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Applying Principles for Active Learning to Promote Student Engagement in Undergraduate Calculus

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This paper reports how principles for active learning within the context of undergraduate calculus were used to inform the design and use of instructional tasks. These principles led to the selection, adaptation and design of instructional activities that invited student engagement and interaction about important concepts and relationships in calculus. To support calculus instructors, weekly one-hour meetings were organized to explicitly discuss and exemplify underlying design principles for active learning, to investigate how these principles were instantiated in the instructional tasks, and to discuss the role of the instructor and students. Our review of instructor feedback and subsequent course enrollment suggests a positive, albeit preliminary, trend toward improved student outcomes in the tertiary calculus sequence. Challenges and necessary supports to achieve classroom environments that support active learning are also discussed.

RATIONALE

Student success in undergraduate mathematics has significant implications for whether they choose to continue into Science, Technology, Engineering, and Mathematics (STEM) majors and future related careers. Even for students who do not choose STEM majors, success in entry-level undergraduate mathematics courses such as calculus can make or break their decision to persist in postsecondary education (Subramaniam, Cates, & Borislava, 2008). In fall 2013, mathematicians and mathematics educators from five U.S. universities began a collaborative process of further analyzing the problem of low performance in introductory mathematics classrooms, determining promising interventions, devising a plan of action, and identifying measures to determine the success of its plans. These activities for improving calculus instruction were organized around principles of Active Learning and involved the mutual support of institutional partners in improving student success and persistence with the undergraduate calculus sequence.

BACKGROUND

Members of this collaborative, the Active Learning in Mathematics Research Action Cluster (ALMRAC), were organized by the Mathematics Teacher Education Partnership (MTEP), a network of over 100 institutions in the United States working to improve mathematics teacher preparation. The ALMRAC uses the Networked Improvement Community design (Bryk, Gomez, Grunow, & LeMahieu, 2015; Martin & Gobstein, 2015) to articulate shared goals, identify common measures, and support progress towards those goals. These shared goals are achieved through Plan-Do-Study-Act (PDSA) cycles that are acted upon locally and reported out in regular in person and virtual meetings among the institutional partners. Even though the ALMRAC currently includes over ten universities who each benefit from each other’s collective contributions, the findings reported in this paper are specific to the resources and activities that occurred at one of the institutions in the ALMRAC, the University of Colorado Boulder.

CONCEPTUAL FRAMEWORK

Active learning incorporates teaching methods and classroom norms that engage students in sense-making activities. Students are encouraged to solve problems, conjecture, experiment, explore, create, and communicate their reasoning in the process of solving mathematics problems, all of which are skills and habits of mind of mathematicians. The active learning approach is aligned with research in teaching mathematics for deeper meaning (Kilpatrick, Swafford, & Findell, 2001; National Mathematics Advisory Panel, 2008). Research has shown that undergraduate students who are involved in active learning techniques can learn more effectively in their classes, resulting in increased achievement and dispositions (Henry, 2010; Freeman et al., 2013; Laursen et al., 2014, Bressoud, & Rasmussen, 2015), particularly so for underrepresented groups (Laursen et al., 2011).

Instructional Design Principles for Active Learning

The ALMRAC developed a set of design principles based on a synthesis of contemporary research on active learning and inquiry-based learning. These principles should be understood as a synergistic set to be applied over the experience of an entire course. Furthermore, the orientation of the application of these principles should be to promote student learning.

Mathematical coherence: While skills and procedural knowledge are a necessary goal of undergraduate calculus, the overarching principle towards the design of courses should be towards key ideas and coherence. Active learning approaches often take more time than lecture, so decisions have to be made to balance time across what are determined to be essential concepts and skills.

Instructional activities: Instructional activities and related questions should promote active construction of meaning, relational reasoning and sense making. Mathematically rich learning tasks are needed to promote reasoning beyond recall, including conceptual and procedural connections, sense making, and relational understanding of mathematical concepts. Tasks should encourage "doing mathematics" and, when possible, involve mathematical problem solving.

Norms for classroom discourse: Even though mathematical discourse should include attending to precision of mathematical language and notation, students also should be encouraged to share "reasoning in process," including partially developed conjectures, explanations and representations of solution strategies. These norms do not develop without intentionality of the instructor and the knowledge and use of instructional methods that promote such norms.

Instructional environment: The classroom norms and organization of the lesson should "support interaction in small groups, whole-class discussion and individual seatwork in accordance with the needs of the learner and the learning task" (Roj-Lindberg, 2001, p. 8). Multiple modes of instruction are warranted and so group work is not the only instructional mode that should be promoted.

