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Essays on Optimal Macroeconomic Stabilization Policy for Developing Economies

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Essays on Optimal Macroeconomic Stabilization Policy for Developing Economies

by

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B.A., Sogang University, 2008

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A thesis submitted to the

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This thesis entitled:
Essays on Optimal Macroeconomic Stabilization Policy for Developing Economies
written by Jongheuk Kim
has been approved for the Department of Economics

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Ufuk Devrim Demirel

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Prof. Martin Boileau

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Prof. Robert McNown

Date _________________

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.
This dissertation studies issues on the macroeconomic stabilization policies in emerging market or developing countries. Specifically, I investigate an impact of the macroeconomic policy regimes and other factors on the fluctuations of business cycles. I investigate issues concerning the procyclical trend of fiscal and monetary policies in a closed economy under imperfectly developed infrastructure, a small open economy case of central banking problem with a restricted financial market accessibility and labor market distortions, and an impact of governmental wage support on consequences of negative external shock. One of the main goal of this dissertation is to investigate how a macroeconomic policy can optimally stabilize the economic volatility and how it can improve a social welfare gain.

In the first chapter, I build a small open economy dynamic stochastic general equilibrium (DSGE) model, and solve a Ramsey policy problem by using a linear-quadratic (LQ) welfare loss function to investigate the optimal monetary policy in developing economies. To capture realistic sides of the region, I add two frictions in the model: An imperfect financial market integration captured by a quadratic financial adjustment cost in a budget constraint of a representative domestic household, and a labor market friction captured by a quadratic labor adjustment cost in a production process of a monopolistic competitive domestic firm. While the financial market friction exacerbates the trade-off between output gap and domestic inflation stabilization faced by policy makers and creates higher level of economic volatility, the labor market friction softens the negative effect of the imperfectly integrated financial market by mitigating the trade-off. I also evaluate alternative monetary policy candidates, and find that a policy emphasizing the domestic inflation stabilization yields higher welfare cost than a policy weighing on the output gap stabilization.

A Rich volume of literature points out that many developing countries have experienced procyclical
macroeconomic policies in recent period while most developed countries have not, but the reason of the phenomenon is still in debate. In the second chapter, I theoretically investigate an optimal fiscal and monetary policy in an economy where an institutional cost associated with public goods influences on economic dynamics and cyclicality of macroeconomic stabilization policies. Based on a simple New Keynesian DSGE model, a real quadratic adjustment cost that is created by a government spending spread between current and efficient level of the public expenditures is invited. This cost captures a negative effect of the newly created institutional cost on trade-off between inflation gap and output gap stabilization encountered by policy authority. As a result, solving Ramsey policy problem with a linear-quadratic welfare loss function, I find that the optimal fiscal and monetary policy tend to be more procyclical and the economy experiences higher level of volatility in the presence of the institutional cost. Comparing alternative monetary policy regimes based on Taylor rule, I find that a forward looking inflation rate targeting rule reduces procyclicality of fiscal and monetary policy and yields a significant improvement in welfare gain, while aggressive stabilization strategy on inflation gap or output gap has no economic merit.

In the last chapter, I investigate the role of real wage changes in the dynamic responses of the optimal macroeconomic policy to the negative foreign demand shocks, where the wage structure is partly affected by a manually operated by a government. To do this, I build a small open economy DSGE model with a sticky price and a monopolistically competitive nontradable sector assumptions. If a government manually supports the domestic consumers by a binding minimum wage which is financed by a lump sum taxation, the optimally determined the real marginal cost in the New Keynesian Phillips Curve (NKPC) is decreased, and thus the economy experiences less exacerbated trade-off between output gap and inflation stabilization faced by a policy maker. Therefore, with the higher level of the real wage support, an economic volatility in key macroeconomic variables from the optimal Ramsey policy problem is more mitigated, and the economy accomplishes more efficient stabilization goal.
Dedication

To my parents.
Acknowledgements

I would like to express my deepest appreciation to my advisor Dr. Ufuk Devrim Demirel, who has been a tremendous mentor for me. I would like to thank him not only for giving me an invaluable insight on economics, but also for encouraging me not to give up. His advice on both research as well as on my career has been priceless. I would also like to thank professor Martin Boileau, professor Robert McNown, professor Philip Graves, and professor Roberto Pinheiro, who served as my committee members even at hardship, made my defense be an enjoyable moment, and gave me brilliant comments and suggestions.

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Words cannot express how grateful I am to my mother, father, and sister for all of the sacrifices that they have made on my behalf. Their prayer for me was what sustained me thus far. Last but not least, I would like to acknowledge my appreciation to Seungmi Park and her family, who were great support for me.
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Chapter 1

Labor Market Friction, Imperfect Financial Market Integration, and Optimal Monetary Policy for Developing Economies

1.1 Introduction

During the last two decades, the imperfect financial integration condition has been relaxed for emerging markets or developing economies. Within this changed international financial market environment, what is an optimal monetary policy for those countries that lack labor market flexibility? According to Frankel (2011) [35], these financial markets have traditionally been less integrated with international financial markets than those of developed countries, but the degree of openness has increased during the last two decades. As shown in Figure 1.1, de facto and de jure barriers to international asset markets prevented Korean domestic consumers from accessing the world financial markets until the early 2000s, but the market has opened significantly since then. However, as noted by Obstfeld (2012) [58], the financial openness of emerging market economies is still low relative to that of developed countries. On the other hand, the labor market in developing countries has been more regulated, or at least, less flexible than in developed countries. Here, relatively powerful labor unions or a high level of regulatory protection for hired workers might have created a higher social cost for adjusting a labor demand allocation in the production sectors. This is not a nominal rigidity problem, which generally appears in business cycle models for developed and developing countries, but a real one, preventing a production sector from flexibly

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1 Obstfeld (2012) [58] uses a metric based on the Grubel–Lloyd index to illustrate the degree of financial market openness, measured as the ratio of gross foreign assets and liabilities to GDP. The definition of the index is $GL = 1 - \frac{|A-L|}{A+L}$, where $A$ denotes assets and $L$ denotes liabilities.
choosing an optimal level for its employment demand in each period. Figure 1.1 shows the number of labor disputes and the workdays missed as a result of these disputes in South Korea. Until 2004, labor friction increased, but since then, the trend has reversed. The focus of this study is on the effect of these two frictions, in developing countries, on economic business cycles and on an optimal monetary policy. The combined effect of these two frictions has not been actively researched in related literature, although the separate effects of the frictions on economic dynamics has been studied quite extensively. One of difficulties hindering academic investigations is a lack of statistical evidence on the interaction between these two frictions. However, theoretical modeling and quantitative simulation analyses can help in this regard. Furthermore, there has been a lengthy debate on the optimal monetary policy structure for developing economies, but as yet, no consensus on a solution. Therefore, an evaluation and comparison of several monetary policy options under a specific economic friction can provide policymakers with much needed insight.

In this study, I develop a small open economy version of the dynamic stochastic general equilibrium (DSGE) model, based on the New Keynesian frameworks. A benchmark New Keynesian DSGE model assumes monopolistic competition and a sticky price setting, which generate money non-neutrality in the short run. Thus, these assumptions enable a monetary policy to be effective on economic dynamics. The small open economy assumption introduces an international dimension in which financial and commodity markets are opened internationally. Thus, the domestic business cycle is linked to foreign exogenous changes, while domestic changes cannot affect the world economy because the size of the domestic economy is assumed to be negligible. Therefore, it is supposed that a domestic country is given world prices and output, as well as world demand for home-produced goods. Then, I add two frictions to the benchmark model to capture a realistic picture of the recent changes within developing economies, as described earlier. The first friction is that of imperfect financial integration, captured by a financial adjustment cost in the budget constraint of representative domestic households. The second is a real labor market friction, cap-

---

2 The benchmark model I use follows the work of Gali and Monacelli (2005) [40], Devereux et al. (2006) [31], Demirel (2009b) [28], and Gali (2008) [39].
tured by a quadratic labor adjustment cost in the production process of a monopolistic competitive
domestic firm. The first friction is popular in New Keynesian literature to represent an imperfection or a fragility in the domestic financial market environment of a small open economy. The linear quadratic form of the financial adjustment cost is widely used in related literature, and here I adopt the form of Demirel (2010) [29]. This quadratic form of a financial imperfection creates interest rate differentials between home and foreign countries, and replicates the recent trends in foreign bond holdings and goods production. This quadratic form is different to the financial cost model used in Benigno (2009) [7] and De Paoli (2009b) [61], which emphasizes the asymmetric asset positions of debtor and creditor countries that largely contribute to the global imbalances problem. This study concentrates on the imperfections in developing countries to determine how the region-specific frictions contribute to the transmission of foreign shocks to a domestic economy, which itself cannot impact the world financial market. Therefore, it does not consider the global imbalance issue, and assumes that the world financial market is perfectly operated. As a result, the asymmetry in the asset positions within a small open economy are beyond the scope of this study. The quadratic model assumes that a domestic agent can access both domestic and foreign currency denominated asset markets, but faces an additional adjustment cost when trading foreign dominated assets. The labor adjustment cost has been favored in related literature since Sargent (1978) [67] introduced the quadratic form of the cost. I simplify the form used in Janko (2008) [43], Chang et al. (2007) [21], and Kehoe and Ruhl (2009) [47] to avoid unnecessary complexity in the calculations. The quadratic labor adjustment cost is created when the current level of labor demand differs from a steady-state level of employment. After successfully introducing these two frictions to the model, I derive some important equations from the linearized competitive equilibrium conditions to interpret the role of the frictions in the economic dynamics. Furthermore, using an appropriate parameterization, I solve a Ramsey policy problem to find the optimal monetary policy, and simulate an equilibrium to obtain impulse responses to various domestic and foreign exogenous shocks. Lastly, I analyze how economic volatility is affected by the frictions, as well as the welfare effect of each monetary policy candidate.
There are two main results from this study. First, the effects of the financial and labor market frictions move in opposite ways. The imperfectly integrated financial market worsens the trade-off between the output gap and domestic inflation stabilization faced by policymakers after introducing the foreign bond holdings differential term to the marginal cost structure of the New Keynesian Phillips Curve. Thus, a policymaker faces a more serious trade-off in the stabilization problem. On the other hand, labor market friction mitigates the negative effect of the financial market friction by reducing the sensitivity of the output gap to changes in domestic inflation and partly muting the effect of domestic productivity shocks on output gap changes. This slow-tempo labor market reallocation effect makes the economy react more sluggishly to exogenous shocks. Hence, increasing financial market friction creates a higher level of economic volatility, but a more sluggish labor market reallocation reduces the instability of the economy. Therefore, the policymakers face a greater trade-off between the output gap and domestic inflation stabilization, because the financial market is less integrated, but this state can be ameliorated by the real labor market distortion. Intuitively, the nominal interest rate is set by the monetary policy authority to optimally stabilize the macroeconomic variables, and is a weighted combination of the output gap and domestic inflation rate. The greater trade-off between the output gap and domestic inflation, caused by the combined effect of the two frictions, directly affects the choice of an optimal monetary policy. Thus, the optimal policy results in a situation in which the economy must bear a higher level of economic instability.

The second result of this study is that different policy parameter values give different results in terms of economic volatility. While a monetary policy that emphasizes domestic inflation yields a higher social welfare cost, a policy that emphasizes output gap stabilization obtains a lower level of welfare loss. Simulation results show that the output gap has a higher level of volatility than does domestic inflation in this economic environment. Thus, if the monetary policymaker emphasizes output gap stabilization, this must achieve a significant reduction in volatility and welfare losses. Therefore, aggressive targeting of the domestic output gap is preferable to targeting domestic inflation.

The main contribution of this study is that it introduces real friction to the labor market in
the DSGE model, along with nominal rigidity problems such as nominal price rigidity and financial adjustment costs, to explain the recent trend in the business cycle of developing countries. Many economists view developing economies as financially fragile. However, these regions also have a high level of misallocation of labor demand or, at least, have a relatively sluggish adjustment in employment demand, which is partly explained by strong regulation in the labor market. When many Asian and Latin American countries were hit by the financial crisis in 1998, the International Monetary Fund (IMF) required that these countries implement several legal reformations of their market structure as an essential prerequisite for their help. One IMF requirement was the liberalization of the labor market. However, because of cultural and social resistance, this constitutional change took longer than expected, and some have yet to change. This study shows that the misallocation of labor demand has a notable effect on the business cycle of a developing economy, and is related to other frictions such as imperfect international financial market accessibility. Many believe that the imperfect financial market integration is a main cause of global imbalances, as effectively argued by Mendoza and Quadrini (2010) [53]. In addition to the body of literature, it is also important to note that in the special economic circumstance in which a labor market is distorted by a real friction, as depicted in this study, the imperfectly integrated financial market condition can change how external shocks are transmitted to the domestic economy. Therefore, a monetary policy should react to these conditions to find an optimal policy rule.

The remainder of this paper is organized as follows. The second section briefly reviews related literature. Here, I discuss several schools of thought on the financial market integration and labor market frictions, not all of which were incorporated into the model in this study. Where relevant, I explain why a theory was excluded from my model, despite its contribution to the body of literature. The third section explains the theoretical DSGE model in detail. This section also qualitatively analyzes the effect of the assumed frictions on the business cycle of the economy and on the decision making of the monetary policy authority. The fourth section explains the parameter values used in the quantitative analysis and notes the impulse responses of the system of equilibrium equations to various types of exogenous shocks. This section also discusses the monetary policy implications.
Finally, the fifth section concludes the paper.

1.2 Literature Review

This study is based on the New Keynesian framework for a small open economy. It assumes monopolistic competition, which gives each firm pricing power and markup revenue, as well as nominal price rigidity, as in the staggered price setting of Calvo (1983) [14]. For an open economy environment, the framework assumes home and foreign final goods are produced from the monopolistic competitive firms and traded internationally. The basic structure of the model in this study starts from the framework of Gali and Monacelli (2005) [40] and Gali (2008) [39]. In the benchmark model, the economy is represented by a relationship between domestic inflation and an output gap, and is analyzed from a welfare implication perspective. According to the model, the main difference between the different monetary policy regimes arises from the level of exchange rate volatility in each. Other seminal works on a monetary policy in an open economy environment include Smets and Wouters (2002) [71] and Devereux et al. (2006) [31], which introduce an imperfect exchange rate pass-through as a main factor that affects monetary policy decision making. The background knowledge on the general characteristics of developing economies in a small open economy was provided by several studies, such as Lane (2003) [50], Frankel (2011) [36], and Frankel et al. (2011) [37]. Lane (2003) [50] argues that there exist significant structural differences between advanced and developing economies, including financial development, foreign currency denominated liabilities, and a time-varying external credit constraint. The study points out that these characteristics in underdeveloped economies can lead to a procyclical macroeconomic stabilization policy, which I consider briefly here as well.

The financial integration and openness of an emerging market economy is one of the most important issues considered in this study. There is a rich volume of literature that has contributed to this topic. As a theoretical approach, a branch of this literature prefers the quadratic form

\[ I \text{ also relied heavily on seminal reference textbooks, such as Woodford (2003) [82], Obstfeld and Rogoff (1996) [59], Gali (2008) [39], and Wickens (2011) [81].} \]
of the financial adjustment cost embodied in a household budget constraint of the maximization problem. For example, Uribe and Yue (2006) [77], Benigno (2009) [7], and Demirel (2009b) [27] design the adjustment cost using this method and successfully introduce an analytically tractable way to express the increasing cost of holding of foreign assets and, thus, an imperfectly open financial market. De Paoli (2009a) [61] makes a small adjustment to the original quadratic form for technical reasons, and shows how different types of asset markets derive various levels of equilibrium conditions. Benigno (2009) [7], De Paoli (2009a) [60], and Faia (2010) [33] adopt a similar approach to the imperfectly integrated financial market, but are slightly different. In the model of De Paoli (2009a) [60], financial friction is defined by an intermediation cost proportional to the level of external debt. The main difference between De Paoli (2009a) [60] and Benigno (2009) [7] is the ownership of the intermediation cost. De Paoli (2009a) [60] assumes the cost belongs to the foreign agent, while Benigno (2009) [7] assigns it to the home country, which means the subsequent welfare gain becomes different. Faia (2010) [33] assumes a borrowing constraint and shows the degree of financial openness by relaxing the level of the external constraint.

Labor market friction is another important issue discussed in this study, and makes it unique in terms of open economy macroeconomics literature. By combining the domestic labor market friction with imperfect financial market integration, this study shows that a macroeconomic stabilization policy should be affected by the combined effect of the two frictions. Most prior studies have tried to use the search and matching friction model pioneered by Mortensen and Pissarides (1994) [56]. To represent a real frictional unemployment environment, some studies, such as Faia (2009) [32] and Faia (2010) [33], try to integrate the matching friction model within the DSGE perspective and to then solve for the welfare cost measure. Ravenna and Walsh (2011) [63] finds that the welfare cost associated with the labor cost created by the search and matching friction model is distinct from the nominal effect on the welfare measure. Walsh (2005) [79] explains that the labor market friction can increase the output response to the monetary policy shock while reducing an inflation response, which departs from the usual Walrasian labor market. In this way, the integrated effect of labor market friction and financial market integration
has drawn some notable academic findings, such as those of Petrosky-Nadeau and Wasmer (2013) [62] and Compolmi and Faia (2011) [25]. Petrosky-Nadeau and Wasmer (2013) [62] uses financial friction as a volatility puzzle solver, combining a job creation cost to create a realistic volatility contributor. Compolmi and Faia (2011) [25] focuses on the European Monetary Union and uses the search and matching friction model to investigate the country-specific inflation dynamics in the study area. The study argues that country-specific labor market friction can cause the idiosyncratic trend in the business cycle. Another branch of labor market friction in the DSGE model is the wage stickiness setting. However, while this assumption has been successful in calibrating realistic data observations, the core characteristic of the nominal rigidity on wage level cannot fully explain the reasonable economic logic on the question drawn in this study. This study uses a quadratic adjustment cost of lagged labor demand in the production function. Sargent (1978) [67] introduces the quadratic adjustment cost for the sectoral reallocation of labor demand, and Kehoe and Ruhl (2008) [46] develops the model for the real business cycle model. The sectoral reallocation of labor demand can be modified easily and understood as an additional cost for a production sector when the present level of employment demand is different from that of the previous period. This type of cost form is heavily discussed in Cooper and Willis (2003) [26], and Chang et al. (2007) [21] and Janko (2008) [43] develop the quadratic labor adjustment cost in a dynamic general equilibrium model approach. This real adjustment cost does not rely on the nominal aspect of labor market frictions, such as wage stickiness, and has a clear edge in the literature on the real frictional labor allocation problem. It also departs from the search and matching friction model, and has the advantage that it does not have to assume a degree of job opening and labor market tightness, which are beyond the scope of the present study.

Empirical research on the economic vulnerability of developing economies helps the reliability of the model in this study. Since this study is motivated by the changed labor market structure in South Korea before and after the financial crisis of 1998, several preceding studies on the event helped to build an appropriate logical background, as well as gain a deeper insight that this study could not capture using theoretical modeling. Braggion et al. (2009) [10] provide a brief statistical
approach to the structural breaking event of the Asian financial crisis. They suggest that the Ko-
rean labor market during the period was inefficient and that the central bank should have reacted
to the distorted market condition. However, their study only concentrates on the inappropriately
determined wage level. Chung et al. (2007) [24] build a traditional New Keynesian small open
economy model with an imperfect exchange rate pass-through in order to evaluate the monetary
policy. Even though it uses before-crisis data, its parameterization method is well defined and
helpful. Kim and Kim (2012) [48] empirically reveals the vulnerability of the Korean economy to
exogenous financial shocks using data from the recent financial crisis between 2008 and 2010. As a
result, it partly provides empirical support for this study.

1.3 Model

The theoretical analysis of the combined effect of labor market friction and imperfect finan-
cial market integration on the business cycles and monetary policy decisions begins by building a
small open economy DSGE model. Here, I follow Gali and Monacelli (2005) [40] as a benchmark
framework for the New Keynesian open economy model. However, since the baseline model assumes
a complete asset market, it cannot fully depict an imperfect financial market with a certain fric-
tion. Therefore, I adopt a quadratic financial adjustment cost to create imperfect financial market
accessibility for a domestic country. In addition I use interest rate differentials between the home
and world economies to replicate the foreign bond holdings in the small open economy, following
the work of Schmitt-Grohe and Uribe (2003) [69] and Demirel (2010) [29]. In this setting, while a
foreign country (world) has no additional cost to access the foreign currency denominated bonds,
the home country pays an additional cost to hold a certain amount of foreign assets. Additionally,
I add two more assumptions for an asymmetric small open economy case. First, home and foreign
countries have different size economies. The economic impact of the home country is assumed to be
negligible compared to that of the world economy. Therefore, the home country is given the foreign
output, consumption, and prices. This assumption makes it possible to observe the response of
a domestic business cycle to exogenous foreign demand and monetary shocks. Second, domestic
households can access both home and foreign currency denominated asset markets, but foreign agents can only access the foreign asset market. This is because the size of the domestic financial market is too small to be significant to the dynamics of the international financial markets. Along with these unique assumptions, I include monopolistic competition and a sticky prices framework, following Calvo (1983) [14] and Yun (1996) [84], to create money non-neutrality and to allow a monetary policy to stabilize economic volatility. Furthermore, the law of one price and purchasing power parity hold. Lastly, this model assumes a cashless economy, following Woodford (2003) [82], because holding cash in a utility function does not offer any improvement to the real side of the economy and, thus, becomes a useless assumption.

1.3.1 Households

Let us consider two connected economies, Home (H) and Foreign (F) countries, which are separately populated with a continuum of agents, and the total population is normalized to one. Home and foreign consumers share the same form of utility function and maximize this utility function given a country-specific budget constraint. The utility function of a representative home agent is given by

$$U(C_t, L_t) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right), \quad (1.1)$$

where $C_t$ refers to the aggregate consumption level at time $t$, $L_t$ denotes the total labor supply of the representative household at time $t$, $\beta$ is a time discounting factor, $\sigma \geq 0$ is the intertemporal elasticity of substitution in private consumption, and $\varphi \geq 0$ is the inverse of the elasticity of labor supply.\footnote{This elasticity is discussed in detail in Christiano et al. (2010) [22]. In their case, $1/\varphi$ can be interpreted as a Frisch labor supply elasticity, which explains the substitution effect with respect to the change of wage rate, assuming that $L_t$ is the number of hours worked by the representative household.} Furthermore, the domestic aggregate consumption level, $C_t$, consists of two parts, namely consumption for home and foreign final goods, and is defined by

$$C_t \equiv [(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}, \quad (1.2)$$
where $\alpha \in [0,1]$ captures the degree of openness to foreign consumption by domestic households, which inversely denotes a home bias preference, and $\eta \geq 1$ is an index of intratemporal elasticity of substitution between home and foreign final goods. Here, $C_{H,t}$ is an index of domestic goods, using the constant elasticity of substitution functional form,

$$C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j)^{\frac{1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon - 1}},$$

where $j \in [0,1]$ denotes the variety of goods, and $\varepsilon \geq 1$ represents the elasticity of substitution among varieties. Then, $C_{F,t}$ is an index of foreign produced (imported) goods, defined by

$$C_{F,t} \equiv \left( \int_0^1 C_{F,t}(j)^{\frac{1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon - 1}}.$$

Note that $\varepsilon$ is common across the consumption of home and foreign goods. This is quite a strong assumption, but since it does not weaken any part of the main argument of this study, I accept it for the sake of simplicity. An aggregate consumption index for a foreign representative household can be similarly defined using an asterisk:

$$C_{t}^* \equiv \left[ (\alpha^*)^{\frac{1}{\eta}} (C_{H,t}^*)^{\frac{\eta - 1}{\eta}} + (1 - \alpha^*)^{\frac{1}{\eta}} (C_{F,t}^*)^{\frac{\eta - 1}{\eta}} \right]^{\frac{1}{\eta - 1}}, \quad (1.3)$$

where $\alpha^* \in [0,1]$ represents the degree of openness to goods produced in the home country, satisfying $\alpha^* = \alpha$, meaning both home and foreign countries have the same degree of openness to each other. Then, $C_{H,t}^*$ and $C_{F,t}^*$ are defined as the amount of consumption by foreign households for goods produced in the home and foreign countries, respectively. Next, price indexes for the commodity markets in the home and foreign countries, based on the above preferences and aggregate consumption indexes, are given by

$$P_t \equiv \left[ (1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad (1.4)$$

and

$$P_t^* \equiv \left[ \alpha P_{H,t}^{1-\eta} + (1 - \alpha) P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad (1.5)$$

respectively. Here, $P_t$ and $P_t^*$ are the home and foreign consumer price indexes (CPI) and $P_{H,t}$ and $P_{F,t}$ are sub-indexes for the home and foreign produced goods consumed in the home country,
respectively. Then, $P^*_H,t$ and $P^*_F,t$ are interpreted as the price indexes of home and foreign produced goods, respectively, expressed in the foreign currency. Each of the four sub-price indexes are expressed by an aggregation, as follows:

\[
P_{H,t} = \left( \int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}, \quad P_{F,t} = \left( \int_0^1 P_{F,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}, \quad (1.6)
\]

\[
P^*_{H,t} = \left( \int_0^1 P^*_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}, \quad P^*_{F,t} = \left( \int_0^1 P^*_{F,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \quad (1.7)
\]

Using the above aggregations, we can solve for the optimal allocation of demand for varieties of goods in the home country:

\[
C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}; \quad C_{F,t}(j) = \left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}. \quad (1.8)
\]

Next, the aggregate total expenditure for the home and foreign goods follow directly from (8):

\[
\int_0^1 P_{H,t}(j)C_{H,t}(j) dj = P_{H,t}C_{H,t}; \quad \int_0^1 P_{F,t}(j)C_{F,t}(j) dj = P_{F,t}C_{F,t}. \quad (1.9)
\]

Now, the optimal allocations of expenditure for home and foreign goods are given by:

\[
C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t. \quad (1.10)
\]

Equation (10) completes the description of optimal expenditure allocations for the intratemporal equilibrium of home households. The optimal allocation of foreign households consumption can be similarly derived, denoted using an asterisk.

