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**CROSS-POLARIZATION LEVEL IN RADIATION
FROM A MICROSTRIP DIPOLE ANTENNA**

by

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Abstract

Cross-polarization level, inherent in radiation from a small horizontal electric dipole (HED) on a flat grounded dielectric substrate, is investigated in detail. The study is directed towards the design of a very low cross-pol level in a linear array of microstrip antenna elements. Field expressions for a microstrip HED are derived in spherical coordinates with respect to the array direction. In particular, two important cases, namely a HED along the array direction (i.e., parallel polarization) and a HED perpendicular to array direction (i.e., perpendicular polarization) are investigated. Extensive numerical examples for the cross-pol levels are given. It is shown that, in general, there are inherent limitations in achieving very low cross-pol levels, especially for the case of parallel polarization.

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1. Introduction

Examination of the possibility of obtaining a -30 dB cross-polarization level in a linear array of microstrip antenna elements motivated the work described in this report. Since the cross-polar side lobes in directions other than that of the beam are reduced by array factor, the cross-polarization level in the direction of the beam is the dominant in calculation of the cross-polar side lobes (i.e., cross-pol level) of a linear array. In the present case, the direction of the beam is assumed to be between 10° to 40° from broadside.

This report describes the investigations of the cross-pol level inherent in the radiation from a small, arbitrarily-oriented, horizontal electric current element on a flat grounded dielectric substrate (Figure 1). Usually, coordinate system selected for deriving the fields of a current element has the z-axis (i.e., reference axis of the spherical coordinate (r, θ_z, ϕ_z)), normal to the substrate [for example, see 1] as seen in Figures 1 and 2. However, for the linear array problem, it is desirable to use a polar coordinate system with reference axis along the array axis, say, the y-axis (Figure 3). This choice of the polar direction, makes it convenient to evaluate co-polar and cross-polar fields with respect to the beam direction. Thus it is desirable to derive the field expressions for the case when the polar-axis is in the plane of the substrate.

In section 2 of this report, based on a directional-cosine formulation, field expressions for a microstrip horizontal electric dipole (HED) are derived in a spherical coordinate with respect to any arbitrarily-oriented axis. In particular, the field expressions with respect to the y-axis (which is along the array direction) are explicitly given.

In section 3, a definition of the cross-polar side lobe level, according to the IEEE standard [2] is given and two important cases, namely, a y-directed HED (i.e., parallel polarization) and an x-directed HED (i.e., perpendicular polarization) are discussed. Also included in this section are the expressions of co-polar and cross-polar fields for an arbitrarily-oriented HED.

In section 4, numerical examples for the co-polar and cross-polar radiation patterns are given and a parametric study of the cross-pol level (for the two different polarizations) is presented. Finally, in section 5, the results are summarized and inherent limitations in achieving very low cross-pol levels are discussed.

2. Formulation

2.1 Derivation of the Radiated Fields

A horizontal electric dipole (HED) is placed on a grounded dielectric slab as shown in Figure 1. The diopole source is of moment p and directed at an angle χ with the x -axis in the x - y plane. The dielectric slab is of thickness d and assumed to be infinitely extended in the x - y plane. As shown in Figure 1, the permittivities in the air and the slab regions are ϵ_0 and ϵ_1 , respectively; the permeability in both media is assumed to be μ_0 .

To find the field expressions due to this HED current source, we employ the z components of two electric and magnetic Hertz vector potentials, Π_e and Π_m . Outside of the source region (i.e., outside $z=0$ plane), we have [3],

$$\bar{E} = \nabla \times \nabla \times (\Pi_e \bar{a}_z) + i \omega \mu_0 \nabla \times (\Pi_m \bar{a}_z) \quad (1.1)$$

$$\bar{H} = \nabla \times \nabla \times (\Pi_m \bar{a}_z) - i \omega \epsilon \nabla \times (\Pi_e \bar{a}_z) \quad (1.2)$$

The potentials Π_e and Π_m satisfy the homogeneous Helmholtz equation

$$(\nabla^2 + k^2) \Pi_{e,m} = 0 \quad z \neq 0 \quad (2)$$

In (1) and (2), $\epsilon = \epsilon_0$ or ϵ_1 and $k = k_0$ or k_1 , depending upon the medium in which the observation point is located.

Equations in (1) can further be reduced to

Fig. 1: An arbitrarily-oriented microstrip dipole antenna.

Fig. 2: Spherical coordinate with respect to z -axis.

Fig. 3(a): Spherical coordinates with respect to the y -axis.

Fig. 3(b): X -directed HED, i.e., polarization perpendicular to array axis.

Fig. 3(c): Y -directed HED, i.e., polarization along the array axis.

$$E_z = -\nabla_t^2 \Pi_e \quad ; \quad \bar{E}_t = \nabla_t \frac{\partial \Pi_e}{\partial z} - i \omega \mu_0 \bar{a}_z \times \nabla \Pi_m \quad (3.1)$$

$$H_z = -\nabla_t^2 \Pi_m \quad ; \quad \bar{H}_t = \nabla_t \frac{\partial \Pi_m}{\partial z} + i \omega \epsilon \bar{a}_z \times \nabla \Pi_e \quad (3.2)$$

where the subscript t denotes to the transverse components with respect to z. Now since

$$\nabla_t \cdot \bar{E}_t = -\frac{\partial}{\partial z} E_z \quad ; \quad \bar{a}_z \cdot \nabla_t \times \bar{E}_t = +i \omega \mu_0 H_z$$

