables included in the TVAR to capture spillover effects are defined in bilateral terms, I need to consider the possibility that the dynamics may be affected by the business cycle in either the U.S. or Canada. Since I also want to allow endogenous regime shifting, the natural candidates – among variables in my specification – for defining business cycle regimes are U.S. and Canadian GDP. I consider several variations: the lagged 1-quarter growth rate and moving averages of 2, 4, 6 and 8 quarters.

The final specification I use for computing impulse responses is the combination of threshold variable, threshold value and delay that minimizes the log determinant of the regression residuals. The maximum delay  $d$  is set at two to coincide with the TVAR lag order and the set of threshold values  $\gamma$  is constrained so that each regime contains at least 38 observations.<sup>2</sup> I implement this constraint by sorting the threshold variable observations from lowest to highest then discarding the first and last 38 observations before estimation. Then for each threshold variable I estimate the model for every combination of delay and threshold value.

A summary of the results from these estimations is presented in Tables 3.1 and 3.2. Among the U.S. threshold variables a 6-quarter moving average of GDP growth, lagged one quarter, provides the best fit for the baseline and real exchange rate specifications. The estimated threshold value is a quarterly growth rate of 0.56%, which corresponds to an annual rate of around 2.26%. For the interest rate specification a 6-quarter moving average is also the best fit, but lagged two quarters, and with an estimated threshold quarterly growth rate of 0.53% (2.13% annual). Using Canadian

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<sup>1</sup> I selected the TVAR lag order based on the Hannan-Quinn Criterion (HQC) for the null linear model. Ivanov and Kilian (2005) find that for impulse response analysis in models with quarterly data, HQC is generally the most accurate lag-order selection criteria.

<sup>2</sup> This minimum number of observations is determined by the number of parameters in each equation (24) plus 10% of the usable sample size ( $\approx 14$ ).

Table 3.1: Specification Tests – U.S. Threshold Variable

		Threshold Variable				
		$\Delta y_{t-d}$	MA(2)	MA(4)	MA(6)	MA(8)
Trade Balance	$\ln  \Sigma_u $	-44.9176	-44.7914	-44.7507	-45.0059	-44.6753
	$d$	2	2	1	1	1
	$\gamma$	0.4309	0.7199	0.6142	0.5611	0.5282
Real Exchange Rate	$\ln  \Sigma_u $	-45.1203	-45.1807	-45.1437	-45.3718	-45.2335
	$d$	2	2	1	1	1
	$\gamma$	0.4309	0.6931	0.5495	0.5611	0.5842
Interest Rate Difference	$\ln  \Sigma_u $	-37.8040	-37.8331	-37.7332	-38.0022	-37.9314
	$d$	2	2	1	2	1
	$\gamma$	0.5169	0.5193	0.5495	0.5282	0.5282

Table 3.2: Specification Tests – Canada Threshold Variable

		Threshold Variable				
		$\Delta y_{t-d}$	MA(2)	MA(4)	MA(6)	MA(8)
Trade Balance	$\ln  \Sigma_u $	-44.7821	-44.9359	-44.6326	-44.6905	-44.7226
	$d$	2	1	2	1	2
	$\gamma$	1.1215	1.0581	0.8573	0.8854	0.9501
Real Exchange Rate	$\ln  \Sigma_u $	-45.1148	-45.1251	-44.9866	-44.9945	-45.1262
	$d$	2	1	1	1	2
	$\gamma$	1.1215	1.0531	0.5188	0.8545	0.9600
Interest Rate Difference	$\ln  \Sigma_u $	-37.8057	-37.8747	-37.7927	-37.9581	-37.8487
	$d$	2	1	2	1	2
	$\gamma$	1.1215	1.0581	0.5427	0.8545	0.8894

threshold variables, the best fit varies for each specification. For the baseline specification, a 2-quarter moving average, lagged one quarter, and a threshold growth rate of 1.0581% (4.3% annual) provides the best fit. For the exchange rate and interest rate specifications, 8- and 6-quarter moving averages, respectively, provide the best fit with estimated threshold growth rates of 0.96% (3.9% annual) and 0.8545% (3.46% annual).

### 3.3.2 Testing for Non-Linearity

Once the threshold variable is chosen I test the null hypothesis of a single-regime linear model versus the alternative of a threshold model with two regimes. This test is complicated by the estimated threshold value  $\gamma$  because this parameter is not present in the null linear model. As a result, the Wald statistic for testing  $A^R = B^R(L) = 0$  has a non-standard distribution. To carry out the test, then, I first estimate the TVAR for each value  $\gamma$  of the threshold variable and calculate the Wald statistic for no difference in regimes. Next, following Hansen (1996) and Andrews and Ploberger (1994), I calculate three test statistics: the sup-Wald statistic, which is the maximum of the Wald statistics calculated for all values of the threshold variable; the exp-Wald, a function of the sum of exponential Wald statistics; and the avg-Wald, the average of the Wald statistics calculated

Table 3.3: Test for Non-linearity with U.S. Threshold Variable

Specification	Threshold Variable	Delay	Wald statistics		
			Sup-	Avg-	Exp-
Trade Balance	MA(6) GDP growth	$d = 1$	173.8668 (0.0010)	110.2984 (0.0175)	82.0599 (0.0010)
Real Exchange Rate	MA(6) GDP growth	$d = 1$	165.3006 (0.0060)	115.8488 (0.0075)	77.7599 (0.0065)
Interest Rate Difference	MA(6) GDP growth	$d = 2$	171.8769 (0.0030)	113.7973 (0.0105)	81.0409 (0.0030)

**Notes:**

The Wald statistics are for a test of the null hypothesis of a linear model versus the alternative of a threshold VAR model with 2 lags, threshold variable and delay parameter indicated. The p-values, given in parentheses, were calculated using the procedures described in Hansen (1996) and Hansen (1999) with 2000 bootstrap replications.

for all values of the threshold variable.<sup>3</sup> Finally, using the bootstrap procedure described in Hansen (1999) I computed 2000 replications of the test statistics. P-values for the test are the percentage of test statistics from the replications that exceeded the test statistics from my estimated TVAR model.

The results of linearity tests using both U.S. and Canadian threshold variables are presented in Tables 3.3 and 3.4. When using U.S. GDP growth as the threshold variable, I can reject the null of a single regime for each specification near or below the 1% level. When Canadian GDP growth is the threshold variable a single regime is rejected near or below the 1% level for the baseline and interest rate specifications. For the exchange rate specification I am unable to reject a linear model at any conventional significance level.

### 3.3.3 Identified Recession Regimes vs. NBER Recession Dates

How do the recession regimes identified by my TVAR model using the U.S. threshold variable compare to the official NBER recession dates? Figure 3.1 shows a plot of the 6-quarter moving

<sup>3</sup> Informally, the avg-Wald statistic tests for alternatives close to the null, the sup-Wald for alternatives very far from the null, and the exp-Wald for intermediate alternatives. Note that by using these test statistics the results of the test do not depend on the estimated threshold value.

Table 3.4: Test for Non-linearity with Canadian Threshold Variable

Specification	Threshold Variable	Delay	Wald statistics		
			Sup-	Avg-	Exp-
Trade Balance	MA(2) GDP growth	$d = 1$	163.3551 (0.0010)	113.2398 (0.0060)	76.9994 (0.0010)
Real Exchange Rate	MA(8) GDP growth	$d = 2$	120.5509 (0.4205)	85.6761 (0.5960)	56.0707 (0.4200)
Interest Rate Difference	MA(6) GDP growth	$d = 1$	163.5146 (0.0115)	107.9670 (0.0330)	76.8743 (0.0120)

**Notes:**

The Wald statistics are for a test of the null hypothesis of a linear model versus the alternative of a threshold VAR model with 2 lags, threshold variable and delay parameter indicated. The p-values, given in parentheses, were calculated using the procedures described in Hansen (1996) and Hansen (1999) with 2000 bootstrap replications.

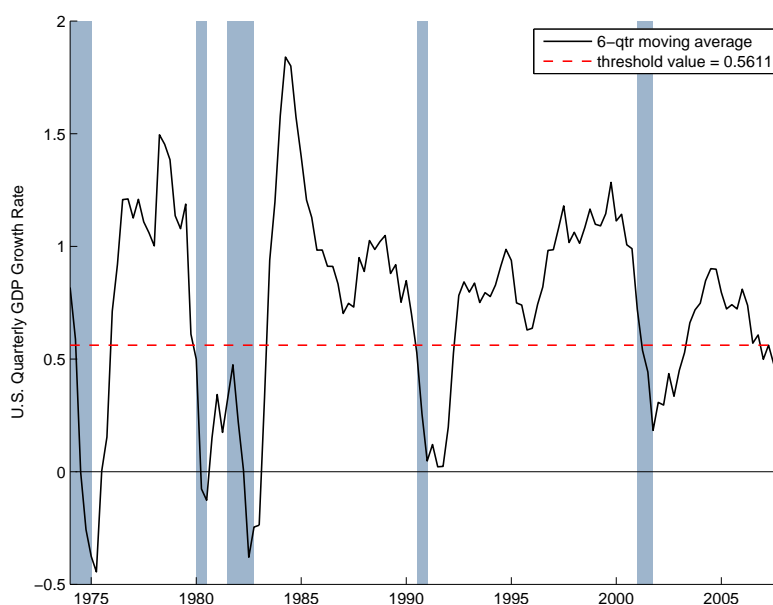


Figure 3.1: TVAR Recessionary Regimes vs. NBER Recession Dates

average of U.S. GDP growth with NBER recession dates shaded in the background. My estimated threshold value of 0.5611 is indicated by a dashed horizontal line. Areas where the GDP growth plot falls below the line correspond to recession regimes identified by the model. Table 3.5 lists the dates of the TVAR recessionary regimes and the NBER recession dates that fall within my sample period of 1974:1 - 2007:4.

While the recessions identified by each overlap for the most part, the TVAR recession regimes are on average twice as long as NBER-identified recessions (just over 8 quarters vs. an average of 4 quarters). The 1980 and 1981-82 NBER recessions are both subsumed in one long TVAR recessionary regime. In the case of the 2000 NBER recession there is no overlap at all – the TVAR model identifies a recessionary regime from 2001 - 2003 instead. In the case of the most recent recession, the TVAR model has the recessionary regime beginning in the first quarter of 2007, rather than the fourth quarter as determined by the NBER.

The timing differences are partly driven by the fact that the NBER dates measure from the peak to the trough of the business cycle, while the TVAR regimes may be interpreted as periods where the economy is operating at less than potential, a condition which may begin before and



Table 3.5: TVAR U.S. Recessionary Regimes vs. NBER Recession Dates

TVAR		NBER	
Recessions	length	Recessions	length
1974 q3 – 1975 q4	6	1974 q1 – 1975 q1	5
1980 q1 – 1983 q2	14	1980 q1 – 1980 q3	3
1990 q3 – 1992 q2	8	1981 q3 – 1982 q4	6
2001 q2 – 2003 q2	9	1990 q3 – 1991 q1	3
2007 q1 – 2007 q4	4	2000 q1 - 2000 q4	4
		2007 q4	1

**Notes:**

TVAR Recession dates are identified by a 6-quarter moving average of U.S. real GDP growth and an estimated threshold value of 0.5611. NBER Recession dates only include those that fall within the sample period of 1974 q1 – 2007 q4.

persist beyond the actual trough of the cycle. In fact, as Figure 3.1 shows, local minimum points for the threshold variable coincide pretty closely to NBER end dates. The longer duration of the TVAR recession regimes is also a consequence of restricting the range of threshold values over which I estimate the model. Unfortunately, this restriction is necessary to identify the model. It is interesting to note, however, that even with this restriction I am able to reject a linear model at the 1% level. This implies that for purposes of evaluating the effects of fiscal policy, NBER business cycle dates may not be the best measure of recessionary regimes. I am unable to make a direct comparison because the 22 NBER recession quarters that fall within my sample are not enough to identify the model.