Next, to explore the intertemporal equilibrium of a representative household, we need to define a budget constraint for the agent. Using equation (9) and the total aggregate consumption expenditure of the home agent, $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$, the budget constraint of the representative home household is given by

\[
P_tC_t + B_{H,t} + \mathcal{E}_t B_{F,t} + \leq R_{t-1}B_{H,t-1} + \mathcal{E}_t R_{t-1}^* B_{F,t-1} + W_t L_t + T_t + \mathcal{E}_t \frac{\Psi_B}{2} (B_{F,t} - B_F)^2 + \int_0^1 \Gamma_t(j) dj, \quad (1.11)
\]

where $B_{H,t}$ and $B_{F,t}$ are the home and foreign currency denominated bonds, respectively, $\mathcal{E}_t$ denotes the nominal exchange rate between the home and foreign currency (relative price of foreign currency
in terms of home currency), \( R_t \) and \( R_t^* \) are the nominal interest rates in the home and foreign countries, respectively, \( W_t \) is a nominal wage, \( B_F \) is the steady-state value of the foreign currency denominated bonds, \( T_t \) is a lump-sum tax or transfer governed by a fiscal authority, and \( \Gamma_t(j) \) represents the profit of firm \( j \). Then, \( \frac{\Psi_B}{2}(B_{F,t} - B_F)^2 \) is a quadratic financial adjustment cost for the domestic household, and the cost is assumed to be a non-zero, positive value when the current foreign bond holding is different to the steady-state value. Furthermore, \( \Psi_B \) is a constant parameter value defined by the degree of the adjustment cost for international borrowing, and captures how the domestic financial market is isolated from the world financial market. For instance, holding the same amount in foreign bonds, the higher adjustment cost associated with \( \Psi_B \) means less perfectly integrated financial markets, or that it is more difficult for the domestic households to access the international financial market. Therefore, \( \Psi_B \) functions as an inverse financial integration indicator. If \( \Psi_B \) approaches zero, or at the steady state, \( B_{F,t} \) is equal to \( B_t \), the domestic economy is assumed to have no financial friction, and the financial markets will be perfectly integrated. This quadratic intermediation cost function delivers an additional cost to domestic households buying foreign assets, and creates an interest rate differential between the home and world economies. This is a main contributor to the amount of foreign bond holdings in the home economy, as well as to changes in the marginal cost structure uniquely built in this model. Note that this type of cost is only associated with the home country agent, since the size of domestic financial market is assumed to be negligible to the foreign (world) economy, based on the small open economy assumption. The negligible size of home economy guarantees the usage of the quadratic form of the cost, and the lack of asymmetry in asset positions between creditor and debtor can be ignored without any significant harm to the logic. Therefore, the foreign representative household faces a different budget constraint:

\[
P_t^*C_t^* + B_{F,t} \leq R_t^*B_{F,t-1} + W_t^*L_t^* + T_t^* + \int_0^1 \Gamma_t^*(j) dj.
\]

While the domestic agent enjoys two different types of assets and can use international risk pooling, the foreign agent is only able to access the foreign currency denominated bonds. This is the result
of the assumption that captures the reality of a small open economy, in which the size of the home financial market is negligible relative to world financial market. Thus, the world’s demand for the home asset can be ignored.

The first-order conditions necessary for equilibrium are given by

\[
\frac{W_t}{P_t} = L^\sigma_t C_t^\sigma \\
\frac{W^*_t}{P^*_t} = L^\sigma_t C^*_t^\sigma
\]

(1.13)

(1.14)

\[
1 = \beta E_t \left[ \frac{R_t}{1 + \Psi_B(B_{F,t} - B_F)} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{E_{t+1}}{E_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right]
\]

(1.15)

\[
1 = \beta E_t \left[ R_t \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{P_t^*}{P_{t+1}^*} \right) \right]
\]

(1.16)

\[
1 = \beta E_t \left[ R_t^* \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{P_t^*}{P_{t+1}^*} \right) \right]
\]

(1.17)

where \(E_t\) is an expectation operator at time \(t\). Equation (13) is interpreted as a labor supply or a real wage determination. Equation (14) is defined similarly for the foreign country. Equation (15) is a home household’s Euler equation for the optimal choice of foreign currency denominated bonds, corresponding to equation (17), which is a foreign agent’s Euler equation for the optimal foreign bonds asset position. Equation (16) states a home household’s Euler equation for the optimal level of home currency denominated bonds. Note that, from equation (15), in the limiting case in which \(\Psi_b\) approaches zero, the Euler equation replicates a frictionless benchmark version. Using the relationship between the overall price levels in the home and foreign countries, \(P_t = E_t P_t^*\), (1.17) can be rewritten in terms of the stream of the home country price level and the nominal exchange rate:

\[
1 = \beta E_t \left[ R_t^* \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{E_{t+1}}{E_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right].
\]

(1.18)

1.3.2 Uncovered Interest Rate Parity, International Risk Sharing, and Terms of Trade

In this subsection, I derive several relations from the previously determined optimal conditions of households, as well as some international macroeconomic definitions. First, from equations (15)
and (16), I find the relationship between two different nominal interest rates:

\[ 1 = E_t \left[ \left( \frac{E_{t+1}}{E_t} \right) \left( \frac{R_t^*}{R_t} \right) \frac{1}{1 + \Psi_B (B_{F,t} - B_F)} \right], \tag{1.19} \]

which is a modified version of an uncovered interest rate parity in a frictional case. According to (19), the difference between home and foreign nominal interest rates is determined by the change of the nominal exchange rate, the foreign bond holding differential, and the degree of the financial adjustment cost. This frictional parity generates a different level of international risk premium from that of the benchmark. More specifically, the difference between the home and foreign nominal interest rate \((R_t - R_t^*)\) will widen if the change in the nominal exchange rate increases or the effect of the financial adjustment cost gets smaller. This means that the partial integration of the financial markets can change the gap between the two interest rates.

Next, let us define the real exchange rate between the home and foreign currencies as the ratio of the two countries’ overall price levels, in which both currencies are denominated domestically,

\[ Q_t \equiv \frac{E_t P_t^*}{P_t}. \tag{1.20} \]

Then combining (15) and (17) gives the relation between the consumption levels of the home and foreign countries in terms of the real exchange rates and the financial adjustment cost for foreign bond holdings:

\[ 1 = E_t \left[ \left( \frac{Q_{t+1}}{Q_t} \right) \left( \frac{C_t^*}{C_t} \right)^{-\sigma} \left( \frac{C_{t+1}}{C_{t+1}} \right)^{-\sigma} \frac{1}{1 + \Psi_B (B_{F,t} - B_F)} \right]. \tag{1.21} \]

According to (21), the adjustment cost for holding foreign assets becomes a factor that partly determines the difference between the changes in consumption of the home and foreign countries. This means that, if \(B_{F,t} > B_F\), the positive effect of \(\Psi_B\) on the difference in foreign bond holdings differential widens the gap between home and foreign consumption. As one of the internationally linked markets becomes more separated, the co-movement of the consumption in both countries would weaken.

Terms of trade is defined as the ratio of the price of imported goods to that of home-produced
goods,

\[ S_t = \frac{P_{F,t}}{P_{H,t}}. \]  \hspace{1cm} (1.22)

In a special case where \( \eta \) is close to unity, the following relation holds:

\[ P_t = P_{F,t}^{(1-\alpha)} \cdot P_{F,t}^\alpha \]
\[ = P_{H,t} \cdot S_t^\alpha. \]  \hspace{1cm} (1.23)

Furthermore, defining an inflation rate from term \( t \) to \( t+1 \) by \( \Pi_t \equiv \frac{P_{t+1}}{P_t} \), the following equation holds:

\[ \Pi_t = \Pi_{H,t} \cdot \Delta S_t^\alpha, \]  \hspace{1cm} (1.24)

where \( \Delta X_t \equiv \frac{X_t}{X_{t-1}} \), for any arbitrary variable, \( X \). Assuming the law of one price, \( P_{F,t} = \mathcal{E}_t P_t^* \), holds, and combining (20) and (22), one can find the relation between the real exchange rate and the terms of trade:

\[ Q_t = S_t^{(1-\alpha)}. \]  \hspace{1cm} (1.25)

Therefore, the international risk-sharing condition (21) can be reorganized in terms of the terms of trade:

\[ 1 = E_t \left[ \left( \frac{S_{t+1}}{S_t} \right)^{(1-\alpha)} \left( \frac{C_t^*}{C_t} \right)^{-\sigma} \left( \frac{C_{t+1}}{C_t^*} \right)^{-\sigma} \left( \frac{1}{1 + \Psi_B (B_{F,t} - B)} \right) \right] 
\]
\[ = E_t \left[ (\Delta S_{t+1})^{(1-\alpha)} (\Delta C_{t+1}^*)^{-\sigma} \left( \frac{1}{1 + \Psi_B (B_{F,t} - B)} \right) \right]. \]  \hspace{1cm} (1.26)

According to (26), intertemporal consumption smoothing differences across the two countries can be determined by the change in terms of trade, the commodity market openness, and the level of home country-specific financial adjustment cost. Specifically, the gap between the current amount of foreign bond holdings and the steady-state level of the bond holdings changes the positive effect of the terms of trade on the international consumption spread differences. For instance, as the financial gap increases (increasing \( (B_{F,t} - B) \) and the value is positive) or the degree of financial inaccessibility worsens (increasing \( \Psi_B \)), the positive effect of the increasing \( \Delta S_{t+1} \) on the level of gap between \( \Delta C_{t+1} \) and \( \Delta C_{t+1}^* \) is alleviated.
Lastly, for the convenience of later analysis, (19) can be rewritten in terms of the real exchange rate or the terms of trade:

\[
1 = E_t \left[ \left( \frac{E_{t+1}}{E_t} \right) \left( \frac{R_t^*}{R_t} \right) \frac{1}{1 + \Psi_B(B_{F,t} - B_F)} \right] \\
= E_t \left[ \left( \frac{Q_{t+1}}{Q_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{R_t^*}{R_t} \right) \frac{1}{1 + \Psi_B(B_{F,t} - B_F)} \right] \\
= E_t \left[ (\Delta S_{t+1})^{(1-\alpha)} (\Pi_{t+1})^{-1} \left( \frac{R_t^*}{R_t} \right) \frac{1}{1 + \Psi_B(B_{F,t} - B_F)} \right].
\] (1.27)

The second equality uses the law of one price and the additional assumption that the foreign overall price, \(P_t^*\), is normalized to 1. The third equality follows directly from (25).

### 1.3.3 Producers

In the production sector of the home economy, many monopolistically competitive firms, indexed by \(j\), produce a slightly differentiated good using labor. A typical firm faces an identical form of a linear quadratic labor adjustment cost when the current labor demand is different from the labor employed at the steady state. This type of cost can possibly create a negative effect on the real output level of each firm by increasing a marginal cost. As a result, the individual output level and aggregate level of national output can be less than the frictionless benchmark economy levels. In addition, the Calvo (1983) [14] staggered price setting is included. Therefore, the current price level is permanently affected by the stream of past price levels.

A typical home country firm uses labor to produce a differentiated final good with a linear technology.

\[
Y_t(j) = A_t N_t(j),
\] (1.28)

where \(Y_t(j)\) is the output level of firm \(j\), \(A_t\) is the exogenous total factor productivity following an AR(1) stochastic process, and \(N_t(j)\) is the labor demand of firm \(j\). In addition, following Kydland and Prescott (1991) [49], Mendoza (1991) [52], and Janko (2008) [43], each firm faces the general form of a quadratic labor adjustment cost if it experiences a difference between the current and
steady-state level of labor demand:

\[ P_t \frac{\Psi_N}{2} \left( \frac{N_t(j)}{N} - 1 \right)^2 N_t(j), \]

where \( \Psi_N \) is a constant parameter value that captures the degree of labor market friction and \( N(j) \) is the steady-state value of \( N_t \). It is clear that as \( \Psi_N \) reaches zero, the production function replicates the benchmark production process of Gali and Monacelli (2005) [40]. A cost minimization problem of a typical firm, \((j)\), in the home country is

\[ \text{Min}_{N_t} W_t N_t(j) + P_t \frac{\Psi_N}{2} \left( \frac{N_t(j)}{N(j)} - 1 \right)^2 N_t(j), \]  

subject to (28). Hereafter, the firm index \( j \) can be dropped in this minimization problem, because every firm faces the same technology and cost structure and faces the same problem. The Lagrangian function is formulated as follows:

\[ L_t = W_t N_t + P_t \frac{\Psi_N}{2} \left( \frac{N_t(j)}{N(j)} - 1 \right)^2 N_t + \lambda_t (Y_t - A_t N_t), \]

where the Lagrangian multiplier, \( \lambda_t \), is a shadow price, the nominal marginal cost for producing one unit of finalized good at time \( t \). The first-order condition is given by

\[ \lambda_t = \frac{1}{A_t} \left[ W_t + P_t \Psi_N \left( \frac{1 - N_t}{N} \right) N_t + P_t \frac{\Psi_N}{2} \left( \frac{N_t(j)}{N} - 1 \right)^2 \right]. \]  

(1.30)

Note that as \( \Psi_N \) converges to zero, the nominal marginal cost also converges to the baseline model’s typical marginal cost, \( \frac{W_t}{A_t} \). Rewriting the above equation and dividing it by \( P_t \), we obtain the real marginal cost condition from the supply side:

\[ MC_t = \frac{1}{A_t} \left[ \frac{W_t}{P_t} + \Psi_N \left( \frac{1 - N_t}{N} \right) N_t + \frac{\Psi_N}{2} \left( \frac{N_t}{N} - 1 \right)^2 \right], \]

(1.31)

where \( MC_t \equiv \frac{\lambda_t}{P_t} \) is defined by the real marginal cost at time \( t \). According to the above condition, if \( N \) is always less than one, which will be discussed in detail in the calibration section, the real marginal cost is permanently higher than the baseline case of the marginal cost. This means that the marginal cost is positively affected by the demand for labor and the constant (positive)
parameter value $\Psi_N$.

Next, following Calvo (1983) [14] and Yun (1996) [84], the model assumes a staggered price setting. A randomly selected portion of producers, $(1 - \theta)$, set a new price level at each period, while the remaining firms, $\theta$, keep their price level as it was in the previous period. Therefore, $\theta$ captures the degree of price rigidity. Let $P_{H,t}(j)$ be the optimal price set by firm $j$ at time $t$. With the staggered price setting described above, $P_{H,t+k}(j) = P_{H,t}(j)$. Then, the problem faced by a typical firm, $j$, is given by

$$\max_{P_{H,t}} E_t \sum_{k=0}^{\infty} \theta^k E_t \left[ \Lambda_{t,t+k} \{ Y_{t,t+k} (P_{H,t} - \lambda_{t+k}) \} \right],$$

subject to the international demand constraints

$$Y_{t,t+k}(j) \leq \left( \frac{P_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} (C_{H,t+k} + C^*_{H,t+k}) \equiv Y^d_{t+k}(P_{H,t}),$$

where $\Lambda_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)$ and $\lambda_{t+k}$ denotes the nominal marginal cost at period $t+k$ with respect to the staggered price setting, $P_{H,t}$, and is determined by the previously derived real wage equation. Note that firm specific index $j$ can be dropped in this problem as well, because all firms use the same price setting, subject to the same marginal cost and the same resource constraint. Furthermore, note that this problem is identical to the price setting of Gali and Monacelli (2005) [40], with the exception of the nominal marginal cost structure. The first-order condition yields

$$\sum_{k=0}^{\infty} \theta^k E_t \left[ \Lambda_{t,t+k} Y_{t,t+k} \left( \frac{P_{H,t}}{\varepsilon - 1} - \lambda_{t+k} \right) \right] = 0. \tag{1.32}$$

Note that in the perfect flexible price setting, $\theta = 0$, the above equation reproduces $P_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \lambda_t$. This can be rearranged using stationary variables, as follows:

$$\sum_{k=0}^{\infty} (\theta \beta)^k E_t \left[ C_{t+k}^{-\sigma} Y_{t,t+k} \frac{P_{H,t}}{P_{t+k}} \left( \frac{P_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{H,t+k}}{P_{H,t-1}} MC_{t+k} \right) \right] = 0, \tag{1.33}$$
where $MC_{t+k}$ is the real marginal cost at time $t+k$, as shown above, and is equal to $\frac{\lambda_{t+k}}{P_{H,t+k}}$. We can now define the new price index of domestically produced goods under the staggered price setting,

$$P_{H,t} = \left[ \theta P_{H,t-1}^{1-\varepsilon} + (1 - \theta)(\frac{P_{H,t}}{P_{H,t-1}})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

(1.34)

$$\leftrightarrow \frac{P_{H,t}}{P_{H,t-1}} = \left[ \theta + (1 - \theta) \left( \frac{P_{H,t}}{P_{H,t-1}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$  

(1.35)

### 1.3.4 Monetary Authority

A fiscal authority organizes a lump-sum tax or transfer. A monetary authority sets the level of the nominal interest rate, following a form of the traditional Taylor rule. The nominal interest rate rule is given by

$$R_t = \left( \frac{P_{H,t}}{P_{t-1}} \right)^{\mu_x} \left( \frac{Y_t}{\bar{Y}} \right)^{\mu_y} (\bar{R})(Z_t)$$

$$= (\Pi_{H,t})^{\mu_x} \left( \frac{Y_t}{\bar{Y}} \right)^{\mu_y} (\bar{R})(Z_t),$$

(1.36)

where $\mu_x$ and $\mu_y$ are policy parameters, weighted by domestic inflation and output changes, respectively. Then, $\bar{Y}$ and $\bar{R}$ are the output and nominal interest rate steady-state values, respectively, and $Z_t$ is an exogenous monetary policy shock, which follows an AR(1) stochastic process.

### 1.3.5 Aggregations, Market Clearing Conditions, and Competitive Equilibrium

The aggregate level of output in the home country is

$$Y_t = \left[ \int_0^1 Y_t(j) \frac{\varepsilon-1}{\varepsilon} dj \right]^{\frac{1}{\varepsilon-1}},$$

(1.37)

and the economy-wide employment is determined by

$$N_t = \int_0^1 N_t(j) dj.$$ \hspace{1cm} (1.38)

The labor market clearing condition is

$$L_t = N_t.$$ \hspace{1cm} (1.39)
Therefore, the aggregate output level is determined by (38) and (39) and is also linear:

\[ Y_t = A_t N_t. \]  

(1.40)

The market clearing condition for each differentiated home final good, \( j \), is given by

\[ Y_t(j) = C_{H,t}(j) + C_{H,t}^*(j). \]  

(1.41)

The world market clearing condition is given by

\[ Y_t^* = C_t^*. \]  

(1.42)

The world output follows an AR(1) stochastic process. Therefore, it is exogenously given to the home country agents since the demand for world output from the home economy is assumed to be negligible in this small open economy setting. The home currency denominated bond market is cleared, such that

\[ B_{H,t} = 0, \]  

(1.43)

and the world bond market is automatically cleared according to Walras’ law.

The home-produced goods market clearing condition (41) can be rewritten as

\[ Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} + \left( \frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} C_{H,t}^* \]

\[ = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \left( \frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} (\alpha) \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \]

\[ = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \right]. \]  

(1.44)

The aggregate supply and demand for home-produced final goods are then calculated by substituting (44) into (37),

\[ Y_t = \left( 1 - \alpha \right) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \]

\[ = \left( 1 - \alpha \right) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \left( \frac{P_{H,t}}{P_t Q_t} \right)^{-\eta} C_t^* \]

\[ = \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \left[ (1 - \alpha) C_t + \alpha (Q_t)^\eta C_t^* \right]. \]  

(1.45)
The law of one price and the definition of the real exchange rate are used in the second step of the above calculation. The above national account states that the overall supply of the domestic output should be equal to the demand from both home and foreign consumers, which depend on the commodity market openness and the price levels of the home country. Furthermore, for the convenience of later discussion, (45) can be rewritten in terms of price levels, private consumption, exogenous foreign demand, and terms of trade:

$$Y_t = (S_t)^{-\eta_\alpha} \left[ (1 - \alpha)C_t + \alpha (S_t)^{\eta(1-\alpha)} C_t^* \right].$$

(1.46)

Note that as $\alpha$ converges to zero, which means the home economy becomes an autarky condition, (45) and (46) converge to the benchmark commodity market clearing condition, $Y_t = C_t$.

Four exogenous variables are defined here:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_A^2$$

(1.47)

$$\log Z_t = \rho_Z \log Z_{t-1} + \epsilon_Z^2$$

(1.48)

$$\log Y_t^* = \rho_Y \log Y_{t-1}^* + \epsilon_Y^2$$

(1.49)

$$\log R_t^* = \rho_A \log R_{t-1}^* + \epsilon_R^2.$$  

(1.50)

A competitive equilibrium is defined by a stream of endogenous variables, $\{C_t, Y_t, L_t, N_t, B_{F,t}, R_t, \frac{W_t}{P_t}, MC_t, \Pi_t, \Pi_{H,t}\}_{t=0}^\infty$, with four exogenous variables, $\{A_t, Z_t, Y_t^*, R_t^*, C_t^*\}_{t=1}^\infty$, that solve (13), (15), (24), (26), (27), (31), (35), (36), (39), (40), (42), (46), and (47) to (50).

1.4 Policy Problem

In this section, I formalize and solve for the Ramsey policy problem to find an optimal monetary policy under the frictions described in the previous section. To do so, I first list the system of log-linearized equations that consist of the optimal allocation equilibrium. Then, I use the system to derive the New Keynesian Phillips Curve (NKPC) and the IS relation in this version of this economy. I also build a simple linear quadratic social welfare loss function as an objective
function for the policy problem. The Ramsey policy problem is then defined by solving the welfare cost function, subject to the NKPC and IS relation.

1.4.1 Linearized System of Equations

To make the problem feasible, the first-order conditions for the optimal allocation equilibrium described in the previous section are log-linearized. The log-linearized equilibrium consists of endogenous variables \( \{ (w/p), n_t, c_t, y_t, b_{F,t}, s_t, \pi_t, \pi_{H,t}, r_t, mc_t \} \), and exogenous stochastic processes \( \{ a_t, z_t, y_t^*, c_t^*, r_t^* \} \) that solve

\[
\left( \frac{w}{p} \right) = \varphi n_t - \sigma c_t \tag{1.51}
\]

\[
0 = (1 - \alpha)E_t [s_{t+1} - s_t] - \sigma E_t [c_{t+1} - c_t] + \sigma E_t [c_{t+1}^* - c_t^*] - B_F \Psi_B b_{F,t} \tag{1.52}
\]

\[
0 = (1 - \alpha)E_t [s_{t+1} - s_t] - E_t [\pi_{t+1}] + r_t^* - r_t + B_F \Psi_B b_{F,t} \tag{1.53}
\]

\[
0 = (r_t - E_t [\pi_{t+1}]) - \sigma E_t [c_{t+1} - c_t] \tag{1.54}
\]

\[
\pi_t = \pi_{H,t} + \alpha E_t [s_{t+1} - s_t] \tag{1.55}
\]

\[
y_t = \frac{\omega_c}{\omega_c + \omega_{c^*}} c_t + \left( \frac{1 - \alpha}{\omega_c + \omega_{c^*}} - \alpha \right) s_t + \frac{\omega_{c^*}}{\omega_c + \omega_{c^*}} c_t^* \tag{1.56}
\]

\[
mc_t = -a_t + \gamma_w \left( \frac{w}{p} \right) + \gamma_n n_t \tag{1.57}
\]

\[
\pi_{H,t} = \beta E_t [\pi_{H,t+1}] + \kappa \hat{m} \hat{c}_t \tag{1.58}
\]

\[
y_t = a_t + n_t \tag{1.59}
\]

\[
\hat{r}_t = \mu \pi_{H,t} + \mu_y \hat{y}_t + z_t \tag{1.60}
\]

\[
y_t^* = c_t^* \tag{1.61}
\]

where \( x_t \approx \frac{X_t - X}{X} \), for any arbitrary variable, \( \omega_c \equiv (1 - \alpha)C, \omega_{c^*} \equiv \alpha \eta S^{\eta(1-\alpha)}C^* \), \( \gamma_w = \frac{W}{(W/w) + \Psi_N(1-N)} \), \( \gamma_n = \frac{2 \Psi_N(1-N)}{(W/w) + \Psi_N(1-N)} \), \( \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \), \( \hat{m} \hat{c}_t = mc_t - mc \), \( mc = -\varepsilon \varepsilon^{-1} \equiv -\mu \), and \( \hat{r}_t \) and \( \hat{y}_t \) will be defined later. Equations (51) to (56) make up the first block of the equilibrium, which is determined from the demand side of the economy. Equation (51) follows directly from (13), the real wage determination equation for domestic households. Equation (52) is a log-linearization result of (26), which represents the international risk-sharing condition. Equation (53) relates the interest
rates of the two economies, derived from (27), and (54) is a simple domestic Euler equation, from (16). Equation (55) is derived from (24), which explains the relationship between CPI inflation, domestic inflation, and the terms of trade of the home country. Equation (56) represents the market clearing condition for the domestic final goods, which is a log-linearized form of (46). Equations (57) and (58) make up the supply block of the economy. The real marginal cost of the economy is determined from (31), and is log-linearized to (57). Equation (58) is derived from the typical domestic firm’s price decision problem, following the staggered price setting of Calvo (1983) [14] and Yun (1996) [84], which is represented in (34) and (35). Equations (57) and (58) determine a modified version of the NKPC of the home economy, which will be derived in the next subsection. Equation (59) is obtained from the aggregate supply equation, (40), and (60) is a log-linearized version of the simple nominal interest rate decision rule set by the monetary authority, defined in (36). Finally, (61) is a market clearing condition for the world commodity market, which is defined by an exogenous stochastic process.