$$\nabla_t \cdot \bar{H}_t = -\frac{\partial}{\partial z} H_z \quad ; \quad \bar{a}_z \cdot \nabla_t \times \bar{H}_t = -i \omega \epsilon E_z$$

the boundary conditions can be translated into the following conditions for E_z and H_z ,

$$H_z|_{z+} = H_z|_{z-} \quad ; \quad E_z'|_{z+} = E_z'|_{z-} \quad (4.1)$$

$$H_z'|_{z+} - H_z'|_{z-} = -\bar{a}_z \cdot \nabla_t \times \bar{J}_s \quad (4.2)$$

$$\frac{ik_0}{\eta_0} (E_z|_{z+} - \epsilon_r E_z|_{z-}) = \nabla_t \cdot \bar{J}_s \quad (4.3)$$

at $z = 0$, and

$$H_z = 0 \quad ; \quad E_z' = 0 \quad (5)$$

at $z = -d$, where $\eta_0 (= 120 \pi \text{ ohms})$ is the free-space characteristic impedance and $\epsilon_r (= \epsilon_1/\epsilon_0)$ is the refractive index of the dielectric substrate.

To solve equation (2) subject to the boundary conditions in (4) and (5), we first define the Fourier transform pair as:

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{f}(\alpha, \beta) e^{-ik_0(\alpha x + \beta y)} d\alpha d\beta \quad (6.1)$$

$$\tilde{f}(\alpha, \beta) = \left(\frac{k_0}{2\pi} \right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{ik_0(\alpha x + \beta y)} dx dy ; \quad (6.2)$$

in transform domain, then, we have,

$$\tilde{\nabla}_t : ik_0(\alpha \bar{a}_x + \beta \bar{a}_y) \quad (7)$$

$$\tilde{E}_z = k_0^2(\alpha^2 + \beta^2) \tilde{\Pi}_e$$

$$\tilde{H}_z = k_0^2(\alpha^2 + \beta^2) \tilde{\Pi}_m$$

where $\tilde{\Pi}_e$ and $\tilde{\Pi}_m$ now satisfy the equation

$$\left(\frac{\partial^2}{\partial z^2} - k_0^2 U_{0,1} \right) \tilde{\Pi}_{e,m} = 0 \quad (8)$$

wherein

$$U_0 = \sqrt{\alpha^2 + \beta^2 - 1} ; \quad (9)$$

$$U_1 = \sqrt{\alpha^2 + \beta^2 - \epsilon_r}$$

$$\text{Re}(U_{0,1}) > 0 ; \text{Im}(U_{0,1}) < 0$$

From (7), (8) and (5), the solutions for E_z and H_z can be constructed as

$$\begin{cases} \tilde{E}_z = E_1 \text{ch} [U_1 k_0(z + d)] \\ \tilde{H}_z = H_1 \text{sh} [U_1 k_0(z + d)] \end{cases} \quad (10)$$

for $-d < z < 0$, and

$$\begin{pmatrix} \widetilde{E}_z \\ \widetilde{H}_z \end{pmatrix} = \begin{pmatrix} E_0 \\ H_0 \end{pmatrix} e^{-U_0 k_0 z} \quad (11)$$

for $z > 0$. We now substitute for E_z and H_z from (10) and (11) into (4.1)-(4.3) and solve for the coefficients $E_{0,1}$ and $H_{0,1}$; this finally yields

$$\begin{cases} E_0 = \frac{-i\eta_0}{k_0} (\widetilde{\nabla}_t \cdot \widetilde{J}_s) \frac{U_1 \operatorname{th}(U_1 k_0 d)}{D_{TM}} \\ H_0 = \frac{1}{k_0} (\bar{a}_z \cdot \widetilde{\nabla}_t \times \widetilde{J}_s) \frac{1}{D_{TE}} \end{cases} \quad (12)$$

and

$$\begin{cases} E_1 = -\frac{U_0}{U_1 \operatorname{sh}(U_1 k_0 d)} E_0 \\ H_1 = \frac{1}{\operatorname{sh}(U_1 k_0 d)} H_0 \end{cases} \quad (13)$$

where

$$\begin{aligned} D_{TE} &= U_0 + U_1 \operatorname{cth}(U_1 k_0 d) \\ D_{TM} &= \epsilon_r U_0 + U_1 \operatorname{th}(U_1 k_0 d) \end{aligned} \quad (14)$$

Equations (10) and (11), via (7) and (6.1), now yield the formal expressions for Π_e and Π_m . In particular, for $z > 0$ one gets

$$\begin{cases} \Pi_e = \frac{-i\eta_0}{k_0^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\alpha^2 + \beta^2} (\widetilde{\nabla}_t \cdot \widetilde{J}_s) \frac{U_1 \operatorname{th}(U_1 k_0 d)}{D_{TM}} e^{-U_0 k_0 z} e^{-ik_0(\alpha x + \beta y)} d\alpha d\beta \\ \Pi_m = \frac{1}{k_0^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\alpha^2 + \beta^2} (\bar{a}_z \cdot \widetilde{\nabla}_t \times \widetilde{J}_s) \frac{1}{D_{TE}} e^{-U_0 k_0 z} e^{-ik_0(\alpha x + \beta y)} d\alpha d\beta \end{cases} \quad (15)$$

