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Abstract

Trapezoidal strip conductor edge shapes, the result of undercutting during the fabrication process of microstrip and other planar lines, are shown to appreciably increase conductor loss, according to a perturbational loss calculation method developed by Lewin and Vainshtein and implemented here for microstrip.

1 Introduction

Loss mechanisms, specifically conductor and dielectric loss, limit the performance of planar lines in microstrip circuits. Dielectric loss, often the smaller of the two loss components, can be readily calculated from the substrate loss tangent and the microstrip geometry. Conductor loss calculations, however, are generally more involved. The present study examines these calculations with emphasis on the effect of microstrip edge shape.

The most general conductor loss calculation is given by the integral

$$P = \frac{1}{2\sigma} \int_V |J|^2 dV. \tag{1}$$

In general, this calculation is too complicated to obtain quick results, so a series of assumptions, approximations, and perturbations are done to simplify the integral. For microstrip and other planar waveguides, the quasi-TEM assumption can be taken at sufficiently low frequencies, thus eliminating all transverse currents and reducing the integral to the product of a length and a two-dimensional integral. The surface impedance boundary condition uses the fact that the fields in a good conductor exponentially decay almost normally from the surface to establish a relationship between the fields at the surface. At this point the integral has been reduced to a contour integral around the surface of the conductors. For typical conductors the perturbation equating the actual surface fields to the fields of the ideal, perfectly conducting case is reasonable. A further perturbation, accurate when the conducting strip is thin compared to its width, assumes that strip thickness can be neglected and the fields of the strip with no thickness can be used. These fields are easier to find, but a problem arises in the loss calculation. The fields of the infinitely thin strip case, which describe the ideal current density, vary as $r^{-1/2}$ near the strip edge, where r is distance from the edge, and this results in a logarithmic divergence when the integration is carried to the edge. This situation has been resolved independently by Lewin [1] and Vainshtein [2] by stopping the integration just short of the edge by a distance Δ dependent on strip geometry, especially strip thickness. These stopping points are obtained by a local quasi-static examination of the fields near the edge in both the actual, nonzero-thickness strip and the infinitely thin strip. Some examples of strip edges with known stopping points ([1],[2]) include the rectangular edge, $\Delta = \frac{t}{290.8}$, and the circular edge, $\Delta = \frac{t}{124.77}$, where t is the thickness of the actual strip.

In this paper, stopping points for several trapezoidal edges, observed as a result of the fabrication process, are derived. These are used to demonstrate the significant effect that undercutting has on conductor loss in a wide range of microstrip configurations.

2 Integration stopping point for 60° trapezoid

A conformal transformation technique can be used to examine the quasi-static fields of a strip under the quasi-TEM approximation [1],[2]. The stream function u and potential v can be related to the coordinates x and y of the cross-section of the strip by an analytic conformal transformation, in this case the Schwarz-Christoffel transformation. The longitudinal surface current, which is the tangential magnetic field on the surface, is proportional to $\partial u/\partial s$, where s is a variable along the surface. Thus the required loss integral can be expressed as

$$P \propto \int |J|^2 ds \propto \int (\frac{\partial u}{\partial s})^2 ds = \int \frac{\partial u}{\partial s} du, \qquad (2)$$

with appopiate limits of integration.

The first step in finding the stopping point for a specific edge shape is to find the Schwarz-Christoffel transformations for both the zero-thickness strip and the actual nonzero-thickness strip. The calculation (2) is done for the actual strip, but for the zero-thickness strip the integration is stopped just short of the edge by the distance Δ. Equating the two losses will yield Δ in terms of the strip geometry.