1.4.2 Open Economy New Keynesian Phillips Curve and IS Relation under Frictions

The marginal cost in (57) can be rewritten in terms of the output, $y_t$, foreign bond holdings, $b_{F,t}$, and the exogenous part of the economy, $a_t$ and $y_t^*$, using (51), (52), (56), (59), and (61):

$$mc_t = -a_t + \gamma_w \left( \frac{w}{P} \right) + \gamma_n n_t$$

$$= (\varphi \gamma_w + \gamma_n) y_t - \sigma \gamma_w c_t - (1 + \varphi \gamma_w + \gamma_n) a_t$$

$$= (\varphi \gamma_w + \gamma_n) y_t - (1 - \alpha) \gamma_w s_t + \gamma_w \Psi_B b_{F,t} - \sigma \gamma_w y_t^* - (1 + \varphi \gamma_w + \gamma_n) a_t$$

$$= \Theta_y y_t + \Theta_b b_{F,t} - \Theta_{y^*} y_t^* - \Theta_a a_t,$$

(1.62)

where $\Theta_s = \frac{(1-\alpha)\omega_c}{\sigma (\omega_c + \omega_c^*)} + \frac{(1-\alpha)\omega_c^*}{\omega_c + \omega_c^*} - \alpha$, $\Theta_y = [(\varphi \gamma_w + \gamma_n) - \Theta_s^{-1}(1 - \alpha) \gamma_w]$, $\Theta_b = \gamma_w \Psi_B B F [1 - \Theta_s^{-1}(1 - \alpha) \frac{\omega_c}{\omega_c + \omega_c^*}]$, $\Theta_{y^*} = \gamma_w [\sigma - \Theta_s^{-1}(1 - \alpha)]$, and $\Theta_a = (1 + \varphi \gamma_w + \gamma_n)$. Equations (51) and (59) are used in the first step in the above calculation to eliminate the real wage and labor, and (52) is implemented in the second step to replace the private consumption with international dimension variables, such
as terms of trade and foreign bond holdings. The last step removes the terms of trade part by inserting (56) into the equation. Note that in an autarky condition, where \( \alpha = 0 \), (62) replicates the log-linearized version of the benchmark marginal cost equation, removing the international part, including \( b_{F,t} \) and \( y_t^* \). Therefore, to see the extreme case of the model, the autarky version of the marginal cost is reduced to

\[
mc_t = [(\varphi - \sigma)\gamma_w + \gamma_n] y_t - (1 + \varphi\gamma_w\gamma_n) a_t.
\]

The complete form of the NKPC in this economy is as follows:

\[
\pi_{H,t} = \beta E_t [\pi_{H,t+1}] + \kappa \left[ \Theta_y \hat{y}_t + \Theta_b \hat{b}_{F,t} - \Theta y^* y_t^* - \Theta a a_t \right],
\]

(1.63)

where \( \hat{y}_t = y_t - y_t^n \) and \( y_t^n \) is the natural rate of output derived from the condition \( mc_t^n = -\mu \), which is specified as

\[
y_t^n = -\Theta^{-1} y - \Theta b_{F,t} + \Theta_y y_t^* + \Theta_y a_t,
\]

and \( \hat{b}_{F,t} = b_{F,t} - b_{F,t}^n \), where \( b_{F,t}^n \) is the natural rate of foreign bond holdings, which is assumed to be zero. According to the above NKPC, the signs of \( \Theta_y \), \( \Theta_b \), \( \Theta y^* \), and \( \Theta a \) all depend on \( \Psi_N \), as well as the steady-state values of \( C, S, \) and \( C^* \). Therefore, it is difficult to find the sole effect of \( \Psi_N \) on the parameters, but it is obvious that the parameters are affected by \( \Psi_N \) to some extent. Thus, \( \Psi_N \) can be a main factor changing the trade-off between the output gap stabilization and inflation stabilization problems faced by monetary policymakers. Its effect on the trade-off between output stabilization and inflation stabilization may be mitigated by the opposite effect of the imperfect financial market integration effect, \( \Psi_B \). To clarify the effect of \( \Psi_B \) on the international part of the curve, \( \Theta y^* \) and \( \Theta b \), we need to observe the sign of the effect of the financial adjustment cost for holding foreign currency denominated bonds, determined by \( 1 - \Theta_s^{-1} (1-\alpha) \frac{\omega_c}{\omega_c + \omega^F} \). If

\[
[1 - \Theta_s^{-1} (1-\alpha) \frac{\omega_c}{\omega_c + \omega^F}]
\]

is positive, \( \Psi_B \) moves the economy in the opposite direction, which means that \( \Psi_B \) worsens the relationship between inflation and the output level if \( \Psi_N \) has a positive effect on economic volatility. Therefore, \( \Psi_B \) also partly determines the level of the transmission mechanism of foreign shocks to the domestic business cycles. Moreover, the partially integrated
financial market assumption, represented by $\Psi_B$, can possibly move the economy in the opposite direction to $\Psi_N$ if $\Theta_b$ has the same sign as $\Theta_y^*$. Thus, it also changes the slope of the curve.

The open economy version of the IS relation is derived from (52), (54), and (56):

$$\hat{y}_t = E_t\hat{y}_{t+1} + \Sigma_y^*E_t(y^*_{t+1} - y^*_t) + \Sigma_b\hat{F}_{t,t} - \sigma_\alpha E_t(\hat{r}_t - \pi_{H,t+1}),$$

(1.64)

where $\Sigma_y^* = \left(\frac{1-\alpha}{\sigma\Theta_s}\right)^{-1} \left[\left(\frac{\sigma\Theta_s - (1-\alpha)}{\sigma\Theta_s}\right) + \frac{\alpha}{1-2\alpha}\right]$, $\Sigma_b = \frac{1}{\sigma} \left(\frac{1-\alpha}{\sigma\Theta_s}\right)^{-1} B_F \Psi_B \left[\alpha + 1 + \Theta_s^{-1}\left(\frac{1-\alpha}{\sigma}\right) \frac{\omega_c}{\omega_c + \omega_c^*}\right]$, and $\sigma_\alpha = \frac{1}{\sigma} \left(\frac{1-\alpha}{\sigma\Theta_s}\right)^{-1} \left(\frac{1-3\alpha}{1-2\alpha}\right)$. Note that as $\alpha$ goes to zero, creating the autarky commodity market condition, both $\Sigma_y^*$ and $\Sigma_b$ go to zero, $\sigma_\alpha$ replicates $\frac{1}{\sigma}$, the closed-economy version parameter, and the IS relation is reduced to that of the benchmark. It is straightforward to see that both $\Sigma_y^*$ and $\Sigma_b$ are positive values. Therefore, they positively influence the sensitivity of the output to the domestic interest rate changes. In this IS relation, as in the case of the NKPC, $\Psi_N$ and $\Psi_B$ participate in determining the value of those parameters.

### 1.4.3 Linear Quadratic Welfare Measure

Following Benigno and Woodford (2006) [8] and De Paoli (2009b) [61], I build a linear-quadratic (LQ) social welfare loss measurement. This welfare loss function has two advantages. First, the LQ form provides a tractable way of calculation by guaranteeing the existence of a local maximum, if parameters are appropriately defined. Second, it gives an easy way to compare alternative policy regimes. The social welfare loss function is derived by

$$W = -\frac{1}{2} \left[\frac{\xi}{\kappa} \pi^2_{H,t} + \Omega_y \hat{y}^2 + \Omega_b \hat{b}_{F,t}^2 - 2\Omega_{y,b} \hat{y} \hat{b}_{F,t} + 2\Omega_{y,\hat{y}} \hat{y} \hat{y}_t^* - 2\Omega_{b,\hat{y}} \hat{b}_{F,t} \hat{y}_t^*\right] + O(\|\xi\|^3) + t.i.p.,$$

(1.65)

where $O(\|\xi\|^3)$ denotes terms with order higher than three in the bound $\|\xi\|$ on the magnitude of the relevant shocks, and $t.i.p.$ represents terms independent of policy variables. A detailed derivation process is given in the appendix. According to (65), unlike the benchmark model in a complete market economy such as Gali and Monacelli (2005) [40], the foreign bond holdings affect the welfare level of the society. Moreover, the degree of financial market openness, $\Psi_B$, is
directly included in $\Xi_b$, which is part of $\Omega_b$, and positively affects the value of those two parameters. Therefore, $\Psi_B$ worsens the effect on the welfare loss function. On the other hand, although the degree of labor market friction, $\Psi_N$, is not directly included in (65), it still has an effect on the value of the welfare function by affecting the steady-state level of key variables.

### 1.4.4 Ramsey Policy Problem

The Ramsey policy problem is defined by maximizing (65) subject to (60), (63), and (64). The linear quadratic approximate solution is defined by the set $\{\hat{y}_t, \pi_{H,t}, \hat{b}_{F,t}, \hat{r}_t\}_{t=0}^\infty$. The Lagrangian equation for the policy problem is given by

$$
L_t = \sum_{t=0}^\infty \beta^t [\mathcal{W} + \chi_{1,t}[\mu_\pi \pi_{H,t} + \mu_y \hat{y}_t + z_t - \hat{r}_t] \\
+ \chi_{2,t} \left[ \beta E_t [\pi_{H,t+1}] + \kappa \left( \Theta_y \hat{y}_t + \Theta_b \hat{b}_{F,t} - \Theta_y^* y_t^* - \Theta_a a_t \right) - \pi_{H,t} \right] \\
+ \chi_{3,t} \left[ E_t \hat{y}_{t+1} + \sum_y \pi_{t+1} \hat{b}_t + \sigma_\alpha E_t (\hat{r}_t - \pi_{H,t+1}) - \hat{y}_t \right],
$$

where $\chi_{1,t}$, $\chi_{2,t}$, and $\chi_{3,t}$ are the shadow prices of the interest rule decision equation, NKPC, and IS relation, respectively. The first-order conditions for the policy problem are as follows:

$$
-\Omega_y \hat{y}_t + \Omega_{y,b} \hat{b}_{F,t} - \Omega_{y,y^*} y_t^* + \chi_{1,t} \mu_y + \chi_{2,t} \kappa \Theta_y - \chi_{3,t} + \beta \chi_{3,t+1} = 0 \\
-\varepsilon \pi_{H,t} + \chi_{1,t} \mu_\pi - \chi_{2,t} + \beta \chi_{2,t+1} + \chi_{3,t} \sigma_\alpha = 0 \\
\Omega_{y,b} \hat{y}_t - \Omega_{b,y^*} y_t^* - \Omega_{b,b} \hat{b}_{F,t} + \chi_{2,t} \kappa \Theta_y + \chi_{3,t} \Sigma_b = 0 \\
\chi_{1,t} + \sigma_\alpha \chi_{3,t} = 0.
$$

The above system of equations are combined with (60), (63), and (65) to find the optimal level of output gap, $\hat{y}_t$, domestic inflation rate, $\pi_{H,t}$, foreign bond holdings, $\hat{b}_{F,t}$, and nominal interest rate, $\hat{r}_t$.

### 1.5 Simulation

In this section, I quantitatively study the model to verify the effect of the two idiosyncratic frictions on the economic dynamics and the optimal monetary policy problem. Furthermore, I
change the monetary policy parameters and observe the changes in the equilibrium to find the optimal weights for domestic inflation and output gap stabilization. As many recent works on the monetary policy of emerging market economies argue, the output gap targeting rule is an interesting topic in central banking literature. This issue motivates the second part of this section. To start with, I adopt established parameter values from several notable studies on economic volatility in emerging markets in the New Keynesian fashion. After that, in a simulation using the Dynare software, I obtain impulse responses of key macroeconomic variables in equilibrium to three types of exogenous stochastic processes: a domestic productivity shock, a domestic monetary policy shock, and a foreign demand shock. Note that the previously defined foreign interest rate shock, $r^*_t$, can be written in terms of a foreign demand shock, $y^*_t$, after some straightforward calculations:

$$r^*_t = \sigma(y^*_{t+1} - y^*_t). \quad (1.67)$$

Therefore, to domestic agents, a foreign interest rate shock can be understood as a weighted foreign demand shock differential. According to the results, while the higher level of financial market friction creates more volatility in the economy in all three responses to the exogenous shocks, the higher labor market friction moves in the opposite direction, thus reducing the economic volatility. Furthermore, if a monetary policy authority targets domestic inflation more aggressively, the economy loses more welfare than when targeting the output gap or the benchmark target.

### 1.5.1 Parameterization

Table 1.1 shows the parameter values used in the simulation. The parameter values that affect the steady-state level of the economy are assigned to those commonly used in small open economy literature. The inverse elasticity of labor supply, $\varphi$, is set to unity, and the intertemporal elasticity of substitution between private consumption, $\sigma$, is set to two, following Demirel (2010) [29]. I adopt the international dimension of parameters from Gali and Monacelli (2005) [40]. The intratemporal elasticity of substitution between home and foreign final goods is set to two and the degree of openness to foreign commodity markets is set to 0.4. The time discounting factor and
the intertemporal elasticity of substitution among final goods are assumed to be identical across countries, and are set to 0.99 and 11, respectively. The degree of price stickiness is supposed to be 0.7, and should be around 0.66 according to many recent New Keynesian works, such as Woodford (2010) [83]. The degree of labor market friction, $\Psi_N$, is set to 0.98, as in Janko (2008) [43], and the financial friction measurement parameter, $\Psi_B$, is set to 0.00042, following Uribe and Yue (2006) [77]. All steady-state values are calculated analytically, and I adopt the benchmark numbers for monetary policy parameters that determine the weights on inflation and output gap targeting, $\mu_\pi$ and $\mu_y$, from Gali (2008) [39]. There are three types of exogenous shocks in this section. For the set of stochastic process parameter values that do not affect the steady-state levels in the model, $\{\rho_A, \rho_Z, \rho_Y, \epsilon_A, \epsilon_Z, \epsilon_Y\}$, I adopt values such that the model replicates key macroeconomic features of the Korean economy represented in Neumeyer and Perri (2005) [57]. The standard deviation of the output and interest rate, and the ratio of these two second moments are targeted. The stochastic process parameters for the domestic productivity shock follow the values of Neumeyer and Perri (2005) [57], the parameters for the domestic monetary policy shock are set similar to Smets and Wouters (2002) [71], and the parameter values for the foreign demand shock is set similar to Gali and Monacelli (2005) [40]. As shown in Table 1.2, the model fails to replicate the actual standard deviations of the output gap and interest rate of the Korean economy, but is relatively more successful in replicating the ratio of the two second moments. The theoretical simulated moments are about ten times higher than the real moments, but the ratio of the output gap to interest rate is within a reasonable distance of the actual ratio.

1.5.2 The Effect of Frictions on the Economic Volatility

Table 1.3 shows the theoretical moments of key macroeconomic variables with different levels of the two frictions. The table compares the baseline model to the case with higher $\Psi_B$ or $\Psi_N$, and shows the changes in social welfare loss in each case. Therefore, the changes in standard deviation or variance explain how those two parameters affect the overall economic volatility and social welfare cost. The table indicates that a higher level of financial adjustment cost creates
higher volatility, and therefore, the economy with the higher level of financial adjustment cost has the higher level of welfare loss. The logic behind the negative effect of the partial financial separation can be shown first in the real marginal cost structure in equation (62). The permanent increase in $\Psi_B$ amplifies the value of $\Theta_b$, the coefficient of $\hat{b}_{F,t}$. This permanent change steepens the slope of the NKPC, and thus exacerbates the trade-off between domestic inflation stabilization and output gap stabilization. This requires that the policymaker sacrifice more to obtain the same level of stabilization as before. Another source of the effect can be found in (65), the LQ welfare loss function. Since $\Psi_B$ permanently increases the value of $\Xi_b$, and participates in many parts of the loss function as well, the welfare cost is directly affected by the change of the parameter and, thus, the cost changes significantly. On the other hand, the higher level of the labor market adjustment cost softens the volatility of the economy and, thus, the economy achieves a lower level of welfare cost. Intuitively, this negative effect of higher financial separation on economic volatility can be understood as the result of imperfect international risk sharing and consumption smoothing that mitigate the potential benefit of financial openness and exaggerate the social cost that the economy must bear. The effect of $\Psi_N$ is not explicitly shown in either the NKPC or the welfare loss function, and thus it is difficult to separate its effect from the effect of the change in steady-state values, which are combined with $\Psi_N$ in all parameters for these two curves. However, from the simulation result, it appears that the parameters in the NKPC, such as $\Theta_y$ or $\Theta_a$, with higher levels of $\Psi_N$, move opposite to $\Theta_b$. Thus, $\Psi_N$ mitigates the effect of $\Psi_B$ on the business cycle stability. For instance, $\Psi_N$ can reduce the absolute value of $\Theta_a$, reducing the overall effect of the domestic productivity shock on the economy, while $\Psi_B$ cannot participate in the change to $\Theta_a$, since it is only involved in $\Theta_b$. Intuitively, the labor market allocation friction makes the economy react more sluggishly, and thus the exogenous shocks becomes less effective under this condition. Figure 1.2 shows the change in variances of the three key variables as the level of $\Psi_B$ and $\Psi_N$ gradually increase. The three variances increase as $\Psi_B$ goes up, but decrease as $\Psi_N$ goes down. Even though we cannot accurately determine the change in the two parameters and the absolute value of the change does not have a significant meaning, it is still clear how the change in the two parameter values affects
the volatility of the economy. In summary, while imperfect financial market integration negatively affects economic volatility, the real labor market adjustment cost mitigates this negative effect by moving the economy in the opposite direction.

### 1.5.3 Alternative Policy Regimes

In this subsection, I change the values of the two policy parameters in (60), $\mu_\pi$ and $\mu_y$, to find the best monetary policy for the economy under the described frictional environment. The original model is assumed to set $\mu_\pi$ to 1.5 and $\mu_y$ to 0.125, following Gali (2008) [39], which takes these values from Taylor (1993) [75]. I suppose two alternative cases: A policy emphasizing domestic inflation targeting, which sets $\mu_\pi$ to 3 while leaving $\mu_y$ unchanged, and a policy emphasizing output gap targeting, setting $\mu_y$ to 0.5, leaving $\mu_\pi$ unchanged. Table 1.4 shows the theoretical moments of key macroeconomic variables and welfare losses of these three cases. According to the results, the economy experiences a higher level of volatility with a higher value of $\mu_\pi$ in its monetary policy, but has lower levels of economic volatility with a higher value of $\mu_y$. This means that a relatively aggressive domestic inflation stabilization regime bears a higher social welfare cost with more unstable economics variables. However, emphasizing the output gap stabilization policy reduces the welfare cost. While targeting inflation targeting achieves a marginal success in stabilizing domestic inflation, targeting the output gap obtains better outcomes in every area other than domestic inflation. Figures 1.3 to 1.5 show the impulse responses of key variables to the different types of shocks. Figure 1.3 represents the responses to a positive domestic productivity shock. The aggressive output gap targeting policy beats the other two regimes in terms of stabilization. A puzzling part of the figure is the change in the nominal interest rate, which decreases even though the output gap increases. This procyclical movement of the interest rate can be interpreted as a countercyclical reaction of the central bank, which focuses more on the deflationary situation and the appreciated domestic currency concern. Figure 1.4 shows the impulse responses to a domestic monetary shock. In this contractionary situation, the output gap targeting is still the best of the three candidates. Figure 1.5 shows the responses to a foreign demand shock. Once again, the output gap targeting
rule is the best of the three. The interest rate again reacts countercyclically to the decreasing output gap, but it helps in stabilizing the inflation situation and solves the depreciation problem.

1.6 Conclusion

This study investigated the effect of frictions in the labor and financial markets on economic volatility, and determined an optimal monetary policy rule under these circumstances. The study also tested alternative monetary policy regimes to find the better policy regime for this economy. The labor demand differential between the current and steady-state levels is assumed to create an additional adjustment cost in a production sector. Furthermore, the financial adjustment cost is added to the budget constraint of the home country consumer when purchasing a foreign currency denominated asset, and represents an additional cost when accessing foreign asset markets and an imperfect integrated financial market. These two frictions change the structure of the real marginal cost in the NKPC, the coefficient values in the IS relation, and the LQ welfare loss function. They also permanently change the slope of the curves, which means that the monetary policymaker should face a worse or softened trade-off between the domestic inflation and output gap stabilization problem. With appropriate parameterization, the imperfectly integrated financial market exacerbates the trade-off in the output gap and domestic inflation stabilization problem and creates a higher level of volatility. However, labor market friction mitigates this negative effect by reducing the level of volatility. The policy implications of these findings are that if the labor market is not fully flexible, as in the reallocation problem in labor demand that creates an additional adjustment cost, the economy is less vulnerable to domestic and foreign shocks in terms of stability. However, the degree of imperfect integration in the financial market can boost the transmission of the shocks by making the economy more sensitive. I also tested alternative monetary policy regimes by changing policy parameters, enabling the central bank to focus its interest rate decision on domestic inflation or the output gap. According to the results, a policy that emphasizes domestic inflation stabilization carries a higher social welfare cost than both the baseline model and the policy emphasizing the output gap stabilization. A relatively volatile output
gap in these economic circumstances may induce this result. Therefore, policymakers facing these two types of friction should consider a policy focusing on domestic output gap stabilization as being optimal.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>Reverse of Elasticity of Labor Supply</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal Elasticity of Substitution in Private Consumption</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Intratemporal Elasticity of Substitution between Home and Foreign Final Goods</td>
<td>2</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Degree of Openness to Foreign Commodity Market</td>
<td>0.4</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time Discounting Factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Intratemporal Elasticity of Substitution among Final Goods</td>
<td>11</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Markup Revenue</td>
<td>1.1</td>
</tr>
<tr>
<td>$\theta$</td>
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<tr>
<td>$Y$</td>
<td>Steady State Value of $Y_t$</td>
<td>0.6324</td>
</tr>
<tr>
<td>$C$</td>
<td>Steady State Value of $C_t$</td>
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</tr>
<tr>
<td>$N$</td>
<td>Steady State Value of $N_t$</td>
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<td>$B_F$</td>
<td>Steady State Value of $B_{F,t}$</td>
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</tr>
<tr>
<td>$\Psi_N$</td>
<td>Degree of Labor Market Friction</td>
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</tr>
<tr>
<td>$\Psi_B$</td>
<td>Degree of Financial Market Friction</td>
<td>0.00042</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>Autoregressive Parameter of Domestic Productivity Shock</td>
<td>0.7</td>
</tr>
<tr>
<td>$\rho_Z$</td>
<td>Autoregressive Parameter of Domestic Monetary Policy Shock</td>
<td>0.9</td>
</tr>
<tr>
<td>$\rho_Y$</td>
<td>Autoregressive Parameter of Foreign Demand Shock</td>
<td>0.86</td>
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<tr>
<td>$\epsilon_A$</td>
<td>Standard Deviation of Domestic Productivity Shock</td>
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<td>$\epsilon_Z$</td>
<td>Standard Deviation of Domestic Monetary Policy Shock</td>
<td>0.01</td>
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<tr>
<td>$\epsilon_Y$</td>
<td>Standard Deviation of Domestic Foreign Demand Shock</td>
<td>0.007</td>
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<tr>
<td>$\mu_{\pi}$</td>
<td>Monetary Policy Parameter for log of Inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>Monetary Policy Parameter for log of Output Gap</td>
<td>0.125</td>
</tr>
<tr>
<td>$R$</td>
<td>Policy Anchor Value of Interest Rate</td>
<td>1.0264</td>
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Table 1.1: Baseline Parameter Values
Table 1.2: Simulated vs. Targeted Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Simulated Value</th>
<th>Estimated Value</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD. DEV. of Output</td>
<td>0.4733</td>
<td>0.0354</td>
<td>0.4379</td>
</tr>
<tr>
<td>STD. DEV of Interest Rate</td>
<td>0.2045</td>
<td>0.0142</td>
<td>0.1903</td>
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<tr>
<td>Ratio of STD. DEV. of Output to STD. DEV. of Interest Rate</td>
<td>2.3144</td>
<td>2.4929</td>
<td>0.1785</td>
</tr>
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</table>

Table 1.3: Theoretical Moments: Changes in $\Psi_B$ and $\Psi_N$ and Contributions to Welfare Losses (HP filter, lambda = 1600)

<table>
<thead>
<tr>
<th>Benchmark $\Psi_B = 0.00042$, $\Psi_N = 0.98$</th>
<th>STD. DEV.</th>
<th>VARIANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Gap ($\hat{y}_t$)</td>
<td>0.4733</td>
<td>0.2240</td>
</tr>
<tr>
<td>Domestic Inflation ($\pi_{H,t}$)</td>
<td>0.1768</td>
<td>0.0312</td>
</tr>
<tr>
<td>CPI Inflation ($\pi_t$)</td>
<td>0.8996</td>
<td>0.8093</td>
</tr>
<tr>
<td>Interest Rate ($r_t$)</td>
<td>0.2045</td>
<td>0.0418</td>
</tr>
<tr>
<td>Terms of Trade ($s_t$)</td>
<td>12.8976</td>
<td>166.3469</td>
</tr>
<tr>
<td>Foreign Bonds Holding Gap ($\hat{b}_{F,t}$)</td>
<td>0.2064</td>
<td>0.8093</td>
</tr>
<tr>
<td>Welfare Loss</td>
<td>.</td>
<td>18.3573</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Financial Adjustment Cost $\Psi_B = 0.001$, $\Psi_N = 0.98$</th>
<th>STD. DEV.</th>
<th>VARIANCE</th>
</tr>
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<tbody>
<tr>
<td>Output Gap ($\hat{y}_t$)</td>
<td>0.4869</td>
<td>0.2370</td>
</tr>
<tr>
<td>Domestic Inflation ($\pi_{H,t}$)</td>
<td>0.1810</td>
<td>0.0328</td>
</tr>
<tr>
<td>CPI Inflation ($\pi_t$)</td>
<td>0.9246</td>
<td>0.8548</td>
</tr>
<tr>
<td>Interest Rate ($r_t$)</td>
<td>0.2092</td>
<td>0.0437</td>
</tr>
<tr>
<td>Terms of Trade ($s_t$)</td>
<td>13.2678</td>
<td>176.0343</td>
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<tr>
<td>Foreign Bonds Holding ($b_{F,t}$)</td>
<td>0.2119</td>
<td>0.4490</td>
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<td>Welfare Loss</td>
<td>.</td>
<td>20.6420</td>
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</table>

<table>
<thead>
<tr>
<th>High Labor Adjusment Cost $\psi_B = 0.00042$, $\psi_N = 1.5$</th>
<th>STD. DEV.</th>
<th>VARIANCE</th>
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<tbody>
<tr>
<td>Output Gap ($\hat{y}_t$)</td>
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<tr>
<td>Domestic Inflation ($\pi_{H,t}$)</td>
<td>0.1607</td>
<td>0.0258</td>
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<tr>
<td>CPI Inflation ($\pi_t$)</td>
<td>0.8187</td>
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<tr>
<td>Interest Rate ($r_t$)</td>
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<td>0.0346</td>
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<tr>
<td>Terms of Trade ($s_t$)</td>
<td>11.7370</td>
<td>137.7572</td>
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<tr>
<td>Foreign Bonds Holding ($b_{F,t}$)</td>
<td>0.1885</td>
<td>0.0356</td>
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<tr>
<td>Welfare Loss</td>
<td>.</td>
<td>16.4562</td>
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</table>
Table 1.4: Theoretical Moments: Alternative Policy Regimes and Contributions to Welfare Losses
(HP filter, lambda = 1600)

<table>
<thead>
<tr>
<th></th>
<th>Benchmark (OPT) $\mu_\pi = 1.5, \mu_y = 0.125$</th>
<th>Emphasis on Domestic Inflation Targeting (DIT) $\mu_\pi = 3, \mu_y = 0.125$</th>
<th>Emphasis on Output Gap Targeting (OGT) $\mu_\pi = 1.5, \mu_y = 0.5$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>STD. DEV. VARIANCE</td>
<td>STD. DEV. VARIANCE</td>
<td>STD. DEV. VARIANCE</td>
</tr>
<tr>
<td>Output Gap ($\hat{y}_t$)</td>
<td>0.4733 0.2240</td>
<td>0.7412 0.5493</td>
<td>0.3325 0.1105</td>
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<tr>
<td>Domestic Inflation ($\pi_{H,t}$)</td>
<td>0.1768 0.0312</td>
<td>0.1678 0.0282</td>
<td>0.1814 0.0329</td>
</tr>
<tr>
<td>CPI Inflation ($\pi_t$)</td>
<td>0.8996 0.8093</td>
<td>1.2849 1.6511</td>
<td>0.6981 0.4873</td>
</tr>
<tr>
<td>Interest Rate ($r_t$)</td>
<td>0.2045 0.0418</td>
<td>0.4089 0.1672</td>
<td>0.0976 0.0095</td>
</tr>
<tr>
<td>Terms of Trade ($s_t$)</td>
<td>12.8976 166.3469</td>
<td>19.9097 396.3976</td>
<td>9.2135 84.8878</td>
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<tr>
<td>Foreign Bonds Holding Gap ($\hat{b}_{F,t}$)</td>
<td>0.2064 0.8093</td>
<td>0.3490 0.1218</td>
<td>0.1319 0.0174</td>
</tr>
<tr>
<td>Welfare Loss</td>
<td>. 18.3573</td>
<td>. 18.9721</td>
<td>. 14.4236</td>
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</tbody>
</table>
Figure 1.1: Characteristic Statistics of South Korea
Figure 1.2: Variances of Key Variables with Changes in Labor Adjustment Cost and Financial Adjustment Cost
Figure 1.3: Impulse Responses to a 1% Positive Domestic Productivity Shock: Comparisons among Alternative Monetary Policy Regimes
Figure 1.4: Impulse Responses to a 1% Positive Domestic Monetary Policy Shock: Comparisons among Alternative Monetary Policy Regimes
Figure 1.5: Impulse Responses to a 1% Positive Foreign Demand Shock: Comparisons among Alternative Monetary Policy Regimes
Chapter 2

Cyclicality of Optimal Stabilization Policy in Developing Countries under Frictions: Role of Institutional Cost Associated with Providing Public Goods

2.1 Introduction

A recent report by Frankel (2011)[36] and Vegh and Vuletin (2012)[78] demonstrate a sharp contrast between industrialized and developing countries in terms of cyclicality of macroeconomic policies. Many of developing countries have experienced a significant level of procyclical fiscal and monetary policies while most developed countries have had acyclical or countercyclical policy regimes since 1960. There has been a rich volume of literature trying to explain this puzzling policy issue, mostly focusing on political economy based theory or microeconomic finance assumptions. Meanwhile, one may wonder if the sharp contrast of cyclicality of macroeconomic policy between developed and developing countries comes from a macroeconomic fundamental difference, such as different level of social cost associated with public goods spending. Can higher level of institutional cost associated with providing public goods be an answer for the procyclical macroeconomic stabilization policy in the fast growing countries? Can that puzzling policy trend be an optimal one? If so, what is the best combination of fiscal and monetary policy to stabilize their business cycle fluctuations? To answer these questions, I investigate the effect of higher institutional cost which is caused from the public expenditure on the procyclical fiscal and monetary policy in developing economies. I further studies to design the optimal fiscal and monetary policy for those countries under this certain friction. To do so, I build a simple new Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with an assumption of a real friction, which is created from a spread
between current and efficient levels of government spending. I then solve a Ramsey policy problem with quadratic welfare loss function to find the optimal package of fiscal and monetary policies under this special economic circumstance. The paper examines how the additional or higher level of institutional cost leads to the procyclical fiscal and monetary policy trend, and that kind of procyclicality is economically rational, which means that it is optimal policy regime under this specific assumption.