It should be noted that the expressions in (15) are valid for any source distribution $\widetilde{J}_s$ on the slab in the x-y plane. For the present case of a dipole source of moment $\mathbf{p}$ which is directed at an angle χ with the x-axis, one can easily show that

$$\widetilde{\nabla_t \cdot \mathbf{J}_s} = i \frac{k_0^3}{4\pi^2} p (\alpha \cos \chi + \beta \sin \chi)$$

$$\bar{\mathbf{a}}_z \cdot \widetilde{\nabla_t \times \mathbf{J}_s} = i \frac{k_0^3}{4\pi^2} p (\alpha \sin \chi - \beta \cos \chi),$$

and therefore, for $z > 0$,

$$\begin{cases} \Pi_e = \frac{\eta_0 P}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\alpha \cos \chi + \beta \sin \chi) \tilde{f}_e(U_0) e^{-U_0 k_0 z + ik_0(\alpha x + \beta y)} \frac{d\alpha d\beta}{U_0} \\ \Pi_m = \frac{P}{4\pi^2} \int \int (\alpha \sin \chi - \beta \cos \chi) \tilde{f}_m(U_0) e^{-U_0 k_0 z + ik_0(\alpha x + \beta y)} \frac{d\alpha d\beta}{U_0} \end{cases} \quad (16)$$

where

$$\tilde{f}_e(U_0) = \frac{1}{1 + U_0^2} \frac{U_0 U_1 \text{th}(U_1 k_0 d)}{D_{\text{TM}}} \quad (17.1)$$

$$\tilde{f}_m(U_0) = \frac{i}{1 + U_0^2} \frac{U_0}{D_{\text{TM}}} \quad ; \quad U_1 = \sqrt{U_0^2 + 1 - \epsilon_r} \quad (17.2)$$

Substituting these results into (3.1) and (3.2) now yields all the components of $\bar{\mathbf{E}}$ and $\bar{\mathbf{H}}$.

2.2 Far-Field Approximation

Recall the identity:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[-k_0 U_0 |z - z'| + ik_0 \alpha(x - x') + ik_0 \beta(y - y')] \frac{d\alpha d\beta}{U_0} = \frac{2\pi}{k_0} \frac{e^{-ik_0 r}}{r} \quad (18)$$

where

$$r = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}$$

$$\simeq r_0 - \frac{x}{r_0} x' - \frac{y}{r_0} y' - \frac{z}{r_0} z' \quad ; \quad \text{for } |\bar{\mathbf{x}}'| \ll |\bar{\mathbf{r}}_0|$$

wherein $r_0 = \sqrt{x^2 + y^2 + z^2}$. Defining the directional-cosines as

$$v_x = \frac{x}{r_0} ; v_y = \frac{y}{r_0} ; v_z = \frac{z}{r_0}$$

with

$$v_x^2 + v_y^2 + v_z^2 = 1$$

in general, we can postulate that for $(k_0x, k_0y, k_0z) > > 1$,

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{f}(\alpha, \beta, U_0) e^{-U_0 k_0 z + i k_0 (\alpha x + \beta y)} \frac{d\alpha d\beta}{U_0} = \frac{2\pi}{k_0} \tilde{f}(\alpha = v_x; \beta = v_y; U_0 = -iv_z) \frac{e^{-ik_0 r_0}}{r_0} \quad (19)$$

Far-field patterns of Π_e and Π_m are derived by applying (19) to the integrals in (16), as

$$\begin{aligned} \Pi_e &\approx \left(\frac{\eta_0 p}{2\pi} \right) \frac{e^{ik_0 r_0}}{k_0 r_0} [(v_x \cos \chi + v_y \sin \chi) \tilde{f}_e(-iv_z)] \\ \Pi_m &\approx \left(\frac{p}{2\pi} \right) \frac{e^{ik_0 r_0}}{k_0 r_0} [(v_x \sin \chi - v_y \cos \chi) \tilde{f}_m(-iv_z)] \end{aligned} \quad (20)$$

In order to obtain the far-field expressions for the electric and magnetic fields, we utilize the fact that to the order of $\frac{1}{k_0 r_0}$, one can replace the operation $\nabla \times$ by $ik_0 \bar{a}_r$ in (1) so that

$$\begin{aligned} \bar{E} &\approx -k_0^2 [(\bar{a}_r \times \bar{a}_r \times \bar{a}_z) \Pi_e + (\bar{a}_r \times \bar{a}_z) \eta_0 \Pi_m] \\ &= \left(\frac{-k_0 \eta_0 p}{2\pi} \right) \frac{e^{ik_0 r_0}}{r_0} [(\bar{a}_r \times \bar{a}_r \times \bar{a}_z)(v_x \cos \chi + v_y \sin \chi) \tilde{f}_e(-iv_z) \\ &\quad + (\bar{a}_r \times \bar{a}_z)(v_x \sin \chi - v_y \cos \chi) \tilde{f}_m(-iv_z)] \end{aligned} \quad (21)$$

The vector products $(\bar{a}_r \times \bar{a}_r \times \bar{a}_z)$ and $\bar{a}_r \times \bar{a}_z$ as well as the directional-cosines can be explicitly expressed in terms of a spherical coordinate system with respect to any axis.