This technique is performed on the trapezoidal edge with a 60° corner as shown in Figure 1, where t is the strip thickness and d is a distance far from the edge. The transformation mapping the outside of the strip to the upper-half plane $v \geq 0$ is given by (similar mappings are given in [3])

$$z(w) = C_1 \int_0^w w^{2/3}(w-1)^{1/3} dw + C_2. \qquad (3)$$

Upon making the substitutions $\alpha = \sqrt[3]{\frac{w-1}{w}}$ for $u > 0$, the top and side surfaces, and $\beta = \sqrt[3]{\frac{w}{w-1}}$ for $u < 0$, the bottom surface, the transformation becomes [4]

$$\begin{aligned} z(w) &= 3C_1[\frac{\alpha}{6(1-\alpha^3)^2} - \frac{\alpha}{18(1-\alpha^3)} - \frac{1}{9}\int \frac{d\alpha}{1-\alpha^3}] + C_2, \quad u > 0, \qquad (4) \\ & \quad 3C_1[\frac{\beta^2}{6(1-\beta^3)^2} - \frac{\beta^2}{9(1-\beta^3)} - \frac{1}{9}\int \frac{\beta d\beta}{1-\beta^3}] + C_2, \quad u < 0. \end{aligned}$$

When the last terms are factored and the integration constants are set to null the integral at $\alpha = 0$ [3], the integrals become

$$\int_0^\alpha \frac{d\alpha}{\alpha^3 - 1} = \frac{1}{3}\int_0^\alpha [\frac{1}{\alpha - 1} + \frac{e^{-4\pi i/3}}{\alpha - e^{2\pi i/3}} + \frac{e^{-8\pi i/3}}{\alpha - e^{4\pi i/3}}] d\alpha \qquad (5)$$

$$= \frac{1}{3}[\ln(1-\alpha) + e^{2\pi i/3}\ln(1-\alpha e^{4\pi i/3}) + e^{4\pi i/3}\ln(1-\alpha e^{2\pi i/3})]$$

and, similarly,

$$\int_0^\beta \frac{\beta d\beta}{\beta^3 - 1} = \frac{1}{3}[\ln(1-\beta) + e^{4\pi i/3}\ln(1-\beta e^{4\pi i/3}) + e^{2\pi i/3}\ln(1-\beta e^{2\pi i/3})]. \quad (6)$$

The final form of the transformation, after solution of constants C_1 and C_2 according to Figure 1, is

$$\begin{aligned} z(w) &= \frac{9t}{\pi}\sqrt[3]{\frac{w-1}{w}}(\frac{w^2}{2} - \frac{w}{6}) + \frac{2te^{\pi i/3}}{\sqrt{3}} \quad (7) \\ &+ \frac{t}{\pi}[\ln(1-\alpha) + e^{2\pi i/3}\ln(1-\alpha e^{4\pi i/3}) + e^{4\pi i/3}\ln(1-\alpha e^{2\pi i/3})], \quad u > 0 \\ z(w) &= \frac{9t}{\pi}(\frac{w}{w-1})^{2/3}(\frac{w^2}{2} - \frac{2w}{3} + \frac{1}{6}) \\ &+ \frac{t}{\pi}[\ln(1-\beta) + e^{4\pi i/3}\ln(1-\beta e^{4\pi i/3}) + e^{2\pi i/3}\ln(1-\beta e^{2\pi i/3})], \quad u < 0. \end{aligned}$$

The corresponding equation for the infinitely thin strip is taken as the limiting form of (7);

$$\begin{aligned} z(w) &= \frac{9t}{\pi}\frac{w^2}{2} \quad (8) \\ x(u) &= \frac{9t}{\pi}\frac{u^2}{2}, \end{aligned}$$

where the second equation is true on the strip.