Main finding of this paper is that the institutional cost associated public goods expenditure deepens the level of procyclical fiscal and monetary policies, and this kind of procyclicality is economically rational and optimal. Moreover, higher level of the effect of the institutional cost on the economy induces the higher level of economics volatility. To be more specific, higher level of the effect of the institutional cost amplifies the effect of exogenous shock on the economy, therefore it worsens trade-off between stabilizing inflation and output gaps. (For example, if there is a negative technology shock, monetary policy maker should increase interest rate to stabilize the price level, while it gives up the volatility of output due to the interest rate change.) It means that, if there is a negative productivity shock to the economy, there should be a pressure on the inflation, and in this situation, for a fiscal authority, since the existence of the higher cost makes a part of government spending and output to be wasted, or to be wasteful (because with the additional cost it is more costly to use a unit of government spending), it is required to more aggressively decrease government spending to stabilize the economy. Another finding of the paper is that, by testing different candidates of Taylor rule type monetary policy based on the welfare loss function criterion, forward looking inflation targeting achieves significant welfare gain while aggressively weighting on normal inflation targeting or output gap targeting has no economic merit. Forward looking inflation targeting enhances policy maker’s ability to protect the economy from exogenous monetary policy shock, which significantly reduces overall economic volatility.

In this closed economy model, in addition to the widely used nominal frictions in new Keynesian model, nominal price rigidity and monopolistic competition in a production sector, I introduce a real friction of the government spending spread which is captured from the fact that many emerging
market or developing economies still have a difficulty in financing cheap public expenditure sources that possibly hampers further economic growth and sustainable stabilization of business cycles.¹ This main assumption is represented in the model as a type of a quadratic form of real adjustment cost which is generated when current level of public expenditure is different from the efficient level of the public expenditure. This real adjustment cost can be interpreted as an institutional cost associated with the public goods spending, and thus the cost negatively affects on the trade-off between inflation gap and output gap stabilization encountered by monetary policy authority. The government spending is considered as a physical public expenditures which is consumed in the form of utility function of representative households, and a degree of the effect of the government spending spread is scaled by a specific parameter (ξ) in the model, which is a key to explain the main findings of this paper.

The effect of overall cost for the public expenditure, combined by parameter value of the government spending spread and the quadratic adjustment cost itself, has an effect on the trade-off between inflation and output gap stabilization policy problem through direct and indirect channels. In the direct channel, the higher value of the adjustment cost steepens the slope of New Keynesian Phillips Curve (NKPC) and IS relation, and amplifies the effect of exogenous productivity shock in these two relations. This results in a worsened trade-off in the relations that policy makers face in their decision making process. The exacerbated trade-off forces policy makers to bear higher level of volatility in key macroeconomic variables and related policy variables to achieve an effective level of inflation in order to stabilize an output, or vice versa. The policy makers also should conduct higher level of changes in the instruments to stabilize the economic fluctuations. This mechanism mainly induces the first two findings of the paper. Additionally, in the indirect channel, the cost from real friction changes weights on each variable in the welfare-based linear quadratic objective function of policy makers in Ramsey problem. In the linear-quadratic form of welfare loss function, changes in degree of the institutional cost can amplify a relative weight on policy variables that

¹ Straub (2008)[72] empirically points out that there is a significant level of deficiencies of infrastructure in many developing countries and the lack of public service is strongly linked to the discouragement of macroeconomic development.
includes public expenditure. With this change in relative importance of each variable in the objective function, policy makers perceive different level of effectiveness of fiscal or monetary policy on the economic dynamics, and decides to conduct more aggressive stabilization policy tool in order to maintain the same level of economic volatility. This structural change also contributes to the main findings.

The main contribution of this paper is that, while it fails to provide a direct causality of procyclical fiscal and monetary policy in developing countries, it still succeeds in proving that the institutional cost associated with the public goods expenditure deepens the abnormal procyclical trend of the macroeconomic policies and the policy is economically optimal. There has been a rich volume of literature on the possible reasons for procyclicality in developing economies, but unfortunately rare chance of global consensus has been driven. This paper suggests that, without considering political economy dimensions such as Talvi and Vegh (2005)[73] or [3], the lack of smooth public goods expenditure, a common feature across the most of developing countries, can endogenously worsens the puzzling tendency of policy regimes. Furthermore, the paper argues that under that kind of economic environment, a procyclical macroeconomic policy is logically optimal, as a possible solution for the puzzling economic phenomena. Another potential contribution of this paper to the related literature is that, the paper opens a new room for a discussion on policy implications of business cycles with an additional cost that captures an imperfect public expenditure structure.

Public investment has been widely studied in development or growth literature as a main driver of economic stimulation, but rarely discussed in business cycle literature. Furthermore, a research on real frictions caused by the imperfectly supplied public goods combined with a nominal rigidity of prices has been little ignored in the field, although the importance of the effect of the combined friction on the economic volatility in many developing countries has been increased. Even though the paper has a limitation of closed economy model that ignores the effect of international dimension such as an effect of exchange rate pass-through or collateral constraint of national debt, this paper still has an edge by providing an insight on the policy implications under circumstances of imperfectly supplied public goods that the policy authority should consider the public spending
spread in order to achieve optimally stabilized macroeconomic variables.

2.2 Literature Review

In this section, I discuss related literature to the key features of the model in this paper. The model mainly focuses on the effect of imperfectly supplied public goods on economic dynamics and policy cyclicality. A real adjustment cost illustrates the gap between current and natural levels of government spendings, which exemplifies the lack of perfectly provided public goods affecting business cycle of the economy. Baier and Glomm (2001)[5], Rioja (1999)[64] and Rioja (2003)[65] examine the effect of development in infrastructure on economic development in neoclassical fashion. Especially Baier and Glomm (2001)[5], putting distortionary taxes in the model, find that the infrastructural development can effectively stimulate the economic growth with appropriate level of elasticity of substitutions between inputs. Azzimonti et al. (2009)[4] build a Ramsey policy problem with alternative technical approaches, to compare welfare losses between commitment and discretion cases when productive public capital is introduced in the model. It shows that welfare loss under discretion relative to the commitment case is minimal. Leeper (2010)[51] build a neoclassical model to find the delayed implementation effect of government investment on the economics growth. The paper reveals that an unanticipated delay of public investment can possibly discourage labor and output growth in short run.

This paper is also interested in a procyclicity of macroeconomic policies. Validity of procyclical fiscal policy has long been an important issue of debate in related literature, while many researchers have tried to find the main determinant of the procycality on the other hand. Papers such as Kaminsky et al. (2004)[45] and Alberola and Montero (2006)[2] empirically demonstrate the recent trend of developing economies that have exhibited procyclicality of important macroeconomic indicators including fiscal and monetary policies. Many papers in the literature have made an effort to validate that kinds of procyclical economic policies with variety of theoretical approaches. Talvi and Vegh (2005)[73] insist that even in an economic boom sustaining budget surplus is costly for some developing countries because there is an ongoing political pressure to spend more tax revenue.
While Ilzetzki (2011) and Alesina and Tabellini (2005) also focus on the political economy side factors on the procyclicality, Tornell and Lane (1999) endogenously solve the unexpectedly increased fiscal redistributions by using the term "voracity effect." Inspired by recent data set, Mendoza and Oviedo (2006) point out that governments in emerging market economies behave like a "tormented insurer," which means that the fiscal authority spends more money on private sector to defend the reduction of variability of revenue as economy enjoys boom, and thus it creates the procyclical fiscal policy regimes in those regions. Upon these findings, Demirel (2010) argues that in a small open economy model with the existence of country spread, optimal stabilization policy is procyclical.

Methodologically this paper aims at finding a mix of optimal fiscal and monetary stabilization policy by using Ramsey problem with linear-quadratic welfare loss function. The paper follows pioneering works of papers such as Benigno and Woodford (2012), Schmitt-Grohe and Uribe (2003), and Schmitt-Grohe and Uribe (2004). The papers enlighten the way of finding both optimal fiscal and monetary policies simultaneously by implementing well-defined Ramsey problems. Especially Benigno and Woodford (2012) provide an ample theoretical background for the benefit of linear-quadratic welfare measure. According to the paper, the functional form gives the enough possibility of unique solution as well as easiness of comparing alternative policy regimes.

### 2.3 Model

Analysis on cyclicality of fiscal and monetary policy regimes starts with a dynamic stochastic general equilibrium model of a new Keynesian economy. Based on the benchmark features of a closed economy new Keynesian model such as staggered price setting following Calvo (1983) and monopolistic competition in production sectors, I add a real quadratic adjustment cost in the economy as one of the main distortions, which is defined by a government spending spread between current and efficient level. This cost captures an institutional cost associated with public goods that hampers efficiently utilized public spending process. The economy consists of households, firms, and governments which complete a competitive equilibrium. A identically populated household
consumes private and public goods and provides inelastic unit labor, firms produces differentiated goods in monopolistic competitive fashion, fiscal authority collects lump sum tax and decides the amount of transfers, and monetary authority sets nominal interest rate as a stabilizing policy.

2.3.1 Households

Identically populated households live infinitely and maximize the discounted expectation of a lifetime utility function. Preferences of a representative household is defined by

\[ U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{G_t^{1-\phi}}{1-\phi} - \chi \frac{L_t(i)^{1+\varphi}}{1+\varphi} \right] \tag{2.1} \]

where \( C_t, G_t, \) and \( L_t \) denote the level of composite private and public consumption and labor supplied, respectively. \( E_t \) is an expectation operator conditional on all information given at time \( t \). For parameters, \( 0 < \beta < 1 \) is time discounting factor, \( \sigma > 0 \) and \( \phi > 0 \) stand for inter-temporal elasticity of substitution of private and public consumption, and \( \varphi > 0 \) is a reverse of an elasticity of labor supply. \( \chi_G \) and \( \chi_L \) are relative weights on public consumption and disutility of labor supply, but I assume that they are normalized by one hereafter for convenience of calculation. The composite private or public consumption is assumed to be a continuum of differentiated goods produced by numerous final goods producers indexed by \( i \in [0,1] \) and defined by

\[ C_t = \left( \int_0^1 C_t(i)^{\frac{1}{1-\sigma}} di \right)^{\frac{1}{\frac{1}{1-\sigma}}} \tag{2.2} \]

\[ G_t = \left( \int_0^1 G_t(i)^{\frac{1}{1-\sigma}} di \right)^{\frac{1}{\frac{1}{1-\sigma}}} \tag{2.3} \]

where \( \delta > 1 \) is an intra-temporal elasticity of substitution between differentiated goods. The labor supply is aggregated by individual labors dedicated to each differentiated production sector:

\[ L_t = \int_0^1 L_t(i) di \tag{2.4} \]

Consumption price index (CPI) is calculated by

\[ P_t = \left( \int_0^1 P_t(i)^{1-\delta} di \right)^{\frac{1}{1-\delta}} \tag{2.5} \]
Furthermore, a representative household’s demand function for each differentiated private good is calculated by

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\delta} C_t \]  \hspace{1cm} (2.6)

The budget constraint for the representative household at period \( t \) is determined by

\[ P_t C_t + B_t \leq W_t L_t + R_{t-1} B_{t-1} + T_t + \Gamma_t \]  \hspace{1cm} (2.7)

where \( B_t \) is nominal bond holdings printed by government, \( R_t \) is the gross interest rate set by monetary policy authority, \( W_t \) denotes the nominal wage for unit amount of labor, \( T_t \) stands for a lump-sum type tax or transfer from government, and \( \gamma_t \) is the the profit of firms since the firms are assumed to be owned by households. To prevent the possibility of Ponzi scheme, the following additional condition is needed:

\[ \lim_{k \to \infty} E_t \left( \prod_{j=0}^{k} \frac{B_{t+k+1}}{R_{t+j}} \right) \geq 0 \]  \hspace{1cm} (2.8)

The household’s problem is defined by the maximization of (2.1) with respect to \( C_t, L_t, \) and \( B_t \), subject to (2.7) and (2.8). The first order necessary conditions are calculated by

\[ \frac{W_t}{P_t} = L_t^\sigma C_t^\sigma \]  \hspace{1cm} (2.9)

\[ 1 = \beta E_t R_t \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \]  \hspace{1cm} (2.10)

Equation (2.9) indicates the mechanism of labor supply or real wage determination. The real wage is determined by weighted combination of labor supply and private consumption. Equation (2.10) is a simple Euler equation that relates inter-temporal consumption streams to future inflation rate, \( \frac{P_{t+1}}{P_t} \) and nominal interest rate, \( R_t \), which are weighted by time discounting factor, \( \beta \). This also represents that the marginal utility for the private consumption at the current period should be equal to the discounted marginal utility of future consumption.

### 2.3.2 Firms

Production sector is assumed to have infinitely many firms indexed by \( i \) on the unit interval \([0, 1]\), and each firm produces a differentiated good in a monopolistically competitive environment.
The each firm has a constant return to scale technology,

$$Y_t(i) = A_t N_t(i)$$  \hspace{1cm} (2.11)

where $Y_t(i)$ is an amount of output for a good $i$, $A_t$ is an economy-wide common productivity shock that follows AR(1) stochastic process which will be defined later, and $N_t(i)$ is an amount of labor demanded for production sector $i$. Cost minimization problem for each firm solves for a nominal marginal cost which is denoted by $MC_t(i)$, to be a function of nominal wage and productivity shock:

$$MC_t(i) = \frac{W_t}{A_t}$$  \hspace{1cm} (2.12)

Furthermore, an aggregate level of labor demanded is a simple sum of each sector’s amount of labor demanded:

$$N_t = \int_0^1 N_t(i) di$$  \hspace{1cm} (2.13)

Following Calvo (1983)[14] and Yun (1996)[84], the model introduces another imperfection of the economy, a staggered price setting. Each firm has a probability of $0 < \theta < 1$ to hold its price at any date. In other words, with the probability $1 - \theta$, a typical firm newly updates its price at each period. $\theta$ is then understood as a degree of price stickiness. Therefore, a single firm’s price $P_t(i)$ is a weighted sum of $P^*_t(i)$, a newly set price at current period, and the price of the previous period, $P_{t-1}(i)$. A price level of each firm set at time $t$ is then given by

$$P_t(i) = (1 - \theta)P^*_t(i) + \theta P_{t-1}(i)$$  \hspace{1cm} (2.14)

At each period, a single firm $i$ encounters a profit maximization problem with respect to $P^*_t(i)$,

$$\max_{P^*_t(i)} \sum_{s=0}^{\infty} E_t\Lambda_{t,t+s}\theta^s Y_{t+s}(i) (P^*_t(i) - MC_{t+s}(i))$$  \hspace{1cm} (2.15)$$
subject to
\[ Y_{t+s}(i) \geq \left( \frac{P^*(i)}{P_{t+s}^*} \right)^{-\delta} Y_{t+s} \] (2.16)
\[ MC_{t+s}(i) = \frac{W_{t+s}}{A_{t+s}} \] (2.17)
\[ Y_t(i) = C_t(i) + G_t(i) \] (2.18)

and (2.2) and (2.3), where \( \Lambda_{t,t+s} \) is a stochastic discount factor defined by
\[ \Lambda_{t,t+s} = \beta^s \left( \frac{P_t}{P_t} \right) \left( \frac{C_t}{C_{t+s}} \right)^{\sigma} \] (2.19)

The first order condition of the maximization problem is reduced to
\[ P_t^*(i) = \frac{\delta}{\delta - 1} \frac{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \left( MC_{t+s}(i) P_{t+s}^{\delta-1} Y_{t+s} \right)}{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \left( P_{t+s}^{\delta-1} Y_{t+s} \right)} \] (2.20)

where \( MC_{t+s}(i) \) denotes a real marginal cost, \( \frac{MC_{t+s}(i)}{P_{t+s}} \). Note that as \( \theta \) converges to zero, i.e., the price goes to the fully flexible state, the equilibrium price level also settles to the benchmark level, \( P_t(i) = \mu MC_{t+s}(i) \), where \( \mu = \frac{\delta}{\delta - 1} \), which can be interpreted as a markup revenue. Since the symmetric equilibrium is assumed, all firms solve identical problems at each period, and thus one can drop \( i \) notation hereafter, such as \( P_t^*(i) = P_t^* \) and \( MC_{t+s}(i) = MC_{t+s} \). Combining CPI definition from (2.5) and (2.14) with the above result gives the clearer version of the inflation rate:
\[ \Pi_t \equiv \frac{P_t}{P_{t-1}} = \left( 1 - \theta \right) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\delta} + \theta \left( \frac{1}{(1-\theta)^{1-\delta} + \theta} \right) \frac{1}{1-\delta} = \left( 1 - \theta \right) \Pi_{t-1}^{1-\delta} + \theta \left( \frac{1}{1-\delta} \right) \] (2.21)

where \( \Pi_t^* \) is defined by \( \frac{P_t^*}{P_{t-1}} \).

2.3.3 Government

There are two policy tools and they are separately operated by two independent authorities, fiscal and monetary policy authorities. A benevolent fiscal authority provides a public expenditure, \( G_t \), to the private sector, collects tax following lump sum fashion, and prints one-period risk free nominal bond, \( B_t \), with price \( R_t \) to finance it. \( G_t \) is an aggregation of \( G_t(i) \) following (3), and thus
the demand function for public good for any variety \( i \) is calculated by

\[
G_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\delta} G_t
\]  

(2.22)

There exists an quadratic adjustment cost if current level of the fiscal spending is different from the efficient level of it. Let \( X_t^n \) be an efficient level of an arbitrary variable \( X_t \) at time \( t \) and it is said to be the state where all prices are fully flexible without any market distortions. The difference, defined by \((G_t - G_t^n)\), is not fully cleared even when the economy reaches at the steady state level. A steady state means all endogenous variables are stable enough so that there is almost no changes on them, still containing one or more market imperfections if the distortions are assumed to exist at the beginning of the economy and continue to have an effect on the economy permanently. Therefore, there is no sound guarantee that the difference, in short term, \( \hat{G}_t \), will be cleared at any steady state level. Furthermore, \( \hat{G}_t^2 \) is a real quadratic adjustment cost departing from the traditional nominal rigidity assumptions such as quadratic capital adjustment cost introduced by Schmitt-Grohe and Uribe (2003)\[69\]. And this real adjustment cost can be interpreted as the difference between the level of government service provided at the current stage and the "desired" level of the public spending, i.e., the efficient level. The real friction occurs when current level of expenditure is not met with the desired level, even including a surplus situation. A budget constraint of the fiscal policy maker is then assumed to be balanced at every period,

\[
P_t G_t + R_{t-1} B_{t-1} = -T_t + B_t - P_t \xi \frac{G_t - G_t^n}{2}
\]  

(2.23)

where \( \xi > 0 \) captures a degree of the effect of the adjustment cost. As \( \xi \) converges to zero, the effect of the government spending gap on the economy becomes negligible. This means that the economy is more independent of the real friction. Potential factors affecting the degree of \( \xi \) is not explicitly demonstrated in this model, but some evidences of the higher level of \( xi \) in developing countries are discussed in several papers such as Talvi and Vegh (2005)\[73\]. In developing countries, because of tax evasion or political corruption, it is hard for a central government to have a fully flexible targeting mechanism to minimize the effect of the real friction between current level
and efficient level of fiscal spending. For instance, in a recession, since developing countries may meet worse situation of tax evasion, they are not able to aggressively cancel the gap immediately. Therefore, the higher level of $\xi$ captures higher level of exogenous factors that amplify the effect of the government spending spread. It is important to note that, the zero value of $\xi$ does not replicate developed countries situation. There should be further consideration and modification of the model to correctly express a developed country version of the economy. Regardless of the value of $x_i$, this model represents the case of developing economies.

On the other side, a monetary authority sets the nominal interest rate, $R_t$, at every period. A simple Taylor rule is implemented as a benchmark one.

$$ R_t = R \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_{\pi}} \left( \frac{Y_t}{Y^*_t} \right)^{\gamma_{\pi}} Z_t $$

where $\gamma_{\pi}$ and $\gamma_{\pi}$ are policy parameters, $R$ is a steady state level real interest rate, and $Z_t$ is an exogenous monetary policy shock which follows AR(1) stochastic process. Therefore, the two idiosyncratic policy authorities choose $\{R_t, G_t, T_t\}_{t \geq 0}$ with uniquely determined $\{B_t\}_{t \geq 0}$.

Maximizing the utility function with respect to $G_t$ subject to (2.23) gives the relation between private and public goods consumption.

$$ G_t^\phi (1 + \xi(G_t - G^*_t)) = C_t^{-\sigma} \tag{2.25} $$

(2.25) says that, with the presence of $\xi(G_t - G^*_t))$, relative price of those two consumption, or marginal rate of substitution, is changed. If one assumes that $G_t > G^*_t$ and $\xi > 0$, marginal utility of public consumption is increased, which means that the cost of unit public spending is higher than frictionless case.

### 2.3.4 Competitive Equilibrium

A competitive equilibrium is a set of endogenous variables $\{C_t, G_t, L_t, N_t, Y_t, B_t, MC_t\}_{t \geq 0}$ with prices $\{\Pi_t, \Pi^*_t, R_t, W_t\}_{t \geq 0}$ and a package of exogenous stochastic processes $\{A_t, Z_t\}$ satisfying (2.8), (2.9), (2.10), (2.12), (2.21), (2.23), (2.24), (2.25), goods market clearing condition,

$$ Y_t = C_t + G_t + \frac{\xi}{2}(\hat{G}_t)^2 \tag{2.26} $$
bond market clearing condition,

\[ B_t = 0 \]  \quad (2.27)

labour market clearing condition,

\[ L_t = N_t \]  \quad (2.28)

the aggregate production,

\[ Y_t = A_t N_t \]  \quad (2.29)

the specification of the common technology shock \( A_t \) which follows AR(1) process

\[ \log A_t = \rho \log A_{t-1} + \epsilon_t^a \]  \quad (2.30)

and exogenous monetary policy shock \( Z_t \) which also follows AR(1) process

\[ \log Z_t = \rho \log Z_{t-1} + \epsilon_t^z \]  \quad (2.31)

2.4 Qualitative Analysis

In this section, I discuss about economic implications of the main assumption of the model described in the above section in detail. From the planner’s problem, I obtain log-linearized versions of important equations that endogenously reflect the distorted effect of imperfectly developed public infrastructure on the economic dynamics. And I also define natural rates of endogenous variables expressed in terms of the exogenous shock and parameter values. After then, I characterize a Ramsey policy problem to obtain an insight on the policy implications. To do so, I construct a linear-quadratic welfare loss function following Woodford (2003) [82] and Benigno and Woodford (2012) [9], that it easily guarantees the uniqueness of solution and it also has a benefit of capability of comparing alternative policy regimes.

2.4.1 Procyclical Economic Policy

Price determination (2.20) can be solved forward and log-linearized that provides a so-called new Keynesian Phillips equation:
\[ \pi_t = \kappa \hat{m}c_t + \beta E_t \pi_{t+1} \]  

(2.32)

where \( \kappa = \frac{(1-\theta)(1-\theta \beta)}{\theta} \), and \( \hat{m}c_t \) is defined by the difference between log deviation of real marginal cost at time \( t \) from its steady state level and the log of its natural level value, \( \log \frac{1}{\mu} \). (2.32) states that the current level of inflation is affected by the real marginal cost, including the effect of monopolistic competition captured by a reverse of markup revenue, and by an expectation of future inflation.