2.2.1 Z-axis as the polar axis

For example, if we choose the reference axis to be the conventional z-axis (Figure 2), we have in the spherical coordinate system $(\bar{a}_r, \bar{a}_{\theta_z}, \bar{a}_{\phi_z})$:

$$\begin{aligned} v_z &= \bar{a}_z \cdot \bar{a}_r = \cos \theta_z \\ v_x &= \bar{a}_x \cdot \bar{a}_r = \sin \theta_z \cos \phi_z \\ v_y &= \bar{a}_y \cdot \bar{a}_r = \sin \theta_z \sin \phi_z \end{aligned} \quad (22)$$

and

$$\begin{aligned} \bar{a}_r \times \bar{a}_r \times \bar{a}_z &= (\bar{a}_r \cdot \bar{a}_z) \bar{a}_r - \bar{a}_z = -(\bar{a}_z \cdot \bar{a}_{\theta_z}) \bar{a}_{\theta_z} \\ \bar{a}_r \times \bar{a}_z &= (\bar{a}_z \cdot \bar{a}_{\phi_z} \times \bar{a}_r) \bar{a}_{\phi_z} = (\bar{a}_z \cdot \bar{a}_{\theta_z}) \bar{a}_{\phi_z} \end{aligned} \quad (23)$$

where $(\bar{a}_z \cdot \bar{a}_{\theta_z}) = -\sin \theta_z$. Insertion of (22) and (23) into (21) results in the two far-zone orthogonal components of $\bar{E}$:

$$\begin{aligned} E_{\theta_z} &\approx -E_0 \sin \theta_z [(v_x \cos \chi + v_y \sin \chi) \tilde{f}_e(-i \cos \theta_z)] \\ E_{\phi_z} &\approx -E_0 \sin \theta_z [(v_x \sin \chi - v_y \cos \chi) \tilde{f}_m(-i \cos \theta_z)] \end{aligned} \quad (24)$$

where $E_0 = \left(\frac{k_0 \eta_0 p}{2\pi} \right) \frac{e^{ik_0 r_0}}{r_0}$. For the special case when $\chi = 0$, i.e., when the dipole is located in the x-direction, the expressions in (24) reduce to:

$$\begin{cases} E_{\theta_z}^x \approx iE_0 \cos \phi_z \cos \theta_z \frac{U_{1\theta_z}}{U_{1\theta_z} - i \epsilon_r \cos \theta \operatorname{cth}(U_{1\theta_z} k_0 d)} \\ E_{\phi_z}^x \approx -iE_0 \sin \phi_z \cos \theta_z \frac{1}{\cos \theta_z + iU_{1\theta_z} \operatorname{cth}(U_{1\theta_z} k_0 d)} \end{cases} \quad (25)$$

where $U_{1\theta} = -i \sqrt{\epsilon_r - \sin^2 \theta}$. The expressions in (25) are identical to those obtained, with a different approach, by Mosig and Gardiol [1].

2.2.2 Y-axis as the Polar Axis

For studying cross-pol level in a linear array, it is convenient to use the corresponding radiation fields expression derived with the polar axis of the spherical coordinates in the array direction, i.e., the y-axis (Figure 3a).

In the spherical coordinate system $(\bar{a}_r, \bar{a}_{\theta_y}, \bar{a}_{\phi_y})$, we have

$$v_y = \bar{a}_y \cdot \bar{a}_r = \cos \theta_y \quad (26)$$

$$v_x = \bar{a}_x \cdot \bar{a}_r = \sin \theta_y \sin \phi_y$$

$$v_z = \bar{a}_z \cdot \bar{a}_r = \sin \theta_y \cos \phi_y$$

and after some vector manipulations

$$\bar{a}_r \times \bar{a}_r \times \bar{a}_z = (-v_z v_y \bar{a}_{\theta_y} - v_x \bar{a}_{\phi_y}) \sqrt{1 - v_y^2} \quad (27)$$

$$\bar{a}_r \times \bar{a}_z = (v_x \bar{a}_{\theta_y} - v_y v_z \bar{a}_{\phi_y}) \sqrt{1 - v_y^2}$$

Substituting (24) and (25) into (21) yields

$$E_{\theta_y} = -E_0 \sin \theta_y [-(v_x \cos \chi + v_y \sin \chi) v_y v_z \hat{f}_e(-iv_z) \quad (28)$$

$$+ (v_x \sin \chi - v_y \cos \chi) v_x \hat{f}_m(-iv_z)]$$

$$E_{\phi_y} = -E_0 \sin \theta_y [-(v_x \cos \chi + v_y \sin \chi) v_x \hat{f}_e(-iv_z)$$

$$- (v_x \sin \chi - v_y \cos \chi) v_y v_z \hat{f}_m(-iv_z)]$$

We now examine two special cases which are of importance in the present study.

(i) *Y-directed HED*, i.e., parallel polarization.

Let $\chi = \frac{\pi}{2}$ in (28); we then have

$$\begin{cases} E_{\theta_y}^y = E_0 \frac{\sin \theta_y \cos \phi_y}{b} [\sin \theta_y \sin^2 \phi_y C_{TE} + \cos^2 \theta_y \cos \phi_y C_{TM}] \\ E_{\phi_y}^y = -E_0 \frac{\sin(2\theta_y)\sin(2\phi_y)}{4b} [C_{TM} - \sin \theta_y \cos \phi_y C_{TE}] \end{cases} \quad (29)$$

where $b = 1 - \sin^2 \theta_y \cos^2 \phi_y$, and

$$\begin{aligned} C_{TE} &= \frac{1}{\sin \theta_y \cos \phi_y + i(\epsilon_r - b)^{1/2} \cot((\epsilon_r - b)^{1/2} k_0 d)} \\ C_{TM} &= \frac{(\epsilon_r - b)^{1/2}}{(\epsilon_r - b)^{1/2} + i\epsilon_r \sin \theta_y \cos \phi_y \cot((\epsilon_r - b)^{1/2} k_0 d)} \end{aligned} \quad (30)$$

(ii) *X-directed HED*, i.e., perpendicular polarization.