The loss of the actual, nonzero-thickness strip is found by integrating dz/dw along the strip surfaces. The parametric equations for these surfaces are $z = x + it$ for the top, $z = x$ for the bottom, and $z = x + iy = \frac{\gamma}{\sqrt{3}} + i\gamma$ for the side, where $\gamma = y$ and is used to parameterize the equation. Thus the power lost on the actual strip is

$$P_1 = \int_{-u_B}^{0} \frac{du}{dx}du + \int_0^1 \frac{2}{\sqrt{3}}e^{\pi i/3}\frac{du}{d\gamma}du + \int_1^{u_T} \frac{du}{dx}du, \quad (9)$$

where $du/dx = du/d\gamma = \frac{\pi}{9t}(u-1)^{-1/3}u^{-2/3}$ from the reciprocal of dx/du, the derivative of (3). Using the same substitutions as before, the first and third terms are reduced to

$$\begin{aligned} \int_1^{u_T} \frac{\pi}{9t}(u-1)^{-1/3}u^{-2/3}du &= \frac{-\pi}{3t}\int_0^{\alpha_o} \frac{\alpha d\alpha}{\alpha^3 - 1} \quad (10) \\ &= \frac{-\pi}{9t}[\ln(\frac{1-\alpha_o}{\sqrt{1+\alpha_o+\alpha_o^2}}) + \sqrt{3}\tan^{-1}(\frac{\sqrt{3}\alpha_o}{2+\alpha_o})] \end{aligned}$$

and

$$\begin{aligned}\int_0^{u_B} \frac{\pi}{9t}(u-1)^{-1/3}u^{-2/3}du &= \frac{-\pi}{3t}\int_0^{\beta_o} \frac{d\beta}{\beta^3-1} \qquad (11)\\ &= \frac{-\pi}{9t}[\ln(\frac{1-\beta_o}{\sqrt{1+\beta_o+\beta_o^2}}) - \sqrt{3}\,\tan^{-1}(\frac{\sqrt{3}\beta_o}{2+\beta_o})],\end{aligned}$$

where $\alpha_o = \sqrt[3]{\frac{u_T-1}{u_T}}$ and $\beta_o = \sqrt[3]{\frac{u_L}{u_L+1}}$, while the second term is simply a beta function and reduces to

$$\int_0^1 \frac{2\pi}{9\sqrt{3}t}e^{\pi i/3}(\gamma-1)^{-1/3}\gamma^{-2/3}d\gamma = \frac{2\pi}{9\sqrt{3}t}\frac{\pi}{\sin(\pi/3)} = \frac{4\pi^2}{27t}. \qquad (12)$$

The corresponding loss calculation on the infinitely thin strip is

$$P_2 = \int_{-u_B}^{-u_\Delta} \frac{du}{dx}du + \int_{u_\Delta}^{u_T} \frac{du}{dx}du = \frac{\pi}{9t}\ln(\frac{u_T u_B}{u_\Delta^2}), \qquad (13)$$

where $\frac{du}{dx} = \frac{\pi}{9tu}$ from (8) and u_Δ is the transformed value of the integration stopping point Δ. When the two values P_1 and P_2 are set equal,

$$\begin{aligned}&\frac{\pi}{9t}[\ln\frac{\sqrt{1+\alpha_o+\alpha_o^2}}{1-\alpha_o} + \ln\frac{\sqrt{1+\beta_o+\beta_o^2}}{1-\beta_o} + \sqrt{3}(\tan^{-1}(\frac{\sqrt{3}\beta_o}{2+\beta_o}) \qquad (14)\\ &\qquad -\tan^{-1}(\frac{\sqrt{3}\alpha_o}{2+\alpha_o})) + \frac{4\pi}{3}] = \frac{\pi}{9t}\ln(\frac{u_T u_B}{u_\Delta^2})\end{aligned}$$

the asymptotic value of the variable Δ is found to be $\Delta = \frac{t}{1243}$, which is the desired stopping point in terms of the actual strip thickness.

The stopping points for the sharper trapezoids with corners of 45° and 30° were also found with a similar technique. For the 45° trapezoid, the stopping point is $\Delta = \frac{t}{20187.6}$, and for the 30° trapezoid, $\Delta = \frac{t}{53997300}$. As the trapezoidal corner becomes sharper, however, the validity of approximations such as the surface impedance boundary condition are subject to question. The applicablility of our results in such a case will be studied elsewhere.