### 3.3.4 Impulse Response Results

In this paper I only consider shocks to U.S. government spending. I make this choice for two reasons: the first is to keep the discussion manageable - to discuss both spending and tax shocks in the non-linear model would involve discussion of the response of five variables to eight shocks to both spending and taxes; the second is that, as I've shown in other work (see Nicar (2012)), the responses to net tax shocks are more sensitive to the identification method than spending shocks. I identify shocks as the innovations from the government spending equation in the TVAR (ordered first) and assume the contemporaneous relationships  $A^E$  and  $A^R$  have a recursive structure determined by the Cholesky decomposition of the residual covariance matrix for each regime. This method is the most straightforward to implement in the TVAR model and for calculation of the non-linear impulse responses. As I've shown in Nicar (2012), with the specification and variable definitions I'm using, this method of identifying shocks results in responses very similar to those found using sign restrictions or the methodology of Blanchard and Perotti (2002).

In order to exploit the features of the non-linear model, I consider eight shocks: a positive and negative one-standard-deviation shock, and a positive and negative two-standard-deviation shock, in each case conditional on occurring in both of the regimes. Responses to each shock were computed for each regime and for specifications using both the U.S. and Canadian threshold

variables. Because the linearity tests for the Canadian threshold variable provided no evidence of non-linearity for the exchange rate specification, I do not present impulse responses for that specification. For each regime, I treat each observation that falls into that regime as a separate “history” and compute 500 replications of the GIRF for that history using the algorithm explained in section 3.2.2. The mean across all replications for that history is saved as an individual realization of the GIRF for that regime. The point estimates and quantiles presented in the tables and graphs for each regime are the mean and quantiles across all realizations for that regime.

Figure 3.2 presents the point estimates for each shock in the recession regime using U.S. GDP growth as the threshold variable and Figure 3.3 shows the point estimates for each shock in the expansion regime. Figures 3.4 and 3.5 present the same using Canadian GDP growth as the threshold variable. From these graphs it appears as though within each regime the responses display the properties of linear impulse responses – *shock linearity* and *shock symmetry* in the terminology of section 3.2.2. I’ll discuss the extent to which my estimated GIRFs differ from linear responses below. I’ll begin the discussion of the impulse response results, however, by looking at a positive one-standard-deviation shock and how the response differs across regimes. This facilitates comparison with other results from the fiscal policy literature since this is the usual type of shock considered.

### **U.S. Threshold Variable**

Table 3.6 shows the responses of U.S. and Canadian GDP to a one dollar increase in U.S. government spending and the responses of the trade balance, real exchange rate, and interest rate differential to a one percent increase. Graphs of the impulse responses are shown in Figures 3.6 and 3.7. The point estimates are the mean response at each horizon across all histories within the regime. The confidence intervals are given by the 2.5% and 97.5% quantiles at each horizon. Somewhat counter-intuitively, (and in contrast to Auerbach and Gorodnichenko (2012)) I find that during a recession a \$1.00 increase in government spending decreases GDP by \$0.24 on impact and the effect remains negative for the first six quarters after the shock. The peak effect occurs

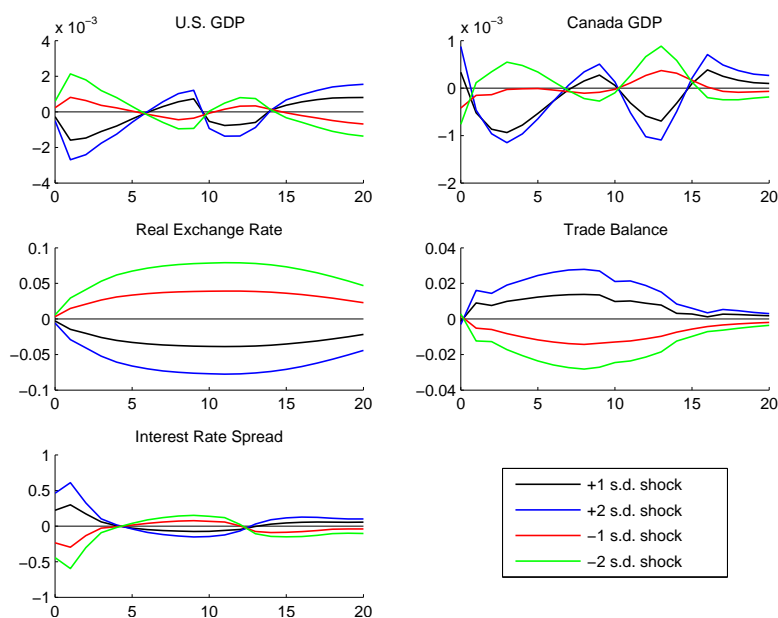


Figure 3.2: TVAR GIRFs – U.S. Recession Regime  
(U.S. GDP growth is threshold variable)

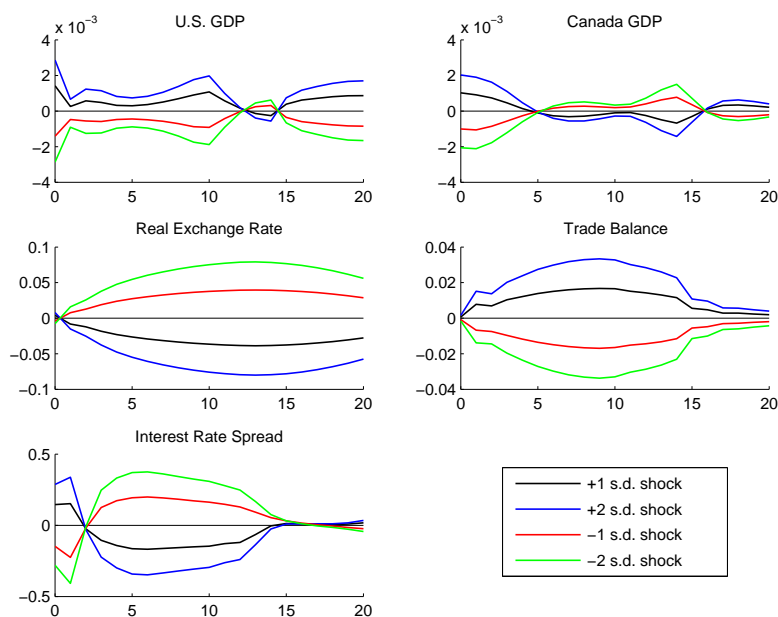


Figure 3.3: TVAR GIRFs – U.S. Expansion Regime  
(U.S. GDP growth is threshold variable)

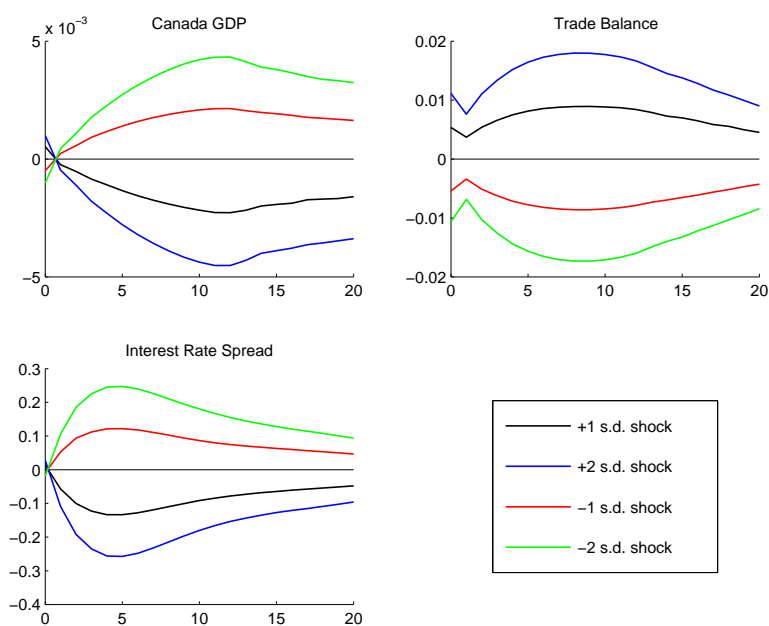


Figure 3.4: TVAR GIRFs – Canada Normal Regime  
(Canada GDP growth is threshold variable)

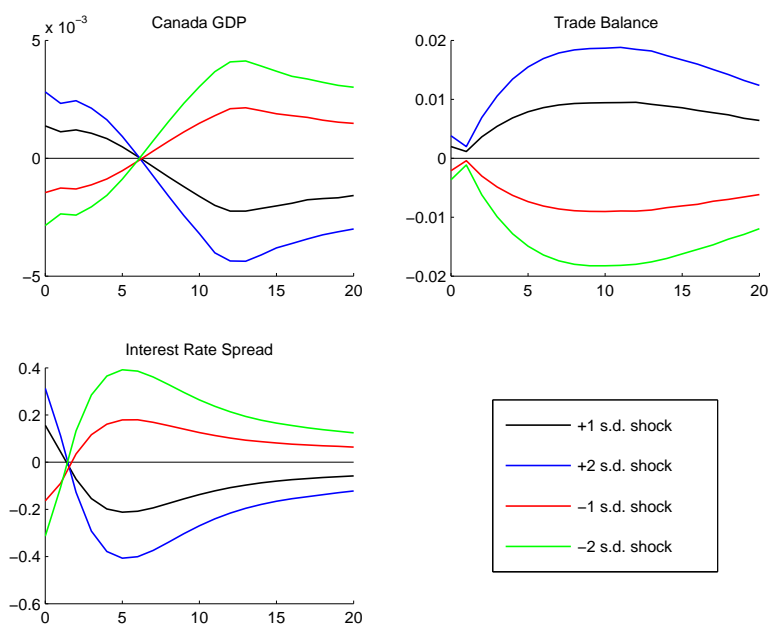


Figure 3.5: TVAR GIRFs – Canada High-Growth Regime  
(Canada GDP growth is threshold variable)

Table 3.6: IRFs by Model and Regime - U.S.

model	impact	4 qrts	8 qrts	12 qrts	20 qrts	Peak	
<b>U.S. GDP (\$)</b>							
Linear	0.93	0.12	-0.18	-0.38	-0.33	0.93	0
GIRF - Rec	-0.24	-0.69	0.49	-0.62	0.69	-1.38	1
GIRF - Exp	1.02	0.23	0.50	0.11	0.62	1.02	0
<b>Canada GDP (CDN \$)</b>							
Linear	0.05	0.03	0.00	-0.04	-0.08	-0.08	20
GIRF - Rec	0.03	-0.07	0.01	-0.06	0.01	-0.09	3
GIRF - Exp	0.08	0.01	-0.02	-0.02	0.02	0.08	0
<b>Trade Balance (%)</b>							
Linear	0.39	0.89	1.04	0.94	0.51	1.04	8
GIRF - Rec	-0.24	1.85	2.30	1.48	0.30	2.30	8
GIRF - Exp	0.09	1.68	2.29	1.98	0.27	2.32	9
<b>Real Exchange Rate (%)</b>							
Linear	0.17	-0.88	-1.71	-2.29	-2.71	-2.72	19
GIRF - Rec	-0.47	-5.04	-6.26	-6.46	-3.63	-6.48	11
GIRF - Exp	0.56	-3.19	-4.66	-5.35	-3.86	-5.38	13
<b>Interest Rate Difference (bps)</b>							
Linear	10.71	-11.22	-17.74	-12.74	-5.25	-17.95	7
GIRF - Rec	37.00	0.63	-11.80	-7.83	9.06	49.97	1
GIRF - Exp	20.15	-19.68	-21.73	-16.75	2.40	-23.28	6

**Notes:**

For U.S. and Canadian GDP, the table gives the response to a one-period, \$1 increase in the level of real U.S. government spending. For the trade balance, real exchange rate and the interest rate difference, the response is to a one-period 1% increase in real U.S. government spending.

in the first quarter with a \$1.38 decrease in GDP. After the initial decrease the the point estimate alternates between positive and negative \$0.65, but the response is not significantly different from zero after the first four quarters. In an expansion, there is a \$1.02 increase in GDP on impact, which is also the peak effect. The effect remains small and positive for most of the rest of the horizon, but not significantly different than zero.