Let $\chi=0$ in (28), then

$$\begin{cases} E_{\theta_y}^x = E_0 \frac{\sin(2\theta_y)\sin(2\phi_y)}{4b} [C_{TE} - \sin \theta_y \cos \phi_y C_{TM}] \\ E_{\phi_y}^x = E_0 \frac{\sin \theta_y \cos \phi_y}{b} [\sin \theta_y \sin^2 \phi_y C_{TM} + \cos^2 \theta_y \cos \phi_y C_{TE}] \end{cases} \quad (31)$$

where C_{TE} and C_{TM} are given by (30).

3. Cross Polarization Level and Co-Polarized Radiation

3.1 Definition of the Cross-Pol Level (CPL)

In accordance with the IEEE Standard Definition of Terms for Antennas [2], we define the cross-polar side lobe level (CPL) as the maximum relative partial directivity (corresponding to the cross-polarization) of a side lobe with respect to the maximum partial directivity (corresponding to the co-polarization) of the antenna. Therefore, with respect to the spherical coordinates (r, θ_y, ϕ_y) , one can write:

$$\text{CPL} = \frac{|E_{\text{cross}}|_{\text{max}}}{|E_{\text{CO}}|_{\text{max}}} \quad (32)$$

where $|E_{\text{CO}}|_{\text{max}}$ and $|E_{\text{cross}}|_{\text{max}}$ are, respectively, the maximum values of co-polar and cross-polar fields for a given value of θ_y and as ϕ_y varies from 0 to 180°.

3.2 Co-pol and Cross-pol for a y-directed HED

For a y-directed HED (Figure 3C), i.e., $\chi = \frac{\pi}{2}$, we have

$$\begin{cases} E_{\text{CO}}^y = E_{\theta_y}^y \\ E_{\text{cross}}^y = E_{\phi_y}^y \end{cases} \quad (33)$$

where $E_{\theta_y}^y$ and $E_{\phi_y}^y$ are given in (29). For the special case of no slab, i.e., as $\epsilon_r \rightarrow 1$, we get

$$\begin{cases} E_{\text{CO}}^y = -iE_0 e^{ik_0 d \sin \theta_y \cos \phi_y} \sin \theta_y \sin(k_0 d \sin \theta_y \cos \phi_y) \\ E_{\text{cross}}^y = 0 \end{cases} \quad (34)$$

i.e., we have a zero cross-pol level. Presence of the substrate ($\epsilon_r > 1$) will increase the CPL.

3.3 Co-pol and Cross-pol for a x-directed HED

For a x-directed HED (Figure 3B), i.e., $\chi = 0$, we have

$$\begin{cases} E_{\text{CO}}^x = E_{\phi_y}^x \\ E_{\text{cross}}^x = E_{\theta_y}^x \end{cases} \quad (35)$$

where $E_{\phi_y}^x$ and $E_{\theta_y}^x$ are given in (31). For the special case of no slab, i.e., as $\epsilon_r \rightarrow 1$, one obtains

$$\begin{cases} E_{\text{CO}}^x = -iE_0 e^{ik_0 d \sin \theta_y \cos \phi_y} \cos \phi_y \sin(k_0 d \sin \theta_y \cos \phi_y) \\ E_{\text{cross}}^x = -iE_0 e^{ik_0 d \sin \theta_y \cos \phi_y} \cos \theta_y \sin \phi_y \sin(k_0 d \sin \theta_y \cos \phi_y) \end{cases} \quad (36)$$

Fig. 4: Elliptical polarization for an arbitrarily-oriented dipole.

$$\theta_0 = \frac{1}{2} \tan^{-1} \left[\frac{2|E_{\theta_y}| |E_{\phi_y}|}{|E_{\phi_y}|^2 - |E_{\theta_y}|^2} \cos(\Delta\alpha) \right] \quad (39)$$

where $\Delta\alpha = \alpha_\theta - \alpha_\phi$.

Therefore, provided that the dipole-orientation, χ , is given, one can determine the co-polar direction, θ_0 , and the maximum values (maximum with respect to ϕ_y) of E_{CO} and E_{cross} , which are needed to calculate the CPL. Conversely, if a desired co-pol direction of θ_0 and value of the CPL, are given, in principle, it is possible to determine the angle χ , i.e., the required orientation of HED.

In deriving the co-polar and cross-polar fields in this work, we have assumed a spherical coordinate with y as the reference axis. However, we note that because the formulation in section 2.2 is in terms of directional cosines v_x , v_y and v_z (see eq. (21)), the discussion is, in fact, independent of the array direction. In other words, one can always assign the dipole-orientation as, say, x and redefine the directional cosines in (21) according to an arbitrary array axis, y' .

4. Numerical Results

For the results presented in this section, the reference-axis (i.e., the array-direction) is assumed to be the y -axis and two cases of diople's orientations are considered.

4.1 y -directed HED

The co-pol ($E_{\theta_y}^y$) and cross-pol ($E_{\phi_y}^y$) radiation patterns for different values of ϵ_r and $k_0 d$ are shown in Figures 5-7. The results are normalized with respect to the maximum value of the co-pol, $E_{\theta_y}^y$, which for all the cases considered here occurs at $\phi_y = 0$. As shown, however, the direction of cross-pol's maximum varies depending on the parameters ϵ_r and $k_0 d$.

Fig. 5: Co-pol and cross-pol radiation patterns for a Y-directed HED;
 $\theta_y = 60^\circ$, $\epsilon_r = 2$, $k_0 d = 0.1$.

