Spring 1-1-2018

Essays in Real Estate Economics

Jordan Michael Martel
University of Colorado at Boulder, jordan.martel@colorado.edu

Follow this and additional works at: https://scholar.colorado.edu/badm_gradetds

Part of the Finance Commons, and the Real Estate Commons

Recommended Citation
https://scholar.colorado.edu/badm_gradetds/7

This Dissertation is brought to you for free and open access by Business Administration at CU Scholar. It has been accepted for inclusion in Business Administration Graduate Theses & Dissertations by an authorized administrator of CU Scholar. For more information, please contact cuscholaradmin@colorado.edu.
Essays in Real Estate Economics

by

Jordan Michael Martel

B.S., Arizona State University, 2011

M.S., University of Colorado at Boulder, 2013

A thesis submitted to the

Faculty of the Graduate School of the

University of Colorado in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

Leeds School of Business

2018
This thesis entitled:
Essays in Real Estate Economics
written by Jordan Michael Martel
has been approved for the Leeds School of Business

Prof. Edward Van Wesep

Prof. Diego Garcia

Prof. Nathalie Moyen

Date

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.
My dissertation studies the role of informational and institutional determinants of house prices and liquidity.

In the first chapter, I quantify the effect of buyers’ uncertainty about a home’s quality on its pricing and liquidity. I first develop a parsimonious model of home sale in which buyers learn about the home’s quality from a walkthrough and from the home’s days-on-market. Quality uncertainty distorts prices and delays trade. As time passes, buyers become pessimistic and sale prices fall. Using transaction-level data from Denver, Charlotte, and Detroit, I structurally estimate counterfactual days-on-market and sale prices for six hundred thousand homes that sold between 2005 and 2015. The costs of quality uncertainty are borne by sellers of high quality homes through mispricing, and by sellers of low quality homes through illiquidity: relative to a market in which home quality were known to buyers, high quality homes take 11 days longer to sell, and sell for 8.0% less, while low quality homes take 47 days longer to sell, and sell for 4.5% more.

In the second chapter, I discuss work co-authored with Edward Van Wesep. We develop a model of asset pricing in which lenders value collateral using appraisals, defined as average selling prices of similar assets in recent transactions. The model features price-predictability, differential pricing for identical assets, “buyer’s and “seller’s markets, and associations between price appreciation, volume, and liquidity. We find support for the model’s predictions in the markets for New York City taxi medallions, used airplanes, commercial and industrial real estate, and single-family homes.

In the third chapter, I investigate the internal finance of customer-owned firms. Customer owned financial institutions originate 11% of home loans. I propose a theory of internal finance for the customer owned firm. I show that its growth, pricing, and capital structure are tied together:
higher sales tomorrow are achieved through higher prices today and lower leverage today. This result does not hold for a shareholder owned firm. I document stylized facts from the credit union industry and find that they are consistent with the theory’s predictions. I discuss empirical implications for other customer owned firms.
Dedication

To my parents, who taught me to seek character before credentials.
Acknowledgements

I am deeply grateful to my adviser, Edward Van Wesep, who shared with me his love of research and who taught me to think like an economist. I am grateful to all of the faculty in the finance division who trained, mentored, and encouraged me. I will forever be in their debt.
Contents

Chapter

1 Quality Uncertainty in Housing Markets

1.1 Introduction ........................................... 1

1.2 The Model ........................................... 6
  1.2.1 Setup ........................................... 6
  1.2.2 Seller Behavior ................................... 7
  1.2.3 Buyer Behavior ................................... 8
  1.2.4 Beliefs .......................................... 8
  1.2.5 Pricing .......................................... 10
  1.2.6 Moments ........................................ 12
  1.2.7 Idiosyncratic Value .............................. 13

1.3 Empirical Facts ...................................... 14
  1.3.1 Data ........................................... 14
  1.3.2 Days-on-Market and the Sale-to-List Ratio ....... 15

1.4 Estimation ........................................... 15
  1.4.1 Heterogeneity ................................... 16
  1.4.2 Estimator ....................................... 17
  1.4.3 Parameter Estimates ............................. 18
  1.4.4 Counterfactuals ................................. 20

1.5 Dynamic Adverse Selection .......................... 21
1.5.1 The Model .................................................. 22
1.5.2 Equilibrium Construction ................................. 23
1.5.3 Discussion .................................................. 25
1.6 Conclusion ..................................................... 26

2 Constrained Asset Prices ................................. 37
  2.1 Introduction ............................................... 37
  2.2 Literature .................................................. 41
  2.3 The Model .................................................. 42
     2.3.1 Setup ............................................... 42
     2.3.2 Constrained Prices ................................. 43
     2.3.3 Naïve Agents ....................................... 45
     2.3.4 Rational Agents .................................... 47
  2.4 Empirical Facts ........................................... 58
     2.4.1 Residential Real Estate ............................. 58
     2.4.2 Alternative Assets .................................. 63
     2.4.3 Extension to Negative Shocks ...................... 65
  2.5 Conclusion ................................................ 66

3 The Internal Finance of Customer Owned Firms .......... 95
  3.1 Introduction ............................................... 95
  3.2 Literature .................................................. 98
  3.3 The Model .................................................. 101
     3.3.1 What’s in a Price? ................................. 104
     3.3.2 Growth, Pricing and Leverage ...................... 108
  3.4 Applications ............................................... 112
     3.4.1 Credit Unions ...................................... 112
     3.4.2 Mutual Insurance Companies ....................... 114
Tables

1.1 **Summary Statistics.** This table presents summary statistics for my sample of parcels. Panel A shows sample means for Denver, Charlotte, and Detroit. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home.

1.2 **Days-on-Market and Sale-to-List.** This table presents regressions of the sale-to-list ratio on days-on-market. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home. For ease of interpretation, the sale-to-list ratio is quoted in points. For example, a 0.02 coefficient on days-on-market means that for each additional day-on-market, the sale-to-list ratio declines by 2 basis points. Standard errors are in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.
1.3 Maximum Likelihood Estimates. This table presents maximum likelihood estimates. The likelihood is defined in Equation 1.20. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home; 1st–4th Quarter denote dummies for the quarter in which the home was listed. For ease of interpretation, I demean log(Age) and log(Age). Panel A displays estimates for the sale-to-list ratio volatility, $\sigma$. Panels B, C, and D correspond to the specifications in Equations 1.16, 1.17, and 1.18 respectively: Panel B displays estimates for the buyer arrival rate, $\lambda$; Panel C displays estimates for the quality premium, $\eta \equiv v_H/v_L$; Panel D displays estimates for the signal-to-noise ratio, $\phi \equiv (v_H - v_L)/\sigma_x$. Standard errors are computed using the asymptotic variance of the maximum likelihood estimator, and are shown in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.

1.4 Days-on-Market Counterfactuals. This table presents the results of hedonic regressions. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home. In the first row of Panels A and B, I estimate the mean days-on-market for each metropolitan area (compare with Table 1.1). In the second row, I estimate the mean days-on-market for an $H$ quality home (Panel A) and an $L$ quality home (Panel B). In the third row, I estimate the mean counterfactual days-on-market for an $H$ quality home (Panel A) and an $L$ quality home (Panel B). In the fourth row, I estimate the percent difference in the mean days-on-market from the counterfactual to the factual. Standard errors are computed using the asymptotic variance of the maximum likelihood estimator, and are shown in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.
1.5 **Sale Price Counterfactuals.** This table presents the results of hedonic regressions. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home. Prices are quoted in thousands of US dollars. In the first row of Panels A and B, I estimate the mean sale price for each metropolitan area (compare with Table 1.1). In the second row, I estimate the mean sale price for an $H$ quality home (Panel A) and an $L$ quality home (Panel B). In the third row, I estimate the mean counterfactual sale price for an $H$ quality home (Panel A) and an $L$ quality home (Panel B). In the fourth row, I estimate the percent difference in the mean sale price from the counterfactual to the factual. Standard errors are computed using the asymptotic variance of the maximum likelihood estimator, and are shown in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.

2.1 **Taxi Medallions.** This table shows results of AR(1) regressions of monthly and annual excess returns for individual and corporate New York City Taxi medallions. Monthly risk-free rates come from Kenneth French’s data library. Return data are from the New York City Taxi and Limousine Commission, and cover the period January, 1992 to December, 2018.

2.2 **Aircraft.** This table shows results of AR(1) regressions of quarterly and annual excess returns for seven types of aircraft. Monthly risk-free rates come from Kenneth French’s data library. Return data are from VREF Aircraft Value Reference, and cover the period 1994 Q4 to 2016 Q4.

2.3 **Multi-Family Residential and Commercial Real Estate.** This table shows results of AR(1) regressions of excess monthly and annual returns for several types of multi-family residential and commercial real estate. Monthly risk-free rates come from Kenneth French’s data library. Data are from Moody’s/Real Capital Analytics, and cover the period December 2000 to December 2015.
2.4 **NYSE Seats.** This table shows results of AR(1) regressions of excess monthly and annual returns for seats on the New York Stock Exchange. Data were graciously provided by Asaf Bernstein, and cover the period January 1888 to December 1925.

3.1 **Loan Growth, Net Interest Margins, and Capital Ratios.** This table displays loan growth regressions. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter $t$ is the aggregate change in loans from quarter $t$ to quarter $t + 1$ divided by the aggregate loans in quarter $t$. The net interest margin in quarter $t$ is the aggregate net interest income in quarter $t$ divided by the aggregate loans in quarter $t$. The capital ratio in quarter $t$ is the aggregate equity capital in quarter $t$ divided by the aggregate loans in quarter $t$. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.
Figures

1.1 **Days-on-Market and Sale-to-List Ratio.** These plots compare the model fit with a LOESS fit. The left shows the empirical PDF of days-on-market. The right plots show the expected sale-to-list ratio for each day-on-market. .......................... 29

1.2 **Dynamic Adverse Selection (Section 1.5).** Panel (a) shows $H$’s utility when she passively lists according to $\bar{p}$. (c) shows her utility when she optimally withdraws at $\pi_H$. Panel (b) shows $L$’s utility when she passively lists according to $\bar{p}$. Panel (d) shows her utility when she optimally capitulates at $\pi_L$. Panel (e), illustrates the equilibrium when $\pi_L < \pi_H$, while Panel (f), illustrates the equilibrium when $\pi_H < \pi_L$. 30

2.1 **Predictability.** ................................................................. 80

2.2 **Prices and Returns.** The top panel plots the total return (price appreciation plus rental income) against the rent-to-price ratio for the 200 largest US metropolitan areas between 2010 and 2013. The coefficient on the rent-to-price ratio is a significant 1.28 ($t = 5.58$). The bottom panel plots price appreciation against the rent-to-price ratio. The coefficient on the rent-to-price ratio is an insignificant .33 ($t = 1.44$). Data are from Zillow. Therefore, lower-priced metro areas earn substantially higher total returns. ................................................................. 82
2.3 **Turnover.** This figure plots the average turnover (percent of homes in a metropolitan area that have changed hands in the last year) against yearly price appreciation for the 200 largest US metropolitan areas between 2010 and 2014. Data are from Zillow. When prices are rising (falling), they rise (fall) faster when turnover is higher. 83

2.4 **Days on Market.** This figure plots the average days on market against yearly price appreciation for the 200 largest US metropolitan areas between 2010 and 2013. Data are from Zillow. Days on market appears to have a floor, which corresponds with the fact that there is a minimum time between when a house is listed for sale and when a sale can close, during which the house is labeled on the market even though it is under contract. Days on market is lower when prices are appreciating faster, consistent with the existence of buyers’ and sellers’ markets. 84

2.5 **Buyers’ and Sellers’ Markets.** This figure plots the time series for the monthly supply of housing inventory, and the time series of house prices, as measured by the Case-Shiller 20-City Composite, between January 1987 and July 2015. Data are from US Census Bureau and Core Logic, respectively. Inventory appears to have a floor, which corresponds with the fact that there is a minimum time between when a house is listed for sale and when a sale can close, during which the house is labeled “inventory” even though it is under contract. Inventory rises when prices are falling, consistent with growth in a seller’s queue during down markets. Inventory is stable at its minimum level with prices are rising, consistent with the lack of a seller’s queue in rising markets. 85

2.6 **Flipping.** This figure plots flipping—defined as the fraction of sales for which the home has sold twice in one year—against yearly price appreciation for 100 US metropolitan areas between 2000 and 2016. Data are from Trulia. When prices are rising, flipping increases. 86
2.7 **Numerical Solution.** This figure shows a numerical solution of the rational model for $p_1 = 200$, $p_0 = 150$, $r = .05$, $\lambda = 1$, $L = \sqrt{p_1/p_0}$, $N = \lambda \log ((p_1 - p_0)(Lp_0 - p_0))/r$, and $C = \lambda L((N - 1)/r - (p_1 - p_0))/(2N^2)$. The top-left plot shows the price paths for the instant adjustment, naïve, and rational models. The top-right plot shows the flow of buyers and sellers to trade. The bottom-left plot shows the buyers’ and sellers’ wait times. The bottom-right plot shows the sizes of the buyers’ and sellers’ queues. Panels (b) through (d) all show paths only for the fully rational model.

2.8 **Queuing.**

2.9 **Appraisal Time Series.** The top-left panel shows the time-series of the purchase-plus-appraisal and purchase only indices between 2000 Q1 and 2016 Q3. The top-right panel shows the distribution of cumulative returns over the period 2011 Q2 to 2016 Q2 for the 100 largest US metropolitan areas for the purchase-plus-appraisal index and the purchase-only indices. The bottom-left panel compares the rate of quarterly appreciation in the purchase-only price index to the gap between the levels of the purchase-only and the purchase-plus-appraisal indices between 2000 Q1 and 2016 Q3. Data come from the FHFA. It is clear that when prices are rising or falling faster, the indices diverge by more.
3.1 **Loan Growth and Net Interest Margins.** This figure plots loan growth against the net interest margin for credit unions and banks. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter $t$ is the aggregate change in loans from quarter $t$ to quarter $t + 1$ divided by the aggregate loans in quarter $t$. The net interest margin in quarter $t$ is the aggregate net interest income in quarter $t$ divided by the aggregate loans in quarter $t$. The coefficient on the net interest margin is significant and positive for credit unions and insignificant for banks (see Table 3.1). The $R^2$ is 27.3% for the credit union regression and 1.0% for the bank regression. The solid line is the regression line for credit unions; the dashed line is the regression line for banks. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.

3.2 **Loan Growth and Capital Ratios.** This figure plots loan growth against the capital ratio for credit unions and banks. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter $t$ is the aggregate change in loans from quarter $t$ to quarter $t + 1$ divided by the aggregate loans in quarter $t$. The capital ratio in quarter $t$ is the aggregate equity capital in quarter $t$ divided by the aggregate loans in quarter $t$. The coefficient on the capital ratio is significant and positive for credit unions and insignificant for banks (see Table 3.1). The $R^2$ is 26.8% for the credit union regression and 4.6% for the bank regression. The solid line is the regression line for credit unions; the dashed line is the regression line for banks. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.
3.3 The Corporation’s (Top) and Cooperative’s (Bottom) Objectives. This figure illustrates the corporation’s and cooperative’s objectives under linear demand. The corporation maximizes the profit \( \Pi(p; c) = (p - c)D(p) \) (the dark rectangle). The cooperative’s members are customers, and so it maximizes the consumer surplus \( \int_0^1 D(s)ds \) (the light triangle), but its members are also owners, and so it maximizes the profit \( \Pi(p; c) = (p - c)D(p) \) (the dark rectangle). It should be clear that \( p > c \) is inefficient in a static setting (although it will be in a dynamic setting).
Chapter 1

Quality Uncertainty in Housing Markets

1.1 Introduction

Many qualities of a home are valued commonly among homebuyers, and uncertain without physical inspection. Examples of “good” qualities include sturdy walls, effective ventilation, and friendly neighbors. Conversely, examples of “bad” qualities include thin walls, poor ventilation, and noisy neighbors. From the existing literature, it is not clear what effect quality uncertainty has on home prices and liquidity. In this paper, I quantify the effect of quality uncertainty on homes’ sale prices and days-on-market.

I first develop a continuous-time model of home sale. A homeowner posts list prices for her home, while homebuyers randomly arrive for a walkthrough, receive noisy signals of quality, and choose whether or not to buy at the list price. Buyers form their beliefs about the home’s quality based on their signals and from the possibility that previous buyers arrived, observed some undesirable quality, and chose not to buy. As time passes, buyers become pessimistic about the home’s quality and prices fall. Under competitive pricing, the model yields a closed-form expression for the joint distribution of days-on-market and sale prices.

I then structurally estimate the model using transaction-level data from Denver, Charlotte, and Detroit. The parameter estimates describe not only the real-world, but a counterfactual in which buyers know home quality with certainty. In the counterfactual, homes take 22 days to sell. In the real-world, high quality homes take 33 days to sell, and sell for 8.0% less than in the counterfactual; low quality homes take 69 days to sell, and sell for 4.5% more than in the
counterfactual. The costs of quality uncertainty are borne by sellers of high quality homes through mispricing, and by sellers of low quality homes through illiquidity.

By incorporating home heterogeneity directly into my structural model, I am able to explore how quality uncertainty varies according to home age and size. I find that the difference between high and low quality is larger for older homes, and that walkthroughs are more informative for older and larger homes. I also document seasonal effects. The difference between high and low quality is smallest—and walkthrough informativeness is highest—for homes listed during the peak homebuying months of April, May, and June.

Seller outcomes have been shown to affect households’ financial decision-making. For example, [36] show that housing illiquidity amplified foreclosure rates and tightened credit constraints during the Great Recession. [42] estimate the welfare loss due to the illiquidity of housing to be a 1.2% reduction in permanent consumption. [18] argues that housing illiquidity may dissuade households from homeownership and other forms of financial risk-taking.

In the model, homes are of either high or low quality. They are identical in terms of those attributes that could be ascertained from an online listing, but differ in those attributes which would require a walkthrough. For example, they were built in the same year, have the same number of beds and baths, and share the same school district, but differ in the durability of appliances, the effectiveness of the heating and cooling system, or the sensibility of layout.

Other examples include how sound travels through walls; how much the temperature fluctuates depending on the time of day or season; whether the neighbors are friendly or unfriendly; whether nearby traffic is heavy or light, and at what times; how major appliances perform during various seasons and conditions. Note that these attributes may even evade inspectors. For example, two inspectors might give the same assessment about the quality of a faucet, but very different assessments about the quality of a heating and cooling system (the faucet being simpler to assess than the heating and cooling system).

In the model, a buyer forms her belief about quality based on (1) a direct signal of quality obtained from her walkthrough, and (2) the home’s days-on-market. Days-on-market signals quality
because the longer the home sits on the market, the greater the likelihood that previous buyers arrived, observed some undesirable quality, and chose not to buy. In the comparative statics and estimation, I restrict attention to competitive prices. However, I show that regardless of equilibrium prices, buyers become more pessimistic about the home’s quality the longer it sits on the market.

Consistent with the model’s predictions, prior work documents that sale-to-list ratios decline with days-on-market [29]. I confirm this fact in my sample. Controlling for home, neighborhood, and time characteristics, I show that the sale-to-list ratio is negatively correlated with days-on-market. The longer a home sits on the market, the less it sells for relative to its list price.

In the model, days-on-market and sale prices are determined by the rate at which buyers arrive for a walkthrough, the difference between high and low quality, and the walkthrough’s informativeness. Walkthroughs are informative if the difference between high and low quality is large, or if the walkthrough clearly distinguishes high from low quality.

The model makes several predictions about sale prices and days-on-market. First, it predicts that days-on-market for high and low quality homes increase with the rate at which buyers arrive for a walkthrough. The more buyers who walk through the home, the more likely it is that one of them walks away with a favorable impression and decides to buy.

Second, it predicts that days-on-market decrease with walkthrough informativeness for high quality homes, and increase with walkthrough informativeness for low quality homes. As walkthroughs become more informative, buyers discern high quality from low quality with greater precision. High quality homes are identified early, and sell early.

Third, the model predicts that buyer arrival affects sale prices non-monotonically. On the one hand, rates of sale increase with the arrival rate of buyers, and homes that sell early sell at higher prices than homes that sell late. On the other hand, beliefs (and hence prices) decrease with the arrival rate of buyers. The higher the arrival rate of buyers, the more likely it is that previous buyers arrived, observed some undesirable quality, and chose not to buy.

Fourth, the model predicts that more informative walkthroughs result in lower sale prices for low quality homes. Surprisingly, increasing walkthrough informativeness may decrease sale prices
for sellers of high quality homes. As walkthroughs become more informative, days-on-market becomes a more important signal of quality. Sellers of high quality homes who are unfortunate, and cannot sell early face lower sale prices than they would otherwise.

These comparative statics inform policy, the key instrument of which is walkthrough (or listing) informativeness [19]. Although I do not perform a general equilibrium welfare analysis, I show that increasing walkthrough informativeness may not be Pareto efficient. Sellers of low quality homes face more days-on-market and lower sale prices, and under certain conditions, sellers of high quality homes also face lower sale prices.

The model has three deep parameters. The first is the buyer arrival rate, which measures how often buyers arrive for a walkthrough. The second is the ratio of the value of a high quality home to the value of a low-quality home, which I call the quality premium. The third is the walkthrough’s informativeness, which measures how much a buyer’s belief about a home’s quality changes because of the walkthrough. I use structural methods to estimate the values of these parameters that can generate observed outcomes.

Under competitive pricing, the model yields a closed-form expression for the joint distribution of days-on-market and sale prices. The closed-form expression allows me to build home heterogeneity directly into the model without sacrificing computational tractability. I allow the model’s deep parameters to vary according to the home’s age and size, and the season in which the home was listed.

I find that the quality premium varies considerably by geography: 7-8% in Denver, 15-16% in Charlotte, and 23-32% in Detroit. In Charlotte and Detroit, the quality premium appears to be lowest during the “hot” homebuying months of late spring and early summer, and lowest during the “cold” months of fall and winter.

Next, I explore how the quality premium varies according to a home’s age and size. I find that adding ten years to the age of a home is associated with a 0.1% increase (in levels) in the quality premium in Denver, a 0.3% increase in Charlotte, and a 0.73% increase in Detroit. These estimates are economically small, but are consistent with the learning mechanism described in this
paper. The evidence regarding size is mixed. In Denver, larger homes are associated with a higher quality premium, while in Charlotte and Detroit, larger homes are associated with a lower quality premium.

I estimate walkthrough informativeness to be 0.97-1.04 (Denver), 0.91-1.01 (Charlotte), and 0.79-0.99 (Detroit). Evidently, walkthroughs appear to be most informative in Denver, and least informative in Detroit. In Charlotte and Detroit, walkthrough informativeness appears to be lowest in late spring and early summer, and highest during fall and winter.

Next, I explore how walkthrough informativeness varies according to a home’s age and size. I only find significant results in Charlotte and Detroit. Adding ten years to the age of a home is associated with a 11% increase (again, in levels) in walkthrough informativeness in Charlotte and a 5% increase in Detroit. Adding 1,000 square feet to a home is associated with a 6% increase in walkthrough informativeness in Charlotte and a 24% increase in Detroit.

The quality uncertainty model assumes that the seller does not know the quality of her home, relative to the quality of similar homes. In practice, sellers attend the open houses of similar homes on the market. If the seller knows the quality of her home, then she may engage in a variety of strategic behaviors. For example, if she knows her home to be of low quality, then she may mimic the list prices of the sellers of high quality homes.

I extend the quality uncertainty model to allow for strategic behavior on the part of sellers. I sketch and analyze an equilibrium in closed-form. Sellers of both high and low quality homes initially pool on the same list price. As in the quality uncertainty model, buyers become more pessimistic about the home’s quality the longer it sits on the market. In contrast to the quality uncertainty model, the seller eventually cuts her list price below the pooling list price, or withdraws her listing.

The literature on the negative relationship between days-on-market and sale prices is growing [101, 47, 77, 8, 60, 56]. I contribute to the literature by quantifying the role of quality uncertainty. I take the causal relationship of days-on-market on sale prices—and sale prices on days-on-market—as given. Using an instrumental variables approach, [29] estimate a negative causal effect of days-on-
market on sale prices, and sale prices on days-on-market, and conclude that “houses staying longer on the market provide negative information to the market.”

In this paper, I formalize a mechanism based on learning, which explains the negative relationship between days-on-market and sale prices. I also estimate a counterfactual in which learning is moot: buyers know homes’ quality with certainty. Estimating this counterfactual poses an econometric challenge because it is “far” from the real-world, and reduced-form estimates are inaccurate. I overcome this challenge via structural estimation.

In terms of modelling and estimation, the papers closest to mine are [19] and [7]. [19] estimates a model of the housing market by maximum likelihood. His counterfactual analysis focuses on the informativeness of listings and real estate agents’ commissions. [7] also estimates a model of the housing market. He shows that information frictions can explain return predictability.

1.2 The Model

In this section, I develop a parsimonious model of home sale. I purposefully abstract from the bidding process, and focus instead on buyers’ learning. The model yields a closed-form expression for the joint distribution of days-on-market and sale prices.

1.2.1 Setup

Time is continuous and indexed by $t$. There is one long-lived seller, and a countably infinite number of short-lived buyers. The seller owns a home of quality $\theta \in \{H, L\}$. She does not know the quality of her home (an assumption I relax in Section 1.5). Let $v_\theta$ denote the value to a buyer of a home of quality $\theta$, where $v_H > v_L > 0$. I define $\eta \equiv v_H/v_L$ to be the quality premium. There are a fraction $\pi_0 \in (0, 1)$ of $H$ quality homes in the market. Let $v$ denote the random variable that takes the value $v_H$ with probability $\pi_0$, and $v_L$ with probability $1 - \pi_0$. Put $\bar{v} \equiv \frac{1}{2}(v_H + v_L)$. At time $t = 0$, the seller puts her home on the market for sale.

Buyers arrive for a walkthrough according to a Poisson process with a constant intensity of $\lambda$ (see [9] for a relaxation of this assumption). Upon arrival, a buyer receives a private, IID signal
$x|\theta$, which is normally distributed with mean $v_\theta$ and variance $\sigma_x^2$. Let $F_\theta$ denote its CDF, and $f_\theta$ its PDF. Define the signal-to-noise ratio to be $\phi \equiv (v_H - v_L)/\sigma_x$, and the likelihood ratio to be $\ell \equiv f^H/f^L$. Note that $\ell$ is strictly increasing. Larger signals are more likely to have come from an $H$ quality home than from an $L$ quality home. The signal-to-noise ratio can be interpreted as the sensitivity of the log-odds to the signal:

$$\log(\ell(x)) = \phi \cdot \frac{x - \bar{v}}{\sigma_x}. \tag{1.1}$$

A signal that is one standard deviation away from $\bar{v}$ changes the log-odds that the home is high quality by $\phi$.

For each $t \geq 0$, let $\pi(t) \in [0,1]$ denote a buyer’s belief that the home is $H$ quality, having arrived at time $t \geq 0$, and prior to having received the signal of quality. $\pi(t)$ is formed without physical inspection of the home. I therefore refer to it as “buyers’ beliefs” or as the “market’s belief.”

### 1.2.2 Seller Behavior

For the purposes of this section, I am agnostic about the seller’s preferences. Let $p(t, \pi(t))$ denote the equilibrium price, and $\mu(t, \pi(t))$ the equilibrium withdrawal rate. As it will appear often, define the sale-to-list ratio to be $\rho(t, \pi) \equiv p(t, \pi(t))/p(0, \pi(0))$. Since the seller is as ignorant of the home’s quality as the buyers, neither the equilibrium price, nor the equilibrium withdrawal rate depend on the home’s quality.

In Section 1.2.4 I show that the rate at which buyers withdraw does not affect beliefs or prices. Intuitively, sellers of high and low quality homes follow the same withdrawal strategy, and hence withdrawal is an uninformative signal of quality. In Section 1.2.5, I specify $p$ in order to investigate comparative statics and to estimate the model.

In Section 1.5, I relax the assumption that sellers do not know the quality of their homes. In the relaxed model, there are two prices, one for the seller of the high quality home, and another for the seller of the low quality. Moreover, sellers of high and low quality homes will choose to
withdraw their listings endogenously; withdrawal becomes an informative signal of quality.

1.2.3 Buyer Behavior

I assume that buyers are short-lived. Having arrived at time \( t \), a buyer buys if and only if her surplus from doing so is positive.

**Assumption 1.2.1 (Buyer Behavior).** *Having arrived at time \( t \geq 0 \), and having received a signal of home quality \( x \), a buyer buys if and only if \( \mathbb{E}_t[v|x] \geq p(t, \pi(t)) \).*

I make Assumption 1.2.1 for parsimony and ease of exposition. A richer model would allow buyers to arrive, receive a signal of quality, and in addition to choosing whether to buy, would allow buyers to choose when to buy.

Homebuyers often monitor a single home for several weeks or months. From a strategic perspective, Assumption 1.2.1 precludes buyers from waiting for lower prices. If they could, buyers would deviate in the present model. I argue that there cannot be an equilibrium in which buyers strategically delay. If buyers arrive, but do not buy, then days-on-market is no longer an informative signal of quality. Beliefs will not decline, and hence prices will not fall. Even if there were non-strategic buyers who arrived, but chose not to buy because their surplus from trade was negative (so that beliefs deteriorated over time), strategic buyers could not commit to trade later. As long as there were more than one strategic buyer, the equilibrium would unwind.

1.2.4 Beliefs

I first discuss how buyers’ beliefs about a home’s quality must evolve. Let \( s \geq 0 \). Having received a signal of quality \( x \in \mathbb{R} \), the buyer’s expected value from purchasing the home is

\[
\mathbb{E}_s[v|x] = \frac{\pi(s)f^H(x)}{\pi(s)f^H(x) + (1 - \pi(s))f^L(x)} \cdot v_H + \frac{(1 - \pi(s))f^L(x)}{\pi(s)f^H(x) + (1 - \pi(s))f^L(x)} \cdot v_L, \tag{1.2}
\]

and the buyer buys if and only if \( \mathbb{E}_s[v|x] > p(s, \pi(s)) \), if and only if

\[
x > x(\pi(s), p(s, \pi)) \equiv \ell^{-1} \left( \frac{1 - \pi(s)}{\pi(s)} \cdot \frac{p(s, \pi(s)) - v_L}{v_H - p(s, \pi(s))} \right). \tag{1.3}
\]
The buyer’s signal must be significantly high for her posterior to justify purchase. During a time period of duration $dt$, trade occurs with probability

$$
\lambda_\theta(\pi(s), p(s, \pi(s)))dt \equiv \lambda dt \cdot \Pr(\mathbb{E}_s[v|x] + \epsilon \geq p(s, \pi(s))|\theta)
$$

(1.4)

$$
= \lambda dt \cdot (1 - F_\theta(x(\pi(s), p(s, \pi(s))))).
$$

(1.5)

With probability $\lambda dt$, a buyer arrives, and with probability $1 - F_\theta(x(\pi(s), p(s, \pi(s))))$, her signal is sufficiently high to justify purchase.

**Lemma 1.2.1.** For each $s \geq 0$ and $\pi \in (0, 1)$, $\lambda > \lambda_H(\pi, p(s, \pi)) > \lambda_L(\pi, p(s, \pi))$.

**Proof.** $0 < F_H(x) < F_L(x)$ for all $x \in \mathbb{R}$. 

The rate at which buyers arrive is strictly greater than the rate at which $H$ quality homes sell, which is strictly greater than the rate at which $L$ quality homes sell. Lemma 1.2.1 follows directly from the fact that $\ell$ is strictly increasing. Higher quality homes are more likely to leave buyers with more favorable signals.

It remains to construct beliefs. Now fix $t > 0$ and define

$$
\Lambda_\theta(t) \equiv \int_0^t \lambda_\theta(\pi(s), p(s, \pi(s)))ds
$$

(1.6)

$$
M(t) \equiv \int_0^t \mu(s, \pi(s))ds.
$$

(1.7)

A $\theta$ quality home remains on the market at time $t$ if and only if it has (1) not yet sold (which occurs at a rate of $\lambda_\theta(t, \pi(t))$, and (2) not yet been withdrawn (which occurs at a rate of $\mu(t, \pi(t))$.

According to Bayes’ rule, buyers’ beliefs are given by

$$
\pi(t) = \frac{\pi_0 e^{-\Lambda_H(t)} \cdot e^{-M(t)}}{\pi_0 e^{-\Lambda_H(t)} \cdot e^{-M(t)} + (1 - \pi_0)e^{-\Lambda_L(t)} \cdot e^{-M(t)}}.
$$

(1.8)

Evidently, withdrawals do not affect beliefs. $H$ and $L$ quality homes are delisted at the same rate.