To replace \( \hat{m}c_t \) with expressions of familiar endogenous variables, I use labor supply equation (2.9), labor market clearing condition (2.28), and (2.29) combined with the specification of the marginal cost (2.12). After some straightforward calculations I express the real marginal cost in terms of \( Y_t \) and \( C_t \):

\[ \hat{MC}_t = (Y_t)^\varphi (C_t)^\sigma (A_t)^{-\varphi+1} \]
\[ = Y_t^\varphi A_t^{\varphi+1} (Y_t - G_t - \xi (\hat{G}_t)^2) \]

(2.33)

By log-linearizing (2.33), one can obtain expression of the log deviation of the real marginal cost in terms of log deviations of output, \( y_t \), government spending, \( g_t \), and the stochastic process, \( a_t \):

\[ \hat{m}c_t = (\varphi + \sigma \frac{Y}{C}) y_t - \sigma \frac{G}{C} (1 + \xi \hat{G}) g_t - (\varphi + 1) a_t \]

(2.34)

Substituting (2.34) into (2.32) with "gap" variable expression, which is defined by the difference between current and natural levels of variable, I provide the modified version of New Keynesian Phillips Curve (NKPC):

\[ \pi_t = \kappa (\lambda_y \hat{y}_t - \lambda_g \hat{g}_t) + \beta E_t \pi_{t+1} - \lambda_a a_t \]

(2.35)

where \( \hat{x}_i \equiv x_t - x_t^n \) for any arbitrary endogenous variable \( x_t \), \( \lambda_y = (\varphi + \sigma \frac{Y}{C}) \), \( \lambda_g = \sigma \frac{G}{C} (1 + \xi \hat{G}) \), and \( \lambda_a = \sigma \frac{G}{C} \xi \hat{G} \left[ \frac{1}{Y} \left( \frac{C \phi}{\sigma} + G \right) + \frac{\phi}{\sigma} \right]^{-1} (1 + \frac{1}{\varphi}) \). \( y_t^n \) and \( g_t^n \) are derived from the social planner's problem in an efficient market environment:

\[ y_t^n = (\varphi + \sigma \frac{Y}{C})^{-1} \left[ \frac{G}{C} g_t^n + (\varphi + 1) a_t \right] \]

(2.36)

\[ g_t^n = \left[ \frac{1}{Y} \left( \frac{C \phi}{\sigma} + G \right) + \frac{\phi}{\sigma} \right]^{-1} (1 + \frac{1}{\varphi}) a_t \]

(2.37)
Detailed calculation of (2.36) and (2.37) is provided in a technical appendix. Note that as $\xi$ goes to zero, (2.35) becomes a benchmark NKPC with government spending and without cost push shock, since $\lambda_a$ converges to zero. But with any value of $\xi$, the real adjustment cost exists and obviously affects the inflation dynamics through cost push shock. As $\xi$ increases, $\lambda_a$ increases, and the trade-off of NKPC is worsened. The other channel of the effect of $\xi \hat{G}$ is captured in $\lambda_g$, the coefficient of $\hat{g}_t$. As $\xi$ goes up, the amount of $\lambda_g$ increases, which creates a steeper slope of NKPC. This means that the policy maker encounters larger trade-off between inflation and government spending gap stabilization. Furthermore, the sign of $\lambda_g$ is determined by $\hat{G}$. $\lambda_g$ is positive if $\hat{G}$ is positive, which means that $G > G^n$, implying that the steady state level of government spending is larger than the efficient level of the spending. This can be interpreted as a boom. In this situation, public spending gap is negatively related with inflation. If $G < \hat{G}$, a possible recession, $\lambda_g$ is negative, and the public spending gap is positively related with the inflation. In either case, the slope of NKPC is steepened.

Monetary policy rule is determined separately. The log-linearized version of benchmark Taylor rule (2.24) is calculated by

$$r_t = r + \gamma_\pi \hat{\pi}_t + \gamma_g \hat{g}_t + z_t$$  

(2.38)

Looking at (2.38), the log-linearized value of interest rate should be determined by a log deviated level of inflation gap, a log deviated output gap and an exogenous monetary policy shock.

Another important macroeconomic equation is a so called IS relation, which can be obtained by log-linearizing the first order necessary condition of household’s problem, (2.10). Substituting economy wide resource constraint into (2.10) to replace $c_t$ with $y_t$ and $g_t$, and using (2.36) and (2.37) to express the log-linearized version of (2.10) with gap variables, it is derived by

$$\hat{y}_t - \eta_y \hat{y}_t = -C1 \frac{Y}{\sigma}(r_t - E_t \pi_{t+1}) + E_t \hat{y}_{t+1} - \eta_y E_t \hat{g}_{t+1} + \eta_y \Delta g^n_{t+1} + \eta_a (E_t a_{t+1} - a_t)$$  

(2.39)
where

\[ \eta_g = \frac{G}{Y}(1 + \xi \hat{G}) \]
\[ \eta_{g,n} = \left( \varphi + \sigma Y \frac{C}{C} \right)^{-1} \left( \frac{\sigma G}{C}(1 + \xi \hat{G}) \right) \]
\[ \eta_a = \left[ \frac{G}{C}(1 + \xi \hat{G}) \left( \frac{1}{Y}(C\phi + G) + \frac{\phi}{\varphi} \right)^{-1} \left( \frac{1}{\varphi} \right) - \left( \varphi + \sigma Y \frac{C}{C} \right)^{-1} (\varphi + 1) \right] \]

and \( \Delta g_{t+1}^n = g_{t+1}^n - g_t^a \). Detailed process of derivation is provided in the technical appendix.

Equation (2.39) indicates that all three parameters \( \eta_g, \eta_{g,n}, \) and \( \eta_a \) are affected by \( \xi \hat{G} \) in some extents. As \( \xi \) increases, values of three parameters also increase, which induce a steeper slope of IS relation. Especially \( \eta_a \) increases with the higher value of \( \xi \), it worsens the trade-off of IS relation. This exacerbated trade-off between variables is clearly captured the amount of \( \xi \hat{G} \), and without \( \xi \hat{G} \), the IS relation obviously comes back to the benchmark case.

Committee of two economic policy authorities simultaneously choose the optimal set \{\( \pi_t, \hat{y}_t, \hat{g}_t, r_t \)\} \( t \geq 0 \) subject to (2.35), (2.38), (2.39) along with \{\( y_t^n, g_t^n \)\} \( t \geq 0 \) that are defined by (2.36) and (2.37), and the stochastic process, (2.30) and (2.31), given \{\( \pi_{-1}, y_{-1}, g_{-1}, r_{-1} \)\}. To solve this problem, I provide a constructed Ramsey policy problem in the next subsections.

### 2.4.2 Linear Quadratic Welfare Measure

I follow Benigno and Woodford (2012)[9] and Woodford (2003)[82] to formulate a linear-quadratic (LQ) welfare loss function from the second order approximation to the utility function of representative household, (2.1), and use it as an objective of macroeconomic stabilization policy. As discussed in Walsh (2010)[80], Gali (2008)[39], and Demirel (2012)[30], LQ welfare loss function has some merits. It not only guarantees an existence of local maximum under convexity assumption and an appropriate set of parameters, but also it provides an advantage of easiness to assess various types of alternative policy regimes measured in terms of social welfare criterion. Approximating to (2.1) and the economy wide resource constraint gives a detail of the welfare criterion, \( \mathbb{W} \)

\[ \mathbb{W}_t = -\frac{1}{2} \left[ \pi_t^2 + \left( \frac{C^2 + (1 - \sigma)}{Y} \right) c_t^2 + (1 - Y)y_t^2 + \left( \frac{\xi \hat{G} + \xi G^2}{Y} \right) g_t^2 + \varphi l_t^2 + y_t a_t + (G \xi \hat{G}) g_t \right] (2.40) \]
(2.40) is called "naive" LQ welfare loss function according to Benigno and Woodford (2012)[9], since the last term is linear, that prevents accurate calculation of economy wide welfare loss because the purpose of LQ function is to capture overall level of variances of key macroeconomic variables. Another reason why it is not the best criterion for the social welfare measure is that it also contains non-policy choice variables, such as \( c_t \) and \( l_t \), and these variables are not much helpful for policy analysis. Substituting two more relations \( c_t = \frac{\gamma}{C}y_t - (1\xi\hat{G})\frac{G}{Y}g_t \) and \( l_t = y_t - a_t \), one can eliminate those two non-policy variables from \( \mathcal{W} \). Therefore, rewriting (2.38) only in purely quadratic terms of policy variables gives a clearer version of the loss function:

\[
\mathcal{W}_t = -\frac{1}{2} \left[ \pi_t^2 + \Theta_y y_t^2 + \Theta_g g_t^2 - \Theta_{y,g} y_t g_t + 2(1 - \varphi) y_t a_t \right]
\]  

(2.41)

where \( \Theta_y = \left[ \left( \frac{C^2 + (1 - \sigma)}{Y} \right) + Y + \varphi - 1 \right] \), \( \Theta_g = \left[ \left( 1 + \xi\hat{G} \right) \frac{G}{Y} \right]^2 \left( 1 + \xi\hat{G} + \xi G^2 + \phi \right) \), and \( \Theta_{y,g} = \left[ \frac{2 Y G}{C^2} (1 + \xi\hat{G}) \right] \).

Note that \( \Theta_g \) and \( \Theta_{y,g} \) contain \( \xi\hat{G} \) with positive signs. If \( \xi \) goes up, \( \Theta_g \) and \( \Theta_{y,g} \) clearly increase while \( \Theta_y \) remains unchanged. This asymmetric changes of parameters influences the relative importance of policy variables in the welfare loss objective function. Relatively increased weights on \( g_t^2 \) and \( y_t g_t \) terms make the policy maker lean more into the government spending variable. This means that, remembering that the increased value of \( \xi \) means the amplified penalty of the government spending spread on the economy, the policy maker perceives that with the increase \( \xi \) the economy will lose more welfare gains from government spending part. This results in an ineffectiveness of fiscal policy with higher level of \( \xi \), because that policy tool becomes less preferred.

### 2.4.3 Optimal Policy Problem

A Ramsey policy problem using LQ approximation is defined by a maximization of the sequence of (2.40) subject to (2.35) and (2.39). The choice set is \( \{ \pi_t, y_t, g_t \}_{t \geq 0} \). \( r_t \) is out of the choice set because it is automatically determined sequentially by (2.38).

\[
max_{\pi_t, y_t, g_t} E_0 \sum_{t=0}^{\infty} \beta^t L_t
\]  

(2.42)
where the formulated Lagrangian equation is given by
\[
L_t = \mathcal{W}_t + \chi_{1,t} (\kappa (\lambda_y \hat{y}_t - \lambda_g \hat{g}_t) + \beta E_t \pi_{t+1} - \pi_t) \\
+ \chi_{2,t} \left( - \frac{C}{2} (r_t - E_t \pi_{t+1}) + E_t \hat{y}_{t+1} - \eta_y E_t \hat{y}_{t+1} + \eta_{g,n} E_t \Delta g^n_{t+1} + \eta_a (E_t a_{t+1} - a_t) - \hat{y}_t - \eta_g \hat{g}_t \right)
\]
(2.43)

and \( \chi_{1,t} \) and \( \chi_{2,t} \) are Lagrangian multipliers or shadow prices for NKPC and IS curve, respectively.

### 2.4.4 Case of Discretion

In the case of full discretion, policy maker encounters a separate policy objective each period, and chooses variables independent of past or future policy regimes. The optimal policy problem under discretion is then modified by
\[
\max_{\pi_t, y_t, g_t} \left[ \mathcal{W}_t + D_{w,t} + \chi_{1,t}^d (\kappa (\lambda_y \hat{y}_t - \lambda_g \hat{g}_t) - \pi_t + D_{1,t}) + \chi_{2,t}^d (- \hat{y}_t + \eta_y \hat{g}_t - \eta_a a_t + D_{2,t}) \right]
\]
(2.44)

where \( \chi_{1,t}^d \) and \( \chi_{2,t}^d \) are the discretion-specific shadow prices, and taking \( D_{w,t}, D_{1,t} \) and \( D_{2,t} \) as given, where \( D_{w,t} = \sum_{s=1}^{\infty} \mathcal{W}_{t+s}, D_{1,t} = E_t [\beta \pi_{t+1} - \lambda_a a_t] \), and
\[
D_{2,t} = E_t \left[ - \frac{C}{2} \pi_{t+1} + \hat{y}_{t+1} - \eta_y \hat{g}_{t+1} + \eta_{g,n} \Delta g^n_{t+1} + \eta_a (a_{t+1} - a_t) \right].
\]
First order conditions are derived as following:

\[
-\pi_t - \chi_{1,t}^d = 0 \quad (2.45)
\]
\[
-\Theta_y y_t - \Theta_{y,g} g_t + 2(1 - \varphi) a_t + \kappa \lambda_y \chi_{1,t}^d - \chi_{2,t}^d = 0 \quad (2.46)
\]
\[
-\Theta_g g_t - \Theta_{g,y} y_t - \kappa \lambda_g \chi_{1,t}^d + \eta_g \chi_{2,t}^d = 0 \quad (2.47)
\]

From the equation (2.45), one can find that a policy inconsistency problem can be arisen. Above equations are reduced to the one to express the Ramsey equilibrium level of \( \pi_t \) in terms of \( y_t, g_t, \) and \( a_t \) in this full discretion case:
\[
\pi_t = \left( 1 - \frac{1}{\eta_y} \right)^{-1} \left[ (\Theta_{y,g} + \Theta_y) y_t - (\Theta_{y,g} + \Theta_y) g_t + 2(1 - \varphi) a_t \right] \quad (2.48)
\]

According to (2.48), regardless of the past history of the policy regimes or future expectation, the monetary policy maker will adopt the notion of information on the fiscal policy decisions as given
and refresh its policy tools at each period. Moreover, since parameters of $\eta_g$, $\Theta_{y,g}$ and $\Theta_g$ contain $\xi\hat{G}$, the level of $\xi$ makes its own effect on the result of $\pi_t$ in this discretion case. The overall effect of $\xi$ is captured by the first two terms of (46), $(1 - \frac{1}{\eta_g})^{-1}[(\Theta_{y,g} + \Theta_y)y_t - (\Theta_{y,g} + \Theta_g)g_t]$. Effect of an additional increase in $\xi$ on $\pi_t$ in (2.46) can be calculated by total derivation of $\pi_t$ with respect to $\xi$. It is derived by

$$\frac{d\pi_t}{d\xi} = (1 - \frac{1}{\eta_g})^{-1}[(\Theta_{y,g} + \Theta_y)(2\hat{G}\frac{YG}{C^2}) - (\Theta_{y,g} + \Theta_g)(2(1 + \xi\hat{G})(\frac{1 + \xi\hat{G}G^2 + \phi}{Y})G\hat{G} + ((1 + \xi\hat{G})^2(\frac{\hat{G} + G^2}{Y})))]$$

The above equation shows the effect of $\xi$ on the equilibrium inflation level, $\pi_t$. If the first term inside the second parenthesis is larger than the second term, $\frac{d\pi_t}{d\xi}$ is positive, which can be interpreted that the additional increase in $\xi$ can positively affect on the inflation rate, and thus on the interest rate through (2.38). Therefore, the level of $\xi$ is a key to the change of interest rate in discretion case.

**2.4.5 Case of Commitment**

Problem of (2.42) and (2.43) can be directly described as a full commitment case. The solutions of the maximization problem can be calculated by the following first order conditions:

$$-\pi_t - \chi_{1,t} + \beta^{-1}\chi_{1,t-1} = 0 \quad (2.49)$$
$$-\Theta_y y_t - \Theta_{y,g} g_t + 2(1 - \varphi)a_t + \kappa\lambda_y\chi_{1,t} - \chi_{2,t} - \beta^{-1}\chi_{2,t-1} = 0 \quad (2.50)$$
$$-\Theta_g g_t - \Theta_{g,y} y_t - \kappa\lambda_y\chi_{1,t} + \eta_y\chi_{2,t} + \beta^{-1}\eta_y\chi_{2,t-1} = 0 \quad (2.51)$$

The above conditions can be reduced to one expression for the $\pi_t$, in terms of current levels and discounted past levels of output, public spending, and the stochastic process deviations:

$$\pi_t = \frac{1}{\kappa(\lambda_y - \lambda_g)}\left[\left(\frac{\Theta_{y,g}}{\eta_g} + \Theta_y\right)(y_t - \beta^{-1}y_{t-1}) - \left(\frac{\Theta_y}{\eta_g} + \Theta_{y,g}\right)(g_t - \beta^{-1}g_{t-1}) + 2(1 - \varphi)(a_t - \beta^{-1}a_{t-1})\right]$$

(2.52)

In the commitment case, unlike the discretion strategy, the effect of variables on $\pi_t$ is one time lagged with discounting factor $\beta$. While policy makers in discretion case should not believe that
his policy decision affects on future economic changes since the inflation is purely independent of past or future period, the policy makers in commitment case should take into account the lagged effect of variables. In addition, note that coefficients on the lagged values of $y_t$ and $g_t$ are slightly different from the discretion case. While in discretion case coefficients are weighted by $(1 - \eta_\eta^{-1})^{-1}$, which includes $\hat{G}$ and is used in IS relation, a commitment case variables are weighted by $\frac{1}{\kappa(\lambda_\pi - \lambda_g)}$, which also includes $\hat{G}$ but it is used in NKPC. Moreover, the effect of the level of $\xi$ can be observed as in the discretion case. Taking total derivative of $\pi_t$ with respect to $\xi$ shows the similar result with the discretion case, arguing the importance of $\xi$ as a determinant of the level of $r_t$, the policy interest rate.

### 2.5 Quantitative Analysis: Commitment Case

In this section, I compute the numerical values of solution from the commitment case with appropriate parameterization. I observe impulse responses of the equilibrium to a 1% positive productivity shock and negative monetary policy shock, and compare derived theoretical moments of the solutions of the model with the real friction to the one without the friction. Furthermore, I test some candidates of Taylor rule based monetary policy with different weights on inflation and output gap targeting stabilization under LQ welfare loss criteria. I compare those policy regimes to find the monetary policy that is most favorable yielding less welfare loss to the economy.

#### 2.5.1 Parameterization

In order to numerically compute the impulse responses of the objective function under optimal commitment stabilization policy to an exogenous stochastic process, I obtain the structural parameters of the described model in the previous sections. Table 2.1 shows the benchmark values of the parameters. First of all, to illustrate the macroeconomic properties of developing economies, I adopt some of the parameters with moderate modification from papers such as Devereux et al. (2006)[31] and Demirel (2010)[29], which consider characterized market imperfections designed for those countries. I follow Demirel (2010)[29] to set up intra-temporal elasticity between private and
public goods as 2, and I assume inverse elasticity of labor supply to be 1.2 following Devereux et al. (2006)[31]. While the correct level of degree of price stickiness is still in debate between leading papers in the area of policy discussion in emerging market economies, it is assumed to be 0.73, which is generally accepted in New Keynesian literature such as Sbordone (2002)[68] which estimates the value of price stickiness under appropriate modeling. The paper also gives a reasonable parameter value of the inverse of labor supply elasticity, which is assumed to be 1.2. Inter-temporal elasticity of substitution between differentiated final goods is set to be 11, which makes the markup revenue for each individual monopolistic competitive firm be 1.1. Steady state values of endogenous policy variables such as C, Y, G, R, and Π are analytically calculated with an assumption of ξ, a key parameter value that determines degree of severity of the institutional cost, that has never been estimated in the related literature. I use Chinese data from 1960 to 2010 to estimate the steady state ratio of government spending to total output to be 0.22, and then adjust the value of ξ such that steady state value of G and Y fit the estimation. As a result, ξ is estimated by 1.1609, and the rest of steady state values are listed on Table 2.1. ξ tells how the economy is affected by the wasteful government spending. For example, ξ converging to zero means that the economy is approaching to the level where less effect of the institutional cost exists in the economy, in relative terms, by improving some exogenous factors such as tax system or political transparency. High level of ξ gives larger effect of the adjustment cost on the economy, which means there will be worse economic condition that amplifies the effect and thus the economy has a long way to go to the ideal level of public expenditures. Two stochastic processes are defined following Demirel (2010)[29], which sets up the autoregressive parameter and productivity shocks such that the model calibrates the results of Adam and Billi (2008)[1] and the history of United States volatility of inflation. The remains of the parameterization are monetary policy parameters, γπ and γy. They are set to be an appropriate level such that the model has a unique local maximum, and modified in the following sections to assess alternative policy regimes. Following Gali (2008)[39], the benchmark value of γπ is varied from 1.5 to 5, and γy is varied from 0.125 to 0.3.
2.5.2 Procyclicality of Fiscal and Monetary Policy

To see how the higher level of the institutional cost associated with the public expenditure leads to more procyclical fiscal and monetary policy, I simulate the Ramsey equilibrium solved in subsection 4.5, and observe impulse responses to 1% positive productivity shock and 1% contractionary monetary policy shock. Benchmark monetary policy parameters are set to be 1.5 for $\gamma_\pi$ and 0.125 for $\gamma_y$. Table 2.2 shows the comparison of theoretical moments between absence in public expenditure real friction ($\xi = 0$) and presence case ($\xi = 1.1609$). Correlation between output gap ($\hat{y}$) and two policy variables ($\hat{\pi}, r$) determines the level of cyclicality of macroeconomic policy trend. If the value of correlation between output gap and public spending gap is positive, then it means that the fiscal policy tends to be procyclical, and an increase in the correlation indicates severer procyclical trend in fiscal policy. Similarly, negative correlation between output gap and interest rate means procyclical monetary policy, and the larger value indicates higher level of procyclical monetary policy trend. From Table 2.2, changes in both correlations validate the hypothesis that the presence of higher level of imperfect infrastructural development deepens the level of procyclicality of fiscal and monetary policy. Noticeably the increment of the level of procyclicality in monetary policy is larger than the one in fiscal policy. This can be interpreted as a result that monetary policy authority tries to compensate wasteful government spending by extremely sacrificing its interest rate policy tool. Moreover, overall economic volatility increases in the presence of $\xi$, and the social welfare loss therefore increases. This result confirms another hypothesis of the paper that the presence of higher effect of the institutional cost associated with public expenditure leads to the higher level of economic volatility. Additionally, correlation between $\hat{y}$ and $\hat{\pi}$ can be interpreted as a approximation to the slope of NKPC, and the result clearly shows that the slope becomes steeper in the presence of $\xi$. Figure 2.1 and 2.2 show impulse responses to 1% positive productivity shock and 1% contractionary monetary policy shock, respectively. In the positive supply shock, price is forced to go down and output gap increases, while abnormal two kinds of macroeconomic policies are well synchronized to promote the stabilization of the economy. In Figure 2.2, presence of the
real friction case (straight line) gives more volatility in all four important endogenous variables. This simulation work cannot entirely explain why the procyclical policy trend is happened in developing countries, but it successfully proves that the additional institutional cost leads to higher level of procyclicality and economic volatility.

2.5.3 Alternative Monetary Policy Discussion

Since many central banks in developing areas are implementing various version of Taylor rule type interest rate rule, it would be interesting to compare different monetary policy regimes and observe key characteristics in the equilibrium under the presence of the real friction to find the better regime that yields lower level of social welfare loss. To do this, I change monetary policy parameter values $\gamma_\pi$ and $\gamma_y$ in the standard Taylor rule (2.38) under the existence of $\xi = 1.1609$. I follow Gali (2008)[39] in varying those values within the range that the unique local maximum is guaranteed. The benchmark value of $\gamma_\pi$ equals 1.5, following Taylor (1993)[75], and $\gamma_y$ is set to be 0.125. First test is to compare the benchmark Taylor rule with aggressive inflation stabilization targeting rule, with $\gamma_\pi = 5$ while $\gamma_y$ unchanged. Next I compare the benchmark Taylor rule with the aggressive weight on output gap stabilization Taylor rule that has higher value in $\gamma_y$ with 0.3. The last test is to compare benchmark one with the forward looking inflation targeting rule which advantage is well explained in Woodford (2003)[82]. This forward looking Taylor rule is defined by

$$r_t = r + \gamma_\pi \tilde{\pi}_t + \gamma_\pi^* E_t \tilde{\pi}_{t+1} + \gamma_y \tilde{y}_t + z_t \quad (2.53)$$

where $\gamma_{\pi}^*$ is a policy parameter for the future inflation rate gap.

Table 2.3 shows some important theoretical moments of two cases with different value in $\gamma_{\pi}^*$. According to Table 2.3, the aggressive inflation rate targeting rule with $\gamma_\pi = 5$ deepens procyclicality of both fiscal and monetary policy. Moreover, overall economic volatility is also increased and it results in the higher level of welfare loss. Figure 2.3 and 2.4 show impulse responses of those two models to the positive productivity shock and negative monetary policy shock. It is interesting to see that the benchmark model has more procyclical fiscal and monetary policy in the response to
the positive real shock while aggressive inflation rate targeting rule has higher level of procyclical fiscal and monetary policy trend. With the higher level of inflation targeting parameter, the policy authority is encouraged to pay more attention on the inflation rate fluctuations at the same rate of infrastructural development, which means the same level of increased cost to use one unit of public spending (public spending becomes more wasteful). In the presence of positive shock to the real sector, this changed scheme is more efficient in terms of stabilizing economy, but in the presence of nominal shock, the scheme is limited and cannot fully function as a efficient stabilizing tool.

Table 2.4 depicts some statistic differences between baseline monetary policy rule and the rule with an aggressively targeted output gap. With higher value of $\gamma_y$, the economy shows severer level of procyclicality as well as higher level of economic volatility. Variances of all four macroeconomic variables are jumped up and the correlations between output gap and two policy variables are also exacerbated. As a result, welfare loss jump almost twice, which means the economy bears higher level of deadweight loss which is measured by economic fluctuations. Figure 2.5 and 2.6 show impulse responses to two different exogenous shocks. Similar to the previous test, aggressive output gap targeting rule experiences higher level of procyclicality of fiscal and monetary policy in response to the productivity shock while it bears less procyclicality in response to the monetary shock. From the previous two tests, inappropriately determined aggressive targeting rule on a specific policy variable can harm the economy in terms of volatility, and the procyclicality could be worsen with the wrongly estimated policy parameters.