Fig. 6: Co-pol and cross-pol radiation patterns for a Y-directed HED;
 $\theta_y = 60^\circ$, $\epsilon_r = 2$, $k_0 d = 0.5$.

Fig. 7: Co-pol and cross-pol radiation patterns for a Y-directed HED;
 $\theta_y = 60^\circ$, $\epsilon_r = 10$, $k_0 d = 0.1$.

Fig. 8: CPL for a Y-directed HED as a function of $\sqrt{\epsilon_r} k_0 d$; $\theta_y = 50^\circ$, $\epsilon_r = 2, 5$ and 10 .

Fig. 10: CPL for a Y-directed HED as a function of $\sqrt{\epsilon_r} k_0 d$; $\theta_y = 70^\circ$, $\epsilon_r = 2, 5$ and 10 .

Fig. 11: CPL for a Y-directed HED as a function of $\sqrt{\epsilon_r} k_0 d$; $\theta_y = 80^\circ$, $\epsilon_r = 2, 5$ and 10 .

Fig. 12: CPL for a Y-directed HED as a function of k_0d ; $\epsilon_r = 2$, $\theta_x = 50^\circ$, 60° , 70° and 80° .

By inserting the values of co-pol and cross-pol's maximums into (32), the cross-pol level is calculated and plotted in Figures 8-11 as a function of k_1d ($= \sqrt{\epsilon_r} k_0d$), and for various values of ϵ_r and θ_y . In general, for $0 < k_1d < 1.5$, the cross-pol level (CPL) decreases linearly as k_1d increases. In addition, the lower the value of ϵ_r , the lower is the CPL. Figure 12 shows the CPL in dB as a function of k_0d and for $\epsilon_r = 2$ and various values of θ_y in degrees. For larger values of θ_y (i.e., closer to broadside direction), the lower values of CPL is achieved. In general, for $k_0d < 1$, the CPL behaves like (Appendix A)

$$\text{CPL} \equiv \frac{|E_{\phi_y}^y|_{\max}}{|E_{\theta_y}^y|_{\max}} \approx \left(\frac{\epsilon_r - 1}{\epsilon_r - \cos^2 \theta_y} \right) \cos \theta_y \quad (40)$$

A more accurate expression is given in Appendix A. As expected, for $\theta_y = 90^\circ$, there is no cross-pol radiation and $(\text{CPL})_{\text{dB}} \rightarrow -\infty$.

These figures show, however, that over the desired range of $50^\circ \leq \theta_y \leq 80^\circ$, no CPL of -30 dB can be achieved for a y-directed HED. We note that because the minimums and maximums in Figures 8-11 correspond to the resonances in the slab and consequently the excitation of surface waves, all of the values of k_0d larger than that of the first minimum should be avoided. In other words, in order to avoid the higher-order surface modes (other than TM_0 mode that always exists), one should have $k_0d \sqrt{\epsilon_r - 1} < \frac{\pi}{2}$.

4.2 x-directed HED

Figures 13-15 show the co-pol ($E_{\phi_y}^x$) and cross-pol ($E_{\theta_y}^x$) radiation patterns of a x-directed HED over a dielectric slab. The corresponding cross-pol levels are shown in Figures 16-20. As opposed to the y-HED case, the CPL in this case initially increases with increasing k_1d (see Figures 12 and 20); furthermore, the higher values of ϵ_r yield lower cross-pol levels.

For this polarization, over the desired range of $50^\circ \leq \theta_y \leq 80^\circ$, the -30 dB CPL can be achieved provided that one chooses a large value of ϵ_r ($\epsilon_r \geq 10$) and small values of k_0d . As

Fig. 13: Co-pol and cross-pol radiation patterns for a X-directed HED;
 $\theta_y = 60^\circ$, $\epsilon_r = 2$, $k_0 d = 0.1$.

Fig. 14: Co-pol and cross-pol radiation patterns for a X-directed HED;
 $\theta_y = 60^\circ$, $\epsilon_r = 2$, $k_0 d = 0.5$.

Fig. 15: Co-pol and cross-pol radiation patterns for a X-directed HED;
 $\theta_y = 60^\circ$, $\epsilon_r = 10$, $k_0 d = 0.1$.

Fig. 16: CPL for a X-directed HED as a function of $\sqrt{\epsilon_r} k_0 d$; $\theta_y = 50^\circ$, $\epsilon_r = 2, 5$ and 10 .

Fig. 17: CPL for a X-directed HED as a function of $\sqrt{\epsilon_r} k_0 d$; $\theta_y = 60^\circ$, $\epsilon_r = 2, 5$ and 10 .

Fig. 18: CPL for a X-directed HED as a function of $\sqrt{\epsilon_r} k_0 d$; $\theta_y = 70^\circ$, $\epsilon_r = 2, 5$ and 10 .

Fig. 19: CPL for a X-directed HED as a function of $\sqrt{\epsilon_r} k_0 y$; $\theta_y = 80^\circ$, $\epsilon_r = 2, 5$ and 10 .

Fig. 20: CPL for a X-directed HED as a function of k_{0d} ; $\epsilon_r = 2$, $\theta_y = 50^\circ$, 60° , 70° and 80° .

derived in Appendix A, for $k_0 d < 1$ and $\epsilon_r > 1$, the CPL for the x-directed HED behaves like

$$\text{CPL} = \frac{|E_{\theta_y}^x|_{\max}}{|E_{\phi_y}^x|_{\max}} \approx \frac{k_1 d}{\sqrt{\epsilon_r}} \cot \theta_y \quad (41)$$

5. Conclusions

Expressions for the co-pol and cross-pol fields for any arbitrarily dipole-orientation and array-direction have been derived. These are given by equations (21), (25) and (28)-(31).