**Lemma 1.2.2.** Buyers’ belief that a home is of high quality is strictly decreasing with the home’s time-on-market: $\pi'(t) < 0$ for all $t \in \mathbb{R}_+$. 

Proof. By Lemma 1.2.1, $\pi' = -\pi(1 - \pi)(\lambda_H(\pi, p(t, \pi)) - \lambda_L(\pi, p(t, \pi))) < 0.$ □

Intuitively, the longer the home has been on the market, the more likely it is that previous buyers arrived, inspected the property, and decided not to buy. Beliefs deteriorate over time. Recall that $p$ was chosen to be arbitrary. It is natural to assume that prices fall with beliefs.

**Corollary 1.2.1.** If $p_\pi < 0$, then the sale-to-list ratio decreases with days-on-market.

Corollary 1.2.1 confirms the idea that the longer a home sits on the market, the less it sells for relative to its list price. In section 1.3.2, I document that on average, sale-to-list ratios decrease with days-on-market.

### 1.2.5 Pricing

Observed prices depend on the respective bargaining powers of buyers and sellers, buyers’ financial constraints, sellers’ knowledge of their local market, and the role of buyers’ and sellers’ agents (to name a few). Rather than model each of these features, I take a reduced-form approach and consider the model under competitive pricing.

**Assumption 1.2.2 (Prices).** *Homes trade at expected value:* $\bar{p}(\pi) \equiv \pi v_H + (1 - \pi)v_L$.

Note that $p$ depends implicitly on $t$ through $\pi$. As beliefs fall, the price falls. Put differently, as beliefs fall, the seller adjusts her price downwards at such a rate that the probability of sale at any moment is constant.

I first discuss how buyers’ beliefs about a home’s quality must evolve. I begin with buyers’ decision rules. Assumption 1.2.2 implies that buyers follow a simple trading rule.

**Lemma 1.2.3.** The buyer buys if and only if $x > \bar{v}$.

Proof. Substitute $\bar{p}(\pi(s))$ for $p(s, \pi(s))$ in Equation 1.3. □

The buyer’s signal must be significantly high for her posterior to justify purchase. From Equation
1.4, a $\theta$ quality home’s days-on-market is exponentially distributed with rate

$$\lambda_{\theta} \equiv \begin{cases} 
\lambda(1 - \Phi(-\phi/2)) & \theta = H \\
\lambda(1 - \Phi(+\phi/2)) & \theta = L 
\end{cases}$$

(1.9)

where $\Phi$ is the standard normal CDF. Note that if $\pi = 0$ or $\pi = 1$, then the rate at which the home sells is $\lambda$. The intuition is as follows. If $\pi = 0$ or $\pi = 1$, then the buyer knows the seller’s type almost surely. Trade occurs with the next arriving buyer. If $0 < \pi < 1$, then trade may not occur with the next buyer who arrives. Under Assumption 1.2.2, the seller posts a list price equal to the buyer’s expected value before having received the signal. Consequently, $f^H(x) = f^L(x)$. The buyer receives a signal that induces her to buy, and a signal that induces her not to buy with equal probability.

The rate at which $H$ and $L$ quality homes sell increases with the rate at which buyers arrive. If buyers’ signals are informative (if $\phi > 0$), then buyers arrive at a greater rate than they trade with $H$, than they trade with $L$: $\lambda > \lambda_H > \lambda_L$.

The days-on-market for a $\theta$-quality home is exponentially distributed with rate $\lambda_{\theta}$. Let $Q_{\theta}$ denote the days-on-market for the $\theta$-quality home:

$$Q_{\theta}(t) \equiv 1 - e^{-\lambda_{\theta} t}.$$  

(1.10)

A $\theta$ quality home sits on the market until time $t$ with probability $1 - Q_{\theta}(t)$. Therefore, the probability that the home is $H$ quality is

$$\pi(t) \equiv \frac{\pi_0 e^{-\lambda_H t}}{\pi_0 e^{-\lambda_H t} + (1 - \pi_0) e^{-\lambda_L t}}.$$  

(1.11)

The list price and sale price (at time $t$) are given by $p(\pi_0)$ and $p(\pi(t))$ respectively. In the estimation, I will estimate the sale-to-list ratio, $p(\pi)/p(\pi_0)$. Because $p$ is linear in $v_H$ and $v_L$, the sale-to-list ratio can be written in terms of the value premium:

$$\bar{\rho}(\pi) = \frac{\pi \eta + (1 - \pi)}{\pi_0 \eta + (1 - \pi_0)}.$$  

(1.12)
1.2.6 Moments

I next consider the expected sale price and days-on-market. Consider a counterfactual in which a home’s quality were known to buyers with certainty. The seller would trade with the first buyer to arrive at the home’s true value, \( SP_\theta^o \equiv v_\theta \). Since the time to the first arrival would be exponential distributed with rate \( \lambda \), the home’s expected days-on-market would be \( DM_\theta^o \equiv \lambda^{-1} \) days.

Next, consider days-on-market and sale prices in the real-world. I first consider days-on-market. Days-on-market is a measure of liquidity in the housing market, and measures the temporal costs of quality uncertainty. Under quality uncertainty, a \( \theta \) quality home’s days-on-market is exponentially distributed with rate \( \Delta_\theta \), and hence its expected days-on-market are

\[
DM_\theta \equiv \frac{1}{\Delta_\theta}
\]  

(1.13) days. From Equations 1.9 and 1.13, the following comparative statics follow.

**Lemma 1.2.4** (Days-on-Market Comparative Statics). \( DM_H \) is strictly decreasing in \( \lambda \), and strictly decreasing in \( \phi \); \( DM_L \) is strictly decreasing in \( \lambda \), and increasing in \( \phi \). Moreover, \( DM^o < DM_L < DM_H \).

*Proof.* See the appendix. 

Homes of both types sell faster as buyers arrive faster. More informative walkthroughs (as measured by \( \phi \)) allow buyers to distinguish \( H \) quality homes from \( L \) quality homes more accurately. Therefore, \( H \) quality homes are more likely to sell, and \( L \) homes are less likely to sell.

A \( \theta \) quality home’s expected sale price is

\[
SP_\theta \equiv \int_0^\infty p(t)dQ_\theta(t).
\]  

(1.14) From Equations 1.9 and 1.14, the following comparative statics follow.

**Lemma 1.2.5** (Sale Price Comparative Statics). \( SP_H \) and \( SP_L \) are strictly decreasing in \( \phi \), and are constant in \( \lambda \). Moreover, \( SP_H < v_H \) and \( SP_L > v_L \).
Proof. See the appendix.

SP_H and SP_L are strictly decreasing in φ because the more informative are walkthroughs, the more negative is time-on-market a signal of quality.

Lemmas 1.2.4 and 1.2.5 have important implications for policy, the key instrument of which is walkthrough (or listing) informativeness [19]. Although I do not perform a general equilibrium welfare analysis, Lemmas 1.2.4 and 1.2.5 suggest that increasing walkthrough informativeness may not be Pareto efficient. Sellers of low quality homes face more days-on-market and lower sale prices. Surprisingly, increasing walkthrough informativeness may decrease sale prices for high quality homes. As walkthroughs become more informative, days-on-market become a more important signal of quality. Sellers of high quality homes who are unfortunate enough to sell late face lower sale prices than they would otherwise.

1.2.7 Idiosyncratic Value

Before leaving this section, I briefly discuss idiosyncratic value. I have assumed throughout that buyers’ preferences are identical. Buyers’ preferences are not identical, and one buyer will obtain utility from a kitchen island, garden bed, or hot tub, while the other will not. Suppose that buyer j’s value from a home of quality θ is

\[ v_{j,θ} = v_θ + ϵ_j, \]  (1.15)

where the ϵ_j are IID normal with mean zero and variance \( σ_ϵ^2 \). The term \( v_θ \) captures the common component of value, while ϵ_j captures the idiosyncratic component of value. Under the assumption that buyers pay their idiosyncratic value, the preceding analysis is unaffected.

Lemma 1.2.6. If \( p_j(π) = p(π) + ϵ_j \), then the buyer buys if and only if \( x > \bar{v} \).

Proof. In Equation 1.3, substitute \( \tilde{p}_j(π(s)) \) for \( p(s, π(s)) \), \( v_{j,H} \) for \( v_H \), and \( v_{j,L} \) for \( v_L \). ■

Lemma 1.2.6 reaches the same conclusion as Lemma 1.2.3. Under Lemma 1.2.6, θ quality homes sell at a rate of \( \Delta_θ \), and hence beliefs and prices are unchanged.
The mechanism described in this paper depends on the observation that buyers care about what other buyers think. Days-on-market signals quality because it signals other buyers' perception of quality. If value were purely idiosyncratic, then days-on-market would have little value in assessing quality.

1.3 Empirical Facts

In Section 1.2, I developed a parsimonious model of home sale based on buyers’ learning. Its central prediction was that the market’s belief that the home is $H$ quality falls with days-on-market. If prices fall with the market’s belief, then prices fall with days-on-market. In this section, I describe my sample, and document that in fact, the longer a home sits on the market, the less it sells for relative to its list price.

1.3.1 Data

I use CoreLogic’s Listings data for the Denver, Charlotte, and Detroit metropolitan areas. The corresponding multiple listing services are REcolorado (Denver), CarolinaMLS (Charlotte), and Realcomp (Detroit). These data are at the transaction-level. To be included in my sample, homes must (1) be listed between 01/01/2005 and 12/31/2015; (2) be more than two-years-old; (3) have 1-6 beds; (4) have 1-6 baths. I require homes to be sufficiently old to mitigate the effects of new construction. For instance, new homes tend to be sold in blocks by homebuilders (not individual owners). Finally, I trim the list price, sale price, sale-to-list ratio, and square-feet at the 1% and 99% levels; I trim days-on-market at the 95% level. The final sample contains 316,760 observations in Denver, 211,012 observations in Charlotte, and 51,265 observations in Detroit. Table 1.1 shows summary statistics for all of the variables that I use in my analysis.

Denver, Charlotte, and Detroit vary according to geography (West, South, and Midwest respectively); mean age of housing stock (33 years, 22 years, and 55 years); mean sale prices ($261k, $196k, and $99k). Moreover, they vary in average home price growth. During my sample period, Denver, Charlotte, and Detroit experienced average annual returns of 3.1%, 2.0%, and -0.4%
respectively (as measured by the S&P/Case-Shiller Home Price Indices). Denver can be viewed as the “hot” market, Charlotte the “neutral” market, and Detroit the “cold” market.

Most relevant to this paper are days-on-market and the sale-to-list ratio. Denver has the highest mean sale-to-list ratio at 99%, followed by Charlotte at 97%, followed by Detroit at 95%. Denver has the lowest mean days-on-market at 45 days. Perhaps surprisingly, Detroit has a lower mean days-on-market than Charlotte: the mean home in Detroit sits on the market for 47 days, while the mean home in Charlotte sits on the market for 64 days.

1.3.2 Days-on-Market and the Sale-to-List Ratio

Corollary 1.2.1 predicts that sale-to-list ratios fall with days-on-market, a finding consistent with the existing literature. In Table 1.2, I document that sale-to-list ratios decline with days-on-market in my sample. I control for home age and size; in columns (1), (3) and (5) I include quarter dummies, while in columns (2), (4), and (6), I include ZIP\times quarter\times year dummies. For ease of interpretation, the sale-to-list ratio is quoted in points. Sellers in Denver and Charlotte lose 1bps per day, while sellers in Detroit lose 4bps per day. In Charlotte and Detroit, older and smaller homes sell at deeper discounts. The pattern flips in Denver. Larger homes sell at deeper discounts. I don’t interpret the negative coefficients on days-on-market in Table 1.2 as evidence of a causal effect. They are simply consistent with the predictions of Section 1.2.

1.4 Estimation

The model can be written in terms of three deep parameters: the rate at which buyers arrive for a walkthrough (\(\lambda\)), the value premium (\(\eta \equiv v_H/v_L\)), and the walkthrough’s signal-to-noise ratio (\(\phi \equiv (v_H - v_L)/\sigma_x\)), which I’ve referred to as the walkthrough’s informativeness. In this section, I estimate these parameters via maximum likelihood following [107] ([19, 20] also estimates a model of housing markets via maximum likelihood, although his model is quite different than mine). I compute standard errors using the asymptotic variance of the maximum likelihood estimator, and apply the delta method when computing standard errors of estimates of functions of the
estimated parameters. While the estimates are interesting in their own right, their greatest value is in estimating counterfactual sale prices and days-on-market.

### 1.4.1 Heterogeneity

The data described in Section 1.3.1 vary along many observable dimensions (age, size, etc.). In the estimation, I allow the deep parameters to vary with observable home characteristics. In theory each home has its own buyer arrival rate, quality premium, and walkthrough informativeness. So many parameters cannot be identified, so I take a reduced-form approach. For each home \( k \), define

\[
\lambda_k \equiv \beta_0^\lambda \log(\text{Age}_k) + \beta_1^\lambda \log(\text{SqFt}_k) + \gamma_k^\lambda
\]

(1.16)

\[
\eta_k \equiv \beta_0^\eta \log(\text{Age}_k) + \beta_1^\eta \log(\text{SqFt}_k) + \gamma_k^\eta
\]

(1.17)

\[
\phi_k \equiv \beta_0^\phi \log(\text{Age}_k) + \beta_1^\phi \log(\text{SqFt}_k) + \gamma_k^\phi
\]

(1.18)

where \( \text{Age}_k \) is the age of home \( k \) at the time of listing in years, \( \text{SqFt}_k \) is the square-footage of home \( k \), and \( q_k \in \{1, 2, 3, 4\} \) is the quarter in which home \( k \) was listed.

I account for heterogeneity in this way because it mitigates the possibility that those attributes that are ascertained from an online listing are not conflated with those attributes which require a walkthrough. For example, a large home may sell faster than a small home—and be interpreted in the quality uncertainty model as an \( H \) quality home—while in fact the large home sells faster because it happens to be in higher demand.

To maintain computational tractability, I omit year and neighborhood dummies in Equations 1.16–1.18. In Table 1.2, I regress the sale-to-list ratio on days-on-market, controlling for age and size. In columns (1), (3) and (5) I include quarter dummies, while in columns (2), (4), and (6), I include ZIP×quarter×year dummies. Replacing the quarter dummies with ZIP×quarter×year dummies increases the R-squared by a factor of 2-3. The coefficient on age in Denver flips sign, suggesting that age is correlated with a ZIP×quarter×year-determinant of the sale-to-list ratio. Otherwise, the coefficients retain their signs and magnitudes between odd and even columns. This
exercise suggests that omitting year and neighborhood dummies in Equations 1.16–1.18 does not dramatically change the effects under study.

Before moving on to the estimation, I develop the following hypotheses:

**Hypothesis 1.** *Buyer arrival should be highest in the second quarter:* $\gamma_2, \gamma_3 > \gamma_1, \gamma_4$.

[82] find that prices and transactions are greater in the second- and third-quarters than in the first- and fourth-quarters. Much of seasonality relates to the academic calendar: parents of school-aged children prefer to move before the school year starts in the fall.

**Hypothesis 2.** *The value premium should increase with age and size:* $\beta^q_0, \beta^q_1 > 0$.

Older homes feature idiosyncrasies from the era in which they were built, and are more likely to have been renovated. Younger homes feature fewer idiosyncrasies, and are more similar to contemporary stock. In a sample of Chicago homes, [54] finds that older and taller homes (as measured by number of stories) are more likely to be renovated. Renovations increase the heterogeneity of the stock. By a similar line of reasoning, larger homes have more features (larger kitchens, more bedrooms and bathrooms), and so the difference in value between high and low quality 5-bed, 3-baths should be larger than the difference in value between high and low quality 1-bed, 1-baths.

**Hypothesis 3.** *The signal-to-noise ratio should increase with age:* $\beta^q_0 > 0$.

Recall that $\phi \equiv (v_H - v_L)/\sigma_x$. Walkthrough informativeness should vary according to the age of the home being inspected. The difference between an old, high-quality home and an old, low-quality could be large. A one-hundred-year-old home could be well-built, or due for an expensive renovation. The marginal minute spent walking through an old home should be quite informative about this difference.

### 1.4.2 Estimator

Let $\pi_k$ denote the belief that home $k$ is $H$ quality, and similarly, $\bar{\rho}_k$ the sale-to-list ratio for home $k$. I assume that the observed sale-to-list ratio, $\rho_k$, is normally distributed with mean
\( \hat{\rho}_k(\pi_k(t_k)) \) and variance \( \sigma^2 \), an assumption motivated by the discussion of idiosyncratic tastes in Section 1.2.7. I estimate the 18 parameters in Equations 1.16–1.18 together with \( \sigma \). I take the fraction of \( H \) quality homes, \( \pi_0 \), to be \( 1/2 \). Therefore, \( H \) quality homes should be interpreted as above median in terms of quality, while \( L \) quality homes should be interpreted as below median in terms of quality. For each home \( k \), I observe the days-on-market, \( t_k \), and sale-to-list ratio, \( \rho_k \). According to the model, \( t_k \) is distributed according to the PDF

\[
\psi(t_k|X_k) \equiv \pi_0 \lambda_k,H e^{-\lambda_k,H t_k} + (1 - \pi_0) \lambda_k,L e^{-\lambda_k,L t_k}.
\]  

Hence, the joint PDF over days-on-market and the sale-to-list ratio is given by the likelihood function

\[
L(\eta, \lambda, \phi, \sigma|X_k) = \varphi(\rho_k; \hat{\rho}_k(\pi_k(t_k)), \sigma) \psi(t_k|X_k)
\]

where \( \varphi \) is the normal PDF whose second and third arguments are the mean and standard deviation respectively. Figure 1.1 shows model and local regression (LOESS) fits for the density of days-on-market (left) and the sale-to-list ratio conditional on days-on-market. Note that LOESS is non-parametric. It fits the data well, but is uninterpretable. The model appears to fit the distribution of days-on-market quite well in each of the three metropolitan areas. It overestimates the sale-to-list ratio for early sales, and underestimates it for later ones.

### 1.4.3 Parameter Estimates

In Table 1.3, I present parameter estimates. In panel A, I estimate the sale-to-list ratio volatility, \( \sigma \). The estimates are 4% in Denver, 5% in Detroit, and 10% in Detroit.

In Panel B, I estimate buyer arrival rates. In the counterfactual, homes sell to the first buyer who arrives. The expected days-on-market before the first buyer arrives is the inverse of the buyer arrival rate. If the buyer arrival rate is 0.1, for example, then the expected days-on-market in the counterfactual is 10 days. I estimate the counterfactual days-on-market to be 18 days in Denver, 27 days in Charlotte, and 21 days in Detroit.
Consistent with Hypothesis 1, the buyer arrival rate is highest in the second quarter. I don’t make any predictions about how the buyer arrival rate varies according to the home’s age and size. However, I find in all three metropolitan areas that the buyer arrival rate is highest for older homes. This finding could be due to an omitted variable, such as distance to downtown, which is correlated both with the home’s age and the buyer arrival rate, but through a channel completely unrelated to quality uncertainty.

In Panel C, I estimate the quality premium. The quality premium varies considerably by geography: 7-8% in Denver, 15-16% in Charlotte, and 23-32% in Detroit. In Charlotte and Detroit, the quality premium appears to be lowest during the “hot” homebuying months of the late spring and early summer, and lowest during the “cold” months of fall and winter.

According to Hypothesis 2, the difference between high and low quality should increase with the age of the home being inspected. I find that adding ten years to the age of a home is associated with a 0.1% increase (in levels) in the quality premium in Denver, a 0.3% increase in Charlotte, and a 0.7% increase in the quality premium. These estimates are economically small, but are consistent with the learning mechanism described in this paper. The evidence regarding size is mixed. In Denver, larger homes are associated with a higher quality premium, while in Charlotte and Detroit, larger homes are associated with a lower quality premium.

In panel D, I estimate walkthrough informativeness. I estimate walkthrough informativeness to be 0.97-1.04 (Denver), 0.91-1.01 (Charlotte), and 0.79-0.99 (Detroit). Evidently, walkthroughs appear to be most informative in Denver, and least informative in Detroit. In Charlotte and Detroit, walkthrough informativeness appears to be lowest during the “hot” months of the late spring and early summer, and highest during the “cold” months of fall and winter.

According to Hypothesis 3, walkthroughs of older homes should be more informative than walkthroughs of newer homes. I only find significant results in Detroit. Adding ten years to the age of a home is associated with a 10.8% increase (in levels) in Charlotte and a 5.1% increase in Detroit. In terms of home size, I only find significant results in Charlotte and Detroit. Adding 1,000 square feet to the home is associated with 5.7% increase (in levels) in walkthrough informativeness.
in Charlotte and a 23.7% increase in Detroit.

1.4.4 Counterfactuals

In Table 1.4, I present mean counterfactual days-on-market as described in Section 1.2.6. In the first row of Panels A and B, I estimate the mean days-on-market for each metropolitan area (compare with Table 1.1: 45 days in Denver, 64 days in Charlotte, and 47 days in Detroit). In the second row, I estimate the mean days-on-market for an H quality home (Panel A) and an L quality home (Panel B): 27/60 days in Denver, 40/85 days in Charlotte, and 32/63 days in Detroit. Evidently, high quality homes sell faster than average, and low quality homes slower than average. In the model, these variables correspond to DM_H and DM_L respectively.

In the third row, I estimate the mean counterfactual days-on-market for an H quality home (Panel A) and an L quality home (Panel B). In the model, the mean counterfactual days-on-market is $\lambda^{-1}$ for both H and L quality homes: 18 days in Denver, 27 days in Charlotte, and 21 days in Detroit. I briefly note that these counterfactual days-on-market are not wholly implausible. In a sample of English transactions, [78] find that 40% of sales are to the first bidder. In the fourth row, I estimate the percent difference in the mean days-on-market from the counterfactual to the real-world. Since the counterfactual days-on-market is the same for homes of both H and L quality, we can attributed a fraction of observed days-on-market to quality uncertainty. By metro, 59% of days-on-market can be attributed to quality uncertainty in Denver, 58% in Charlotte, and 56% in Detroit.

In Table 1.5, I present mean counterfactual sale prices as described in Section 1.2.6. Table 1.5 follows the format of Table 1.4. In the first row of Panels A and B, I estimate the mean sale price for each metropolitan area (compare with Table 1.1: $261k in Denver, $190k in Charlotte, and $95k in Detroit). In the second row, I estimate the mean sale price for an H quality home (Panel A) and an L quality home (Panel B): $262k/$261k in Denver, $193k/$190k in Charlotte, and $95k/$95k in Detroit. In the model, these variables correspond to SP_H and SP_L respectively. The estimate for SP_L in Charlotte is concerning, as it is higher than the mean sale price (a moment
to which I don’t match the model).

In the third row, I estimate the mean counterfactual sale price for an $H$ quality home (Panel A) and an $L$ quality home (Panel B): $274k/255k$ in Denver, $210k/182k$ in Charlotte, and $109k/88k$ in Detroit. In the model, the mean counterfactual sale price is $v_H$ for an $H$ quality home and $v_L$ for an $L$ quality home. As a fraction of total housing value, $\bar{v}$, sellers of $\theta$ quality homes claim $v_\theta/\bar{v}$ of value. Under quality uncertainty, sellers of $\theta$ quality homes claim $\text{SP}_\theta/\bar{v}$ of value. Since $\text{SP}_H < v_H$ and $\text{SP}_L > v_L$, quality uncertainty transfers value from sellers of $H$ quality homes to sellers of $L$ quality homes. The value of the transfer is $\text{SP}_L - v_L$, which I estimate to be 2% in Denver, 4% in Charlotte, and 7% in Detroit. Sellers of $H$ quality homes lose more than sellers of $L$ quality homes gain. There is a deadweight loss. In the fourth row, I estimate the percent difference in the mean sale price from the counterfactual to the real-world.

1.5 Dynamic Adverse Selection

The preceding analysis assumed that the seller does not know the quality of her home. If she does know the quality of her home, then she may engage in strategic behavior. As a primitive example, suppose that sellers of $H$ quality homes list their homes for $v_H$. Then sellers of $L$ quality homes gain by mimicking the sellers of $H$ quality homes, and also listing their homes for $v_H$.

Though sellers know the idiosyncrasies of their homes, they may not know the quality of their home relative to the quality of similar homes. For example, a seller may feel that her 2-bed, 2-bath condo has poor natural lighting, and conclude that her home is $L$ quality, despite the fact that the other 2-bed, 2-bath condo for sale in her neighborhood has even poorer natural lighting. That being said, sellers may learn about the quality of their own homes relative to the quality of similar homes. The seller of the 2-bed, 2-bath condo can attend an open house at the other 2-bed, 2-bath condo, and learn that her condo actually has better natural lighting.

In this section, I develop a model of dynamic adverse selection and sketch an equilibrium. The equilibrium features two periods. In the first period, trade proceeds as in the quality uncertainty model: sellers of $H$ and $L$ quality homes sell their homes at rates of $\lambda_H$ and $\lambda_L$, and at a price of
In the second period, either the seller of the $H$ quality home or the seller of the $L$ quality home decides to cut the price to $v_L$ or withdraw the listing.

### 1.5.1 The Model

The seller and buyers are risk-neutral and discount the future at a rate of $r > 0$. The seller owns a home of quality $\theta \in \{H, L\}$, which is now her private information. Let $k_\theta$ denote the value to the seller of a home of quality $\theta$. I refer to the seller of the $\theta$ quality home as “$\theta$.” I assume that there are gains from trade ($k_\theta < v_\theta$) and that an $H$ quality home is worth more to the seller than an $L$ quality home ($k_H > k_L$). Following the literature, I say that the static lemons condition (SLC) is satisfied if

$$k_H > v_L.$$  

(SLC)

It guarantees that there is a $\pi_0 \in (0, 1)$ such that $\pi_0 v_H + (1 - \pi_0) v_L < k_H$. In other words, there is a belief below which $H$ prefers not to trade at the expected value, and the market breaks down.

At each moment, the seller chooses a probability distribution over prices, and a probability with which to withdraw her listing. To keep the analysis simple, I focus on equilibria in which $H$ and $L$ initially pool on the price $\bar{p}$ and then choose a time at which to either (1) cut the price to $v_L$ or (2) withdraw the listing.

I will often consider the case in which the seller obtains $v_H$ or $v_L$ from the next buyer who arrives. For each $\eta \in \{H, L\}$, define

$$\tilde{v}_\theta^\eta \equiv \frac{\lambda}{\lambda + r} \cdot v_\eta + \frac{r}{\lambda + r} \cdot k_\theta. \quad (1.21)$$

$\tilde{v}_\theta^\eta$ is the expected present value to the seller. Her home sells at a rate of $\lambda$. At the time of sale, she receives the sale price $v_L$. Before the time of sale, she receives flow utility $r k_\theta$ from owning the home.

The seller can either withdraw her listing and obtain $k_\theta$, or cut her price to $v_L$ and obtain $\tilde{v}_\theta^L$. Given a set of parameters, one of these choices is preferable to the other. For $L$, $\tilde{v}_L^L > k_L$ and hence cutting the price to $v_L$ is preferred. For $H$, $\tilde{v}_H^L > k_H$ if and only if the SLC is violated. If
the SLC is violated, then cutting the price to \( v_L \) is preferred, whereas if the SLC is satisfied, then withdrawing the listing is preferred.

### 1.5.2 Equilibrium Construction

Initially, \( H \) and \( L \) pool on \( \bar{p} \). Buyers’ beliefs evolve as in the quality uncertainty model:

\[
\pi' \equiv \Pi(\pi) = -\pi(1 - \pi)(\Delta_H - \Delta_L).
\]  

(1.22)

The flow of utility to the seller of type-\( \theta \) (\( ru(\pi) \)) equals the instantaneous probability of trade \( (\lambda_{\theta} \bar{p}(\pi) - u_\theta(\pi)) \), plus the flow of utility from owning the home \( (r k_\theta) \), plus the marginal value of improving beliefs \( (u'_{\theta}(\pi) \Pi(\pi, p)) \). The seller’s Bellman equation is therefore

\[
ru(\pi) = \lambda_{\theta}(\bar{p}(\pi) - u_\theta(\pi)) + rk_\theta + u'_{\theta}(\pi)\Pi(\pi).\]  

(1.23)

Equation 1.23 is readily solved by the integrating factor method. Put \( \gamma_{\theta} \equiv \frac{\lambda_{\theta} + r}{\lambda_H - \lambda_L} \), and suppose that for some \( \pi^{o}_{\theta} \in (0, 1) \) and \( u^{o}_{\theta} \in \mathbb{R} \), we have that \( u_{\theta}(\pi^{o}_{\theta}) = u^{o}_{\theta} \). Then the utility of the seller whose home is of quality \( \theta \in \{H, L\} \) is given by

\[
u_{\theta}(\pi) = \left[1 - \frac{\pi^o_{\theta}}{\pi}\right]^{\gamma_{\theta}} \left\{ \frac{\lambda_{\theta} \gamma_{\theta}^{\gamma_{\theta}} \xi^d \xi}{\pi^o_{\theta}} \left(1 - \xi\right)^{\gamma_{\theta} - 1} + \frac{u^{o}_{\theta}}{\lambda_{\theta}^{\gamma_{\theta}} - 1} \left(1 - \frac{\pi^o_{\theta}}{\pi}\right)^{\gamma_{\theta}} u^{o}_{\theta} \right\}.
\]  

(1.24)

Suppose that the seller sets her price to \( \bar{p} \) and takes no further action. Define

\[
u^{\eta}_{\theta} \equiv \frac{\lambda_{\theta}}{\lambda_{\theta} + r} \cdot v_{\eta} + \frac{r}{\lambda_{\theta} + r} \cdot k_\theta.
\]  

(1.25)

Then her utility profile is given by Equation 1.24, where \( \pi^{o}_{\theta} \) and \( u^{o}_{\theta} \) are chosen such that \( u_{\theta}(0^+) = \nu^{L}_{\theta} \) and \( u_{\theta}(1^-) = \nu^{H}_{\theta} \). I plot the utility profiles for \( H \) and \( L \) in Figures 1.2a and 1.2b. It transpires that these utility profiles cannot appear in equilibrium. Figure 1.2a shows that \( u_H \) eventually falls below \( H \)’s reservation value of \( k_H \), so that she prefers to withdraw her listing. Figure 1.2b shows that \( u_L \) eventually falls below \( L \)’s reservation value of \( \nu^{L}_{\theta} \), so that she prefers to cut her price to \( v_L \).

Given that the seller cannot set her price to \( \bar{p} \) without further action, I compute the optimal time to cut the price to \( v_L \) or withdraw the listing (actions which I refer to as stopping). Define
\( u_\theta \equiv \max \{ \tilde{v}_L^\theta, k_\theta \} \) (the value of the preferred action). During the first period, beliefs strictly decrease (Equation 1.22), so to choose an optimal time at which to stop is equivalent to choosing an optimal belief, \( \pi_\theta \in (0, 1) \), at which to stop. Differentiating Equation 1.23 with respect to \( \pi \), I find that if \( u'_\theta(\pi) = 0 \), then \( u''_\theta(\pi) > 0 \). Therefore, \( u_\theta \) is quasiconvex on \((0, 1)\), and I can obtain the optimal stopping belief by value matching (\( u_\theta(\pi_\theta) = u_\theta \)) and smooth pasting (\( u'_\theta(\pi_\theta) = 0 \)):

\[
\pi_\theta \equiv \frac{u_\theta - \tilde{v}_L^\theta}{\tilde{v}_H^\theta - \tilde{v}_L^\theta}.
\] (1.26)

Figures 1.2c and 1.2d illustrate how \( H \)'s and \( L \)'s utility profiles change under optimal stopping.

The equilibrium which I now sketch depends on the ordering of \( \pi_H \) and \( \pi_L \) (whether \( H \) wants to stop before \( L \), or \( L \) before \( H \)). I specify off-equilibrium path beliefs to be those for which any deviation is attributed to the seller of the home \( L \) quality. I consider the cases \( \pi_H > \pi_L \), and \( \pi_H < \pi_L \) in turn.

**Case I.** Suppose that \( \pi_H > \pi_L \). At time \( t_H^+ \equiv \pi^{-1}(\pi_H^+) \), \( H \) stops. If the SLC is satisfied, then she withdraws her listing. Knowing that \( H \) has withdrawn her listing, buyers believe any remaining home to be \( L \) quality: \( \pi(t_H^+) = 0 \). If the SLC is violated, then \( H \) cuts her price to \( v_L \) and trades with the next buyer who arrives. Again, buyers believe any remaining home to be \( L \) quality: \( \pi(t_H^+) = 0 \). I illustrate Case I in Figure 1.2e.