3 Application to loss calculations

When the perturbation of an infinitely thin strip is taken with the other previously described approximations, the power lost on the strip portion of a microstrip (Figure 2) of unit longitudinal length can be written as [5],[6]:

$$P = R_s[\int_{-w/2+\Delta}^{w/2-\Delta} \frac{1}{2}J_z^2(x)dx + \int_{-w/2}^{w/2} 2\delta J_z^2(x)dx], \qquad (15)$$

where J_z is the total current density profile, singular at the edges, δJ_z is a nonsingular term representing twice the difference between the surface currents of the bottom and top of the strip, and R_s is the real part of the surface impedance. The nonsingular term can be found from the integral equation

$$\delta J_z(x) = \frac{h}{\pi} \int_{-w/2}^{w/2} \frac{J_z(x')dx'}{(x-x')^2 + 4h^2} \tag{16}$$

in terms of the total current J_z, and so (15) can be completely solved in terms of J_z. When the closed-form expression for this current given by Kobayashi [7] is used for J_z, the first term of (15) is reduced to closed-form, and the second term must be approximated numerically. A comparison of conductor loss for a variety of different edge shapes has been done in Figure 3 for a wide range of microstrip geometries. The conductor loss on the strip is the only part being considered since it often dominates the ground plane loss in a practical microstrip configuration.

When the thickness of the strip approaches the width, the increase in loss due to edge shape rises dramatically, as seen in Figure 3. This is not attributed to an invalid Δ, since the stopping point does not increase much in the thick strip limit, but the result can be explained by the inaccuracy of the infinitely thin perturbation. For practical microstrip configurations, however, our assumptions are valid, and the effect of edge shape is seen to be important. The results here compare favorably with a quasi-static moment method study by Chryssomallis *et.al.*[8], who found that for the specific case of $w/h = 0.101$ and $t/h = 0.011$, the microstrip with a circular-edged strip will have 15% less conductor loss than the microstrip with rectangular edges. For the same configuration, the present technique predicts a 10.7% decrease in loss for a circular cross section.

4 Conclusion

Microstrip conductor loss can be calculated quickly by a perturbation method which includes the effect of strip edge shape. The work of Lewin and Vainshtein has been extended here to implement the technique on microstrip. The required stopping points in the loss integrations have been found here for a variety of trapezoidal strip edges. Loss calculations have been done for these and other edges, and it is apparent that the edge shape has a significant effect on conductor loss. Although results here tend to overestimate the effect due to breakdown of approximations, conductor loss due to edge shape increases as the edge shape gets sharper and the strip gets narrower or thicker.

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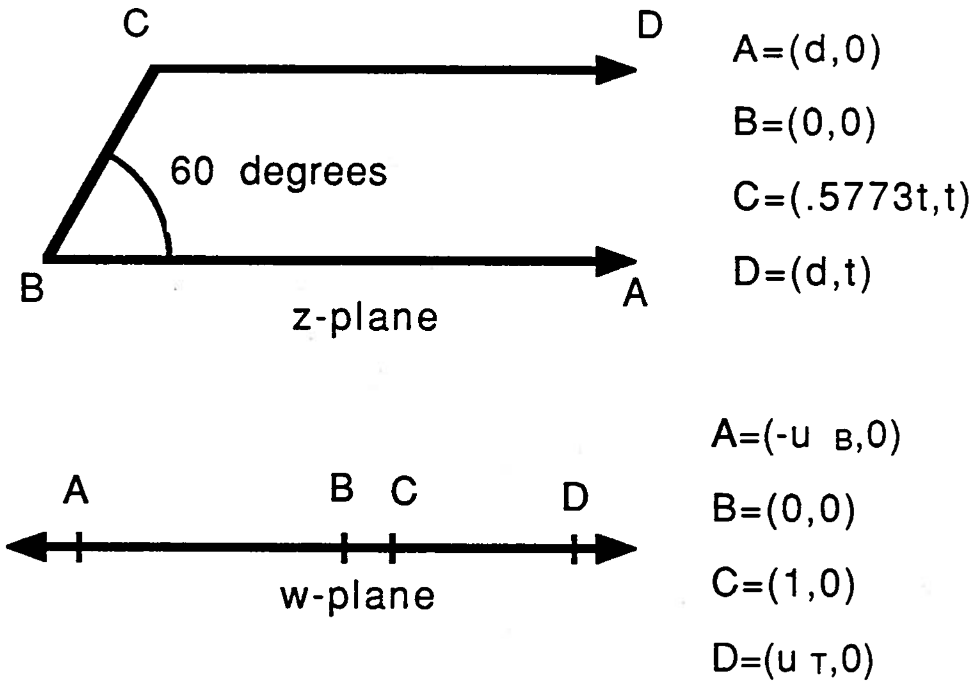