It is found that for a y-directed HED a relatively low cross-pol level may be achieved by using small values of ϵ_r and $k_0 d \sqrt{\epsilon_r - 1} < \frac{\pi}{2}$. For example, for $\epsilon_r = 2$, $k_0 d = 1$ and over the desired range of the beam-direction, $50^\circ \leq \theta_y \leq 80^\circ$, one gets: $-12\text{dB} \leq \text{CPL} \leq -25\text{dB}$ (see Figure 12). To obtain lower CPL, smaller values of ϵ_r must be used.

For the x-directed HED, however, a -30 dB (or lower) CPL over the whole range of the beam-direction can be achieved by using a large value of ϵ_r (i.e., $\epsilon_r \gtrsim 10$) and a small value of $k_0 d$ (Figures 16-19).

In general, the cross-pol radiation cannot be completely eliminated. This stems from the fact that even for the case of an air-filled (i.e., $\epsilon_r = 1$) microstrip (x-directed) dipole antenna, the cross-polarized field (as defined by (36)) always exists. The effect of the dielectric substrate ($\epsilon_r > 1$), as compared to the case of $\epsilon_r = 1$, is to increase the cross-pol level in the y-directed HED and to decrease it in the x-directed HED configurations.

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APPENDIX A

In this appendix, we derive the approximate expressions for the cross-pol level (CPL) of a microstrip dipole antenna. These expressions are derived for a y-directed HED when $\sqrt{\epsilon_r} k_0 d \ll 1$ and for a x-directed HED when $\sqrt{\epsilon_r} k_0 d \ll 1$ and $\epsilon_r \gg 1$.

(I) Y-directed HED.

From (33) and (29), we have

$$E_{CO}^y = E_0 \frac{\sin\theta_y \cos\phi_y}{b} [\sin\theta_y \sin^2\phi_y C_{TE} + \cos^2\theta_y \cos\phi_y C_{TM}] \quad (A.1)$$

$$E_{cross}^y = -E_0 \frac{\sin(2\theta_y) \sin(2\phi_y)}{4b} [C_{TM} - \sin\theta_y \cos\phi_y C_{TE}] \quad (A.2)$$

where $b = 1 - \sin^2\theta_y \cos^2\phi_y$, and

$$C_{TE} = \frac{1}{\sin\theta_y \cos\phi_y + i(\epsilon_r - b)^{1/2} \cot((\epsilon_r - b)^{1/2} k_0 d)} \quad (A.3)$$

$$C_{TM} = \frac{(\epsilon_r - b)^{1/2}}{(\epsilon_r - b)^{1/2} + i \epsilon_r \sin\theta_y \cos\phi_y \cot((\epsilon_r - b)^{1/2} k_0 d)} \quad (A.4)$$

For $k_1 d = \sqrt{\epsilon_r} k_0 d \ll 1$, we may write

$$\cot((\epsilon_r - b)^{1/2} k_0 d) \sim \frac{1}{(\epsilon_r - b)^{1/2} k_0 d}$$

and therefore the expressions in (A.3) and (A.4) can be approximated by

$$\begin{cases} C_{TE}^0 = -ik_0 d \\ C_{TM}^0 = \frac{(\epsilon_r - b)k_0 d}{(\epsilon_r - b)k_0 d + i \epsilon_r \sin\theta_y \cos\phi_y} \end{cases} \quad (A.5)$$

Substitution of (A.5) into (A.1) and (A.2) now yields

$$E_{CO}^y \doteq E_0 \frac{\sin\theta_y \cos\phi_y}{b} \left[-ik_0 d \sin\theta_y \sin^2\phi_y + \cos^2\theta_y \cos\phi_y C_{TM}^0 \right] \quad (A.6)$$

$$E_{cross}^y \doteq -E_0 \frac{\sin(2\theta_y) \sin(2\phi_y)}{4b} \left[C_{TM}^0 + ik_0 d \sin\theta_y \cos\phi_y \right] \quad (A.7)$$

In order to calculate the CPL, we need to have the maximum values of $|E_{CO}^y|$ and $|E_{cross}^y|$ as ϕ_y varies from 0 to π . We note that $|E_{CO}^y|_{\max}$, at least for small $k_0 d$, always occurs at $\phi_y = 0$; then from (A.6) one gets

$$|E_{CO}^y|_{\max} \approx E_0 \left(\frac{\epsilon_r - \cos^2\theta_y}{\epsilon_r} \right) k_0 d \quad (A.8)$$

In deriving (A.8) from (A.6), it is assumed that θ_y is not close to zero; this is indeed consistent with the assumption that $50^\circ \leq \theta_y \leq 80^\circ$.