The last test is comparing benchmark interest rate rule with the forward looking inflation targeting rule, as shown in (2.53). This interest rate decision rule is actually not a pure forward looking policy rule, because it combines targeting an expectation of future inflation rate gap and present value of inflation rate gap. However, this combination would give an insight how the economy reacts to the shocks in the imperfect environment where a policy maker has much better control on one of the policy variables. Table 4 shows differences in theoretical moments between benchmark rule and forward looking inflation gap targeting rule. First of all, procyclicality of fiscal and monetary policy is reduced. Specifically the amount of reduction in procyclicality of monetary policy is significant.
This result can be interpreted as a situation that the better information on the path of inflation can gives a way to reduce the procyclical trend of the policy. On the other hand, overall variances of the economy is also decreased significantly, and thus the economy experiences much less social welfare loss. The weight on the policy parameter of future inflation rate maybe too strong, but the result still clearly tells that the forward looking inflation targeting rule is optimal among all candidates.

2.6 Conclusion

I build a closed economy new Keynesian model Calvo type staggered price setting. In addition to the traditional modeling, real quadratic adjustment cost is invited, that captures an additional institutional cost associated with public goods. From the model, that social cost leads to the severer procyclicality of fiscal and monetary policy. Moreover, it also exacerbates economic volatility relative to the frictionless version of the model. It fails to clearly show the reason of procyclicality of fiscal and monetary policy in developing economies, but it still validates that the newly added institutional cost deepens the abnormal policy trend in the rapidly growing regions. The paper also shows that forward looking inflation rate targeting rule improves economy in terms of reducing economic volatility and it also reduces procyclicality of fiscal and monetary policy. The policy rule is suboptimal among all candidates because a rule determined by the Ramsey policy is always optimal, but the test still has an edge in providing some insights in practical way.

There are additional notable limitations in this paper. First, the model assumes that the fiscal authority collects taxes only in lump sum fashion to finance its spending. But in reality, as mentioned by Tanzi and Zee (2000)[74], most developing countries experience the trend that large portion of their tax structure is consumption or income taxes which are known as distortionary. Therefore, it must be worthwhile to look carefully at the change in the model if any kind of distortionary tax is introduced. Second, since many developing or emerging market economies are heavily dependent of international trade and foreign capital flows, opening up the international dimensions of the model should be interesting and more than encouraging. While monetary policies of most developing
countries are influenced by the exchange rate in some extent, introducing exchange rate pegging option in the group of alternative policy candidates and comparing it with the closed economy version Taylor rule maybe also interesting. As Gali and Monacelli (2008)[41] tries for the case of monetary union in Europe, an special case of developing countries can be treated in open economy version of model. Third, as Frankel (2010)[35] points out, besides the institutional cost and heavy dependence on international trades, another characterized fact about developing countries can be substantial in policy decision making such that the fact which they still suffer from political instability or central bank independence problem. In many of those countries, central bank is under pressure of fiscal or other political institutions and thus the central bank cannot optimally choose its own policy regime independently. Related to this topic, inconsistency problem of discretion is still common across the countries. The model of this paper ignores those realities and they should be reconsidered. Another interesting possible future work is recently changing trend of the procyclic in developing economies. According to Frankel (2011)[36], during the last decade, 24 out 73 developing countries made a historic shift from procyclical trend to countercyclical tendency of their policy regimes. This should be related with the previously mentioned limitation of the model such as the international dimension of policy decision making, since the most of those countries have experienced an opening of their financial markets or significant change in international capital flows in the recent decade.
Table 2.1: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi )</td>
<td>Reverse of Elasticity of Labour Supply</td>
<td>1.2</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Inter-temporal Elasticity of Substitution in Private Consumption</td>
<td>2</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Inter-temporal Elasticity of Substitution in Public Consumption</td>
<td>2</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Time Discount Factor</td>
<td>0.99</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Intra-temporal Elasticity of Substitution between Differentiated Goods</td>
<td>11</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Markup Revenue</td>
<td>1.1</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Degree of Price Stickiness</td>
<td>0.73</td>
</tr>
<tr>
<td>( Y )</td>
<td>Steady State Value of ( Y_t )</td>
<td>1.4968</td>
</tr>
<tr>
<td>( G )</td>
<td>Steady State Value of ( G_t )</td>
<td>0.7485</td>
</tr>
<tr>
<td>( C )</td>
<td>Steady State Value of ( C_t )</td>
<td>0.7485</td>
</tr>
<tr>
<td>( L )</td>
<td>Steady State Value of ( L_t )</td>
<td>1.4968</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Degree of Severeness of Imperfectly Developed Infrastructure</td>
<td>[0, 1.1609]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Coefficient of AR(1) process</td>
<td>0.9</td>
</tr>
<tr>
<td>( \varepsilon^a )</td>
<td>Standard Deviation of Productivity Shock</td>
<td>0.8125</td>
</tr>
<tr>
<td>( \gamma_{\pi} )</td>
<td>Policy Parameter for log of Inflation</td>
<td>[1.5, 5]</td>
</tr>
<tr>
<td>( \gamma_{\gamma} )</td>
<td>Policy Parameter for log of Output Gap</td>
<td>[0.125, 0.3]</td>
</tr>
<tr>
<td>( R )</td>
<td>Policy Anchor Value of Interest Rate</td>
<td>1.0264</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>Steady State Value of ( \Pi_t )</td>
<td>1.0161</td>
</tr>
</tbody>
</table>
Table 2.2: Theoretical Moments: Without or with Real Frictions in Government Spending Difference: $\xi = 0$ versus $\xi = 1.1609$ (HP filter, lambda = 1600)

<table>
<thead>
<tr>
<th></th>
<th>$\xi = 0$</th>
<th>STD. DEV.</th>
<th>Corr($\hat{y}_{..}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\hat{\pi}$)</td>
<td>0.0756</td>
<td>0.1278</td>
<td></td>
</tr>
<tr>
<td>Output Gap ($\hat{y}$)</td>
<td>0.2533</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Public Spending Gap ($\hat{g}$)</td>
<td>0.3316</td>
<td>0.9963</td>
<td></td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.5656</td>
<td>-0.8861</td>
<td></td>
</tr>
<tr>
<td>Welfare Loss = 2.5863</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\xi = 1.1609$</th>
<th>STD. DEV.</th>
<th>Corr($\hat{y}_{..}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\hat{\pi}$)</td>
<td>0.0730</td>
<td>0.1511</td>
<td></td>
</tr>
<tr>
<td>Output Gap ($\hat{y}$)</td>
<td>0.2701</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Public Spending Gap ($\hat{g}$)</td>
<td>0.3435</td>
<td>0.9966</td>
<td></td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.5712</td>
<td>-0.9551</td>
<td></td>
</tr>
<tr>
<td>Welfare Loss = 2.68556</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.3: Theoretical Moments: Standard ($\gamma_\pi = 1.5$) versus Aggressive ($\gamma_\pi = 5$) Inflation Stabilization Strategy. (HP filter, lambda = 1600)

<table>
<thead>
<tr>
<th>$\gamma_\pi = 1.5$</th>
<th>STD. DEV.</th>
<th>Corr($\hat{y}_{..}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\hat{\pi}$)</td>
<td>0.0730</td>
<td>0.1511</td>
</tr>
<tr>
<td>Output Gap ($\hat{y}$)</td>
<td>0.2701</td>
<td>1.0000</td>
</tr>
<tr>
<td>Public Spending Gap ($\hat{g}$)</td>
<td>0.3435</td>
<td>0.9966</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.5712</td>
<td>-0.9551</td>
</tr>
<tr>
<td>Welfare Loss = 2.68556</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma_\pi = 5$</th>
<th>STD. DEV.</th>
<th>Corr($\hat{y}_{..}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\hat{\pi}$)</td>
<td>0.0471</td>
<td>-0.0944</td>
</tr>
<tr>
<td>Output Gap ($\hat{y}$)</td>
<td>0.2850</td>
<td>1.0000</td>
</tr>
<tr>
<td>Public Spending Gap ($\hat{g}$)</td>
<td>0.3557</td>
<td>0.9982</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.6011</td>
<td>-0.9798</td>
</tr>
<tr>
<td>Welfare Loss = 2.88252</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>
Table 2.4: Theoretical Moments: Standard ($\gamma_y = 0.125$) versus Aggressive ($\gamma_y = 0.3$) Output Gap Stabilization Strategy. (HP filter, lambda = 1600)

<table>
<thead>
<tr>
<th>$\gamma_y = 0.125$</th>
<th>STD. DEV.</th>
<th>Corr($\tilde{y}_\cdot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\tilde{\pi}$)</td>
<td>0.0730</td>
<td>0.1511</td>
</tr>
<tr>
<td>Output Gap ($\tilde{y}$)</td>
<td>0.2701</td>
<td>1</td>
</tr>
<tr>
<td>Public Spending Gap ($\tilde{g}$)</td>
<td>0.3435</td>
<td>0.9966</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.5712</td>
<td>-0.9551</td>
</tr>
<tr>
<td>Welfare Loss = 2.68556</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma_y = 0.3$</th>
<th>STD. DEV.</th>
<th>Corr($\tilde{y}_\cdot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\tilde{\pi}$)</td>
<td>0.0162</td>
<td>-0.1913</td>
</tr>
<tr>
<td>Output Gap ($\tilde{y}$)</td>
<td>0.3831</td>
<td>1</td>
</tr>
<tr>
<td>Public Spending Gap ($\tilde{g}$)</td>
<td>0.4808</td>
<td>0.9998</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.7776</td>
<td>-0.9993</td>
</tr>
<tr>
<td>Welfare Loss = 5.24278</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>
Table 2.5: Theoretical Moments: With or without Forward Looking Inflation Gap Stabilization Parameter: $\gamma^* = 0$ versus $\gamma^* = 1$ (HP filter, lambda = 1600)

<table>
<thead>
<tr>
<th>$\gamma^* = 0$</th>
<th>STD. DEV.</th>
<th>Corr((\hat{y},))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\hat{\pi}$)</td>
<td>0.0730</td>
<td>0.1511</td>
</tr>
<tr>
<td>Output Gap ($\hat{y}$)</td>
<td>0.2701</td>
<td>1</td>
</tr>
<tr>
<td>Public Spending Gap ($\hat{g}$)</td>
<td>0.3435</td>
<td>0.9966</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.5712</td>
<td>-0.9551</td>
</tr>
<tr>
<td>Welfare Loss = 2.68556</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma^* = 1$</th>
<th>STD. DEV.</th>
<th>Corr((\hat{y},))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Gap ($\hat{\pi}$)</td>
<td>0.0749</td>
<td>-0.4881</td>
</tr>
<tr>
<td>Output Gap ($\hat{y}$)</td>
<td>0.0925</td>
<td>1</td>
</tr>
<tr>
<td>Public Spending Gap ($\hat{g}$)</td>
<td>0.1002</td>
<td>0.9764</td>
</tr>
<tr>
<td>Interest Rate ($r$)</td>
<td>0.3668</td>
<td>-0.8310</td>
</tr>
<tr>
<td>Welfare Loss = 0.261654</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>
Figure 2.1: Impulse Response to 1% Positive Productivity Shock: With ($\xi = 1.1609$) or without ($\xi = 0$) Real Frictions in Government Spending
Figure 2.2: Impulse Response to 1% Positive Monetary Policy Shock: With ($\xi = 1.1609$) or without ($\xi = 0$) Real Frictions in Government Spending
Figure 2.3: Impulse Response to 1% Positive Productivity Shock: Standard ($\gamma_\pi = 1.5$) versus Aggressive ($\gamma_\pi = 5$) Inflation Gap Stabilization
Figure 2.4: Impulse Response to 1% Positive Monetary Policy Shock: Standard ($\gamma_\pi = 1.5$) versus Aggressive ($\gamma_\pi = 5$) Inflation Gap Stabilization
Figure 2.5: Impulse Response to 1% Positive Productivity Shock: Standard ($\gamma_y = 0.125$) versus Aggressive ($\gamma_y = 0.3$) Output Gap Stabilization
Figure 2.6: Impulse Response to 1% Positive Monetary Policy Shock: Standard ($\gamma_y = 0.125$) versus Aggressive ($\gamma_y = 0.3$) Output Gap Stabilization
Figure 2.7: Impulse Response to 1% Positive Productivity Shock: With ($\gamma^* = 1$) or without ($\gamma^* = 0$) Forward Looking Inflation Gap Stabilization
Figure 2.8: Impulse Response to 1% Positive Monetary Policy Shock: With $(\gamma^{*}_{\pi}=1)$ or without $(\gamma^{*}_{\pi}=0)$ Forward Looking Inflation Gap Stabilization
Chapter 3

Effect of Wage Support by Government on Economic Volatility and Optimal Stabilization Policy

3.1 Introduction

A sudden stop is defined, by related notable studies such as Calvo and Reinhart (1999)[19], as a sudden decrease in foreign capital inflows. Since a capital inflow is calculated by a sum of current account deficit and foreign reserves, the sudden reversal of foreign capital flows damages both an output and a financial vulnerability, and it also exacerbates an unemployment rate and a relative price volatility. This phenomenon has been a main topic in the related literature since many emerging market or developing countries have experienced the similar pattern of economic shifts during economic crises in various times. A notable example of the phenomenon observed in those regions would be Mexican financial crisis in 1994 or Asian and Latin American financial crisis in 1998 and 1999. While significant academic achievement on the possible causality of the historic economic turmoil has been done in the literature, there has been still an ongoing debate on a policy implication of the aftermath of the economic crisis in developing countries. Braggion et al. (2009)[10] is a milestone study for the monetary policy implication which is aimed to stabilize the economic volatility after a sudden stop comes. Another important stabilization policy discussion regarding the phenomenon is Caballero and Krishnamurthy (2004)[12], which considers an inflation targeting as a possible remedy for the consequential event of the financial crisis. While these papers efficiently argue that the main monetary policy tool has been made an enough effect on the business cycles in some countries, an effect of a structural changes in real wages on the consequence of the
sudden stops, or similar pattern of economic crisis has been a little ignored because of its minor role as a macroeconomic policy tool. In fact, there are strong statistical motivations to be interested in the role of a wage support by a government such as a increasing minimum wages in a specific period with a specific regional conditions. First, there is an intriguing linkage between an inflation volatility and a real minimum wage level, which suggests that an increasing minimum wage level maybe helpful in stabilizing price volatility after sudden stops or similar crisis caused by a sudden external changes. Figure 3.1 shows a different path of inflation volatility of two countries which kept dramatically different minimum wage regimes. South Korea, which experienced a financial crisis in 1998 and has consistently increased its real minimum wage level for the last 25 years, was able to avoid a high volatile price changes after the crisis. On the other hand, Mexico, which had the similar crisis during 1994, never increased its real minimum wage level for the same span of period, and it experienced relatively high volatile inflation fluctuations after the crisis. While the minimum wage controlling would not be the most favorable main fiscal policy tool to stabilize the economic volatility during and after the economic crisis, it must be studied further to recognize to what extent this wage controlling method can help the main macroeconomic stabilization policies in the special situation of crisis. Second, there is an ongoing question of what should be the optimal level of minimum wage to fully stabilize an inflation volatility after severe fluctuation of macroeconomic variables is experienced. Figure 3.2 compares different level of real minimum wage changes and their linkage to the macroeconomic variables. Chile has kept the lower growth rate of the real minimum wage than South Korea for the last 25 years and the country was successful in suppressing the price fluctuations after the sudden stops in 1999, but it failed to stabilize the higher inflation volatility around the new financial crisis in 2008. International and domestic economic environments should be different between those two different times, but it is still interesting to wonder if there is any specific level of optimal minimum wage level to fully stabilize the volatility of some key macroeconomic variables after the crisis. Third, if there is a specific link between minimum wage control and price volatility, it is also worth to question if the relationship only matters for emerging market or developing countries. Figure 3.3 tells that the possible effect of
the minimum wage on the economic dynamics is limited to the developed economies, by showing that US data has no seemingly correlated linkage between the special fiscal regime and the business cycles. US never experienced any type of economic crisis similar to a sudden stop or the other pattern of crisis caused by a sudden foreign shocks during the past 25 years, and the data never shows any meaningful effect of the minimum wage levels on the economic dynamics during any type of external crisis. It is still unknown and very difficult to sort out an independent effect of the minimum wage on the business cycles because the regime has never been considered as a main macroeconomic stabilization policy tool in those developing countries during the crisis, and it only has been used for a supplemental purpose as a part of social security system. But it is important to investigate the effect of the minimum wage on the inflation stabilization process after the economy experiences a dramatic foreign demand changes such as a sudden stop, and to figure out how much it is helpful to support the monetary policy to accomplish its major goal of price and output stabilization in the uniquely characterized economic downturn.

To answer those questions, I build a Small Open Economy (SOE) Dynamic Stochastic General Equilibrium (DSGE) model with New Keynesian fashion. In addition to assumptions of a price rigidity and a monopolistically competitive nontradable sector, I add a partially supported wage structure by a government. A government sets a fixed level of a portion of the wage level that positively affects the labor income and finances it by a lump sum fashioned taxation. This simple wage structure cannot fully capture a realistic model of the minimum wage, but it still successfully illustrates the effect of the minimum wage on the overall wage level and the economy wide marginal cost structure. In this structure, the fixed part of the wage directly changes the overall income of the domestic households, but it does not affect the optimal labor demand and wage determined in a production sector. Minimizing a distorting effect of the government operation on the economic equilibrium, the model shows the effect of the changing government wage support on the economic dynamics of macroeconomic variables in a Ramsey policy equilibrium. I build a linear-quadratic (LQ) social welfare loss measurement and use it as an objective function of the Ramsey policy problem. The LQ form of the welfare loss function has a merit in terms of tractability. As a result,
I find that the increasing government wage support in the form of minimum wages can reduce the economic volatility of key macroeconomic variables, by ameliorating the trade-off between output gap and inflation stabilizations faced by a policy maker. With this reduced trade-off in New Keynesian Phillips Curve (NKPC) the economy can achieve the goal of stabilization more successfully. The economic intuition behind this result is that, a permanent increase in a certain portion of the labor income preserves higher level of real purchasing power of the domestic agents and it gives them a more room to keep their original level of consumption and production even if there comes a sudden, unexpected negative foreign demand shock. This positive preservation effect overcomes a negative effect of the increasing lump sum taxation in this model, and thus the overall stabilization effect of the economy is improved. This model lacks an ability to answer the second question drawn in the previous paragraph, so that it cannot find the optimal level of minimum wage to fully stabilize the economy since the minimum wage level is fixed and given in the problem. It also fails to answer the question of what should be the main difference between developed and developing countries if the same level of the government wage support program is introduced. Those two questions should be sought in the future development of the paper.

One main contribution of the paper is that, it introduces a kind of wage manipulation by government in DSGE literature and tries to find the optimal policy regime in that environment. Benassy (1995)[6] brings the minimum wage discussion into the macroeconomics context and Gali (1996)[38] briefly discusses the role of it in Real Business Cycle fashion, but there has not been an any other notable works studying the role of government-lead wage structural changes in the consequences of the foreign-originated crisis in developing countries. The importance of the minimum wage has never been ignored in labor economics literature with microeconomic scope, but it is also important in the context of business cycle and sudden stops literature. This paper shows that, while the statistical background is remained as a weak part, the aggressive wage support program by the government possibly reduces the macroeconomic volatility by a significant level.
3.2 Literature Review

There has been a rich volume of literature on a sudden stop. Majority of the research has been focused on the causality of the sudden foreign capital inflows decrease motivated by Mexican crisis in 1994 and the financial crisis in Asia and Latin America in 1998 and 1999. The international dimensions of the developing countries naturally became strong candidates for the main driver of the macroeconomic collapse. While Cavallo and Frankel (2008) focus on the negative effect of an openness to trade on the severity of the sudden stop, Joyce and Nabar (2009) look the history of emerging markets from 1976 to 2002 to find out that the financial openness made those countries more vulnerable to the sudden capital reversals. Similarly, Mendoza and Smith (2006) points out that financial friction such as collateral constraint in those countries could be a possible trigger. Another potential reasons for the sudden stops are liability dollarization argued by Chue and Cook (2008), Calvo (2006) and Calvo (2002), fiscal institutional problem argued by Calvo (2003), and asymmetric information problem pointed out by Rothenberg and Warnock (2011).

The result of the sudden stop is characterized by sudden decrease in output, severe depreciation in real exchange rate, and high volatile price fluctuations. Kehoe and Ruhl (2009) tries to replicate the observed movement of the macroeconomic variables by inviting several restrictions in their growth model, such as labor sectoral reallocation or various types of intermediate goods production process. Relative price volatility is studied by Calvo et al. (2006), which empirically confirms the relevance of sudden stops and price volatility.

Study on policy implications of the aftermath of the sudden stops has been relatively little considered in the literature, due to the difficulty of the empirical analysis on the identification of the effect of macroeconomic policies on the consequences of the sudden stops. Several theoretical works on a monetary policy after sudden stops are notable, such as Braggion et al. (2009) which points out that the increasing interest rate in South Korea during the crisis was logically appropriate by showing the procyclical policy mitigated the distortions created by a binding collateral constraint,
and thus it was a welfare improving. On the other hand, Caballero and Krishnamurthy (2004)[11] suggests that the countercyclical time-contingent inflation targeting monetary policy is optimal in the dimension of sudden stops where time inconsistency problem is relatively more significant.

Role of minimum wage on the business cycles are rarely studied in the recent New Keynesian literature, since the body of school mainly focuses on the role of interest rate controlling in which a money non-neutrality is a strong feature as an economic environment. But in Real Business Cycle (RBC) literature, the supplemental policy regime is considered a positive rather than negative supporting tool for the economic dynamics. Cahuc and Michel (1996)[13] argues that setting minimum wage above the steady state wage level of an unskilled labor would increase demands for a skilled labor, giving the unskilled labor a motivation to improve its labor efficiency, and thus the economy shifts to the trade-friendly labor market structure and it finally induces higher level of economic growth. Flug and Galor (1986)[34] also points out that in the unique environment of developing countries where the real wage of unskilled labor is sticky downwards, the effective minimum wage control roles as a distortion that changes a composition between unskilled and skilled labor forces and thus it affects the trade pattern in an open economy and economic dynamics.

In sum, the effect of the minimum wage on the economic dynamics in an environment where the sudden stops come or the money neutrality assumption is broken has not been clearly investigated, and there is a room for this paper to have a potential contribution to the related literature.

3.3 Model

To see the effect of an increase in minimum wages on business cycles in an open economy, I build a small open economy DSGE model with nominal price rigidity in a nontradable sector. A small open economy, called domestic country, is assumed to be enough small to be ignored by the world economy, which is the other side of the model. Therefore, the domestic country is given world price and output as an exogenous variables. The world economy ignores an economic activity of the domestic country because it is too small to make any worldwide change. Each part of the economy consists of three parts: Households, firms, and a government. A representative household consumes
tradable and nontradable final goods and a firm uses a labor as an input. The nontradable sector is monopolistically competitive, which gives an each firm in the sector a small price power at each period. The tradable sector is assumed to be perfectly competitive and the price is totally flexible. The model includes a structure of binding minimum wage. The wage level is the same across sectors, so that if the government sets the binding minimum wage which is manually determined such as a parameter, it increases the overall wage level directly. By changing the binding minimum wage level, the wage and price level of the economy are affected and the change finally makes a different transmission process of foreign exogenous demand shocks to the domestic economy. I solve Ramsey policy problem from the competitive equilibrium conditions, and find an optimal policy values which are used to determine the effect of the binding minimum wage on the equilibrium of the economy.

3.3.1 Households

There are two types of goods produced in each country: Tradable and nontradable goods. Composite final goods are aggregated by Cobb-Douglass fashion:

\[ C_t = \frac{(C_{T,t})^\gamma (C_{N,t})^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \]  

(3.1)

where \( C_t \) is a final goods consumption, \( C_{T,t} \) is an amount of consumption for the tradable goods and \( C_{N,t} \) is an amount of consumption for the nontradable goods. In the similar way, an overall price level (CPI) is defined by prices of tradable and nontradable goods.

\[ P_t = (P_{T,t})^\gamma (P_{N,t})^{1-\gamma} \]  

(3.2)

where \( P_t \) is an overall price level, \( P_{T,t} \) is a price level of tradable goods and \( P_{N,t} \) is a price level of nontradable goods. \( P_{T,t} \) is directly linked to the world tradable goods price level and a nominal exchange rate:

\[ P_{T,t} = P_{T,t}^* \epsilon_t \]  

(3.3)
where \( P^*_T,t \) denotes the foreign (world) price level of tradable goods and \( E_t \) stands for a nominal exchange rate. Hereafter, any variable with asterisk mark is defined by a foreign variable. By normalizing \( P^*_T,t \) by unity, the domestic price level of tradable goods is simply the same with the level of nominal exchange rate.

The demand functions for tradable and nontradable goods are then calculated:

\[
C_{T,t} = \gamma \left( \frac{P_{T,t}}{P_t} \right)^{-1} C_t \quad (3.4)
\]

\[
C_{N,t} = (1 - \gamma) \left( \frac{P_{N,t}}{P_t} \right)^{-1} C_t \quad (3.5)
\]

The foreign country’s composite consumption, price level, and the demand functions are similarly defined and calculated with asterisk mark. The demand function for each differentiated nontradable good is derived by

\[
C_{N,t}(j) = \left( \frac{P_{N,t}(j)}{P_{N,t}} \right)^{-\varepsilon} C_{N,t}
\]

where \( j \in (0, 1) \) is an index for single firm in the nontradable sector, \( \varepsilon \) denotes a substitution of elasticity between the differentiated goods in the sector. The domestic households are assumed to access to both domestic and foreign currencies denominated nominal bonds, \( B_{H,t} \) and \( B_{F,t} \), respectively, with fixed returns of interest, \( R_t \) and \( R^*_t \), respectively. They are also assumed to receive a wage by providing an inelastic labor from one of tradable and nontradable sectors, and own firms in both tradable and nontradable sectors. Therefore, a budget constraint for the represent household is given by

\[
P_tC_t + B_{H,t} + \mathcal{E}_t B_{F,t} \leq R_{t-1} B_{H,t-1} + \mathcal{E}_t R^*_{t-1} B_{F,t-1} + (W_t \cdot \bar{W})(L_{T,t} + L_{N,t}) - T_t + \Gamma_{T,t} + \int_0^1 \Gamma_{N,t}(j) dj \quad (3.6)
\]

where \( W_t \) is a nominal wage in both sectors sectors, \( \bar{W} \) is a fixed minimum wage supported by a government, \( L_{T,t} \) and \( L_{N,t} \) are an amount of labor supplied to tradable and nontradable sectors, respectively, \( T_t \) is a lump sum transfer from a government, and \( \Gamma_{T,t} \) and \( \int_0^1 \Gamma_{N,t}(j) dj \) are total profits of tradable and nontradable firms. Note that, a consumer receives a total wage \( W_t \cdot \bar{W} \) at
each period $t$, which consists of two margins, $W_t$, paid by a firm, and $\bar{W}$, paid by a government. This simple wage structure illustrates a reality that a strictly set minimum wage by government increases overall wage level, while that forced part of the wage structure is partly subsidized by the government spending. The government finances the total minimum wage spending $\bar{W}(L_{T,t} + L_{N,t})$ by levying a lump sum tax $T_t$. According to the constraint, the baseline value of the minimum wage is 1, which means that the minimum wage does not increase or decrease the wage level paid to the consumers. If the minimum wage level is 1.1, the overall wage level is increased by 10%. If the minimum wage level is set to be 0.9, the overall wage level is decreased by 10%. Therefore, by controlling the minimum wage level, the government can directly change the overall wage level finally paid to the domestic consumers. It is weird to see that the minimum wage is expressed by a proportional value, but this type of modeling is designed for the tractability, and this does not harm the original intuition of the role of the minimum wage on the wage structure and the effect of it on the entire model of this paper. If a government understands the fact that the increasing minimum wage directly affects the increasing overall wage level and wants to do it, the government precisely determine the increasing amount of wage level and manually control it.