Figure 1: Geometries of the conformal transformation.

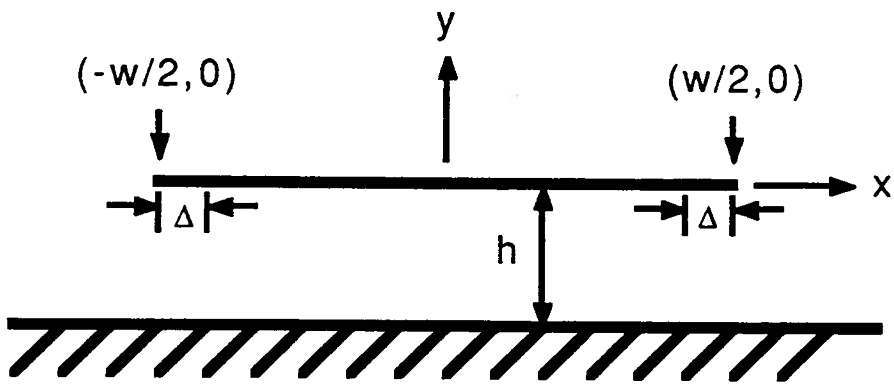

Figure 2: Geometry for the infinitely thin microstrip.

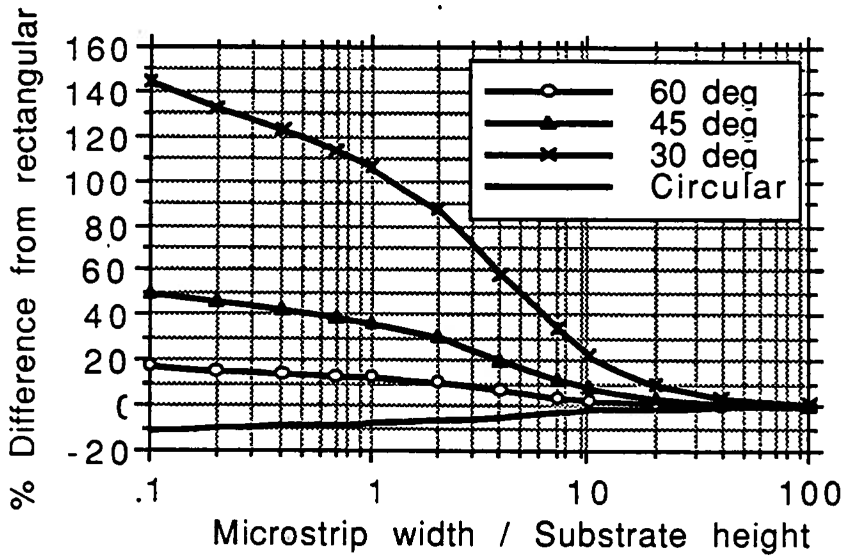

Figure 3: Conductor power lost for a variety of microstrip edge shapes and configurations (thickness t=10 um), given in percentage difference from the rectangular-edged strip.