**Case II.** Suppose that \( \pi_H < \pi_L \). At time \( t_L^+ \equiv \pi^{-1}(\pi_L^+) \), \( L \) is indifferent between receiving \( \bar{p}(\pi_L) \) at a rate of \( \Delta_L \), and \( v_L \) at a rate of \( \lambda \). She mixes between setting her price to \( \bar{p}(\pi_L) \) and \( v_L \): over any time interval of length \( dt \) and for some \( \beta \geq 0 \), she sets her price to \( v_L \) with probability \( \beta dt \), and \( \bar{p}(\pi_L) \) with probability \( 1 - \beta dt \). Equivalently, the arrival of the time at which \( L \) cuts her price to \( v_L \) is exponentially distributed with rate \( \beta \). Therefore, beliefs evolve according to

\[
\pi'(t_L^+) = -\pi_L(1 - \pi_L)(\Delta_H - \Delta_L - \beta).
\] (1.27)

She mixes at precisely the rate that keeps beliefs fixed at \( \pi_L \), namely, \( \beta = \Delta_H - \Delta_L \). Mixing at any rate \( \geq \lambda_H - \lambda_L \) is payoff equivalent. Mixing at any rate \( < \lambda_H - \lambda_L \) causes beliefs to continue to deteriorate. I illustrate Case II in Figure 1.2f.
1.5.3 Discussion

Before leaving this section, I briefly summarize the implications of the dynamic adverse selection model, and compare them to the implications of the quality uncertainty model (Section 1.2). In both Case I and Case II, there is a time before which the distribution of days-on-market and sale prices are identical to those of the quality uncertainty model.

In Case I, there are two possibilities. First, the seller of the \( H \) quality home might find herself better-off withdrawing her listing. Second, she might find herself better-off cutting her price to \( v_L \). In either case, \( \pi(t^+_H) = 0 \), and hence the rate at which homes sell jumps discontinuously to \( \lambda \), and sale prices fall discontinuously to \( v_L \). Compare with the quality uncertainty model, in which the rate at which homes sell is bounded strictly above by \( \lambda \), and sale prices are bounded strictly below by \( v_L \). In Case II, sellers of \( L \) quality homes mix between \( \bar{p}(\pi_L) \) and \( v_L \), while sellers of \( H \) quality homes sell their homes for \( \bar{p}(\pi_L) \). Sale prices fall discontinuously at time \( t^+_L \). As in Case I, the rate at which homes sell jumps discontinuously at time \( t^+_L \).

It is tempting to search for separating equilibria in which sellers of high quality homes signal their home’s quality by delaying trade (their homes are of better quality, and are therefore less burdensome to live in). In fact, [58] develop a model of dynamic adverse selection with search, and find that when initial beliefs are low (high), beliefs tend to increase (decrease) over time. Their framework admits the possibility that sellers of high quality assets (such as houses) can signal their quality by rejecting buyers’ offers.

Separating equilibria cannot be supported in my framework. Following a standard argument in the dynamic adverse selection literature, suppose that in equilibrium, \( L \) stops at a time \( \tau_L \geq 0 \), and \( H \) stops at time \( \tau_H > \tau_L \). Since buyers’ beliefs must obey Bayes’ law on the equilibrium path, \( \pi(\tau^+_L) = 1 \). But then \( L \) deviates by waiting, by which she obtains a payoff of \( \tilde{v}_L^H \), which strictly exceeds her equilibrium payoff of \( \tilde{v}_L^L \).
1.6 Conclusion

Housing accounts for half of household wealth. This paper quantifies the effect of uncertainty about a home’s quality on its sale price and days-on-market. I structurally estimate days-on-market and sale price counterfactuals for six-hundred thousand homes that sold between 2005 and 2015 in three disparate metropolitan areas. I find that relative to a counterfactual in which a home’s quality were known by buyers with certainty, high quality homes take 11 days longer to sell, and sell for 8.0% less, while low quality homes take 47 days longer to sell, and sell for 4.5% more. These estimates suggests that quality uncertainty exacerbates the underlying mispricing and illiquidity of housing.

The model demonstrates that the negative relationship between the sale-to-list ratio and days-on-market—documented in both the literature and in my sample—can be explained by Bayesian learning. If buyers care about what other buyers thought of a home, then they become more pessimistic about the home’s quality the longer the home sits on the market: previous buyers arrived for a walkthrough, observed some undesirable quality, and chose not to buy. Consistent with this explanation, I find that quality uncertainty increases with the home’s age and size.
Appendix: Proofs

For ease of exposition, let the symbol “±” be “+” when \( \theta = H \), and “−” when \( \theta = L \) (and \( ± \equiv -(±) \)); let the symbol “≤” be “<” when \( \theta = H \), and “>” when \( \theta = L \). Let \( \varphi \) denote the standard normal PDF.

**Proof of Lemma 1.2.4.** The two comparative statics are

\[
D_\lambda DM_\theta = -\lambda^{-1}\lambda_\theta^{-1} < 0 \tag{1.28}
\]

\[
D_\phi DM_\theta = \mp \frac{\lambda}{2} \varphi \left( \mp \frac{\phi}{2} \right) \lambda_\theta^{-2} \leq 0 \tag{1.29}
\]
as desired. Moreover, \( \lambda_\theta = \lambda \left( 1 - \Phi \left( \mp \frac{\phi}{2} \right) \right) < \lambda \), and

\[
\lambda_L = \lambda \left( 1 - \Phi \left( \mp \frac{\phi}{2} \right) \right) < \lambda \left( 1 - \Phi \left( -\frac{\phi}{2} \right) \right) = \lambda_H \tag{1.30}
\]
from which it follows that \( DM_\circ < DM_H < DM_L \).

**Proof of Lemma 1.2.5.** Put \( q_\theta \equiv Q'_\theta \) and \( \hat{p} \equiv \bar{p} \circ \pi \). Note that

1. \( \lim_{t \downarrow 0} \hat{p}(t)q_\theta(t)t = \lim_{t \uparrow \infty} \hat{p}(t)q_\theta(t)t = 0 \tag{1.31} \)

2. \((\Delta_H - \Delta_L)^{-1} > \pm \frac{1}{2} \Delta_\theta^{-1} \).

\[
[D_\phi \hat{p}](t) = (v_H - v_L)[D_\phi \pi](t) \tag{1.33}
\]

\[
= -\pi(t)(1 - \pi(t)) \cdot \lambda \varphi(\phi/2)(v_H - v_L)t \tag{1.34}
\]

\[
= (\Delta_H - \Delta_L)^{-1} \cdot \lambda \varphi(\phi/2)(v_H - v_L) \cdot \pi'(t)t \tag{1.35}
\]

\[
= (\Delta_H - \Delta_L)^{-1} \cdot \lambda \varphi(\phi/2) \cdot \hat{p}'(t)t, \text{ and} \tag{1.36}
\]

\[
[D_\phi q_\theta](t) = \pm (1 - \lambda_\theta t)e^{-\lambda_\theta t} \cdot \frac{1}{2} \varphi(\phi/2) \tag{1.37}
\]

\[
= \pm \frac{1}{2} \Delta_\theta^{-1} \cdot \lambda \varphi(\phi/2) \cdot (q_\theta(t) + q_\theta'(t)t). \tag{1.38}
\]
Hence

\[ D_\phi SP_\theta = \int_0^\infty ([D_\phi \hat{p}](t) q_\theta(t) + \hat{p}(t) [D_\phi q_\theta](t)) dt \]  

\[ = \lambda \varphi(\phi/2) \cdot \int_0^\infty ((\Delta_H - \Delta_L)^{-1} \hat{p}'(t) q_\theta(t) + \hat{p}(t) q_\theta(t) + \hat{p}(t) q_\theta'(t) t) dt \]  

\[ < \pm \lambda \varphi(\phi/2) \cdot \int_0^\infty (\hat{p}'(t) q_\theta(t) + \hat{p}(t) q_\theta(t) + \hat{p}(t) q_\theta'(t) t) dt \]

\[ = 0 \]  

where the second to last line follows from Equation 1.32 and the fact that \( \hat{p}' < 0 \), and the last line from Equation 1.31. Note that \( D_\lambda \Delta_\theta = \lambda^{-1} \Delta_\theta \),

\[ [D_\lambda P](t) = (v_H - v_L)[D_\lambda \pi](t) \]  

\[ = -\pi(t)(1 - \pi(t)) \cdot \lambda^{-1}(\Delta_H - \Delta_L)(v_H - v_L)t \]  

\[ = \lambda^{-1}(v_H - v_L)\pi'(t)t \]  

\[ = \lambda^{-1} \cdot P'(t)t, \text{ and} \]

\[ [D_\lambda q_\theta](t) = (1 - \lambda_\theta t)e^{-\lambda_\theta t} \cdot \lambda^{-1} \Delta_\theta \]  

\[ = \lambda^{-1} \cdot (q_\theta(t) + q_\theta'(t)t) \].

Hence

\[ D_\lambda SP_\theta = \int_0^\infty ([D_\phi P](t) q_\theta(t) + P(t) [D_\phi q_\theta](t)) dt \]  

\[ = \lambda^{-1} \cdot \int_0^\infty (P'(t)q_\theta(t)t + P(t)q_\theta(t) + P(t)q_\theta'(t)t) dt \]  

\[ = 0. \]

where the last line follows from Equation 1.31. For each \( t > 0, \pi(t) \in (0,1) \), and hence \( v_L < p(\pi(t)) < v_H \), and hence \( v_L < SP_\theta < v_H \).
Figure 1.1. **Days-on-Market and Sale-to-List Ratio.** These plots compare the model fit with a LOESS fit. The left shows the empirical PDF of days-on-market. The right plots show the expected sale-to-list ratio for each day-on-market.
Figure 1.2. **Dynamic Adverse Selection (Section 1.5)**. Panel (a) shows $H$’s utility when she passively lists according to $\bar{p}$. (c) shows her utility when she optimally withdraws at $\pi_H$. Panel (b) shows $L$’s utility when she passively lists according to $\bar{p}$. Panel (d) shows her utility when she optimally capitulates at $\pi_L$. Panel (e), illustrates the equilibrium when $\pi_L < \pi_H$, while Panel (f), illustrates the equilibrium when $\pi_H < \pi_L$. 
Table 1.1. **Summary Statistics.** This table presents summary statistics for my sample of parcels. Panel A shows sample means for Denver, Charlotte, and Detroit. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home.

### Panel A: Sample Means

<table>
<thead>
<tr>
<th></th>
<th>Denver</th>
<th>Charlotte</th>
<th>Detroit</th>
</tr>
</thead>
<tbody>
<tr>
<td>List Price ($k)</td>
<td>264.8</td>
<td>196.0</td>
<td>98.7</td>
</tr>
<tr>
<td>Sale Price ($k)</td>
<td>260.9</td>
<td>189.5</td>
<td>94.8</td>
</tr>
<tr>
<td>Sale-to-List (%)</td>
<td>98.7</td>
<td>96.6</td>
<td>95.2</td>
</tr>
<tr>
<td>Days-on-Market</td>
<td>44.6</td>
<td>63.8</td>
<td>47.2</td>
</tr>
<tr>
<td>Age (years)</td>
<td>33.4</td>
<td>21.9</td>
<td>55.1</td>
</tr>
<tr>
<td>Square Feet</td>
<td>1,936.2</td>
<td>1,953.6</td>
<td>1,385.3</td>
</tr>
<tr>
<td>#Listings</td>
<td>316,760</td>
<td>211,012</td>
<td>51,265</td>
</tr>
</tbody>
</table>

### Panel B: Percentiles

<table>
<thead>
<tr>
<th></th>
<th>Denver</th>
<th>Charlotte</th>
<th>Detroit</th>
</tr>
</thead>
<tbody>
<tr>
<td>List Price ($k)</td>
<td>165</td>
<td>239</td>
<td>335</td>
</tr>
<tr>
<td>Sale Price ($k)</td>
<td>164</td>
<td>235</td>
<td>330</td>
</tr>
<tr>
<td>Sale-to-List (%)</td>
<td>97</td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>Days-on-Market</td>
<td>7</td>
<td>25</td>
<td>66</td>
</tr>
<tr>
<td>Age (years)</td>
<td>12</td>
<td>29</td>
<td>47</td>
</tr>
<tr>
<td>Square Feet</td>
<td>1,303</td>
<td>1,791</td>
<td>2,389</td>
</tr>
<tr>
<td>#Listings</td>
<td>316,760</td>
<td>211,012</td>
<td>51,265</td>
</tr>
</tbody>
</table>
Table 1.2. **Days-on-Market and Sale-to-List.** This table presents regressions of the sale-to-list ratio on days-on-market. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home. For ease of interpretation, the sale-to-list ratio is quoted in points. For example, a 0.02 coefficient on days-on-market means that for each additional day-on-market, the sale-to-list ratio declines by 2 basis points. Standard errors are in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.

<table>
<thead>
<tr>
<th></th>
<th>Sale-to-List (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Denver</td>
</tr>
<tr>
<td>Days-on-Market</td>
<td>-0.019***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>log(Age)</td>
<td>-0.057***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>log(Square Feet)</td>
<td>-0.476***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
</tr>
<tr>
<td>Quarter × ZIP × Year × Quarter</td>
<td>×</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>7.01%</td>
</tr>
<tr>
<td>#Listings</td>
<td>633,506</td>
</tr>
</tbody>
</table>
Table 1.3. **Maximum Likelihood Estimates.** This table presents maximum likelihood estimates. The likelihood is defined in Equation 1.20. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home; 1st–4th Quarter denote dummies for the quarter in which the home was listed. For ease of interpretation, I demean log(Age) and log(Age). Panel A displays estimates for the sale-to-list ratio volatility, $\sigma$. Panels B, C, and D correspond to the specifications in Equations 1.16, 1.17, and 1.18 respectively: Panel B displays estimates for the buyer arrival rate, $\lambda$; Panel C displays estimates for the quality premium, $\eta \equiv v_H/v_L$; Panel D displays estimates for the signal-to-noise ratio, $\phi \equiv (v_H - v_L)/\sigma_x$. Standard errors are computed using the asymptotic variance of the maximum likelihood estimator, and are shown in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.

<table>
<thead>
<tr>
<th>Panel A: Sale-to-List Ratio Volatility ($\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Buyer Arrival Rate ($\lambda$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>log(Age)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log(SqFt)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1st Quarter</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2nd Quarter</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3rd Quarter</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4th Quarter</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
### Panel C: Quality Premium ($\eta \equiv v_H/v_L$)

<table>
<thead>
<tr>
<th></th>
<th>Denver</th>
<th>Charlotte</th>
<th>Detroit</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Age)</td>
<td>0.003***</td>
<td>0.007***</td>
<td>0.040***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>log(SqFt)</td>
<td>0.012***</td>
<td>-0.020***</td>
<td>-0.193***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>1st Quarter</td>
<td>1.068***</td>
<td>1.155***</td>
<td>1.302***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>2nd Quarter</td>
<td>1.070***</td>
<td>1.148***</td>
<td>1.231***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>3rd Quarter</td>
<td>1.074***</td>
<td>1.163***</td>
<td>1.320***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>4th Quarter</td>
<td>1.082***</td>
<td>1.159***</td>
<td>1.287***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.009)</td>
</tr>
</tbody>
</table>

### Panel D: Signal-to-Noise Ratio ($\phi$)

<table>
<thead>
<tr>
<th></th>
<th>Denver</th>
<th>Charlotte</th>
<th>Detroit</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Age)</td>
<td>-0.019</td>
<td>0.237***</td>
<td>0.282***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.006)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>log(SqFt)</td>
<td>-0.023</td>
<td>0.112***</td>
<td>0.328***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>1st Quarter</td>
<td>1.039***</td>
<td>0.958***</td>
<td>0.819***</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.015)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>2nd Quarter</td>
<td>1.020***</td>
<td>1.010***</td>
<td>0.989***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.015)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>3rd Quarter</td>
<td>0.990***</td>
<td>0.914***</td>
<td>0.790***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>4th Quarter</td>
<td>0.986***</td>
<td>0.960***</td>
<td>0.809***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.015)</td>
<td>(0.026)</td>
</tr>
</tbody>
</table>
Table 1.4. **Days-on-Market Counterfactuals.** This table presents the results of hedonic regressions. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home. In the first row of Panels A and B, I estimate the mean days-on-market for each metropolitan area (compare with Table 1.1). In the second row, I estimate the mean days-on-market for an *H* quality home (Panel A) and an *L* quality home (Panel B). In the third row, I estimate the mean counterfactual days-on-market for an *H* quality home (Panel A) and an *L* quality home (Panel B). In the fourth row, I estimate the percent difference in the mean days-on-market from the counterfactual to the factual. Standard errors are computed using the asymptotic variance of the maximum likelihood estimator, and are shown in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.

<table>
<thead>
<tr>
<th></th>
<th>Denver</th>
<th>Charlotte</th>
<th>Detroit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean (H&amp;L)</strong></td>
<td>44.616***</td>
<td>63.777***</td>
<td>47.188***</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.125)</td>
<td>(0.199)</td>
</tr>
<tr>
<td><strong>Estimated</strong></td>
<td>26.511***</td>
<td>39.463***</td>
<td>31.574***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td><strong>Counterfactual</strong></td>
<td>18.375***</td>
<td>26.776***</td>
<td>20.870***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>Percent Difference</strong></td>
<td>0.443***</td>
<td>0.467***</td>
<td>0.508***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Denver</th>
<th>Charlotte</th>
<th>Detroit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean (H&amp;L)</strong></td>
<td>44.616***</td>
<td>63.777***</td>
<td>47.188***</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.125)</td>
<td>(0.199)</td>
</tr>
<tr>
<td><strong>Estimated</strong></td>
<td>59.892***</td>
<td>85.143***</td>
<td>62.617***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td><strong>Counterfactual</strong></td>
<td>18.375***</td>
<td>26.776***</td>
<td>20.870***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>Percent Difference</strong></td>
<td>2.260***</td>
<td>2.211***</td>
<td>2.019***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>
Table 1.5. **Sale Price Counterfactuals.** This table presents the results of hedonic regressions. Age is the age of the home at the time of listing in years; SqFt is the square-footage of the home. Prices are quoted in thousands of US dollars. In the first row of Panels A and B, I estimate the mean sale price for each metropolitan area (compare with Table 1.1). In the second row, I estimate the mean sale price for an \( H \) quality home (Panel A) and an \( L \) quality home (Panel B). In the third row, I estimate the mean counterfactual sale price for an \( H \) quality home (Panel A) and an \( L \) quality home (Panel B). In the fourth row, I estimate the percent difference in the mean sale price from the counterfactual to the factual. Standard errors are computed using the asymptotic variance of the maximum likelihood estimator, and are shown in parentheses. *, **, and *** denote statistical significance at the ten, five, and one percent levels.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ((H&amp;L))</td>
<td>260.870***</td>
<td>189.517***</td>
<td>94.781***</td>
</tr>
<tr>
<td></td>
<td>(0.238)</td>
<td>(0.259)</td>
<td>(0.406)</td>
</tr>
<tr>
<td>Estimated</td>
<td>262.422***</td>
<td>192.674***</td>
<td>96.366***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>274.247***</td>
<td>209.780***</td>
<td>108.885***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>-0.043***</td>
<td>-0.082***</td>
<td>-0.115***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Sale Price for L-Quality Home</th>
<th>Denver</th>
<th>Charlotte</th>
<th>Detroit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ((H&amp;L))</td>
<td>260.870***</td>
<td>189.517***</td>
<td>94.781***</td>
</tr>
<tr>
<td></td>
<td>(0.238)</td>
<td>(0.259)</td>
<td>(0.406)</td>
</tr>
<tr>
<td>Estimated</td>
<td>260.555***</td>
<td>190.085***</td>
<td>94.725***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>255.304***</td>
<td>182.198***</td>
<td>88.439***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>0.021***</td>
<td>0.043***</td>
<td>0.071***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>
Chapter 2

Constrained Asset Prices

2.1 Introduction

In determining the terms of a secured loan, lenders estimate the value of the asset which serves as collateral. These estimates—termed appraisals—are based on recent transaction prices of similar assets. In this paper, we construct a model in which lenders use backward-looking appraisals as an estimate of the liquidation value of the asset. We show implications for price adjustment, return predictability, volume, supply-demand imbalances, and a number of other variables. We also document new stylized facts consistent with the model’s predictions in the markets for used airplanes, New York City taxi medallions, commercial real estate, and residential real estate.

Our model makes six empirical predictions: (1) constraints on offer and sale prices determine the “speed limit” of price changes in a market. Because speed limits are likely to be similar from month-to-month and year-to-year within a market, returns are highly predictable; (2) because different markets have different speed limits, prices and appreciation rates can vary substantially across markets and markets with higher earnings-to-price ratios exhibit substantially lower, or substantially higher total returns even when cash flows, discount rates, and growth rates are identical; (3) changes in agents’ valuations generate trade. As the rate of change in agents’ valuations increases, trade increases. Faster trade allows faster appreciation, so markets with more change in valuations across agents will experience faster price changes; (4) in rapidly appreciating markets, queues of buyers form, so that liquidity on the buyers’ side is severely diminished (this is a “seller’s market,” in which selling is easy, buying is hard, and prices are rising); (5) in rapidly appreciating markets,
“investment” buyers (also known as “flippers”) enter. These are not natural owners of the asset. They enter to take advantage of predictable returns.

Three aspects of a market are necessary for our model to apply. (1) most purchases must be financed with debt and lenders must be limited in their willingness to lend more than a transaction-based appraisal. (2) the market must be illiquid. If transactions are frequent, then the price adjusts quickly and the model does not make any empirical predictions. (3) there must be frictions in the rental market for the asset. This is to prevent a few deep-pocketed investors from bidding prices to their long-run level without the need for debt and then renting the asset to constrained buyers. We call markets possessing these three properties *appraisal-affected*.

In the market for single-family homes, most buyers finance their purchases with loans. In the US, it takes on average four months to sell a home. Finally, homeowners incur substantial costs in renting their homes, such as maintenance and vacancy. After introducing the model, we delve into our model’s predictions in both of these markets, as well as those for used aircraft and commercial real estate.

The model predicts that appraisal-affected markets feature return-predictability. Figure 2.1 plots contemporaneous and future quarterly or monthly returns for six assets. There is clear return-predictability for aircraft, taxi medallions, single family homes, and commercial real estate. For comparison, Figure 2.1 also provide similar plots for large stocks and for seats on the New York Stock Exchange. Neither stocks nor seats are typically purchased with collateralized debt, and returns to neither are predictable at the monthly level.

In order to investigate the other six predictions of the model, we focus on the market for single family homes. We do so because this market is large – valued at $25 trillion in 2016 – and because the data are higher quality. As anyone who has been through the process of purchasing a house knows, lenders use appraisals based on recent sales of comparable homes as the basis of their estimate of value. Indeed, this is codified into law: real estate loans of $250,000 or more require an appraisal, and the United States Congress authorizes The Appraisal Foundation to write the Uniform Standards of Professional Appraisal Practice. According to the Appraisal Foundation,
“often the primary approach to develop an opinion of value for a residential property, the sales comparison approach utilizes recent sales of comparable properties” (emphasis added).

In spite of the results in this paper, requiring appraisals may be consistent with efficient contracting. Banks, government sponsored entities (Fannie Mae and Freddie Mac), mortgage insurers, and mortgage-backed security buyers need some measure of the value of a property against which the loan amount can be compared. As originators have a clear incentive to inflate the value, some objective constraint on valuations must be chosen. Unless a buyer has enough cash to make up the difference, her ability to pay a given price for a house is limited by prices of recent transactions. This makes residential real estate an excellent laboratory to test our model’s implications.

The model predicts that identical assets can trade at different prices. Figure 2.2 shows total returns on rental real estate versus rent-to-price ratios for 200 major metro areas covered by Zillow. In the data, prices range from as low as 5 times rents to as high as 25 times rents. In the stock market, variation like this in Price-to-Earnings ratios is associated with variation in expected growth rates in earnings: prices are high relative to earnings because earnings are expected to grow. As we see in Figure 2.2, while it is true that more expensive cities experience faster price appreciation, they actually experience lower total returns. This is consistent with the model’s fifth implication: total returns can be substantially higher for lower priced assets.

The model predicts that assets for which the natural owner changes frequently experience faster appreciation. This is because more change in natural ownership causes more trade, which allows for more recent comparable prices. As prices are generally increasing, more recent prices mean higher comparable prices, which allows for a higher offer price. Figure 2.3 plots the relationship between turnover – the fraction of homes in a neighborhood that changed hands in a given year – and price appreciation. As the model suggests, in the domain in which prices are increasing, higher turnover is associated with faster appreciation.

The model predicts that liquidity on the buyer’s side will be diminished when prices are rising. Figure 2.4 shows the average number of days on market for a property, compared to the market’s annual price appreciation. The existence of “sellers’” and “buyers’” markets is clear: when
prices are increasing quickly, houses sell quickly. Figure 2.5 plots the value of the Case-Shiller 10 City Composite from 1987 to 2015, along with the supply of unsold housing inventory (normalized by monthly sales). It is clear that inventories are low and constant when prices are appreciating and grow as prices level out and then fall.

The model predicts the presence of investors who occupy the market during periods of rapid appreciation but not at other times. In the single-family residential real estate market, in last 15 years, this behavior has been called “flipping” (in large part due to a television show about how to make money doing so called “Flip This House”). Figure 2.6 shows that flipping—defined as the fraction of sales for which the home has sold twice in one year—is positive correlated with price appreciation.

We model the case in which buyers are constrained and prices are rising, but the argument likely holds for the case in which sellers are constrained and prices are falling. In this case, a buyer’s market emerges, in which case buyers are able to quickly find an asset to buy, but sellers take longer to find a buyer.

We show that the model’s core empirical prediction of price-predictability is present in several disparate markets: residential real estate, commercial and industrial real estate, taxi medallions, and airplanes. This prediction is not borne out in stocks or seats on the NYSE, markets which are not subject to the core constraint defining the model.

Our theory has implications for monetary policy. Central banks respond to the onset of recessions by cutting interest rates—with the intent of spurring investment—and much of that response must come from investment in residential real estate. Low interest rates spur investment more if prices of real estate rise more quickly in response to low rates. Our model establishes a limit on price increases driven by the requirement/structure of appraisals. The effectiveness of monetary policy is therefore limited by appraisal constraints. The ability to relax these constraints could be a useful additional tool for policy-makers looking to respond to recessions.

The model also has welfare implications. During the period in which prices are rising, assets will be mis-allocated. Some owners who should sell retain the asset in order to capture the expected
price appreciation. Others who should buy will be unable to find a seller. If the price were able
to adjust immediately, then the asset would always be held by the person who most values it.
Especially if we believe that owner-occupancy serves some good, then reducing the strength of
appraisal constraints can improve welfare.

2.2 Literature

Return predictability in real estate was identified in [21] and has been highly studied the-
oretically and empirically since then. While there are models of momentum that apply to assets
generally (e.g., [55]), a substantial theoretical literature has arisen to explain momentum in real
estate specifically. [86], [53], [6], [106], [43], and [3] offer search models which exhibit momentum.
[96], [99], [39], and [40] offer alternative explanations. Most similar to our model, [87] note that
appraisals are backward-looking and that they are biased downward in periods of price apprecia-
tion, but they do not formally model the price process resulting from an appraisal constraint. [39],
[24], and [43] also document long-run reversals in returns on housing. Our baseline model cannot
account for this fact, though [55] and [40] can. A natural extension would allow for the supply of
the asset to respond to prices – e.g., the supply of housing would increase if prices were high enough
to justify new construction. We suspect that this would allow for (partial) reversal of prices, but
do not model it.

The relationship between rent-price ratios and total returns has previously been documented
by [22]. The existence of hot and cold markets has been well documented, and several theories have
attempted to explain them, both at the seasonal level (e.g., [82]), and at lower frequencies (e.g.,
[82], [17]).

An extensive literature has studied the effects of constraints on market participants and the
effect on prices and efficiency. This literature is too large to fully discuss here, but a few examples
are in order. [79] discusses the effect of short-sale constraints on asset prices, and [51] analyze the
effects of leverage constraints. A substantial literature arose in the early 1990s surrounding the
failure of the Consumption Capital Asset Pricing Model (CCAPM) to account for equity returns:
analyze the effects of short-sale, leverage, and solvency constraints, and argue that these can reconcile the CCAPM with stock price return data. [68] disagrees. [35] analyze asset pricing when investors face margin requirements. Other forms of constraints on investors have been studied as well. For example, [32] analyze a setting in which foreign ownership is constrained. We do not provide a thorough literature review on these models here.

More recently, a small literature has developed documenting the inefficient use of “comparable” transaction prices in setting prices. For example, the market for syndicated lending appears subject to excessive reliance on comparable prices. [81] show this for debt at separate firms. [28] find an overuse of a firm’s past borrowing rates in setting rates for a new loan. In our model, we establish that using comparables to appraise assets has real effects on market quality, though we do not show that the practice is inefficient in a broader sense. There is clearly a good reason to use comparables, especially in illiquid markets with hard-to-value assets.

2.3 The Model

2.3.1 Setup

Time is continuous and indexed by $t \in [0, T]$, for some $T > 0$. There is a unit mass of an asset (e.g. houses, shares in a firm, taxi medallions, airplanes, etc.). There is a mass $N > 1$ of risk-neutral, penniless agents who discount the future at a rate of $r > 0$. They have random utility for the asset. Consider a particular agent. She receives flow utility $v - x$ from owning the asset, where $v > 0$ is common and $x$ is idiosyncratic and uniformly distributed over $[0, N]$. Note that there is a greater mass of assets than agents. Agents can own no more than one asset. Let $p(t)$ denote the price of the asset at time $t$. Initially, agents with types $x \in [0, 1]$ are endowed with assets and the remainder do not own the asset. Agents of all types are allowed to trade at no cost. Absent a change in value, $v$, or the discount rate, $r$, the demand for the asset is the mass of agents for whom the surplus from purchase is positive:

$$D(p) = N \cdot \Pr \left( \frac{v - x}{r} > p \right) = v - rp. \tag{2.1}$$
Equating supply and demand, one finds the equilibrium price

\[ p^* = \frac{v - 1}{r}. \]  

(2.2)

We induce trading by randomly shocking agents’ types. Over a time interval of length \( dt \), a mass \( \lambda dt \) of agents are shocked. Their new types are uniformly distributed on \([0, N]\). Because (1) types are initially uniformly distributed on \([0, N]\), (2) shocks occur for all types equally often, and (3) new types are uniformly distributed on \([0, N]\), types remain uniformly distributed on \([0, N]\).

Suppose the asset is a house, and that there is a unit mass of houses which constitute a town. A shock to a type might be a good new job opportunity. If the opportunity is in the town, then the agent will want to move to the town, so long as house prices are not too high. If the opportunity is elsewhere, then she will want to live elsewhere. These facts are true regardless of where her previous job was located.

We consider an unanticipated shock to the asset’s value. The shock may affect value, \( v \), the discount rate, \( r \), or both. At \( t = 0 \), the fair price changes from \( p_0 \) to \( p_1 \).

### 2.3.2 Constrained Prices

The basis of this manuscript is the following assumption:

**Assumption 2.3.1.** Lenders do not lend buyers more than recent transaction prices.

Let \( \phi(t, p) \) denote the trade flow. The window over required comps, \( w \), the average price in recent transactions, \( \bar{p} \), and the transaction price, \( p \), are jointly defined by

\[ C = \int_{t-w(t)}^{t} \phi(s, p) ds \]  

(2.3)

\[ \bar{p}(t) = \frac{1}{C} \int_{t-w(t)}^{t} p(s) \phi(s, p(s)) ds \]  

(2.4)

\[ p(t) = \min\{L\bar{p}(t), p_1\} \]  

(2.5)

The following Lemma gives us an ordinary differential equation for the price dynamics.
Lemma 2.3.1. If $\tau(t) < 0$ for all $t \in [0,T)$, then the price evolves according to

$$p'(t) = \frac{L}{C} \cdot (p(t) - p_0) \cdot \phi(t,p)$$

subject to $p(0) = Lp_0$.