Instructional decisions: The choices made in lesson design and adaptation should favor the perspective of the learners. Student reasoning should be monitored and responded to using various formative assessment methods. This requires instructor knowledge of formative assessment techniques and their practical application in undergraduate mathematics (e.g., anticipating potential student misconceptions and planning activities to address those misconceptions).

STUDY CONTEXT

The way in which first semester calculus is taught at University of Colorado Boulder is as follows. Four times per week a lecture is prepared and delivered by mathematics instructors, which include faculty, graduate research assistants, and adjunct faculty. For the recitation, which is usually offered once a week, another doctoral student is assigned as a teaching assistant. For the past 15 years, calculus lectures and recitations have been organized into small sections with no more than 35 students, resulting in 18-24 sections of first semester calculus and 10-16 sections of second semester calculus each semester. Even though we found some success with the undergraduate learning assistant (ULA) program in promoting active learning in the recitation (Webb, Stade & Grover, 2014), the lecture remained largely teacher-directed with little to no student-to-student interaction or other observed aspects of active learning.

To apply these design principles in the "lectures," we leveraged resources that were developed to support the implementation of the ULA program. These ULA activities were created for the use of groups of students in the Thursday recitations. The characteristics of the tasks that made them more group worthy (Boaler, 2006) included emphasis on important mathematics concepts, reduced scaffolding to engage students in sustained problem solving, prompts that promoted student reasoning beyond recall, and questions that were challenging and open enough to evoke student-to-student interaction. These recitation activities were designed and used over a period of 14 semesters, resulting in a substantial library of instructional resources from which we could draw upon (see http://math.colorado.edu/activecalc) to infuse ALM across a greater proportion of calculus instruction.

In spring 2014 we decided to utilize these ULA resources in the four days that were designated for lecture. Knowing that a greater emphasis on ALM would require more instructional time we developed a scope and sequence for calculus topics each semester that favored mathematical coherence, one of the design principles for ALM. We found that the time allocated for review at the beginning and end of the course could be reduced dramatically allowing for more time for sustained investigations of important calculus concepts. We also considered which topics were central or tangential to the primary goals of each semester of calculus.

Application of Design Principles in Instructional Resources

As argued by Heibert and Grouws (2007), mathematics teaching can be understood as part of a system. Instructors' decisions and actions are influenced by personal and contextual factors that include the instructors' prior experiences, his or her beliefs about teaching and learning, available resources, expectations articulated in the curriculum, and the norms of the department, among other factors. In situations where teaching expertise is new and emergent, and time for lesson preparation relatively constrained, instructional resources have great sway on instructors' decisions with respect to planning and instruction.

For many undergraduate calculus courses in the United States, the primary instructional resource is often a textbook that includes presentations of mathematical skills and concepts, and a practice set of tasks to be completed as assigned work outside of class (i.e., home work). At the University of Colorado Boulder, a textbook is still used in calculus courses however it is paired with other instructional materials that were originally designed for use in the ULA recitations. Additional instructional resources were also developed to support active learning in just over half of the designated lecture sessions. We identified these additional activities, designed to be completed by pairs or groups of students, as either *TACTivities* or *projects*. What occurs with the other half of the lecture sessions is left up to the discretion of the instructor, and often includes teacher-directed lessons that may or may not involve some degree of ALM.

Tactile + Activities = TACTivities

Inspired by Angie Hodge (an ALMRAC colleague from the University of Nebraska at Omaha), at a colloquium she facilitated at University of Colorado Boulder she had participants work with sets of cards with various representations (e.g., graphs, equations, descriptive sentences, solutions, etc). We observed the that quality and extent of participant discourse that was elicited by this activity was significant. As outcome of that colloquium we decided to partner with Prof. Hodge in the design of many other TACTivities for calculus. The characteristics of these TACTivities generally included two or more different types of mathematical representations printed on cut cardstock that could be organized to suggest either fulfillment of a complete set, or a categorization scheme that could be justified by students. For example, Figure 1 shows a portion of the Definite Integral Dominos TACTivity. As students touched and moved cards to pair representations, they would discuss their reasons for doing so. Often this would elicit peer feedback either affirming or countering the decision to pair the representations on different sides of the cards.