The utility function of the domestic households is defined by

$$U(C_t, L_{T,t}, L_{N,t}) = E_0 \sum_{t=0}^{\infty} \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(L_{T,t} + L_{N,t})^{(1+\phi)}}{1+\phi} \right)$$

(3.7)

where $\beta$ is a time discounting factor, $\sigma \geq 0$ is an intertemporal elasticity of substitution in private consumption, and $\phi \geq 0$ denotes a reverse elasticity of labor supply. The utility function of the foreign agent is defined identically with asterisk mark to the endogenous variables. The budget constraint of the foreign agent is defined by

$$P_t^* C_t^* + B_{F,t}^* \leq R_{t-1}^* B_{F,t-1}^* + W_t^* L_t^* + T_t^* + \Gamma_t^*$$

(3.8)

Note that for the small open economy assumption, the foreign (world) economy ignores the domestic financial market activity and thus it does not consume the domestic nominal bond holdings.
The first order conditions of consumers’ problems are calculated as following:

\[
\frac{(W_t \cdot \bar{W})}{P_t} = (L_{T,t} + L_{N,t})^\phi C_t^\sigma \\
1 = \beta R_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \Pi_{t+1} \right)^{-1} \tag{3.10}
\]

\[
1 = \beta R^*_t \left( \frac{E_{t+1}}{E_t} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \Pi^*_{t+1} \right)^{-1} \tag{3.11}
\]

\[
1 = \beta R^*_t \left( \frac{C^*_t}{C^*_{t+1}} \right)^{-\sigma} \left( \Pi^*_{t+1} \right)^{-1} \tag{3.12}
\]

where \( \Pi_t = \frac{P_t}{P_{t-1}} \) and \( \Pi^*_t = \frac{P^*_t}{P^*_{t-1}} \). (3.9) is a real wage determination or labor supply equation, (3.10) and (3.11) are an Euler equation for home country. (3.11) is affected by a foreign interest rate, which is assumed to be exogenous and stochastic to be defined later. (3.12) is an Euler equation for the foreign economy, and both foreign consumption and inflation rate are assumed to be exogenous to the domestic economy. Therefore, (3.13) describes the relation between those exogenous stochastic processes.

### 3.3.2 Firms

There are two types of firms in the domestic country: Tradable and nontradable firms. While the nontradable sector is monopolistically competitive with differentiated goods produced, the tradable sector is supposed to be perfectly competitive.

** Tradable Sector** A single tradable sector firm produces goods by using labor with linear technology,

\[
Y_{T,t} = A_{T,t} N_{T,t} \tag{3.13}
\]

where \( Y_{T,t} \) is a non-differentiated final tradable good, \( A_{T,t} \) is a productivity shock which follows the stochastic process,

\[
\log A_{T,t} = \rho_T \log A_{T,t-1} + \mu_{T,t} \tag{3.14}
\]
where $0 \leq \rho_T < 1$ and $\mu_{T,t} \sim N(0, \sigma_T^2)$, and $N_{T,t}$ is a labor demand for tradable goods production. A single tradable sector firm faces a profit maximization problem,

$$\max_{N_{T,t}} \sum_{t=s}^{\infty} \Lambda_{t,s} (P_{T,t}Y_{T,t} - W_{T,t}N_{T,t})$$  \hspace{1cm} (3.15)

subject to (3.15), where $\Lambda_{t,s} = \beta^{t-s} \left( \frac{C_t}{C_s} \right)^{-\sigma} \left( \frac{P_t}{P_s} \right)^{-1}$, a stochastic discount factor. The first order condition is calculated by

$$\frac{W_t}{P_{T,t}} = A_{T,t}$$  \hspace{1cm} (3.16)

**Nontradable Sector**  A single nontradable sector firm faces the similar production technology and the profit maximization problem. The production technology is linear:

$$Y_{N,t}(j) = A_{N,t}N_{N,t}(j)$$  \hspace{1cm} (3.17)

where $Y_{N,t}(j)$ is a differentiated final nontradable good of firm $j$, $A_{N,t}$ is a generally influential productivity shock which follows the stochastic process,

$$\log A_{N,t} = \rho_N \log A_{N,t-1} + \mu_{N,t}$$  \hspace{1cm} (3.18)

where $0 \leq \rho_N < 1$ and $\mu_{N,t} \sim N(0, \sigma_N^2)$, and $N_{N,t}$ is a labor demand for nontradable goods production. A marginal cost for a typical nontradable sector $j$, $MC_{N,t}(j)$, is derived by

$$MC_{N,t}(j) = \frac{W_t}{A_{N,t}}$$  \hspace{1cm} (3.19)

A price rigidity assumption is invited in this sector, following Calvo (1983)[14] and Yun (1996)[84] type staggered price setting. A randomly selected portion of producers $(1 - \theta)$ freshly sets a new price level at each period, while $\theta$ of firms keep their old price level at the same level with the last period. Therefore, $\theta$ captures a degree of price rigidity. Let $\overline{P_{N,t}(j)}$ be a price set optimally by a firm $j$ at time $t$. With the staggered price setting described above, $P_{N,t+k}(j) = \overline{P_{N,t}(j)}$. Then, a typical firm $j$’s problem is given by

$$\max_{P_{N,t}(j)} \sum_{k=0}^{\infty} \theta^k E_t \left[ A_{N,t}^k \{ Y_{N,t+k}^k(j) \left( \overline{P_{N,t}(j)} - MC_{N,t+k}(j) \right) \} \right]$$
subject to the international demand constraints

\[ Y^k_{N,t}(j) \leq \left( \frac{P_{N,t}}{P_{N,t+k}} \right)^{-\varepsilon} Y^k_{N,t} \]  

(3.20)

where \( \Lambda^k_{N,t} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right) \) and \( MC_{N,t+k} \) denote a stochastic discounting factor and a nominal marginal cost at period \( t + k \) with respect to the staggered price setting \( P_{H,t} \), respectively.

Note that a firm specific index \( j \) can be dropped in this problem as well, because every single firm uses the same price setting subject to the same marginal cost and the same resource constraint.

First order condition yields

\[ \sum_{k=0}^{\infty} \theta^k E_t \left[ \Lambda^k_{N,t} Y^k_{N,t} \left( P_{N,t} - \frac{\varepsilon}{\varepsilon - 1} MC_{N,t+k} \right) \right] = 0 \]  

(3.22)

Note that in the perfect flexible price setting, \( \theta = 0 \), above equation reproduces \( P_{N,t} = \frac{\varepsilon}{\varepsilon - 1} MC_t \).

It can be rearranged with stationary variables,

\[ \sum_{k=0}^{\infty} (\theta \beta)^k E_t \left[ C_{N,t+k}^{-\sigma} Y^k_{N,t} \frac{P_{N,t}}{P_{N,t+k}} \left( \frac{P_{N,t}}{P_{N,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{N,t+k}}{P_{N,t-1}} MC_{N,t+k} \right) \right] = 0 \]  

(3.23)

One can now define the new price index of domestically produced goods under the staggered price setting,

\[ P_{N,t} = \left[ \theta P_{N,t-1}^{1-\varepsilon} + (1 - \theta) \left( \frac{P_{N,t}}{P_{N,t-1}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \]  

(3.24)

\[ \leftrightarrow \Pi_{N,t} \equiv \frac{P_{N,t}}{P_{N,t-1}} = \left[ \theta + (1 - \theta) \left( \frac{P_{N,t}}{P_{N,t-1}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \]  

(3.25)

### 3.3.3 Competitive Equilibrium

Remaining part of this section is describing market clearing conditions, undefined exogenous processes, and defining competitive equilibrium. First of all, the nontradable goods market clearing condition is given by

\[ Y_{N,t} = (1 - \gamma) \left( \frac{\xi_t}{P_{N,t}} \right)^{\gamma} C_t \]  

(3.26)

\[ = A_{N,t} N_t \]  

(3.27)
Domestic and foreign bonds markets are cleared every period,

\[ B_{H,t} = 0 \]  
\[ B_{F,t} + B_{F,t}^* = 0 \]

and both productions sectors has matched labor supply and demand at each term,

\[ L_{T,t} = N_{T,t} \]  
\[ L_{N,t} = N_{N,t} = \int_0^1 N_{N,t}(j) dj \]

Balance of payment constraint is calculated by

\[ \gamma C_t = \left( \frac{E_t}{P_{N,t}} \right)^{(1-\gamma)} (R_{t-1}^* B_{F,t-1} - B_{F,t} + Y_{T,t}) \]

Next, I specify two types of external shocks to the domestic economy, \( R_t^* \) and \( C_t^* \). The foreign price changes, the other exogenous variables to the domestic market, is automatically determined by (3.14). The foreign interest rate shock is defined by

\[ \log R_t^* = \rho_R \log R_{t-1}^* + \mu_{R,t} \]

where \( 0 \leq \rho_R < 1 \) and \( \mu_{R,t} \sim N(0, \sigma_R^2) \) and the foreign consumption (production) shock is defined by

\[ \log C_t^* = \rho_C \log C_{t-1}^* + \mu_{C,t} \]

where \( 0 \leq \rho_C < 1 \) and \( \mu_{C,t} \sim N(0, \sigma_C^2) \). The reduced form of competitive equilibrium is defined by a set of endogenous variables, \( \{ C_t, L_{T,t}, L_{N,t}, N_{T,t}, N_{N,t}, Y_{T,t}, Y_{N,t}, B_{H,t}, B_{F,t}, MC_{N,t}\}_{t=0}^\infty \) with a set of prices, \( \{ E_t, W_t, P_t, P_{N,t}, P_{N,t}, \Pi_{N,t}\}_{t=0}^\infty \), and a set of exogenous variables, \( \{ A_{T,t}, A_{N,t}, R_t^*, C_t^*, \Pi_{N,t}\}_{t=0}^\infty \), which solves equations (3.9) to (3.12), (3.13), (3.14), (3.16), (3.18), (3.19), (3.20), (3.22), (3.24) to (3.32), (3.33) and (3.34).

### 3.4 Qualitative Analysis

In this section, I derive a log-linearized system of equations that characterizes the competitive equilibrium, which is reduced to several important relations such as New Keynesian Phillips Curve
(NKPC) or IS relation. To solve an optimal policy problem, I derive a linear-quadratic (LQ) welfare loss function from the domestic household’s utility function and the other equations. Then I derive first order conditions from the well defined Ramsey policy problem.

3.4.1 Log-linearized System of Equations

First of all, let us define a relative price of tradable goods $P_{T,t}$ to the price of nontradable goods, $P_{N,t}$, by $Q_t \equiv \frac{P_{T,t}}{P_{N,t}}$. Then the CPI price index can be expressed by

$$P_t = Q_t^\gamma P_{N,t} \quad (3.35)$$

The total production of tradable and nontradable goods can be log-linearized by

$$y_{T,t} = a_{T,t} + n_{T,t} \quad (3.36)$$
$$y_{N,t} = a_{N,t} + n_{N,t} \quad (3.37)$$

where $x_t = \frac{X_t - X}{X}$ for any arbitrary endogenous variable $X_t$, and $X$ means a zero inflation steady state value of $X_t$. The market clearing conditions of the nontradable and tradable sectors, (3.26) and (3.32), are also log-linearized to find the relationship between two outputs in the domestic country.

$$y_{T,t} = -q_t + y_{N,t} \quad (3.38)$$

where I use the market clearing conditions of home and foreign asset holdings and $\frac{Y_T}{(R-1)}B_F + Y_T$ approximates to unity. Next, the real marginal cost in the nontradable sector is expressed by

$$\frac{MC_{N,t}}{P_{N,t}} = \frac{W_t}{P_{N,t}A_{N,t}} = \left( \frac{(N_{T,t} + N_{N,t})^\theta C_t^\sigma}{W} \right) \frac{Q_t^\gamma}{A_{N,t}} \quad (3.39)$$
Combining (3.36), (3.37), (3.38), and (3.39), the log-linearized version of the real marginal cost in the nontradable sector is calculated by

\[
\hat{m}_{cN,t} \equiv m_{cN,t} - p_{N,t} - \log \mu \\
= \phi \gamma n_{T,t} + \phi(1 - \gamma)n_{N,t} + \sigma c_t + \gamma q_t - \bar{w} - a_{N,t} \\
= \phi \gamma y_{T,t} + (\phi(1 - \gamma) + \sigma)y_{N,t} + \gamma(1 - \sigma)q_N - \bar{w} - \sigma \gamma a_{T,t} - (\phi(1 - \gamma) + 1)a_{N,t} \\
= (\phi + \sigma) y_{N,t} + \gamma(1 - \sigma)(1 - \phi)q_t - \bar{w} - \sigma \gamma a_{T,t} - (\phi(1 - \gamma) + 1)a_{N,t} 
\]

(3.40)

where \(\mu = \frac{\varepsilon}{\varepsilon - 1}\), \(\bar{w} = \log \bar{W}\), and I use an estimation \(\frac{N_n}{N_{t+1}} = \gamma\). The price decision equation in the nontradable sector (3.23) can be solve forward and log-linearized by

\[
\pi_{N,t} = \kappa \hat{m}_{cN,t} + \beta E_t \pi_{N,t+1} 
\]

(3.41)

where \(\kappa = \frac{(1 - \theta)(1 - \beta \theta)}{\beta}\). Next equations explain natural rates of three endogenous variables which will be helpful in deriving NKPC and IS relation with a "gap" variable, which is defined by \(\tilde{x}_t = x_t - x_n^t\), where \(x_n^t\) is a natural rate of \(x_t\). "Natural rate" means an equilibrium variable in an economy where no economic friction exists.

\[
q^n_t = a_{T,t} - a_{N,t} 
\]

(3.42)

\[
y^n_{N,t} = \left(1 + \frac{\gamma(1 - \sigma)}{\sigma}\right) q^n_t 
\]

(3.43)

\[
y^n_{T,t} = (1 - \gamma)q^n_t 
\]

(3.44)

The detailed derivations are provided in the appendix. By combining (3.41) with (3.40), (3.42), and (3.43), the baseline NKPC is derived by

\[
\pi_{N,t} = \kappa (\Omega_y \tilde{y}_{N,t} + \Omega_q \tilde{q}_t - \bar{w} - \Omega_t a_{T,t} - \Omega_n a_{N,t}) + \beta E_t \pi_{N,t+1} 
\]

(3.45)

where \(\Omega_y = \phi + \sigma\), \(\Omega_q = \gamma(1 - \sigma)(1 - \phi)\), \(\Omega_t = \Omega_y \left(\frac{1 - \gamma(1 - \sigma)}{\sigma}\right) - \gamma(1 + \phi)\), and \(\Omega_n = (\phi(1 - \gamma) + 1) - \gamma - \Omega_y \left(\frac{1 - \gamma(1 - \sigma)}{\sigma}\right)\). Note that as \(\gamma\) goes to zero, the equation replicates a closed economy version of the NKPC. Also note that, \(\bar{w}\) diminishes the trade-off between nontradable inflation and output gap stabilization as long as \(\Omega_y\) is positive value, and it also softens the trade-off between nontradable
inflation and the relative price changes, \( \hat{q}_t \), as long as \( \Omega_q \) is positive value. Therefore, if \( \Omega_y \) and \( \Omega_q \) are positive, \( \bar{w} \) behaves an economic volatility ameliorating role and thus the trade-off faced by a policy maker is also mitigated. The demand side of the economy can be summarized by so called IS relation, which can be started by a log-linearization of the household’s Euler equations, (3.10) and (3.11), combining with (3.12):

\[
0 = \log \beta + r_t - \sigma(c_{t+1} - c_t) - \pi_{t+1}
\]

\[
= \log \beta + r^*_t + (1 - \sigma\gamma)(q_{t+1} - q_t) - \sigma(y_{t+1} - y_t)
\]

\[
= \log \beta + \sigma(y^*_t - y^*_{t+1}) + \pi^*_{t+1} + (1 - \sigma\gamma)(q_{t+1} - q_t) - \sigma(y_{N,t+1} - y_{N,t})
\]

\[
= \log \beta + r_t - \sigma \left( 1 + \frac{\gamma(1 + \sigma)}{1 - \sigma\gamma} \right) (\Delta y^*_{N,t+1}) + \frac{\sigma\gamma(1 + \sigma)}{1 - \sigma\gamma} (\Delta y^*_{t+1}) - \pi_{N,t+1} + \frac{\sigma\gamma}{1 - \sigma\gamma} \pi^*_{t+1}
\]

(3.46)

where \( \Delta x_{t+1} \equiv x_{t+1} - x_t \). By combining with (3.42) and (3.43), the above equation can be expressed with "gap" variables:

\[
0 = \log \beta + r_t - \Omega_{yy}(\Delta y^*_{N,t+1}) + \frac{\sigma\gamma(1 + \sigma)}{1 - \sigma\gamma} (\Delta y^*_{t+1}) - \pi_{N,t+1} + \frac{\sigma\gamma}{1 - \sigma\gamma} \pi^*_{t+1} - \Omega_{y^*}(\Delta a_{T,t+1} - \Delta a_{N,t+1})
\]

(3.47)

where \( \Omega_{yy} = \sigma \left( 1 + \frac{\gamma(1 + \sigma)}{1 - \sigma\gamma} \right) \), \( \Omega_{y^*} = \Omega_{yy} \left( \frac{1 - \gamma(1 - \sigma)}{\sigma} \right) \). Note that, as tradable sector weigh parameter \( \gamma \) goes zero, the IS relation replicates a typical closed economy IS curve. Also note that, the output gap in nontradable sector is positively affected by the interest rate, foreign demands, but it is negatively affected by domestic inflation rate and combined negative productive shocks. The other two important equations which consist of the log-linearized equilibrium conditions are a relation between home and foreign output changes and a relation between tradable and nontradable output gaps.

\[
\hat{y}_{N,t} = y^*_t + \left( \frac{1 - \gamma(1 - \sigma)}{\sigma} \right) \hat{q}_t - \left( \frac{1 - \gamma(1 - \sigma)}{\sigma} \right) (1 - \gamma)(a_{T,t} - a_{N,t})
\]

(3.48)

\[
\hat{y}_{T,t} = (-\hat{q}_t + \hat{y}_{N,t} + \frac{1 - \gamma}{\sigma} (a_{T,t} - a_{N,t}))
\]

(3.49)
3.4.2 Linear Quadratic Welfare Loss Function

Following Benigno and Woodford (2012)[9] and Woodford (2003)[82], I build a linear-quadratic social welfare loss measurement. This LQ type welfare loss function has a merit in two dimensions. First, by using LQ welfare loss function, it is convenient to find a guaranteed locally maximized unique solution. Second, it is also convenient to use for comparing different types of policy tool alternatives. The welfare loss function in this economy is derived by

\[
W = -\frac{1}{2}\left(\pi_{N,t}^2 + \Sigma_q \hat{q}_t^2 + \Sigma_n \hat{y}_{N,t}^2 + \Sigma_t \hat{y}_{T,t}^2 + (1 - \gamma)\gamma \hat{y}_{N,t} \hat{y}_{T,t} + \left(\frac{1 - \gamma(1 - \sigma)}{\sigma}\right) \hat{y}_{N,t} \hat{y}_{N,t}
\]

\[
+ (1 - \gamma)\hat{y}_{N,t} a_{N,t} + \gamma \hat{y}_{T,t} a_{T,t} + 2\gamma \Sigma_q \hat{q}_t (a_{N,t} - a_{T,t}) + \gamma \left(\frac{1 - \gamma(1 - \sigma)}{\sigma}\right) \Sigma_n \hat{y}_{N,t} (a_{N,t} - a_{T,t})
\]

\[
+ \gamma \left(\frac{1 - \gamma}{\sigma}\right) \Sigma_t \hat{y}_{T,t} (a_{N,t} - a_{T,t}) + O(\|\xi\|^3) + t.i.p.
\]

(3.50)

where \(\Sigma_n = (1 - \gamma)((1 - \gamma) + 2)\), \(\Sigma_q = \left(\frac{1 - \gamma(1 - \sigma)}{\sigma}\right)^2 + \gamma(1 - \gamma)\), \(\Sigma_t = \gamma(\gamma + 2)\), \(O(\|\xi\|^3)\) denotes terms that are of order higher than third, in the bound \(\|\xi\|\) on the magnitude of the relevant shocks, and \(t.i.p\). represents terms independent of policy variables. A detailed derivation is given in the appendix. Note that every coefficient on each part of (3.50) is positive, and thus the social welfare loss is increased by increasing variance or covariance of the policy variables.

3.4.3 Ramsey Policy Problem

A Ramsey policy problem is defined by a maximization of the welfare loss function, (3.50), with respect to the policy variables \(\{\pi_{N,t}, \hat{q}_t, \hat{y}_{N,t}, \hat{y}_{T,t}, r_t\}\), subject to the NKPC, (3.45), IS relation, (3.47), and two other relations, (3.48) and (3.49). Lagrangian equation for the policy problem is
given by

\[
L_t = \sum_{t=0}^{\infty} \beta^t [\mathcal{W} + \chi_{1,t} [\kappa (\Omega_y \hat{y}_{N,t} + \Omega_q \hat{q}_t - \bar{w} - \Omega_n a_{T,t} - \Omega_n a_{N,t}) + \beta E_t \pi_{N,t+1} - \pi_{N,t}] \\
+ \chi_{2,t} \log \beta + r_t - \Omega_y (\Delta \hat{y}_{N,t+1}) + \frac{\sigma \gamma (1 + \sigma)}{1 - \sigma \gamma} (\Delta \hat{y}_{t+1} - \pi_{N,t+1} + \frac{\sigma \gamma}{1 - \sigma \gamma} \pi_{t+1}^* - \Omega_y (\Delta a_{T,t+1} - \Delta a_{N,t+1})) \\
+ \chi_{3,t} \left[ y_t^* + \left(1 - \gamma (1 - \sigma) \right) \hat{q}_t - \left(1 - \gamma (1 - \sigma) \right) (1 - \gamma) (a_{T,t} - a_{N,t}) - \hat{y}_{N,t} \right] \\
+ \chi_{4,t} \left[ (-\hat{q}_t + \hat{y}_{N,t} + \frac{1 - \gamma}{\sigma} (a_{T,t} - a_{N,t})) - \hat{y}_{T,t} \right] \\
\right]
\]  

(3.51)

where \( \chi_{1,t}, \chi_{2,t}, \chi_{3,t} \) and \( \chi_{4,t} \) are the shadow prices for NKPC, IS relation, the foreign-home outputs relation, and tradable-nontradable outputs relation, respectively. The first order conditions of the Ramsey policy problem is derived by

\[
-\pi_{N,t} - \chi_{1,t} + \beta \chi_{1,t+1} - \chi_{2,t} = 0 \tag{3.52}
\]
\[
\Sigma_t \hat{y}_{T,t} - \frac{\gamma (1 - \gamma)}{2} \hat{y}_{N,t} - \frac{\gamma}{2} a_{T,t} - (1 - \gamma) \Sigma_t (a_{N,t} - a_{T,t}) - \chi_{3,t} = 0 \tag{3.53}
\]
\[
-\Sigma_q \hat{q}_t \gamma \Sigma_q (a_{N,t} - a_{T,t}) + \kappa \Omega_q \chi_{1,t} + \left(1 - \gamma (1 - \sigma) \right) \chi_{3,t} - \chi_{4,t} = 0 \tag{3.54}
\]
\[
\chi_{2,t} = 0 \tag{3.55}
\]
\[
-\Sigma_n \hat{y}_{N,t} - \frac{\gamma}{2} \hat{y}_{T,t} - \frac{1 - \gamma (1 - \sigma)}{\sigma} y_t^* - \left(1 - \gamma \right) \frac{a_{N,t}}{\sigma} - \left(1 - \gamma (1 - \sigma) \right) \Sigma_n (a_{N,t} - a_{T,t}) \\
+ \kappa \Omega_y + \sigma \left(1 + \gamma (1 + \sigma) \right) \chi_{2,t} - \beta \sigma \left(1 + \gamma (1 + \sigma) \right) \chi_{2,t+1} - \chi_{3,t} + \chi_{4,t} = 0 \tag{3.56}
\]

(3.45), (3.47), (3.48), (3.49), and (3.52) to (3.56) consist of the complete system of equations of the Ramsey policy problem.

### 3.5 Quantitative Analysis

In this section, I observe impulse responses of some important macroeconomic variables at the competitive equilibrium to the previously defined exogenous shocks. The purpose of this work is that, by changing the binding minimum wage in the model, one can clearly and numerically observe how the transmission of the external shock is varied. In the model, the baseline value
of the minimum wage is 1, which means that the minimum wage does not increase or decrease the wage level paid to the consumers. If the minimum wage level is 1.1, the overall wage level is increased by 10%. If the minimum wage level is set to be 0.9, the overall wage level is decreased by 10%. Therefore, by controlling the minimum wage level, the government can directly change the overall wage level finally paid to the domestic consumers.

3.5.1 Parameterization

Table 3.1 shows the list of baseline parameter values. Time discounting factor is set to be 0.99 as normal in the literature, elasticity of labor supply and intertemporal elasticity of substitution in composite consumption are set to be unity and two, respectively, by following Demirel (2010)[29]. I also follow the same paper to set the share of tradable goods in composite consumption to be 0.45. Price stickiness parameter $\theta$ is set to be 0.66. There are two types of domestic productivity shocks, and they are separately defined. I follow Demirel (2010)[29] to define the specifications of the both productivity shocks in different sectors. I follow [39] and Gali and Monacelli (2005)[40] to define two international dimensions of shocks, foreign interest rate shocks and foreign demand shocks.