We refer to the differential equation in Lemma 2.3.1 as the pricing equation. Because $\phi$ depends on $p$, the dynamics for $p$ may be non-linear. Prices appreciate faster when they have greater flexibility ($L$ is large) and require fewer comps ($C$ is small), when the difference between the current price and the old price is large ($p(t) - p_0$ is large), and when the trade flow is high ($\phi(t,p)$ is large). As a corollary, we compute the cumulative trade flow:

**Corollary 2.3.1.** The cumulative trade flow is given by

$$\int_0^t \phi(s)ds = \frac{C}{L} \log \left( \frac{p(t) - p_0}{Lp_0 - p_0} \right).$$

Fewer trades mean fewer comps, which means a longer adjustment-period; more trades mean more comps, which means a shorter adjustment-period. The length of the adjustment period decreases with the trade flow at such a rate that cumulative trade at the time of adjustment is constant. Since the trade flow is strictly positive, we immediately obtain the following corollary.

**Corollary 2.3.2.** $p$ is strictly increasing.

There is a time $\bar{t} \in (0,T)$ at which $p(\bar{t}) = p_1$. There are three periods of interest in the model: the pre-shock period, during which the fair price is $p_0$; the adjustment period, during which the price constraint binds; the post-adjustment period, during which the price is $p_1$. Note that, if $p_1 \leq Lp_0$, there is no period of adjustment. If $p_1 > Lp_0$, then there is a period during which the price is constrained.

Before leaving this section, we make note of two implications from the model. First, price changes are predictable. Prices do not change for $t < 0$ or $t > \bar{t}$. Adjustment is a smooth process: if one knows prices increased by, for example, 10% in the last $\Delta t$, then one knows that they will
increase something in the neighborhood of 10% in the next $\Delta t$, so long as that does not take the time past $\tilde{t}$.

Second, prices and appreciation rates can vary substantially among assets that appear to offer identical cash flows and risks. This is true within a single asset over time: the relevant asset pricing variables $V$ and $r$ are constant during the adjustment period, yet prices are changing. This means that, at different points in the adjustment period, the same asset will have different prices and different total returns. This is also true across assets. The model only has a single asset, but presumably the economy would have more than one. For two assets with identical values of $V$ and $r$, but different parameters $L_H, L_L, C$, or $\lambda$, appreciation and depreciation rates will differ. Apparently equivalent assets will feature different pricing and returns.

2.3.3 Naïve Agents

In this section, we begin with a model in which agents are naïve, in that they decide whether to trade based only on the flow utility from owning the asset and the flow disutility from paying for the asset, and not price appreciation due to the appraisal constraint.

The advantage of beginning with this non-fully-rational model is that it is considerably simpler to solve than the full model, yet still provides a substantial share of the empirical implications that the full model yields.

During a time interval of length $dt$, a mass $\lambda dt$ of agents are shocked, a fraction $1/N$ of whom are owners, a fraction $(N-1)/N$ of whom are shocked to $(1,N]$. Similarly, a fraction $(N-1)/N$ of shocked agents are non-owners, a fraction $1/N$ of whom are shocked to $[0,1]$. The flow of sellers and buyers are equal, so the market clears at a volume of

$$\phi_0 \equiv \lambda \cdot \frac{1}{N} \cdot \frac{N-1}{N}$$

Assumption 1 states that a buyer’s ability or willingness to pay is limited by a mass $C$ of recent sale prices ("comps"). Over the period $[t-C/\phi_0, t]$, a mass $C$ of trades occur. Therefore, $w(t) = C/\phi_0$
and
\[ \bar{p}(t) = \frac{1}{C} \int_{t-C\phi_0^{-1}}^{t} p(s)\phi_0 ds. \] (2.9)

The price is \( p_0 \) for \( t < 0 \), jumps to \( Lp_0 \) at \( t = 0 \), evolves according to
\[ p(t) = p_0 + (L - 1)p_0 \exp \left\{ \frac{L}{C} \cdot \phi_0 \cdot t \right\} \] (2.10)
for \( t \in (0, \bar{t}) \), where
\[ \bar{t} = \frac{C}{L\phi_0} \cdot \log \left( \frac{p_1 - p_0}{Lp_0 - p_0} \right), \] (2.11)
and is \( p_1 \) for \( t \in [\bar{t}, \infty) \).

With no pricing constraint the actual price jumps immediately, as shown in the lightly dashed line in the top-left panel of panel of Figure 2.7. Under Assumption 2.3.1, if \( p_0 \ll p_1 \), and if agents trade as soon as their types/valuations are shocked, then the price will gradually adjust from \( p_0 \) to \( p_1 \), as shown in the heavily dashed line in the top-left panel of Figure 2.7.

Comparative statics can be obtained easily and, even though the model is not yet fully rational, these comparative statics establish several implications. Consider the speed of adjustment:
\[ p'(t) = \frac{L(L - 1)p_0\phi_0}{C} \exp \left\{ \frac{L}{C} \cdot \phi_0 \cdot t \right\}. \] (2.12)
First, it is increasing with the trading intensity, \( \phi_0 \). More trading means more recent comps. As prices are increasing over time, more recent comps imply higher average selling prices off of which an appraisal is based, and therefore higher current appraisals. This weakens the appraisal constraint, allowing faster appreciation. Second, the speed of adjustment, \( p'(t) \), is increasing with the flexibility in appraisals, \( L \). Third, the speed of adjustment is decreasing with the required comps \( C \). Naturally, the more comps that are required, the farther back in time the appraiser must look. As prices are increasing over time, looking farther back for comps reduces the average price in an appraisal, and therefore reduces the speed of appreciation.

The time to adjustment, \( \bar{t} \), is decreasing with the size of the adjustment \( p_1 - p_0 \). When shocks are larger, the amount of adjustment must be larger, and therefore the time to adjustment must be
larger. The speed of adjustment does not depend on \( p_1 - p_0 \), so any gap between \( p_1 \) and \( p_0 \) must be covered by the time to adjustment, \( \bar{t} \). This has important empirical implications: \textit{prices that are more “out of equilibrium” will not necessarily experience faster price appreciation. Instead, they will experience longer sustained price appreciation.} The time to adjustment decreases also with \( L \). As the appraisal process is more flexible (or buyers have more cash), appreciation is faster (see (2) above) and therefore full adjustment comes more quickly.

2.3.4 Rational Agents

In this section, we develop a model with fully rational agents, in that both buyers and sellers form expectations of future price changes when they decide whether or not to trade. While buyers and sellers are still limited by the appraisal constraints, they are also aware that during the adjustment period, prices are not at their long-run levels.

Specifically, if prices are rising and the buyer’s constraint binds, both buyers and sellers are aware that prices will rise in the future, making buying more attractive and selling less attractive. This will cause excess demand for the asset, and a queue of buyers will form. This also will prompt “investor demand,” in which agents buy the asset purely to earn a profit during the adjustment period and sell when adjustment is complete, at time \( t = \bar{t} \). Further, some owners who would prefer to sell the asset if prices were not constrained, instead retain it.

2.3.4.1 Model Assumptions

We make several technical assumptions that allow the model to be solved analytically.

\textbf{Assumption 2.3.2.} \textit{Agents are shocked at most once.}

Assumption 2.3.2 will be helpful because prices will not be static in this model. An agent’s beliefs about her future type will affect her buying and selling decisions. For our purposes, it is simplest to assume that she is shocked only once. Once shocked, she does not need to worry about future shocks when making her buying and selling decisions. We note that this assumption could be
loosened to “agents are shocked at most once during the adjustment period”, and the results would follow. We do not like this alternative assumption, because the adjustment period is endogenous, but it should be clear that we are only ruling out multiple shocks to an investor’s valuation during periods of adjustment, which may be short.

**Assumption 2.3.3.** *During the adjustment period, agents can trade at most once.*

We make Assumption 2.3.3 for tractability. We note that this assumption still allows agents to make multiple trades, so long as only one is during the adjustment period (for example, buyers are allowed to buy the asset for its appreciation and then sell as soon as appreciation is complete).

**Assumption 2.3.4.** *Agents do not trade before they are shocked.*

Assumption 2.3.4 is helpful for the same reason as Assumption 2.3.2: buying and selling decisions that depend on the possibility of future shocks make agents’ objectives complicated enough that the model becomes intractable. If they act as though their types will not change, the model can be solved.

**Assumption 2.3.5.** *If there are more buyers (sellers) than sellers (buyers) at any given time, and prices are unable to adjust to equate the two, then buyers (sellers) are assigned the asset according to a “first-in-first-out” (FIFO) rationing rule.*

We will see that Assumption 2.3.1 will prevent prices from fully adjusting instantaneously. Prior to full adjustment, there will be a supply-demand imbalance. In order to derive an analytical solution for the price trajectory, we assume a FIFO assignment rule. We note that nearly any reasonable rule would yield the same price path as we find – we simply need to choose one, and FIFO is simple. Other rules would quantitatively affect the size of buyer or seller queues, but this aspect of the model is not meant for anything beyond qualitative analysis.
2.3.4.2 Parameter Restrictions

We make three parameter restrictions for tractability. First, the price adjustment is sufficiently flexible:

\[ L > \sqrt{\frac{p_1}{p_0}}. \] (2.13)

Equation 2.13 implies that the price trajectory satisfies an ordinary differential equation instead of a more cumbersome delay differential equation. As solving the delay differential equation is not feasible, we make this assumption for tractability.

Second, the population is sufficiently large:

\[ N > \frac{\lambda}{r} \log \left( \frac{p_1 - p_0}{Lp_0 - p_0} \right). \] (2.14)

Agents are shocked at most once. Equation 2.14 restricts parameters so that there are agents to shock up until time \( T \). Mathematically, it guarantees that \( \lambda T < N \).

Third, the mass of required comps is sufficiently small:

\[ C < \frac{\lambda L}{N^2} \cdot \left( \frac{N - 1}{r} - (p_1 - p_0) \right). \] (2.15)

While we assume that buyers are constrained in what the can pay, the constraint cannot be too severe. If \( C \) is very large or \( L \) is close to unity, then this constraint binds so severely that the price adjusts too slowly to model. For example, if \( C \) is very large, then the average price over comps will stay close to \( p_0 \) even as a long time passes, because most comps will come from the pre-shock period \( t < 0 \). Prices will not move much above \( Lp_0 \) before we run out of agents to shock. The same argument follows for \( L \) close to unity.

2.3.4.3 Discussion

Once the shock occurs at time \( t = 0 \), all agents rationally anticipate the coming price increase. This makes fewer shocked owners willing to sell. We assume that the population is sufficiently large (Equation 2.14). This means that, even knowing that prices will appreciate, some owners will be shocked to types where the utility of owning, \( v - x \), is low enough that they will still sell. This
generates a low-but-positive flow of sellers. The flow of sellers is therefore less than $\phi_0$, which is the flow when agents are naïve.

Non-owners are also shocked. Of this group, those in $[0, 1]$ will want to buy the asset and hold it until the game ends at time $T$. Those with types in $[1, N]$ may also want to purchase the asset in order to capture the coming price appreciation, and they will sell at the adjustment time $\bar{t}$, after which the price is $p_1$. We call these types “flippers,” because they buy specifically for expected price appreciation, not because they are the most natural owners. The flow of buyers to trade is therefore greater than $\phi_0$.

As should be clear, this means that there will be an imbalance between buyers and sellers during the adjustment period. At time $t = 0$, a queue of buyers forms, and it continues to grow up until the point where the number of buyers in the queue equals the aggregate future flow of sellers up to time $\bar{t}$. The queue ceases to form then, because getting in the queue at that time is equivalent to waiting until $\bar{t}$ to trade.

During this time, the ownership of the asset will not be efficient. Some owners who are shocked and now have types in $[1, N]$ continue to hold the asset for its appreciation. Some non-owners who are shocked and have new types in $[1, N]$ experience the shock early enough to buy the asset during the adjustment period, and become investors. Meanwhile, some non-owners who are shocked into $[0, 1]$ experience the shock too late in the adjustment period to buy the asset before adjustment is complete, and are stuck waiting out the adjustment.

At time $\bar{t}$, prices reach $p_1$ and stop appreciating. At this time, all owners with types in $[1, N]$ sell the asset and all non-owners with types in $[0, 1]$ buy the asset. The asset allocation to agents is efficient and, after this time, trade flows are identical to the naïve model – buyers and sellers trade as soon as they are shocked.

### 2.3.4.4 Buyer and Seller Queues and Trade Flows

Let $N(t)$ denote the mass of non-owners who have yet to be shocked, $\mathcal{N}(t)$ the mass of non-owners who have been shocked, but have yet to queue to buy, and $\mathcal{B}(t)$ the mass of non-owners who
have queued to buy. Some non-owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\varphi_B(t)$. Others find it optimal to queue some time after being shocked. They enter the queue at a rate of $\varphi_B(t)$. Define the total flow of non-owners to trade to be

$$\varphi_B(t) \equiv \varphi_B(t) + \varphi_B(t).$$

(2.16)

Similarly, let $O(t)$ denote the mass of owners who have yet to be shocked, $O(t)$ the mass of owners who have been shocked, but have yet to queue to sell, and $S(t)$ the mass of owners who have queued to sell. Some owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\varphi_S(t)$. Others find it optimal to queue some time after being shocked. They enter the queue at a rate of $\varphi_S(t)$. Define the total flow of non-owners to trade to be

$$\varphi_S(t) \equiv \varphi_S(t) + \varphi_S(t).$$

(2.17)

Figure 2.8 illustrates these stocks and flows.

Having defined notation to track agents throughout the market, we now turn our attention to the trade flow, $\phi(t)$. When buyers queue—$S = 0$ and $B > 0$—the trade flow is the seller flow: $\phi(t) = \varphi_S(t)$. Conversely, when sellers queue—$S > 0$ and $B = 0$—the trade flow is the buyer flow: $\phi(t) = \varphi_B$. When $S(t) > 0$ and $B(t) > 0$, the trade flow is undefined. Suppose that $B(t) > S(t) > 0$. Then the mass $S(t)$ of sellers can match with a mass $S(t)$ of buyers, leaving a mass $B(t) - S(t)$ unmatched. A positive mass of the asset changes hands instantaneously, so the trade flow is “infinite.” Formally, define

$$\mathcal{T} \equiv \{t \in [0, T] \mid \varphi_S(t) > 0 \text{ and } S(t) = 0, \text{ or } \varphi_B(t) > 0 \text{ and } B(t) = 0\}$$

(2.18)

and let the trade flow, $\phi : \mathcal{T} \to (0, \infty)$, be given by

$$\phi(t) \equiv \begin{cases} 
\varphi_S(t) & \text{if } S(t) = 0 \\
\varphi_B(t) & \text{if } B(t) = 0 
\end{cases}.$$

(2.19)

For $t < 0$, define $\phi(t) \equiv \phi_0$, the trade flow in the naïve equilibrium. As it will be useful in the
exposition that follows, define the cumulative trade flow after the shock at time $t = 0$ to be
\[ \Phi(t) = \int_0^t \phi(s) ds. \]  (2.20)
Note that since $\phi > 0$, $\Phi$ is invertible.

Let $w_S$ and $w_B$ denote sellers’ and buyers’ queuing times respectively. If a seller enters the selling queue at time $t$, then she sells at time $t + w_S(t)$; if a buyer enters the buying queue at time $t$, then he buys at time $t + w_B(t)$. Agents exit their queues at a rate equal to the trading intensity $\phi$. Under Assumption 2.3.5, $w_S$ and $w_B$ can be defined implicitly as
\[ \int_t^{t+w_S(t)} \phi(s) ds = S(t); \]  (2.21)
\[ \int_t^{t+w_B(t)} \phi(s) ds = B(t). \]  (2.22)
When, for example, a buyer enters the buyers’ queue, there is a mass $B(t)$ of buyers ahead of her. $w_B(t)$ is exactly the amount of time it will take for all of the buyers ahead of her to exit the queue by buying from sellers.

2.3.4.5 Equilibrium Definition

Definition 2.3.1 (Admissible Price Paths). A price path $p$ is said to be admissible if there is $\bar{t} \in [0,T]$ such that $p(0) = Lp_0$, $p(t) = p_1$ for all $t \geq \bar{t}$, and $p$ is twice continuously differentiable in $t$. Let $P$ denote the set of all admissible price paths.

Definition 2.3.1 says that, if prices are rising, a price path must start at $Lp_0$ at time 0, end at $p_1$ at time $\bar{t}$, and remain at $p_1$ for all times thereafter. Moreover, it must be twice continuously differentiable between times 0 and $\bar{t}$.

For $j \in \{S, B\}$, define $\tilde{t}_j \in (0, \bar{t}]$ to be the time after which
\[ w_j(t) = \bar{t} - t \text{ for all } t \in [\tilde{t}_j, \bar{t}). \]  (2.23)
Such a $\tilde{t}$ must always exist, as $\tilde{t}$ may equal $\bar{t}$. It may not equal 0 as $w_j(0) = 0$. We will refer to $\tilde{t}$ as the trade horizon, as it is the last date on which a non-owner (owner) can enter the buyers’ (sellers’) queue and buy (sell) the asset for less than $p_1$ (more than $p_1$).
Suppose, for example, that $B(t) > S(t) = 0$ at time $t$. A buyer who enters the buyers’ queue will have to wait until time $t$ to buy. She is indifferent between entering the queue at time $t$ (when there is wait of $\bar{t} - t$), and entering the queue at time $\bar{t}$ (when there is no wait). The existence and uniqueness of $\bar{t}$ is necessary for the solution of the non-owners’ problem.

**Definition 2.3.2** (Admissible Wait Times). Fix $j \in \{S, B\}$ and $p \in P$. A wait time $w_j$ is said to be admissible if there is $\bar{t} \in [0, \bar{t}]$ such that $w_j(0) = 0$, $w_B(t) = \bar{t} - t$ for all $t \in [\bar{t}_j, \bar{t}]$, $w_j(t) = 0$ for all $t \in [\bar{t}, T]$, and $w_j$ is twice continuously differentiable over $(0, \bar{t}_j)$. Let $W_j$ denote the set of all admissible wait times.

Consider (for concreteness) the buyers’ wait time. Definition 2.3.2 says that a wait time (whether it be for sellers or buyers) must start at 0 at time 0. Once it reaches $\bar{t}_B(t)$, it decreases at a rate of $-1$ until time $\bar{t}$, at which time the market clears. Moreover, it must be twice continuously differentiable between times 0 and $\bar{t}_j$ (note that $w_B$ has kinks at times $\bar{t}_j$ and $\bar{t}$).

Given a price path $p \in P$, a sellers’ wait time $w_S \in W_S$, and a buyers’ wait time $w_B \in W_B$, we define the owner’s and non-owner’s problems to be

$$V_O(t, x) \equiv \max_{t_s \in [t, T]} \int_t^{t_s + w_S(t_s)} (v - x) e^{-rs} ds + p(t_s + w_S(t_s)) e^{-r(t_s + w_S(t_s))} + V_N(t_s + w_S(t_s), x) \quad (2.24)$$

$$V_N(t, x) \equiv \max_{t_b \in [t, T]} -p(t_b + w_B(t_b)) e^{-r(t_b + w_B(t_b))} + V_O(t_b + w_B(t_b), x). \quad (2.26)$$

The shocked non-owner chooses a time $t_b$ at which to enter the buying queue. At time $t_b + w_B(t_b)$, he pays the sale price $p(t_b + w_B(t_b))$ and receives the flow utility $v - x$ from $t_b + w_B(t_b)$ until he sells the asset. The shocked owner chooses a time $t_s$ at which to enter the selling queue. The queue may be empty, in which case she proceeds directly to trade. She receives a flow utility of $v - x$ from the asset until $t_s + w_S(t_s)$. At time $t_s + w_S(t_s)$, she receives the sale price $p(t_s + w_S(t_s))$.

By Assumption 2.3.3, agents can trade at most once during the adjustment period. We
implement this assumption in the following way. Define
\[
\tau_s(t) \equiv \max\{t + w_S(t), \bar{t}\} \tag{2.28}
\]
\[
\tau_b(t) \equiv \max\{t + w_B(t), \bar{t}\}. \tag{2.29}
\]
The owner’s and non-owner’s problems can then be written as
\[
V_O(t, x) \equiv \max_{t_s \in [t, \bar{t}]} \int_t^{t_s + w_S(t_s)} (v - x)e^{-rs}ds + p(t_s + w_S(t_s))e^{-r(t_s + w_S(t_s))} + V_N(\tau_b(t), x) \tag{2.30}
\]
\[
V_N(t, x) \equiv \max_{t_b \in [t, \bar{t}]} -p(t_b + w_B(t_b))e^{-r(t_b + w_B(t_b))} + \int_{t_b + w_B(t_b)}^{\tau_b(t_b)} (v - x)e^{-rs}ds + V_O(\tau_s(t), x). \tag{2.31}
\]
An owner who chooses to sell during the period of adjustment becomes a non-owner, but cannot re-purchase until time \(\bar{t}\). Similarly, a non-owner who chooses to buy during the period of adjustment becomes an owner, but cannot re-sell until time \(\bar{t}\). Between the time that he buys, and \(\bar{t}\), he receives the flow utility \(v - x\) from the asset.

**Definition 2.3.3 (Equilibrium).** An equilibrium is a price path \(p \in P\), a sellers’ wait time \(w_S \in W_S\), and a buyers’ wait time \(w_B \in W_B\), such that \(p\) satisfies
\[
p(t) = \begin{cases} 
L\bar{p}(t) & \text{if } w_S(t) > 0 \text{ or } w_B(t) > 0 \\
p_1 & \text{otherwise}
\end{cases} \tag{2.34}
\]
where \(\phi\) is the trade flow implied by the solutions to the owner’s and non-owner’s problems under \(p, w_S, \text{ and } w_B\), and \(w(t) = \Phi^{-1}(\Phi(t) - C)\).

For ease of notation, define \(E = P \times W_S \times W_B\), so that an equilibrium can be described as a triple \((p, w_S, w_B) \in E\).

### 2.3.4.6 Auxiliary Definitions

Agents will take different actions depending on their types. Our first step is to partition the type-space \([0, N]\) into different actions. Fix some price trajectory \(p \in P\). There is some \(\bar{t} \in (0, T)\)
such that \( p(t) = p_1 \) for all \( t \geq \bar{t} \). Define \( z(t, \bar{t}) \equiv \exp(-r(\bar{t} - t)) \) and

\[
\bar{x}(t, p) \equiv 1 + r \left( \frac{p_1 - p(t)}{1 - z(t, \bar{t})} \right). \tag{2.35}
\]

\( \bar{x} \) is the type of the owner who is indifferent between selling at time \( t \) and time \( \bar{t} \):

\[
\int_{t}^{\bar{t}} (v - \bar{x}(t))e^{-rs}ds + p_1e^{-r\bar{t}} = p(t)e^{-rt} \tag{2.36}
\]

In the rest of the paper, we will use the shorthand “\( \bar{x}(t) \)” for “\( \bar{x}(t, p) \).” The dependence of \( \bar{x} \) on \( p \) should be understood.

Next, define

\[
\bar{y}(t, p) \equiv \bar{x}(t, p) + p(t) - rp(t). \tag{2.37}
\]

\( \bar{y}(t) \) denotes the non-owner who’s indifferent between queuing to buy now and queuing to buy later. He is the non-owner for whom the marginal benefit of owning equals the marginal cost of foregone price appreciation:

\[
v - \bar{y}(t) = -p(t)(\rho(t) - r). \tag{2.38}
\]

where \( \rho(t) \equiv p'(t)/p(t) \). As with \( \bar{x} \), we will use the shorthand “\( \bar{y}(t) \)” for “\( \bar{y}(t, p) \).” We have the following facts about \( \bar{x} \) and \( \bar{y} \):

**Lemma 2.3.2.** If \( p \in P \), then for all \( t \in [0, \bar{t}] \), \( \bar{x}'(t) > 0 \), \( \bar{y}'(t) > 0 \), and \( \bar{x}(t) > \bar{y}(t) > 1 \).

These facts will be useful in establishing the existence of an equilibrium. In the meantime, the two facts about \( \bar{x} \), together with Assumptions 2.13 – 2.15, ensure that \( \lim_{t \uparrow \bar{t}} p'(t) < N - 1 \) and guarantee that \( \bar{x}(t) \in (0, N) \) for all \( t \in (0, \bar{t}) \).

We now characterize the subset of \( \mathcal{E} \), which will be the focus of our discussion.

\[
\mathcal{E}_0 \equiv \{ (p, w_S, w_B) \in \mathcal{E} \mid p'' > rp' \text{ for all } t \in (0, \bar{t}), \text{ and } w_j(t) > -1 \text{ for all } t \in (0, \bar{t}_j), j \in \{B, S\} \}. \tag{2.39}
\]

To ease the exposition, define the price \( u(t) \equiv p(t) - p_0 \), and the parameters

\[
u_0 \equiv LP_0 - p_0, \alpha \equiv \frac{\lambda L}{C N^2}, \beta \equiv \frac{N - 1}{r}. \tag{2.40}\]
2.3.4.7 Equilibrium

We begin by determining the equilibrium price path after a positive shock to the fair price. The space of equilibria is large, so it will be difficult to rule out the possibility of multiple equilibria. We will instead show that, within $E_0$, there exist equilibria, and each such equilibrium has the same price path.

Specifically, we will look for equilibria with two properties. The more important, and more restrictive, is that we require that the price path be sufficiently convex: $p''(t) > rp'(t)$.

As it turns out, the condition $p''(t) > rp'(t)$ results in threshold strategies in which owners either sell immediately when they are shocked, or wait until adjustment is complete to sell. If it is optimal to own over the period of length $dt$ at time $t$, then it makes sense to own over the period of length $dt$ at time $t' > t$. This generates a threshold strategy.

The other property that we require is that the wait time for buyers and sellers is greater than $-1$ for any time periods in which a queue is forming: $w'_j(t) > -1$ for all $t \in (0, \hat{t}_j), j \in \{B, S\}$. This is sufficient (but far from necessary) for the buyers’ and sellers’ problems to be quasi-concave and quasi-convex, respectively. In words, we assume that arriving in a queue at a later time does not enable agents to trade sooner: the length of the queue, measured in units of time, cannot shrink by more than one unit of time every unit of time. Given FIFO, this requirement is satisfied so long as there is a positive flow arriving to buy and sell at all times up to the point where a queue is so long that, upon entering, an agent will not exit before adjustment is complete.

Lemma 2.3.3. If $(p, w_S, w_B) \in E_0$, then the trade flow induced by $(p, w_S, w_B)$ is

$$\phi(t, p) = \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(t, p)}{N},$$

(2.41)

Moreover, sellers do not wait to trade: $w_S = 0$.

Since $\phi(t) = \Phi'(t)$, we can write the trade flow in Lemma 2.3.3 as

$$d\Phi = \lambda dt \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(t, p)}{N}.$$
During a time increment $dt$, a mass $\lambda dt$ of agents are shocked, of whom a fraction $1/N$ are owners, of whom a fraction $(N - \bar{x}(t))/N$ are shocked to $[\bar{x}(t), N]$. Recall that $\bar{x}(t)$ is the type of the agent who is indifferent between selling at times $t$ and $\tilde{t}$. Therefore, all agents for whom $x > \bar{x}(t)$ will sell at time $t$, while agents for whom $x \leq \bar{x}(t)$ will sell at time $\tilde{t}$.

Given an equilibrium, Lemma 2.3.3 says that the trade flow is determined by the rate at which owners are shocked to $[\bar{x}(t), N]$. Let $p^* : [0, T] \times (0, T) \rightarrow \mathbb{R}$ be given by

$$p^*(t; \tilde{t}) \equiv p_0 + \frac{z(t, \tilde{t})^\alpha (\beta - \Delta p)(1 - z(t, \tilde{t}))^\alpha \Delta p}{\int_{z(t, \tilde{t})}^{1} s^\alpha (\beta - \Delta p - 1)(1 - s)^{\alpha \Delta p - 1} ds}, \tag{2.43}$$

for $t \in [0, \tilde{t}]$, and $p(t, \tilde{t}) = p_1$ for $t \in [\tilde{t}, T]$. $\tilde{t}$ is the unique solution of $p^*(0; \tilde{t}) = Lp_0$ (the existence and uniqueness of $\tilde{t}$ is shown in the proof of Proposition 2.3.1).

**Proposition 2.3.1.** There is an equilibrium $(p, w_S, w_B) \in \mathcal{E}_H$; each equilibrium has $p = p^*$.

The top-left panel of Figure 2.7 shows a numerical solution of the fully rational model. The dashed lines show the solution under naïve trade flow, while the solid lines show the solution under rational trade flow. The naïve trade flow trajectory converges to the new price faster than the fully rational trade flow trajectory.

Similar results hold for the rate of appreciation, price predictability, differential pricing of assets with the same cash flows and discount rates, and differential total returns based on asset prices, as in the naïve model.

The first new implication from the rational model is that we observe sellers’ markets, in which sellers are able to sell the asset whenever they wish, but buyers must queue and wait to purchase. In real estate, it is common for both participants and brokers to talk about sellers’ and buyers’ markets, terms and concepts which are foreign to standard models of financial assets. While our model alone cannot produce this fact, we will argue that an equivalent model, in which interest rates rise rather than fall, would produce queues of sellers rather than queues of buyers. Combining the two would yield the relationship that sale times are shorter when markets are appreciating and longer when they are depreciating. These facts are consistent neither with standard asset pricing theories, nor with the naïve model in this paper. Only the fully rational model can produce it. We
see this in the bottom-left panel of Figure 2.7, which displays the time between when a buyer or seller queues and when they can transact. For sellers, the wait time is always zero. For buyers, it is zero before and after adjustment, but during adjustment, it is considerable longer.

For completeness, we provide the following comparative statics.

**Corollary 2.3.3.** The time to adjustment, $\bar{t}$, is decreasing with the shock intensity, $\lambda$; decreasing with appraisers’ flexibility, $L$; increasing with the required comps, $C$; increasing with the size of the adjustment, $\Delta p$.

Rationality does not solve the problems that arise in the naïve model. Indeed, it makes them worse, as it lengthens the time to adjustment.

**Lemma 2.3.4.** The time to adjustment in the fully rational trading model, $\bar{t}_s$, is greater than the time to adjustment in the naïve trading model, $\bar{t}_n$.

### 2.4 Empirical Facts

#### 2.4.1 Residential Real Estate

To illustrate the effect of appraisals in residential housing, we plot data provided by the Federal Housing Finance Agency (FHFA), a government agency responsible for overseeing aspects of the secondary mortgage market in the US. They maintain two house price indices which we reproduce in this paper. One index is, like the S&P Case-Shiller indices, built using only transaction data. The other uses a combination of transaction prices and appraisals.

The top-left panel of Figure 2.9 plots the values of these two indices from January, 2000, until January, 2016. Especially after 2006, the index which includes appraisals clearly lags the purchase-only index. This is consistent with the idea that appraisals are backward-looking, as we assume in this paper, and that the resulting gap between appraisals and market prices can be large. In the top-right panel of Figure 2.9, we focus on the period of rapidly increasing prices during recovery from the Great Recession – the second quarter of 2011 to the second quarter of 2016. For the 100 largest cities in the US, we plot the cumulative increases over that five year period in the
purchase-only and purchase-plus-appraisal indices. Cities are ordered, from left to right, by the lowest cumulative return to highest, using the averages of the two indices. It should be clear that the facts that appraisals lag price increases, and that the gap is larger where returns have been larger, are true for essentially every city – there is not much variation from city to city.

It is also clear from the figure that the gap between appraisals and market prices is larger when prices are rising or falling faster, but to make it more so, we document this explicitly in the bottom-left panel of Figure 2.9, which shows a scatter comparing the rate of monthly appreciation in the purchase-only price index to the gap between the levels of the purchase-only and the purchase-plus-appraisal indices. It is clear that when prices are rising or falling faster, the indices diverge by more.

2.4.1.1 Return Predictability

Appraisals lag prices. In order to evaluate whether this is true in practice, we use data provided by the Federal Housing Finance Agency (FHFA), a government agency responsible for overseeing aspects of the secondary mortgage market in the US. They maintain two house price indices which we reproduce in this paper. One index, like the S&P Case-Shiller indices, is built using transaction prices. The other uses a combination of transaction prices and appraisals.

The top-left panel of Figure 2.9 plots the values of these two indices from January, 2000, until January, 2016. Especially after 2006, the index which includes appraisals clearly lags the purchase-only index. This is consistent with the idea that appraisals are backward-looking, as we assume in this paper, and that the resulting gap between appraisals and market prices can be large.

2.4.1.2 Rent-to-Price Ratio

Prices and appreciation rates can vary substantially, both over time, for a given asset whose fundamentals do not change, and across apparently identical assets at the same time. Figure 2.2 presents two scatter-plots comparing the rent-price ratio to price appreciation (top) and to total returns (bottom) for rental real estate in 200 metro areas in the years 2010-2013, as calculated by
Zillow. The first thing to notice is that there is substantial variation in the price of rental housing, as prices regularly range from five to 25 times annual rents. Panel A shows that the variation in price appreciation between properties whose prices – relative to rents – differ by a factor of five, have only a 6% difference in annual price appreciation. As shown in panel B, this appreciation is not nearly enough to account for the lower rents that accrue to expensive properties. This figure suggests substantial variation in the pricing of similar properties.