On impact and for the next two quarters the responses for the two regimes are significantly different from each other in the sense that their confidence intervals do not overlap. After the first two quarters there is significant overlap in the confidence intervals for the two responses, and after

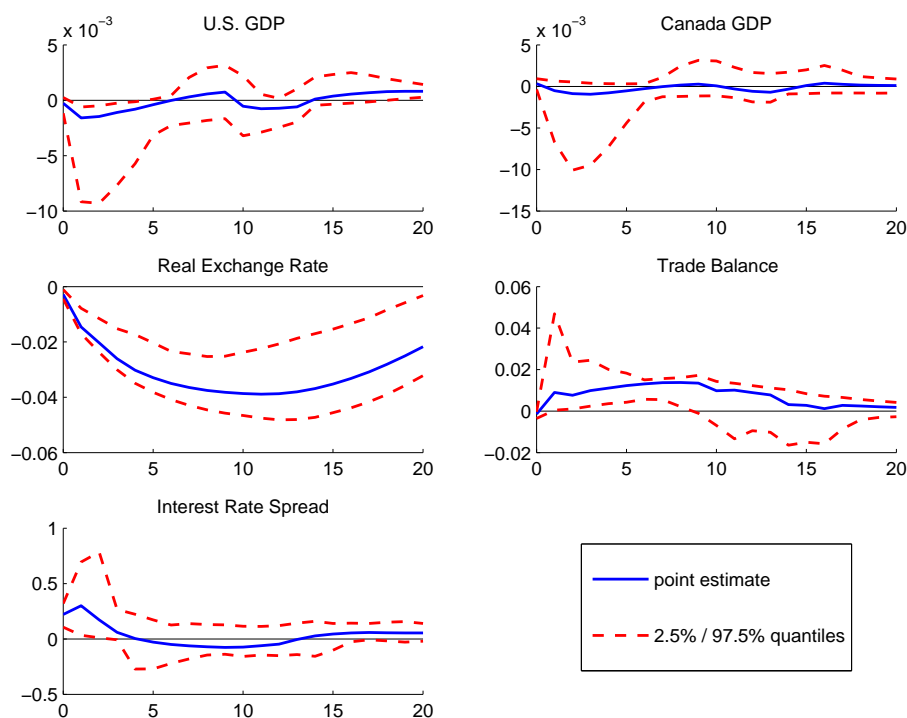


Figure 3.6: TVAR GIRFs – U.S. Recession Regime

Response to a positive 1 std. error shock to U.S. Gov't Spending.

about six quarters the point estimates for the two regimes track each other pretty closely, though neither is significantly different than zero.

The estimated responses of Canadian GDP to U.S. spending shocks are small in both recessions and expansions. When the U.S. is in recession, a \$1.00 increase in U.S. government spending has a peak effect of decreasing Canadian GDP by \$0.09 (Canadian), occurring three quarters after the shock, and in an expansion the peak effect is an increase in GDP of \$0.08 on impact. In the recession regime, the response is never significantly different from zero, but for the expansion regime the effect is significant on impact and for the following quarter. The responses for the two regimes are significantly different from each other on impact and the following two quarters, but afterwards the two confidence intervals overlap completely.

The response of the trade balance, measured as the ratio of U.S. exports to imports, is positive and almost the same magnitude across the two regimes, at least for the first four quarters after the shock. During a recession the trade balance drops slightly (-0.25%) on impact but the

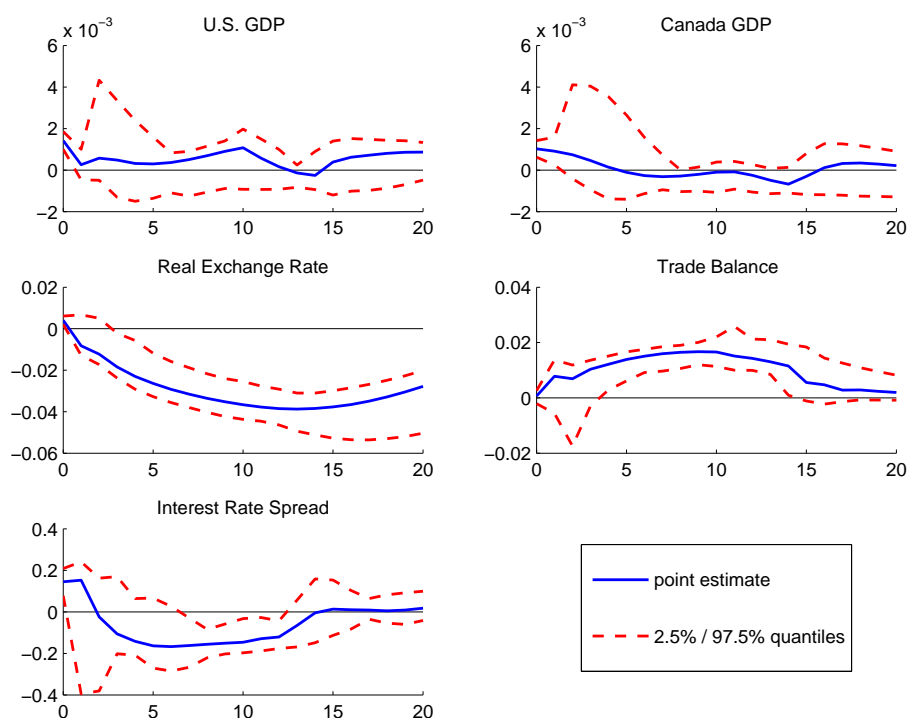


Figure 3.7: TVAR GIRFs – U.S. Expansion Regime

Response to a positive 1 std. error shock to U.S. Gov't Spending.



effect increases sharply to a positive 1.52% increase in the first quarter. The peak effect occurs in the eighth quarter at 2.30%, gradually decreasing to 0.30% twenty quarters after the shock. The response is significantly different from zero for the first through ninth quarters. In an expansion the effect on impact is slightly positive at 0.09% and is almost identical to the recession response for the first four quarters becoming a bit larger thereafter. The peak effect is a 2.32% increase in the ninth quarter, after which the effect decreases more slowly than in the recession regime but ends in nearly the same place at 0.27%. The response becomes significant between the fourth and sixteenth quarters. Although the response during expansion appears to be a bit larger and more persistent than during a recession, there is significant overlap in the confidence intervals for the two regimes for most of the response horizon.

The response of the real exchange rate tells a similar story as the trade balance: the magnitude and direction of the response is very close across the two regimes. In the recession regime a positive spending shock leads to a 0.47% depreciation of the exchange rate on impact, continuing to a peak depreciation of 6.48% eleven quarters after the shock. The exchange rate then appreciates, ending up 3.63% below its level before the shock. The response is significant for the entire horizon. In an expansion the exchange rate appreciates by 0.56% upon impact but then quickly depreciates by 1.70% in the first quarter, and eventually reaches a peak depreciation of 5.38% in the thirteenth quarter. As in the recession regime, it then appreciates back to 3.86% below its level before the shock. The response is significant on impact and after the fourth quarter. The responses across regimes are significantly different from each other on impact, but afterwards the confidence intervals overlap.

The initial response of the spread between U.S. and Canadian short-term interest rates to a spending shock is positive in both recessions and expansions. There is an increase of 37 basis points on impact in the recessions regime, peaking at a 50 basis point increase a quarter later. The response becomes negative after five quarters, reaching a decrease of around 12 basis points relative to the pre-shock spread, then positive again at fourteen quarters, ending at 9 basis points above the pre-shock spread. The response is only significant for the first four quarters however. In

an expansion there's an increase of 20 basis points on impact, then the effect becomes negative at three quarters, peaks at a decrease of 23 basis points in the sixth quarter, and becomes positive again after fifteen quarters. The response is significant on impact and for quarters eight through thirteen. As with the real exchange rate, the responses for the two regimes are significantly different on impact, but not thereafter.

### **Canada Threshold Variable**

As discussed in section 3.3.1, the estimated threshold values using Canadian GDP growth as the threshold variable correspond to annual growth rates of 4.3% and 3.5% for the trade balance and interest rate specifications, respectively. These estimates imply that the non-linearities present in the model do not correspond to “recession” and “expansion” regimes as with the U.S. threshold variable but are perhaps better characterized as “normal” and “high-growth” regimes. Table 3.7 shows the responses of Canadian GDP, the trade balance and the interest rate spread to a one dollar increase in U.S. government spending when Canadian GDP growth is used to define the two regimes. Graphs of the impulse responses are shown in Figures 3.8 and 3.9.

The response on impact of Canadian GDP to a \$1.00 U.S. spending shock is positive in both the normal and high-growth regimes. In the normal regime, however, the response becomes negative in the first quarter, falling from an increase of \$0.05 on impact to a peak decrease of \$0.20 in the twelfth quarter following the shock. The response is significantly different than zero after the fourth quarter. The positive effect is more persistent in the high-growth regime, starting at an increase of \$0.10 on impact and remaining positive until the seventh quarter. The peak effect is a decrease of \$0.17 in the twelfth quarter, but the response is not significant for the fourth through tenth quarters. The responses are significantly different from each other through the first six quarters. Relative to the model with the U.S. threshold variable, the responses are about the same magnitude on impact, but larger and more persistent thereafter.

For the trade balance, the response has the same shape and similar magnitudes across regimes: an increase (from the U.S. perspective) on impact followed by a short dip then a much larger increase

Table 3.7: IRFs by Model and Regime - Canada

model	impact	4 qrts	8 qrts	12 qrts	20 qrts	Peak	
<b>Canada GDP (CDN \$)</b>							
Linear	0.05	0.03	0.00	-0.04	-0.08	-0.08	20
GIRF - Rec	0.05	-0.10	-0.17	-0.20	-0.14	-0.20	12
GIRF - Exp	0.10	0.06	-0.06	-0.17	-0.12	-0.17	12
<b>Trade Balance (%)</b>							
Linear	0.39	0.89	1.04	0.94	0.51	1.04	8
GIRF - Rec	0.82	1.15	1.37	1.29	0.69	1.37	9
GIRF - Exp	0.26	0.90	1.22	1.25	0.84	1.25	12
<b>Interest Rate Difference (bps)</b>							
Linear	10.71	-11.22	-17.74	-12.74	-5.25	-17.95	7
GIRF - Rec	1.82	-19.01	-15.62	-11.11	-6.81	-19.01	4
GIRF - Exp	19.25	-24.36	-21.48	-13.27	-7.12	-26.05	5

**Notes:**

For Canadian GDP, the table gives the response to a one-period, \$1 increase in the level of real U.S. government spending. For the trade balance and the interest rate difference, the response is to a one-period 1% increase in real U.S. government spending.