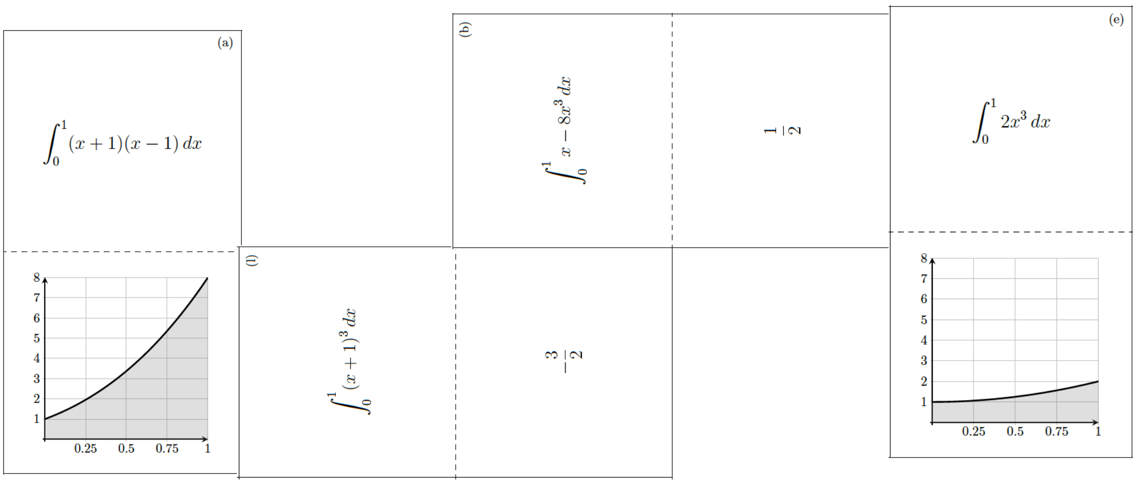

Figure 1: Partial solution of the Definite Integral Domino TACTivity

The other reason to design these TACTivities was that we found they required "low instructional overhead." Our mathematics department's commitment to small section calculus requires many sections – and many instructors! The majority of calculus instructors at University of Colorado Boulder are doctoral students. But even at universities where calculus is taught in large lectures there are usually a multitude of recitations which are also typically led by doctoral students. The need for ALM resources with low instructional overhead is due to the relative limited teaching experience of calculus instructors, compared to say that offered by a secondary mathematics licensure program with multiple courses that are connected to field experiences. Given the constraints of the undergraduate context for calculus, these new resources could not require professional development beyond the weekly one-hour meetings for calculus instructors. These activities, therefore, were designed to be easy to launch – i.e., they were somewhat intuitive for students as to how to proceed with minimal guidance from the instructor. As instructors used these TACTivities, as student discussions about the representations emerged instructors would be able to hear and observe students' ideas and conceptions and use that information as they interacted with specific groups or facilitated a whole class debrief of the activity.

A different example of a TACTivity, one that students encounter on the first day of Calculus I, is the Function Placemat. We felt it was important to start off the semester by establishing norms for classroom discourse, student interaction and sense-making and so this activity was designed as a review activity of pre-calculus concepts involving different characteristics of functions. As shown in Figure 2, small cards are organized on "placemats" of 15 different mathematical statements. These placemats are used with a set of 44 small cards that include various mathematical definitions and conventions used to describe advanced algebraic and trigonometric functions.

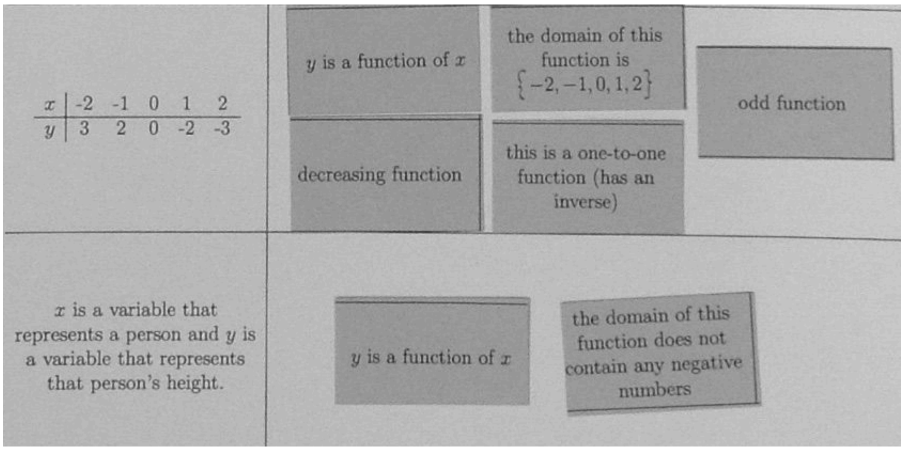

Figure 2: Partial solution for the Function Placemat

As with the Integral Dominos and the rest of the TACTivities, it is expected that instructors have a limited role. The instructors priority is on attending to productive student-to-student interaction. Instructors organize the class into pairs or small groups, provide limited guidance to invite students to begin, and then circulate among the class to observe or listen to proposed moves and justifications.