2.4.1.3 Turnover

Markets with a higher natural rate of change in the identity of the optimal owner experience faster appreciation or depreciation. This is because trade generates recent comparable prices which are less stale, allowing prices to move more quickly toward their new fair values. Figure 2.2 presents a scatter-plot of price appreciation versus turnover for 200 metro areas over the years 2010-2014, where turnover is the fraction of homes that sold in the prior year. Also presented are lines-of-best-fit for the regions in which appreciation is positive and in which it is negative. When prices are increasing, higher turnover is associated with faster appreciation, as the model suggests that it should be. When prices are decreasing, higher turnover is associated with faster depreciation, as the model suggests that it should be.

2.4.1.4 Sellers’ Markets

First, buyers’ and sellers’ markets emerge when prices are rising or falling, respectively. In a buyer’s market, prices are rising, queues of buyers form, and the time between when a typical buyer starts searching for a house and when she purchases one is high. Sellers are able to sell relatively quickly. In a seller’s market, prices are falling, queues of sellers form, and the time-on-market for a house is high. It is difficult to find good data for average time-to-buy a house. While we have many anecdotal examples of quickly appreciating markets in which bidding wars are common (or in which buyers approach owners whose houses aren’t even on the market, in the hopes of making an offer that the owners cannot refuse).
We can, however, observe average days-on-market for sellers, and the length of the sellers’ queue. Figure 2.4 is a scatter-plot comparing the average days on market for houses which sell in a given year with the price appreciation that year. Periods of increasing prices should feature no seller queues and short periods between a house being listed and sold.

In the model, when prices are appreciating, there is no seller queue and sales take place as soon as houses are listed. In practice, even if a house goes under contract on the day it is listed, it still takes 30-60 days to close, yielding a practical minimum of average days-on-market of somewhere in the neighborhood of at least 45 days, depending on local market rules. As the real estate market also features search and as some sellers might initially overprice their houses, average days-on-market will typically be higher than 60 days, even in a seller’s market (see [70]).

Periods of decreasing prices should experience seller queues, yielding higher days-on-market for sold homes. Figure 2.4 shows this rather clearly, and there appears to be a floor of average days-on-market in the neighborhood of 90 days, reached when appreciation hits 10% per year.

Another way to measure the seller queue is inventories, typically measured as the ratio of current houses for sale divided by monthly sales (to account for seasonal variation in inventories and longer-term growth in the size of the market). In Figure 2.5, we plot the inventory-to-sales ratio from January 1987 to January 2016, as well as the Case-Shiller 10-City Composite index. There is always some inventory of unsold homes on the market, simply because some search is required in real estate, there is time between a house going under contract and the sale closing, and because some people choose unreasonable list prices. This minimum inventory appears to be in the neighborhood of four months worth of homes. As Figure 2.5 shows, inventories oscillate around six months of supply while prices are stable. Beginning in 1997, prices begin a steady rise, and inventories fall to four months supply, staying remarkably stable until September 2005. This is a seller’s market, in which prices are predictably increasing and the seller’s queue is as close to empty as appears empirically possible.

By the summer of 2006, nine months later, prices flatten and begin to fall, falling in earnest until May 2009. The seller’s queue reaches a peak four months prior, and reaches a new normal
of eight months supply, two months later. Prices begin rising in earnest again in February 2012, precisely the same month in which inventories reach their new normal. It should be immediately clear that (1) periods of rising prices are those in which inventories are low and stable, (2) periods of falling prices are those in which inventories grow, and (3) the end of a period of declining prices is associated with a substantial reduction/elimination of excess inventories. All three of these predictions are consistent with the model, in which (1) sellers’ markets are associated with rising prices and no seller’s queue, (2) buyers’ markets feature a growing queue of sellers, and (3) queues evaporate when prices reach their new long-run levels.

2.4.1.5 Flippers

In a seller’s market, some purchasers are investors. These people are not the most natural owners for a property, but enter the market to enjoy the expected price appreciation. Investors exit the market when price adjustment is complete. There is no good database of investor-owners – simply looking at non-owner-occupied property is not sufficient, as it is not efficient for everyone to their own residences. However, we offer two pieces of evidence that, during periods of rapid price appreciation, many owners only intend to own the property during the period of expected appreciation. First, anecdotal evidence suggests that the rate at which homeowners choose to rent out their homes after they move has increased substantially since the housing crisis of 2007-2010.¹

There has also been a substantial increase in the ownership of single family rentals by Wall Street investment funds, like BlackStone.² The six largest buyers have purchased $28 billion on properties since the crisis. This welfare loss suggests room for policy interventions. Figure 2.6 shows that flipping—defined as the fraction of sales for which the home has sold twice in one year—is positive correlated with price appreciation.

¹ http://money.cnn.com/2014/06/17/real_estate/homeowner-landlords/
² http://www.wsj.com/articles/a-onetime-housing-skeptic-plans-1-billion-bet-on-homes-1475619217
2.4.2 Alternative Assets

We analyze the prices of assets other than single family real estate. We show that the basic prediction of return-predictability holds for aircraft, commercial real estate, and and taxi medallions. These assets share little in common beyond the fact that they are all often purchased with debt, collateralized by the asset itself. We conclude by showing that price-predictability is not inherent to all assets. Seats on the New York Stock Exchange, which share a number of important properties with taxi medallions, do not exhibit price-predictability.

2.4.2.1 Taxi Medallions

If you want to drive a taxi in New York City, you must either own a medallion or rent one from somebody else. There is an active market for both individual and corporate (rental) medallions, and medallion transactions are recorded by the New York City Taxi & Limousine Commission. We were able to acquire the data using a Freedom of Information Act request, and have monthly transaction volume and average prices from 1947 to 2016.

An individual or company purchasing a medallion can finance the purchase with a loan, cash, or some combination of the two. If the medallion purchase is financed with a loan, then it may be constrained by the appraisal process just as in residential real estate.

As we see in Table 2.1 medallion prices are predictable. They are less predictable than single-family homes, suggesting that the constraint may be weaker for medallions than for houses, but they are clearly predictable.

2.4.2.2 Aircraft

We repeat the analysis using indices for airplane prices. Aircraft price indices were provided by VREF, an aggregator of aircraft transaction data and order books. VREF creates price indices by calculating average transaction prices for all equivalent planes within a class-vintage at the quarterly level. For example, the “Light Single” class is comprised of transaction data for the Tiger AA5B, the Beechcraft C23 Sundowner, the Cessna 172P, the Cardinal, the Piper Warrior, and the
Archer, all from the late 1960s. The index is simply average transaction prices of these airplanes, adjusted for condition (e.g., the age of key components), over time. The vintage is not changing over time, so prices tend to decline as the aircraft age. That said, prices of the smallest and least expensive planes have been considerably more stable than, say, the prices of cars. Note that these data are for much smaller and older airplanes than those studied by [11, 12, 13]. Our data cover planes which are often owned by individuals or firms which lease them to individuals.

As we see in Figure 2.1, airplane prices are predictable. Table 2.2 splits the sample into seven classes of plane, which range from very inexpensive (the Light Single class described above, whose average selling price in the fourth quarter of 2016 is approximately $45 thousand) to more expensive (the Large Jet class had an average selling price in the fourth quarter of 2016 of over $2.5 million). Regardless of class, there is substantial predictability.

2.4.2.3 Commercial Real Estate

We also make use of price indices for commercial, industrial, and multi-family residential property provided by Moody’s/Real Capital Analytics for the years 2000 to 2015. As we see in Figure 2.1, commercial (including industrial and multi-family residential) real estate returns are predictable. In Table 2.3, we report estimates from AR(1) regressions on returns.

2.4.2.4 NYSE Seats

Perhaps predictability is present for all but the most liquid financial assets for some reason other than our model. Are there assets similar to medallions, for example, which do not exhibit predictability? If so, why don’t they?

An investment house wanting to trade on the floor of the New York Stock Exchange (NYSE) is required to own a seat. As seats are in limited supply, a firm wanting to trade must buy one from another firm willing to sell. These seats are similar to taxi medallions, in that both a seat and a medallion give the bearer the right to engage in some profitable business – trading stock or driving a taxi. Those profits are heterogeneous (some traders and drivers are better than others),
generating heterogeneous valuations for the assets and, therefore, trade. The main differences, for
the purpose of our model, are that investment houses do not need to borrow to afford a seat. We
would therefore expect that seats on the NYSE would not exhibit the same pricing patterns as taxi
medallions. Indeed, they do not.

The predictability of returns on NYSE seats has been thoroughly studied in [91], who finds
no predictability. We also analyze monthly closing prices from 1880 to 1925, a period prior to the
period studied by [91] for which we have closing price data (and not just monthly high-low data).

As we see in Table 2.4 we find no return predictability. While a seat on the NYSE is, in many
important ways, similar to a taxi medallion, their investor bases are not. As the model predicts, it
is not the asset which drives predictability, but rather constraints on the investor base.

2.4.3 Extension to Negative Shocks

As noted above, our model only discusses the effect of limits to what buyers are able to pay
for an asset, and therefore only has novel predictions for markets in which the fair price of an asset
rises. We believe, however, that the model can apply equally well to settings in which there are
limits to what sellers are willing to accept and the fair price falls. Continuing with the leading
example of this paper of the market for real estate, suppose that an owner pays $100,000 for a
house with a $10,000 down payment. After a year, she owes $88,000 on the house, but the fair
price has fallen to $75,000. She may understand that $75,000 is the new fair price but, if she is
financially constrained, she may not have the $13,000 to send to the bank in the case of a short
sale.

Banks sometimes accept that sellers must sell the house and repay less than the outstanding
principal value of the mortgage. If the alternative is default, then a short sale may be more
profitable than costly foreclosure proceedings. It is likely that, as house prices fall, the amount
that a bank would accept on a sale would depend critically on recent comparable transactions. This

---

3 We would like to thank David Gross for the idea of looking at the pricing of NYSE seats.
4 These data were courteously provided by Asaf Bernstein.
example, therefore, would suggest an unwillingness (on the part of the lender) to allow a sale for substantially less than the $88,000 owed on the house, and a relationship between that level and the level of comparable transactions.

If we believe that a similar argument to that which we apply to buyers would apply to sellers as well, then the implications of the model should hold when the fair price falls. A lower fair price would cause an increase in the number of shocked owners wanting to sell, and a decrease in the number of shocked non-owners wanting to buy. The result would be a seller queue; that is, increased inventory on the market. A prolonged, predictable decline in real estate prices would ensue. Some owners would have to sell, because their disutilities of owning would be prohibitively high (perhaps because of unemployment), and these sales would lower comparable prices for others, thus lowering the prices at which they would sell. Eventually, prices would reach a new, lower, level.

2.5 Conclusion

We have developed a simple model of asset prices under the assumption that lenders are unwilling to lend buyers substantially more than prices recent transactions. We have argued that legal, agency, or behavioral constraints are all possible sources of this inability or unwillingness. Our model generates return predictability, differential pricing for identical assets, and higher returns on lower priced assets. Markets which experience higher turnover in the optimal owner of the asset experience more rapid conversion to the long-run price. Perhaps surprisingly, when agents are rational and anticipate these pricing anomalies, each problem becomes more severe. When prices are rising, a seller’s market emerges: sellers are able to sell their assets rapidly, but buyers need a long time to locate a seller.
Proofs

For $a, b > 0$, define

$$m(z) = \frac{z^a(1 - z)^b}{\int_z^1 t^{a-1}(1 - t)^{b-1}dt}.$$  \hspace{1cm} (2.44)

In [71], it is shown that

1. $\lim_{z \uparrow 1} m(z) = b$;
2. $\lim_{z \uparrow 1} m(z) = (a + b)b/(1 + b)$;
3. $m$ is strictly increasing;
4. $m$ is strictly convex if and only if $a > 1$.

**Proof of Lemma 2.3.1.** First, differentiate the comparables equation:

$$C = \int_{w(t)}^t \phi(s)ds \Rightarrow \phi(t) - \phi(w(t))\tau'(t) = 0.$$  \hspace{1cm} (2.45)

Next, differentiate the appraiser’s equation:

$$p(t) = \frac{L}{C} \int_{w(t)}^t p(s)\phi(s)ds. \Rightarrow p'(t) = \frac{L}{C} \cdot (p(t)\phi(t) - p(w(t))\phi(w(t))\tau'(t))$$  \hspace{1cm} (2.46)

$$= \frac{L}{C} \cdot (p(t) - p(w(t)))\phi(t)$$  \hspace{1cm} (2.47)

$$= \frac{L}{C} \cdot (p(t) - p_0) \cdot \phi(t)$$  \hspace{1cm} (2.48)

where the last line follows because $w(t) < 0$ and hence $p(w(t)) = p_0$.

**Proof of Corollary 2.3.1.** Observe that

$$\frac{L}{C} \int_0^t \phi(s)ds = \int_0^t \frac{u'(s)}{u(s)} \cdot ds = \log(u(s))\bigg|_0^t = \log\left(\frac{u(t)}{u_0}\right).$$  \hspace{1cm} (2.49)

as desired.

**Proof of Corollary 2.3.2.** By Corollary 2.3.1, we have that for $t > 0$

$$0 < \frac{L}{C} \int_0^t \phi(s)ds = \log\left(\frac{u(t)}{u_0}\right) \Rightarrow u(t) > u_0 \Rightarrow u'(t) = \frac{L}{C} \cdot u(t) \cdot \phi(t, p) > 0,$$  \hspace{1cm} (2.50)

where we’ve used the fact that $\phi > 0$ by definition.
Proof. We prove each fact in order.

(1) \( p_0 \leq p(t) \leq p_1 \) by definition. Therefore,

\[
\bar{x}(t) = 1 + r \left( \frac{p_1 - p(t)}{1 - z(t, t)} \right) > 1
\]

as desired.

(2) Fix \( t \in (0, \bar{t}) \), and let \( t' \in (t, \bar{t}) \). Define

\[
\eta(s, \bar{x}(t)) \equiv v - \bar{x}(t) + p'(s) - rp(s).
\]

Since \( p'' > rp \) on \((0, \bar{t})\), \( \eta(\bullet, \bar{x}(t)) \) is strictly increasing (in its first argument). By the definition of \( \bar{x} \), we have that

\[
0 = \int_t^{\bar{t}} \eta(s, x(t))e^{-rs}ds;
\]

\[
0 = \int_t^{\bar{t}} \eta(s, \bar{x}(t'))e^{-rs}ds > \int_t^{\bar{t}} \eta(s, \bar{x}(t'))e^{-rs}ds,
\]

where the second line follows because \( \eta(\bullet, \bar{x}(t')) \) is strictly increasing in its first argument and it changes sign between \( t' \) and \( \bar{t} \). Observe that

\[
0 < \int_t^{\bar{t}} (\eta(s, x(t)) - \eta(s, \bar{x}(t')))e^{-rs}ds = \int_t^{\bar{t}} (\bar{x}(t') - \bar{x}(t))e^{-rs}ds.
\]

Taking \( t' \to t \), we conclude that \( \bar{x}'(t) \geq 0 \).

(3) \( \bar{y}(t) = v + p'(t) - rp(t) = v - 1 + 1 + p'(t) - rp(t) = r(p_1 - p(t)) + 1 + p'(t) \) from which the result follows.

(4) \( \bar{x}(t) > \bar{y}(t) \). Since \( \bar{y} \) is increasing,

\[
0 = \int_t^{\bar{t}} (v - \bar{x}(t) + p'(s) - rp(s))e^{-rs}ds
\]

\[
= \int_t^{\bar{t}} (\bar{y}(s) - \bar{x}(t))e^{-rs}ds
\]

\[
> \int_t^{\bar{t}} (\bar{y}(t) - \bar{x}(t))e^{-rs}ds,
\]

which implies that \( \bar{x}(t) > \bar{y}(t) \).
Proof of Lemma 2.3.3. The goal of the proof is to show that the flow of buyers to trade always exceeds the flow of sellers to trade, and hence the trade flow is the seller flow.

Objectives. Fix \( t \in [0, T] \) and \( x \in [0, N] \). Recall that the shocked owner chooses a time \( t_s \) to enter the sellers’ queue, while the shocked non-owner chooses a time \( t_b \) to enter the buyers’ queue:

\[
V_O(t, x) = \max_{t_s \in [t, T]} \int_t^{t_s + w_S(t_s)} (v - x)e^{-rs} ds + p(t_s + w_S(t_s))e^{-r(t_s + w_S(t_s))} + V_N(\tau_s(t_s), x) \tag{2.59}
\]

\[
V_N(t, x) = \max_{t_b \in [t, T]} -p(t_b + w_B(t_b))e^{-r(t_b + w_B(t_b))} + \int_{t_b + w_B(t_b)}^{\tau_b(t_b)} (v - x)e^{-rs} ds + V_O(\tau_b(t_b), x) \tag{2.60}
\]

where

\[
\tau_s(t_s) \equiv \max\{t_s + w_S(t_s), \bar{t}\} \tag{2.61}
\]

\[
\tau_b(t_b) \equiv \max\{t_b + w_B(t_b), \bar{t}\} \tag{2.62}
\]

For tractability, we’ve assumed that agents can only trade once before time \( \bar{t} \). They may trade as often as they like thereafter. Let \( f_O(\bullet; t, x), f_N(\bullet; t, x) : [0, T] \rightarrow \mathbb{R} \) be given by

\[
f_O(t_s; t, x) = \int_t^{t_s + w_S(t_s)} (v - x)e^{-rs} ds + p(t_s + w_S(t_s))e^{-r(t_s + w_S(t_s))} \tag{2.63}
\]

\[
f_N(t_b; t, x) = -p(t_b + w_B(t_b))e^{-r(t_b + w_B(t_b))} + \int_{t_b + w_B(t_b)}^{\tau_b(t_b)} (v - x)e^{-rs} ds \tag{2.64}
\]

so that

\[
V_O(t, x) = \max_{t_s \in [t, T]} f_O(t_s; t, x) + V_N(\tau_s(t_s), x) \tag{2.65}
\]

\[
V_N(t, x) = \max_{t_b \in [t, T]} f_N(t_b; t, x) + V_O(\tau_b(t_b), x). \tag{2.66}
\]

It will be useful to split the owner’s and non-owner’s problems into the pre-adjustment period, \([0, \bar{t}]\), and the post-adjustment period, \([\bar{t}, T]\).
Post-Adjustment. Suppose that \( t \in [\tilde{t}, T] \). Then for all \( s \in [t, T] \), \( p(s) = p_1 \), \( \omega_B(s) = 0 \), and \( \omega_S(s) = 0 \). The owner’s and non-owner’s problems are

\[
V_O(t, x) = \max_{t_s \in [t, T]} f_O(t_s; t, x) + V_N(t_s, x) \tag{2.69}
\]
\[
V_N(t, x) = \max_{t_b \in [t, T]} f_N(t_b; t, x) + V_O(t_b, x). \tag{2.70}
\]

The owner’s value is

\[
V_O(t, x) = \begin{cases} 
\int_t^\infty (v - x) e^{-rs} ds & \text{if } x < 1 \\
p_1 e^{-rt} & \text{otherwise}
\end{cases} \tag{2.71}
\]

She can either hold the asset forever, in which case she receives \( v - x \) in perpetuity, or sell the asset, in which case she receives \( p_1 \). The non-owner’s value is

\[
V_N(t, x) = V_O(t, x) - p_1 e^{-rt}. \tag{2.72}
\]

It is the owner’s value, less the price \( p_1 \) of becoming an owner.

Pre-Adjustment. Suppose that \( t \in [0, \tilde{t}] \), and let \( t_s \in (0, \tilde{t}_s) \) and \( t_b \in (0, \tilde{t}_b) \), so that \( \omega^\prime_S(t_s) > 0 \) and \( \omega^\prime_B(t_b) > 0 \). In what follows, primes denote differentiation with respect to the first argument:

\[
f_O'(t_s; t, x) = (v - x + p'(t_s + \omega_S(t_s))) - rp(t_s + \omega_S(t_s))(1 + \omega^\prime_S(t_s)) e^{-r(t_s + \omega_S(t_s))} \tag{2.73}
\]
\[
f_N'(t_b; t, x) = -(v - x + p'(t_b + \omega_B(t_b))) - rp(t_b + \omega_B(t_b))(1 + \omega^\prime_B(t_b)) e^{-r(t_b + \omega_B(t_b))}. \tag{2.74}
\]

If \( f_O'(t_s; t, x) = 0 \), then

\[
f_O''(t_s; t, x) = (p''(t_s + \omega_S(t_s))) - rp'(t_s + \omega_S(t_s))(1 + \omega^\prime_S(t_s))^2 e^{-r(t_s + \omega_S(t_s))} \tag{2.75}
\]
\[
- \left[ \frac{\omega^\prime_S(t_s)}{1 + \omega^\prime_S(t_s)} - r(1 + \omega^\prime_S(t_s)) \right] f_O'(t_s; t, x) \tag{2.76}
\]
\[
= (p''(t_s + \omega_S(t_s))) - rp'(t_s + \omega_S(t_s))(1 + \omega^\prime_S(t_s))^2 > 0. \tag{2.77}
\]
Hence, \( f_O(\bullet; t, x) \) is strictly quasiconvex on \([0, \bar{t}_s]\). If \( f'_N(t_b; t, x) = 0 \), then
\[
f''_N(t_b; t, x) = -p''(t_b + w_B(t_b)) - rp'(t_b + w_B(t_b))(1 + w'_B(t_b))^2 < 0.
\]
Hence, \( f'_N(\bullet; t, x) \) is strictly quasiconcave on \([0, \bar{t}_b]\).

The Owner’s Problem. The owner’s problem is to
\[
\max_{t_s \in [t, \bar{t}]} f_O(t_s; t, x) + V_N(\bar{t}, x).
\]
Since \( f_O \) is strictly quasiconvex, on \([0, \bar{t}_s]\), and constant on \([\bar{t}_s, \bar{t}]\), \( t^*_s(t, x) \in \{t, \bar{t}\} \). By definition, \( \bar{x}(t + w_S(t)) \) is the type who is indifferent between queuing to sell at time \( t \) and time \( \bar{t} \): \( f_O(\bar{t}; t, x) = f_O(t; t, x) \). Therefore,
\[
t^*_s(t, x) = \begin{cases} 
  t & \text{if } x > \bar{x}(t + w_S(t)) \\
  \bar{t} & \text{otherwise} 
\end{cases}
\]
The seller flow is
\[
\varphi_S(t) = \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(t + w_S(t))}{N}.
\]
for \( t \in [0, \bar{t}_s] \), and \( \varphi_S(t) = \phi_0 \) for \( t \in [\bar{t}_s, \bar{t}] \). During a time increment \( dt \), a mass \( \lambda dt \) of agents are shocked, of which a fraction \( 1/N \) are owners, of which a fraction \((N - \bar{x}(t + w_S(t)))/N\) are shocked to \([\bar{x}(t + w_S(t)), N]\). For \( t \in (\bar{t}, \bar{t}] \), \( \varphi_B(t) = \phi_0 \).

The Non-Owner’s Problem. The non-owner’s problem is to
\[
\max_{t_b \in [t, \bar{t}]} f_N(t_b; t, x) + V_O(\bar{t}, x).
\]
Observe that \( f'_N(t_b; t, x) > 0 \) if and only if \( x > \bar{y}(t_b + w_B(t_b)) \). For \( t \in [0, \bar{t}] \), there are three categories of non-owners.
(1) Non-owners for whom \( f_N'(t) < 0 \) (i.e., non-owners for whom \( x < \bar{y}(t + w_B(t)) \)) find it optimal to queue immediately because their objectives are quasiconcave. Therefore,

\[
\varphi_B(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{\bar{y}(t + w_B(t))}{N}.
\]

(2.85)

During a time increment \( dt \), a mass \( \lambda dt \) of agents are shocked, of which a fraction \( (N - 1)/N \) are non-owners, of which a fraction \( \bar{y}(t + w_B(t))/N \) are shocked to \([0, \bar{y}(t + w_B(t))]\).

(2) Non-owners for whom \( f_N'(t) < 0 \) (i.e., non-owners for whom \( x > \bar{y}(t + w_B(t)) \)) find it optimal to wait to queue:

\[
o(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{N - \bar{y}(t + w_B(t))}{N}.
\]

(2.86)

During a time increment \( dt \), a mass \( \lambda dt \) of agents are shocked, of which a fraction \( (N - 1)/N \) are non-owners, of which a fraction \( (N - \bar{y}(t + w_B(t)))/N \) are shocked to \([\bar{y}(t + w_B(t)), N]\).

(3) Some non-owners who found it optimal to wait to queue now find it optimal to queue.

Consider the mass \( N(t) \) of non-owners who have been shocked but have yet to queue to buy. Since \( \bar{y} \) is increasing, these non-owners are distributed uniformly on \([\bar{y}(t), N]\). Over a time increment \( dt \), a mass \( d\bar{y}(t + w_B(t))(1 + w_B'(t)) \) now find it optimal to queue to buy:

\[
\varphi_B(t) \cdot dt = \frac{d\bar{y}(t + w_B(t))(1 + w_B'(t))}{N - \bar{y}(t + w_B(t))} \cdot N(t).
\]

(2.87)

Therefore, \( N(t) \) solves the initial value problem

\[
N'(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{N - \bar{y}(t + w_B(t))}{N} - \frac{\bar{y}'(t + w_B(t))(1 + w_B'(t))}{N - \bar{y}(1 + w_B'(t))} \cdot N(t)
\]

subject to \( N(0) = 0 \), which can be readily solved using the method of integrating factors:

\[
N(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{N - \bar{y}(t + w_B(t))}{N} \cdot t
\]

(2.89)

and hence,

\[
\varphi_B(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{t\bar{y}'(t + w_B(t))(1 + w_B'(t))}{N}.
\]

(2.90)
Therefore, the total flow of non-owners to trade is
\[
\varphi_B(t) = \varphi_B(t) + \varphi_{LB}(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{\bar{y}(t + w_B(t)) + t\bar{y}'(t + w_B(t))(1 + w_B'(t))}{N}.
\] (2.91)

Fortuitously, \(\varphi_B\) integrates without much effort:
\[
\int_0^t \varphi_B(s) ds = \lambda \cdot \frac{N - 1}{N} \cdot \int_0^t \frac{\bar{y}(s + w_B(s)) + s\bar{y}'(s + w_B(s))(1 + w_B'(s))}{N} ds
\]
\[
= \lambda \cdot \frac{N - 1}{N} \cdot \left[ \frac{s\bar{y}(s + w_B(s))}{N} \right]_0^t
\]
\[
= \lambda \cdot \frac{N - 1}{N} \cdot \frac{t\bar{y}(t + w_B(t))}{N}.
\] (2.92)

Equivalently,
\[
\frac{L}{C} \int_0^t \varphi_B(s) ds = \alpha \beta r t\bar{y}(t + w_B(t)).
\] (2.95)

For \(t \in (\tilde{t}_b, \bar{t})\), \(\varphi_B(t) = \phi_0\).

**Buyers and Sellers.** Note that
\[
\varphi_B(0) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{\bar{y}(0)}{N} \geq \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{y}(0)}{N} > \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(0)}{N} = \varphi_B(0).
\] (2.96)

where the first inequality follows since \(\bar{y}(0) \geq 1\), and the second since \(\bar{x}(0) > \bar{y}(0)\). Put \(\tilde{t} = \min\{\tilde{t}_s, \tilde{t}_b\}\). Now \(\bar{y}\) is strictly increasing, \(w_B(t) \geq -1\), and hence \(\varphi_B\) is strictly increasing. On the otherhand, \(\bar{x}\) is increasing, and hence \(\varphi_S\) is decreasing. Since \(\varphi_B(0) > \varphi_S(0)\), we have \(\varphi_B > \varphi_S\) on \([0, \tilde{t}]\) (and hence \(\phi = \varphi_S\), \(\mathcal{S} = 0\), and \(\mathcal{B} > 0\) on \([0, \tilde{t}]\)). But then \(\tilde{t} = t_b\). For all \(t \in [\tilde{t}_b, \tilde{t}]\), \(w_B(t) = \tilde{t} - t\), and hence
\[
\mathcal{B}(t) = \int_t^{t + w_B(t)} \phi(s) ds = \int_t^{\tilde{t}} \phi(s) ds > 0
\] (2.97)
by the definitions of \(\tilde{t}_b\) and \(w_B\). Hence, \(\phi = \varphi_S\), and \(\mathcal{S} = 0\) on \([\tilde{t}_b, \tilde{t}]\). We conclude that \(\phi = \varphi_S\) on \([0, \tilde{t}]\).

**Proof of Proposition 2.3.1.** We consider the price path, sellers’ wait time, and the buyers’ wait time in order.
Price Path (Existence 1). We first verify the following facts about $p^*$:

**Fact 1:** Since $v$ is strictly increasing (Item 3), $v(0) = 0$, $v(1) = \alpha \Delta p$, and $\alpha u_0 < \alpha \Delta p$, there is a unique $z_0 \in (0, 1)$ such that $v(z_0) = \alpha u_0$. Put $\bar{t} = -r^{-1} \log(z_0)$, so that $z_0 = z(0, \bar{t})$. Hence,

$$p(0) = p_0 + \alpha^{-1} v(z(0, \bar{t})) = p_0 + \alpha^{-1} v(z_0) = p_0 + u_0 = L p_0.$$  \hfill (2.98)

**Fact 2:** By Corollary 2.3.1,

$$\frac{L}{C} \int_0^{\bar{t}} \phi(s) \, ds = \log \left( \frac{p_1 - p_0}{p_0 - p_0} \right) < \frac{p_1 - p_0}{L p_0 - p_0} - 1 = \frac{p_1 / p_0 - 1}{L - 1} - 1 = \frac{L^2 - 1}{L - 1} - 1 = L$$

(2.99)

which implies that $\int_0^{\bar{t}} \phi(s) \, ds < C$.

**Fact 3:** By Item 4, $v$ is convex, and hence $v(z) < v(1) z = \alpha \Delta p z$ for all $z \in (0, 1)$. Therefore,

$$\bar{t} = -\frac{1}{r} \cdot \log(z(0, \bar{t})) < -\frac{1}{r} \cdot \log \left( \frac{v(z(0, \bar{t}))}{\alpha \Delta p} \right) = -\frac{1}{r} \cdot \log \left( \frac{\alpha u_0}{\alpha \Delta p} \right) = \frac{1}{r} \cdot \log \left( \frac{\Delta p}{u_0} \right) < T$$

(2.100)

where the last line follows by Assumption 2.15.

**Fact 4:** $p^*(\bar{t}, \bar{t}) = p_0 + \alpha^{-1} v(z(\bar{t}, \bar{t})) = p_0 + \alpha^{-1} v(1) = p_0 + \Delta p = p_1$.

**Fact 5:** By Item 1,

$$p'(\bar{t}) = \alpha^{-1} r z(\bar{t}, \bar{t}) v'(z(\bar{t}, \bar{t})) = \alpha^{-1} r \cdot \frac{\alpha \Delta p ((\beta - \Delta p) + \alpha \Delta p)}{\alpha (1 + \alpha \Delta p)} = \frac{\alpha \Delta p}{1 + \alpha \Delta p} \cdot (N - 1) < N - 1.$$  \hfill (2.101)

**Fact 6:** Note that $p'(t) = \alpha^{-1} r z(t, \bar{t}) v'(z(t, \bar{t}))$. By Item 4 and Assumption 2.14,

$$p''(t) = \alpha^{-1} r^2 z(t, \bar{t}) v'(z(t, \bar{t})) + \alpha^{-1} r^2 z(t, \bar{t})^2 v''(z(t, \bar{t})) > \alpha^{-1} r^2 z(t, \bar{t}) v'(z(t, \bar{t})) = rp'(t).$$

(2.102)

We conclude that $p^*$ is admissible, and satisfies the assumptions of Lemma 2.3.3. Two facts follow immediately from Lemma 2.3.3. First, the trade flow is

$$\phi(t) = \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(t)}{N},$$  \hfill (2.103)
and second, $w^*_S = 0$.