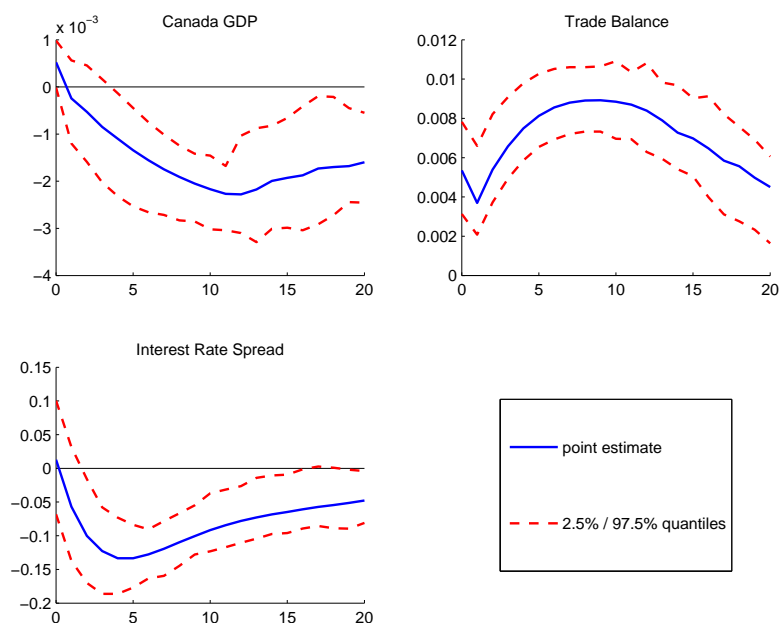


Figure 3.8: TVAR GIRFs – Canada Normal Regime

Response to a positive 1 std. error shock to U.S. Gov't Spending.

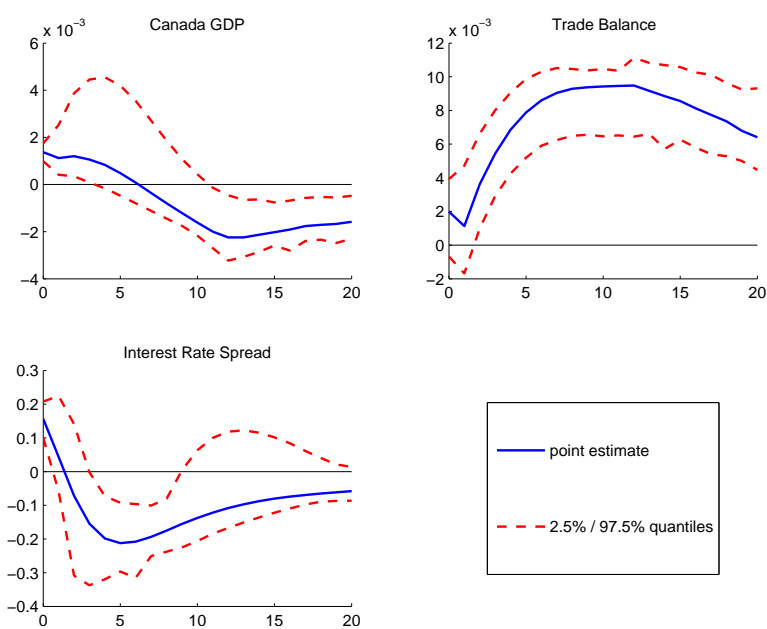


Figure 3.9: TVAR GIRFs – Canada High-Growth Regime  
 Response to a positive 1 std. error shock to U.S. Gov't Spending.

and gradual decrease. The response on impact is larger in the normal regime – 0.86% increase vs. 0.25% – but the effect peaks at a lower level – 1.37% vs. 1.25% – and decreases faster than in the high-growth regime. In each regime the responses are significantly different from zero across most of the horizon. The responses are significantly different from each other on impact but afterwards the confidence intervals overlap. The responses are in the same direction but of a somewhat smaller magnitude than when U.S. GDP growth is the threshold variable.

The initial response of the spread between U.S. and Canadian short-term interest rates to a spending shock is positive in both the normal and high-growth regimes: 1.82 basis points and 19.25 basis points, respectively. For both, however, the response becomes negative within two quarters, peaking at a 19 basis point decrease in the normal regime and a 26 basis point decrease in the high-growth regime. The responses are significantly different from each other on impact but not thereafter. In the normal regime the response is significantly different from zero from quarters two through sixteen while in the high-growth regime the response is significant on impact and for quarters three through eight.

### **TVAR vs. Linear Model**

#### ***History Dependence***

For many of the responses above there is a significant difference between regimes only at impact or for at most a couple of quarters after. If there is no difference in the response across regimes for most of the response horizon, how do the estimates compare to a strictly linear model? Figures 3.10 (U.S. threshold variable) and 3.11 (Canada threshold variable) show the point estimates for both regimes plotted alongside the IRFs from a linear VAR with the same variables and lag order. I've also plotted the two-standard-error asymptotic confidence band for the linear IRFs.

With the U.S. threshold variable, the most notable qualitative difference occurs in the response of U.S. GDP. The linear and expansion regime responses are both positive and of similar magnitude on impact and for the first four quarters. The recession response is initially much lower than the linear response, falling outside the linear confidence band. While the linear response de-

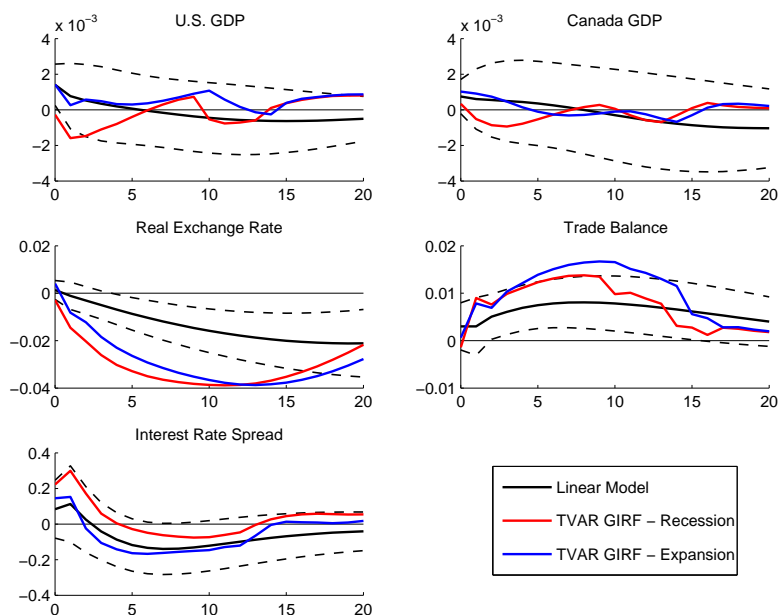


Figure 3.10: Linear vs. Non-Linear IRFs – U.S.

Response to a positive 1 std. error shock to U.S. Gov't Spending.  
Dashed lines are 2 std. error confidence bands for the linear model IRFs.

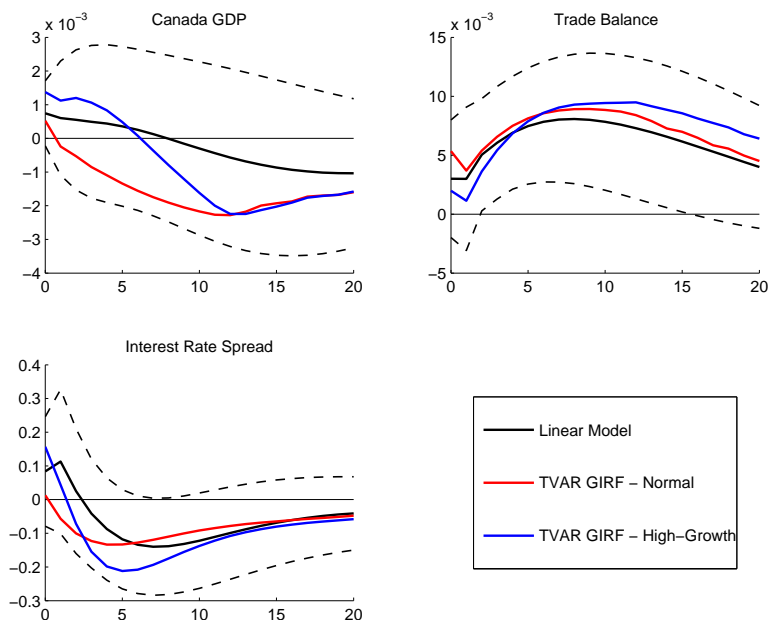


Figure 3.11: Linear vs. Non-Linear – Canada

Response to a positive 1 std. error shock to U.S. Gov't Spending.  
Dashed lines are 2 std. error confidence bands for the linear model IRFs.

creases for the full twenty quarter horizon, the recession and expansion regimes both show increases in GDP after about four quarters. But after the first quarter for the recession regime, and for the whole horizon for the expansion regime, the GIRFs fall within the confidence band for the linear model.

The response of Canadian GDP is similar for the linear model and the GIRFs from both regimes of the non-linear model. The GIRFs also fall well within the confidence band for the linear model. For the interest rate difference as well, the impulse responses for both the recession and expansion regime have the same general shape as the linear response and both fall within the linear model's confidence band.

Relative to the responses from a linear model, the trade balance and real exchange rate responses move in the same direction – a depreciation of the exchange rate and an improvement in the U.S. trade balance – but the magnitude of the response is larger than the linear model in both cases. Moreover, the differences appear to be significant as the GIRF responses fall near or outside the boundary of the linear model confidence bands.

With the Canadian threshold variable the biggest difference from the linear model comes in the response of Canadian GDP, where the decline across both regimes of the non-linear model is sharper and more persistent than the linear model response. Both of the GIRFs fall well within the confidence bands for the linear response. For the trade balance there appears to be almost no difference between the linear response and the response in either of the regimes from the non-linear model. The same hold for the response of the interest rate difference.

### ***Shock Symmetry***

Shock symmetry means that means that the response to positive and negative shocks have the same magnitude but with opposite signs. Tables 3.8 and 3.9 show, for each regime and variable, the amount by which the magnitude of the response to a positive shock exceeds the magnitude of response to a negative shock. A positive number indicates that a positive shock has larger effect than a negative shock at that horizon, and vice versa.

With the U.S. threshold variable, the recession regime shocks show some evidence of asym-

Table 3.8: Shock Symmetry of GIRFs - U.S. Threshold Variable

	shock size	impact	4 qtr	8 qtr	12 qtr	20 qtr	peak	quarter
<b>recession regime</b>								
US GDP (\$)	\$ 1	0.04	0.50	0.11	0.35	0.10	0.72	2
	\$ 2	-0.04	0.20	0.03	0.24	0.08	0.27	2
Canada GDP (CDN \$)	\$ 1	-0.01	0.08	0.00	0.03	0.00	0.09	3
	\$ 2	0.01	0.02	0.01	0.02	0.00	0.03	2
Trade Balance (%)	1 %	-0.05	0.16	-0.10	-0.39	-0.03	-0.71	14
	2 %	0.02	0.09	-0.03	-0.22	-0.04	-0.34	14
Real Exchange Rate (%)	1 %	-0.05	-0.12	-0.09	-0.07	-0.19	-0.19	20
	2 %	0.00	-0.08	-0.13	-0.15	-0.23	-0.24	19
Interest Rate Difference (bps)	1 %	-2.30	-0.76	0.04	7.27	2.39	-11.73	13
	2 %	1.57	-0.41	-0.07	3.75	-0.28	-6.42	13
<b>expansion regime</b>								
US GDP	\$ 1	0.01	-0.11	-0.01	0.09	0.01	-0.15	1
	\$ 2	0.00	-0.05	-0.01	0.03	0.01	-0.09	1
Canada GDP	\$ 1	0.00	-0.01	0.00	-0.01	0.00	0.01	12
	\$ 2	0.00	-0.01	0.00	0.00	0.00	-0.01	1
Trade Balance	1 %	-0.03	0.05	-0.02	-0.01	0.00	0.16	1
	2 %	0.00	0.01	-0.02	-0.02	-0.02	0.09	1
Real Exchange Rate	1 %	0.02	-0.12	-0.15	-0.12	-0.11	-0.15	9
	2 %	0.01	0.00	0.04	0.06	0.11	0.11	19
Interest Rate Difference	1 %	-0.32	-4.28	-3.63	-1.18	-0.91	-10.11	1
	2 %	0.42	-2.32	-1.52	-0.50	-0.66	-4.84	1

**Notes:**

Each entry shows the amount by which the magnitude of the response to a positive shock exceeds (+), or is less than (-), the magnitude of response to a negative shock.