Expanded Use of ULA Projects

To support more extensive infusion of ALM in both recitation and lecture sessions, we repurposed instructional resources originally designed for ULA supported recitations. These projects are worksheet-like in format and include a sequence of prompts designed to support exploration of calculus concepts, calculus in realistic contexts, and other goals identified for each course. Questions are posed to evoke student conversation about mathematical relationships: the prompts are written with varying degrees of openness to provide a balance of instructional scaffolding to support student-led inquiry requiring little or no instructor input. As with the TACTivities, the instructor role with these projects is to support student engagement and persistence in challenging tasks and to have students draw upon the support of peers as they make sense of the prompts and develop justifiable responses.

Professional Development to Support Active Learning

Recognizing the limited opportunities mathematics doctoral students have to investigate theories and methods of mathematics pedagogy, we focused the one hour weekly meetings to address the following topics: theories of student learning, ways of supporting student engagement in open ended tasks, translation of instructional materials into lesson plans, research on IBL and active learning, and assessing different forms of mathematical reasoning (Webb, 2009). For some of the sessions we also modeled how to use the TACTivities and projects so that calculus instructors could experience first-hand the role of the instructor and students, and the importance of specific features of the activity. At the very least, we wanted instructors to experience new ways of learning familiar topics and to value student-centered pedagogical approaches.

FINDINGS

Recent results from our application of principles for ALM have demonstrated positive trends in student evaluations of the experience, and in the improvement of student persistence in course taking from Calculus 1 to Calculus 2. Each of these findings are suggestive of positive shifts in student engagement and achievement.

Instructor Feedback

To better understand instructors' conceptions and use of ALM activities we developed a brief survey for instructors to complete after they used activities with students. Instructors (n = 16) were asked to rate student behavior on a scale of 1 (Low), 2 (Typical), 3 (High) for four categories. The summary of this survey for the *Function Placemat* TACTivity is presented below:

Category	Mean (SD)
Level of student frustration	1.56 (0.63)
Student requests for help	1.81 (0.66)
Student engagement	2.69 (0.48)
Student-to-student interaction	2.75 (0.45)

Table 1: Instructor Feedback for *Function Placemat* activity

Our primary goal in applying principles for active learning was to improve student engagement and interaction. For this activity (and many others that were developed) instructors were in strong agreement that these aspects of student behavior were higher than typically observed, while the level of student frustration and student requests for help were lower than typically observed.

Shifts in Student Achievement and Persistence

To examine whether or not the implementation of ALM resources in first semester calculus had any impact on student enrollment in second semester calculus, we reviewed the institutional data for calculus enrollment. To enroll in Calculus II students must have earned a C- in Calculus I or equivalent course at another institution. Therefore, student success in Calculus I has a direct impact on students' ability to continue to take courses in the calculus sequence. Table 2 highlights an increased shift in student enrollment from fall semester Calculus I to spring semester Calculus 2, an increase from 50.1 percent to 62.2 percent. Even though there could be other confounding factors that have contributed to this trend, it is likely that the implementation of ALM activities that were initiated in Calculus I in Fall 2014 may have contributed to this improved rate of persistence. This 12% shift resulted in the need to add two additional sections of Calculus II in Spring 2015.

School Year	Calculus I (Fall)	Calculus II (Spring)	Fall to Spring Persistence
2012-13	720	372	51.7%
2013-14	767	384	50.1%
2014-15	696	433	62.2%

Table 2: Student enrollment from Calculus I (fall) to Calculus II (spring)

Related to this improved persistence rate is a parallel drop in the percentage of students with end-of-course marks of D (below passing), Fail or Withdraw. The DFW rate for Calculus I in Fall 2013, prior to the ALM infusion, was 28.6%. The Fall 2015 semester DFW rate for Calculus I showed a reduction to 21.5%.

DISCUSSION

With respect to the design principles for ALM, applying the principle of *mathematical coherence* involves course level decisions regarding the selection and sequencing of topics in addition to more focused attention to how important concepts are developed in specific activities. The design of *instructional activities* should support a range of mathematical reasoning from the students' point of view. *Classroom discourse* is a goal that requires explicit attention in professional development and in the design of resources. Yet, explicit attention is not enough. Calculus instructors should have opportunities to experience how productive student-to-student discourse is supported through the use of ALM instructional resources so that their mathematical experiences with calculus can include instances of active learning in calculus. This helps to exemplify an *instructional environment* that is supportive of active learning and offers calculus instructors experiential evidence of the characteristics and value of student-centered *instructional decisions.*

We recognize that sustained ALM in undergraduate calculus involves more than instructional resources. Sustained support also requires a combination of: 1) guidance for instructors using ALM, 2) engagement of instructional leaders, 3) faculty awareness of strategies for local institutional change, and 4) institutional incentives and resources. We argue, however, that positive results in the short term can be realized with low instructional overhead resources designed to support student engagement and interaction, which can later motivate institutional commitments for ALM.

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