**Price Path (Existence II and Uniqueness).** We claim that there is only one price path that satisfies the pricing equation under $\phi$, namely, $p^*$. By Lemma 2.3.1, and the definitions of $\alpha$, $\beta$, and $\bar{x}$, we obtain

$$p'(t) = \frac{L}{C} \cdot (p(t) - p_0) \cdot \phi(t) = \alpha r \cdot (p(t) - p_0) \cdot \left( \beta - \left( \frac{p_1 - p(t)}{1 - z(t, \bar{t})} \right) \right).$$

(2.104)

Using the transformation $u = p - p_0$, and the definition of $\Delta p$, we further obtain

$$u'(t) = \alpha r \cdot u(t) \cdot \left( \beta - \left( \frac{\Delta p - u(t)}{1 - z(t, \bar{t})} \right) \right).$$

(2.105)

Define $z^{-1}(z, \bar{t}) \equiv \bar{t} + r^{-1} \log(z)$, and $v(z) \equiv \alpha u(z^{-1}(z, \bar{t}))$, and observe that

$$v'(z) = \frac{\alpha u'(z^{-1})}{rz} \quad \text{(2.106)}$$

$$= \frac{\alpha u(z^{-1})}{z} \cdot \left( \alpha \beta - \frac{\alpha \Delta p - \alpha u(z^{-1})}{1-z} \right) \quad \text{(2.107)}$$

$$= \left( \frac{\alpha \beta}{z} - \frac{\alpha \Delta p}{z(1-z)} \right) \cdot \alpha u(z^{-1}) + \left( \frac{1}{z(1-z)} \right) \cdot (\alpha u(z^{-1}))^2 \quad \text{(2.108)}$$

$$= \left( \frac{\alpha (\beta - \Delta p)}{z} - \frac{\alpha \Delta p}{z(1-z)} \right) \cdot \alpha u(z^{-1}) + \left( \frac{1}{z(1-z)} \right) \cdot (\alpha u(z^{-1}))^2 \quad \text{(2.109)}$$

$$= X(z) \cdot (\alpha u(z^{-1})) + Y(z) \cdot (\alpha u(z^{-1}))^2 \quad \text{(2.110)}$$

$$= X(z) \cdot v(z) + Y(z) \cdot v(z)^2 \quad \text{(2.111)}$$

By Corollary 2.3.2, the unique solution is

$$v(z(t, \bar{t})) = \frac{z(t, \bar{t})^{\alpha(\beta-\Delta p)}(1-z(t, \bar{t}))^{\alpha \Delta p}}{\int_{z(t, \bar{t})}^{1} s^{\alpha(\beta-\Delta p)-1}(1-s)^{\alpha \Delta p-1} ds} \quad \text{(2.112)}$$

where $v(z(t, \bar{t})) = v(1) = \alpha \Delta p$, and hence

$$p(t; \bar{t}) \equiv p_0 + \frac{z(t, \bar{t})^{\alpha(\beta-\Delta p)}(1-z(t, \bar{t}))^{\alpha \Delta p}}{\int_{z(t, \bar{t})}^{1} s^{\alpha(\beta-\Delta p)-1}(1-s)^{\alpha \Delta p-1} ds} \quad \text{(2.113)}$$

as desired. Finally, we show that there exists a wait time satisfying FIFO.

**Sellers’ Wait Time.** Evidently, $w^*_S = 0$ is admissible. It has a trade horizon of $\bar{t}_s = \bar{t}$, and satisfies $w'_S > -1$ on $(0, \bar{t}_s)$. 


**Buyers’ Wait Time.** Given that the price path is \( p^* \), we compute the buyers’ wait time, \( w^*_B \), and verify that it is admissible and satisfies the assumptions of Lemma 2.3.3. By construction,

\[
\int_t^{t+w_B(t)} \phi(s)ds = B(t) = \int_0^{t} (\varphi_B(s) - \phi(s))ds \Rightarrow \int_t^{t+w_B(t)} \phi(s)ds = \int_0^{t} \varphi_B(s)ds.
\]

We establish the following facts:

**Fact 1:** There exists a \( w_B \) and \( \tilde{t}_b \). Using the calculation of the cumulative buyer flow and Corollary 2.3.1, we obtain

\[
\log(u(t + w_B(t))) - \log(u_0) = \alpha \beta r \tilde{y}(t + w_B(t)).
\]

Define

\[
F(t, w) \equiv \log(u(t + w)) - \log(u_0) - \alpha \beta r \tilde{y}(t + w).
\]

Since \( \tilde{y} > 1 \), and \( u(t) < u_n(t) \), we have that

\[
F(t, 0) = \log \left( \frac{u(t)}{u_0} \right) - \tilde{y}(t) \log \left( \frac{u_n(t)}{u_0} \right) < \log \left( \frac{u(t)}{u_0} \right) - \log \left( \frac{u_n(t)}{u_0} \right) = \log \left( \frac{u(t)}{u_n(t)} \right) < 0.
\]

Put \( g(t) \equiv F(t, \tilde{t} - t) = \log(\Delta p/u_0) - \alpha \beta r \tilde{y}(t) \). Now \( g(0) = \log(\Delta p/u_0) > 0 \), and

\[
g(\tilde{t}) = F(\tilde{t}, 0) = \log(\Delta p) - \log(u_0) - \alpha \beta r \tilde{y}(\tilde{t}) = \alpha \beta r \tilde{t}_n - \alpha \beta r \tilde{y}(\tilde{t}) < 0.
\]

\( g \) is affine, so it has a unique zero in \((0, \tilde{t})\), namely, \( \tilde{t}_b \). So for \( t \in [0, \tilde{t}_b] \), \( F(t, 0) < 0 \) and \( F(t, \tilde{t} - t) = g(t) > 0 \). So there exists \( w \in (0, \tilde{t} - t) \) such that \( F(t, w) = 0 \). That \( w \) is twice continuously differentiable follows immediately from the fact that \( F \) is also twice continuously differentiable.

**Fact 2:** \( w'_B > -1 \) on \((0, \tilde{t}_b)\). Observe that

\[
\int_0^{t+w_B(t)} \phi(s)ds = \int_0^{t} \varphi_B(s)ds \Rightarrow \phi(t + w_B(t))(1 + w_B'(t)) = \varphi_B(t) \Rightarrow w'_B(t) > -1.
\]

We conclude that \((p^*, 0, w^*_B)\) is the unique equilibrium in \( E_0 \).
Proof of Lemma 2.3.4. According to proposition 2.3.1, the sophisticated equilibrium has stopping time
\[ \bar{t}_s = -r^{-1} \log(v^{-1}(\alpha u_0)). \] (2.114)

The naïve equilibrium has stopping time
\[ \bar{t}_n = \frac{1}{\alpha \beta r} \log(u_0^{-1} \Delta p). \] (2.115)

Since \( v \) is strictly convex, \( \alpha u_0 = v(z_0) < \alpha \Delta p z_0 = \alpha \Delta p v^{-1}(\alpha u_0) \), and hence
\[ t_s - t_n = -r^{-1} \log(v^{-1}(\alpha u_0)) - \alpha^{-1} \beta^{-1} r^{-1} \log(u_0^{-1} \Delta p) \] (2.116)
\[ = -r^{-1} \log(v^{-1}(\alpha u_0)) + \alpha^{-1} \beta^{-1} r^{-1} \log((\alpha u_0)(\alpha \Delta p)^{-1}) \] (2.117)
\[ > -r^{-1} \log(v^{-1}(\alpha u_0)) - \beta^{-1}(\beta - \Delta p)r^{-1} \log(m^{-1}(\alpha u_0)) \] (2.118)
\[ = -\alpha^{-1} \beta^{-1} r^{-1}(\alpha \beta + \alpha(\beta - \Delta p)) \log(v^{-1}(\alpha u_0)) > 0 \] (2.119)
as desired. ■

Proof of Lemma 2.3.3. Recall that \( \bar{t} \) is the unique solution to
\[ p(0, \bar{t}) = p_0 + \alpha^{-1} m(z(0, \bar{t})). \] (2.120)

Equivalently,
\[ \alpha u_0 = \frac{z_0^\alpha (1 - z_0)^{\alpha \Delta p}}{\int_{z_0}^1 s^{\alpha (\beta - \Delta p) - 1}(1 - s)^{\alpha \Delta p - 1} ds} \] (2.121)
where \( z_0 = z(0, \bar{t}) \). We call this equation the stopping equation. For ease of notation, put \( a \equiv \alpha(\beta - \Delta p) \) and \( b \equiv \alpha(\beta - \Delta p) \), so that
\[ \alpha u_0 = \frac{z_0^a (1 - z_0)^b}{\int_{z_0}^1 s^{a-1}(1 - s)^{b-1} ds}. \] (2.122)

Taking logs, we obtain
\[ \log(\alpha u_0) + \log \left( \int_{z_0}^1 s^{a-1}(1 - s)^{b-1} ds \right) = a \log(z_0) + b \log(1 - z_0). \] (2.123)

In what follows, we’ll show that \( z_0 \) is decreasing with \( \Delta p \) and increasing with \( \alpha \).
\( z_0 \) is decreasing with \( \Delta p \). In what follows, primes denote differentiation with respect to \( \Delta p \).

Differentiating the stopping equation with respect to \( \Delta p \), we obtain

\[
\left( -\alpha \log(z_0) + a\frac{z'_0}{z_0} + \alpha \log(1 - z_0) - b\frac{z'_0}{1 - z_0} \right) \left( \int_{z_0}^{1} s^{a-1}(1 - s)^{b-1}ds \right)
= -z'_0 z_0^{a-1}(1 - z_0)^{b-1} + \alpha \int_{z_0}^{1} (\log(1 - s) - \log(s))s^{a-1}(1 - s)^{b-1}dz
\]

(2.124)

Equivalently,

\[
\left( a\frac{z'_0}{z_0} - b\frac{z'_0}{1 - z_0} \right) z_0^{a}(1 - z_0)^{b} - (\alpha \log(z_0) - \alpha \log(1 - z_0)) \left( \int_{z_0}^{1} s^{a-1}(1 - s)^{b-1}dz \right)
= -z'_0 z_0^{a-1}(1 - z_0)^{b-1} + \alpha \int_{z_0}^{1} (\log(1 - s) - \log(s))s^{a-1}(1 - s)^{b-1}dz.
\]

(2.126)

Collecting terms, we obtain

\[
(a(1 - z_0) - b z_0 + \alpha u_0) z'_0 = \alpha \int_{z_0}^{1} \log \left( \frac{z_0}{s} \cdot \frac{1 - s}{1 - z_0} \right) s^{a-1}(1 - s)^{b-1}dz.
\]

(2.128)

Now \( a(1 - z_0) - b z_0 + \alpha u_0 = z_0(1 - z_0)m'(z_0)/m(z_0) > 0 \), and \( z_0(1 - s) < s(1 - z_0) \). Therefore, \( z'_0 > 0 \) as desired.

\( z_0 \) is increasing with \( \alpha \). In what follows, primes denote differentiation with respect to \( \alpha \). Differentiating the stopping equation with respect to \( \alpha \), we obtain

\[
\left( (\beta - \Delta p) \log(z_0) + a\frac{z'_0}{z_0} + \Delta p \log(1 - z_0) - b\frac{z'_0}{1 - z_0} - \frac{1}{\alpha} \right) \int_{z_0}^{1} s^{a-1}(1 - s)^{b-1}ds
= -z'_0 z_0^{a-1}(1 - z_0)^{b-1} + \int_{z_0}^{1} ((\beta - \Delta p) \log(s) + \Delta p \log(1 - s))s^{a-1}(1 - s)^{b-1}ds.
\]

(2.129)

Equivalently,

\[
\left( a\frac{z'_0}{z_0} - b\frac{z'_0}{1 - z_0} \right) z_0^{a}(1 - z_0)^{b} + (a \log(z_0) + b \log(1 - z_0) - 1) \frac{1}{\alpha} \int_{z_0}^{1} s^{a-1}(1 - s)^{b-1}ds
= -z'_0 z_0^{a-1}(1 - z_0)^{b-1} + \frac{1}{\alpha} \int_{z_0}^{1} (a \log(s) + b \log(1 - s))s^{a-1}(1 - s)^{b-1}ds.
\]

(2.130)
Collecting terms, we obtain

\[
(a(1 - z_0) - bz_0 + \alpha u_0) z'_0 = \frac{1}{\alpha} \int_{z_0}^{1} \left( a \log \left( \frac{s}{z_0} \right) + b \log \left( \frac{1-s}{1-z_0} \right) + 1 \right) s^{a-1}(1-s)^{b-1} ds \tag{2.133}
\]

\[
> \frac{1}{\alpha} \int_{z_0}^{1} \left( b \log \left( \frac{1-s}{1-z_0} \right) + 1 \right) s^{a-1}(1-s)^{b-1} ds \tag{2.134}
\]

\[
> \frac{1}{\alpha} z_0^{a-1}(1-z_0)^{b-1} \int_{z_0}^{1} \left( b \log \left( \frac{1-s}{1-z_0} \right) + 1 \right) \left( \frac{1-s}{1-z_0} \right)^{b-1} ds \tag{2.135}
\]

\[
= 0. \tag{2.136}
\]

Again, \(a(1 - z_0) - bz_0 + \alpha u_0 = z_0(1 - z_0)m'(z_0)/m(z_0) > 0\). Therefore, \(z'_0 > 0\) as desired.

\(z_0\) is increasing with \(u_0\). In what follows, primes denote differentiation with respect to \(u_0\). \(m\) doesn’t depend explicitly on \(u_0\). Therefore,

\[
\alpha u_0 = m(z_0) \Rightarrow \alpha = m'(z_0)z'_0 \Rightarrow z'_0 > 0. \tag{2.137}
\]

**Results.** We’ve established that \(z_0\) is decreasing with \(\Delta p\) and increasing with \(\alpha\). We can now conclude that

\[
\frac{\partial \bar{t}}{\partial \lambda} = \frac{d \bar{t}}{dz_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \lambda} = -\frac{r}{z_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{L}{CN^2} \leq 0; \tag{\lambda}
\]

\[
\frac{\partial \bar{t}}{\partial L} = \frac{d \bar{t}}{dz_0} \cdot \left( \frac{\partial z_0}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial L} + \frac{\partial z_0}{\partial u_0} \cdot \frac{\partial u_0}{\partial L} \right) = -\frac{r}{z_0} \cdot \left( \frac{\partial z_0}{\partial \alpha} \cdot \frac{\lambda L}{CN^2} + \frac{\partial z_0}{\partial u_0} \cdot p_0 \right) \leq 0; \tag{L}
\]

\[
\frac{\partial \bar{t}}{\partial C} = \frac{d \bar{t}}{dz_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial C} = \frac{r}{z_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{\lambda L}{C^2 N^2} \geq 0; \tag{C}
\]

\[
\frac{\partial \bar{t}}{\partial \Delta p} = \frac{d \bar{t}}{dz_0} \cdot \frac{\partial z_0}{\partial \Delta p} = -\frac{r}{z_0} \cdot \frac{\partial z_0}{\partial \Delta p} \geq 0; \tag{\Delta p}
\]

as desired.
Figure 2.1. Predictability.

Aircraft

Taxi Medallions

Single Family Homes

Commercial Real Estate

S&P 500

NYSE Seats

R-squared values for each graph:
- Aircraft: r-squared = 49.34%
- Taxi Medallions: r-squared = 6.32%
- Single Family Homes: r-squared = 49.08%
- Commercial Real Estate: r-squared = 86.10%
- S&P 500: r-squared = 0.48%
- NYSE Seats: r-squared = 0.05%
**Predictability.** This figure plots future versus current returns for aircraft (top-left), taxi medallions (top-right), single family homes (middle-left), multi-family residential and commercial real estate (middle-right), the S&P 500 (bottom-left), and seats on the NYSE (bottom-right). These are month-over-month returns, with the exception of aircraft, which are only available quarter-over-quarter. Data on aircraft cover the period 1994 Q4 to 2016 Q4, and are provided by VREF. Data on taxi medallions cover the period January 1992 to December 2018, and are provided by the New York City Taxi and Limousine Commission. Data on single family residential real estate cover the period January 2000 to June 2016, and are obtained from S&P Core-Logic. Data on multi-family and commercial real estate cover the period December 2000 to December 2015, and are obtained from Moody’s/Real Capital Analytics. Data on the S&P 500 cover the period January 1987 to December 2015, and are obtained from CRSP. Data on NYSE seats cover the period 1880 to 1925, and were collected from the New York Times by Asaf Bernstein.
Figure 2.2. **Prices and Returns.** The top panel plots the total return (price appreciation plus rental income) against the rent-to-price ratio for the 200 largest US metropolitan areas between 2010 and 2013. The coefficient on the rent-to-price ratio is a significant $1.28 (t = 5.58)$. The bottom panel plots price appreciation against the rent-to-price ratio. The coefficient on the rent-to-price ratio is an insignificant $.33 (t = 1.44)$. Data are from Zillow. Therefore, lower-priced metro areas earn substantially higher total returns.
Figure 2.3. **Turnover.** This figure plots the average turnover (percent of homes in a metropolitan area that have changed hands in the last year) against yearly price appreciation for the 200 largest US metropolitan areas between 2010 and 2014. Data are from Zillow. When prices are rising (falling), they rise (fall) faster when turnover is higher.
Figure 2.4. **Days on Market.** This figure plots the average days on market against yearly price appreciation for the 200 largest US metropolitan areas between 2010 and 2013. Data are from Zillow. Days on market appears to have a floor, which corresponds with the fact that there is a minimum time between when a house is listed for sale and when a sale can close, during which the house is labeled on the market even though it is under contract. Days on market is lower when prices are appreciating faster, consistent with the existence of buyers’ and sellers’ markets.
Figure 2.5. **Buyers’ and Sellers’ Markets.** This figure plots the time series for the monthly supply of housing inventory, and the time series of house prices, as measured by the Case-Shiller 20-City Composite, between January 1987 and July 2015. Data are from US Census Bureau and Core Logic, respectively. Inventory appears to have a floor, which corresponds with the fact that there is a minimum time between when a house is listed for sale and when a sale can close, during which the house is labeled “inventory” even though it is under contract. Inventory rises when prices are falling, consistent with growth in a seller’s queue during down markets. Inventory is stable at its minimum level with prices are rising, consistent with the lack of a seller’s queue in rising markets.
Figure 2.6. **Flipping.** This figure plots flipping—defined as the fraction of sales for which the home has sold twice in one year—against yearly price appreciation for 100 US metropolitan areas between 2000 and 2016. Data are from Trulia. When prices are rising, flipping increases.
Figure 2.7. **Numerical Solution.** This figure shows a numerical solution of the rational model for $p_1 = 200$, $p_0 = 150$, $r = 0.05$, $\lambda = 1$, $L = \sqrt{p_1/p_0}$, $N = \lambda \log ((p_1 - p_0)(Lp_0 - p_0))/r$, and $C = \lambda L((N - 1)/r - (p_1 - p_0))/(2N^2)$. The top-left plot shows the price paths for the instant adjustment, naïve, and rational models. The top-right plot shows the flow of buyers and sellers to trade. The bottom-left plot shows the buyers’ and sellers’ wait times. The bottom-right plot shows the sizes of the buyers’ and sellers’ queues. Panels (b) through (d) all show paths only for the fully rational model.
Figure 2.8. Queuing.
Queuing. $N(t)$ is the mass of non-owners who have yet to be shocked, $\mathcal{N}(t)$ is the mass of non-owners who have been shocked, but have yet to queue to buy, and $\mathcal{B}(t)$ is the mass of non-owners who have queued to buy. Some non-owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\varphi_B(t)$. Others find it optimal to queue some time after being shocked. They enter the waiting state at a rate of $n(t)$. They enter the queue at a rate of $\varphi_B(t)$. $O(t)$ is the mass of owners who have yet to be shocked, $\mathcal{O}(t)$ the mass of owners who have been shocked, but have yet to queue to sell, and $\mathcal{S}(t)$ is the mass of non-owners who have queued to sell. In the case of an upward adjustment (top), it will transpire that $\mathcal{S}(t) = 0$. Some owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\varphi_S(t)$. Others find it optimal to queue some time after being shocked. They enter the waiting state at a rate of $o(t)$. They enter the queue at a rate of $\varphi_S(t)$. non-owners become owners at a rate of $\phi$; owners become non-owners at the same rate. In the case of a downward adjustment (bottom), it will transpire that $\mathcal{B}(t) = 0$. Some non-owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\varphi_B(t)$. Others find it optimal to queue some time after being shocked. They enter the waiting state at a rate of $n(t)$. They enter the queue at a rate of $\varphi_B(t)$. Owners become non-owners at a rate of $\phi$; non-owners become owners at the same rate.
Figure 2.9. **Appraisal Time Series.** The top-left panel shows the time-series of the purchase-plus-appraisal and purchase only indices between 2000 Q1 and 2016 Q3. The top-right panel shows the distribution of cumulative returns over the period 2011 Q2 to 2016 Q2 for the 100 largest US metropolitan areas for the purchase-plus-appraisal index and the purchase-only indices. The bottom-left panel compares the rate of quarterly appreciation in the purchase-only price index to the gap between the levels of the purchase-only and the purchase-plus-appraisal indices between 2000 Q1 and 2016 Q3. Data come from the FHFA. It is clear that when prices are rising or falling faster, the indices diverge by more.
Table 2.1. **Taxi Medallions.** This table shows results of AR(1) regressions of monthly and annual excess returns for individual and corporate New York City Taxi medallions. Monthly risk-free rates come from Kenneth French’s data library. Return data are from the New York City Taxi and Limousine Commission, and cover the period January, 1992 to December, 2018.

### Panel A: Monthly Excess Return\(_{t+1}\)

<table>
<thead>
<tr>
<th></th>
<th>Individual</th>
<th>Corporate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return(_t)</td>
<td>0.511***</td>
<td>0.498***</td>
</tr>
<tr>
<td></td>
<td>(0.127)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0001</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Observations</td>
<td>123</td>
<td>123</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.118</td>
<td>0.130</td>
</tr>
</tbody>
</table>

*Note:* \(\ast p<0.1; \ast\ast p<0.05; \ast\ast\ast p<0.01\)

### Panel B: Annual Excess Return\(_{t+1}\)

<table>
<thead>
<tr>
<th></th>
<th>Individual</th>
<th>Corporate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return(_t)</td>
<td>−0.094</td>
<td>0.194</td>
</tr>
<tr>
<td></td>
<td>(0.378)</td>
<td>(0.370)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>Observations</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.009</td>
<td>0.038</td>
</tr>
</tbody>
</table>

*Note:* \(\ast p<0.1; \ast\ast p<0.05; \ast\ast\ast p<0.01\)
Table 2.2. **Aircraft.** This table shows results of AR(1) regressions of quarterly and annual excess returns for seven types of aircraft. Monthly risk-free rates come from Kenneth French’s data library. Return data are from VREF Aircraft Value Reference, and cover the period 1994 Q4 to 2016 Q4.

<table>
<thead>
<tr>
<th>Panel A: Quarterly Excess Return&lt;sub&gt;t+1&lt;/sub&gt;</th>
<th>Turbo Prop</th>
<th>Light Single</th>
<th>Complex Single</th>
<th>Light Twin</th>
<th>Pressurized Piston Twin</th>
<th>Light Jet</th>
<th>Large Jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.641***</td>
<td>0.413***</td>
<td>0.445***</td>
<td>0.514***</td>
<td>0.615***</td>
<td>0.624***</td>
<td>0.661***</td>
</tr>
<tr>
<td>Constant</td>
<td>−0.001</td>
<td>−0.002</td>
<td>−0.002</td>
<td>−0.002*</td>
<td>−0.001</td>
<td>−0.008**</td>
<td>−0.008**</td>
</tr>
<tr>
<td>Observations</td>
<td>87</td>
<td>87</td>
<td>87</td>
<td>86</td>
<td>87</td>
<td>87</td>
<td>86</td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.411</td>
<td>0.170</td>
<td>0.198</td>
<td>0.266</td>
<td>0.389</td>
<td>0.389</td>
<td>0.436</td>
</tr>
<tr>
<td>Note:</td>
<td>*p&lt;0.1; **p&lt;0.05; ***p&lt;0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Annual Excess Return&lt;sub&gt;t+1&lt;/sub&gt;</th>
<th>Turbo Prop</th>
<th>Light Single</th>
<th>Complex Single</th>
<th>Light Twin</th>
<th>Pressurized Piston Twin</th>
<th>Light Jet</th>
<th>Large Jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.415*</td>
<td>0.212</td>
<td>0.413*</td>
<td>0.511**</td>
<td>0.518**</td>
<td>0.620***</td>
<td>0.596***</td>
</tr>
<tr>
<td>Constant</td>
<td>−0.009</td>
<td>−0.008</td>
<td>−0.007</td>
<td>−0.009</td>
<td>−0.006</td>
<td>−0.030</td>
<td>−0.034</td>
</tr>
<tr>
<td>Observations</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>21</td>
<td>22</td>
<td>22</td>
<td>21</td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.171</td>
<td>0.045</td>
<td>0.170</td>
<td>0.254</td>
<td>0.273</td>
<td>0.394</td>
<td>0.355</td>
</tr>
<tr>
<td>Note:</td>
<td>*p&lt;0.1; **p&lt;0.05; ***p&lt;0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.3. **Multi-Family Residential and Commercial Real Estate.** This table shows results of AR(1) regressions of excess monthly and annual returns for several types of multi-family residential and commercial real estate. Monthly risk-free rates come from Kenneth French’s data library. Data are from Moody’s/Real Capital Analytics, and cover the period December 2000 to December 2015.

<table>
<thead>
<tr>
<th></th>
<th>Apartment</th>
<th>Retail</th>
<th>Industrial</th>
<th>Office</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Excess Return</strong></td>
<td>0.923***</td>
<td>0.846***</td>
<td>0.820***</td>
<td>0.884***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.040)</td>
<td>(0.043)</td>
<td>(0.035)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0005)</td>
<td>(0.0005)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>179</td>
<td>179</td>
<td>179</td>
<td>179</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.852</td>
<td>0.716</td>
<td>0.673</td>
<td>0.781</td>
</tr>
<tr>
<td><strong>Note:</strong></td>
<td>*p&lt;0.1; **p&lt;0.05; ***p&lt;0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Apartment</th>
<th>Retail</th>
<th>Industrial</th>
<th>Office</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Excess Return</strong></td>
<td>0.452*</td>
<td>0.619**</td>
<td>0.473*</td>
<td>0.332</td>
</tr>
<tr>
<td></td>
<td>(0.249)</td>
<td>(0.223)</td>
<td>(0.245)</td>
<td>(0.265)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.034</td>
<td>0.016</td>
<td>0.016</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.027)</td>
<td>(0.026)</td>
<td>(0.038)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.203</td>
<td>0.371</td>
<td>0.223</td>
<td>0.108</td>
</tr>
<tr>
<td><strong>Note:</strong></td>
<td>*p&lt;0.1; **p&lt;0.05; ***p&lt;0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.4. **NYSE Seats.** This table shows results of AR(1) regressions of excess monthly and annual returns for seats on the New York Stock Exchange. Data were graciously provided by Asaf Bernstein, and cover the period January 1888 to December 1925.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Monthly Excess Return t+1</th>
<th>Panel B: Annual Excess Return t+1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Seat Price</td>
<td>Seat Price</td>
</tr>
<tr>
<td>Excess Return t</td>
<td>−0.022 (0.047)</td>
<td>−0.101 (0.169)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.007* (0.004)</td>
<td>0.093* (0.047)</td>
</tr>
<tr>
<td>Observations</td>
<td>449</td>
<td>37</td>
</tr>
<tr>
<td>R²</td>
<td>0.0005</td>
<td>0.010</td>
</tr>
<tr>
<td>Note:</td>
<td>*p&lt;0.1; **p&lt;0.05; ***p&lt;0.01</td>
<td>*p&lt;0.1; **p&lt;0.05; ***p&lt;0.01</td>
</tr>
</tbody>
</table>
Chapter 3

The Internal Finance of Customer Owned Firms

3.1 Introduction

Two firms operate in identical, but independent markets. They face identical demands for their goods and identical costs of production. One firm is owned by shareholders—the “corporation.” It maximizes profit. The other firm is owned by its customers—the “cooperative.” It maximizes total surplus, as its owners are both purchasers of its good and suppliers of its capital. How do growth, pricing, and capital structure compare between these two firms? I show that in the cooperative—unlike the corporation—they are fundamentally linked and should covary in predictable ways.

Suppose, for concreteness, that a corporate bank observes an opportunity to grow its loan book next quarter. Perhaps a new source of information will reduce the riskiness of making loans—like credit reports—or perhaps a new technology will lower the cost of day-to-day transactions—like smartphone apps. When next quarter rolls around, the bank will offer lower loan rates to bring more applicants through the door. But it won’t offer this quarter’s loan applicants lower (or higher) rates. This quarter’s rates are set to maximize this quarter’s profit. They don’t tell us anything about credit reports or smartphone apps. What if the bank is running low on equity? Can it use this quarter’s rates to shore up its balance sheet? It can’t. It already makes as much profit as it can. If it needs more equity, it will have to get it somewhere else.

Suppose, on the other hand, that a cooperative bank observes an opportunity to grow its loan book next quarter. Like the corporate bank, its costs will fall. Now the cooperative bank
would happily set loan rates so that it makes no profit.\textsuperscript{1} If it has ample equity, it will do just this. But what if it doesn’t have ample equity? Unlike the corporate bank, the cooperative bank can do better than offer its “first best” rate. It can offer positive-profit rates, retain the earnings and use them to finance next quarter’s loan growth. The cooperative bank balances the gains from loan growth next quarter against the deadweight loss from profit this quarter.

In this paper, I show that the cooperative’s growth, prices, and leverage will covary in predictable ways. I show that higher sales next quarter are achieved through higher prices this quarter and lower leverage this quarter. In insurance-speak, higher policy growth is achieved through a higher insurance margin and a higher solvency ratio. In bank-speak, higher loan growth is achieved through a higher net interest margin and a higher capital ratio.

While this paper’s contribution is primarily theoretical, I document a number of stylized facts about US credit unions, which are cooperative banks. I find that industry loan growth is positively correlated with the industry net interest margin and positively correlated with the industry capital ratio. These correlations don’t hold for traditional, shareholder owned banks and are consistent with the theory’s predictions.

Cooperatives appear most commonly in insurance—as mutual insurance companies—and banking—as credit unions, mutual savings banks, and agricultural credit associations. Mutual insurance companies are responsible for 35% of consumer insurance in the US and 27% of consumer insurance worldwide. They manage $7.7 trillion in assets.\textsuperscript{2} Credit unions are responsible for 10% of consumer banking in the US.\textsuperscript{3} Approximately 8% of adults worldwide are members of a credit union and collectively they manage $1.8 trillion in assets in 105 countries.\textsuperscript{4} A non-negligible amount of banking services are provided by mutual savings banks, and agricultural credit associations. Cooperatives appear outside of the financial sector as well, most notably in electricity provision. Rural electricity cooperatives supply 13% of US electricity and hold $150 billion in assets.\textsuperscript{5} Despite

\textsuperscript{1} Profit creates a deadweight loss for its customer-owners.
\textsuperscript{3} Credit Union Report Year-End 2014. Credit Union National Association.
\textsuperscript{4} 2014 Statistical Report. World Council of Credit Unions.
their economic importance, customer owned firms receive little theoretical or empirical attention.\textsuperscript{6} In this paper, I show that they should defy some basic theories of corporate finance and industrial organization.

The model works as follows. There are two firms: the shareholder owned corporation and the customer owned cooperative. The corporation maximizes profit while the cooperative maximizes total surplus. Firms operate for two periods and then liquidate. They face identical demands for their goods and identical costs of production. In the first period, they observe an opportunity to grow—a \textit{technology shock}—which lowers their costs of doing business. Firms use the prices of their goods to manage their sales and internal funds.

The model has three working assumptions. The first assumption is that firms enjoy market power. They can increase sales by lowering prices and decrease sales by raising prices. In the model, I assume that firms are monopolists. This isn’t true in reality, but it’s a good benchmark.

The second assumption is that firms face capital requirements. There is only so much leverage that regulators, creditors or investors can stomach. In the model, firms must hold a minimum ratio of equity-to-sales. If “sales” is off-putting, recall that in an insurance company, “sales” means “policies” (a liability) and in a bank, “sales” means “loans” (an asset) or “deposits” (a liability). Equity-to-sales is a measure of leverage. I don’t impose a capital structure \textit{per se}. Firms can hold equity in excess of the minimum.

The third assumption is that firms rely on internal funds for equity. The cooperative can’t raise external equity. If it did, it would cede control to non-customers. It would no longer be “customer owned.” There’s nothing stopping the corporation from raising external equity. For the sake of a clean comparison, I assume that it can’t. I show that differences in behavior are driven by differences in ownership and not by differences in financial constraints.