Table 3.9: Shock Symmetry of GIRFs - Canada Threshold Variable

	shock size	impact	4 qtr	8 qtr	12 qtr	20 qtr	peak	quarter
<b>normal regime</b>								
Canada GDP (CDN \$)	1 s.d.	0.00	0.01	0.00	0.01	0.00	0.01	12
	2 s.d.	0.00	0.00	0.01	0.01	-0.01	0.01	12
Trade Balance (%)	1 s.d.	-0.01	0.06	0.05	0.08	0.04	0.09	13
	2 s.d.	0.04	0.07	0.05	0.05	0.05	0.07	3
Interest	1 s.d.	0.59	2.06	1.19	0.57	0.19	2.06	4
Rate Difference (bps)	2 s.d.	0.70	0.93	0.36	-0.10	0.18	0.93	4
<b>high-growth regime</b>								
Canada GDP	1 s.d.	-0.01	0.00	-0.01	-0.01	-0.01	-0.02	11
	2 s.d.	0.00	0.00	0.00	-0.01	0.00	-0.01	11
Trade Balance	1 s.d.	-0.02	0.08	0.05	0.07	0.03	0.10	1
	2 s.d.	0.01	0.04	0.03	0.03	0.03	0.06	1
Interest	1 s.d.	-0.96	5.13	2.75	0.79	-0.80	-6.41	1
Rate Difference	2 s.d.	-0.03	0.95	0.71	0.15	-0.15	1.04	5

**Notes:**

Each entry shows the amount by which the magnitude of the response to a positive shock exceeds (+), or is less than (-), the magnitude of response to a negative shock.

metric effects. For U.S. GDP the effect of a positive shock is consistently larger than the effect of a negative shock and the difference is at times significant. For example, the peak difference occurs in the second quarter after the shock, when a one-dollar positive shock to government spending decreases U.S. GDP by \$0.72 more than a negative shock of the same size would increase it (a \$1.27 decrease vs. a \$0.55 increase). For a two-dollar shock, a positive shock has a peak difference of \$0.27. For Canadian GDP positive shocks consistently show larger effects as well, but the magnitudes of the differences are small: \$0.09 and \$0.03 for one- and two-dollar shocks respectively.

For the trade balance and real exchange rate the results are reversed; positive shocks consistently have smaller effects than negative shocks. The peak asymmetry for the trade balance is a difference of 0.71 percentage points in response to a 1% shock – approximately a third of the 2.30% peak effect of spending on the trade balance in the recession regime. For the real exchange rate the peak asymmetry is 0.24 percentage points in response to a 2% shock, which is much smaller relative to the almost 13% peak effect that would result from a perfectly symmetric shock. For the interest rate difference there is no pattern in the asymmetries, but the peak effect is relatively large – the magnitude of the response to a positive 1% shock is around 12 basis points smaller than the response to negative 1% shock, compared to a peak effect of 50 basis points in response to the positive shock.

In the expansion regime there is no pattern to the asymmetries and in general the magnitudes are much smaller. The peak effect for U.S. GDP is a \$0.15 lower response to positive shocks relative to negative ones and for Canadian GDP the peak difference is only \$0.01. For the real exchange rate the asymmetry is almost the same as for the recession regime while much smaller for the trade balance. The interest rate difference is the exception here, with positive shocks showing consistently lower effects than negative ones. The peak asymmetry of 10 basis points is almost half the peak response to spending shocks in the expansion regime.

Using Canadian GDP as the threshold variable, I do not find any significant asymmetries. There are no consistent patterns and the magnitudes of the differences are very small for all variables and for both regimes.

Table 3.10: Shock Linearity of GIRFs - U.S. Threshold Variable

	shock direction	impact	4 qtr	8 qtr	12 qtr	20 qtr	peak	quarter
<b>recession regime</b>								
US GDP (\$)	positive	-0.04	-0.30	-0.09	-0.07	-0.05	-0.46	2
	negative	0.11	0.31	0.06	0.15	0.00	0.45	2
Canada GDP (CDN \$)	positive	0.02	-0.06	0.00	-0.01	0.01	-0.07	2
	negative	-0.01	0.04	0.00	0.01	0.00	0.05	3
Trade Balance (%)	positive	0.04	-0.07	0.06	0.18	-0.09	0.36	14
	negative	-0.09	0.06	-0.08	-0.16	-0.06	-0.39	14
Real Exchange Rate (%)	positive	0.01	0.06	0.00	-0.04	0.12	0.12	20
	negative	-0.09	-0.02	0.06	0.12	0.21	0.21	19
Interest Rate Difference (bps)	positive	2.88	1.33	-0.22	-4.23	-1.36	-4.23	12
	negative	-4.86	0.63	0.01	2.82	3.97	-7.20	13
<b>expansion regime</b>								
US GDP	positive	0.02	0.13	-0.01	-0.09	-0.03	0.13	4
	negative	0.03	0.00	-0.01	-0.01	-0.03	0.10	3
Canada GDP	positive	0.00	0.02	0.00	0.01	0.00	0.02	4
	negative	0.00	0.01	0.00	-0.01	-0.01	0.01	4
Trade Balance	positive	0.04	-0.05	-0.01	-0.02	0.00	-0.07	2
	negative	-0.01	0.01	-0.02	-0.01	0.04	-0.08	3
Real Exchange Rate	positive	-0.03	0.23	0.36	0.35	0.28	0.37	9
	negative	-0.01	-0.01	-0.03	-0.01	-0.17	-0.17	20
Interest Rate Difference	positive	-0.50	2.16	1.02	-0.16	-0.09	-2.57	2
	negative	-1.97	-1.76	-3.21	-1.51	-0.58	-5.94	1

**Notes:**

Each entry shows the amount by which the magnitude of the response to a \$2.00 (or 2%) shock exceeds (+) or is less than (-) twice the magnitude of the response to a \$1.00 (or 1%) shock.

***Shock Linearity***

Shock linearity means that the relationship between response magnitudes is strictly proportional to the size of the shock – doubling the size of the shock doubles the size of the response. Tables 3.10 and 3.11 show, for each regime and variable, the amount by which the magnitude of the response to \$2.00 (or 2%) shock exceeds twice the magnitude of response to a \$1.00 (or 1%) shock. A positive number indicates that the response increases more than proportionately with the size of the shock, while a negative number indicates a less than proportionate response.

Table 3.11: Shock Linearity of GIRFs - Canada Threshold Variable

	shock direction	imapct	4 qtr	8 qtr	12 qtr	20 qtr	peak	quarter
<b>normal regime</b>								
Canada GDP (CDN \$)	positive	-0.01	0.01	0.01	0.00	0.02	0.02	20
	negative	0.00	-0.01	0.00	0.00	0.00	-0.01	4
Trade Balance (%)	positive	0.07	0.04	0.02	-0.02	0.00	0.07	0
	negative	-0.04	0.02	0.02	0.04	-0.02	-0.04	0
Interest Rate Difference (bps)	positive	0.31	-1.80	-0.82	-0.38	0.01	-1.84	3
	negative	0.08	0.46	0.85	0.97	0.03	1.23	11
<b>high-growth regime</b>								
Canada GDP	positive	0.01	0.00	0.00	-0.01	-0.01	-0.02	15
	negative	0.00	-0.02	0.01	-0.01	0.00	-0.02	2
Trade Balance	positive	-0.02	-0.04	-0.03	-0.07	-0.06	-0.07	12
	negative	-0.08	0.03	0.03	0.00	-0.05	-0.08	0
Interest Rate Difference	positive	-0.01	-2.50	-1.48	-0.07	0.86	3.40	1
	negative	-1.87	5.86	2.59	1.21	-0.44	-10.19	1

**Notes:**

Each entry shows the amount by which the magnitude of the response to a \$2.00 (or 2%) shock exceeds (+) or is less than (-) twice the magnitude of the response to a \$1.00 (or 1%) shock.

For the U.S. threshold variable, there aren't any consistent patterns to the results. In the recession regime there is little evidence of non-proportional responses for Canadian GDP and the real exchange rate. For U.S. GDP, the trade balance and the interest rate difference there are moderate effects. For U.S. GDP the response increases less than proportionately for positive shocks, but more than proportionately for negative shocks, with peak differences that are nearly the same at around \$0.45. For the trade balance the situation is reversed, with responses to positive shocks increasing more than proportionately. The peak difference for both positive and negative shocks is around 0.36 percentage points. For the expansion regime, there is little evidence of significant differences in the responses.

For the Canada threshold variable there is, once again, little evidence that responses are not proportional to the size of shocks across variables or regimes.

### **3.4 Conclusion**

My results in this paper show that allowing for nonlinearities in the responses to U.S. government spending shocks produces some significant differences from the responses from a linear model. Using tests for non-linear behavior I am able to reject the null of a linear model for specifications using both U.S. and Canadian GDP growth as a threshold variable. The estimated threshold value for the U.S. corresponds reasonably to a division between recession and expansion regimes. For Canada however, the estimated threshold variable more closely corresponds to normal and high-growth regimes.

When U.S. GDP growth is used to define recessionary and expansionary periods I find significant differences on impact and for several quarters in the responses of U.S. and Canadian GDP, the trade balance, the real exchange rate, and short-term interest rate differentials across regimes. The difference in responses across regimes is generally not significant after a few quarters. The response of U.S. GDP in recessionary periods and the responses of the trade balance and real exchange rate in both recessionary and expansionary periods show significant differences from linear responses in terms of the direction of effect (U.S. GDP) and the magnitude (trade balance and real exchange

rate). One issue that I need to explore more is the counterintuitive direction of responses of U.S. GDP across regimes: negative in the recession regime and positive in the expansion regime.

In addition to differences in direction and magnitude across regimes, I find evidence of other non-linearities in the responses, particularly during recessionary regimes. U.S. GDP responds more to positive spending shocks in a recession than to negative ones. However, the response to positive shocks increases less than proportionately to the size of the shock, while for negative shocks the reverse is true. For the trade balance and real exchange rate, positive shocks have smaller effects than negative ones during a recession. The trade balance response also increases more than proportionately to the size of positive shocks and less to negative shocks. In the expansionary regime there is little evidence of asymmetry or non-proportional responses.

Using Canadian GDP growth as the threshold variable, I find significant differences across regimes for the response of Canadian GDP, the trade balance, and the interest rate differential. These differences are only significant on impact, however. In addition, there is little evidence of other non-linearities in the responses in either regime. Aside from the effects on impact, there is little difference in the non-linear and linear responses for the trade balance and interest rate differential. For Canadian GDP, the response during the normal regime becomes negative much earlier than in a linear model and the magnitude is larger. In the high-growth regime, the response is larger than in a linear model for several quarters, becomes negative sooner, and then has a larger negative response than in the linear model.