To obtain the maximum of $|E_{cross}^y|$, we let

$$\frac{\partial |E_{cross}^y|}{\partial \phi_y} = 0 \quad (A.9)$$

By neglecting the terms of the order $(k_0 d)^4$ and $(k_0 d)^2 \cos^2\phi_y$ (since, we expect $(\phi_y)_{\max}$ to be close to $\frac{\pi}{2}$), (A.9) leads to the equation:

$$Z^2 + 2 \left(\frac{\epsilon_r - 1}{\epsilon_r \sin\theta_y} k_0 d \right)^2 Z - \left(\frac{\epsilon_r - 1}{\epsilon_r \sin\theta_y} k_0 d \right)^2 = 0 \quad (A.10)$$

where $Z = \cos^2\phi_y$. Solving (A.10) for Z , yields

$$Z = \frac{1}{\epsilon_r^2 \sin^2\theta_y} \left\{ - (k_0 d)^2 (\epsilon_r - 1)^2 + \left[(k_0 d)^2 \epsilon_r^2 (\epsilon_r - 1)^2 \sin^2\theta_y + (k_0 d)^4 (\epsilon_r - 1)^4 \right]^{1/2} \right\}$$

or, for $\sin\theta_y$ not very small,

$$\begin{aligned}\cos^2\phi_y = Z &\approx \left(\frac{\epsilon_r - 1}{\epsilon_r \sin\theta_y}\right) k_0 d \left[1 - \left(\frac{\epsilon_r - 1}{\epsilon_r \sin\theta_y}\right) k_0 d\right] \\ &\approx \left(\frac{\epsilon_r - 1}{\epsilon_r \sin\theta_y}\right) k_0 d\end{aligned}\quad (\text{A.11})$$

Also,

$$\sin^2\phi_y = 1 - Z \approx 1 - \left(\frac{\epsilon_r - 1}{\epsilon_r \sin\theta_y}\right) k_0 d \quad (\text{A.12})$$

Substituting for $\cos\phi_y$ and $\sin\phi_y$ from (A.11) and (A.12) into (A.7) finally yields

$$|E_{\text{cross}}^y|_{\text{max}} \approx E_0 \left(\frac{\epsilon_r - 1}{\epsilon_r}\right) k_0 d \left[1 - \frac{2(\epsilon_r - 1)}{\epsilon_r \sin\theta_y} k_0 d\right]^{1/2} \cos\theta_y \quad (\text{A.13})$$

or to the order of $(k_0 d)^2$,

$$|E_{\text{cross}}^y|_{\text{max}} \approx E_0 \left(\frac{\epsilon_r - 1}{\epsilon_r}\right) k_0 d \cos\theta_y \quad (\text{A.14})$$

By inserting (A.8) and (A.14) into (32), we finally get

$$(\text{CPL})_y \approx \left(\frac{\epsilon_r - 1}{\epsilon_r - \cos^2\theta_y}\right) \cos\theta_y \quad (\text{A.15})$$

A more accurate expression for CPL can be obtained if one uses the expression in (A.13) for $|E_{\text{cross}}^y|_{\text{max}}$.

$$(\text{CPL})_y \approx \left(\frac{\epsilon_r - 1}{\epsilon_r - \cos^2\theta_y}\right) \left[1 - \left(\frac{\epsilon_r - 1}{\epsilon_r \sin\theta_y}\right) k_0 d\right] \cos\theta_y \quad (\text{A.16})$$

II. X-Directed HED

From (35) and (31), we have

$$E_{CO}^x = E_0 \frac{\sin\theta_y \cos\phi_y}{b} [\sin\theta_y \sin^2\phi_y C_{TM} + \cos^2\theta_y \cos\phi_y C_{TE}] \quad (A.17)$$

$$E_{cross}^x = E_0 \frac{\sin(2\theta_y) \sin(2\phi_y)}{4b} [C_{TE} - \sin\theta_y \cos\phi_y C_{TM}] \quad (A.18)$$

For $\sqrt{\epsilon_r} k_0 d \ll 1$ and $\epsilon_r \gg 1$, C_{TE} and C_{TM} in (A.3) and (A.4) can be approximated by

$$\begin{cases} C_{TE}^0 \approx -ik_0 d \\ C_{TM}^0 \approx \frac{k_0 d}{k_0 d + i \sin\theta_y \cos\phi_y} \end{cases} \quad (A.19)$$

Substitution from (A.19) into (A.17) and (A.18) gives

$$E_{CO}^x \approx E_0 \frac{\sin\theta_y \cos\phi_y}{b} k_0 d \left[\frac{\sin\theta_y \sin^2\phi_y}{k_0 d + i \sin\theta_y \cos\phi_y} - i \cos^2\theta_y \cos\phi_y \right] \quad (A.20)$$

$$E_{cross}^x \approx -iE_0 \frac{\sin(2\theta_y) \sin(2\phi_y)}{4b} \frac{(k_0 d)^2}{k_0 d + i \sin\theta_y \cos\phi_y} \quad (A.21)$$

Since the maximum value of $|E_{CO}^x|$ occurs at $\phi_y = 0$, therefore from (A.20) we obtain

$$|E_{CO}^x|_{\max} \approx E_0 (k_0 d) \sin\theta_y \quad (A.22)$$

In order to obtain $|E_{cross}^x|_{\max}$, we let

$$\frac{\partial |E_{cross}^x|}{\partial \phi_y} = 0 \quad (A.23)$$

this leads to an approximate solution

$$\sin^2 \phi_y \approx 1 - \frac{k_0 d}{\sin \theta_y} \quad (A.24)$$

Therefore, from (A.21) and (A.24), for $\frac{k_0 d}{\sin \theta_y} \ll 1$, we get

$$|E_{\text{cross}}^x|_{\text{max}} \approx E_0(k_0 d)^2 \cos \theta_y \quad (\text{A.25})$$

Substitution from (A.22) and (A.25) into (32), finally yields

$$(\text{CPL})_x \approx k_0 d \cot \theta_y = \frac{k_1 d}{\sqrt{\epsilon_r}} \cot \theta_y \quad (\text{A.26})$$

where $k_1 = (\epsilon_r)^{1/2} k_0$.