The first set of results concerns the behavior of prices. I show that the corporation’s prices never depend on the technology shock. For sufficiently large technology shocks, the cooperative’s

\textsuperscript{6} The basic contracting issues of customer ownership have been studied by several authors (see section 3.2). What remains undeveloped is an operational theory of the customer owned firm.
prices are *increasing* in the technology shock.

The second set of results concerns the covariation of growth, pricing and leverage. For sufficiently large technology shocks, the cooperative’s growth will covary positively with its prices and negatively with its leverage. The corporation’s growth will vary with the technology shock, but it won’t *covary* with its prices or leverage.

The theory proposed in this paper offers a novel explanation for the sensitivity of cooperatives’ investment to cash flow.\(^7\) The standard explanation says that financially constrained firms turn positive cash flow directly back into investment. The explanation in this paper is quite different. It says that cooperatives observe investment opportunities and adjust their (expected) cash-flow by adjusting their prices.

The theory also offers a novel example of a dynamic pricing problem. In the model, the corporation’s problem is time-separable. The corporation can choose today’s prices without regard for the future. The cooperative’s problem is not. It adjusts today’s prices according to future growth opportunities. That pricing should be a dynamic problem in insurance or banking deserves pause.

As demonstrated by [16], a durable goods monopolist may have an incentive to intertemporally price discriminate, in which case her problem problem is “dynamic” (it’s not time-separable). But if she can lease the good instead of selling it, her problem is static again (it’s time-separable). Lending is leasing cash. Insurance is leasing a state contingent claim to cash. Borrowers can refinance. Policyholders can surrender their policies and obtain new ones. I should not expect Bulow’s intertemporal price discrimination in the financial sector.

### 3.2 Literature

It has long been postulated that cooperatives should, because of their peculiar organization, offer competitive prices (see, for example, [30]). The implication is powerful: a monopolist may enjoy economies of scale, but society suffers a deadweight loss; Bertrand competitors may offer competitive prices, but miss out on economies of scale. The cooperative skirts both costs. It

\(^7\) I don’t develop a Q-theory for the cooperative, so I only conjecture that this channel is at-play.
can offer competitive prices while enjoying economies of scale. As noted by [46], many insurance mutuals in nineteen century America were formed precisely to avoid paying the rates offered by corporate carriers.

Also noted by [46] is the reality that contracting is costly when ownership lies with customers. Whether customer owned firms offer prices that are consistent with their ownership is an empirical question, and a challenging one at that. Consider a firm that operates for one period (and one period only). If it shows a profit, then I can conclude that its prices are inconsistent with customer ownership. Now suppose that it operates for more than one period. If it still shows a profit, then I can’t be so sure that it’s prices are inconsistent with customer ownership. Maybe it needed to grow, creating a need for profits. A dynamic perspective is essential. This paper takes a step toward answering the “pricing and ownership” question by providing a dynamic theory of the customer owned firm, with particular emphasis on those found in insurance and banking.

A substantial literature beginning with [34, 33], [45, 46], [72, 73] and [93, 94, 92] explores the basic contracting problems of customer ownership. Broadly, this literature argues that customer ownership reduces informational costs inherent to some businesses. By making its policyholders owners, for example, a mutual insurance company attenuates moral hazard by tying its policyholders’ welfare to its own.

A parallel, empirical literature studies the effects of customer ownership on firm policies. [84] examines the impact of customer ownership on costs, profitability, risk and growth in the savings and loan industry. She finds that savings and loan associations are less profitable and grow slower than their stock association counterparts. [74] conduct a similar study in the property/casualty market. They examine the impact of ownership on lines-of-business specialization, line-of-business concentration and geographic concentration. More recently, [85] examine the survivability of savings banks in the presence of competition.

What seems absent from this literature is an applied theory of corporate finance and industrial organization for customer owned firms. Because of their ownership, they rely heavily—if not

---

8 I’m assuming that profit is deterministic, as it will be in the model.
exclusively—on internal funds for growth. Internal finance is an inherently dynamic problem, in the sense that today’s decisions affect tomorrow’s availability of funds. The static models that kickstarted the literature are insufficient to explain real-world growth, pricing and capital structure decisions.

The predictions of the current paper are consistent with a number of recent findings. [109] finds that when regulation of the life insurance industry changed from federal to state hands, mutual life insurance companies survived in states with less stringent capital requirements. The theory presented in this paper predicts that capital requirements are a limiting factor for the growth of mutual insurance companies. [88], who study the transmission of financial shocks through bank networks, find that loan growth in US credit unions is highly sensitive to their capital ratios, a relationship that will emerge from the theory in this paper. This paper strongly complements the recent empirical work of [adelino2014investment], who examine investment policies of non-profit hospitals. The authors point out that non-profits dominate the healthcare sector, which accounts for 15% of the US economy. Non-profits do not share objectives with the profit-maximizing, shareholder-owned firms of standard theory. The standard theory of corporate finance simply doesn’t apply. While this paper considers customer owned firms—such as mutual insurance companies and credit unions—many of the insights can be translated to the problem of the non-profit, which also has incentives for growth and also relies on internal funds for equity. Complimenting their findings, I explore a novel mechanism behind the investment cash-flow sensitivity.

That customer ownership should affect pricing dates back to [30], who argued that a cooperative should offer competitive prices, even if it enjoyed market power. The importance of a dynamic theory for customer owned firms cannot be understated. Cooperatives regularly make profits, which are justified on the grounds of “risk management” and “growth.” Asking an empirical question, like “do credit unions offer rates consistent with customer ownership?” requires an understanding of their internal finance. To the best of my knowledge, this is the first paper to theoretically investigate the effect of a customer friendly objective on pricing by benchmarking to the behavior of a profit-maximizer.
Several attempts have been made at a dynamic theory of customer owned financial institutions. [1] presents a dynamic model of a credit union, but obtains qualitative results only. [95] present an ad-hoc model of credit union capital management for estimation purposes. [90] present a dynamic model of a credit union, where members’ concern for the viability of the institution drive rate decisions. None of these models explicitly benchmark to a shareholder owned, profit-maximizing firm.

3.3 The Model

I consider two types of firms: the cooperative, whose ownership lies with its customers, and the corporation, whose ownership lies with shareholders (who are not customers). For ease of exposition, I model firm behavior using the standard model of a single-product, two-period monopolist. I make two adjustments to the standard model. First, I adjust the cooperative’s objective to reflect its customers’ ownership. Second, I assume that firms face capital requirements, as they would if they operated in the insurance or banking industries. To examine the effect of customer ownership on growth, pricing and capital structure, I benchmark my results for the cooperative against those of the corporation, which faces capital requirements but has the usual objective of profit maximization.

There are four periods, $t = 0, 1, 2, 3$. Firms are capitalized in period zero. They operate in periods one and two. They liquidate in period three. There is no discounting.

Firms face constant marginal costs $c_t > 0$ for $t \in \{1, 2\}$. Growth emerges as a feature of the model because of an exogenous shock to marginal costs. Put $\delta := (c_1 - c_2)/c_1$, so that $\delta$ measures the percentage by which marginal costs fall between the first and second periods. For example, if $\delta = 1/2$, then the marginal cost falls by 50%. I will refer to $\delta$ as the technology shock. The technology shock can be interpreted as an innovation in the transaction technology (e.g. the Internet, smartphone apps), risk assessment (e.g. actuarial tables, credit reports) or knowledge (e.g. learning-by-insuring, learning-by-lending).
Firms face a twice continuously differentiable demand \( D \), which is defined for strictly positive prices. \( D \) satisfies \( D > 0, D' < 0, \) and \( D'' > 0 \). Put \( P := D^{-1} \). Define the elasticity and curvature of \( D \) to be \( \epsilon(p) := -pD'(p)/D(p) \) and \( \sigma(p) := -pD''(p)/D'(p) \) respectively. \( D \) is such that for all \( p < 0 \),

\[
(A1) \quad \sigma(p) < 2\epsilon(p),
\]

\[
(A2) \quad \sigma(p) \leq 1 + \epsilon(p),
\]

\[
(A3) \quad \int_{-\infty}^{\infty} D(s)ds < \infty, \text{ and}
\]

\[
(A4) \quad \lim_{\rho \to \infty} \rho D(\rho) = 0.
\]

\(A1\) guarantees the strict quasiconcavity of one-period profit (and hence the uniqueness of the profit-maximizing price). \(A2\)—without too much loss of generality—simplifies the proof of Proposition 3.3.2. \(A3\) guarantees the existence of the consumer surplus. \(A4\) guarantees the existence of a profit-maximizing price. Exponential demand and constant elasticity demand (with elasticity strictly greater than one) satisfy \(A1 - A4\). Linear demand doesn’t, but it results in particularly well-posed problems for both the cooperative and corporation, as the objectives and constraints are all concave.

The first assumption deals with firms’ market power.

**Assumption 3.3.1. Firms are monopolists.**

Reality is more complicated. Mutual insurance companies compete with a plethora of carriers. Credit unions compete with just about anyone who makes home and auto loans or issues credit cards. To model this rich ecosystem of financial institutions would cloud the basic intuition of the theory. For simplicity and clarity, I assume that firms are monopolists and leave the task of developing a richer theory to future work.

Define the profit \( \Pi(p; c) := (p - c)D(p) \) and the consumer surplus \( S(p) := \int_{-\infty}^{\infty} D(s)ds \). As it will appear frequently, define the monopoly price to be \( p^m(c) := \arg\max_p \Pi(p; c) \). \( p^m(c) \) exists and is unique (see Lemma 3.7.2 in the appendix).

The next assumption deals with firms’ sources of equity.
Assumption 3.3.2. A firm’s internal funds are its only source of equity.

Assumption 3.3.2 might seem strong, but it turns out to be natural for the cooperative. Suppose a firm needs equity. By equity, I mean the residual claim. The residual claim, because of its riskiness, gives its holder two rights: the right to control the firm and the right to appropriate the firm’s profits. These rights constitute ownership [46]. If the firm is a cooperative, then it can’t issue residual claims to non-customers. If it did, it would no longer be customer owned. Now it can issues residual claims to its customers, but the practice is uncommon.9

There’s no reason why the corporation can’t issue equity. I make Assumption 3.3.2 for the corporation so that I can make a clean, clear and fair comparison with the cooperative. All of the paper’s main results go through without it.

To fix ideas, I assume that firms pay dividends upon liquidation. Again, this assumption doesn’t change the paper’s main results. It just relieves us from the burden of deriving a dividend policy.

Assumption 3.3.2 implies that a firm’s stock of equity, \( m_t \), evolves according to

\[
m_{t+1} = \Pi(p_t; c_t) + m_t.
\]

(3.1)

The firm makes profits and increases its stock of equity by retaining earnings. The stock is neither depleted through dividend payments nor replenished through equity issuance.

The last assumption deals with firms’ capital structures.

Assumption 3.3.3. Firms face capital requirements.

Whether to appease regulators, creditors, or investors, firms must holds some minimum fraction of equity in their capital structure. They are free to hold a larger fraction. The assumption can be interpreted in a variety of ways, depending on the nature of the business. In insurance and banking, firms face state imposed capital requirements. In non-financial industries, the “capital requirement” amounts to “risk management.” I will defer discussion of real-world capital requirements to Section 3.4.

9 See [69] and Section 3.4.
Formally, firms face a capital requirement of the form

\[ kD(p_t) \leq m_t \]  

for \( k \in (0, 1) \). Taken literally, CR\( t \) says that firms must hold a fraction \( k \) of sales in equity. Recall that in an insurance company, “sales” means “policies” (a liability); in a bank, “sales” means “loans” (an asset) or “deposits” (a liability), so the capital requirement is a constraint on leverage.

I assume that the corporation maximizes profit. For the cooperative, I adopt the objective postulated by [30]: the cooperative maximizes total surplus, which is the sum of consumer surplus and profit.\(^{10}\) The total surplus has a number of desirable properties. First, it clearly reflects the fact that the cooperative’s members are both customers (consumer surplus) and owners (profit). Second, the optimal price is the marginal cost, so the cooperative is a natural non-profit. What Enke’s objective lacks in specificity, it makes up for in clarity, elegance and generality.

The timeline is as follows:

**Period 0**: Owners learn \( c_1 \) and capitalize the firm with equity \( m_1 > 0 \).

**Period 1**: Owners learn \( c_2 \) and set price \( p_1 \).

**Period 2**: Owners earn profit \( \Pi(p_1; c_1) \) and set price \( p_2 \).

**Period 3**: Owners earn profit \( \Pi(p_2; c_2) \) and pay liquidating dividend \( m_3 \).

To widen the scope of the model’s applications, I don’t attempt to model the capitalization process in period 0. For the remainder of the paper, I treat \( m_1 \) as if it were exogenous.

### 3.3.1 What’s in a Price?

In this section, I define the cooperative’s and corporation’s problems. I show that they always have solutions and I present the paper’s core result: the cooperative’s first period price is increasing in the technology shock, while the corporation’s first period price is constant.

\(^{10}\) Because firms are assumed to be monopolists (Assumption 3.3.1), the industry profit and the firm profit are the same.
Formally, the cooperative’s problem is to
\[
\max_{p > 0} \int_{p_1}^{\infty} D(s) ds + \int_{p_2}^{\infty} D(s) ds + m_3
\]  
subject to
\[
kD(p_1) \leq m_1, \quad \text{(CR1)}
\]
\[
kD(p_2) \leq m_2 = m_1 + \Pi(p_1; c_1) \quad \text{and} \quad \text{(CR2)}
\]
\[
m_3 = \Pi(p_1; c_1) + \Pi(p_2; c_2) + m_1.
\]  

The cooperative operates in the first and second periods, offering favorable prices to its customer-owners. In the third period, it pays a liquidating dividend. By substituting \( m_3 \), I see that it maximizes total surplus each period.

The solution to the cooperative’s unconstrained problem is to offer the marginal cost price each period. Any other price creates a deadweight loss for its customer-owners. The solution to its constrained problem is far more nuanced and is the subject of this section.

It’s worth pointing out how growth, pricing and capital structure enter the cooperative’s problem. The role of pricing should be evident (prices are the choice variables). Growth is the percentage by which sales increase between the first and second periods. Capital structure comes from the capital requirements. If the capital requirement binds, then the firm is very levered; if it doesn’t, then the firm is less levered. At the heart of the paper’s results is the profit term in the second period capital requirement. It represents the firm’s internal funds. By adjusting this term in the first period, the firm adjusts the amount of growth that it can support in the second period.

Fortunately, the cooperative’s problem admits solutions under the assumptions made thus far.

**Lemma 3.3.1.** There is a solution to the cooperative’s problem. Moreover, it satisfies the Kuhn-Tucker conditions.

Throughout the paper, I benchmark the cooperative’s behavior to that of the corporation.
The corporation’s problem is to

\[
\max_{p>0} m_3 = \Pi(p_1; c_1) + \Pi(p_2; c_2) + m_1 \tag{3.5}
\]

subject to

\[
kD(p_1) \leq m_1 \quad \text{(CR1)}
\]

and it also admits solutions.

**Lemma 3.3.2.** There is a solution to the corporation’s problem. Moreover, it satisfies the Kuhn-Tucker conditions.

Our first result deals with the corporation’s prices.

**Proposition 3.3.1.** The corporation’s first period price is constant with respect to the technology shock.

Proposition 3.3.1 says that the corporation sets today’s price to maximize today’s profit. It may need additional equity to support tomorrow’s growth, but today’s prices won’t be of any help.

It’s instructive to look at the first-order conditions for \( p_1 \). If the first period capital requirement binds, then \( p^*_1 = P(m_1/k) \), which is constant with respect to \( \delta \). Now suppose that the first period capital requirement doesn’t bind. Let \( \lambda \) denote the Langrange multiplier on the second period capital requirement. The corporation’s first-order condition reads

\[
0 = \Pi'(p^*_1; c_1) + \lambda \Pi'(p^*_1; c_1) \tag{Corporation}
\]

Since \( \lambda \geq 0 \), I have that \( \Pi'(p^*_1; c_1) = 0 \). \( p^*_1 \) doesn’t depend on \( \lambda \) and \( \lambda \) is the only variable tying \( p^*_1 \) to \( \delta \), so \( p^*_1 \) doesn’t depend on \( \delta \) either. In fact, \( p^*_1 = p^m(c_1) \). The corporation’s first period price is constant with respect to the technology shock. Increasing its internal funds (B) is consistent with maximizing profit (A).

Now that I have a benchmark in place, let’s turn our attention back to the cooperative. Before proceeding, I segregate cooperatives into those that are *adequately capitalized* and those
that are poorly capitalized. This segregation has no economic importance and is meant only to ease the exposition.

**Definition 3.3.1.** The cooperative is adequately capitalized if \( m_1 > kD(c_1 - k) \) and poorly capitalized otherwise.\(^{11}\)

The corporation’s pricing problem is time-separable. It sets today’s price to maximize today’s profit, without regard for tomorrow’s growth opportunity. This is always true, regardless of the state of its internal funds. With this result in mind, the following result about the cooperative is surprising.

**Proposition 3.3.2.** If the cooperative is adequately capitalized, then for sufficiently large technology shocks \( \delta > 1 - P(m_1/k)/c_1 \), its first period price is strictly increasing in the technology shock. Its first period price is constant for all other technology shocks. If the cooperative is poorly capitalized, then its first period price is non-decreasing in the technology shock.

Proposition 3.3.2 says that the cooperative adjusts today’s price according to tomorrow’s growth opportunity. Its pricing problem is *not* time-separable. Today’s prices depend on information about the future.

The first-order conditions are again informative. Let \( \lambda \) be as before. The cooperative’s first-order condition reads:

\[
0 = \underbrace{S'(p_1^*)}_{A} + \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}_{B} + \lambda \underbrace{\Pi'(p_1^*; c_1)}}_{(Cooperative)}

(recall that \( S \) is the consumer surplus). Now, \( p_1^* \) depends on \( \lambda \) and so \( p_1^* \) depends on \( \delta \). Increasing its internal funds (\( \text{B} \)) is *inconsistent* with maximizing first period total surplus (\( \text{A} \)), although it is consistent with maximizing second period total surplus. Unlike the corporation, the cooperative faces a trade-off. Increasing its internal funds creates slack in the second period capital requirement—which makes growth possible—but it also creates a deadweight loss.

The proof is slightly more involved than the argument that I’ve just made, but the result is deep. I said that \( S \) is the consumer surplus, but any function that depends on \( p_1 \) will tie the

\(^{11}\)I have to assume that the minimum equity-to-sales ratio is sufficiently small (\( k < c_1 \)).
first period price to the technology shock. Put differently, the cooperative’s first period price will depend on the technology shock, even if I’ve misspecified its objective.

### 3.3.2 Growth, Pricing and Leverage

Propositions 3.3.1 and 3.3.2 suggest that while the corporation’s problem is time-separable, the cooperative’s is not. The cooperative adjusts today’s prices according to tomorrow’s growth opportunity. The result is theoretically interesting, but it doesn’t lend itself to predictions about the real world.

In what follows, I show that the cooperative’s growth is positively related to prices and negatively related to leverage.

In this model, the technology shock is the natural source of variation, so I index all variables by $\delta$. Define growth to be

$$\text{Growth}(\delta) = \gamma(\delta) := \frac{D(p^*_2(\delta)) - D(p^*_1(\delta))}{D(p^*_1(\delta))}. \quad (3.7)$$

It is the percentage by which sales increase between the first and second periods. If, for example, $D$ is the demand for insurance policies, then Growth is policy growth. If $D$ is the demand for loans, then Growth is loan growth.

Define the profit margin to be

$$\text{ProfitMargin}(\delta) = \mu(\delta) := p^*_1(\delta) - c_1. \quad (3.8)$$

In insurance and banking, ProfitMargin is the insurance margin and the net interest margin respectively. Note that $c_1$ is fixed, so ProfitMargin will vary one-for-one with $p^*_1$.

In the model, the natural measure of leverage is the ratio of equity-to-sales. If $D$ is the demand for insurance policies, then the equity-to-sales ratio is the solvency ratio. If $D$ is the demand for loans—as it is in my data work—the equity-to-sales ratio is the equity-to-loans ratio or the capital ratio. I will use the banking nomenclature. Define the capital ratio to be

$$\text{CapitalRatio}(\delta) = \kappa(\delta) := \frac{m_1}{D(p^*_1(\delta))}. \quad (3.9)$$
Leverage and the capital ratio are inverses: the higher the leverage, the lower the capital ratio and the higher the capital ratio, the lower the leverage.

The first implication of the model is that the cooperative’s growth is increasing with its profit margin.

**Proposition 3.3.3** (Cooperative). If the cooperative is adequately capitalized, then for sufficiently large technology shocks \((\delta > 1 - P(m_1/k)/c_1)\), its growth is strictly increasing with its profit margin. Its profit margin is constant for all other technology shocks. If the cooperative is poorly capitalized, then its growth is non-decreasing with its profit margin.

*Proof.* Fix \(\delta > 0\). Consider first the adequately capitalized cooperative. If \(\delta > 1 - P(m_1/k)/c_1\), then there is an open interval of technology shocks around \(\delta\) in which the second period capital requirement binds, but the first period capital requirement does not (see Lemma 3.7.12 in the appendix). The second period capital requirement reads

\[
kD(p_2^*(\delta)) = m_1 + \Pi(p_1^*(\delta); c_1) = m_1 + (p_1^*(\delta) - c_1)D(p_1^*(\delta)). \tag{3.10}
\]

Rewriting it in terms of growth and the profit margin, I obtain

\[
\gamma(\delta) = \frac{m_1}{kD(\mu(\delta) + c_1)} + \frac{\mu(\delta)}{k} - 1. \tag{3.11}
\]

Differentiating \(\gamma\) with respect to \(\delta\), I obtain

\[
\frac{d\gamma}{d\delta} = -\frac{m_1D'(\mu + c_1)}{kD(\mu + c_1)^2} \cdot \frac{d\mu}{d\delta} + \frac{1}{k} \cdot \frac{d\mu}{d\delta}. \tag{3.12}
\]

Proposition 3.3.2 yields

\[
\frac{d\mu}{d\delta} = \frac{dp_1^*}{d\delta} > 0 \tag{3.13}
\]

so I can safely write

\[
\frac{d\gamma}{d\mu} = \frac{d\gamma}{d\delta} \cdot \frac{d\mu}{d\delta} = -\frac{m_1D'(\mu + c_1)}{kD(\mu + c_1)^2} + \frac{1}{k} > 0. \tag{3.14}
\]

as desired. If \(\delta \leq 1 - P(m_1/k)/c_1\), then neither the first nor the second period capital requirements bind (see, again, Lemma 3.7.12 in the appendix). In fact, \(p_1^* = c_1\) so the profit margin is constant.
Suppose that the cooperative is poorly capitalized. I can no longer appeal to Lemma 3.7.12, but the arguments made for the adequately capitalized cooperative can be carried over to the poorly capitalized cooperative. If the first period capital requirement binds, then \( p_1^* = P(m_1/k) \). If neither the first nor the second period capital requirements bind, then \( p_1^* = c_1 \). In either case, the first period price is constant with respect to the technology shock—and hence the profit margin is constant with respect to the technology shock. If the second period capital requirement binds, but the first period capital requirement does not, then I can appeal to the argument for the adequately capitalized cooperative to conclude that growth is strictly increasing with the profit margin.

In section 3.4, I will take Proposition 3.3.3 to the data. Suppose I ran the regression

\[
Growth_t = \alpha + \beta ProfitMargin_t + \epsilon_t.
\]

Proposition 3.3.3 says that either \( ProfitMargin_t \) is constant—in which case the regression suffers from multicollinearity—or \( Growth_t \) is positively correlated with \( ProfitMargin_t \), in which case \( \beta \) will load with a positive sign.

As a benchmark, note the following result.

**Proposition 3.3.4.** The corporation’s profit margin is constant.

*Proof.* This is a direct consequence of Proposition 3.3.1.

The second implication of the model is that the cooperative’s growth is decreasing with its leverage (equivalently, increasing with its capital ratio). The intuition is a bit subtle. I know from Proposition 3.3.2 that the cooperative’s first period price is increasing in the technology shock. If the cooperative observes a growth opportunity but doesn’t have ample equity, it will raise its first period price, make a profit, retain its earnings and use them to support growth. By raising its first period price, it lowers its first period sales, which lowers its first period leverage (equivalently, raises its first period capital ratio). If the cooperative is a bank—for example—then expanding its first period net interest margin means contracting its first period loan book. It will appear less levered in the first period.
Proposition 3.3.5. If the cooperative is adequately capitalized, then for sufficiently large technology shocks \((\delta > 1 - P(m_1/k)/c_1)\), its growth is strictly increasing with its capital ratio. Its capital ratio is constant for all other technology shocks. If the cooperative is poorly capitalized, then its capital ratio is non-decreasing with its capital ratio.

Proof. The proof follows the proof of Proposition 3.3.3. Consider first the adequately capitalized cooperative. Rewriting the second period capital requirement in terms of growth and the capital ratio, I obtain

\[
\gamma(\delta) = \frac{\kappa(\delta)}{k} + \frac{P(m_1/\kappa)}{k} - 1. \tag{3.16}
\]

Differentiating \(\gamma\) with respect to \(\delta\), I obtain

\[
\frac{d\gamma}{d\delta} = \frac{1}{k} \cdot \frac{d\kappa}{d\delta} - \frac{P'(m_1/\kappa)}{k\kappa^2} \cdot \frac{d\kappa}{d\delta} \tag{3.17}
\]

Proposition 3.3.2 yields

\[
\frac{d\kappa}{d\delta} = -\frac{m_1 D'(p_1^*)}{D(p_1^*)^2} \cdot \frac{dp_1^*}{d\delta} > 0 \tag{3.18}
\]

so I can safely write

\[
\frac{d\gamma}{d\kappa} = \frac{d\gamma}{d\delta} \bigg/ \frac{d\kappa}{d\delta} = \frac{1}{k} - \frac{P'(m_1/\kappa)}{k\kappa^2} > 0 \tag{3.19}
\]

as desired. If \(\delta \leq 1 - P(m_1/k)/c_1\), then neither the first nor the second period capital requirements bind (see, again, Lemma 3.7.12 in the appendix). \(p_1^* = c_1\) so the capital ratio is constant. The argument for the poorly capitalized cooperative follows the one contained in the proof of Proposition 3.3.3.

Again, I can think about Proposition 3.3.5 in terms of a regression:

\[
\text{Growth}_t = \alpha + \beta \text{CapitalRatio}_t + \epsilon_t. \tag{3.20}
\]

Proposition 3.3.3 says that either \(\text{CapitalRatio}_t\) is constant—in which case the regression suffers from multicollinearity—or \(\text{Growth}_t\) is positively correlated with \(\text{CapitalRatio}_t\), in which case \(\beta\) will load with a negative sign.

As a benchmark, note the following result.
Proposition 3.3.6. The corporation’s capital ratio is constant.

The profit margin and capital ratio emerge as the key determinants of the cooperative’s growth. It’s natural to ask how growth covaries with both the profit margin and the capital ratio. Doing so, I obtain the following elegant result:

Proposition 3.3.7. If the cooperative is adequately capitalized, then for sufficiently large technology shocks \((\delta > 1 - P(m_1/k)/c_1)\), the cooperative’s growth can be written as a linear function of its profit margin and capital ratio.

Proof. The proof follows the proof of Proposition 3.3.3. Rewriting the second period capital requirement in terms of growth, the profit margin and the capital ratio, I obtain

\[
\gamma(\delta) = k^{-1} \mu(\delta) + k^{-1} \kappa(\delta) - 1.
\]  

(3.21)
as desired.

At first blush, it might seem that Propositions 3.3.3, 3.3.5, and 3.3.7 are just restatements of the second period capital requirement. This is not the case. The corporation’s second period capital requirement may bind, but variation in its profit margin and its capital ratio will not translate into variation in its growth.

3.4 Applications

In this section, I discuss customer owned firms to which the theory can be applied. For each type of firm, I assess the extent to which Assumptions 3.3.1 (market power), 3.3.2 (internal finance) and 3.3.3 (capital requirements) are satisfied. In the case of credit unions, I document two stylized facts that are consistent with Propositions 3.3.3 and 3.3.5.

3.4.1 Credit Unions

Credit unions are savers’ cooperatives. They are owned by the people from whom they accept deposits, known as members. Like any other bank, credit unions sell deposits to savers and buy
loans from borrowers. Borrowers are typically nominal savers and enjoy control rights comparable to those of savers.\textsuperscript{12} Members who borrow are offered low rates on loans while members who save are offered high rates on deposits. Members’ control rights are similar to those of the shareholders of large, publicly held corporations. The Federal Credit Union Act requires boards “...to be elected annually by and from the members....” In the event of a voluntary liquidation, credit unions distribute their equity to their members through a liquidating dividend.

US credit unions hold $1.1 trillion in assets.\textsuperscript{13} They hold 9.5\% of consumer savings ($971.2 billion) and originate 10.3\% of installment credit ($345.6 billion).\textsuperscript{14} They originate 11\% of home loans and 17\% of auto loans.\textsuperscript{15} Over 100 million US adults are members of a credit union.\textsuperscript{16}

The theory proposed in this paper assumes that internal funds—amassed through earnings retention—are the cooperative’s only source of equity. Credit unions satisfy this assumption well. The Federal Credit Union Act states that credit unions “...do not issue capital stock...[and]...must rely on retained earnings to build net worth.”

US credit unions face capital requirements under federal law. They must maintain a capital asset ratio of 6\%. They are also evaluated using more contemporary Risk Based Capital measures.

Because of data availability, I use the US credit union industry as a laboratory to test the theory’s predictions. Data on credit unions comes from the NCUA call reports; data on traditional banks comes from the FDIC call reports. Observations are industry aggregates for the first quarter of 2003 through the last quarter of 2012. Figure 3.1 plots loan growth against the net interest margin. For both samples I plot a regression line. The point estimates and the standard errors can be found in Table 3.1. For credit unions, loan growth is positively and significantly related

\textsuperscript{12} To illustrate the point, consider the following allegory. Bonnie and Clyde choose to become members of a credit union. They each make a minimum deposit of—say—$5. Credit unions operates under a \textit{one-member-one-vote rule}. So Bonnie receives the right to exactly one vote and Clyde receives the right to exactly one vote. After opening their accounts, Bonnie deposits $9,995 and Clyde borrows $10,005. Bonnie is a net $10,000 depositor and Clyde is a net $10,000 borrower. Bonnie could care less about the rate on loans and Clyde could care less about the rate on deposits, but each has one vote just the same.

\textsuperscript{13} 2017 Statistical Report. World Council of Credit Unions.
\textsuperscript{14} Credit Union Report Year-End 2014. Credit Union National Association.
\textsuperscript{16} 2017 Statistical Report. World Council of Credit Unions.
to the net interest margin, a finding consistent with Proposition 3.3.3. I’m unable to document a relationship for banks. This finding is either consistent with Proposition 3.3.4 or a symptom of high variance.

A similar story emerges for leverage. Figure 3.1 plots loan growth against the industry capital ratio. Again, point estimates and the standard errors can be found in Table 3.1. For credit unions, loan growth is positively and significantly related to the capital ratio (negatively related to leverage), a finding consistent with Proposition 3.3.5. I’m unable to document a relationship for banks. It is unclear whether this result is consistent with Proposition 3.3.6.

Finally, I attempt to test Proposition 3.3.7. Neither the net interest margin nor the capital ratio load. The two covariates have a correlation of .84 (compare with .46 for banks) so it is likely that the regression suffers from multicollinearity.

**3.4.2 Mutual Insurance Companies**

Mutual insurance companies are policyholders’ cooperatives. They are owned by the people to whom they underwrite insurance policies, also known as members. Members can vote for directors and have a claim to the company’s equity upon dissolution. Mutual insurance companies can—and do—pay non-obligatory dividends.\(^{17}\)

In 2012, US mutual insurance companies enjoyed a 34.5% market share (28.7% for life and 39.2% for non-life); globally, mutual insurance companies enjoyed a 26.7% market share (25.0% for life and 28.9% for non-life). The two largest mutual life insurance companies are New York Life (4.8% market share) and MassMutual (2.8% market share) while the two largest mutual home/auto insurance companies are State Farm (20.3% for home, 18.7% for auto) and Liberty Mutual (6.6% for home, 5.0% for auto).\(^{18}\)

Like their credit union cousins, mutual insurance companies rely heavily on retained earnings for equity.\(^{19}\) They are state regulated. Most of the statutory capital requirements specify the

---


amount of initial equity needed to be chartered. That being said, state regulators use Risk Based Capital measures as part of their ongoing oversight role. The National Association of Insurance Commissioners (NAIC) has developed “model laws” to unify insurance regulation through the US.