## Chapter 4

### Does the Effect of Fiscal Policy in Canada Differ in Recessions and Expansions?

#### 4.1 Introduction

The recent recession has resulted in increased research into the effects of fiscal policy, most of which has focused on the United States. While this focus is understandable given the role of the U.S. in triggering the recession and its status as the world's largest economy, the recession was a global one and a number of countries instituted fiscal stimulus measures to combat the recession. In this paper I examine the effects of fiscal policy in the U.S.'s small open-economy neighbor to the north: Canada. In doing so I also consider whether the effect differs based on the state of the business cycle.

#### 4.2 Methodology

Traditional impulse responses functions are calculated by estimating model parameters for a given sample period and then using this set of parameters for forecasting iteratively one step ahead at a time. A potential problem with this approach is that any misspecification error will be compounded as the forecast horizon increases. Jordà (2005) proposes an alternative technique, which he calls "local projection," that involves estimating a new set of impulse response coefficients at each horizon. Consider a vector of endogenous variables  $y_t$ . The impulse response coefficients are obtained by estimating the following equation (abstracting from constants and other exogenous variables):

$$y_{t+h} = B_1^h y_t + B_2^h y_{t-1} + \cdots + B_p^h y_{t-p+1} + u_{t+h}, \quad h = 1 \dots H \quad (4.1)$$

where  $p$  is the lag length. Note that the superscripts on the  $B$  matrices refer to the forecast horizon and are not exponents and  $B_1^0 = I$  for  $h = 0$ . The impulse responses at horizon  $h$  are the elements of the matrix  $B_1^h$ . The timing notation illustrates the relationship between local projection IRFs (LP IRFs) and traditional IRFs calculated from VAR coefficients. When  $h = 0$ , the system in (4.1) coincides with a VAR. This implies that for the first period after the shock (and trivially on impact) the IRFs will be the same for both methods. But for  $h > 1$ , i.e. starting with the second period of the forecast horizon, they will generally not be the same.

Jordà (2005) uses a Monte Carlo experiment to show that LP IRFs are more robust to misspecification of the model relative to the true data generation process, including lag length, than traditional IRFs. Thus the LP approach is particularly attractive when the sample size is relatively small, since the additional of additional lags leads to less precise estimates. A downside to the approach from a practical standpoint is that you lose more observations than when estimating a VAR. For example, with a VAR( $p$ ),  $p$  are lost as pre-sample values. For LP, you need to save  $p + H + 1$  sample values.

## 4.3 Empirical Analysis

### 4.3.1 Data and Specification

I estimate the impulse responses using quarterly data for Canada for the time period 1975Q2 - 2010Q4. I limit the analysis to the effects of changes in aggregate real government spending per capita on the following variables: real GDP per capita, the GDP deflator inflation rate, the interest rate on long-term government bonds, and the real effective exchange rate. In an alternate specification I include real net exports instead of the exchange rate. Government spending, GDP, the exchange rate and net exports in natural logs. I estimate each equation in levels, including two lags of the endogenous variables, a constant, and a linear time trend.



To allow for different effects across the business cycle, I modify equation (4.1) as follows:

$$y_{t+h} = F(z_t)B_1^{R,h}y_t + [1 - F(z_t)]B_1^{E,h}y_t \quad (4.2)$$

$$+ F(z_t)B_2^{R,h}y_{t-1} + [1 - F(z_t)]B_2^{E,h}y_{t-1} + u_{t+h} \quad (4.3)$$

$$F(z_t) = \frac{\exp(-\gamma z_t)}{1 + \exp(-\gamma z_t)}, \quad \gamma > 0$$

where the matrix  $B_1^{i,h}$ ,  $i \in R, E$  gives the impulse responses at horizon  $h$  in recession ( $R$ ) and expansion ( $E$ ). The variable  $z_t$  is an index of the state of the business cycle, normalized to have zero mean and unit variance.  $F(\cdot)$  is a function describing the transition from one regime to the next and the parameter  $\gamma$  determines the smoothness of the transition between the regimes.  $F(\cdot)$  takes on values between zero and one, which indicate the likelihood that the economy is in recession. For  $z_t$  I use an eight-quarter moving average of the quarterly growth rate of real GDP. I set  $\gamma$  equal to 1.3 so that that the economy spends about 16 percent of the time in a recessionary regime, with a recessionary regime defined as  $F(z_t) > 0.8$ . This is consistent with the actual fraction of recessionary quarters in Canada over my sample period. Figure 4.1 shows a plot of  $F(z_t)$  against recession dates for the Canadian economy.<sup>1</sup>

### 4.3.2 Results

#### *VAR vs. Local Projection IRFs*

I begin by comparing the responses to changes in government spending estimated by linear local projections and traditional IRFs from a linear VAR(2). In both cases government spending shocks are identified by a Cholesky decomposition with government spending ordered first. Graphs of the impulse responses can be found in Figure 4.2. In the graphs, the shaded area shows the 95% asymptotic confidence interval for the linear VAR and the dashed lines show the same confidence

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<sup>1</sup> The recession dates are those identified by the Economic Cycles Research Institute (ECRI) using a methodology similar to that used to identify business cycle dates in the U.S. See <http://www.businesscycle.com>.

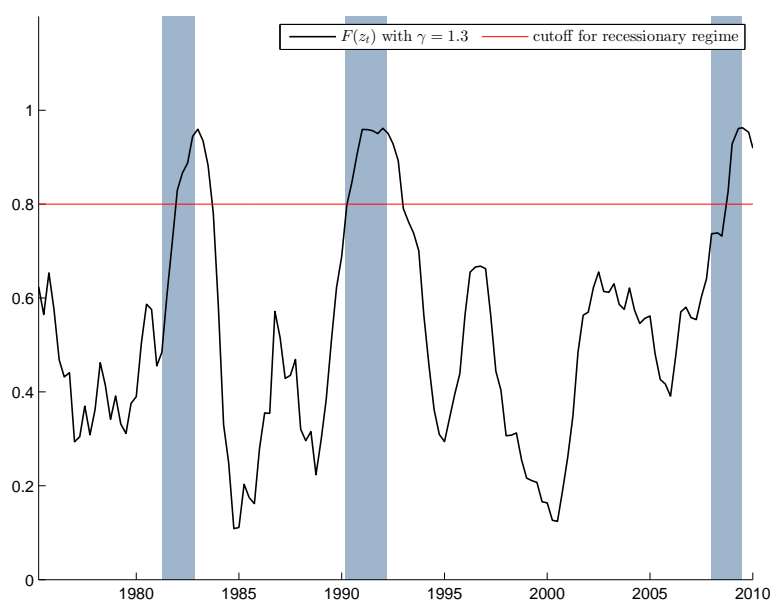


Figure 4.1: Identified Recessionary Regimes vs. Canadian Recession Dates

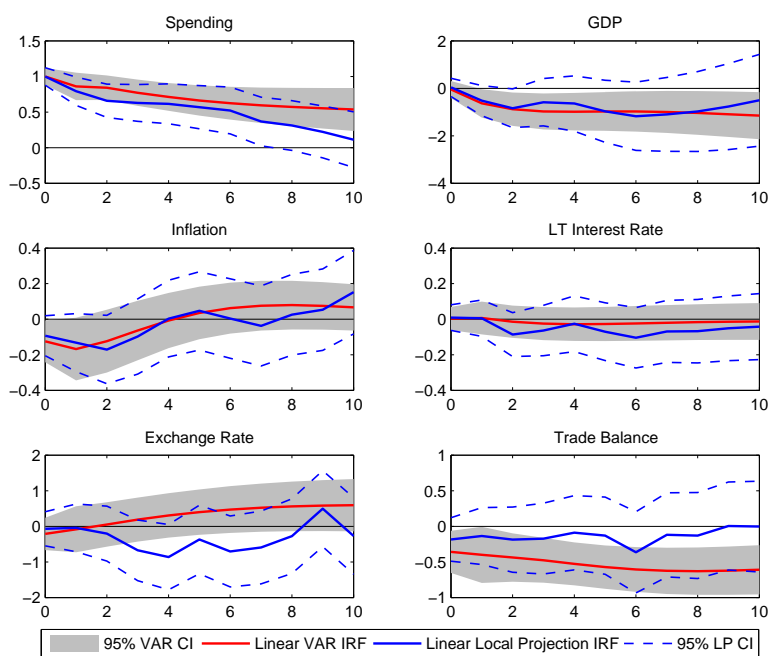


Figure 4.2: VAR IRF vs. Local Projection IRF

Each graph displays the response to a positive 1% shock to government spending.

interval for the LP IRFs.<sup>2</sup> A comparison of peak, average and cumulative responses across all specifications is given in Table 4.1, while estimates for the entire response horizon are detailed in Table 4.2.

An apparent difference between the two is that the VAR IRFs are smoother, by nature of how they are calculated, than the LP IRFs. The LP IRF confidence intervals are also slightly wider. For the response of spending, GDP, the inflation and interest rates the estimates are nearly the same across the entire ten-quarter response horizon. In each case the LP IRF response falls within the VAR IRF's 95% confidence interval. Going forward, I will use the LP responses as the reference unless they differ significantly from the linear VAR responses. The estimates show that Canadian GDP increases by \$0.04 in response to a \$1.00 dollar increase in government spending, but the effect is not significantly different from zero. Thereafter, the effect is negative and significant at the 10% level for much of the response horizon. Because of the narrower confidence bands on the

<sup>2</sup> The confidence intervals for the LP IRFs are calculated according to a formula given in Jordà (2009).

Table 4.1: Multipliers across models and regimes

	Peak			Average	Cumulative
	point est.	std. err.	quarter		
<b>GDP (\$)</b>					
linear VAR	-1.15**	(0.50)	(10)	-0.89	-1.26
linear LP	-1.18*	(0.72)	(6)	-0.73	-1.39
LP - Recession	-1.75**	(0.48)	(5)	-1.30	-3.69
LP - Expansion	-0.86**	(0.35)	(7)	-0.35	-0.84
<b>Inflation (% pts)</b>					
linear VAR	-0.17*	(0.09)	(1)	-0.01	-0.01
linear LP	-0.17*	(0.10)	(2)	-0.02	-0.04
LP - Recession	-0.15**	(0.06)	(2)	-0.01	-0.02
LP - Expansion	-0.20**	(0.07)	(2)	-0.01	-0.04
<b>Interest Rate (% pts)</b>					
linear VAR	-0.03	(0.05)	(4)	-0.02	-0.02
linear LP	-0.10	(0.08)	(6)	-0.05	-0.10
LP - Recession	-0.16**	(0.05)	(2)	-0.10	-0.27
LP - Expansion	-0.13**	(0.05)	(6)	-0.06	-0.15
<b>Exchange Rate (%)</b>					
linear VAR	0.59*	(0.37)	(10)	0.31	0.44
linear LP	-0.86*	(0.46)	(4)	-0.33	-0.62
LP - Recession	-1.11**	(0.22)	(4)	-0.48	-1.36
LP - Expansion	1.02**	(0.34)	(9)	-0.14	-0.33
<b>Trade Balance (%)</b>					
linear VAR	-0.63**	(0.17)	(8)	-0.53	-0.76
linear LP	-0.36	(0.28)	(6)	-0.14	-0.26
LP - Recession	-0.33*	(0.18)	(6)	-0.05	-0.14
LP - Expansion	-0.32**	(0.16)	(6)	-0.05	-0.12