3.5.2 Role of Minimum Wage on Business Cycles

Figure 3.4 shows the changes in standard deviations of the key variables as the value of $\bar{w}$ changes from 0.5 to 1.5. The benchmark value of $\bar{w}$ is set to be unity. If $\bar{w}$ is 1.1, the wage is increased by 10% by the minimum wage increment forced by the government. Figure 3.4 clearly shows that, for all five variables, economic volatility is decreased by the increasing real wage, supported by the minimum wage raises. CPI inflation rate and the output gap in tradable sector show a relatively higher volatility which is reasonable since the observed result is gathered from the exogenous foreign output (demand) shock, and thus the terms including tradable goods react more sensitively. Figure 3.5 shows the impulse responses of key macroeconomic variables to the 1% foreign negative demand shock. These five graphs also support the argument that the increasing
minimum wage can reduce the volatility of the economy when sudden decrease in foreign demand for the domestic tradable goods. It is interesting to see that when minimum wage is increased by 50%, the relative price of tradable sector to the one of nontradable sector positively reacts to the negative shock while the nontradable sector is decreased. CPI inflation still negatively reacts to the shock, but the volatility is much reduced in the case of positive minimum wage increasing.

3.6 Conclusion

This paper investigates the role of change in real wages on the economic dynamics and optimal policy decision making where a portion of the wage structure is forced to operated manually by a government. This model structure is designed to illustrate the effect of a fixed level of minimum wage on the overall wage level and the marginal cost that permanently change the reaction of the economy to the unexpected external foreign demand changes. Increasing minimum wage reduces the real marginal cost by amount of changes in the overall wage increment, and thus it ameliorates the trade-off between output gap and inflation stabilization faced by a policy maker. This alleviated trade-off problem can lower the level of volatilities in key macroeconomic variables in the Ramsey policy equilibrium. Economic intuition of this simulation result is that, the increasing minimum wage can preserve a certain (more) amount of real purchasing power of the domestic agents even if the public support is governed by a lump sum taxation, and this increasing purchasing power enables the domestic economy to persist with the original pattern of economic activities more consistently.

There are several significant limitation in this paper. First, wage structure should be considered more seriously. It must show the effect of the minimum wage on the overall wage level and the marginal cost structure more clearly, and it also must explain more clearly about the role of the minimum wage on the business cycles. Second, staggered wage assumption would be helpful to explain the puzzled part of the paper. In many developing countries, a nominal wage rigidity has been found as a major economic characteristics and this phenomenon could help the role of minimum wage in a clearer view.
Figure 3.1: Statistical Comparison between South Korea and Mexico during the Different Time of Crisis
Figure 3.2: Statistical Comparison between South Korea and Chile during the Different Time of Crisis
Figure 3.3: Statistical Comparison between South Korea and US
Figure 3.4: Changes in Standard Deviations of Key Variables with respect to the Change in $\bar{w}$
Figure 3.5: Impulse Responses of Key Variables to Foreign Demand Shock
Table 3.1: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Estimated Value</th>
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<tr>
<td>$\phi$</td>
<td>Reverse of Elasticity of Labor Supply</td>
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<tr>
<td>$\sigma$</td>
<td>Intertemporal Elasticity of Substitution in Private Consumption</td>
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<tr>
<td>$\gamma$</td>
<td>Share of Tradable Goods in Composite Consumption</td>
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</tr>
<tr>
<td>$\beta$</td>
<td>Time Discounting Factor</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>Degree of Price Stickiness</td>
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<tr>
<td>$\rho_T$</td>
<td>Autoregressive Parameter of Domestic Tradable Sector Productivity Shock</td>
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</tr>
<tr>
<td>$\rho_N$</td>
<td>Autoregressive Parameter of Domestic Nontradable Sector Productivity Shock</td>
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</tr>
<tr>
<td>$\rho_R$</td>
<td>Autoregressive Parameter of Foreign Interest Rate Shock</td>
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<td>$\rho_C$</td>
<td>Autoregressive Parameter of Foreign Demand Shock</td>
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<tr>
<td>$\sigma_T$</td>
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Bibliography


Appendix A

Mathematical Appendix in Chapter 1

A.1 Derivation of Linear-Quadratic Welfare Loss Function

In this appendix, I derive a second-order approximation to the representative home consumer’s utility function, (1.1), which is used as an objective function in the Ramsey policy problem in section 4. To begin with, I use the following Taylor series approximation rule in this entire appendix:

\[
\frac{X_t - X}{X} \simeq x_t + \frac{1}{2}x_t^2 + O(\|\xi\|^3) \tag{A.1}
\]

It is also useful to know the following knowledge:

\[
\sigma = -\frac{U_{CC}}{U_C}C \tag{A.2}
\]

\[
\varphi = \frac{U_{LL}}{U_L}L \tag{A.3}
\]

where \(U_X \equiv \frac{\partial U(\cdot)}{\partial X}\) and \(U_{XX} \equiv \frac{\partial^2 U(\cdot)}{\partial X^2}\) for any arbitrary variable \(X\). The first term in (1.1) can be second-order approximated by

\[
\frac{C_t^{1-\sigma}}{1 - \sigma} \simeq \frac{C_t^{1-\sigma}}{1 - \sigma} + U_CC(c_t + \frac{1}{2}c_t^2) + \frac{1}{2}U_{CC}C^2c_t^2 + O(\|\xi\|^3)
\]

\[
= \frac{C_t^{1-\sigma}}{1 - \sigma} + U_CC(c_t + \frac{1}{2}(1 - \sigma)c_t^2) + O(\|\xi\|^3) \tag{A.4}
\]
The second term in (1.1) is approximated by

\[
\frac{L_{t+1}^{1+\varphi}}{1 + \varphi} \approx \frac{L_{t}^{1+\varphi}}{1 + \varphi} + U_{L}L(l_{t} + \frac{1}{2}l_{t}^{2}) + \frac{1}{2}U_{LL}L^{2}l_{t}^{2} + O(\|\xi\|^{3})
\]

\[
= \frac{L_{t}^{1+\varphi}}{1 + \varphi} + U_{L}L(l_{t} + \frac{1}{2}(1 + \varphi)g_{t}^{2}) + O(\|\xi\|^{3})
\]

\[
= \frac{L_{t}^{1+\varphi}}{1 + \varphi} - UC\tilde{C}(l_{t} + \frac{1}{2}(1 + \varphi)l_{t}^{2}) + O(\|\xi\|^{3}) \tag{A.5}
\]

In the last step I use \(\frac{U_{L}}{U_{C}} = MPN = -YN\) and \(Y = C\). So far, the second-order approximated utility function is rewritten by

\[
\frac{U_{C} - U}{U_{C}C} = \left(c_{t} + l_{t} + \frac{1}{2}((1 - \sigma)c_{t}^{2} + (1 + \varphi)l_{t}^{2})\right) + O(\|\xi\|^{3}) + \text{t.i.p.} \tag{A.6}
\]

Next, from the the risk sharing condition, (1.52), and the market clearing condition, (1.56), one can obtain the following relation:

\[
\hat{c}_{t} = \Xi_{y}y_{t} - \Xi_{b}\tilde{b}_{F,t} + \Xi_{y}^{*}y_{t}^{*} \tag{A.7}
\]

where

\[
\Xi_{y} = \left(\frac{\omega_{c}}{\omega_{c} + \omega_{c}^{*}} + \frac{\sigma}{1 - \alpha}\left(\frac{(1 - \alpha)\omega_{c}^{*}}{\omega_{c} + \omega_{c}^{*} - \alpha}\right)\right)^{-1}
\]

\[
\Xi_{b} = \frac{B_{F}\Psi_{B}}{(1 - \alpha)}\left(\frac{(1 - \alpha)\omega_{c}^{*}}{\omega_{c} + \omega_{c}^{*}} - \alpha\right)\left(\frac{\omega_{c}}{\omega_{c} + \omega_{c}^{*}} + \frac{\sigma}{1 - \alpha}\left(\frac{(1 - \alpha)\omega_{c}^{*}}{\omega_{c} + \omega_{c}^{*} - \alpha}\right)\right)^{-1}
\]

\[
\Xi_{y}^{*} = \left(\frac{\omega_{c}^{*}}{\omega_{c} + \omega_{c}^{*}} + \frac{\sigma}{1 - \alpha}\left(\frac{(1 - \alpha)\omega_{c}^{*}}{\omega_{c} + \omega_{c}^{*} - \alpha}\right)\right)\left(\frac{\omega_{c}}{\omega_{c} + \omega_{c}^{*}} + \frac{\sigma}{1 - \alpha}\left(\frac{(1 - \alpha)\omega_{c}^{*}}{\omega_{c} + \omega_{c}^{*} - \alpha}\right)\right)^{-1}
\]

Furthermore, \(L_{t} = N_{t}\) is also written in terms of variables included in NKPC and IS relation, and approximated by,

\[
\tilde{n}_{t} = \tilde{y}_{t} + k_{t} \tag{A.8}
\]

where \(k_{t} = \log_{1}^{\frac{1}{\varepsilon}}\left(\frac{P_{H,t}^{\varepsilon}(j)}{P_{H,t}}\right)\). Combining (A.7) and (A.8) with (A.6) and applying two lemmas in the appendix of [40] and the chapter 6 of [82], the welfare loss function is represented by,

\[
\mathcal{W} \equiv \frac{U_{C} - U}{U_{C}C} = \frac{1}{2} \left[k_{t} + \Omega_{y}\tilde{y}_{t}^{2} + \Omega_{b}\tilde{b}_{F,t}^{2} - 2\Omega_{y}\tilde{y}_{t}\tilde{y}_{t}^{*} - 2\Omega_{b}\tilde{b}_{F,t}\tilde{b}_{F,t}^{*} + 2\Omega_{y}\tilde{b}_{F,t}\tilde{y}_{t}^{*} - 2\Omega_{b}\tilde{b}_{F,t}\tilde{y}_{t}^{*}\right] + O(\|\xi\|^{3}) + \text{t.i.p.}
\]

\[
= \frac{1}{2} \left[\frac{\varepsilon}{\kappa}p_{H,t}^{2} + \Omega_{y}\tilde{y}_{t}^{2} + \Omega_{b}\tilde{b}_{F,t}^{2} - 2\Omega_{y}\tilde{b}_{F,t}\tilde{b}_{F,t} + 2\Omega_{y}\tilde{y}_{t}\tilde{b}_{F,t}^{*} - 2\Omega_{b}\tilde{b}_{F,t}\tilde{y}_{t}^{*}\right] + O(\|\xi\|^{3}) + \text{t.i.p.} \tag{A.9}
\]
where

\[
\begin{align*}
\Omega_y &= (1 - \sigma)\Xi_y^2 + (1 + \varphi) \\
\Omega_b &= (1 - \sigma)\Xi_b^2 \\
\Omega_{y,b} &= 2(1 - \sigma)\Xi_y\Xi_b \\
\Omega_{y,y^*} &= 2(1 - \sigma)\Xi_y\Xi_{y^*} \\
\Omega_{b,y^*} &= 2(1 - \sigma)\Xi_{y^*}\Xi_b
\end{align*}
\]

which corresponds to (1.65).
Appendix B

Mathematical Appendix in Chapter 2

B.1 Efficient Level Equilibrium

In order to have natural rates of output and government spending as a log-linearized form, one needs to solve a competitive equilibrium problem under complete market environment. A social planner’s problem is given by the maximization of the utility function, (2.1), such that the economy-wide budget constraint,

$$C_t + G_t = A_t L_t \tag{B.1}$$

First order necessary conditions are calculated and log-linearized by

$$\varphi l^n_t - a_t = -\sigma c^n_t = -\phi g^n_t \tag{B.2}$$

Note that the second equality comes from the efficient level equilibrium condition that marginal utility of private consumption should be equal to marginal utility of public consumption. The economy-wide budget constraint is also log-linearized by

$$a_t + l^n_t = \frac{C}{Y} c^n_t + \frac{G}{Y} g^n_t \tag{B.3}$$

Combining (B.2) and (B.3) to remove $l^n_t$ and $c^n_t$ and express $g^n_t$ in terms of $a_t$, the exogenous variable, one can obtain the natural level of government spending given by

$$g^n_t = \left[ \frac{1}{Y} \left( \frac{C \phi}{\sigma} + G \right) + \frac{\phi}{\varphi} \right]^{-1} (1 + \frac{1}{\varphi}) a_t \tag{B.4}$$

which corresponds to (2.37).

To achieve a efficient level of output, $y^n_t$, setting (2.34) to be zero, which means that in the efficient
level the real marginal cost should be zero since there is neither price rigidity nor imperfect competition. And substituting (B.4) into the modified equation, one can express $y_t^n$ in terms of $a_t$ and the other parameters.

$$y_t^n = (\varphi + \sigma Y C) - 1 \left[ \sigma C g_t^n + (\varphi + 1)a_t^n \right]$$  \hspace{1cm} (B.5)

which corresponds to (2.36).

### B.2 Derivation of IS relation

In this part, I show the detailed calculation of deriving IS equation. Log-linearizing the Euler equation (2.10) gives a log deviation version of relationship between consumption stream and inflation changes:

$$-\sigma c_t = (r_t - \pi_{t+1}) - \sigma c_{t+1}$$  \hspace{1cm} (B.6)

To replace $c_t$ and $c_{t+1}$ with terms of $y_t$ and $g_t$, one needs to log-linearize economy wide resource constraint, $Y_t = C_t + G_t + \xi \frac{G_t}{2}(G_t - G_t^n)^2$ and rewrite it with the expression of $c_t$,

$$c_t = \frac{C}{Y} y_t + \frac{G}{C}(1 + \xi \hat{G}) g_t$$  \hspace{1cm} (B.7)

Substituting (B.7) into (B.6) gives an expression for $y_t, g_t, r_t,$ and $\pi_{t+1},$

$$y_t - \frac{G}{Y}(1 + \xi \hat{G}) g_t = -\frac{C}{\sigma Y}(r_t - \pi_{t+1}) + y_{t+1} + \frac{G}{Y}(1 + \xi \hat{G}) g_{t+1}$$  \hspace{1cm} (B.8)

The remaining part is to express (B.8) with "gap" variables, which is defined by $\hat{x}_t = x_t - x_t^n$ for any arbitrary variable $x$. Substituting definitions of $y_t^n$ and $g_t^n$ from (2.36) and (2.37) into (B.8) provides an expression for $\hat{y}_t$ and $\hat{g}_t$:

$$\hat{y}_t + (\varphi + \sigma Y C) - 1 \left[ \sigma C (1 + \xi \hat{G}) \right] g_t^n + (\varphi + \sigma Y C) - 1 (\varphi + 1)a_t - \frac{G}{Y}(1 + \xi \hat{G}) \hat{g}_t$$

$$- \frac{G}{Y}(1 + \xi \hat{G})(\frac{1}{Y}(\frac{C \phi}{\sigma} + G) + \frac{\phi}{\varphi}) - 1 (1 + \frac{1}{\varphi}) a_t$$

$$= -\frac{1}{\sigma Y}(r_t - \pi_{t+1}) + \hat{y}_{t+1} + (\varphi + \sigma Y C) - 1 \left[ \sigma C (1 + \xi \hat{G}) \right] \hat{g}_{t+1} + (\varphi + \sigma Y C) - 1 (\varphi + 1)a_{t+1}$$

$$- \frac{G}{Y}(1 + \xi \hat{G}) \hat{g}_{t+1} - \frac{G}{Y}(1 + \xi \hat{G})(\frac{1}{Y}(\frac{C \phi}{\sigma} + G) + \frac{\phi}{\varphi}) - 1 (1 + \frac{1}{\varphi}) a_{t+1}$$  \hspace{1cm} (B.9)
Rewriting (B.9) provides new Keynesian IS equation with the government spending under frictions:

\[
\hat{y}_t - G Y (1 + \xi \hat{G}) \hat{g}_t + G Y (1 + \xi \hat{G}) \hat{g}_{t+1} \\
= -\frac{1}{\sigma} (r_t - E_t \pi_{t+1}) + \hat{y}_{t+1} + (\varphi + \sigma \frac{Y}{C})^{-1} \left( \sigma \frac{G}{Y} (1 + \xi \hat{G}) (\hat{y}_{t+1} - \hat{g}_n) \right) \\
- \left[ \frac{G}{Y} (1 + \xi \hat{G}) \left( \frac{C \phi}{\sigma} + G \right) + \phi \hat{g}_{t+1} (1 + \frac{1}{\varphi}) - (\varphi + \sigma \frac{Y}{C})^{-1} (\varphi + 1) \right] (a_{t+1} - a_t) \tag{B.10}
\]

which corresponds to (2.39) with following definitions of parameters.

### B.3 Derivation of Linear-Quadratic Welfare Loss Function

In this part, I derive the second order approximation to the utility function of a representative consumer to find the appropriate value of linear-quadratic welfare loss function, shown in (2.40) and (2.41). To do this job, I use the following Taylor series expansion

\[
\frac{X_t - X}{X} \simeq x_t + \frac{1}{2} x_t^2 + O(\| \varepsilon_a \|^3) \tag{B.11}
\]

for any arbitrary endogenous variable \( X_t \). First step is to get second order approximation to the utility function, (2.1). It is useful to use the knowledge of (2.64) and the following three additional convenient relations:

\[
\begin{align*}
\sigma &= -\frac{U_{CC}}{U_C} C \\ 
\phi &= -\frac{U_{GG}}{U_G} G \\ 
\varphi &= \frac{U_{LL}}{U_L} L 
\end{align*} \tag{B.12-14}
\]

where \( U_X \equiv \frac{\partial U(\cdot)}{\partial X} \) and \( U_{XX} \equiv \frac{\partial^2 U(\cdot)}{\partial X^2} \). After second order approximation, the first term in (2.1) can be rewritten by

\[
\begin{align*}
\frac{C_t^{1-\sigma}}{1-\sigma} &\simeq \frac{C_t^{1-\sigma}}{1-\sigma} + U_CC(c_t + \frac{1}{2} c_t^2) + \frac{1}{2} U_{CC} C^2 c_t^2 + O(\| \varepsilon_a \|^3) \\
&= \frac{C_t^{1-\sigma}}{1-\sigma} + U_CC(c_t + \frac{1}{2} (1-\sigma) c_t^2) + O(\| \varepsilon_a \|^3) \tag{B.15}
\end{align*}
\]
The second term can be expressed in the similar way,

\[
\frac{G_t^{1-\phi}}{1-\phi} \approx \frac{G_t^{1-\phi}}{1-\phi} + U_C G(g_t + \frac{1}{2} g_t^2) + \frac{1}{2} U_C^2 G_t^2 + O(\| \varepsilon_a \|^3)
\]

\[
= \frac{G_t^{1-\phi}}{1-\phi} + U_C G(g_t + \frac{1}{2} (1-\phi) g_t^2) + O(\| \varepsilon_a \|^3)
\]

\[
= \frac{G_t^{1-\phi}}{1-\phi} + U_C \frac{G}{C} (g_t + \frac{1}{2} (1-\phi) g_t^2) + O(\| \varepsilon_a \|^3)
\] (B.16)

The last line in (B.16) is derived by the efficient level equilibrium condition \( U_C = U_G \). The third term for labor disutility is approximated by

\[
\frac{L_t^{1+\varphi}}{1+\varphi} \approx \frac{L_t^{1+\varphi}}{1+\varphi} + U_L L_l + \frac{1}{2} U_LL^2 l_t^2 + O(\| \varepsilon_a \|^3)
\]

\[
= \frac{L_t^{1+\varphi}}{1+\varphi} + U_L L_l + \frac{1}{2} (1+\varphi) g_t^2 + O(\| \varepsilon_a \|^3)
\]

\[
= \frac{L_t^{1+\varphi}}{1+\varphi} + U_C \frac{L}{C} (l_t + \frac{1}{2} (1+\varphi) l_t^2) + O(\| \varepsilon_a \|^3)
\] (B.17)

To express the last line I use the condition that the marginal rate of substitution between private consumption and labor should be equal to the marginal product of labor. Combining (B.15), (B.16), and (B.17), the welfare measure at time \( t \), \( V_t \), is calculated by

\[
V_t = U_C \left[ c_t + \frac{G}{C} g_t - \frac{L}{C} l_t + \frac{1}{2} (1-\sigma) c_t^2 + \frac{1}{2} (1-\phi) g_t^2 + \frac{1}{2} (1+\varphi) l_t^2 \right] + t.i.p. + O(\| \varepsilon_a \|^3) \] (B.18)

where \( t.i.p. \) denotes "terms independent of policy." To remove the linear terms in (B.18) and replace \( l_t \) with other variables, I obtain the log-linearized version of the second order approximations of economy wide resource constraint, \( Y_t = C_t + G_t + \xi \left( G_t - G_t^0 \right)^2 \) and the technology of the economy, \( Y_t = A_t N_t \).

\[
Y_t g_t + \frac{1}{2} g_t^2 = \frac{1}{2} \pi^2 + (1+\xi \hat{G})G_t + \frac{1}{2} (1+\xi \hat{G} + \xi G^2) g_t^2 + Cc_t + \frac{1}{2} C^2 c_t^2 \] (B.19)

\[
l_t = y_t + \frac{1}{2} g_t^2 - y_t a_t - \frac{1}{2} l_t^2 \] (B.20)

Substituting (B.19) and (B.20) into (B.18) gives

\[
V_t = U_C \left[ -\frac{1}{2 \overline{Y}} (1+\xi \hat{G} + \xi G^2) g_t^2 - \frac{1}{2 \overline{Y}} C_t^2 - \frac{G}{\overline{Y}} \xi \hat{G} g_t + \frac{1}{2} Y g_t^2 - \frac{1}{2} g_t^2 + y_t a_t + \frac{1}{2} l_t^2 + \frac{1}{2 \overline{Y}} (1-\sigma) c_t^2 \right.
\]

\[
+ \frac{1}{2 \overline{Y}} (1-\phi) g_t^2 - \frac{1}{2} (1+\phi) l_t^2 \]
which is equivalent to

\[
V_t = U_C \left[ \frac{C^2}{Y} + \frac{(1 - \sigma)}{Y} c_t^2 + (Y - 1) y_t^2 + 2 y_t a_t + \left( -\frac{(1 + \xi \hat{G})}{Y} + \frac{(1 - \sigma)}{Y} \right) g_t^2 - \varphi_t^2 \right] + t.i.p. + O(\|\varepsilon_a\|^3)
\] (B.21)

which corresponds to (2.40).
Appendix  C

Mathematical Appendix in Chapter 3

C.1  Derivation of Natural Rates of $q_t$, $y_{N,t}$, $y_{T,t}$

A "natural" variable is defined by a variable at a state where no friction exists in the economy. In the case of this paper, since the only economic friction is a price rigidity in the nontradable sector, the fully flexible prices in both sectors are assumed to derive natural rates of log-linearized version of key variables. First of all, in the frictionless economy, the marginal cost of firm $j$ in the nontradable sector equals to the price of a perfectly competitive sector:

$$MC_{N,t}(j) = P_{N,t} = \frac{W_t}{A_{N,t}}$$

The price of the tradable sector is the same in the previous sector:

$$P_{T,t} = \frac{W_t}{A_{T,t}} = \mathcal{E}_t$$

Equalizing the above two equations, a natural rate of the relative price of tradable goods to the nontradable goods is derived by

$$Q^n_t = \frac{P_{T,t}}{P_{N,t}} = \frac{A_{T,t}}{A_{N,t}}$$  \hspace{1cm} (C.1)

Log-linearizing (C.1) gives a natural rate of $q_t$:

$$q^n_t = a_{T,t} - a_{N,t}$$  \hspace{1cm} (C.2)

Next, combining (3.11) and (3.12), and substituting (C.1) into it, a natural rate of private consumption has a relation with foreign consumption and the relative price:

$$C^n_t = \xi C^*_t (Q^n_t)^\frac{1}{\sigma}$$  \hspace{1cm} (C.3)
Log-linearizing (C.3) gives
\[ c_t^n = y_t^* + \left( \frac{1 - \gamma}{\sigma} \right) q_t^n \] (C.4)

Log-linearizing the nontradable goods market clearing condition (3.26) and combining it with (C.4) gives a natural rate of \( y_{N,t} \):
\[ y_{N,t}^n = \gamma q_t^n + c_t^n \]
\[ = \gamma q_t^n + \left( \frac{1 - \gamma}{\sigma} \right) q_t^n \]
\[ = \left( \frac{1 - \gamma(1 - \sigma)}{\sigma} \right) q_t^n \] (C.5)

Latly, the market clearing condition (3.32) and substituting (3.29), (C.4), and (C.5) into it gives a natural rate of \( y_{T,t} \):
\[ y_{T,t}^n = (-q_t^n + y_{N,t}) \]
\[ = (1 - \gamma) q_t^n \] (C.6)

### C.2 Derivation of Linear-Quadratic Welfare Loss Fuction \( W \)

In this appendix, I derive a linear-quadratic welfare loss function by doing a second-order approximation to the representative home consumer’s utility function and the other important equations. This loss function is used as an objective function in the Ramsey policy problem in section 4. To begin with, I use the following simple rule in this entire appendix:
\[ \frac{X_t - X}{X} \simeq x_t + \frac{1}{2} x_t^2 + O(\| \xi \|^3) \] (C.7)

It is also useful to know the following knowledge:
\[ \sigma = -\frac{U_{CC}}{U_C} C \] (C.8)
\[ \phi = \frac{U_{LL}}{U_L} L \] (C.9)
where $U_X \equiv \frac{\partial U(.)}{\partial X}$ and $U_{XX} \equiv \frac{\partial^2 U(.)}{\partial X^2}$ for any arbitrary variable $X$. The domestic consumer’s utility function is approximated by

$$\frac{U_C - U}{U_{CC}} = \left( c_t + l_{N,t} + l_{T,t} + \frac{1}{2}(1 - \sigma)c_t^2 + \frac{1}{2}(1 + \phi)(l_{N,t} + l_{T,t})^2 \right) \tag{C.10}$$

To eliminate the first-order terms in (C.10), I use the following identities:

$$c_t + \frac{1}{2}c_t^2 = y_{N,t} + \frac{1}{2}y_{N,t}^2 - \gamma(q_t + \frac{1}{2}q_t^2) \tag{C.11}$$

$$l_t = (1 - \gamma)y_{N,t} + \gamma y_{T,t} - \frac{1}{2}l_t^2 + \frac{1}{2}(\gamma)y_{T,t}^2 + \left( \frac{1 - \gamma}{2} \right)y_{N,t}^2 - (1 - \gamma)y_{N,t}a_{N,t} - \gamma y_{T,t}a_{T,t} \tag{C.12}$$

Substituting (C.11) and (C.12) into (C.10) gives an identical equation with (3.50).