3.4.3 The Farm Credit System

The Farm Credit System is a network of cooperative banks known as Agricultural Credit Associations (ACAs). ACAs are borrowers’ cooperatives. They are owned by the people to whom they make loans. Unlike credit unions, ACAs don’t have saver-members. Members finance their loans by issuing bonds through Farmer Mac. ACA members—like credit union and mutual insurance company members—can vote for directors.

In 2014, the Farm Credit System was responsible for 42.5% ($135 billion) of total farm debt. The Farm Credit System is regulated by the Farm Credit Administration, which has adopted capital requirements “comparable to the Basel III framework.” ACAs are unique among cooperative banks in that they require borrowers to purchase “at-risk stock.” This practice violates Assumption 3.3.2 and should attenuate the effects predicted by Propositions 3.3.3 and 3.3.5. That being said, it does not seem that borrower stock completely covers ACA equity needs. In 2014, ACAs financed sizable 17% of their assets with retained earnings.

3.4.4 Rural Electricity Cooperatives

This paper has focused on cooperatives in the financial sector. Cooperatives appear in other industries as well. One of the most interesting and economically important examples comes from rural electricity provision. Rural electricity cooperatives supply 13% of US electricity (kllowatt-hours) and serve 42 million Americans. Rural electricity cooperatives don’t face capital requirements like banks do, but equity still seems to be important. In 2010, the industry maintained a capital-asset ratio of 30%.

---

21 2014 Annual Report on the Farm Credit System, Farm Credit Administration.
3.5 Conclusion

Despite their economic significance, customer owned firms have received little theoretical or empirical attention. This paper presents a simple theory of internal finance for the customer owned firm. For lack of a better phrase, these firms “…don’t do it for the money”[100]. It shows that a customer friendly objective ties together the firm’s growth, pricing and capital structure. It shows that high sales growth is achieved through high prices and low leverage. Underlying this result is a dynamic pricing problem. The price of the firm’s good depends on future growth opportunities. These results stand in stark contrast to traditional theories of corporate finance and industrial organization.

3.6 Appendix A: The Cooperative’s Objective

This section contains an informal derivation of the cooperative’s objective. It is by no means the only derivation. For simplicity and clarity, I use a random utility framework in which members have linear utility and random valuations for the cooperative’s good. Suppose there is a unit mass of members indexed by \( s \) with unit demand for the good. Member \( s \) has valuation \( v_{st} \sim F(v) \) in period \( t \), where \( F \) is a twice continuously differentiable distribution. \( F \) satisfies \( F'' < 0 \). Moreover, its elasticity, \( \epsilon_F(v) \), and curvature, \( \sigma_F(v) \), satisfy \( \sigma_F(v) < \min\{2\epsilon_F(v), 1 + \epsilon_F(v)\} \). Valuations are independent across members and across time. The cooperative operates for \( T \) periods, after which it dissolves.

I will focus on the case in which the profits made from a member’s purchase are rebated back to her pro-rata through the liquidating dividend. Consider member \( s \). If she purchases the good, then she obtains utility \( v_{st} - p_t \) from her purchase and \( \beta T (p_t - c_t) \) from her dividend. Assumption 3.3.2 implies that \( p_t \geq c_t \). If \( p_t < c_t \), then the cooperative would be raising equity from members upon dissolution. Her net utility is \( v_{st} - c_t \). If she doesn’t purchase the good, then she obtains zero utility. Therefore, she purchases if \( v_{st} > (1 - \beta)p_t + \beta c_t \equiv \tilde{p}_t \) and doesn’t purchase otherwise.

Member \( s \) buys from the cooperative if and only if \( v_{st} \geq \tilde{p}_t \). The expected lifetime utility of
members $s$, as a function of prices, is

$$E[u_s(p)] := \sum_{t=1}^{T} \left( (v_t - p_t)1_{v_t \geq p_t} + \beta^t (p_t - c_t)1_{v_t \geq p_t} \right)$$  \hspace{1cm} (3.22)$$

Put $D = 1 - F$. Members $s$ can be identified by her valuations: $(v_1, v_2, \ldots, v_T) := v$.

Aggregate member utility is

$$\int_{R_{++}^T} u(p; v) dF(v_1) \ldots dF(v_T) = \sum_{t=1}^{T} \left( \int_{0}^{\infty} (v_t - p_t)1_{v_t \geq p_t} dF(v_t) + \int_{0}^{\infty} (p_t - c_t)1_{v_t \geq p_t} dF(v_t) \right)$$  \hspace{1cm} (3.23)$$

$$= \sum_{t=1}^{T} \left( \int_{p_t}^{\infty} dF(v_t) + \int_{p_t}^{\infty} dF(v_t) \right)$$  \hspace{1cm} (3.24)$$

$$= \sum_{t=1}^{T} \left( \int_{0}^{\infty} D(v_t) dv_t + (p_t - c_t)D(p_t) \right)$$  \hspace{1cm} (3.25)$$

which is simply a time-separable, $T$-period extension of the one-period objective postulated by [30].

### 3.7 Appendix: Proofs

This section contains proofs omitted from the text.

#### 3.7.1 A Few Definitions

As they will appear frequently, put

$$\bar{p}_1 := \max\{P(m_1/k), p^m(c_1)\},$$  \hspace{1cm} (3.26)$$

$$\underline{p}_1 := \max\{P(m_1/k), c_1\},$$  \hspace{1cm} (3.27)$$

$$\bar{p}_2 := \max\{P(m_1/k + \Pi(\bar{p}_1; c_1)/k), p^m(c_2)\},$$  \hspace{1cm} (3.28)$$

$$\underline{p}_2 := \max\{P(m_1/k + \Pi(\underline{p}_1; c_1)/k), c_2\}.$$  \hspace{1cm} (3.29)$$

It will transpire that solutions to both the cooperative’s problem and the corporation’s problem live in the box $B := [\underline{p}_1, \bar{p}_1] \times [\underline{p}_2, \bar{p}_2]$. That $\underline{p}_1 \leq \bar{p}_1$ follows immediately from the fact that $p^m(c_1) > c_1$. 


That $p_2 \leq \bar{p}_2$ will follow from Lemma 3.7.4. Put

$$g^1(p) := m_1 - kD(p_1), \quad (3.30)$$

$$g^2(p) := m_1 + \Pi(p_1; c_1) - kD(p_2). \quad (3.31)$$

In terms of $g^1$ and $g^2$, the constraint set for both the cooperative's problem and the corporation's problem is given by

$$C := \{ p \in \mathbb{R}^2_{++} \mid g^1(p) \geq 0, g^2(p) \geq 0 \}. \quad (3.32)$$

The cooperative's problem is equivalent to

$$\max_{p \in C} f^{S+\Pi}(p) := S(p_1) + \Pi(p_1; c_1) + S(p_2) + \Pi(p_2; c_2) \quad (3.33)$$

and the corporation's problem is equivalent to

$$\max_{p \in C} f^\Pi(p) := \Pi(p_1; c_1) + \Pi(p_2; c_2). \quad (3.34)$$

I will frequently use the definitions $\delta := (c_1 - c_2)/c_1$ and $p^{na}(c) := \arg\max_p \Pi(p; c)$ and the assumptions $A1$ – $A4$. Definitions and assumptions not contained in this appendix can be found in the body of the text.

### 3.7.2 General Results

**Lemma 3.7.1.** $\Pi(\bullet; c)$ is strictly quasiconcave.

**Proof.** If $p > 0$ is such that $0 = \Pi'(p; c) = D(p) + (p - c)D'(p)$, then

$$\Pi''(p; c) = 2D'(p) + (p - c)D''(p)$$

$$= 2D'(p) - \frac{D(p)D''(p)}{D'(p)} \quad (3.35)$$

$$= D'(p) \left( 2 - \frac{D(p)D''(p)}{D'(p)^2} \right) \quad (3.36)$$

$$= D'(p) \left( 2 - \frac{\sigma(p)}{\epsilon(p)} \right) > 0 \quad (3.37)$$

where the last line follows by (A1). I conclude that $\Pi(\bullet; c)$ is strictly quasiconcave.
Lemma 3.7.2. \( p^m(c) \) exists and is unique.

Proof. \( \Pi'(c; c) = D(c) > 0 \), so there is some \( p_0 > c \) such that \( \Pi(p_0; c) > 0 = \Pi(c; c) \). Now \( \lim_{p \to \infty} pD(p) = 0 \) (A4) and \( \lim_{p \to \infty} D(p) = 0 \) (a direct consequence of A3), so \( \lim_{p \to \infty} \Pi(p; c) = 0 \): there is some \( p_1 > c \) such that \( \Pi(p; c) < \Pi(p_0; c) \) for all \( p > p_1 \). \( p_m(c) \), if it exists, is an element of \([c, p_1]\). But \( \Pi(p; c) \) is continuous, so it achieves \( p_m(c) \) on \([c, p_1]\) (Extreme Value Theorem). The uniqueness of \( p_m(c) \) follows by the quasiconcavity of \( \Pi(\bullet; c) \).

Lemma 3.7.3. Let \( c > 0 \). \( c \) is the unique maximizer of \( S + \Pi(\bullet; c) \).

Proof. Let \( p > 0, p \neq c \). Then

\[
S(p) + \Pi(p; c) = \int_D(s)ds + (p - c)D(p)
\]

\[
= \int_c^p D(s)ds + \int_c^p D(s)ds + (p - c)D(p)
\]

\[
= \int_c^p D(s)ds - \int_c^p (s - c)D'(s)ds
\]

\[
< \int_c^p D(s)ds = S(c) + \Pi(c; c).
\]

Use integration by parts to obtain the third line; use the fact that \( D' < 0 \) to obtain the fourth.

Lemma 3.7.4. If \( p_1 \leq p_1 \leq \bar{p}_1 \), then \( \Pi(p_1; c_1) \geq \Pi(p_1; c_1) \).

Proof. If \( p_1 = c_1 \), then \( p_1 \geq p_1 = c_1 \) and hence \( \Pi(p_1; c_1) \geq 0 = \Pi(c_1; c_1) = \Pi(p_1; c_1) \). If \( p_1 = p_1 = P_m(c_1) \), then the result follows immediately. If \( p_1 = P_m(c_1) \) and \( p_1 = P^m(c_1) \), then \( p_1 \leq p_1 \leq p_m(c_1) \) and hence

\[
\Pi(p_1; c_1) \geq \min\{\Pi(p_1; c_1), \Pi(p_m(c_1); c_1)\} = \Pi(p_1; c_1)
\]

by the strict quasiconcavity of \( \Pi(\bullet; c_1) \) and the definition of \( p^m(c_1) \).

Lemma 3.7.5. Let \( p \in C, t \in \{1, 2\} \). If \( p_t > \bar{p}_t \), then \( \Pi(p_t; c_t) > \Pi(p_t; c_t) \).
Proof. If $\bar{p}_t = p^m(c_t)$, then $p_t > \bar{p}_t = p^m(c_t)$ and hence $\Pi(\bar{p}_t; c_t) = \Pi(p^m(c_t); c_t) > \Pi(p_t; c_t)$. If $\bar{p}_t \neq p^m(c_t)$, then $p_t > \bar{p}_t > p^m(c_t)$ and hence

$$\Pi(\bar{p}_t; c_t) > \min\{\Pi(p_t; c_t), \Pi(p^m(c_t); c_t)\} = \Pi(p_t; c_t) (3.44)$$

by the strict quasiconcavity of $\Pi(\bullet; c_t)$ and the definition of $p^m(c_t)$. ■

Lemma 3.7.6. Let $p \in C$, $t \in \{1, 2\}$. If $p_t > \bar{p}_t$, then $S(\bar{p}_t) + \Pi(\bar{p}_t; c_t) > S(p_t) + \Pi(p_t; c_t)$.

Proof. Use Lemma 3.7.5 and the fact that $S$ is strictly decreasing. ■

Lemma 3.7.7. Let $p \in C$, $t \in \{1, 2\}$. If $p_t < \underline{p}_t$, then $p_1 = c_t$.

Proof. Suppose $p_t \neq c_t$. If $t = 1$, then $p_1 < \underline{p}_t = P(m_1/k) \leq p_1$ (recall that $p \in C$). I have a contradiction. If $t = 2$, then $p_2 < \underline{p}_2 = P(m_1/k + \Pi(\bar{p}_1; c_1)/k)$. Now $p \in C$, so $p_1 \geq P(m_1/k)$ and $p_2 \geq P(m_1/k + \Pi(p_1; c_1)/k)$. If $\bar{p}_1 = p^m(c_1)$, then $\Pi(\bar{p}_1; c_1) = \Pi(p^m(c_1); c_1) \geq \Pi(p_1; c_1)$. If $\bar{p}_1 \neq p^m(c_1)$, then $p_1 \geq P(m_1/k) = \bar{p}_1 > p^m(c_1)$ and hence

$$\Pi(\bar{p}_1; c_1) \geq \min\{\Pi(p_1; c_1), \Pi(p^m(c_1); c_1)\} = \Pi(p_1; c_1) (3.45)$$

by the strict quasiconcavity of $\Pi(\bullet; c_1)$ and the definition of $p^m(c_1)$. Therefore,

$$p_2 < \underline{p}_2 = P(m_1/k + \Pi(\bar{p}_1; c_1)/k) \leq P(m_1/k + \Pi(p_1; c_1)/k) \leq p_2. (3.46)$$

I again have a contradiction. ■

Lemma 3.7.8. Let $p \in C$, $t \in \{1, 2\}$. If $p_t < \underline{p}_t$, then $\Pi(\underline{p}_t; c_t) > \Pi(p_t; c_t)$.

Proof. $\underline{p}_t = c_t$ (Lemma 3.7.7) so $p_t < \underline{p}_t = c_t < p^m(c_t)$ and hence

$$\Pi(p_t; c_t) > \min\{\Pi(p_t; c_t), \Pi(p^m(c_t); c_t)\} = \Pi(p_t; c_t) (3.47)$$

by the strict quasiconcavity of $\Pi(\bullet; c_t)$ and the definition of $p^m(c_t)$. ■

Lemma 3.7.9. Let $p \in C$, $t \in \{1, 2\}$. If $p_t < \underline{p}_t$, then $S(p_t) + \Pi(p_t; c_t) > S(p_t) + \Pi(p_t; c_t)$.

Proof. $\underline{p}_t = c_t$ (Lemma 3.7.7) so $p_t < \underline{p}_t = c_t$. The result follows by Lemma 3.7.3. ■
Lemma 3.7.10. Let $p^* \in C$ be a solution to either the cooperative’s problem or the corporation’s problem. The constraint qualification holds at $p^*$.

Proof. The derivative of $(g^1, g^2)$ has full rank at $p^*$:

\[
\begin{vmatrix}
g_1^1(p^*) & g_1^2(p^*) \\
g_1^2(p^*) & g_2^2(p^*)
\end{vmatrix}
= \begin{vmatrix}
-kD'(p_1^*) & 0 \\
\Pi'(p_1^*; c_1) & -kD'(p_2^*)
\end{vmatrix}
= k^2 D'(p_1^*) D'(p_2^*) > 0
\] (3.48)

so the constraint qualification holds.

Lemma 3.7.11. Let $p^* \in C$ be a solution to either the cooperative’s problem or the corporation’s problem. If $g^1(p^*) = 0$, then $m_1 < D(c_1 - k)$.

Proof. If $g^1(p^*) = 0$, then $m_1 = kD(p_1^*)$. $g^2(p^*) \geq 0$ implies that

\[
0 < kD(p_2^*) \leq m_1 + \Pi(p_1^*; c_1)
\] (3.49)

\[
= kD(p_1^*) + (p_1^* - c_1)D(p_1^*)
\] (3.50)

\[
= (k + p_1^* - c_1)D(p_1^*),
\] (3.51)

which implies that $p_1^* > c_1 - k$. Hence, $m_1 = kD(p_1^*) < kD(c_1 - k)$.

3.7.3 The Cooperative

Proof of Lemma 3.3.1. I will start by showing that solutions, if they exist, live in the box $B$: for each $p \in C \cap B^c$, there is a $\bar{p} \in C \cap B$ such that $f^{\Pi+S}(\bar{p}) > f^{\Pi+S}(p)$. Fix $p \in C \cap B^c$. Each element of $B$ is strictly positive in each of its coordinates. Moreover, $g^1(p, \bullet) \geq 0$, $g^2(\bar{p}, \bullet) \geq 0$ and $g^1(p_1, \bullet) \geq 0$ by construction. Consider each of the following cases:

1. Suppose $p_1 > \bar{p}_1$ and $p_2 > \bar{p}_2$. Observe that $(\bar{p}_1, \bar{p}_2) \in C \cap B$:

\[
g^2(\bar{p}_1, \bar{p}_2) = m_1 + \Pi(\bar{p}_1; c_1) - kD(\bar{p}_2)
\] (3.52)

\[
\geq m_1 + \Pi(\bar{p}_1; c_1) - kD(P(m_1/k + \Pi(\bar{p}_1)/k))
\] (3.53)

\[
= \Pi(\bar{p}_1; c_1) - \Pi(p_2^*; c_1) \geq 0
\] (3.54)

(Lemma 3.7.4). Apply Lemma 3.7.6 twice to obtain $f^{\Pi+S}(\bar{p}_1, \bar{p}_2) > f^{\Pi+S}(p_1, p_2)$. 


(2) Suppose \( p_1 > p_\bar{1} \) and \( p_2 < p_\bar{2} \). Observe that \((p_1, p_2) \in C \cap B:\)

\[
g^2(p_1, p_2) = m_1 + \Pi(p_1; c_1) - kD(p_2) \geq m_1 + \Pi(p_1; c_1) - kD(P(m_1/k + \Pi(p_1; c_1)/k)) = 0.
\]

Apply Lemmas 3.7.6 and 3.7.9 to obtain \( f^{\Pi+S}(\bar{p}_1, p_2) > f^{\Pi+S}(p_1, p_2). \)

(3) Suppose \( p_1 < p_\bar{1} \) and \( p_2 > p_\bar{2} \). Observe that \((p_1, p_2) \in C \cap B:\)

\[
g^2(p_1, p_2) = m_1 + \Pi(p_1; c_1) - kD(p_2) \geq m_1 + \Pi(p_1; c_1) - kD(P(m_1/k + \Pi(p_1; c_1)/k)) = 0.
\]

Apply Lemmas 3.7.9 and 3.7.6 to obtain \( f^{\Pi+S}(p_1, \bar{p}_2) > f^{\Pi+S}(p_1, p_2). \)

(4) Suppose \( p_1 < p_\bar{1} \) and \( p_2 < p_\bar{2} \). Observe that \((p_1, p_2) \in C \cap B:\)

\[
g^2(p_1, p_2) = m_1 + \Pi(p_1; c_1) - kD(p_2) \geq m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0
\]

(Lemma 3.7.8). Apply Lemma 3.7.9 twice to obtain \( f^{\Pi+S}(p_1, p_2) > f^{\Pi+S}(p_1, p_2). \)

(5) Suppose \( p_1 > p_\bar{1} \) and \( p_2 \leq p_\bar{2} \). Observe that \((p_1, p_2) \in C \cap B:\)

\[
g^2(p_1, p_2) = m_1 + \Pi(p_1; c_1) - kD(p_2) \geq m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0
\]

(Lemma 3.7.5). Apply Lemma 3.7.6 to obtain \( f^{\Pi+S}(p_1, p_2) > f^{\Pi+S}(p_1, p_2). \)

(6) Suppose \( p_1 < p_\bar{1} \) and \( p_2 \leq p_\bar{2} \). Observe that \((p_1, p_2) \in C \cap B:\)

\[
g^2(p_1, p_2) = m_1 + \Pi(p_1; c_1) - kD(p_2) \geq m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0
\]

(Lemma 3.7.8). Apply Lemma 3.7.9 to obtain \( f^{\Pi+S}(p_1, p_2) > f^{\Pi+S}(p_1, p_2). \)
(7) Suppose $p_1 \leq p_1 \leq \bar{p}_1$ and $p_2 > \bar{p}_2$. Observe that $(p_1, \bar{p}_2) \in C \cap B$:

$$g^2(p_1, \bar{p}_2) = m_1 + \Pi(p_1; c_1) - kD(\bar{p}_2)$$

$$\geq m_1 + \Pi(p_2; c_1) - kD(m_1/k + \Pi(p_2; c_1)/k)) \tag{3.66}$$

$$= \Pi(p_1; c_1) - \Pi(p_2; c_1) \geq 0 \tag{3.67}$$

(Lemma 3.7.4). Apply Lemma 3.7.6 to obtain $f^{II+S}(p_1, \bar{p}_2) > f^{II+S}(p_1, p_2)$.

(8) Suppose $p_1 \leq p_1 \leq \bar{p}_1$ and $p_2 < \bar{p}_2$. Observe that $(p_1, p_2) \in C \cap B$:

$$g^2(p_1, p_2) = m_1 + \Pi(p_1; c_1) - kD(p_2)$$

$$> m_1 + \Pi(p_1; c_1) - kD(p_2) = g^2(p_1, p_2) \geq 0. \tag{3.69}$$

Apply Lemma 3.7.9 to obtain $f^{II+S}(p_1, p_2) > f^{II+S}(p_1, p_2)$.

The solution, if it exists, must be an element of $B$. Put $C := C \cap B$. $C$ is non-empty because $(\bar{p}_1, \bar{p}_2) \in C$. $C$ is closed because it is defined by weak inequalities. $C$ is bounded because it lives in the box $B$. So $C$ is compact. Now $f^{S+II}$ is continuous. The Extreme Value Theorem tells us that $f^{S+II}$ attains its maximum on $C$, which I’ve shown is its maximum on $C$. I have a solution, the objective and constraints are twice continuously differentiable and I know that the constraint qualification holds (Lemma 3.7.10). I appeal to Kuhn-Tuckers’ Theorem: there are multipliers satisfying the Kuhn-Tucker conditions at the solution.

Lemma 3.7.12. Suppose that the cooperative is adequately capitalized. Then its first period capital requirement does not bind. Its second period capital requirement binds if and only if $\delta > 1 - P(m_1/k)/c_1$.

Proof. Let $p^* \in C$ be a solution to the cooperative’s problem (Lemma 3.3.1 guarantees that $p^*$ exists). Because the cooperative is adequately capitalized, I have that $m_1 > kD(c_1 - k)$ and so Lemma 3.7.11 guarantees that $g^1(p_1^*) > 0$. Put differently, the first period capital requirement doesn’t bind. If $\delta > 1 - P(m_1/k)/c_1$, then I have that $m_1 < kD((1 - \delta)c_1) = kD(c_2)$ and so Lemma
3.7.3 guarantees that \( g^2(p^*) = 0 \). The second period capital requirement binds. Conversely, if \( \delta \leq 1 - P(m_1/k)/c_1 \), then I have that \( m_1 \geq kD(c_2) \) and Lemma 3.7.3 guarantees that \( g^2(p^*) = 0 \). The second period capital requirement doesn’t bind. ■

Proof of Proposition 3.3.2. Suppose that the cooperative is adequately capitalized. If \( \delta > 1 - P(m_1/k)/c_1 \), then Lemma 3.7.12 guarantees that there’s an open interval of technology shocks around \( \delta \) in which the second period capital requirement binds, but its first period capital requirement doesn’t. In this neighborhood, the first-order conditions are:

\[
0 = -D(p_1^*) + \Pi'(p_1^*; c_1) + \lambda \Pi'(p_1^*; c_1) 
\tag{3.70}
\]

\[
0 = -D(p_2^*) + \Pi'(p_2^*; c_2) - \lambda kD'(p_2^*) 
\tag{3.71}
\]

\[
kD(p_2^*) = m_1 + \Pi(p_1^*; c_1). 
\tag{3.72}
\]

Eliminating \( \lambda \) yields

\[
kD(p_1^*) = (k + p_2^* - c_2)\Pi'(p_1^*; c_1) 
\tag{3.73}
\]

\[
kD(p_2^*) = m_1 + \Pi(p_1^*; c_1). 
\tag{3.74}
\]

Now \( kD(p_1^*) > 0 \), so \( \Pi'(p_1^*; c_1) \neq 0 \) and I can safely write

\[
p_2^* = c_2 - k + \frac{kD(p_1^*)}{\Pi'(p_1^*; c_1)}. 
\tag{3.75}
\]

Using the second period capital requirement, I can write

\[
P(m_1/k + \Pi(p_1^*; c_1)/k) = c_2 - k + \frac{kD(p_1^*)}{\Pi'(p_1^*; c_1)}. 
\tag{3.76}
\]

Rearranging, I obtain

\[
c_2 = \phi(p_1^*) := k + P(m_1/k + \Pi(p_1^*; c_1)/k) - \frac{kD(p_1^*)}{\Pi'(p_1^*; c_1)}. 
\tag{3.77}
\]

All that’s left to show is the invertability of the right-hand-side. Put \( \psi(p_1^*) := D(p_1^*)/\Pi(p_1^*; c_1) \).
Differentiating $\psi$, I obtain

$$
\psi'(p_1^*) = (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*)\Pi'(p_1^*; c_1) - D(p_1^*)\Pi''(p_1^*; c_1))
= (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*)(D(p_1^*) + (p_1^* - c_1)D'(p_1^*)) - D(p_1^*)(2D'(p_1^*) + (p_1^* - c_1)D''(p_1^*)))
= (\Pi'(p_1^*; c_1))^{-2}((p_1^* - c_1)(D'(p_1^*))^2 - D(p_1^*)D'(p_1^*)
- (p_1^* - c_1)D(p)D''(p_1^*))
= (\Pi'(p_1^*; c_1))^{-2}((D'(p_1^*))^2((p_1^* - c_1)(1 - D(p)D''(p_1^*))
/ (D'(p_1^*))^2 - D(p_1^*)/D'(p_1^*))
= (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*))^2((p_1^* - c_1)(1 - \sigma(p_1^*)/\epsilon(p_1^*)) + p_1^*/\epsilon(p_1^*))
= (\Pi'(p_1^*; c_1))^{-2}(D'(p_1^*))^2(p_1^* - c_1)(\epsilon(p_1^*) - \sigma(p_1^*) + p_1^*)/\epsilon(p_1^*)
\geq (\Pi'(p_1^*; c_1))^{-2}((D'(p_1^*))^2(- (p_1^* - c_1) + p_1^*)/\epsilon(p_1^*))
= (\Pi'(p_1^*; c_1))^{-2}c_1(D'(p_1^*))^2/\epsilon(p_1^*)
> 0,
$$

(3.80)

(3.81)

(3.82)

(3.83)

(3.84)

(3.85)

(3.86)

having used the facts that $p_1^* \geq c_1$ (Lemma 3.3.1) and $\sigma(p) \leq 1 + \epsilon(p)$ (A2). Together with the fact that $P' < 0$, I conclude that $\phi' < 0$ by the inverse function theorem. Finally,

$$
\frac{dp_1^*}{d\delta} = \frac{dp_1^*}{dc_1} \cdot \frac{dc_1}{d\delta} = -c_1 \phi'(p_1^*) > 0
$$

(3.87)

so that the first period price is strictly increasing in the technology shock.

If $\delta \leq 1 - P(m_1/k)/c_1$, then Lemma 3.7.12 says that neither the first nor the second period capital requirements bind. In fact, $p_1 = c_1$ so the first period price is constant with respect to the technology shock.

Suppose that the cooperative is poorly capitalized. I can no longer appeal to Lemma 3.7.12, but the arguments made for the adequately capitalized cooperative can be carried over to the poorly capitalized cooperative. If the first period capital requirement binds, then $p_1^* = P(m_1/k)$. If neither the first nor the second period capital requirements bind, then $p_1^* = c_1$. In either
case, the first period price is constant with respect to the technology shock. If the second period capital requirement binds, but the first period capital requirement doesn’t, then I can appeal to the argument for the adequately capitalized cooperative to conclude that the first period price is strictly increasing in the technology shock.

3.7.4 The Corporation

Proof of Lemma 3.3.2. The proof is identical to that of Lemma 3.3.1, except that Lemma 3.7.5 should be used in place of Lemma 3.7.6, Lemma 3.7.8 should be used in place of Lemma 3.7.9 and $f^{\Pi}$ should be used in place of $f^{S+\Pi}$.

Proof of Proposition 3.3.1. Let $p^* \in C$ be a solution to the corporation’s problem and let $\lambda \geq 0$ denote the Lagrange multiplier on the constraint $g^2(p) \geq 0$. If $g^1(p^*) = 0$, then $p_1^* = P(m_1/k)$. If $g^1(p^*) > 0$, then the condition for $p_1$ is $0 = (1 + \lambda)\Pi'(p_1^*; c_1)$, which implies that $p_1^* = p^m(c_1)$. □
Figure 3.1. **Loan Growth and Net Interest Margins.** This figure plots loan growth against the net interest margin for credit unions and banks. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter $t$ is the aggregate change in loans from quarter $t$ to quarter $t+1$ divided by the aggregate loans in quarter $t$. The net interest margin in quarter $t$ is the aggregate net interest income in quarter $t$ divided by the aggregate loans in quarter $t$. The coefficient on the net interest margin is significant and positive for credit unions and insignificant for banks (see Table 3.1). The $R^2$ is 27.3% for the credit union regression and 1.0% for the bank regression. The solid line is the regression line for credit unions; the dashed line is the regression line for banks. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.
Figure 3.2. **Loan Growth and Capital Ratios.** This figure plots loan growth against the capital ratio for credit unions and banks. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter $t$ is the aggregate change in loans from quarter $t$ to quarter $t+1$ divided by the aggregate loans in quarter $t$. The capital ratio in quarter $t$ is the aggregate equity capital in quarter $t$ divided by the aggregate loans in quarter $t$. The coefficient on the capital ratio is significant and positive for credit unions and insignificant for banks (see Table 3.1). The $R^2$ is 26.8% for the credit union regression and 4.6% for the bank regression. The solid line is the regression line for credit unions; the dashed line is the regression line for banks. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.
Figure 3.3. **The Corporation’s (Top) and Cooperative’s (Bottom) Objectives.** This figure illustrates the corporation’s and cooperative’s objectives under linear demand. The corporation maximizes the profit $\Pi(p; c) = (p - c)D(p)$ (the dark rectangle). The cooperative’s members are customers, and so it maximizes the consumer surplus $\int_0^1 D(s) ds$ (the light triangle), but its members are also owners, and so it maximizes the profit $\Pi(p; c) = (p - c)D(p)$ (the dark rectangle). It should be clear that $p > c$ is inefficient in a static setting (although it will be in a dynamic setting).
<table>
<thead>
<tr>
<th></th>
<th>Credit Unions</th>
<th></th>
<th>Banks</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Net Interest Margin</td>
<td>4.537***</td>
<td>2.585</td>
<td>2.427</td>
<td>6.143</td>
<td>(4.361)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.200)</td>
<td>(2.179)</td>
<td>(4.004)</td>
<td></td>
<td>(4.361)</td>
<td></td>
</tr>
<tr>
<td>Capital Ratio</td>
<td>0.943***</td>
<td>0.490</td>
<td></td>
<td>-0.298</td>
<td>-0.456*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.252)</td>
<td>(0.457)</td>
<td></td>
<td>(0.220)</td>
<td>(0.244)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.032**</td>
<td>-0.140***</td>
<td>-0.092</td>
<td>-0.007</td>
<td>0.080*</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.041)</td>
<td>(0.058)</td>
<td>(0.056)</td>
<td>(0.040)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>Observations</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>R²</td>
<td>27.3%</td>
<td>26.8%</td>
<td>29.5%</td>
<td>1.0%</td>
<td>4.6%</td>
<td>9.5%</td>
</tr>
<tr>
<td>F Statistic</td>
<td>14.290***</td>
<td>13.939***</td>
<td>7.748***</td>
<td>0.367</td>
<td>1.840</td>
<td>1.936</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.1. **Loan Growth, Net Interest Margins, and Capital Ratios.** This table displays loan growth regressions. An observation is a quarter between the first quarter of 2003 and the fourth quarter of 2012. Loan growth in quarter $t$ is the aggregate change in loans from quarter $t$ to quarter $t+1$ divided by the aggregate loans in quarter $t$. The net interest margin in quarter $t$ is the aggregate net interest income in quarter $t$ divided by the aggregate loans in quarter $t$. The capital ratio in quarter $t$ is the aggregate equity capital in quarter $t$ divided by the aggregate loans in quarter $t$. Data on credit unions comes from the NCUA call reports; data on banks comes from the FDIC call reports.
Bibliography