Table 4.2: IRFs across models and regimes

Model	quarter										
	0	1	2	3	4	5	6	7	8	9	10
	<b>GDP (\$)</b>										
linear VAR	-0.05	-0.63**	-0.88**	-0.97**	-0.98**	-0.98**	-0.98**	-1.00**	-1.04**	-1.09**	-1.15**
linear LP	0.04	-0.52*	-0.84**	-0.59	-0.64	-0.97*	-1.18*	-1.10*	-0.98	-0.77	-0.51
LP - Recession	-0.34*	-1.15**	-1.56**	-1.40**	-1.44**	-1.75**	-1.71**	-1.50**	-1.41**	-1.19**	-0.89*
LP - Expansion	0.33*	-0.15	-0.43*	-0.04	-0.15	-0.50*	-0.84**	-0.86**	-0.61*	-0.39	-0.17
	<b>Inflation (% pts)</b>										
linear VAR	-0.12**	-0.17*	-0.12*	-0.06	-0.01	0.04	0.06	0.08	0.08	0.07	0.07
linear LP	-0.09*	-0.13*	-0.17*	-0.10	0.00	0.05	0.00	-0.04	0.03	0.05	0.15
LP - Recession	-0.07	-0.09*	-0.15**	-0.08	-0.01	0.04	0.02	-0.01	0.08	0.06	0.13*
LP - Expansion	-0.15**	-0.19**	-0.20**	-0.12*	0.05	0.09*	0.03	-0.01	0.04	0.09*	0.20**
	<b>Interest Rate (% pts)</b>										
linear VAR	0.00	0.01	-0.01	-0.02	-0.03	-0.03	-0.02	-0.02	-0.02	-0.01	-0.01
linear LP	0.01	0.01	-0.09*	-0.06	-0.03	-0.07	-0.10	-0.07	-0.07	-0.05	-0.04
LP - Recession	-0.01	-0.03	-0.16**	-0.13**	-0.09*	-0.14**	-0.15**	-0.09*	-0.09*	-0.07*	-0.08*
LP - Expansion	0.03	0.01	-0.07*	-0.06*	-0.04	-0.09**	-0.13**	-0.10**	-0.10**	-0.08*	-0.04
	<b>Exchange Rate (%)</b>										
linear VAR	-0.21	-0.08	0.05	0.19	0.31	0.40	0.47*	0.52*	0.56*	0.58*	0.59*
linear LP	-0.07	-0.04	-0.20	-0.67*	-0.86*	-0.37	-0.71*	-0.59	-0.28	0.50	-0.27
LP - Recession	-0.12	-0.17	-0.37*	-0.88**	-1.11**	-0.56**	-0.76**	-0.76**	-0.49*	0.37*	-0.43*
LP - Expansion	-0.02	-0.07	-0.35	-0.81**	-0.85**	-0.23	-0.57*	-0.16	0.35	1.02**	0.19
	<b>Trade Balance (%)</b>										
linear VAR	-0.36**	-0.40**	-0.44**	-0.48**	-0.53**	-0.57**	-0.61**	-0.62**	-0.63**	-0.62**	-0.61**
linear LP	-0.18	-0.14	-0.19	-0.17	-0.09	-0.13	-0.36	-0.12	-0.13	0.01	-0.00
LP - Recession	0.04	0.10	0.05	0.03	0.05	-0.12	-0.33*	-0.15	-0.14	-0.05	-0.04
LP - Expansion	-0.21*	-0.11	-0.04	0.08	0.10	0.04	-0.32**	-0.07	-0.09	0.07	0.02

linear VAR IRFs, the effect is significant at the 5% level except on impact. The peak effect is a decrease in GDP of \$1.18 in the sixth quarter after the shock. The immediate and peak response of inflation is a decrease of 0.17 percentage points, significant at the 10% level. The effect remains negative and significant for the next two quarters, but thereafter is close to zero and not significant. The effect on the interest rate is slightly negative, peaking at a decrease of 0.10 percentage points, but is never significantly.

For the exchange rate and trade balance there's more divergence between the VAR and LP impulse responses. In the VAR model, the response of the exchange rate is small and negative on impact but the effect becomes positive after a quarter, peaking at an appreciation of 0.59 percentage points in the tenth quarter after the shock. The peak effect is significant at the 10% level. In the LP model, the effect is negative over the entire horizon peaking at a depreciation of 0.86 percentage points. The peak effect occurs earlier, in the fourth quarter, and is significant at the 10% level. In addition, the LP response falls outside the 95% confidence interval for the VAR response in quarters three through eight. For the trade balance, the VAR response is persistently negative and significant at the 95% level, with a peak decrease of 0.63 percentage points in the eighth quarter after the shock. The LP response is also negative but smaller in magnitude, often twice as small, with a peak decrease of .36 percentage points, but the effect is never significant. The LP response does, however, fall outside the VAR 95% confidence band after the fourth quarter.

### ***Recession vs. Expansion Regimes***

Figure 4.3 shows the impulse responses, along with 95% confidence bands, for the recession and expansion regimes. Figures 4.4 and 4.5 show the recession and expansion regimes compared to the linear LP responses. The most significant difference across regimes, and relative to the linear model, is in the response of GDP. In the recessionary regime, GDP falls on impact by \$0.34 in response to a \$1.00 increase in government spending, while in the expansionary regime it increases by \$0.33. In both cases the response on impact is significant at the 10% level. In a recession, the response continues to be negative, peaking at a decrease of \$1.75 in the fifth quarter after the shock.

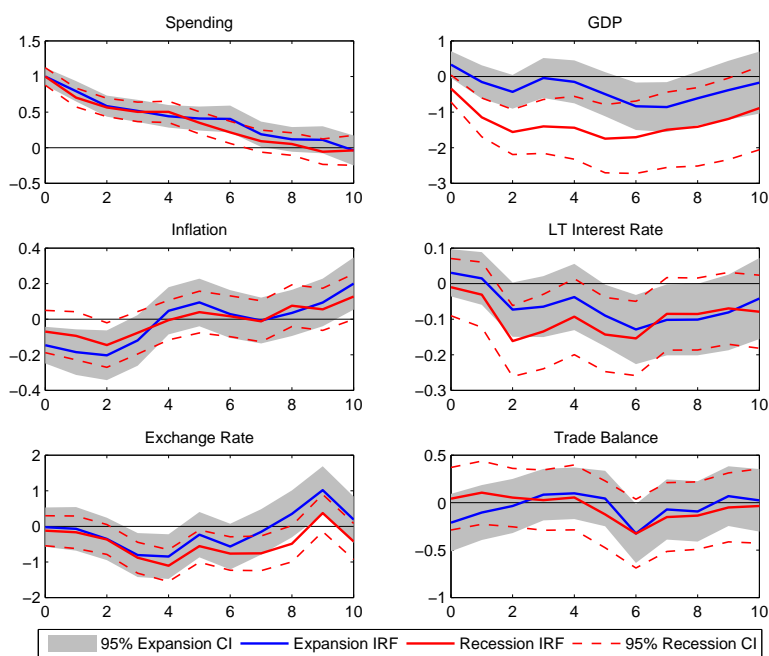


Figure 4.3: Recession Response vs. Expansion Response

Each graph displays the response to a positive 1% shock to government spending.

Thereafter it increases through the tenth quarter, ending at a decrease of \$0.89. The response is significant over the entire horizon, and on impact through the third quarter there is no overlap in the confidence intervals for the recession and expansion responses. The recession response is more negative and significantly different than the response from the linear model as well, on impact and for the following two quarters. In the expansion regime, after the initial positive response, the effect becomes negative, peaking at a decrease of \$0.86 in the seventh quarter. Relative to the response from the linear model, the expansion response is less negative but never falls outside the linear model's 95% confidence interval.

For inflation, the response in both the recession and expansion regimes is qualitatively the same as the linear model – a small decrease which dissipates within three quarters. In both regimes the effect is significantly different than zero through two quarters after the shock, but the difference across regimes is never more than 0.10 percentage points and is not significant. Quantitatively, the response in both recessions and expansions is nearly the same as the response in the linear model.

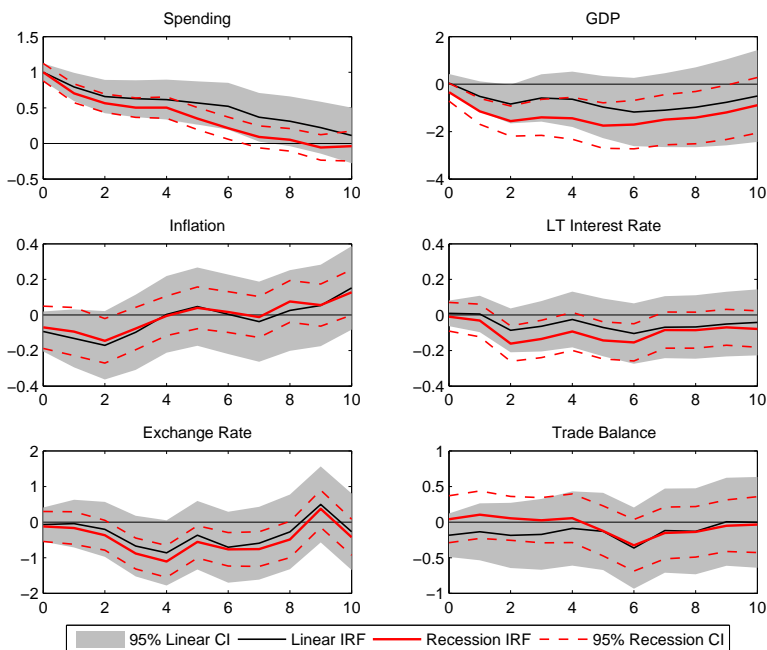


Figure 4.4: Recession Response vs. Linear Response

Each graph displays the response to a positive 1% shock to government spending.

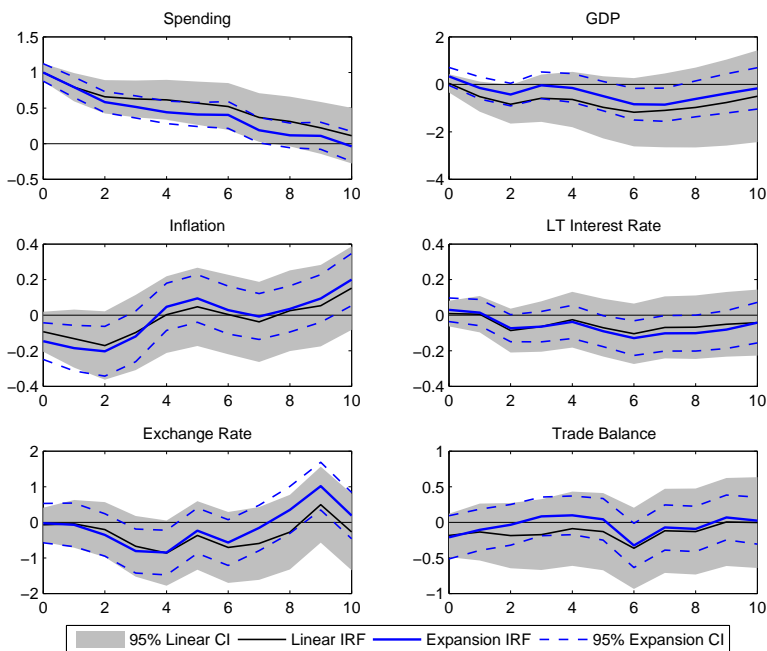


Figure 4.5: Expansion Response vs. Linear Response

Each graph displays the response to a positive 1% shock to government spending.



For the interest rate, the response in recessions and expansions follows the same pattern, with the response in the recession regime more negative through six quarters. The difference between the two regimes is only significant when the recession response peaks at -0.16 percentage points in the second quarter. This peak, while larger in magnitude than the linear response, still falls within the 95% confidence interval for the linear response. The response in the expansion regime is nearly identical to the linear response.

The response of the exchange rate in recessions and expansions is nearly zero on impact and negative for the first three quarters after the shock. Thereafter, the recession response remains negative, falling outside the expansion confidence interval after the seventh quarter. In contrast, the expansion response becomes positive and peaks at an appreciation of 1.02% in the ninth quarter after the shock. The recession response is nearly identical to the linear response, while the expansion response is more positive after the sixth quarter. Neither the recession nor the expansion responses fall outside the 95% confidence interval for the linear response.

There is virtually no effect on the trade balance on impact in the recession regime, while in the expansion regime there is a decrease of 0.21%, which is significant at the 10% level. In the recession regime the effect hovers around zero but becomes negative after the fifth quarter, with a peak effect of -0.32% in the sixth quarter after the shock. Though the expansion response begins negative, it returns to near zero after two quarters and is nearly the same as the recession response thereafter. Neither response ever falls outside the 95% confidence interval for the linear response.

#### 4.4 Conclusion

In this paper I have estimated the response of several Canadian macroeconomic series to shocks in government spending using local projections. These responses are more robust to misspecification of the empirical model than traditional IRFs calculated from a VAR. I found, however, that in most cases there was no significant difference in the LP responses and VAR IRFs. The exceptions were the real exchange rate and the trade balance. Then, using local projection IRFs, I estimated the response in recession and expansion. With the notable exception of GDP, I found

that the responses across regimes was generally not significant in a statistical or qualitative sense and not statistically different from the linear LP response. For GDP I found that the response to government spending shocks is negative in both recessions and expansions, but the magnitude of the response in recession is significantly larger. The recession response is significantly different, and larger, than the response in the linear LP model as well.

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## Appendix A

### Data Appendix

#### A.1 Data and Tests for Chapter 2

##### Variable Descriptions and Sources

Data for the U.S. variables are from the U.S. Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA). Bilateral trade data are from the BEA's International Transactions Accounts. Foreign data is from the International Monetary Fund's International Financial Statistics Database. U.S. NIPA and IMF data are seasonally adjusted by the source. International trade are seasonally adjusted by me using the U.S. Census Bureau's X12-ARIMA.<sup>1</sup> Variable names and definitions are presented in Table A.1.

All real variables except the real exchange rates are calculated as nominal variables deflated by the appropriate GDP deflator. U.S. real net taxes are from NIPA Table 3.1 and calculated as the sum of Current Tax Receipts (line 2), Contributions for Government Social Insurance (line 7) and Current Transfer Receipts (line 11) minus the sum of Current Transfer Payments (line 17), Interest Payments (line 22) and Subsidies (line 25). Interest rate differentials are calculated as the U.S. rate minus the foreign rate. Ex-post real interest rates are calculated by subtracting the year-over-year CPI inflation rate. The real bilateral exchange rate is calculated as:

$$\text{Nominal Exchange Rate (units of foreign / \$ U.S.)} \times \frac{\text{US CPI}}{\text{Foreign CPI}}.$$

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<sup>1</sup> I used the IRIS Toolbox for Matlab for this. <https://code.google.com/p/iris-toolbox-project/>.

Table A.1: Variable Definitions

Variable	Definition
usrgdp	Natural log of U.S. Real GDP per capita
usrgs	Natural log of U.S. Real Government Consumption and Investment per capita
usrnt	Natural log of U.S. Real Net Taxes per capita
cargdp	Natural log of Canada Real GDP per capita
jprgdp	Natural log of Japan Real GDP per capita
ukrgdp	Natural log of U.K. Real GDP per capita
castrd	U.S./Canada short-term real interest rate differential
jpstrd	U.S./Japan short-term real interest rate differential
ukstrd	U.S./U.K. short-term real interest rate differential
carxr	Natural log of U.S./Canada bilateral real exchange rate
jprxr	Natural log of U.S./Japan bilateral real exchange rate
ukrxr	Natural log of U.S./U.K. bilateral real exchange rate
catb	Natural log of U.S./Canada real bilateral trade balance
jptb	Natural log of U.S./Japan real bilateral trade balance
uktb	Natural log of U.S./U.K. real bilateral trade balance

The trade balance is calculated as the log ratio of exports to imports. In order to maintain consistent orders of magnitude across variables, all bilateral trade balances and the real exchange rates for Canada and the U.K. were multiplied by 10.

### Unit Root Tests

Each variable was tested for the presence of a unit root using the Dickey-Fuller GLS and KPSS tests. For the DF-GLS the null hypothesis is a unit root. For the KPSS test the null hypothesis is stationarity. The tests were conducted in Stata using the `dfgls` and `kpss` commands.

Table A.2: Unit Root Tests

Variable	DF-GLS	KPSS
<i>with trend</i>		
<code>usrgdp</code>	reject at 5%	fail to reject at 10%
<code>usrqs</code>	fail to reject at 10%	reject at 1%
<code>usrnt</code>	fail to reject at 10%	reject at 1%
<code>cargdp</code>	fail to reject at 10%	reject at 1%
<code>jprgdp</code>	fail to reject at 10%	reject at 1%
<code>ukrgdp</code>	fail to reject at 10%	reject at 1%
<i>no trend</i>		
<code>catb</code>	fail to reject at 10%	reject at 1%
<code>jptb</code>	fail to reject at 10%	fail to reject at 10%
<code>uktb</code>	fail to reject at 10%	reject at 10%
<code>carxr</code>	fail to reject at 10%	reject at 1%
<code>jprxr</code>	fail to reject at 10%	reject at 1%
<code>ukrxr</code>	fail to reject at 10%	reject at 1%
<code>castrd</code>	fail to reject at 10%	reject at 10%
<code>jpstrd</code>	fail to reject at 10%	fail to reject at 10%
<code>ukstrd</code>	reject at 1%	reject at 1%

### Cointegration Tests

Given the evidence of non-stationarity in most of the variables, each specification was tested for cointegration using the Johansen methodology. The tests were conducted in Stata using the `vecrank` command. Each specification included U.S. government spending, net taxes, GDP, and foreign GDP, with the trade balance, real exchange rate and interest rate differential rotated in as the fifth variable.



Table A.3: Cointegration Tests

Specification	Test	$r$	Test Statistic	5% Critical Value
<b>Canada</b>				
Trade Balance	Trace	0	73.92	68.52
		1	44.44*	47.21
	Max. Eigenvalue	0	29.48*	33.46
		1	26.59	27.07
Exchange Rate	Trace	0	83.37	68.52
		1	44.68*	47.21
	Max. Eigenvalue	0	38.70	33.46
		1	24.02*	27.07
Rate Spread	Trace	0	75.22	68.52
		1	38.39*	47.21
	Max. Eigenvalue	0	36.82	33.46
		1	21.06*	27.07
<b>Japan</b>				
Trade Balance	Trace	0	68.94	68.52
		1	39.79*	47.21
	Max. Eigenvalue	0	29.15*	33.46
		1	17.29	27.07
Exchange Rate	Trace	0	71.22	68.52
		1	39.62*	47.21
	Max. Eigenvalue	0	31.60*	33.46
		1	19.34	27.07
Rate Spread	Trace	0	77.88	68.52
		1	44.37*	47.21
	Max. Eigenvalue	0	33.51	33.46
		1	25.76	27.07
<b>U.K.</b>				
Trade Balance	Trace	0	75.46	68.52
		1	33.21*	47.21
	Max. Eigenvalue	0	42.24	33.46
		1	17.24*	27.07
Exchange Rate	Trace	0	75.26	68.52
		1	30.29*	47.21
	Max. Eigenvalue	0	45.00	33.46
		1	16.35*	27.07
Rate Spread	Trace	0	70.07	68.52
		1	29.86*	47.21
	Max. Eigenvalue	0	40.21	33.46
		1	17.56*	27.07

## VAR Specification

Although there are some ambiguities in the cointegration test results, most specification seem to have one cointegrating relationship. Therefore, the models are estimated in levels rather than first differences. VAR specifications were chosen as the minimum lag length for which the residuals were uncorrelated.

## A.2 Data and Tests for Chapter 3

### Variable Descriptions and Sources

The variables used in Chapter 3 are the same as those used in Chapter 2. See description in section A.1, above.

### Unit Root Tests

See unit root tests in section A.1, above.

### Cointegration Tests

Each specification included U.S. government spending, GDP, and Canadian GDP, with the trade balance, real exchange rate and interest rate differential rotated in as the fourth variable.

Table A.4: Cointegration Tests

Specification	Test	$r$	Test	5%
			Statistic	Critical Value
Trade Balance	Trace	0	43.58*	68.52
		1	17.36	47.21
	Max. Eigenvalue	0	26.22*	33.46
		1	12.33	27.07
Exchange Rate	Trace	0	56.09*	68.52
		1	19.10	47.21
	Max. Eigenvalue	0	36.99	33.46
		1	11.68*	27.07
Rate Spread	Trace	0	53.65*	68.52
		1	19.13	47.21
	Max. Eigenvalue	0	34.52	33.46
		1	15.84*	27.07

## VAR Specification

Although there are some ambiguities in the cointegration test results, most specification seem to have one cointegrating relationship. Therefore, the models are estimated in levels rather than first differences. VAR specifications were chosen as the minimum lag length for which the residuals were uncorrelated.

### A.3 Data and Tests for Chapter 4

#### Variable Descriptions and Sources

Data are from Statistics Canada, except the long-term interest rate which is from the IMF's International Financial Statistics. All real variables are calculated as nominal variables deflated by the GDP deflator. The GDP deflator inflation rate is calculated year-over-year.

Table A.5: Variable Definitions

Variable	Definition
<code>cargs</code>	Natural log of Canada Real Government Consumption and Investment per capita
<code>cargdp</code>	Natural log of Canada Real GDP per capita
<code>deflinfl</code>	Canada GDP deflator inflation rate
<code>ltintrate</code>	Nominal interest rate on long-term government bonds
<code>canx</code>	Natural log of real Net Exports
<code>carxr</code>	Natural log of Real Effective Exchange Rate

#### Unit Root Tests

Table A.6: Unit Root Tests

Variable	DF-GLS	KPSS
<i>with trend</i>		
<code>cargs</code>	fail to reject at 10%	reject at 1%
<code>cargdp</code>	fail to reject at 10%	reject at 1%
<i>no trend</i>		
<code>deflinfl</code>	fail to reject at 10%	reject at 1%
<code>ltintrate</code>	fail to reject at 10%	reject at 1%
<code>canx</code>	fail to reject at 10%	fail to reject at 10%
<code>carxr</code>	fail to reject at 10%	reject at 5%

## Cointegration Tests

Each specification includes Canada government spending, GDP, GDP deflator inflation rate, and long-term interest rate, with real effective exchange rate and trade balance rotated in as the fifth variable.

Table A.7: Cointegration Tests

Specification	Test	$r$	Test	5%
			Statistic	Critical Value
Exchange Rate	Trace	0	76.84	68.52
		1	41.36*	47.21
	Max. Eigenvalue	0	35.48	33.46
		1	23.11*	27.07
Trade Balance	Trace	0	85.21	68.52
		1	39.72*	47.21
	Max. Eigenvalue	0	45.49	33.46
		1	23.41*	27.07

## VAR Specification

For both specifications the tests indicate a single cointegrating relationship. Therefore, the models are estimated in levels rather than first differences. VAR specifications were chosen as the minimum lag length for which the residuals were uncorrelated.